STRESS AND STRAIN

1.0 STRESS

Stress is a quantity that describes the magnitude of force that causes deformation. It is also said to be the force per unit area.

$$\sigma = \frac{F}{A}$$

Understanding stress is essential for designing structures, analysing materials, and ensuring safety in engineering applications.

1.0.1 BASIC TYPES OF STRESS

The two basic types of stresses are the **tensile stress** and **compressive stress**. These two are collectively called **Normal stress** because they act perpendicularly to the surface applied.

However, there are stresses that act parallel to the surface. This is called Tangential stress or Shear stress

1.0.1.1 TENSILE STRESS

This stress occurs when a material is subjected to a force that tends to stretch or elongate it.

CHARACTERISTICS

i. Tensile stress is positive in calculation when the material is being pulled out

ii. It leads to lengthening of the material along the applied force

iii. Commonly encountered in scenarios like stretching a rubber band or pulling a rope.

1.0.1.2 COMPRESSIVE STRESS

Compressive stress occurs when a material is subjected to a force that tends to compress or shorten it.

CHARACTERISTICS

i. Compressive stress is negative when the material is being pushed together.
ii. It leads to shortening of the material along the applied force
iii. Commonly encountered in scenarios like supporting columns or pressing down on a spring.

1.0.2 CLASSIFICATION OF STRESSES BASED ON DIRECTION

Forces could act either perpendicularly or parallel to the object thereby creating different kinds of stresses.

1.0.2.1 NORMAL STRESSES

Normal stresses include Longitudinal stress and bulk stress. These stresses act perpendicularly (normal or at 90 degrees) to the surface of interaction.
The general formula for all normal stresses is given as

$$\sigma_N = \frac{F}{A}$$

1.0.2.1.1 LONGITUDINAL STRESS

This is the stress associated with a change in length.

$$\sigma_l = \frac{F}{A}$$

1.0.2.1.2 BULK STRESS

This is the stress associated with the change in volume of the material

$$\sigma_B = \Delta P = \frac{F}{A}$$

1.0.2.2 LATERAL STRESSES

These are all the stresses acting parallel to the surface of the body. This type of stress causes sliding or shearing effects. These include shear stress (AKA tangential stress).

1.0.2.2.1 SHEAR STRESS

This stress occurs when forces act tangentially or parallel to a surface, causing sliding or twisting effects.

For example, when you slide your hand across the top of a desk, the resistance you feel is a shear stress between your hand and the desk.

These occur when we shear a material (like cutting or bending it). It also occurs when we twist a cylindrical object e.g. wringing out a wet cloth.

1.1 STRAIN

This describes as the amount of formation produced by a certain force or stress. It is mathematically defined as the ratio of its new size to its old size. It is the percentage change in the material’s length, area, or volume.

1.1.1 TYPES OF STRAIN

There are many types of strains and more will be seen as we study more complex objects. The most common types of strains are

1.1.1.1 LONGITUDINAL STRAIN

This is the strain associated with the change in length of the material. In a 2d object, it is the horizontal stress.

$$\varepsilon_l = \varepsilon_x = \frac{\Delta l}{l_o}$$

1.1.1.2 LATERAL STRAIN

This refers to the change in width or height of a material due to applied stress. In an object it is the vertical stress

$$\varepsilon_W = \varepsilon_y = \frac{\Delta W}{W_o}$$

1.1.1.3 SHEAR STRAIN

This occurs when adjacent layers of a material slide past each other

$$\varepsilon_\gamma = \frac{\Delta x}{h}$$

Δx is the displacement parallel to the applied force

h is the thickness of the material

1.1.1.4 BULK STRAIN

This accounts for the change in volume of a material

$$\varepsilon_B = \frac{\Delta V}{V_o}$$

1.1.1.5 VOLUMETRIC STRAIN

This is related to the sum of longitudinal and lateral stains

$$\varepsilon_V = \varepsilon_y + \varepsilon_x$$

NOTE:

The ability of load-bearing object to resist applied load is called Strength, and the actual strength of structures must exceed the required strength to support forces.

The ratio of the actual strength to the required strength is called the **factor of safety**, n,

$$n = \frac{actual\,strength}{required\,strength}$$

1.2 HOOKE’S LAW

Hooke’s Law, also known as the law of elasticity, was discovered by the English scientist Robert Hooke in 1660. It describes the behavior of elastic materials when subjected to deformation.

This law states that **“The force applied to an elastic material is directly proportional to the extension produced by the material”**.

$F \propto e$

$F = ke$

k in the equation above is the constant of proportionality. It is called the **Force constant** or the **Stiffness Constant** or **Elastic constant** in the material

It can also stated as the **“The stress on a body is directly proportional to the strain produced”**.

$stress \propto strain$

$\sigma \propto \varepsilon$

Usually, young’s modulus is applied to just longitudinal stress (σ_x) and longitudinal strain (σ_x) though not all the time.

1.3 MODULI

Depending of the type of stress and strain, different kinds of Moduli(plural of modulus) can be produced.

1.3.1 YOUNG’S MODULUS

$\sigma = E\varepsilon$

$E = \frac{\sigma}{\varepsilon}$

$E = \frac{F}{A} \div \frac{\Delta l}{l_o}$

$E = \frac{F l_o}{A \Delta l}$

When a material behaves elastically and exhibits a linear relationship between stress and strain, it is called a **linearly elastic material**.

Within the elastic limit, stress (σ) is directly proportional to the strain (ε)

(E) is the Young's modulus of elasticity, which is a material property.

Example: Imagine stretching a spring. As long as the deformation remains within the elastic limit, the force applied is directly proportional to the resulting elongation.

1.3.2 BULK MODULUS

Applying Hooke's law,

$$Bulk\,stress \propto Bulk\,strain$$

$$\Delta P \propto \frac{\Delta V}{V_o}$$

$$\Delta P = -B\frac{\Delta V}{V_o}$$

B in the equation above is the constant of proportionality known as Bulk's Modulus.

It has a unit Pascal or Newton per square meter.

The equation above can be modified as

$$B = \frac{-V_o \Delta P}{\Delta V}$$

We include the minus sign because an increase in pressure corresponds to a decrease in volume.

The reciprocal of the Bulk's modulus is known as compressibility and is denoted as C.

$$C = \frac{1}{B}$$

$$C = \frac{-\Delta V}{V_o \Delta P}$$

1.3.3 MODULUS OF RIGIDITY

The ratio $\frac{(shear\ stress)}{(shear\ strain)}$ is called the modulus of rigidity.

$G = \frac{\tau}{\varphi}$, $(unit = N/mm^2)$

1.4 PRINCIPLE OF SUPERPOSITION

Engineering materials are usually subjected to several or multiple load actions, and the system of forces causes deformation that must be determined, which is the basis of principle of superposition.

The principle states that, where several loads are acting together on an elastic material, the resultant strain (or effective strain), is the sum of the strains cause by each load acting separately on the body.

Ie, $\varepsilon = \sum_{i=1}^{n} \varepsilon = \varepsilon_1 + \varepsilon_2 + \ldots + \varepsilon_n$

The Principle of Superposition applies to linear systems and states that the net effect of multiple forces acting on a body is the sum of the individual effects of each force.

MATHEMATICAL REPRESENTATION

For a system with multiple forces, the total force $F_T$is the vector sum of all individual forces: $F_T = F_1 + F_2 + \ldots + F_n$. Similarly, the total displacement x_T is the sum of the individual displacements caused by each force:$x_T = x_1 + x_2 + \ldots + x_n$

Example: Consider a beam subjected to several point loads. The deflection at any point due to these loads can be determined by adding up the deflections caused by each load individually.

1.5 POISSON'S RATIO

According to hooke's law for elastic material, stress is directly proportional to strain, thereby suggesting an increase in length for tensile stress.

But contrarily, for a possible increase in length, and there must be a compensation for equilibrium of mass, according to conservation of matter.

Therefore, to compensate for an increase in length in the longitudinal direction (length), there must be a reduction in the lateral direction (width) complementarily to balance the mass equilibrium. The magnitude of the lateral strain is described by the quantity known as Poisson's ratio (γ)

Poisson's ratio $\gamma = \frac{-(lateral\,strain)}{(longitudinal\,strain)}$

The negative sign indicates an equilibrium action, an increase in longitudinal length will require a decrease in the lateral dimension

$-\gamma \times \varepsilon_x = \varepsilon_y$

From youngs modulus,

$E = \frac{\sigma_x}{\varepsilon_x}$

$\varepsilon_x = \frac{\sigma_x}{E}$

Thus, lateral strain, $\varepsilon_y = -\gamma \frac{(\sigma_x)}{E}$ (for uni-axial stress)

1.6 UNIAXIAL AND BI-AXIAL STRESSES

A body could experience normal stress in any direction and in multiple directions. The following definitions are necessary to be known

1.6.1 UNIAXIAL STRESS

This refers to a specific type of stress in which a material experiences forces acting along only one direction. Uniaxial stress occurs when an object or material is subjected to an external force that acts along a single axis (direction).
It is a simplified stress state where the material experiences either tension (stretching) or compression (shortening) along that single axis.
For example, either stress in the longitudinal direction, σ_x , or stress in the lateral direction, σ_y .

1.6.1.1 POISSON'S RATIO FOR UNIAXIAL STRESSES

Lateral strain, $\varepsilon_y = -\gamma \frac{(\sigma_x)}{E}$

Longitudinal strain, $\varepsilon_x = -\gamma \frac{\sigma_y}{E}$

1.6.2 BI-AXIAL STRESS

This occurs when a material sample is simultaneously stressed along two perpendicular axes. Unline uniaxial stress, which acts along a single direction, biaxial stress considerss both in-plane directions. σ_x and σ_y . These two are both normal forces either tensile or compressive stresses.

1.6.1.1 POISSON'S RATIO FOR UNIAXIAL STRESSES

x-direction, $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\gamma \sigma_y}{E} = \frac{1}{E}(\sigma_x - \gamma \sigma_y)$

y-direction, $\varepsilon_y = \frac{\sigma_y}{E} - \frac{\gamma \sigma_x}{E} = \frac{1}{E}(\sigma_y - \gamma \sigma_x)$