Main Title = 40

Sub title = 32

ELECTRIC FIELD AND ELECTROSTATICS

Electro – Electricity

Statics – Stationary

Therefore, electrostatics is the study (and properties) of stationary -electrical- charges (charges at rest) (positive and negative).

Conventionally, like charges (like positive and positive or negative and negative) repel (i.e. they move away from each other) while unlike charges (like positive and negative or negative and positive) attract (i.e. they come together). Charges of the same sign repel each other while charges of opposite signs attract each other.

It should be noted that all bodies or objects contain charges. It was **Benjamin Franklin (1706 – 1790)** that first referred to electric charges as positive and negative. Accord to Franklin, a body can be positively charged or negatively charged.

A body is said to have a positive charge if it contains excess positive charges (more positive charges than negative charges) or deficient negative charges. Likewise, a body is said to be negatively charged if it has excess negative charges and deficient positive charges. However, a body is said to be neutral if has an equal number of positive and negative charges.

DISTRIBUTION OF CHARGES ON A BODY

According to the ice-pail experiment, the net charge on a body is always found outside the body.

Any excess charge on a conductor resides on the surface of the conductor and not in the conductor; remember excess charges.
For a uniform body like a spherical body, the charges are distributed uniformly outside the body and are in are orderly manner.
For a pear shaped body or a pointed body, the charges are concentrated at the pointed end (i.e. the surface charge density is maximum at the pointed end)
The concept of charge distribution on bodies is used in lightning conductors.

METHODS OF CHARGING BODIES

Charging by contact: This involves putting a charged body in contact with a neutral body. In this process, a neutral body is placed in contact with a charged body. During contact, charges will flow from the charged body to the uncharged (the neutral body) until the two bodies have equal magnitude of the same charge. After the separation of the two bodies it will be observed that body bodies have equal magnitude and have the same type of charge (either positive or negative).

The two bodies should always be placed on insulating stands in order to prevent the flow of charge from the body to the earth or from the earth to the object.

Charging by friction: This involves rubbing two bodies against each other in order to produce charges

Negative charges can be produced by rubbing ebonite rod with fur. The fur becomes positively charged and the ebonite (hard rubber) rod is then negatively charged.

Positive charges can be produced by rubbing glass rod with silk. The glass acquires a net positive charge.

In charging by friction, the net charge is zero (that is the amount of charge lost is equal to the amount of charge gained)

Charging by induction: In charging by induction there is no contact between the some steps are taken which include
A body called the induced body (or induced charge) is being charged by another object called the inducing body (or inducing charge)

Step 1: To make a body negatively charged, a positively charged rod is brought near the body. Since unlike charges attract and like charges repel, the positive charges (in the body) are repelled from the rod while the negative charges are attracted to the rod. However, the rod must not touch the body

Step 2: Earthling is carried out by touching the body making negative charges to flow into the body to neutralize the positive charges

Step 3: The finger is then removed

Step 4: The rod is also removed thereby leaving the body negatively charged.

To make the body positively charged, a negative rod is used.
It can be observed that after the induction process, both the induced charge and the inducing charge have equal magnitude but opposite charges.
This process is also used in charging the gold leaf (or gold foil) electroscope

INSTRUMENTS USED AND STUDIED IN ELECTROSTATICS

LIGHTNING CONDUCTOR

These are conductors mainly made of copper which are used (a lot in houses and buildings) to transfer excess charges from the atmosphere (during lightning or thunder storms) to the earth’s crust.
These conductors help to prevent lightning damages on electric materials.
A lightning conductor consists of a copper strip with sharply pointed end projecting above the building to be protected while the lower end is connected to a metal plate buried below the earth.

If a positively charged cloud passes over the building, negative electric charges are induced at the tip of the lightning conductor, thereby forming a dipole with the cloud. The high electric field thus set up between the cloud and the tip of the conductor results in the ionization of the atoms of the air, with negative ions drifting towards the cloud and thereby neutralizing the positive charges.
Lightning is caused by a storm cloud which is highly charged as a result of the rubbing action between the cloud and the air (charging by friction). The sudden flow of charges between the cloud and the earth results in a lightning flash.

GOLD LEAF ELECTROSCOPE

This is a device used for detecting the nature of charges (whether positive or negative charges) on a body. It comprises a metal cap, a thin conducting rod and a gold leaf all enclosed in a glass window. Before a gold leaf electroscope can be used it must first be charged.
The gold foil is charged by the process of induction. A charged body is placed close to the cap of the electroscope and then the cap is earthed. The electroscope can also be charged by contact.
When a body is placed on a gold leaf electroscope and the gold leaf diverges, it implies that the body and the electroscope have identical charges. However, once the body is placed on the gold leaf and the gold leaf collapses, it indicates that the body and opposite charges.
However, if the body placed on the electroscope is neutral, the gold leaf will also collapse.

Charge of Electroscope	Charge of body	Effect on Gold leaf
$+ve$	$+ve$	$-Divergence$
$+ve$	$-ve$	$-Collapse$
$-ve$	$+ve$	$-Collapse$
$-ve$	$-ve$	$-Divergence$
$+ve$	$Neutral$	$-Collapse$
$-ve$	$Neutral$	$-Collapse$

USES OF THE GOLD LEAF ELECTROSCOPE

1. They are used to detect the nature of charges
2. They are used for comparing the magnitude of charges
3. They can also be employed as volt meters

Proof plane

This device is used for transferring large amount of charges from one body to another

Electrophorus

This device is used for securing large amount of similar charges by induction.

Faraday's Net

This device is used for comparing the magnitude of charges between the order and inner part of a hollow conductor

Capacitor

This is a passive device used for storing charges

COULOMB'S LAW OF ELECTROSTATICS

The French Physicist Charles Coulomb(1736 – 1806) studied the magnitude of this force using a torsion balance. The torsion balance is used to measure very small forces.

This law states that:

In free space, the electrostatic force (of attraction or repulsion) between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of their distance apart.

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{K q_1 q_2}{r^2}$$

But

$$K = \frac{1}{4\pi\varepsilon_o}$$

$$\frac{1}{4\pi\varepsilon_o} = 9\times10^9\,N\,m^2\,C^{-2}$$

Also,

ε_o Is called the permittivity of free space; for charges located in vacuum,

$$\varepsilon_o = 8.85\times10^{-12}C^2N^{-1}m^{-2}$$

$$F = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1q_2}{r^2}$$

In vector notation,

$$F_{12} = \frac{K\,q_1q_2}{|r_{12}|^2}\cdot\widehat{r_{12}}$$

But

$$\hat{r}_{12} = \frac{r_1r_2}{|r_{12}|} = \frac{r_1 - r_2}{|r_1 - r_2|}$$

The force usually has signs (positive or negative). If both charges are of the same sign, there will be a repulsive force. Therefore the force will be -ve

The total force in vector notation is the sum of all the forces

$$F_0 = F_{01} \pm F_{02} \pm F_{03}$$

Note the following:

1. Every charge has vector electric field around it which causes attraction/repulsion
2. The direction of the field (electric) around any charge object is the direction in which the positive charge will move if it is placed close to the object
3. As seen above, electric fields obey the inverse square law

ELECTRIC FIELDS

Electric fields are defined as regions where electric forces can be experienced.

The electric lines of forces can be used to show the direction of the electric field.

A line of force is an imaginary line representing a field of force such as an electric or magnetic field, such that the tangent at any point is the direction of the field vector at that point.

For a positively charged body, the (arrows of the) lines point outwards while for a negatively charged body, they point inwards.

If the lines are close together and parallel, that indicates a strong field.

The force that an object experiences in an electric field is dependent on the charge (q) of the electric field.

Note the following:

1. The electric lines of force are drawn such that the magnitude of the electric field is proportional to the number of lines crossing a unit area perpendicular to the lines
2. The tangent of the lines of force at every point gives the direction of the field at that point
3. The lines of force are continuous and they start on positive charges and end only on negative charges
4. Lines of force do not touch or intersect

The force experienced by a charge in an electric field is directly proportional to the magnitude of the charge

$F \propto q$

$F = Eq$

E in the equation above is known as the **electric field intensity** or **electric field strength**.

It can be defined as the ratio of the Force experienced in the field to the electric charge of the field.

$$E = \frac{F}{q}$$

Similarly in vector notation

$$\vec{E} = \frac{\vec{F}}{q}$$

$q_1 = q_2 = q$

$$F = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q \times q}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q^2}{r^2}$$

But

$F = Eq$

Therefore,

$$Eq = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q^2}{r^2}$$

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$$

ELECTRIC FIELD OF CONTINUOUS CHARGE DISTRIBUTION

To evaluate the electric field by continuous charge distribution, we first divide the total charge distribution (q) into small elements, each of which contains a small charge (Δq). We can calculate the electric field with respect to one of the these small elements

$$\Delta E = \frac{k\,\Delta q}{r^2}\hat{r}$$

$$E = k \lim_{\Delta q \to 0} \sum_i \frac{\Delta q}{{r_1}^2} = k \int \frac{dq}{r^2}\hat{r}$$

If a charge Q is uniformly distributed along a line of length l, the linear charge density $\lambda$is defined by

$$\lambda = \frac{Q}{l}$$

If we consider a small section of that rod of length dx, the amount of charge there is dq. It shows that charge depends on the length of the material somehow :-)

$$dq = \lambda\, dx$$

$E = \frac{kq}{r^2}$For a small charge

$$dE = k\,\frac{dq}{x^2}$$

The unit will be Coulombs per metres

Similarly, the surface charge density is

$$\sigma = \frac{Q}{A}$$

Also, if the charge Q is uniformly distributed throughout a volume, V the volume charge density ρ is defined by

$$\rho = \frac{Q}{V}$$

MOTION OF A CHARGED PARTICLE IN A UNIFORM ELECTRIC FIELD

When a particle of charge q and mass m is placed in an electric field, the charge experiences a force. This force can cause the body to accelerate according to Newton's second law of motion

$$F = qE = ma$$

$$a = \frac{Eq}{m}$$

If E is constant in magnitude and direction, that is uniform, then the acceleration a is constant. If the particle is positively charged, then its acceleration is in the direction of the electric field. If the particle is negatively charged, then its acceleration is in the direction opposite the electric field.

Consider an electron – which is negatively charged – travelling with a speed of $v_e$entering a uniform electric field (at x = y = 0), the field being at right angle to the velocity.

$$F = qE = m_e a$$

So, looking at the motion of the electron, you'll see that the electric field is directly parallel to the y axis. As the electron enters the field, it will be moving horizontally since its motion is perpendicular to the electric field. It will therefore have no vertical component of the velocity. However, when it enters, it tends to move faster towards the positive side therefore accelerating vertically. It also doesn't have a horizontal acceleration.

The initial velocity

$$v = v_x + v_y$$

$$v_y = 0$$

$$v = v_x$$

$$a_x = 0$$

$$a_y = a$$

Taking a look at the specific movements

$$y = v_{ey} t + \frac{1}{2} a_y t^2$$

$$y = 0t + \frac{1}{2} \frac{qE}{m_e} t^2$$

$$y = \frac{1}{2} \frac{qE}{m_e} t^2$$

Considering the horizontal movements:

$$x = v_{ox} t + \frac{1}{2} a_x t^2$$

$$x = vt$$

$$t = \frac{x}{v}$$

Substituting that in the vertical equation, we get

$$y=\frac{1}{2}\frac{qE}{m_e v_e^{2}}x^2$$

From the above, we can see that the movement of an electron in a field forms a parabola. You'll notice that the equation is in the form:

$$y=kx^2$$

ELECTRIC FLUX

The electric flux is defined as the **product of the electric field intensity E and the A** perpendicular to the field. It is proportional to the number of electric field lines penetrating a surface. For an electric field that is uniform in both magnitude and direction, the electric flux $\Phi_E$is defined as

$$\Phi_E=EA\cos\theta$$

$E\cos\theta$ is the component of E along the perpendicular to the area

$$\Delta\Phi_E=E\Delta A\cos\theta=E\cdot\Delta A$$

The SI unit of Electric flux is newton metres squared per coulomb $N m^2 C^{-1}$

In more general situations, the electric field may very over a surface. If the surface is divided up into a large number of elements, each of area ΔA. If the element of area {%DELTA}A is crossed by an electric field E in the direction which makes an angle %theta with the normal to the area, then the electric flux crossing the area is given by

$$\Delta\Phi=E\Delta A\cos\theta=E\cdot\Delta A$$

$$\Phi_E=\sum\Delta\Phi_E=\oint E\cdot dA=\oint E\cos\theta\, dA$$

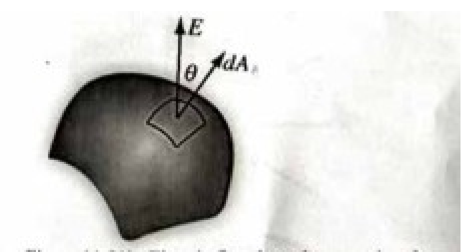

Figure 11.31b: Electric flux through a curved surface

GAUSS'S LAW

Karl Friedrich Gauss (1777 – 1855) developed a relation between electric charge and electric field. The relation is usually referred to as Gauss's law.

GAUSS'S LAW states that the net flux through any closed surface is

$\Phi_E = \oint E \cdot dA = \frac{Q_{in}}{\varepsilon_0}$

Q_{in} is the net (or total) charge inside the surface. The position and distribution of the charge is irrelevant. Any charge outside the surface must not be used in the calculation of electric field. The charge that is outside may affect the distribution of the electric field lines but it will not affect the total number of electric charges entering or leaving the surface.

GAUSSIAN SURFACE

A Gaussian surface is a closed surface in which the flux of an electric field is calculated. It is an arbitrary closed surface $A = \delta V$

Properties of Gaussian Surfaces

1. Must be closed: It has an inside and an outside e.g. A balloon, a box etc. If you look at a cup, it is not a closed surface
2. Its an imaginary surface. We are assuming it to be there
3. Symmetry is important: We match our symmetry with our Gaussian surface

A Gaussian surface is a closed surface around a charge and it is a 3d surface. Since 3d surfaces are difficult to draw we use 2d drawings

Consider a single isolated charge. The Gaussian surface for this point charge is an imaginary sphere of radius r centred on the charge. Since the imaginary sphere is symmetrical about the charge at its centre, we know that E must have the same magnitude at any point on the surface and that E points radially outward parallel to dA

Note that the surface area of a sphere of radius r is $4\pi r^2$ and the magnitude of E is the same at all points on the Gaussian spherical surface.

$\frac{Q}{\varepsilon} = \oint E \cdot dA = E\left(4\pi r^2\right)$

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$

The reverse is also the case:

$\oint E \cdot dA = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} dA = \frac{Q}{4\pi\varepsilon_0 r^2} 4\pi r^2 = \frac{Q}{\varepsilon_0}$

Note the following:

The net flux through any closed surface is independent of the shape of that surface

The net flux through any closed surface surrounding a point charge q is given by $\frac{q}{\varepsilon_0}$

The net electric flux through a closed surface that surrounds no charge is zero

APPLICATIONS OF GAUSS' LAW

1. We can apply Gauss' law in the determination of electric field if the charge distribution is known and if it has enough symmetry.
2. If on the other hand, we know the field, we can apply it in determining the charge distribution.

Note: Gauss' law is valid for any charge distribution and for any closed surface

To do the above mentioned applications, we have to:

choose a gaussian surface that is symmetry for the electric field to be considered as being constant or we must choose a surface so that the flux through the part of the surface is zero

3. Application of Gauss' law in thin spherical conductor
4. Electric field inside and outside a non-conducting sphere

ELECTRIC POTENTIAL

This can be defined as the work done in moving a unit charge from infinity to the point in consideration. It can therefore also be defined as the work per unit charge.

A test charge q, placed in an electric field E will experience a force. When the test charge is moved in the field by some external agent, the work done by the field on the charge is equal to the negative work done by the the external agent causing the displacement.

The work done by the external agent in moving the test charge q, from point A to point B along an arbitrary path in an electric field is therefore given as

$$W(A \to B)_{ag} = -\int_A^B F \cdot dl$$

or

$$W(A \to B)_{ag} = -q_o \int_A^B E \cdot dl$$

The equation is valid because the force is a conservative force. A force is said to be conservative if the work done by or against it in moving an object is independent of the object's path. This means that the work done by a conservative force depends only on the initial and final positions of an object.

Work done by the electric field: $W(A \to B)_E$ is the negative of the work done against the field (i.e. by an external constraint)

$$W(A \to B)_E = -W(A \to B)_{ag}$$

$$W(A \to B)_E = q_o \int_A^B E \cdot dl$$

We all know that…

$$Energy = Work\ Done$$

Change in potential energy ΔU is equal to the work done on the particle by the electric field through a potential difference $V_{BA}(V_A - V_A)$

$$W = qV$$

$$\Delta U = U_B - U_A = -W(A \to B)_E$$

$$\Delta U = -q_o \int_A^B E \cdot dl = q_o(V_B - V_A)$$

From the above, we can see that change in potential is change in potential per unit charge. But change in potential is work done

Therefore $V = \frac{W}{q}$

$$(V_B - V_A) = -\int_A^B E \cdot dl$$

For a uniform electric field we have:

$$(V_B - V_A) = -\int_A^B E \cdot dl = -E(l_B - l_A) = -Ed$$

The above formula only applies if the electric field is uniform.

$$d = l_B - l_A$$

The distance d, is parallel to the field lines

$$(V_B - V_A) = -\int_A^B E \cdot dl = -\int_A^B \frac{kQ}{r^2} \cdot dl = kQ\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

POTENTIAL DUE TO A POINT CHARGE

This can also be referred to as **absolute** potential. The absolute potential of a point is the work done against electrical forces in carrying a unit (positive) charge from infinity to that point.

The potential difference between two points A and B in a field created by a point charge depends only on the radial coordinates r_A and r_B of the points. It is a convention to choose the reference of electric potential to be zero at $r_A = \infty$.

$$V_B = \frac{kQ}{r_B}$$

In general, for any arbitrary point, the electric potential at r relative to infinity is given as

$$V = \frac{kQ}{r}$$

When Q is positive,V will also be positive

FOR A CHARGED SPHERE

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r}$$

You'll notice that the whole spherical conductor is at the same potential

Inside a sphere, no work is done because $E = 0$

Electric potential (V) can also be expressed as the product of the electric field intensity (E) and the distance (d)

$$V = Ed$$

$$E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$$

And

$$d = r$$

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2} \times r$$

$$V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r}$$

Also,

$$V = \int Edl$$

$$dV = Edl$$

Therefore,

$$E = \frac{dV}{dl}$$

For two or more cases, the principle of superposition can be used

$$V = V_1 + V_2 + \ldots + V_n$$

$$V = K\frac{q_1}{r_1} + K\frac{q_2}{r_2} + K\frac{q_n}{r_n}$$

$V = K \sum \frac{q_i}{r_i}$

ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTION

dV related to dq

EQUIPOTENTIAL SURFACES

This is also called and **equivalent surface**. An equipotential surface is a surface or volume over which the potential is constant. The surface of a conductor is an equipotential surface. The space inside a hollow charged conductor is also an equipotential volume.

An equivalent surface is an imaginary surface on which all points have the same potential. In an equivalent surface the work done in moving a charge from one point to another is zero.

For equipotential surfaces

$E = 0$, $dl = 0$ or $\cos\theta = 0$ where $\theta = 90$meaning that the electric field is perpendicular to the equipotential surface

Here, q is the magnitude of the charge

Here, V is the electric potential

$V = \frac{W}{q}$

From the above, it can also be defined as the work done per unit charge

$q = \frac{W}{V}$

The potential difference from a point (A) to another point (B) is the work done against electrical forces in carrying a unit (positive) charge from A to B.

$V_B - V_A = \frac{W}{q}$

$W = q(V_B - V_A)$

Since work done is the product of force and distance,

$W = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q^2}{r^2} \times r$

$W = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_1 q_2}{r}$

UNIFORM ELECTRIC FIELD BETWEEN TWO PLATES

The electric field between plates is the area or space where the plates' charges influences can be seen. For example, if a charged particle is placed near any charged plate, the plate exerts an electric force to attract or repel the charged particle.

If the two parallel plates are oppositely and uniformly charged, then each plate carries an equal charge density allowing the electric field between the two plates to be uniform. An electric field between two plates needs to be uniform. Therefore, charges must be equally distributed on the two plates.

Once a plate is charged, it carries either negative or positive charges. It creates an electric field around it that may attract or repel other electric particles by exerting an electric force. Gauss's law gives the electric field flux through a closed surface (such as a charged plate) as product of the electric field vector standing perpendicular to the surface's area multiplied by the area of that surface. A two charged and parallel plates will create a uniform electric field when both carry equally distributed opposite charges. Still, if the plates have the same charge (positive or negative), the electric field equals zero. The lines are vectors perpendicular to both plates for a uniform electric field.

The magnitude of the electric field of two parallel plates is given by the formula $E=V/d$
The motion's acceleration of a particle placed in the electric field E is $a=qV/md$

QUESTIONS

1. Calculate the distance between point charges q1=26uC and q2=47.0uC, if the magnitude of the electrostatic force is 5.7N,. Answer: 138.9cm
2. Two charges, magnitude 2 times 10^-6C each are 60cm apart. Find the magnitude of the force exerted by these charges on a third charge of magnitude 4 times 10^-6C that is 50cm away from each of the first two charges

Amswer: F = 0.46

F = sqrt {{F31 rsup 2}+{F32 rsup 2}}

3. At what distance will the repulsive force between two electrons have a magnitude of 1N

r = 1.52 times 10^-14

4. How many excess electrons must be placed on each of two small spheres spaced 3cm apart, if the force of repulsion between the spheres is to be 10^-19

5. What is the total positive charge in Coulombs, of all protons in 1 mol of Hydrogen atoms?

6. Two positively charged spheres have a combined charge of 4.0 times 10^-8. Calulate the charge on each sphere if they are repelled with a force of 27 times 10^-5N when placed 0.1m apart. Tips: q1 = total charge – q2. Answer:

q1 = 1 times 10^-4 or 3 times 10^-4

q2 = 1 times 10^-4 or 3 times 10^-4

7. Compare the electric and gravitational forces of attraction between the electrons and the proton in a hydrogen atom assuming that their distance of separation is 5.3 times 10^-11

Answer: 2.2 times 10^39

8. Two charges q1 = 500uC and q2 = 100uC are located on the xy-plane at the position ri1 = 3j and r2 = 4i. Find the force exerted on q2. Answer: 18N

9. Calculate the resultant force on the charge q1 due to the other charges. Given that all the six charges have the same magnitude q = 20nC, a =3.0cm theta = 30. (q1 = q2 = q4 = -2nC)

(q3 = q6 = 2nC)

10. What is the magnitude of a point charge whose electric field 50cm away has a magnitude of 2.0N/C

11. What is the electric fields E experienced by a charge of magnitude 5nC at a point where the electic force is 2.0 times 10^-4N in the x-direction? If an electron is placed in the field, what force will be exerted on it