**APPLICATIONS OF LINEAR DIFFERENTIAL EQUATIONS**

**7.0 NEWTON’S LAW OF COOLING**

– Temperature of the object at time t

– Temperature of the surroundings

– Original temperature of the object

– time

– A constant to be found in the question

**7.1 PROOF OF NEWTON’S LAW**

From the above equation, it can be sait that as the object approaches the surrounding temperature, the rate of temmperature change decreases.

At t=0,

Putting that back into the original equation

**7.1.1 QUESTIONS**

It takes 12mins for an object at 100C to cool to 80C in a room at 50C. How much longer will it take for its temperature to decrease to 70C.

Answer: 9.408mins

**7.2 Exponential growth and Decay Calculus, Relative Growth Rate, Differential equations**

The above implies that the population grows at a rate that is proportional to the population size.

– Relative growth rate

At

**7.3 MIXING PROBLEMS**

A vat with 500 gallons of bear contains 4% alcohol (by volume). Beer with 7% alcohol is pumped into the vat at a rate of 5gal/min and the mixture is pumpedout at the same rate.

a. What is the amount of alcohol after an hour

b. What is the percentage of alcohol after an hour.

Dividing through by -0.01

On solving with the initial conditions ,

**7.4 SPRING**

The general formula for the second order differential equation for a spring system is

To solve the homogeneous equation,

But

Using initial conditions,

1. An object stretches a spring 6 inches in equilibrium

a. Setup and solve a DE for its motion

b. Find the displacement given it is initially displaced by 18 inches with a velocity of 3ft/s

Assume there’s no damping

From the question, there is no damping and there is no statement about an external force. So our general equation, , can be reduced to

But

From the question

On solving, ,

2. A 10kg mass is attached to a spring with . The mass is given and an initial velocity of 2m/s upwards with an exteral force of . The resistance due to damping is

7.5 FIRST ORDER R-C CIRCUITS

7.6 FIRST ORDER R-L CIRCUITS

**7.7** **SECOND ORDER R-L-C CIRCUITS**

RLC Circuits are used as tuning circuits in radio communications

Are used as voltage multipliers

7.7.1 SERIES RLC CIRCUITS

A voltage source, a key, a resistor, an inductor and a capacitor are connected in series.

Before time t=0 i.e. t<0, the switch is open and therefore no enenergy is stored in the elements.

At time t = 0, the switch is closed.

Applying KVL

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The current flowing through the circuit is the same as the current flowing through the capacitor

Dividing through by LC

Now, we have gotten a second order linear differential equation.

The complementary function is the transient response of the circuit.

The Particular Integral is the steady state response of the circuit.

For the transient response

The characteristic equation is given as:

In an RLC circuit, L and C induce some kind of oscillation in the circuit. The resistor has a tendency to dampen/suppress the oscillation.

The oscillation generated, is given as

The frequencty of this oscillation is known as the natural frequency.

Damping Factor is the normalized damping coefficient and this defines the circuit responds to different excitations.

Cases to consider:

Case 1: , Overdamped Response

It takes time to reach its maximum value slowly because the response is sluggish

In that case,

Case 2, , Critically damped Response:

Here it is fast to reach its maximum value

Roots will be negative, real and equal

Case 3: , Under damped Response

For this it reaches a maximum response, and then there is a slight oscillation and then gets to rest.

Roots will be coomplex conjugate

The solution is given as

Case 4:

Roots will be imaginary

The solution is given as

7.7.2 PARALLEL RLC CIRCUIT

A current source is connected in parallel with a resistor, inductor and capacitor.

**7.8 RADIOACTIVE DECAY**

A certain radioactive material is known to decay at a rate proportional to the amount present. If initally there is 50mg of the material present and after two hours it is observed that the material has lost 10 % of its original mass. Find

I. the mass of the material after four hours and

ii. The time at which the material has decayed to ½ its original mass

A. 40.5mg,t