Lichtenberg Figures

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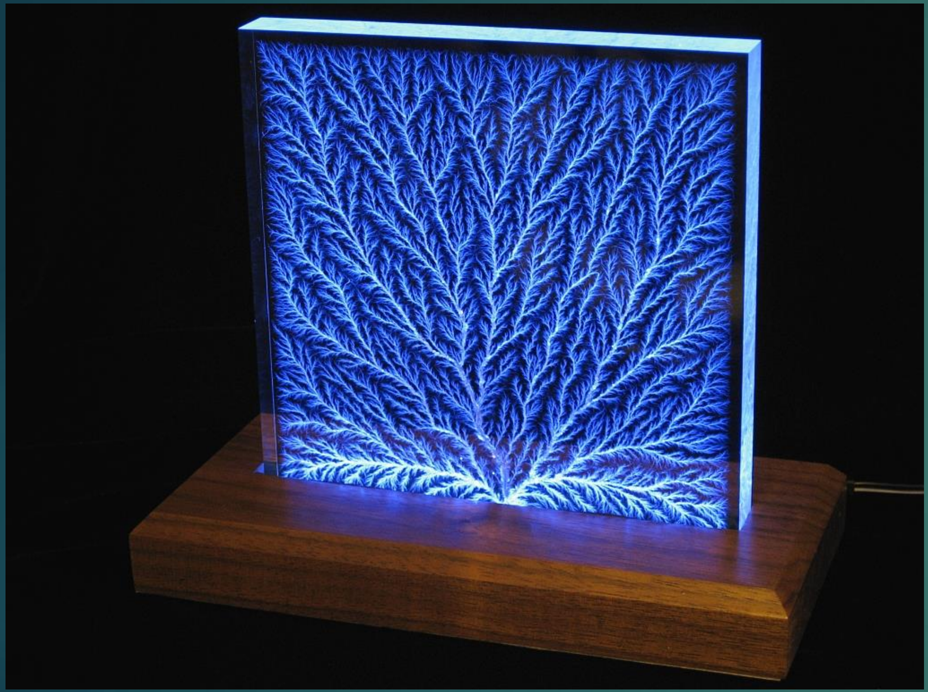
General Overview

- Phenomena resulting from electric discharge.
- Forms 2D and 3D shapes.
- Can be simulated using a variety of methods.

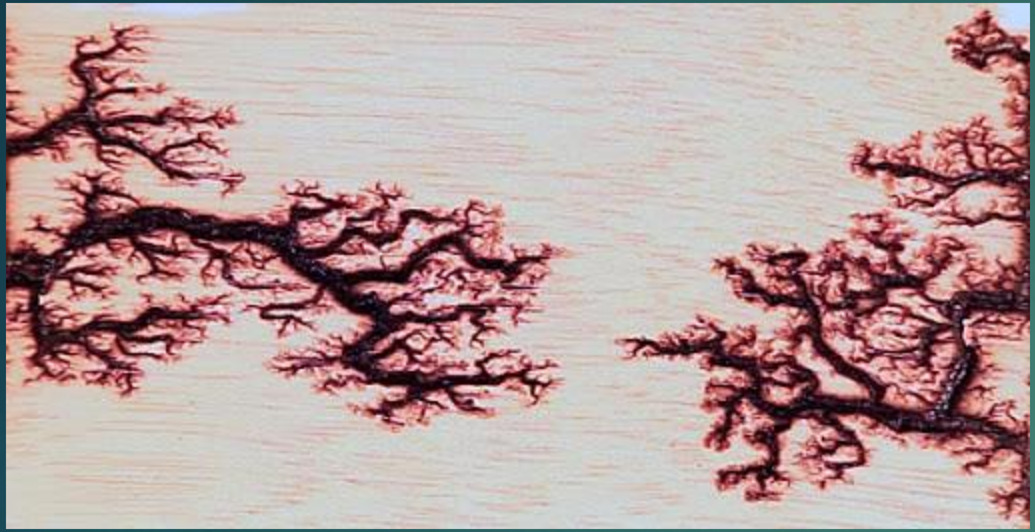
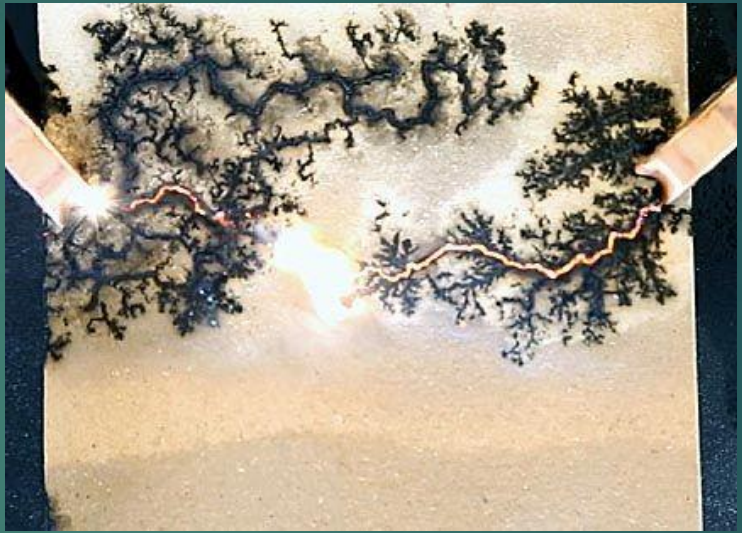
Materials

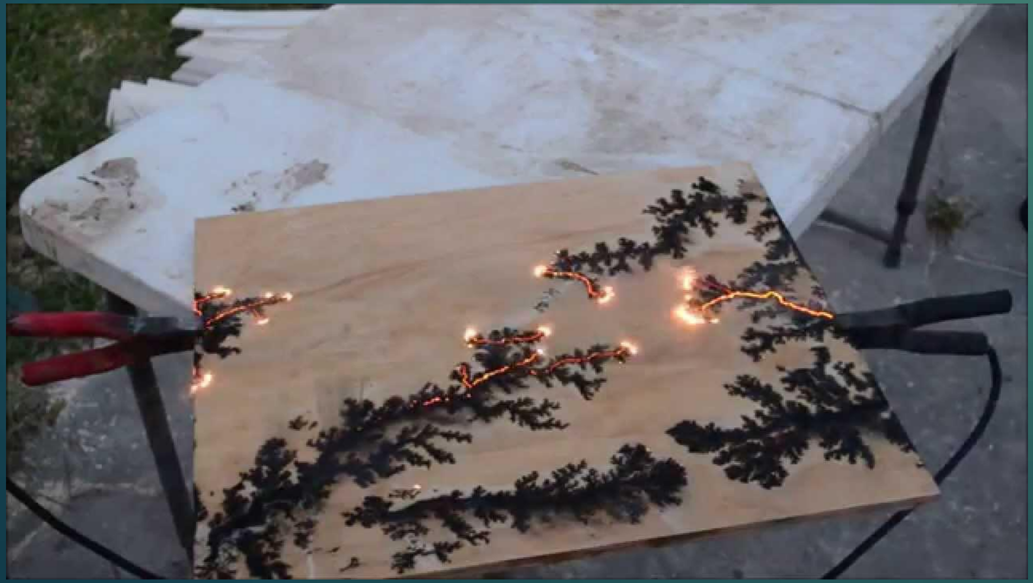
- Acrylic: 2D/3D discharge. Produced by first irradiating acrylic.
- Wood: Prepared surface with two electrodes.
- Skin: Result of lightning strikes.
- Resin/Glass Surfaces: Original demonstration, uses dust.

Acrylic



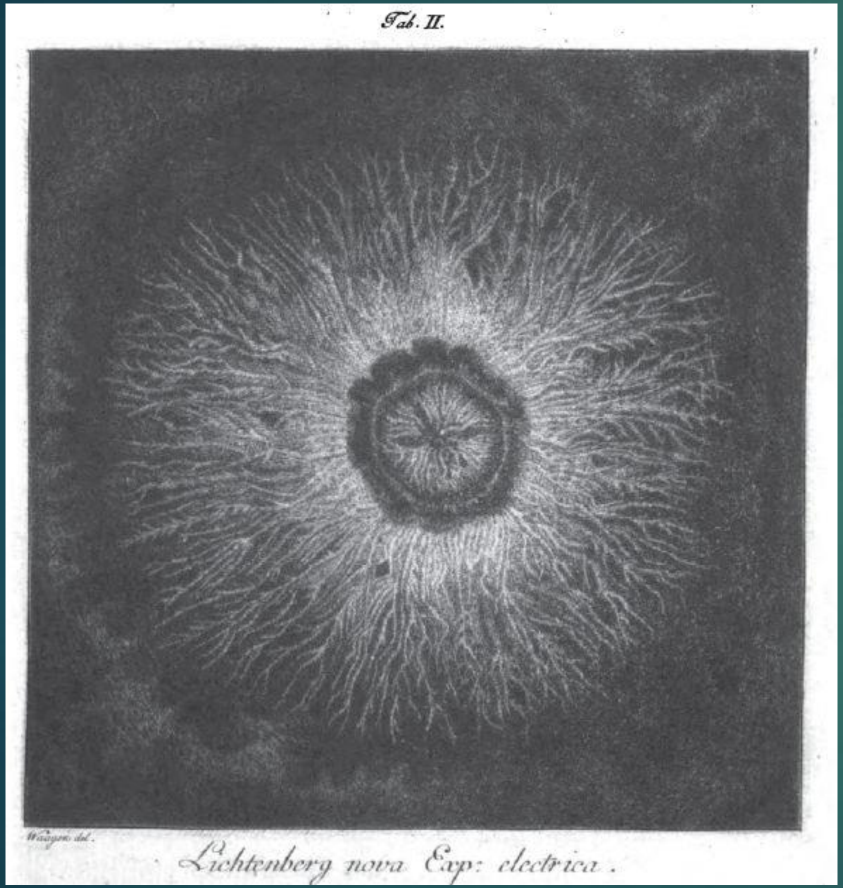
Wood



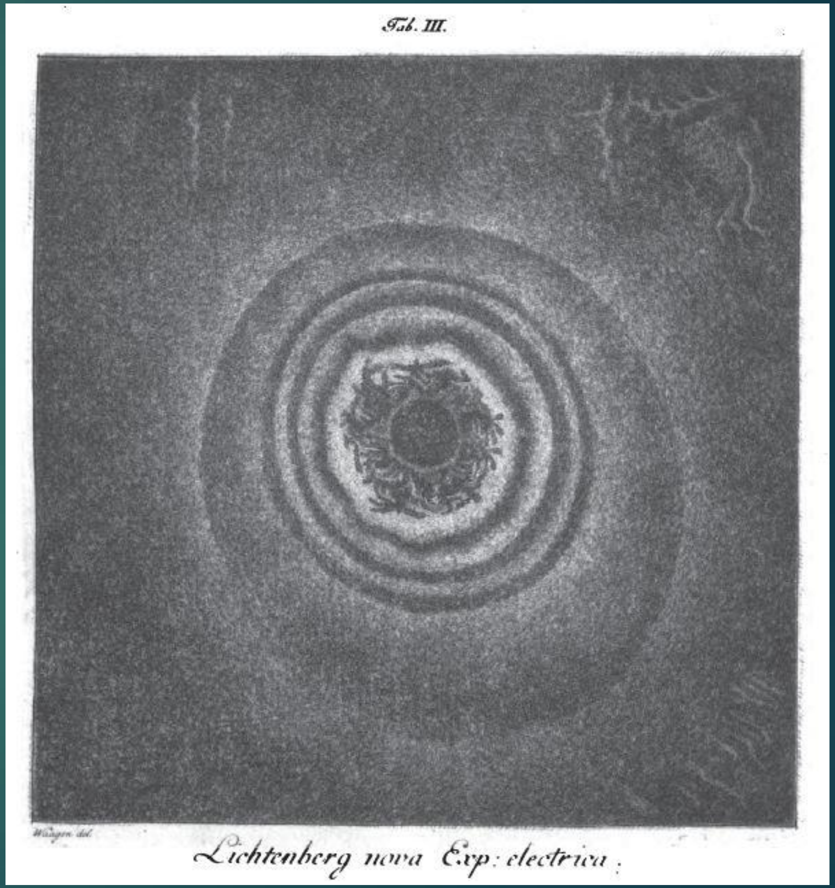

Skin

Insulating Surfaces



Positive

- Pockets of charge generated by static electricity.
- Trapped because of insulating surface.
- Positive and negative electrodes result in different images.



Negative

Practical Applications

- Generate Lichtenberg figures as art by specifying initial mask
- Calculate susceptibility of material to fail.
- Understand how lightning discharges affect circuits and the body.
- Protecting key infrastructure in electricity grid.

Physics

- Result from high voltage discharges.
- In acrylic, high voltage accelerator is used.
- Masking can be performed in Acrylic and Wood
- Trees are formed due to massive voltage differentials.

Computational Techniques

- Random walk: Monte Carlo simulation by allowing particles to move freely
- Random walk with DBM/DLA: Dielectric Breakdown Model/Diffusion Limited Aggregation

Random Walk

- Free motion of particles.
- Directions may be limited or skewed.
- Particles may interact with each other.
- Computationally expensive when particles interact.

Diffusion Limited Aggregation

- Uses Brownian motion of particles to form trees.
- Forms fractal symmetry.

Dielectric Breakdown Model

- Combination of DLA to incorporate electric potentials
- Requires calculating electric field at each point in model where there is a particle.
- Computationally expensive but more realistic.

Techniques used to solve

- Monte Carlo Simulation
- Numpy vector operations
- Cloud Computing - Azure

Mathematics

Diffusion-Limited Aggregation

Dielectric Breakdown Mathematical Sketch

In an electric field, work must be done to move a charged particle from one point to another.

If $W((x_1,y_1),(x_2,y_2))$ is the work to move a particle of charge q_0 from a point (x_2,y_2) to a point (x_1,y_1), the electric potential relative to (x_1,y_1) is the function f which assigns to each point (x_2,y_2) the number $f((x_2,y_2)) = W((x_1,y_1),(x_2,y_2))/q_0$.

The potential is related to the strength of the electric field around (x_1,y_1).

Like simple DLA, the dielectric breakdown model is constructed on a grid of square cells c(i,j).

Select a large circle, S, on the grid.

Assign to the potential f the value 1 at each cell on this circle, and the value 0 to the cell c(0,0) in the center of the circle.

The boundary conditions of the problem are that the potential keeps these values at these locations.

Consistent with these conditions, the value of the potential is determined at each cell of the grid (by a discrete version of Laplace's equation - details are not necessary here) and then for each of the four cells c(-1,0), c(1,0), c(0,-1), and c(0,1) adjacent to c(0,0), the growth probability at c(a,b) is defined as

$$p(a,b) = f(c(a,b))^t(f(c(a-1,b))^t + f(c(a+1,b))^t + f(c(a,b-1))^t + f(c(a,b+1))^t)$$

One of these four points is selected at random, with probabilities given by the p(a,b).

Together with the point c(0,0), this point forms the beginning of the DBM cluster.

Compute the potential again with boundary conditions f = 1 on the same large circle S, and f = 0 on the cluster.

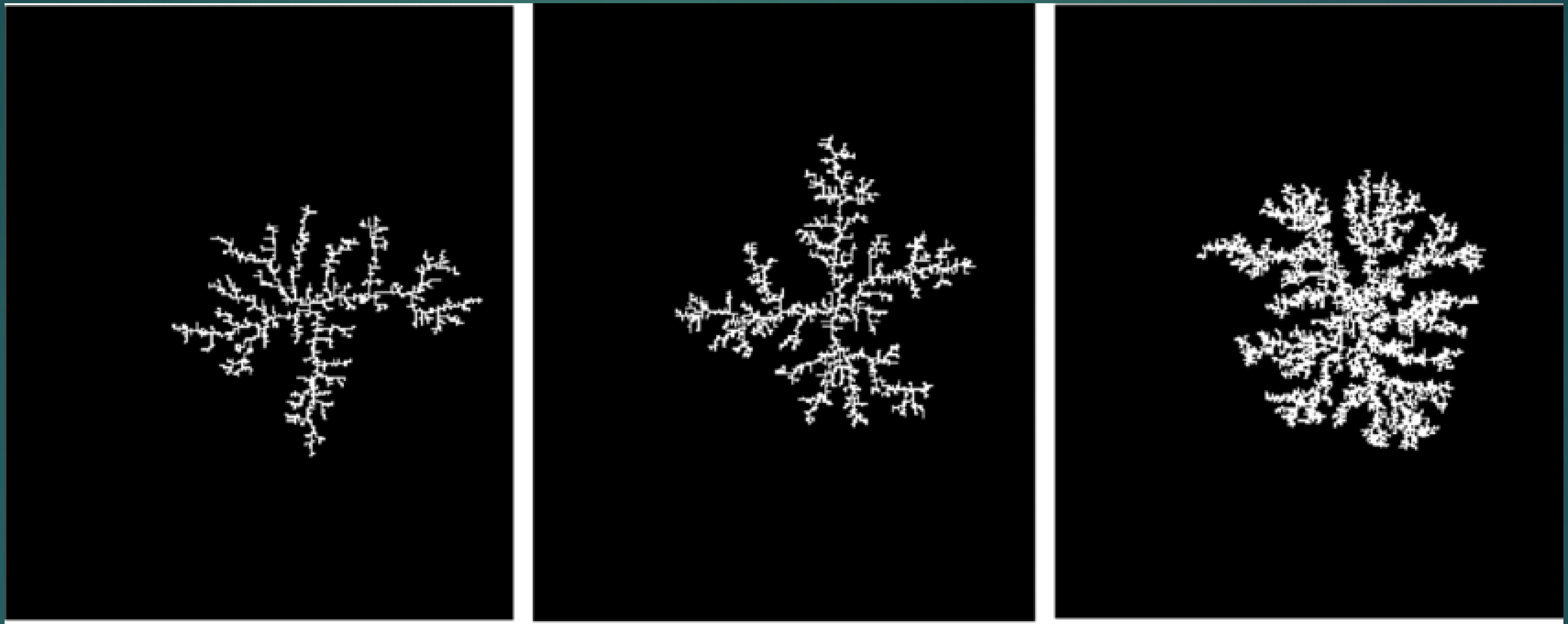
Compute the growth probability for each of the 6 cells adjacent to the cluster and add another cell randomly, according to these probabilities.

Continuing in this way grows a DBM cluster.

Return to DLA.

See Jacobi Method from in class
https://users.math.yale.edu/public_html/People/frame/Fractals/Panorama/Physics/DLA/DBM/DBM2.html

Stickiness

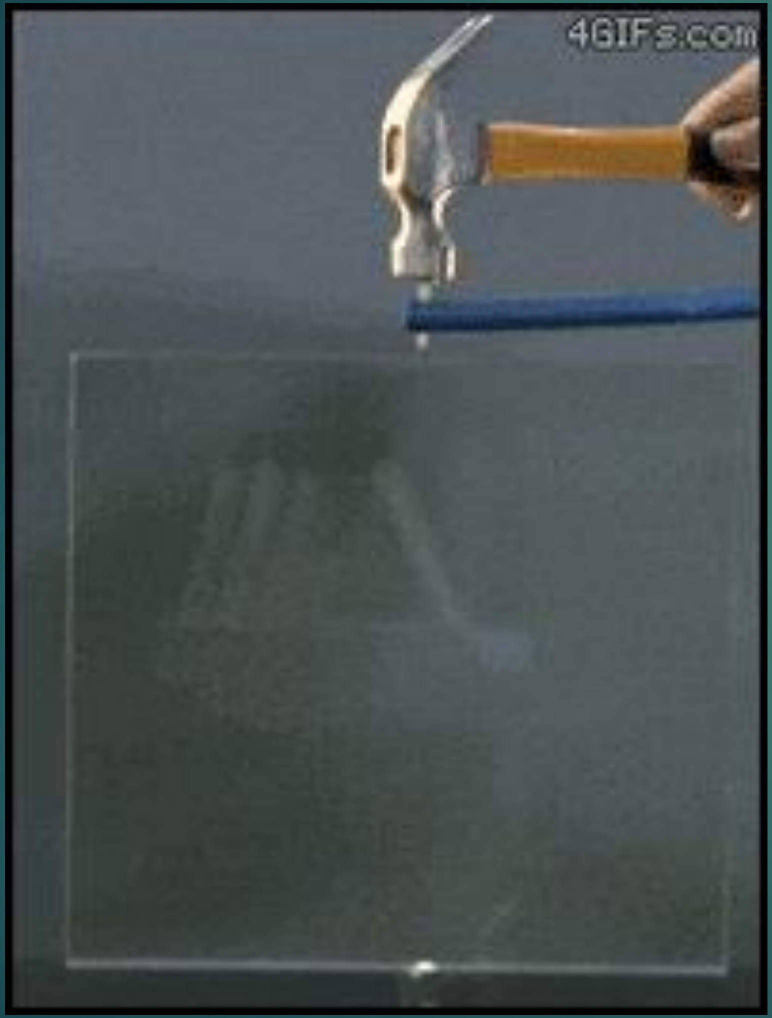


(a) $R_c = 100$ e $S = 1$ (b) $R_c = 100$ e $S = 0.5$ (c) $R_c = 100$ e $S = 0.1$

Results

- Simulations are not physically accurate due to not incorporating Dielectric Breakdown Model.
- Plausible results match expectations.
- Branching and fractal behavior is exhibited.
- Impact of stickiness factor can be seen.

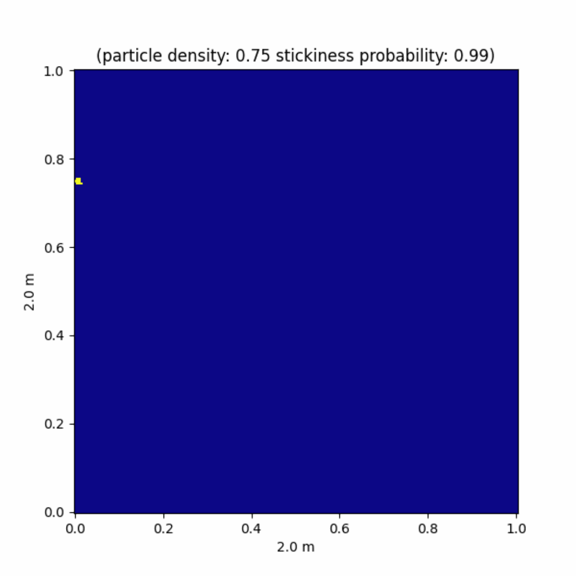
Real World

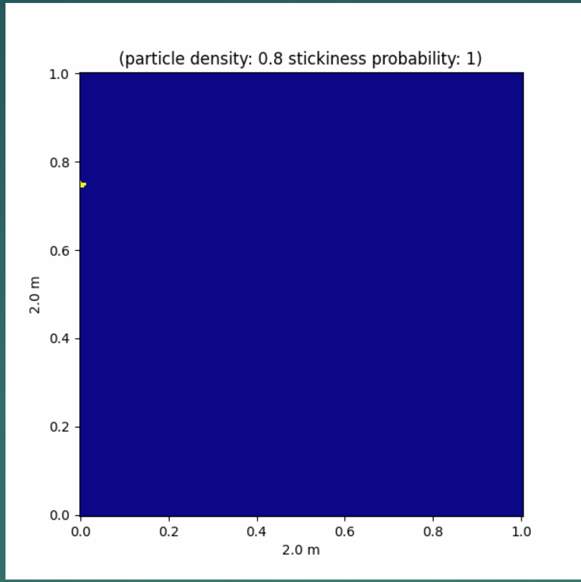


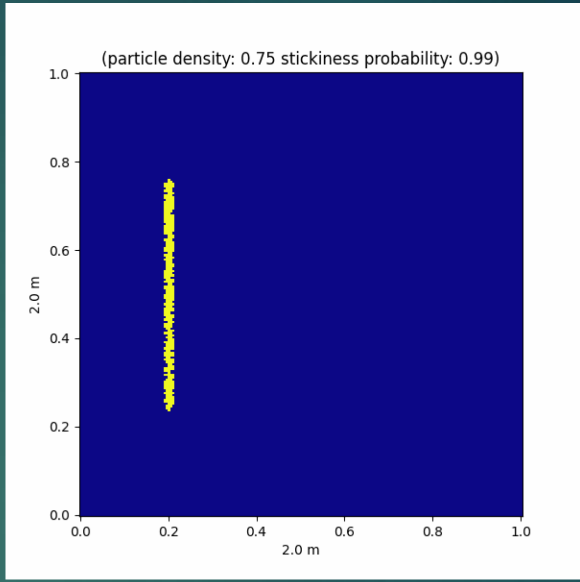
configuration.yaml

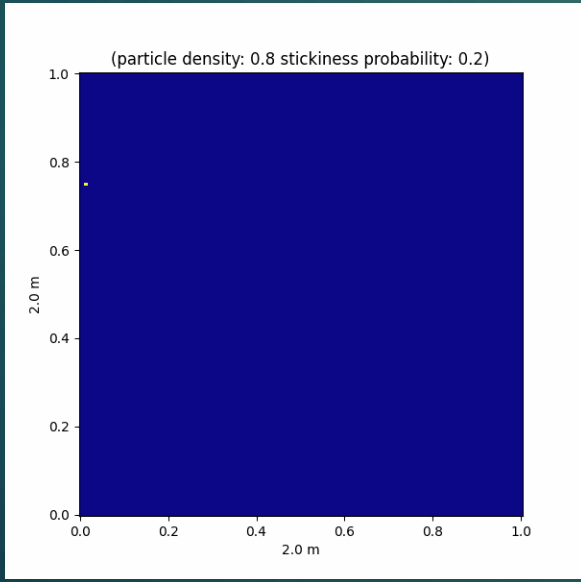
configuration.yaml

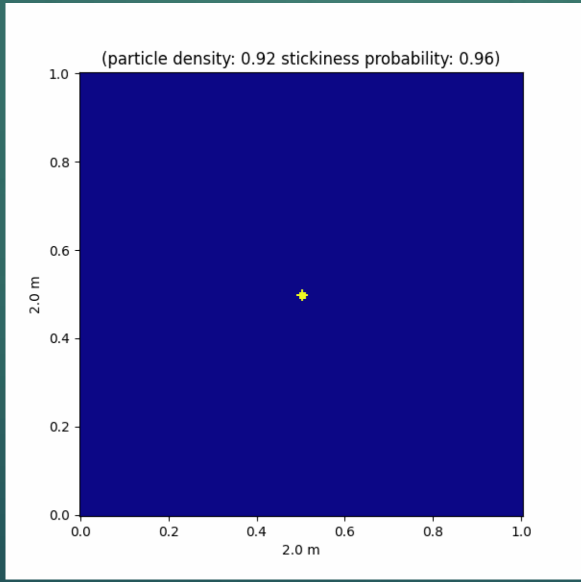
```yaml
# Lichtenberg Configuration Options

#################################
## Graph Configuration Options ##
#################################

# Points per meter (.01 = 1 point per centimeter).
point_to_meter_ratio: .01

# Point resolution in x axis.
grid_resolution_x_points: 500

# Point resolution in y axis.
grid_resolution_y_points: 500

# Initial particle(s) position mask.
initial_particles: [ [50,250],[450,250]]

#####################################
## Animation Configuration Options ##
#####################################

# Number of time steps to run program for.
timesteps: 1000

# Interval at which to capture frame at.
capture_interval: 100

# Delay in milliseconds between frames.
frame_delay_ms: 100

# Delay in milliseconds before animation loops.
repeat_delay_ms: 1000

#####################################
## Simulation Configuration Options ##
#####################################

# Density of particles used for simulation, (0 = No particles)
initial_particle_density: .75 # Good value is ~.5

# Probability that particle will stick (0 = Never sticks)
stickiness_probability: .99
```

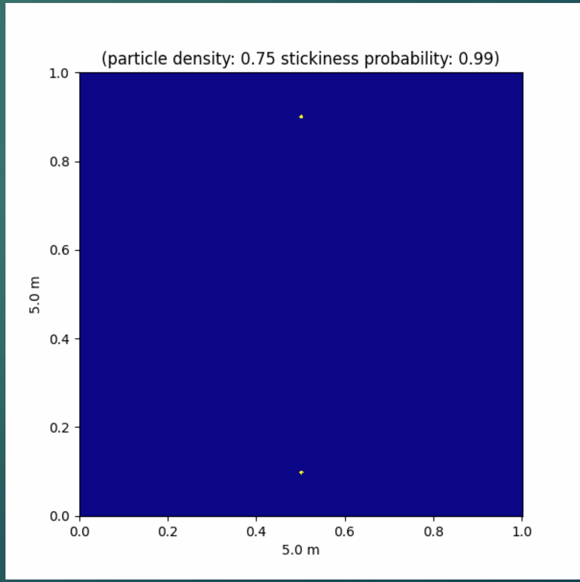

(particle density: 0.75 stickiness probability: 0.99)
2.0 m
2.0 m


(particle density: 0.8 stickiness probability: 1)
2.0 m
2.0 m


(particle density: 0.75 stickiness probability: 0.99)
2.0 m
2.0 m


(particle density: 0.8 stickiness probability: 0.2)
2.0 m
2.0 m


(particle density: 0.92 stickiness probability: 0.96)
2.0 m
2.0 m


(particle density: 0.75 stickiness probability: 0.99)
5.0 m
5.0 m

Future Research

- Implement DBM to physically render correctly.
- Improve random particle movement to speed up code.
- Implement multithreading to render images as already setup to do so.
- Use ability to display layers to visualize electric field strength.
- Implement masking to modify conductivity of surfaces.

Questions?