INTRODUCTION TO DIFFERENTIAL EQUATIONS

A differential equation is one which contains a function and its derivative in the same equation.
It could be said to be an equation with at least one derivative

$$\frac{dy}{dx}+x=y$$

$$3\frac{d^2y}{dx^2}+5\frac{dy}{dx}-y=3$$

A differential equation is a relationship between an independent variable x, a dependent variable y and at least one derivative of y. $\left(\frac{dy}{dx}\right)$

$$\frac{dy}{dx}+p(x)y=q(x)$$

NB:

$y=dependent\ vaiable \rightarrow a\ function\ of\ x$

$p(x)=a\ function\ of\ x$

$q(x)=another\ function\ of\ x$

$\frac{dy}{dx}=The\ derivative$

1.1 TYPES OF DIFFERENTIAL EQUATIONS

1. Ordinary Differential Equations: This is one in which the unknown function (y) depends only on one independent variable (x).

$$\frac{dy}{dx}=5x+4$$

$$y\frac{d^2y}{dx^2}+4x\frac{dy}{dx}=y^4$$

2. Partial Differential Equations. This is one in which the unknown functions depends on at least two independent variables.

1.2 FORMATION OF A DIFFERENTIAL EQUATION

A differential equation is formed when arbitrary constants are eliminated from a given function. Omo me seff no unders

1.3 ORDER AND DEGREE OF A DIFFERENTIAL EQUATION

The order of a DE is the highest derivative appearing in the equation. It is also called **differential coefficient**.
The degree of a DE is the power of the highest order is the degree

a. $\left(\frac{dy}{dx}\right)^6+\frac{d^2y}{dx^2}=y^{20}$. This will have an order of 2 and a degreee of 1.
The order of a differential equation indicates how many initial conditions are needed to find a unique solution.

1.4 INITIAL VALUE PROBLEM AND BOUNDARY VALUE PROBLEM

In initial value problem (IVP), we are given the value of the function $y(x)$ and its derivative $y'(x)$ at the same point (initial point)
$y(0)=x_1$ and $y'(0)=x_2$. For initial value problem, we are given a differential equation and an initial condition. For a first order equation, you will need one initial condition. For a second order equation, you will need two separate initial conditions.

Boundary Value Problems: Here we are given the value of function $y(x)$ at two different points i.e. $y(a)=x_1$, $y(b)=x_2$

1.5 VERIFYING SOLUTIONS

Verify that $x=c_1e^{-3t}+c_2e^{2t}$ is a solution of $x''+x'-6x=0$. Find c_1 and c_2 such that $x(0)=0$, $x'(0)=-10$.
Answer: Therefore, $x=c_1e^{-3t}+c_2e^{2t}$is a solution of the differential equation $x''+x'-6x=0$
$c_2=-2$
$c_1=2$

1.6 LINEAR AND NON-LINEAR DIFFERENTIAL EQUATIONS

A linear differential equation is one where the degree is 1.

$$a_n \frac{d^n y}{dx^n} + \ldots + a_1(x)\frac{dy}{dx} + a_o(x)\, y = g(x)$$

1.7 CONDITIONS FOR LINEAR DIFFERENTIAL EQUATION

1. In front of y and its derivatives must only be pure functions of x and never of y or other variables.
2. The powers of y and its derivatives must be 1. (Degree of 1)
3. g(x) must also be a function of x and neither y or its derivative should be there.

AUTONOMOUS DIFFERENTIAL EQUATIONS

A differential equation is called autonomous if it does not depend on the independent variable. For example,

$\frac{dx}{dt} = -x(x-1)^2$, you'll see that in the expression, there is no expression of t

NOTE: It does not change as time (independent variable) changes

Critical points: These are points where the derivative is equal to zero.

If $\frac{dx}{dt} = 0$at $x = x_o$, x_o is called a critical point

After that, we may check whether the critical points are stable or not:

Example, find the critical points and classify them as stable, half stable or unstable.

$$\frac{dx}{dt} = -x^2(x+1)(x-2)$$

For critical point, $-x^2(x+1)(x-2) = 0$

$x = 0, -1, 2$

PROMPTS

1. Hi, I want you to answer like a [Millionaires name]. Use all the knowledge of [their name] and any and every available information about how [name] thinks. My challenge is: []. now provide me a detailed 500-word answer with 3 action points

2.