Hybrid Conversion Matrix/Full-Wave Simulation for Periodically Time-Varying Electromagnetics

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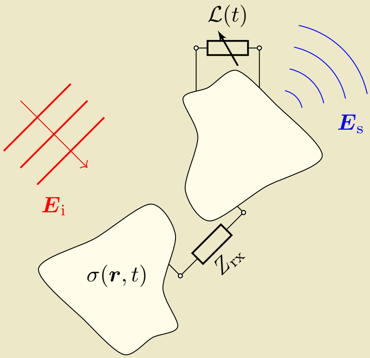
Motivation



Lumped or distributed periodically time-varying loads (or small signal approximation of pumped nonlinear loads)

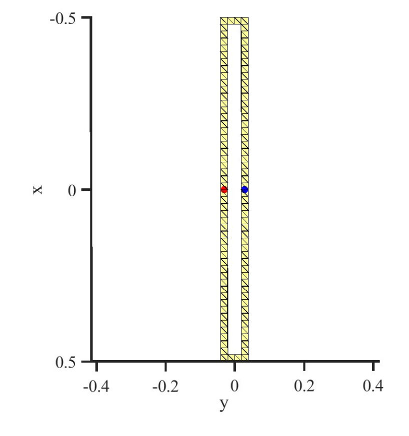
Lumped sources or incident waves

Scattering and radiation

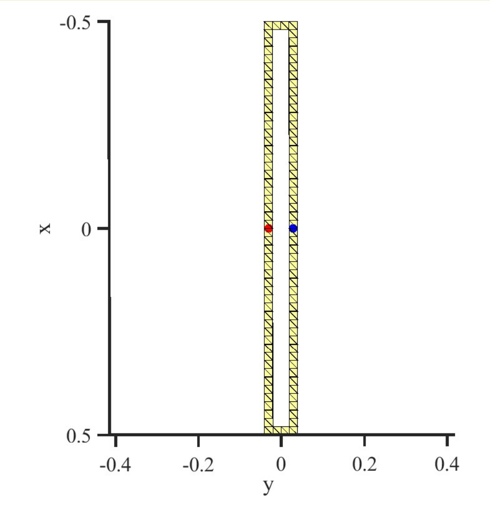
Time-domain modeling is slow, especially for large sweeps

Outline

- Download tutorial files now:
- Hybrid CM/full wave modeling walkthrough
    A. Example problem (thanks to Carey Buxton)
    B. Time-varying load representation
    C. Interface with LTI model elements
    D. Output



A.1-2 Example Problem



Signal frequency
$f_s = 1\ \mathrm{MHz}$
Length = 1 m
gap = 6 cm
Feed point (50 Ω)
Switch

- Swept switching frequency
- 50% duty cycle
- $R_{\mathrm{on}} = 0.3\ \Omega$
- $R_{\mathrm{off}} = 100\ \mathrm{k}\Omega$

A.3 LTI comparison

Same antenna without switching

- Switch replaced by short circuit: $P_{rx} = 15\mathrm{nW}$
- Switch replaced by open circuit: $P_{rx} = 7.2\mathrm{nW}$

Power received by optimal effective aperture for this form factor

- External lossless tuning $P_{rx} = 10.3\mathrm{W}$

B.1 Periodically Time-Varying Loads

Periodically time-varying resistance $r(t)$ with fundamental frequency $\omega_p \rightarrow$ Fourier series

$$r(t) = \sum_{k=-\infty}^{\infty} R_k e^{jk\omega_p t}$$

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Sinusoidal source at frequency $\omega_s \rightarrow$ harmonic frequencies $\omega_k = \omega_s + k\omega_p$
Voltage and current on time-varying resistance $\rightarrow$ *modulated* Fourier series

$$\{v, i\}(t) = e^{j\omega_s t} \sum_{k=-\infty}^{\infty} \{V_k, I_k\} e^{jk\omega_p t}$$

B.1 Periodically Time-Varying Loads

Time-varying load converts between harmonic frequencies

$$V_\ell = \sum_{k=-\infty}^{\infty} R_{\ell-k} I_k \quad \rightarrow \quad \hat{\mathbf{V}} = \hat{\mathbf{R}}\hat{\mathbf{I}}$$

[1] Maas 2003.

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Truncate to finite max harmonic order K: conversion matrix[1]

$$\begin{bmatrix} V_{-K} \\ V_{-K+1} \\ \vdots \\ V_{K-1} \\ V_K \end{bmatrix} = \begin{bmatrix} R_0 & \cdots & R_{-2K} \\ \vdots & \ddots & \vdots \\ R_{2K} & \cdots & R_0 \end{bmatrix} \begin{bmatrix} I_{-K} \\ I_{-K+1} \\ \vdots \\ I_{K-1} \\ I_K \end{bmatrix}$$

[1] Maas 2003.

B.1 Other Basic Time-Varying Components

Conductance $g(t)$:

$$\hat{\mathbf{I}} = \hat{\mathbf{G}}\hat{\mathbf{V}}, \quad \hat{\mathbf{G}} = \hat{\mathbf{R}}^{-1}$$

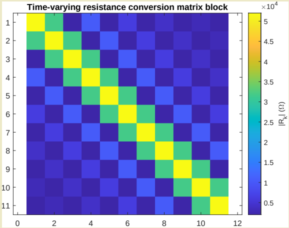
Inductance $L(t)$ or capacitance $C(t)$:

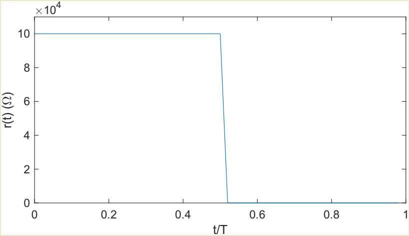
$$\hat{\mathbf{V}} = j\hat{\mathbf{\Omega}}\hat{\mathbf{L}}\hat{\mathbf{I}} \quad \hat{\mathbf{I}} = j\hat{\mathbf{\Omega}}\hat{\mathbf{C}}\hat{\mathbf{V}} \tag{1}$$

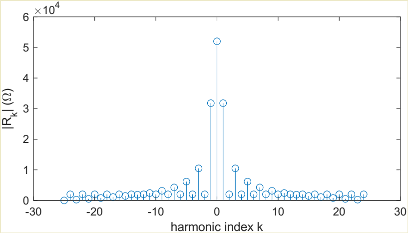
$$\hat{\mathbf{\Omega}} = \mathrm{diag}[\omega_{-K}, \cdots, \omega_K], \quad \omega_k = \omega_s + k\omega_p \tag{2}$$

B.2-3 Implement Time-Varying Load

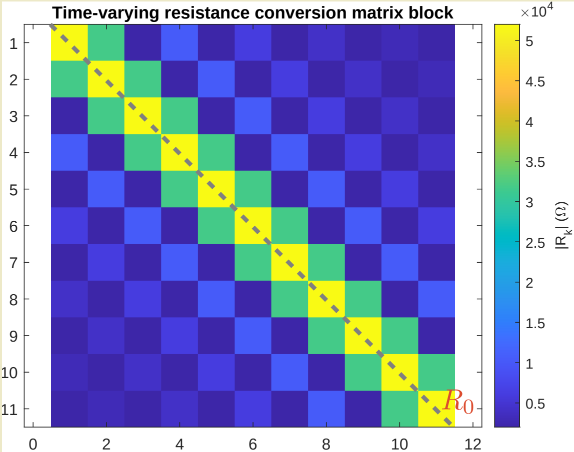
Compute Fourier series coefficients analytically or by FFT
Fill conversion matrix for time-varying load with coefficients







B.3 Time-varying load conversion matrix structure



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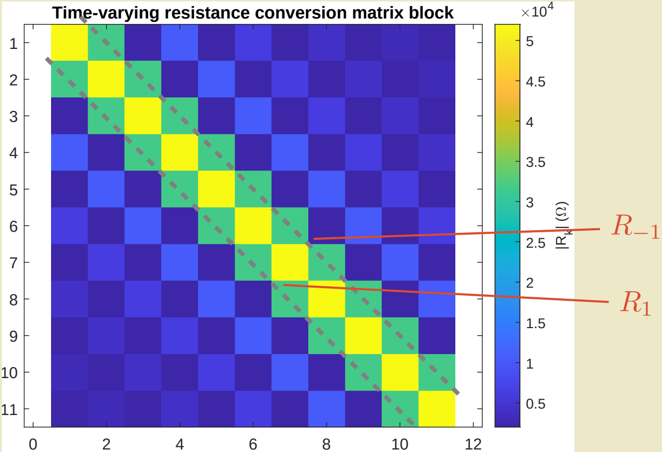


B.3 MoM/Multiport System with Time-Varying Load

Each pair of ports or MoM elements (i, j) has an associated conversion matrix $\hat{\mathbf{R}}_{ij}$ for conversion between harmonic frequencies and ports

$$\begin{bmatrix} \hat{\mathbf{V}}_1 \\ \vdots \\ \hat{\mathbf{V}}_N \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{R}}_{11} & \cdots & \hat{\mathbf{R}}_{1N} \\ \vdots & \ddots & \vdots \\ \hat{\mathbf{R}}_{N1} & \cdots & \hat{\mathbf{R}}_{NN} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{I}}_1 \\ \vdots \\ \hat{\mathbf{I}}_N \end{bmatrix}$$

Can swap indexing and group by frequency instead if desired

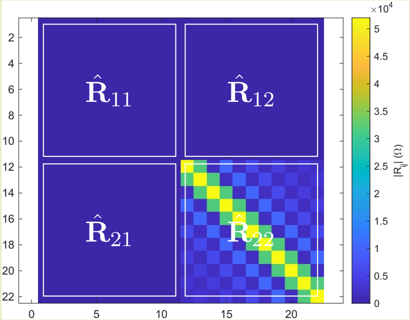
Increases MoM Z matrix dimension by a factor of $(2K + 1)$: compression recommended if discrete loading

B.3 MoM/Multiport System with Time-Varying Load

Compressed system has 2 ports: load (1) and switch (2)

$\hat{\mathbf{R}}_{11} = \hat{\mathbf{0}}$ because no time-varying load on port 1

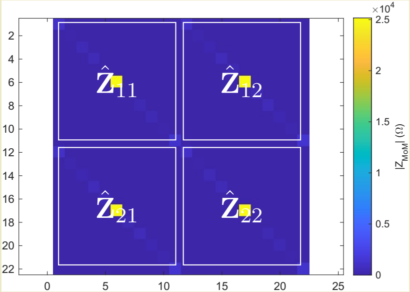
$\hat{\mathbf{R}}_{12} = \hat{\mathbf{R}}_{21} = \hat{\mathbf{0}}$ because frequency conversion effect is localized at port 2



C.2 "Conversionified" LTI Model Z matrix

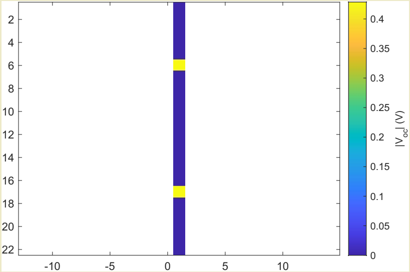
Compressed MoM Z matrix equivalent to port impedance matrix from any solver

Block (i, j) describes conversion between harmonic frequencies and ports (i, j)



C.3 Excitation Voltage

Populate elements of excitation voltage vector corresponding to excitation frequency (usually 0th harmonic)
RX: open circuit voltage at each port due to incident plane wave
TX: lumped source



D. Output Calculation

Element-wise multiplication keeps power in individual harmonic frequencies separate

$$P_k = \frac{1}{2}\mathrm{Re}\{I_k^* V_k\}$$

```
I = (ZMoM+ZL+R)\Voc;                       % solve for multifrequency port currents
P(:,fdex) = 0.5*real(conj(I).*(ZL*I));     % load power
```

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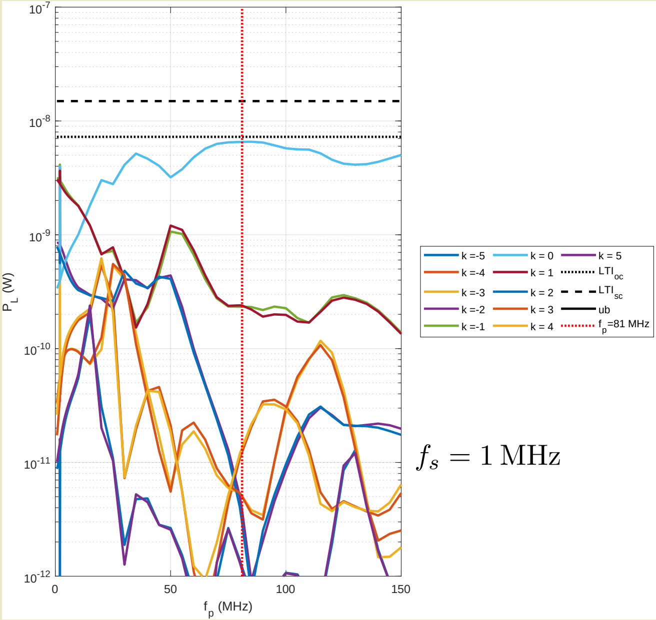
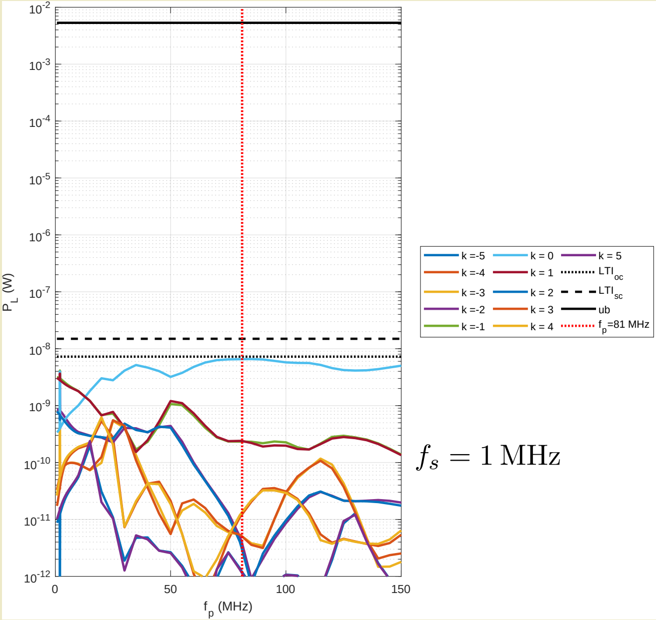
D. Output: Multi-Harmonic Response vs. Pump Frequency



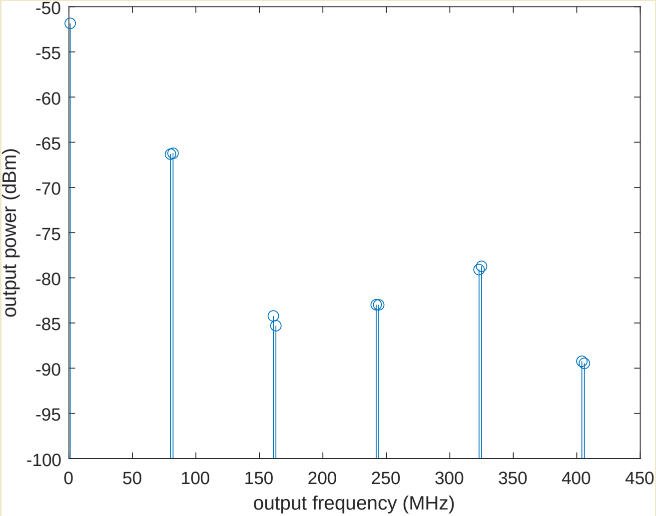
$$f_s = 1\,\mathrm{MHz}$$

D. Output: Multi-Harmonic Response vs. Pump Frequency



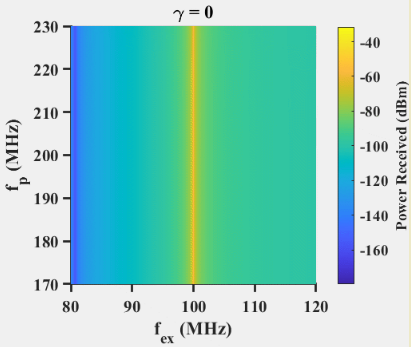
$$f_s = 1\,\mathrm{MHz}$$

D. Output Spectrum for Pump Frequency = 81 MHz



Discussion

- Conversion matrices model periodically time-varying linear devices
- Hybridization with MoM models lumped or distributed time varying loading on radiating or scattering structures
- Walked through example CMMoM calculation



Thank you

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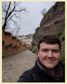

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