STRESSES IN THIN CYLINDER

A thin cylinder in one that has the diameter at least 20 times the thickness.

$d > 20t$

TYPES OF STRESS IN A 3D BODY

σ_L- Longitudinal stress: Related with length

σ_h – Hoop / Circumferential stress: Related with Diameter

σ_r – radial stress

These stresses are perpendicular to each other.

In a thin cylinder, the radial stress is negligible

However for the others, the equations are:

$$\sigma_h = \frac{Pd}{2t}$$

$$\sigma_L = \frac{Pd}{4t}$$

$$\sigma_L = \frac{1}{2}\sigma_h$$

P – Pressure

d – Diameter

t – Thickness

STRAIN

Types of strain

Usually when there is a stress, it is accompanied with a strain.

Uni-axial - $\varepsilon = \frac{\Delta L}{L}$

Bi-axial - $\varepsilon_x = \frac{\sigma_x - \nu\sigma_y}{E}$

Now expressing strain in terms of

$$\varepsilon_L = \frac{\sigma_L - \nu \sigma_h}{E}$$

$$\varepsilon_h = \frac{\sigma_h - \nu \sigma_L}{E}$$

But recall that…

$$\sigma_h = \frac{Pd}{2t}$$

$$\sigma_L = \frac{Pd}{4t}$$

$$\varepsilon_L = \frac{\sigma_L - \nu \sigma_h}{E}$$

$$\varepsilon_L = \frac{\frac{Pd}{4t} - \nu \frac{Pd}{2t}}{E}$$

$$\varepsilon_L = \frac{\frac{Pd}{4t} - 2\nu \frac{Pd}{4t}}{E}$$

$$\varepsilon_L = \frac{Pd}{4tE}(1 - 2\nu)$$

Also,

$$\varepsilon_L = \frac{\Delta L}{L}$$

$$\varepsilon_h = \frac{Pd}{4tE}(2 - \nu)$$

Also,

$$\varepsilon_h = \frac{\Delta D}{D}$$

$$\frac{\Delta D}{D} = \frac{Pd}{4tE}(2 - \nu)$$

$$\Delta D = \frac{Pd}{4tE}(2 - \nu)d$$

$$\Delta D = \frac{P d^2}{4tE}(2 - \nu)$$

Also,

$$\frac{\Delta L}{L}=\frac{Pd}{4tE}(1-2v)$$

$$\Delta L=\frac{Pd}{4tE}(1-2v)L$$

$$\frac{\Delta V}{V}=2\varepsilon_h+\varepsilon_L$$

$$\frac{\Delta V}{V}=2\left[\frac{Pd}{4tE}(1-2v)\right]+\left[\frac{Pd}{4tE}(2-v)\right]$$

$$\Delta V=\frac{Pd}{4tE}(5-4V)V$$

$$V_i=\frac{\pi d^2}{4}\times L_i$$

$$V_f=\frac{\pi d_f^2}{4}\times L_f$$

Sample questions.

1. Calculate ΔD, ΔL and ΔV of a thin cylinder of diameter $100\,cm$, thickness is $1\,cm$, length(L) is 5m and pressure (P) is 3N/mm^2. Young's modulus of $2\times10^5\,N/mm^2$ and poisson ratio is 0.3

A force is anything that causes motion and is defined by its line of action

For equilibrium,

$$\sum F_x=0$$

$$\sum F_y=0$$

$$\sum M=0$$

$$M=Fd\sin\theta$$

Moment of a force can be defined as the product of the force and the perpendicular distance

After getting the reactions, move on to showing the shear force diagram.It usually starts at zero and ends at zero to show that it is in equilibrium

$$Engineering\ strain = \frac{+\Delta l}{l_0}$$