KINETICS

Note that pounds is a unit of weight and not mass

There are four ways of solving problems in Kinetics

1. Method of Impulse/Momentum
2. Method of Inertia
3. Method of Energy/Workdone
4. Method of Conservation of Energy

METHOD OF INERTIA

$\sum F = ma$

$\sum F_x = m a_x$

$\sum F_y = m a_y$

$Impulse = \int F\, dt$

METHOD OF IMPULSE/MOMENTUM

$\sum F\, t = m(v - u)$

$\sum F\, t = P_2 - P_1$

ENERGY/WORK DONE

When using this method, we need to take into consideration the values of F (force), x – x_o (displacement), v, t

$KE = \frac{1}{2} m v^2$

Work done from A to B = Kinetic Energy of B – Kinetic Energy of A

$W_{A \to B} = KE_B - KE_A$

Work done = Force times displacement

$W = \sum F(x - x_o)$

$\sum F(x - x_o) = \frac{1}{2} m v_B^{\ 2} - \frac{1}{2} m v_A^{\ 2}$

The value of Work done can also be expressed as

$W = \Delta\, Potential\, Energy$

$W_{A \to B} = PE_B - PE_A$

The potential energy can be expressed in terms of gravitational and elastic potential energy

$PE = PE_g + PE_e$

$PE_g = mgh$

$PE_e = Energy\, stored \in the\, string = \frac{1}{2} k e^2$

METHOD OF CONSERVATION OF ENERGY

From the method of conservation of mechanical energy, the total energy is constant

$PE + KE = constant$

Recall that

$W_{A \to B} = KE_B - KE_A$

$W_{A \to B} = PE_B - PE_A$

Therefore,

$PE_B - PE_A = KE_B - KE_A$

$PE_B + KE_A = KE_B + PE_A$

$PE_{gB} + PE_{eB} + KE_A = KE_B + PE_{gA} + PE_{eA}$

QUESTIONS

1. A 20lb collar slides without friction along a vertical rod as shown. The spring attached to the collar has an un-deformed length of 4in and a constant of 3lb/in. If the collar is released from rest in position 1. Determine its velocity after it has moved 6in in to position 2

Writing out values

Force constant, k = 3

$x_o = 4\, inch$

$x_i = 8\, inch$

At point 1.

$Total\, Energy = KE + PE$

$0 + PE$

$PE_1 = PE_{g1} + PE_{e1}$

$PE_{g1} = mgh = Wh = 20 \times 6 = 120\, lb\, in$

$PE_g = 10\, lb\, ft$

$PE_e = \frac{1}{2} k e^2$

$e = x_i - x_o$

$e = 8 - 4 = 4\, inch$

$PE_e = \frac{1}{2} \times 3 \times 4^2$

$PE_e = 24\, lb\, inch$

$12\, inch = 1\, ft$

$PE_e = 2\, lb\, ft$

$PE = PE_g + PE_e$

$PE = 10 + 2\, lb\, ft$

$PE = 12\, lb\, ft$

At point two, the body has moved to its final position and it is at its highest velocity and the gravitational potential energy will be 0. However, it will still have some elastic potential energy.

$$PE = PE_g + PE_e$$
$$PE = 0 + \frac{1}{2}k(10-4)^2$$
$$PE = 0 + \frac{1}{2}k(6)^2$$
$$PE = 0 + \frac{1}{2} \times 3 \times 36$$
$$PE = 54\, lb\, inch$$
$$PE = \frac{54}{12}\, lb\, ft$$
$$PE = 4.5\, lb\, ft$$
$$Total\, energy = PE + KE$$
$$12 = 4.5 + KE$$
$$KE = 7.5$$
$$KE = \frac{1}{2} m v^2$$
$$7.5 = \frac{1}{2} \frac{20\, lb}{32.2} v^2$$
$$24.15 = v^2$$
$$v = \sqrt{24.15}$$
$$v = 4.9143 \frac{ft}{s}$$

2. A 1.5kg collar is attached to a spring and slides without friction along a circular road in a **horizontal plane**. The spring has an un-deformed length of 150mm and constant k=400N/m. Knowing that the collar is in equilibrium at A, and is given a slight push to get it moving, determine the velocity of collar

a. as it passes through B

b. as it passes through C

Solution:

Whenever you hear horizontal plane, note that your value of $PG_g = 0$. For any point in the motion

m = 1.5kg

$$x_o = 150\, mm = 0.15\, m$$

The collar is at equilibrium at A therefore it only has potential energy. Also, at A, its length is

$$x_A = 175 + 125 + 125$$
$$x_A = 425\, mm$$
$$x_A = 0.425\, m$$
$$e = x_A - x_o$$
$$e = 0.425 - 0.15$$
$$e = 0.275$$

$PE_{eA} = \frac{1}{2} k e^2$

$PE_{eA} = \frac{1}{2} \times 400 \times 0.275^2$

$PE_{eA} = 15.125\, J$

$Total\, Energy = PE + KE$

$Total\, Energy = 15.125\, J$

At Point B,

$x_B{}^2 = 125^2 + (125 + 175)^2$

$x_B^2 = 15625 + 90000$

$x_B^2 = 105625$

$x_B = \sqrt{105625}$

$x_B = 325\, mm$

$x_B = 0.325\, m$

$e = x_B - x_o$

$e = 0.325 - 0.15$

$e = 0.175\, m$

3. A 500g collar can slide without friction on the curved rod BC in a horizontal plane. Knowing that the undefined length of the spring is 80mm, and $k = 400\, kN/m$, determine:

a. the velocity that the collar should be given at A to reach B with 0 velocity;

b. the velocity of the collar when it eventually reaches C

4. A thin circular rod is supported in a **vertical plane** by a bracket at A. Attached to the bracket and loosely wound around the rod is a spring of constant $k = 3\, lb\, ft$ and un deformed length equal to the arc circle AB. An 8oz collar C, **not attached** to the spring can slide without friction along the rod. Knowing the collar is released from rest when $\theta = 30$, determine

a. Max height above point B reached by the collar

b. Max speed of the collar

Note: 16oz = 1lb

Original length of the string = length of AB

length of arc, $l = \frac{\theta}{360} \times 2\pi r$

$l = \frac{90}{360} \times 2\pi \times 12\, inch$

$l = \frac{\pi}{2} ft$

At point c, the height of the collar will be $r\cos\theta$. Our datum line (lowest point) is $b = r$

The height, $h = r - r\cos\theta$

$h = 12\, inch - 12\cos(30)\, inch$

$h = 12 - 12 \times \frac{\sqrt{3}}{2}$

$h = 12 - 6\sqrt{3}$

$h = 1.608\, inch$

$h = 0.134\, ft$

At point C,
$v = 0$, $KE = 0$
$PE = PE_g + PE_e$
$PE_e = \frac{1}{2} k e^2$
$e = x - x_o$
$x = \frac{\theta}{360} \times 2\pi r$
$x = \frac{90 - 30}{360} \times 2\pi\, 12\, inch$
$x = \frac{60}{360} \times 24\pi$
$x = 4\pi\, inch$
$x = \frac{4}{12} \pi\, ft$
$x = \frac{\pi}{3}$
$e = \frac{\pi}{3} - \frac{\pi}{2}$
$e = \left| \frac{-\pi}{6} \right|$
$e = \frac{\pi}{6}$
$PE_e = \frac{1}{2} 3 \left(\frac{\pi}{6} \right)^2$
$PE_e = 0.41\, lb\, ft$
{PE rsub g} = Wh
$PE_g = 0.5 \times 0.134$
$PE_g = 0.067\, lb\, ft$
$PE = PE_e + PE_g$
$PE = 0.41 + 0.067$
$PE = 0.477$
$Total\, energy = KE + PE$
$= 0 + 0.477$
$E = 0.477$

At maximum height,
v = 0, KE = 0
Collar is not moving with spring. It’ll move alone (no string attached).
For that reason, it will not have elastic potential energy
$PE_g = Wh = Total\, energy$
$0.477 = 0.5\, h$
$h = \frac{0.477}{0.5}$
$h = 0.954\, ft$

c. At minimum height, {PE rsub g} = 0, h = 0. That means maximum velocity is at point B.
At maximum KE, PE = 0
KE = total energy

$$\frac{1}{2}mv^2=0.477$$

$$\frac{1}{2}\times\frac{0.5}{32.2}\times v^2=0.477$$

$$v^2=61.4376$$

$$v=7.838$$

5. An elastic cord is stretched between two points A and B, located 16 inches apart in the same horizontal plane when stretched directly between A and B, the tension is 10lb. The cord is then stretched until its midpoint C has moved through 6 inches to C, a force of 60lb is required to hold the cord at C. A 0.2lb pellet is placed at C′ and the cord is released. Determine the speed of the pellet as it passes through C

NEED EXPLANATION AGAIN

METHOD OF IMPULSE/MOMENTUM
An automobile weighing 400lb is driven down a 5 degree incline at a speed of 60m/h. When the breaks are applied, causing a constant total breaking force (applied by the road on the tires) of 1500lb. Determine the time required for the automobile to stop.

$$\sum Ft=P_2-P_1$$

$$\sum F=W\sin\theta-Break\,force$$

{} = 4000{sin{%theta}} – 1500
{} = 348.62 – 1500

$$\sum F=-1151.38$$