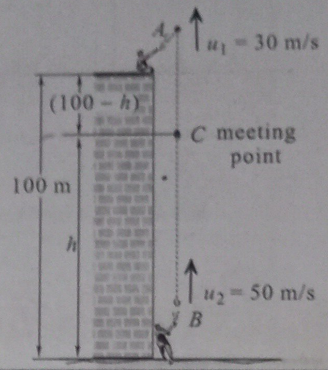
11



Kinematics of Particles - I Rectilinear Motion

***Kinematics of Particles** is the study of geometry of translation motion without reference to the cause of motion. Force and mass are not considered.*

In this chapter, we shall study translation motion of a particle considering its position, displacement, velocity, acceleration and time. Particle cannot have rotational or general plane motion. The two major point of chapter are rectilinear motion and curvilinear motion.

11.1 Rectilinear Motion

If the particle is moving along straight path then it is called a *rectilinear motion*. For example, a train moving on a straight track, a stone released from the top of tower, etc.

Position

Position means the location of a particle with respect to a fixed reference point say origin O. The sketch (given below) shows position of particle at A as $S_A = 4$ m and at B as $S_B = -3$ m.

Example

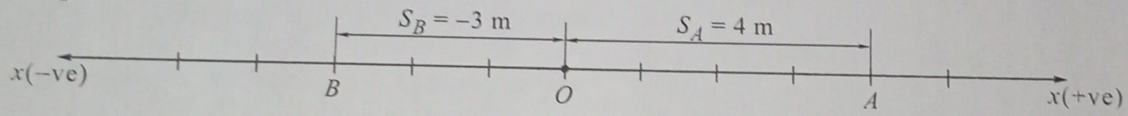


Fig. 11.1-i

Displacement

Displacement is a change in position of the particle. It is the difference between final position and initial position. It is a vector quantity connecting the initial position to the final position.

Example

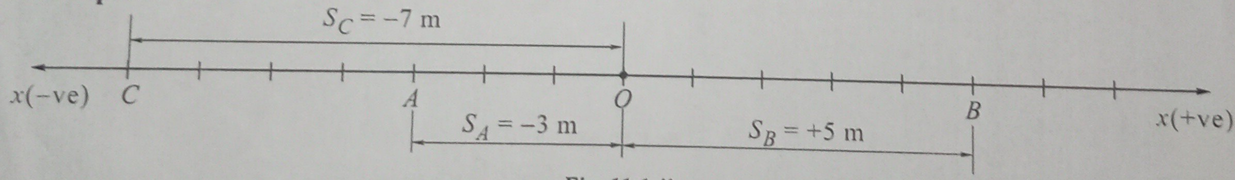


Fig. 11.1-ii

Initial position of particle is A ($S_A = -3$ m). It moves to position B ($S_B = +5$ m) and finally to position C ($S_C = -7$ m).

$$\therefore \text{ Displacement of a particle} = \text{Final position} - \text{Initial position}$$

$$S = S_C - S_A = -7 - (-3) = -4 \text{ m} = 4 \text{ m} (\leftarrow)$$

Thus, displacement depends only on initial and final position of the particle and its value may be positive or negative.

Distance

Distance is the total path travelled by a particle from initial position to final position. It is a scalar quantity. For example, refer to Fig. 11.1-ii,

$$d = AO + OB + BO + OC = 3 + 5 + 5 + 7$$

$$\therefore\ d = 20 \text{ m}$$

If the particle is moving along the straight line in the same direction then the distance covered is equal to displacement.

Velocity

The rate of change of displacement with respect to time is called velocity. It is a vector quantity. If s is the displacement in time t, then the average velocity

$$v = \frac{s}{t}$$

Velocity of a particle at a given instant is called *instantaneous velocity* and is given by the limiting value of the ratio s/t at time t when both s and t are very small. Let δs be the small displacement in a small interval time δt.

The instantaneous velocity $v = \lim\limits_{\delta t \to 0} \dfrac{\delta s}{\delta t}$

$$v = \frac{ds}{dt}$$

The S.I. unit of velocity is m/s.

Conversion of kilometer per hour (kmph)

$$1 \text{ kmph} = \frac{1 \times 1000}{60 \times 60} = \frac{5}{18} \text{ m/sec}$$

For example, $v = 54 \text{ kmph} = 54 \times \dfrac{5}{18} \text{ m/sec}$

$$v = 15 \text{ m/sec}$$

Speed

The rate of change of distance with respect to time is called speed. It is a scalar quantity. *The magnitude of velocity is also known as speed.*

Example

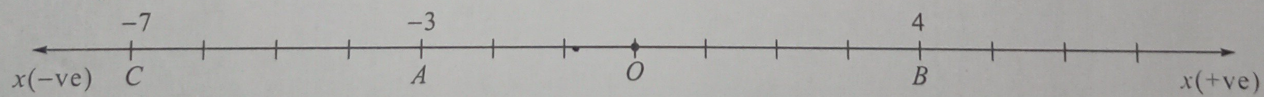


Fig. 11.1-iii

Consider a particle moving from A to B in time $t_1 = 4$ sec then from B to C in time $t_2 = 6$ sec. Find the average velocity and the average speed.

$$\text{Average velocity} = \frac{\text{Change in displacement}}{\text{Time interval}} = \frac{-7-(-3)}{4+6} = \frac{-4}{10}$$

$$\text{Average velocity} = 0.4 \text{ m/sec } (\leftarrow)$$

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time interval}} = \frac{3+4+4+7}{4+6} = \frac{20}{10}$$

$$\text{Average speed} = 2 \text{ m/sec}$$

Acceleration

The rate of change of velocity with respect to time is called acceleration.

The instantaneous acceleration $a = \lim_{\delta t \to 0} \frac{\delta v}{\delta t}$

$$a = \frac{dv}{dt}$$

S.I. unit of acceleration 'a' is m/s^2

The acceleration may be positive or negative. *Positive* acceleration is simply called *acceleration* and *negative* acceleration is called *retardation* or *deceleration*.

Positive acceleration means magnitude of velocity increases w.r.t. time and particle moves faster in positive direction. Negative acceleration means particle moves slowly in the positive direction or moves faster in the negative direction.

In other words, if the acceleration is in the direction of velocity then velocity increases and if the direction of acceleration is in opposite to the direction of velocity then velocity decreases.

Acceleration can also be expressed as

1. $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}$
2. $a = \frac{dv}{dt} = \frac{vdv}{ds}$

11.2 Equations of Motion

1. **Motion with uniform (constant) velocity :**

$$\text{Displacement} = \text{Velocity} \times \text{Time}$$

$$s = v \times t$$

2. **Motion with uniform (constant) acceleration :** Consider a particle moving with constant acceleration.

Let u be the initial velocity, v be the final velocity and t be the time interval.

Acceleration is the rate of change of velocity with respect to time.

$$a = \frac{v-u}{t}$$

$$v = u + at \quad \text{..... (11.1)}$$

$$\text{Displacement } s = \text{Average velocity} \times \text{Time}$$

$$s = \left(\frac{u+v}{2}\right)(t) \quad \text{..... (11.2)}$$

Substituting the value of v from Eq. (11.1) in Eq. (11.2)

$$s = \left(\frac{u+u+at}{2}\right) \times t$$

$$s = ut + \frac{1}{2}at^2 \qquad \text{..... (11.3)}$$

From Eq. (11.1)

$$t = \frac{v-u}{a}$$

Substituting in Eq. (11.2)

$$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \frac{v^2-u^2}{2a}$$

$$v^2 = u^2 + 2as \qquad \text{..... (11.4)}$$

Thus, equations of motion of a particle moving with a constant acceleration are

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

3. **Motion with variable acceleration**

Motion with variable acceleration : If the rate of change of velocity is not uniform then it is called *variable acceleration motion*.

When the variation of acceleration or velocity or displacement with respect to time is known, we can solve such problems by differentiation or by integration with boundary conditions

$$a = \frac{dv}{dt} = v\frac{dv}{ds} \; ; \; v = \frac{ds}{dt}$$

4. **Vertical motion under gravity :** This is the special case of motion with uniform acceleration.

Equations of motion of a particle moving under gravity are

$$v = u + gt$$

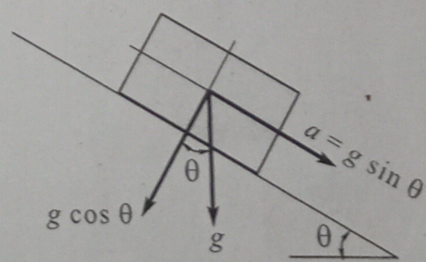
$$h = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh$$

where $g = 9.81 \text{ m/s}^2 (\downarrow)$ and $g = -9.81 \text{ m/s}^2 (\uparrow)$.

5. **Motion along an inclined plane under gravity :** If a block is sliding by its self-weight on a frictionless inclined plane then its constant acceleration is given as

$$a = g \sin\theta$$



11.3 Sign Convention

- The kinematics quantities such as displacement, velocity and acceleration are vector quantities.
- Therefore, we should use the proper sign convention while solving the problems.
- We shall consider the initial direction of motion as positive sign for displacement, velocity and acceleration.
- But retardation will be negative in initial direction of motion.

Example 1

A ball is thrown vertically up.

Sign Convention

- Initial direction of motion is upwards ($\uparrow$).
- Therefore, upward direction ($\uparrow$) will be positive.
- Velocity in upward direction ($\uparrow$) will be positive.
- Displacement in upward direction ($\uparrow$) will be positive.
- Acceleration due to gravity in upward direction ($\uparrow$) (retardation) will be negative (i.e., $g = -9.81 \text{ m/s}^2$).

Example 2

A ball is thrown vertically down.

Sign Convention

- Initial direction of motion is downwards ($\downarrow$).
- Therefore, downward direction ($\downarrow$) will be positive.
- Velocity in downward direction ($\downarrow$) will be positive.
- Displacement in downward direction ($\downarrow$) will be positive.
- Acceleration due to gravity in downward direction ($\downarrow$) will be positive (i.e., $g = 9.81 \text{ m/s}^2$).

Example 3

A car starting from rest moves towards right.

Sign Convention

- Initial direction of motion is towards right ($\rightarrow$).
- Therefore, all kinematic quantities, direction towards right ($\rightarrow$) will be positive but retardation will be negative.

Example 4

A car starting from rest moves towards left.

Sign Convention

- Initial direction of motion is towards left ($\leftarrow$).
- Therefore, all kinematic quantities, direction toward left ($\leftarrow$) will be positive but retardation will be negative.

11.4 Motion Curves

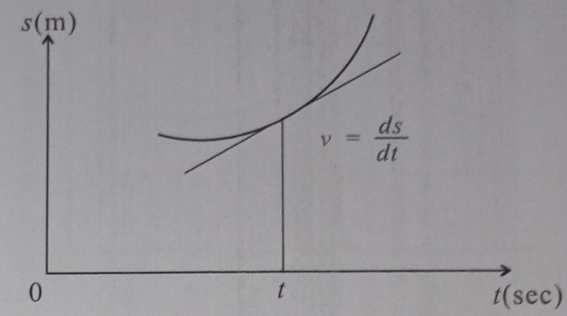
Motion curves are the graphical representation of displacement, velocity and acceleration with time.

1. **Displacement-Time Curve (s-t curve)**

In displacement-time curve, time is plotted along x-axis (abscissa) and displacement is plotted along y-axis (ordinate).

The velocity of particle at any instant of time t is the slope of s-t curve at that instant.

$$v = \frac{ds}{dt} \text{ (slope)}$$

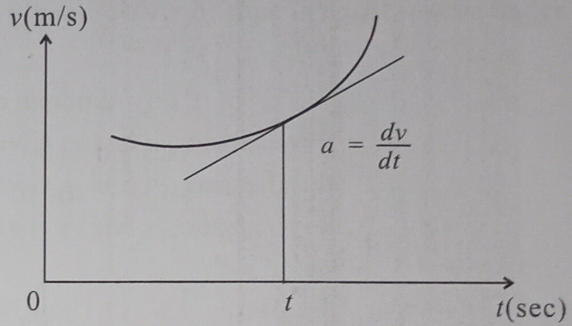


2. **Velocity-Time Curve (v-t curve)**

In velocity-time curve, time is plotted along x-axis (abscissa) and velocity is plotted along y-axis (ordinate).

(a) The acceleration of a particle at any instant of time t is the slope of v-t diagram at that instant.

$$a = \frac{dv}{dt} \text{ (slope)}$$



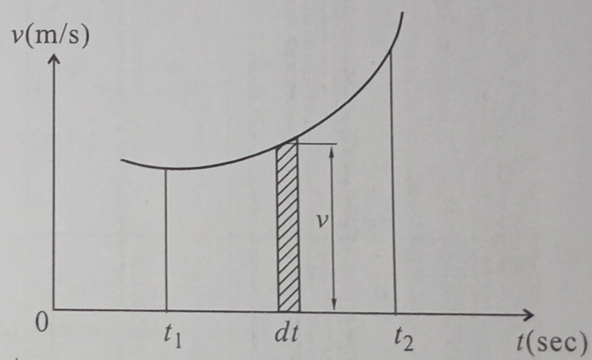
(b) Let the particle's position be s_1 at time t_1 and s_2 at time t_2. From v-t curve, we have

area of the elemental strip $= vdt$

$\therefore \int_{t_1}^{t_2} vdt$ represents the entire area under v-t curve between time t_1 and t_2.

$$\therefore \int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} vdt$$

$$\therefore s_2 - s_1 = \text{Area under } v\text{-}t \text{ curve}$$



$\therefore$ Change in displacement = Area under v-t curve

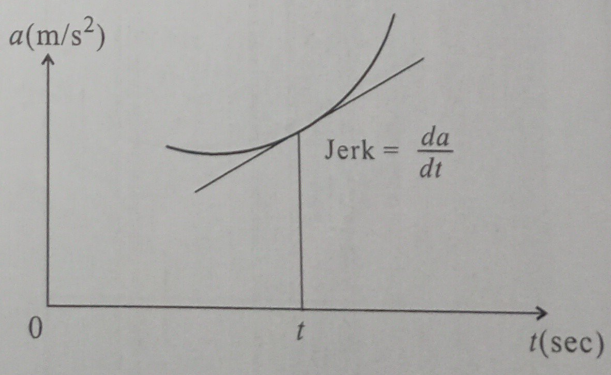
Thus, change in displacement of a particle in given interval of time is equal to area under v-t curve during the same interval of time.

3. **Acceleration-Time Curve (a-t curve)**

(a) In acceleration-time curve, time is plotted along x-axis (abscissa) and acceleration is plotted along y-axis (ordinate).

The slope of a-t curve is the jerk.

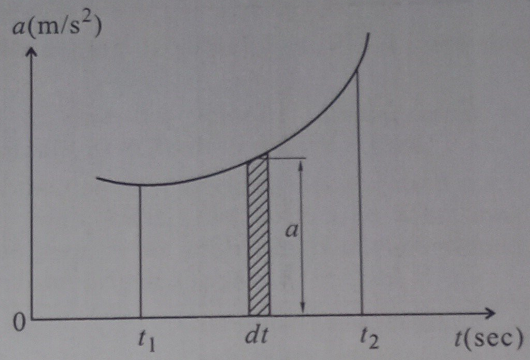
$$\text{Jerk} = \frac{da}{dt} \text{ (slope)}$$



(ii) Let the particles velocity be v_1 at time t_1 and v_2 at time t_2. From a-t curve, we have

Area of the elemental strip $= adt$

$\therefore \int_{t_1}^{t_2} adt$ represents the entire area under a-t curve between time t_1 and t_2.



$$\therefore \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} adt$$

$\therefore$ $v_2 - v_1$ = Area under a-t curve

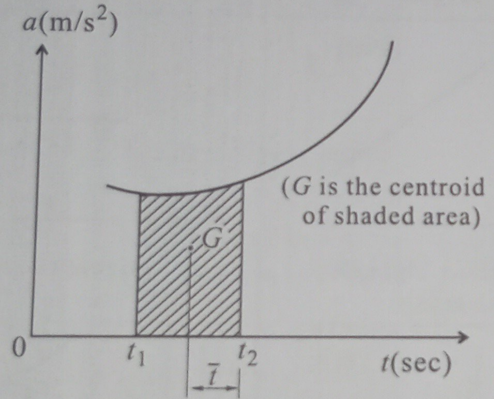
$\therefore$ Change in velocity = Area under a-t curve

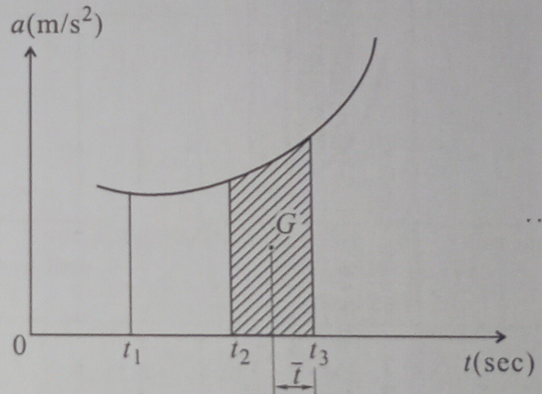
Thus, change in velocity of a particle in a given interval of time is equal to area under a-t curve during the same interval of time.

(iii) For finding displacement use moment area method.

- At time t_1 particles position and velocity is s_1 and v_1, respectively.
- At time t_2 particles position and velocity is s_2 and v_2, respectively.
- At time t_3 particles position and velocity is s_3 and v_3, respectively.

So on and so forth.





.....

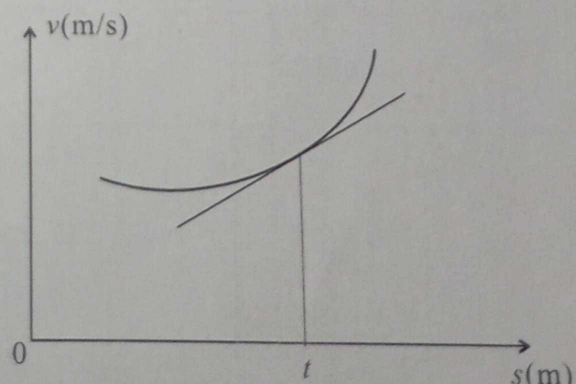
$s_2 - s_1 = v_1(t_2 - t_1) +$ (Area under a-t curve between t_1 and t_2) $(\bar{t})$

$s_3 - s_2 = v_2(t_3 - t_2) +$ (Area under a-t curve between t_2 and t_3) $(\bar{t})$

so on and so forth.

4. Velocity-Displacement Curve (v-s curve)

In velocity-displacement curve, displacement is plotted along x-axis (abscissa) and velocity is plotted along y-axis (ordinate).



From v-s curve, we have

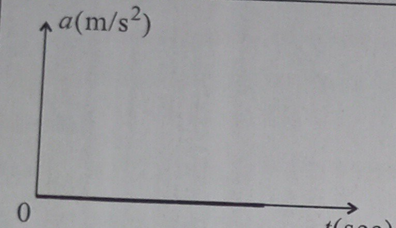
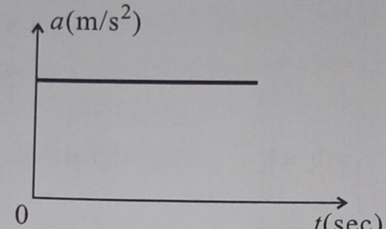
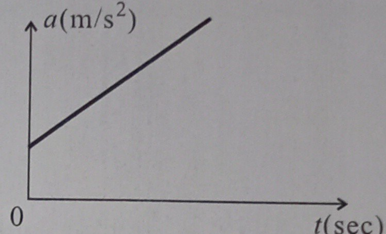
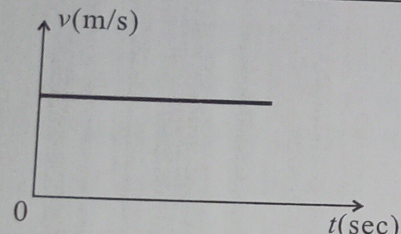
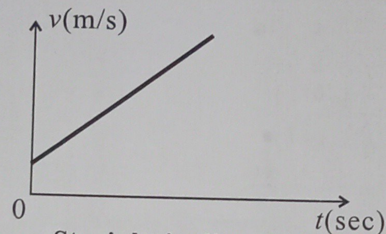
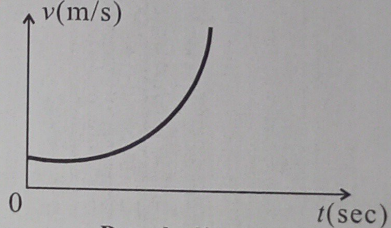
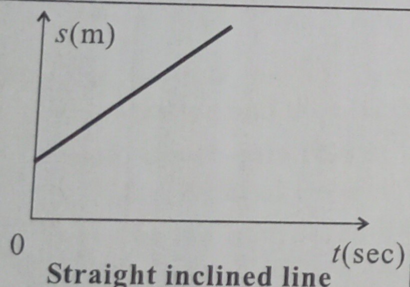
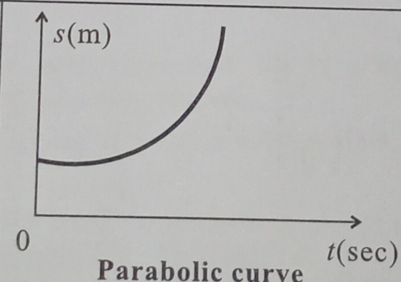
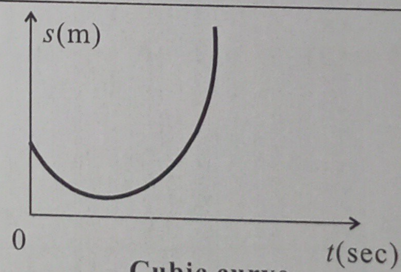
$$\text{slope} = \frac{dv}{ds}$$

We know, $$a = v\frac{dv}{ds}$$

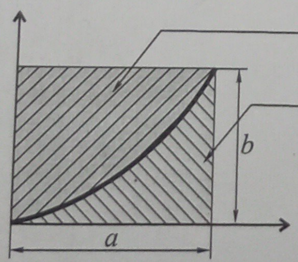
$$\therefore a = (v)\,(\text{slope of } v\text{-}s \text{ curve})$$

11.4.1 Important Relations of Motion Curve

1. If the acceleration is a polynomial of degree n, the degree of velocity is $(n + 1)$ and the degree of displacement is $(n + 2)$.
2. If the equation of displacement is given, then to find the velocity, displacement is differentiated w.r.t. time and to find acceleration, velocity is differentiated w.r.t. time.
3. If the equation of acceleration is given, then to find the velocity, integrate acceleration w.r.t. time and to find displacement, integrate velocity w.r.t. time. The sufficient conditions required to find the constants of integration will be give in the problem. If not mentioned then we can assume that the particle starts from rest from origin.

Uniform Velocity Motion Curve	Uniform Acceleration Motion Curve	Variable Acceleration Motion Curve
 **Zero acceleration**	 **Uniform acceleration degree = 0**	 **Straight inclined line degree = 1**
 **Uniform velocity**	 **Straight inclined line degree = 1**	 **Parabolic curve degree = 2**
 **Straight inclined line degree = 1**	 **Parabolic curve degree = 2**	 **Cubic curve degree = 3**

Area bounded by curve



$$A_1 = \frac{nab}{n+1}$$

$$A_2 = \frac{ab}{n+1}$$

$A = A_1 + A_2 = ab$

n = Curve of degree of polynomial

11.5 Solved Problems Based on Rectilinear Motion with Uniform (Constant) Acceleration and Uniform (Constant) Velocity

Problem 1

A particle starts moving along a straight line with initial velocity of 25 m/s, from O under a uniform acceleration of -2.5 m/s^2. Determine

(i) velocity, displacement and the distance travelled at $t = 5$ sec,

(ii) how long the particle moves in the same direction ? What are its velocity, displacement and distance covered then ?

(iii) the instantaneous velocity, displacement and the distance covered at $t = 15$ sec,

(iv) the time required to come back to O, velocity displacement and distance covered then and

(v) instantaneous velocity, displacement and distance covered at $t = 25$ sec.

Solution

Given : $u = 25$ m/s, $a = -2.5$ m/s^2, O is the origin.

(i) $t = 5$ sec. Refer to Fig. 11.1(a)

$$v = u + at$$
$$v = 25 + (-2.5) \times 5$$
$$v = 12.5 \text{ m/s} \quad (\rightarrow)$$

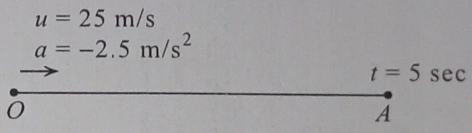


Fig. 11.1(a)

Now, $s = ut + \frac{1}{2}at^2$

$$s = 25 \times 5 + \frac{1}{2} \times (-2.5) \times 5^2$$
$$s = 93.75 \text{ m} \ (\rightarrow)$$

Since velocity is positive, the particle is moving in the same direction and therefore displacement is equal to the distance travelled.

$\therefore \ d = s = 93.75$ m. ***Ans.***

(ii) The particle moves in the same direction till it comes to rest because of negative acceleration and then its direction will reverse.

At the above-said instant velocity of particle will be zero

$v = 0$ (Point of reversal)

Let t be the time taken by the particle to move in the same direction [Refer to Fig. 11.1(b)], we have

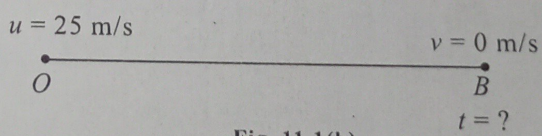


Fig. 11.1(b)

$$v = u + at$$
$$0 = 25 + (-2.5)t$$

$t = 10$ sec ***Ans.***

For displacement, we have

$$s = ut + \frac{1}{2}at^2$$
$$s = 25 \times 10 + \frac{1}{2} \times (-2.5) \times 5^2$$
$$s = 218.75 \text{ m} \ (\rightarrow)$$

As particle has not reversed its direction, we have

Displacement = Distance travelled

$s = d = 218.75$ m. **Ans.**

(iii) $t = 15$ sec. Refer to Fig. 11.1(c).

$v = u + at$

$v = 25 + (-2.5) \times 15 = -12.5$ m/s

$v = 12.5$ m/s $(\leftarrow)$

Now, $s = ut + \frac{1}{2}at^2$

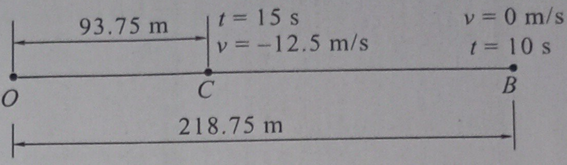


Fig. 11.1(c)

$s = 25 \times 15 + \frac{1}{2} \times (-2.5) \times 15^2 = 93.75$ m $(\rightarrow)$

Particle is moving from O to B and then from B to C in $t = 15$ sec.

$\therefore$ Distance travelled $d = OB + BC \quad (BC = OB - OC)$

$d = 218.75 + (218.75 - 93.75)$

$d = 343.75$ m **Ans.**

(iv) Let t be the time taken by particle to reach origin. Refer to Fig. 11.1(d).

$\therefore$ Displacement = 0

$s = ut + \frac{1}{2}at^2$

$0 = 25 \times t + \frac{1}{2} \times (-2.5) \times t^2$

$t = 20$ sec

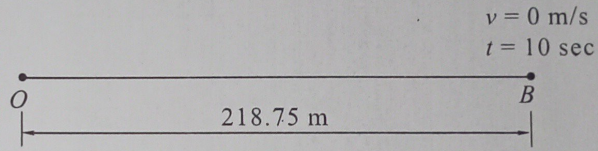


Fig. 11.1(d)

$v = u + at$

$v = 25 + (-2.5) \times 20 = -25$ m/s

$v = 25$ m/s $(\leftarrow)$

Distance covered $d = OB + BO = 218.75 + 218.75$

$d = 437.5$ m **Ans.**

(v) At $t = 25$ sec. Refer to Fig. 11.1(e).

$v = u + at$

$v = 25 + (-2.5) \times 25$

$v = -37.5$ m/s

$v = 37.5$ m/s $(\leftarrow)$ **Ans.**

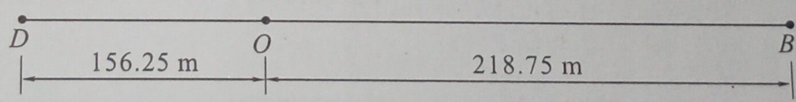


Fig. 11.1(e)

Now, $s = ut + \frac{1}{2}at^2$

$s = 25 \times 25 + \frac{1}{2} \times (-2.5) \times 25^2 = -156.25$ m

$s = 156.25$ m $(\leftarrow)$ **Ans.**

Distance covered $d = OB + BO + OD = 218.75 + 218.75 + 156.25$

$d = 593.75$ m **Ans.**

Problem 9

A burglar's car had a start with an acceleration 2 m/s^2. A police vigilant came in a van to the spot at a velocity of 20 m/s after 3.75 seconds and continued to chase the burglar's car with uniform velocity. Find the time in which the police van will overtake the burglar's car.

Solution

Given : Burglar's car $\Rightarrow t = 0 \,;\, a = 2 \text{ m/s}^2$

Police van $\Rightarrow t = 3.75 \text{ sec} \,;\, v = 20 \text{ m/s (constant)}$

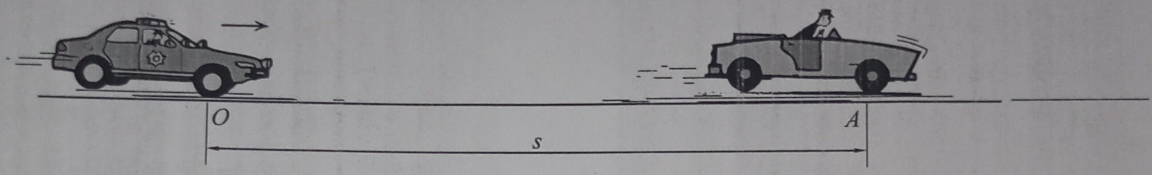


Fig. 11.9

(i) Let t be the time duration for motion of burglar's car. Motion of police van starts after 3.75 sec from the given spot. Therefore, time interval will be $(t - 3.75)$ seconds.

Motion of Burglar's car
(constant acceleration)

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times 2 \times t^2$$

$$s = t^2 \qquad \text{..... (I)}$$

Motion of Police Van
(constant velocity)

$$s = v \times t$$

$$s = 20(t - 3.75) \qquad \text{..... (II)}$$

(ii) Equating Eqs. (I) and (II), we have

$$t^2 = 20(t - 3.75)$$

$$t^2 = 20t - 75$$

$$t^2 - 20t + 75 = 0$$

Solving the quadratic equation, we get

$t = 5$ sec and $t = 15$ sec

(iii) In the beginning, the velocity of burglar's car is less than police van. Therefore, at $t = 5$ sec the police van overtakes burglar's car.

Since burglar's car is moving with constant acceleration, thus as time progresses velocity of the car also increases, but velocity of the police van remains the same. $\therefore$ At $t = 15$ seconds burglar's car will overtake police van.

11.6 Solved Problems Based on Rectilinear Motion Under Gravity

Problem 10

A stone is dropped from the top of a tower. When it has fallen a distance of 10 m, another stone is dropped from a point 38 m below the top of the tower. If both the stones reach the ground at the same time, calculate

(i) the height of the tower and

(ii) the velocity of the stones when they reach the ground.

Solution

(i) **Motion of Stone (1)**

$$u_1 = \sqrt{2 \times 9.81 \times 10}$$

$$u_1 = 14 \text{ m/s}$$

$$h - 10 = u_1 t + \frac{1}{2} \times 9.81 \times t^2 \qquad \text{..... (I)}$$

(ii) **Motion of Stone (2)**

$$h - 38 = u_1 t + \frac{1}{2} \times 9.81 \times t^2 \qquad \text{..... (II)}$$

(iii) Solving Eqs. (I) and (II)

$$h = 57.62 \text{ m}$$

$$t = 2 \text{ sec}$$

$$v_1 = \sqrt{2 \times 9.81 \times h}$$

$$v_1 = \sqrt{2 \times 9.81 \times 57.62}$$

$v_1 = 33.62$ m/s (↓) ***Ans.***

$$v_2 = \sqrt{2 \times 9.81 \times (h - 38)}$$

$$v_2 = \sqrt{2 \times 9.81 \times 19.62}$$

$v_2 = 19.62$ m/s (↓) ***Ans.***

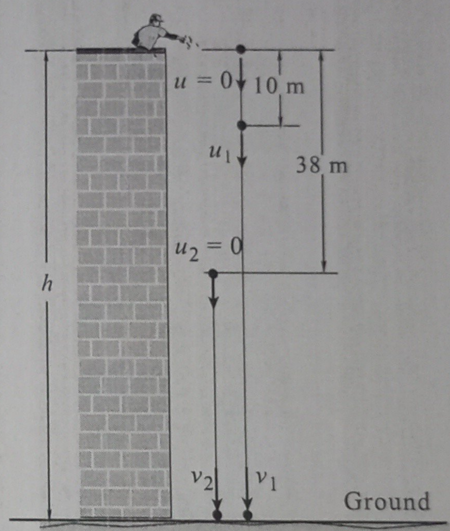


Fig. 11.10

roblem 11

A stone is dropped from a balloon at an altitude of 600 m. How much time is required for the stone to reach the ground if the balloon is

(i) Ascending with a velocity of 10 m/s,

(ii) Descending with a velocity of 10 m/s,

(iii) Stationary and

(iv) Ascending with a velocity of 10 m/s and an acceleration of 1 m/s^2 (Neglect the air resistance).

Solution : (i) Balloon is ascending with a velocity of 10 m/s

Method I

(a) **Motion from A to B (↑)**

Initial velocity of balloon (stone) $= u_b = u_s = 10$ m/s (↑) ;
Final velocity of stone $= v = 0$; $g = -9.81$ m/s^2
Time $t = t_1$; Displacement $h = h_1$.

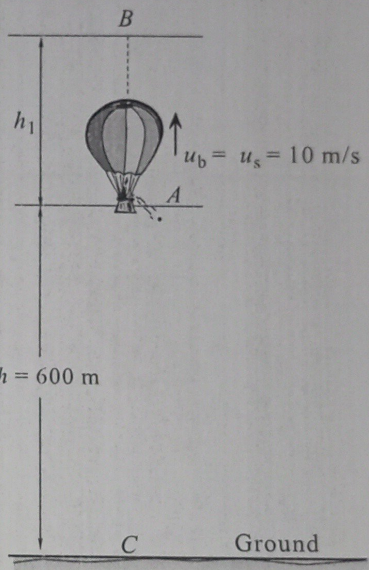


Fig. 11.11(a)

$$v = u + gt$$
$$0 = 10 + (-9.81)(t_1)$$
$$t_1 = 1.02 \text{ sec}$$
$$h_1 = ut + \frac{1}{2}gt^2$$
$$h_1 = 10 \times 1.02 + \frac{1}{2}(-9.81) \times (1.02)^2$$
$$h_1 = 5.1 \text{ m}$$

(b) **Motion from B to C (↓)**

Initial velocity of stone $u_s = 0$;
Displacement $h = 600 + 5.1 = 605.1$ m ;
Time $t = t_2$.

$$h = ut + \frac{1}{2}gt^2$$
$$605.1 = 0 \times t_2 + \frac{1}{2} \times 9.81 \times (t_2)^2$$
$$t_2 = 11.11 \text{ sec}$$

Total time $t = t_1 + t_2 = 1.02 + 11.11$

$t = 12.13$ sec ***Ans.***

Method II : Consider initial position A and final position C

Initial velocity of balloon (stone) $= u_b = u_s = 10$ m/s (↑) ; $g = -9.81$ m/s^2
Displacement $h = -600$ m ; Time $t = t$.

$$h = ut + \frac{1}{2}gt^2$$
$$-600 = 10 \times t + \frac{1}{2}(-9.81) \times t^2$$

$$4.905t^2 - 10t - 600 = 0$$

$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4.905)(-600)}}{2 \times 4.905}$$

t = 12.13 sec ***Ans.***

(ii) Balloon is descending with a velocity of 10 m/s (↓)

Initial velocity of balloon (stone)

$u_b = u_s = 10$ m/s (↓) ;

Displacement h = 600 m ; g = 9.81 m/s^2

Time $t = t$.

$$h = ut + \frac{1}{2} gt^2$$

$$600 = 10 \times t + \frac{1}{2} \times 9.81 \times t^2$$

$$4.905\, t^2 + 10\, t - 600 = 0$$

Solving quadratic equation, we get

t = 10.09 sec ***Ans.***

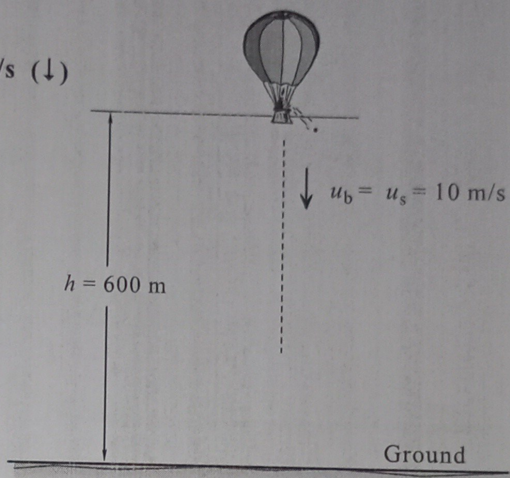


Fig. 11.11(b)

(iii) Balloon is stationary

Initial velocity of balloon (stone)

$u_b = u_s = 0$

Displacement h = 600 m ; g = 9.81 m/s^2

Time $t = t$.

$$h = ut + \frac{1}{2} gt^2$$

$$600 = 0 + \frac{1}{2} \times 9.81 \times t^2$$

t = 11.06 sec ***Ans.***

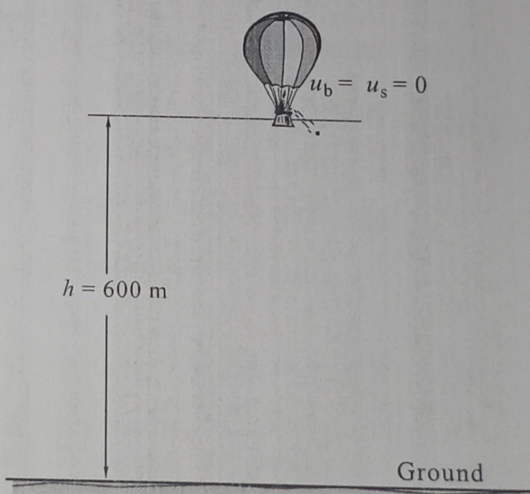


Fig. 11.11(c)

(iv) Balloon ascending with a velocity of 10 m/s and an acceleration of 1 m/s^2.

As long as the stone is attached to balloon, acceleration is influencing increase in velocity of balloon and stone simultaneously. At the instant when stone is dropped from balloon, gravity takes over and its motion is unaffected by acceleration of balloon. So, stone simply carries the instantaneous velocity of balloon.

Therefore, cases (i) and (iv) are similar.

$$h = ut + \frac{1}{2} gt^2$$

$$-600 = 10 \times t + \frac{1}{2}(-9.81) \times t^2$$

$$4.905\, t^2 - 10\, t - 600 = 0$$

t = 12.13 sec ***Ans.***

roblem 12

A body A is projected vertically upwards from the top of a tower with a velocity of 40 m/s, the tower being 180 m high. After t seconds, another body B is allowed to fall from the same point. Both the bodies reach the ground simultaneously. Calculate t and the velocities of A and B on reaching the ground.

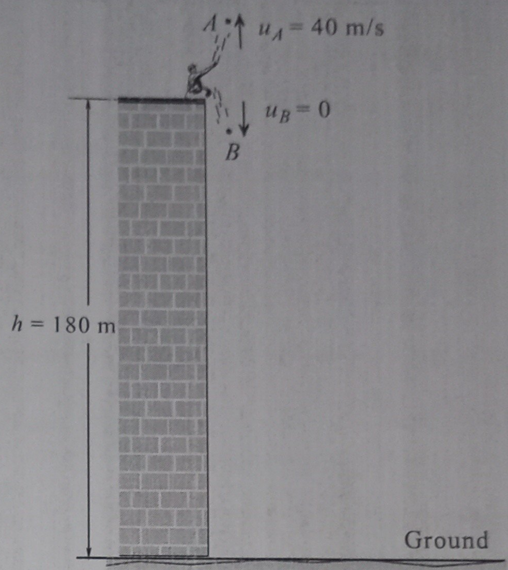


Fig. 11.12

Solution

(i) Motion of Body A

$$u = u_A = 40 \text{ m/s } (\uparrow);$$
$$h = -180 \text{ m}; \ g = -9.81 \text{ m/s}^2$$
$$t = t_A$$
$$h = ut + \frac{1}{2} g t^2$$
$$-180 = 40 t_A + \frac{1}{2}(-9.81) \times (t_A)^2$$
$$4.905 t_A^2 - 40 t_A - 180 = 0$$
$$t_A = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(4.905)(-180)}}{2 \times 4.905}$$
$$t_A = 11.38 \text{ sec}$$

Motion of Body B

$$u = u_B = 0 \ (\downarrow)$$
$$h = 180 \text{ m}; \ g = 9.81 \text{ m/s}^2$$
$$t = t_B$$
$$h = ut + \frac{1}{2} g t^2$$
$$180 = 0 + \frac{1}{2}(9.81) \times (t_B)^2$$
$$t_B = 6.06 \text{ sec}$$

(ii)

$$v = u + gt$$
$$v_A = u_A + g t_A$$
$$v_A = 40 + (-9.81) \times (11.38)$$
$$v_A = -71.64 \text{ m/s}$$
$$v_A = 71.64 \text{ m/s } (\downarrow)$$

$$v = u + gt$$
$$v_B = u_B + g t_B$$
$$v_B = 0 + (9.81) \times (6.06)$$
$$v_B = 59.45 \text{ m/s } (\downarrow)$$

(iii) $t = t_A - t_B = 11.38 - 6.06$

$t = 5.32$ sec ***Ans.***

oblem 13

A ball is thrown vertically upwards at 30 m/s from the top of a tower 100 m high. Five seconds later another ball is thrown upwards from the base of the tower along the same vertical line at 50 m/s. Find when and where both balls will meet and their instantaneous velocity then.

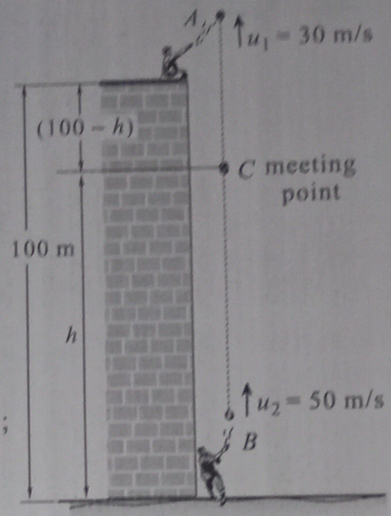


Fig. 11.13

Solution

(i) Motion of first ball from A to C

$u = u_1 = 30$ m/s (↑) ;

$h = -(100 - h)$;

$g = -9.81$ m/s^2

$t = t$

$$h = ut + \frac{1}{2} gt^2$$

$$-(100 - h) = 30t + \frac{1}{2}(-9.81) \times t^2$$

$$h = 30t - 4.905t^2 + 100 \quad \text{..... (I)}$$

Motion of second ball from B to C

$u = u_2 = 50$ m/s (↑) ;

$h = h$;

$g = -9.81$ m/s^2

$t = (t - 5)$

$$h = ut + \frac{1}{2} gt^2$$

$$h = 50(t - 5) + \frac{1}{2}(-9.81) \times (t - 5)^2$$

$$h = 50(t - 5) - 4.905(t - 5)^2 \quad \text{..... (II)}$$

(ii) Equating Eqs. (I) and (II)

$$30t - 4.905t^2 + 100 = 50(t - 5) - 4.905(t - 5)^2$$

$$30t - 4.905t^2 + 100 = 50t - 250 - 4.905(t^2 - 10t + 25)$$

$$30t - 4.905t^2 + 100 = 50t - 250 - 4.905t^2 + 49.05t - 122.625$$

$$69.05t = 472.625$$

$t = 6.85$ seconds ***Ans.***

(iii) From Eq. (I), we get

$$h = 30 \times 6.85 - 4.905 \times (6.85)^2 + 100$$

$h = 75.35$ m (Meeting point of balls from the ground)

(iv) Velocity of first ball

$v = u + gt$

$v = 30 + (-9.81) \times (6.85)$

$v = -37.52$ m/s

$v = 37.52$ m/s (↓) ***Ans.***

Velocity of second ball

$v = u + gt$

$v = 50 + (-9.81) \times (6.85 - 5)$

$v = 31.85$ m/s (↑) ***Ans.***

1.25 If **A** = 2i + 3j and **B** = 3i + 5j, then **A** · **B** is ________.
1.26 If **P** = 3i − 2j and **Q** = 6i + 3j, then **P** × **Q** is ________.
1.27 What do you mean by unit vectors ?

CHAPTER 2

KINEMATICS OF PARTICLES—RECTILINEAR MOTION

2.1 INTRODUCTION

Kinematics is that branch of dynamics which deals with the geometry of motion and which relates displacement, velocity, acceleration and time without reference to the (ause of the motion. ,In this chatJter, kinematics of particles with rectilinear motion '1.14 llle discussed. We know that a particle is a body whose physical dimensions are ry small C'Ompared to the radius of curvature of its path as in the case of rockets, ojectiles, spacecraft etc. The motion of these bodies should be characterised by he motion of their mass centres and any rotation of the bodies should be neglected so as to consider them as particles.

2.2 RECTILINEAR MOTION OF PARTICLES

When a particle moVes along a straight line path, rectilinear motion occurs. By specifying the particle's position, velocity and acceleration, the kinematics of the motion is characterised.

Position Consider a particle located at point P as shown in Fig. 2.1. O is a fixed origin which is intended to define the position P of the particle at any instant of time t. The distance s from O to P is measured and recorded with a plus or minus -sign according to whether the particle is located to the right of the origin or to the left of the origin. The distance s, with the appropriate sign, completely defines the position of the particle and it is called the position coordinate of the particle considered. Since motion is along a straight line path, the position coordinate has the properties of a vector and hence may be termed as position vector.

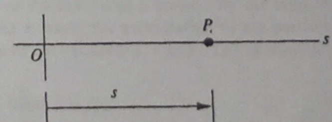


Fig. 2.1 Position

Displacement Let the particle at P move to P' at time $t + \Delta t$ as shown in Fig. 2.2. The change in the position coordinate during the time interval Δt is termed as displacement Δs of the particle. Δs is positive if the final position of the particle is to the right of its initial position and is negative if it is to the left of its initial position.

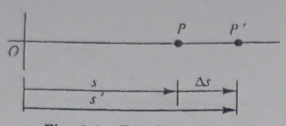


Fig. 2.2 Displacement

The distance travelled should not be confused with the displacement of a particle because, the distance travelled is a scalar quantity and which represents the total length of path traversed by the particle while displacement is a vector quantity.

Velocity The average velocity of the particle over the time interval Δt (Fig. 2.3) may be defined as the quotient of the displacement Δs and the time interval Δt.

$$\text{Average velocity, } v_{av} = \frac{\Delta s}{\Delta t}$$

If smaller and smaller values of Δt are taken, the magnitude of Δs becomes smaller and smaller and consequently the instantaneous velocity v of the particle at the instant t may be obtained as

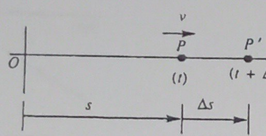


Fig. 2.3 Velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \quad \text{(or)}$$

$$v = \frac{ds}{dt} \tag{2.1}$$

Velocity is positive if the displacement is positive and is negative if displacement is negative and hence velocity is also a vector quantity. The magnitude of velocity is known as speed expressed in units of m/s.

Acceleration The average acceleration of the particle over the time interval Δt (Fig. 2.4) may be defined as the quotient of the change in velocity Δv and the time interval Δt.

$$\text{Average acceleration, } a_{av} = \frac{\Delta v}{\Delta t}$$

If smaller and smaller values of Δt are taken, the magnitude of Δv becomes smaller and smaller and consequently the instantaneous acceleration a of the particle at the instant t may be obtained as

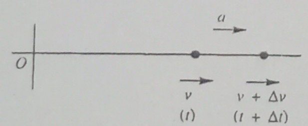


Fig. 2.4 Acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad \text{(or)}$$

$$a = \frac{dv}{dt} \tag{2.2}$$

Substituting $v = \frac{ds}{dt}$ in Equation (2.2), $a = \frac{d^2 s}{dt^2}$ (2.3)

Also acceleration can either be positive or negative. When the speed of the particle decreases, the particle is said to be decelerating as shown in Fig. 2.5.

The unit of measurement of acceleration and deceleration will be m/s^2.

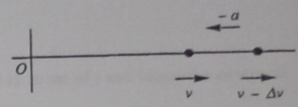


Fig. 2.5 Deceleration

From Equation (2.1)

$$dt = \frac{ds}{v}$$

substituting this value of dt in Equation (2.2)

$$a = \frac{dv}{\left(\frac{ds}{v}\right)}$$

$$\therefore a = v\frac{dv}{ds} \quad \text{or}$$

$$a\, ds = v\, dv \tag{2.4}$$

Motion Curves The curves which are plotted with the position coordinate, the velocity and the acceleration against the time t are called as motion curves. Typical motion curves are shown in Fig. 2.6. We should note that the particle does not move along any of these curves; the particle moves in a straight line path.

We have $v = \frac{ds}{dt}$. Hence the slope of the s-t curve at any given time will be equal to the value of v at that time since the derivative of a function measures the slope of the corresponding curve. Also, the slope of the v-t curve will be equal to the value of 'a' since $a = \frac{dv}{dt}$.

2.3 DETERMINATION OF THE MOTION OF A PARTICLE

It is customary to specify the conditions of the motion of a particle by the type of acceleration that the particle possesses. As a result, the acceleration of the particle may be expressed in terms of one or more of the variables s, v and t. Let us consider the three common classes of motion.

(a) Acceleration given as a function of time, $a = f(t)$.

We have,

$$a = \frac{dv}{dt}$$

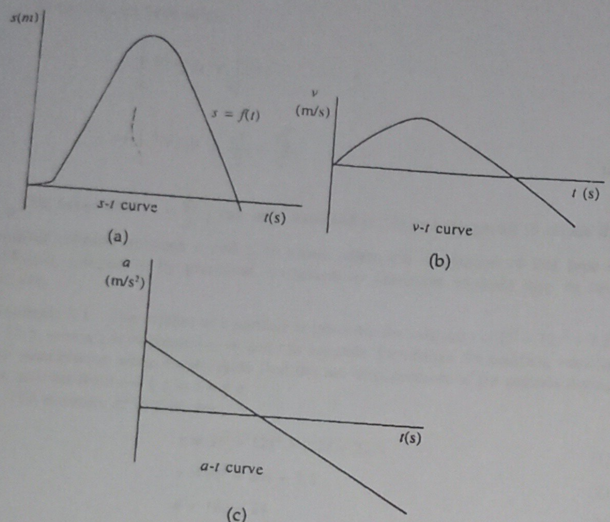


Fig. 2.6 Motion curves

$$\Rightarrow \quad f(t) = \frac{dv}{dt}$$

Separating the variables and integrating,

$$\int dv = \int f(t)\, dt$$

If $v = v_o$ at $t = 0$ and $v = v$ at $t = t$ then,

$$\int_{v_o}^{v} dv = \int_0^t f(t)\, dt$$

$$\Rightarrow \quad v - v_0 = \int_0^t f(t)\, dt$$

$$\therefore \; v = v_0 + \int_0^t f(t)\, dt \qquad (1)$$

Also, we have

$$v = \frac{ds}{dt}$$

From (1), v is a function of t and hence separating the variables and integrating,

$$\int_{s_0}^{s} ds = \int_0^t v\, dt$$

$$\Rightarrow \quad s - s_0 = \int_0^t v\, dt \qquad \text{or}$$

$$s = s_0 + \int_0^t v\, dt$$

The position coordinate s is thus obtained in terms of t and hence the motion is completely defined.

(b) Acceleration given as a function of velocity, $a = f(v)$

We have

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad f(v) = \frac{dv}{dt}$$

$$= v\frac{dv}{ds} \qquad \left[\because v = \frac{ds}{dt},\ dt = \frac{ds}{v}\right]$$

Now separating the variables and integrating,

$$\int_{s_0}^{s} ds = \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

$$\Rightarrow \quad s - s_0 = \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

$$\therefore \quad s = s_0 + \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

The relation between s and v is thus obtained without explicit reference to t which characterises the motion of the particle.

(c) Acceleration given as a function of displacement, $a = f(s)$

We have

$$a\, ds = v\, dv \qquad \text{[From Equation (2.4)]}$$

$$\Rightarrow f(s)\, ds = v\, dv$$

Integrating on both sides,

$$\int_{s_0}^{s} f(s)\,\mathrm{d}s = \int_{v_0}^{v} v\,\mathrm{d}v$$

$$\Rightarrow \int_{s_0}^{s} f(s)\,\mathrm{d}s = \frac{v^2}{2} - \frac{v_0^2}{2} \qquad (2)$$

The value of v $\left(v = \frac{\mathrm{d}s}{\mathrm{d}t}\right)$ can be substituted in (2) and integrated to obtain the desired relation between s and t. In cases where the integration of this type is difficult, integration by graphical, numerical or computer methods may be employed.

Example 2.1 The motion of a particle is given by the relation $s = 3t^3 - 12t^2 + 7.5t + 22.5$, where s is expressed in m and t in seconds. Determine the position, velocity and acceleration when $t = 4s$. Also find the net displacement of the particle during the interval from $t = 1\ s$ to $t = 4\ s$.

The equation of motion are

$$s = 3t^3 - 12t^2 + 7.5t + 22.5 \qquad (1)$$

$$v = 9t^2 - 24t + 7.5 \qquad (2)$$

$$a = 18t - 24 \qquad (3)$$

(i) Position when $t = 4\ s$

Substituting $t = 4\ s$ in (1)

$$s = 3 \times 4^3 - 12 \times 4^2 + 7.5 \times 4 + 22.5$$
$$= 52.5 \text{ m} \qquad \text{(Ans.)}$$

(ii) Velocity when $t = 4\ s$

Substituting $t = 4\ s$ in (2)

$$v = 9 \times 4^2 - 24 \times 4 + 7.5$$
$$= 55.5 \text{ m/s} \qquad \text{(Ans.)}$$

(iii) Acceleration when $t = 4\ s$

Substituting $t = 4\ s$ in (3)

$$a = 18 \times 4 - 24$$
$$= 48 \text{ m/s}^2$$

(iv) Net displacement during $t = 1s$ to $t = 4s$

$$\Delta s = s_4 - s_1$$

$$s_4 = 3 \times 4^3 - 12 \times 4^2 + 7.5 \times 4 + 22.5$$
$$= 52.5 \text{ m}$$
$$s_1 = 3 \times 1^3 - 12 \times 1^2 + 7.5 \times 1 + 22.5$$
$$= 21.0 \text{ m}$$
$$\therefore \quad \Delta s = 31.5 \text{ m} \qquad \text{(Ans.)}$$

The values of s, v and a are plotted against the time t as shown in Fig. 2.7 to help visualise the motion.

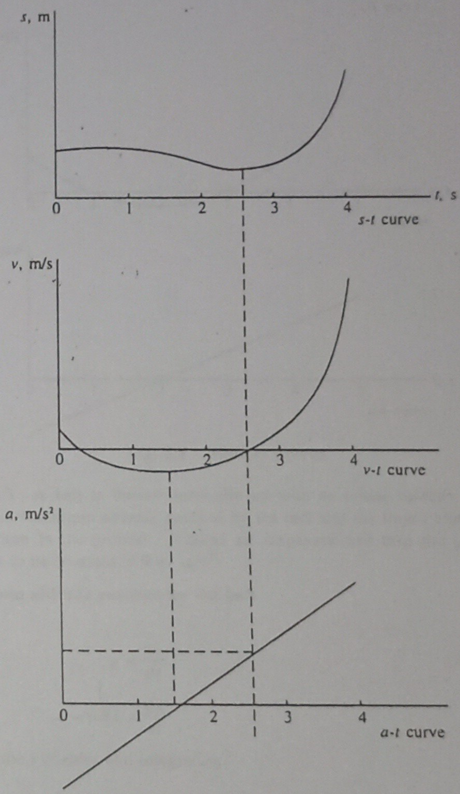


Fig. 2.7 Motion curves

Example 2.2 The motion of a particle is defined by the relation $s = 1.5\,t^3 - 9\,t^2 + 13.5\,t + 7.5$, where s is expressed in metres and t in seconds. Determine (a) the time at which velocity is zero, (b) the position, acceleration and total distance travelled when $t = 5\ s$. Also, plot the motion curves.

The equations of motion are

$$s = 1.5\,t^3 - 9\,t^2 + 13.5\,t + 7.5 \qquad (1)$$

$$v = 4.5\,t^2 - 18\,t + 13.5 \qquad (2)$$

and

$$a = 9\,t - 18 \qquad (3)$$

(a) Time at which $v = 0$

From (2)

$$v = 4.5\,t^2 - 18\,t + 13.5$$

$$\Rightarrow \quad 0 = 4.5\,t^2 - 18\,t + 13.5$$

$$\therefore \quad t = 1\ s \quad \text{and} \quad t = 3\ s \qquad \text{(Ans.)}$$

Hence at $t = 1\ s$ and $t = 3s$, $v = 0$

(b) Position, acceleration and total distance travelled when $t = 5\ s$.

(i) Position

$$s_5 = 1.5 \times 5^3 - 9 \times 5^2 + 13.5 \times 5 + 7.5 \ [\text{using (1)}]$$

$$= 37.5\ \text{m} \qquad \text{(Ans.)}$$

(ii) Acceleration

$$a_5 = 9 \times 5 - 18 \ [\text{using (3)}]$$

$$= 27\ \text{m/s}^2 \qquad \text{(Ans.)}$$

(iii) Total distance travelled

Time t (s)	*Displacement* s (m)	*Distance travelled* (m)
0	7.5	6.0
1	13.5	3.0
2	10.5	3.0
3	7.5	6.0
4	13.5	24.0
5	37.5	
Total distance travelled		42.0 m

The motion of the particle can be very well visualised form the motion curves shown in Fig. 2.8.

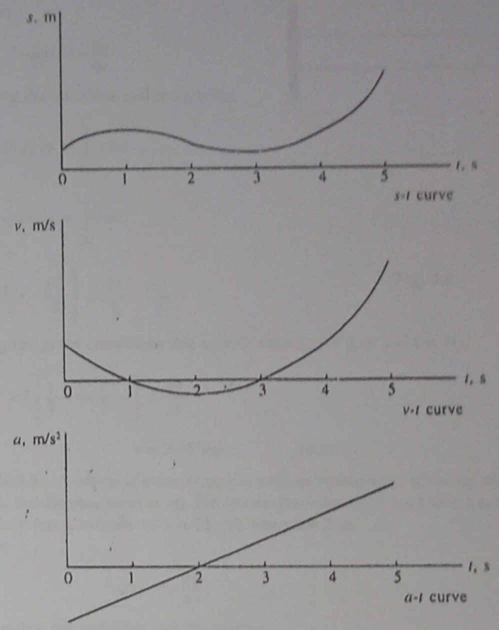


Fig. 2.8 Motion curves

Example 2.3 A ball is thrown vertically up with an initial velocity of 25 m/s. Calculate the maximum altitude reached by the ball and the time t after throwing for it to return to the ground. Neglect air resistance and take the gravitational acceleration to be constant at 9.81 m/s^2.

(a) Maximum altitude reached by the ball

We have,

$$a = \frac{dv}{dt}$$

$$\Rightarrow -9.81 = \frac{dv}{dt}$$

Separating the variables and integrating,

$$\int_{v_0 = 25}^{v} dv = -\int_0^t 9.81\,dt$$

$$v - 25 = -9.81\,t$$

$$\therefore \quad v = 25 - 9.81\,t \qquad (1)$$

We know that

$$v = \frac{ds}{dt}$$

Let s_y be the elevation of the ball above the ground. Hence

$$v = \frac{ds_y}{dt}$$

$$\Rightarrow \quad 25 - 9.81\,t = \frac{ds_y}{dt}$$

Separating the variables and integrating

$$\int_{s_0}^{s_y} ds_y = \int_0^t (25 - 9.81\,t)\,dt$$

$$s_y - s_0 = 25\,t - \frac{9.81\,t^2}{2}$$

$$\therefore \quad s_y = s_0 + 25\,t - 4.905\,t^2 \qquad (2)$$

When the ball reaches its highest altitude, we have $v = 0$.
Substituting into (1), we obtain

$$25 - 9.81\,t = 0$$

$$\therefore \quad t = 2.55 \text{ s}$$

Substituting $t = 2.55$ s in (2),

$$s_y = 0 + 25 \times 2.55 - 4.905 \times 2.55^2$$

$$= 31.85 \text{ m} \qquad \text{(Ans.)}$$

(b) Time taken by the ball to return to the ground

When the ball reaches the ground, $s_y = 0$
Substituting into (2), we obtain

$$25\,t - 4.905\,t^2 = 0$$

$$\therefore \quad t = 5.10 \text{ s} \qquad \text{(Ans.)}$$

Example 2.4 A ball hanging from the end of a cable as shown in Fig. 2.9 has an acceleration given by $a(s) = -2\,s$ m/s^2. (s is the displacement of the ball in m) Determine the velocity of the ball when $s = 1$ m if the ball is released from rest when $s = -2$ m.

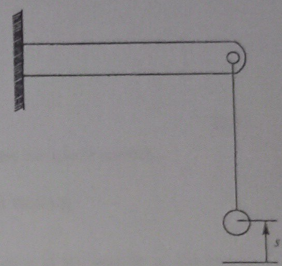


Fig. 2.9

We have,

$$a(s) = v\frac{dv}{ds}$$

Separating the variables and integrating,

$$\int_{s_0}^{s} a(s)\,ds = \int_{v_0}^{v} v\,dv$$

$$\Rightarrow \int_{s_0}^{s} -2s\,ds = \int_{v_0}^{v} v\,dv$$

$$\Rightarrow -2\left[\frac{s^2}{2} - \frac{s_0^2}{2}\right] = \frac{v^2}{2} - \frac{v_0^2}{2}$$

Using the given conditions that $v_0 = 0$ when $s_0 = -2$ m and $s = 1$m,

$$-2\left[\frac{1}{2} - \frac{(-2)^2}{2}\right] = \frac{v^2}{2}$$

$$\therefore \quad v = 2.45 \text{ m/s} \qquad \text{(Ans.)}$$

Example 2.5 A block of mass m moves with an acceleration given by $a(s) = -2\,s$ m/s^2 (s is the displacement in m). Determine the velocity of the block when $s = 8$ m if the block has a velocity of $v = 12$ m/s when $s = 2$ m.

We have

$$a(s) = v\frac{dv}{ds}$$

Separating the variables and integrating,

$$\int_{s_0}^{s} a(s)\,ds = \int_{v_0}^{v} v\,dv$$

$$\Rightarrow \int_{s_0}^{s} -2s\,ds = \int_{v_0}^{v} v\,dv$$

$$\Rightarrow -2\left[\frac{s^2}{2} - \frac{s_0^2}{2}\right] = \left[\frac{v^2}{2} - \frac{v_0^2}{2}\right]$$

Using the given conditions that $v_0 = 12$ m/s when $s_0 = 2$ m and $s = 8$ m,

$$-2\left[\frac{8^2}{2} - \frac{2^2}{2}\right] = \left[\frac{v^2}{2} - \frac{12^2}{2}\right]$$

$$v = 4.90 \text{ m/s} \qquad \text{(Ans.)}$$

Example 2.6 A block is attached between two springs whose coils are very close together. The acceleration of the block is given by $a(s) = -(3s^2 + s)$ where a is in m/s^2 and s is in m. Determine the maximum displacement of the block if it has a velocity of 2 m/s when $s = -1$ m.

We have

$$a(s) = v\frac{dv}{ds}$$

Separating the variables and integrating

$$\int_{s_0}^{s} a(s)\,ds = \int_{v_0}^{v} v\,dv$$

$$\Rightarrow \int_{s_0}^{s} -(3s^2 + s) = \int_{v_0}^{v} v\,dv$$

$$\Rightarrow -\left[\frac{3s^3}{3} + \frac{s^2}{2}\right]_{s_0}^{s} = \frac{v^2}{2} - \frac{v_0^2}{2}$$

$$\Rightarrow -\left[(s^3 + 0.5s^2) - (s_0^3 + 0.5s_0^2)\right] = 0.5(v^2 - v_0^2)$$

Using the given conditions that $v_0 = 2$ m/s when $s_0 = -1$m,

$$-\left[(s^3 + 0.5s^2) - ((-1)^3 + 0.5(-1)^2)\right] = 0.5(v^2 - 2^2)$$

$$\Rightarrow -\left[s^3 + 0.5s^2 + 0.5\right] = 0.5v^2 - 2$$

When the block attains maximum displacement, $v = 0$

$\therefore$ $-s^3 - 0.5s^2 - 0.5 = -2$

$\Rightarrow s^3 + 0.5s^2 - 1.5 = 0$

$\therefore$ $s = 1$ m (Ans.)

2.4 UNIFORM RECTILINEAR MOTION

The motion of a particle in which the acceleration a of the particle is zero for every value of t is called uniform rectilinear motion. As such, the velocity v will be constant in case of uniform rectilinear motion.

i.e., $v = \dfrac{ds}{dt} = \text{constant}$

Separating the variables and integrating

$$\int_{s_0}^{s} ds = v\int_{0}^{t} dt$$

$$\Rightarrow s - s_0 = vt$$

$$s = s_0 + vt \qquad (2.5)$$

uation (2.5) is valid only if the velocity of the particle is constant.

UNIFORMLY ACCELERATED RECTILINEAR MOTION

otion of a particle in which the acceleration a of the particle is constant is be uniformly accelerated rectilinear motion.

$$a = \frac{dv}{dt} = \text{constant}$$

ting the variables and integrating,

$$\int_{v_0}^{v} dv = a\int_{0}^{t} dt$$

$$\Rightarrow v - v_0 = at \qquad (2.6)$$

$$v = v_0 + at$$

uting $v = \dfrac{ds}{dt}$ in Equation (2.6)

$$\frac{ds}{dt} = v_0 + at$$

gain separating the variables and integrating

$$\int_{s_0}^{s} ds = \int_{0}^{t} (v_0 + at)\,dt$$

$$\Rightarrow s - s_0 = v_0 t + \frac{at^2}{2}$$

$$s = s_0 + v_0 t + \frac{1}{2}at^2 \qquad (2.7)$$

Equation (2.4)

$$v\,dv = a\,ds$$

grating both sides, we get

$$\int^{v} dv = a\int^{s} ds$$

$$\Rightarrow \frac{1}{2}\left(v^2 - v_0^2\right) = a\,(s - s_0)$$

or

$$v^2 = v_0^2 + 2a\,(s - s_0) \tag{2.8}$$

The relations derived are necessarily restricted to the special case where the acceleration is constant. A common example of this motion occurs when a body falls freely toward the earth. If air resistance is neglected and the distance of fall is short, then the constant downward acceleration of the body when it is close to the earth is approximately 9.81 m/s^2.

Example 2.7 A ball is thrown vertically upward from the top of a 20 m high building. Knowing that it strikes the ground 3 s after release, determine (a) the speed with which the ball was thrown upward, (b) the speed with which the ball strikes the ground.

The motion of the ball is said to be uniformly accelerated since it falls freely toward the earth after throwing upward.

We have

$$s = s_0 + v_0 t + \frac{1}{2} at^2$$

$$s_0 = 20 \text{ m}$$

$$s = 0 \text{ at } t = 3\text{s}$$

$$\therefore \quad 0 = 20 + v_0 \times 3 + \frac{1}{2}(-9.81) \times 3^2$$

$$\therefore \quad v_0 = 8.05 \text{ m/s} \uparrow \qquad \text{(Ans.)}$$

(v_0 is the speed with which the ball was thrown upward)

Also, we have,

$$v = v_0 + at$$

$$= 8.05 + (-9.81) \times 3$$

$$= -21.38 \text{ m/s}$$

Hence,

$$v = 21.38 \text{ m/s} \downarrow \qquad \text{(Ans.)}$$

(v is the speed with which the ball strikes the ground)

Example 2.8 A vehicle covers a distance of 300 m in 25 s with a constant acceleration of 0.5 m/s^2. Determine (a) its initial velocity, (b) its final velocity and (c) the distance travelled during the first 8 s.

(a) Initial velocity of the vehicle

We have

$$s = s_0 + v_0 t + \frac{1}{2} at^2$$

$$\Rightarrow \quad 300 = 0 + v_0 \times 25 + \frac{1}{2} \times 0.5 \times 25^2$$

$$\therefore \quad v_0 = 5.75 \text{ m/s} \qquad \text{(Ans.)}$$

(b) Final velocity of the vehicle

$$v = v_0 + at$$

$$= 5.75 + 0.5 \times 25$$

$$= 18.25 \text{ m/s} \qquad \text{(Ans.)}$$

(c) Distance travelled during the first 8 s

$$s = s_0 + v_0 t + \frac{1}{2} at^2$$

$$= 0 + 5.75 \times 8 + \frac{1}{2} \times 0.5 \times 8^2$$

$$= 62 \text{ m} \qquad \text{(Ans.)}$$

Example 2.9 A car accelerates uniformly from a speed of 30 km/h to a speed of 75 km/h in 5 s. Determine the acceleration of the car and also the distance travelled during the 5 s.

$$\text{Initial velocity, } v_0 = 30 \text{ km/h} = \frac{30 \times 1000}{60 \times 60} = 8.33 \text{ m/s}$$

$$\text{Final velocity, } v = 75 \text{ km/h} = \frac{75 \times 1000}{60 \times 60} = 20.83 \text{ m/s}$$

$$\text{Acceleration, } a = \frac{v - v_0}{t} = \frac{20.83 - 8.33}{5}$$

$$= 2.5 \text{ m/s}^2 \qquad \text{(Ans.)}$$

We have

$$v^2 = v_0^2 + 2a\,(s - s_0)$$

$$\Rightarrow 20.83^2 = 8.33^2 + 2 \times 2.5\,(s - 0)$$

$$\therefore \quad s = 72.9 \text{ m} \qquad \text{(Ans.)}$$

Example 2.10 Two vehicles approach each other in opposite lanes of a straight horizontal roadway as shown in Fig. 2.10. At time $t = 0$, the vehicles have the speeds and positions shown in the figure. Find the time and positions at which the vehicles meet if both continue to move with constant speed.

Let s_A be the displacement of vehicle A after t seconds. Similarly s_B be the displacement of vehicle B after t seconds. Since both the vehicles move with constant speed, their motion is said to be uniform rectilinear motion.

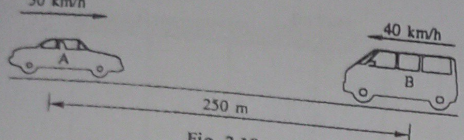


Fig. 2.10

As such

$$s_A = (s_0)_A + (v_0)_A\, t$$

$$(s_0)_A = 0$$

$$(v_0)_A = 50 \text{ km/h} = 13.89 \text{ m/s}$$

$$s_A = 13.89\, t$$

$$s_B = (s_0)_B + (v_0)_B\, t$$

$$(s_0)_B = 0$$

$$(v_0)_B = 40 \text{ km/h} = 11.11 \text{ m/s}$$

$$s_B = 11.11\, t$$

From Fig. 2.11

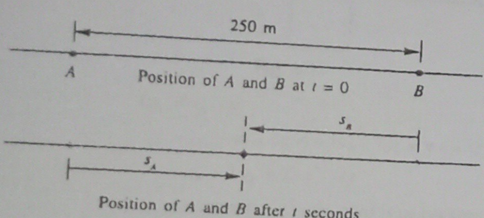


Fig. 2.11

$$s_A + s_B = 250 \text{ m}$$

$$\Rightarrow 13.89\, t + 11.11\, t = 250$$

$$t = 10 \text{ s} \qquad \text{(Ans.)}$$

H...ce

$$s_A = 138.9 \text{ m} \qquad \text{(Ans.)}$$

$$s_B = 111.1 \text{ m} \qquad \text{(Ans.)}$$

...6 MOTION OF SEVERAL PARTICLES

...re are occasions when motion of several particles occurs independently along ... ame line. In such cases, independent equations of motion have to be written for ... particle and if possible the displacements should be measured from the same

...gin and in the same direction. Also, time should be considered from the same ...tial instant for all particles.

Let us consider two particles A and B moving along the same straight line as ...own in Fig. 2.12.

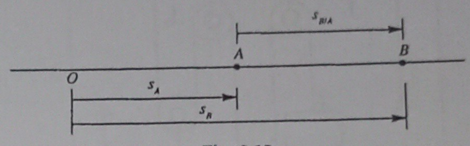


Fig. 2.12

The difference between the position coordinate of B (s_B) and that of A (s_A) will ...fine the relative position coordinate of B with respect to A ($s_{B/A}$). Since the ...ition coordinates of A and B are measured from the same origin.

$$s_{B/A} = s_B - s_A \quad \text{or} \quad s_B = s_A + s_{B/A} \tag{2.9}$$

If the position of B is to the right of A, $s_{B/A}$ will be positive and negative if B is to left of A.

If we consider the velocities of A and B, the relative velocity of B with respect to ... ($v_{B/A}$) may be defined as the rate of change of the relative position coordinate of B ...h respect to A ($s_{B/A}$). Thus, by differentiating Equation (2.9), we get

$$v_{B/A} = v_B - v_A \quad \text{or} \quad v_B = v_A + v_{B/A} \tag{2.10}$$

If B is observed from A to move in the positive direction, ($v_{B/A}$) will be positive ... negative when observed to move in the negative direction.

When the acceleration is concerned, the relative acceleration of B with respect to ... ($a_{B/A}$) may be defined as the rate of change of the relative velocity of B with ...pect to A ($v_{B/A}$). Thus, by differentiating Equation (2.10), we get

$$a_{B/A} = a_B - a_A \quad \text{or} \quad a_B = a_A + a_{B/A} \tag{2.11}$$

...tion of connected particles In case of connected particles, the motion of one ...ticle will depend upon the corresponding motion of another or of several other ...ticles. Such motions are termed as dependent motions. For example, let us ...sider the system of two interconnected particles A and B as shown in Fig. 2.13. The position of block A is specified by s_A and the position of block B by s_B. The ...l length of the cord is given by,

$$L = L_{AC} + L_{CD} + L_{DE} + L_{EF} + L_{FG}$$

$$L_{AC} = s_A - y$$

$$L_{CD} = \pi r \qquad (r = \text{radius of the pulley})$$

$$L_{DE} = s_B - 2y$$

$$L_{EF} = \pi r$$

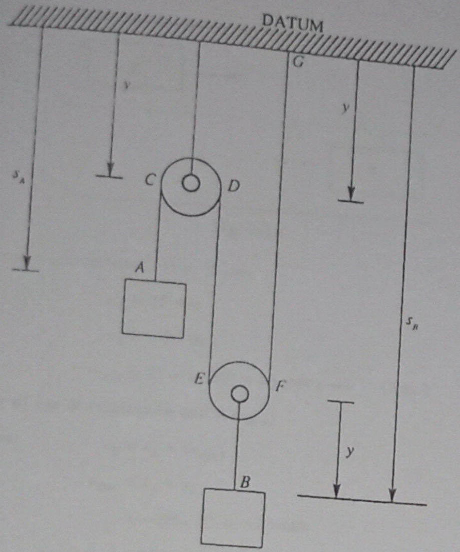


Fig. 2.13

$$L_{FG} = s_B - y$$

$$L = (s_A - y) + (\pi r) + (s_B - 2y) + \pi r + (s_B - y)$$

$$= s_A + 2 s_B + 2\pi r - 4y$$

$$= \text{constant} \qquad (\because \text{ the cord is of constant length})$$

Since the lengths of the portions of cord CD and EF wrapped around the pulleys and also the distance y are constant, we may write,

$$s_A + 2 s_B = \text{constant} \tag{1}$$

The system of Fig. 2.13 is said to have one degree of freedom since only one variable, either s_A or s_B, is needed to specify the positions of all parts of the system. From the relation $s_A + 2s_B$ = constant, it shall be stated that if s_A is given an increment Δs_A, i.e., if block A is lowered by an amount Δs_A the coordinate s_B will attain an increment $\Delta s_B = -\frac{1}{2} \Delta s_A$; that is, block B will rise by half the same amount.

Now, let us consider the system of three interconnected particles as shown in Fig. 2.14

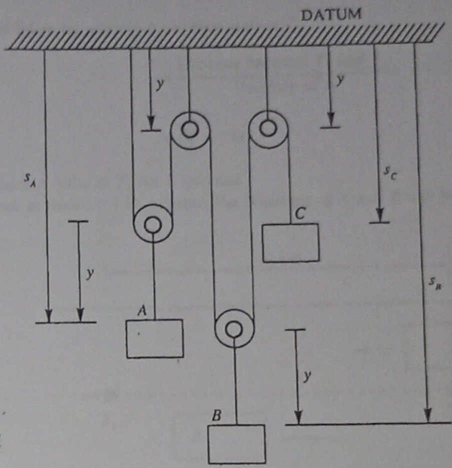


Fig. 2.14

In this case, the total length of the cord is given by

$$L = (s_A - y) + \pi r + (s_A - 2y) + \pi r + (s_B - 2y) + \pi r + (s_B - 2y) + \pi r + s_C - y$$

$$= \text{Constant } (\because \text{ the cord is of constant length})$$

Hence, we may write

$$2s_A + 2s_B + s_C = \text{constant} \tag{2}$$

The system shown in Fig. 2.14 is said to possess two degrees of freedom since two of the coordinates may be chosen arbitrarily.

Differentiating (2) with respect to t,

$$2 v_A + 2 v_B + v_C = 0 \tag{3}$$

Differentiating (3) with respect to t,

$$2 a_A + 2 a_B + a_C = 0 \tag{4}$$

Example 2.11 Car A travels at a speed of 25 m/s and car B travels at a speed of 20 m/s in the directions shown in Fig. 2.15. Determine (a) the velocity of car A relative to car B and (b) the velocity of car B relative to car A.

(a) Velocity of car A relative to car B ($v_{A/B}$)

We have $\qquad v_A = v_B + (v_{A/B})$

$$\therefore \quad v_{A/B} = v_A - v_B$$

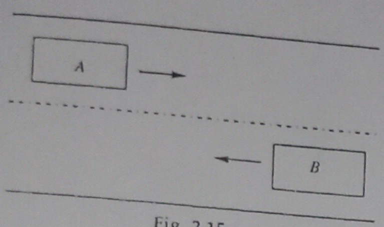


Fig. 2.15

With the sign convention [→] as (+)ve,

$$v_A = 25 \text{ m/s}$$

$$v_B = -20 \text{ m/s}$$

Hence $\quad v_{A/B} = 25 - (-20) = 45 \text{ m/s } (\rightarrow)$ (Ans.)

(b) Velocity of car B relative to car A ($v_{B/A}$)

We have, $\quad v_B = v_A + (v_{B/A})$

$\therefore \quad v_{B/A} = v_B - v_A$

$$= -20 - 25 = -45 \text{ m/s}$$

Hence $\quad v_{B/A} = 45 \text{ m/s } (\leftarrow)$ (Ans.)

Example 2.12 Two vehicles travel between two stations 50 km apart. Both vehicles start at the same time from the same station. The first vehicle travels at 50 km/h while the second vehicle travels at 30 km/h. If the first vehicle halts in the second station for 5 minutes and then returns with the same speed, determine where the two vehicles will meet.

Let the two stations be designated as T_1 and T_2 and the vehicles be A and B as shown in Fig. 2.16.

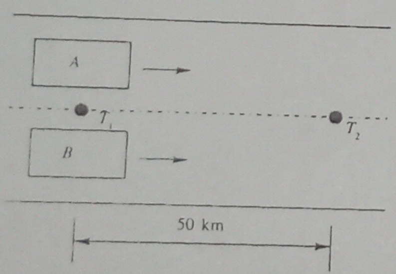


Fig. 2.16

Time taken by vehicle A to reach station T_2

$$= \frac{\text{Distance between } T_1 \text{ and } T_2}{\text{Velocity of } A}$$

$$= \frac{50}{50} = 1 \text{ hr}$$

Vehicle A halts at T_2 for 5 minutes.

Hence at time $t = 1$ hr + 5 mts, the positions of A and B will be as shown in Fig. 2.17.

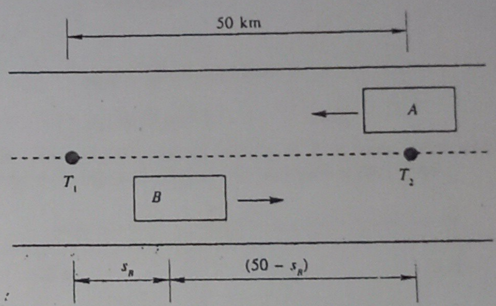


Fig. 2.17

$$s_B = v_B\, t$$

$$v_B = 30 \text{ km/h}$$

$$t = \left(1 + \frac{5}{60}\right) \text{hr}$$

$$= 1.083 \text{ hr}$$

$\therefore \quad s_B = 32.49$ km

$$50 - s_B = 17.51 \text{ km}$$

Let the vehicles meet at x km from T_2 as shown in Fig. 2.18.

We shall write

$$\frac{17.51 - x}{v_B} = \frac{x}{v_A}$$

$$\Rightarrow \frac{17.51 - x}{30} = \frac{x}{50}$$

$\therefore \quad x = 10.94$ km (Ans.)

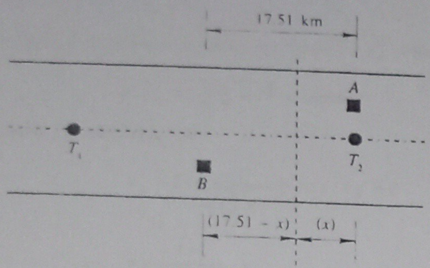


Fig. 2.18

The two vehicles will meet at a distance of 10.94 km from T_2 or at 39.06 km from T_1.

Example 2.13 If the velocity of block A shown in Fig. 2.19 moving up is increasing at the rate of 0.50 m/s each second, determine the acceleration of B.

Let the displacement of block B be s_B and that of A be s_A as shown in Fig. 2.20. The total length of the cable may be written as $2\,s_B + s_A$ which is constant.

i.e. $$2\,s_B + s_A = \text{Constant} \qquad (1)$$

Differentiating (1) with respect to t,

$$2\,v_B + v_A = 0 \qquad (2)$$

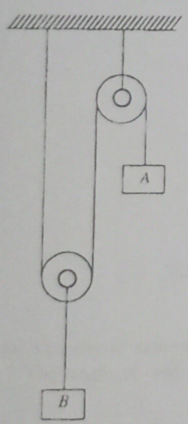


Fig. 2.19

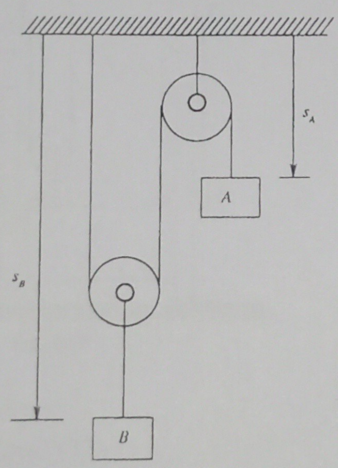


Fig. 2.20

Differentiating (2) with respect to t,

$$2\,a_B + a_A = 0$$

$$\therefore \quad a_B = -\frac{a_A}{2}$$

Given that the velocity of block A is increasing at the rate of 0.5 m/s each econd.

i.e. $$a_A = 0.5 \text{ m/s/s}$$
$$= 0.5 \text{ m/s}^2$$

Hence

$$a_B = -\frac{0.5}{2} = -0.25 \text{ m/s}^2$$

$$\therefore \quad a_B = 0.25 \text{ m/s}^2 \downarrow \qquad \text{(Ans.)}$$

xample 2.14 Determine the upward velocity of A in terms of the downward elocity of B shown in Fig. 2.21. Neglect the diameters of the pulleys.

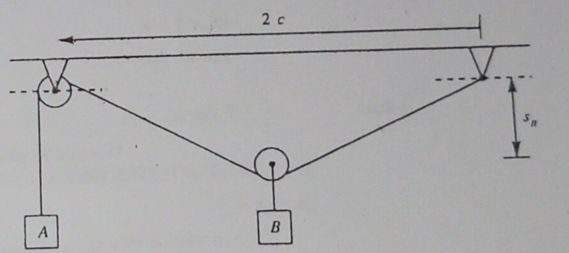


Fig. 2.21

Let the displacement of block A be s_A as shown in Fig. 2.22.

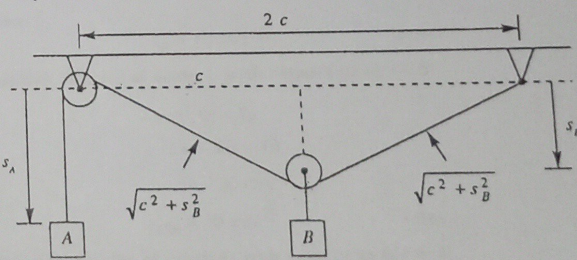


Fig. 2.22

The total length of the cable is given by $s_A + 2\sqrt{c^2 + s_B^2}$ which is constant.

$$\text{i.e., } s_A + 2\sqrt{s_B^2 + c^2} = \text{constant} \qquad (1)$$

Differentiating (1) with respect to t,

$$v_A + 2 \cdot \frac{1}{2\sqrt{s_B^2 + c^2}} \cdot 2\, s_B \cdot v_B = 0$$

$$\Rightarrow v_A = \frac{-2\, s_B}{\sqrt{s_B^2 + c^2}} \cdot v_B$$

$$\therefore \quad v_A = \frac{2\, s_B}{\sqrt{s_B^2 + c^2}}\, v_B \uparrow \qquad \text{(Ans.)}$$

Example 2.15 The block B shown in Fig. 2.23 moves downward with a constant velocity of 10 m/s. Determine (a) the velocity of the cable C, (b) the velocity of the block D, (c) the relative velocity of the cable C with respect to the block B and (d) the relative velocity of the block D with respect to the block B.

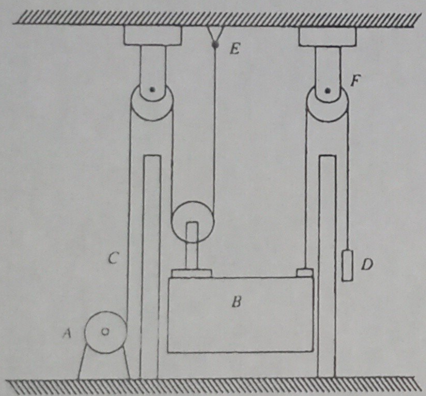


Fig. 2.23

(a) Velocity of cable C

The length of cable ACBE, (Fig. 2.24)

$$l_1 = s_C + 2\, s_B \qquad (1)$$

Differentiating (1) with respect to t,

$$0 = v_C + 2\, v_B$$

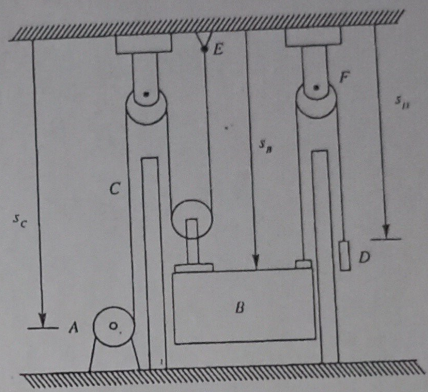


Fig. 2.24

$$v_C = -2\, v_B$$
$$= -2 \times 10$$
$$= -20 \text{ m/s}$$

Hence

$$v_C = 20 \text{ m/s} \uparrow \qquad \text{(Ans.)}$$

) Velocity of block D

The length of cable BFD, (Fig. 2.24)

$$l_2 = s_B + s_D \qquad (2)$$

Differentiating (2) with respect to t,

$$0 = v_B + v_D$$
$$v_D = -v_B = -10 \text{ m/s}$$

Hence,

$$v_D = 10 \text{ m/s} \uparrow \qquad \text{(Ans.)}$$

) Relative velocity of cable C with respect to block B

$$v_{C/B} = v_C - v_B$$
$$= -20 - 10$$
$$= -30 \text{ m/s}$$
$$v_{C/B} = 30 \text{ m/s} \uparrow \qquad \text{(Ans.)}$$

l) Relative velocity of block D with respect to block B

$$v_{D/B} = v_D - v_B$$