**ODE WITH CONSTANT COEFFICIENTS**

**4.1 INTRODUCTION**

Given the general form of a second order linear differential equation:

, , are usually continuous functions of x. In that case, it is called a differential equation with variable coefficients

If , we have a homogeneous linear equation

If , we have a non-homogeneous linear equation

If , , amd are constants, it will give an ODE with constant coefficients equation

**4.2 SOLVING HOMOGENEOUS EQUATIONS WITH CONSTANT COEFFICIENTS**

A typical solution of this is:

On replacing the equation, we have:

Next we solve for r:

Now you need to consider three cases when solving for r:

Case 1:

Here, You will have two values for r i.e. and

The general solution for this case is:

Case 2: ,

Here you get one value for r

The general solution is given as

y =

Case 3:

Here you would have to use the quadratic formula to solve.

You will also get two values of r

The general solution will be

**4.3 SOLVING NONHOMOGENEOUS EQUATIONS WITH CONSTANT COEFFICIENTS**

**4.3.1 METHOD OF UNDETERMINED COEFFICIENTS**

For non homogeneous 2nd ORDER Differential equations

This is a nonhomogeneous general equation.

To solve we have the general solution

is the particular solution of the non-homogeneous equation

Next, we write the homogeneous equation of the differential equation

The solution of this homogeneous equation is the value of .

Questions

1. Solve the equations .

First step is to solve the homogeneous equation of the differential equation.

On solving, the general solution is

Next, we look at the degree of the function on the right.

Since the power of the function on the right is 2 , the general solution for this non-homogeneous equation will have a general form

Then if we find the following derivatives:

Next we substitute the equations into the general form

6A = 1

Multiplying through by 6

Multipying through by 18

But,

2. Solve the differential equation

Answers

3. Solve the differential equation

Answers:

Note for help that

4. Solve the differential equation

Looking at this, would normally be . However if you use this, and will cancel out and we will have and we can’t do anything with that.

So we will use

5.

For , we are going to have two values: and . This is because for the homogeneous equation solution, the function on the right is a summation of two functions and

So for the first solution,

**4.3.2 METHOD OF VARIATION OF PARAMETERS**

First you solve the homogeneous equation

On solving we see that

When use the method of variation of parameters, our solution to the non-homogeneous equation will be

When using variation of parameters, and from the homogeneous equation will be converted to functions and . .

Next we write the condition that we want to achieve

Next we find and and we plug it into the expression.

Using the above gotten equation and the other equation , we can solve for and . Then we integrate to get and .

Then we plug that into and we get the solution of the non-homogeneous equation

Recall from our condition that

On solving

Solving by system of equations:

On adding,

Recall that: