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ИП-811

Вариант - 13

$\boxed{1}$ Вычислить: $(1-i)^4/(-2+i2\sqrt{3})^6$,

$\sin(1+2i)$

(1) $\frac{(1-i)^4}{(-2+i2\sqrt{3})^6} =$

1) $(1-i)^4$

$z^n = |z|^n(\cos(n\varphi) + i\sin(n\varphi))$

$\varphi = \operatorname{arctg}\frac{y}{x}$ $\quad x = 1$

$y = -1$

$\varphi = \operatorname{arctg}\left(\frac{-1}{1}\right) = -\frac{\pi}{4}$

$$|Z| = \sqrt{x^2 + y^2}$$

$$|Z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$Z^n = \sqrt{2}^4 \left(\cos\left(4 \cdot \frac{-\pi}{4}\right) + i \sin\left(4 \cdot \frac{-\pi}{4}\right)\right) =$$

$$= 4\left(\cos(-\pi) + i \sin(-\pi)\right) =$$

$$= 4(-1 + 0i) = -4$$

2) $(-2 + i2\sqrt{3})^6$

$$Z^n = |Z|^n \left(\cos(n\varphi) + i \sin(n\varphi)\right)$$

$$\varphi = \pi + \operatorname{arctg} \frac{y}{x}$$

$$|Z| = \sqrt{x^2 + y^2}$$

$x = -2$

$y = 2\sqrt{3}$

$$\varphi = \pi + \operatorname{arctg}\left(\frac{2\sqrt{3}}{-2}\right) = \pi + \operatorname{arctg}(-\sqrt{3}) = \frac{2\pi}{3}$$

$$|z| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{16} = 4$$

$$z^n = 4^6\left(\cos\left(6 \cdot \frac{2\pi}{3}\right) + i\sin\left(6 \cdot \frac{2\pi}{3}\right)\right) =$$

$$= 4^6\left(\cos(4\pi) + i\sin(4\pi)\right) =$$

$$= 4^6(1 + 0i) = 4^6$$

$$= \frac{-4}{4^6} = -4^{-5}$$

Ответ: -4^{-5}

2 $\sin(1+2i)$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\sin(1+2i) = \frac{e^{i(1+2i)} - e^{-i(1+2i)}}{2i} = \frac{e^{i-2} - e^{-i+2}}{2i} =$$

$$= \frac{e^{-2}(\cos 1 + i\sin 1) - e^{2}(\cos(-1) + i\sin(-1))}{2i} \cdot \frac{i}{i} =$$

$$\frac{ie^{-2}\cos 1 - e^{-2}\sin 1 - ie^{2}\cos 1 - e^{2}\sin 1}{-2} =$$

$$= \frac{\sin 1\,(e^{2} + e^{-2})}{2} + i\,\frac{\cos 1\,(e^{2} - e^{-2})}{2} =$$

$$= \sin 1 \cdot ch\,2 + i\cos 1 \cdot sh\,2$$

<u>Ответ:</u> $\sin 1\, ch\,2 + i\cos 1\, sh\,2$