**FIRST ORDER LINEAR DIFFERENTIAL EQUATIONS**

First Order ODEs

- Types and Techniques of solution of first order ODE’s

\* Integration Methods

- Direct Integration

- Separation of Variables

- Integrating Factor

\* Substitution Methods

- Direct Substitution

- Homogeneous 1st order equations

- Bernoullis’s Equations

- Exact Differential Equations

- Numerical methods for solving ODEs

\* Euler’s Method

\* Improved Euler’s Method

\* Runge-Kutta Method

\* Picard’s iterative method

- Physical applications of first order ODE.

**2.0 INTRODUCTION**

What is a first order linear differential equation.

This is a linear differential equation where the highest power of the derivative is 1 (first order).y(x) is

**2.1 DIFFERENTIAL FORM OF LINEAR ODE**

If

The general standard form for a first order ODE in the function

Sample questions.

Write the differential equation

**2.2 STANDARD FORM OF FIRST ORDER LINEAR ODE**

**2.3 METHODS OF SOLVING LINEAR DIFFERENTIAL EQUATIONS**

1. Direct Integration

2. Means of Separating Variables

3. Integrating Factor Method

4. Subsitution Methods

5. Picard’s Iterative Method

**2.3.1 DIRECT INTEGRATION METHOD**

You have an independent variable x and the function, y, is in terms of x

If , is a function of the independent variable only

Examples:

1. Solve the initial value problem

By integrating,

On solving for the initial problem,

2. Solve the IVP

Integrate with respect to y

Using IC(Initial Conditions)

**2.3.2 METHOD OF SEPARATING VARIABLES**

This method is used to solve separable differential equations.

You’ll see that the function of x and y, is actually a product of the **function of x** and the **function of y**.

Given the function:

**2.3.2.1 STEPS**

1. **Separate variables**: On one side, we want only **y-variables** and **dy**. On the other side, we want only **x-variables** and **dx**’s.

2. Integrate both sides.

3. Add the constant of integration to the x-side of the integrals

4. Solve and make y the subject of the formula

The formula you get is called the **general solution**. If you are given an initial value, , then you find the **specific solution**

Solve the following

1. . Given the initial position , then you should find the specific solution

Answers: and

2. At point Answer:

3. At

4.

5. Solve , : General Solution:

Final solution:

6. Solve Implicit solution:

**2.3.3 INTEGRATING FACTOR METHOD**

This is also used to solve the first order linear differential equations.

1. Write the equation in the **standard form** the first order linear differential equation.

2. Identify the functions p(x) and q(x)

3. Determine the integrating factor

4. Multiply through by the integrating factor.

5. This can be reduced to the form:

6. On integrating,

7. Write the general solution

Solve the following questions

1. Answer:

2. Answer:

3.

4. Answer:

5.

6.

**2.3.4 SUBSTITUTION METHODS**

**2.3.4.1 DIRECT SUBSTITUTION**

Let u = “inside function”

Rewrite the DE using the new variable

For example, Solve:

,

Let

**2.3.4.2 HOMOGENEOUS EQUATIONS**

A homogeneous equation is an equation that can be written in the form:

A homogeneous differential equation is a type of differential equation that can be written in the form , where f is a function of the ratio . This means that both sides of the equation are homogeneous functions of the same degree of x and y.

Steps to solve:

1. Let

Therefore,

2. Rewrite the equation in v and x variables

3. Solve for v first, and then use y=vx to solve for y.

Examples:

1. ,

Dividing by xy

Let

But

Using

Example 1:

This is a homogeneous differential equation, since both sides are homogeneous functions of degree one. We can solve it by substituting and separating the variables.

Example 2:

**2.3.4.3 The Bernoulli’s Equation**

This is an extension of the first order linear equation

If , and you use the integrating factor method

If , , there are two ways to go about this:

a. , then we use the integrating factor method.

b. , then use separation method

If , dividing by

Then use the subsitution:

Taking the derivatives,

From the chain rule

Where

This can be seen to be the first order linear equation in v

Then we solve for v using the integrating factor method

Use to find y.

Example: Solve

Dividing by

Substituting

→ 1st order linear in v

Multiplying through by x

On integrating

Using

**2.4 EXACT AND ALMOST EXACT DIFFERENTIAL EQUATIONS**

**2.4.1 EXACT DIFFERENTIAL EQUATITIONS**

The standard form of a differential equation is written as

It can also be written in the form by dividing through by dx

if , then the equation is called EXACT differential equation, and there is a function

, such that and .

Solve the function, for y. Then the equation of y is the solution

**2.4.1.1 METHODS OF SOLVING**

There are two major methdos to solving the exact differential equations

1. Solution Method:

2. Shortcut Method

**2.4.1.1.1 SOLUTION METHOD**

Integrate with respect to x.

Since it is coming from a partial derivative, there is a possibility that there will be a function of x that will be considered as a constant, so we add it back to the equation when integrating.

Next, differentiate with respect to y

Next we want to find the equation g(y). Therefore, set and then find , then integrate to find

Substituting

Solve for y, if possible. This is because there may not be a solution for y. If there is, that will be the implicit

**2.4.1.1.2 SHORTCUT METHOD**

Integrate with respect to x

Then you get the function

Integrate with respect to y

f(x,y) – Write common terms and uncommon terms once

f(x,y)= C

Solve for y, if possible.

Examples:

Solve:

Since,, the equation is an exact differential equation.

This means there must be a function z = f(x, y) = C such that

Integrate with respect to x

Differentiate with respect to y.

It can be seen that

Now we integrate with respect to y

Since there is no way you can solve for y, the implicit solution is

**2.4.2 ALMOST EXACT EQUATION**

If , then the equation is not exact so…

We will use the integrating factor method to solve.

There are two cases to consider:

1. **The MNN case**: If is a function of x only, then the integrating factor is given as

2. **The NMM case**: If is a function of y only, then the integrating factor is given as

Then if you multiply the standard form by the integrating factors, you get…

→ This is going to form an exact equation.

Solve:

Since they are not the same, it is not an exact equation.

→ Since this is not a function of a single variable, we cannot use this expression.

→ This is a function of y only

Multiply by the integrating factor

Since this is exact, we use the shortcut method.

On integrating

On integrating with respect to y:

If you look at both equations you’ll see that the term is common to both.

So we write the common terms first

Next we write the uncommon terms to both

Since this is not easy to solve for y, this is the implicit solution for y.

**2.5** **NUMERICAL METHODS FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS**

1. Euler’s Method

2. Improved Euler’s Method

3. Runge-Kutta Method

4. Picard’s Iteration Method

**2.5.1 EULER’S METHOD**

You are going to look at the slopes at each point and use the tangent lines to approximate the next point.

,

h → gap between x values

We are given the slope at every point (the derivatives), and the initial values

Example:

Approximate y(2) using eulers method if given that

, ,

Since the step is 0.5 and we are looking for x=2

So similarly, we are looking for , , and

When , ,

, ,

**2.5.2 RUNGE-KUTTA METHOD**

This method produces the best result

with

For

This is called the fourth order Runge-Kutta (RK) Method

ODE45 Algorithm in Matlab use an improved version of this method

**2.5.3 PICARDS ITERATION METHOD**

Questions:

1. Solve by Picard method. Find successive approximation. Solve up to fourth order of initial value problem , .

Solution:

,

Put n=0,

For n=1

Omo this thing long oh. So person wan dey solve this thing for exam. God forbid oh. Well… let’s continue

For n=2

For n=3

Thanks for watching. Please subscribe and don’t forget to hit the like button. Lmaoo