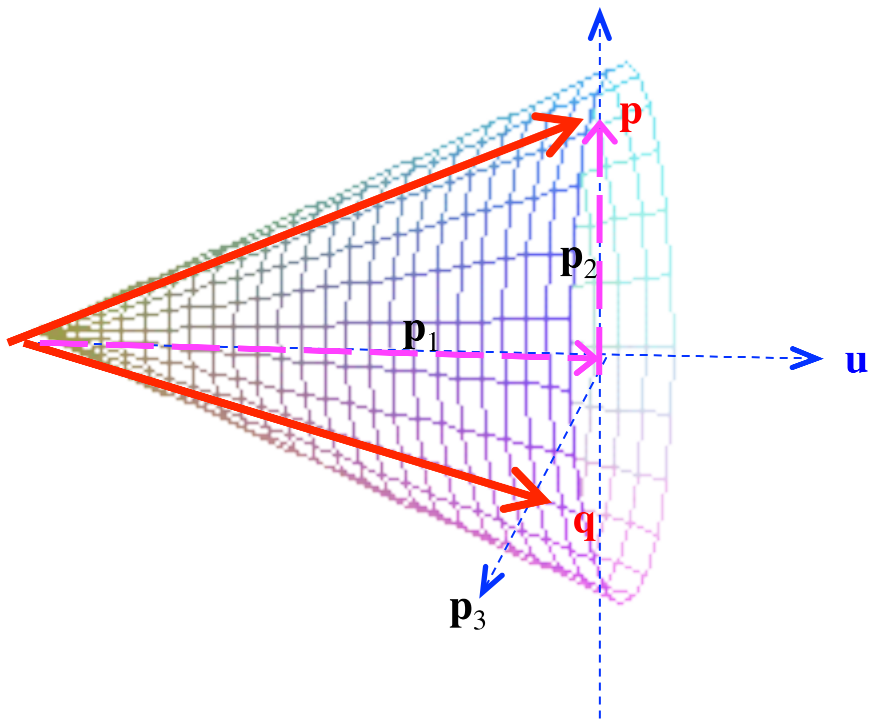
Derivation of general rotation matrix $\mathbf{R}(\varphi, \mathbf{u})$

(operation that rotates vector **p** about axis **u** by angle φ to new position **q**)



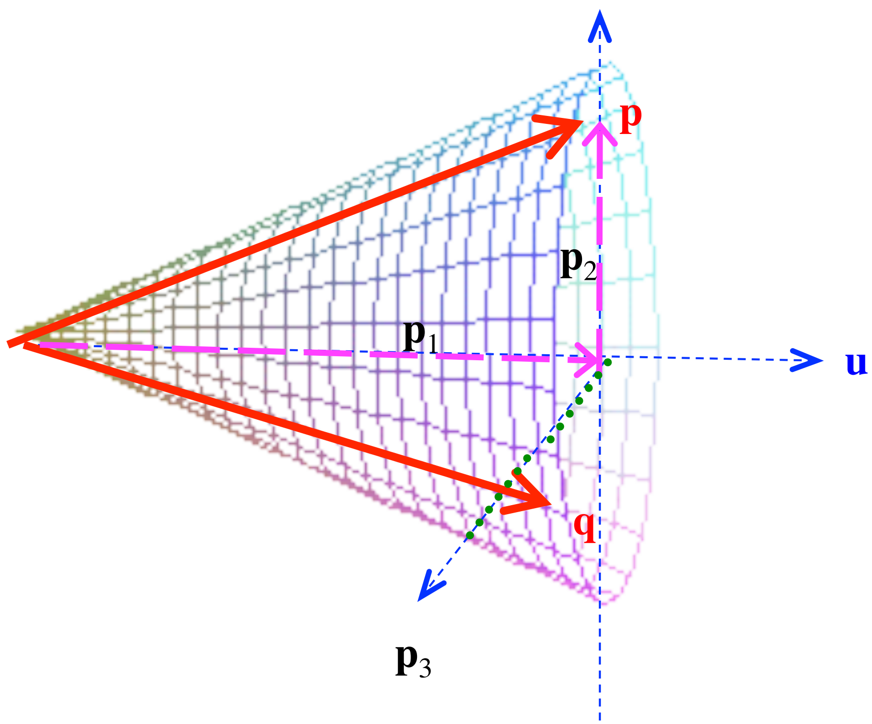
$$\mathbf{p}_1 = (\mathbf{p} \cdot \mathbf{u})\mathbf{u}$$

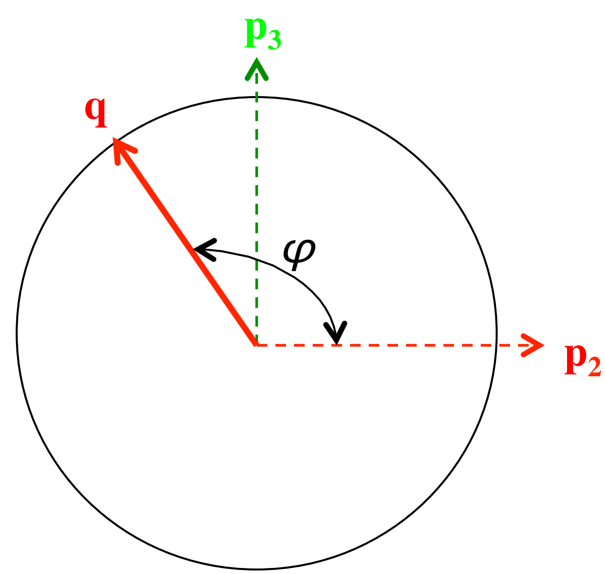
$$\mathbf{p}_2 = \mathbf{p} - \mathbf{p}_1$$

$$\mathbf{p}_3 = \mathbf{p}_1 \times \mathbf{p}_2$$

Position **q** can be expressed as a sum of the vectorial components of **p**.

$$\mathbf{q} = \mathbf{p}_1 + \mathbf{p}_2 \cos \varphi + \mathbf{p}_3 \sin \varphi = \mathbf{R}(\varphi,\mathbf{u})\, \mathbf{p}$$





The vectorial components of **p** can also be expressed in terms of the elements of **p** = (p_x, p_y, p_z) and **u** = ($u_1, u_2\ \ u_3$).

$$\mathbf{p}_1 = \left(\mathbf{p} \cdot \mathbf{u}\right)\mathbf{u}$$

$$\mathbf{p}_1 = \left(p_x u_1 + p_y u_2 + p_z u_3\right)\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\mathbf{p}_1 = \begin{bmatrix} p_x u_1^2 + p_y u_1 u_2 + p_y u_1 u_3 \\ p_x u_1 u_2 + p_y u_2^2 + p_z u_2 u_3 \\ p_x u_1 u_3 + p_z u_2 u_3 + p_z u_3^2 \end{bmatrix}$$

$$\mathbf{p}_1 = \begin{bmatrix} u_1^2 & u_1 u_2 & u_1 u_3 \\ u_1 u_2 & u_2^2 & u_2 u_3 \\ u_1 u_3 & u_2 u_3 & u_3^2 \end{bmatrix}\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

The vectorial components of **p** can also be expressed in terms of the elements of **p** = (p_x, p_y, p_z) and **u** = ($u_1, u_2 \;\; u_3$).

$$\mathbf{p}_2 = \mathbf{p} - \mathbf{p}_1$$

$$\mathbf{p}_2 = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - \begin{bmatrix} u_1^2 & u_1u_2 & u_1u_3 \\ u_1u_2 & u_2^2 & u_2u_3 \\ u_1u_3 & u_2u_3 & u_3^2 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

$$\mathbf{p}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - \begin{bmatrix} u_1^2 & u_1u_2 & u_1u_3 \\ u_1u_2 & u_2^2 & u_2u_3 \\ u_1u_3 & u_2u_3 & u_3^2 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

$$\mathbf{p}_2 = \begin{bmatrix} 1-u_1^2 & -u_1u_2 & -u_1u_3 \\ -u_1u_2 & 1-u_2^2 & -u_2u_3 \\ -u_1u_3 & -u_2u_3 & 1-u_3^2 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

The vectorial components of **p** can also be expressed in terms of the elements of **p** = (p_x, p_y, p_z) and **u** = $(u_1, u_2\ \ u_3)$.

$$\mathbf{p}_3 = \mathbf{p}_1 \times \mathbf{p}_2$$

$$\mathbf{p}_3 = \left[\begin{bmatrix} u_1^2 & u_1u_2 & u_1u_3 \\ u_1u_2 & u_2^2 & u_2u_3 \\ u_1u_3 & u_2u_3 & u_3^2 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \right] \times \left[\begin{bmatrix} 1-u_1^2 & -u_1u_2 & -u_1u_3 \\ -u_1u_2 & 1-u_2^2 & -u_2u_3 \\ -u_1u_3 & -u_2u_3 & 1-u_3^2 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \right]$$

$\cdots$

$\cdots$

General rotation matrix $\mathbf{R}(\varphi, \mathbf{u})$

(used to effect the rotation of a vector **p** about an arbitrary unit vector **u** through an angle of magnitude φ to a new position **q**)

$$\mathbf{R}(\varphi,\mathbf{u}) = \begin{bmatrix} \cos\varphi + (1-\cos\varphi)u_1^2 & u_1u_2(1-\cos\varphi) - u_3\sin\varphi & u_1u_3(1-\cos\varphi) + u_2\sin\varphi \\ u_1u_2(1-\cos\varphi) + u_3\sin\varphi & \cos\varphi + (1-\cos\varphi)u_2^2 & u_2u_3(1-\cos\varphi) - u_1\sin\varphi \\ u_1u_3(1-\cos\varphi) - u_2\sin\varphi & u_2u_3(1-\cos\varphi) + u_1\sin\varphi & \cos\varphi + (1-\cos\varphi)u_3^2 \end{bmatrix}$$

$$\mathbf{q} = \mathbf{p}_1 + \mathbf{p}_2\cos\varphi + \mathbf{p}_3\sin\varphi = \mathbf{R}(\varphi,\mathbf{u})\mathbf{p}$$