The Charge-to-Mass Ratio of Electrons

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The ratio of e, the charge of an electron, to m, the mass of an electron, was found using an electron beam and a magnetic field. A magnetic field, generated by two Helmholtz coils, was used to bend the electron beam into a circle. The current and voltage used to create the beam, along with the diameter of the resultant electron beam circle, are sufficient to calculate the e/m ratio. Our best data determines this value to be $(-1.697 \pm 0.117)x10^{-11}(C/kg)$*. This value has a* 3.56% *error from the accepted value of* $-1.759x10^{-15}(C/kg)$*. [1]*

I. Introduction/Theory

The motivation for this lab was to use a magnetic field to alter the path of electrons in order to calculate the ratio between an electron's charge, e, and its mass, m. An e/m apparatus allows us to create a beam of electrons with known energy. The e/m apparatus utilizes two Helmholtz coils to create a magnetic field and therefore alter the path of the electron beam. The force vector on the electron, is calculated via

$$\overrightarrow{F} = -e(\overrightarrow{v} x \overrightarrow{B}) \tag{1}$$

where e is the charge of the electron, $\overrightarrow{v}$ is the velocity vector of the electron, and $\overrightarrow{B}$ is the magnetic field vector.

The current creating the magnetic field through the coils can be altered, as can the voltage used to create the electron beam. These parameters are altered to shape the electron beam into a circle. While in a circle, the centripetal force on the electron is

$$F_{cent} = \frac{mv^2}{R} \tag{2}$$

where R is the radius of the circle created by the electron beam and v is the speed of the electrons. Equating this equation with the magnitude of the force due to the magnetic field, $F = evB$, the ratio e/m becomes

$$\frac{e}{m} = \frac{v}{BR} \tag{3}$$

Since the voltage that creates the electron beam is known, the energy of each electron can be calculated via $eV = \frac{1}{2}mv^2$, where V is the voltage. This equation can be solved for v and plugged into Equation (3) to give

$$\frac{e}{m} = \frac{2V}{B^2R^2} \tag{4}$$

This is the equation that we will use to calculate e/m, since we will be able to measure R, V, and calculate B in Tesla via

$$B = Ix7.8x10^{-4} \tag{5}$$

in the direction perpendicular to the plane of the Helmholtz coils. The number of coils in the Helmholtz coils is found using the equation

$$B = \frac{\mu_0 r^2 NI}{(r^2 + l^2/4)^{3/2}} \tag{6}$$

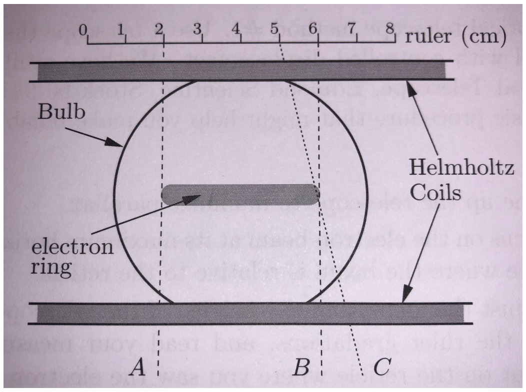

Figure 1: *Fig. 1: Top down view of the two Helmholtz coils parallel with each other and the bulb in between.*

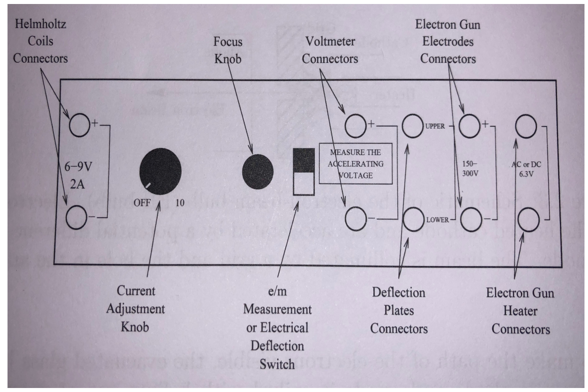

Figure 2: *Fig. 2: The inputs for the e/m apparatus. The e/m Measurement Switch is set in the e/m position.*

The constant μ_0, known as the magnetic permeability of free space, is $4\pi x 10^{-7}(tesla \cdot meter/ampere)$ [2]. The number of coils, N, can be solved since the parameters I, r, and l are known. The value l is the distance between the two coils.

II. Experimental Arrangement and Procedure

The primary tool used for this experiment was the Kent e/m Experimental Apparatus Model Tg-13. The apparatus, shown in Fig. (1) uses two Helmholtz coils to induce a magnetic field perpendicular to the plane of the Helmholtz coils and through the bulb. The transparent glass EB-bulb is where the electron beam is created and observed. A ruler is placed behind one of the Helmholtz coils so that, using an Edmond Scientific Telescope with a stand, one can measure the radius of the created electron ring. The telescope is placed about $15cm$ in front of the H-coil that does not have the ruler directly behind it. The electron ring is visible because the bulb has helium gas at a pressure of $10^{-2}mmHg$. The electrons in the ring collide with the Helium atoms and excite them. The decay of these excited atoms gives off a green halo that is easily identifiable.

In order to operate the e/m apparatus, various power supplies are needed. A GW Laboratory DC Power Supply Model GPS-1850 is plugged in to the Helmholtz Coils Connectors visible in Fig. (2). The low voltage $(5-15V)$ GW DC power supply provides a high current $(0.5-1.5A)$ to go through the Helmholtz coils and create a magnetic field. The current provided is measured on a DMM multimeter. This current was set at various levels throughout the experiment to lower statistical uncertainty.

On the apparatus, there is a Current Adjustment Knob that was set to give the lowest resistance. The resistance at its lowest point was 115 ± 0.1 Ohms as measured by the DMM multimeter. For the rest of the experiment, the DMM multimeter measured the current through the Helmholtz coils.

The high voltage $(100-300V)$ power supply used was a PASCO SF-9585A voltage supply with a current around $50mA$. The PASCO provided DC current to create the electron beam via the Electron Gun Electrodes Connectors in Fig. (2). These voltage values were also altered to reduce statistical uncertainty in our calculated value for e/m. This DC current was measured with a GDM multimeter via the Voltmeter Connectors. To test the uncertainty of the PASCO voltage reading, it was connected to the GDM multimeter directly. At a reading of $300 \pm 1V$ on the PASCO, the multimeter measured $299.75 \pm .03V$. This suggests a minimum uncertainty of the PASCO DC to continue as

$\pm 1V$.

The PASCO also provided AC current at $6V$ to heat the electron gun via the Electron Gun Heater Connectors. The GDM multimeter was used to measure the AC voltage produced by the PASCO at $6V$ when connected to the Electron Gun Heater jacks. A value of $6.419 \pm 0.005V$. This gives an uncertainty in the PASCO AC current to be at least $0.5V$. With the PASCO connected to provide both DC current and AC current, the GPS power supply to power the Helmholtz coils, an green electron beam in the shape of a circle was successfully created inside the bulb.

In order to measure the magnetic field due to objects other than the H-coils, a Bell 260 Gaussmeter with a Hall Probe was used. The Gaussmeter helped to correct our calculation for e/m by revealing that the magnetic field due to the H-coils was not uniform in the plane of the electron ring.

III. Data, Analysis, and Results

Once the apparatus was warmed up and showing a visible green electron ring with $150V$ powered to the Electron Gun Electrodes, the direction of the electrons was a counterclockwise circle. Using Equation (1), we deduced that the magnetic field was pointing towards us and our telescope.

A preliminary measurement of e/m was made by measuring the diameter of the electron ring with the current applied to the H-coils at $0.7 \pm 0.001A$ and the voltage applied to the gun electrodes at $120.09 \pm 0.01V$. These values were chosen to give a large, yet bright electron ring to make measuring the diameter easier. The electron ring diameter was measured using the outsides of the rings and again using the insides of the ring. The average of the two was used as the ring's diameter. The diameter was also measured twice, once by each lab member and the average was taken. The final average diameter was measured to be $12.05 \pm .10cm$. This diameter, after converted to meters and divided by two to get the radius, is plugged into Equation (4). The voltage, V, in

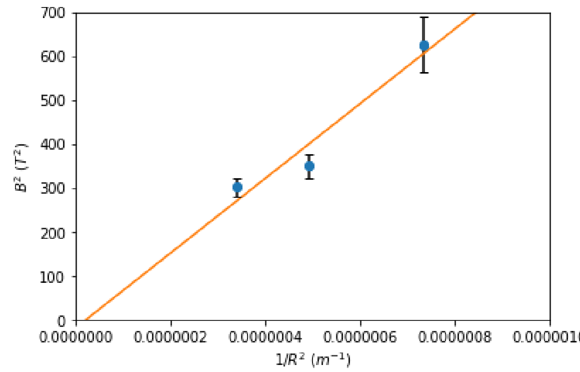

Figure 3: *The graph of B^2 vs $1/R^2$ at a constant voltage of $100.00 \pm 0.01V$. The $1/R^2$ error bars are too small to display.*

Equation (4) can be plugged in, and B is found by plugging in the current, I, into Equation (5). The preliminary value for e/m is calculated to be $(2.22 \pm 0.32)x10^{11}(C/kg)$.

The Gaussmeter was used to detect interfering magnetic fields not created by the apparatus. For the Gaussmeter, the 0 gauss range was when the probe was pointing straight downwards toward the ground. The maximum gauss range, at $0.71 \pm 0.01G$ was when the probe was pointing straight upward. This is likely from the Earth's magnetic field. Using the Gaussmeter, we found that the closest object to the apparatus set up that caused significant interference was our cell phones. They produced larger than $1G$ of interference. For the more precise measurements of e/m, we made sure to keep our cell phones away from the table.

The more precise measurements of e/m consisted of 8 measurements total. One lab partner measured the radius for the first 4 measurements, and the other partner measured the radius for the last 4 measurements. The first 4 measurements were done by altering the current provided to the H-coils while keeping the accelerating voltage at $100.00 \pm 0.01V$. The e/m values were calculated for each setup using Equation (4). The standard deviation for these values is $0.19x10^{11}(C/kg)$. The data is plotted with a line of best fit in Fig. (3). One

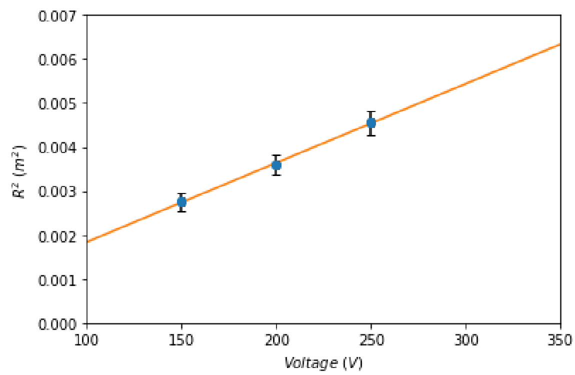

Figure 4: *The graph of R^2 vs V at a constant current of $1.000 \pm 0.001A$. The voltage errorbars are too small to display.*

data point was removed to increase the accuracy of our e/m value. The slope of the line represents the e/m value because the graph is B^2 vs $1/R^2$. The slope on this graph is $(1.70 \pm 0.19)x10^{-11}(C/kg)$.

The other 4 measurements were recorded using a constant current of $1.000 \pm 0.001A$ and altering the accelerating voltage. The standard deviation for these values is $0.02x10^{11}(C/kg)$. This standard deviation is much lower and may be due to the fact that one lab partner was more accurate in their measurements of the radius than the other partner. The data, again with taking out one outlier, is plotted in Fig. (4). The slope of the line is $(1.83 \pm 0.02)x10^{11}(C/kg)$. A χ^2 value was not found for these graphs because they only contain 3 data points with 2 degrees of freedom. This would make the χ^2 value much higher and less useful than wanted because of the lack of data.

In order to test the accuracy of the magnetic field produced by the H-coils, the Gaussmeter was used to measure the magentic field at the center of where the bulb would be. The results are found in Table (1).

Using these values for the magnetic field but converted to Tesla, they are plugged in to Equation (6) to solve for the number of coils. The distance between the two coils is $0.15 \pm 0.01m$. The radius of the coil is $.145 \pm 0.005m$. This gives an average value for the number of

Table 1: *The B values in 10G for the first column at various currents.*

Current (A)	B (10G)
0.500 ± 0.001	0.41 ± 0.01
0.600 ± 0.001	0.51 ± 0.01
0.700 ± 0.001	0.58 ± 0.01
0.800 ± 0.001	0.64 ± 0.01
0.900 ± 0.001	0.72 ± 0.01

coils to be 135.0 coils with a standard deviation of 3.47 coils.

The uniformity of the magnetic field was tested using the Gaussmeter probe at various points along the path the electron ring was at. At a current of $0.900 \pm 0.001A$ flowing through the H-coils, the max magnetic field was found to be 0.72 ± 0.05 (10G) near the top right position of the ring. The lowest reading was near the bottom left position of the ring at 0.59 ± 0.05 (10G). These results suggest the average magnetic field responsible for creating the electron ring is lower than Equation (5) predicts. The average magnetic field between the maximum and minimum fluctuations predicts a new magnetic field equation of $B = Ix7.22x10^{-4}$ (Tesla) instead of Equation (5). Using this equation instead, our values for e/m only increase in error percentage. A more sophisticated setup of measurements of the magnetic field fluctuations inside the bulb must be taken to increase accuracy.

IV. Conclusion

The preliminary value for e/m, $(2.22 \pm 0.32)x10^{11}(C/kg)$, has a percent error from the accepted value of 26.0%. The error is 14.3% of the calculated value. This error is very significant and likely due to the fact that only two measurements of the diameter were made, and that the current and voltage were kept constant.

The 4 measurements where the voltage was kept constant yielded an e/m value of $(1.70 \pm 0.19)x10^{-11}(C/kg)$. This value has a 3.56% error with the actual value. The measurements with constant current yielded a value of $(1.83 \pm 0.02)x10^{11}(C/kg)$ with a percent error of 3.77%. These errors are much smaller than the preliminary e/m value because more measurements were taken with varied parameters. Between these two, our most accurate measurement was

$$\frac{e}{m} = (1.70 \pm 0.19)x10^{-11}(C/kg)$$

The lab may improved by taking more measurements of the magnetic field in the vicinity of where the electron ring would be while the H-coils are on. These measurements would give better accuracy to the error in the H-coils provided magnetic field. Additionally, measuring the electron ring diameter could be improved by removing all sources of light from the room so the ring is more identifiable. However, even with the interference, it is convenient that a ratio of e/m can be found to under 5% uncertainty. The e/m value, combined with the h/e value from the previous lab, can help to measure the actual value of e with the help of the Rydberg constant R from a future lab.

References

[1] Mass-to-charge ratio Wikipedia entry en.wikipedia.org/wiki/Mass-to-charge_ratio

[2] G. Wang, Physics 18L Lab Manual (2018).