SIMPLE AC CIRCUITS

Alternating Current is defined as the current that varies periodically with time. It can be defined as an electric current that reverses its direction many times a second at regular intervals, typically used in power supplies.

Alternating current is a type of current in which the direction of the flow of electrons switches back and forth at regular intervals or cycles. Current flowing in power lines and normal household electricity that comes from a wall outlet is alternating current.

An alternating current let's say starts from zero goes to maximum in one direction and comes back to zero then goes to a maximum (amplitude) in the opposite direction.

The instantaneous current (I) (the current at any instance in the motion) is expressed as

$$I = I_o \sin\theta$$

$$\theta = \omega t$$

$$I = I_o \sin\omega t$$

$$\omega = 2\pi f$$

$$I = I_o \sin 2\pi f t$$

I_oIs the maximum current or peak current or amplitude current.

Similarly, for voltages

$$V = V_o \sin\theta$$

$$V = V_o \sin\omega t$$

$$V = V_o \sin 2\pi f t$$

The maximum frequency of an AC is 60Hz. The standard range of home Alternating currents is from 50Hz to 60Hz

DIRECT CURRENT

This is a current that flows in one direction only unless the terminals are reversed then they flow in the opposite direction. A direct current has a frequency of zero. This is because the frequency is the number of cycles per second and since direct current moves straight and does not go in cycles, it can't have a frequency.

PULSATING CURRENT

This is like a direct current that has a more recurring or less regular variations in magnitude. This is a DC in usually produced by an AC by a half-wave rectifier or a full-wave rectifier (This is more commonly used) with a frequency that is 1 or 2 times that of the AC depending on the rectification method. This is direct current that changes in value but never changes in direction. It is also called pulsating direct current (PDC) or pulsed direct current (PDC)

VARIABLE CURRENT

This is the type of current that flows in earbuds that we listen to

ROOT MEAN SQUARE VALUE (RMS VALUE)

The root mean square value of an alternating current is defined as the value of the alternating current that has the same heating or lightning effect as that of a direct current.

The root mean square value is also called the DC equivalent.

It can be expressed in terms of the peak current as follows

$$I_{rms}=\sqrt{\frac{(I_o)^2}{2}}$$

$$I_{rms}=\frac{\sqrt{(I_o)^2}}{\sqrt{2}}$$

$$I_{rms}=\frac{I_o}{\sqrt{2}}$$

$$I_o=I_{rms}\sqrt{2}$$

From

$$I_{rms}=\frac{I_o}{\sqrt{2}}$$

On rationalizing,

$$I_{rms}=\frac{I_o\sqrt{2}}{2}$$

Similarly, for voltages

$$V_{rms}=\sqrt{\frac{(V_o)^2}{2}}$$

$$V_{rms}=\frac{\sqrt{(V_o)^2}}{\sqrt{2}}$$

$$V_{rms}=\frac{V_o}{\sqrt{2}}$$

$$V_o=V_{rms}\sqrt{2}$$

From

$$V_{rms} = \frac{V_o}{\sqrt{2}}$$

On rationalizing,

$$V_{rms} = \frac{V_o \sqrt{2}}{2}$$

TYPES OF AC CIRCUITS

1. Resistive circuit: This is defined as a circuit containing a generator and a resistor only

The current flowing in the circuit is expressed as

$$I = \frac{V}{R}$$

$$V_R = IR$$

The voltage across the resistor and the current across it are in phase

2. Inductive circuit: This is a circuit containing generator and an inductor only. When alternating current flows in a coil, a rapidly changing magnetic field is set up around the coil. This induced emf according to Lenz's law tends to oppose the current giving rise to it.
   This effect called self induction offers a resistance to the current build up in an AC circuit so that the current lags behind the imposed emf i.e. the current and emf are out of phase

The current across the inductor is expressed as

$$I = \frac{V}{X_L}$$

$$V_L = I X_L$$

X_L Is called the inductive reactance or the reactance of the inductor

It is like the resistance of the inductor. The inductive reactance is defined as the measure to the opposition to the flow of current offered by the inductor. Its unit is ohms

$$X_L = \omega L$$

$$\omega = 2\pi f$$

$$X_L = 2\pi f L$$

Therefore,

$$I = \frac{V_L}{2\pi f L}$$

$$V_L = 2\pi f I L$$

$V_L -$ This is the voltage across the Inductor

L is the inductance of the inductor.

Vectorially, the voltage across the inductor leads the current by $90\,°\text{or}\frac{\pi}{2}rads$. It can also be said that the current lags behind the voltage by 90 degrees.

Capacitive Circuit: This is a circuit with a capacitor and a generator only.

The current flowing in the circuit is expressed as

$$I=\frac{V_C}{X_C}$$

$$V_C=I\,X_C$$

$$V_c=\frac{q}{C}=V_o\sin\omega t$$

X_C −This is called the capacitive reactance. It is defined as the measure of the opposition to the flow of current offered by the capacitor. Its unit is the ohms

$$X_C=\frac{1}{\omega c}$$

$$\omega=2\pi f$$

$$X_C=\frac{1}{2\pi fc}$$

$$I=\frac{V_C}{\left(\frac{1}{2\pi fc}\right)}$$

$$I=V_c 2\pi fc$$

Also,

$$V_C=I\,X_C$$

$$V_c=\frac{I}{2\pi fc}$$

The current across the capacitor leads the voltage by 90 degrees or the voltage lags behind the current by 90 degrees.

<u>RLC CIRCUITS</u>

This is a resistive, inductive and capacitive circuit.

The direction of the current flowing in the circuit can be obtained by combining the phase diagrams of the three elements. On the super positioning of the these diagrams

From the above diagram,

ϕ −This is the phase angle or the angle of lead of lag

Let EMF be V

From Pythagoras' theorem,

$$H^2=A^2+O^2$$

$$V^2 = (V_R)^2 + (V_L - V_C)^2$$

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$V_R = IR$$

$$V_L = I X_L$$

$$V_C = I X_C$$

$$V = \sqrt{(IR)^2 + (I X_L - I X_C)^2}$$

$$V = \sqrt{I^2 R^2 + I^2 (X_L - X_C)^2}$$

$$V = \sqrt{I^2 \left[R^2 + (X_L - X_C)^2\right]}$$

$$V = I \sqrt{R^2 + (X_L - X_C)^2}$$

But,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$E = IZ$$

$$I = \frac{E}{Z}$$

Z in the equation above is called the impedance of the circuit. Impedance is defined the measure of opposition to the flow of current in a circuit offered by all the (three) circuit elements. Its unit is ohms.

From the diagram,

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

RESONANCE IN AC CIRCUIT

This is a phenomenon whereby the inductive reactance equals the capacitive reactance,

At resonance,

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega c}$$

$$\omega l \times \omega c = 1$$

$$\omega^2 Lc = 1$$

$$\omega^2 = \frac{1}{Lc}$$

$$\omega = \sqrt{\frac{1}{Lc}}$$

$$\omega = \frac{1}{\sqrt{Lc}}$$

But,

$$\omega = 2\pi f$$

Therefore at resonance,

$$\omega = 2\pi f_r$$

$$2\pi f_r = \frac{1}{\sqrt{Lc}}$$

$$f_r = \frac{1}{2\pi\sqrt{Lc}}$$

Also,

$$f_r = \frac{1}{\sqrt{4\pi^2 Lc}}$$

The following conditions occur at resonance.

At resonance,

$$X_L = X_C$$

Multiplying both sides by current (I)

$$I X_L = I X_C$$

$$V_L = V_C$$

Since $X_L = X_C = X$,

And $Z = \sqrt{R^2 + (X_L - X_C)^2}$, we therefore have

$$Z = \sqrt{R^2 + (X - X)^2}$$

$$Z = \sqrt{R^2 + 0^2}$$

$$Z = \sqrt{R^2}$$

At resonance,

$$Z = R$$

At resonance,

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$\tan\phi = \frac{V - V}{V_R}$$

$$\tan\phi = \frac{0}{V_R}$$

$$\tan\phi = 0$$

Therefore at resonance,

$$\phi=0$$

$$P_f=\cos\phi$$

$$P_f=\cos 0$$

At resonance,

$$P_f=1$$

In conclusion, at resonance,

$$X_L=X_C$$

$$V_L=V_C$$

$$Z=R$$

$$\phi=0$$

$$P_f=1$$

The rate at which the current flowing is maximum

<u>POWER IN AC CIRCUITS</u>

The following formulae are used in calculating the power in AC circuits

$$P=\left(I_{rms}\right)^2 R$$

But

$$I_{rms}=\sqrt{\frac{\left(I_o\right)^2}{2}}$$

Therefore,

$$P=\left(\sqrt{\frac{\left(I_o\right)^2}{2}}\right)^2 R$$

$$P=\frac{I_o{}^2 R}{2}$$

$$P=V_{rms} I_{rms}\cos\phi$$

$$\left(I_{rms}\right)^2 R=V_{rms} I_{rms}\cos\phi$$

$$I_{rms} R=V_{rms}\cos\phi$$