GEG 219 (Ordinary Differential Equations) 2units

COURSE OUTLINE

1. Introduction to Differential Equations

- Linear dependence

- Classification of Ordinary Differential Equations

\* Order

\* Degree

\* Linearity.

2. First Order ODEs

- Types and Techniques of solution of first order ODE’s

- Picard’s iterative method

- physical applications of first order ODE.

3. Theory and solutions of higher order linear equations; physical applications.

4. Ordinary differential equations with constant coefficients

- Methods of undetermined coefficients

-Variation of parameters

- D-Operator.

5. Linear Differential Equations with Variable coefficients.

6. Cauchy-Euler’s equations

7. Systems of linear operations

8. Properties of linear operations

9. Series solution

8. First order non-linear equations

- Autonomous

- Equidimensional

- Scale-invariant

**INTRODUCTION TO DIFFERENTIAL EQUATIONS**

A differential equation is one which contains a function and its derivative in the same equation.

It could be said to be an equation with at least one derivative

A differential equation is a relationship between an independent variable x, a dependent variable y and at least one derivative y.

NB:

**TYPES OF DIFFERENTIAL EQUATIONS**

1. Ordinary Differential Equations: This is one in which the unknown function (y) depends only on one independent variable (x).

2. Partial Differential Equations. This is one in which the unknown functions depends on at least two independent variables.

**FORMATION OF A DIFFERENTIAL EQUATION**

A differential equation is formed when arbitrary constants are eliminated from a given function.

**ORDER AND DEGREE OF A DIFFERENTIAL EQUATION**

The order of a DE is the highest derivative appearing in the equation. It is also called differential coefficient.

The degree of a DE is the power of the highest order is the degree

a. . This will have an order of 2 and a degreee of 1.

The order of a differential equation indicates how many initial conditions are needed to find a unique solution.

**INITIAL VALUE PROBLEM AND BOUNDARY VALUE PROBLEM**

In initial value problem, we are given the value of the function and its derivative at the same point (initial point)

and . For initial value problem, we are given a differential equation and an initial condition. For a first order equation, you will need one initial condition. For a second order equation, you will need two separate initial conditions.

Boundary Value Problems: Here we are given the value of function y(x) at two different points i.e. ,

**VERIFYING SOLUTIONS**

Verify that is a solution of . Find and such that , .

Answer: Therefoe, is a solution of the differential equation

**LINEAR AND NON-LINEAR DIFFERENTIAL EQUATIONS**

A linear differential equation is one where the degree is 1.

**CONDITIONS FOR LINEAR DIFFERENTIAL EQUATION**

1. In front of y and its derivatives must only be pure functions of x and never of y or other variables.

2. The powers of y and its derivatives must be 1. (Degree of 1)

3. g(x) must also be a function of x and neither y or its derivative should be there.

**FIRST ORDER LINEAR DIFFERENTIAL EQUATIONS**

This is a linear differential equation where the highest power of the derivative is 1 (first order).y(x) is

**DIFFERENTIAL FORM OF LINEAR ODE**

if

The general standard form for a first order ODE in the function

Sample questions.

Write the differential equation

**METHODS OF SOLVING LINEAR DIFFERENTIAL EQUATIONS**

1. Direct Integration

2. Integrating Factor Method

3. Means of Separating Variables

4. Picard’s Iterative Method

**INTEGRATING FACTOR METHOD**

1. Write the equation in the standard form

2. Identify the functions p(x) and q(x)

3. Determine the integrated factor

4. Write the general solution

Solve the following questions

1. Answer:

2. Answer:

3.

4. Answer:

**METHOD OF SEPARATING VARIABLES**

1. Separate variables. On one side, we want only y-variables and dy. On the other side, we want only x-variables and dx’s.

2. Integrate both sides

3. Add the constant of integration to the x-side of the integrals

4. Solve and make y the subject of the formula

The formula you get is called the **general solution**. If you are given an initial value, y(1)=2, then you find the specific solution

Solve the following

1. . Givne the initial position , then you should find the specific solution

Answers: and

2. At point Answer:

3. At

4.

HOMOGENEOUS EQUATIONS

A homogeneous differential equation is a type of differential equation that can be written in the form , where f is a function of the ration . This means that both sides of the equation are homogeneous functions of the same degree of x and y.

Example 1:

This is a homogeneous differential equation, since both sides are homogeneous functions of degree one. We can solve it by substituting y=vx and separating the variables.

Example 2:

LINEAR DEPENDENCE

Linear Dependence is used as a method of solution for 2nd Order Ordinary Differential Equations (ODE)

Given two non-zero functions and defined in the equation below

(1)

Notice that and will make equation (1) to be true for all values of regardless of the functions that we use.

Now, if we can find non-zero values of and ( and ) for which equation (1) will still be true i.e. (eqn. (1)) for all values of , then we call the two functions ( and ) linearly dependent functions.

On the other hand, if the only two constants for which eqn.(1) is true are and , then we call the functions linearly independent functions.

How to determine whether functions are Linearly Dependent or not (Linearly Independent)

Wroskian Method

Let and be differentiable functions and and be differentials of and then they are Linearly Dependent if the non-constants and with

Or