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Q1.

$$\dfrac{n\geq 0 \Rightarrow I[0/r, 0/i, 1/p] \qquad \{I[0/r,0/i,1/p]\}\ r:=0\ \{I[0/i,1/p]\}}{\{n\geq 0\}\ r:=0\ \{I[0/i,1/p]\}}$$

$$\dfrac{\{n\geq 0\}\ r:=0\ \{I[0/i,1/p]\} \qquad \{I[0/i,1/p]\}\ i:=0\ \{I[1/p]\}}{\{n\geq 0\}\ r:=0;\ i:=0\ \{I[1/p]\}}$$

$$\dfrac{\{n\geq 0\}\ r:=0;\ i:=0\ \{I[1/p]\} \qquad \{I[1/p]\}\ p:=1\ \{I\}}{\{n\geq 0\}\ r:=0;\ i:=0;\ p:=1\ \{I\}}$$

$$\dfrac{I\wedge i\neq n \Rightarrow I[2*p/p, r+p/r, i+1/i] \qquad \{I[2*p/p, r+p/r, i+1/i]\}\ r:=r-p;\ p:=2*p;\ r:=r+p;\ i:=i+1\ \{I\}}{\{I\wedge i\neq n\}\ r:=r-p;\ p:=2*p;\ r:=r+p;\ i:=i+1\ \{I\}}$$

$$\dfrac{\{I\}\ \text{while } i\neq n \text{ do } (r:=r-p;\ p:=2*p;\ r:=r+p;\ i:=i+1)\ \{I\wedge i=n\} \qquad \{I\wedge i=n\} \Rightarrow \{r=2^n-1\}}{\{I\}\ \text{while } i\neq n \text{ do } (r:=r-p;\ p:=2*p;\ r:=r+p;\ i:=i+1)\ \{r=2^n-1\}}$$

$$\dfrac{\{n\geq 0\}\ r:=0;\ i:=0;\ p:=1\ \{I\} \qquad \{I\}\ \text{while } i\neq n \text{ do } (r:=r-p;\ p:=2*p;\ r:=r+p;\ i:=i+1)\ \{r=2^n-1\}}{\{n\geq 0\}\ r:=0;\ i:=0;\ p:=1;\ \text{while } i\neq n \text{ do } (r:=r-p;\ p:=2*p;\ r:=r+p;\ i:=i+1)\ \{r=2^n-1\}}$$

The remaining constraints:

1. $n\geq 0 \Rightarrow I[0/r, 0/i, 1/p]$
2. $I\wedge i=n \Rightarrow r=2^n-1.$
3. $I\wedge i\neq n \Rightarrow I[2*p/p, r+p/r, i+1/i]$

let $I = p=2^i \wedge r=2^i-1 \wedge i\leq n$

1. $n\geq 0 \Rightarrow 1=2^0 \wedge 0=2^0-1 \wedge 0\leq n$

1. $n\geq 0 \Rightarrow 1=1 \wedge 0=0 \wedge 0\leq n$

2. $I\wedge i=n \Rightarrow r=2^n-1.$

2. $p=2^i \wedge r=2^i-1 \wedge i\leq n \wedge i=n \Rightarrow r=2^n-1$

2. $p=2^n \wedge r=2^n-1 \Rightarrow r=2^n-1$

3. $I\wedge i\neq n \Rightarrow I[2*p/p, r+p/r, i+1/i].$

3. $p=2^i \wedge r=2^i-1 \wedge i\leq n \wedge i\neq n \Rightarrow 2p=2^{i+1} \wedge r+p=2^{i+1}-1 \wedge i+1\leq n$

3. $p=2^i \wedge r=2^i-1 \wedge i<n \Rightarrow p=2^i \wedge r+p=2^{i+1}-1 \wedge i\leq n-1$

All constraints are valid.

The program is correct.

Q4.b.

The inductive invariant is (r = x + c) and ((c<=y) and (0<=c))