MECHANICAL PROPERTIES OF SOLIDS

External forces can be applied to a solid to produce a change in the shape or volume of the solid. This change in shape or volume is known as deformation. In some cases, when the deforming forces are removed, solids return to their original shapes and volumes.

Elasticity is the ability of a material to regain its original length (size), shape and volume upon the removal of the deforming force (a load). Such kinds of solids (Materials that undergo elasticity) are called elastic solids or elastic materials or simply elastic.

The deformation of this form is said to be an elastic deformation. This type of deformation is reversible. Once the forces are no longer applied, the object returns to its original shape and size

Plasticity is the inability of a material to retain its original size and shape upon the removal of the load (deforming forces). Plastic deformation shape in a is the process in which enough force is applied on the metal or plastic to cause the object to change in size or shape in a way that is not reversible. In other words, the changes are permanent even when the force is removed, the material will not go back to its original shape. Sometimes it is referred to as Plasticity.

Plasticity is the kind of deformation that can be conducted under control circumstances. Both the deformation of plastics and the deformation of metals involve changes in the makeup of the material itself e.g. Metals that undergo this process of plastic deformation experience a condition known as dislocation. As force is exerted on the material, the material reaches a point known as the yield strength. When this point is reached , the pattern of the molecules that make up the metal begin to shift. The end result is that the molecules rearrange themselves in a pattern that is shaped by the external force placed on the object.

Another type of deformation is FRACTURE.

This type of deformation is also irreversible. A break occurs after the material has reached the end of the elastic and plastic limits. At this point, forces accumulate until they are sufficient to cause a fracture. All materials will eventually fracture if sufficient forces are applied.

HOOKE’S LAW

This states that The force applied to an elastic material is directly proportional to the extension provided that the elastic(or proportional) limit is not exceeded.

k in the equation above is the constant of proportionality. It is called the FORCE CONSTANT or the STIFFNESS CONSTANT or proportionality constant or elastic constant in the material

For two situations,

STRESS AND STRAIN

Stress is defined as the force per unit area

Tensile Strain: This is defined as extension per unit length.

Hooke’s law also states that the stress is directly proportional to the strain if the proportional limit is not exceeded.

Is the constant of proportionality known as the Young’s Modulus or the modulus of elasticity for the deformation.

But

Therefore,

From Hooke’s law,

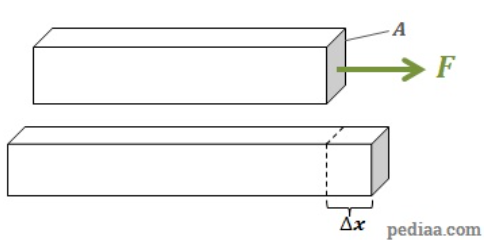
From linear expansion,

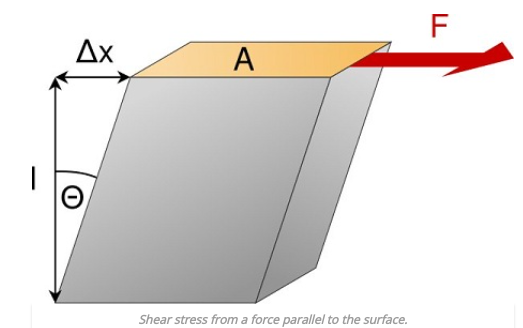
F in the equation above is known as

The force that opposes expansion

The Young’s modulus can also be gotten from the slope of the graph of Tensile Stress against Tensile Strain

Generally, in tensile stress, the force is applied perpendicular to the body. This tensile force causes an increase in length of the material. The opposite of the tensile force is the compression force. The two are all applied perpendicularly.



However, the shear stress is different. In shear stress the force is applied parallel to one of the faces of the result of this is causes one surface of an object to displace with respect to the surface opposite to it.

When you cut a piece of paper with a pair of scissors, you are giving a shear stress to the paper. One side of the scissors attempt to pull the paper in one direction, the other side of the scissors attempt to pull the paper in the other direction.

MODULUS OF RIGIDITY

This is shear modulus and also called clef modulus. Modulus of rigidity (n) can be defined as the ratio of sheer stress to sheer strain

Sheer modulus is the elastic modulus for the deformation which takes place when the force is applied parallel to one face of the object while the opposite face is held fixed by another equal force.

The bigger the shear modulus, the more rigid the material is since for the same change in horizontal distance, strain, you will need a bigger force (stress). This is why the shear modulus is sometimes called the modulus of rigidity

If is very small, then where is in radians

Shear Modulus and the Speed of Shear Waves in Solids

Sound waves in solids are composed of compression waves (just as in gases and liquids), and a different type of sound wave called a [shear wave](https://en.wikipedia.org/wiki/S-wave), which occurs only in solids. Shear waves in solids usually travel at different speeds, as exhibited in [seismology](https://en.wikipedia.org/wiki/Seismology). The speed of compression waves in solids is determined by the medium's [compressibility](https://en.wikipedia.org/wiki/Compressibility), [shear modulus](https://en.wikipedia.org/wiki/Shear_modulus) and density. The speed of shear waves is determined only by the solid material's shear modulus and density.

BULK MODULUS

This is just as young’s modulus. It is a measure of the resistance of a bar to deformation along with its length and shear modulus the resistance of a body to shearing forces applied along one face, the bulk modulus of a body is a measure of its resistance to compression

A solid object can be compressed by applying the same compressional stress to all its phases

Bulk stress or volume stress is the same as pressure in definition.

It is defined mathematically as

Bulk stress produces a reduction in the volume of the object to give what is known as bulk strain.

Applying Hooke’s law,

B in the equation above is the constant of proportionality known as Bulk’s Modulus.

It has a unit Pascal or Newton per square meter.

The equation above can be modified as

We include the minus sign because an increase in pressure corresponds to a decrease in volume.

The reciprocal of the Bulk’s modulus is known as compressibility and is denoted as C

Bulk Modulus and Speed of Sound Waves

Sound waves are an example of longitudinal mechanical waves. A mechanical wave is a wave that propagates as an oscillation of matter. This is why we say sound needs a material medium for its propagation. As we said before, individual particles performing simple harmonic motion do so about their equilibrium positions. As such, they do not travel far along the direction of propagation of energy.

As a sound wave moves from one point to another, particles of air perform simple harmonic motion, i.e., they vibrate back and forth in the same direction and the opposite direction of energy transport. The vibration (back and forth) of the particles in the direction of energy transport creates regions in which particles of the medium are pressed together and other regions where the particles are spread apart, compression and rarefaction.

Sound waves are generated via compression and rarefaction. Consider a tube of length much greater than the width. Assume a piston is attached to the left end. Pushing the piston in compresses the fluid next to the piston. This fluid layer in turn passes this compression onto other fluid layers farther down the tube. The result is a compression pulse. If we now withdraw the piston, the fluid pressure and density in front of the piston drops, sending a rarefaction down the length of the tube. If the piston vibrates to and fro, then a continuous train of compression and rarefaction travels down the tube.

Applying Newton’s laws to the fluid element when it is entering the compressional zone, we have

where is the cross-sectional area of the tube.

This is balanced by the inertial force . The mass of the fluid element is

and the acceleration

Balancing these two forces,

Therefore, ,

Or .

Let be the volume of the fluid element before entering the compressional zone and be the change in volume when it is in the compressional zone. Then,

Hence, we conclude that

The term on the right is the bulk modulus of elasticity, , of the fluid. Notice that is itself negative since it is a compression. Hence, is positive. Thus, the velocity of the longitudinal pulse in the medium is

The above analysis applies to pulses of any shape and to extended wave trains.

For a gaseous medium, we can express the Bulk modulus in terms of the undisturbed gas pressure as , where is the ratio of specific heats for the gas.

For a solid thin rod, bulk modulus is replaced by the Young’s modulus and for an extended solid, we have to make use of the shear modulus (a measure of elastic resistance to tangential or shearing force) and the bulk modulus.

The velocity of sound in some media:

|  |  |  |
| --- | --- | --- |
| Medium | Temperature | Speed |
| Air  Water  Copper | 0  15  20 | 330  1450  3560 |

You would observe that the larger these moduli, the higher the speed of sound in the medium. This is why sound travels fastest in solids as they are the most difficult to compress. Seen another way, the molecules are so close the sound wave is easily transmitted from one to the other. The speed of sound in liquids is much faster than the speed of sound in gases. In air, about 340 m/s, in water, about 1,400 m and in iron, about 5,000 m/s. In diamond (443 GPa, 3.51 g/cm3), an extremely compact material, sound travels at about 11,000 m/s.

ENERGY STORED IN AN ELASTIC MATERIAL

The linear portion of the force against extension graph provides very useful info.

The slope of the graph gives the force constant.

The area under the graph gives the energy stored in the elastic material or the work done on the elastic material.

But

Therefore

But