DYNAMICS AND KINEMATICS OF PARTICLES

A key thing in dynamics is that the body already has acceleration
Three things can occur when force is applied to a body

1. Motion (The body can be placed)
2. Deformation (Compression or elongation of the object)
3. It can break

Dynamics can be divided into two namely:

Kinematics: The study of the geometry of motion. Kinematics is used to relate displacement. Velocity and acceleration and time, without reference to the cause of the motion. Kinetics can therefore be simply defined as the study of motion.

Kinetics: The study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

In Kinetics we will be talking about **rectilinear motion**, and **curvilinear motion**.

Rectilinear motion can have some of these properties

1. Constant acceleration
2. Constant or uniform velocity
3. Non-constant acceleration

Uniform motion → Constant velocity
Uniformly accelerated motion → Constant acceleration
For a not constant acceleration, it can be solved with

1. Equations
2. Graphical solutions

Curvilinear motion
This involves calculating motion parameters around curves
Circular motion
Polar coordinate
Cylindrical coordinate

KINETICS
This involves trying to relate force and motion. There are four ways that this can be done

1. Inertia
2. Energy and work done method
3. Impulse method (Law of conservation of linear momentum)

4. Law of conservation of energy

TERMS USED IN KINEMATICS

1. Position: This is usually denoted as **X** and it is defined as the location of an object in space at an instance. When we say space we are talking about planes (e.g. Cartesian planes). The

2. Initial position of a particle: Denoted as X_o, is defined as the position of the particle at start of motion. It is **constant** and therefore never changes.
Given an equation $x=t^2+4t+4$, the initial position will be 4 because it doesn't depend on time. The constant of the equation will be your initial position.

3. Displacement: This is defined as the change in position of a particle. It is a vector quantity and therefore pays attention to direction.
Displacement = final position – initial position (Irrespective of the path)
That means if you start from a position and come back to that position, you have no displacement even if you may have covered a distance
Displacement $D=X_B-X_A$

4. Distance: This is the change in position of a body in one direction. When calculating the total distance covered, the method I use is to calculate is to add the distances between two positions till the particular time. For example, if we are to calculate the total distance covered by 4 seconds. We calculate the positions at each time from t=0 to that point. Then we find the gap (absolute value of the displacement) between each time i.e. the distance between t=0 and t=1, gap between t=1 and t=2 etc. After that, we add the values together to get the total distance covered.

5. Average velocity: This pays attention to distance. It basically means in one second, how much distance you can cover. The average velocity of a particle over the time interval %delta t is defined as the quotient displacement Δx and the time interval Δt

$$V_{av}=\frac{X_f-X_o}{t_f-t_i}$$

$$Average\ velocity=\frac{\Delta x}{\Delta t}$$

$$V_{av}=\frac{v_1+v_2}{2}$$

The SI units for x and t are meters and seconds respectively
The US customary units are feet and seconds respectively

6.Instantaneous velocity. This is defined as the velocity at an instance. To find this velocity, we want our change in time to be equal to zero

$$Instantaneous\ velocity = \lim_{\Delta t \to 0} Average\ velocity$$

$$v_{inst} = \lim_{\Delta t \to 0} v_{av}$$

$$Instantaneous\ velocity, v = \frac{dx}{dt}$$

7. Initial velocity is the velocity at the start of the journey denoted by v_o. It is also **constant**.

The velocity v – known as the speed of the particle – can be positive or negative. It is positive when x is positive and vice-versa.

8. Acceleration: This is by how much you can increase your speed in a second.
9. Average acceleration: This is the time rate of change of velocity

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

10. Similarly to instantaneous acceleration

$$a_{inst} = \lim_{\Delta t \to 0} a_{av}$$

$$a_{inst} = \frac{dv}{dt}$$

$$v = \frac{dx}{dt}$$

$$dt = \frac{dx}{v}$$

$$a = \frac{dv}{dt}$$

$$a = \frac{dv}{\frac{dx}{v}}$$

$$a = v\frac{dv}{dx}$$

Even though acceleration and velocity are vectors, since we are dealing with rectilinear motion where the particle has a known and fixed direction, we only need to specify the sense (+ve or -ve) and the magnitude (scalar quantity).

Acceleration can be positive or negative. A positive value indicates that velocity increases. This may mean that the particle is moving faster in the positive direction or moving slowly in the negative direction. In both cases Δv is positive.

11. Deceleration: The term deceleration is used to refer to acceleration when the speed of the particle decreases; the particle is then moving more slowly.

In physics or sciences when a big object is considered a particle, that means calculation done on that object is done without regard to its size.

We will look at uniform motion and uniform accelerated motion.

A particle moving along a straight line is said to be in **rectilinear motion**. At a given time t, the particle will occupy a position on the straight line – usually with respect to an origin –. The position/distance can either be positive or negative depending on the position relative to the origin. So if it is on the left, then it is negative. However, if the position is on the right of the origin, then it is positive.

When the position coordinate x of a particle is known for every time t, we say that the motion of the particle is known. The timetable of the motion can be given in the form of an equation in x and t, such as x = 6t^2 – t^3, or in the form of a graph of x versus t.

Examples:

Consider a particle moving in a straight line and assume that its position is defined by the equation

$$x = 6t^2 - t^3$$

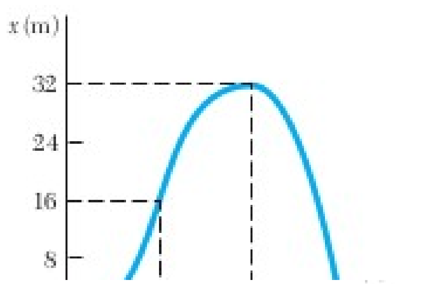
where t is expressed in seconds and x in metres. The instantaneous velocity – velocity at any time t – is obtained by differentiating x with respect to t

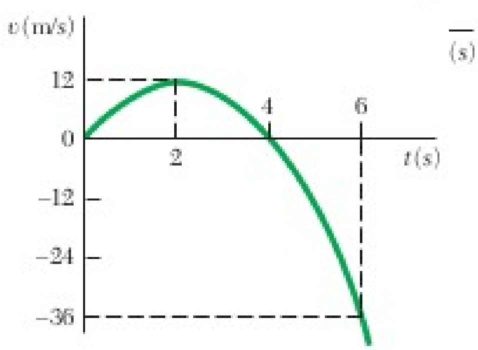
$$v = \frac{dx}{dt} = 12t - 3t^2$$

Acceleration a, is obtained by differentiating velocity.

$$A = \frac{dv}{dt} = 12 - 6t$$

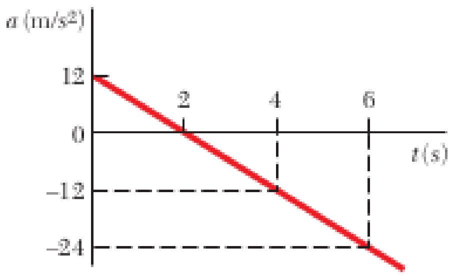
Note the following **motion curves**.





Displacement-time graph

Velocity time graph



A study of the three motion curves of Fig 11.6 shows that the motion of the particle from t=0 to t=infinite can be divided into 4 phases

1. The particle starts from the origin x=0, with no velocity but with a positive acceleration. Under this acceleration, the particle gains a positive velocity and moves in the positive direction. From t =0 to t = 2s, x, v and a are all positive

2. At t=2s, the acceleration is zero; the velocity has reached its maximum value. From t = 2 to t = 4, v is positive but a is negative; the particles moves in the positive direction but reduces in velocity.

3. At t = 4s, the velocity is zero; the position coordinate x has reached its maximum value. From then on, both v and a are negative; the particle is accelerating and moves in the negative direction with increasing speed.

4. At t=6s, the particle passes through the origin; its coordinate x is then zero while the total distance traveled since the beginning of the motion is 64m. For values of t larger than 6s, x, v and a will all be negative. The particle keeps moving in the negative direction away from O faster and faster

DETERMINATION OF THE MOTION OF A PARTICLE
When $a=0$ => Uniform motion
When $a\neq 0$ => Uniform accelerated motion

Motion is not always defined by a relation between x and t. Acceleration can be expressed in different forms and depending on what causes the acceleration. The acceleration of the motion of a body tied to a spring will depend on factors relating to the elongation of the spring.

$$v=\frac{dx}{dt}$$

$$dt = \frac{dx}{v}$$

$$a = \frac{dv}{dt}$$
$$a = \frac{dv}{\frac{dx}{v}}$$
$$a = v\,\frac{dv}{dx}$$

Three common classes of motion

1. When acceleration is given as a function of time, f(t). We will like to have an integral of that function with respect to time.

$$a = \frac{dv}{dt} = f(t)$$
$$dv = f(t)\,dt$$
$$\int dv = \int f(t)\,dt$$

$$\int_{v^o}^{v} dv = \int_0^t f(t)\,dt$$
$$v - v_o = \int_0^t f(t)\,dt$$

Similarly for the velocity

$$v = \frac{dx}{dt}$$
$$dx = v\,dt$$
$$\int_{x_o}^{x} dx = \int_0^t v dt$$
$$x - x_o = \int_0^t v dt$$

v_o and x_o are the initial velocity and initial position respectively. They are the initial conditions of the motion

2. When acceleration is given as a function of x f(x)

$$a = v\,\frac{dv}{dx}$$
$$a = f(x)$$
$$v\,dv = f(x)\,dx$$

For velocity

$$v = \frac{dx}{dt} = v(x)$$

3. As a function of v

$f(v) = \frac{dv}{dt}$

$f(v) = v\frac{dv}{dx}$

$dt = \frac{dv}{f(v)}$

$dx = \frac{vdv}{f(v)}$

UNIFORM RECTILINEAR MOTION

This is motion where there is no acceleration and velocity is constant

$\frac{dx}{dt} = v$

$dx = vdt$

$x - x_o = vt$

UNIFORM ACCELERATED RECTILINEAR MOTION.

Here, acceleration is constant

$\frac{dv}{dt} = a$

$dv = adt$

QUESTIONS

1. The position of a particle which moves along a straight line is defined by the relation.

$X = t^3 - 6t^2 - 15t + 40$, where x is expressed in feet and t in seconds. Determine:

a. The time at which the velocity will be zero. Answer: t=5

b. The position and distance traveled by the particle at that. Answer: position = -60ft, distance traveled = 100ft

c. The acceleration of the particle at that time. +18

d. Distance traveled by the particle from t=4 to t=6. Answer: 18ft

2. A ball is tossed with a velocity of 10m/s directed vertically upward from a window located 20m above the ground. Knowing that the acceleration of the ball is constant an

a. the velocity v and elevation y of the ball above the ground at any time t. Ans: $v = 10 - 9.81t$, $y = 20 + 10t - 4.905t^2$

b. The highest elevation reached by the ball and the corresponding value: Ans: $t = 1.019s$ $y = 25.1m$

c. The time when the ball will hit the ground and the corresponding velocity: t=3.28s, v=-22.2

d. Draw the v-t and y-t curves

3. The brake mechanism used to reduce recoil in certain types of guns consists essentially of a piston attached to the barrel and moving in a fixed cylinder filled with oil. As the barrel recoils with an initial velocity v0, the piston moves and oil is forced through orifices in the piston, causing the piston and the barrel to decelerate at a rate proportional to their velocity; that is,

a 5 2kv. Express
(a) v in terms of t,
(b) x in terms of t,
(c) v in terms of x.
d. Draw the corresponding motion curves.

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4. The acceleration of a particle is directly proportional to the square of the time t. When t = 0, the particle is at x = 24 m. Knowing that at t = 6 s, x = 96 m and v = 18 m/s, express x and v in terms of t.
If you look at the question, you'll see that we were given a few values.

$$a \propto t^2$$
$$a = k t^2$$
$$v = \int a\,dt$$
$$\int dv = \int k t^2 dt$$
$$\int_{18}^{v} dv = \int_{6}^{t} k t^2 dt$$
$$v - 18 = \left.\frac{k t^3}{3}\right|_{6}^{t}$$
$$v - 18 = \frac{k t^3}{3} - \frac{k 6^3}{3}$$
$$v - 18 = \frac{k t^3}{3} - \frac{216 k}{3}$$
$$v = \frac{k t^3}{3} - \frac{216 k}{3} + 18$$

On integrating, you'll s get the value of the position
int from{24} to{x} {v}

The reason that we used the value when t=0, is so that we can get the equation for x
$$x - 24 = 18 t + \frac{1}{3} k \left(\frac{1}{4} t^4 - -216 t \right)$$
3v-54 = {} - 216k
Looking at the values they gave us in the question, we can start that as our base values in the integration
$$v = \frac{k t^3}{3} + c$$

QUESTIONS

1. A train is moving in the west direction with a velocity 15m/s. A monkey runs on the roof of the train against its motion with a velocity 5m/s with respect to the train. Take the motion along the west as positive. What is the velocity of the train with respect to the monkey.
When you see with respect to something, that is the relative motion
$$v_{m/t} = -5$$

$v_{m/t} = v_m - v_t$
$-5 = v_m - 15$
$v_m = 10$
$v_{t/m} = v_t - v_m$
$v_{t/m} = 15 - 10 = 5\, m/s$.

2. The greatest possible acceleration or deceleration a train may have is “a” and its maximum speed is “v”. Find the minimum and maximum time in which the train can get from one station to the next if the total distance is S.

We assume that there will be a motion in the form
acceleration → constant velocity → deceleration
The initial and final sides of the motion will be similar since the acceleration and deceleration values are the same.

For the first side
$v = a t_1$
$t_1 = \frac{v}{a}$
$s_1 = ut + \frac{1}{2} a t^2$
$s_1 = 0 + \frac{1}{2} a t_1^{\ 2}$
$s_1 = \frac{1}{2} a \frac{v}{a}^2$

For the second side,
It is at constant velocity
$v_2 = v$
$s_2 = vt$
$s_2 = v t_2$
$t_2 = \frac{s_2}{v}$
$s_2 = \frac{s}{2}$

First, there is no limit on the speed of the train, so minimum time will be when the train will accelerate for t over 2 and decelerate for t over 2. So time taken is calculated by $S = ut + \frac{1}{2} a t^2$, where u=0, $displacement = \frac{s}{2}$, $time = \frac{t}{2}$

Maximum acceleration and deceleration is same. So time for reaching the maximum speed is same as time for reaching the zero speed from maximum speed.

$t_1 = \frac{v}{a} = t_3$

$s = ut + \frac{1}{2} a t^2$

$s_1 = s_3 = \frac{1}{2} a \frac{v^2}{a^2}$

For the part of constant velocity,

$s_2 = v \times t_2$

$t = t_1 + t_2 + t_3$

$t = \frac{v}{a} + \frac{s}{v} - \frac{v}{a} + \frac{v}{a}$

$t = \frac{s}{v} + \frac{v}{a}$

3. A man X drops a stone from the fifteen floor of the building. A man Y ascending in an elevator at a constant speed v=10m/s passed the fifteen floor just as the stone is released.

a. Find the acceleration of the stone with respect to man X and Y. Answer 9.8m/s, 9.8m/s

b. Find the position of the stone relative to man Y at 3 seconds.

Distance moved by the stone in 3 sec

$x = v_o t + \frac{1}{2} a t^2$

$x = 44\, m$ downward

Distance moved by the elevator in 3 secs = 10×3 = $30\, m$

The position with respect to the man y = 44 + 30 = 74m

c. Find the velocity of the stone relative to man Y at 3 seconds.

Now velocity of stone at 3 sec = 9.8×3 = $29\, m/s$

Velocity of elevator = 10m/s (upward)

So velocity of stone with respect to Y

$29 + 10$

$39\, m/s$

C. Find the position of the stone as seen by the Man X at time $t = 2s$

For X, position of stone is given by

$a = g = 9.8\, m/s^2$

$x = ut + \frac{1}{2} a t^2$

$x = 0 + \frac{1}{2} \times 9.8 \times 4$

= 19.6m

d. Find the velocity of the stone as seen by the Man X at time $t = 2s$

$v = u + at$

$v = at$

$v = 9.8 \times 2$

$v = 19.6\, m/s$

4. A particle moves in a straight line with acceleration described by equation given below $a=mx-\frac{v_o^{\,2}}{x_o}$. If the initial velocity and displacement are $(v_o, 0)$ and at any time t_o velocity and displacement are $(0, x_o)$ the value of the constant m is?

$$a=mx-\frac{v_o^{\,2}}{x_o}$$

$$v\frac{dv}{dx}=mx-\frac{v_o^{\,2}}{x_o}$$

$$v\,dv=\left[mx-\frac{v_o^{\,2}}{x_o}\right]dx$$

$$\int v\,dv=\int\left[mx-\frac{v_o^{\,2}}{x_o}\right]dx$$

$$\left.\frac{v^2}{2}\right|_{v_o}^{0}=\left[m\frac{x^2}{2}-\left(\frac{v_o^{\,2}}{x_o}\right)x\right]_0^{x_o}$$

$$\frac{-v_o^{\,2}}{2}=m\frac{x_o^{\,2}}{2}-v_o^{\,2}$$

$$\frac{v_o^{\,2}}{2}=m\frac{x_o^{\,2}}{2}$$

$$m=\frac{v_o^{\,2}}{x_o^{\,2}}$$

5. Two objects are released off a building from the same height. Object A is dropped and hits the ground in time t. Object B is thrown upward with a velocity of v_o and hits the ground in a time 3t. Assume air resistance is negligible, write an expression for v_o.

6. A car starts from rest and accelerates uniformly to a speed of 72km/h over a distance of 500m. If a further acceleration raises the speed to 90km/h in 10 seconds, find the acceleration and the further distance moved. Leaving your answer in meters per square second.

$$a=\frac{\Delta v}{\Delta t}$$

$$v_2=90\,km/h=\frac{90\times1000}{3600}\,m/s=25\,m/s$$

$$v_1=72\,km/h=\frac{72\times1000}{3600}\,m/s=20\,m/s$$

$$\Delta v=v_2-v_1$$

$$25-20$$

$$5\,m/s$$

$$a=\frac{\Delta v}{t}$$

$a = \frac{5}{10}$

$a = 0.5 m/s^2$

7. A 100m long train crosses a man traveling at 5km/hr. In opposite direction in 7.2 seconds then the velocity of the train is:
Given velocity of man $v_m = 5 kmph$
let v_t be the velocity of the train.
Velocity of the train with respect to the man

$v_{tm} = v_t - v_m$

But v_t and v_m are in opposite direction

$v_{tm} = v_t + v_m = v_t + 5$

$time = \frac{length\ of\ train}{velocity\ of\ train\ wrt\ man}$

$t = \frac{l}{v_{tm}}$

$\frac{100}{v_t + 5} = 7.2 s$

$v_t + 5 = \frac{100}{7.2}$

To convert back to km/h

$v_t + 5 = \frac{100}{7.2} \times \frac{18}{5}$

$v_t + 5 = 50$

$v_t = 45 km/h$

If a car is traveling at 8m/s accelerates uniformly at 2m/s^2. Its velocity after 5 seconds will be

A ball is thrown vertically upward from the 12-m level in an elevator shaft with an initial velocity of 18 m/s. At the same instant an open-platform elevator passes the 5-m level, moving upward with a constant velocity of 2 m/s.
Determine
(a) when and where the ball will hit the elevator,
(b) the relative velocity of the ball with respect to the elevator when the ball hits the elevator.

SOLUTION

Motion of Ball. Since the ball has a constant acceleration, its motion is uniformly accelerated. Placing the origin O of the y axis at ground level and choosing its positive direction upward, we find that the initial position is $y_o = +12 m$, the initial velocity is $v_o = +18 m/s$, and the acceleration is $a = -9.81 m/s^2$. Substituting these values in the equations for uniformly accelerated motion, we write

$v_B = v_o + at$

$v_B = 18 - 9.81 t$

$y_B = y_o + v_o t + 12 a t^2$

Motion of Elevator: Since the elevator has a constant velocity, its motion is uniform. Again placing the origin O at the ground level and choosing the positive direction upward, we note that $y_o = +5 m$and write.

$v_E = 12 m/s$

$y_E = y_o + v_E t$

$y_E = 5 + 12 t$

Ball Hits Elevator: We first note that the same time t and the same origin O were used in writing the equations of motion of both the ball and the elevator. We see from the figure that when the ball hits the elevator,

$y_E = y_B$

Substituting for y_E and y_B from (2) and (4) into (5), we have

$5 + 2t = 12 + 18t - 4.905 t^2$

$t = 20.39 s t = 3.65 s$

Take note of this

1.$a = \frac{dv}{dt}$, $v = \frac{dx}{dt}$

$dt = \frac{dv}{a}$, $dt = \frac{dx}{v}$

$a\,dx = v\,dv$

2. $a = \frac{dv}{dt}$, $F = ma$

$F = m \frac{dv}{dt}$

$Fdt = mdv$

If you integrate,

$\int_0^t F\,dt = mv_2 - mv_1$

ERRATIC MOTION

It simply means that the position-time dependence can’t be fully characterized by one equation but need to include more than 1 equation that apply to different time periods. These functions are called piece wise functions.

Chances are that the velocity function and the acceleration function are piece wise functions.