**ERRORS IN NUMERICAL METHODS/COMPUTATIONS**

1.1 INTRODUCTION

Numerical methods, while powerful, are not immune to errors. Understanding these errors is crucial for interpreting the results and ensuring the accuracy of your calculations.

The methods of numerical analysis are finite processes and a numerical result is an approximate value of the unknown exact result, except for the rare cases where the exact answer is sufficiently simple rational number and here we can use a numerical method that gives the exact answer.

### 1.2 CLASSIFICATION OF ERRORS

1.2.1 EXPERIMENTAL ERRORS

There are errors of given data probably arising from measurements. There are two main types of errors that arise in numerical computations:

Trucation Errors: These are errors corresponding to the facts that a finite or infinite sequence of computational steps necessary to produce an exact result is truncated prematurely after a certain number of steps. These errors depend on computational methods and they are discussed individually with each method. This error arises from simplifying a mathematical problem to make it solvable by a numerical method. For example, approximating a continuous function with a discrete set of points or representing an infinite series with a finite number of terms introduces truncation error. The magnitude of this error depends on the chosen method and the level of approximation used.

Rounding Errors or Round-off errors: These are errors arrising from the process of rounding off during computation. In automatic digital computers,

This error occurs due to the finite precision of computer arithmetic. Computers can only store numbers with a limited number of digits. When performing operations on numbers with more digits than the computer can store, some digits are rounded off or truncated. This rounding off can accumulate throughout a series of calculations, leading to a final result that deviates slightly from the true value.

### Error Analysis and Propagation:

Error analysis is the process of estimating the magnitude and impact of errors in numerical computations. It helps us understand the reliability of our results and choose appropriate numerical methods:

If is an approximate value whole exact value is a,

Absolute error = approximation – exact value

Approximation = True value + Error

Relative Error, this is the ratiio of the absolute error to the actual value

Correction,

Error bound for

**Estimating Round-off Error**

Understanding the number of significant digits carried by your computer system allows you to estimate the potential round-off error. Techniques like investigating the number of bits used for calculations or utilizing libraries with higher precision can help mitigate this error.

**Truncation Error Analysis**

Different numerical methods have different inherent truncation errors. Analyzing the properties of the chosen method (e.g., order of accuracy) helps estimate the magnitude of this error. There's often a trade-off between computational complexity and truncation error; simpler methods might have larger truncation errors.

**Error Propagation**

Errors can propagate through a series of calculations, potentially amplifying the overall error in the final result. Understanding how errors from each step accumulate is crucial. Some methods are more prone to error propagation than others.

By understanding and analyzing errors, we can choose appropriate numerical methods, control the level of accuracy, and interpret our results with confidence.

Fixed point system: The numbers are represented with a fixed nmber of dcimal places e.g. 62.358, 0.013

Floating point system: In this system, a fixed number of significant figures are represented e.g. ,

Practice: Use the quadratic formula to solve the equation:

WRITING DISTRIBUTIONS

Distributions are written as

a is the initial point

h is the difference between the numbers

b is the final number.

ASSIGNMENTS

1. Compute a 2 decimal table of , and find out how the rounding error is distributed

2. Compute a 3 decimal table of , and find how the rounding error is distributed

3. Explain the difficulties in obtaining a 4-place table by rounding off the radius in a 5-place table and 6-piece table

4. If and are error bounds of bar and , show that is an error bound for the sume

5. Show that in number 4 is an error bound for the difference and illustrate in general that cannot be replaced by a smaller number

## Subtopic 1.2: Errors in Numerical Computations

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### Types of Errors:

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* **Truncation Error:** This error arises from simplifying a mathematical problem to make it solvable by a numerical method. For example, approximating a continuous function with a discrete set of points or representing an infinite series with a finite number of terms introduces truncation error. The magnitude of this error depends on the chosen method and the level of approximation used.

### Error Analysis and Propagation:

Error analysis is the process of estimating the magnitude and impact of errors in numerical computations. It helps us understand the reliability of our results and choose appropriate numerical methods:

* **Estimating Round-off Error:** Understanding the number of significant digits carried by your computer system allows you to estimate the potential round-off error. Techniques like investigating the number of bits used for calculations or utilizing libraries with higher precision can help mitigate this error.
* **Truncation Error Analysis:** Different numerical methods have different inherent truncation errors. Analyzing the properties of the chosen method (e.g., order of accuracy) helps estimate the magnitude of this error. There's often a trade-off between computational complexity and truncation error; simpler methods might have larger truncation errors.
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