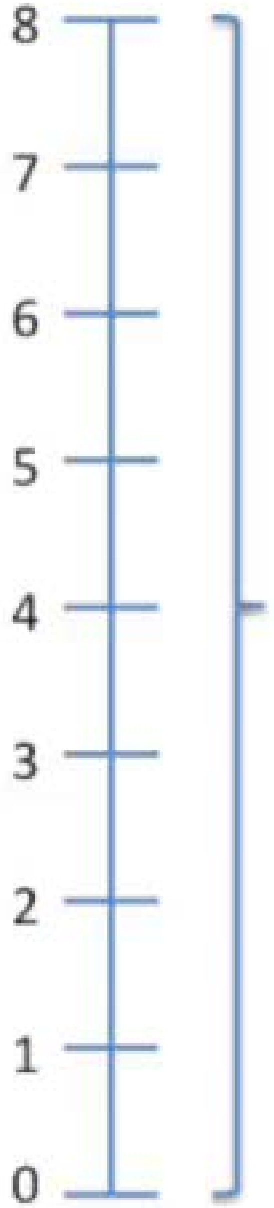
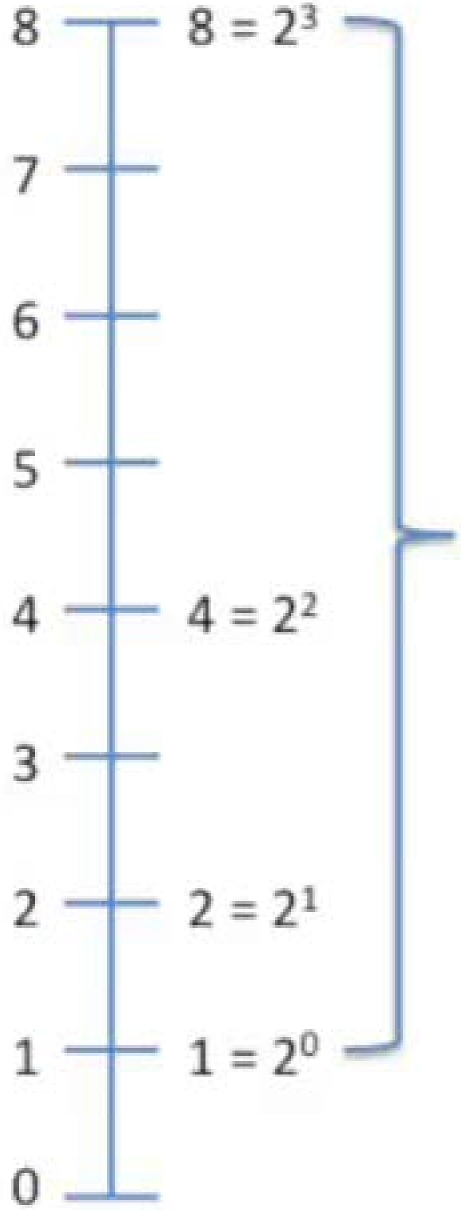
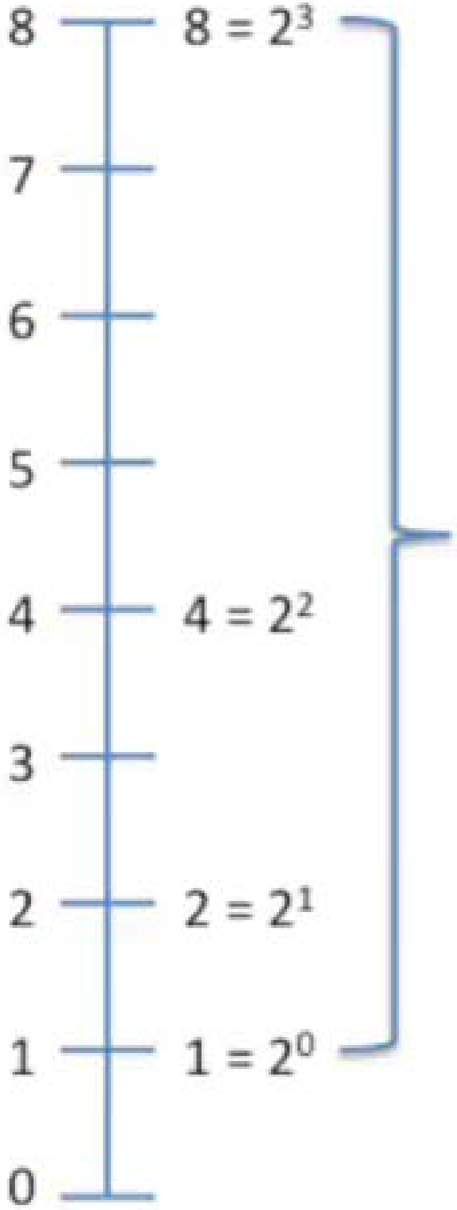


An ordinary number line (going from 0 to 8).

No big deal!



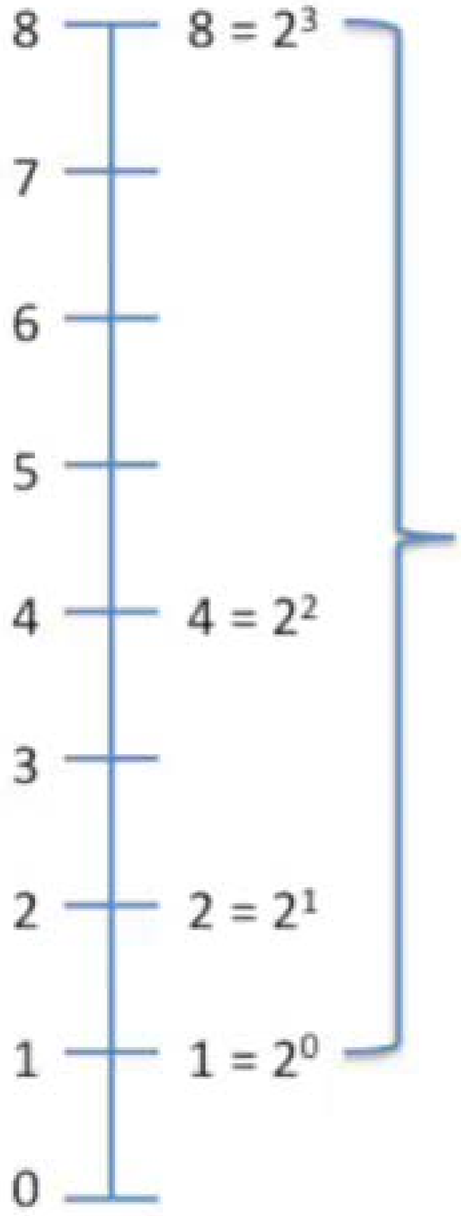
8, 4, 2, and 1 are easily rewritten as powers of 2.



8, 4, 2, and 1 are easily rewritten as powers of 2.

$$\left.\begin{aligned} 7 &= 2^{2.8} \\ 6 &= 2^{2.6} \\ 5 &= 2^{2.3} \end{aligned}\right\}$$

Other numbers can be written as powers of 2, it just isn't as neat and tidy.

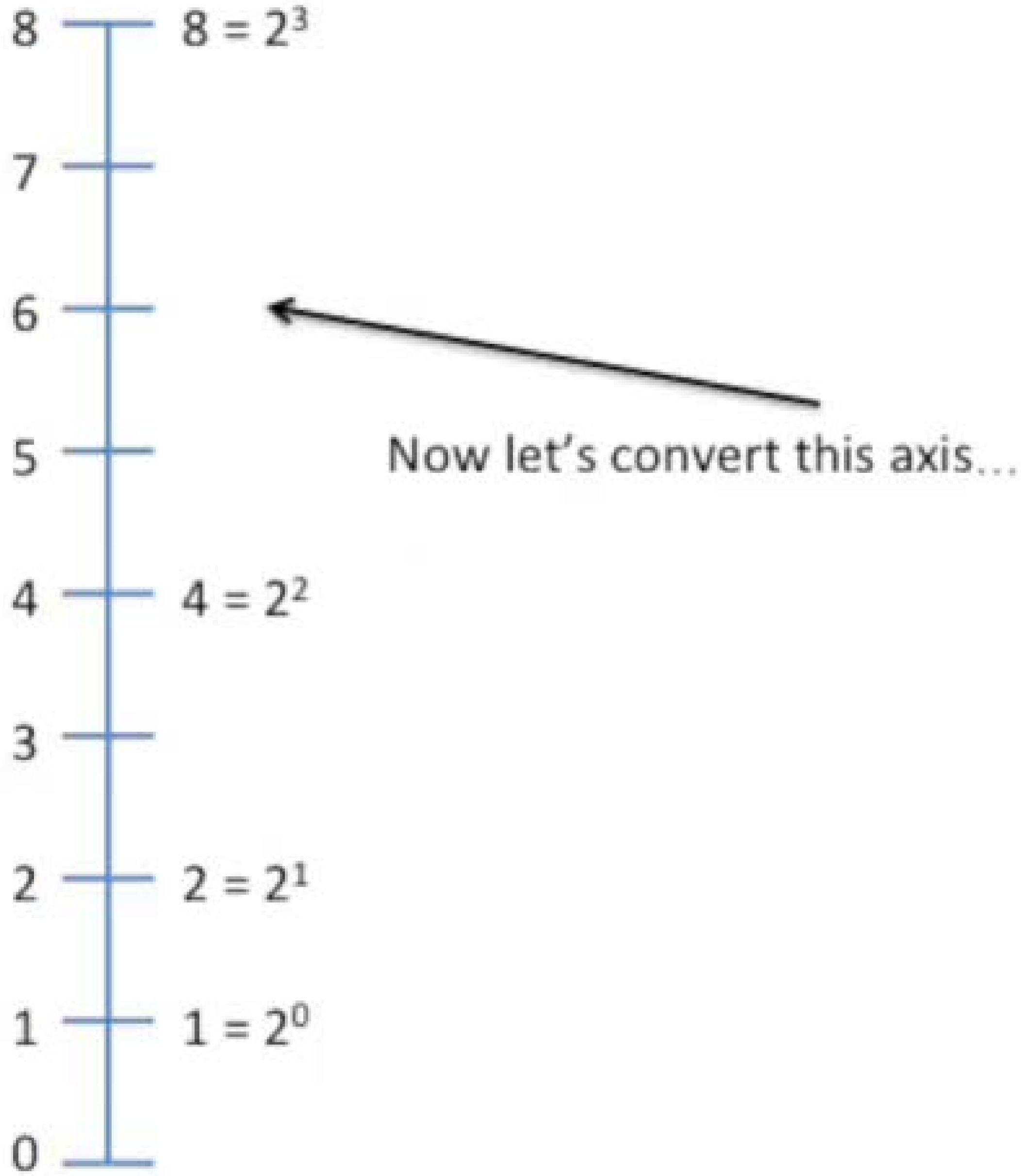


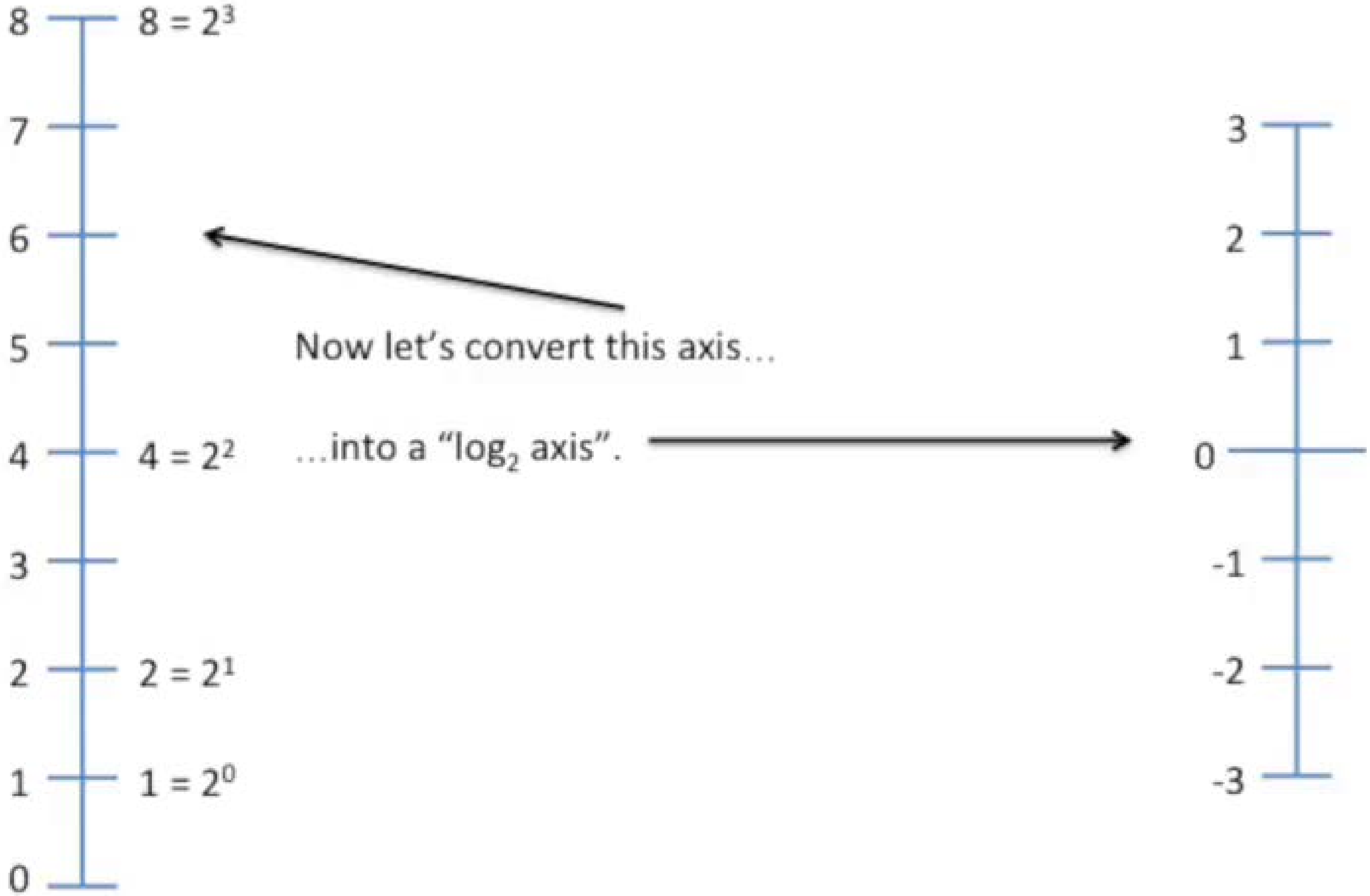
8, 4, 2, and 1 are easily rewritten as powers of 2.

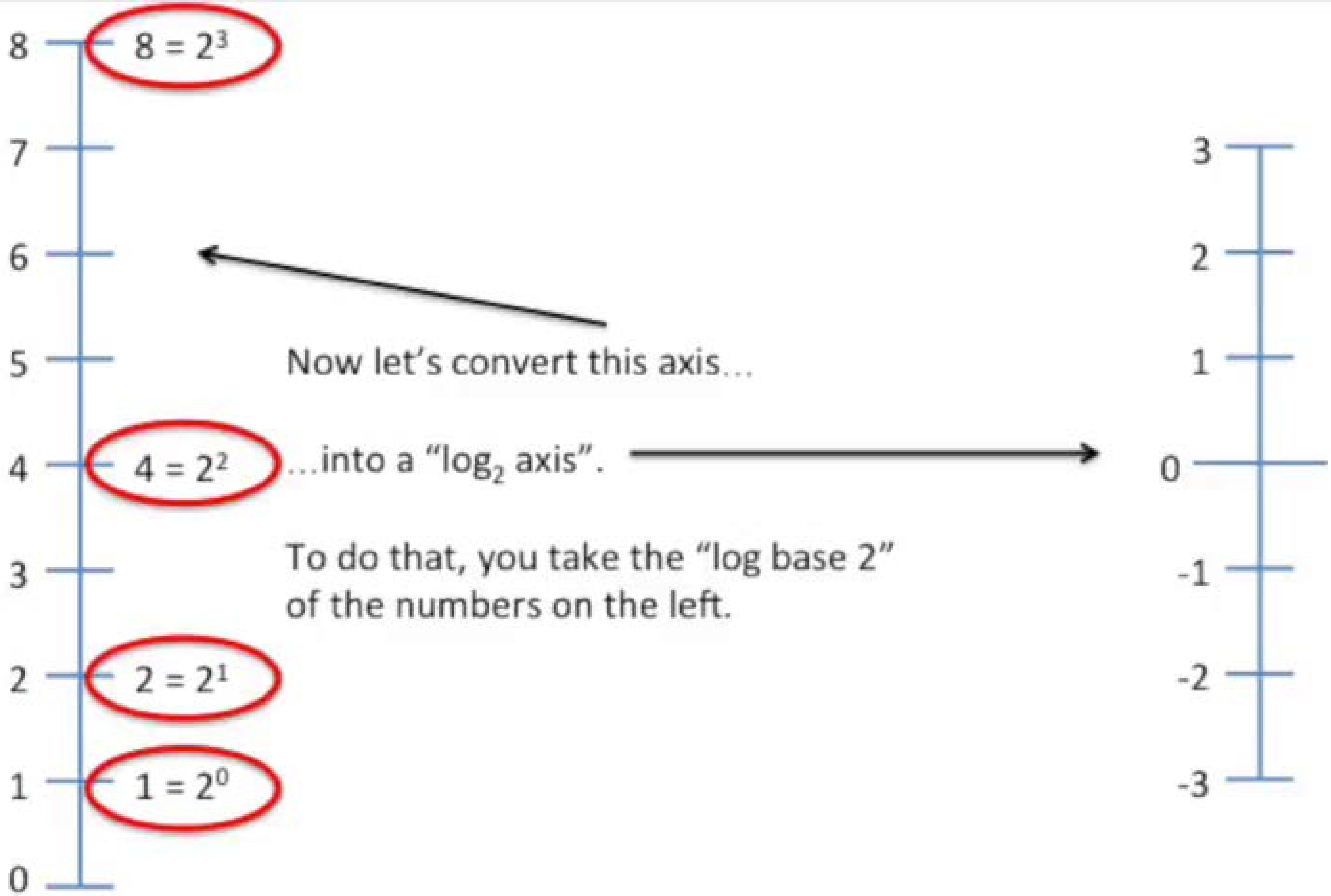
$7 = 2^{2.8}$
$6 = 2^{2.6}$
$5 = 2^{2.3}$

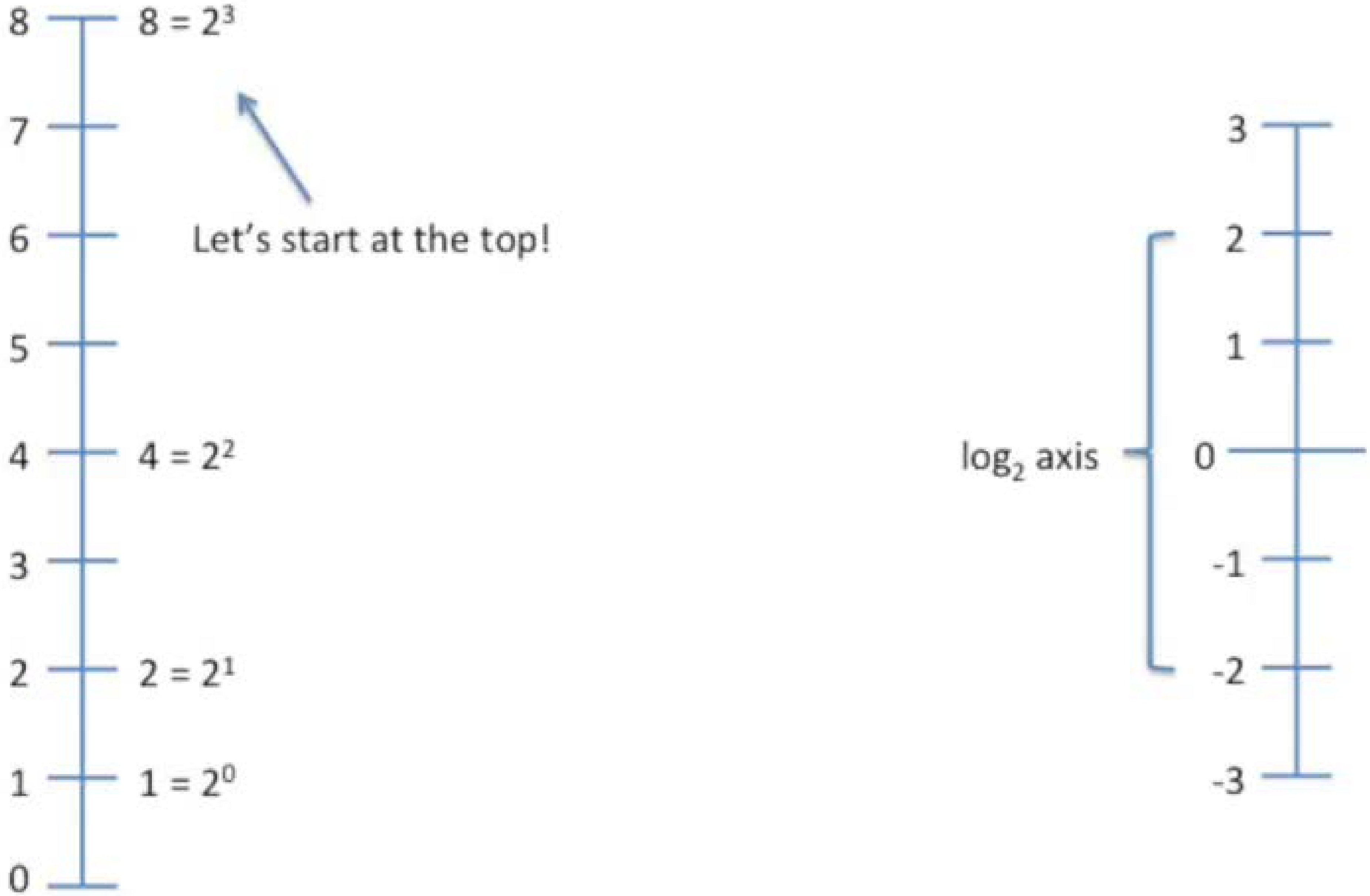
Other numbers can be written as powers of 2, it just isn't as neat and tidy.

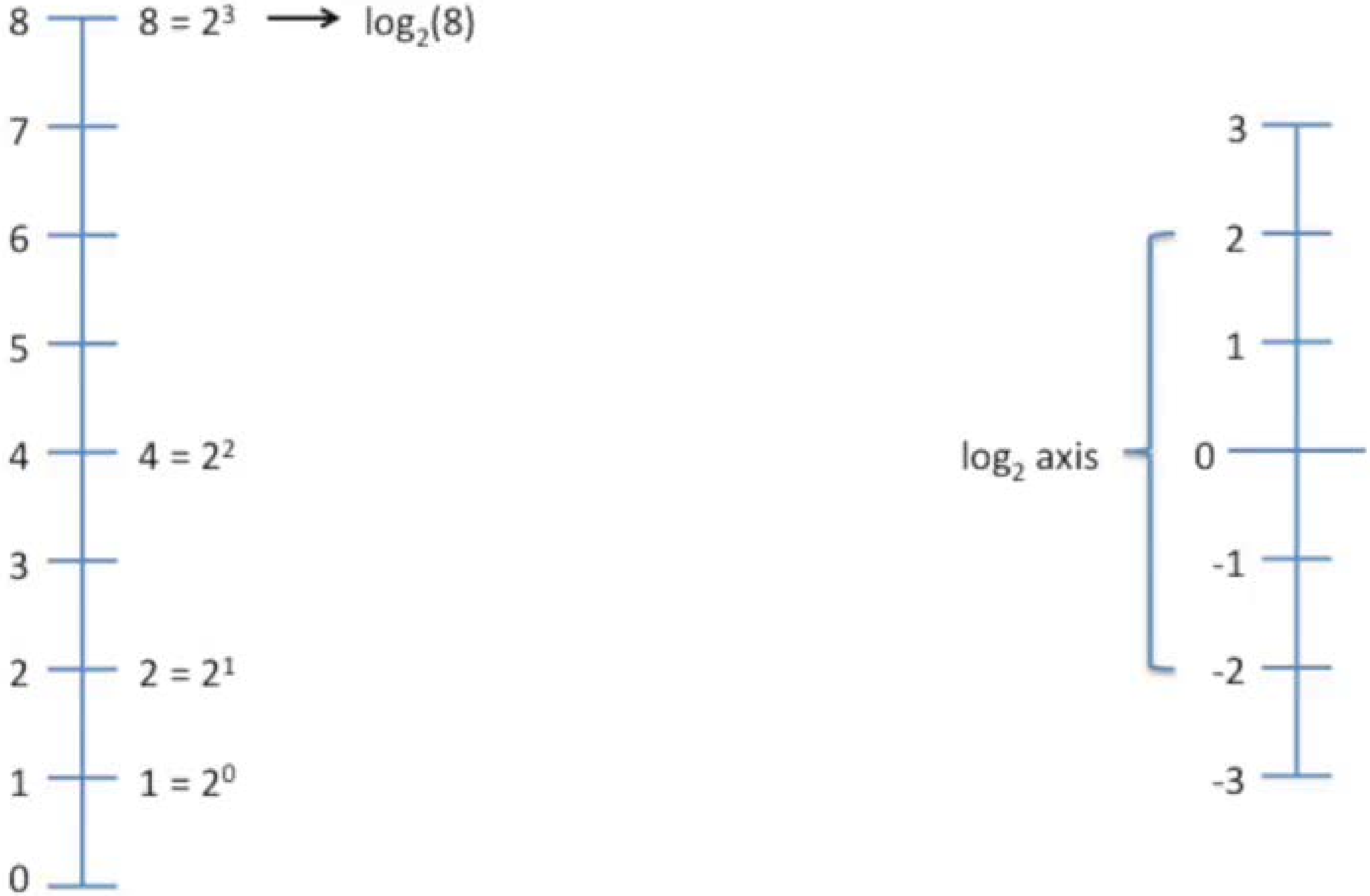
$3.14 = 2^{1.65}$ Even pi can be written as a power of 2.


8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
Now let's convert this axis…

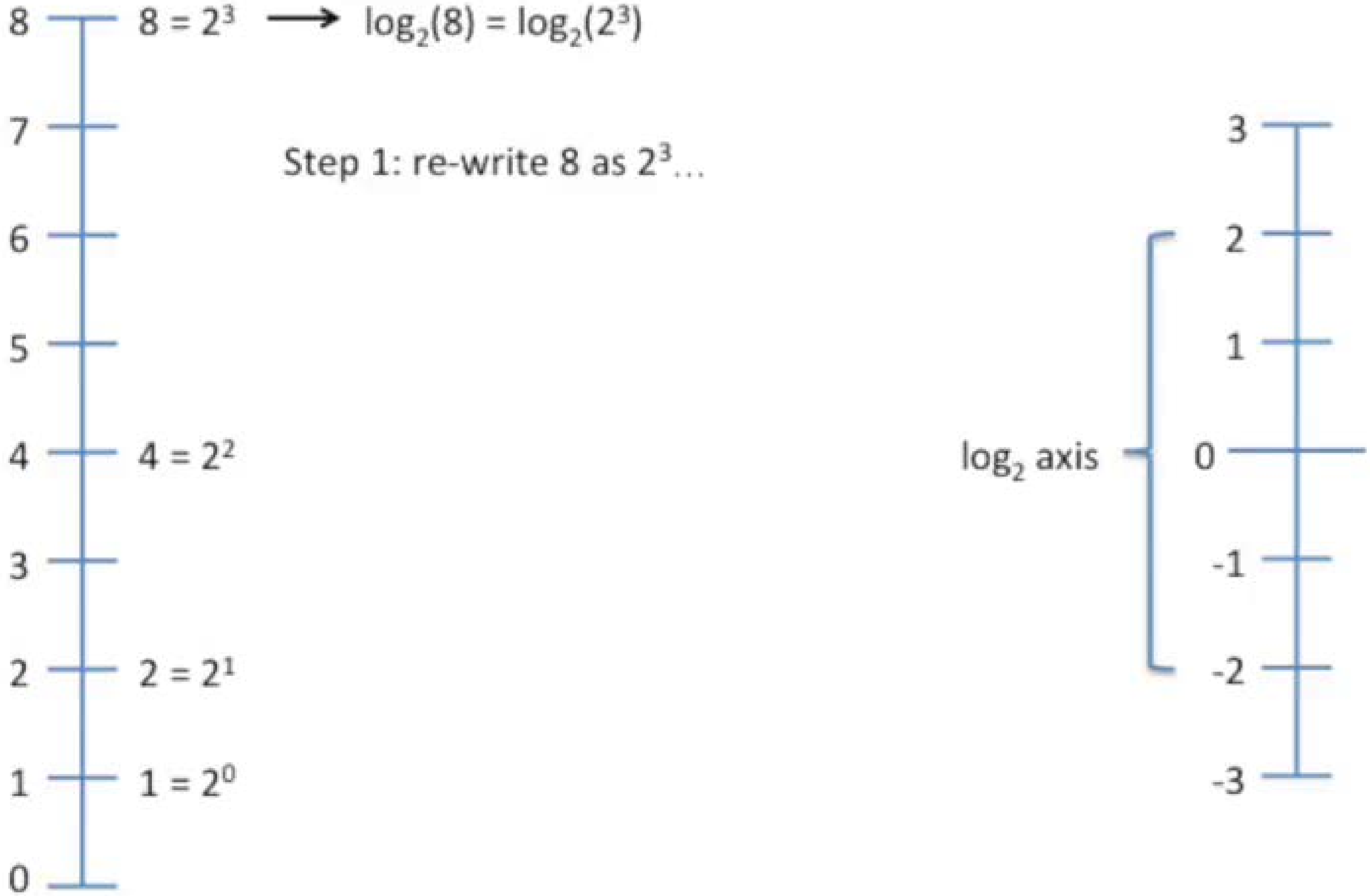

8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
Now let's convert this axis…
…into a "log2 axis".
3
2
1
0
-1
-2
-3


8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
Now let's convert this axis…
…into a "log$_2$ axis".
To do that, you take the "log base 2" of the numbers on the left.
8
7
6
5
4
3
2
1
0
3
2
1
0
-1
-2
-3

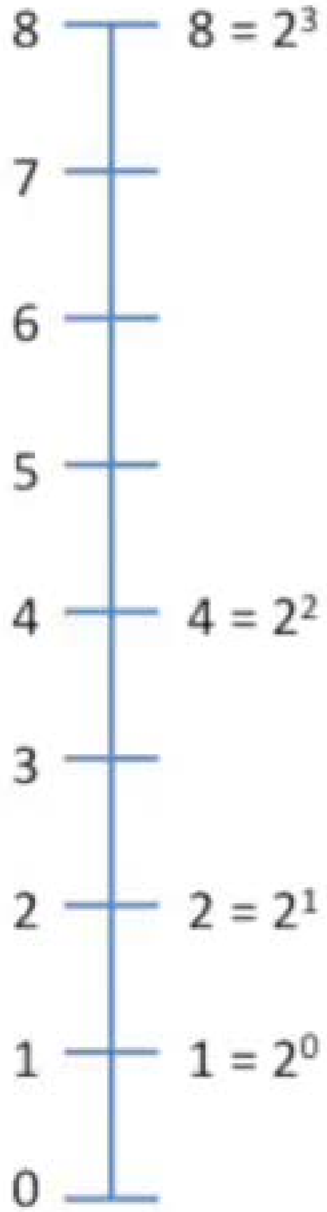

8
7
6
5
4
3
2
1
0
8 = 2^3
Let's start at the top!
4 = 2^2
2 = 2^1
1 = 2^0
3
2
1
log2 axis
0
-1
-2
-3



8
7
6
5
4
3
2
1
0
8 = 2^3
log_2(8)
4 = 2^2
2 = 2^1
1 = 2^0
3
2
1
log_2 axis
0
-1
-2
-3



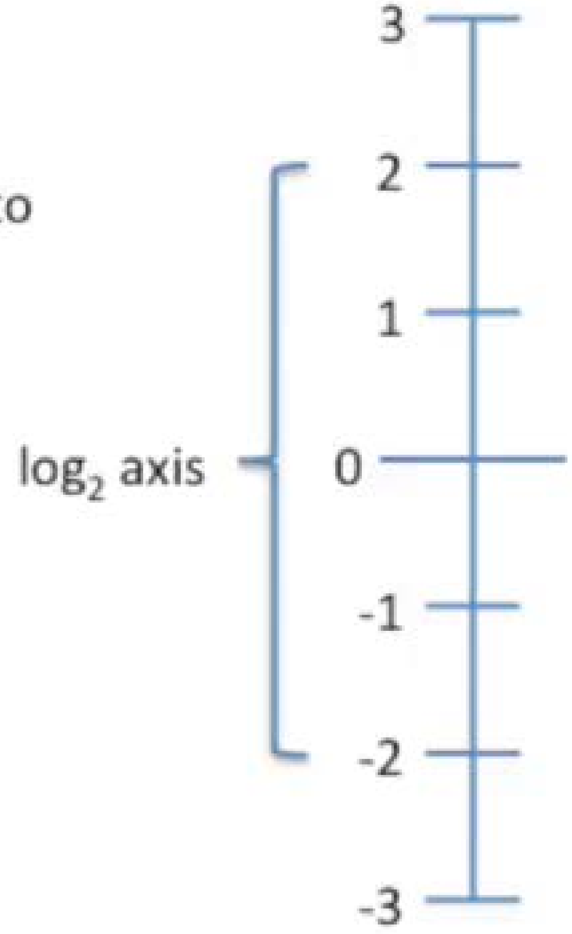
8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
log_2(8) = log_2(2^3)
Step 1: re-write 8 as 2^3…
log_2 axis
3
2
1
0
-1
-2
-3

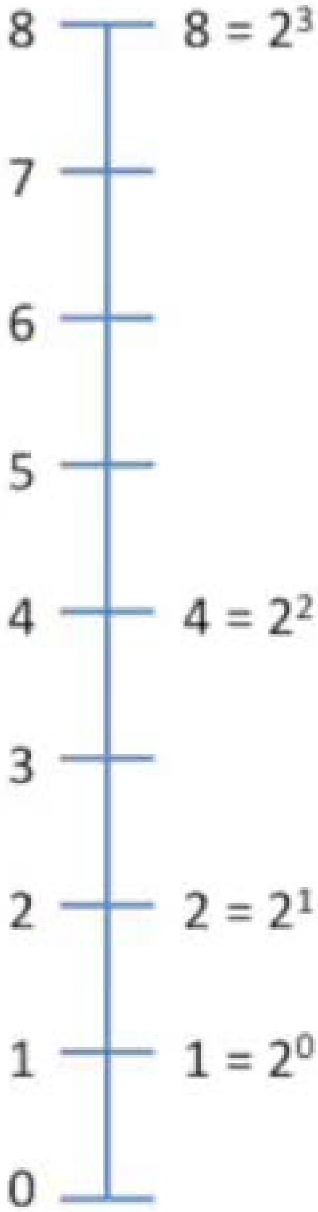


$\longrightarrow \log_2(8) = \log_2(2^3)$

Step 1: re-write 8 as 2^3…

$\log_2(2^3)$ is just the exponent you need to raise 2 by in order to get 8.



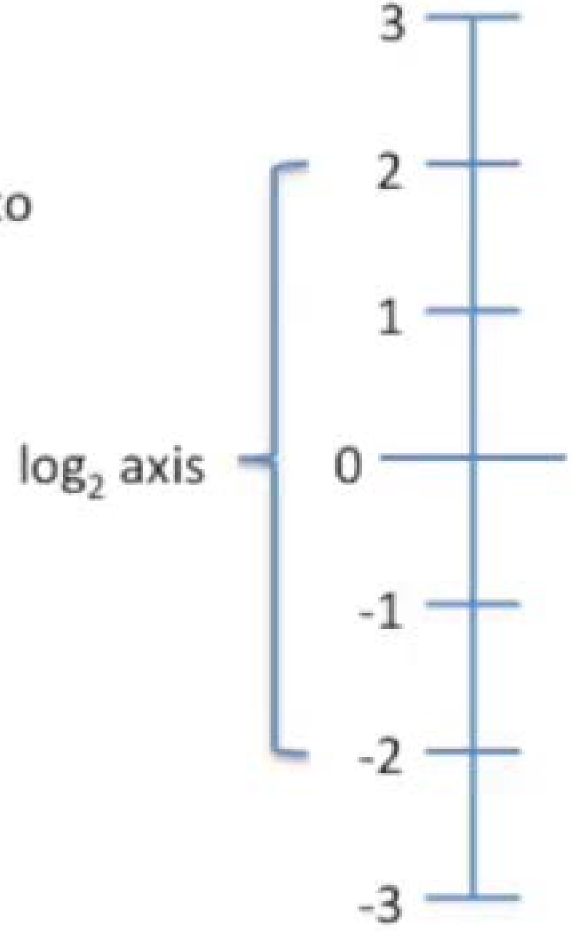


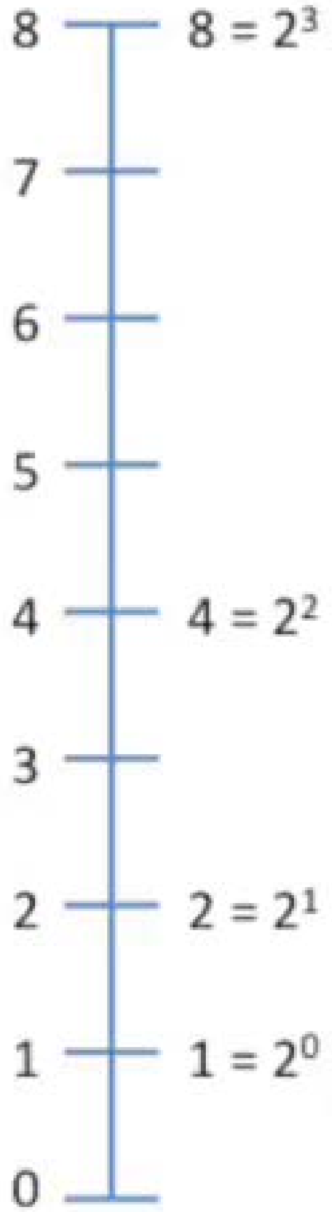
$\longrightarrow \log_2(8) = \log_2(2^3) = 3$

Step 1: re-write 8 as 2^3…

$\log_2(2^3)$ is just the exponent you need to raise 2 by in order to get 8.

In this case, the exponent is 3, so $\log_2(2^3) = 3$.





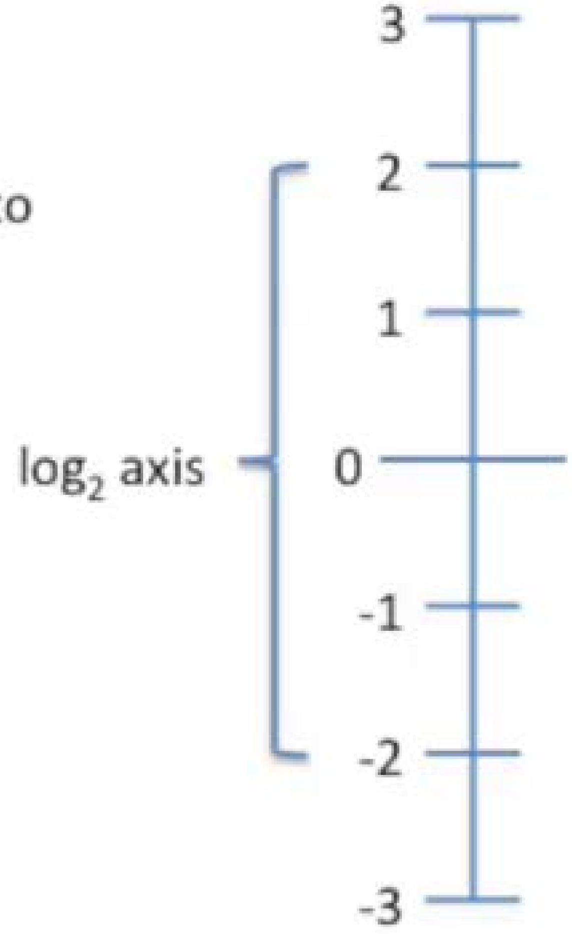
$\longrightarrow \log_2(8) = \log_2(2^3) = 3$

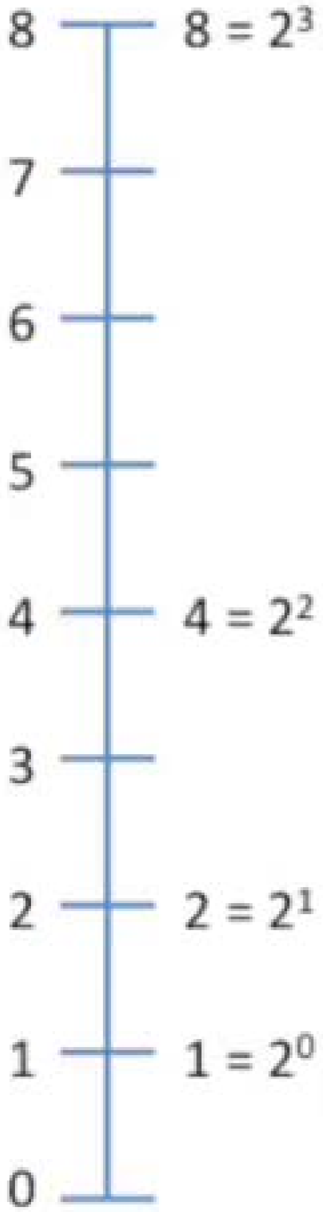
Step 1: re-write 8 as 2^3…

$\log_2(2^3)$ is just the exponent you need to raise 2 by in order to get 8.

In this case, the exponent is 3, so $\log_2(2^3) = 3$.

The log function just isolates the exponent.





$\longrightarrow \log_2(8) = \log_2(2^3) = 3$

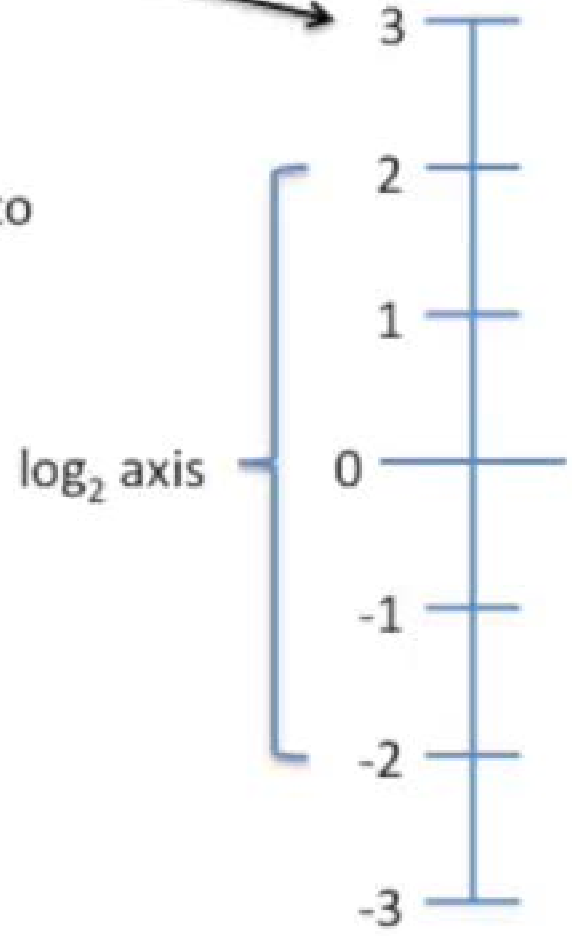
3 is the $\log_2$ equivalent of 8.

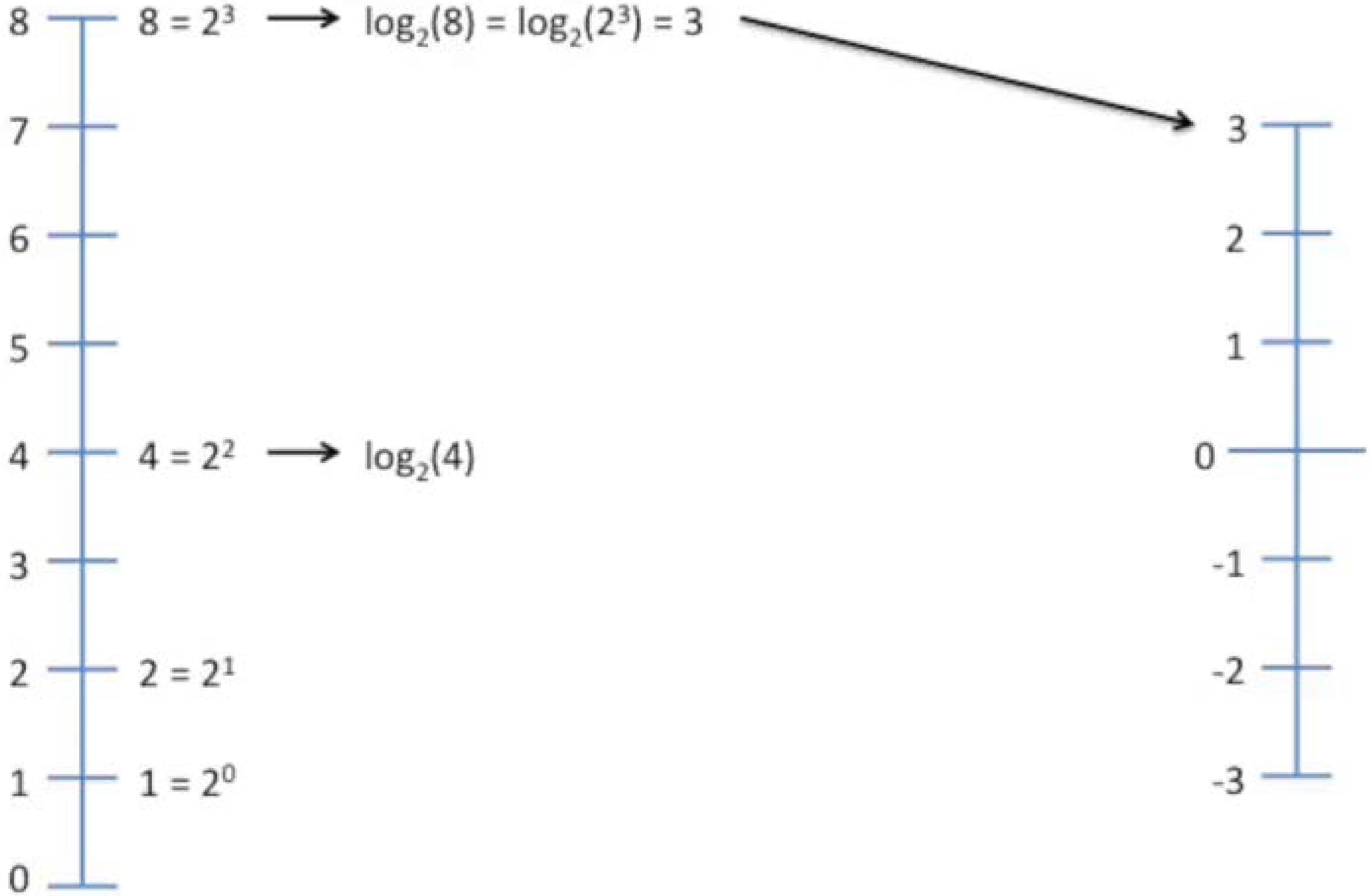
Step 1: re-write 8 as 2^3…

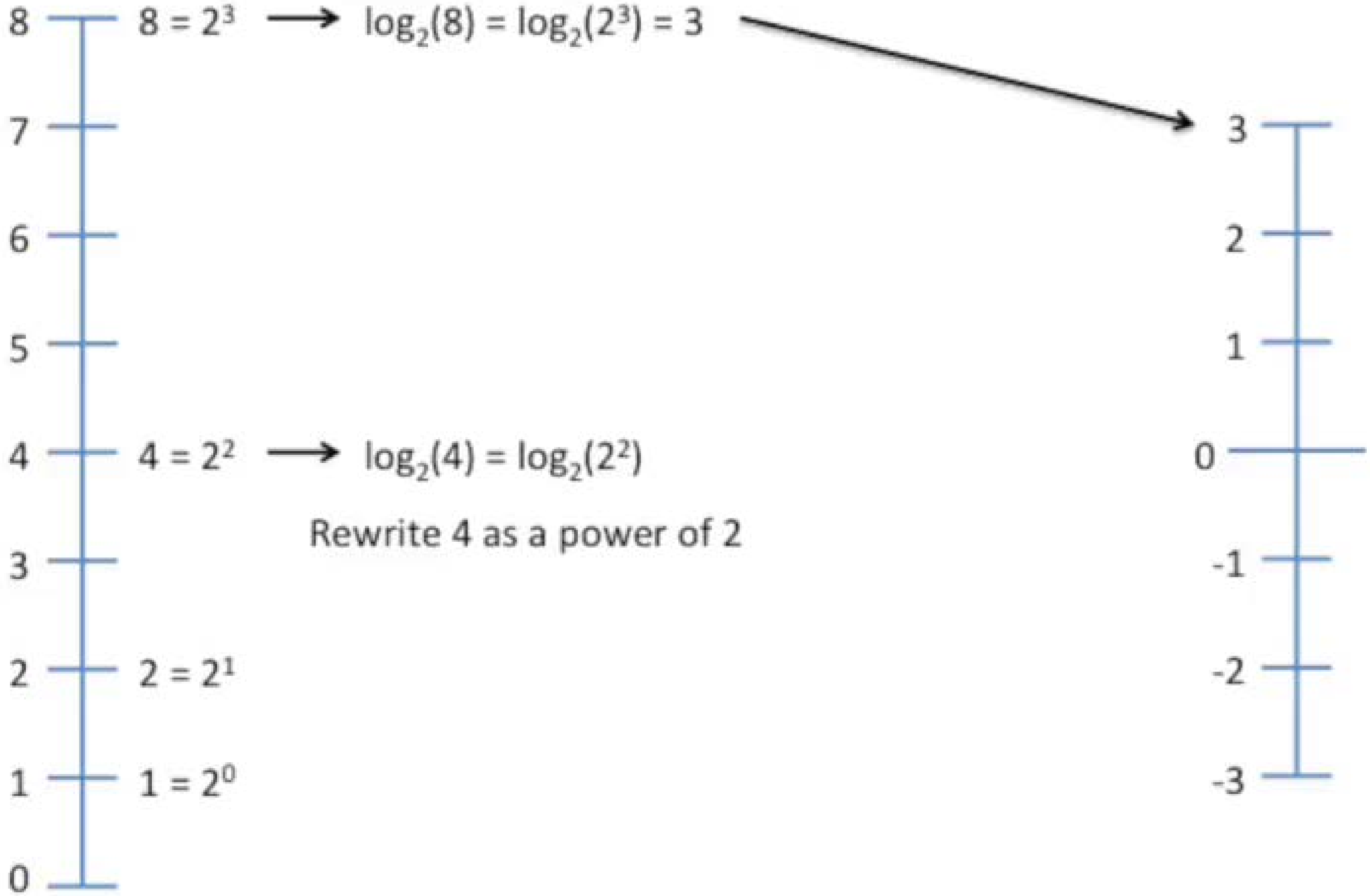
$\log_2(2^3)$ is just the exponent you need to raise 2 by in order to get 8.

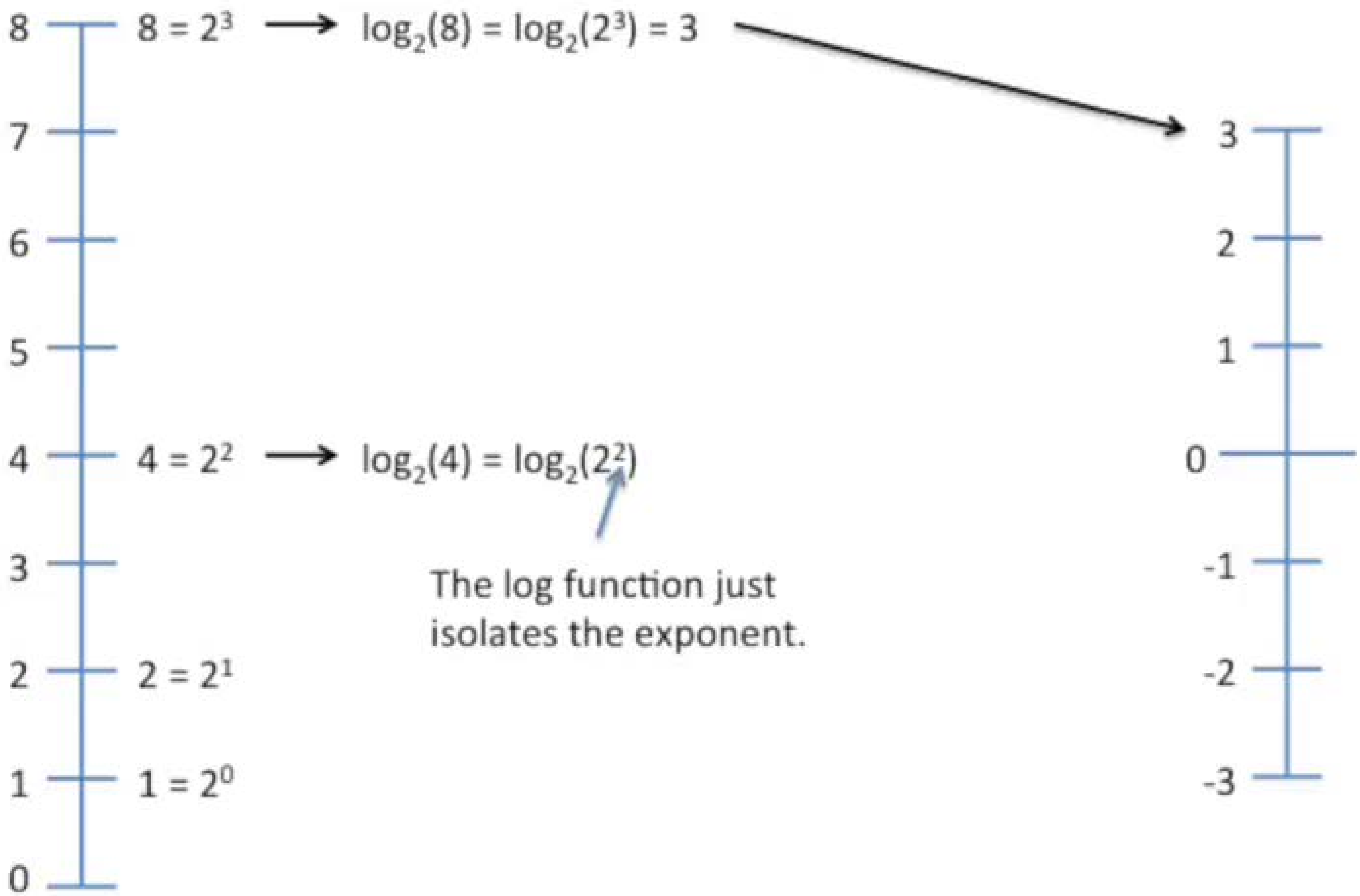
In this case, the exponent is 3, so $\log_2(2^3) = 3$.

The log function just isolates the exponent.

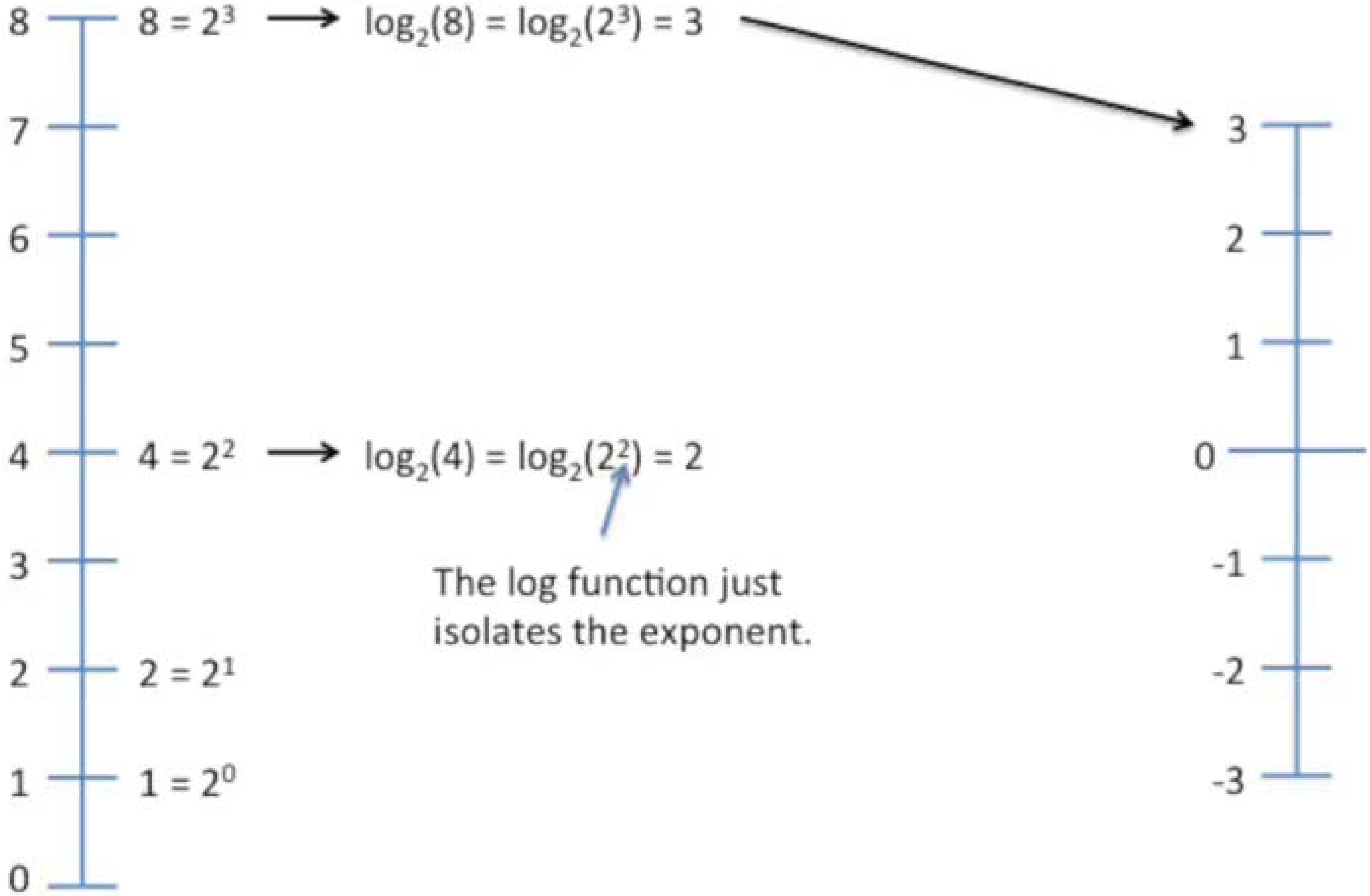


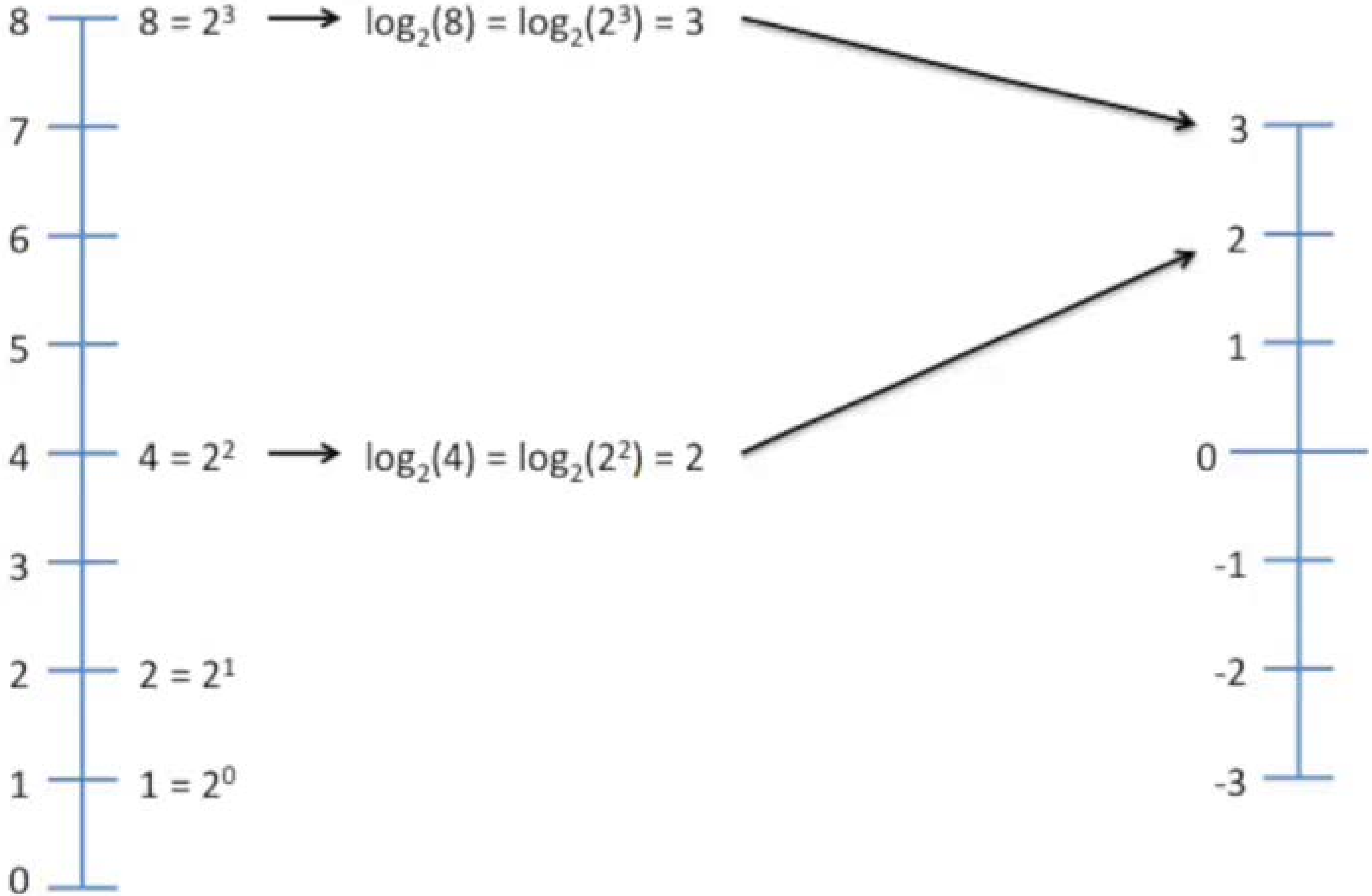

8
7
6
5
4
3
2
1
0
8 = 2^3
log2(8) = log2(2^3) = 3
4 = 2^2
log2(4)
2 = 2^1
1 = 2^0
3
2
1
0
-1
-2
-3


8
7
6
5
4
3
2
1
0
$8 = 2^3$
$\log_2(8) = \log_2(2^3) = 3$
$4 = 2^2$
$\log_2(4) = \log_2(2^2)$
Rewrite 4 as a power of 2
$2 = 2^1$
$1 = 2^0$
3
2
1
0
-1
-2
-3

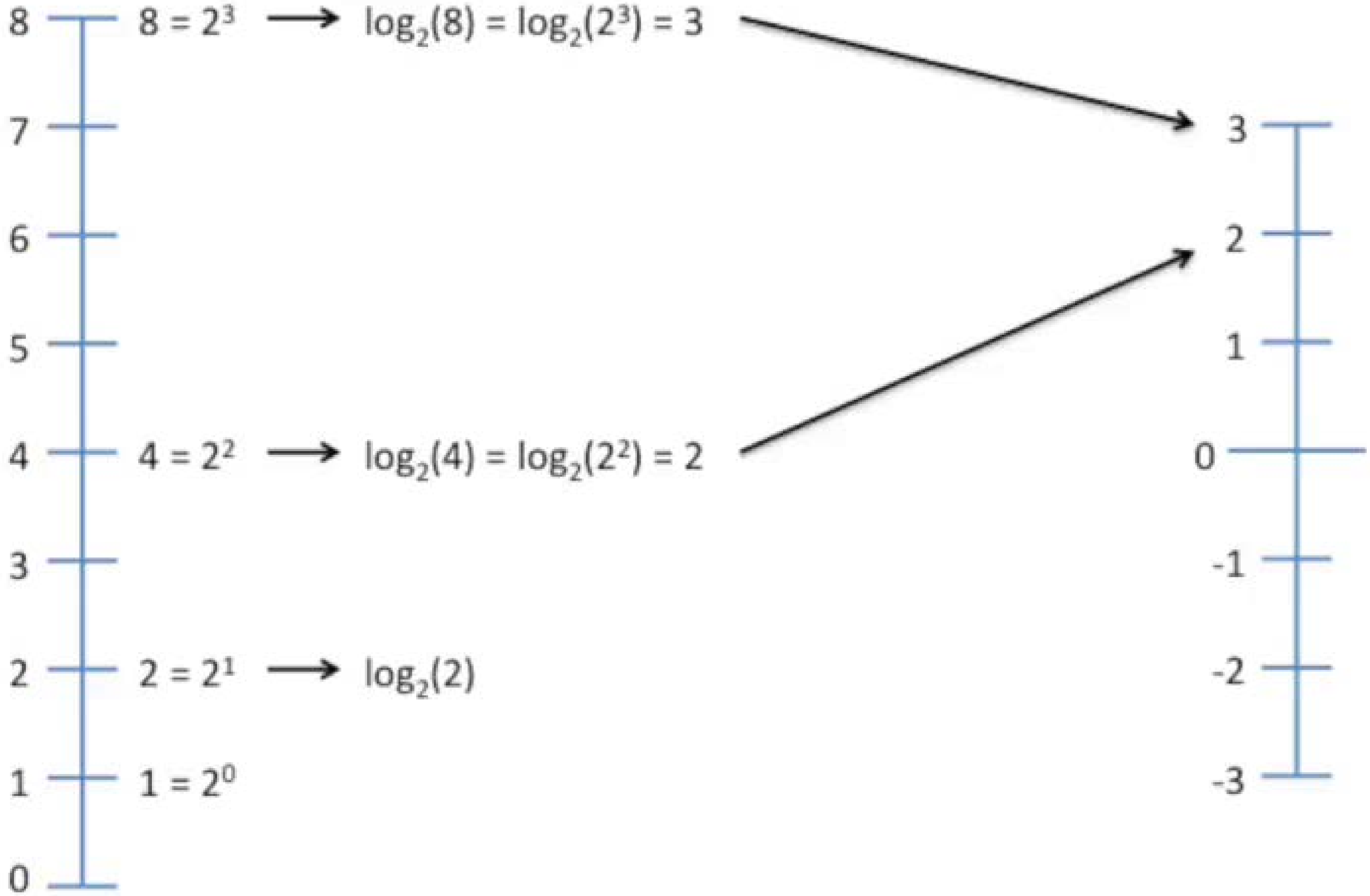


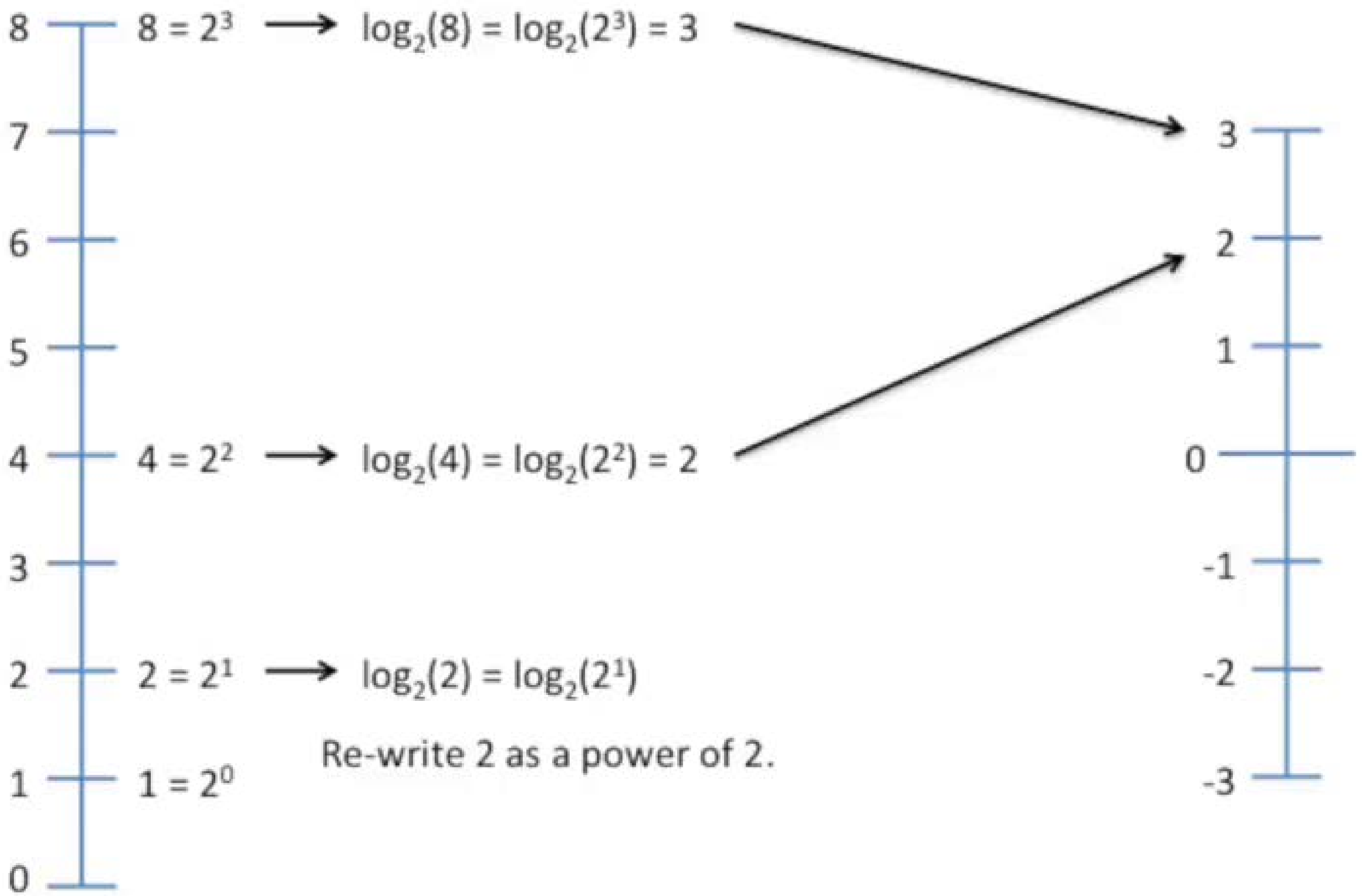
8
7
6
5
4
3
2
1
0
8 = 2³
log₂(8) = log₂(2³) = 3
4 = 2²
log₂(4) = log₂(2²)
2 = 2¹
1 = 2⁰
The log function just
isolates the exponent.
3
2
1
0
-1
-2
-3


$8 = 2^3$
$\log_2(8) = \log_2(2^3) = 3$
$4 = 2^2$
$\log_2(4) = \log_2(2^2) = 2$
The log function just
isolates the exponent.
$2 = 2^1$
$1 = 2^0$
8
7
6
5
4
3
2
1
0
3
2
1
0
-1
-2
-3

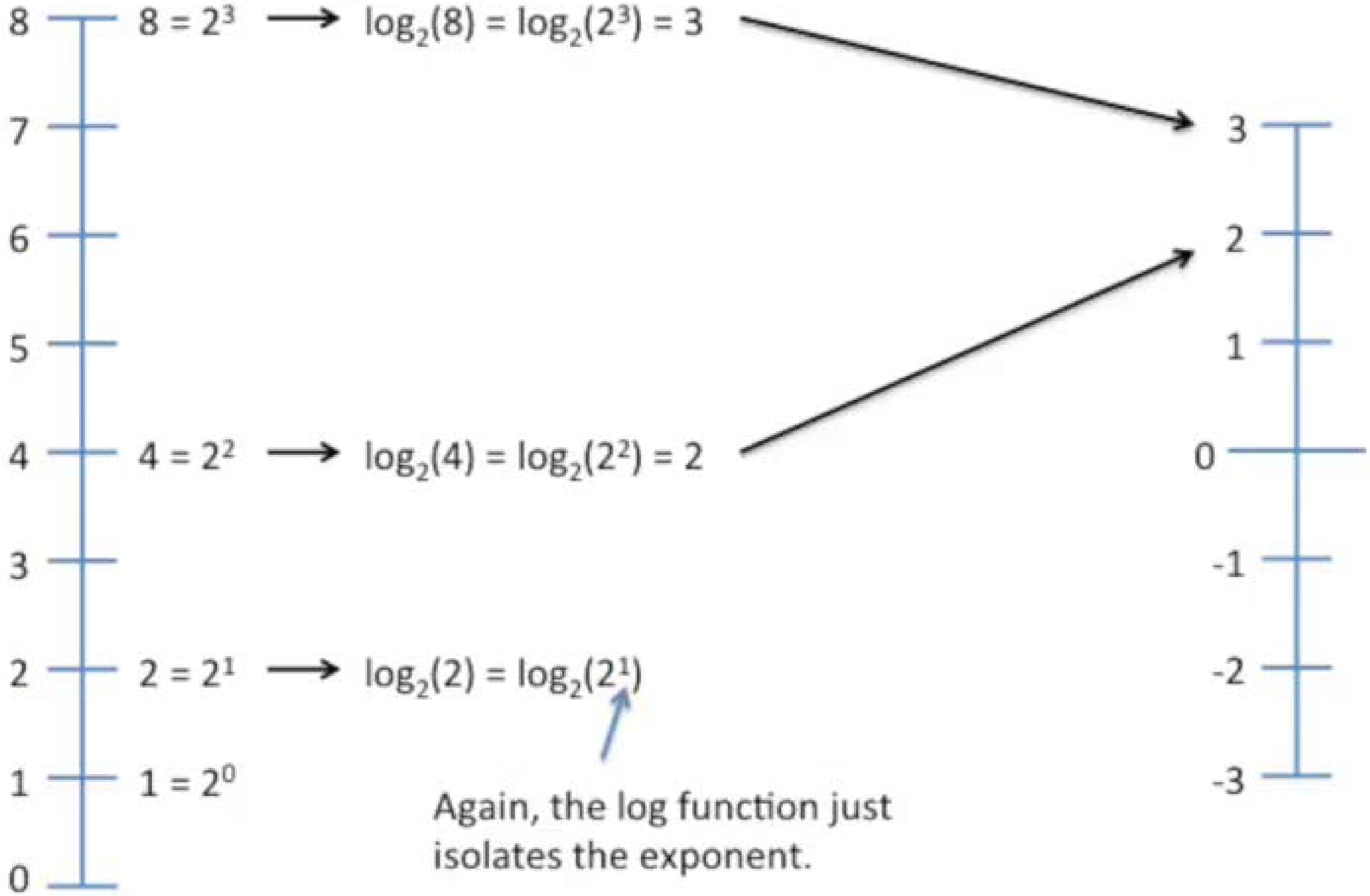


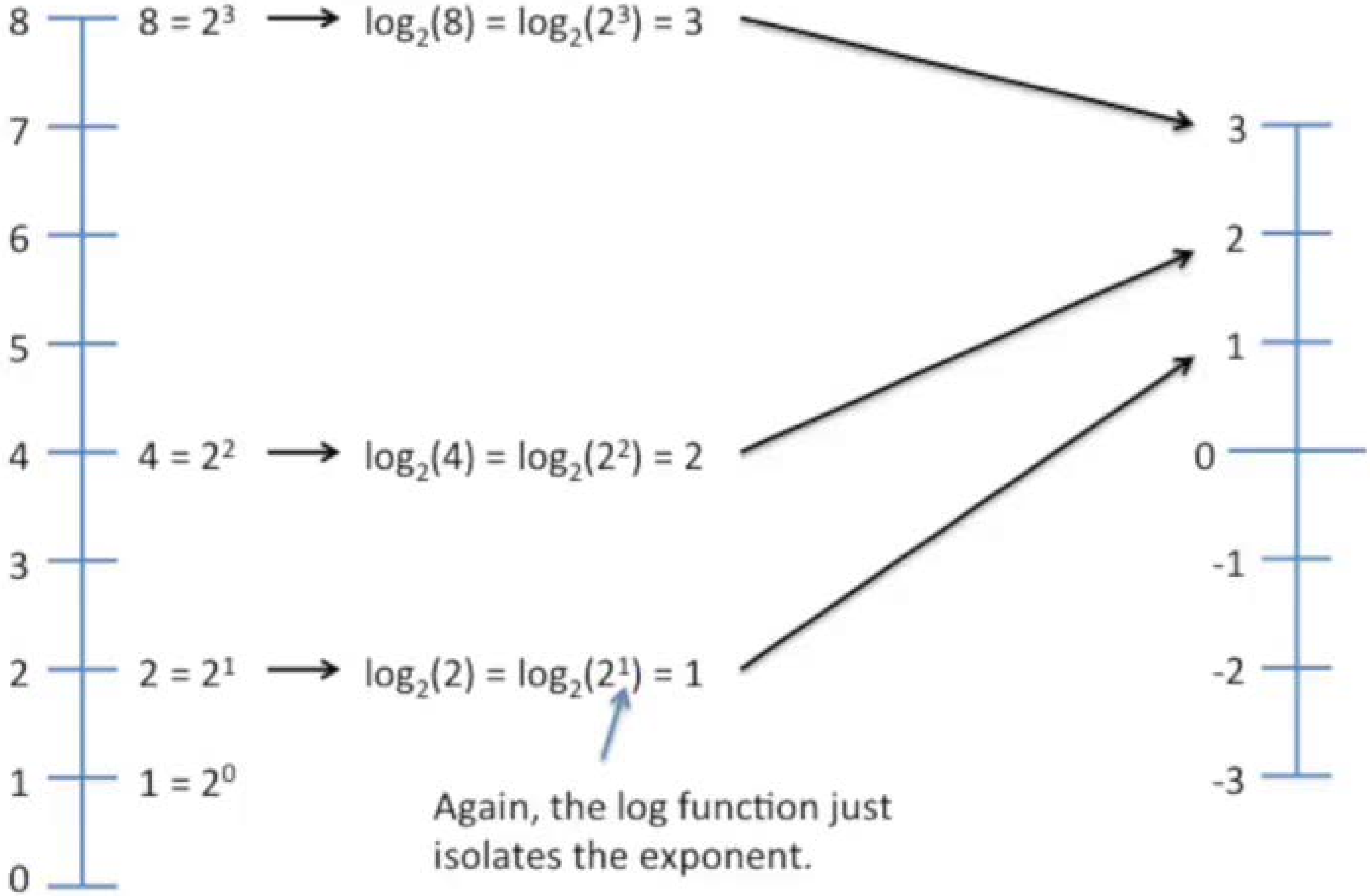
8
7
6
5
4
3
2
1
0
$8 = 2^3$
$\log_2(8) = \log_2(2^3) = 3$
$4 = 2^2$
$\log_2(4) = \log_2(2^2) = 2$
$2 = 2^1$
$1 = 2^0$
3
2
1
0
-1
-2
-3


8
7
6
5
4
3
2
1
0
$8 = 2^3$
$4 = 2^2$
$2 = 2^1$
$1 = 2^0$
$\log_2(8) = \log_2(2^3) = 3$
$\log_2(4) = \log_2(2^2) = 2$
$\log_2(2)$
3
2
1
0
-1
-2
-3

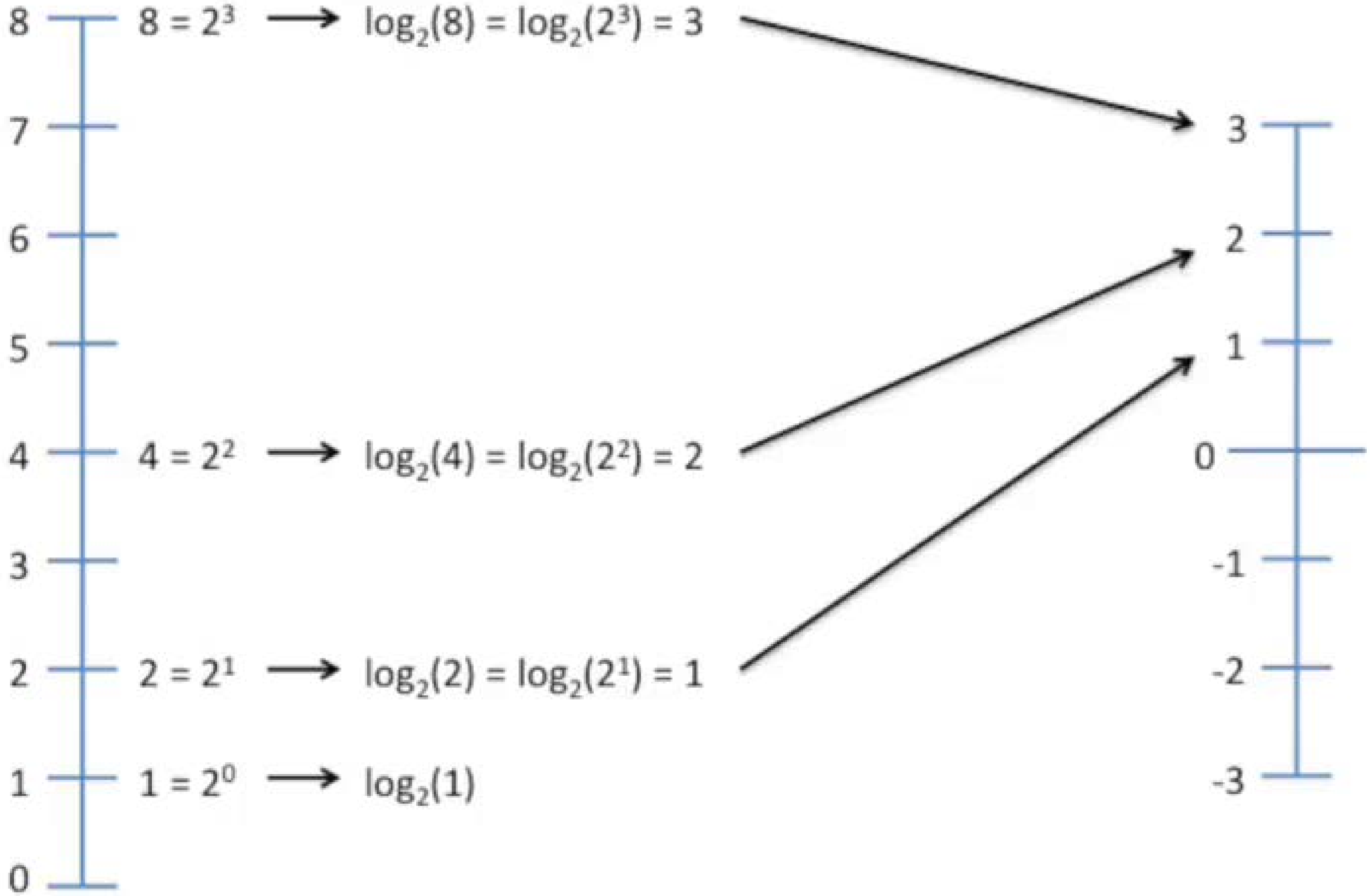


8
7
6
5
4
3
2
1
0
$8 = 2^3$
$\log_2(8) = \log_2(2^3) = 3$
$4 = 2^2$
$\log_2(4) = \log_2(2^2) = 2$
$2 = 2^1$
$\log_2(2) = \log_2(2^1)$
$1 = 2^0$
Re-write 2 as a power of 2.
3
2
1
0
-1
-2
-3

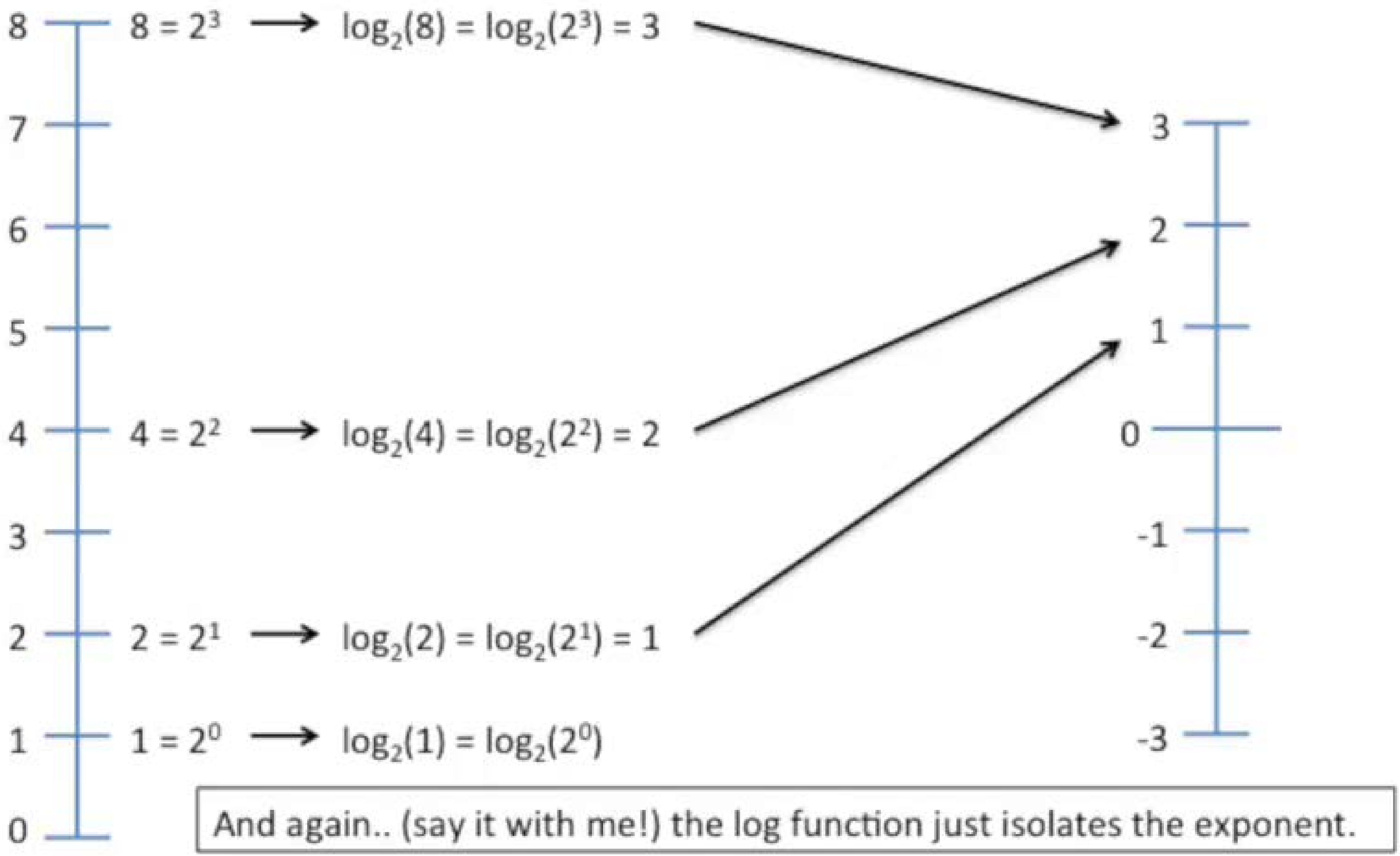

8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
log_2(8) = log_2(2^3) = 3
log_2(4) = log_2(2^2) = 2
log_2(2) = log_2(2^1)
Again, the log function just isolates the exponent.
3
2
1
0
-1
-2
-3

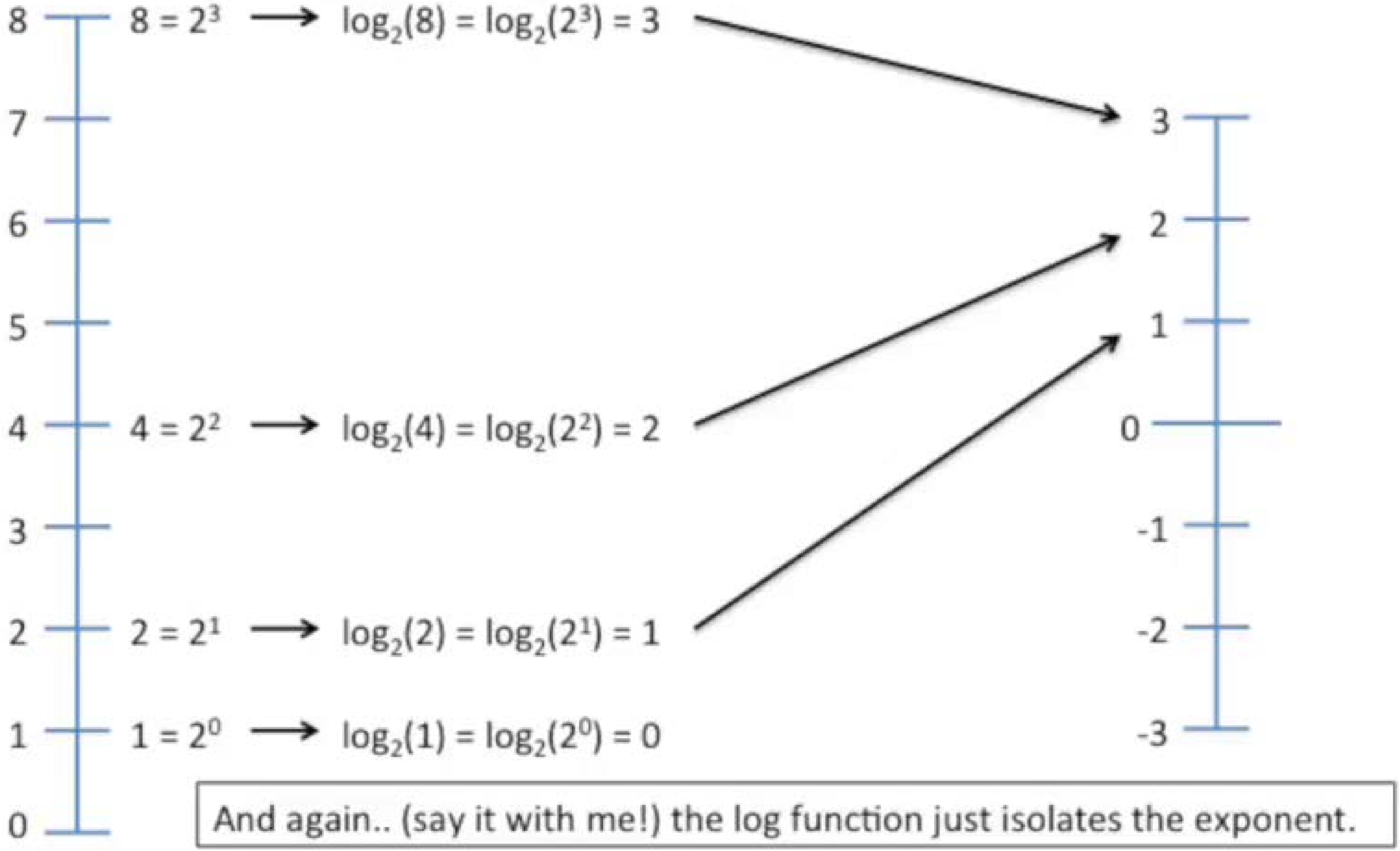


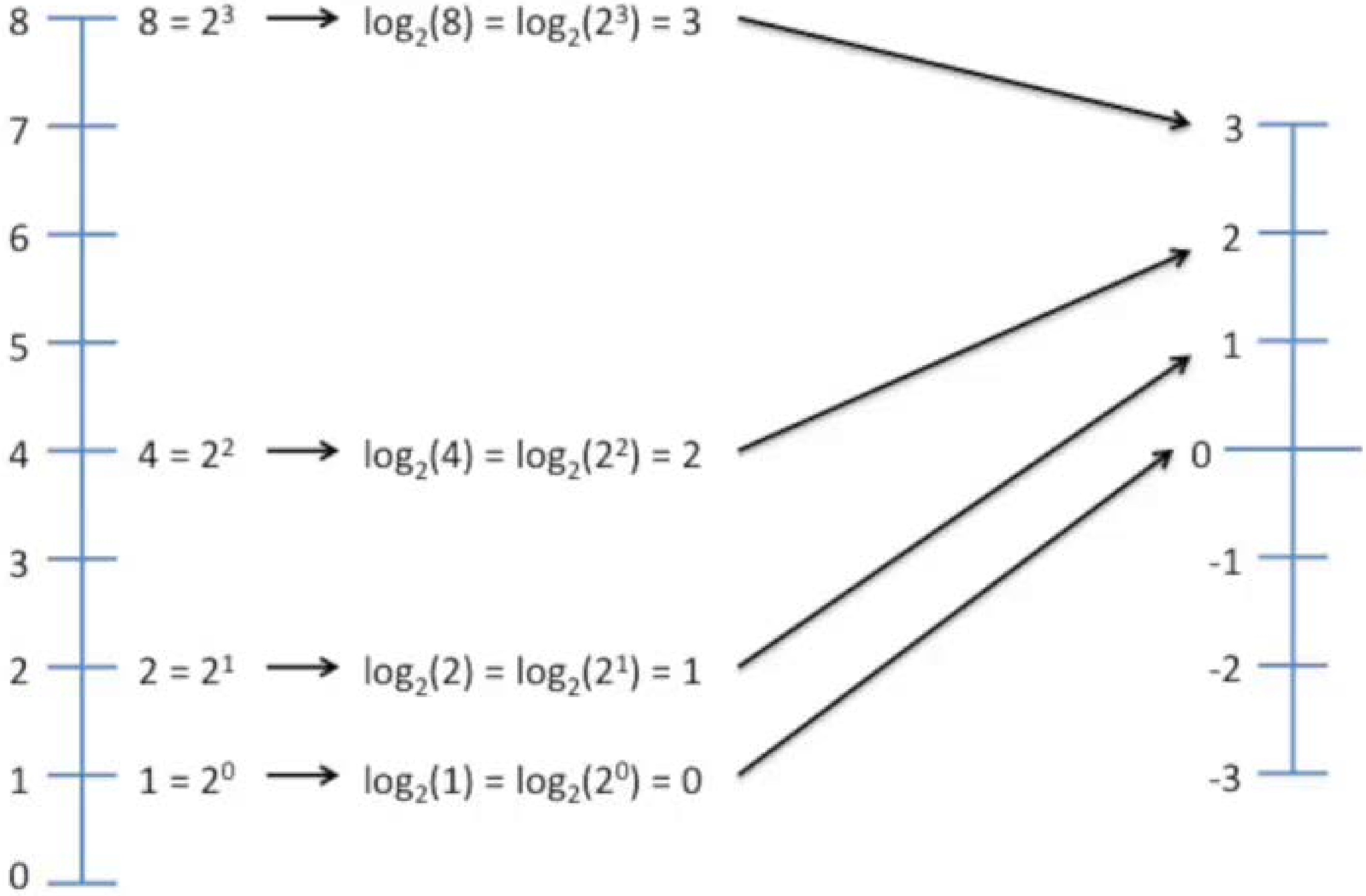
8
7
6
5
4
3
2
1
0
8 = 2³ → log₂(8) = log₂(2³) = 3
4 = 2² → log₂(4) = log₂(2²) = 2
2 = 2¹ → log₂(2) = log₂(2¹) = 1
1 = 2⁰
3
2
1
0
-1
-2
-3
Again, the log function just
isolates the exponent.

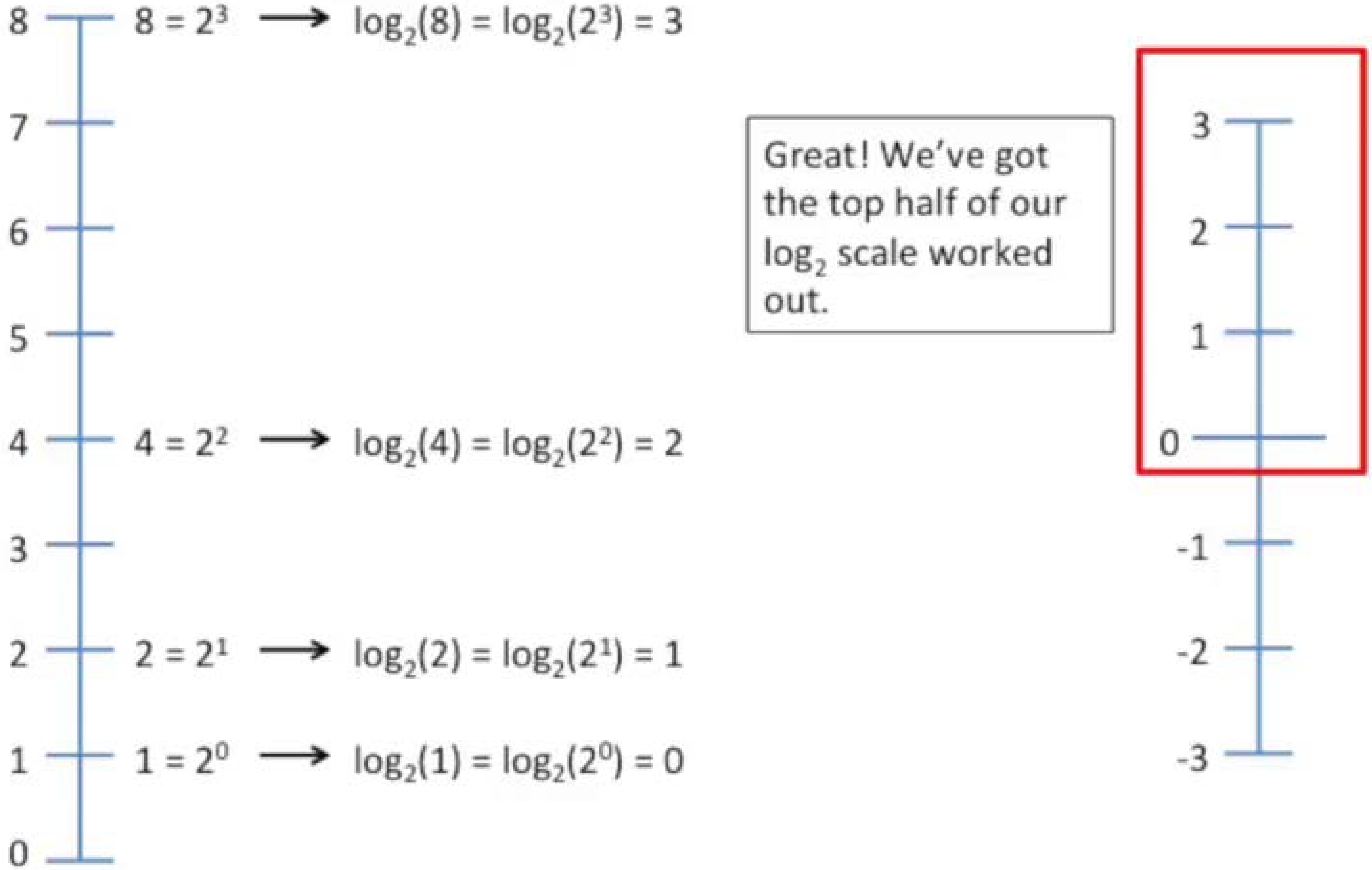


8
7
6
5
4
3
2
1
0
8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
log₂(8) = log₂(2³) = 3
log₂(4) = log₂(2²) = 2
log₂(2) = log₂(2¹) = 1
log₂(1)
3
2
1
0
-1
-2
-3

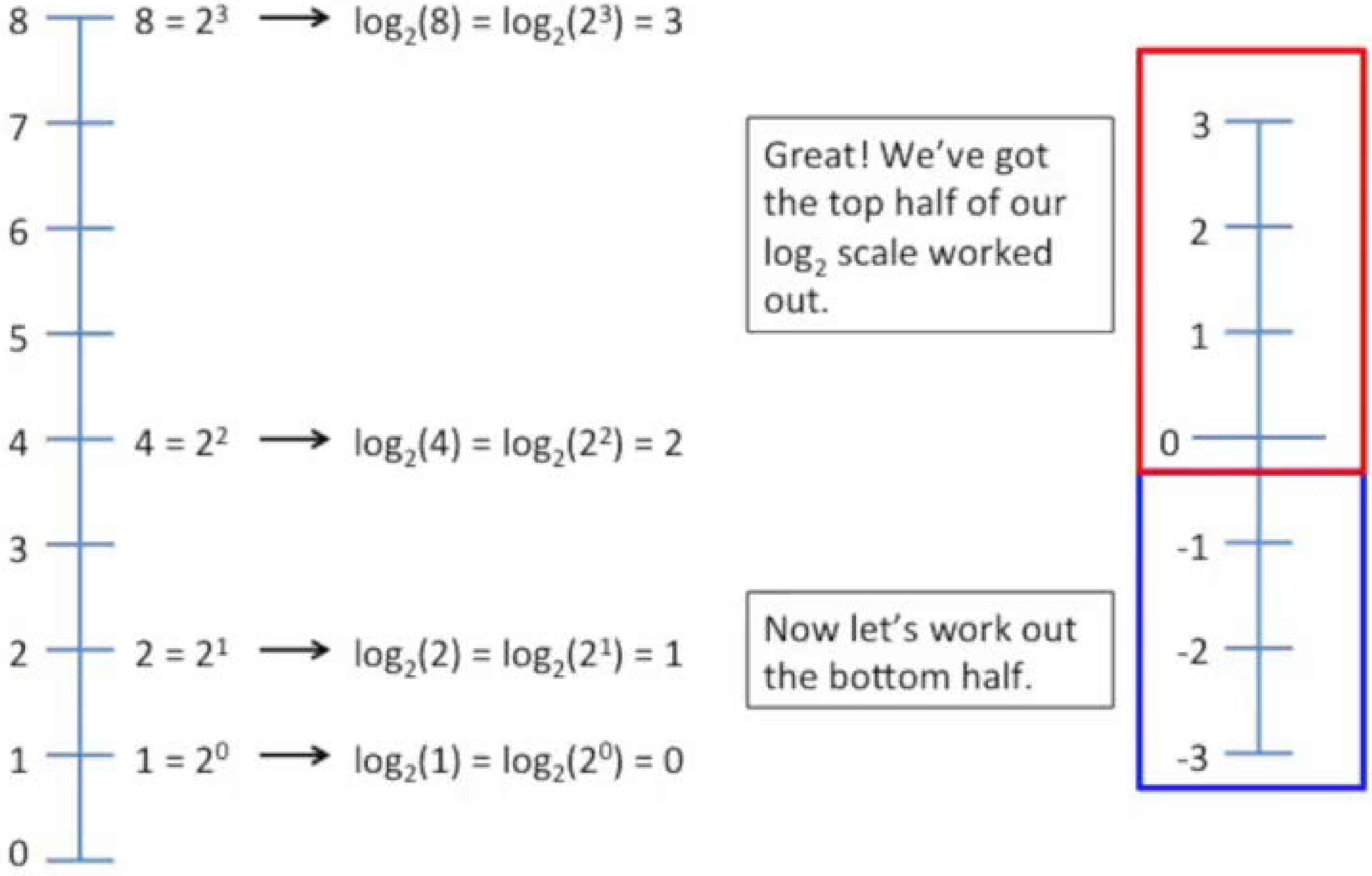

8
7
6
5
4
3
2
1
0
8 = 2^3
log_2(8) = log_2(2^3) = 3
4 = 2^2
log_2(4) = log_2(2^2) = 2
2 = 2^1
log_2(2) = log_2(2^1) = 1
1 = 2^0
log_2(1) = log_2(2^0)
3
2
1
0
-1
-2
-3
And again.. (say it with me!) the log function just isolates the exponent.


8
7
6
5
4
3
2
1
0
8 = 2^3 → log_2(8) = log_2(2^3) = 3
4 = 2^2 → log_2(4) = log_2(2^2) = 2
2 = 2^1 → log_2(2) = log_2(2^1) = 1
1 = 2^0 → log_2(1) = log_2(2^0) = 0
3
2
1
0
-1
-2
-3
And again.. (say it with me!) the log function just isolates the exponent.

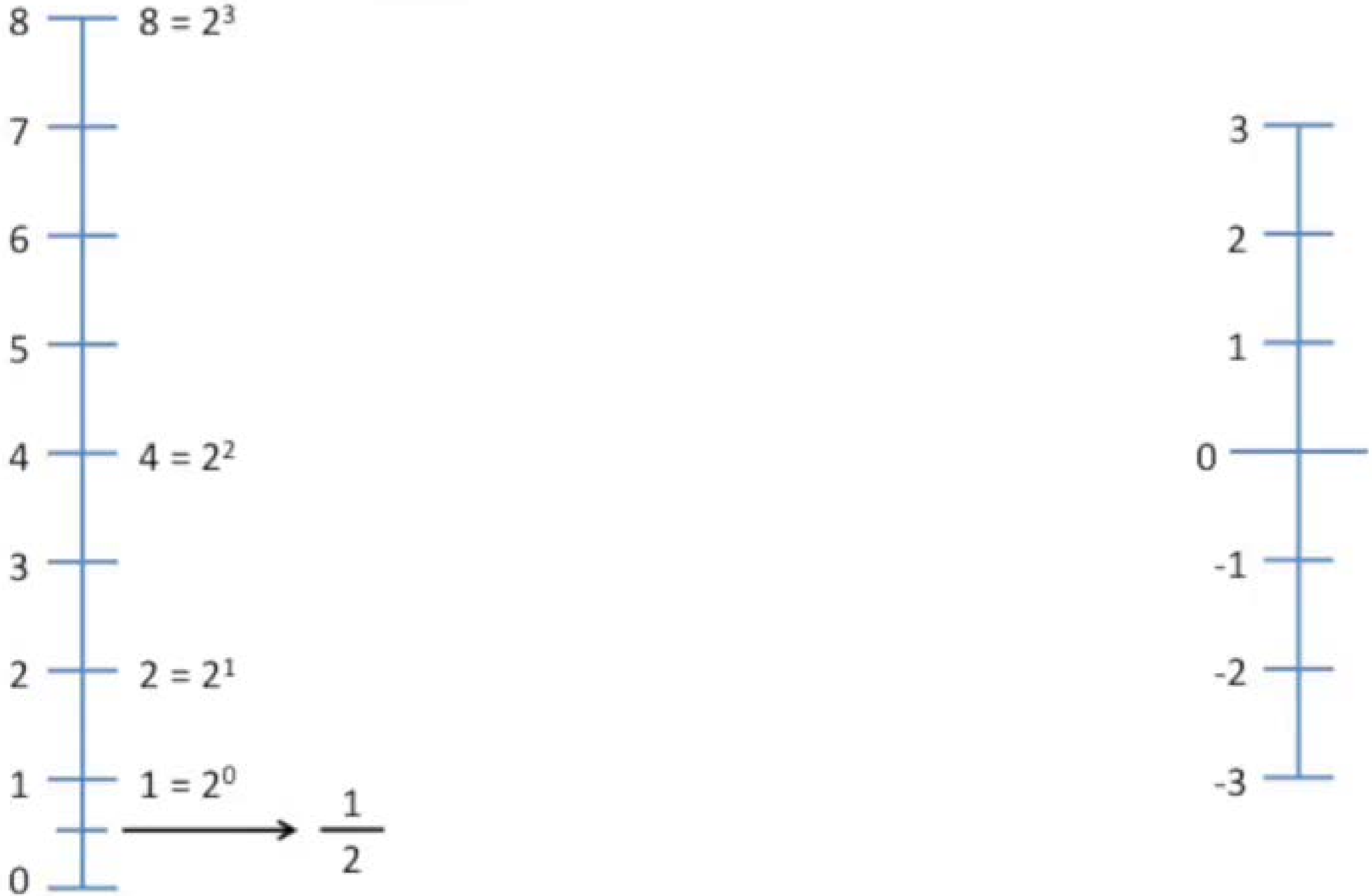

8
7
6
5
4
3
2
1
0
8 = 2³ → log₂(8) = log₂(2³) = 3
4 = 2² → log₂(4) = log₂(2²) = 2
2 = 2¹ → log₂(2) = log₂(2¹) = 1
1 = 2⁰ → log₂(1) = log₂(2⁰) = 0
3
2
1
0
-1
-2
-3

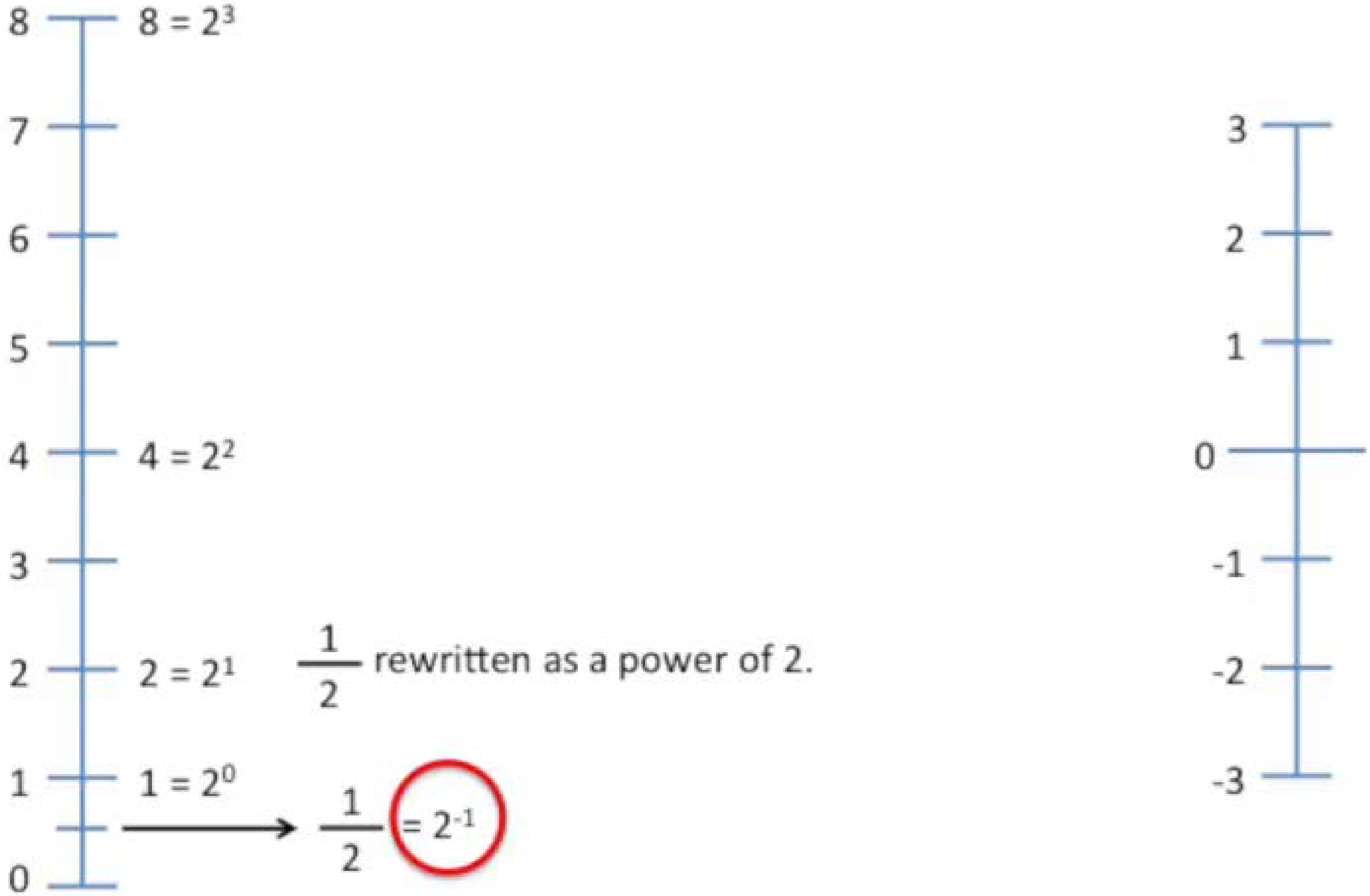


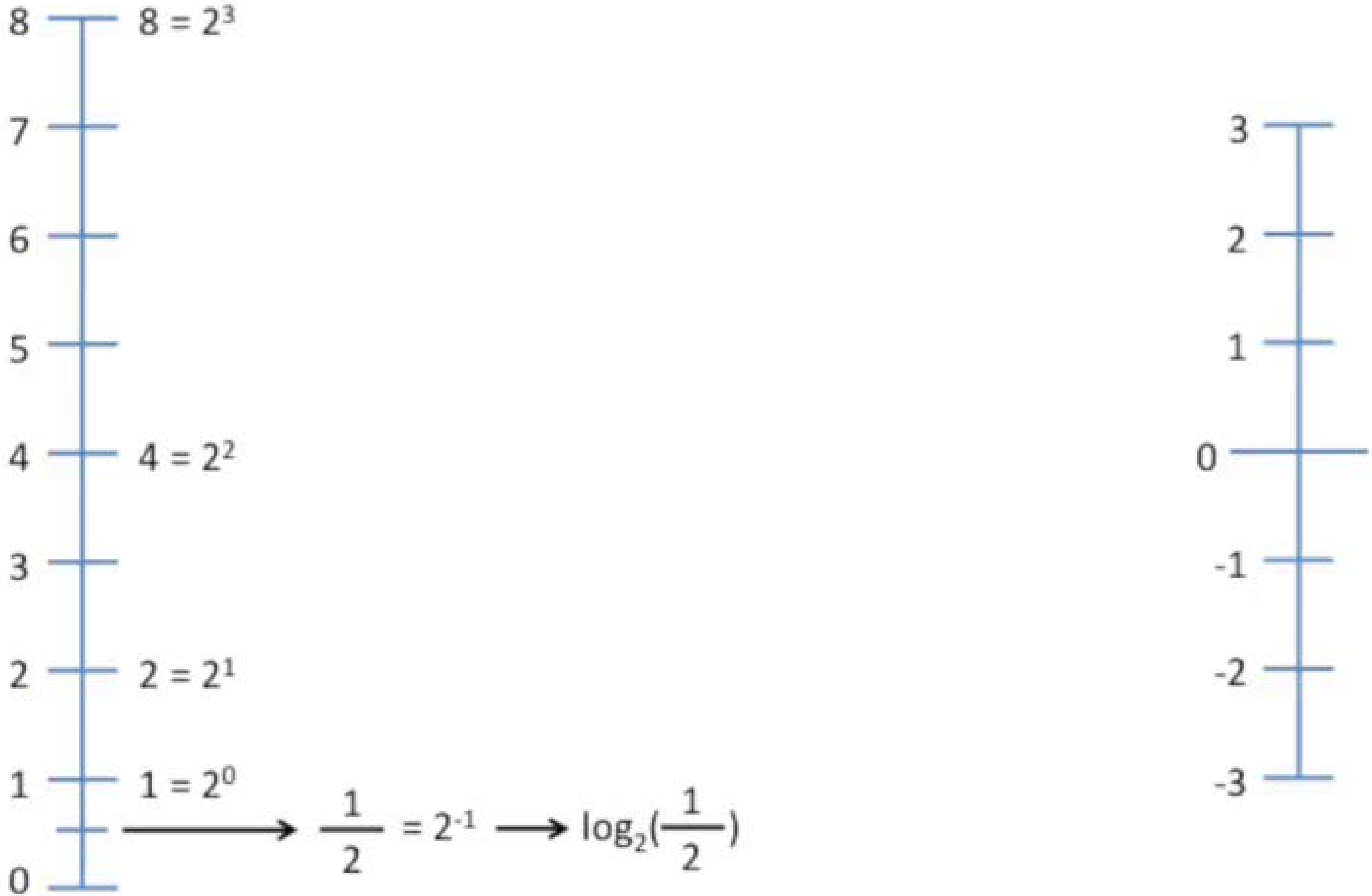
8
7
6
5
4
3
2
1
0
8 = 2³ → log₂(8) = log₂(2³) = 3
4 = 2² → log₂(4) = log₂(2²) = 2
2 = 2¹ → log₂(2) = log₂(2¹) = 1
1 = 2⁰ → log₂(1) = log₂(2⁰) = 0
Great! We've got the top half of our log₂ scale worked out.
3
2
1
0
-1
-2
-3



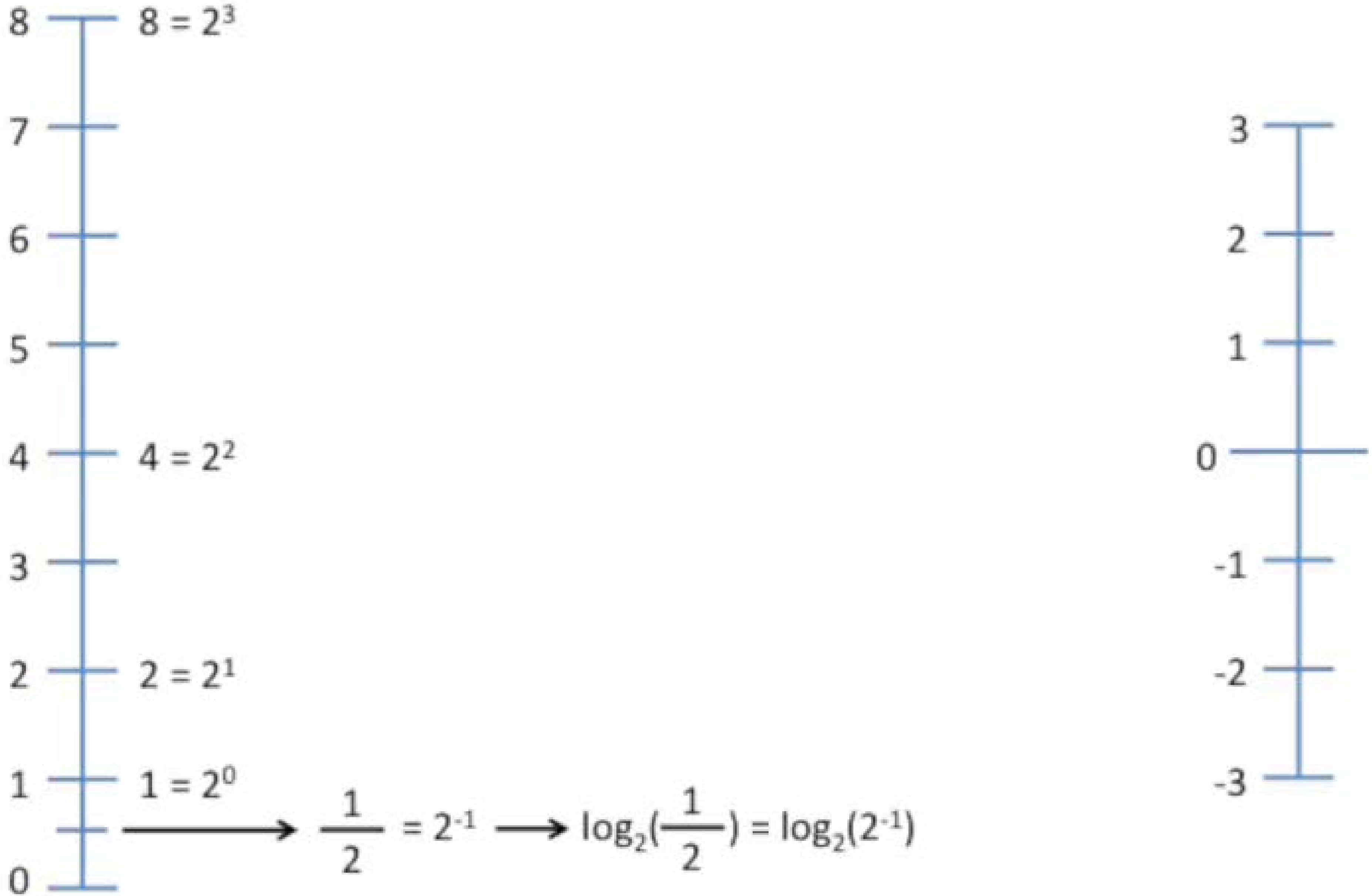
8 = 2^3 → $\log_2(8) = \log_2(2^3) = 3$
4 = 2^2 → $\log_2(4) = \log_2(2^2) = 2$
2 = 2^1 → $\log_2(2) = \log_2(2^1) = 1$
1 = 2^0 → $\log_2(1) = \log_2(2^0) = 0$
8
7
6
5
4
3
2
1
0
Great! We've got the top half of our $\log_2$ scale worked out.
Now let's work out the bottom half.
3
2
1
0
-1
-2
-3


8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
1/2
3
2
1
0
-1
-2
-3

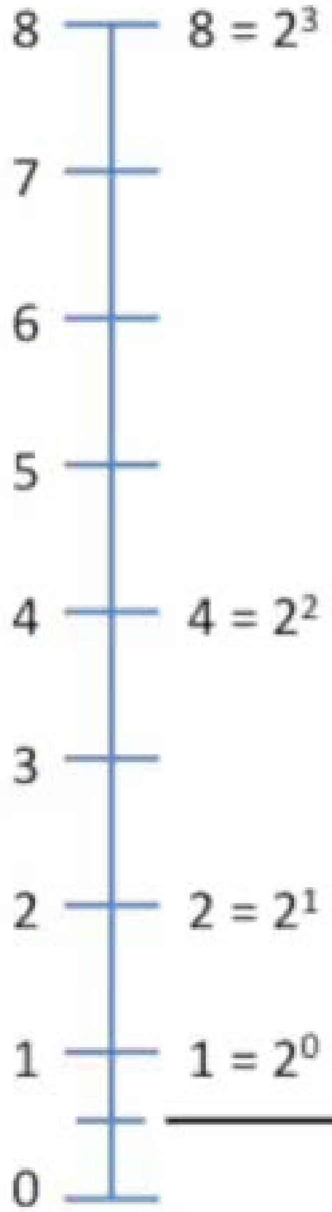

8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
1/2 rewritten as a power of 2.
1/2 = 2^-1
3
2
1
0
-1
-2
-3



8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
1/2 = 2^-1
log_2(1/2)
3
2
1
0
-1
-2
-3

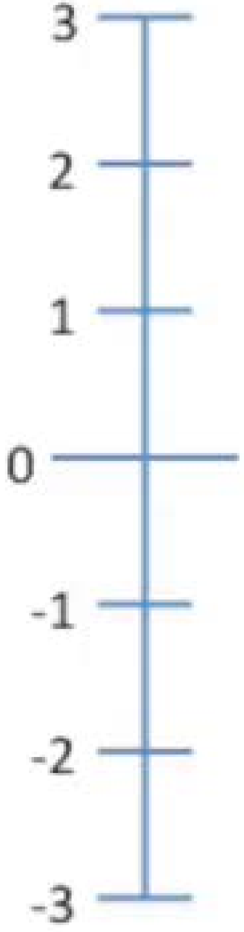


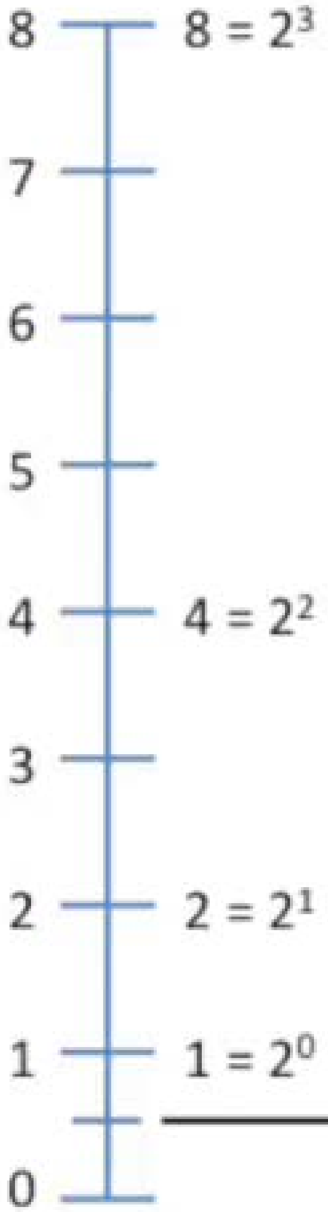
8
7
6
5
4
3
2
1
0
$8 = 2^3$
$4 = 2^2$
$2 = 2^1$
$1 = 2^0$
$\frac{1}{2} = 2^{-1}$
$\log_2(\frac{1}{2}) = \log_2(2^{-1})$
3
2
1
0
-1
-2
-3



Guess what the log function is just about to do!!!

$$\rightarrow \frac{1}{2} = 2^{-1} \longrightarrow \log_2\left(\frac{1}{2}\right) = \log_2(2^{-1})$$

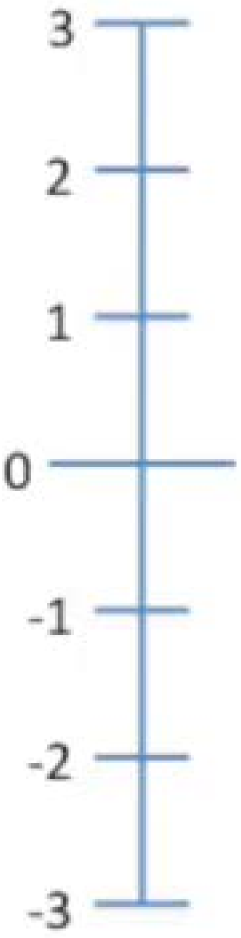


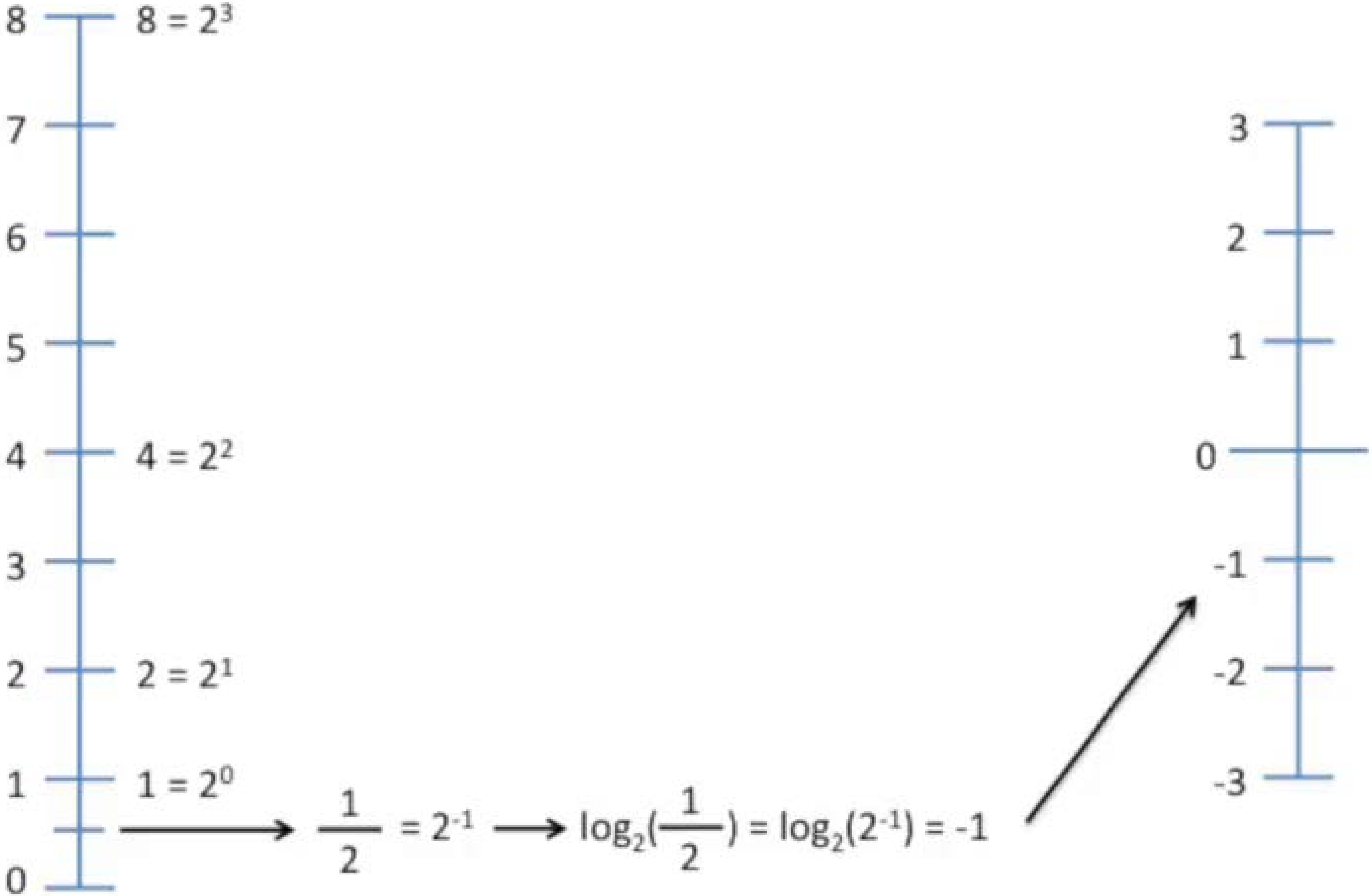


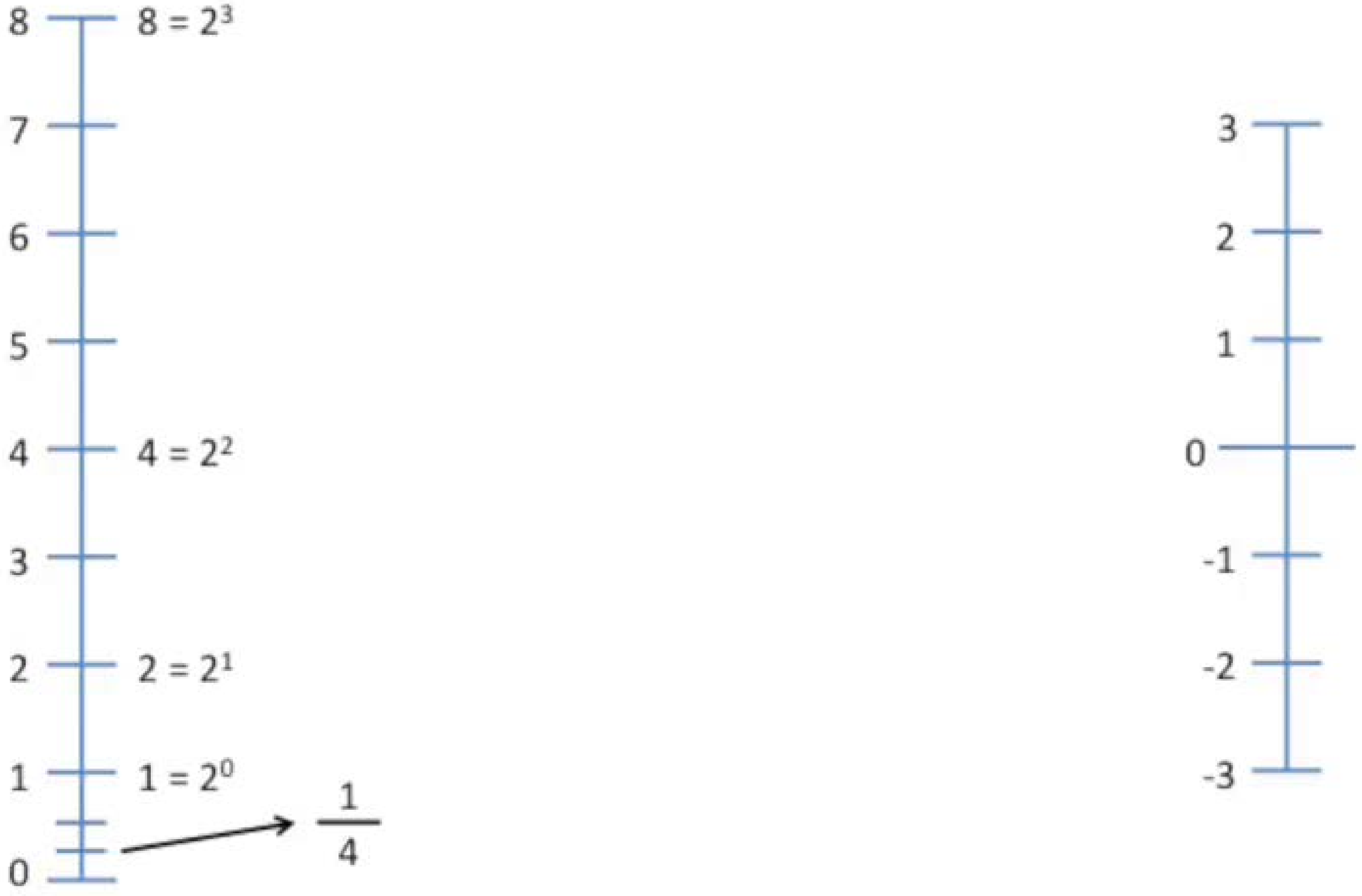
Guess what the log function is just about to do!!!

Isolate the exponent.

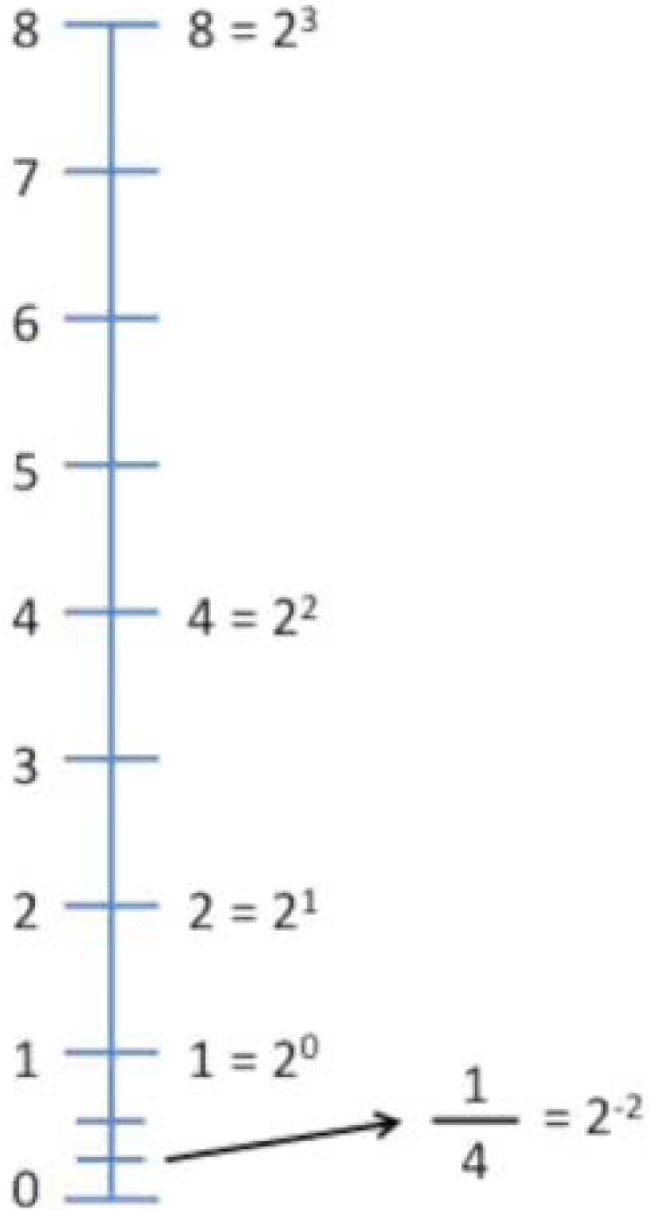
$$\frac{1}{2} = 2^{-1} \longrightarrow \log_2\left(\frac{1}{2}\right) = \log_2(2^{-1})$$

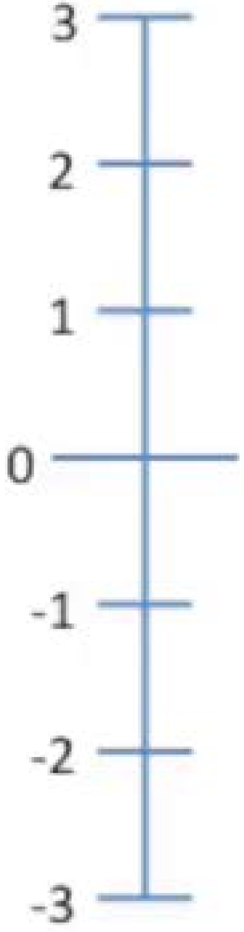


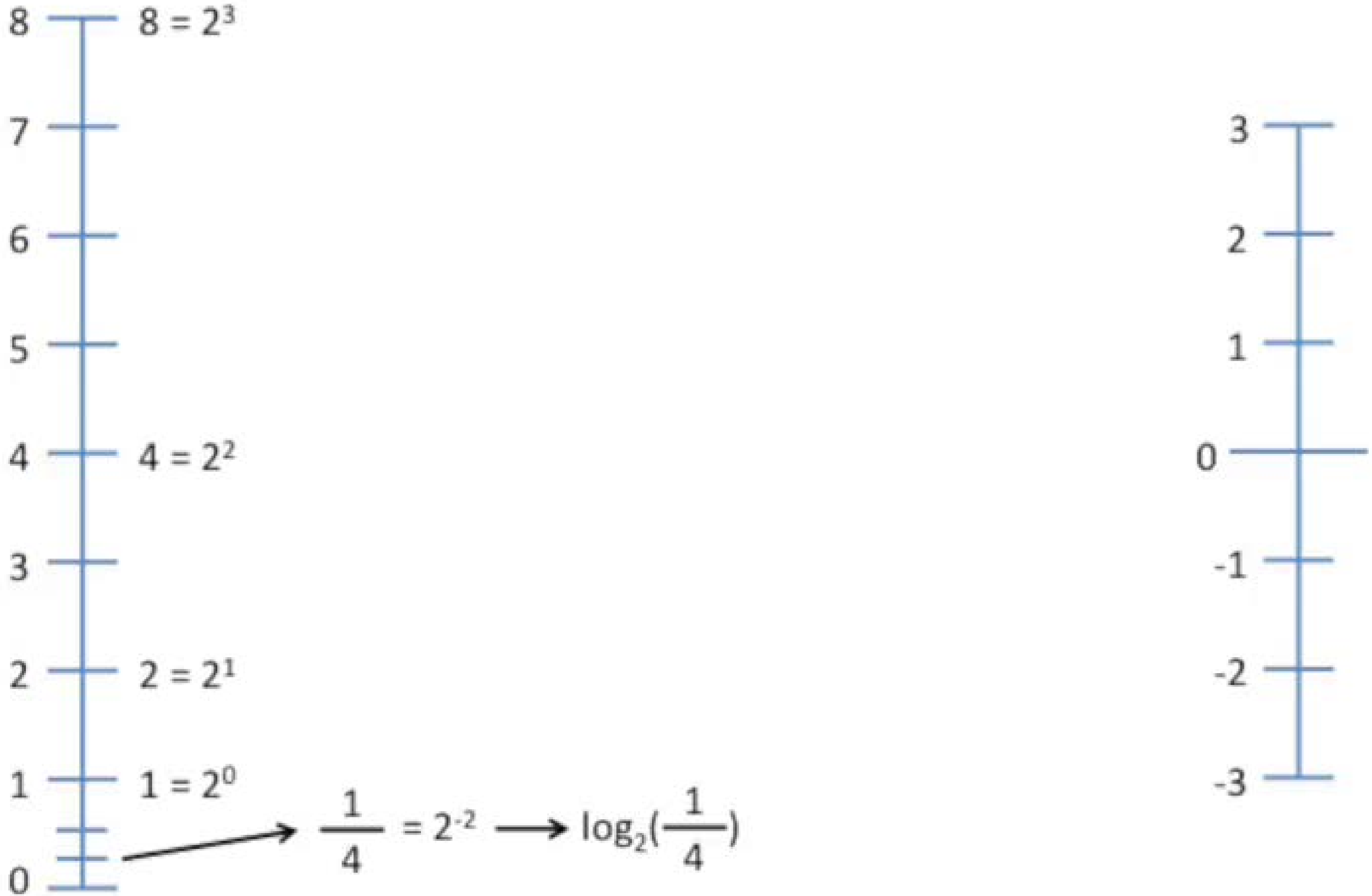

8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
1/2 = 2^-1
log_2(1/2) = log_2(2^-1) = -1
3
2
1
0
-1
-2
-3



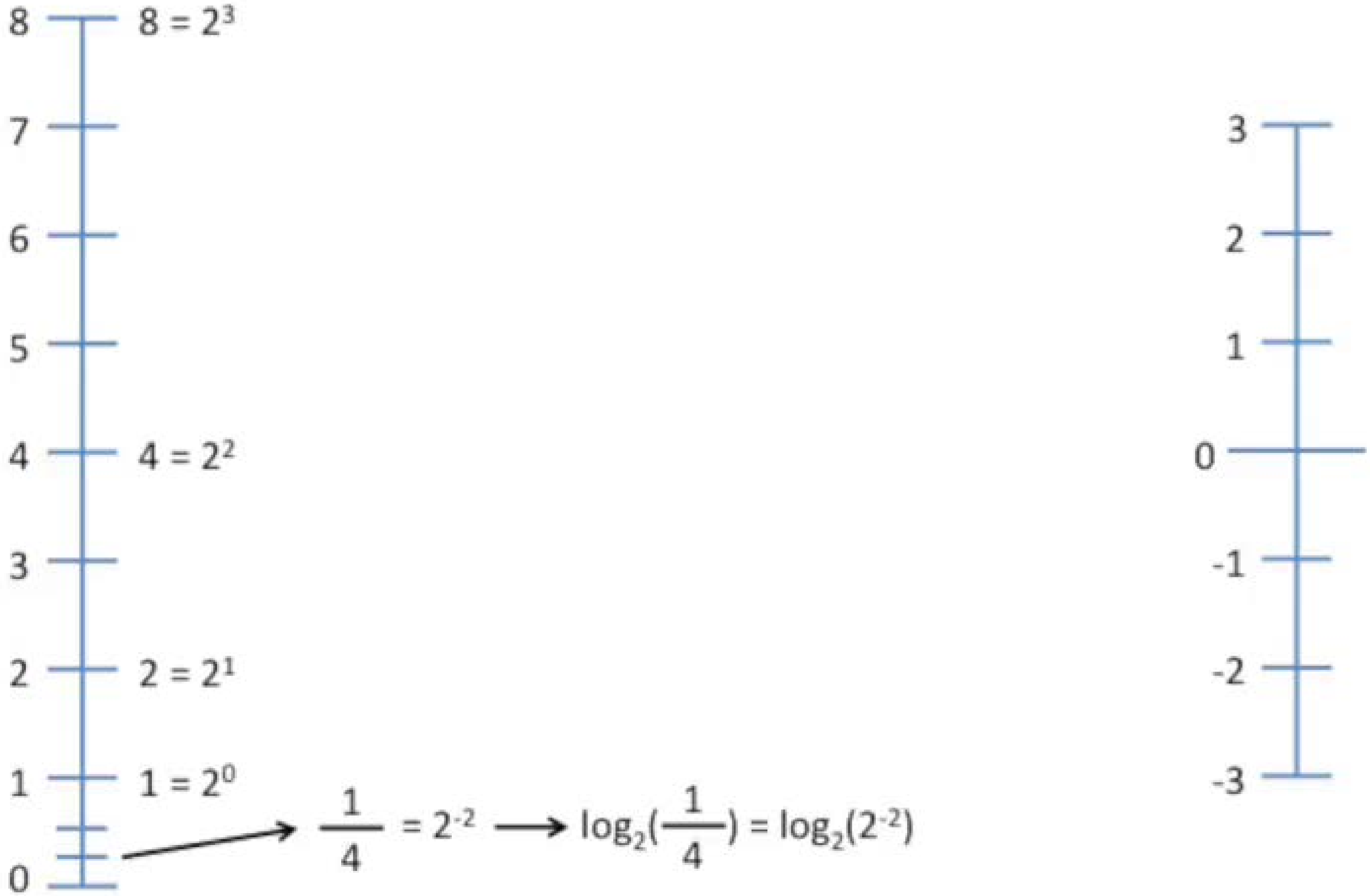
8
8 = 2^3
7
6
5
4
4 = 2^2
3
2
2 = 2^1
1
1 = 2^0
0
1/4
3
2
1
0
-1
-2
-3


8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
1/4 = 2^-2

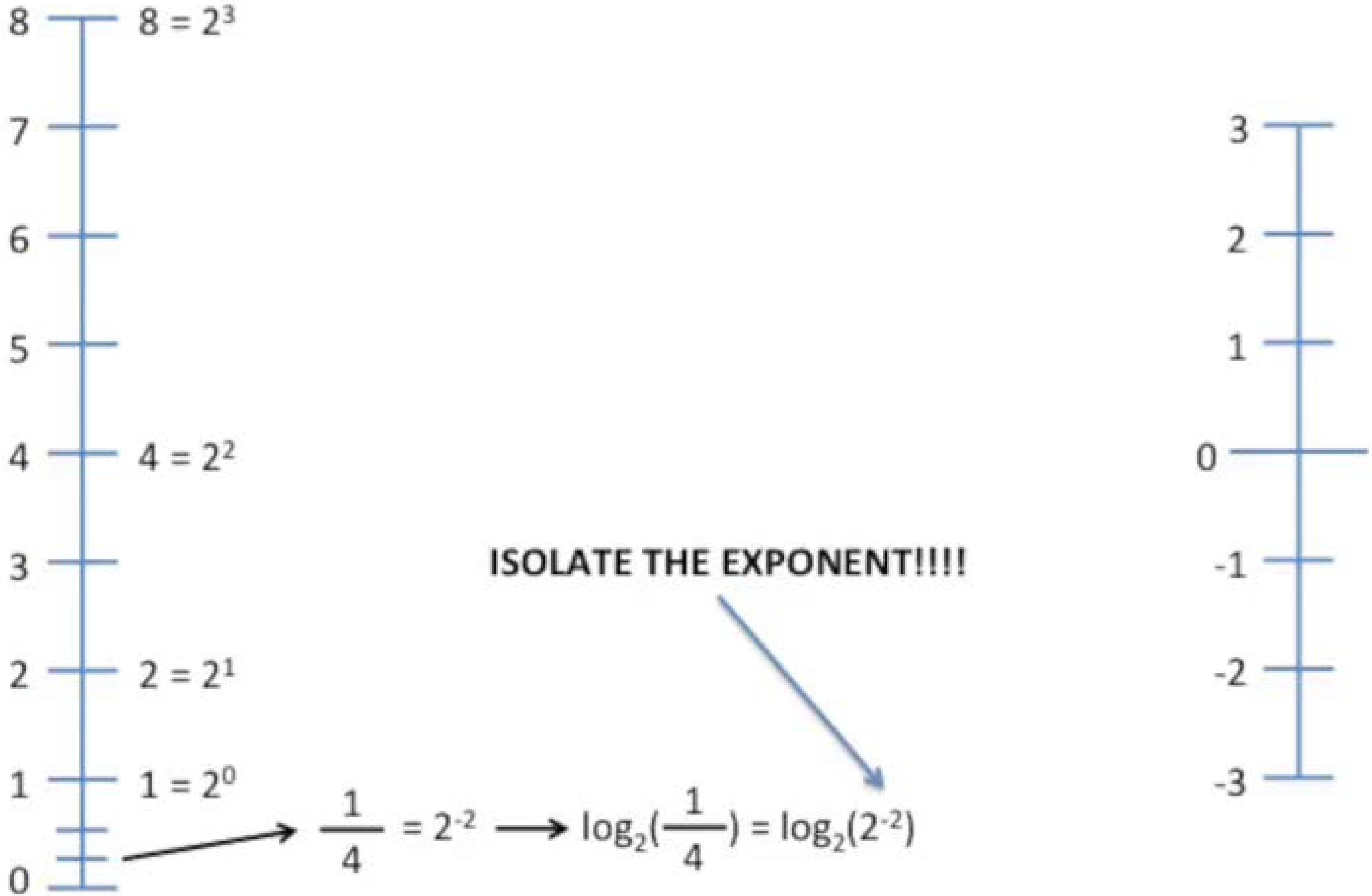

3
2
1
0
-1
-2
-3

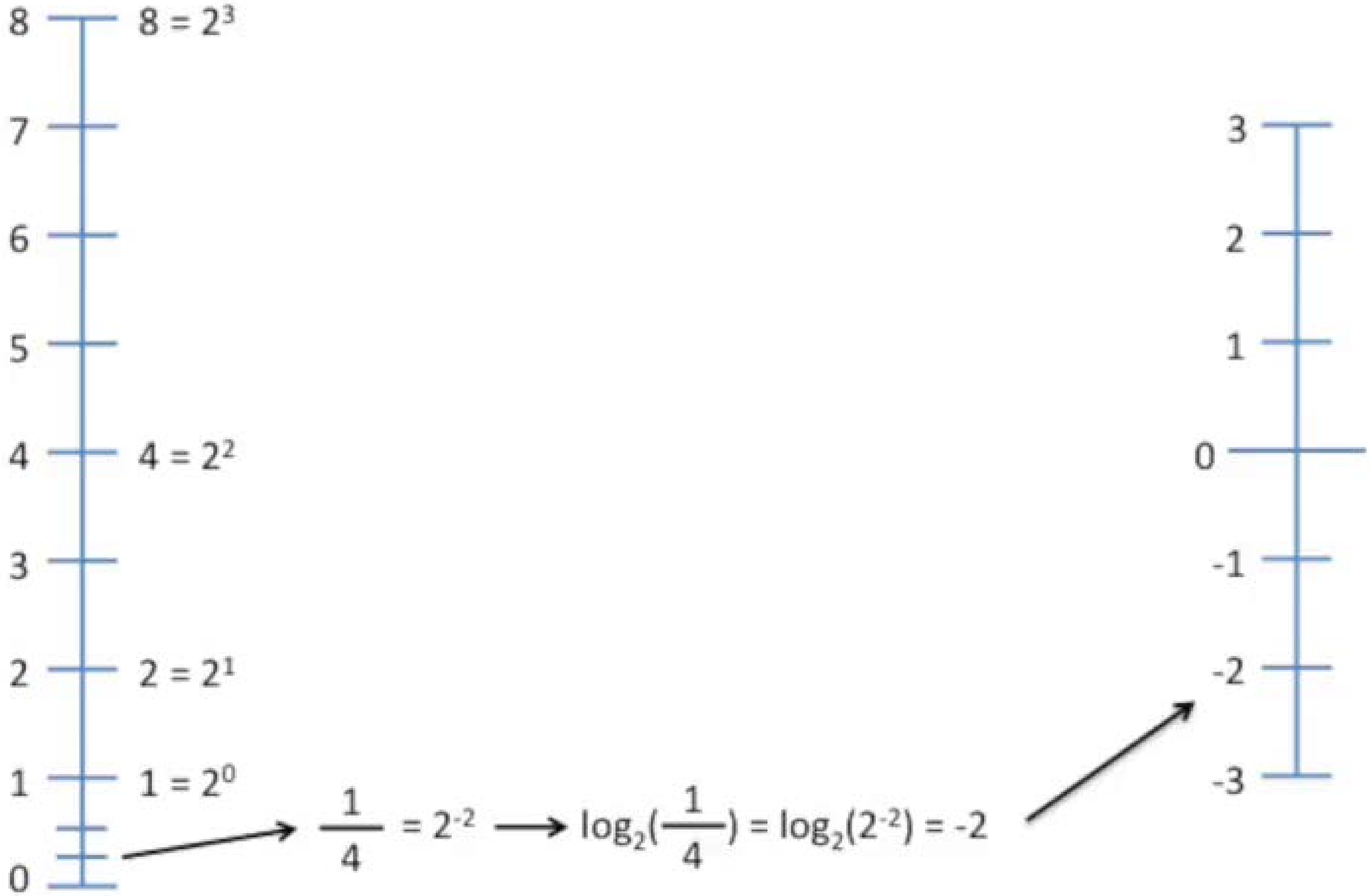


8
7
6
5
4
3
2
1
0
$8 = 2^3$
$4 = 2^2$
$2 = 2^1$
$1 = 2^0$
$\frac{1}{4} = 2^{-2} \longrightarrow \log_2(\frac{1}{4})$
3
2
1
0
-1
-2
-3

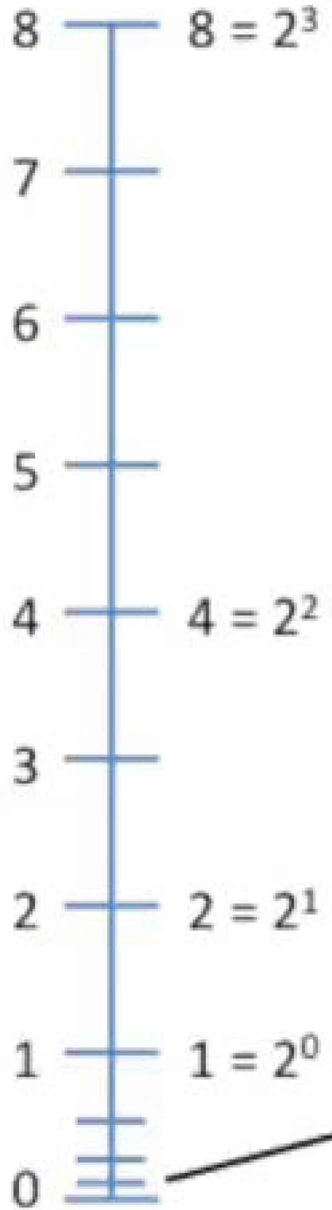


8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
1/4 = 2^-2
log_2(1/4) = log_2(2^-2)
3
2
1
0
-1
-2
-3


8
7
6
5
4
3
2
1
0
$8 = 2^3$
$4 = 2^2$
$2 = 2^1$
$1 = 2^0$
ISOLATE THE EXPONENT!!!!
$\frac{1}{4} = 2^{-2} \longrightarrow \log_2(\frac{1}{4}) = \log_2(2^{-2})$
3
2
1
0
-1
-2
-3



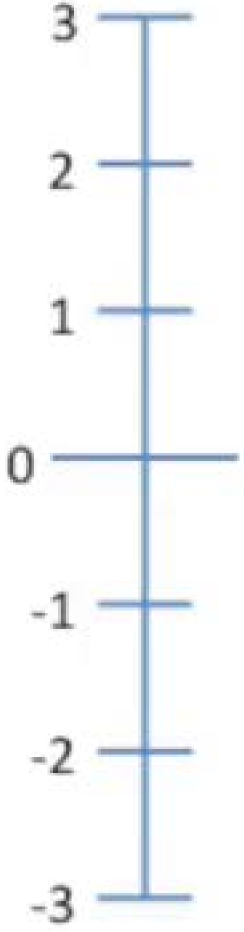
8
7
6
5
4
3
2
1
0
$8 = 2^3$
$4 = 2^2$
$2 = 2^1$
$1 = 2^0$
$\frac{1}{4} = 2^{-2}$
$\log_2(\frac{1}{4}) = \log_2(2^{-2}) = -2$
3
2
1
0
-1
-2
-3

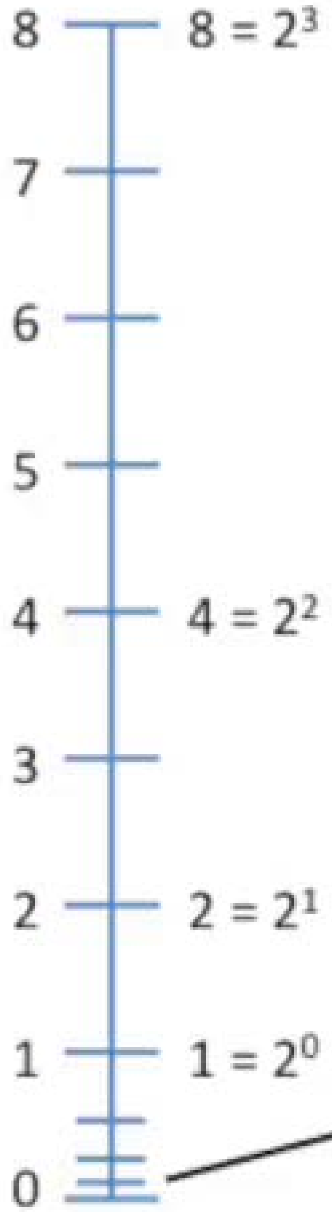


You get the idea...

The log function isolates the exponent.

$$\frac{1}{8} = 2^{-3} \longrightarrow \log_2\left(\frac{1}{8}\right) = \log_2(2^{-3}) = -3$$





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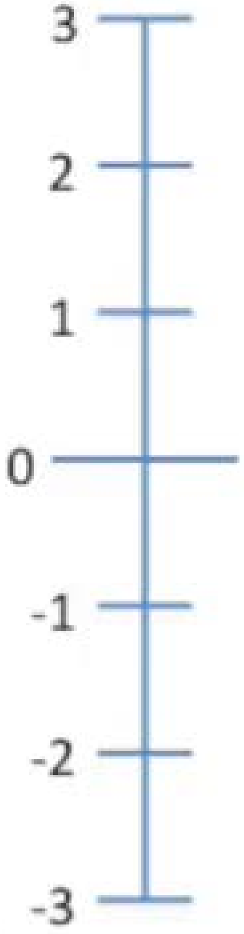
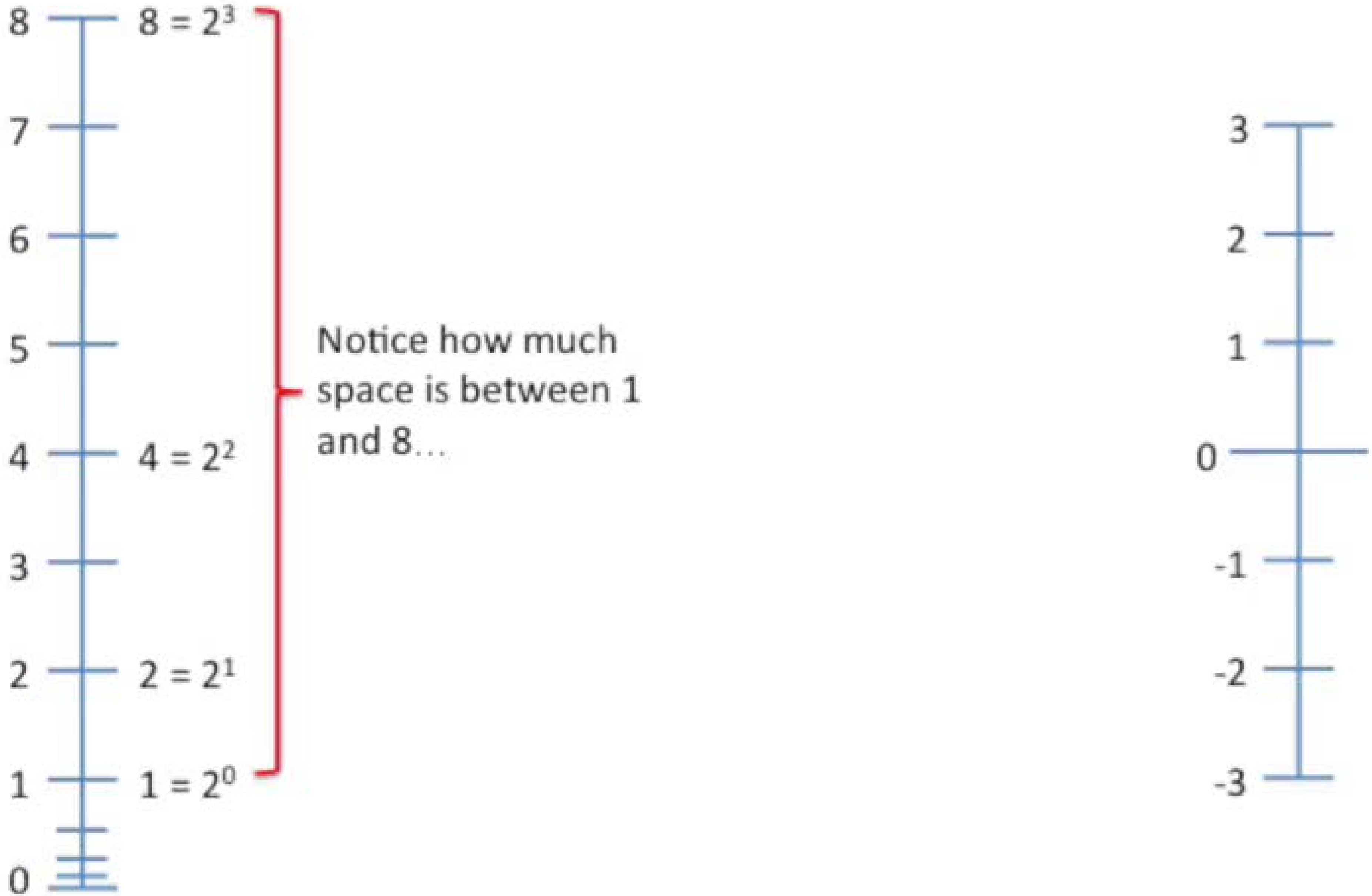
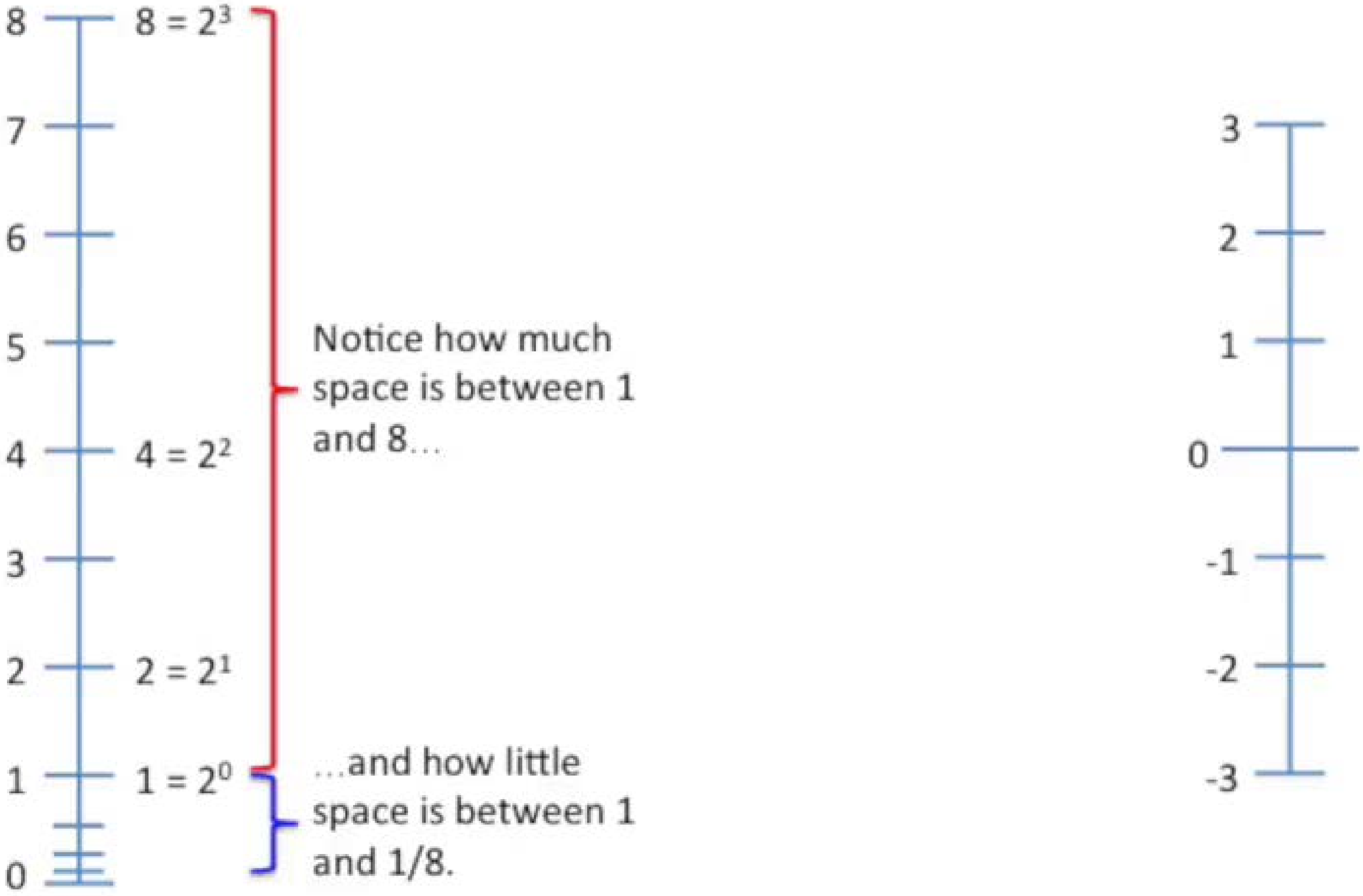
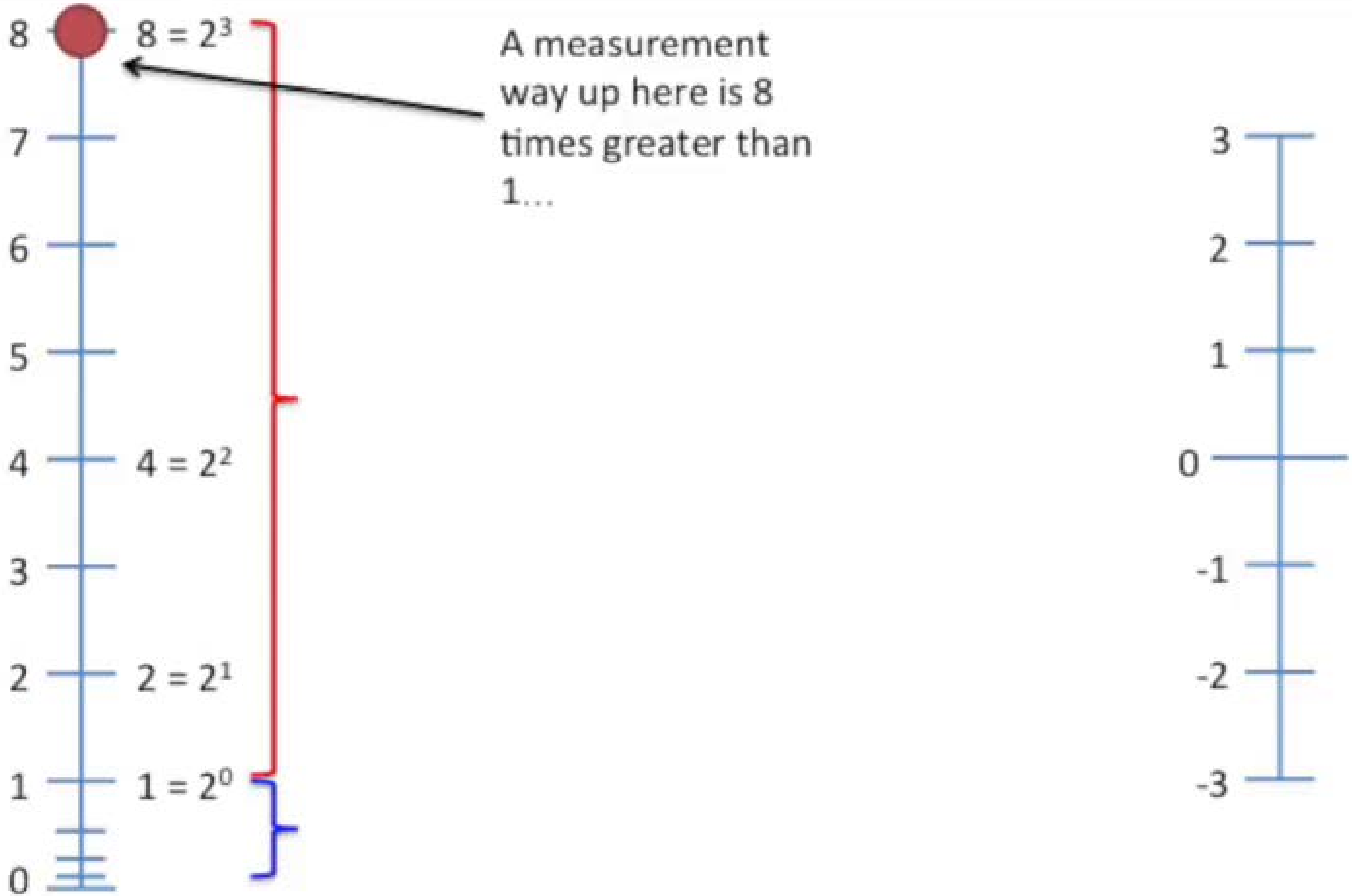
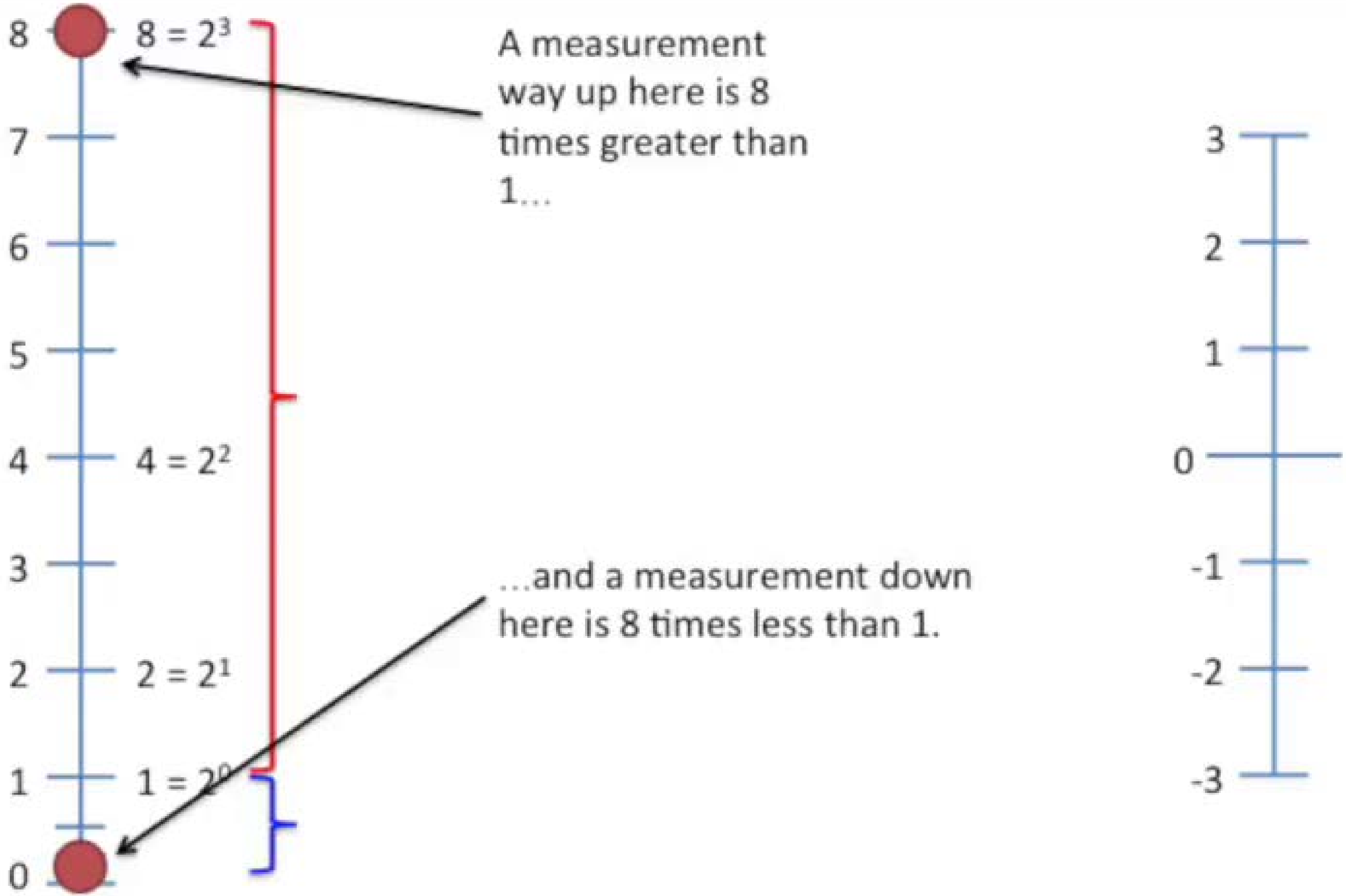


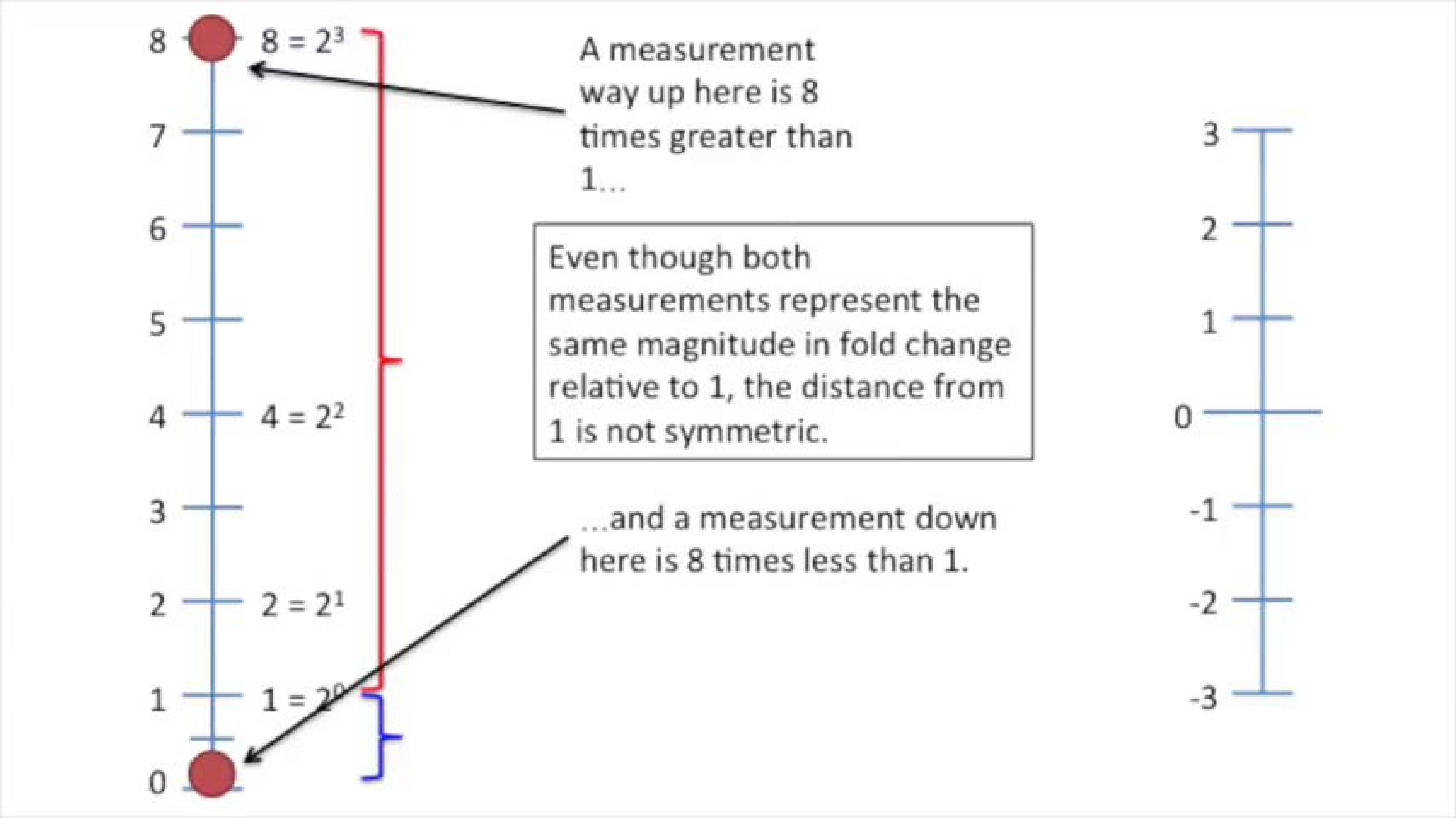

8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
Notice how much space is between 1 and 8…
3
2
1
0
-1
-2
-3

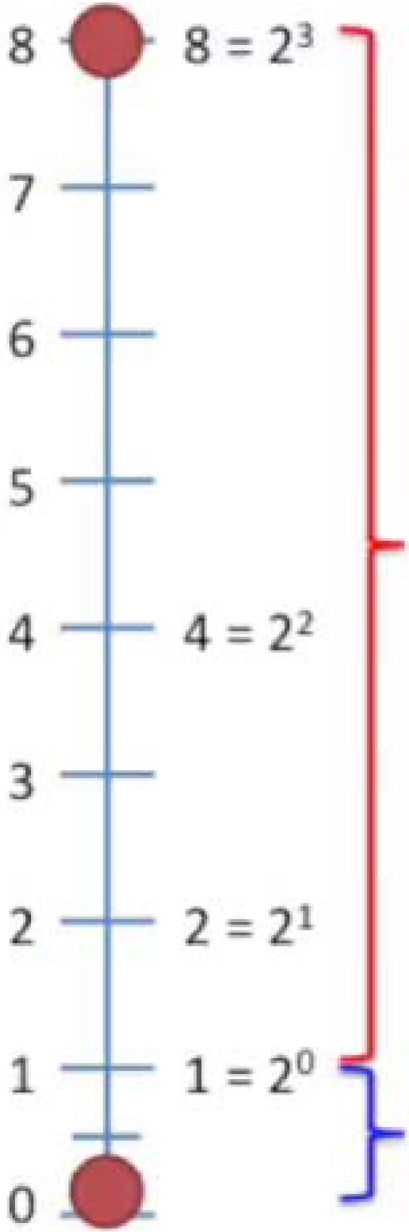


8
7
6
5
4
3
2
1
0
8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
Notice how much space is between 1 and 8…
…and how little space is between 1 and 1/8.
3
2
1
0
-1
-2
-3

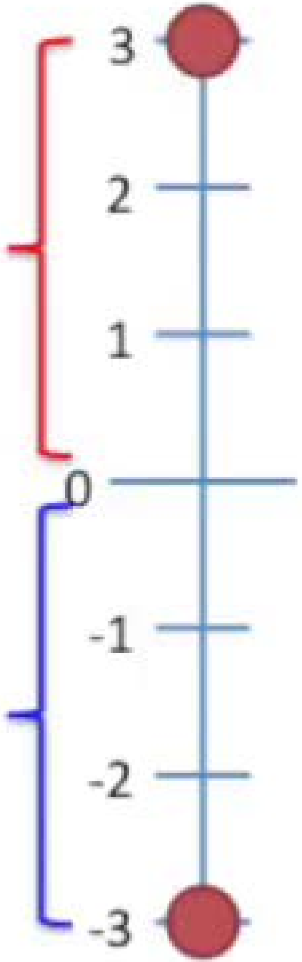

8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
A measurement way up here is 8 times greater than 1…
8
7
6
5
4
3
2
1
0
3
2
1
0
-1
-2
-3

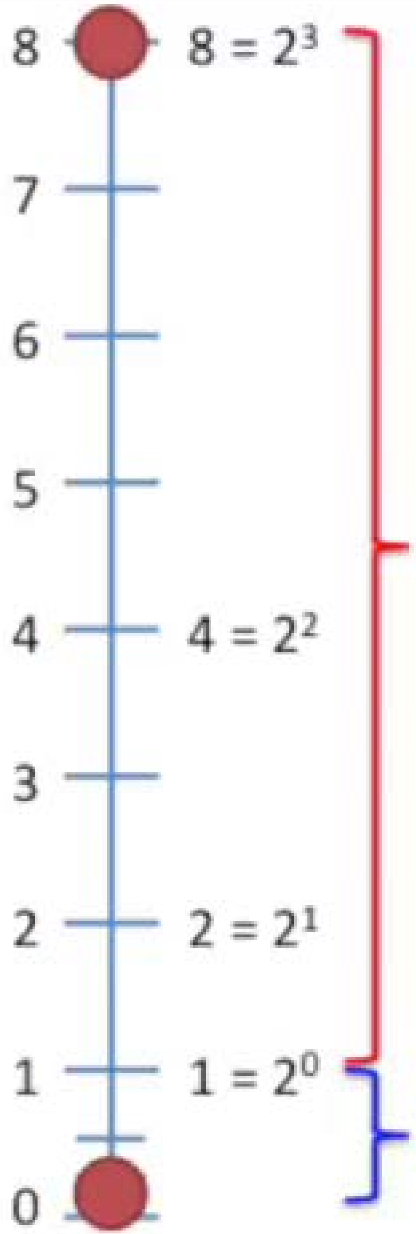

8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
A measurement way up here is 8 times greater than 1…
…and a measurement down here is 8 times less than 1.
3
2
1
0
-1
-2
-3


8
8 = 2^3
A measurement way up here is 8 times greater than 1…
7
6
Even though both measurements represent the same magnitude in fold change relative to 1, the distance from 1 is not symmetric.
5
4
4 = 2^2
3
…and a measurement down here is 8 times less than 1.
2
2 = 2^1
1
1 = 2^0
0
3
2
1
0
-1
-2
-3



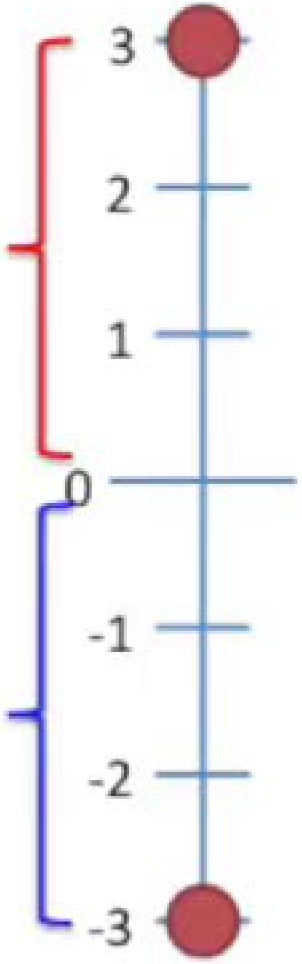
In contrast, the magnitude is equidistant on the log axis.

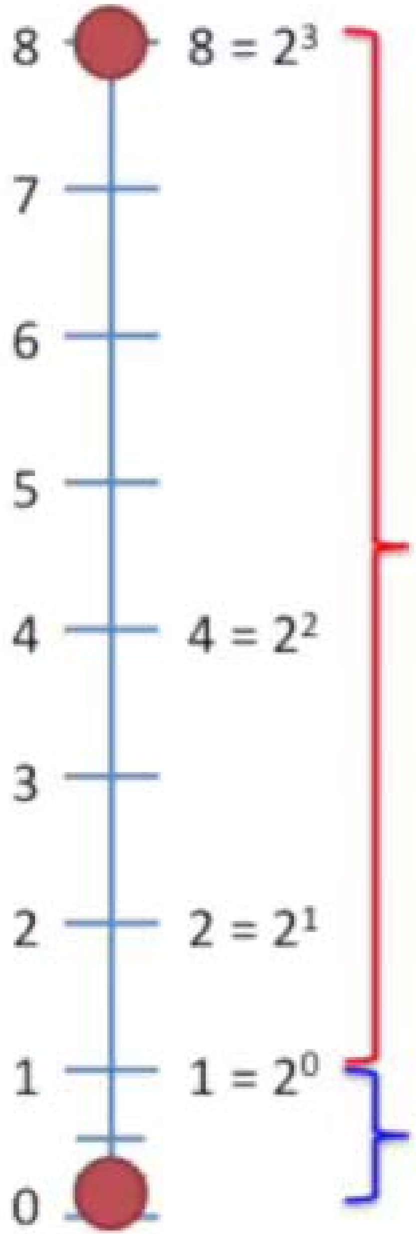




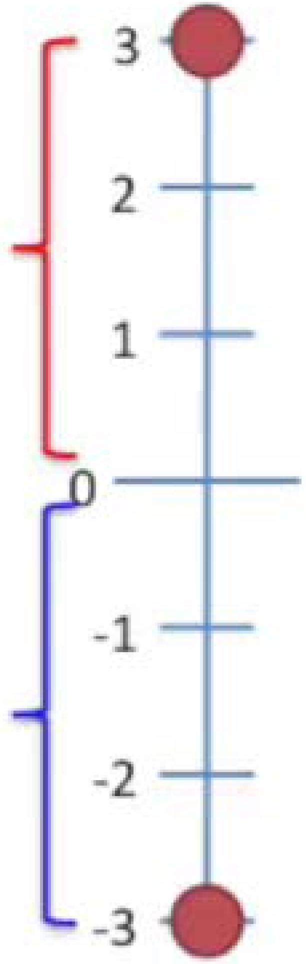
In contrast, the magnitude is equidistant on the log axis.

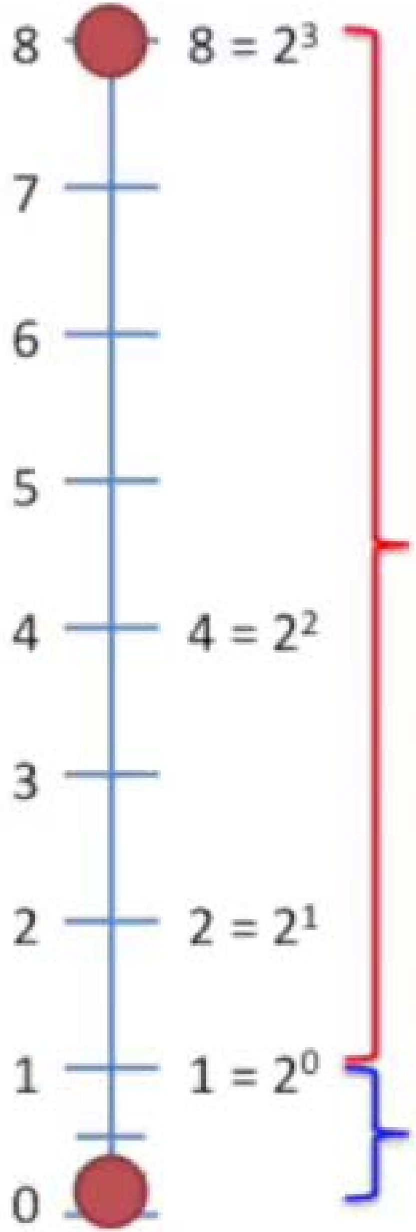
This is why fold changes should always be plotted on log axes.





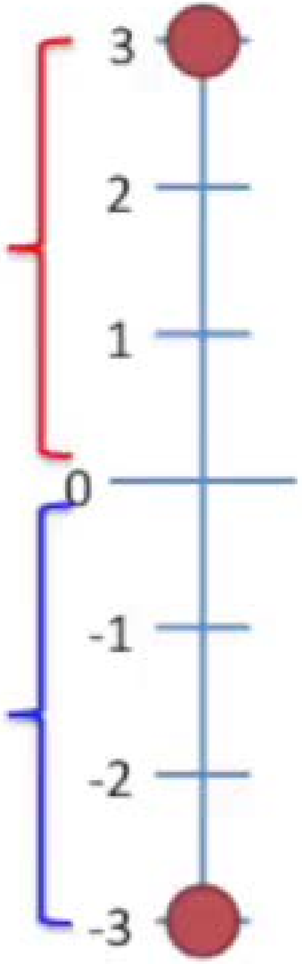
Take home message so far…

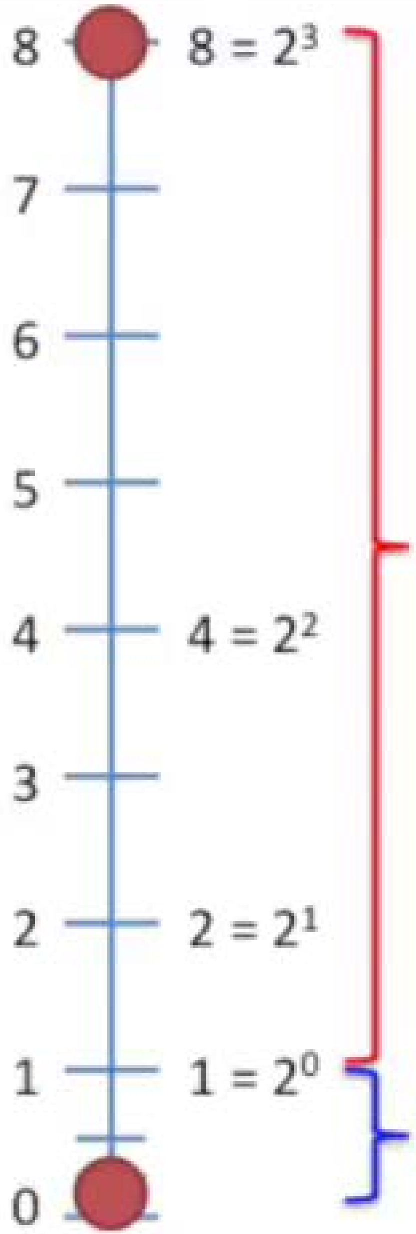




Take home message so far…

1) "logs" isolate exponents.

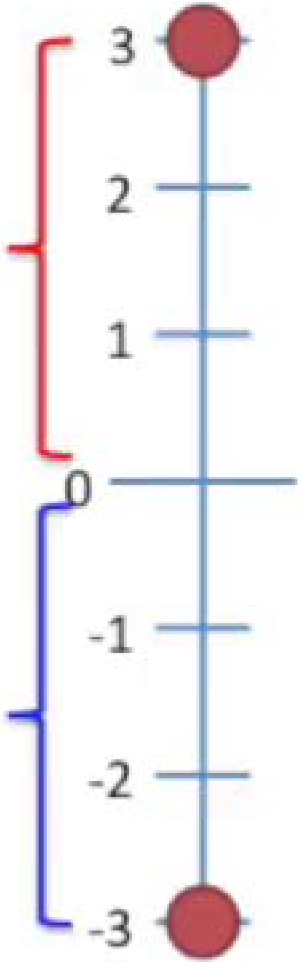


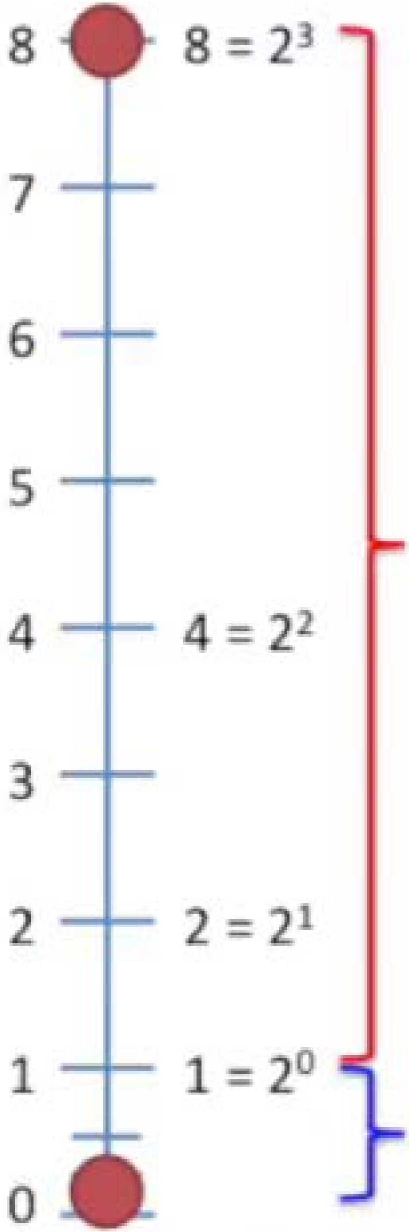


Take home message so far…

1) **“logs” isolate exponents.**

$$\log_2(8) = \log_2(2^3) = 3$$



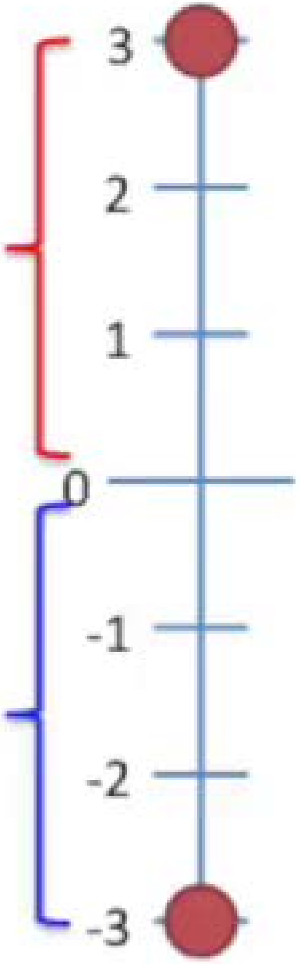


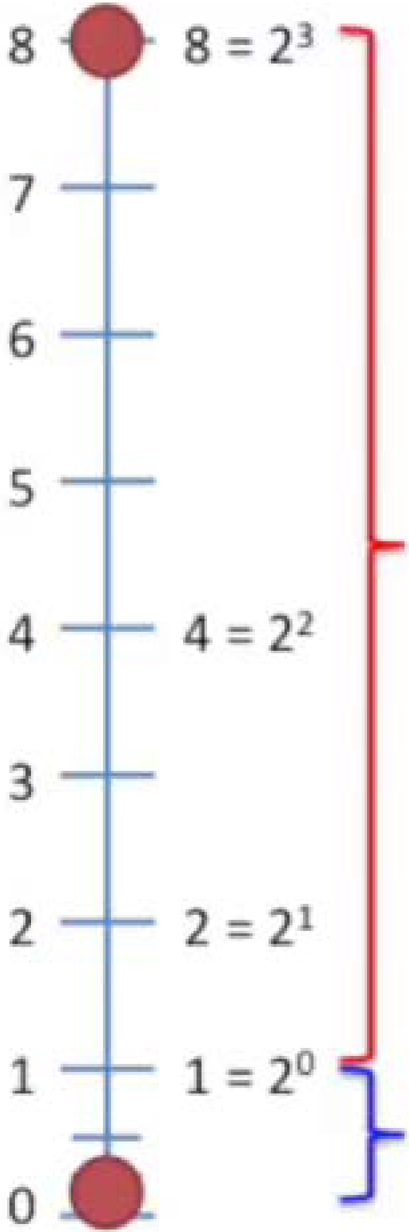
Take home message so far…

1) "logs" isolate exponents.

$$\log_2(8) = \log_2(2^3) = 3$$

2) Use a log scale/axis when talking about fold change. This puts positive and negative fold changes on a symmetric scale.





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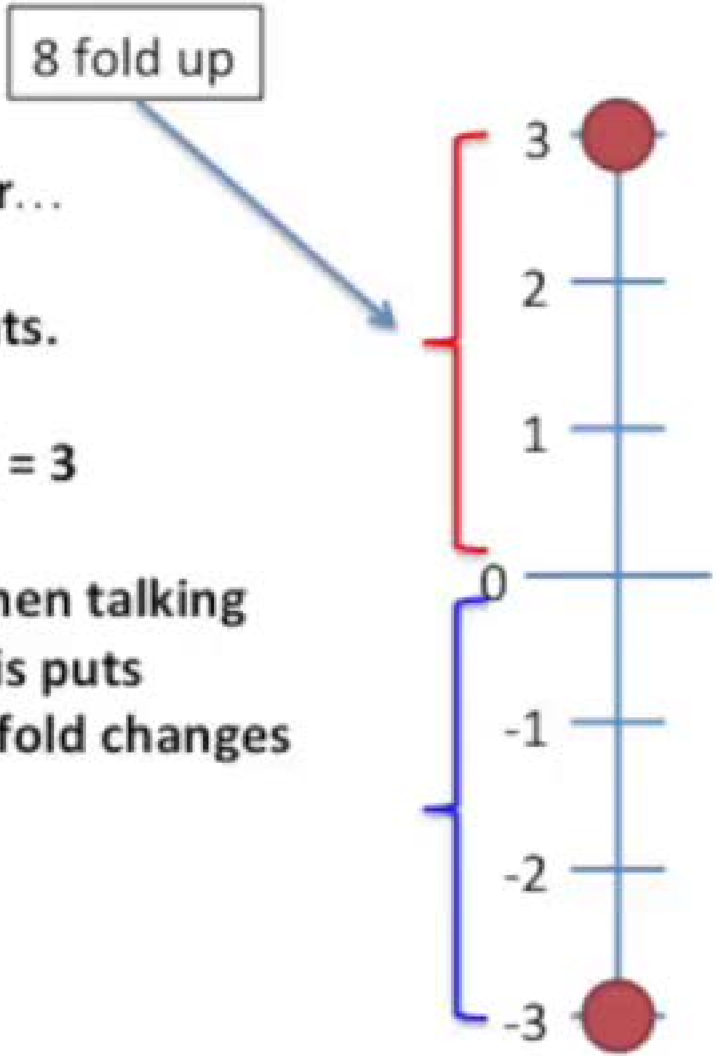

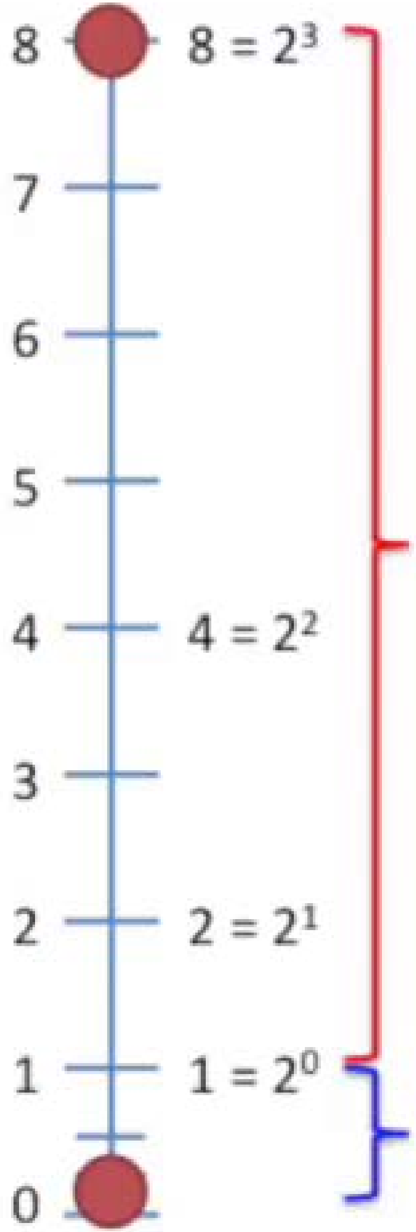

Take home message so far…

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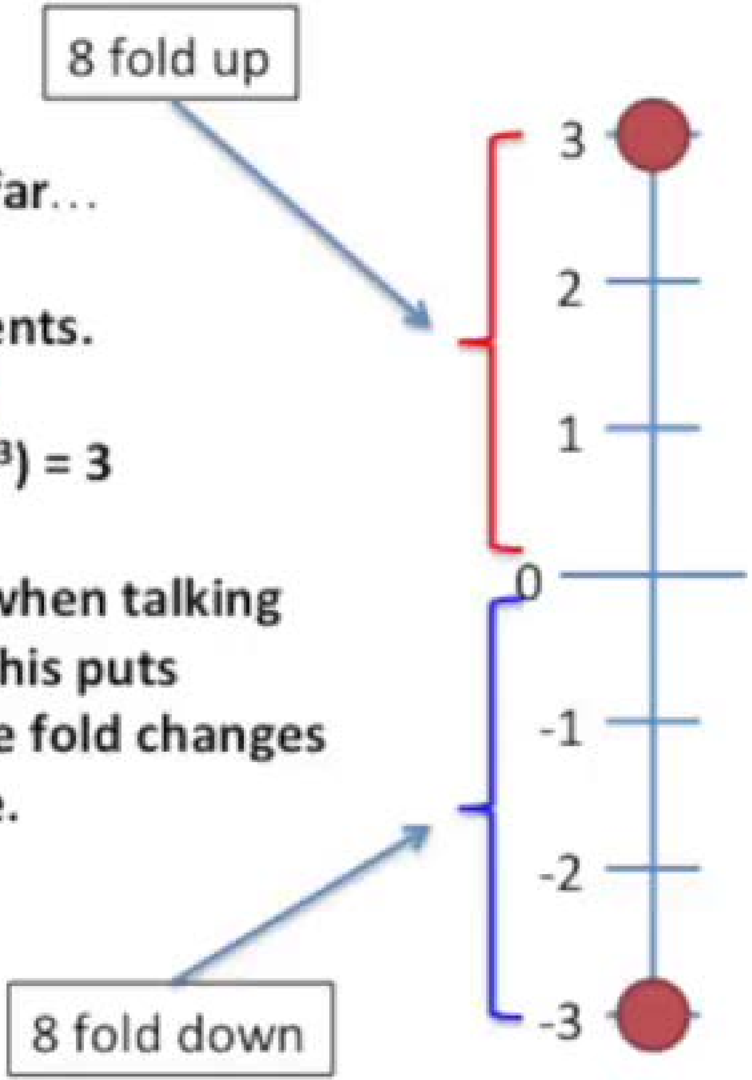
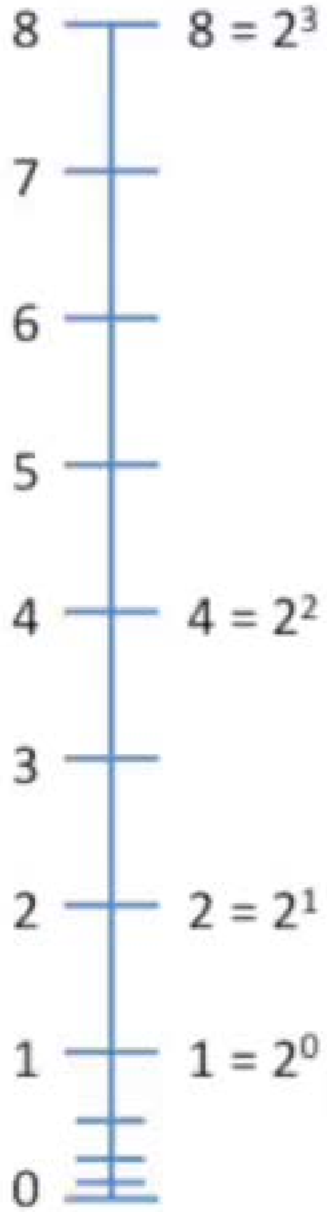
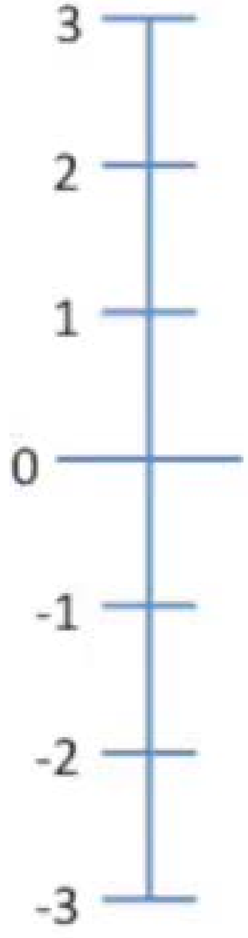
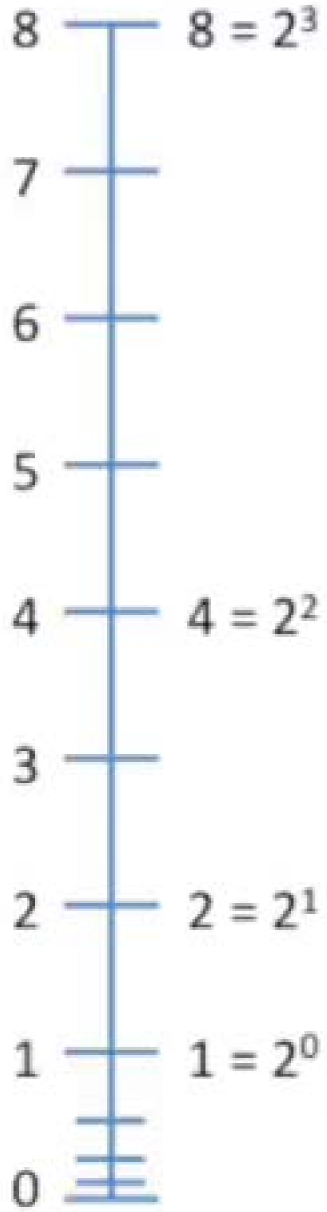


Here’s the weird thing...

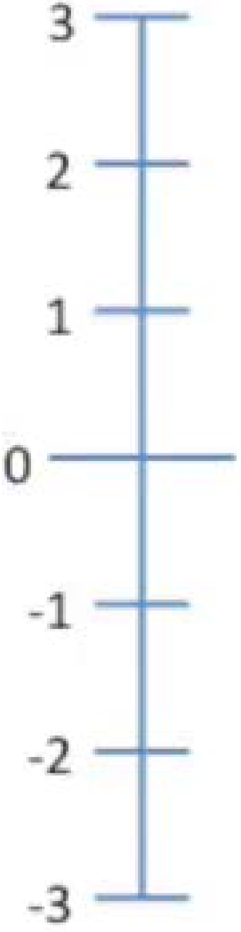


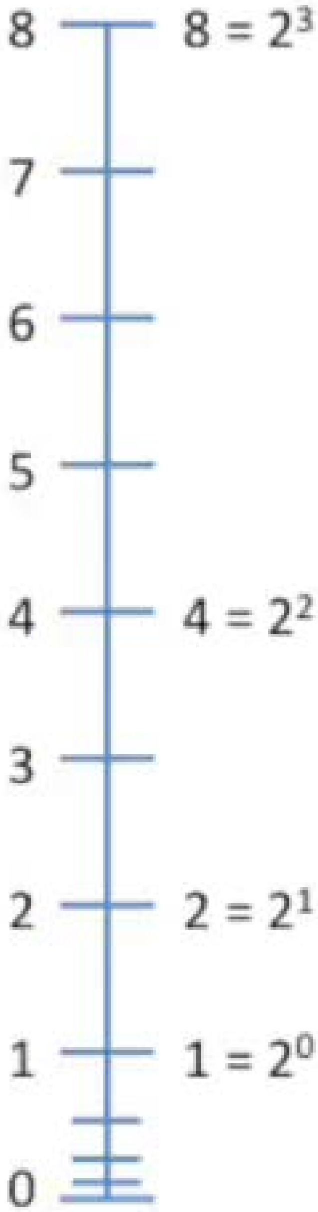




Here's the weird thing...

...there is no value for $\log_2(0)$.





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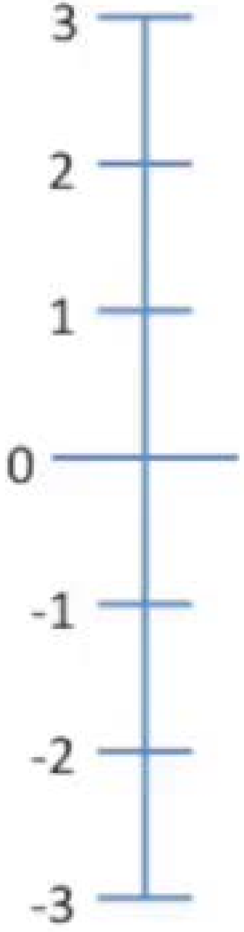
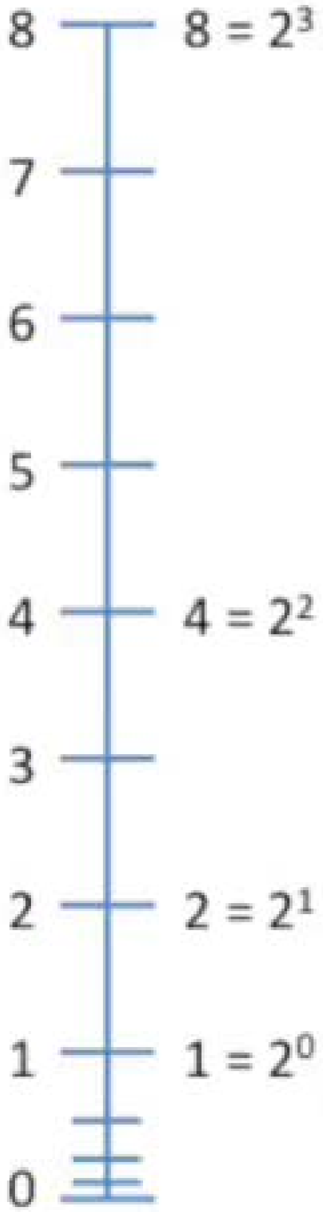
This is because there is no exponent, ***x***, for 2 such that...

$$2^x = 0$$





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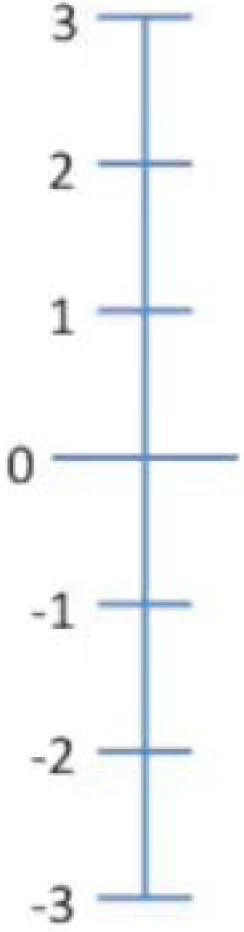
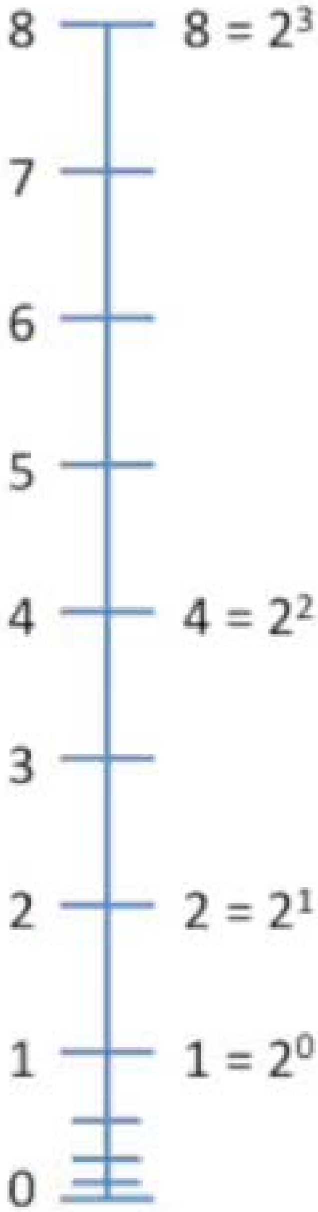
If we set ***x*** to -1000, we just get $\dfrac{1}{2^{1000}}$





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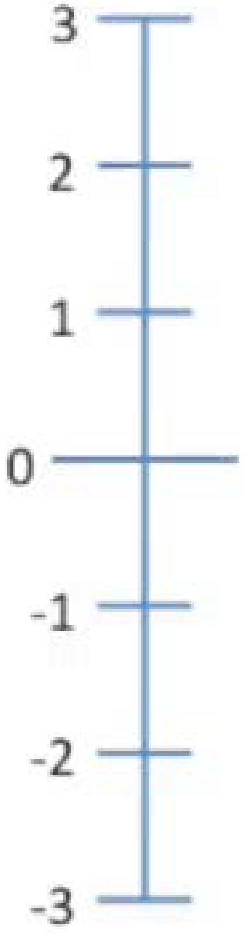
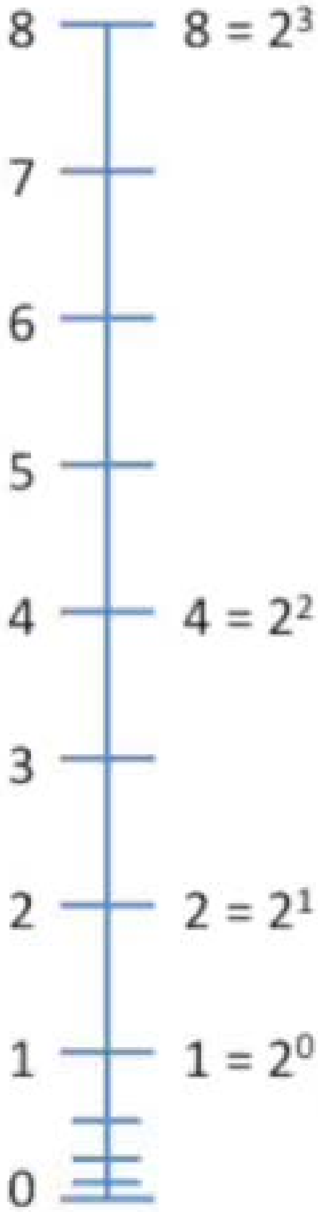
This is a tiny number, but it's still greater than 0.





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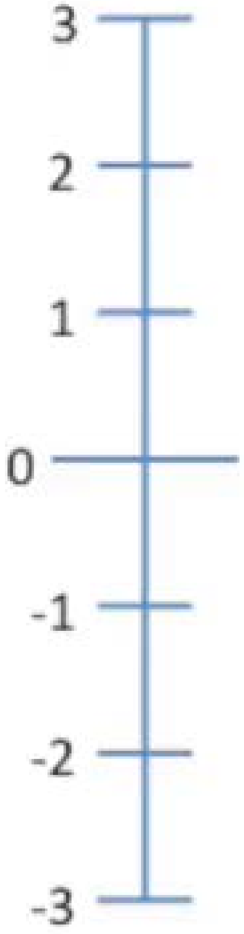
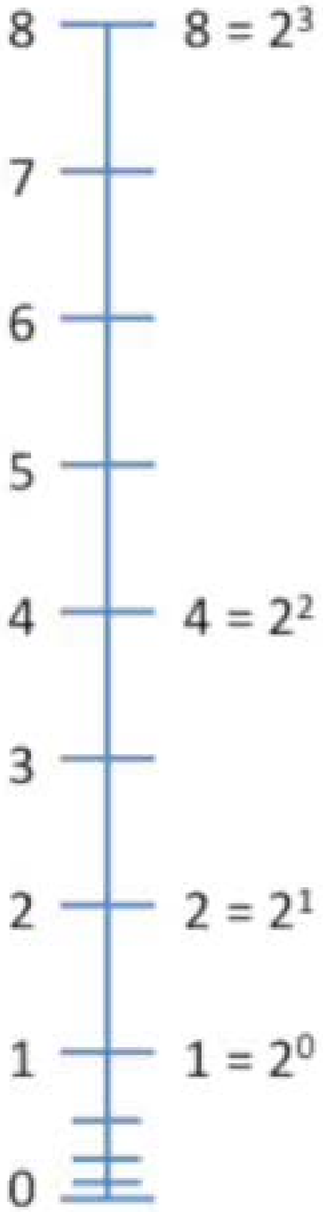
If we set ***x*** to -1,000,000, we just get $\frac{1}{2^{1,000,000}}$





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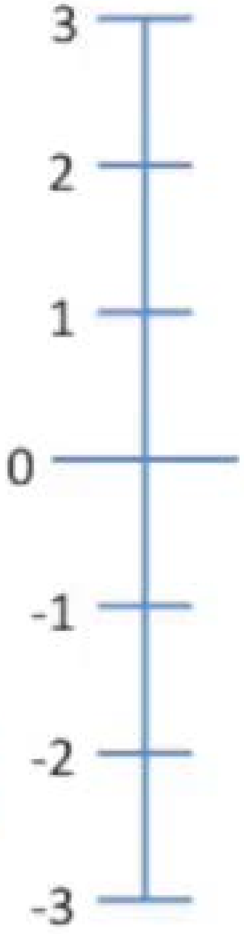
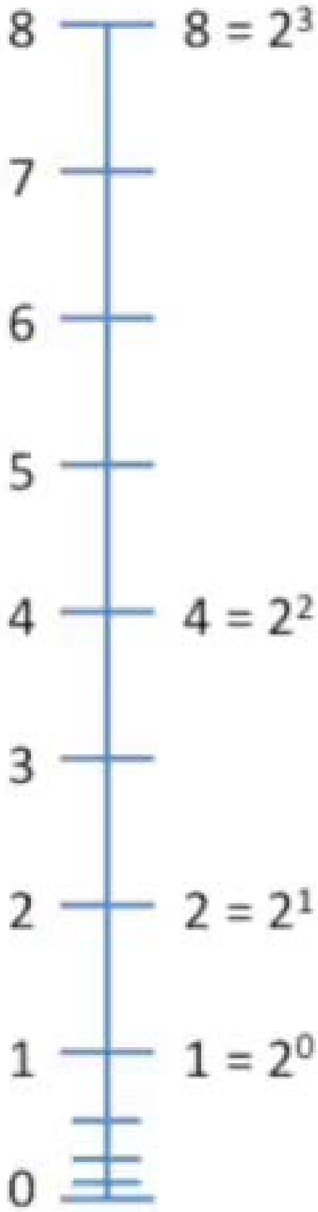
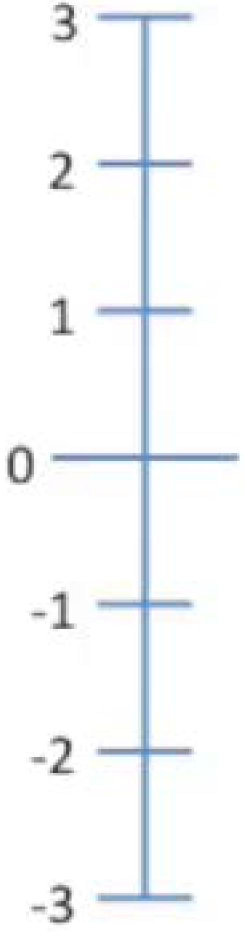
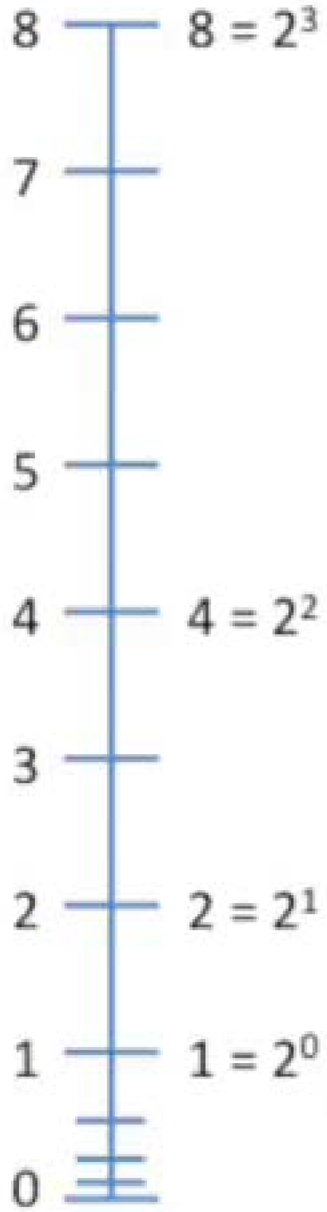
This is a super tiny number, but it's still greater than 0!!!





R, a programming language that a lot of people use to do stats, defines $\log_2(0)$ = -Infinity





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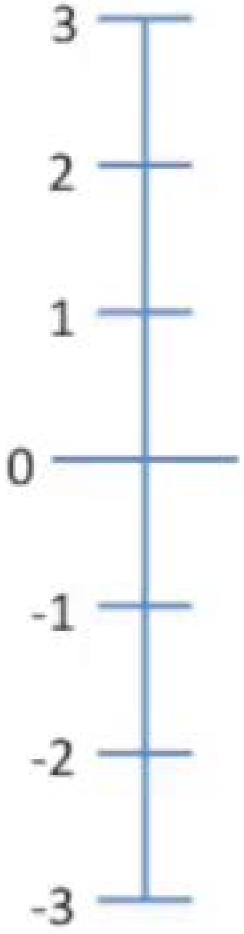
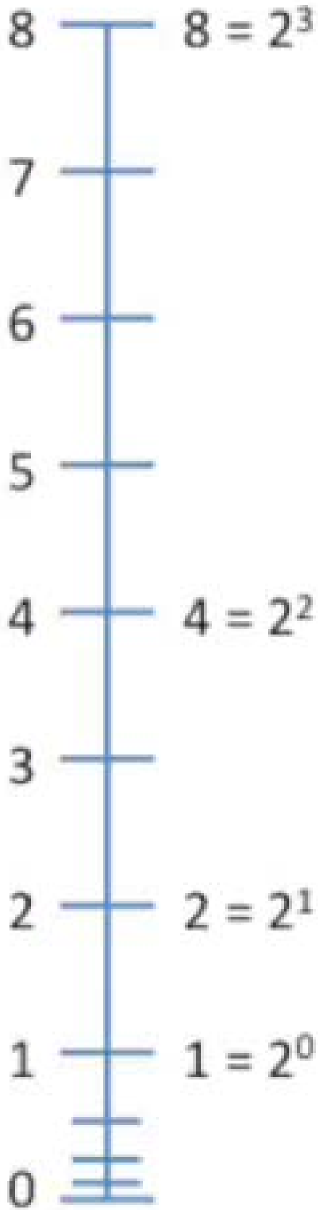
Intuitively, this makes sense since…





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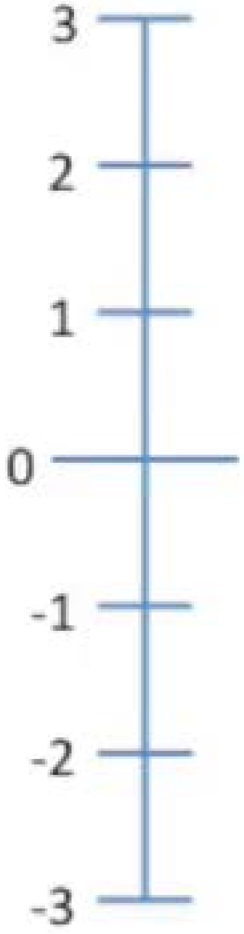
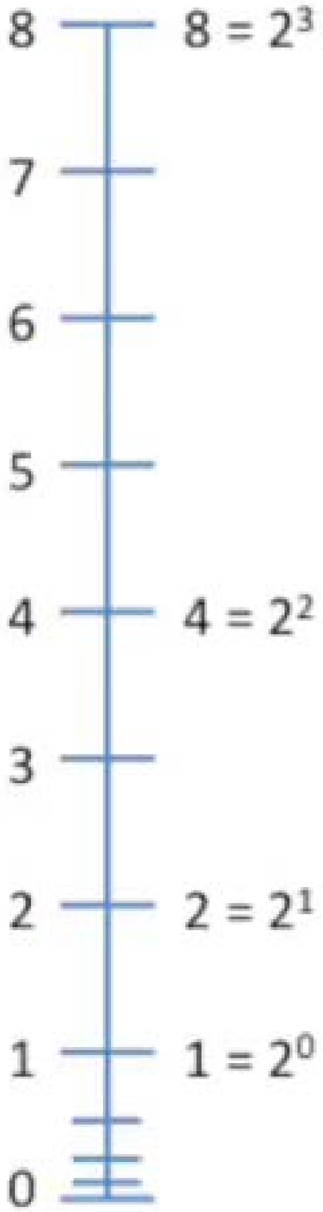
$\frac{1}{2^{\text{Infinity}}}$ = the smallest number you can imagine

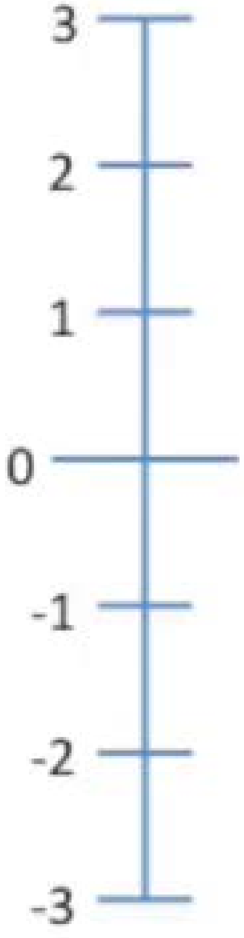


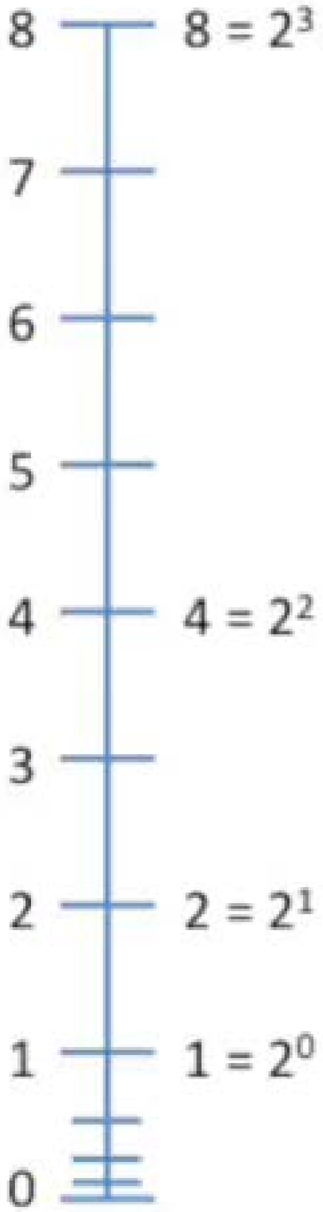


R, a programming language that a lot of people use to do stats, defines $\log_2(0)$ = -Infinity

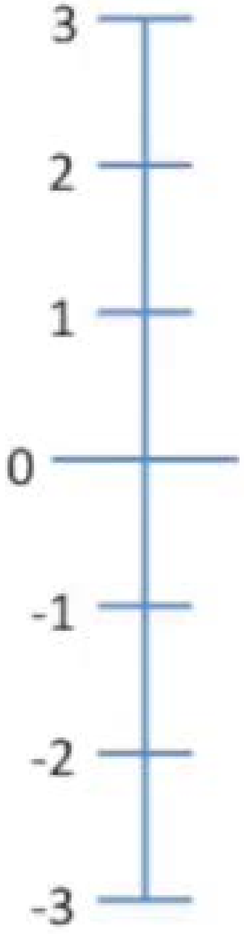
Intuitively, this makes sense since…

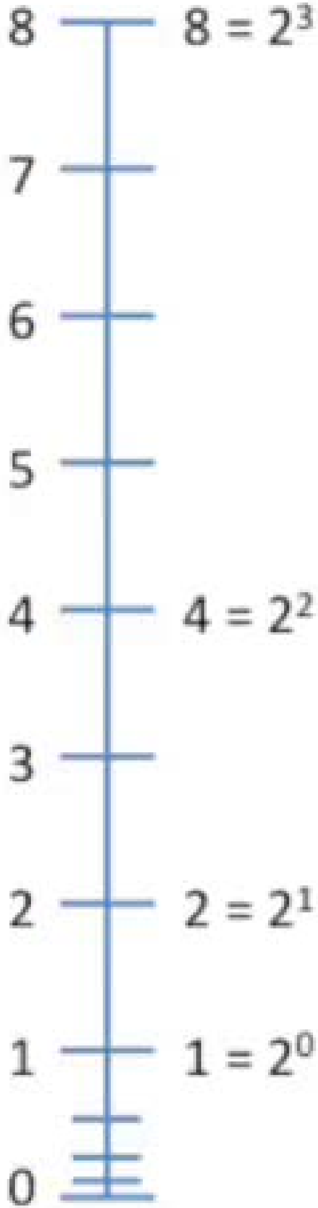
$$\frac{1}{2^{\text{Infinity}}} = \text{the smallest number you can imagine} = 0$$





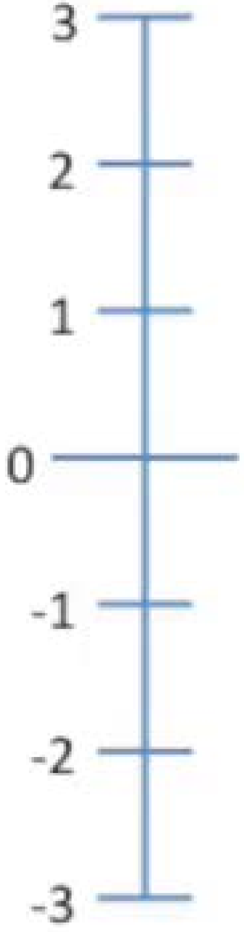
Here's an example of when using $\log_2$ is super cool

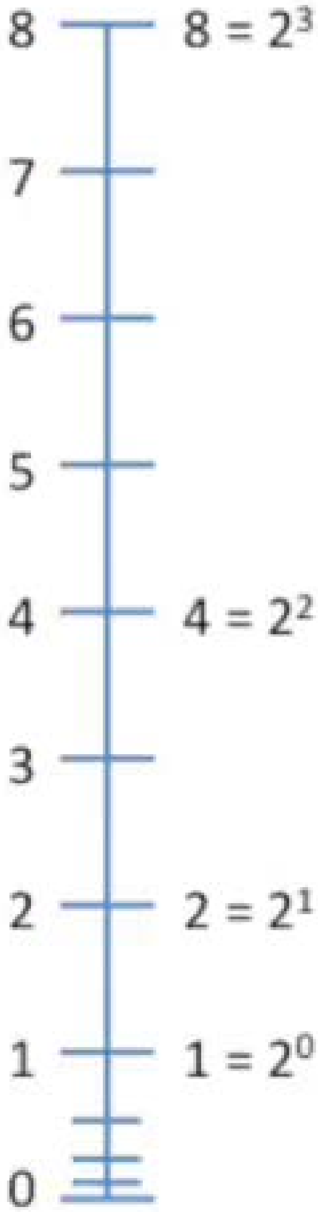




Here's an example of when using $\log_2$ is super cool

qPCR or Real-Time PCR.

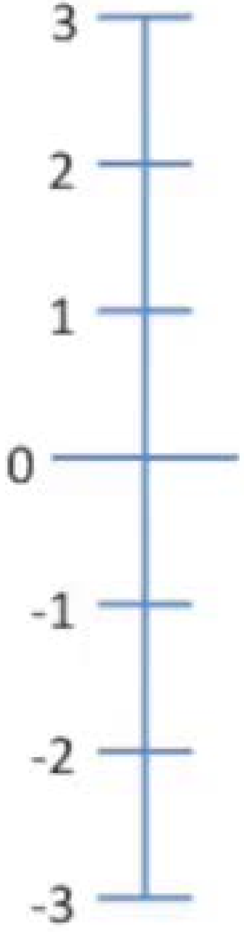


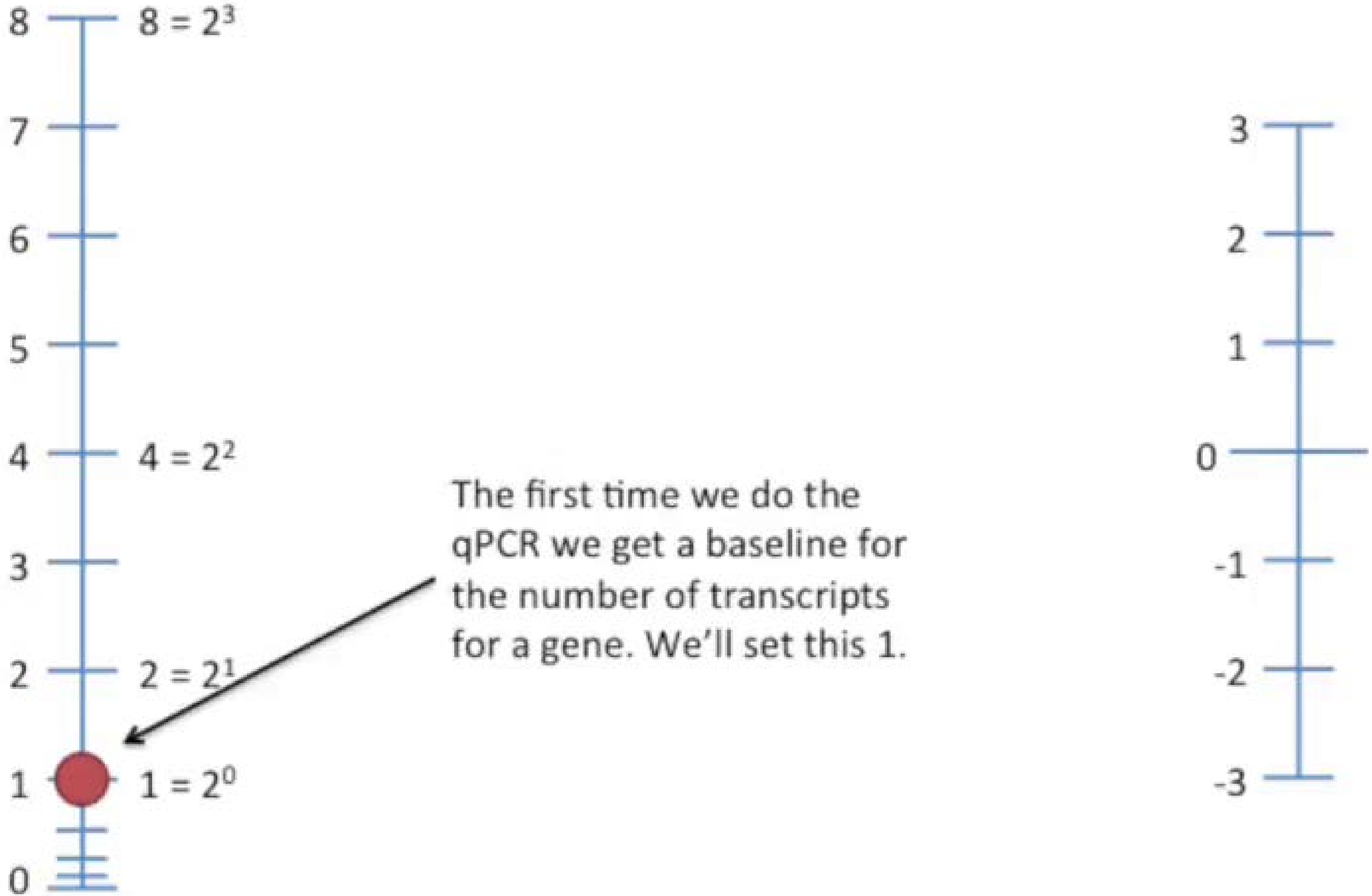


Here's an example of when using $\log_2$ is super cool

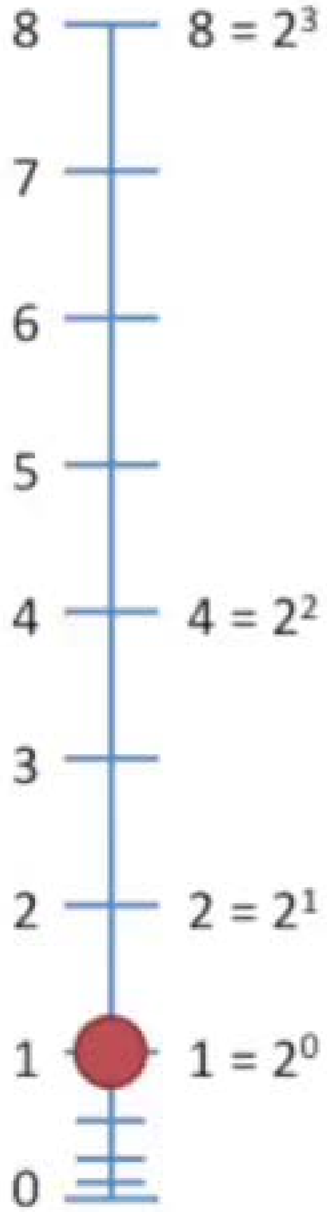
qPCR or Real-Time PCR.

Log_2 makes sense with this data because each time the machine goes through a cycle, the number of PCR products doubles.



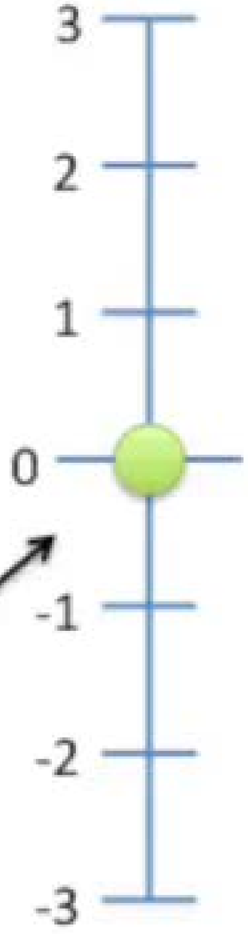


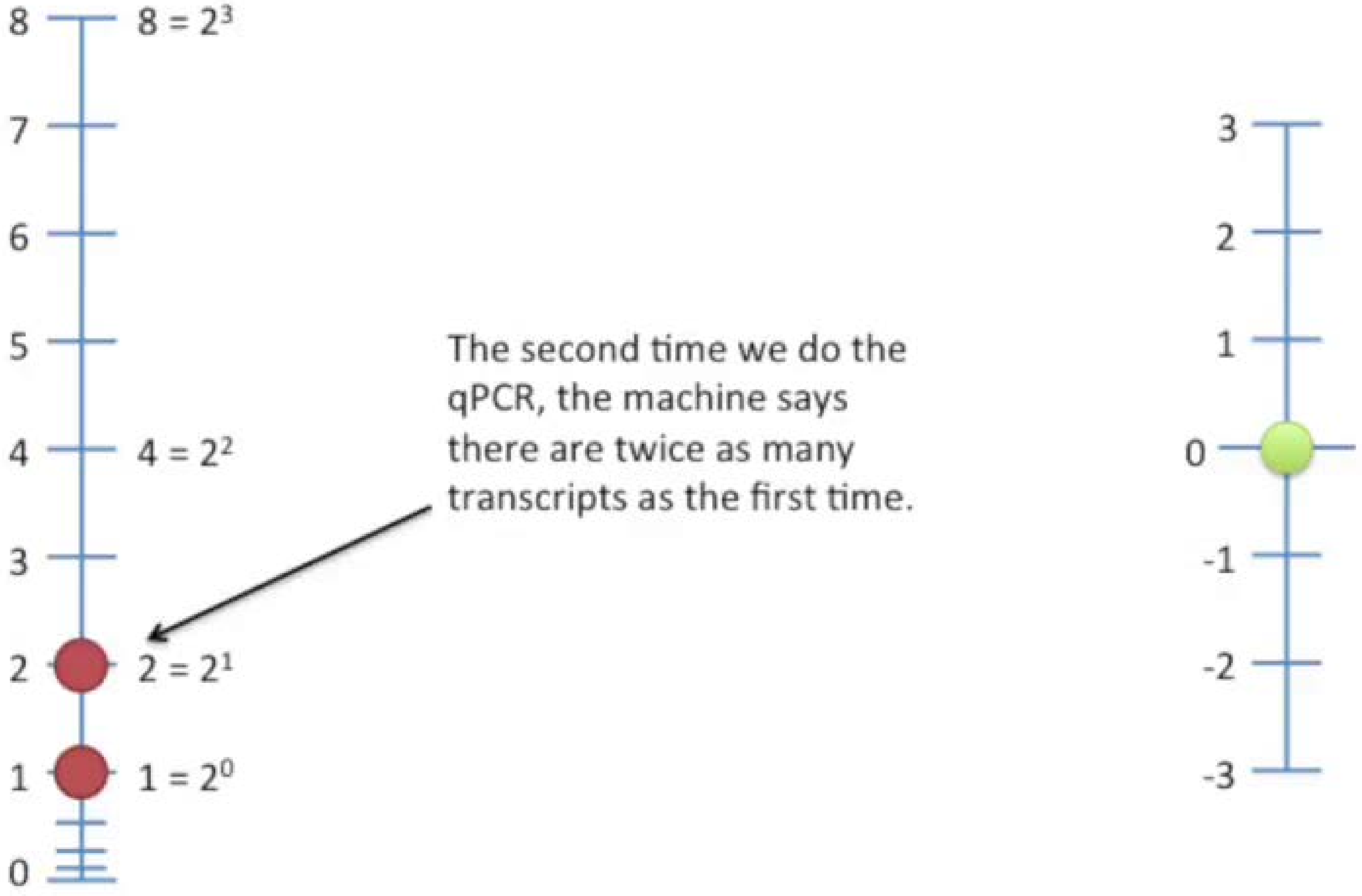
8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
The first time we do the qPCR we get a baseline for the number of transcripts for a gene. We'll set this 1.
3
2
1
0
-1
-2
-3



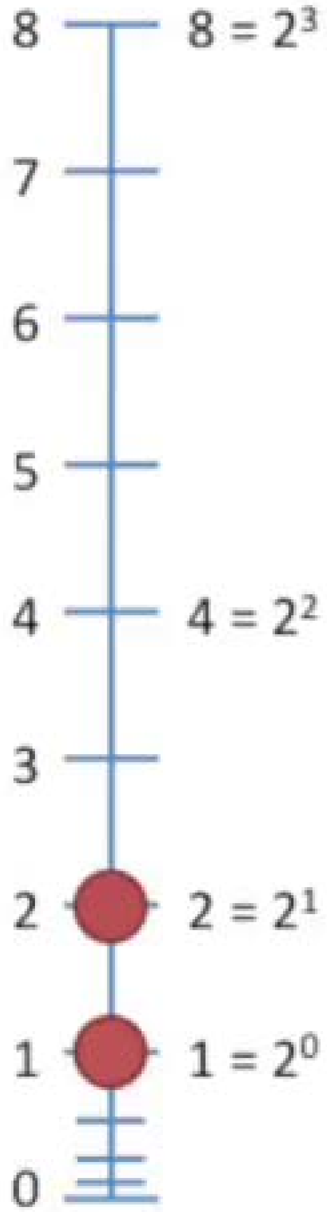
The first time we do the qPCR we get a baseline for the number of transcripts for a gene. We'll set this 1.

In $\log_2$ land, 0 becomes our baseline.



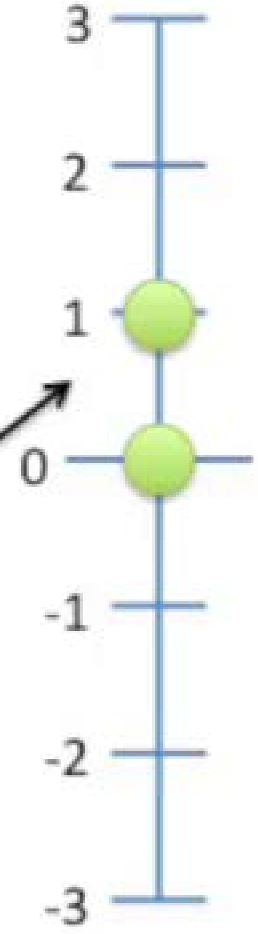


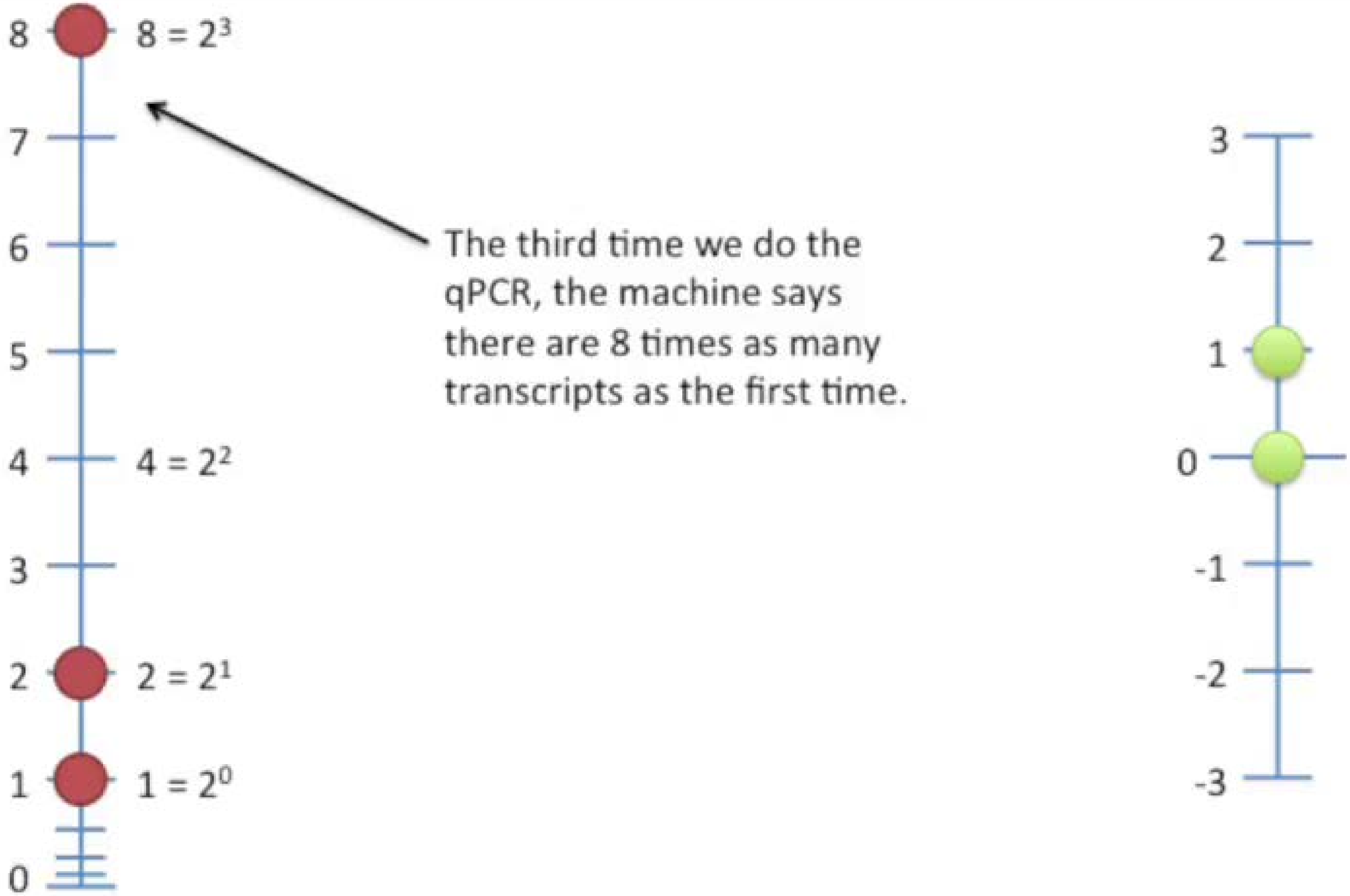
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
8
7
6
5
4
3
2
1
0
The second time we do the qPCR, the machine says there are twice as many transcripts as the first time.
3
2
1
0
-1
-2
-3

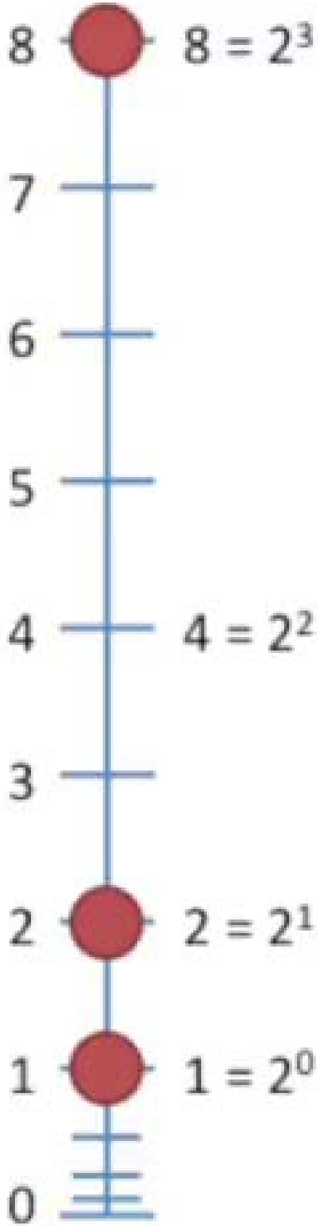


The second time we do the qPCR, the machine says there are twice as many transcripts as the first time.

In other words, the difference between the first and second runs was 1 cycle in the machine.

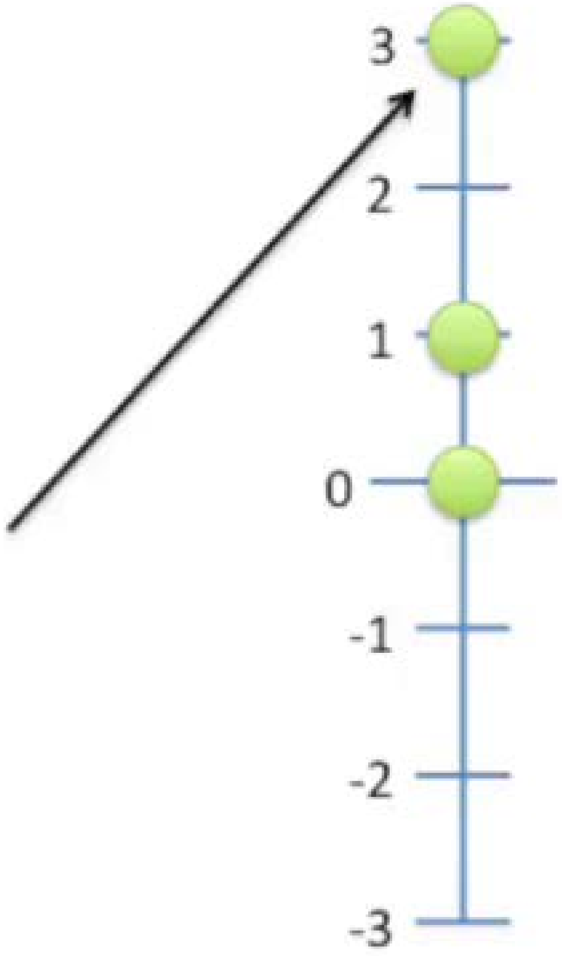


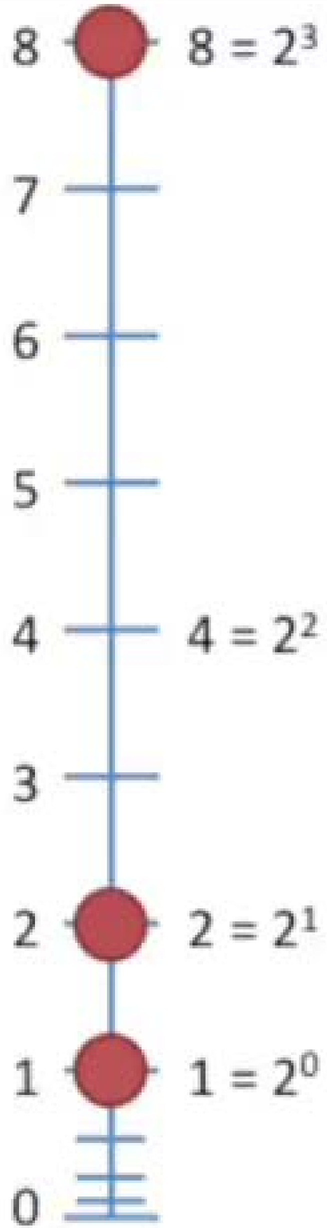

8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
8
7
6
5
4
3
2
1
0
The third time we do the qPCR, the machine says there are 8 times as many transcripts as the first time.
3
2
1
0
-1
-2
-3



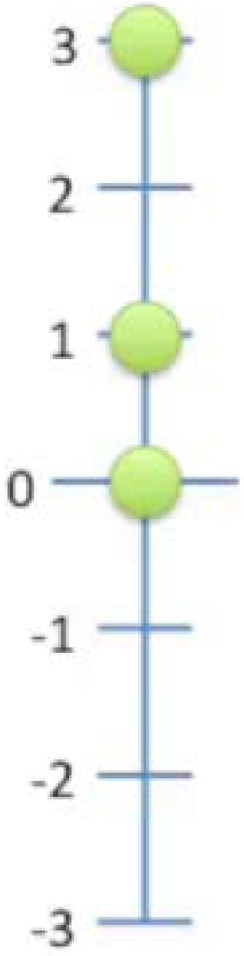
The third time we do the qPCR, the machine says there are 8 times as many transcripts as the first time.

In other words, there was a difference of 3 cycles between the first run and this last run.

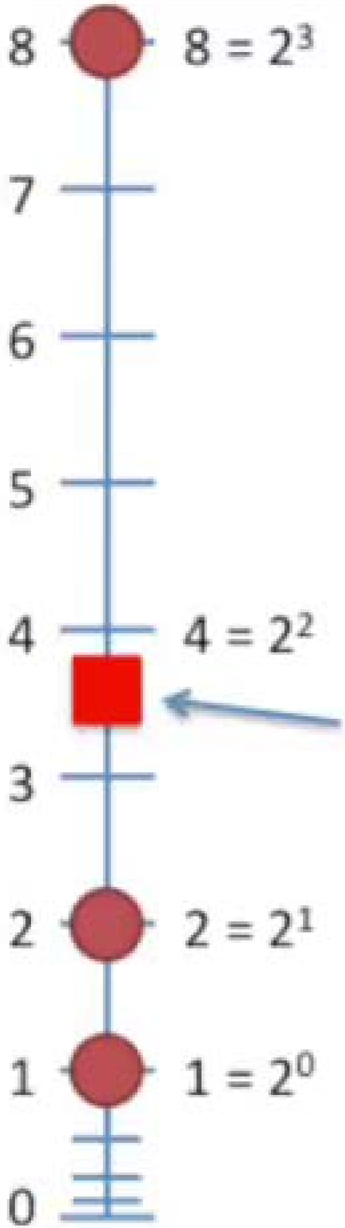




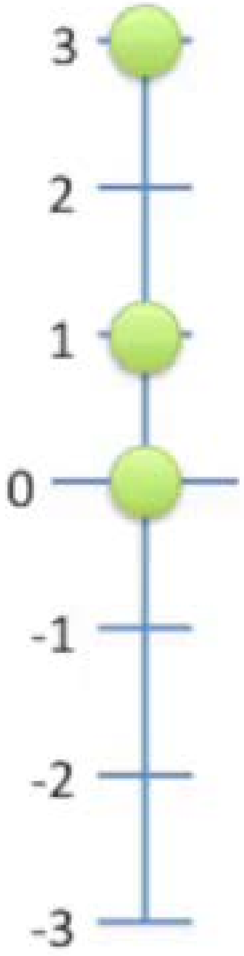
Now, here's a question:
What's the average of the
three runs?

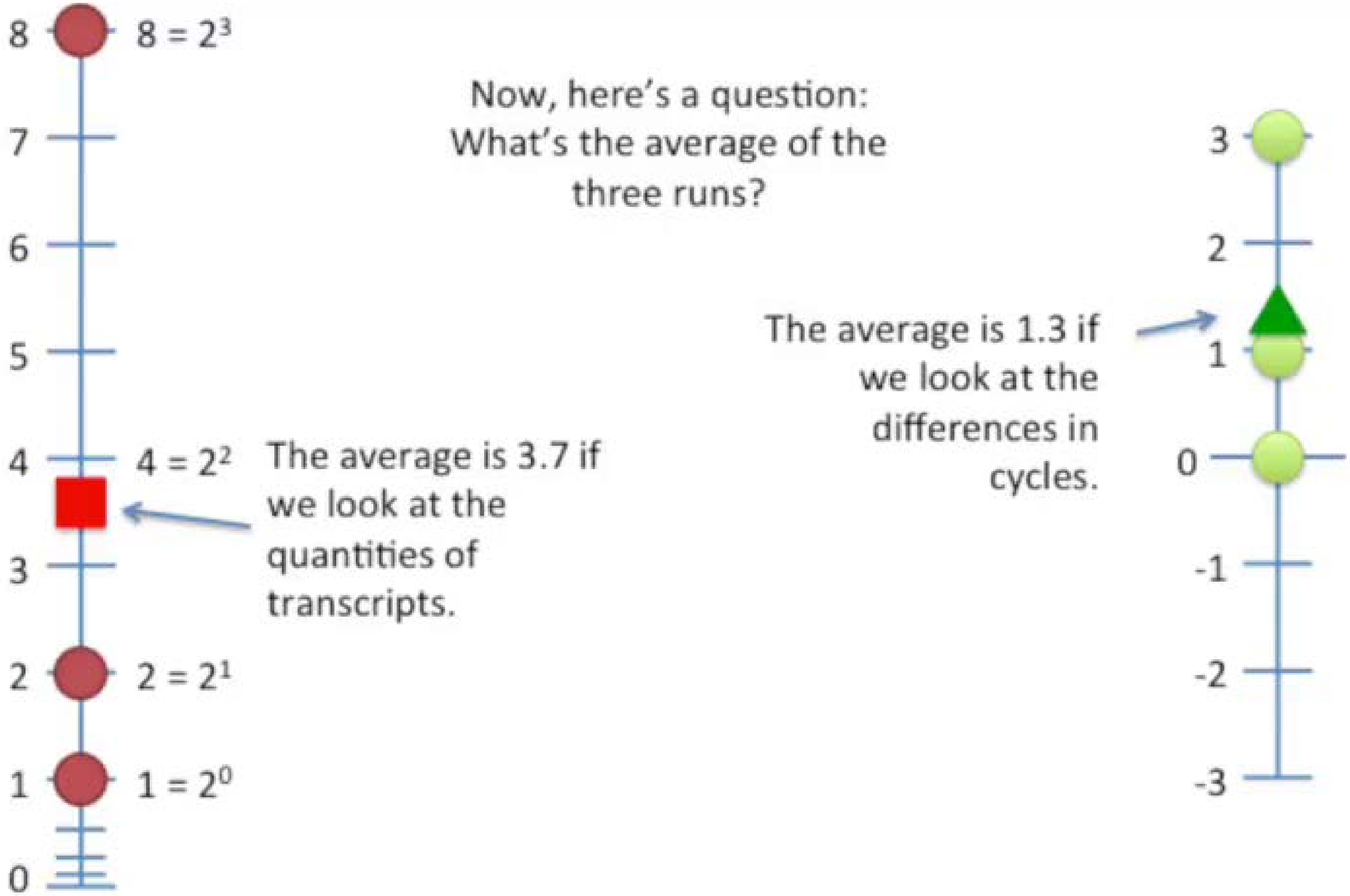


Now, here’s a question:
What’s the average of the three runs?

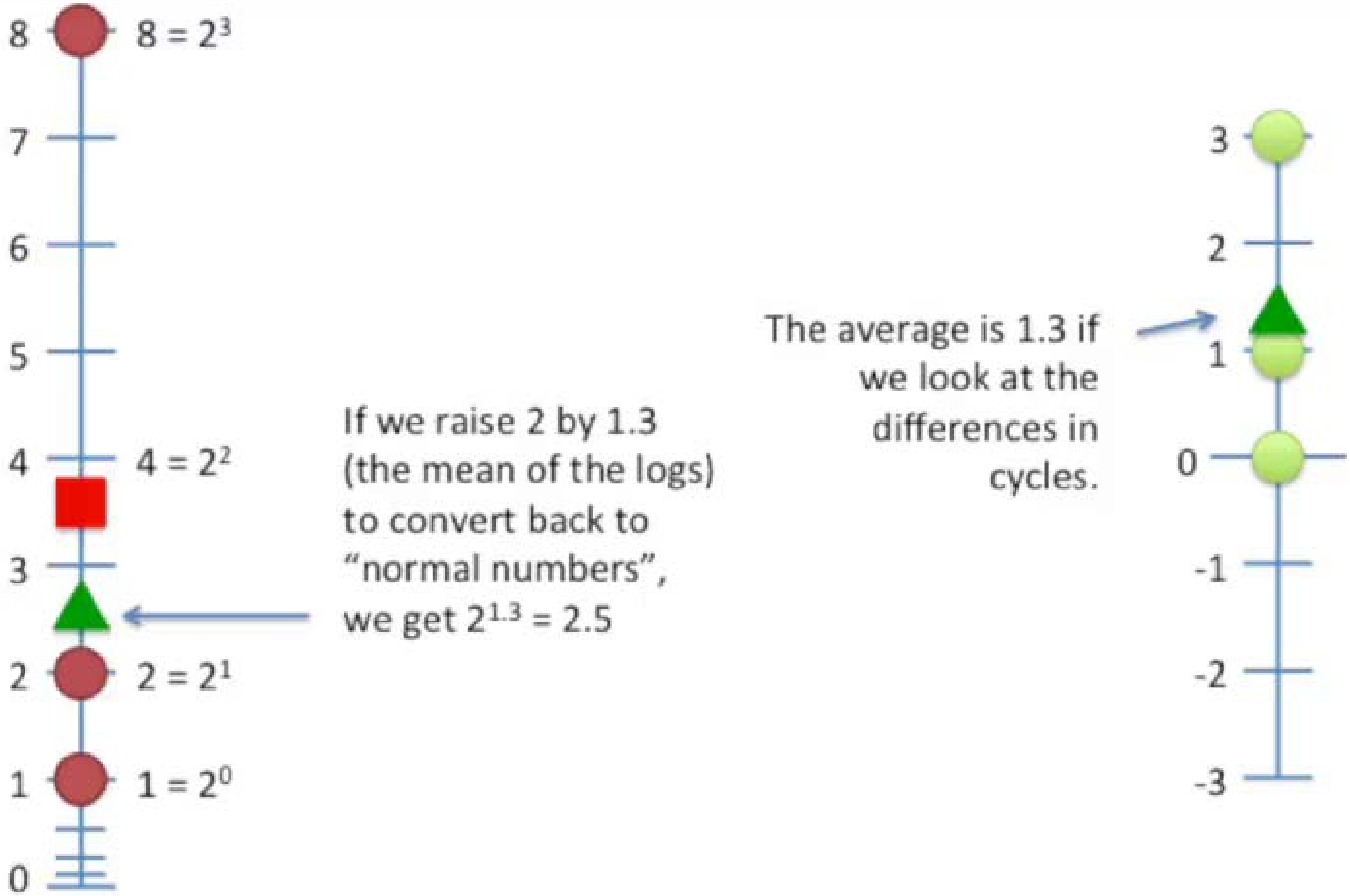


The average is 3.7 if we look at the quantities of transcripts.

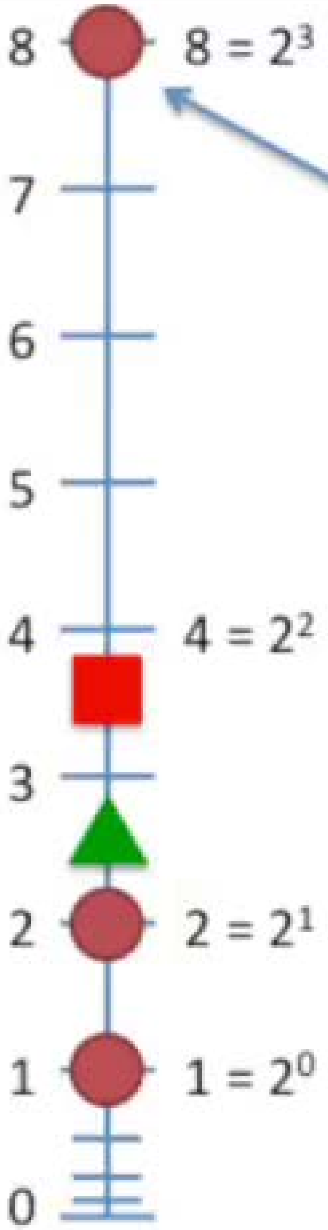




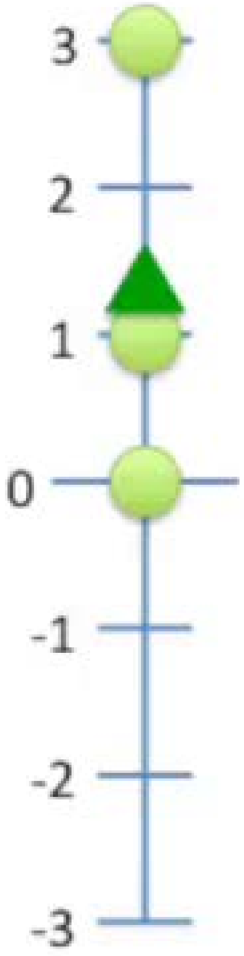
Now, here's a question:
What's the average of the three runs?
8
7
6
5
4
3
2
1
0
8 = 2³
4 = 2²
2 = 2¹
1 = 2⁰
The average is 3.7 if we look at the quantities of transcripts.
3
2
1
0
-1
-2
-3
The average is 1.3 if we look at the differences in cycles.

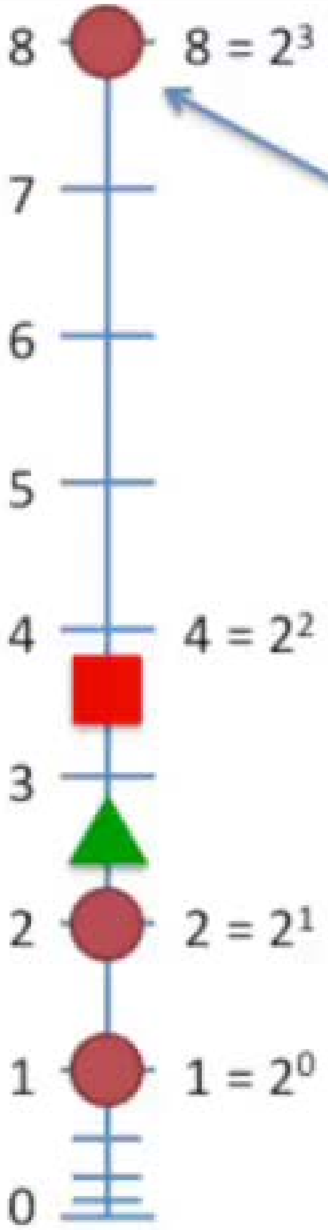


8
7
6
5
4
3
2
1
0
8 = 2^3
4 = 2^2
2 = 2^1
1 = 2^0
If we raise 2 by 1.3 (the mean of the logs) to convert back to "normal numbers", we get 2^1.3 = 2.5
The average is 1.3 if we look at the differences in cycles.
3
2
1
0
-1
-2
-3



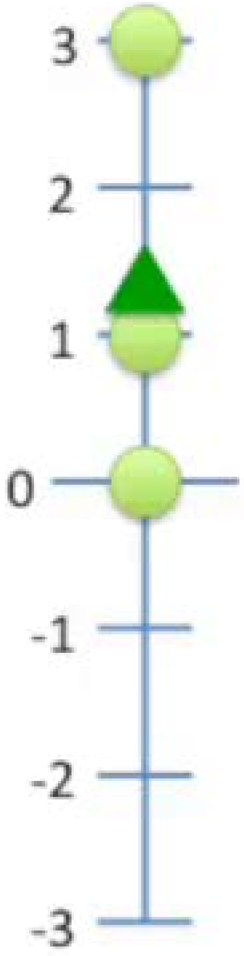
The mean calculated with the logs is less than the one calculated with "normal numbers" because it wasn't as swayed by the third run (when we got 8 times as many transcripts).

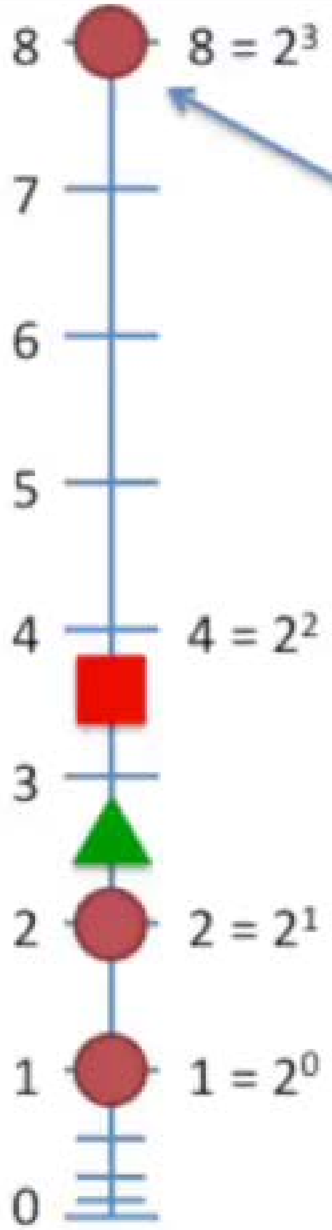




The mean calculated with the logs is less than the one calculated with “normal numbers” because it wasn’t as swayed by the third run (when we got 8 times as many transcripts).

That 3rd run is a bit of an outlier, and the mean of logs, called the “Geometric Mean”, is more robust to the effects of outliers than the mean of “normal numbers”.

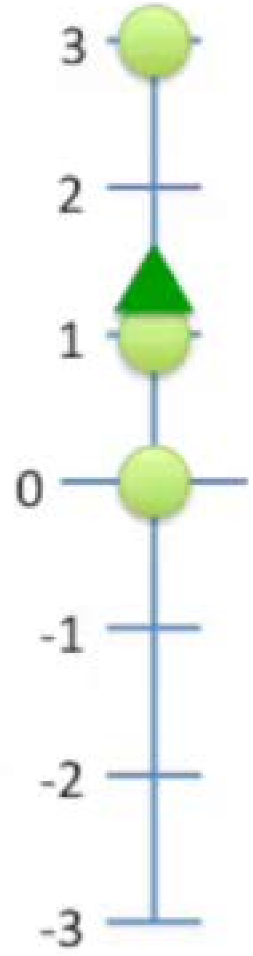




The mean calculated with the logs is less than the one calculated with "normal numbers" because it wasn't as swayed by the third run (when we got 8 times as many transcripts).

That 3rd run is a bit of an outlier, and the mean of logs, called the "Geometric Mean", is more robust to the effects of outliers than the mean of "normal numbers".

The Geometric Mean is useful for qPCR since every cycle doubles the amount of transcripts – This makes it easy to get outliers!



Now that we know all about log scales, let's (really briefly) talk about arithmetic with logs.

Let’s start with
normal
multiplication:

$$2 \times 4 = 8$$

Let's start with normal multiplication:

$$2 \times 4 = 8$$

$$\downarrow$$

$$2^1 \times 2^2 = 2^3$$

We already saw how 2, 4 and 8 can be re-written as powers of 2.

Let's start with normal multiplication:

$$2 \times 4 = 8$$

Multiplying numbers…

$$2^1 \times 2^2 = 2^3$$

$$2^{1+2} = 2^3$$

Let's start with normal multiplication:

$$2 \times 4 = 8$$

Multiplying numbers…

$$2^1 \times 2^2 = 2^3$$

…is the same as adding their exponents (after converting the numbers to powers of 2).

$$2^{1+2} = 2^3$$

Let's start with normal multiplication:

$2 \times 4 = 8$

$\downarrow$

$2^1 \times 2^2 = 2^3$

$\downarrow$

$2^{1+2} = 2^3$

This works, even when the numbers aren't power of 2 friendly…

$3 \times 5 = 15$

Let's start with normal multiplication:

$$2 \times 4 = 8$$

$$\downarrow$$

$$2^1 \times 2^2 = 2^3$$

$$\downarrow$$

$$2^{1+2} = 2^3$$

This works, even when the numbers aren't power of 2 friendly…

$$3 \times 5 = 15$$

$$\downarrow$$

$$2^{1.6} \times 2^{2.3} = 2^{3.9}$$

Re-write the numbers as powers of 2.

Let's start with normal multiplication:

$$2 \times 4 = 8$$

$$2^1 \times 2^2 = 2^3$$

$$2^{1+2} = 2^3$$

This works, even when the numbers aren't power of 2 friendly…

$$3 \times 5 = 15$$

$$2^{1.6} \times 2^{2.3} = 2^{3.9}$$

$$2^{1.6+2.3} = 2^{3.9}$$

Add the exponents together.

Let's start with normal multiplication:

$$2 \times 4 = 8$$

$$\downarrow$$

$$2^1 \times 2^2 = 2^3$$

$$\downarrow$$

$$2^{1+2} = 2^3$$

This works, even when the numbers aren't power of 2 friendly…

$$3 \times 5 = 15$$

Once again, multiplying numbers…

$$\downarrow$$

$$2^{1.6} \times 2^{2.3} = 2^{3.9}$$

$$\downarrow$$

$$2^{1.6+2.3} = 2^{3.9}$$

Let's start with normal multiplication:

$$2 \times 4 = 8$$

$$2^1 \times 2^2 = 2^3$$

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$$2 \times 4 = 8$$

$$\downarrow$$

$$2^1 \times 2^2 = 2^3$$

$$\downarrow$$

$$2^{1+2} = 2^3$$

This works, even when the numbers aren't power of 2 friendly…

$$3 \times 5 = 15$$

$$\downarrow$$

$$2^{1.6} \times 2^{2.3} = 2^{3.9}$$

$$\downarrow$$

$$2^{1.6+2.3} = 2^{3.9}$$

Now let's talk about $\log_2(2x4)$ and $\log_2(3x5)$

Let's start with normal multiplication:

$2 \times 4 = 8$

$2^1 \times 2^2 = 2^3$

This works, even when the numbers aren't power of 2 friendly…

$3 \times 5 = 15$

$2^{1.6} \times 2^{2.3} = 2^{3.9}$

I'll give you a hint…logs just isolate exponents.

$\boxed{2^{1+2} = 2^3 \qquad 2^{1.6+2.3} = 2^{3.9}}$

Now let's talk about $\log_2(2 \times 4)$ and $\log_2(3 \times 5)$

$\log_2(2x4)$ and $\log_2(3x5)$

$$2 \times 4 = 8$$

$$\downarrow$$

$$2^1 \times 2^2 = 2^3$$

$$\downarrow$$

$$2^{1+2} = 2^3$$

We'll start with the "power of 2" friendly numbers.

$\log_2(2x4)$ and $\log_2(3x5)$

$$2 \times 4 = 8$$

$$\downarrow$$

$$2^1 \times 2^2 = 2^3$$

$$\downarrow$$

$$2^{1+2} = 2^3$$

$$\log_2(2 \times 4) = \log_2(8) \longleftarrow$$

First we just wrap everything up in $\log_2$ functions.

$\log_2(2x4)$ and $\log_2(3x5)$

$$2 \times 4 = 8$$

$$\downarrow$$

$$2^1 \times 2^2 = 2^3$$

$$\downarrow$$

$$2^{1+2} = 2^3$$

$$\log_2(2 \times 4) = \log_2(8)$$

$$\downarrow$$

$$\log_2(2^1 \times 2^2) = \log_2(2^3) \leftarrow$$

Second, we re-write the numbers as powers of 2.

$\log_2(2x4)$ and $\log_2(3x5)$

$2 \times 4 = 8$

$\downarrow$

$2^1 \times 2^2 = 2^3$

$\downarrow$

$2^{1+2} = 2^3$

$\log_2(2 \times 4) = \log_2(8)$

$\downarrow$

$\log_2(2^1 \times 2^2) = \log_2(2^3)$

$\downarrow$

$\log_2(2^{1+2}) = \log_2(2^3)$ $\longleftarrow$ Add the exponents together.

$\log_2(2x4)$ and $\log_2(3x5)$

$$2 \times 4 = 8 \qquad \log_2(2 \times 4) = \log_2(8)$$

$$\downarrow \qquad \downarrow$$

$$2^1 \times 2^2 = 2^3 \qquad \log_2(2^1 \times 2^2) = \log_2(2^3)$$

$$\downarrow \qquad \downarrow$$

$$2^{1+2} = 2^3 \qquad \log_2(2^{1+2}) = \log_2(2^3) \longleftarrow$$

Add the exponents together.

$$\downarrow$$

$$1 + 2 = 3 \longleftarrow$$

The log function isolates the exponents.

$\log_2(2x4)$ and $\log_2(3x5)$

$2 \times 4 = 8$

$2^1 \times 2^2 = 2^3$

$2^{1+2} = 2^3$

$\log_2(2 \times 4) = \log_2(8)$

$\log_2(2^1 \times 2^2) = \log_2(2^3)$

$\log_2(2^{1+2}) = \log_2(2^3)$

$1 + 2 = 3$

This shows that the log of multiplied numbers…

$\log_2(2x4)$ and $\log_2(3x5)$

$2 \times 4 = 8$

$2^1 \times 2^2 = 2^3$

$2^{1+2} = 2^3$

$\log_2(2 \times 4) = \log_2(8)$

$\log_2(2^1 \times 2^2) = \log_2(2^3)$

$\log_2(2^{1+2}) = \log_2(2^3)$

$1 + 2 = 3$

This shows that the log of multiplied numbers…

… is just the sum of their exponents.

$\log_2(2\text{x}4)$ and $\log_2(3\text{x}5)$

$$3 \times 5 = 15$$

$$\downarrow$$

$$2^{1.6} \times 2^{2.3} = 2^{3.9}$$

$$\downarrow$$

$$2^{1.6 + 2.3} = 2^{3.9}$$

Now let's look at numbers that are not "power of 2" friendly.

$\log_2(2x4)$ and $\log_2(3x5)$

$3 \times 5 = 15$

$\downarrow$

$2^{1.6} \times 2^{2.3} = 2^{3.9}$

$\downarrow$

$2^{1.6 + 2.3} = 2^{3.9}$

$\log_2(3 \times 5) = \log_2(15)$ ← Wrap everything up in $\log_2$ functions.

$\log_2(2x4)$ and $\log_2(3x5)$

$3 \times 5 = 15$

$\log_2(3 \times 5) = \log_2(15)$

$\downarrow$

$2^{1.6} \times 2^{2.3} = 2^{3.9}$

$\log_2(2^{1.6} \times 2^{2.3}) = \log_2(2^{3.9})$ $\leftarrow$ Re-write the numbers as powers of 2.

$\downarrow$

$2^{1.6 + 2.3} = 2^{3.9}$

$\log_2(2x4)$ and $\log_2(3x5)$

$3 \times 5 = 15$

$\log_2(3 \times 5) = \log_2(15)$

$\downarrow$

$2^{1.6} \times 2^{2.3} = 2^{3.9}$

$\log_2(2^{1.6} \times 2^{2.3}) = \log_2(2^{3.9})$

$\downarrow$

$2^{1.6+2.3} = 2^{3.9}$

$\log_2(2^{1.6+2.3}) = \log_2(2^{3.9})$ $\leftarrow$ Add the exponents together.

$\log_2(2x4)$ and $\log_2(3x5)$

$3 \times 5 = 15$	$\log_2(3 \times 5) = \log_2(15)$
$\downarrow$	$\downarrow$
$2^{1.6} \times 2^{2.3} = 2^{3.9}$	$\log_2(2^{1.6} \times 2^{2.3}) = \log_2(2^{3.9})$
$\downarrow$	$\downarrow$
$2^{1.6 + 2.3} = 2^{3.9}$	$\log_2(2^{1.6 + 2.3}) = \log_2(2^{3.9})$
	$\downarrow$
	$1.6 + 2.3 = 3.9 \longleftarrow$ Isolate the exponents.

$\log_2(2x4)$ and $\log_2(3x5)$

$3 \times 5 = 15$

$2^{1.6} \times 2^{2.3} = 2^{3.9}$

$2^{1.6 + 2.3} = 2^{3.9}$

$\log_2(3 \times 5) = \log_2(15)$

$\log_2(2^{1.6} \times 2^{2.3}) = \log_2(2^{3.9})$

$\log_2(2^{1.6 + 2.3}) = \log_2(2^{3.9})$

$1.6 + 2.3 = 3.9$

Again, we see that the log of multiplied numbers…

$\log_2(2x4)$ and $\log_2(3x5)$

$3 \times 5 = 15$ | $\log_2(3 \times 5) = \log_2(15)$

$2^{1.6} \times 2^{2.3} = 2^{3.9}$ | $\log_2(2^{1.6} \times 2^{2.3}) = \log_2(2^{3.9})$

$2^{1.6 + 2.3} = 2^{3.9}$ | $\log_2(2^{1.6 + 2.3}) = \log_2(2^{3.9})$

$1.6 + 2.3 = 3.9$

Again, we see that the log of multiplied numbers…

… is just the sum of their exponents.

We just saw how **multiplication becomes addition** with logs.

We just saw how **multiplication becomes addition** with logs.

Now let's see how **division becomes subtraction**.

$$\frac{2}{4} = \frac{1}{2}$$

Taking the log of division turns it into subtraction.

Start with normal division.

$$\frac{2}{4} = \frac{1}{2}$$

Taking the log of division turns it into subtraction.

$$\downarrow$$

$$\frac{2^1}{2^2} = \frac{2^0}{2^1}$$

Re-write everything as powers of 2.

Taking the log of division turns it into subtraction.

$$\frac{2}{4} = \frac{1}{2}$$

$$\downarrow$$

$$\frac{2^1}{2^2} = \frac{2^0}{2^1}$$

$$\downarrow$$

$$2^1 \text{ x } \frac{1}{2^2} = 2^0 \text{ x } \frac{1}{2^1} \longleftarrow$$ Re-write the division as multiplication.

$$\frac{2}{4} = \frac{1}{2}$$

$$\frac{2^1}{2^2} = \frac{2^0}{2^1}$$

$$2^1 \times \frac{1}{2^2} = 2^0 \times \frac{1}{2^1}$$

$$2^1 \times 2^{-2} = 2^0 \times 2^{-1}$$

Taking the log of division turns it into subtraction.

Re-write the fractions by flipping the sign on the exponents.

Taking the log of division turns it into subtraction.

$$\frac{2}{4} = \frac{1}{2}$$

$$\downarrow$$

$$\frac{2^1}{2^2} = \frac{2^0}{2^1}$$

$$\downarrow$$

$$2^1 \times \frac{1}{2^2} = 2^0 \times \frac{1}{2^1}$$

$$\downarrow$$

$$2^1 \times 2^{-2} = 2^0 \times 2^{-1}$$

$$\downarrow$$

$2^{1-2} = 2^{-1}$ $\leftarrow$ Now we subtract the exponents.

$$\frac{2}{4} = \frac{1}{2}$$

$$\downarrow$$

$$\frac{2^1}{2^2} = \frac{2^0}{2^1}$$

$$\downarrow$$

$$2^1 \times \frac{1}{2^2} = 2^0 \times \frac{1}{2^1}$$

$$\downarrow$$

$$2^1 \times 2^{-2} = 2^0 \times 2^{-1}$$

$$\downarrow$$

$$2^{1-2} = 2^{-1}$$

Taking the log of division turns it into subtraction.

OK – here's a pop quiz…

Taking the log of division turns it into subtraction.

$$\frac{2}{4} = \frac{1}{2}$$

$$\downarrow$$

$$\frac{2^1}{2^2} = \frac{2^0}{2^1}$$

$$\downarrow$$

$$2^1 \times \frac{1}{2^2} = 2^0 \times \frac{1}{2^1}$$

$$\downarrow$$

$$2^1 \times 2^{-2} = 2^0 \times 2^{-1}$$

$$\downarrow$$

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If we wrapped $\log_2$ functions around everything, what would happen?

$$\log_2\left(\frac{2}{4}\right) = \log_2\left(\frac{1}{2}\right)$$

Taking the log of division turns it into subtraction.

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… we end up with subtraction.

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In Summary

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- Logs just isolate exponents – no big deal!

$$\log_2(8) = \log_2(2^3) = 3$$

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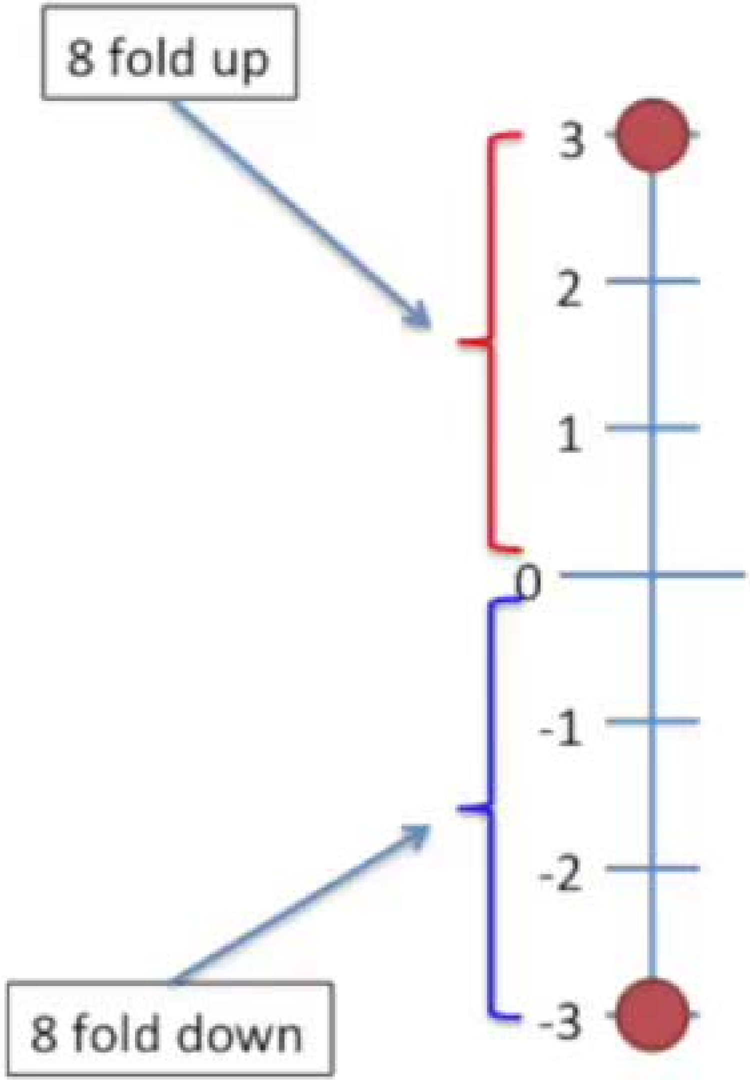
- Log scales a great for plotting "fold change"



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$$\log_2(2 \times 4) = \log_2(2^1 \times 2^2)$$
$$= 1 + 2 = 3$$

In Summary

- Logs just isolate exponents – no big deal!
- Log scales a great for plotting "fold change"
- The mean of logs, aka The Geometric Mean, is great for log based data (i.e. when something is doubling every unit of time) and is less sensitive to outliers.
- The log of multiplication = adding exponents.
- The log of division = subtracting exponents. $\log_2(\frac{2}{4}) = \log_2(\frac{2^1}{2^2}) = 1 - 2$

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Remember '*e*' from math class? I don't. In theory it's "natural", but I always forget why. However, just know that it is a number, like pi, but in this case it is approximately 2.7

I mention it because it is the $\log_e$ is often the default. I use it all the time even though I'm not sure what *e* is. I just know that all logs work the same, so it doesn't matter.

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- What we've talked about applies to all logs
  - $\log_{10}$
  - $\log_e$
  - $\log_{\text{whatever makes sense given your data.}}$