Confidence interval for difference of two means; independent samples, variances unknown but assumed to be equal

Apples example

NY apples		LA apples	
$	3.80	$	3.02
$	3.76	$	3.22
$	3.87	$	3.24
$	3.99	$	3.02
$	4.02	$	3.06
$	4.25	$	3.15
$	4.13	$	3.81
$	3.98	$	3.44
$	3.99		
$	3.62		

	NY	LA
Mean	$ 3.94	$ 3.25
Std. deviation	$ 0.18	$ 0.27
Sample size	10	8

Pooled variance formula

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$$

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	NY	LA
Mean	$ 3.94	$ 3.25
Std. deviation	$ 0.18	$ 0.27
Sample size	10	8

Pooled variance formula

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2} = \frac{(10-1)0.18^2 + (8-1)0.27^2}{10+8-2}$$

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$ 3.99	
$ 3.62	

	NY	LA
Mean	$ 3.94	$ 3.25
Std. deviation	$ 0.18	$ 0.27
Sample size	10	8

Pooled variance 0.05

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2} = \frac{(10-1)0.18^2 + (8-1)0.27^2}{10+8-2} = 0.05$$

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$ 3.99	
$ 3.62	

	NY	LA
Mean	$3.94	$3.25
Std. deviation	$0.18	$0.27
Sample size	10	8

Pooled variance	0.05
Pooled std	0.22

$$(\bar{x} - \bar{y}) \pm t_{n_x+n_y-2,\alpha/2} \sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}$$

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$$

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$ 4.25	$ 3.15
$ 4.13	$ 3.81
$ 3.98	$ 3.44
$ 3.99	
$ 3.62	

	NY	LA
Mean	$ 3.94	$ 3.25
Std. deviation	$ 0.18	$ 0.27
Sample size	10	8

Pooled variance	0.05
Pooled std	0.22

$$(\bar{x}-\bar{y}) \pm t_{n_x+n_y-2,\alpha/2}\sqrt{\frac{s_p^2}{n_x}+\frac{s_p^2}{n_y}}$$

$$(\bar{x}-\bar{y}) \pm z_{\alpha/2}\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}$$

$$s_p^2 = \frac{(n_x-1)s_x^2+(n_y-1)s_y^2}{n_x+n_y-2}$$

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Apples example

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$ 4.02	$ 3.06
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$ 4.13	$ 3.81
$ 3.98	$ 3.44
$ 3.99	
$ 3.62	

	NY	LA
Mean	$ 3.94	$ 3.25
Std. deviation	$ 0.18	$ 0.27
Sample size	10	8

Pooled variance	0.05
Pooled std	0.22

$$(\bar{x} - \bar{y}) \pm \boxed{t_{n_x+n_y-2,\alpha/2}} \sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}$$

The degrees of freedom are equal to the total sample size minus the number of variables.

$$n_x + n_y - 2$$

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$$

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$ 3.80	$ 3.02
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$ 4.02	$ 3.06
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$ 4.13	$ 3.81
$ 3.98	$ 3.44
$ 3.99	
$ 3.62	

	NY	LA
Mean	$ 3.94	$ 3.25
Std. deviation	$ 0.18	$ 0.27
Sample size	10	8

Pooled variance	0.05
Pooled std	0.22

$$(\bar{x}-\bar{y}) \pm t_{n_x+n_y-2,\alpha/2}\sqrt{\frac{s_p^2}{n_x}+\frac{s_p^2}{n_y}}$$

The degrees of freedom are equal to the total sample size minus the number of variables.

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$$s_p^2 = \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x+n_y-2}$$

Confidence interval for difference of two means; independent samples, variances unknown but assumed to be equal

Apples example

NY apples	LA apples
$ 3.80	$ 3.02
$ 3.76	$ 3.22
$ 3.87	$ 3.24
$ 3.99	$ 3.02
$ 4.02	$ 3.06
$ 4.25	$ 3.15
$ 4.13	$ 3.81
$ 3.98	$ 3.44
$ 3.99	
$ 3.62	

	NY	LA
Mean	$ 3.94	$ 3.25
Std. deviation	$ 0.16	$ 0.27
Sample size	10	8

Pooled variance	0.05
Pooled std	0.22

$$(\bar{x} - \bar{y}) \pm \boxed{t_{n_x+n_y-2,\alpha/2}} \sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}}$$

The degrees of freedom are equal to the total sample size minus the number of variables.

$$n_x + n_y - 2 = 10 + 8 - 2 = 16$$

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$$

d.f. / α	0.1	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.306	1.690	2.030	2.438	2.724
40	1.303	1.684	2.021	2.423	2.704
50	1.299	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
inf.	1.282	1.645	1.960	2.326	2.576
CI	80%	90%	95%	98%	99%

$$n_x + n_y - 2 = 10 + 8 - 2 = 16$$

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Mean	$ 3.94	$ 3.25
Std. deviation	$ 0.18	$ 0.27
Sample size	10	8
Pooled variance	0.05	
Pooled std	0.22	
95% t-stat	2.12	

$$(\bar{x}-\bar{y}) \pm t_{n_x+n_y-2,\alpha/2}\sqrt{\frac{s_p^2}{n_x}+\frac{s_p^2}{n_y}} = (3.94-3.25) \pm 2.12\sqrt{\frac{0.05}{10}+\frac{0.05}{8}}$$

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Pooled variance 0.05

Pooled std 0.22

95% t-stat 2.12

$$(\bar{x}-\bar{y}) \pm t_{n_x+n_y-2,\alpha/2}\sqrt{\frac{s_p^2}{n_x}+\frac{s_p^2}{n_y}} = (3.94-3.25) \pm 2.12\sqrt{\frac{0.05}{10}+\frac{0.05}{8}}$$

$$\mathrm{CI}_{95\%} = (0.47, 0.92)$$

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Pooled variance	0.05	
Pooled std	0.22	
95% t-stat	2.12	

Takeaway:

Apples in NY are much more expensive than in LA

$$(\bar{x} - \bar{y}) \pm t_{n_x+n_y-2,\alpha/2}\sqrt{\frac{s_p^2}{n_x} + \frac{s_p^2}{n_y}} = (3.94 - 3.25) \pm 2.12\sqrt{\frac{0.05}{10} + \frac{0.05}{8}}$$

$$CI_{95\%} = (0.47, 0.92)$$