FLUID PROPERTIES
&
INTRODUCTION TO FLUID STATICS

MEG 222 Week 2 Lecture

Lecture Learning Outcomes

At the end of this lecture, you will be able to,

- Discuss why bulk modulus, vapor pressure and surface tension are important fluid properties
- Derive the expression for the pressure at various locations in a fluid at rest for both incompressible and compressible fluids

More Fluid Properties

Compressibility in Fluids

How easily can the volume (and as a result density) of a given mass of fluid change when there is a change in pressure?

This question can be simplified as "How compressible is this fluid?"

Bulk Modulus E_v, is used to characterize the compressibility of a fluid and it is defined as,

$$E_v = -\frac{dp}{dV/V}$$

More Fluid Properties

Bulk Modulus

$$E_v = -\frac{dp}{dV/V}$$

dp is the differential change in pressure needed to create a differential change in volume. The negative sign is included because an increase in pressure will cause a decrease in volume.

A decrease in volume will result in an increase in density.

The bulk modulus also called bulk modulus of elasticity can also be expressed as

$$E_v = \frac{dp}{d\rho/\rho}$$

More Fluid Properties

Bulk Modulus

Large values of bulk modulus indicate that the fluid is relatively incompressible, i.e. a large pressure change is required to create a small change in volume.

Liquids are generally considered as incompressible in practical engineering

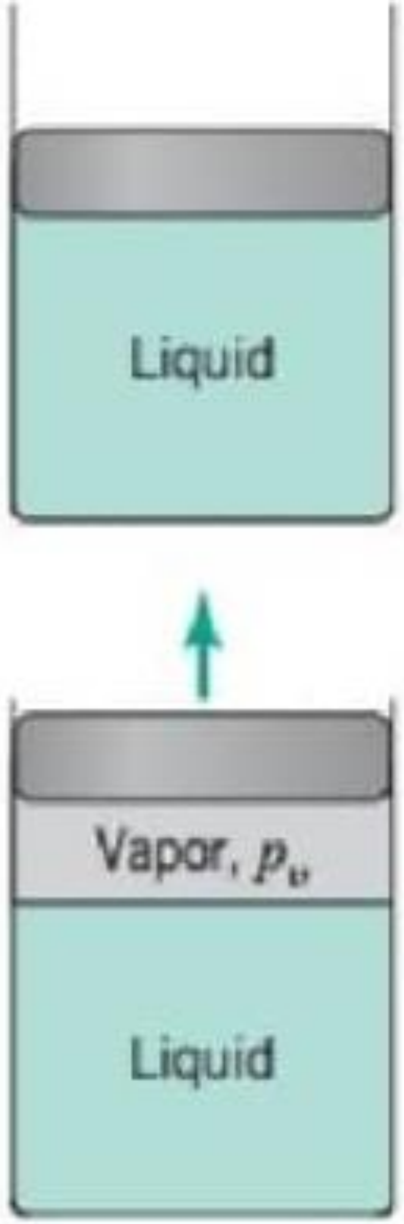
Vapor Pressure

Liquids such as water and gasoline evaporate when place in a container open to the atmosphere.

Evaporation takes place because the liquid molecules at the surface have sufficient momentum to overcome the intermolecular cohesive forces and escape to the atmosphere.

More Fluid Properties

If a lid on a completely liquid-filled closed container is raised without letting air in, a pressure will develop in the space as a result of the vapor formed by the escaping molecules.



Vapor Pressure

When an equilibrium condition is reached, the vapor is said to be saturated and the pressure the vapor exerts on the liquid surface is called vapor pressure.

More Fluid Properties

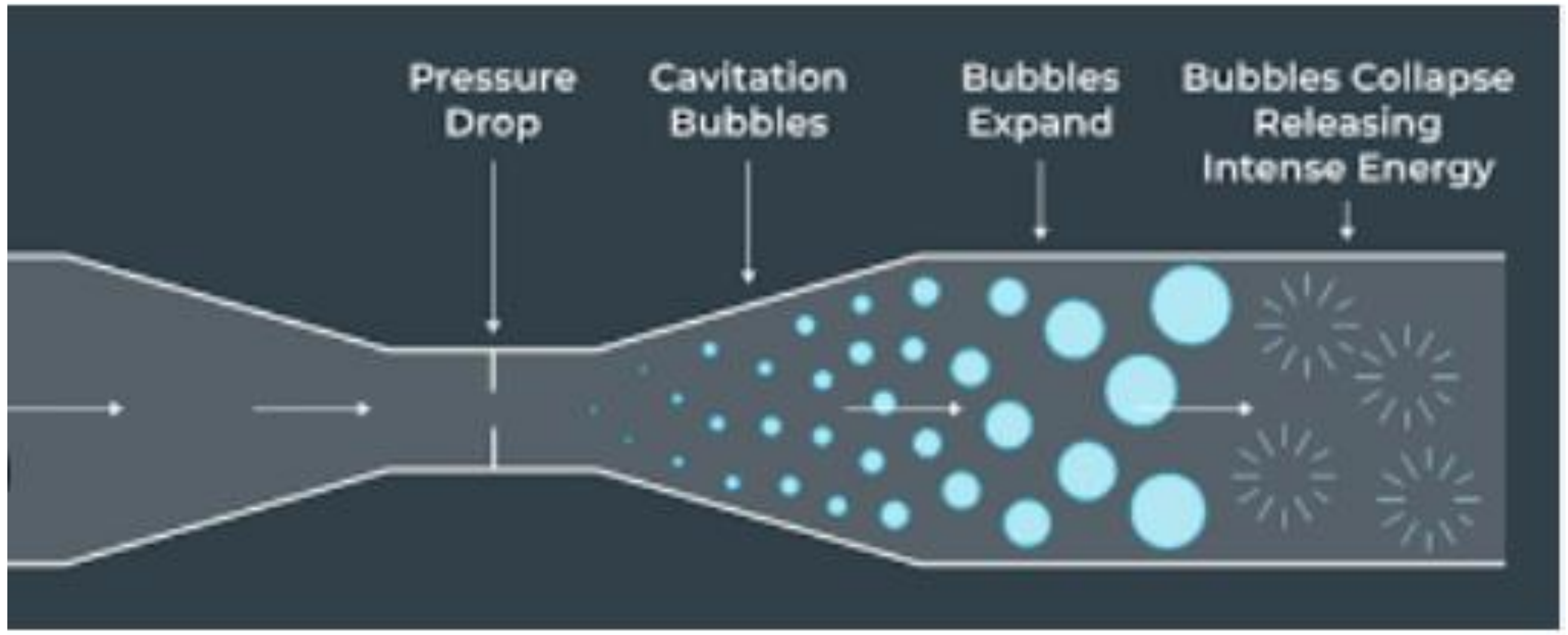


Fig. 1: Cavitation resulting from pressure drop

https://www.tribonet.org/wiki/cavitation/

- **An important reason why vapor pressure is a property of interest in fluid mechanics is boiling**
- **Boiling, which is the formation of vapor bubbles within a mass of fluid is initiated when the absolute pressure in the fluid reaches the vapor pressure**
- **In flowing fluids, it is possible to develop very low pressure due to the fluid motion and if pressure is lowered to vapor pressure, boiling will occur.**
- **When vapor bubbles that are formed are swept along with the flowing fluid to regions of higher pressure, they collapse suddenly. This phenomenon is called cavitation**

More Fluid Properties

Fig. 2: Damage to a blade as a result of cavitationhttps://www.tribonet.org/wiki/cavitation/

Fig. 3: Floating razor blade

- These collapsed vapor bubbles have enough intensity to cause structural damage

Surface Tension

- Forces develop at the interface between a liquid and a gas or between two immiscible liquids that cause the surface to behave as if it were a 'skin' or 'membrane'

- This explains some commonly observed phenomena e.g. floating of a razor blade on water as shown in Figure 3

- Surface tension is a phenomenon in physics when the surface of a liquid in contact with another phase (solid, liquid or gas) causes the surface of the liquid to act like an elastic sheet

More Fluid Properties

Fig. 4: Water droplet lying on a damask fabric

- At liquid–air interfaces, surface tension results from the greater attraction of liquid molecules to each other (due to cohesion) than to the molecules in the air (due to adhesion)

- One common phenomena associated with surface tension is the rise or fall of a liquid in a capillary tube

Surface tension is the intensity of the molecular attraction per unit length along any line in the surface. (N/m)

It depends on temperature and the fluid it is in contact with at the interface

More Fluid Properties

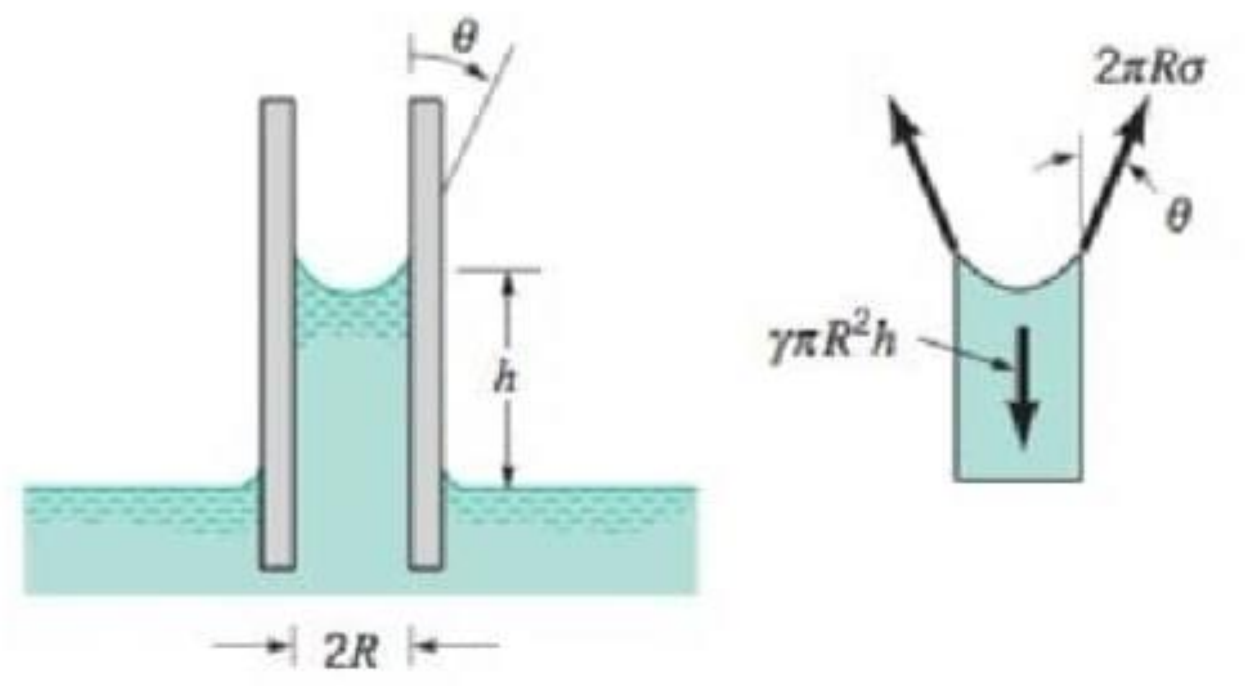


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Fig. 5: Effect of capillary action in small tubes (a) rise of column for a liquid that wets the surface (b) Free body diagram for calculating column height

- If a small open tube is inserted into water, the water level in the tube will rise above the water level outside the tube
- Note that for this case, we have a liquid-gas-solid interface

- There is adhesion between the wall of the tube and the liquid molecules which is strong enough to overcome the cohesive force of the liquid molecules
- As a result of this adhesion, the liquid is pulled up the tube wall. Hence, it wets the solid surface
- The height h is governed by the value of the surface tension σ, the tube radius R, the specific weight of the liquid γ and the angle of contact θ between the fluid and the tube
- From the free body diagram, the vertical force due to the surface tension is $2\pi R\sigma\cos\theta$

More Fluid Properties

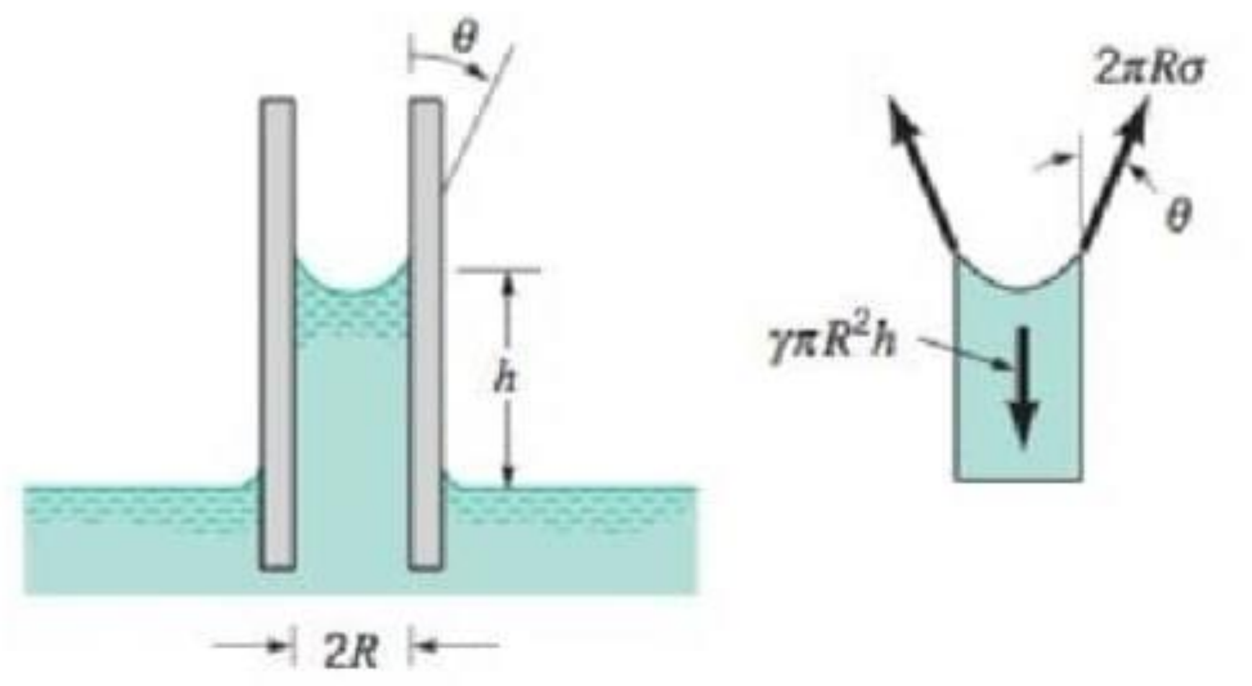

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Fig. 5: Effect of capillary action in small tubes (a) rise of column for a liquid that wets the surface (b) Free body diagram for calculating column height

- For equilibrium, the vertical force due to surface tension must be equal to the weight of the fluid $\gamma\pi R^2 h$

$$2\pi R\sigma\cos\theta = \gamma\pi R^2 h$$

- The height of the fluid is therefore,

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

- The expression above shows that the height is inversely proportional to the tube radius

- As tube radius decreases, the capillary action becomes more pronounced

- For a clean glass, $\theta = 0°$

Students should research on important applications of surface tension as a fluid property

More Fluid Properties

WORKED EXAMPLE

What diameter of clean glass tubing is required so that the rise of water at $20℃$ with $\sigma = 0.0728\ N/m, \gamma = 9.789\ kN/m^3$ in a tube due to capillary action is less than $1.0\,mm$?

$$h = \frac{2\sigma\cos\theta}{\gamma R}; R = \frac{2\sigma\cos\theta}{\gamma h} = 0.0149\,m$$

Minimum required tube diameter is $2R = 29.8\,mm$

PRACTICE PROBLEM

1. **A capillary tube of internal diameter 0.21 mm is dipped into a liquid whose density is 0.79 g cm^{-3} . The liquid rises in this capillary to a height of 6.30 cm. Calculate the surface tension of the liquid. (g = 980 cm sec^{-2}).**
2. **How high will sap rise in a plant if the capillaries are 0.01 mm diameter, the density of the fluid is 1.3 g cm^{-3} and its surface tension 0.065 Nm^{-1} . (g = 981 cm s^{-2})**

Introduction to Fluid Statics

Fluid statics is the study of an important class of fluid problems in which the fluid is either at rest or moving in such a manner that there is no relative motion between adjacent particles

For this class of problems, there will be no shearing stresses in the fluid, and the only forces acting on the fluid particles will be due to pressure

Introduction to Fluid Statics

Focus of Interest

- Investigation of pressure and its variation in a fluid at rest
- Effect of pressure in submerged surfaces

Pressure

Normal force per unit area at a given point acting on a given plane within the fluid mass of interest

Pressure at a Point

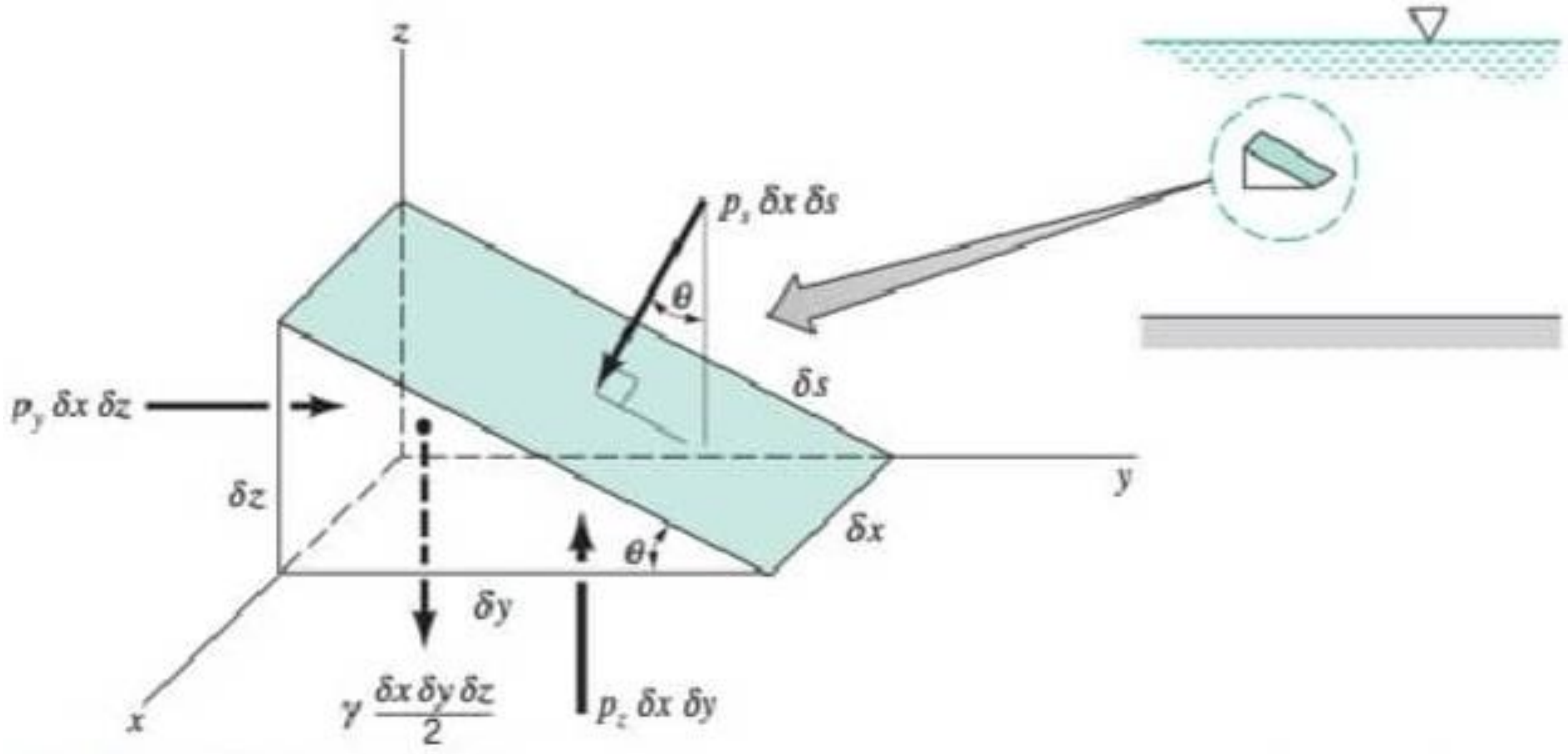


Fig. 6: Forces on an arbitrary wedge-shaped element of fluid

- Even though our focus is on fluids at rest, a general analysis that allows the fluid element to accelerate will be done
- The assumption of zero shear stresses will still hold as long as the fluid element moves as a rigid body i.e. no relative motion between adjacent fluid elements
- Also for simplicity, the forces in the x direction are not shown

Pressure at a Point

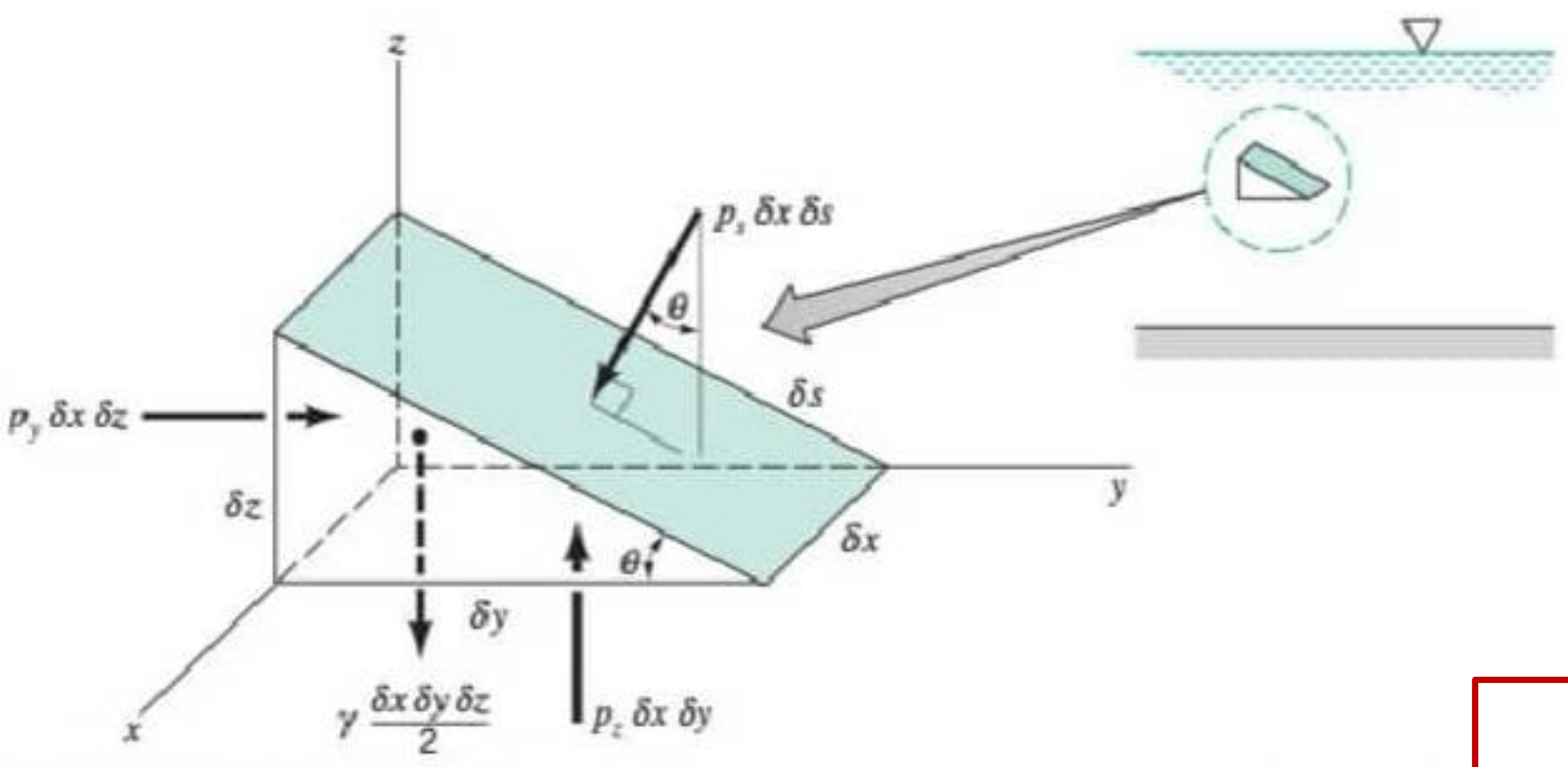


Fig. 6: Forces on an arbitrary wedge-shaped element of fluid

- **We begin our analysis from Newton's second law ($\boldsymbol{F} = m\boldsymbol{a}$)**
- **Applying this law to the forces acting on the x-z and x-y planes gives the sum of forces acting in the y- and z-directions respectively**

$$\sum F_y = p_y\delta x\delta z - p_s\delta x\delta s\ \sin\theta = \rho\frac{\delta x\delta y\delta z}{2}a_y$$

$$\sum F_z = p_z\delta x\delta y - p_s\delta x\delta s\ \cos\theta - \gamma\frac{\delta x\delta y\delta z}{2} = \rho\frac{\delta x\delta y\delta z}{2}a_z$$

Pressure at a Point

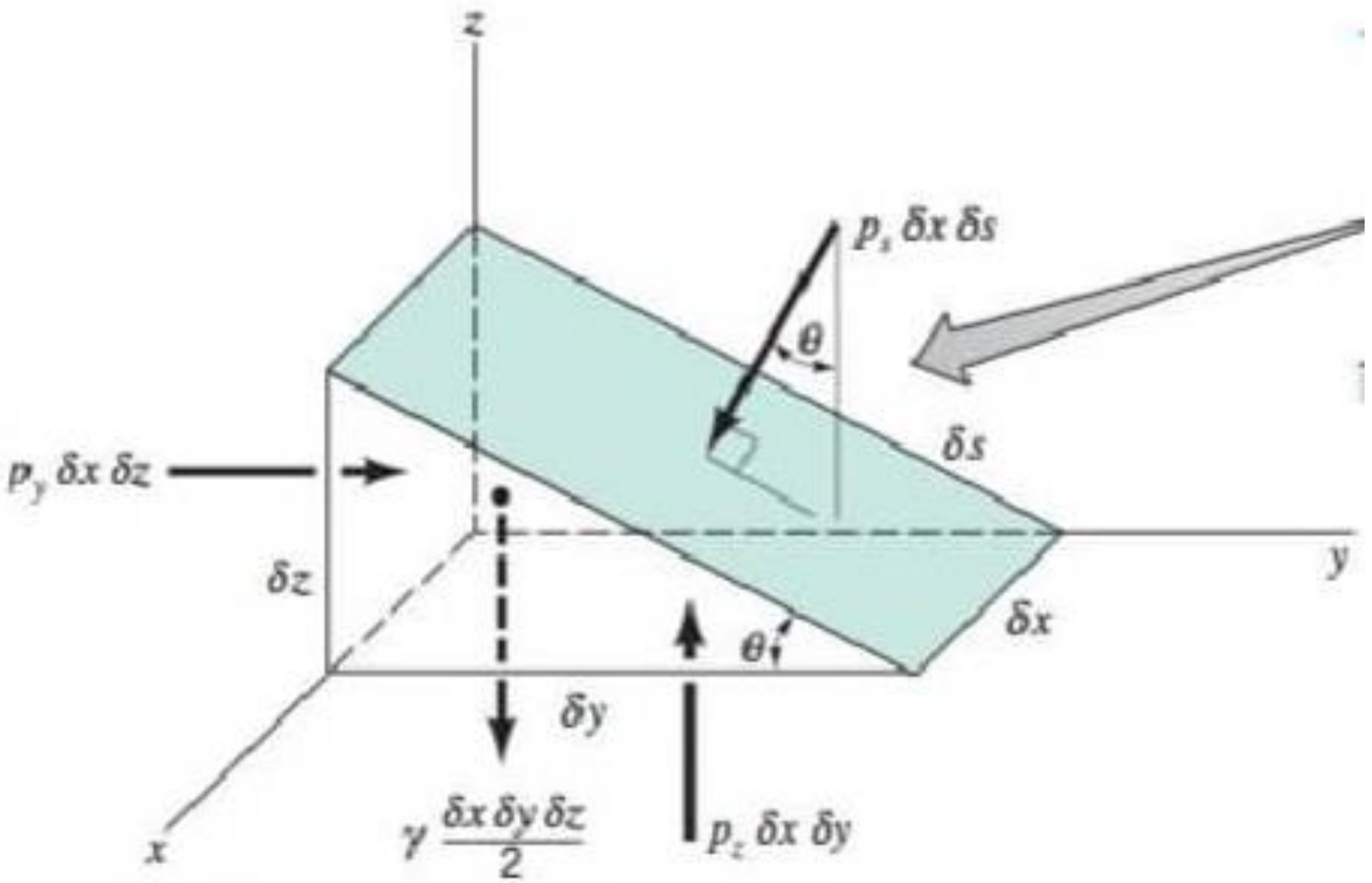
- The average pressures on the faces are p_s, p_y and p_z while ρ and γ are the fluid specific weight and density respectively. The accelerations are a_y and a_z.

- From the geometry of the wedge, it follows that $\delta y = \delta s \cos\theta$ and $\delta z = \delta s \cos\theta$

- Therefore, the equations of motion can be rewritten as

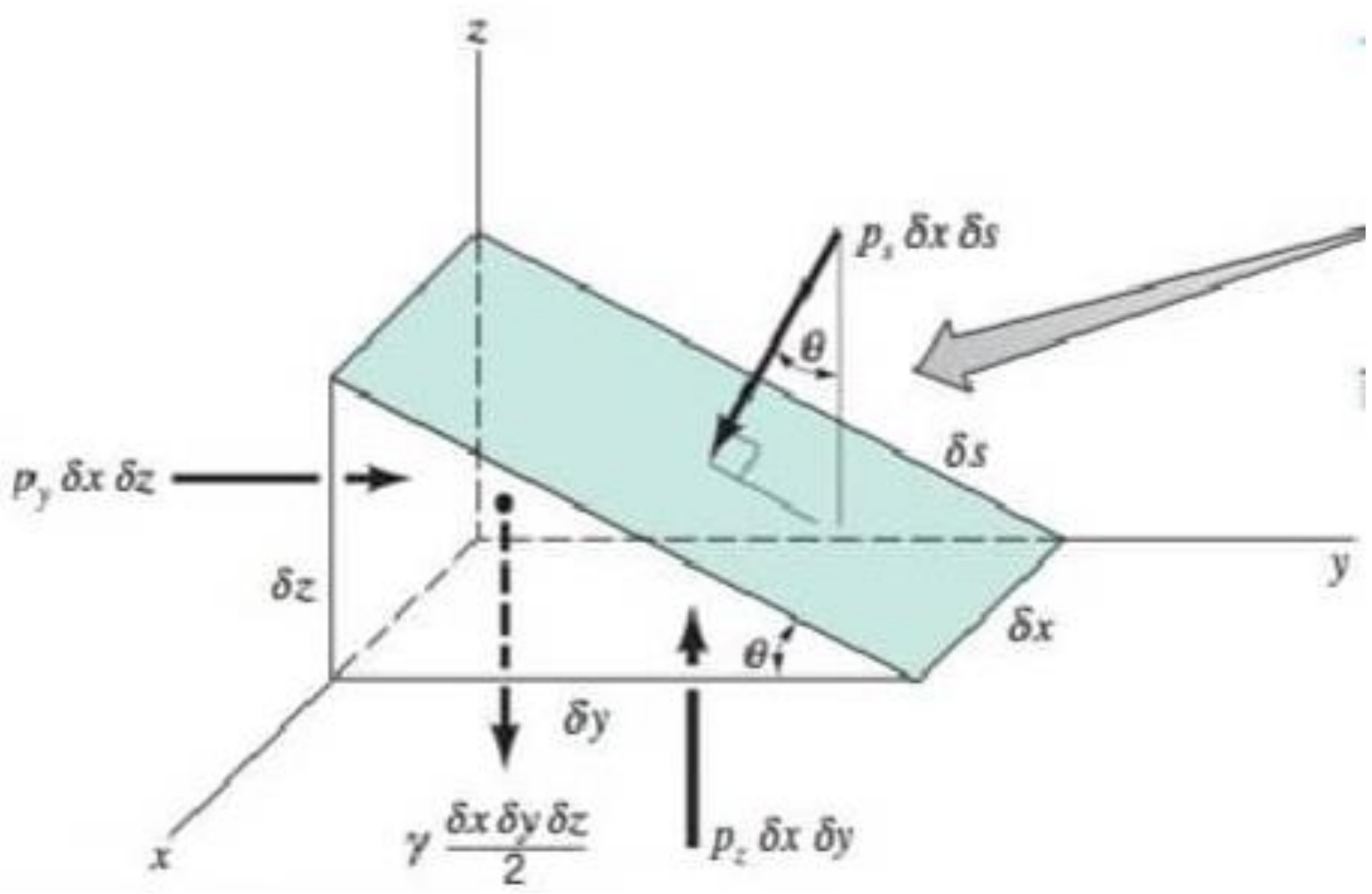
$$p_y - p_s = \rho a_y \frac{\delta y}{2}$$

$$p_z - p_s = (\rho a_z + \gamma)\frac{\delta z}{2}$$



- Considering pressure at a point, we take the limit as $\delta x, \delta y, \delta z$ tends to zero.
- This gives $p_y = p_s$ and $p_z = p_s$ or simply $p_s = p_y = p_z$

Pressure at a Point



- **The equality of the pressures $p_s = p_y = p_z$ shows that the pressure at a point in a fluid at rest or in motion is independent of direction as long as there are no shearing stresses present.**

- **This important statement is called the Pascal's Law**

- **The next important question is, How does pressure in a fluid in which there are no shearing stresses vary from point to point to point?**

Basic Equation for Pressure Field

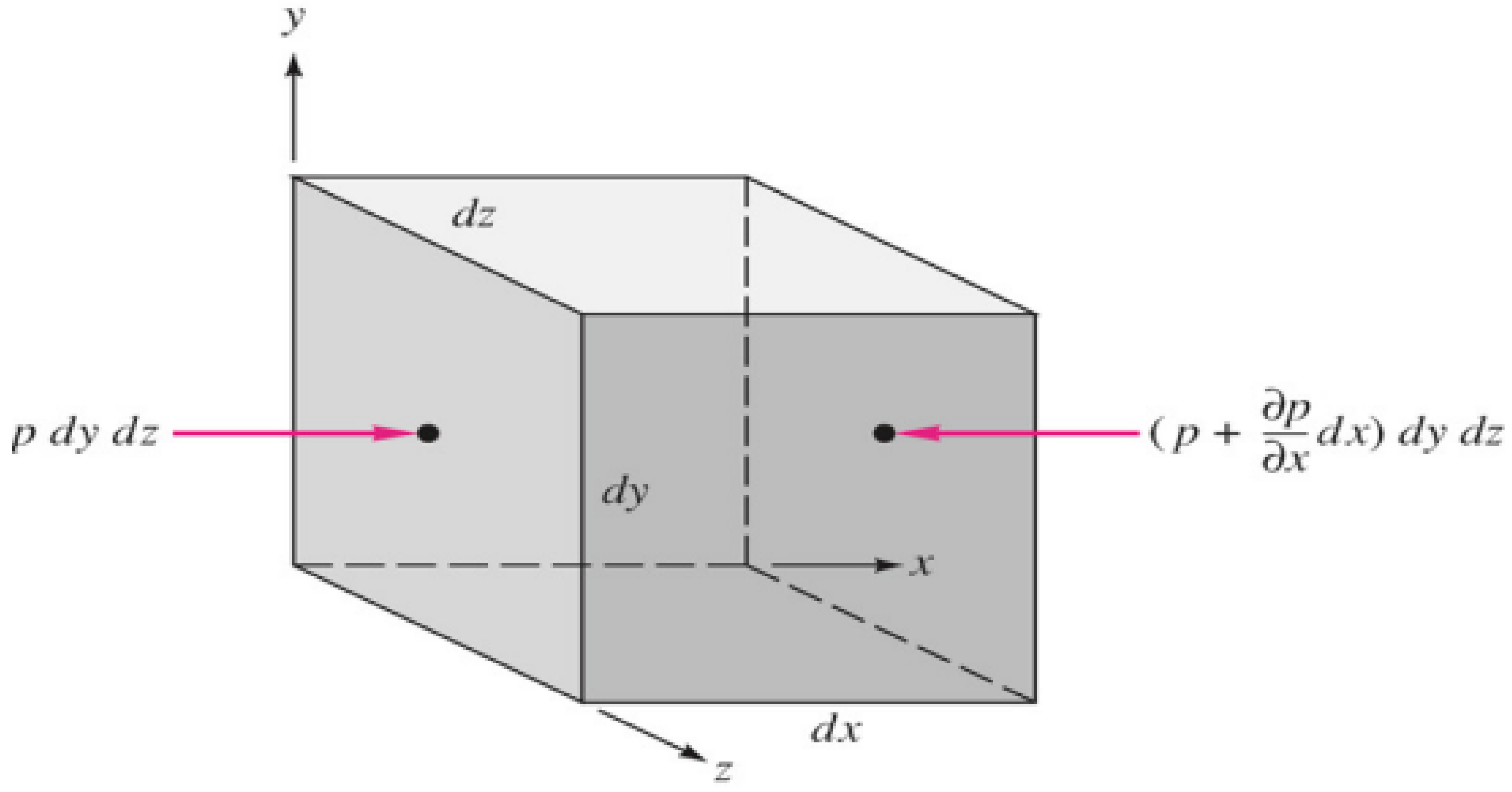


Fig. 7: Forces acting on the fluid element in the x-direction

- Let’s consider a small rectangular element of fluid removed from some arbitrary position within the mass of fluid of interest

- The size of the fluid element is dx, dy, dz

- The net surface force acting on the fluid element in the x-direction is given by

$$dF_x = pdydz - \left(p + \frac{\partial p}{\partial x} dx\right) dydz = -\frac{\partial p}{\partial x} dxdydz$$

Basic Equation for Pressure Field

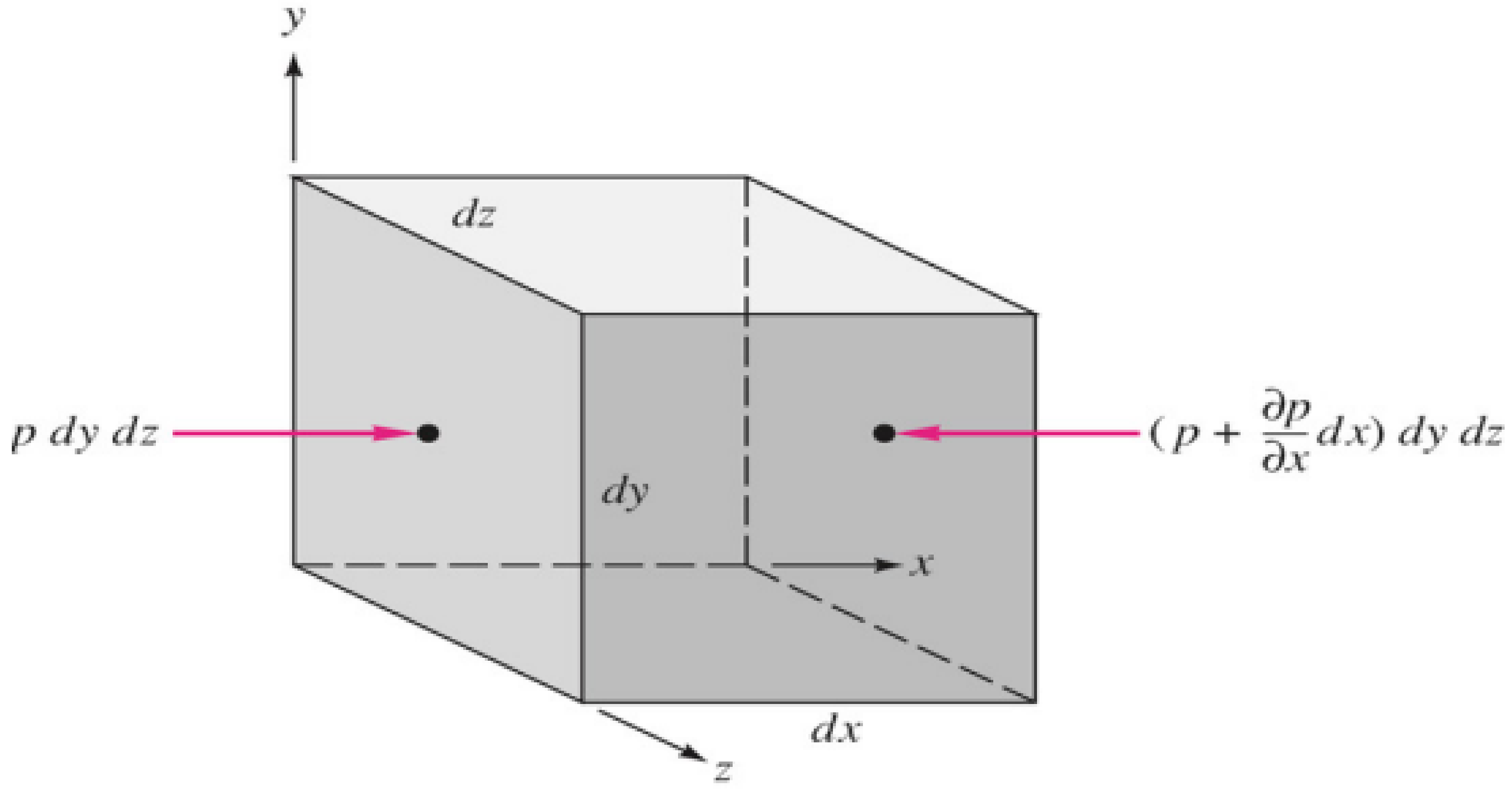


Fig. 7: Forces acting on the fluid element in the x-direction

- Similarly, for the y- and z-directions

$$dF_y = pdxdz - \left(p + \frac{\partial p}{\partial y} dy\right) dxdz = -\frac{\partial p}{\partial y} dxdydz$$

$$dF_z = pdxdy - \left(p + \frac{\partial p}{\partial z} dz\right) dxdy = -\frac{\partial p}{\partial z} dxdydz$$

- All the forces defined above are surface forces

- There are also body forces that can act on a fluid. E.g. forces due to gravity, electric fields, magnetic fields etc.

Basic Equation for Pressure Field

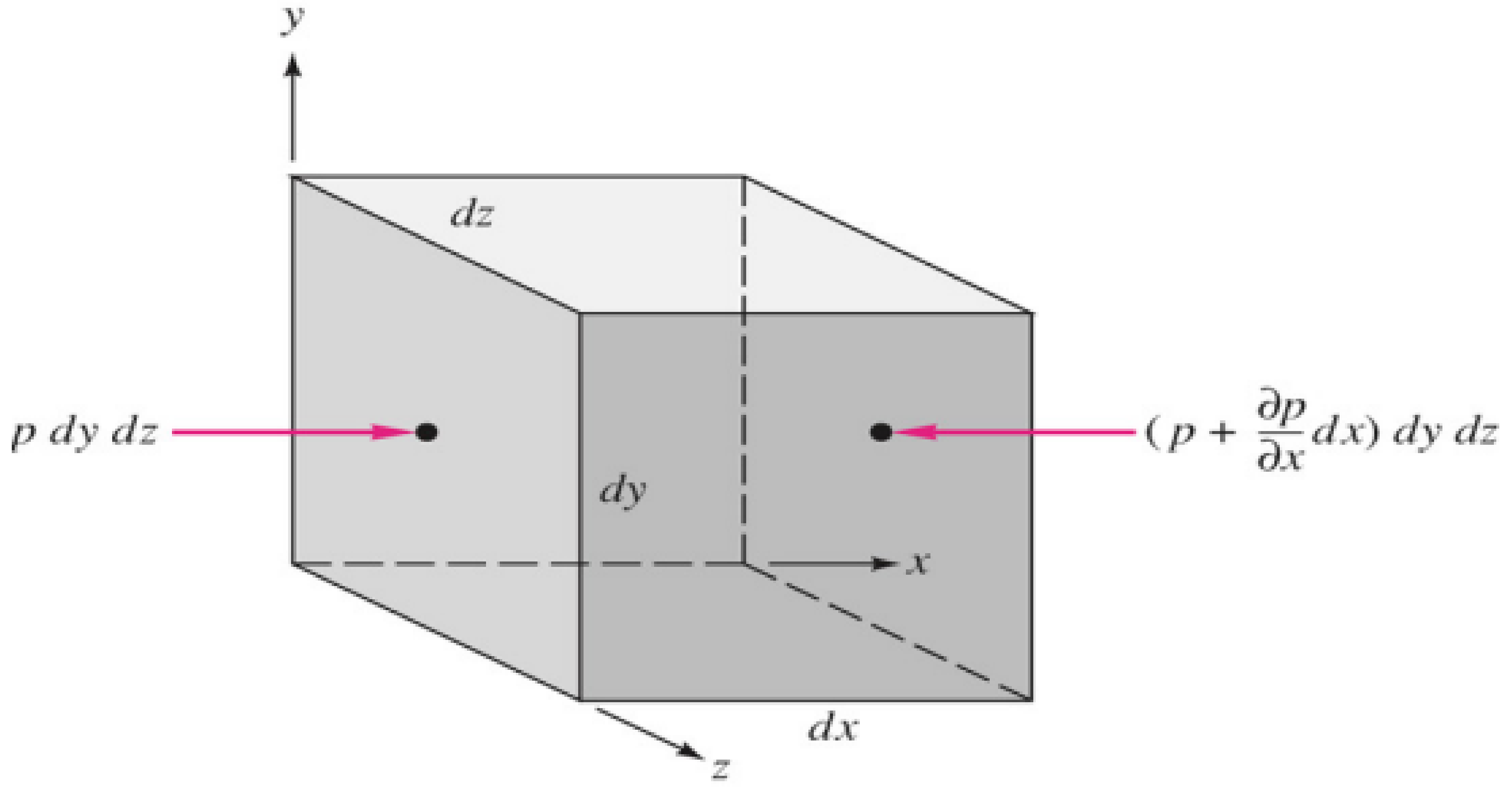


Fig. 7: Forces acting on the fluid element in the x-direction

- **For this fluid element, the body force is assumed to be as a result of gravity which is the weight of the fluid**

$$d\boldsymbol{F}_b = -\gamma dxdydz\hat{\boldsymbol{j}}$$

- **The resultant surface force acting on the fluid element can be expressed as**

$$d\boldsymbol{F}_s = dF_x\hat{\boldsymbol{\imath}} + dF_y\hat{\boldsymbol{\jmath}} + dF_z\widehat{\boldsymbol{k}}$$

$$= -\left(\frac{\partial p}{\partial x}\hat{\boldsymbol{\imath}} + \frac{\partial p}{\partial y}\hat{\boldsymbol{\jmath}} + \frac{\partial p}{\partial z}\widehat{\boldsymbol{k}}\right) dxdydz = -\nabla p dxdydz$$

- ∇p **is called the pressure gradient**

Basic Equation for Pressure Field

- **Applying the Newton's second law to the fluid element**

$$\sum d\boldsymbol{F} = d\boldsymbol{F}_s + d\boldsymbol{F}_b = dm \times \boldsymbol{a}$$

$$-\nabla p dxdydz - \gamma dxdydz\hat{\boldsymbol{j}} = \rho dxdydz \times \boldsymbol{a}$$

$$\boxed{-\nabla p - \gamma\hat{\boldsymbol{j}} = \rho\boldsymbol{a}}$$

- **This is the general equation of motion of a fluid in which there are no shearing stresses**

For a fluid at rest $\nabla p + \gamma\hat{\boldsymbol{j}} = 0$ **or**

$$\frac{\partial p}{\partial y} = -\gamma$$

This is the fundamental equation for fluids at rest and can be used to determine how pressure changes with elevation. The negative sign shows that the pressure decreases as we move upward for a fluid at rest.

Pressure Variation for Incompressible Fluid

- **Changes in specific weight of a fluid are caused by either a change in density or the acceleration due to gravity**

- **For most engineering applications changes in acceleration due to gravity is assumed to be negligible**

- **Also, for liquids, variation in density is usually negligible. Therefore for incompressible fluids, pressure variation with elevation is**

$$\int_{p_1}^{p_2} dp = -\gamma \int_{y_1}^{y_2} dy$$

$$p_1 - p_2 = \gamma(y_2 - y_1) = \gamma h$$

Pressure Variation for Incompressible Fluid

This shows that for an incompressible fluid at rest pressure varies linearly with depth.

$$p_1 = p_2 + \gamma h$$

This equation gives the **hydrostatic pressure distribution** within a fluid at rest

Pressure difference can be specified by the difference in height.

This is called PRESSURE HEAD

$$h = \frac{p_1 - p_2}{\gamma}$$

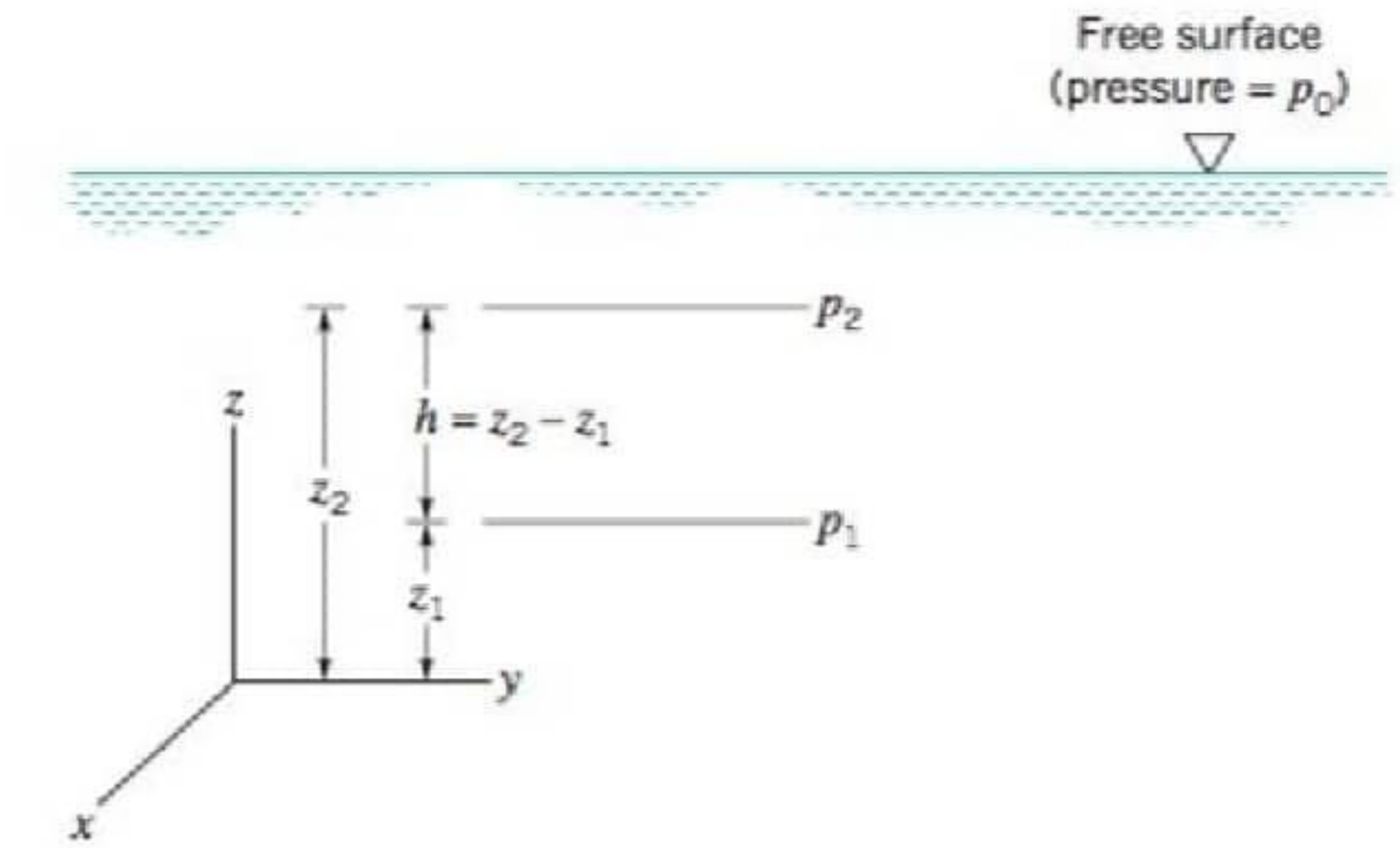


Fig. 8: Pressure variation in a fluid at rest with a free surface

Pressure Variation for Incompressible Fluid

For a liquid with a free surface, it is convenient to use this surface as a reference plane. If we let $p_2 = p_0$, it follows that the pressure p at any depth h below the free surface can be obtained by the equation

$$p = p_0 + \gamma h$$

Pressure in an incompressible fluid at rest open to a free surface is depends on the depth of the fluid relative to a reference plane.

It is not influenced by the size or shape of the container holding the fluid.

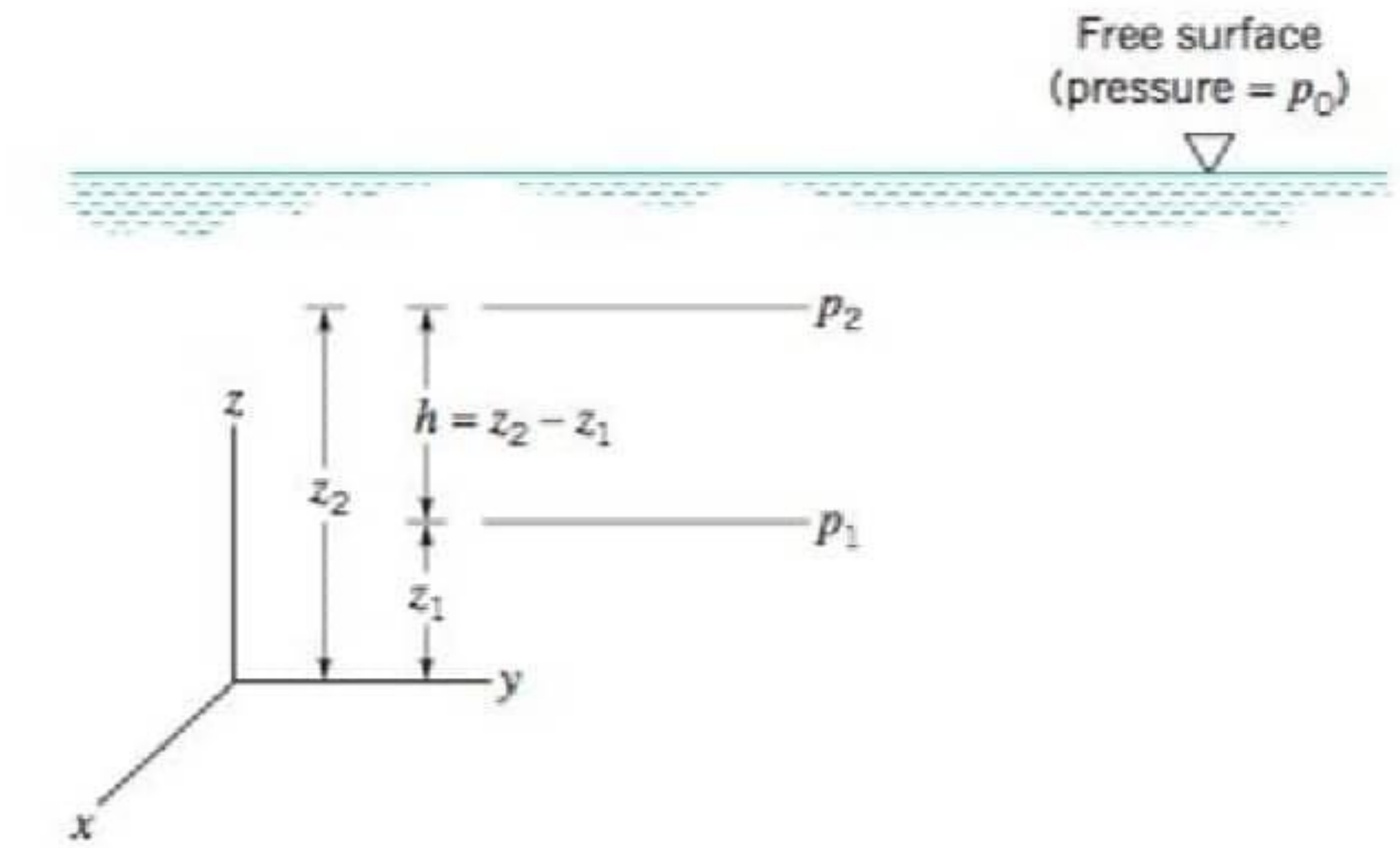
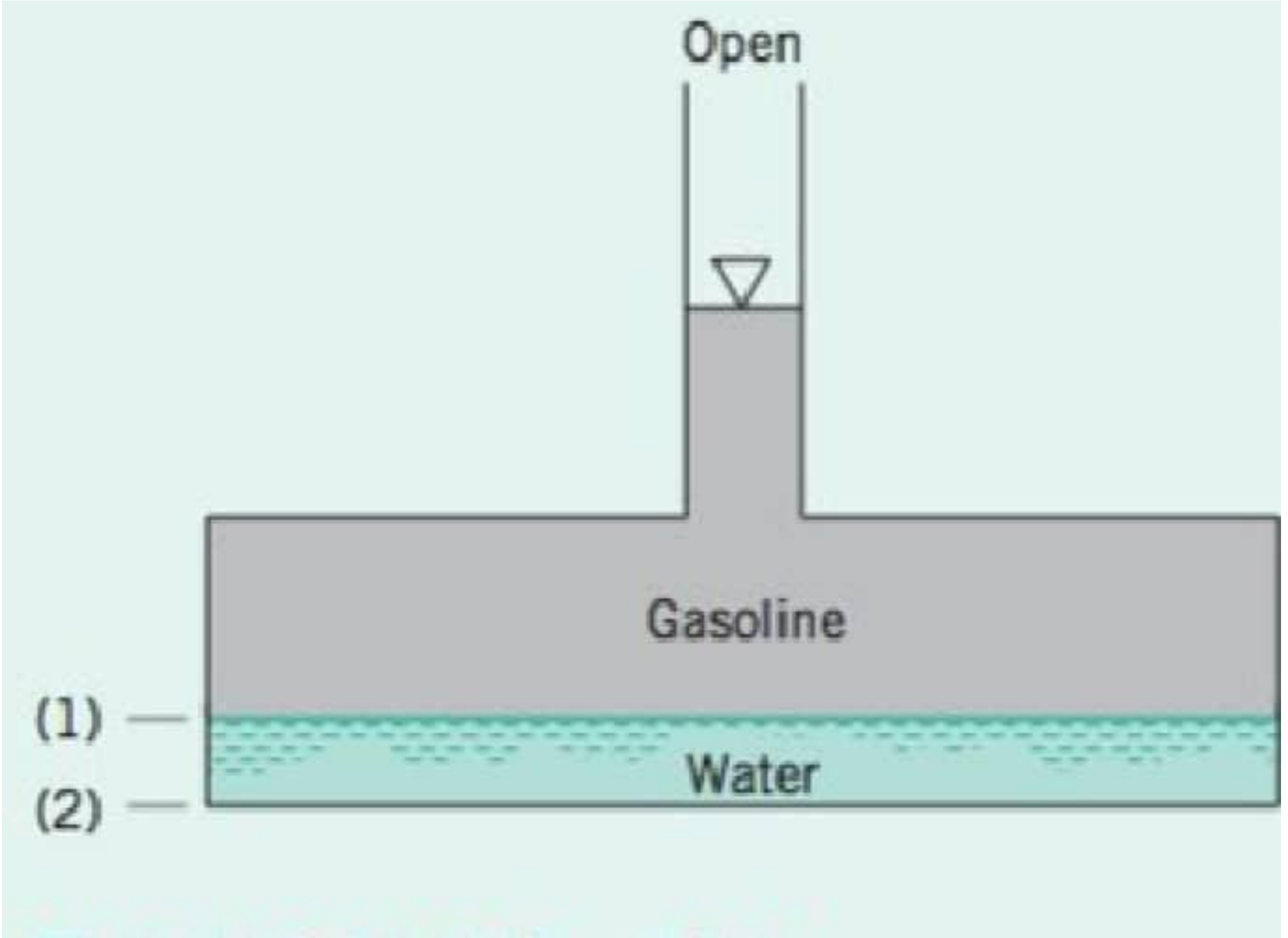


Fig. 8: Pressure variation in a fluid at rest with a free surface

Pressure Variation for Incompressible Fluid

WORKED EXAMPLE

Because of a leak in a buried gasoline storage tank, water has seeped in to the depth as shown in the figure. The specific gravity of the gasoline is 0.68. If the depth $h_2 - h_1 = 1\,m$ and $h_1 - h_{open} = 5.2\,m$, determine the pressure at the gasoline-water interface and at the bottom of the tank. (Take the gauge pressure at the free surface as $p_0 = 0\,Pa$



At the water-gasoline interface, $p_1 = p_0 + \gamma_G h$; $\gamma_G = \rho_G \times g = SG_G \times \rho_w \times g = 680 \times 9.81 = 6{,}670.8\ N/m^3$

Therefore the pressure at interface, $p_1 = 6670.8 \times 5.2 = 34.69\,kPa$

At the bottom of the tank, $p_2 = p_1 + \gamma_1 h$; $\gamma_w = \rho_w \times g = 1000 \times 9.81 = 9810\ N/m^3$

Therefore the pressure at bottom of the tank, $p_2 = 34{,}690 + (9810 \times 1) = 44.5\,kPa$

Pressure Variation for Compressible Fluid

- **Gases such as air, oxygen, and nitrogen are classified as compressible fluids which means there will be a variation in γ. But γ of gases are small when compared with those of liquids**

- **This means that pressure gradient in the vertical direction in compressible fluids will be correspondingly. Therefore, the effect of elevation changes on the pressure in stationary gases can be neglected.**

- **The equation of state for an ideal gas is**

$$p = \rho RT;\ \rho = p/RT\ ;\ \frac{dp}{dy} = -\gamma$$

Pressure Variation for Compressible Fluid

- Using a combination of the relationships gives

$$\frac{dp}{dy} = -\frac{pg}{RT}$$

- By separation of variables

$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_{y_1}^{y_2} \frac{dy}{T}$$

- For isothermal conditions, assuming that the temperature has a constant value T_0

$$p_2 = p_1 exp\left[-\frac{g(y_2 - y_1)}{T_0}\right]$$

Practice Exercises

1. An unknown immiscible liquid seeps into the bottom of an open oil tank. Some measurements indicate that the depth of the unknown liquid is 1.5 m and the depth of the oil with specific weight of $8.5\ kN/m^3$, floating on top is 5.0 m. A pressure gauge connected to the bottom of the tank reads 65 kPa. What is the specific gravity of the unknown liquid?

2. What pressure, expressed in Pascals will a skin diver be subjected to at a depth of 50 m in seawater

SUMMARY

1. More Fluid Properties
2. Pressure at a Point
3. Basic Equation for Pressure Field
4. Pressure Variation for Incompressible Fluid
5. Pressure Variation for Compressible Fluid
6. Worked Examples & Practice Exercises

Next Lecture

- **Pressure Measurement**

✓ **Mercury Barometer**
✓ **Piezometer Tube**
✓ **U-Tube Manometer**
✓ **Inclined-Tube Manometer**
✓ **Mechanical and Electronic Pressure Measuring Devices**