**APPLICATIONS OF DOT PRODUCTS**

The dot product can be used in different places like

1. Vector Equation of a line

2. Equation of a straight line passing through two given points

3. Perpendicular distance between a point and a line

4. Point of intersection (Where the two lines have the same value)

4. Shortest distance between two skew lines

5. Equation of a plane

6. Equation of a plane with three given points

**VECTOR EQUATION OF STRAIGHT LINES**

Equation of a line passing through a point () and parallel to a vector ()

Diagram

The diagram above depicts a straight line passing through point () with position vector () and is parallel to the vector

This is because parallel vectors are scalar multiples of each other hence

This equation is known as a straight line vector

The equation of a line can be expressed in different forms which include the Cartesian form, the parametric form and the normal form.

You’ll notice that points

So on expanding the equation ,

On resolution, you should get the Cartesian equations below

When resolved, they can be transformed into parametric equations with the

By equating I, j and k components, on both sides the vector equation of the straight line through P1 and parallel to a leads to equations called **parametric equations**.

And also Cartesian equations shown below

Example: Determine the vector equation of the straight line passing through the point with position vector and parallel to the vector

Answer:

Example: Determine the vector equation of a straight line passing through the point (5, 6, -3) and parallel to the vector (2, 8, 4)

Obtain the equation of the line passing through the points A () and parallel to the vector ()

Verify if the point (-23, 5, 8) lies on the line

To verify that, we input the values into the equation

Recall that

Since we got the same value of t for all equations, therefore the point lies on the line.

Find the coordinates of the point on the line with x-coordinate of 3

Therefore, the coordinates of the point are

Find the co-ordinates of the point on the line each distant 5 units from A

Let the points be B and C

Find the equation of a line passing through the point A(-1,0,3) and parallel to the vector u(5,3,-4). Find the point where this line intersects the xy-plane

At the point where it intersects the x-y-plane,

Therefore the point is

EQUATION OF A STRAIGHT LINE PASSING THROUGH TWO GIVEN POINTS

In this case, nothing really changes; we are simply going to evaluate the parallel vector to the straight line .

Equation of a straight line passing through two points () and () is expressed as

Example: Determine the vector equation of the straight line passing through the points and

Answer:

Example: Determine the vector equation of a straight line passing through the points and

PERPENDICULAR DISTANCE OF A POINT FROM A STRAIGHT LINE

The diagram depicts a straight line passing through points A and B and is parallel to the vector b. The line … can be seen to be at a perpendicular distance from point c

The distance d from a point c to a line that passes through r1 and r2 is given as

Where

SHORTEST DISTANCE BETWEEN TWO SKEW LINES

Skew lines are non-parallel lines that do not intercept. However, to both lines, there is a common perpendicular. The shortest distance is given as that perpendicular

The distance (d) between two lines and is given as

Example, determine the perpendicular distance between two skew lines where

Answer:

Example: Determine the distance between the lines and

Answer: 8.39841254841

Find the shortest distance between the two lines A and B where:

EQUATION OF A PLANE

A plane in space is completely specified if we know one point in it, together with a vector which is perpendicular to the plane called the normal vector

Finding the equation of a plane with a given point and a (normal) vector

In a diagram, we have a point on the plane. The vector n, (the normal vector) is perpendicular to . Also, we have a position vector (vector that starts from the origin) to . There is another point P on the plane and a position vector r that goes to P. When the diagram is drawn it will be seen that there is another vector from to P which we will call (r - )

The normal vector (n) is perpendicular to . Since they are perpendicular, their dot product is 0.

At the end of the calculation, we will see that we have an equation like

Determine the vector equation and hence the Cartesian equation of the plane passing through the point with position vector and perpendicular to the vector

Solution:

Ans:

Find the equation of a plane passing through the point with a normal

Find the equation of the plane through the point (1,-1,2) and normal to 2i-3j+4k

FINDING THE EQUATION OF A PLANE WITH THREE GIVEN POINTS:

In order to write the equation of a plane or define a plane, we need a point on a plane and a vector perpendicular to the plane. This perpendicular vector is called the normal vector.

If you are given 3 points, we can define the plane using these points but we don’t have the normal vector.

Find an equation of the plane that passes through the points P (2, 1, 4), Q (4, -2, 7) and R (5, 3, -2)

Let

Let

The normal vector will be the cross products of the vectors a and b.

From there, we can find the equation of the plane and any of the given points P, Q or R.

Also note that if we have three points P (ap, bp, cp), Q(aq, bq, cq) and R(ar, br, cr) the equation of the plane can be expressed as

Determine the Cartesian equation of the plane passing through the three points (0, 2, -1), (3, 0 , 1) and (-3, -2, 0)

POINT OF INTERSECTION BETWEEN A LINE AND A PLANE

We have a plane and a line intersects it at P (x, y, z) coordinate. If we are given a parametric equation e.g. (x = 3 + 4t, y = 5 – 2t, z = 4 + 7t) and the equation of the plane e.g. (2x + 4y – z = 1).

What we have to first do is to substitute the parameters of the parametric equation into the equation of the plane

We get, t = 3.

We substitute the value of t into the parametric equation

So,

So, y = 5 – 2t = 5 – 2(3) = -1

So, z = 4 + 7t = 4 + 7(3) = 25

Therefore, the point of intersection P is P (15, -1, 25) between a line and the plane.

Therefore, the steps to follow are:

1. Find the parametric equations of a line

2. Find the equation of the plane

3. Pass the parametric equations into the equations of the plane to get the value of t

4. After getting the value of t, put it back into the equation of the line and then you will get the points of intersection

At what point does the line passing through P(2, 1, 3) and Q(5, 2, 1) intersect the plane x – 3y – 5z = 4

First, to define a line, we have to define a vector. The vector is given as (a, b, c):

Next, we define parametric equations. The general parametric equations are

are any of the given points P or Q. After solving, we get t = 2 and we can continue solving to get our answer

All the equations above can be expressed in a general form

If we are given the equation of a straight line as , and a plane as

Then,

The value of the point t(x, y, z) is the point of intersection

Determine the point of intersection of the plane, whose vector equation is and the straight line passing through the point , which is parallel to the vector .

First we find the equation of the line

NB: When given the equation of a plane, the values of the normal vector can be gotten from the coefficients of the equation. Given the equation ax + by +cz = d as the equation of a plane, the normal vector of the plane is n = <a, b, c> and these values are called the direction numbers.

**ANGLE BETWEEN TWO (NON-PARALLEL) PLANES**

The angle between two planes is also the angle between the normal vectors of the two planes.

Given two planes 2x – 3y + 4z = 5 and 3x + 5y – 2z = 7, Find the angle between the two planes;

First, you have to find their normal:

So, n1 = <2, -3, 4> and n2 = <3, 5, -2>

PERPENDICULAR DISTANCE BETWEEN A POINT AND A PLANE

A plane has an equation of . There is a point on the plane. The position vector of this point is . A line passes through this point and this line is perpendicular to the plane. The equation of this line is

Reason being that since the line is perpendicular to the plane, it will be a multiple of the normal because it will be parallel to the normal and the line passes through a point P\_1.

There is another point where the line meets the plane. The point has a position vector

Its magnitude , will be the perpendicular distance (p) between the point P1 from the plane

The perpendicular distance between a point () and the plane is expressed as

Also note that from the above, we have

Determine the perpendicular distance, p, of the point () from the plane whose Cartesian equation is .

Find the distance from point () to the plane that passes through the point () and has a normal

Equation of the plane

LINE OF INTERSECTION OF TWO PLANES

Suppose we are given two non-parallel planes whose vector equations are:

Their line of intersection will be perpendicular to and also perpendicular to , since these are the normal to the planes

And recall that a line that is perpendicular to two different vectors is parallel to the cross product of the two vectors. Thus, this line of intersection of two planes is parallel to , and then we get the vector equation