ALTERNATING CIRCUITS

1.0 Topics to Cover
- Alternating Current
- rms
- mean
- form factor
- peak factor
- phasor diagram.

1.1 ALTERNATING CURRENT

Alternating Current is defined as the current that varies periodically with time. It can be defined as an electric current that reverses its direction many times a second at regular intervals, typically used in power supplies.

Alternating current is a type of current in which the direction of the flow of electrons switches back and forth at regular intervals or cycles. Current flowing in power lines and normal household electricity that comes from a wall outlet is alternating current.

An alternating current let's say starts from zero goes to maximum in one direction and comes back to zero then goes to a maximum (amplitude) in the opposite direction.

1.1.1 INSTANTANEOUS CURRENT

The instantaneous current (I) (the current at any instance in the motion) is expressed as

$i = I_o \sin\theta$

$\theta = \omega t$

$i = I_o \sin\omega t$

$\omega = 2\pi f$

$i = I_o \sin 2\pi f t$

I_o Is the maximum current or peak current or amplitude current.

Similarly, for voltages

$V = V_o \sin\theta$

$V = V_o \sin\omega t$

$V = V_o \sin 2\pi f t$

The maximum frequency of an AC is 60Hz. The standard range of home Alternating currents is from 50Hz to 60Hz

1.2 RMS VALUE

The r.m.s. value of an alternating current is defined as that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

The root mean square value of an alternating current is defined as the value of the alternating current that has the same heating or lightning effect as that of a direct current.

The root mean square value is also called the **DC equivalent**.

It is also known as the **effective** or **virtual value** of an alternating current, the former term being used more extensively.

The rms value of symmetrical sinusoidal alternating current can be calculated using any of the following two methods: midordinate method and analytical method. However for symmetrical but non-sinusoidal waves, the mid-ordinate method is found to be more convenient to use

The rms value of an alternating current is of considerable importance in practice, because the ammeters and voltmeters record the rms value of alternating current and voltage respectively. In electrical engineering work, unless indicated otherwise, the values of the given current and voltage are always the rms values

The mean of the squares of the instantaneous values of current over one complete cycle is

$$ms=\int_0^{2\pi}\frac{i^2\,d\theta}{(2\pi-0)}$$

The square root of this value is:

$$rms=\sqrt{\int_0^{2\pi}\frac{i^2\,d\theta}{(2\pi-0)}}$$

Where, $i=I_o\sin\omega t$

$$I_{rms}=\sqrt{\frac{I_o^{\ 2}}{2\pi}\int_0^{2\pi}\sin^2\theta\,d\theta}$$

$$\sin^2\theta=\frac{1-\cos 2\theta}{2}$$

$$I_{rms}=\sqrt{\frac{I_o^{\ 2}}{4\pi}\int_0^{2\pi}(1-\cos 2\theta)\,d\theta}$$

$$I_{rms}=\sqrt{\frac{I_o^{\ 2}}{2}}=\frac{I_o}{\sqrt{2}}=0.707\,I_o$$

1.2.2 AVERAGE HEATING EFFECT PRODUCED IN ONE CYCLE

$$P=I^2R=\left(\frac{I_o}{\sqrt{2}}\right)^2R=\frac{1}{2}I_o^{\ 2}R$$

1.2.3 RMS VALUE OF A COMPLEX WAVE

The rms value of a complex current wave is equal to the square root of the sum of the squares of the rms values of its individual components.

For example, suppose a current having the equation $i=12\sin\omega t+6\sin(3\omega t-\pi/6)+4\sin(5\omega t+\pi/3)$ flows through a resistor of R ohms. The rms value of the complex wave is

$$I_{rms}=\sqrt{(12/\sqrt{2})^2+(6/\sqrt{2})^2+(4/\sqrt{2})^2}=9.74\,A$$

1.3 AVERAGE VALUE OF ALTERNATING CURRENT

The average value of an alternating current is defined as that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time. RMS is the same that transfers heat, while average value is transferring same charge.

In the case of a symmetrical alternating current (i.e. one whose two half-cycles are exactly similar, whether sinusoidal or nonsinusoidal), the average value over a complete cycle is zero. Hence, in their case, the average value is obtained by adding or integrating the instantaneous values of current over one half-cycle only. But in the case of an unsymmetrical alternating current (like half-wave rectified current) the average value must always be taken over the whole cycle.

Average value can be determined using either mid-ordinate method or analytical method (More research on this later…)

$$I_{av}=\int_0^{\pi}\frac{i\,d\theta}{\pi-0}=\frac{I_o}{\pi}\int_0^{\pi}\sin\theta\,d\theta$$

On solving

$$I_{av}=\frac{2I_o}{\pi}=\frac{I_o}{\frac{\pi}{2}}$$

$$I_{av}=0.637\,I_o$$

Note that rms value is always greater than average value except in the case of a rectangular wave when both are equal

1.4 FORM FACTOR

The form factor K_f for sinusoidal alternating current is defined as the ratio of rms value to average value. That is

$$K_f=\frac{rms}{average}=\frac{0.707\,I_o}{0.637\,I_o}=1.1$$

Similarly, for voltage

$$K_f=\frac{rms}{average}=\frac{0.707\,V_o}{0.637\,V_o}=1.1$$

The knowledge of form factor will enable the rms value to be found from the arithmetic mean value and vice-versa

1.5 AMPLITUDE FACTOR

This is also called crest factor or peak factor.

For sinusoidal current (AC), the crest or peak or amplitude factor is defined as the ratio of maximum value to the rms value of the current. That is

$$K_a=\frac{maximum}{rms}=\frac{I_o}{I_o/\sqrt{2}}=\sqrt{2}=1.414$$

Knowledge of this factor is of importance in dielectric insulation testing, because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage. The knowledge is also necessary when measuring iron losses, because the iron loss depends on the value of maximum flux