**SETS**

A set is a collection of well-defined objects. A set can be presented by listing out its values. E.g. A set may also be called a collection, family or an aggregate.

The goal of set theory is to formalize intuitive concepts so that we can extend good sense of reasoning to more complex situations.

{1, 2, 3, 4, 5, 6, 7}

{1, 3, 5, and 10}

{Bola, Ade, Jeff}

**AXIOMATIC SET THEORY**

1. Theorems: These are rules which require proofs
2. Axioms: These are statements on which these theorems are based on. Therefore, axiomatic set theory is based on a handful of axioms; Axioms are also defined as basic statements from which we can deduce the properties of sets

**BASIC SET CONCEPTS**

1. Membership or elements: A value is said to be an element of a set if it is inside the set.

If , then

, meaning x is an element (or a member) of A

Also said as x belongs to A

, meaning y is not an element of A

1. Description: This refers to expressing set elements as English alphabets {a, b, c, x, y, z}
2. Equal and Non-Equal sets: Two sets A and B are equal, written as A=B, if every element which belongs to A also belongs to B and vice versa i.e they have the same elements.
3. Non-Equal sets: Two sets A and B are not equal, written as A≠B, this is the negation of A=B.
4. Specification: This refers to the way a set is expressed. This refers to specifying a condition or a property that characterizes the elements of a set. These conditions are only fulfilled by the members of the set. This is also called set-builder form.

a. Expressions

Which are both interpreted as B is a set of x, such that x is an even integer and x > 0.

The above denotes a set B whose elements are positive integers and you can see that the set built itself from the condition hence set-builder form

Here, a letter usually 𝑥, is used to denote a typical member of the set. The colon or slash is read as “such that” and the comma is read as “and”.

b. Enumeration: This refers to listing out the elements in a set as such;

A = {a, b, c, d, e, f}

**Note**: We describe a set by listing its elements only if the set contains few elements; otherwise, we describe a set by the property which characterize its elements.

**SET NOTATION**

This refers to the way a set is represented and interpreted.

The name of a set is usually written in capital letters whereas the children (or elements) are written in small letters.

For example, a set A with elements a, b, c d, e and f

A = {a, b, c, d, e, f}

Recall that the multiplication sign is like the expansion of brackets.

If A and B are any two non-empty sets, then

If , then

If , then

If , then

For any sets A, B, C and D

For sets A, B and C

Let A and B be non-empty sets having n elements in common, the and have elements in common

**BASIC SET OPERATIONS**

1. Subset and Super set: This is also called inclusion. For two given sets A and B, if all the elements of A can be found in B, we say A is a subset of B or A is included in B. We can also say that B is a super (or super set) of A.

A is a proper subset of B

A is a subset of B

The above means that all the elements of A are in B

Example,

𝑃 = {𝑎, 𝑏, 𝑔, 𝑘, 𝑓}, 𝑄 = {𝑘, 𝑎, 𝑓, 𝑟, 𝑔, 𝑏, 𝑡 }

Since every, this means,

𝐴 = {1, 7, 8, 9}, 𝐵 = {1, 3, 9, 7, 10, 12}.

Since 8 is an element of A i.e.

8∈𝐴 𝑎𝑛𝑑 8∉𝐵⟹𝐴⊈𝐵.

NOTE: The null (empty) set is included in every set

. What that means a null set is a subset of all sets

Cardinality of a set: This refers to the number of items in the set. It is represented as n.

This is interpreted as the cardinality of P is 6 since there are 6 elements in the set

The cardinality of a set A, is denoted as #A or /A/, is the number of members or objects contained in the set.

Example, 𝑃 = {𝑎, 𝑏, 𝑡, 𝑘, 𝑗, 𝑤}⟹/𝑃/ = 6

1. Union:

The set of all objects belonging to at least one of a number of sets is called the union of such sets. We write for two sets A and B,

.

Simply said, a union refers to all the elements in the given sets

An element 𝑥 belongs to the union

𝐴∪𝐵 𝑖𝑓 𝑥∈𝐴 𝑜𝑟 𝑥 ∈𝐵

Hence, every element in A is a subset of the union; 𝑎𝑙𝑠𝑜 𝑒𝑣𝑒𝑟𝑦 𝑒𝑙𝑒𝑚𝑒𝑛𝑡 𝑖𝑛

i.e.𝐴⊆𝐴∪𝐵 𝑎𝑛𝑑 𝐵⊆𝐴∪𝐵

1. Intersection: This refers to a set where all its values are common to the main sets involved.

Every element 𝑥 𝑖𝑛 𝐴∩𝐵 belongs to both A and B. Hence, and thus, 𝐴∩𝐵 is a subset of A and of B;

𝐴∩𝐵⊆𝐴 𝑎𝑛𝑑 𝐴∩𝐵⊆𝐵

1. Asymmetric difference: For two given sets A and B, the asymmetric difference is the set which its elements are only in A and not in B.

The Asymmetric difference of A and B written as {A\B} or {A - B} or {A~B}

𝐴∖𝐵 = {𝑥∣𝑥∈𝐴; 𝑥∉𝐵}.

𝐵∖𝐴 = {𝑥∣𝑥∈𝐵; 𝑥∉𝐴}.

1. Symmetric Difference: This can be referred to as the union of the asymmetric differences of two given sets. For any two sets, the symmetric difference is the set of all objects which belong in A alone or B alone, but not to both.

or reads: A symmetric 𝐴 difference B.

In other words, it is a set containing elements that can be found in A only or B only and never in the two

Complement of A set: The complement of a set A denoted as is the set that is in the universal set but not in A.

1. Universal: A universal set in a given operation is the set that contains all the elements that can be found in any of the given sets. All sets under investigation in any application of set theory are assumed to be contained in some large fixed set called the universal set or universe, denoted by 𝑈 or . All sets in a given set operation are subsets of the universal set
2. Empty Set: This is also called a null set. It is a set that has no value or no element. Its symbol is
3. Disjoint Set: Two sets are said to be disjoint if they do not have any members in common that is to say their intersection is empty
4. Finite Sets: This is a set which has a countable number of elements.
5. Non-Finite: These are sets that have an uncountable number of elements. For Example
6. Singleton: This is a set with only one element

**SET PROPERTIES**

1. All sets in a given operation are subsets of the universal set
2. The null set is a subset of every given set
3. Every set is a subset of itself and is equal to itself.
4. If 𝐴⊆𝐵; and 𝐵⊆𝐴, then A and B have the same elements, i.e. A=B. 𝐼𝑓 𝐴 = 𝐵, 𝑡ℎ𝑒𝑛 𝐴⊆𝐵 𝑎𝑛𝑑 𝐵⊆𝐴, since every set is a subset of itself.
5. The number of subsets that a set can have is . Here, n is the cardinality of the set

**ALGEBRA OF SET OR SET LAWS**

1. Idempotent law:
2. Commutative law:
3. Associativity law:
4. Distributive Law:
5. De Morgan’s Law:
6. Involution Law:
7. Law of absorption:
8. Distributing intersection over union:
9. Distributing union over intersection:

**SET IDENTITIES**

1. Complementation Laws:

**DUALITY OF A SET**

If A is an equation of set algebra, the dual of E written as can be gotten by replacing an operator with its opposite i.e.

For example, the dual of

Is

If any equation E is an identity, then its dual 𝐸 is \* also an identity.

**FAMILY OF SETS**

A family of sets is a collection of sets 𝑓, ∋𝑓 = 𝐴1, 𝐴2, 𝐴3, ….., 𝐴𝑛 { }.

For a family of sets, we create an index set I = {1, 2, 3, 4, 5 … n}

The value of n is the number of sets in the family

Here, X represents the sets and that operation above represents the union of all the sets

**UNION**

The union of a family of set

The above statement means the x is an element of the union such that there exists a set A that is an element of the family f and there exists a value of x that is an element of A

That is a collection of all 𝑥'𝑠 which belong to at least one set in the family.

If a family of set is non-empty,

Basically, it represents all the elements in all the sets

**INTERSECTION**

The intersection of a family of set⟹

**DIFFERENCE**

For difference relation of family of set 𝑆,

The above expression means that the difference between the family (S) and the union of sets of the family is equal to the intersection between the family and each corresponding set.

So first, we solve and we get different sets and then we find the intersection of these newly formed sets . That will then give us the the difference between the family and the sets

Theorem: prove that

Proof:

let

Similarly,

let

From 1 and 2, 𝑘⊂𝑧; 𝑧⊂𝑘⟹𝑘 = 𝑧

**INDEXED SET**

Let where is a set of positive integers divisible by n

Obviously,

Where, are positive integers divisible by both 2 and 3.

Observe that

⟹ where 6 is the L.C.M. of 2 and 3

Note; We use the LCM and not the product of the numbers

Generally,

Where, 𝑚 𝑖𝑠 𝑡ℎ𝑒 𝐿𝐶𝑀. 𝑜𝑓 𝑛1, 𝑛2, 𝑛3, …, 𝑛𝑘.

**PARTITION OF A SET**

Let 𝑆 be a nonempty set. A partition of 𝑆 is a subdivision of 𝑆 into *NON-OVERLAPPING*, *NON-EMPTY* subsets. Precisely, a partition of 𝑆 is a collection

𝑃 = of non-empty subsets of such that 𝑆

Each 𝑎∈𝑆 belongs to one of the 𝐴𝑖.

The sets are mutually disjoint; that is, (for all sets)

, 𝑡ℎ𝑒𝑛 .

The partition of a set can be defined as a set which contains sets that fulfil the above conditions i.e. all the elements of the main set must be in one of the partitions, all the elements in the set have to be in one of the partitions. Also there should be no repition of any of the elements of the following sets i.e. all the sets have to be disjoint

In JavaScript terms, it means an array containing individual subsets of the main set with each subset having no intersection with the other and the union of these subsets should be equal to the main set.

In other words, a partition is a collection of

Non-empty

Mutually disjoint subsets

Whose union is the given set.

The subsets in a partition are called cells, blocks or parts. Thus, each belongs to 𝑎∈𝑆 (each element of S) exactly one of the cells.

The figure below is a Venn diagram of a partition of the rectangular set 𝑆 of points into five cells: 𝐴1, 𝐴2, 𝐴3, 𝐴4, 𝐴5.

Example, consider the following collections of subsets of

𝑆 = {1, 2, 3, 4, 5, 6, 7, 8, 9}:

𝑃1 = [{1, 3, 5}, {2, 6}, {4, 8, 9}]

𝑃2 = [{1, 3, 5}, {2, 4, 6, 8}, {5, 7, 9}]

𝑃3 = [{1, 3, 5}, {2, 4, 6, 8}, {7, 9}]

is not a partition of since 7∈𝑆 (7 which is an element of S) does not belong to any of the subsets, blocks or cells.

is also not a partition of since the intersection of the cells {1, 3, 5} 𝑎𝑛𝑑 {5, 7, 9} is not disjoint. 𝑎𝑛𝑑

On the other hand, is a partition of S because?

Every element in 𝑆 belongs

Notice: the union of 𝑃

The sets (cells of the partition) 𝐴1, 𝐴2𝑎𝑛𝑑 𝐴3 are mutually disjoint i.e. 𝐴1 ∩ 𝐴2∩ 𝐴3 = ϕ

Remark: given a partition of a set 𝑆, any element is called a *representative* of the cell,

And a subset 𝐵 𝑜𝑓 𝑆 is called a system of representatives, if 𝐵 contains exactly one element of each of the cells of 𝑃.

Note: 𝐵 = {1, 2, 7} is a *system* *of representatives* of the partition 𝑃 in the example 3 above.

**POWER SET (𝐴)\_**

The power set of a non-empty set 𝐴 is a collection of all possible subsets of the set 𝐴.

Let 𝐴 = {1, 3, 5}

(𝐴) = [{1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}, {1, 3, 5}, {ϕ}]

**CARDINALITY OF A POWER SET**

Generally, the cardinality of a power set (𝐴) of a set 𝐴is denoted as #(𝐴) 𝑜𝑟 /𝑃(𝐴)/ is equal to

**SET OF NUMBERS**

1. REAL NUMBERS, ℝ: The set of real numbers, together with its properties, constitute the real number system. The set of real numbers is dense.
2. INTEGERS, ℤ An integer is a positive or negative whole

number{……-4,-3,-2,-1,0,1,2,3,4,…}.

Closure properties:

if 2 elements 𝑎, 𝑏∈𝑍,

𝑎 + 𝑏 = 𝑐, 𝑐 ∈ 𝑍

. The set of integers is closed under the usual addition.

𝑎 − 𝑏 = 𝑑, 𝑑 ∈ 𝑍,

The set of integers is closed under the usual subtraction.

𝑎. 𝑏 = 𝑒, 𝑒 ∈ 𝑍, the set of integers is closed under the usual multiplication.

𝑎/𝑏 = 𝑓, 𝑒 ∉ 𝑍, the set of integers is not closed under quotient operations.

𝑎/𝑏 = 𝑓, 𝑒 ∈ 𝑍 . 𝑓𝑜𝑟 𝑠𝑜𝑚𝑒 𝑣𝑎𝑙𝑢𝑒𝑠

(c) RATIONAL NUMBERS, ℚ

Since

𝑥 + 𝑦 = 𝑍 ∈ 𝑄, closure holds under addition;

𝑥 − 𝑦 = 𝑡 ∈ 𝑄, closure holds under subtraction;

𝑥. 𝑦 = 𝑘 ∈ 𝑄, closure holds under multiplication;

𝑥/𝑦 = 𝑗 ∈ 𝑄, closure holds under quotient;

(d) NATURAL NUMBERS, ℕ

This is a set of all positive integers.

(e) PRIME NUMBERS, 𝑁𝑝

= {2, 3, 5, 7, 11, 13}

⊂ 𝑁 ⊂ 𝑍 ⊂ 𝑄 ⊂ 𝑅

(f) IRRATIONAL NUMBERS, 𝑄'

If is our universe 𝑅 of discourse,

𝑄' = 𝑅/𝑄 ⟹𝑄 ∪ 𝑄' = 𝑅

(g) COMPLEX NUMBERS,ℂ

8 = 2. 3 + 0. 7𝑖 ∈ 𝐶

Generally, ⊂ 𝐶.

Finally, the chain of inclusion of sets of numbers is thus,

⊂ 𝑁 ⊂ 𝑍 ⊂ 𝑄 ⊂ 𝑅 ⊂ 𝐶.

**INCLUSION-EXCLUSION PRINCIPLE**

If A and B are finite and disjoint sets (their intersection is empty).

Then, 𝐴∪𝐵 is also finite and

Given any set A, the universal set 𝑈 is a disjoint union of A and its complement,

Here,

Therefore,

Suppose A and B are finite sets. Then,

This reads number of elements in A that is not in B.

Recall, (𝐴∪𝐵) = (𝐴) + (𝐵), if 𝐴∩𝐵 = ∅.

Suppose sets A and B are not disjoint, that means, then 𝐴∩𝐵≠∅ 𝐴∩𝐵 𝑎𝑛𝑑 𝐴∪𝐵 are finite. Then,

𝑛(𝐴∪𝐵) = 𝑛(𝐴) + 𝑛(𝐵) − 𝑛(𝐴∩𝐵)

The principle states that to find the number of elements in A or B (or both). First, add (𝐴) and (𝐵) [inclusion] and then subtract (𝐴∩𝐵) [exclusion], since the elements in 𝐴∩𝐵 were counted twice.

For three sets A, B, C; suppose A, B, C are finite;

𝐴∪𝐵∪𝐶 is also finite, then

QUESTIONS

If there are 20 male students in a class of 35 students, how many female students are there?

Solution: let A=set of number of male students. Recall

From

An archery class A contains 35 students, and 15 of them are also in a bowling class. How many students are in class A who are not in class B?

Solution

This is a problem of asymmetric difference of sets A and B.

Recall,

If there are 110 students in a college dormitory:

30 students are on list A;

35 students are on list B;

20 students are on both lists.

Find the number of students:

on list A or B

on exactly one of the two lists,

c on neither list.

The number of students in list

Solution:

A or B is the union of A and B.

𝑛(𝐴∪𝐵) = 𝑛(𝐴) + 𝑛(𝐵) − 𝑛(𝐴∩𝐵)

=30+35-20=45.

This means number in A not in B +number B not in A. Make use of the asymmetric difference formula:

Thus, there are 10+15=25 students on exactly one of the two lists.

The number of students not in both lists is the set 𝐴'∩𝐵'. 𝑈

By De Morgan’s law, 𝐴' ∩ 𝐵' = (𝐴∪𝐵)'

𝑛(𝐴' ∩ 𝐵') = 𝑛[(𝐴∪𝐵)'] = 𝑛(𝑈) − 𝑛(𝐴∪𝐵)

=110-45=65.

In a class of 120 mathematics students; 65 study French, 45 study German, 42 study Russian, 20 study French and German, 25 study French and Russian, 15 study German and Russian and 8 study all the languages.

Let F, G, and R denote the sets of students studying French, German and Russian respectively.

Find the number of students studying at least one of the three languages i.e.

Fill in the correct number of students in each of the eight regions of the Venn diagram.

Find the number of students studying

Exactly one language

Exactly two languages.

Solutions:

Those who study German and Russian only =15-8=7.

Those who study French and Russian only =25-8=17.

Those who study French and German only =20-8=12

Those who study Russian only =42-17-8-7=10

Those who study German only=45-12-8-7=18

Those who study French only=65-12-8-17=28

Those who did not study any of the 3 languages =120-100=20

=28+18+10=56

=12+17+7=36.

**PROVING SET THEOREMS**

Theorem 1: If A and B are pairwise disjoint (i.e. 𝐴 ∩ 𝐵 = ∅), Prove that 𝐴∖𝐵 = 𝐴.

Proof:

Theorem 2: Prove that 𝐴∖𝐵 = 𝐴∩𝐵'.

Proof:

Similarly:

Theorem 3: Prove .

Proof: since from theorem 2,

Then,

(𝑓𝑟𝑜𝑚 𝑖𝑛𝑑𝑒𝑚𝑝𝑜𝑡𝑒𝑛𝑐𝑦 𝑎𝑛𝑑 𝑐𝑜𝑚𝑝𝑙𝑒𝑚𝑒𝑛𝑡 𝑙𝑎𝑤𝑠)

Alternatively,

Let

𝑓𝑟𝑜𝑚 𝑖𝑛𝑑𝑒𝑚𝑝𝑜𝑡𝑒𝑛𝑐𝑦 𝑎𝑛𝑑 𝑐𝑜𝑚𝑝𝑙𝑒𝑚𝑒𝑛𝑡 𝑙𝑎𝑤𝑠

∴{𝑦 ∈ (𝐴 ∩ ∅)} = ∅

∴(𝐴 ∩ 𝐵)∩(𝐴∖𝐵) = ∅)

Theorem 4: De-Morgan’s law (complement law)

If A and B are sets, show that (𝐴∪𝐵)' = 𝐴' ∩ 𝐵'.

Proof:

Let

Let

'

From 1 and 2,

Theorem 5: Prove that 𝐴∩𝐴' = ∅

Proof:

Theorem 6: Prove that (𝐴')' = 𝐴

Proof:

Theorem 7: Prove that ∅' = 𝑈

Proof:

Theorem 8: Prove that 𝐴∪𝐴' = 𝑈

Proof:

Theorem 9: Prove that 𝑈' = ∅

Proof:

Theorem 10: Prove that (𝐴∖𝐶) ∩ (𝐵∖𝐶) = (𝐴∩𝐵)∖𝐶

Proof:

Let

Similarly,

Let