SOLVING LINEAR EQUATIONS WITH MATRICES

1. Crammer rule
2. Gaussian Elimination method
3. Gauss Jordan Elimination method

GAUSSIAN ELIMINATION METHOD

In this method, you will have to reduce (or eliminate) values by making them 0

The first row shouldn't change
The first column of row 2 becomes 0
The first and second columns of row three become 0

Given a simultaneous equations
$x+y-z=-2$
$2x-y+z=5$
$-x+2y+2z=1$
First, you express the system of linear equations in a matrix augmented form

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

To be able to eliminate values, we will need to perform operations with rows and apply either these changes to either the second or third row depending on the element that we want to make 0

The convention to doing this is the convention:
When reducing any row or column, that row or column should come first in the reduction formula before other rows.
For example, if we want to change the first element in row 2 to 0,
$R_2 \rightarrow R_2 - 2R_1$
You'll notice that we change the values of the whole row

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

Similarly, to eliminate the first item in the third row, we can say that:

$R_3 \rightarrow R_3 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ -1 & 2 & 2 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right]$$

Now to eliminate the second column of the third row, we can say once again

$R_3 \rightarrow R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

From this, we can say

$4z = 8$

$z = \frac{8}{4}$

$z = 2$

$-3y + 3z = 9$

$-3y + 3(2) = 9$

$-3y + 6 = 9$

$-3y = 3$

$y = -1$

$x + y - z = -2$

$x - 1 - 2 = -2$

$x - 3 = -2$

$x = 1$

GAUSS JORDAN ELIMINATION METHOD

Similar to the Gaussian elimination method, we have a matrix with 0s on the bottom. However, the difference to this is that, you also have zeros on the top as well.

You will be having a matrix of the form:

$$\left[\begin{array}{ccc|c} a & 0 & 0 & ax \\ 0 & b & 0 & by \\ 0 & 0 & c & cz \end{array}\right]$$

This can further be reduced to the the “Reduced Row Echelon Form”

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \end{array}\right]$$

Try the following

1.

$x - 4y - 2z = 21$

$2x + y + 2z = 3$

$3x+2y-z=-2$
Answer: (3, -5, 1)

2.
$7x-4y=12$
$-4x+12y-6z=0$
$-6y+14z=0$

3.
$x+2y+3z=-4$
$2x+6y-3z=33$
$4x-2y+z=3$

4.
x – y + 7z = 3
x + 4y + z = 8
x + 3y – 3z = 2