CHAPTER 3

KINEMATICS OF PARTICLES—CURVILINEAR MOTION

3.1 INTRODUCTION

In this chapter, kinematics of particles with curvilinear motion will be dealt with. The motion of a particle along a curved path is said to be curvilinear motion. It is better to formulate the particle's position, velocity and acceleration using vector analysis since the path is often represented in three dimensions. The general aspects of curvilinear motion, derivatives of vector function, rectangular components of velocity and acceleration and motion relative to a frame in translation will be covered in detail in this chapter. Also tangential and normal components as well as radial and transverse components will be dealt with.

3.2 POSITION VECTOR, VELOCITY AND ACCELERATION

Position vector Consider a particle located at point P on a space curve as shown in Fig. 3.1. The position of the particle measured from a fixed point O is designated by the position vector $\mathbf{r}$.

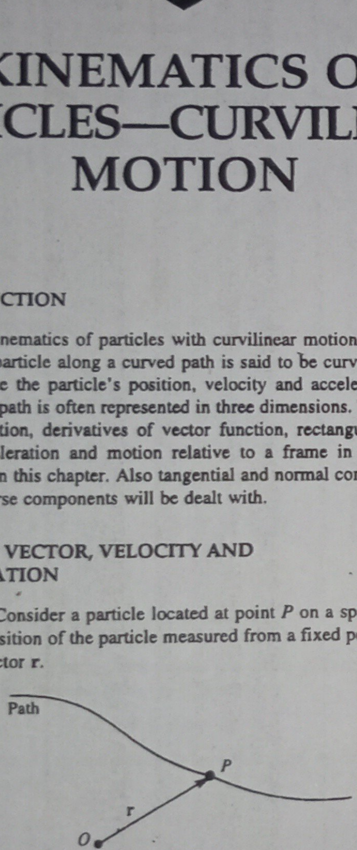


Fig. 3.1 Position vector

The position vector $\mathbf{r}$ will be a function of time t since its magnitude and direction changes as the particle moves along the space curve.

Let the particle reach a new position P' during a small time interval Δt as shown in Fig. 3.2. The position vector of the particle's new position is given by

$$\mathbf{r}' = \mathbf{r} + \Delta\mathbf{r}$$

The difference between the two position vectors shall be defined as the displacement $\Delta\mathbf{r}$ of the particle during the time interval Δt.

$$\therefore \quad \Delta\mathbf{r} = \mathbf{r}' - \mathbf{r}$$

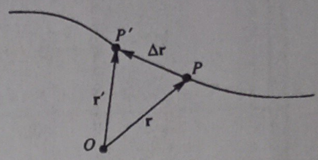


Fig. 3.2

Velocity The average velocity of the particle during the time interval Δt may be defined as the quotient of $\Delta\mathbf{r}$ and Δt. Thus,

$$\mathbf{v}_{av} = \frac{\Delta\mathbf{r}}{\Delta t}$$

If we let Δt approach zero, $\Delta\mathbf{r}$ will approach the tangent to the curve at point P. The velocity obtained by this will be the instantaneous velocity $\mathbf{v}$. Thus,

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t}$$

or

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \tag{3.1}$$

The direction of $\mathbf{v}$ will be at a tangent to the path at P as shown in Fig. 3.3 since $d\mathbf{r}$ is at a tangent to the curve at point P.

The magnitude of $\mathbf{v}$ is called the speed v.

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

or

$$v = \frac{ds}{dt} \tag{3.2}$$

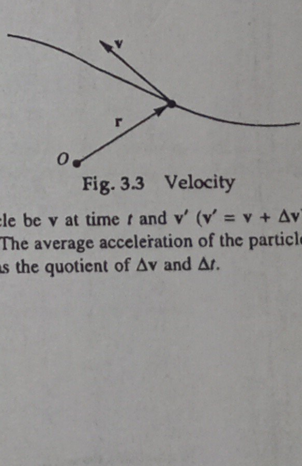


Fig. 3.3 Velocity

Where Δs is the magnitude of $\Delta\mathbf{r}$.

Acceleration Let the velocity of the particle be $\mathbf{v}$ at time t and $\mathbf{v}'$ ($\mathbf{v}' = \mathbf{v} + \Delta\mathbf{v}$) after a time interval Δt as shown in Fig. 3.4. The average acceleration of the particle during the time interval Δt may be defined as the quotient of $\Delta\mathbf{v}$ and Δt. Thus,

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t}$$

The time rate of change of velocity can be better understood by plotting the velocity vectors **v** and **v'** with their tails located at a fixed point O' and their tips at points along the dotted curve as shown in Fig. 3.5. This curve which describes the locus of points for the tip of the velocity vector is called a hodograph. The instantaneous acceleration may be obtained by letting $\Delta t \to 0$ so that $\Delta \mathbf{v}$ becomes tangent to the hodograph and hence **a** acts tangent to the hodograph as shown in Fig. 3.6.

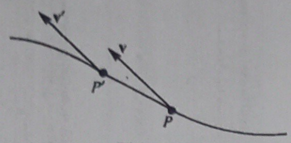


Fig. 3.4

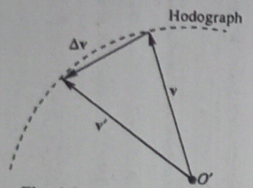


Fig. 3.5 Hodograph

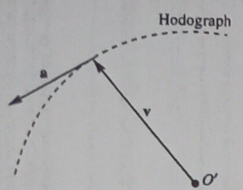


Fig. 3.6 Acceleration

Thus,

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

or

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} \tag{3.3}$$

Using Equation (3.1)

$$\mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} \tag{3.4}$$

3.3 DERIVATIVES OF VECTOR FUNCTIONS

Consider a vector **P** which varies with respect to a scalar quantity such as time t. A change $\Delta\mathbf{P}$ in **P** as time changes from t to $(t + \Delta t)$ is,

$$\Delta \mathbf{P} = \mathbf{P}(t + \Delta t) - \mathbf{P}(t)$$

Dividing throughout by Δt and letting Δt approach zero, we define the derivative of the vector function $\mathbf{P}(t)$

$$\frac{d\mathbf{P}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{P}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\mathbf{P}(t + \Delta t) - \mathbf{P}(t)}{\Delta t} \tag{3.5}$$

Let us consider the rectangular components of the vector function $\mathbf{P}(t)$.

$$\mathbf{P}(t) = P_x\,\mathbf{i} + P_y\,\mathbf{j} + P_z\,\mathbf{k},$$

where P_x, P_y and P_z are functions of time.
Then,

$$\frac{d\mathbf{P}}{dt} = \lim_{\Delta t \to 0} \frac{(P_x + \Delta P_x)\,\mathbf{i} + (P_y + \Delta P_y)\,\mathbf{j} + (P_z + \Delta P_z)\,\mathbf{k} - P_x\,\mathbf{i} - P_y\,\mathbf{j} - P_z\,\mathbf{k}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta P_x\,\mathbf{i} + \Delta P_y\,\mathbf{j} + \Delta P_z\,\mathbf{k}}{\Delta t}$$

$$\therefore \quad \frac{d\mathbf{P}}{dt} = \frac{dP_x}{dt}\mathbf{i} + \frac{dP_y}{dt}\mathbf{j} + \frac{dP_z}{dt}\mathbf{k} \tag{3.6}$$

The following operations are valid with respect to vector functions

$$\frac{d}{dt}(\mathbf{P} + \mathbf{Q}) = \frac{d\mathbf{P}}{dt} + \frac{d\mathbf{Q}}{dt} \tag{3.7}$$

$$\frac{d}{dt}(\mathbf{P} \cdot \mathbf{Q}) = \frac{d\mathbf{P}}{dt} \cdot \mathbf{Q} + \mathbf{P} \cdot \frac{d\mathbf{Q}}{dt} \tag{3.8}$$

$$\frac{d}{dt}(\mathbf{P} \times \mathbf{Q}) = \frac{d\mathbf{P}}{dt} \times \mathbf{Q} + \mathbf{P} \times \frac{d\mathbf{Q}}{dt} \tag{3.9}$$

$$\frac{d}{dt}(f\mathbf{P}) = f\frac{d\mathbf{P}}{dt} + \frac{df}{dt}\mathbf{P} \tag{3.10}$$

(Where f is a scalar function of t.)

3.4 RECTANGULAR COMPONENTS OF VELOCITY AND ACCELERATION

If at a given instant the particle is located at point $P(x, y, z)$ along a curve as shown in Fig. 3.7, its location is defined by the position vector **r** having components $r_x = x$, $r_y = y$ and $r_z = z$.
As such,

$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k} \tag{3.11}$$

Where x, y and z are functions of t.
The first time derivative of **r** yields the velocity **v** of the particle.

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$$

or

$$\mathbf{v} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j} + \dot{z}\,\mathbf{k} \tag{3.12}$$

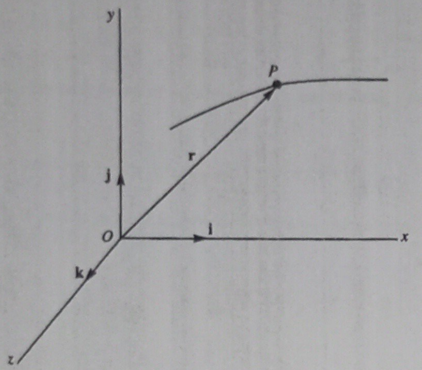


Fig. 3.7

The acceleration of the particle shall be obtained by taking the time derivative of **v**.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j} + \frac{d^2z}{dt^2}\mathbf{k}$$

or

$$\mathbf{a} = \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j} + \ddot{z}\,\mathbf{k} \qquad (3.13)$$

From Equations (3.12) and (3.13), the scalar components of the velocity and acceleration are

$$v_x = \dot{x} \qquad v_y = \dot{y} \qquad v_z = \dot{z} \qquad (3.14)$$

$$a_x = \ddot{x} \qquad a_y = \ddot{y} \qquad a_z = \ddot{z} \qquad (3.15)$$

To study the free-flight motion of a projectile, rectangular components of velocity and acceleration are frequently used. Let us consider the motion of a projectile shown in Fig. 3.8.

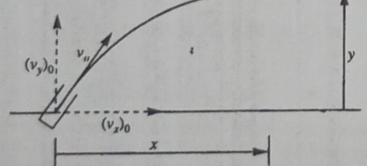


Fig. 3.8 Motion of a projectile

When air resistance is neglected, the only force acting on the projectile is its weight, which creates a constant downward acceleration of approximately $a_y = a = 9.81$ m/s^2.

Hence the components of the acceleration are

$$a_x = \ddot{x} = 0$$

$$a_y = \ddot{y} = -a = -9.81 \text{ m/s}^2$$

$$a_z = \ddot{z} = 0$$

If the components of the initial velocity of the projectile are $(v_x)_0$, $(v_y)_0$, $(v_z)_0$, then

$$v_x = \dot{x} = (v_x)_0$$

$$v_y = \dot{y} = (v_y)_0 - at$$

$$v_z = \dot{z} = (v_z)_0$$

which are obtained by integrating the components of acceleration.

Similarly the position coordinates will be obtained by integrating the components of velocity.

$$s_x = (s_x)_0 + (v_x)_0\, t$$

$$s_y = (s_y)_0 + (v_y)_0\, t - \frac{1}{2}at^2$$

$$s_z = (s_z)_0 + (v_z)_0\, t$$

If the path of the projectile is defined in the x-y plane, the equations of motion reduce to

$$v_x = (v_x)_0 \qquad v_y = (v_y)_0 - at \qquad v_z = 0 \qquad \because (s_x)_0 = (s_y)_0 = (s_z)_0 = 0$$

$$s_x = (v_x)_0\, t \qquad s_y = (v_y)_0\, t - \frac{1}{2}at^2 \quad s_z = 0 \qquad \text{and } (v_z)_0 = 0$$

Example 3.1 The motion of a particle is given by the equations $x = 3\,t^3 - 12\,t^2$ and $y = 4.5\,t^2 - 18\,t$ where x and y are expressed in metres and t in seconds. Determine the velocity and acceleration when (a) $t = 2$ s and (b) $t = 3$ s.

The equations of motion are

$$x = 3\,t^3 - 12\,t^2 \qquad y = 4.5\,t^2 - 18\,t$$

$$\therefore \quad v_x = 9\,t^2 - 24\,t \qquad \text{and} \qquad v_y = 9\,t - 18$$

$$a_x = 18\,t - 24 \qquad a_y = 9$$

(a) At t = 2 s,

$$v_x = -12 \text{ m/s} \qquad v_y = 0 \qquad \therefore v = 12 \text{ m/s} \leftarrow \qquad \text{(Ans.)}$$

$$a_x = 12 \text{ m/s}^2 \qquad a_y = 9 \text{ m/s}^2 \qquad \therefore a = 15 \text{ m/s}^2 \angle 37^\circ \qquad \text{(Ans.)}$$

(b) At t = 3 s,

$v_x = 9$ m/s $\quad v_y = 9$ m/s $\quad \therefore\ v = 12.73$ m/s $\angle 45°$ (Ans.)

$a_x = 30$ m/s^2 $\quad a_y = 9$ m/s^2 $\quad \therefore\ a = 31.32$ m/s^2 $\angle 17°$ (Ans.)

Example 3.2 The position vector of a point which moves along a space curve is given by

$$\mathbf{r} = \left[\frac{t^3}{3} - \frac{3t^2}{4}\right]\mathbf{i} + \left[\frac{t^4}{24}\right]\mathbf{j} + \left[\frac{2t^3}{3} - \frac{3t^2}{2}\right]\mathbf{k}$$

Where **r** is in metres and t is in seconds. Determine the velocity **v** and the **a** acceleration when $t = 2$ s.

(a) Velocity v

We know that,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

$$= \frac{d}{dt}\left\{\left[\frac{t^3}{3} - \frac{3t^2}{4}\right]\mathbf{i} + \left[\frac{t^4}{24}\right]\mathbf{j} + \left[\frac{2t^3}{3} - \frac{3t^2}{2}\right]\mathbf{k}\right\}$$

$$= \left[\frac{3t^2}{3} - \frac{6t}{4}\right]\mathbf{i} + \left[\frac{4t^3}{24}\right]\mathbf{j} + \left[\frac{6t^2}{3} - \frac{6t}{2}\right]\mathbf{k}$$

$$= (t^2 - 1.5\,t)\,\mathbf{i} + (0.17\,t^3)\,\mathbf{j} + (2\,t^2 - 3\,t)\,\mathbf{k}$$

When $t = 2$ s,

$$\mathbf{v} = \mathbf{i} + 1.36\,\mathbf{j} + 2\,\mathbf{k}\ \text{(m/s)} \qquad \text{(Ans.)}$$

(b) Acceleration a

We know that

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

$$= \frac{d}{dt}\left[(t^2 - 1.5t)\,\mathbf{i} + (0.17t^3)\,\mathbf{j} + (2t^2 - 3t)\,\mathbf{k}\right]$$

$$= (2\,t - 1.5)\,\mathbf{i} + (0.51\,t^2)\,\mathbf{j} + (4t - 3)\,\mathbf{k}$$

When $t = 2$ s,

$$\mathbf{a} = 2.5\,\mathbf{i} + 2.04\,\mathbf{j} + 5\,\mathbf{k}\ (\text{m/s}^2) \qquad \text{(Ans.)}$$

Example 3.3 A projectile is fired from a weapon with an initial velocity of 200 m/s at an angle of 20° with the horizontal as shown in Fig. 3.9. Neglecting air resistance, find (a) the maximum height reached by the projectile and the time taken to

reach this height, (b) the duration of the flight and the distance from the weapon at which the projectile strikes the ground.

Let us consider the vertical and horizontal motion of the projectile separately.

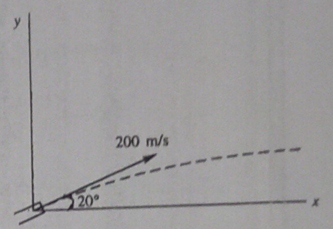


Fig. 3.9

Vertical Motion

The vertical motion will be uniformly accelerated motion. Let the initial vertical velocity be $(v_y)_0$ as shown in Fig. 3.10.

$$(v_y)_0 = 200 \sin 20°$$

$$= 68.4 \text{ m/s}$$

$$a = -9.81 \text{ m/s}^2$$

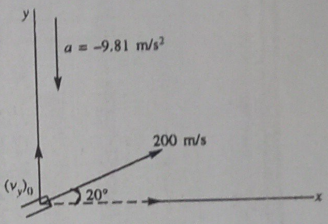


Fig. 3.10

The equations of uniformly accelerated motion are:

$$(v_y) = (v_y)_0 + at$$

$$\Rightarrow \quad v_y = 68.4 - 9.81\,t \qquad (1)$$

$$s_y = (v_y)_0\,t + \frac{1}{2}\,a\,t^2$$

$$\Rightarrow \quad s_y = 68.4\,t - 4.905\,t^2 \qquad (2)$$

$$(v_y)^2 = (v_y)_0^2 + 2a\left[s_y - (s_y)_0\right] \Rightarrow \qquad v_y^2 = 4678.6 - 19.62\,s_y \quad (3)$$

Horizontal Motion

The horizontal motion will be uniform motion. Let the initial horizontal velocity be $(v_x)_0$ as shown in Fig. 3.11.

$$(v_x)_0 = 200 \cos 20°$$

$$= 187.9 \text{ m/s}$$

We have the equation of uniform motion as

$$s_x = (s_x)_0 + (v_x)_0\,t$$

$$= 187.9\,t \qquad (4)$$

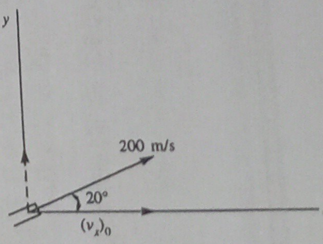


Fig. 3.11

(a) Maximum height reached and time taken to reach maximum height

When the maximum height is reached,

$$v_y = 0$$

Substituting $v_y = 0$ in (1),

$$68.4 - 9.81\, t = 0$$

$$\therefore \quad t = 6.97 \text{ s} \quad \text{(Ans.)}$$

Substituting the value of t in (2)

$$s_y = 68.4 \times 6.97 - 4.905 \times 6.97^2$$

$$\Rightarrow \quad s_y = 238.45 \text{ m} \quad \text{(Ans.)}$$

The above result may be checked using $v_y = 0$ in (3),

$$\Rightarrow \quad 0 = 4678.6 - 19.62\, s_y$$

$$\therefore \quad s_y = 238.45 \text{ m} \quad \text{(Ans.)}$$

(b) Duration of flight and horizontal distance

When the projectile strikes the ground, $s_y = 0$

Substituting $s_y = 0$ in (2),

$$0 = 68.4\, t - 4.905\, t^2$$

$$\Rightarrow \quad t\,(68.4 - 4.905\, t) = 0$$

$$t = 0, \; 13.94 \text{ s} \quad \text{(Ans.)}$$

The first value corresponds to the initial time. The projectile strikes the ground after 13.94 s after firing.

The horizontal distance from the weapon at which the projectile strikes the ground shall be found using $t = 13.94$ s in (4)

$$\therefore \quad s_x = 187.9 \times 13.94$$

$$s_x = 2619.3 \text{ m.} \quad \text{(Ans.)}$$

Example 3.4 A projectile is fired from A at an angle of 30° with the horizontal as shown in Fig. 3.12. Determine the initial velocity v_A at which the projectile is fired to hit the target at B.

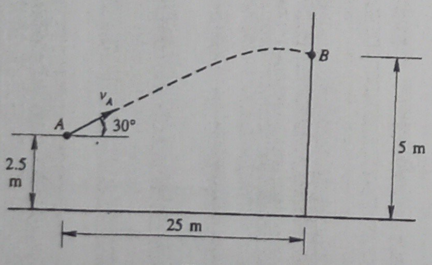


Fig. 3.12

Let us consider the horizontal and the vertical motion separately.

Horizontal Motion

Let the initial horizontal velocity at A be $(v_x)_0$ as shown in Fig. 3.13.

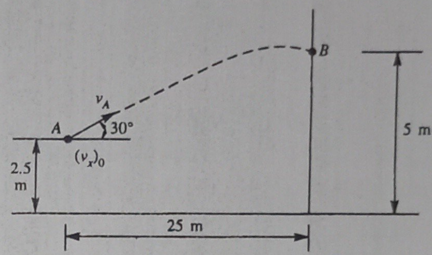


Fig. 3.13

$$(v_x)_0 = v_A \cos 30°$$

$$= 0.87\, v_A$$

The horizontal motion will be uniform motion.

As such,

$$s_x = (s_x)_0 + (v_x)_0\, t$$

$$\Rightarrow \quad 25 = 0.87\, v_A\, t \qquad (1)$$

Vertical Motion

Let the initial vertical velocity at A be $(v_y)_0$ as shown in Fig. 3.14.

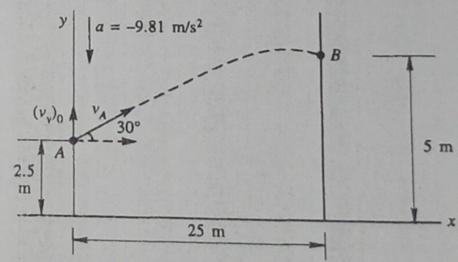


Fig. 3.14

$$(v_y)_0 = v_A \sin 30°$$

$$= 0.5\, v_A$$

The vertical motion will be uniformly accelerated motion.
As such,

$$s_y = (s_y)_0 + (v_y)_0\, t + \frac{1}{2}\, at^2$$

$$s_y = (s_y)_0 + 0.5\, v_A\, t - 4.905\, t^2 \qquad (2)$$

When the projectile hits the target at B,

$$s_y = 5 \text{ m}$$

From (1)

$$v_A = \frac{28.74}{t}$$

Substituting the value of v_A in (2)

$$5 = 2.5 + 0.5 \times \frac{28.74}{t} \times t - 4.905\, t^2$$

$$\Rightarrow \quad t = 1.56 \text{ s}$$

Hence,

$$v_A = 18.42 \text{ m/s} \qquad \text{(Ans.)}$$

Example 3.5 The aeroplane shown in Fig. 3.15 is flying horizontally with a constant speed of 60 m/s at an altitude of 900 m. If the pilot drops a package with the same horizontal speed of 60 m/s, determine the velocity when the package hits the target B and the angle θ at which he must sight the target so that when the package is released it falls and strikes the target.

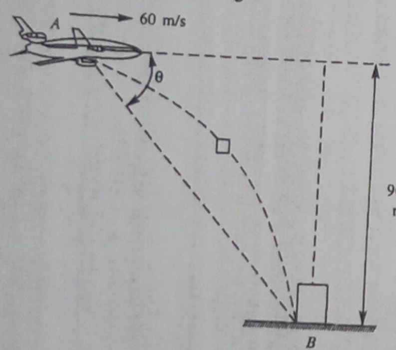


Fig. 3.15

The equation of motion in the vertical direction is

$$s_y = (s_y)_0 + (v_y)_0\, t + \frac{1}{2}\, at^2$$

Using

$$s_y = 0,\ (s_y)_0 = 900 \text{ m} \quad \text{and} \quad (v_y)_0 = 0,$$

$$0 = 900 - \frac{1}{2} \times 9.81 \times t^2$$

$$\Rightarrow \quad t = 13.55 \text{ s}$$

The component of velocity in the vertical direction is given by,

$$v_y = (v_y)_0 + at$$

$$= -9.81\, t$$

$$= -9.81 \times 13.55$$

$$= -132.93 \text{ m/s}$$

$$v_x = (v_x)_0 + (v_x)_p \quad [(v_x)_p \rightarrow \text{horizontal velocity at which the package is dropped}]$$

$$= 60 + 60$$

$$= 120 \text{ m/s}$$

The velocity when the package hits the target shall be found from

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{120^2 + 132.93^2}$$

$$\therefore \quad v = 179 \text{ m/s} \qquad \text{(Ans.)}$$

The direction of the velocity shall be found from

$$\tan\theta = \frac{v_y}{v_x}$$

$$= \frac{132.93}{120}$$

$$= 1.11$$

Hence,

$$\theta = 47.9^\circ \qquad \text{(Ans.)}$$

3.5 MOTION RELATIVE TO A FRAME IN TRANSLATION

The motion of a particle has been determined using a single fixed frame of reference in the preceding section. In cases where the path of motion for a particle is complicated, it will be better to analyse the motion using two or more frames of reference. Among these frames, one should be designated as a fixed frame of reference and the others which are not attached to this frame should be defined as moving frames of reference.

$$= -10 - 10$$

$$= -20 \text{ m/s}$$

$$v_{DB} = 20 \text{ m/s} \uparrow \qquad \text{(Ans.)}$$

Short Answer Questions

2.1 In kinematics, the geometry of motion is considered without reference to the cause of the motion. State true or false.
2.2 Define a particle.
2.3 What is rectilinear motion?
2.4 What do you mean by the term position coordinate?
2.5 Explain the term displacement.
2.6 What is the distinction between the distance travelled and the displacement of a particle ?
2.7 Explain briefly the average velocity and instantaneous velocity.
2.8 State clearly the difference between average acceleration and instantaneous acceleration.
2.9 When do you say that a particle is decelerating?
2.10 What are motion curves?
2.11 The slope of the s-t curve at any given time will be equal to the velocity v at that time. How?
2.12 How is the motion of a particle characterised?
2.13 What is a uniform rectilinear motion?
2.14 Give the relation between displacement, velocity and time in case of a uniform rectilinear motion.
2.15 Briefly describe uniformly accelerated rectilinear motion.
2.16 List out any two relations derived from uniformly accelerated motion of a particle.
2.17 What is relative position coordinate and relative velocity?
2.18 Define relative acceleration.
2.19 Give two examples for motion of several particles.
2.20 Briefly explain the term dependent motion.

Exercise Problems

2.1 The motion of a particle is given by the relation $s = t^3 - 6t^2 + 16t - 8$, where s is expressed in metres and t in seconds. Determine the position, velocity and acceleration when $t = 4$ s. Also draw the motion curves.

Ans.

$$s = 24 \text{ m}$$

$$v = 16 \text{ m/s}$$

$$a = 12 \text{ m/s}^2$$

2.2 The acceleration of a particle is given by the relation $a = -3$ m/s². If $v = 12$ m/s and $s = 0$ when $t = 0$, determine the velocity, position and distance travelled when $t = 6$ s.

Ans.

Velocity, $v = -6$ m/s

Position, $s = 18$ m

Distance travelled = 30 m

2.3 The position of a particle is given by the relation $s = 1.5t^3 - 9t^2 - 22.5t + 60$, where s is expressed in metres and t in seconds. Determine (a) the time at which the velocity will be zero, (b) the position and distance travelled by the particle at that time, (c) the acceleration of the particle at that time and (d) the distance travelled by the particle from $t = 4$ s to $t = 6$ s.

Ans.

(a) $t = 5$ s
(b) Position = -90 m
Distance travelled = -150 m
(c) $a = 27$ m/s²
(d) 27 m

2.4 A stone is thrown vertically upward from the top of a 30 m high building with a velocity of 15 m/s. Taking the acceleration of stone as 9.81 m/s² and taking that as constant, determine (a) the velocity v and elevations s_y of the stone above the ground at any time t, (b) the maximum altitude reached by the stone and (c) time when the stone strikes the ground.

Ans.

(a) $v = 15 - 9.81t$, $s_y = 30 + 15t - 4.905t^2$
(b) $(s_y)_{max} = 41.47$ m
(c) $t = 4.44$ s

2.5 A particle moves along a horizontal straight line with an acceleration $a = -ks$. Determine (a) the value of k such that $v = 20$ m/s when $s = 0$ and $s = 4$ m when $v = 0$, (b) the velocity of the particle when $s = 3$ m.

Ans.

$k = 25$ (s^{-2})

$v = 13.23$ (m/s)

2.6 A bicyclist has an acceleration of 3.6 m/s². If he starts from rest, determine his velocity and position at $t = 8$ s.

Ans.

$v = 28.8$ m/s

$s = 115.2$ m

2.7 A ball is thrown vertically upward with a velocity of 18 m/s. Two seconds later, another ball is thrown upward with a velocity of 13.5 m/s. At what position above the ground will they meet?

Ans.

Both the balls will meet at 8.9 m above the ground.

2.8 Two vehicles approach each other in opposite lanes of a straight horizontal

Curvilinear Motion

If a particle is moving along curved path then it is said to perform ***curvilinear motion***.

12.3 Position, Velocity and Acceleration for Curvilinear Motion

Position

Consider the motion of a particle along a curved path, as shown in Fig. 12.3-i. It is represented by a position vector $\overline{r}$ which is drawn from the origin 'O' of the fixed reference axis to particle 'P'. The line OP is called *position vector*. As the particle will move along the curved path, the value of $\overline{r}$ will go on changing.

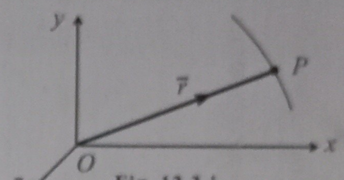


Fig. 12.3-i

Velocity

Consider after a short interval of time, Δt particle has occupied new position P' simultaneously the position vector $\overline{r}$ will change to $\overline{r}'$.

The vector joining P and P' is the change in position vector Δr during the time interval Δt.

$\therefore$ The average velocity $v = \dfrac{\Delta \overline{r}}{\Delta t}$

For very small interval of time $\Delta t \to 0$

Instantaneous velocity at P is $\lim\limits_{\Delta t \to 0} \dfrac{\Delta \overline{r}}{\Delta t}$

$$\therefore \; v = \frac{dr}{dt}$$

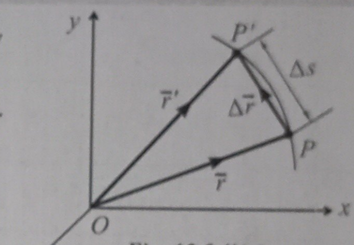


Fig. 12.3-ii(a)

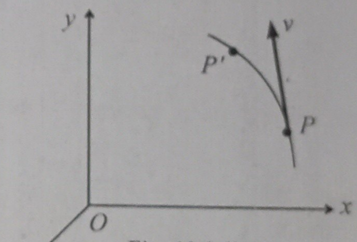


Fig. 12.3-ii(b)

Here during a small interval of time Δt, the particle moves a distance Δs along the curve.

$\therefore$ The magnitude of velocity called speed is given by relation

$$\text{Speed} = |\overline{v}| = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Note : In curvilinear motion, velocity of particle is always tangent to the curved path at every instant.

Acceleration

As the direction of velocity is continuously changing instant to instant in curvilinear motion it is responsible to develop acceleration also at every instant.

Consider the velocity of the particle at P to be v and at position P' be v'.

$\therefore$ Average acceleration $a = \dfrac{\Delta \overline{v}}{\Delta t}$

For very small interval of time $\Delta t \to 0$

Instantaneous velocity at P is $\lim\limits_{\Delta t \to 0} \dfrac{\Delta \overline{v}}{\Delta t}$

$$\therefore \; a = \frac{dv}{dt}$$

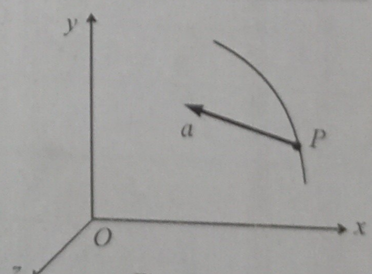


Fig. 12.3-iii

In rectilinear motion displacement, velocity and acceleration are always directed along the path of particle. Whereas in curvilinear motion it change its direction instant to instant. Therefore, the analysis of curvilinear motion is done by considering different components system. There are two methods for analysis in terms of different component system.

(1) Curvilinear motion by Rectangular Component System.

(2) Curvilinear motion by Tangential and Normal Component System.

12.4 Curvilinear Motion by Rectangular Component System

If a particle is moving along a curved path, its motion can be splitted into x, y and z direction as independently performing rectilinear motions.

Thus for curvilinear motion we can have a relation as follows.

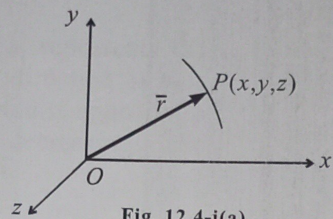


Fig. 12.4-i(a)

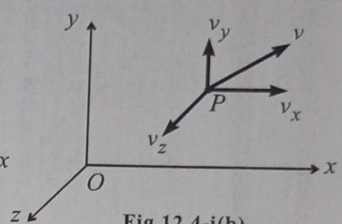


Fig.12.4-i(b)

Vector Form	$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$	$\bar{v} = \frac{d\bar{r}}{dt} = v_x\bar{i} + v_y\bar{j} + v_z\bar{k}$	$a = \frac{dv}{dt} = a_x\bar{i} + a_y\bar{j} + a_z\bar{k}$
Magnitude	$r = \sqrt{x^2 + y^2 + z^2}$	$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$	$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Direction is given by the relation

$$\cos\alpha = \frac{x}{r} = \frac{v_x}{v} = \frac{a_x}{a}\ ;\ \cos\beta = \frac{y}{r} = \frac{v_y}{v} = \frac{a_y}{a}\ ;\ \cos\gamma = \frac{z}{r} = \frac{v_z}{v} = \frac{a_z}{a}\ ;$$

While dealing with coplanar motion we can consider that the particle is moving in xy plane. Its rectangular component system will be as follows.

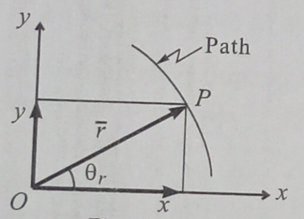


Fig. 12.4-ii(a)

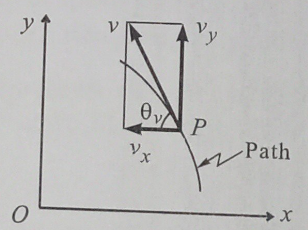


Fig. 12.4-ii(b)

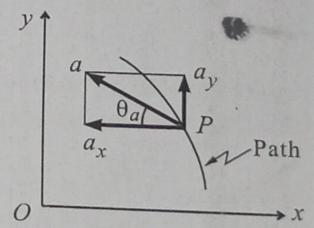


Fig. 12.4-ii(c)

Vector Form	$\bar{r} = x\bar{i} + y\bar{j}$	$\bar{v} = v_x i + v_y j$	$a = a_x\bar{i} + a_y\bar{j}$
Magnitude	$r = \sqrt{x^2 + y^2}$	$v = \sqrt{v_x^2 + v_y^2}$	$a = \sqrt{a_x^2 + a_y^2}$
	$\tan\theta_r = \frac{y}{x}$	$\tan\theta_v = \frac{v_y}{v_x}$	$\tan\theta_a = \frac{a_y}{a_x}$

The radius of curvature is calculated by following relation:

$$\rho = \left| \frac{(v_x^2 + v_y^2)^{3/2}}{v_x a_y - v_y a_x} \right|$$

urvilinear Motion by Tangential and Normal Component System

rvilinear motion the acceleration is splitted into two ponents, one along tangential direction (a_t) and other along normal direction (a_n).

$$\therefore \bar{a} = a_t \bar{e}_t + a_n \bar{e}_n$$

$$\text{Magnitude } a = \sqrt{a_t^2 + a_n^2}$$

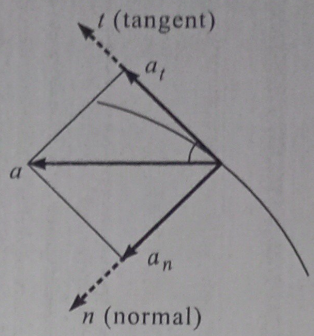


Fig. 12.5-i

Tangential component of acceleration a_t represents the rate of change of speed of a particle. The direction of a_t is along the velocity if speed is increasing, and is opposite to the direction of velocity if speed is decreasing. The direction of velocity is always tangential.

For constant speed in curvilinear motion

$$a_t = \frac{dv}{dt} = 0$$

and following equation of motion is applicable

$$v = u + a_t t$$

$$s = ut + \frac{1}{2} a_t t^2$$

$$v^2 = u^2 + 2a_t s$$

where s is the distance covered along curved path.

u is initial speed and v is final speed and

a_t is the component of acceleration along tangential direction.

Normal component of acceleration a_n represents the change in direction of motion and is always directed towards the centre of curvature of the path. It is also called as *centripetal acceleration*.

$$\text{Magnitude of } a_n = \frac{v^2}{\rho}$$

where v is the speed at the instant and ρ is the radius of curvature.

If curve is following a path defined by $y = f(x)$ then

$$\rho = \left| \frac{\left[1 + \left(\frac{dy}{dt}\right)^2\right]^{3/2}}{\frac{d^2y}{dt^2}} \right|$$

Direction of velocity is always tangential

$$\therefore \text{ Slope } = \frac{dy}{dx} = \tan\theta$$

$$\therefore v_x = v\cos\theta \text{ and } v_y = v\sin\theta$$

olved Problems Based on Curvilinear Motion

lem 19

point moves along the path $y = \frac{1}{3}x^2$ with a constant speed of 8 m/s. What are the x and y components of the velocity when $x = 3$? What is the acceleration of the point when $x = 3$?

Solution

Given : v = 8 m/s is constant;

$$a_t = 0 \qquad a_t = \frac{dv}{dt} = 0$$

$$\therefore a_n = a \qquad [\because a_t = 0]$$

$$\therefore a_n = \frac{v^2}{\rho}$$

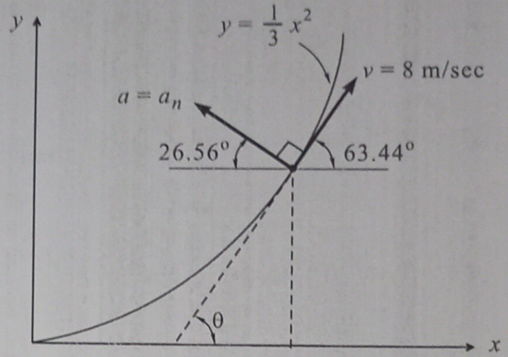


Fig. 12.19

$$y = \frac{1}{3}x^2$$

$$\frac{dy}{dx} = \frac{2}{3}x$$

$$\left(\frac{dy}{dx}\right)_{x=3} = \frac{2}{3} \times 3 = 2$$

$$\frac{d^2y}{dx^2} = \frac{2}{3}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=3} = \frac{2}{3} \qquad \textit{Ans.}$$

$$\therefore \rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right| = \left| \frac{\left[1 + (2)^2\right]^{3/2}}{\frac{2}{3}} \right|$$

$$\rho = 16.77 \text{ m}$$

$$a_n = \frac{v^2}{\rho} = \frac{(8)^2}{16.77} = 3.82 \text{ m/s}^2$$

$$\tan\theta = \left(\frac{dy}{dx}\right)_{x=3} = 2$$

$$\theta = 63.44°$$

$$v_x = v\cos\theta = 8\cos 63.44 = 3.58 \text{ m/s} \qquad \textit{Ans.}$$

$$v_y = v\sin\theta = 8\sin 63.44 = 7.15 \text{ m/s} \qquad \textit{Ans.}$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0 + (3.82)^2}$$

$$a = 3.82 \text{ m/s}^2 \quad (26.56°) \qquad \textit{Ans.}$$

20

…cle moves in the x-y plane with velocity components $v_x = 8t - 2$ and $v_y = 2$. If it passes …gh the point $(x, y) = (14, 4)$ at $t = 2$ seconds, determine the equation of the path traced by particle. Find also the resultant acceleration at $t = 2$ seconds.

…olution

Given : $v_x = 8t - 2$; $v_y = 2$

$$\frac{dx}{dt} = 8t - 2 \; ; \qquad \frac{dy}{dt} = 2$$

Integrating we get

$$\therefore x = 4t^2 - 2t + c_1$$

At t = 2 seconds, $x = 14$ m

$$14 = 4(2)^2 - 2(2) + c_1$$

$$c_1 = 2$$

$$\therefore x = 4t^2 - 2t + 2$$

$$y = 2t + c_2$$

At t = 2 seconds, $y = 4$ m

$$4 = 2(2) + c_2$$

$$c_2 = 0$$

$$y = 2t$$

$$x = (2t)^2 - 2t + 2 = y^2 - y + 2 \qquad (\because y = 2t)$$

$x = y^2 - y + 2$ is the equation of path. ***Ans.***

(Any equation of path does not have time)

$$\therefore \bar{v} = (8t - 2)\,\bar{i} + 2\,\bar{j}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = 8\,\bar{i} \text{ m/s}^2$$

$\therefore \bar{a} = 8\,\bar{i}$ m/s^2 ***Ans.***

Problem 21

A rocket follows a path such that its acceleration is given by $\bar{a} = (4\;i + t\;j)$ m/s^2 at $\bar{r} = 0$, it starts from rest. At $t = 10$ seconds. Determine **(i)** speed of the rocket, **(ii)** radius of curvature of its path and **(iii)** magnitude of normal and tangential components of acceleration.

Solution

(i) $\bar{a} = 4\,i + t\,j$ at $\bar{r} = 0, \bar{v} = 0, t = 0$

Integrating, we get

$$\bar{v} = 4\,t\,i + \frac{t^2}{2}j + c_1$$

At $t = 0, v = 0 \;\therefore\; c_1 = 0$

$$\bar{v} = 4\,t\,i + \frac{t^2}{2}j$$

At t = 10 seconds

$$\bar{a} = 4\,i + 10\,j$$

$$a = \sqrt{4^2 + 10^2}$$

$a = 10.77$ m/s^2 ***Ans.***

$$\bar{v} = 40\,i + 50\,j$$

$$v = \sqrt{40^2 + 50^2}$$

$v = 64.03$ m/s^2 ***Ans.***

4 and $a_y = 10$

adius of curvature ρ

$$\rho = \frac{(v_x^2 + v_y^2)^{3/2}}{v_x a_y - v_y a_x} = \frac{(40^2 + 50^2)^{3/2}}{40 \times 10 - 50 \times 4}$$

$\rho = 1312.64$ m **Ans.**

(iii) Component of normal acceleration a_n

$$a_n = \frac{v^2}{\rho} = \frac{64.03^2}{1312.64}$$

$a_n = 3.123$ m/s^2 **Ans.**

Component of tangential acceleration a_t

$$a_t = \sqrt{a^2 - a_n^2} = \sqrt{10.77^2 - 3.123^2}$$

$a_t = 10.31$ m/s^2 **Ans.**

Problem 22

A car travels along a vertical curve on a road, the equation of the curve being $x^2 = 200y$ (x-horizontal and y-vertical distances in m). The speed of the car is constant and equal to 72 km/hr. **(i)** Find its acceleration when the car is at the deepest point on the curve and **(ii)** What is the radius of curvature of the curve at this point ?

Solution

$x^2 = 200y$

$\therefore\ y = \frac{x^2}{200}$

$v = 72$ km/hr $= 72 \times \frac{5}{18}$ m/sec

$v = 20$ m/s (constant)

Now, $a_t = 0$

$\therefore\ y = \frac{x^2}{200}$

$$\frac{dy}{dx} = \frac{1}{200}(2x)$$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=0} = 0\ ; \quad \left(\frac{d^2y}{dx^2}\right)_{\text{at } x=0} = \frac{1}{100}$$

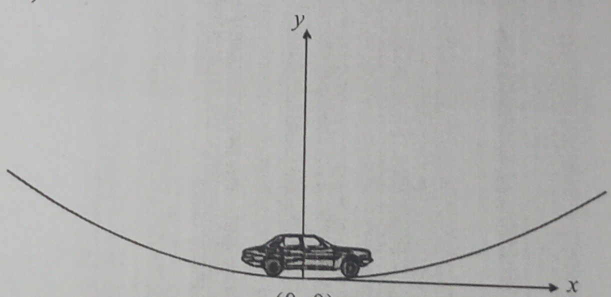


Fig. 12.22

$$\therefore\ \rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right| = \left| \frac{\left[1 + (0)^2\right]^{3/2}}{\frac{1}{100}} \right|$$

$\rho = 100$ m **Ans.**

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{100}$$

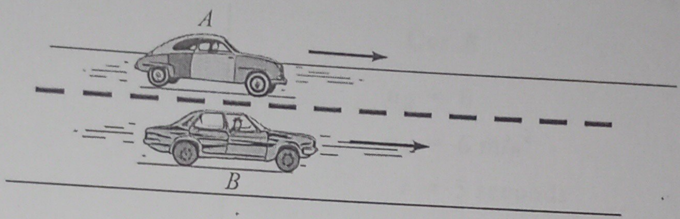
$\therefore\ a_n = 4$ m/s^2 **Ans.**

ATIVE MOTION

nerally, a moving body is observed by a person who is at rest. Considering the observers position at rest we are developing fixed axis reference. Such a set of fixed axes is defined as *absolute* or *Newtonian* or *inertial frame of reference*. For most of the moving bodies, the Earth is regarded as fixed although Earth itself is moving in space. Motion referred with such fixed axis is called an *absolute motion*, which we have already dealt in previous discussion.

However, if the axes reference is attached to a moving object then such motion is termed as *relative motion*. It means person in moving object is observing another object which is also in motion.

Example 1



Cars A and B are moving in the same direction on road parallel to each other. Car A is moving with a speed of 60 km/hr and car B is moving with 80 km/hr (These are the absolute speeds of the cars). Car A in relation to car B is moving backward with speed 20 km/hr whereas car B in relation to car A is moving forward with speed of 20 km/hr. Observation of drivers of car A and car B w.r.t. each is developing relative motion between them.

Example 2

If a pilot of fighter plane wants to target another moving plane, then relative motion analysis is must.

12.7 Relative Motion Between Two Particles

Consider two particles A and B moving on different path as shown in Fig. 12.7-i. Here xOy is the fixed frame of reference. Therefore, absolute position of A is given by $r_A = OA$ and of B is $r_B = OB$. Therefore, relative position of B w.r.t. A is written as $r_{B/A}$.

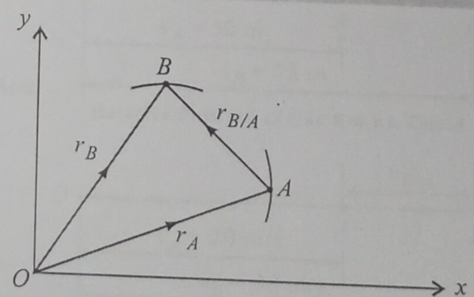


Fig. 12.7-i

By triangle law of vector addition, we have

$$r_A + r_{B/A} = r_B$$

$\therefore$ Relative position of B w.r.t. A $\quad r_{B/A} = r_B - r_A$ ..... (I)

Differentiating Eq. (I) w.r.t. t, we have

Relative velocity of B w.r.t. A $\quad v_{B/A} = v_B - v_A$ ..... (II)

Further differentiating Eq. (II) w.r.t. t, we have

Relative acceleration of B w.r.t. A $\quad a_{B/A} = a_B - a_A$ ..... (III)

...ved Problems Based on Relative Motion

Problem 23

Two cars A and B start from rest from point O at the same instant and travel towards right along a straight road as shown in Fig. 12.23. Car A moves with an acceleration of 4 m/s^2 and car B moves with an acceleration of 6 m/s^2. Find relative position, velocity and acceleration of car B w.r.t. car A 5 seconds from the start.

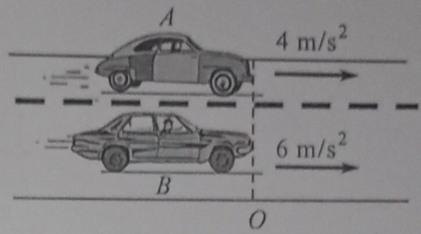


Fig. 12.23

Solution

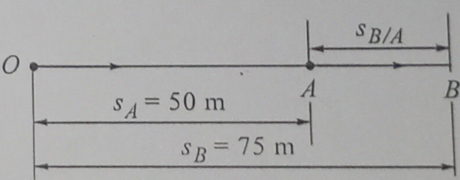
Car A	**Car B**
$u_A = 0$	$u_B = 0$
$a_A = 4 \text{ m/s}^2$	$a_B = 6 \text{ m/s}^2$
$t = 5$ seconds	$t = 5$ seconds
$v_A = u_A + a_A t$	$v_B = u_B + a_B t$
$v_A = 0 + 4 \times 5$	$v_B = 0 + 6 \times 5$
$v_A = 20 \text{ m/s}$	$v_B = 30 \text{ m/s}$
$s_A = u_A t + \frac{1}{2} a_A t^2$	$s_B = u_B t + \frac{1}{2} a_B t^2$
$s_A = 0 + \frac{1}{2} \times 4 \times 5^2$	$s_B = 0 + \frac{1}{2} \times 6 \times 5^2$
$s_A = 50 \text{ m}$	$s_B = 75 \text{ m}$

Relative position of car B w.r.t. car A

$s_{B/A} = s_B - s_A$ | $s_{B/A} = s_B - s_A$

$s_{B/A} = 75\,i - 50\,i$ | $s_{B/A} = 75 - 50$

$s_{B/A} = 25\,i \text{ m}$ | $s_{B/A} = 25 \text{ m } (\rightarrow)$ ***Ans.***



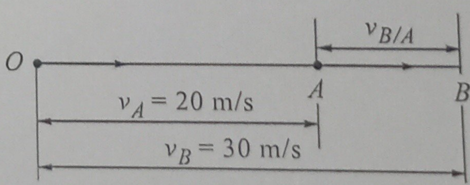
Relative Position of Car B w.r.t. Car A

Relative velocity of car B w.r.t. car A

$v_{B/A} = v_B - v_A$ | $v_{B/A} = v_B - v_A$

$v_{B/A} = 30\,i - 20\,i$ | $v_{B/A} = 30 - 20$

$v_{B/A} = 10\,i \text{ m/s}$ | $v_{B/A} = 10 \text{ m } (\rightarrow)$ ***Ans.***



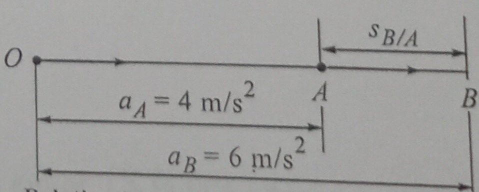
Relative Velocity of Car B w.r.t. Car A

Relative acceleration of car B w.r.t. car A

$a_{B/A} = a_B - a_A$ | $a_{B/A} = a_B - a_A$

$a_{B/A} = 6\,i - 4\,i$ | $a_{B/A} = 6 - 4$

$a_{B/A} = 2i \text{ m/s}^2$ | $a_{B/A} = 2 \text{ m/s}^2 (\rightarrow)$ ***Ans.***



Relative Acceleration of Car B w.r.t. Car A

...nt O in Fig. 12.24(a), a ship A travels in the North making an angle of 45° to the West ...velocity of 18 km/hr and ship B travels in the East with a velocity of 9 km/hr. Find the ...e velocity of ship B w.r.t. ship A.

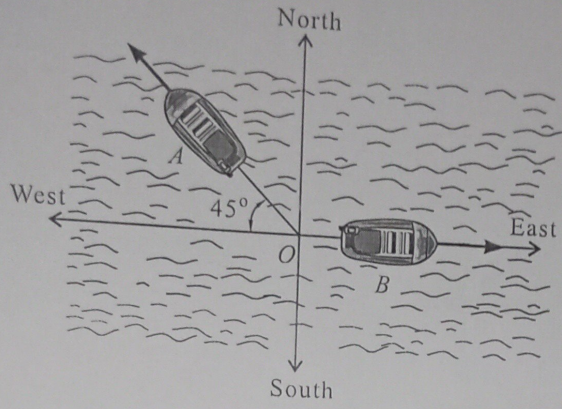


Fig. 12.24(a)

Solution

Refer to Fig. 12.24(b).

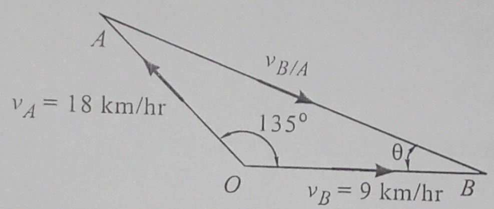


Fig. 12.24(b)

Consider the triangle law.

By cosine rule, we have

$$v_{B/A} = \sqrt{v_A^2 + v_B^2 - 2(v_A)(v_B)\cos 135^\circ}$$

$$v_{B/A} = \sqrt{18^2 + 9^2 - 2(18)(9)\cos 135^\circ}$$

$$v_{B/A} = 25.18 \text{ km/hr}$$

By sine rule, we have

$$\frac{v_{B/A}}{\sin 135^\circ} = \frac{v_A}{\sin\theta}$$

$$\frac{25.18}{\sin 135^\circ} = \frac{18}{\sin\theta}$$

$$\therefore \theta = 30.36^\circ$$

Relative velocity of ship B w.r.t. ship A is $v_{B/A} = 25.18$ km/hr $\left(\measuredangle 30.36^\circ\right)$ ***Ans.***

...em 25

...ure 12.25(a) shows cars A and B ... a distance of 35 m. Car A moves with a constant speed of 36 kmph and car B starts from rest with an acceleration of 1.5 m/s^2. Determine the relative (i) position, (ii) velocity and (iii) acceleration of car B w.r.t. car A 5 seconds after car A crosses the intersection.

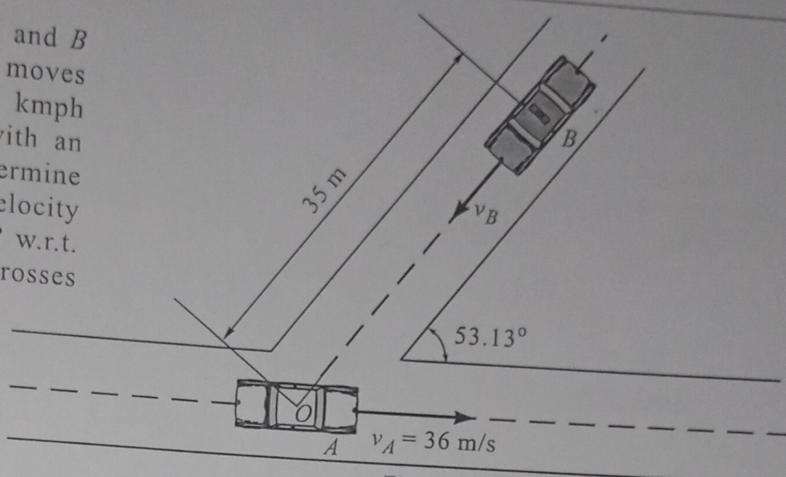


Fig. 12.25(a)

Solution

Car A (Uniform velocity)

$u_A = 10$ m/s, $t = 5$ seconds

$a_A = 0$

$s_A = u_A t$

$s_A = 10 \times 5$

$s_A = 50$ m

Car B (Uniform acceleration)

$u_B = 0$, $a_B = 1.5$ m/s^2, $t = 5$ seconds

Displacement of car B

$$s = u_B t + \frac{1}{2} a_B t^2$$

$$s = 0 + \frac{1}{2} \times 1.5 \times 5^2 = 18.75 \text{ m}$$

Initial distance from O is 35 m

Position of car B w.r.t. O after 5 seconds

$s_B = 35 - 18.75$

$s_B = 16.25$ m

$v_B = u_B + a_B t = 0 + 1.5 \times 5$

$v_B = 7.5$ m/s

(i) **Relative position of car B w.r.t. car A**

Consider the triangle law.

By cosine rule, we have

$$s_{B/A} = \sqrt{s_A^2 + s_B^2 - 2(s_A)(s_B)\cos 53.13^\circ}$$

$$s_{B/A} = \sqrt{50^2 + 16.25^2 - 2(50)(16.25)\cos 53.13^\circ}$$

$$s_{B/A} = 42.3 \text{ m}$$

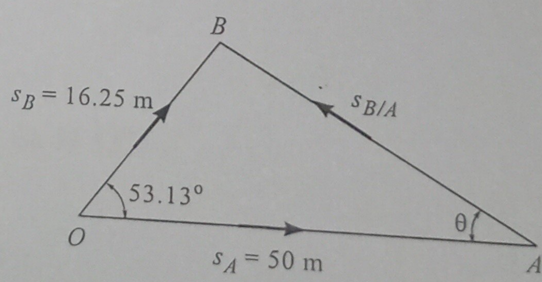


Fig. 12.25(b)

By sine rule, we have

$$\frac{s_A}{\sin\theta} = \frac{s_{B/A}}{\sin 53.13^\circ}$$

$$\frac{16.25}{\sin\theta} = \frac{42.3}{\sin 53.13^\circ} \quad \therefore \theta = 17.9^\circ$$

$\therefore$ Relative position of car B w.r.t. car A will be $s_{B/A} = 42.3$ m $\left(\underline{17.9^\circ}\right)$ ***Ans.***

Relative velocity of car B w.r.t. car A

Consider the triangle law.

By cosine rule, we have

$$v_{B/A} = \sqrt{v_A^2 + v_B^2 - 2(v_A)(v_B)\cos 126.87^\circ}$$

$$v_{B/A} = \sqrt{10^2 + 7.5^2 - 2(10)(7.5)\cos 126.87^\circ}$$

$$v_{B/A} = 15.69 \text{ m/s}$$

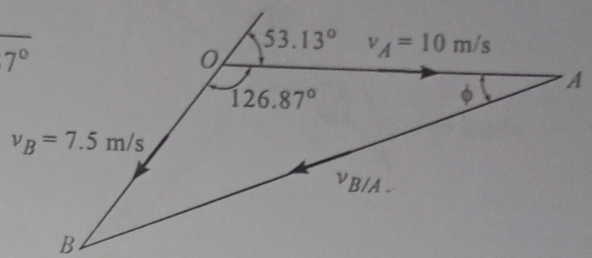


Fig. 12.25(c)

By sine rule, we have

$$\frac{v_{B/A}}{\sin 126.87^\circ} = \frac{v_B}{\sin\phi}$$

$$\frac{15.69}{\sin 126.87^\circ} = \frac{7.5}{\sin\phi} \qquad \therefore \phi = 22.48^\circ$$

$\therefore$ Relative velocity of car B w.r.t. car A will be $v_{B/A} = 15.69$ m/s (22.48°) *Ans.*

(iii) Relative acceleration of car B w.r.t. car A

$$a_{B/A} = a_B - a_A = 1.5 - 0$$

$$a_{B/A} = 1.5 \text{ m/s}^2 \ (53.13^\circ)$$ *Ans.*

Alternate Method

(i) Relative position of car B w.r.t. car A

$$s_A = 50\, i \text{ and } s_B = 16.25\cos 53.13^\circ\, i + 16.25\sin 53.13^\circ\, j$$

$$s_{B/A} = s_B - s_A$$

$$s_{B/A} = (16.25\cos 53.13^\circ\, i + 16.25\sin 53.13^\circ\, j) - 50\, i = -40.25\, i + 13\, j$$

Magnitude

$$s_{B/A} = \sqrt{(40.25)^2 + (13)^2}$$

$$s_{B/A} = 42.3 \text{ m}$$

Direction

$$\tan\theta = \frac{13}{40.25} \qquad \therefore \theta = 17.9^\circ$$

$$\therefore s_{B/A} = 42.3 \text{ m } (17.9^\circ)$$ *Ans.*

(ii) Relative velocity of car B w.r.t. car A

$$v_A = 10\, i \text{ and } v_B = -7.5\cos 53.13^\circ\, i - 7.5\sin 53.13^\circ\, j$$

$$v_B = -4.5\, i - 6\, j$$

$$v_{B/A} = v_B - v_A$$

$$v_{B/A} = (-4.5\, i - 6\, j) - (10\, i) = -14.5\, i - 6\, j$$

Magnitude

$$v_{B/A} = \sqrt{(14.5)^2 + (6)^2}$$

$$v_{B/A} = 15.69 \text{ m/s}$$

Direction

$$\tan\phi = \frac{6}{14.5} \qquad \therefore \phi = 22.48^\circ$$

$$\therefore v_{B/A} = 15.69 \text{ m/s } (22.48^\circ)$$ *Ans.*

26

A is travelling along a straight ay, while a truck B is moving g a circular curve of 150 m radius. e speed of car A is increased at the te of 1.5 m/s^2 and the speed of truck B is being decreased at the rate of 0.9 m/s^2. For the position shown in Fig. 12.26(a), determine the velocity of A relative to B and the acceleration of A relative to B. At this instant the speed of A is 75 km/hr and that of B is 40 km/hr.

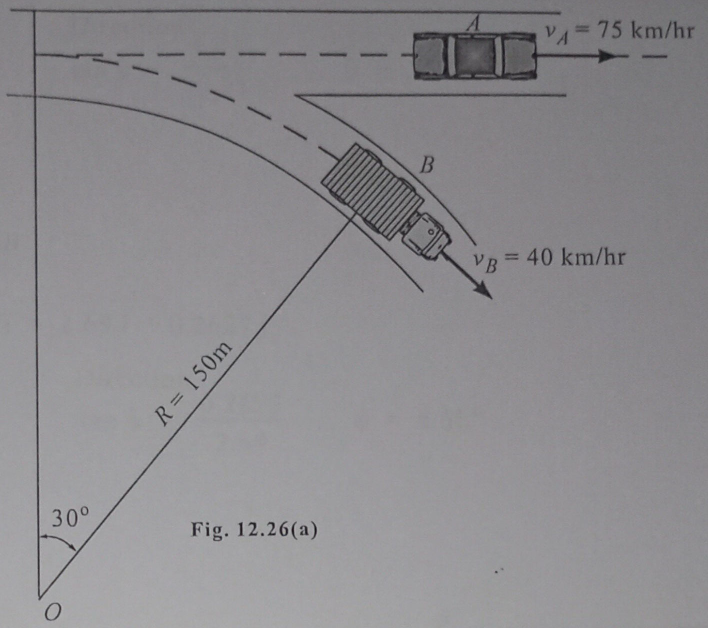


Fig. 12.26(a)

Solution

(i) **Motion of car A**

$$v_A = 75 \text{ km/hr} = 20.83 \text{ m/s}$$

$$\mathbf{v}_A = 20.83\, i$$

$$\mathbf{a}_A = 1.5\, i$$

Motion of truck B

$$v_B = 40 \text{ km/hr} = 11.11 \text{ m/s } (30^\circ)$$

$$\mathbf{v}_B = 11.11 \cos 30^\circ\, i - 11.11 \sin 30^\circ\, j$$

$$\mathbf{v}_B = 9.622\, i - 5.55\, j$$

Tangential component of acceleration,

$$a_t = 0.9 \text{ m/s}^2 \ (30^\circ)$$

Normal component of acceleration,

$$a_n = \frac{v^2}{\rho} = \frac{11.11^2}{150}$$

$$a_n = 0.823 \text{ m/s}^2 \ (60^\circ)$$

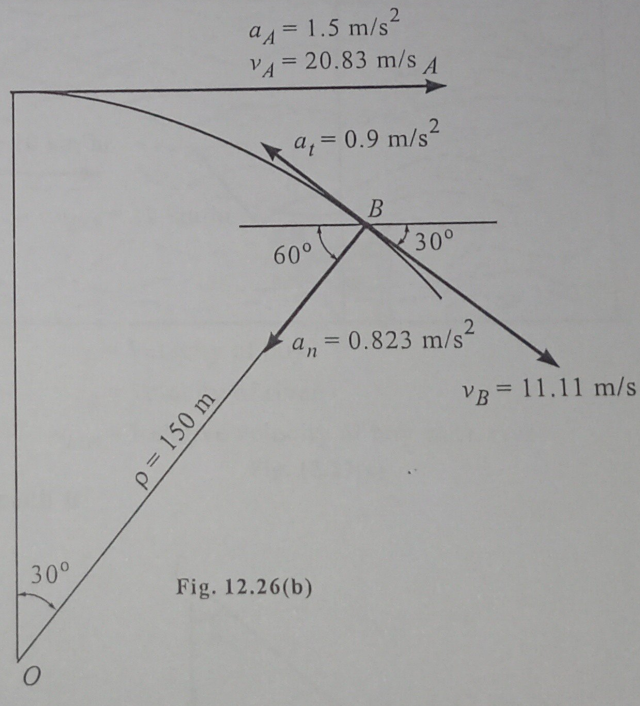


Fig. 12.26(b)

$$\mathbf{a}_B = (-0.9 \cos 30^\circ - 0.823 \cos 60^\circ)\, i + (0.9 \sin 30^\circ - 0.823 \sin 60^\circ)\, j$$

$$\mathbf{a}_B = -1.190\, i - 0.267\, j$$

(ii) **Relative velocity of A w.r.t. B**

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$\mathbf{v}_{A/B} = 20.83\, i - (9.622\, i - 5.55\, j) = 11.21\, i + 5.55\, j$$

gnitude

$v_{A/B} = \sqrt{(11.21)^2 + (5.55)^2}$

$v_{A/B} = 12.51$ m/s

$\therefore v_{A/B} = 12.51$ m/s $(\measuredangle 26.36^\circ)$ ***Ans.***

Direction

$\tan\theta = \dfrac{5.55}{11.21}$ $\therefore \theta = 26.36^\circ$

(iii) Relative acceleration of A w.r.t. B

$a_{A/B} = a_A - a_B$

$a_{A/B} = 1.5\,i - (-1.190\,i - 0.267\,j) = 2.69\,i + 0.2627\,j$

Magnitude

$a_{A/B} = \sqrt{(2.69)^2 + (0.2627)^2}$

$a_{A/B} = 2.7 \text{ m/s}^2$

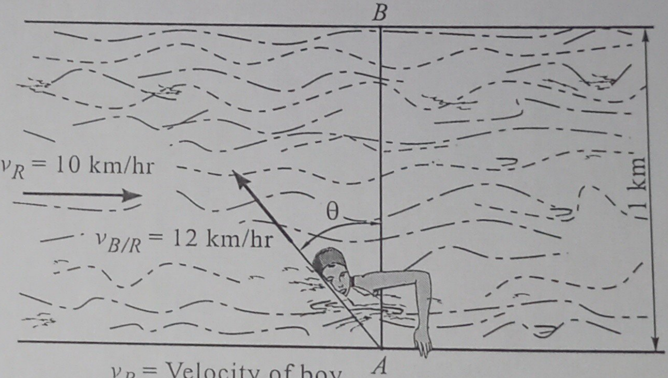
$\therefore a_{A/B} = 2.7 \text{ m/s}^2$ $(\measuredangle 5.58^\circ)$ ***Ans.***

Direction

$\tan\phi = \dfrac{0.2627}{2.69}$ $\therefore \phi = 5.58^\circ$

Problem 27

A boy wants to swim across a river of 1 km width which is flowing at 10 km/hr. The boy wants to reach the other side of bank B and so swims at 12 km/hr at an angle θ, as shown in Fig. 12.27(a). Determine **(i)** the angle θ at which the boy should swim to reach B, **(b)** the time taken to reach B and **(c)** if the boy is swimming straight at $\theta = 0$ where would he have landed on the opposite bank and how much time is required.



v_B = Velocity of boy

v_R = Velocity of river

$v_{B/R}$ = Relative velocity of boy w.r.t. river

Fig. 12.27(a)

Solution

Refer to Fig. 12.27(b).

(i) Angle θ at which the boy should swim to reach B

$\sin\theta = \dfrac{10}{12}$ $\therefore \theta = 56.44^\circ$ ***Ans.***

(ii) The time taken to reach B

By Pythagoras theorem, we have

$v_B = \sqrt{12^2 - 10^2}$

$\therefore v_B = 6.633$ km/hr $(\uparrow)$

Displacement = Velocity × Time

$\therefore \text{Time} = \dfrac{1}{6.633} \times 3600$

$t = 542.74$ seconds

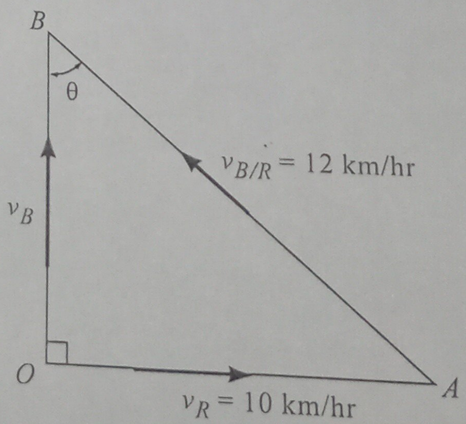


Fig. 12.27(b)