IMPROPER INTEGRALS

An improper integral is one in which the upper or lower limit of the integral is infinity

For example,

$$\int_1^\infty \frac{1}{x}\,dx$$

CONVERGENCE AND DIVERGENCE

1. If you get a finite number when you solve an improper integral, the integral is **convergent**.
2. If you get an infinite or non existent number like infinity, the integral is **divergent**.

Solving the above example, we follow the following steps

1. Replace infinity with some variable (t) $\int_1^t \frac{1}{x}\,dx$

2. Find the limit of the new expression as t tends to infinity $\lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx$

3. Next we evaluate the limit

$$\lim_{t\to\infty} \ln x\Big|_1^t$$
$$\lim_{t\to\infty} \ln t - \ln 1$$
$$\lim_{t\to\infty} \ln t - 0$$
$$\lim_{t\to\infty} \ln t$$
$$¿\infty$$

Therefore the improper integral $\int_1^\infty \frac{1}{x}\,dx$, is divergent

Example 2: $\int_1^\infty \frac{1}{x^2}\,dx$

$$\int_1^t \frac{1}{x^2}\,dx$$
$$\lim_{t\to\infty} \int_1^t \frac{1}{x^2}\,dx$$
$$\lim_{t\to\infty} \left[\frac{-1}{x}\Big|_1^t \right]$$
$$\lim_{t\to\infty} \left[\frac{-1}{t} - \frac{-1}{1} \right]$$

¿ $\frac{-1}{\infty}+1$

¿ $0+1$

¿ 1

Therefore, the improper integral is **convergent**.

INTEGRAL OF IMPROPER P-SERIES

Given an integral

$$\int_1^\infty \frac{1}{x^p}\,dx$$

If $p \le 1$, then the improper integral is divergent

If $p \ge 1$, then the improper integral is convergent

For example, if we take a look at the integral of

$$\int_1^\infty \frac{1}{(3x+1)^2}\,dx$$

You'll see that when we expand the denominator, the highest power of x will be 2. This is greater than 1. Hence, we can conclude that it is convergent