Universal relations after GW170817

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(Dated: October 6, 2018)

I. INTRODUCTION

A. Executive Summary

II. NEUTRON STAR TIDAL DEFORMABILITY

We begin by reviewing how one can extract internal structure information of NSs via GW measurement. In the presence of a neighboring tidal field $\mathcal{E}_{ij}$, such as the binary NS system found in GW170817, NSs tidally deform away from sphericity and acquire a non-vanishing quadrupole moment Q_{ij} that is characterized by the *tidal deformability* λ [1]: This process is characterized by the *tidal deformability*, λ [2, 3]:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}. \tag{1}$$

Such tidal deformability can be made dimensionless as

$$\Lambda \equiv \frac{\lambda}{M^5}, \tag{2}$$

and can be calculated via the following expression [2–4]:

$$\begin{aligned}\Lambda = &\frac{16}{15}(1-2\bar{C})^2[2+2\bar{C}(y_{\rm R}-1)-y_{\rm R}] \\ &\times \{2\bar{C}[6-3y_{\rm R}+3\bar{C}(5y_{\rm R}-8)] \\ &+4\bar{C}^3[13-11y_{\rm R}+\bar{C}(3y_{\rm R}-2)+2\bar{C}^2(1+y_{\rm R})] \\ &+3(1-2\bar{C})^2[2-y_{\rm R}+2\bar{C}(y_{\rm R}-1)]\ln(1-2\bar{C})\}^{-1}.\end{aligned} \tag{3}$$

Here $\bar{C} \equiv M/R$ is the stellar compactness with M and R representing the NS mass and radius. $y_{\rm R} \equiv y(R)$ with $y(r) \equiv rh'(r)/h(r)$, where a prime stands for taking a derivative with respect to the radial coordinate r. h represents the quadrupolar part of the (t,t) component of the metric perturbation that satisfies the following differential equation:

$$\begin{aligned}&h'' + \left\{\frac{2}{r} + \left[\frac{2m}{r^2} + 4\pi r(p-\epsilon)\right]e^{\lambda}\right\}h' \\ &+ \left\{4\pi\left[5\epsilon + 9p + (p+\epsilon)\frac{d\epsilon}{dp}\right]e^{\lambda} - \frac{6}{r^2}e^{\lambda} - \left(\frac{d\nu}{dr}\right)^2\right\}h = 0\end{aligned} \tag{4}$$

with background metric coefficients $e^{\nu} = g_{tt}$ and $e^{\lambda} = (1-2m/r)^{-1} = g_{rr}$, while p and ϵ represent pressure and energy density respectively.

The above differential equation can be solved as follows. First, one needs to prepare unperturbed background solutions by choosing a specific EoS, or $p(\epsilon)$, and solve a set of Tolman-Oppenheimer-Volkoff (TOV) equations with a chosen central density (or pressure) and appropriate boundary conditions. The stellar radius is determined from $p(R) = 0$ while the mass is determined by matching the interior solution to the Schwarzschild metric. Having such solutions at hand, one then plugs them into Eq. (4) and solve it with the boundary condition $y(0) = 2$ [2].

Because there are two NSs in a binary, two tidal deformabilities Λ_1 and Λ_2 associated with each star enter in the gravitational waveform. However, extracting such parameters independently is challenging due to the strong correlation between them[1]. Thus, what one can measure is the dominant tidal parameter in the waveform, which corresponds to the weighted average tidal deformability given by [1]

$$\tilde{\Lambda} = \frac{16}{13}\frac{(1+12q)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}, \tag{5}$$

where $q \equiv m_2/m_1$ is the mass ratio between two stars.

III. SPECTRAL REPRESENTATIONS OF NEUTRON STAR EQUATIONS OF STATE

The structure of a NS and its tidal interactions in a binary system rely heavily on the underlying EoS of nuclear matter.

IV. UNIVERSAL RELATIONS

A. I-Love-Q relations

B. Binary love relations

V. CONSTRAINTS ON NEUTRON STAR EQUATION OF STATE

VI. CONCLUSION AND DISCUSSION

ACKNOWLEDGMENTS

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[1] One way to cure this problem is to use universal relations between them [5, 6].