WAVES

A wave can be defined as a disturbance that travels through a medium and transfers energy from one point to another without causing any permanent displacement of the medium itself
Waves can be classified based on:
1. Whether they require material media
2. The direction of the vibration

Based on whether they require material mediums
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1. **Mechanical Waves**: These are waves that require material mediums for their propagation. They possess mechanical energy. Also, they can't travel in vacuum or free space.
The sound wave is a typical example of a mechanical wave.

2. **Electromagnetic waves**: These are the waves that do not require material media for their propagation. They usually travel in vacuums.
They also have oscillating electric and magnetic properties hence the name electromagnetic waves.
The major examples are Radio waves, Infrared ways, Visual light, Ultraviolet rays, X-rays and Gamma rays.
These can be put in the acronym "RIVUX-G" for easy remembrance.

Based on the direction of their vibration

Based on the direction of their vibration, waves can be classified into

1. **Transverse waves**: A transverse wave is one which travels perpendicularly (i.e. at 90 degrees) to its vibration. The major examples are light waves, water waves, waves produced by strings (which are sound waves) and all electromagnetic waves.
2. **Longitudinal waves**: These are waves that travel parallel or in the same direction as their vibration. The major examples are sound waves (except the one produced in strings). The speed of compressional waves in other materials (such as liquids and solids) is given by:

$$speed = \sqrt{\frac{modulus}{density}}$$

$$v = \sqrt{\frac{Y}{\rho}}$$

If the material is in the form of a bar (or solid), the young's modulus is used.
If the material is a liquid, the bulk's modulus is used.

TERMS USED IN WAVES

1. **Wave front**: A wave front is a surface over which particles vibrate in the same phase.
It could be defined as a surface containing points affected in the same way by a wave at a given time. When identical waves having a common origin travel through a homogeneous medium, the corresponding crests and troughs at any instant are in phase i.e. they have completed identical fractions of their cyclic motion and any surface drawn through all the points of the same phase will constitute a wave front.
The wave front is the front of the wave or the same point on each wave.
In physics, the wave front of a time-varying field is the set (locus) of all points where the wave has the same phase of the sinusoid. For waves propagating in a one-dimensional medium, the wave fronts are usually single points, in two-dimensional media, they are curves and in three-dimensional media, they are surfaces.
2. **Crest**: This can be defined as the highest (or maximum) point in a wave motion.
3. **Trough**: This is the lowest point in a wave motion
4. **Amplitude (A)**: This is usually represented as A. It is defined as the maximum displacement of a (wave) particle from the equilibrium position.
5. **Cycle or oscillation (n)**: This is defined as the movement of the wave touching the maximum (crest) and the minimum (trough) points.
6. **Period (T)**: This is defined as the time taken for one oscillation.

$$Period = \frac{Time\ taken}{One\ oscillation}$$

$$Period = \frac{Time\ taken\ for\ total\ wave\ motion}{Total\ number\ of\ cycles\ of\ the\ wave\ motion}$$

$$Period = \frac{Time\ taken}{Number\ of\ oscillations}$$

The above formula is the standard one

$$T = \frac{t}{n}$$

7. **Frequency (f)**: This can be defined as the number of oscillations made per second. The unit of frequency is hertz (Hz)

$$frequency = \frac{number\ of\ oscillations}{time\ taken\ for\ oscillations}$$

$$f = \frac{n}{t}$$

On multiplying (T) and (f)

$$T \times f = \frac{t}{n} \times \frac{n}{t}$$

$$Tf = 1$$

$$f = \frac{1}{T}$$

8. **Wavelength**(λ): This can be defined as the distance between two successive crests or two successive troughs. In longitudinal waves, it is defined as the distance between a successive compression or the distance between two successive rarefactions.The wavelength of a wave can also be defined as the distance covered by the wave in one oscillation.

9. **Wave speed (v)**: This is defined as the distance travelled by the wave per unit time.

$$speed = \frac{distance\ travelled}{time}$$

$$v = \frac{x}{t}$$

For one oscillation,

$$x = \lambda$$

$$t = T$$

Therefore,

$$v = \frac{\lambda}{T}$$

$$v = \lambda \times \frac{1}{T}$$

But

$$f = \frac{1}{T}$$

Therefore,

$$v = \lambda \times f$$

$$v = f\lambda$$

10. **Angular Frequency** (ω): This is related to linear frequency (f) in the form $\omega = 2\pi f$

But

$$f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T}$$

10. **Wave constant (k)**: This is also called wave number. This is expressed mathematically as

$$k = \frac{2\pi}{\lambda}$$

If we divide the angular frequency by the wave constant

$$\frac{\omega}{k} = \frac{2\pi}{T} \div \frac{2\pi}{\lambda}$$

$$\frac{\omega}{k} = \frac{2\pi}{T} \times \frac{\lambda}{2\pi}$$

$$\frac{\omega}{k} = \frac{\lambda}{T}$$

But,

$$v = \frac{\lambda}{T}$$

$$\frac{\omega}{k} = v$$

$$v = \frac{\omega}{k}$$

The velocity is sometimes referred to as the phase velocity

11. **Phase angle**(ϕ): This can be defined as the angular displacement between two waves that are traveling in the same direction.
Mathematically, it is the product of the wave constant and the distance covered.

$$\phi = kx$$

$$\phi = \frac{2\pi}{\lambda} x$$

$$\phi = \frac{2\pi x}{\lambda}$$

Therefore, for one oscillation, $\phi = 360$, For a half oscillation

12. **Vertical displacement (y)**: This can be defined as the movement of the wave in a vertical direction.
The general equation of a progressive wave is given as:

$$y = A\sin(kx - \omega t)$$

$(kx - wt)$ = Phase of the wave
Or

$$y = A\sin(\omega t - kx)$$

$y(x) = A\sin\frac{2\pi}{\lambda}x$. This equation applies when x = 0, $x = \lambda$, $x = 2\lambda$ and so (since $\sin 0 = \sin\pi = \sin 2\pi$), hence waveform repeats itself every wavelength
However, at some time t, when the value of $x \neq 0$. The current position is $(x - vt)$

$$y(x, t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

Taking the first,

$$y = A\sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

And

$$\omega = 2\pi f$$

$$y = A\sin\left(\frac{2\pi}{\lambda}x - 2\pi f t\right)$$

$$y = A\sin 2\pi\left(\frac{x}{\lambda} - \frac{ft}{1}\right)$$

$$y = A\sin 2\pi\left(\frac{x - f\lambda t}{\lambda}\right)$$

$$y = A\sin\frac{2\pi}{\lambda}(x - f\lambda t)$$

But

$$v = f\lambda$$

And

$$k = \frac{2\pi}{\lambda}$$

$$y = A\sin k(x - vt)$$

Also,
$y = A \sin(kx - \omega t)$
But
$\phi = kx$
$y = A \sin(\phi - \omega t)$
If the wave is traveling to the left axis $y = A \sin[k(x + vt)]$

Therefore the equation of the wave can be generally written as
$y = A \sin(kx \pm \omega t + \phi)$
if $y = 0$ at $t = 0$, $x = 0$, then $\phi = 0$
Also, $v = \frac{dy}{dt}$
$y = A \sin(kx - \omega t)$
$v = \omega A \cos(kx - \omega t)$
$a = \frac{dv}{dt}$
$a = -\omega^2 A \sin(kx - \omega t)$

PRINCIPLE OF THE SUPERPOSITION OF WAVES

This principle states that when two or more waves cross at a point, the displacement at that point is equal to the sum of displacements of the individual waves. The individual wave displacements may be positive or negative. If the displacements are vectors, then the sum is calculated by vector addition.
It could also be said as
The superposition principle states that when two or more waves overlap in space, the resultant disturbance is equal to the algebraic sum of individual disturbances.
This principle explains why we get higher amplitude in constructive interference and lower amplitude in destructive interference.
When two wave pulses are started along a rope from opposite ends, the waves will meet, pass through each other and continue their motion as though nothing has happened. The resultant wave observed when the two overlap is the algebraic sum of individual amplitudes.
When two or three people are talking in the same room, we can distinguish the individual voices even if they speak simultaneously. The sound of each person’s voice isn’t disturbed by the other person’s voice. Also, if all of them speak at the same time, the maximum sound (or noise) will be heard. This effect of combined sound is an example of superposition and an example of interference.

$y = y_1 + y_2 + y_3$
Recall that y is the vertical displacement.

GENERAL PROPERTIES OF WAVES

1. Reflection: This is defined as the bouncing back of waves when they hit a medium. The original waves are called the incident waves while the waves that are sent back are called the reflected waves. All waves undergo reflection. The reflection of sound waves is called echo.

2. Refraction: This can be defined as the apparent change in the direction of waves when they travel from one medium to another. During refraction, the **frequency remains unchanged** (i.e. the frequency is constant) but the **direction, velocity and wavelength change**. The amount of refraction depends on the refractive indices (singular-index) of the two media (mediums). The angle of refraction increases if a wave goes from a denser medium to a less dense medium (e.g. from water to air) and the angle of refraction reduces if the wave moves from a less dense medium to a denser medium.

3. Diffraction: This can be defined as the spreading out of a wave when it passes through a narrow medium. It can also be defined as the bending of waves through a narrow opening. Generally, waves travel on straight lines but if they get in contact with a narrow aperture, they bend round the aperture. The amount of diffraction depends on the size of the aperture and the wavelength of the wave.
The smaller the size of the aperture, the higher the amount of diffraction
The longer the wavelength, the higher the amount of diffraction

4. Polarization: This is a property only specific to electromagnetic waves. A wave is said to be plain polarized if its vibration occurs on a single plane. The practical application of polarization is a Polaroid which is used in sunglasses to reduce the intensity of sunlight and also to eliminate reflected light.
Polarization is also used to determine the concentration of a sugar solution.

INTERFERENCE

This can be defined as the superposition of waves. It can also be defined as the interaction of two waves when the sources of the waves are very close to each other. Consider two wave pulses of the same amplitude and frequency on a cord programming along the x-axis in opposite directions. Therefore, $y_1 = A\sin(kx - \omega t)$, $y_2 = A\sin(kx + \omega t)$. From the principle of the superposition of waves,

$y = y_1 + y_2$

$y = [2A\sin kx]\cos\omega t$

The resulting amplitude is

$A_r = 2A\sin kx$

The resulting wave is known as a **standing wave**. It is not a traveling wave but an oscillation that has a position-dependent amplitude. A standing wave does not transfer energy from one end to the other.

CONDITIONS FOR INTERFERENCE

1. The sources of the waves must be coherent.
2. The frequencies of the waves must be the same
3. The amplitude of the waves must be approximately equal.

TYPES OF INTERFERENCE

There are two major types of interference

1. **Constructive Interference**: In constructive interference, the waves match crest for crest and trough for trough (one on the other) i.e. the two waves are in phase. Constructive interference is also called additive interference because a higher amplitude is usually obtained. For maximum constructive interference, the phase angle is 0. The positions where the standing wave amplitude is maximum are called antinodes, A. They occur

$kx=0$

$kx=\left(n+\frac{1}{2}\right)\pi$, for n = 0, 1, 1

But $k=\frac{2\pi}{\lambda}$

$$x=\left(n+\frac{1}{2}\right)\frac{\lambda}{2}$$

2. **Destructive Interference**: In this form of interference, the two waves match crest for trough and trough for crest i.e. they are not in phase. Destructive interference consists of two waves traveling in opposite directions and are combined together. Destructive interference is also called subtractive interference because a lower amplitude is obtained when the two waves are super imposed.
For a maximum destructive interference, the phase angle is 180 degrees. At times in the motion of the wave such as $t=\frac{1}{4}T$, we will have destructive inteference. The positions where the standing wave amplitude vanishes are called nodes. They occur when

$kx=n\pi$

$$x=n\frac{\lambda}{2}$$

Distance between successive nodes or antinodes

$$NN=AA=\frac{\lambda}{2}$$

APPLICATIONS OF INTERFERENCE

1. Colors seen in soap bubbles
2. Colors seen in oil that is mixed with water or oil on the road

QUESTIONS

1. A sinusoidal wave train is moving along a string $y(x,t)=0.001\sin(62.8x+31.4t)$. The wavelength, frequency and period are respectively:

Solution: $y=A\sin(kx-\omega t)$

2. Find the equation of a wave if half a wave occupies 0.5cm and twice the wave is exactly 4s

$$\frac{\lambda}{2}=0.5\,cm$$
$$\lambda=1\,cm$$
$$2T=4\,s$$
$$T=2\,s$$
$$y=A\sin 2\pi\left(\frac{x+vt}{\lambda}\right)$$
$$y=A\sin 2\pi\left(\frac{x+vt}{1}\right)$$
$$y=A\sin 2\pi\left(x+\frac{\lambda t}{T}\right)$$
$$y=A\sin 2\pi\left(\frac{2x+t}{2}\right)$$
$$y=A\sin \pi(2x+t)$$

3. Calculate the tension in a string if the equation of a wave on a string of linear mass density 0.04kg/m is given by: $y=0.02\sin 2\pi\left(\frac{t}{0.04}-\frac{x}{0.5}\right)$

4. A string of mass 2.5kg is under tension of 200N, the length of the stretched string is 20m. If the transverse jerk is struck at one end, how long does it take for the disturbance to reach the other end?

5. A stretched string of length 1m is fixed at both ends, having a mass of 2g and under a tension of 20N. If plucked at a point situated at 20cm from one end,find the frequency at which it vibrates