CAPACITANCE AND DIELECTRICS

A capacitor is a passive device that stores electrical energy (or electrical charges) temporarily in an electric field. This means it is a storage device. It is also called a condenser. Basically it is an arrangement of any two conductors separated by an insulator (called dielectric)

If a voltage V is applied across the metal plates of a capacitor, the two plates become charged. One of the plates will acquire a negative charge (-Q) and the other will acquire an equal amount of positive charge (+Q)

It is found that the charge on each capacitor is proportional to the potential difference between the conductors

$Q \propto V$

$Q = CV$

where c is called the capacitance of the capacitor. The capacitance of a capacitor is its ability to store charges. The unit of capacitance is coulomb per volt and this unit is also called the Farad (F).

CAPACITANCE

This is defined as the ability of a materia to store charges (when its plates are at different potentials). It is defined as the ratio of the charge stored to the voltage

$c = \frac{q}{V}$

FACTORS THAT AFFECT CAPACITANCE

In general,
1. Size of the conductors
2. Shape of the conductors
3. Relative position of the two conductors
4. Dielectric that separates conductors

For a parallel plate capacitor
1. Area of the plate. $c \propto A$
2. Distance between the plates: $c \propto \frac{1}{d}$
3. Nature of the dielectric

TYPES OF CAPACITORS

1. Parallel plate capacitor
2. Cylindrical capacitor
3. Concentric spherical capacitor
4. Isolated sphere

PARALLEL PLATE CAPACITORS

A parallel plate capacitor consists of two parallel metallic plates of equal area separated by a distance d. When battery terminals of potential difference V are connected to the plates, one acquires a negative charge while the other acquires an equal amount of positive charge

The electric field strength E between the plates is given by.

$Flux = Electric\ field\ strength \times Area$

$\Phi = E\,A$

$E = \frac{\Phi}{A}$

From Gauss' law

$\Phi = \frac{Q}{\varepsilon}$

$E = \frac{Q}{\varepsilon\,A}$

But surface charge density, $\sigma = \frac{Q}{\varepsilon}$

$E = \frac{\sigma}{\varepsilon}$

And

$V = Ed$

$E = \frac{Q}{\varepsilon\,A}$

$V = \frac{Q\,d}{\varepsilon\,A}$

$\frac{Q}{V} = \frac{\varepsilon_o A}{d}$

But $C = \frac{Q}{V}$

$C = \frac{\varepsilon_o\,A}{d}$

If a dielectric is introduced,

$C = \frac{K\,\varepsilon_o\,A}{d}$

From the above it can be seen that capacitance can be increased by

1. Increasing the area of the plates
2. Decreasing the distance between the plates
3. Increasing the dielectric constant by changing the material.

CYLINDRICAL CAPACITORS

A cylindrical capacitor consists of an inner conductor that is a cylinder of radius $R_b$and coaxial outer conductor with inner radius R_a

Both cylinders are of equal length

The length of these cylinders is greater than the gap between them

$l \gg R_a - R_b$

When connected to a battery, with one cylinder (let's say the inner one) have a +ve charge and the other having a negative charge

We know that the electric field intensity outside a long wire has a magnitude

$$E = \frac{linear\,charge\,density}{2\pi\varepsilon_o R}$$

$$E = \frac{\lambda}{2\pi\varepsilon_o R}$$

$$E = \frac{Q}{2\pi l\varepsilon_o R}$$

R = R

Potential difference,

$$V = V_b - V_a = -\int_a^b E\cdot dl = -\int_a^b E\,dl\cos\theta = -\int_{R_a}^{R_b} E(-dR)\cos(180)$$

$$\theta = Angle\,between\,E\,a\,nd\,dR$$

$$E = \frac{\lambda}{2\pi\varepsilon_o R}$$

$$V = -\int_{R_a}^{R_b} \frac{Q}{2\pi\varepsilon_o lR}(-dR)\cos(180)$$

$$V = -\frac{Q}{2\pi\varepsilon_o l}\int_{R_a}^{R_b}\frac{-dR}{R}(-1)$$

$$V = -\frac{Q}{2\pi\varepsilon_o l}\int_{R_a}^{R_b}\frac{dR}{R}$$

$$V = -\frac{Q}{2\pi\varepsilon_o l}\ln R\Big|_{R_a}^{R_b}$$

$$V = -\frac{Q}{2\pi\varepsilon_o l}\ln\frac{R_b}{R_a}$$

$$V = \frac{Q}{2\pi\varepsilon_o l}\ln\left(\frac{R_b}{R_a}\right)^{-1}$$

$$V = \frac{Q}{2\pi\varepsilon_o l}\ln\frac{R_a}{R_b}$$

C = Q over V

$$C = \frac{Q}{\frac{Q}{2\pi\varepsilon_o l}\ln\frac{R_a}{R_b}}$$

$$C = \frac{Q\times 2\pi\varepsilon_o l}{Q\times\ln\frac{R_a}{R_b}}$$

$$C = \frac{2\pi\varepsilon_o l}{\ln\frac{R_a}{R_b}}$$

CONCENTRIC SPHERICAL CAPACITOR

A spherical conductor consists of two thin concentric spherical conducting shells of radii r_a and r_b
The inner sphere carries a positive charge +Q
The electric field outside a uniformly charged conducing sphere is

$$E=\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$

$$V_{ba}=V_b-V_a=-\int_a^b E\cdot dl=-\frac{Q}{4\pi\varepsilon_o}\int_{r_a}^{r_a}\frac{1}{r^2}dr=\frac{Q}{4\pi\varepsilon_o}\left(\frac{r_a-r_b}{r_a r_b}\right)$$

$$C=\frac{Q}{V}=\frac{4\pi\varepsilon_o r_a r_b}{r_a-r_b}$$

Note the following table to remember the capacitance of different values

Object Name	Electric Field Strength	Voltage (Potential Difference)	Capacitance	Specific Notes
Parallel Plate Capacitor	$E=\frac{Q}{\varepsilon A}$	$V=\frac{Qd}{\varepsilon A}$	$C=\frac{\varepsilon_o A}{d}$	
Cylindrical Capacitor	$E=\frac{\lambda}{2\pi\varepsilon_o R}$ or $E=\frac{Q}{2\pi l\varepsilon_o R}$	$V=\frac{Q}{2\pi\varepsilon_o l}\ln\frac{R_a}{R_b}$	$C=\frac{2\pi\varepsilon_o l}{\ln\frac{R_a}{R_b}}$	
Concentric Spherical Capacitor	$E=\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$	$V=\frac{Q}{4\pi\varepsilon_o}\left(\frac{r_a-r_b}{r_a r_b}\right)$	$C=\frac{Q}{V}=\frac{4\pi\varepsilon_o r_a r_b}{r_a-r_b}$	This is a capacitor with one inner ball(or sphere) inside another ball(sphere)

CAPACITORS IN SERIES AND PARALLEL

Capacitors can be connected in different ways. The two most common ways are in series and parallel

CAPACITORS IN SERIES

In series connection, capacitors are connected one to another. Charges therefore flow from one positive plate of c1 to the negative plate of c1 then to the positive plate of c2 etc.
The net charge is zero
The charge on all the plates is equal. The total capacitance or equivalent capacitance$\left(C_{eq}\right)$

$Q=C_{eq}V$

V = Q over C
The potential differences on the capacitors are V1+V2+V3. The total potential difference applied across the capacitors V is

V = V1+V2+V3
For Each capacitor
Q = C1V1 + C2V2 +C3V3

$$V=V_1+V_2+V_3$$

$$\frac{Q}{C_{eq}}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3}$$

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

So the following should be noted of capacitors in series
1. They have the same charge
2. The total voltage is the sum of voltage in each capacitor
3. $V=V_1+V_2+V_3$
4. $\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$

CAPACITORS IN PARALLEL

For capacitors in parallel,
1. The have the same voltage
2. The total charge is the sum of charges in each capacitor
3. $Q=Q_1+Q_2+Q_3$
4. $C=C_1+C_2+C_3$

ENERGY STORED IN A CAPACITOR

The energy stored in a capacitor is equal to the work done in charging the capacitor.
The work done to increase the charge on an uncharged capacitor to Q when a potential difference V is across the capacitor is

$$W=\int_0^Q V\,dq$$

But $V=\frac{Q}{C}$

$$W=\frac{1}{C}\int_0^Q q\,dq$$

On integrating

$$W=\frac{1}{C}\frac{q^2}{2}$$

$$W=\frac{1}{2}\frac{q^2}{C}$$

$$E=\frac{1}{2}\frac{q^2}{C}$$

But $Q=CV$

$$E=\frac{1}{2}\frac{(CV)^2}{C}$$
$$E=\frac{1}{2}\frac{C^2V^2}{C}$$
$$E=\frac{1}{2}C\,V^2$$
$$E=\frac{1}{2}Q\,V$$

But, $V=Ed$
$$E=\frac{1}{2}C\,V^2$$
$$E=\frac{1}{2}C\,(Ed)^2$$
$$E=\frac{1}{2}C\,E^2\,d^2$$

For a parallel plate capacitor, $C=\frac{\varepsilon_o\,A}{d}$
$$E=\frac{1}{2}\varepsilon_o\,E^2\,A\,d$$
Volume = Ad
$$E=\frac{1}{2}\varepsilon_o\,E^2\,Vol.$$
$$Energy\,density=\frac{Energy}{Volume}$$
$$u=\frac{E}{Vol}=\frac{1}{2}\varepsilon_o\,E^2$$
Note the major formulae:

1. $E=\frac{1}{2}\frac{q^2}{C}$
2. $E=\frac{1}{2}C\,V^2$
3. $E=\frac{1}{2}Q\,V$
4. $E=\frac{1}{2}C\,E^2\,d^2$
5. $E=\frac{1}{2}\varepsilon_o\,E^2\,A\,d$

GRAPHS IN THE STUDY OF CAPACITORS

Also, in a graph of charge (q) against voltage or potential difference (V), the slope gives the capacitance of the capacitor and the area under the graph gives the energy stored in the capacitor.

USES OF CAPACITORS

1. For tuning in radio receivers
2. Filters in power supplies
3. To eliminate sparking in automobile ignition systems/switches
4. For timing circuits
5. For storing energy in electronic flashes\They are used for storing charges

They are also used for storing electrical energy
They are used for establishing desired electric (field) configuration
They can be used for creating electronic time delays
They are employed in induction coils to prevent electric sparks
They are used in the inverter and the UPS (Uninterrupted power supply) for storing energy
They can be used for lowering the value of current flowing in a circuit
They are essential components of radios, TVs and computers.

DIELECTRICS

A dielectric is a non-conducting material such as glass, rubber or waxed paper placed between the plates of the capacitor in order to alter the capacitance value of the capacitor.

Every material has a dielectric constant K and the capacitance is directly proportional to the dielectric constant. The dielectric constant is also called relative permittivity or permittivity constant.
The dielectric constant of a vacuum is 1. That of air is 1.0006 and that of glass is 5.

The dielectric constant can be defined as

$$Dielectric\,constant=\frac{Capacitance\,when\,the\,material\,is\,used\,as\,a\,dielectric}{Capacitance\,of\,the\,capacitor\in a\,vacuum}$$

$$K=\frac{c_m}{c_v}$$

Dielectric constant can also be expressed as

$$Dielectric\,constant=\frac{permitivity\,of\,the\,material}{Permi\,t\,ivity\,of\,free\,space}$$

$$K=\frac{\varepsilon}{\varepsilon_o}$$

1. When a dielectric material is inserted between the plates of a capacitor, the capacitance increases by a factor K, called the dielectric constant. The capacitance is given as
$C=K\,C_o$
2. The dielectric constant has no unit. It is the characteristic of a given material
3. If ε_o is the permittivity of free space and ε is the permittivity of the dielectric, then
$$K=\frac{\varepsilon}{\varepsilon_o}$$
4. The energy density becomes
$$u=\frac{1}{2}\varepsilon E^2$$

5. At constant charge, the voltsge he will decrease
$V_o = KV$
6. If the potential difference is kept unchanged, we can say that the charge increases at a constant voltage
$Q = K Q_o$
6. The electric field strength also decreases by a factor of K
$E_o = EK$
The net field induced in the dielectric is given as
$E = E_o - E_{ind}$
$E_{ind} = E_o - E$
$E = \frac{E_o}{K}$
$E_{ind} = E_o \left(1 - \frac{1}{K}\right)$
Similarly, for induced Charge and induced surface charge density
$\sigma_{ind} = \sigma_o \left(1 - \frac{1}{K}\right)$
$Q_{ind} = Q_o \left(1 - \frac{1}{K}\right)$

<u>CHARACTERISTICS OF A GOOD DIELECTRIC</u>

1. K is also called the relative permittivity of the dielectric material.
2. Dielectric strength: This is the maximum electric field intensity that a pure material can withstand under ideal conditions without breaking down

NB: LEARN DIPOLE CONCEPTS AND DIPOLES.

QUESTIONS
1. What is capacitance?
2. What is a capacitor?
3. State the factors that affect the capacitance of a capacitor
4. Mention 3 types of capacitors and give the formulae for capacitance for each
5. A 1uF and a 2uF capacitor are connected in series across a 1000volt supply line. Find the charge on each capacitor and the voltage across each. Answers: $Q = 6.67 \times 10^{-4} V_1 = 667\,V V_2 = 333\,V$