COMPLEX NUMBERS

AGENDA

1. The real and complex numbers

2. Representations and Algebra of Complex numbers

3. Complex Functions

4. Roots of Unity

5. De Moivre’s Theorem and Applications

Complex numbers can be expressed in different forms:

1. Cartesian form or rectangular form

2. Polar form

3. Exponential form

Complex number in rectangular form

To graph a complex number in rectangular form, the x-axis is the real axis (that is the values for a) while the y-axis will be the imaginary axis (that is the values for b)

The absolute value of a complex number is given as

The value of this absolute value is always positive

Trying to draw this as a right angled triangle, the hypotenuse will be the absolute value of z.

The angle between |z| and a is

Complex number in polar form

When represented in polar form graphically, we have to take note of the r-values. These r values are usually the circles in the graph. The closest circle has an r value of 1, the second closest has an r-value of 2 and so on.

Trying to draw this as a right angled triangle, the hypotenuse will be r.

Trying to convert from rectangular form to polar form

Multiplying complex numbers in polar form

Quotient of Two Complex Numbers in Polar Form

De Moivre’s Theorem – Finding the nth power of a complex number

De Moivre’s Theorem – Finding Complex roots

Solve the following questions:

1. Plot each complex number

A. z = 4 + 3i

B. z = -2 – 3i

C. z = 2

D. z = -3

E. z = 4i

F. Z = -3i

2. Calculate the absolute value of each complex number shown below

A. z = 3 + 4i Answer: 5

B. z = 4 – 6i Answer:

C. z = 3 Answer: 3

D. z = -4 Answer: 4

E. z = -8i Answer: 8

3. Write the complex number in polar form

A.

Soln:

%theta = {}{()}

To convert it to radian forms,

B.

Soln:

Notice that even though the value of b is negative, we used it as a positive value.

When the graph is drawn, it can be seen that the value of r is the fourth quadrant.

However, in our calculation, we want the angle with the positive x-axis

That will be .

So from the above, it can be noted that the angle used for calculation depends on the reference angle and the position of the reference angle.

For quadrant 1

For quadrant 2

For quadrant 3

For quadrant 4

Therefore the answer for this will be given as

You can still decide to change it to radians

We often use a shorthand version to denote polar form

For the last example

,

It can be expressed like

C. z = -3 + 5i. Answer:

D. z = 3. This can be easily solved:

If z = 3, then r = 3

Then if we place this on the graph, we will see that the angle is 0. Therefore if given just one value, the angle will either be 0, 90, 180 or 270

E z = -4

For this, r = 4 and

F. z = -2i Answer: r = 2, %theta = 270

G. z = 5i Answer: r = 5, %theta = 90

4. Write the complex number in rectangular form

This can easily be done by plugging in the values

A. Answer: z = 4i

B. Answer:

C. Answer:

5. Write the complex number in rectangular form. Round your answer to the nearest hundredth

A. This calculation should be done in radian mode in your calculator

Answer: To the nearest hundredth.

6. Find the product of the two complex numbers. Write the answer in polar form

A.

Answer:

B.

Answer:

C.

Answer:

D.

z = 3-4i

z\_2 = 5+12i

Answer: r = 65,

7. Find the quotient {z1 over z2} of the complex numbers shown below. Write the final answer in polar form

A.

z1 = 12[{cos{80}}+i{sin{80}}]

z2 = 3[{cos{30}} + i{sin{30}}]

Answer: 4[{cos{50}}+i{sin{50}}]

B.

z1 = 15[{cos{%pi over 5}}+i{sin{%pi over 5}}]

z2 = 3[{cos{%pi over 10}}+i{sin{%pi over 10}}]

Answer: 5[{cos{%pi over 10}} + i{%pi over 10}]

C.

z1 = 5[{cos{60}} + i{sin{60}}]

z2 = 35[{cos{190}} + i{sin{190}}]

{} = {1 over 7}[{cos{-130}} + i{sin{-130}}]

{} = {1 over 7}[{cos{230}} + i{sin{230}}]

8. Find the quotient z1/z2 of the complex numbers shown below. Write the final answer in polar form using an angle between 0 and 360 degrees

z1 = {3{sqrt{3}}} – {3}i

z2 = -1 + {sqrt{3}}i

z = 3L210

Complex Numbers in Exponential Form

In the polar form of complex numbers,

According to the Euler’s Formula

r is the distance between the polar coordinates and the pole (or origin) (0, 0)

%theta is said as the argument of z and it is written as

In polar form, we get an infinite number of possible exponential form of a given complex number.

Each %theta differs by a multiple of

Like…

Therefore, the exponential form can be written as

If %theta = {{2{%pi}} over 3}, then

From this questions

For every value of theta, you will get the same thing

z1, z2, z3, z4 … z5 will all represent the same complex number

MULTIPLICATIVE INVERSE

In exponential form:

z = r{e rsup {i{%theta}}}

then,

This is the multiplicative index in exponential form

Now, by Euler’s formula

Multiplication in exponential form

Let

FUNCTIONS OF COMPLEX VARIABLES

Given that

z = x + yi

w = f(z)={z rsub 2}

w = {(x + yi) rsup 2}

w = {x rsup 2} + 2xyi – {y rsup 2}

w = ({x rsup 2} - {y rsup 2}) + 2xyi

You’ll see that the answer we got for w has the variables of the input value in it.

A complex function f, is a pair of real functions put together. Our goal is to do calculus on these functions. Our complex functions have to be differentiable. Their derivatives have to be deffined.

HOLOMORPHIC FUNCTIONS

Analytic or Regular functions

f(z) is holomorphic in a region R of the complex plane if it has a derivative at every point in R

From our knowledge in differentiation that

For a limit to exist and for the function to be differentiable,

f(x) = {x rsup 2}

You’ll notice that the functions of real numbers are usually drawn on the x-y plane and they are represented with lines. Therefore the lim from the left and right is required for the function to be differentiable

However, for complex functions, it is on a 3d-plane and for it to be differentiable, the limit from all sides have to be equal

Given that f(z) = {z rsup 3}