# UNIVERSITY OF LAGOS

**DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING**

## LECTURE NOTE SERIES ON

## EEG 222 (FUNDAMENTALS OF ELECTRICAL ENGINEERING)

**SERIES 2**

**LECTURER:**

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# RECALL: COURSE OUTLINE

***SECTION A***

- Emf Generation - single phase
- Root mean square (rms)
- Form factor
- Peak factor
- Phasor diagram

## ROOT-MEAN-SQUARE (RMS) VALUE

- The r.m.s. value of an alternating current is defined as that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.
- It is also known as the **effective** or **virtual** value of an alternating current, the former term being used more extensively.
- The rms value of symmetrical sinusoidal alternating current can be calculated using any of the following two methods: mid-ordinate method and analytical method. However for symmetrical but non-sinusoidal waves, the mid-ordinate method is found to be more convenient to use

## Analytical Method

- The standard form of a sinusoidal alternating current is

$$i = I_m sin\omega t = I_m sin\theta \qquad (1)$$

- The mean of the squares of the instantaneous values of current over one complete cycle is (even the value over half a cycle can be used)

$$= \int_0^{2\pi} \frac{i^2 \, d\theta}{(2\pi - 0)}$$

- The square root of this value is

$$= \sqrt{\left(\int_0^{2\pi} \frac{i^2 \, d\theta}{2\pi}\right)}$$

Hence the rms value of the alternating current is

$$I_{rms} = \sqrt{\left(\int_0^{2\pi} \frac{i^2 d\theta}{2\pi}\right)} = \sqrt{\left(\frac{I_m^2}{2\pi} \int_0^{2\pi} \sin^2\theta \, d\theta\right)}$$

But $\cos 2\theta = 1 - 2\sin^2\theta$, which implies that $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

Therefore,

$$I_{rms} = \sqrt{\left(\frac{I_m^2}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\,d\theta\right)} = \sqrt{\left(\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{2\pi}\right)}$$

$$= \sqrt{\frac{I_m^2}{2}} = \boxed{\frac{I_m}{\sqrt{2}} = 0.707\, I_m}$$

Thus,

$$rms\ value\ of\ current = 0.707\ X\ (maximum\ value\ of\ current)$$

- The rms value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the rms value of alternating current and voltage respectively. In electrical engineering work, unless indicated otherwise, the values of the given current and voltage are always the rms values.

- It should be noted that the average heating effect produced during one cycle is

$$= I^2R = (I_m/\sqrt{2})^2 R = \frac{1}{2} I_m^2 R$$

**R.M.S VALUE OF A COMPLEX WAVE**

- The rms value of a complex current wave is equal to the square root of the sum of the squares of the rms values of its individual components.
- For example, suppose a current having the equation

$$i = 12 \sin \omega t + 6 \sin (3\omega t - \pi/6) + 4 \sin (5\omega t + \pi/3)$$

flows through a resistor of R ohm. The rms value of the complex wave is

$$I_{rms} = \sqrt{[(12/\sqrt{2})^2 + (6/\sqrt{2})^2 + (4/\sqrt{2})^2]} = 9.74 \text{ A}$$

# AVERAGE VALUE

- The average value of an alternating current is defined as that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time
- In the case of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or non-sinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.
- Average value can be determined using either mid-ordinate method or analytical method

## Analytical Method

- Using the standard equation of an alternating current ($i = I_m sin\omega t = I_m sin\theta$), the average value is expressed as

$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{(\pi - 0)} = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\left| -\cos\theta \right|_0^{\pi} = \frac{I_m}{\pi}\left| +1-(-1) \right| = \frac{2I_m}{\pi} = \frac{I_m}{\pi/2} = \frac{\text{twice the maximum current}}{\pi}$$

$$I_{av} = I_m / \tfrac{1}{2}\pi = 0.637\, I_m$$

- Thus,

*Average value of current = 0.637 X (maximum value of current)*

- Note that rms value is always greater than average value except in the case of a rectangular wave when both are equal

# FORM FACTOR

- The form factor $K_f$ for sinusoidal alternating current is defined as the ratio of rms value to average value. That is

$$K_f = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{0.707\, I_m}{0.637\, I_m} = 1.$$

- In the case of sinusoidal alternating voltage also, the form factor is

$$K_f = \frac{0.707\, E_m}{0.637\, E_m} = 1.11$$

- The knowledge of form factor will enable the rms value to be found from the arithmetic mean value and vice-versa

# CREST OR PEAK OR AMPLITUDE FACTOR

- For sinusoidal current, the crest or peak or amplitude factor is defined as the ratio of maximum value to the rms value of the current. That is

$$K_a = \frac{\text{maximum value}}{\text{r.m.s. value}} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414$$

- Similarly, the peak factor for sinusoidal voltage is

$$K_a = \frac{E_m}{E_m/\sqrt{2}} = 1.414$$

- Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux.

## PHASOR OR VECTOR DIAGRAM

- It is cumbersome to continuously handle the instantaneous values in the form of equations of waves like $e = E_m \sin \omega t$ conventional method is to employ vector method of representing these sine waves. These vectors may then be manipulated instead of the sine functions to achieve the desired result.
- In fact, vectors are a shorthand for the representation of alternating voltages and currents and their use greatly simplifies the problems in ac work
- A vector is a physical quantity which has magnitude as well as direction. Such vector quantities are completely known when particulars of their magnitude, direction and the sense in which they act, are given. They are graphically represented by straight lines called vectors.

- The length of the line represents the magnitude of the alternating quantity, the inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.
- The alternating voltages and currents are represented by such vectors rotating counter-clockwise with the same frequency as that of the alternating quantity
- In Figure 1, OP is such a vector which represents the maximum value of the alternating current and its angle with X axis gives its phase. Let the alternating current be represented by the equation $e = E_m \sin \omega t$
- It will be seen that the projection of OP and Y-axis at any instant gives the instantaneous value of that alternating current

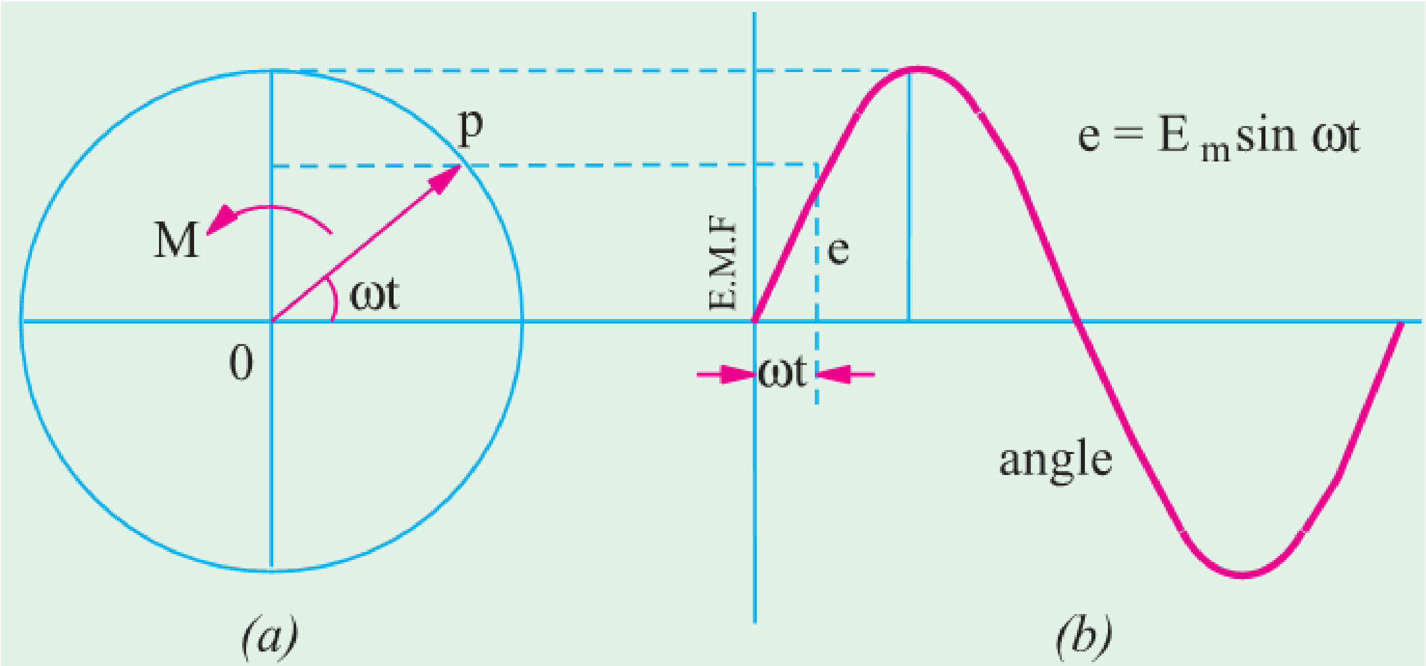


**Figure 1:**

$$OM = OP \sin \omega t \ \text{ or } \ e = OP \sin \omega t = E_m \sin \omega t$$

It should be noted that a line like OP can be made to represent an alternating voltage of current if it satisfies the following conditions

i. Its length should be equal to the peak or maximum value of the sinusoidal alternating current to a suitable scale.

ii. It should be in the horizontal position at the same instant as the alternating quantity is zero and increasing.

iii. Its angular velocity should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.