**ODE WITH VARIABLE COEFFICIENTS**

**0.1 CAUCHY-EULER EQUATIONS**

These are also called **Homogeneous linear differential equations**.

The general form is:

Note the following of homogeneous linear differential equations

1. A differential equation is called Homogeneous Linear Differential Equation with variable coefficients if the powers of x are equal to the orders of the dervative associated with them.

2. The dependent variable y and its derivatives with respect to the independent variable x appear in their first degree and are not multiplied together.

3. These DE are also known as Cauchy-Euler equations.

**0.1.1 EXAMPLES**

This is homogeneous because the order of the derivatives and the power of x preceding it are the same.

These two above are not homogeneous because the powers of the x and the order are not the same.

**0.2 METHODS OF SOLVING**

1. Reduction of Orders

2. D-factor method

**0.2.1 REDUCTION OF ORDERS**

Given is a solution to, find the general solution of the differential equation.

The general solution is a sum of the unique independent variables of the two possible solutions and of the differential equation:

The other solution will be a product of a function of t and the first solution

On substituting to these into the differential equation,

Dividing through by

Let ,

Recall,

Now the first solution is . The second solution

The unique functions in the first one and the second one are

, common to both and which is in the second solution.

**0.2.2 D-OPERATOR METHOD**

Cauchy-Euler equations can be easily converted to equations with constant coefficients by changing the independent variable by the transformation

So from the above it can be seen and

Similarly,

Steps to solving:

1. Check if the equation is homogeneous or not

2. Make it homogeneous if not

3. Substitute

4. Obtained differential equation will be a linear differential equation with constant coefficients in terms of D.

5. Find the complementary function (CF) and the particular integral(PI)

6. Find the general solution,

7. Finally, substitute and

QUESTIONS

1. Solve

On substituting,

,

We have

→

The auxiliary equation is given as

On solving this,

twice

The general solution for this solution is given as

Since it is also a homogeneous equation with constant coefficients.

Substituting back,

2. Solve

On changing to the D-operator,

The Auxiliary Equation is

3.

The auxiliary function:

y = CF + PI

4.

Answer:

EQUATION REDUCIBLE TO HOMOGENEOUS FORM

This can be reduced to homogeneous differential equation with constant coefficients by substituting: