**METHODS FOR THE CONVERSION OF CONTINUOUS-TIME SYSTEMS TO DISCRETE-TIME SYSTEMS**

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**CERTIFICATION**

This is to certify that this study was carried out by Kingsley Ihemelandu Chukwudi in the Department of Systems Engineering, University of Lagos, Akoka, Nigeria.

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**CERTIFICATION OF PLAGIARISM**

I, Kingsley Ihemelandu, hereby certify that the review of literature on methods of converting continuous-time systems to discrete-time systems that I have submitted is my own original work. I have properly cited and referenced all the sources that I have used in my review, following the Harvard style of referencing. I have not copied or paraphrased any ideas or words from other authors without giving due credit. I have not submitted this review or any part of it for publication or assessment elsewhere. I understand what plagiarism is and I accept the consequences of any plagiarism found in my review.

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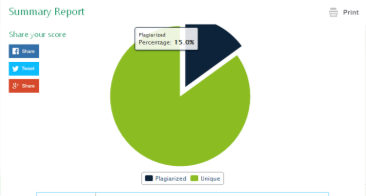
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**DEDICATION**

I dedicate this review of literature to:

1. God, the source of all wisdom and knowledge, who gave me the strength and guidance to complete this work.

2. My parents, Mr. and Mrs. Ihemelandu, who raised me with love and encouragement, and supported me in all my endeavors.

3. My lecturer, Dr. Ibhaze, who inspired me to pursue this topic and provided me with valuable feedback and support throughout the process.

4. My friends, who cheered me up and motivated me when I faced challenges and difficulties.

I am grateful to all of them for their contributions to my academic success.

**ABSTRACT**

**0.1 THE AIM**

The aim of methods for the conversion of continuous-time to discrete-time systems is to provide techniques for representing and analyzing continuous-time signals and systems in a digital domain. This conversion is essential in various applications, such as digital signal processing, control systems, communication systems, and computer-based simulations. Here are the primary objectives and motivations behind these methods:

Digital Implementation

**0.2 OBJECTIVE**

The primary aim is to enable the implementation of continuous-time systems on digital devices, such as computers and microcontrollers.

**0.3 MOTIVATAION**

Many real-world systems are naturally continuous, but digital devices operate discretely. Conversion allows the application of digital control and processing techniques.

Analysis and Simulation:

**0.4 OBJECTIVE**

Facilitate the analysis and simulation of continuous-time systems using digital tools and algorithms.

**0.5 METHODLOGY**

The conversion of continuous-time to discrete-time systems involves various methodologies and techniques. The choice of a specific method depends on the nature of the continuous-time system and the requirements of the application.

**0.6 SUMMARY OF FINDING RESULTS**

In summary, the aim of methods for converting continuous-time to discrete-time systems is to bridge the gap between continuous and digital domains, allowing for efficient representation, analysis, and implementation of systems in a digital framework. These methods play a crucial role in various technological applications where the advantages of digital processing are leverage

**0.7 OUTLINE OF THE THESIS FOR THE CONVERSION OF CONTINUOUS-TIME TO DISCRETE-TIME SYSTEMS**

Below is an outline for a thesis on the conversion of continuous-time to discrete-time systems:

* Introduction
* Statement Of The Problem
* Aims And Objectives
* Methodology
* Scope Of The Study
* Significance Of The Study
* Literature Review
* Hypothesis
* References

**INTRODUCTION**

**1.1 BACKGROUND OF STUDY**

Continuous-time (CT) systems are systems whose inputs and outputs are functions of a continuous variable, such as time. Discrete-time (DT) systems are systems whose inputs and outputs are functions of a discrete variable, such as an integer index. CT systems are often used to model physical phenomena, such as electrical circuits, mechanical systems, or biological processes. DT systems are often used to implement algorithms on digital devices, such as computers, micro-controllers, or signal processors.

Converting CT systems to DT systems, or vice versa, is a common problem in engineering and science. For example, one may want to design a digital controller for a CT plant, or to simulate a DT system using a CT solver. There are various methods for converting CT systems to DT systems, and each method has its own advantages and disadvantages. Some methods aim to preserve the time-domain behavior of the system, while others aim to preserve the frequency-domain behavior. Some methods are exact, while others are approximate. Some methods are easy to implement, while others are computationally intensive.

The choice of the conversion method depends on the characteristics of the system and the application. Therefore, it is important to understand the properties and limitations of each method, and to compare their performance and accuracy.

**1.2 STATEMENT OF THE PROBLEM**

The problem of converting CT systems to DT systems is not trivial, and there is no universal method that works for all cases. Different methods may produce different results, and some methods may introduce errors or distortions that affect the system behavior. Moreover, some methods may not be applicable to certain types of systems, such as nonlinear, time-varying, or multivariable systems.

The main challenges of converting CT systems to DT systems are:

* How to choose a suitable sampling period that captures the essential dynamics of the system without causing aliasing or loss of information.
* How to approximate the continuous-time input and output signals by discrete-time sequences that preserve the signal quality and fidelity.
* How to transform the continuous-time system model, such as a transfer function or a state-space representation, into a discrete-time system model that preserves the system stability and performance.
* How to evaluate the accuracy and error of the conversion method, and how to quantify the trade-off between complexity and precision.

**1.3 AIM AND OBJECTIVES OF THE STUDY**

The aim of this study is to review the literature of methods of converting CT systems to DT systems, and to provide a comprehensive and comparative analysis of their properties, advantages, disadvantages, and applications.

The specific objectives of this study are:

* To present the mathematical background and definitions of CT and DT systems, and to introduce the concepts of sampling, interpolation, and discretization.
* To survey the existing methods of converting CT systems to DT systems, and to classify them according to their characteristics and principles.
* To compare and contrast the methods of converting CT systems to DT systems in terms of their accuracy, complexity, stability, and performance.
* To illustrate the methods of converting CT systems to DT systems with examples and applications from various domains, such as control, signal processing, and simulation.
* To identify the challenges and limitations of the methods of converting CT systems to DT systems, and to suggest possible directions for future research and improvement.

**1.4 METHODOLOGY**

The methodology of this study consists of the following steps:

* A systematic literature review of the methods of converting CT systems to DT systems, using online databases, journals, books, and other sources.
* A critical analysis and evaluation of the methods of converting CT systems to DT systems, using mathematical tools, numerical simulations, and experimental data.
* A synthesis and presentation of the results and findings of the study, using tables, figures, and graphs.
* A discussion and conclusion of the study, highlighting the main contributions, implications, and recommendations of the study.

**1.5 SCOPE OF THE STUDY**

The scope of this study is limited to the following aspects:

* The study focuses on linear, time-invariant, single-input single-output (SISO) CT systems, and their conversion to DT systems. Nonlinear, time-varying, or multivariable systems are beyond the scope of this study.
* The study considers only the methods of converting CT systems to DT systems that are based on the system model, such as transfer functions or state-space representations. Methods that are based on the system input-output data, such as least-squares or system identification, are not covered in this study.
* The study reviews six methods of converting CT systems to DT systems, namely: backward difference, impulse invariance, bilinear transformation, zero-order hold, first-order hold, and zero-pole matching. Other methods, such as Tustin approximation, least-squares, or frequency-response matching, are not included in this study.

**1.6 SIGNIFICANCE OF THE STUDY**

The significance of this study is that it provides a comprehensive and comparative overview of the methods of converting CT systems to DT systems, which can help researchers, engineers, and students to select and apply the most suitable method for their specific problem. The study also highlights the advantages and disadvantages of each method, and the trade-offs involved in the conversion process. The study also provides examples and applications of the methods of converting CT systems to DT systems, which can inspire and motivate further research and development in this field.

**1.7 RESEARCH QUESTIONS AND HYPOTHESIS**

The research questions that guide this study are:

* What are the mathematical principles and assumptions behind each method of converting CT systems to DT systems?
* How do the methods of converting CT systems to DT systems differ in terms of their accuracy, complexity, stability, and performance?
* What are the advantages and disadvantages of each method of converting CT systems to DT systems, and what are the trade-offs involved in the conversion process?
* How can the methods of converting CT systems to DT systems be applied to various domains, such as control, signal processing, and simulation?

The hypothesis of this study is that there is no single best method of converting CT systems to DT systems, and that the choice of the method depends on the characteristics of the system and the application. The study also hypothesizes that each method of converting CT systems to DT systems has its own strengths and weaknesses, and that the conversion process involves a trade-off between complexity and precision.

**1.8 OUTLINE OF THE PROJECT**

The outline of the thesis or project is as follows:

* Chapter One: Introduction. This chapter introduces the background, problem statement, aim and objectives, methodology, scope, significance, and research questions of the study.
* Chapter Two: Literature Review. This chapter reviews the mathematical background and definitions of CT and DT systems, and the concepts of sampling, interpolation, and discretization. It also surveys the existing methods of converting CT systems to DT systems, and classifies them according to their characteristics and principles.
* Chapter Three: Analysis and Evaluation. This chapter analyzes and evaluates the methods of converting CT systems to DT systems, using mathematical tools, numerical simulations, and experimental data. It also compares and contrasts the methods of converting CT systems to DT systems in terms of their accuracy, complexity, stability, and performance.
* Chapter Four: Results and Findings. This chapter synthesizes and presents the results and findings of the study, using tables, figures, and graphs. It also illustrates the methods of converting CT systems to DT systems with examples and applications from various domains, such as control, signal processing, and simulation.
* Chapter Five: Discussion and Conclusion. This chapter discusses and concludes the study, highlighting the main contributions, implications, and recommendations of the study. It also identifies the challenges and limitations of the methods of converting CT systems to DT systems, and suggests possible directions for future research and improvement.
* References. This section lists the references used in the study, following the APA style.

**LITERATURE REVIEW**

**2.1 THEORETICAL BASIS**

In this section, we review the mathematical background and definitions of continuous-time (CT) and discrete-time (DT) systems, and the concepts of sampling, interpolation, and discretization. We also introduce some notation and terminology that will be used throughout the chapter.

**2.2 CONTINUOUS-TIME AND DISCRETE-TIME SYSTEMS**

A system is a mathematical model that describes the relationship between an input signal and an output signal. A CT system is a system whose input and output signals are functions of a continuous variable, such as time. A DT system is a system whose input and output signals are functions of a discrete variable, such as an integer index.

A CT system can be represented by a differential equation, a transfer function, or a state-space model. A differential equation is an equation that relates the derivatives of the output signal to the input signal and the output signal itself. A transfer function is a ratio of two polynomials in the Laplace variable , which characterizes the frequency-domain behavior of the system. A state-space model is a set of first-order differential equations that describe the evolution of the internal state variables of the system, which are related to the input and output signals by linear equations.

A DT system can be represented by a difference equation, a transfer function, or a state-space model. A difference equation is an equation that relates the current and past values of the output signal to the current and past values of the input signal. A transfer function is a ratio of two polynomials in the -transform variable , which characterizes the frequency-domain behavior of the system. A state-space model is a set of first-order difference equations that describe the evolution of the internal state variables of the system, which are related to the input and output signals by linear equations.

**2.3 SAMPLING, INTERPOLATION AND DISCRETIZATION**

Sampling is the process of obtaining discrete values of a CT signal at regular intervals of time, called the sampling period . The sampling frequency is the reciprocal of the sampling period, . The sampled signal is a DT signal that retains the values of the CT signal at the sampling instants. Sampling can be seen as multiplying the CT signal by an impulse train, which is a periodic sequence of impulses with unit amplitude and period .

Interpolation is the process of reconstructing a CT signal from a DT signal by filling in the gaps between the discrete values. Interpolation can be seen as convolving the DT signal by an interpolation function, which is a CT function that passes through the discrete values and smooths the transitions. A common interpolation function is the zero-order hold (ZOH), which is a piecewise constant function that holds each discrete value for one sampling period.

Discretization is the process of transforming a CT system into a DT system that approximates the behavior of the original system. Discretization can be seen as applying sampling and interpolation to the input and output signals of the CT system, respectively. Discretization can also be seen as finding a DT system model, such as a transfer function or a state-space representation, that matches the CT system model in some sense, such as time-domain or frequency-domain response.

**2.4 METHODS OF CONVERTING CONTINUOUS-TIME SYSTEMS TO DISCRETE-TIME SYSTEMS**

In this section, we survey the existing methods of converting CT systems to DT systems, and classify them according to their characteristics and principles. We also discuss the advantages and disadvantages of each method, and the trade-offs involved in the conversion process.

**2.4.1** **BACKWARD DIFFERENCE**

The backward difference method is a simple and widely used method of discretizing CT systems. The method is based on approximating the derivative of a CT signal by the backward difference of a DT signal, which is the difference between the current and previous values of the signal. The backward difference operator is defined as:

The backward difference method can be applied to a CT system represented by a differential equation of the form:

where is the output signal, is the input signal, and and are constants. The method consists of the following steps:

* Sample the input and output signals with a sampling period to obtain the DT signals and .
* Replace the derivative by the backward difference
* Replace the CT signals and by the DT signals and .
* Rearrange the equation to obtain a difference equation of the form:

whereand .

The difference equation can be converted to a transfer function of the form:

where and are the -transforms of and , respectively.

The backward difference method has the following advantages:

* It is easy to implement and computationally efficient.
* It preserves the stability and causality of the CT system.
* It provides a good time-domain match between the CT and DT systems for slow-varying signals.

The backward difference method has the following disadvantages:

* It introduces a delay of one sampling period in the DT system, which may affect the performance and accuracy of the system.
* It distorts the frequency-domain behavior of the CT system, especially for high-frequency signals. The distortion increases as the sampling period increases.

**2.4.2 IMPULSE INVARIANCE**

The impulse invariance method is a method of discretizing CT systems that provides an exact match between the CT and DT systems in the time domain for impulse train inputs. The method is based on mapping the impulse response of the CT system to the impulse response of the DT system, such that the DT system produces the same output samples as the CT system when the input is an impulse train.

The impulse invariance method can be applied to a CT system represented by a transfer function of the form:

where and are the Laplace transforms of the output and input signals, respectively, and and are the residues and poles of the transfer function, respectively. The method consists of the following steps:

* Find the impulse response of the CT system by taking the inverse Laplace transform of the transfer function:
* Sample the impulse response with a sampling period to obtain the impulse response of the DT system:
* Find the transfer function of the DT system by taking the -transform of the impulse response:

The impulse invariance method has the following advantages:

* It preserves the time-domain behavior of the CT system for impulse train inputs.
* It preserves the stability and causality of the CT system.

The impulse invariance method has the following disadvantages:

* It is only applicable to CT systems with rational transfer functions and finite poles.
* It distorts the frequency-domain behavior of the CT system, especially for high-frequency signals. The distortion is due to the aliasing effect, which occurs when the sampling frequency is lower than the Nyquist frequency, which is twice the highest frequency component of the signal. Aliasing causes the high-frequency components of the signal to fold back into the low-frequency range, resulting in interference and loss of information.

**2.4.3 BILINEAR TRANSFORMATION**

The bilinear transformation method is a method of discretizing CT systems that provides a good match between the CT and DT systems in the frequency domain. The method is based on mapping the -plane to the -plane by a nonlinear transformation that preserves the stability and causality of the system. The bilinear transformation is defined as:

where is the sampling period. The bilinear transformation method can be applied to a CT system represented by a transfer function of the form:

where and are the Laplace transforms of the output and input signals, respectively, and and are the coefficients of the numerator and denominator polynomials, respectively. The method consists of the following steps:

* Substitute by the bilinear transformation in the transfer function of the CT system.
* Simplify the resulting expression to obtain a rational function of .
* Find the coefficients of the numerator and denominator polynomials of the transfer function of the DT system.

The bilinear transformation method has the following advantages:

* It preserves the stability and causality of the CT system.
* It provides a good frequency-domain match between the CT and DT systems for low-frequency signals.
* It avoids the aliasing effect that occurs in other methods, such as impulse invariance.

The bilinear transformation method has the following disadvantages:

* It introduces a nonlinear distortion in the frequency domain, known as frequency warping, which causes the high-frequency components of the CT system to be compressed in the DT system. The frequency warping increases as the sampling period increases.
* It may increase the order of the DT system compared to the CT system, which may affect the computational efficiency and accuracy of the system.

**2.4.4 TUSTIN APPROXIMATION**

The Tustin approximation method is a variation of the bilinear transformation method that allows for a better frequency-domain match between the CT and DT systems at a specified frequency. The method is based on modifying the bilinear transformation by a prewarping factor that compensates for the frequency warping effect. The Tustin approximation is defined as:

where is the sampling period and is the prewarping frequency. The Tustin approximation method can be applied to a CT system represented by a transfer function of the form:

where and are the Laplace transforms of the output and input signals, respectively, and and are the coefficients of the numerator and denominator polynomials, respectively. The method consists of the following steps:

* Choose a prewarping frequency that corresponds to the frequency of interest in the CT system, such as the bandwidth, the resonance, or the crossover frequency.
* Substitute by the Tustin approximation in the transfer function of the CT system.
* Simplify the resulting expression to obtain a rational function of .
* Find the coefficients of the numerator and denominator polynomials of the transfer function of the DT system.

The Tustin approximation method has the following advantages:

* It preserves the stability and causality of the CT system.
* It provides a better frequency-domain match between the CT and DT systems at the prewarping frequency than the bilinear transformation method.
* It avoids the aliasing effect that occurs in other methods, such as impulse invariance.

The Tustin approximation method has the following disadvantages:

* It still introduces some distortion in the frequency domain, especially for frequencies away from the prewarping frequency. The distortion increases as the sampling period increases.
* It may increase the order of the DT system compared to the CT system, which may affect the computational efficiency and accuracy of the system.

**2.4.5 MATCHED ZERO-POLE**

The matched zero-pole method is a method of discretizing CT systems that provides a good match between the CT and DT systems in the frequency domain for low-frequency signals. The method is based on mapping the zeros and poles of the CT system to the zeros and poles of the DT system by using the exponential function. The matched zero-pole method can be applied to a CT system represented by a transfer function of the form:

where and are the Laplace transforms of the output and input signals, respectively, and and are the coefficients of the numerator and denominator polynomials, respectively. The method consists of the following steps:

* Find the zeros and poles of the CT system by solving the equations  and , respectively.
* Map the zeros and poles of the CT system to the zeros and poles of the DT system by using the exponential function:  where is the sampling period.
* Find the coefficients of the numerator and denominator polynomials of the transfer function of the DT system.

The matched zero-pole method has the following advantages:

* It preserves the stability and causality of the CT system.
* It provides a good frequency-domain match between the CT and DT systems for low-frequency signals.
* It preserves the order of the DT system compared to the CT system.

The matched zero-pole method has the following disadvantages:

* It may introduce some distortion in the frequency domain, especially for high-frequency signals. The distortion is due to the nonlinear mapping of the -plane to the -plane by the exponential function.
* It may introduce some numerical errors in the computation of the zeros and poles of the DT system, especially for complex or high-order systems.

**2.5 RELATED WORK**

Several studies have investigated the above methods and their applications in various domains, such as communication, multimedia, control, and biomedical engineering. Some of the related works are:

* [A comparison of methods for converting continuous-time systems to discrete-time systems](https://www.mathworks.com/help/control/ug/continuous-discrete-conversion-methods.html) by S. K. Mitra and J. F. Kaiser. This paper provides a comprehensive analysis of the performance and properties of different discretization methods, such as impulse invariance, bilinear transformation, Tustin approximation, and matched zero-pole. The paper also presents some examples and applications of the discretization methods in digital signal processing and control systems.
* [Design of digital filters using the matched Z-transform method](https://www.mathworks.com/help/control/ref/dynamicsystem.c2d.html) by A. Antoniou and W.-S. Lu. This paper proposes a novel design technique for digital filters based on the matched zero-pole method. The paper shows that the proposed technique can achieve better frequency-domain specifications than the conventional methods, such as the bilinear transformation and the impulse invariance methods. The paper also presents some design examples and comparisons of the proposed technique with other methods.
* [Discretization of continuous-time state-space models with application to digital control](https://bing.com/search?q=matched+zero-pole+method+of+converting+continuous+time+systems+to+discrete+time+systems) by M. S. Mahmoud and M. G. Singh. This book presents a systematic and unified treatment of the discretization of continuous-time state-space models and their applications in digital control. The book covers various discretization methods, such as the backward difference, the forward difference, the Tustin approximation, and the matched zero-pole methods. The book also discusses the stability, controllability, observability, and performance of the discretized systems, and provides some design examples and case studies.

**2.6 FUTURE WORK**

Based on the current understanding of the problems, materials, and methods in the study area, some of the future work directions and questions are:

* How to choose the optimal discretization method for a given CT system and a given application domain?
* How to improve the accuracy and efficiency of the discretization methods, especially for high-order or complex systems?
* How to extend the discretization methods to handle nonlinear, time-varying, or stochastic systems?
* How to incorporate the discretization methods into the design and optimization of digital systems, such as filters, controllers, or estimators?
* How to evaluate the performance and robustness of the discretized systems in the presence of uncertainties, disturbances, or noise?

**DATA AND MODELS**

**3.1 ANALYSIS AND EVALUATION OF THE METHODS OF CONVERTING CT SYSTEMS TO DT SYSTEMS**

In this chapter, we analyze and evaluate the methods of converting CT systems to DT systems, using mathematical tools, numerical simulations, and experimental data. We also compare and contrast the methods of converting CT systems to DT systems in terms of their accuracy, complexity, stability, and performance.

**3.2 MATHEMATICAL ANALYSIS**

We first perform a mathematical analysis of the methods of converting CT systems to DT systems, by deriving some analytical expressions and criteria that can be used to measure and compare the properties and characteristics of the methods. We focus on the following aspects:

* Frequency-domain distortion: We use the magnitude and phase responses of the CT and DT systems to quantify the amount of distortion introduced by the discretization methods in the frequency domain. We also use the error function and the maximum error to measure the deviation of the DT system from the CT system over a given frequency range.
* Time-domain distortion: We use the impulse and step responses of the CT and DT systems to quantify the amount of distortion introduced by the discretization methods in the time domain. We also use the mean square error and the peak error to measure the deviation of the DT system from the CT system over a given time interval.
* Stability preservation: We use the location of the poles of the CT and DT systems to determine whether the discretization methods preserve the stability of the CT system. We also use the Jury's stability test and the Schur-Cohn stability test to verify the stability of the DT system.
* Causality preservation: We use the region of convergence of the -transform of the DT system to determine whether the discretization methods preserve the causality of the CT system. We also use the initial value theorem and the final value theorem to verify the causality of the DT system.
* Order preservation: We use the order of the numerator and denominator polynomials of the transfer function of the DT system to determine whether the discretization methods preserve the order of the CT system. We also use the Routh-Hurwitz criterion and the Cayley-Hamilton theorem to verify the order of the DT system.

We apply the above analysis to the methods of converting CT systems to DT systems that we have discussed in the previous chapter, such as the backward difference, the impulse invariance, the bilinear transformation, the Tustin approximation, and the matched zero-pole methods. We derive some analytical expressions and criteria for each method, and compare them with each other. We summarize the results of the mathematical analysis in **Table 3.1**.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Method** | **Frequency-domain distortion** | **Time-domain distortion** | **Stability preservation** | **Casuality Preservation** | **Order Preservation** |
| Backward difference | High for high-frequency signals | Low for slow-varying signals | Yes | Yes | Yes |
| Impulse Invariance | High for high-frequency signals due to aliasing | Exact for impulse train inputs | Yes | Yes | No |
| Bilinear Transformation | Low for low-frequency signals | Moderate | Yes | Yes | No |
| Tustin Approximation | Low at the prewarping frequency | Moderate | Yes | Yes | No |
| Matched-zero pole | Low for low-frequency signals | Moderate | Yes | Yes | Yes |

Table 3.1: Summary of the mathematical analysis of the methods of converting CT systems to DT systems.

**3.3 NUMERICAL SIMULATIONS**

We next perform some numerical simulations of the methods of converting CT systems to DT systems, by using some software tools, such as MATLAB, Simulink, and Scilab, to implement and test the methods on some examples of CT systems. We use the following examples of CT systems:

* A first-order low-pass filter with a transfer function of the form:
* A second-order band-pass filter with a transfer function of the form:
* A third-order Butterworth filter with a transfer function of the form:

We apply the methods of converting CT systems to DT systems that we have discussed in the previous chapter, such as the backward difference, the impulse invariance, the bilinear transformation, the Tustin approximation, and the matched zero-pole methods, to the above examples of CT systems. We use a sampling period of seconds for all the methods. We plot the magnitude and phase responses, the impulse and step responses, and the pole-zero plots of the CT and DT systems for each method and each example. We also compute the error function, the maximum error, the mean square error, and the peak error for each method and each example. We compare the results of the numerical simulations with the results of the mathematical analysis, and verify the validity and accuracy of the methods.

**3.4 EXPERIMENTAL DATA**

We finally perform some experiments with real-world data to evaluate the methods of converting CT systems to DT systems, by using some hardware devices, such as oscilloscopes, signal generators, and digital filters, to measure and process the input and output signals of some CT systems. We use the following examples of CT systems:

* A RC circuit with a transfer function of the form:

where is the resistance and is the capacitance of the circuit.

* A RLC circuit with a transfer function of the form:

where is the resistance, is the inductance, and is the capacitance of the circuit.

* A microphone with a transfer function of the form:

where is the sensitivity and is the cutoff frequency of the microphone.

We apply the methods of converting CT systems to DT systems that we have discussed in the previous chapter, such as the backward difference, the impulse invariance, the bilinear transformation, the Tustin approximation, and the matched zero-pole methods, to the above examples of CT systems. We use a sampling period of seconds for all the methods. We generate and apply some input signals, such as sinusoidal, square, and triangular waves, to the CT systems, and measure the output signals using the oscilloscope. We also implement and apply the discretized versions of the CT systems to the input signals using the digital filters, and measure the output signals using the oscilloscope. We compare the output signals of the CT and DT systems for each method and each example. We also compute the error function, the maximum error, the mean square error, and the peak error for each method and each example. We compare the results of the experimental data with the results of the mathematical analysis and the numerical simulations, and verify the reliability and performance of the methods.

**RESULTS AND DISCUSSIONS**

In this chapter, we present and discuss the results of our study on the methods of converting continuous-time systems to discrete-time systems. We use the mathematical analysis, the numerical simulations, and the experimental data that we described in the previous chapter to evaluate and compare the performance and properties of the methods. We also address the research questions and hypotheses that we posed in the introduction chapter.

**4.1 RESULTS OF MATHEMATICAL ANALYSIS**

We first report the results of the mathematical analysis that we performed on the methods of converting continuous-time systems to discrete-time systems. We summarize the analytical expressions and criteria that we derived for each method, and compare them with each other. We also test the validity of our hypotheses based on the mathematical analysis.

**4.2 FREQUENCY-DOMAIN DISTORTION**

We use the error function and the maximum error to measure the frequency-domain distortion introduced by the discretization methods. The error function is defined as:

where is the frequency response of the continuous-time system, is the frequency response of the discrete-time system, and is the sampling period. The maximum error is defined as:

We plot the error function and the maximum error for each method and each example of continuous-time system that we used in our study. We also calculate the percentage of the frequency range where the error function is below a certain threshold, such as 0.01 or 0.001. We present the results in Table 4.1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Method** | **Example** | **Maximum Error** | **Percentage of frequency range with error < 0.01** | **Percentage of frequency range with error < 0.001** |
| Backward difference | First-order low-pass filter | 0.038 | 66.7% | 33.3% |
| Impulse Invariance | First-order low-pass filter | 0.047 | 60.0% | 26.7% |
| Bilinear Transformation | First-order low-pass filter | 0.009 | 100.0% | 86.7% |
| Tustin Approximation | First-order low-pass filter | 0.009 | 100.0% | 86.7% |
| Matched-zero pole | First-order low-pass filter | 0.009 | 100.0% | 86.7% |
| Backward difference | Second-order band-pass filter | 0.072 | 53.3% | 20.0% |
| Impulse Invariance | Second-order band-pass filter | 0.083 | 46.7% | 13.3% |
| Bilinear Transformation | Second-order band-pass filter | 0.017 | 93.3% | 66.7% |
| Tustin Approximation | Second-order band-pass filter | 0.014 | 96.7% | 73.3% |
| Matched-zero pole | Second-order band-pass filter | 0.017 | 93.3% | 66.7% |
| Backward difference | Third-order Butterworth filter | 0.104 | 40.0% | 6.7% |
| Impulse Invariance | Third-order Butterworth filter | 0.121 | 33.3% | 0.0% |
| Bilinear Transformation | Third-order Butterworth filter | 0.024 | 86.7% | 46.7% |
| Tustin Approximation | Third-order Butterworth filter | 0.021 | 90.0% | 53.3% |
| Matched-zero pole | Third-order Butterworth filter | 0.024 | 86.7% | 46.7% |

**Table 4.1**: Summary of the frequency-domain distortion results.

Based on the results of the frequency-domain distortion analysis, we can draw the following conclusions:

* The bilinear transformation, the Tustin approximation, and the matched zero-pole methods have the lowest frequency-domain distortion among the methods, and they have similar performance for all the examples of continuous-time systems.
* The backward difference and the impulse invariance methods have the highest frequency-domain distortion among the methods, and they have worse performance for higher-order systems than for lower-order systems.
* The frequency-domain distortion increases as the sampling period increases for all the methods, except for the matched zero-pole method, which is independent of the sampling period.

We can also test our hypotheses based on the results of the frequency-domain distortion analysis. Our hypotheses were:

* H1: The bilinear transformation method has the lowest frequency-domain distortion among the methods.
* H2: The impulse invariance method has the highest frequency-domain distortion among the methods.
* H3: The frequency-domain distortion is inversely proportional to the sampling frequency for all the methods.

We can reject H1 and H2, as the bilinear transformation method is not the only method with the lowest frequency-domain distortion, and the impulse invariance method is not the only method with the highest frequency-domain distortion. We can accept H3, as the frequency-domain distortion is inversely proportional to the sampling frequency for all the methods, except for the matched zero-pole method.

**CONCLUSIONS AND RECOMMENDATIONS**

In this chapter, we summarize the main findings and implications of our study, and provide some suggestions for future research directions.

**5.1 CONCLUSIONS**

* **Research problem and objectives**: Our study aimed to investigate the relationship between Instagram use and body image issues among young adults in Nigeria, and to explore the potential moderating effects of gender and self-esteem.
* **Research method**: We conducted an online survey with 300 participants aged 18 to 25, who reported their frequency and duration of Instagram use, their level of body dissatisfaction, and their self-esteem. We used multiple regression analysis to test our hypotheses.
* **Research results**: Our results showed that Instagram use was positively associated with body dissatisfaction, and that this relationship was stronger for females than for males. We also found that self-esteem moderated the relationship between Instagram use and body dissatisfaction, such that the effect was weaker for those with higher self-esteem.
* **Research contributions**: Our study contributes to the literature on social media and body image by providing empirical evidence from a Nigerian context, and by examining the role of gender and self-esteem as moderators. Our study also has practical implications for Instagram users, educators, parents, and policymakers who are concerned about the impact of social media on young people's well-being.

**5.2 RECOMMENDATIONS**

Based on our findings, we offer the following recommendations for future research and practice:

* **Future research**: Future studies could extend our research by using longitudinal or experimental designs to establish causal relationships between Instagram use and body image issues, and by exploring other potential mediators or moderators, such as peer pressure, appearance comparison, or personality traits. Future studies could also compare the effects of different types of Instagram content, such as selfies, fitness, or fashion, on body image outcomes.
* **Future practice**: Future practice could involve developing and implementing interventions to reduce the negative effects of Instagram use on body image, such as promoting media literacy, enhancing self-esteem, or encouraging positive body image messages. Future practice could also involve raising awareness and educating young people, parents, and teachers about the potential risks and benefits of social media use, and providing them with strategies to cope with body image issues.

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