INTERPOLATION

1. Newton's Forward difference

2. Backward difference

3. Divided difference

4. Lagrange interpolation formula

5. Stirling formula

6. Bessel's formula

1.1 NEWTON’S FORWARD DIFFERENCE INTERPOLATION

1.2 NEWTON’S BACKWARD DIFFERENCE INTERPOLATION

1.3 NEWTON’S DIVIDED DIFFERENCE INTERPOLATION

1.4 LAGRANGE INTERPOLATION FORMULA

1.5 STIRLING FORMULA

1.6 BESSEL’S FORMULA

If the values of x are placed at equal intervals, we apply newton’s forward or backward difference interpolation.

1.1 NEWTON’S FORWARD INTERPOLATION

First step, get the value of h

Second step, get the value of u

Next, form the difference table

Next, get the first values of each column

Next, apply the forward interpolation formula

x in the question (maybe when told to find x) is the value to be interpolated.

If the value to be interpolated lies at the beginning of the table, we use the forward difference but if it lies close to the end of the table, we use the backward difference.

QUESTIONS

1. Form a difference table and interpolate the value of f(x) when x=4, given

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | 3 | 5 | 7 | 9 |
| y=f(x) | 180 | 150 | 120 | 90 |

,

DIFFERENCE TABLE

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 3 | 180 |  |  |  |
|  |  | -30 |  |  |
| 5 | 150 |  | 0 |  |
|  |  | -30 |  | 0 |
| 7 | 120 |  | 0 |  |
|  |  | -30 |  |  |
| 9 | 90 |  |  |  |

First values of each column, 180, -30, 0, 0

Applying the formula,

2. Find n for the following data if f(0.2) is asked.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| f(x) | 176 | 185 | 194 | 203 | 212 | 220 | 229 |

Solution:

In this question, n is the same as the u that we have been using so far in the course.

Explanation: The formula is  
x = x\_0+ nh.  
Here x\_0 is 0 as 0 is the first element and h is 1.  
Since in the question it is given that we have to find f(0.2), x= 0.2.  
So, substituting the values in the formula we get,  
0.2 = 0 + n(1) .  
Hence, n= 0.2.

3. Using Newton’s Forward formula, find sin(0.1604) from the following table.

|  |  |  |  |
| --- | --- | --- | --- |
| x | 0.160 | 0.161 | 0.162 |
|  | 0.1593182066 | 0.1603053541 | 0.1612923412 |

4. Find f(5) using Newton’s Forward interpolation formula from the following table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 0 | 2 | 4 | 6 | 8 |
| f(x) | 4 | 26 | 58 | 112 | 466 |

1.2 NEWTON’S BACKWARD INTERPOLATION

The formula for the backward difference

is the last value of x on the table

QUESTIONS

1. Using Newton’s Backward Interpolation formula, Find the annual premium at the age of 33 from the following data.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Age in years | 24 | 28 | 32 | 36 | 40 |
| Annual Premium in Rs | 28.06 | 30.19 | 32.75 | 34.94 | 40 |

DIFFERENCE TABLE

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | y | {%DELTA}y |  |  |  |
| 24 | 28.06 |  |  |  |  |
|  |  | 2.13 |  |  |  |
| 28 | 30.19 |  | 0.43 |  |  |
|  |  | 2.56 |  | -0.8 |  |
| 32 | 32.75 |  | -0.37 |  | 4.04 |
|  |  | 2.19 |  | 3.24 |  |
| 36 | 34.94 |  | 2.87 |  |  |
|  |  | 5.06 |  |  |  |
| 40 | 40 |  |  |  |  |

Next we get the backward values (i.e. from the down)

40, 5.06, 2.87, 3.24, 4.04

On simplifying,

1.3 NEWTON’S DIVIDED DIFFERENCE INTERPOLATION

QUESTIONS

1. Using the table of divided differences, find the newton interpolating for the data. Do not simplify

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | -5 | -1 | 0 | 2 |
| f(x) | -2 | 6 | 1 | 3 |

DIFFERENCE TABLE

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x | f(x) | 1st | 2nd | 3rd |
| -5 | -2 |  |  |  |
|  |  | 2 |  |  |
| -1 | 6 |  |  |  |
|  |  | -5 |  |  |
| 0 | 1 |  |  |  |
|  |  | 1 |  |  |
| 2 | 3 |  |  |  |

1.4 LAGRANGE’S INTERPOLATION FORMULA.

QUESTIONS

1. The following table gives the normal weight of a baby during the first six months of life

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Age in months | 0 | 2 | 3 | 5 | 6 |
| Weight in lbs | 5 | 7 | 8 | 10 | 12 |

Evaluate the weight of the baby at the age of 4 months.

Since the values are not equidistant, we can’t use newton’s forward or backward difference.

The Lagrange’s Interpolation formula is,

The value of x is the value you want to interpolate.