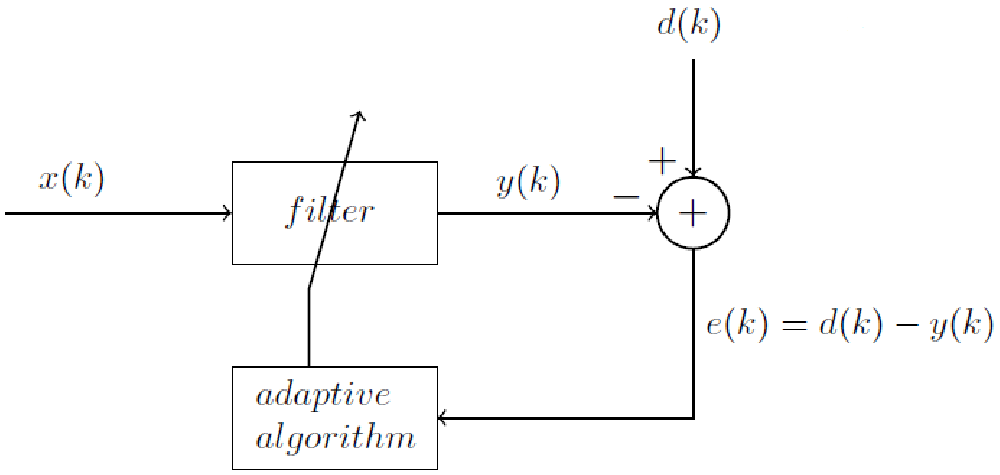
E5ADSB Exercise 2 – The Wiener Filter

Below are a few exercises in different applications of the Wiener filter

KPL, 2018-07-04

Exercise 1

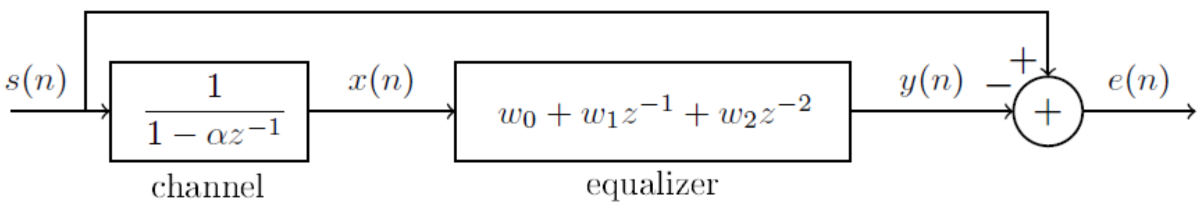
Consider an adaptive filtering setup where the input signal is given by $x(k) = \frac{1}{2}\cos(0.3k)$ and the desired signal is given by $d(k) = \sin(0.3k)$.



1. The adaptive filter is an FIR filter. How long must the filter be in order to filter $x(k)$ to $d(k)$?
2. Discuss how the filter should work. What is the purpose of the filter?
3. Calculate the optimum Wiener filter.
4. Investigate the magnitude and phase response of the optimum filter and explain how the filter works.

Exercise 2 – Channel equalization

This is a little exercise in channel equalization (inverse filtering). Consider the setup below. The input signal (data symbols) is assumed to be stationary white noise.



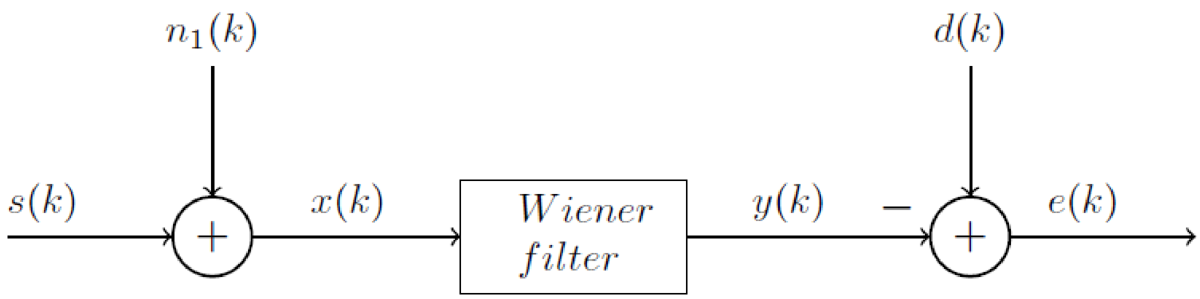
In channel equalization the goal is to find a transfer function that compensates for the linear distortion caused by the channel.

1. Find the autocorrelation matrix $\mathbf{R_x}$ of the input signal to the equalizer and also find the cross-correlation vector $\mathbf{p_{dx}}$ between the desired output and the input to the equalizer.
2. Calculate the optimum set of filter coefficients for the equalizer (the Wiener solution).
3. What is the minimum mean square error (MMSE) of the equalizer output?
4. Discuss how the desired signal could be obtained in practice.

Exercise 3 – Signal Prediction

In prediction the goal is to predict future samples of a signal. In this exercise one sample ahead.

Consider the setup below where a sinusoidal signal, $s(k)$, have been corrupted (polluted) by noise, $n_1(k)$.



The sinusoidal signal is given by $s(k) = \sin(0.3k + \theta)$ where θ is some value between 0 and 2π, and the noise $n_1(k)$ is generated by an AR(1) process with the difference equation $n_1(k) = -0.2n_1(k-1) + n(k)$ where $n(k)$ is Gaussian white noise with zero mean value and a variance of 0.2.

The goal is to predict future values (one sample ahead) of the sinusoidal signal through a Wiener filter. That is $d(k) = s(k+1)$.

1. Use a first order Wiener FIR filter (M=2). Calculate the autocorrelation matrix $\mathbf{R_x}$ and the cross-correlation vector $\mathbf{p_{dx}}$. Find the optimum set of filter coefficients $\mathbf{w_o}$.
2. Simulate the system in Matlab and compare the predicted signal and the actual signal.
3. Does the prediction improve if the filter order is increased to a second order filter (M=3)?
4. Try to use a second order Wiener filter (M=3) to predict two samples ahead.
   That is $d(k) = s(k+2)$.