DYNAMICS AND

KINEMATICS OF PARTICLES

Dynamics can be divided into two namley:

Kinematics: The study of the geometry of motion. Kinematics is used to relate displacement. Velocity and acceleration and time, without reference to the cause of the motion.

Kinetics: The study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

In physics or sciences when a big object is considered a particle, that means calculation done on that object is done without regard to its size.

We will look at uniform motion and uniform accelerated motion.

A particle moving along a straight line is said to be in **rectilinear motion**. At a given time t, the particle will occupy a position on the straight line – usually with respect to an origin –. The position/distance can either be positive or negative depending on the position relative to the origin. So if it is on the left, then it is negative. However, if the position is on the right of the origin, then it is positive.

When the position coordinate x of a particle is known for every time t, we say that the motion of the particle is known. The timetable of the motion can be given in the form of an equation in x and t, such as x = 6t^2 – t^3, or in the form of a graph of x versus t.

The average velocity of a particle over the time interval %delta t is defined as the quotient displacement %delta x and the time interval %delta t

Average velocity = delta x over delta t

The SI units for x and t are metres and seconds respectively

The US customary units are feet and seconds respectively

Instantaneous velocity = lim from %delta → 0 {Average velocity}

Instantaneous velocity, v = dx/dt

The velocity v – known as the speed of the particle – can be positive or negative. It is positive when x is positive and vice-versa.

Average acceleration = delta v over delta t

Its units are metres per square second or feet per square second

Instantaneous acceleration, a = dv/dt

= d^2x/dt^2

Even though acceleration and velocity are vectors, since we are dealing with rectilinear motion where the particle has a known and fixed direction, we only need to specify the sense (+ve or -ve) and the magnitude (scalar quantity).

Acceleration can be positive or negative. A positive value indicates that velocity increases. This may mean that the particle is moving faster in the positive direction or moving slowly in the negative direction. In both cases delta v is positive.

The term deceleration is used to refer to acceleration when the speed of the particle decreases; the particle is then moving more slowly.

v = dx/dt

dt = dx/v

a = dv/dt

a = dv div dx/v

a = v dv/dx

Examples:

Consider a particle moving in a straight line and assume that its position is defined by the equation

x = 6t^2 – t^3

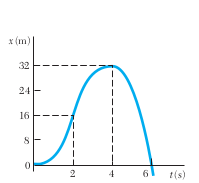
where t is expressed in seconds and x in metres. The instantaneous velocity – velocity at any time t – is obtained by differentiating x with respect to t

v = dx/dt = 12t – 3t^2

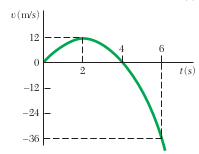
Acceleration a, is obtained by differentiating velocity.

A = dv/dt = 12 – 6t

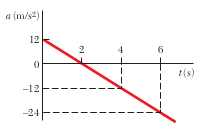
Note the following **motion curves**.



Displacement-time graph



Velocity time graph



A study of the three motion curves of Fig 11.6 shows that the motion of the particle from t=0 to t=infinite can be divided into 4 phases

1. The particle starts from the origin x=0, with no velocity but with a positive acceleration. Under this acceleration, the particle gains a positive velocity and moves in the positive direction. From t =0 to t = 2s, x, v and a are all positive

2. At t=2s, the acceleration is zero; the velocity has reached its maximum value. From t = 2 to t = 4, v is positive but a is negative; the particles moves in the positive direction but reduces in velocity.

3. At t = 4s, the velocity is zero; the position coordinate x has reached its maximum value. From then on, both v and a are negative; the particle is accelerating and moves in the negative direction with increasing speed.

4. At t=6s, the particle passes through the origin; its coordinate x is then zero while the total distance traveled since the beginning of the motion is 64m. For values of t larger than 6s, x, v and a will all be negative. The particle keeps moving in the negative direction away from O faster and faster

DETERMINATION OF THE MOTION OF A PARTICLE

When a = 0 => Uniform motion

When a /= 0 => Uniform accelerated motion

Motion is not always defined by a relation between x and t. Acceleration can be expressed in different forms and depending on what causes the acceleration. The acceleration of the motion of a body tied to a spring will depend on factors relating to the elongation of the spring.

v = dx/dt

dt = dx/v

a = dv/dt

a = dv div dx/v

a = v dv/dx

Three common classes of motion

1. When acceleration is given as a function of time, f(t). We will like to have an integral of that function with respect to timetable

a = dv/dt = f(t)

dv = f(t)dt

int {dv} = int {f(t)dt}

int from{v rsup o} to{v} {dv} = int from{0} to{t} {f(t) dt}

v – v\_o = int from{0} to{t} {f(t) dt}

Similarly for the velocity

v = dx/dt

dx = vdt

int from{x\_o} to {x} = int from{0} to{t} {vdt}

x – x\_o = int from{0} to{t} {vdt}

v\_o and x\_o are the initial velocity and initial position respectively. They are the initial conditions of the motion

2. When acceleration is given as a function of x f(x)

a = v dv/dx

a = f(x)

v dv = f(x)dx