

Spectral Lines and the Rydberg Constant

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The Rydberg constant, R_H , was found using a spectrometer to measure the angles of the diffracted Hydrogen spectral lines. Our best value for R_H was calculated to be $10.980 \pm 0.009(\mu\text{m}^{-1})$. The R_H value along with our best values for h/e from lab 1, $(3.74 \pm 0.02) \times 10^{-15}(\text{V}/\text{Hz})$ [1], and e/m from lab 2, $(-1.70 \pm 0.12) \times 10^{11}(\text{C}/\text{kg})$ [2], allows us to calculate values for e , h , and m . The calculated values are $e = (1.35 \pm 0.06) \times 10^{-19}(\text{C})$, $h = (5.07 \pm 0.09) \times 10^{-34}(\text{m}^2\text{kg}/\text{s})$, and $m = (7.97 \pm 0.08) \times 10^{-31}(\text{kg})$. The respective errors for these values are 15.4%, 23.5%, and 12.5%.

I. INTRODUCTION/THEORY

THE motivation for this lab was to use a hydrogen tube and a spectrometer to measure the different angles of different colored spectral lines. A diffraction grating was used to split the light rays into different angles based on their wavelength. The angle of diffraction not only depended on the wavelength of the light, but also on the spacing of slits on the diffraction grating. The spacing of slits can be found via

$$d \sin \theta = m\lambda \quad (1)$$

where d is the spacing, θ is the angle of the diffracted light from the center, m is the order number, and λ is the wavelength. A sodium light with known spectral lines at 589.59nm [3] was used to find a solution for d .

If the angle of diffraction is known along with the spacing, Equation (1) can be used to solve for the wavelength. The Rydberg and Ritz formula is used to calculate R_H

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \quad (2)$$

since for hydrogen Z , the number of protons, is known to be 1 and the values for n_2^2 and n_1^2 are known for the respective color of the spectral line. The visible spectral lines for our experiment and their accepted values were the red line at 656.3nm , the green line at 486.1nm and the blue line at 434.0nm [4]. If R_H , e/m , and h/e are all known, then e , m , and h can be solved with

$$R_H = e^4 m_e / 8c\epsilon_0^2 h^3 \quad (3)$$

where ϵ_0 is the vacuum permittivity constant, $8.85419 \times 10^{-12}(\text{C}/(\text{V} \cdot \text{m}))$ [5].

II. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The precision spectrometer was tool of choice for measuring the diffraction angles. A light source was placed about 1 inch from the slit on the spectrometer. blocks of varying height were used under the spectrometer and light source in order to align the slit with the light source. The light flows through the slit and collimator and hit the diffraction grating in the center of the spectrometer. The light rays then

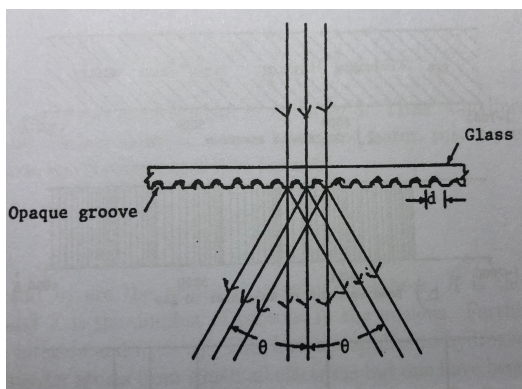


Figure 1: The effects of the diffraction grating on incident light rays, where d is the slit spacing and θ is the angle of diffraction. [3]

diffract as seen in Figure (1). Once diffracted, the light rays travel to the telescope and then to the eye. Figure (2) gives an accurate depiction of this process.

The collimator is the tube closest to the light source and contains the slit on its end. The slit width could be widened to increase brightness but at a cost of the resolution. For our experiment, we tended to use a narrower slit in favor of resolution over brightness. The slit width we used was a fine line much less than $1/2\text{mm}$.

The diffraction grating was lined up manually to be perpendicular to the incoming light rays. This created a source of error because there was no instrument used to measure the accuracy of its perpendicular alignment.

The telescope utilized cross hairs in the middle of the tube so that the viewer could align the spectral lines with the cross hair lines. The telescope was focused so that the resolution of the cross hair was at its maximum. In order to get the spectral lines into view of the telescope, we manually moved the telescope with our hands. Once in view, the a fine adjustment screw on the telescope would move its view in a much more controlled and small range. This adjustment screw was used to accurately place the cross hairs on the spectral line. The zeroth order angle, where the slit was seen, was first measured so that other angles were calibrated to this zero. Additionally, a black

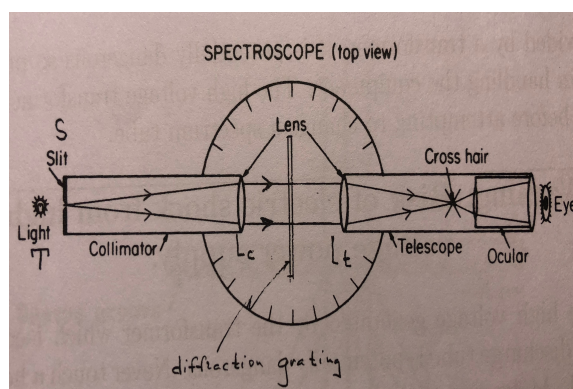


Figure 2: Top down view of the precision spectrometer. L_t is the telescope and L_c is the collimator. [3]

cloth was placed over the spectrometer to prevent ambient light from interfering with our measurements.

The extreme accuracy of the spectrometer, which has an error of ± 1 minute of arc, is due to its vernier scale. The vernier scale allowed for measurements beyond the degree scale and into the minutes of arc scale. However, the vernier scale was quite small on the spectrometer and required a magnifying glass to visualize. The small nature of the scale, along with its sometimes worn out and dirty lines, made it difficult for me and my lab partner to record the exact minute line that lines up with the degree line.

The first light used was a sodium light in order to gather a value for the slit spacing d . An unknown bulb was also used as a light source, which we show in *Data, Analysis, and Results* to be Mercury. A hydrogen bulb was used to identify R in Equation (2).

III. DATA, ANALYSIS, AND RESULTS

Once the sodium light source was warmed up for 10 minutes, the spectrometer was used to measure the diffraction angles for 1st and 2nd order spectral lines. The first order spectral lines were measured to be $21^\circ 8' \pm 1'$ on one side and $21^\circ 13' \pm 1'$ on the other. The difference between the two reveals an error of about $5'$ in the perpendicular alignment of the diffrac-

Table 1: The mystery gas's measured spectral line wavelengths vs the accepted wavelengths of mercury. The order is yellow, turquoise, blue, then violet. [6]

Measured λ nm	Accepted λ nm	Error (%)
585.9 ± 0.5	579.1	1.18
490.6 ± 0.5	491.6	0.21
435.4 ± 0.5	435.8	0.60
407.1 ± 0.5	404.7	0.10

tion grating.

The second order spectral lines were measured to calculate a value for d . The second order spectral line was measured to be at $44^\circ 49' \pm 1'$. Using this angle for θ , 589.59nm for λ , and 2 for m in Equation (1), d is 1673 ± 3 nm. This value of the slit spacing gives a value of 597 ± 1 (lines/mm) with an error of 1.01% from the given value of 590lines/mm. However, it is better to continue using the value my partner and I achieved because the error was in measuring the sodium spectral lines, so this error likely exists in measuring the hydrogen lines as well. It will give us more accurate wavelengths because it is incorporated in both d and the angle of diffraction.

The mystery bulb had visible spectral lines of yellow, blue/turquoise, violet, and blue. For blue/turquoise, blue, and violet, often there were a series of lines close to each other but the brightest one was chosen for each color. The measured wavelengths juxtaposed with the accepted values are shown in Table (1).

The hydrogen tube was powered by a transformer at around 45V. The visible spectral lines were red/orange, green and blue. For each color, there are multiple lines of various brightness. The best measurements did not always come from the brightest line but rather from the line with the finest resolution. To obtain a good resolution the slit was closed to the point where the lines were barely still visible.

We measured the first order lines for each color on both sides. The second order lines were too dim to see for our bulb, even after widening the slit. The wavelengths are calculated using Equation (1). Equation (2) is used

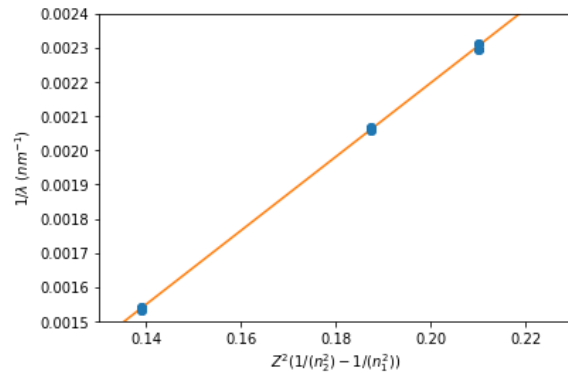


Figure 3: The inverse of the wavelength vs the Balmer series difference. The error bars are too small to display.

i, j	$\lambda_{i,j}/\lambda_{i,j+1}$ Expected	$\lambda_{i,j}/\lambda_{i,j+1}$ Measured
2, 3	1.35	1.35
2, 4	1.12	1.12

Table 2: The ratios of $\lambda_{i,j}$ to $\lambda_{i,j+1}$ for the expected values vs the measured values.

to plot the points on a graph as in Figure (3). Both first order values for each color were used as a data point. The inverse of the wavelengths are able to be plotted as a function of the n_1 and n_2 values because they are already known. For every color n_2 is 2, for red/orange n_1 is 3, for green n_1 is 4, and for blue n_1 is 5. [4]

The slope of the line of best fit in Figure (3) represents the R_H value calculated from the data. The R_H value is $10.78 \pm 0.04(\mu\text{m}^{-1})$. The normalized χ^2 value is 58.2. The χ^2 is much higher than 1, which is likely due to the extremely small uncertainty compared to the differences in measured and expected values. The R_H value improves to $10.98 \pm 0.08(\mu\text{m}^{-1})$ when using the best first order data point for each color instead of both data points. The χ^2 value for the line of best fit improves to 22.5 using these points only.

In order to validate the n transitions of these wavelengths, Table (2) shows the expected ratios between different quantum transition levels n . Since the values are equal, the Balmer formula is verified.

Using Equation (3) and the values of e/m

and h/e from labs 1 and 2, e becomes

$$e = (1.35 \pm 0.06) \times 10^{-19} (\text{C})$$

The value for m_e is calculated to be

$$m_e = (7.97 \pm 0.08) \times 10^{-31} (\text{kg})$$

The value for h is

$$h = (5.07 \pm 0.09) \times 10^{-34} (\text{m}^2 \text{kg/s})$$

These values are the primary purpose of the past 3 labs.

IV. CONCLUSION

The R_H value of $10.78 \pm 0.04 (\mu\text{m}^{-1})$ using 6 data points has a percent error of the accepted value, $10.97 (\mu\text{m}^{-1})$, of 1.69%. This value is larger than expected and could be due to a variety of reasons. For one, the hydrogen bulb we used was dimmer when compared to the others. This could cause useful spectral lines to be hidden. Additionally, the vernier scale was difficult to read due to dirt and wear on the lines. A misread of minutes could cause a large error. The R_H value of $10.98 \pm 0.08 (\mu\text{m}^{-1})$ when using the 3 best data points has an error of 0.11%. This error is much better but requires specific selection of only 3 data points.

The calculated value for e , $(1.35 \pm 0.06) \times 10^{-19} (\text{C})$, has an error of 15.4% from the expected value of $1.602 \times 10^{-19} (\text{C})$ [7]. The calculated value for m_e , $(7.97 \pm 0.08) \times 10^{-31} (\text{kg})$, has an error of 12.5% from the accepted value of $9.109 \times 10^{-31} (\text{kg})$ [7]. Lastly, the calculated value for h , $(5.07 \pm 0.09) \times 10^{-34} (\text{m}^2 \text{kg/s})$, has an error of 23.5% from the accepted value of $6.626 \times 10^{-34} (\text{m}^2 \text{kg/s})$ [8]. These errors are much higher than the 5% range. This is likely due to the cumulative errors in the three measured values, h/e , e/m , and R .

The lab may improved by removing all possible light sources in the room. This should allow for a minimum slit width because the light should be bright enough with no interference. Additionally, remeasuring the zeroth order after each measurement would make the other measurements more accurate. I noticed

that after a series of measurements, when we remeasured the zeroth order, it would change slightly.

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