

# Quicksort Algorithm: Implementation, Analysis and Randomization

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**GitHub Link:** <https://github.com/kchemutai/Quicksort-Analysis>

## Quicksort-Analysis

**Quick Sort implantation:** quick\_sort.py

**Randomized quick sort Implementation:** randomized\_quick\_sort.py

### Performance Analysis

#### *Time Complexity Analysis*

**Best Case:  $O(n \log n)$**  Occurs when the pivot splits the array into two roughly equal halves at every recursive step.

**Average Case:  $O(n \log n)$**  On average, the pivot divides the array into two reasonably balanced partitions.

**Worst Case:  $O(n^2)$**  Happens when the pivot is always the smallest or largest element, leading to highly unbalanced partitions (e.g., already sorted arrays in deterministic Quicksort).

#### *Space Complexity*

In-place Implementation:  $O(\log n)$  (stack space for recursive calls). Additional Overheads: If not in-place, extra space is required for temporary arrays during partitioning.

### Summary of Empirical Analysis Results

+ Code
+ Text
Copy to Drive

1s
# Run comparisons
results = []
for size in input\_sizes:
 for dist\_name, generator in distributions.items():
 arr = generator(size)
 deterministic\_time = measure\_time(quicksort, arr)
 randomized\_time = measure\_time(randomized\_quicksort, arr)
 results.append({
 "Input Size": size,
 "Distribution": dist\_name,
 "Deterministic Time (s)": deterministic\_time,
 "Randomized Time (s)": randomized\_time
 })

# Print results
results\_df = pd.DataFrame(results)
print(results\_df)

	Input Size	Distribution	Deterministic Time (s)	Randomized Time (s)
0	10	random	0.000034	0.000032
1	10	sorted	0.000020	0.000024
2	10	reverse_sorted	0.000018	0.000026
3	100	random	0.000319	0.000350
4	100	sorted	0.000237	0.000369
5	100	reverse_sorted	0.000246	0.000321
6	1000	random	0.004604	0.006429
7	1000	sorted	0.003950	0.006851
8	1000	reverse_sorted	0.004638	0.024917
9	10000	random	0.090188	0.091728
10	10000	sorted	0.032288	0.053671
11	10000	reverse_sorted	0.042785	0.063005

The empirical analysis of deterministic and randomized Quicksort reveals the following insights:

### ***Small Input Sizes (10 elements):***

Both deterministic and randomized Quicksort have negligible runtimes ( $< 0.0001$  seconds). There is minimal performance difference between the two approaches, with slight variations due to randomness in pivot selection.

### ***Medium Input Sizes (100 elements):***

Both deterministic and randomized Quicksort show a marginal increase in runtime compared to smaller inputs. Randomized Quicksort is slightly slower than deterministic Quicksort for sorted and reverse-sorted inputs due to the overhead of random pivot selection.

### ***Large Input Sizes (1000 elements):***

For random and sorted inputs, deterministic Quicksort performs marginally faster than randomized Quicksort. For reverse-sorted inputs, randomized Quicksort significantly underperforms, indicating that while randomization reduces the likelihood of the worst-case scenario, it may still encounter performance issues in certain cases.

### ***Very Large Input Sizes (10,000 elements):***

Deterministic Quicksort outperforms randomized Quicksort consistently for all input distributions. For sorted and reverse-sorted distributions, the deterministic approach exhibits better runtime efficiency, highlighting its stability under specific conditions. Randomized Quicksort takes longer to process reverse-sorted inputs, though it performs similarly to deterministic Quicksort for random distributions.

### ***Overall Trends:***

Randomized Quicksort tends to introduce additional overhead due to pivot randomization, making it slightly slower in practice compared to deterministic Quicksort for smaller input sizes and sorted/reverse-sorted distributions. Deterministic Quicksort is more predictable and generally faster, particularly for large input sizes and structured data distributions.

### ***Performance Breakdown by Input Distribution:***

***Random Distribution:*** Both algorithms perform similarly, with randomized Quicksort occasionally being slower. ***Sorted Distribution:*** Deterministic Quicksort consistently outperforms randomized Quicksort.

***Reverse-Sorted Distribution:*** Deterministic Quicksort is significantly faster, especially for larger input sizes.

***Key Observations: Randomized Quicksort*** is effective in avoiding the worst-case scenario of  $O(n^2)$  but introduces slight overhead due to randomness.

***Deterministic Quicksort*** shows better practical performance across most cases and is more stable for sorted and reverse-sorted inputs.

These results align with theoretical expectations. While randomization helps reduce worst-case likelihood, deterministic Quicksort offers better overall runtime efficiency in practical scenarios.