MATRICES

This is the arrangement of objects or items in rows (horizontal) and columns (vertical)

TYPES OF MATRICES

1. Square
2. Diagonal
3. Scalar
4. Unit or Identity. It is usually represented with the letter I
5. Null
6. Column
7. Row
8. Upper triangle
9. Lower triangle
10. An involutory matrix is a type of non-singular square matrix whose product with itself is equal to the identity matrix of the same order. For that matrix, $A^2=I$. Involutory matrices can be seen as the square roots of the identity matrix

Also, for an involutory matrix, $A=A^{-1}$

$|A|^2=1$

$|A|^2=\pm 1$

11. **Singular matrix**: This is a matrix that has no determinant or the determinant is 0

$$\begin{bmatrix} a\,b \\ c\,d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$$

12. Identity matrix: This is a square matrix where all the values of the diagonal are all ones. A matrix multiplied by an identity matrix of the same order is equal to the matrix

$A \times I=A$

13. Orthogonal Matrix: This is one in which the transpose of the matrix is equal to the inverse.

TERMS USED IN MATRICES

1. **Transpose of a matrix**: Given a matrix A, the transpose of that matrix, represented as A^Tcan be obtained by interchanging rows for columns and vice versa
2. **Symmetric matrix**: A symmetric matrix is one which is the same as its transpose

$A=A^T$

The sum of A and A^T will give a symmetric matrix

$A+A^T=symmetric$

3. Skew Symmetric matrix: A skew symmetric matrix is a square matrix that is equal to the negative of its transpose matrix

$A=-A^T$

The sum of A and $-A^T$ will give a symmetric matrix

$A-A^T=skew\ symmetric$

EXPRESSING A MATRIX IN SYMMETRIC AND SKEW-SYMMETRIC FORM

$(A+A^T)+(A-A^T)=2A$

$A=\frac{1}{2}\left[(A+A^T)+(A-A^T)\right]$

The symmetric part of a matrix is therefore expressed as

$\frac{1}{2}(A+A^T)$

The skew symmetric part of a matrix is also expressed as

$\frac{1}{2}(A-A^T)$

NB: If a matrix is both Symmetric and Skew Symmetric, then it is a **zero matrix**.

TRANSPOSE OF A MATRIX

The transpose of a matrix A denoted by A^T can be gotten from interchanging the rows and columns of the matrix.

PROPERTIES OF THE TRANSPOSE OF A MATRIX

1. If A has an order of $m \times n$, its transpose will have an order of $n \times m$
2. A skew matrix is one in which $A^T = A$
3. A skew symmetric matrix is one that $A^T = -A$
4. A square complex matrix whose transpose is equal to the matrix with every entry replaced by its complex conjugate (denoted with an overline) is called a Hermitian matrix ()
5. Skew-Hermitian matrix
6. Unitary matrix
7. $(A^T)^T = A$
8. $(A+B)^T = A^T + B^T$Transpose respects addition
9. $(AB)^T = B^T \cdot A^T$
10. $(ABCDE)^T = E^T D^T C^T B^T A^T$
11. $(cA)^T = c(A^T)$where c is a scalar
12. $det(A^T) = det(A)$
13. $(A^T)^{-1} = (A^{-1})^T = A^{-T}$
14. $A(A^T)$is a square symmetric matrix

MINOR OF A MATRIX

The minor of a matrix can be found by canceling out the row and column containing that element. The minor of a 3 by 3 matrix can also be found similarly. However, in this case, we have to deal with determinants. That is to say, we find the determinant after we have canceled out the row and column containing that element.

CO-FACTOR OF A MATRIX

The co-factor of a matrix can be found by

1. Finding the minor

2. Applying the sign conventions: for the sign convention, it is a minus raised to the position of the element (that is the row and column of the element)
For example, given a matrix A. When finding the co-factor the element A_{11} will have a sign $-^{1+1}$. That is $-^{2}$. That will be a positive sign.

ADJOINT OF A MATRIX

This is the transpose of the co-factor of a matrix

INVERSE OF A MATRIX

Given a matrix A, the inverse of the matrix can be gotten from the formula:

$$A^{-1}=\frac{Adjoint}{Determinant}$$

Only a square matrix can have an inversely
A singular matrix is one that has a determinant of 0
A singular matrix has no inverse and its inverse is therefore undefined
If $A\times B=I$, then $B=A^{-1}$

$$A^{-1}=\frac{I}{A}$$

Also $A^{0}=I$

ORDER OF MATRIX

2 order matrix: Any square matrix of order 2 will be

$$A=\begin{bmatrix} k\,0 \\ 0\,k \end{bmatrix}$$

The determinant of A

$$|A|=k^{2}$$

3 order matrix

$$A=\begin{bmatrix} k\,0\,0 \\ 0\,k\,0 \\ 0\,0\,k \end{bmatrix}$$

The determinant of A, $|A|=k^{3}$
Similarly, $|2A|=8k^{3}$

RANK OF A MATRIX
The order of highest ordered non-zero minor (The determinant value of square submatrix) is called the rank of the matrix

The rank of a matrix is the number of linearly independent rows or equivalently the number of independent columns. Only a zero matrix or null matrix has a rank of zero
Examples:

1. $A=\begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 4 \\ 4 & 1 & -3 \end{bmatrix}$

If you change this to row echelon form, you'll see that the number of non-zero rows in the row-echelon form is 3. That will be the power of the matrix

Properties:

The rank of null matrix is taken as zero

The highest rank of the non-singular matrix is its order

The rank of the singular matrix is less than its order

If A is $m \times n$matrix then Rank of matrix A, $\rho(A) \le min\{m,n\}$

If A is a 3×4 real matrix

PROPERTIES OF DETERMINANTS

1. If $A=\begin{bmatrix} a\,c \\ b\,d \end{bmatrix}$and $B=\begin{bmatrix} b\,d \\ a\,c \end{bmatrix}$. Notice that the rows were changed

$det(B)=-det(A)$

2. If A is an $n \times n$ matrix and $B=kA$ (multiplying by a scalar), then

$det(B)=k^n det(A)$

This property can also be modified to state that n = number of rows multiplied by the scalar

3. If A and B are two $n \times n$matrices, Then

$det(AB)=det(A) \times det(B)$

4. For a matrix A,

$det(A)=det(A^T)$

$det(A^{-1})=\frac{1}{det(A)}$

Note the following

$(AB)^T=B^T \cdot A^T$

$BA \neq AB$

APPLICATIONS OF MATRICES

1. Solving linear equations (simultaneous equations)
2. Finding the area of the triangle
3. Encoding
4. Decoding
5. Mathematics puzzles
6. Games
7. Information like credit card number
8. Optics
9. Economics

QUESTIONS

1. Given that $B=\begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$, find B^4(the answer is a matrix)

2. If $A=\begin{vmatrix} 122 \\ 210 \\ 302 \end{vmatrix}$ and $B=\begin{vmatrix} 0 & 23 \\ 2 & 45 \\ - & 213 \end{vmatrix}$, find $\left(A^T+B\right)^T$

3. If $Q=\begin{bmatrix} 2\lambda-3 \\ 02\ 5 \\ 11\ 3 \end{bmatrix}$, then A^{-1}exists only if

a. $\lambda \neq -2$
b. $\lambda \neq 2$
c. None of the above
d. $\lambda = -2$
e. $\lambda = 2$

4. Given that $A=\begin{vmatrix} 1-10 \\ 2\ 3\ 5 \\ 0\ 1\ 2 \end{vmatrix} B=\begin{vmatrix} 020 \\ 120 \\ 235 \end{vmatrix}$, find $AB+A^T$

5. Evaluate $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ -(b+c-a) & -(c+a-b) & -(a+b-c) \end{vmatrix}$

6. If $A=\begin{vmatrix} 102 \\ 210 \\ 312 \end{vmatrix}$ and $B=\begin{vmatrix} 0 & 23 \\ 1 & 45 \\ - & 213 \end{vmatrix}$, find $\left(A^T+B^T\right)^T$

7. If $A=\begin{vmatrix} 102 \\ 210 \\ 312 \end{vmatrix}$ and $B=\begin{vmatrix} 0 & 23 \\ 1 & 45 \\ - & 213 \end{vmatrix}$, find $(A+B)^T$

8. If $A=\begin{vmatrix} 102 \\ 210 \\ 312 \end{vmatrix}$ and $B=\begin{vmatrix} 0 & 23 \\ 1 & 45 \\ - & 213 \end{vmatrix}$, find $\left(A^T+B\right)^T$

9. If $H=\begin{bmatrix} 3\ 1 \\ -12 \end{bmatrix}$, then $14H^{-1}$ is given by

10. If A is a square matrix of order 3 and $|A|=5$, then the value of $|2A'|$is? (Finding a value) Answer is 40.

11. If A is an involuntary matrix and I is a unit matrix of the same order, then $(1-A)(1+A)$ is?

$(I-A)(I+A)=I^2-IA+AI-A^2$

$I-A+A-I$

0

12. A is a 3 by 4 real matrix and Ax = B is an inconsistent system of equations. The highest possible rank of A is? Answer: 2

13. The symmetric part of the matrix $A=\begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 2 \\ 2 & -2 & 7 \end{bmatrix}$

AGENDA

1. Matrices, Types
2. Determinants
3. Applications of Matrices
4. Basic Matrix Theory and Algebra
5. Systems of Linear Equations
6. Elementary Row-reduction
7. Types and Methods of Solution
8. Echelon Form
9. Introduction to systems of inequalities and linear programming