IBHAZE KEY POINT

video 01

Signal: An abstraction of any measurable quantity that is a function of one or more independent variables such as time or space.

E.G: Alternating current (A.C)

Note: DC is not a signal.

A signal could either be a

:- single variable - $f(x)$, $f(t)$

:- Multi variable - $F(x_1, x_2)$

- $f(t_1, t_2, t_3, t_4)$

Classification of Signal

- Continous time signal.

The signal $x(t)$ has a value for every time t.

- Discrete time signals:

Only present at discrete points in time

SYSTEMS: An abstraction of anything that takes an input signals, operate on it and produces an output signals.

A system is meaningless without signals.

Eg

i/p signal → system → o/p signal

Continous & Discrete

1

describing LTI

Linear time Invariant (LTI) system.

An LTI combines the properties of linearity and time invariance.

→ linearity: Additivity & Superimposition.

(1)

Given $x_1(t)$ → system → $y_1(t)$

& $x_2(t)$ → system → $y_2(t)$

linearity $x_1(t) + x_2(t)$ → system → $y_1(t) + y_2(t)$

Given $x(t)$ → system → $y(t)$

(2) $x(t-\gamma)$ → system → $y(t-\gamma)$

The properties that describe the signal does not change w.r.t time

LTI

$x(t)$ → LTI system $h(t)$ → $y(t) = x(t) * h(t)$

$x(t)$ → Input signal

$y(t)$ Output

$h(t)$ Impulse response

4 Concept under Consideration

- The description of signals as function of frequency
- Investigating how systems respond to input of different ~~signal~~ frequencies.
- Providing tools for switching between time domain and frequency domain representation
- Determination of which domain is suitable for a particular problem.

Problems in Signals & System.

Q3 - Analysis ~~System~~ Problem

Sol i/p signal → [System] → o/p signal

✗

- Synthesis problem

Input ~~Syst~~ signal
Output signal
System ✱

Video 02

→ Analog (CTS)
→ Digital (DTS)
→ Real signal
→ Complex signal
→ Deterministic signal
→ Random signal

Even Signals → These are signals that are Symmetric about the y-axis

$x(t)$ - axis.

$$x(-t) = x(t)$$
$$x[-n] = x[n]$$

Time reversal: is that operation on signals that multiplies the time scale of the signal by some Parameter α. Where $\alpha = -1$.

This Performs folding operation.

Odd Signal → This Signal is antiSymmetrical about the $x(t)$ axis.

An odd Signal must be zero at time $t = 0$.

$$x(-t) = -x(t)$$
$$x[-n] = -x[n]$$

Q1] Show that the Product of 2 even Signal is even

Soln

let Signal 1 be $x_1(t)$ and signal 2 be $x_2(t)$. If the Product of the two Signals is $x(t)$ then ↓

$$x(t) = x_1(t) + x_2(t)$$

Provided that $x_1(t)$ and $x_2(t)$ are even signals, then

$$x(-t) = x_1(-t) \cdot x_2(-t)$$
$$= x_1(t) \cdot x_2(t)$$

for an even signal, $x(-t) = x(t)$

Hence, $x(t)$ is Even.

Q2] Show that the Product of 2 odd Signals is an even Signal.

Let $x(t) = x_1(t) + x_2(t)$

if $x_1(t)$ and $x_2(t)$ are both odd

then $x(t) = x_1(-t) \cdot x_2(-t)$

Recall, $x(-t) = -x(t)$

$$x(-t) = [-x_1(t)] \cdot [-x_2(t)]$$
$$x(-t) = x_1(t)\, x_2(t)$$
$$x(-t) = x(t)$$

Hence, $x(t)$ is even.

Q_3. Show that the product an even and an odd signal is an odd signal

Soln

let $x(t) = x_1(t) \cdot x_2(t)$

If $x_1(t)$ is even and $x_2(t)$ is odd

then $x(-t) = x_1(-t) \cdot x_2(-t)$

$= x_1(t) \cdot [-x_2(t)]$

$= -x_1(t) \cdot x_2(t)$

$x(-t) = -x(t)$

Hence, $x(t)$ is odd.

Complex symmetric signal –

Signal whose original signal $x(t)$ is the same as the complex conjugate of the time reversed version of the original signal

given signal $x(t)$

time reversed $x(-t)$

complex conjugate $x^*(-t)$

If $x(t)$ is $x^*(-t)$

$$x(t) = x^*(-t)$$

$x(t) = x_1(t) + j\,x_2(t)$

$x(-t) = x_1(-t) + j\,x_2(-t)$

$x^*(-t) = x_1(-t) - j\,x_2(-t)$

For complex symmetry $[x(t) = x^*(-t)]$

$x_1(t) = x_1(-t)$ → Real is even

$x_2(t) = -x_2(-t)$ → Im is odd.

For complex anti-symmetry

$x(t) = -x^*(-t)$

$x_1(t) + j\,x_2(t) = -x_1(-t) + j\,x_2(-t)$

$x_1(t) = -x_1(-t)$ → real is odd.

$x_2(t) = x_2(-t)$ → Im is odd.

Periodic signal → $x(t) = x(t+T)$

$\forall t$

$x(t)$ is periodic if there is a positive

T_0 → The smallest value of T

CTS → T, T_0

DTS → N, N_0

Energy signal: A signal is said to be an energy signal if and only if the total energy is finite.

$0 < E < \infty$; $P = 0$

Energy signal must be absolute ~~signal~~ Integrable

$$\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$$

Power signal: $x(t)$ is a power signal if and only if the average power (P) is finite

$0 < P < \infty$; $E = \infty$.

* Periodic signals are power signals.

• NEMP → Neither Energy Nor Power Signal: Any given signal that doesn't obey the principles for Energy or Power Signal.

$$0 \not< E \not< \infty$$
$$0 \not< P \not< \infty$$

Q1 Determine if the following are Energy, Power or NEMP Signals

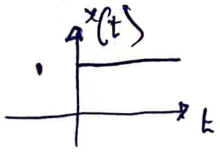
(i) $x(t) = e^{-at} u(t), \; a > 0$

Evaluating the Energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E = \int_{-\infty}^{\infty} |e^{-at} u(t)|^2 \, dt$$

but $u(t) =$



Unit step signal only has values for $t > 0$ such that

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

this changes the boundaries of integration to ~~0<E<~~

$$\int_0^{\infty} |e^{-at}|^2 \, dt$$

$$= \int_0^{\infty} e^{-2at} \, dt$$

$$= \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty}$$

because $a > 0$

$$e^{-\infty} = 0$$

$$= 0 - \left(-\frac{1}{2a}\right) = \boxed{\frac{1}{2a}}.$$

Since E is finite,

$x(t) \Rightarrow$ Energy Signal

(ii) $x(t) = A\cos(\omega_0 t + \varphi)$

$x(t)$ is a periodic signal with Period $T_0 = \frac{2\pi}{\omega_0}$

We know that periodic signals are Power Signals. To verify,

$$P_0 = \frac{1}{T_0} \int_0^{T_0} |x(t)|^2 \, dt$$

$$P = \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} A^2 \cos^2(\omega_0 t + \varphi) \, dt$$

$$= \frac{A^2 \omega_0}{2\pi} \int_0^{2\pi/\omega_0} \frac{1}{2}\left[1 + \cos(2\omega_0 t + 2\varphi)\right] dt$$

$$= \frac{A^2}{2} < \infty$$

Since Average Power is finite,

$x(t)$ is a Power Signal

iii $x(t) = t\,u(t)$

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \int_0^{\infty} t^2 dt$$

$$= \lim_{T\to\infty} \frac{(T/2)^3}{3}$$

$$= \infty$$

Since E is not finite, $x(t)$ is not an energy signal

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^{T/2} t^2 dt$$

$$P = \lim_{T\to\infty} \frac{1}{T} \frac{(T/2)^3}{3} = \infty$$

$x(t)$ Not a Power Signal

Thus, $x(t) \rightarrow$ NENP Signal

(iv) $x[N] = (-0.5)^n u[n]$

$$E = \sum^{\infty} |x[n]|^2$$

recall

$$\sum_{n=0}^{\infty} k^n = \lim_{n\to\infty} \sum_{n=0}^{N-1} k^n = \lim_{n\to\infty} \frac{1-k^N}{1-k}$$

$$= \frac{1}{1-k}$$

Thus

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty} |(-0.5)^n u[n]|$$

$$= \sum_{n=-\infty}^{\infty} |-(0.5)^n|^2 = \sum_{n=0}^{\infty} 0.5^{2n}$$

$$= \sum_{n=0}^{\infty} 0.25^n = \frac{1}{1-0.25}$$

$$= \frac{1}{0.75}$$

$$= \frac{4}{3}$$

$x[n]$ is an energy signal

(v) Show that if $x(t)$ is Periodic with fundamental period T_0, then the normalized average power of $x(t)$ defined by $P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$ is the same as the average power of $x(t)$ over any interval of length T_0 given by $P = \frac{1}{T_0} \int_0^{T} |x(t)|^2 dt$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

If $T = kT_0$

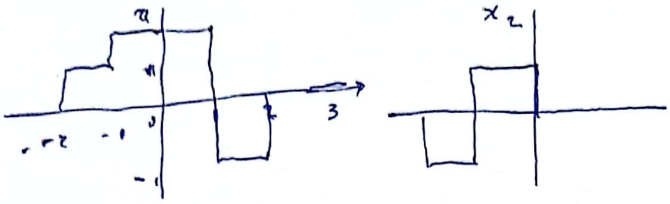
as $k \to \infty$ $T \to \infty$

{Since T_0 is constant}

$$P = \lim_{k\to\infty} \frac{1}{kT_0} k \int_0^{T_0} |x(t)|^2 dt$$

Video 3

Addition, Substraction, multiplication of the following



t	-2	-1	0	1	2
x_1	1	2	2	-1	0
x_2	-1	2	0	4	0
+	0	4	2	3	0
-	2	0	2	-5	0
×	-1	4	0	-4	0

Time scaling: The time scale is multiplied by some parameter $\beta = 1/p$ to achieve

i) signal compression: $x(t) \rightarrow \boxed{\beta = 1/p}$

$\rightarrow y(t) = x(\beta)t$

$= x\left(\frac{t}{p}\right) \forall\, p > 1$

ii) signal expansion: (opposite)

Time reversal → Reflection / folding.

→ Amplitude scaling

- Signal amplification
- Signal attenuation

Time shifting → This is the movement to the right or left of the time scale.

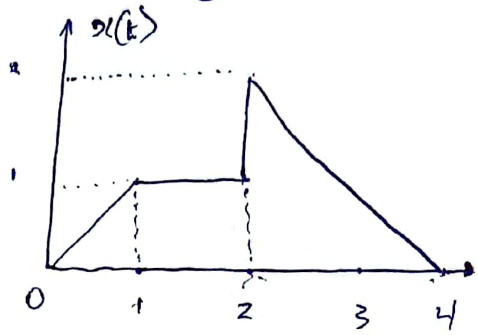
— signal advance (left shifting) $[\alpha = +ve]$

Multi Priority

Time reversal	Amplitude reversal
Time shift	Amplitude shift
Time scale	Amplitude scale

Waveform representation.

Q. Provide the mathematical representation of the signal waveform shown



$x(t) = 0 + \left[\frac{1-0}{1-0}\right] r(t-0) - \left(\frac{1-0}{1-0}\right) r(t-1)$

$+ (2-1)\, u(t-2) - \left(\frac{2-0}{4-2}\right) r(t+\cancel{3})$

$+ \left(\frac{2-0}{4-2}\right) r(t-4)$

$x(t) = r(t) - r(t-1) + u(t-2) - r(t-\cancel{3})$

$+ r(t-4)$

$x(t) = t\,u(t) - (t-1)\,u(t-1) + u(t-2)$

$-(t-3)\,u(t-\cancel{3}) + (t-4)\, r(t-4)$

Video 04

→ CTS receives and delivers CT Signals.
→ DTS receives & delivers DT Signals

→ Static System: Memoryless → Present output depends only on Present input.

→ Dynamic System → Output depends on Past, Present & future.

→ Causal depends only on Present and Past input But not future

→ Non Causal → Depends also on future.
→ Anti-Causal → Depends only on future

→ linear System → Superposition
↳ law of additivity
↳ law of Homogeneity

→ Time Invariant: A delay or advance in the input Signal Causes the Same or equivalent time Shift in the output.

delay → $x(t-\tau) \rightarrow \square \rightarrow y(t-\tau)$
Advance → $x(t+\tau) \rightarrow \square \rightarrow y(t+\tau)$

You Cannot Scale a time invariant System.

→ Stable System → BIBO
↳ If ~~the~~ a bounded output results from an equivalent bounded input at any given instant of time.

LTI → Additivity
homogenity
Time invariant

→ ZIR: The output when the input is set to Zero.

→ ZSR: Output when input is applied.

Representing system in frequency domain
$x(t) = e^{st}$

$e^{st} \rightarrow$ [CT LTI] $\rightarrow y(t)$

$$y(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(t)\,dt$$

$$x(t) = e^{st}$$

$$y(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} e^{st}\,dt$$

$$y(t) = \frac{1}{T}\left[\frac{1}{s}e^{st}\right]_{t-T/2}^{t+T/2}$$

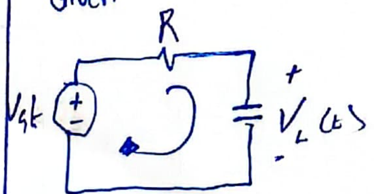
$$y(t) = \frac{1}{sT}\left[e^{st+sT/2} - e^{st-sT/2}\right]$$

$$y(t) = \frac{e^{st}}{sT}\left[e^{sT/2} - e^{-sT/2}\right]$$

$$\lambda = \frac{1}{sT}\left[e^{sT/2} - e^{-sT/2}\right]$$

$$y(t) = \lambda x(t)$$

Ex 2.
Given



Note: CT systems are modelled using differential equations.

By KVL
$-V_s(t) + i(t)R + V_c(t) = 0$

$i(t)R + V_c(t) = V_s(t)$ —(1)

but $i(t) = \frac{C\,dV_c(t)}{dt}$ —(2)

Put (2) into (1)

$$CR\frac{dV_c(t)}{dt} + V_c(t) = V_s(t)$$

if $x(t) = V_s(t)$

$y(t) = V_c(t)$

$$CR\frac{dy(t)}{dt} + y(t) = x(t)$$

↳ First order ODE

Systems are modelled using Differential eqn

(b) integrate (2)

$$\int_{-\infty}^{t} i(\tau)\,d\tau = C\int_{-\infty}^{t} \frac{dV_c(t)}{dt}$$

$$\int_{-\infty}^{t} i(t)\,dt = CV_c(t) \quad —(5)$$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt \quad —(6)$$

Put (6) into (1)

$$i(t)R + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = V_s(t)$$

$V_s(t) \to x(t)$

$i(t) = y(t)$

$$y(t)R + \frac{1}{C}\int_{-\infty}^{t} y(t)\,dt = x(t)$$

$$y(t) + \frac{1}{RC}\int_{-\infty}^{t} y(t)\,dt = \frac{1}{R}x(t)$$

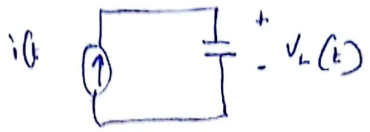
Differentiate

$$\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{R}\frac{dx(t)}{dt} \quad (8)$$

(a) → rep the eqn in terms of v not (i.t)

(b)

Q3: Determine the type of system



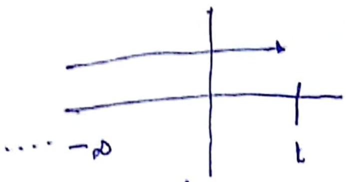
$x(t) = i(t)$

$y(t) = V_c(t)$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

(1) It is a memory system (a dynamic system)

This is because it output depends on past and present input from the limit of integration



we integrate from $-\infty$ to point t.

[This is practical because capacitors are storage element]

(2) It is a causal system

This is because it doesn't depend on the future input. From the the limit of integration

(3) It is a realistic system because it is causal and stable.

→ It has bounded output for all bounded input as the integral would always be finite for finite values [stable]

It is causal as defined above

Test for linearity

$x(t) = k_1 x_1(t) + k_2 x_2(t)$

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} [k_1 x_1(t) + k_2 x_2(t)]\, dt$$

$$y(t) = k_1 \left[\frac{1}{C}\int_{-\infty}^{t} x_1(t)\,dt\right]$$

$$+ k_2 \left[\frac{1}{C}\int_{-\infty}^{t} x_2(t)\,dt\right]$$

↳ It obeys additivity and homogeinity.

The system is linear.

Test for time invariance

$Y_1(t) = x(t - t_k)$

$$Y_1(t) = \frac{1}{C}\int_{-\infty}^{t} x(t - t_k)\,dt$$

$$= \frac{1}{C}\int_{-\infty}^{t-k} x(\alpha)\,d\alpha$$

$$= y(t - t_k)$$

The shift in time input is reflected in the output

This is a time invariant system.

∴ The system is a continous time, linear time invariant system.

Summary: The above system is

1. Dynamic [Past & Present]
2. Stable [BIBO]
3. Causal [No future]
4. Realistic [Stable & Causal]
5. CT LTI [linear and time-invariant]