INTEGRATION

This can be seen as the anti derivatives of something.

Take a look at the power rule of differentiation:

$$\frac{d}{dx}\left[x^n\right]=n\,x^{n-1}$$

You'll see that the power rule for integration will be

$$\int \left[x^n\right]dx=\frac{x^{n+1}}{n+1}+c$$

If we find the derivative of x^3, it will be

$$\frac{d}{dx}\left[x^3\right]=3\,x^2$$

Now, if we find the integral of $3\,x^2$, we will have

$$\int 3\,x^2\,dx=\frac{3\,x^3}{3}+c$$

1: Find the anti derivative of

1. x^4 Answer: $\frac{x^5}{5}+c$
2. $\frac{x}{4}$ Answer: $\frac{x^2}{8}+c$
3. $8\,x^3$ Answer: $2\,x^4+c$

2. Find the integral of 4

$$\int 4\,dx=4\,x+c$$

$$\int 5\,dy=5\,y+c$$

3. Find $\int (7\,x-6)dx$ Answer: $\frac{7\,x^2}{2}-6\,x+c$

DEFINITE INTEGRALS

The process by which we evaluate the anti derivatives comes from thew fundamental theorem of calculus.

A function represented with f(x) – small f – while the anti-derivative F(x) – capital F –

One of the theorems says, the integral from a to b of a function f(x) where this function is continuous on a closed interval [a, b] is given below.

$$\int_a^b f(x)\,dx=F(b)-F(a)$$

You should note that
$\int f(x)dx = F(x) + c$

Example 7: $\int_{1}^{2} 6x^2 dx$

$\int_{1}^{2} 6x^2 dx = 2x^3 + c\Big|_1^2$

$[2(2)^3 + c] - [2(1)^3 + c]$ # The c will cancel out

14

METHODS OF SOLVING INTEGRATION

1. Direct method
2. Substitution method
3. Integration by Parts
4. Integration by Partial Fractions
5. Integration by trigonometry

METHOD OF SUBSTITUTION

Given this
$\int e^{8x} dx$
We can say, let
$u = 8x$

$\frac{du}{dx} = 8$

Making dx the subject of the formula,

$dx = \frac{du}{8}$

$\int e^{8x} dx = \int e^u \frac{du}{8}$

$\frac{1}{8}\int e^u du = \frac{1}{8} e^u + c$

$\frac{1}{8} e^{8x} + c$

Looking at another example,
$\int 4x e^{x^2} dx$
Let
$u = x^2$
$du = 2x dx$

$dx = \frac{du}{2x}$

$\int 4x e^{u} \frac{du}{2x}$
$2\int e^{u} du$
$2e^{u}+c$
$But\ u=x^2$
$2e^{x^2}+c$

Solve
$\int \frac{1}{x^2} dx$ Answer: $\frac{-1}{x}+c$
$\int \frac{1}{x^3} dx$
$\int \frac{8}{x^4} dx$

Using the u substitution
Typically you want to make “u” the stuff that is more complicated.
When using the substitution method, you want to make eliminate every value of x when you are substituting the u
$\int \frac{1}{(4x-3)^2} dx$ Answer: $\frac{-1}{4(4x-3)}+c$
$\int 4x[x^2+5]^3 dx$Answer: $\frac{1}{2}[x^2+5]^4+c$
$\int x^3 e^{x^4} dx$Answer: $\frac{1}{4}e^{x^4}+c$
$\int \frac{x^3}{(2+x^4)^2} dx$Answer: $\frac{-1}{4(2+x^4)}+c$
$\int \sin^4(x)\cos(x) dx$Let u = sin (x). Answer: $\frac{1}{5}\sin^5(x)+C$
$\int \sqrt{5x+4}\, dx$Answer: $\frac{2}{15}[5x+4]^{\frac{3}{2}}+c$
$\int x\sqrt{3x+2}\, dx$
Let u = 3x + 2
If you do it the normal way, you’ll see that you’ll get a value like
$dx=\frac{du}{3}$
And the value 3 won’t be able to cancel out the outstanding x.
So in this situation where it both expressions, they have the same power. That is in x and {3x + 2}, you’ll have to solve for x
$u=3x+2$
$x=\frac{u-2}{3}$
$\int x\sqrt{3x+2}\, dx$
$\int \frac{u-2}{3}\sqrt{u}\frac{du}{3}$

$$\frac{2}{45}[3x+2]^{5/2} - \frac{4}{27}[3x+2]^{3/2} + c$$

$$\int 8x\sqrt{40-2x^2}\,dx$$

EXPONENTIAL INTEGRATION

Recall that, given a function

$\frac{d}{dx}\left(e^{f(x)}\right)=f'(x)\cdot e^{f(x)}$

For the anti-derivatives,

$\int e^{f(x)}=\frac{e^{f(x)}}{f'(x)}+c.$

This applies if and only if the function f(x) is a linear function like ax+b or something.
For example

$\int e^{x}dx=e^{x}+c$

$\int e^{5x}dx=\frac{e^{5x}}{5}+c$

INTEGRATION TO THE FORM

$\int \frac{f'(x)}{f(x)}dx=\ln\left(f(x)\right)+c$

$\int \frac{1}{x}dx=\ln x+c$

Example

$\int \frac{\ln x}{x} dx$

$\int \ln x \frac{1}{x} dx$

Proving it by the method of substitution

Let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\, du$

$\int \ln x \frac{1}{x} dx = u \cdot \frac{1}{x} \cdot x\, du$

$\int u\, du$

$\frac{u^2}{2}$

But $u = \ln x$

$\frac{\ln^2 x}{2}$

When you don’t know what to do again, just use integration by parts. Don’t follow me oh :-)