Using Fourier series to compare abnormalities in electrocardiograms

Introduction

I have always been fascinated by mathematics, especially in topics where there are real life applications. It is on one of the purest fields and is used in almost all the other subjects. One such example is Fourier series which are heavily used in fields like quantum mechanics, electrical engineering, and signal processing. They are used to convert any oscillating function into a series of sins and cosines. While deciding what kind of functions to base my exploration on, I remembered a time from my childhood, when my father had a heart attack. This revelation pushed me to research on heartbeats and how doctors study them to identify abnormalities in the heartbeat. An electrocardiograph (ECG) is used to monitor and graph heartbeats. They measure the voltage produced by the heart, and since heartbeats also create an oscillating pattern, it is implied that ECGs also create periodic patterns of the heartbeat.

Aim

The aim of this investigation is to apply Fourier series to a sinus rhythm function and compare the coefficients of that series to other standard anomalies observed in ECGs.

Fourier series

A Fourier series is an infinite sum of trigonometric functions that can be manipulated so that they can represent any periodic function. It normally only consists of sinusoidal and co-sinusoidal functions with varying amplitudes. This begs the question, why only sines and cosines are used. The answer to that question lies in the linear independence of these function. A group of functions are considered to be linearly independent if and only if no such function in that group can be represented by a combination of other functions in that group. This is the case with sines and cosines since they cannot be defined in terms of the other function i.e., sine(ax+b) cannot be defined using a sum of cosine(cx+d).

Two functions, $c_1F(x)$ and $c_2G(x)$ are linearly independent if:

$c_1F(x) + c_2G(x) = 0$, Where c_1 and c_2 are scalar quantities

This equation can only be satisfied when $c_i = 0$ for $i = 1 \, and \, 2$

We can use this equation to demonstrate why sine cosine are linearly independent. Assume F(x)=sin(x) and G(x)=cos(x). Therefore, the linear combination of these two functions is:

$$c_1 \sin(x) + c_2 \cos(x) = 0$$

Taking $x = 0$

$$c_1(0) + c_2(1) = 0$$

$$c_2 = 0$$

Now taking $x = \frac{\pi}{2}$,

$$c_1(1) + c_2(0) = 0$$

$$c_1 = 0$$

As seen, $c_1 = c_2 = 0$, this directly implies that sine and cosine are linearly independent functions.

Now that we have proved the linear independence of the functions used in the Fourier series, we can derive some important general formulas used in the series that will help with the calculations done ahead.

For a function $y = f(x)$ with a period 2L and defined in the interval $[-L, L]$, the Fourier series for the function is defined as follows:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{L}\right)\right) + \sum_{n=1}^{\infty}\left(b_n \sin\left(\frac{n\pi x}{L}\right)\right)$$

where,

a_0 (Mean Value Term) $= \frac{1}{L}\int_{-L}^{L} f(x)\, dx$

a_n (Coefficient of the cosine functions) $= \frac{1}{L}\int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx$

b_n (Coefficient of the sine functions) $= \frac{1}{L}\int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx$

a_n and b_n are commonly known as the first and the second coefficients of the Fourier series respectively. They both start with a_0 and b_0, but since b_0 is equal to sin (0), which is equal to 0 it is excluded from the series.

To derive the first coefficient of the series, it is important to start from the general formula for the cosine functions of the series:

$$f(x) = \sum_{n=0}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right)\right)$$

Multiply each side by $\cos\left(\frac{m\pi x}{L}\right)$, $m \in \mathrm{N}$

$$f(x) \times \cos\left(\frac{m\pi x}{L}\right) = \sum_{n=0}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right)\right) \times \cos\left(\frac{m\pi x}{L}\right)$$

Integrating each side over a period of 2L

$$\int_{2L} \left(f(x) \times \cos\left(\frac{m\pi x}{L}\right)\right) dx = \int_{2L} \left(\sum_{n=0}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right)\right) \times \cos\left(\frac{m\pi x}{L}\right)\right) dx$$

Then the following identity can be used to convert the function into a combination of cosines: $\cos(a) \times \cos(b) = \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b)$

$$\int_{2L} \left(f(x) \times \cos\left(\frac{m\pi x}{L}\right)\right) dx = \frac{1}{2}\sum_{n=0}^{\infty} a_n \int_{2L} \left(\cos\left((m+n)\frac{\pi x}{L}\right) + \cos\left((m-n)\frac{\pi x}{L}\right)\right) dx$$

The period of $cos\left((m+n)\frac{\pi x}{L}\right)$ is $\frac{2L}{m+n}$, which means the frequency of the function is $(m+n)$, and since the function has $(m+n)$ complete waves over 2L, the end result will equal 0 since all the positive and negative parts will cancel each other out. The same thing happens with $\cos\left((m-n)\frac{\pi x}{L}\right)$ which has $(m-n)$ number of oscillations. The previous statement is only true when $m \neq n$ but the scenario changes when $m = n$. In this case, $\cos\left((m-n)\frac{\pi x}{L}\right)$ equals to 1 and therefore:

$$\int_{2L} \left(f(x) \times \cos\left(\frac{m\pi x}{L}\right)\right) dx = \frac{1}{2} a_m \int_{2L} 1\, dx$$

Solving the integral results in:

$$\int_{2L} \left(f(x) \times \cos\left(\frac{m\pi x}{L}\right)\right) dx = \frac{1}{2} a_m \times 2L$$

Since m and n are just variables it is the same as saying:

$$a_n = \frac{1}{L}\int_{2L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

The second coefficient can be derived using the same steps except we multiply both the side with $\sin\left(\frac{m\pi x}{L}\right)$.

Background information

There are three distinct parts in a normal person's ECG: P wave, QRS complex and the T wave. They are created when different parts of the heart depolarize in the distinct pattern, this depolarization creates electric signals that are recorded in the ECG machine. The depolarization of the atria creates the P wave, then the ventricles depolarize creating the QRS complex and finally the repolarization of the ventricles form the T wave. A normal sinus rhythm looks something like this:

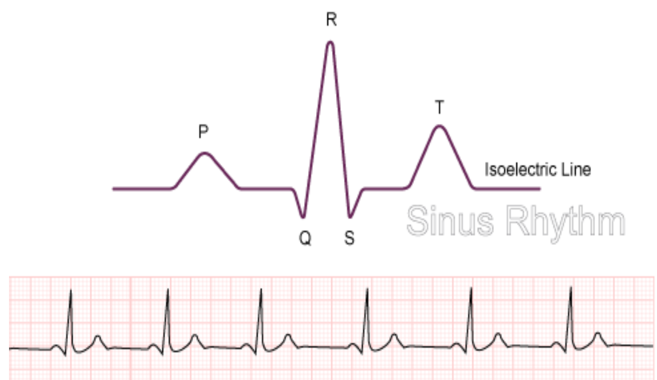


Figure 1- ECG of a sinus rhythm (https://www.nottingham.ac.uk/nursing/practice/resources/cardiology/function/sinus_rythm.php)

Majority of the values for the amplitudes and duration of the waves are taken from internet sites mentioned in the bibliography. A few assumptions were made for the values that are not to be found on any site. By observing some ECGs of normal healthy people, I approximated certain values for the different parts of the waveform that were not available. The following table lists all the durations and amplitudes of the different parts of the ECG.

Table 1

ECG wave intervals	duration(ms)	amplitude (mV)
P wave	80	0.25
PQ segment	40	0
QRS complex	100	Q = -0.2
		R = 2.8
		S = -0.26
RT segment	75	0
T wave	150	0.45

The assumptions made were for PQ segment, amplitude of s, durations of Q and S and the duration of ST segment. The durations of both, Q and S in the QRS complex are assumed to be 20 milliseconds.

Since the whole waveform in an ECG is rather smooth and irregular, another set of assumptions were made in order to simplify the whole waveform which can be represented in a graphical manner such it can be a function of mV in terms of ms (milliseconds). Firstly, both the P and T waves are assumed to be parabolic functions with the x coordinate of the vertex halfway between its roots and the QRS complex is assumed to be a combination of straight lines. P wave started from the origin and all both, PQ and ST segments lied on the x axis.

A rough sketch of the waveform is as follows:

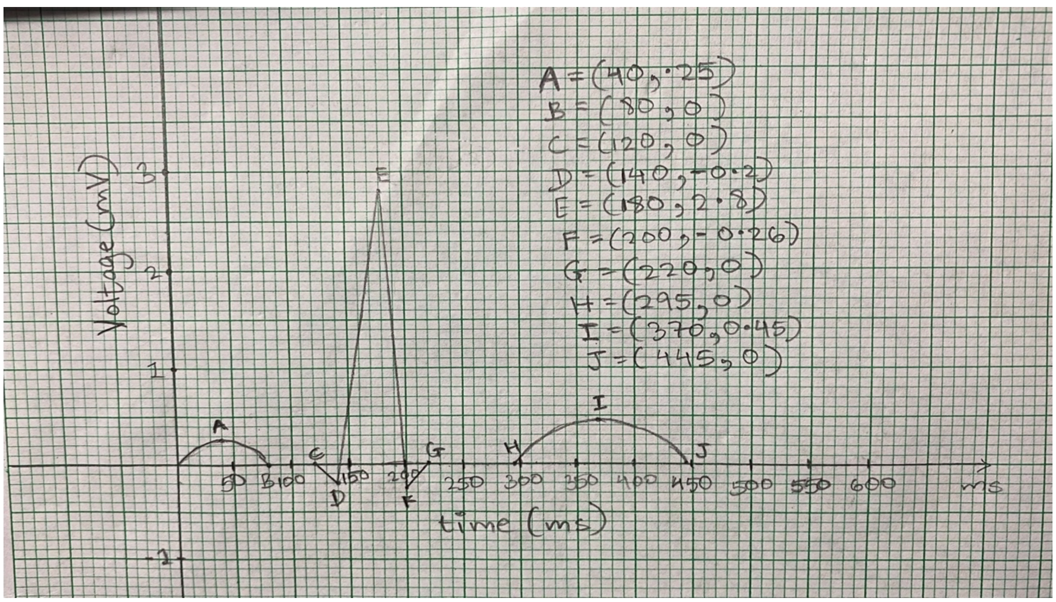


Figure 2

Methodology

To create a Fourier series for the sinus rhythm, first it must be converted into a function. Using the assumptions made earlier we can represent the waveform as a piecewise function and then apply Fourier series to that function. Then apply Fourier series to some other anomalies whose function will be derived beforehand. Lastly compare the coefficients of the anomalies to the coefficients of a sinus rhythm to differentiate the anomaly more efficiently.

Determining the functions

P wave:

The P wave is assumed to be a parabola which has the general equation:
$y = ax^2 + bx + c$, where "a", "b" and "c" are numerical coefficients. The sign of "a" determines whether the function will have a maximum or a minimum point and "c" is the y intercept of the function. Since the P wave has a maximum point at (40,0.25) "a" has a negative sign i.e., concave down. The coordinates of the zeros are (0,0) and (80,0). The vertex of a quadratic formula is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.
So, using that formula and the general formula for quadratic functions we can determine a, b and c.

$\frac{-b}{2a} = 40$ $\therefore b = -80a$[1] Substituting the coordinates (0,0) of the roots in the general formula, we get $0 = a(0)^2 + b(0) + c$ $c = 0$	Then substitute the coordinates of the vertex: $0.25 = a(40)^2 + b(40) + 0$ $0.25 = 1600a + 40b$ $0.25 = 1600a + 3200a$ (Substitute [1]) $a = -1.5625 \times 10^{-5}$ $b = 0.0125$

The equation of the P wave is:

$$y = (-1.5625 \times 10^{-5})x^2 + 0.0125x$$

QRS complex:

The QRS complex can be divided into 4 distinct linear functions and the equations of those lines can be derived using the coordinates lying on the line and the gradient of the line.

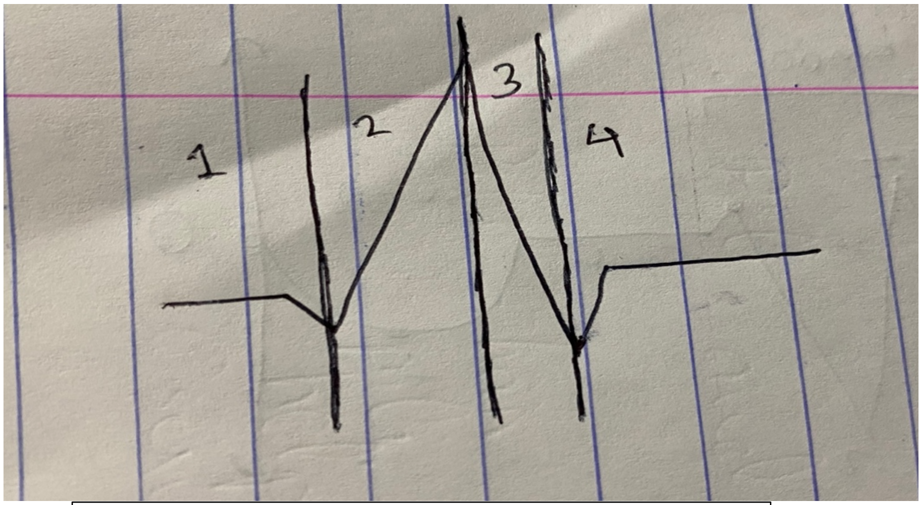


Figure 3

Equation of the first part:

We can use the x intercept of the line along with the amplitude of Q to derive the equation of the line. The coordinates are (120,0) and (140, -0.2).

To calculate the gradient of the line we can use the following formula:

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

$$m = \frac{-0.2 - 0}{140 - 120} = -0.01$$

Then using $(y - y_0) = m(x - x_0)$ and inputting one of the coordinates we get:

$$y - (-0.2) = m(x - 140)$$

$$y = (-0.01)x + 1.2$$

For the remaining parts of the QRS complex, the same process and be used and therefore:

Equation of the 2nd Part: $y = 0.075x - 10.7$
Equation of the 3rd Part: $y = (-0.153)x + 30.34$
Equation of the 4th Part: $y = 0.013x - 2.86$

T wave:

Again, the same process that we used to derive the equation of the P wave can be used to get an equation of the T wave. Using these three coordinates- (370,0.45), (295,0) and (445,0) and inputting them in the general formula for the quadratic equation, we get three simultaneous equations that can be solved using technology:

$$0.45 = 136900a + 370b + c$$
$$0 = 87025a + 295b + c$$
$$0 = 198025a + 445b + c$$

Solving them gives us the values for a, b and c:

$$a = -8 \times 10^{-5}$$
$$b = 0.0592$$
$$c = -10.502$$

Therefore, the equation is:

$$y = (-8 \times 10^{-5})x^2 + 0.0592x - 10.502$$

Equation of the sinus rhythm:

The combined piecewise function of the sinus rhythm is as follows:

$$f(x) = \begin{cases} (-1.5625 \times 10^{-5})x^2 + (0.0125)x & 0 \leq x \leq 80 \\ 0 & 80 \leq x \leq 120 \\ (-0.01)x + 1.2 & 120 \leq x \leq 140 \\ 0.075x - 10.7 & 140 \leq x \leq 180 \\ (-0.153)x + 30.34 & 180 \leq x \leq 200 \\ 0.013x - 2.86 & 200 \leq x \leq 220 \\ 0 & 220 \leq x \leq 295 \\ (-7.8125 \times 10^{-5})x^2 + (7.8125 \times 10^{-2})x - 19.0313 & 295 \leq x \leq 445 \\ 0 & 445 \leq x \leq 1000 \end{cases}$$

__**Application of Fourier series**__

Mean value term (a_0):

The formula for mean value term is:

$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\, dx$

Here I am taking L as 500 since the period of one heartbeat is 1000 ms which is also the period the sinus rhythm. Therefore:

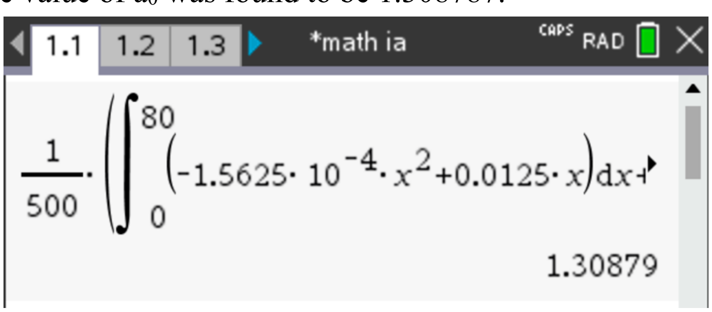
$$a_0 = \frac{1}{500}\int_{-500}^{500} f(x)\,dx$$

Since our piecewise function starts from 0 and ends at 1000, we can change the limits of the above formula so that the limits match with the piecewise function I just derived. This is possible since the waveform of a sinus rhythm is periodic, and the value will be the same no matter what interval is being used. As a result, instead of calculating the value over an interval of [-500,500] we can calculate it over an interval of [0, 1000] and the value won't get affected.

$$a_0 = \frac{1}{500}\int_{-500}^{500} f(x)\,dx = \frac{1}{500}\int_{0}^{1000} f(x)\,dx$$

$$\boxed{\begin{aligned} a_0 = \frac{1}{500}\Bigg(\int_0^{80} &[(-1.5625\times 10^{-5})x^2 + (0.0125)x]dx + \int_{120}^{140}[(-0.01)x + 1.2]dx \\ &+ \int_{140}^{180}[0.075x - 10.7]dx + \int_{180}^{200}[(-0.153)x + 30.34]\,dx \\ &+ \int_{200}^{220}[0.013x - 2.86]dx \\ &+ \int_{295}^{445}[(-8\times 10^{-5})x^2 + (0.0592)x - 10.502]\,dx\Bigg) \end{aligned}}$$

Using technology, the value of a_0 was found to be 1.308787.



First coefficient (a_n)

Formula used:

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\int_{0}^{2L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$\boxed{\begin{aligned} a_n = \frac{1}{500}\Bigg(&\int_0^{80}[(-1.5625\times10^{-5})x^2+(0.0125)x]\cos\frac{n\pi x}{500}dx+\int_{120}^{140}[(-0.01)x \\ &+1.2]\cos\frac{n\pi x}{500}dx+\int_{140}^{180}[0.075x-10.7]\cos\frac{n\pi x}{500}dx \\ &+\int_{180}^{200}[(-0.153)x+30.34]\cos\frac{n\pi x}{500}dx+\int_{200}^{220}[0.013x-2.86]\cos\frac{n\pi x}{500}dx \\ &+\int_{295}^{445}[(-8\times10^{-5})x^2+(0.0592)x-10.502]\cos\frac{n\pi x}{500}dx\Bigg)\end{aligned}}$$

Second coefficient (b_n)

Formula used:

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\int_{0}^{2L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$\boxed{\begin{aligned} b_n = \frac{1}{500}\Bigg(&\int_0^{80}[(-1.5625\times10^{-5})x^2+(0.0125)x]sin\frac{n\pi x}{500}dx+\int_{120}^{140}[(-0.01)x \\ &+1.2]sin\frac{n\pi x}{500}dx+\int_{140}^{180}[0.075x-10.7]sin\frac{n\pi x}{500}dx \\ &+\int_{180}^{200}[(-0.153)x+30.34]sin\frac{n\pi x}{500}dx+\int_{200}^{220}[0.013x-2.86]sin\frac{n\pi x}{500}dx \\ &+\int_{295}^{445}[(-8\times10^{-5})x^2+(0.0592)x-10.502]sin\frac{n\pi x}{500}dx\Bigg)\end{aligned}}$$

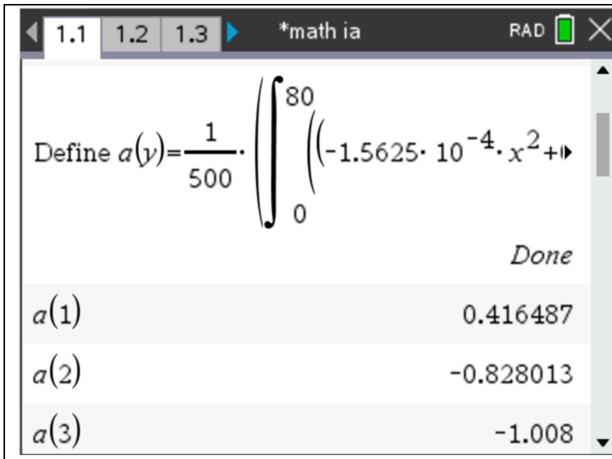
Once the formulas for the coefficients of the Fourier series are derived, now I can distinguish the Fourier coefficients of a normal ECG with abnormal ECGs. It is important to compare their coefficients and analyze any patterns between them to understand the difference between them. Therefore, the first 10 terms of both the first and the second coefficient are listed below.

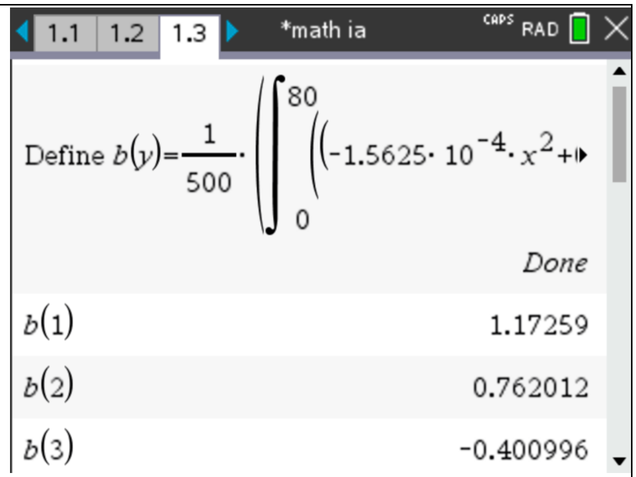
Analysis

Table 2

n	a_n	b_n
1	0.416486973	1.17258646032
2	-0.828012636	0.76201151622
3	-1.007999400	-0.40099593980
4	-0.027708606	-1.13951838005
5	1.112120787	-0.43490327930
6	0.788621099	0.88408011891
7	-0.488937378	1.02373166844
8	-1.091562599	-0.03666347831
9	-0.367530636	-0.99792302936
10	0.778697453	-0.68034176037

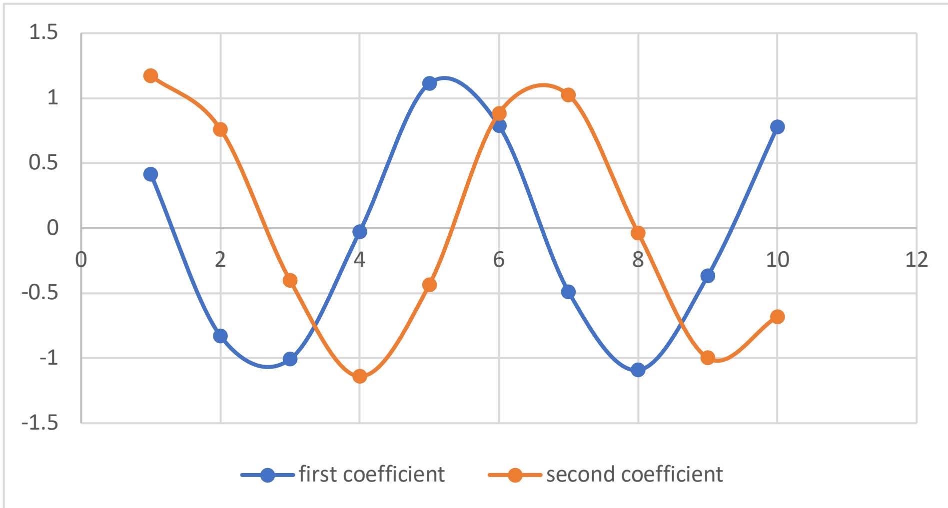
All the values were calculated using technology (I used my GDC TI-nspire cx ii). First, I defined the whole formula as a function of the coefficient in terms of n. Since all the integrals had constant values, the only value varying was n. Then I just varied n till 10.





Using the values in table 2, a line graph was plotted of the magnitude of the coefficient vs the term number of the coefficient to better see the trends between the data points.

Graph 1



At the first glimpse, both the lines approximately represent trigonometric functions due to their oscillating shape. The curve of the first coefficient is very similar to the function -sin(x) and the curve of the second coefficient represents the function cos(x). Even though the functions are not perfectly symmetrical, both the curves have an amplitude slightly greater than 1 and have a period of approximately equal to 5 terms. When viewed from a different perspective, the curve of the first coefficient when horizontally translated 2 units to the right represents the curve for the second coefficient.

Equations for the Fourier series of a normal ECG

$$f(x) = \frac{1.30878}{2} + \left((0.416486973)\cos\left(\frac{\pi x}{500}\right)\right) + \left((-0.828012636)\cos\left(\frac{2\pi x}{500}\right)\right) + \ldots + \left(a_n cos\left(\frac{n\pi x}{500}\right)\right) + \left((1.17258646032)\sin\left(\frac{\pi x}{500}\right)\right) + \left((0.76201151622)\sin\left(\frac{2\pi x}{500}\right)\right) + \ldots + \left(b_n sin\left(\frac{n\pi x}{500}\right)\right)$$

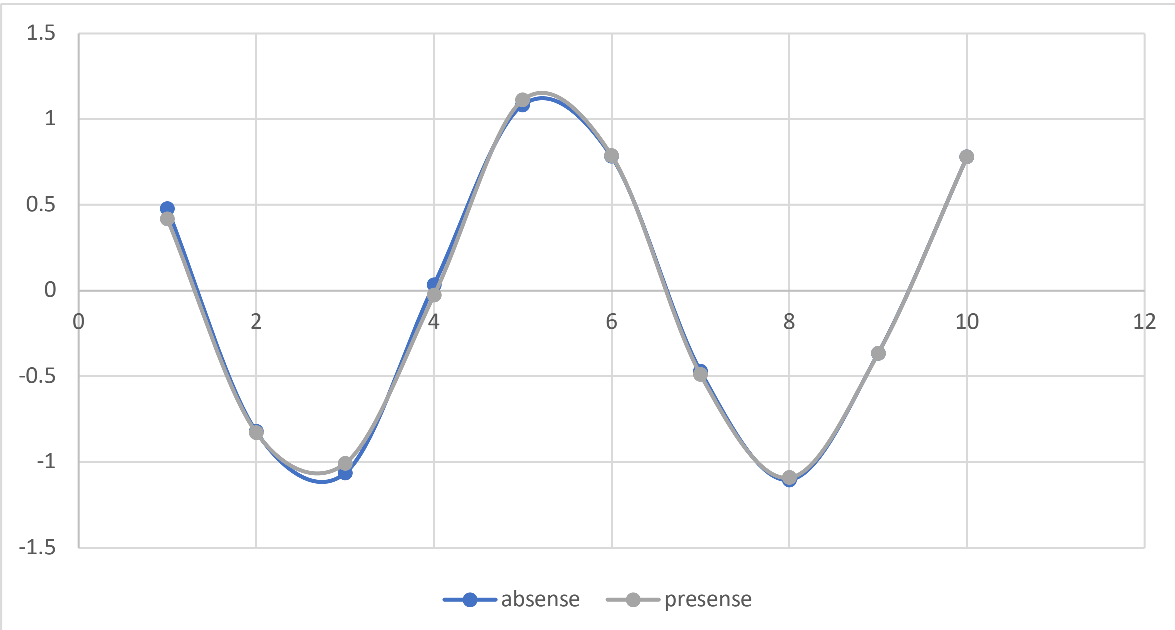
Abnormalities in ECGs:

Absence of T wave

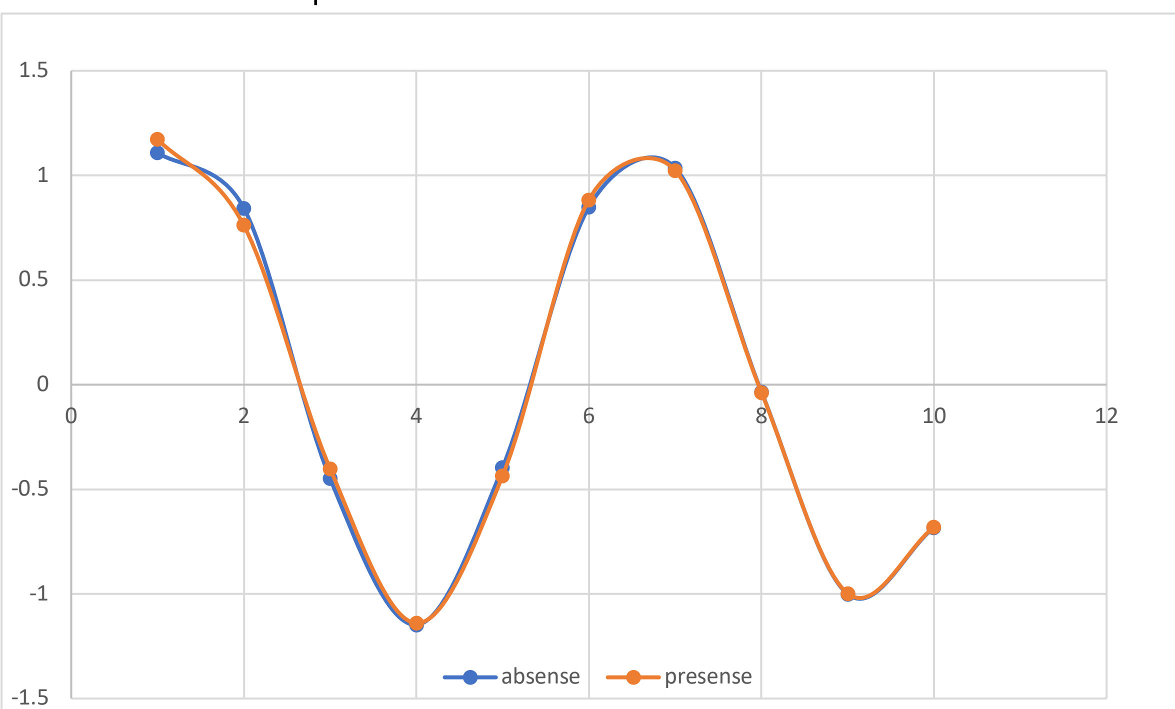
This anomaly is caused when there is disruption in the repolarization of the one of the segments of the heart. Flattened T waves normally represent Ischemia or hypokalemia. In such cases the heart is unable to repolarize properly before the atrial depolarization of the next cycles, therefore the heart is not able to pump adequate amount of blood since it stays contracted.

To derive the piecewise function of the following abnormality, I just replaced the function for T wave with 0.

Graph 2-first coefficient with and without T wave



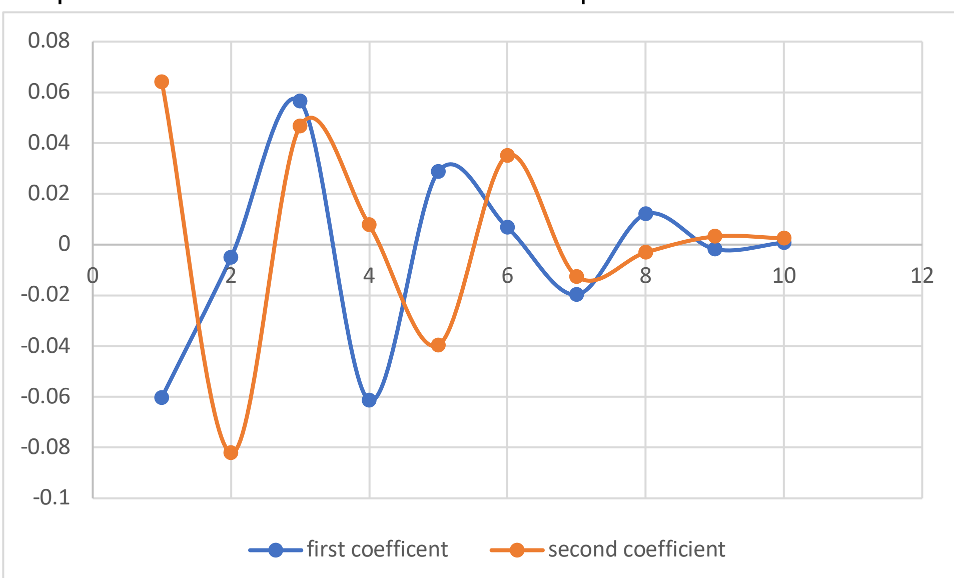
Graph 3-second coefficient with and without T wave



As visible in graph 1 and 2, the curves for the first and second coefficients are very similar for absent and present T waves. For graph 1 after the 6^{th} term the coefficients become undistinguishable. On the other hand, for graph 2 the terms start to look similar after the 7^{th} term. The magnitude of the first coefficient is greater when the T wave is present for terms 1 and 4. It is smaller than the coefficient for absent t wave for terms 3 and 6. Similarly, for graph 2 the second coefficient for present T wave is greater than that of absent T wave for terms 1, 3, and 6 and is smaller for terms 2, 5 and 7. A major characteristic of graph 2 is the deviation of the data points for term 1 and 2.

Since the coefficients for both, absent and present T wave are so similar, it can get hard to differentiate them because after 6 terms the graphs look identical. Therefore, to distinguish between absent and present T wave a graph of deviation of the coefficients against the term number is formed.

Graph 4- deviation of the coefficients for present and absent T wave



The shape of both the curves is very similar, the difference being the curve for the second coefficient can be viewed as an inverted curve for the first coefficient. The deviations observed for both the coefficients are very similar as the difference between the coefficients for present and absent T wave keep oscillating between positive and negative values. Another useful characteristic of the graph is that the curve for the second coefficient can be viewed as the curve for the first coefficient translated one unit to the left and reflected on the x axis. The maximum deviation is observed at term 1 for both the coefficients and then they decrease as the term number increases. Such deviation graphs can be more useful than graphs of coefficient vs term number as they help differentiate between an anomaly and a normal sinus rhythm.

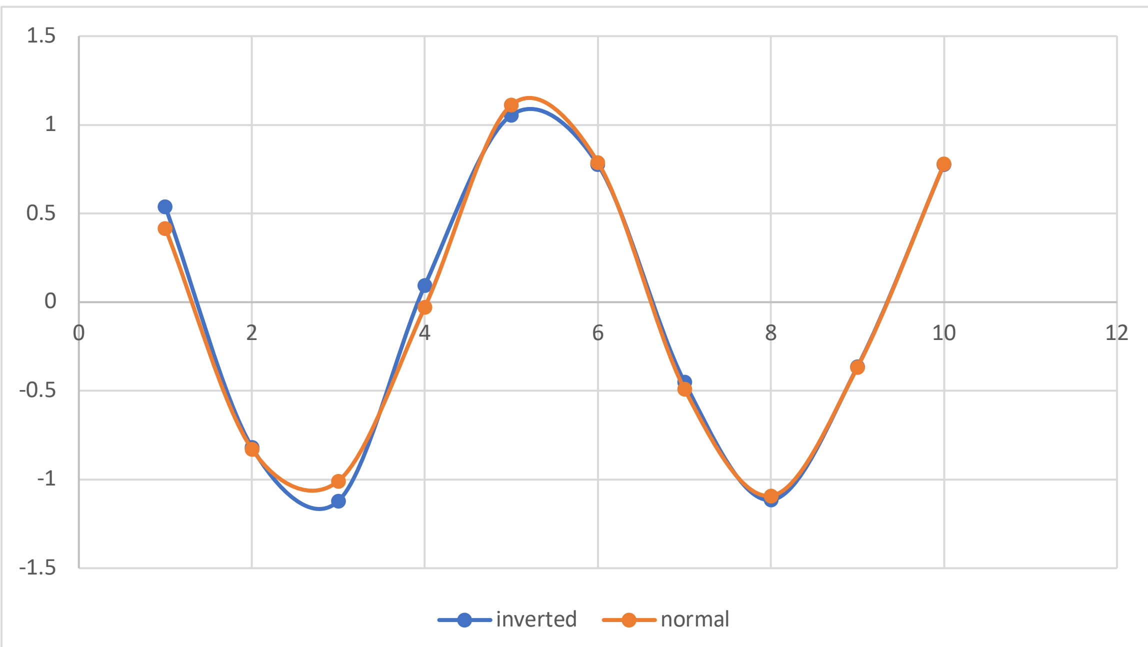
Inverted T wave

One of the many conditions my dad faced when he experienced heart attack was myocardial ischemia and bundle branch blocking. When diagnosed by both these conditions, an inverted T wave is present in the person's ECG.

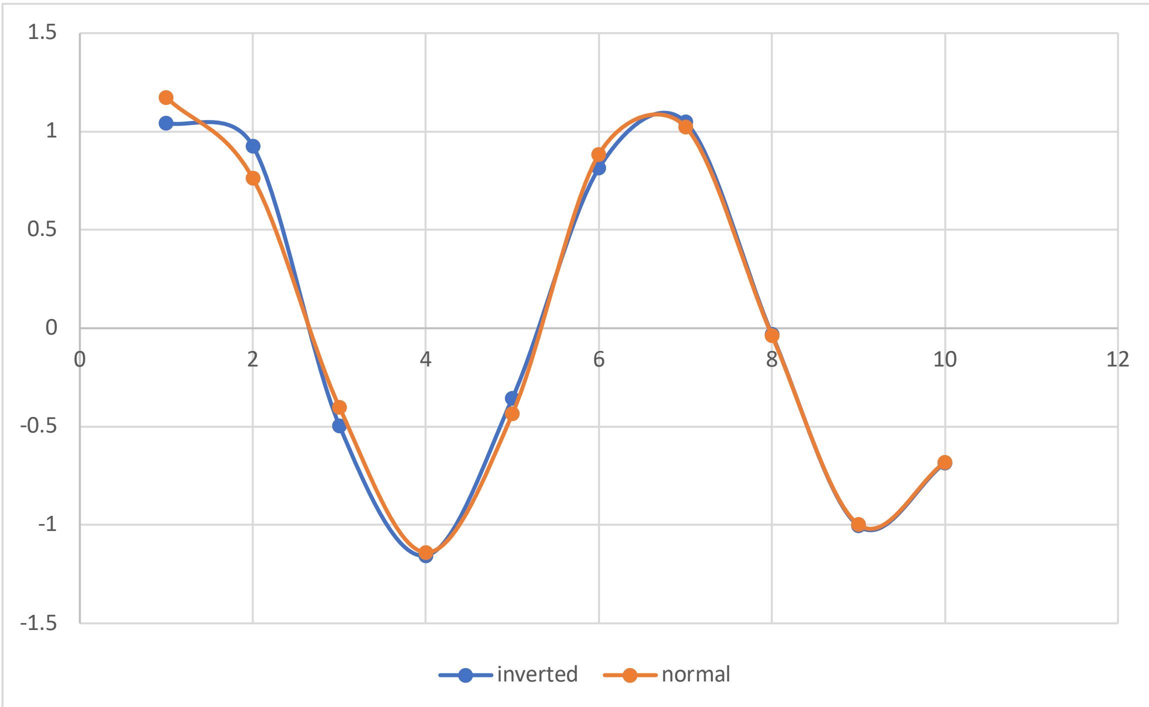
To create a function for this abnormality, it was assumed that the inverted T wave was a perfect reflection on the x axis of a normal T wave in sinus rhythm. To account for this reflection, the whole function is multiplied by -1.
So, the equation of the T wave is:

$$(8 \times 10^{-5})x^2 - (0.0592)x + 10.502$$

Graph 5- first coefficient of a normal and an inverted T wave



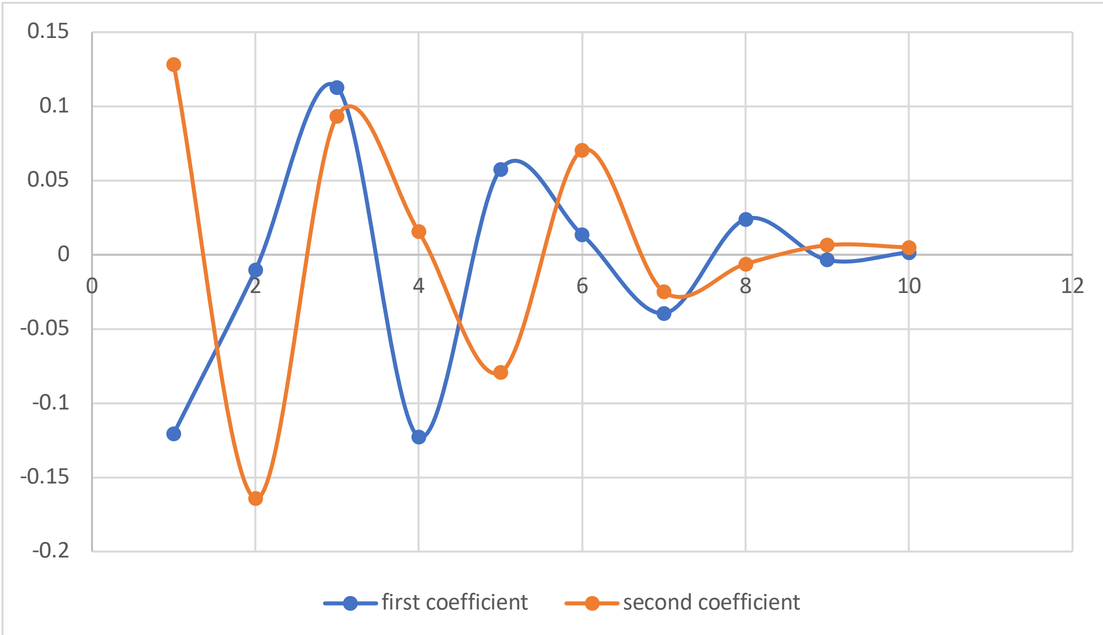
Graph 6- second coefficient of normal and inverted T wave



The shape of the curve for both first and second coefficients for inverted T wave are very similar to the curves for normal T wave. The same patterns between the two Fourier series are observed as showcased in absent T wave. As in graph 2, the first coefficient for absent and normal T wave for term 2 are very similar and the same trend is observed for graph 5 where the values for term 2 are almost identical. Again, for graph 5 the terms become more or less the same after term 6 and for graph 6 the coefficients become similar after term 7, the same pattern observed for absent t wave. One another characteristic worth mentioning is the fact that the pattern of the deviations is similar but the magnitude of the deviation between the inverted T wave and normal T wave is much greater than compared to absent T wave.

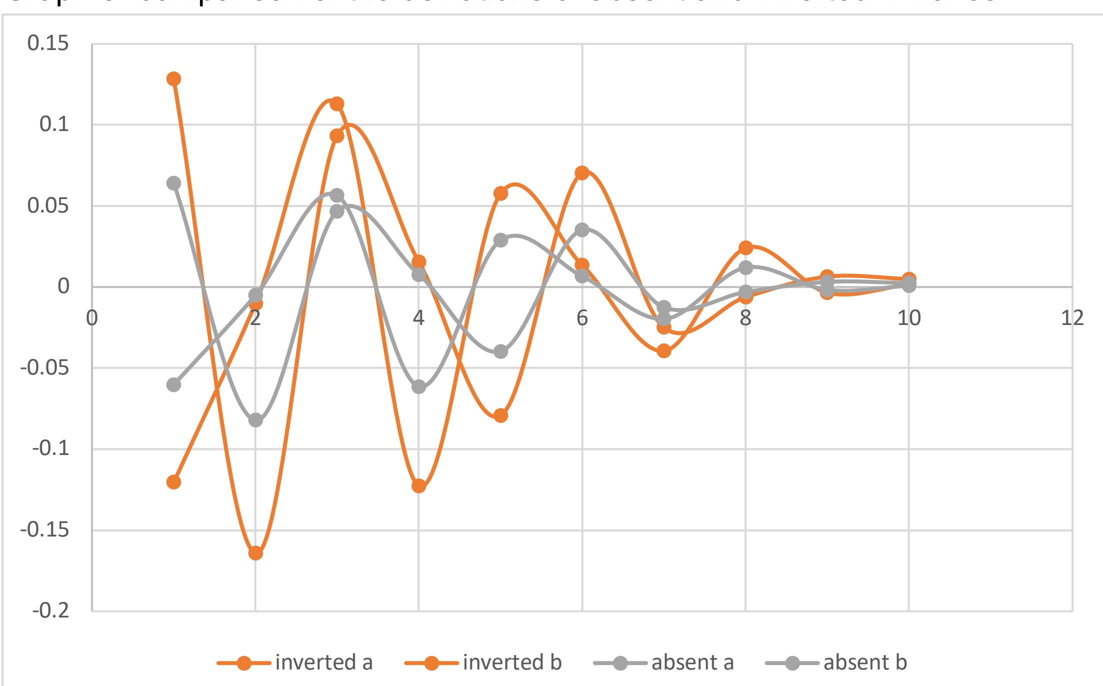
Furthermore, a graph of deviation of the two coefficients can be plotted to better understand the difference between the anomalies.

Graph 7- deviations of the first and second coefficients for inverted T wave



Again, the maximum deviation is observed at term 1 for both the coefficients and they decrease as the term number increases. The same pattern as observed in the graph of deviation for absent T wave are visible here, the only difference being the magnitude of the deviation. Evidently, the deviations for an inverted T wave are much greater than absent T wave, this makes them very easy to differentiate when looking at their Fourier coefficients.

Graph 8- comparison of the deviations of absent and inverted T waves



As previously discussed, the shapes of the curves for first and second coefficients for the deviation of inverted and absent T waves mimic each other. The deviation for both the anomalies decreases and will eventually become 0 as predicted by the graph. The shape of the absent T wave curve is very similar to that of the inverted T wave curve. Also, difference between the deviations of the absent and inverted T wave curve seems to have a scale factor equal to two as the curves for absent T wave seem to be vertically stretched to represent the curves for the inverted T wave deviations. The increase in deviation is partly because the sinus rhythm undergoes changes from an absent to an inverted T wave. This proves that the graphs of deviations of the coefficients of the Fourier series can be used to detect and identify anomalies in ECG diagrams.

Limitations

One of the major limitations in this investigation is the assumptions made to either simply the process of deriving the function of the sinus rhythm or due to lack of information available about the amplitude and durations of different intervals. In addition to that ECGs normally have a very irregular shape with asymmetrical P and T waves and overall is a very smooth curve which was not the case in my investigation since we assumed it was a combination of parabolas and straight lines. These assumptions should be taken care of when presenting an ECG through a Fourier series. Furthermore, one improvement that could be done to improve this investigation would be to derive graphically accurate functions of the ECG.

Conclusion

The methodology used in this investigation could be used by doctors to identify and differentiate various abnormalities when it becomes hard to do so. Specifically, graphs 7 and 8 can be used to detect the type of cardiac abnormality present in the patient’s ECG. Moreover, this method can be converted into a code so that a machine can directly input the durations and amplitudes of the ECG into a calculator and performs the necessary operations on those values to create a Fourier series out of it and automatically compare the coefficients of that series to other sinus rhythms to identify the defect. Such an advancement would make reading ECGs much more efficient since it avoids human error made while reading them manually. To conclude this investigation, I believe that this study has proven that utilizing this method of using sinus rhythms and converting them into Fourier series is credible enough to be used by doctors all over the world and aid the field of medicine.

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