**MATHEMATICAL INDUCTION**

Mathematical Induction is a mathematical technique which is used to prove if a statement, formula or a theorem is true for every natural number.

It is also defined as a technique for proving some statement is true for all positive integers.

It uses the dominoes principle. Imagine an infinite line of dominoes starting at a point, Suppose the following is true

You can knock over the first domino

Every domino can be knocked over by its predecessor

Then you can knock over the entire string of dominos

This technique involves two steps

Base Step: It proves that a statement is true for the initial value e.g. n = 0. The base case does not necessarily begin with n = 0 but often with n = 1 and possibly with any fixed natural number n = N, establishing the truth of the statement for all natural numbers n >= N

Inductive step: This proves that if the statement is true for the nth iteration (or number n), then it is also true for the (n+1)th iteration (or number n+1),

Basis Step: Check if the f(1) is true

Inductive step: Show that if f(k) is true for some integer k>=1, then f(k+1) is also true

Verify that f(1) is true

Assume that f(k) is true for some integer k>=1

Show that f(k+1) is also true

MORE EXPLANATION

If a proposition is true for the number n = 1 and if it can be shown that if the proposition is true for n, it will also be true for n+1, then the proposition is true for all natural numbers

QUESTIONS

Use mathematical induction to prove that

For all positive integers

Solution:

Let be

First, we show that is true

For the basis case:

For the first value: 1 we assume that n = 1

Second step, we assume that n = k,

Then,

Next, we try to show that the statement is true for n = k+1 based on the above assumption

Since we are assuming that is true, we can do this

We replaced the equation with the value on the right

If we can prove that the above equation is true, then we have done a mathematical induction

Since the denominators are the same, we can prove the equation with only the numerators

Since both equations on the right and left side are the same we

We have showed that the original equation is true for n = k+1 based on the assumption that it is n=k

Use the principle of mathematical induction to show that the statements are true for all natural numbers

Check these out too

Prove that for all positive integers

Prove that is divisible by 5 for all integers n

Solution

For ,

, true

For n = k

, here m is a multiple

For

, here, L is a multiple also

But,

Recall that we assumed that m was a positive integer and sure is also a positive integer. Therefore L is also a positive integer. Since L is also a positive integer, it has to be a multiple

Prove that

Prove that:

Prove that for any natural numbers n,

Use mathematical induction to prove De Moivre’s theorem

Prove that any positive integer number n, is divisible by 3

Prove that for n = 1, n = 2 and use the mathematical induction to prove that for “n” a positive integer greater than two

Use mathematical induction to show that

Use induction to prove that

Use mathematical induction to show that, for any integer :

Use mathematical induction to show that

Prove that for n a positive integer greater than or equal to 4.

Solution:

Basis step:

The above is true

Consider the Fibonacci sequence defined by the relation , and fro

Compute

Use mathematical induction to show that for

is true

List the importance of mathematical induction in engineering algebra

Using mathematical induction, solve that for all

is true

Given the sequence -1, 0, 4, 8, 12, …, Obi devises a formula for the sum of n terms of the sequence. His formula is

. Is the formula correct? Show work to support your answer

Solution: Testing for k = 1

For an arbitrary number k,

For ,

Find in terms of , if and