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Q1 a) There are 4 different paths.

Path 1: 1, 2, 3, 4, 9, 11, 12 ,13, 17
Path 2: 1, 2, 3, 4, 9, 11, 15, 16, 17
Path 3: 1, 2, 6, 7, 9, 11, 12, 13, 17
Path 4: 1, 2, 6, 7, 9, 11, 15, 16, 17

b) Path1:

Edge	Symbolic State (PV)	Path Condition (PC)
1→2	x ↦ X0 , y ↦ Y0	true .
2→3	x ↦ X0 , y ↦ Y0	X0 + Y0 > 15
3→4	x ↦ X0 + 7 , y ↦ Y0	X0 + Y0 > 15
4→9	x ↦ X0 + 7 , y ↦ Y0 - 12	X0 + Y0 > 15
9→11	x ↦ X0 + 9 , y ↦ Y0 - 12	X0 + Y0 > 15
11→12	x ↦ X0 + 9 , y ↦ Y0 - 12	X0+Y0 > 15 ∧ 2(X0+9+Y0-12) > 21
12→13	x ↦ 3(X0+9) , y ↦ Y0 - 12	X0+Y0 > 15 ∧ 2(X0+9+Y0-12) > 21
13→17	x ↦ 3(X0+9) , y ↦ 2(Y0-12)	X0+Y0 > 15 ∧ 2(X0+9+Y0-12) > 21

Path2

Edge	Symbolic State (PV)	Path Condition (PC)
1→2	x l→ X0, y l→ Y0	true
2→3	x l→ X0, y l→ Y0	X0 + Y0 > 15
3→4	x l→ X0 + 7, y l→ Y0	X0 + Y0 > 15
4→9	x l→ X0 + 7, y l→ Y0 -12	X0 + Y0 > 15
9→11	x l→ X0 + 9, y l→ Y0 - 12	X0 + Y0 > 15
11→15	x l→ X0 + 9, y l→ Y0 - 12	X0 + Y0 > 15 ^ 2 (X0 + 9 + Y0 -12) <= 21
15→16	x l→ 4(X0 + 9), y l→ Y0 - 12	X0 + Y0 > 15 ^ 2 (X0 + 9 + Y0 -12) <= 21
16→17	x l→ 4(X0 + 9), y l→ 3*(Y0-12) + 4(X0+9)	X0 + Y0 > 15 ^ 2(X0 + 9 + Y0 -12) <= 21

Path3:

Edge	Symbolic State (PV)	Path Condition (PC)
1→2	x l→ X0, y l→ Y0	true
2→6	x l→ X0, y l→ Y0	X0 + Y0 <= 15
6→7	x l→ X0, y l→ Y0 + 10	X0 + Y0 <= 15
7→9	x l→ X0 - 2, y l→ Y0 + 10	X0 + Y0 <= 15
9→11	x l→ X0, y l→ Y0 + 10	X0 + Y0 <= 15
11→ 12	x l→ X0, y l→ Y0 + 10	X0 + Y0 <= 15 ^ 2 (X0 + Y0 + 10) > 21
12→13	x l→ 3X0, y l→ Y0 + 10	X0 + Y0 <= 15 ^ 2 (X0 + Y0 + 10) > 21
13→17	x l→ 3X0, y l→ 2(Y0 + 10)	X0 + Y0 <= 15 ^ 2 (X0 + Y0 + 10) > 21

Path4:

Edge	Symbolic State (PV)	Path Condition (PC)
1→2	x |→ X0, y |→ Y0	true
2→6	x |→ X0, y |→ Y0	X0 + Y0 <= 15
6→7	x |→ X0, y |→ Y0 + 10	X0 + Y0 <= 15
7→9	x |→ X0 - 2, y |→ Y0 + 10	X0 + Y0 <= 15
9→11	x |→ X0, y |→ Y0 + 10	X0 + Y0 <= 15
11→15	x |→ X0, y |→ Y0 + 10	X0 + Y0 <= 15 ^ 2 (X0 + Y0 + 10) <= 21
15→16	x |→ 4X0, y |→ Y0 + 10	X0 + Y0 <= 15 ^ 2 (X0 + Y0 + 10) <= 21
16→17	x |→ 4X0, y |→ 3(Y0 + 10) + 4X0	X0 + Y0 <= 15 ^ 2 (X0 + Y0 + 10) <= 21

C): Path 1 is feasible : X0 = 8, Y0 = 8.
X0 + Y0 = 16 > 15 ^ 2*(8+9+8-12) = 26 > 21

Path 2 is not feasible: the path condition is X0 + Y0 > 15 ^ 2(X0 + 9 + Y0 -12) <= 21 which can be simplified as: X0 + Y0 > 15 ^ X0 + Y0 <= 13.5. This condition is always false.

Path 3 is feasible: X0 = 8, Y0 = 7.
X0 + Y0 = 15 <= 15 ^ 2*(8+7+10) = 50 > 21

Path 4 is feasible: X0 = 0 , Y0 = 0
X0 + Y0 = 0 < 15 ^ 2*(0+0+10) = 20 < 21

Q2: a) CNF: $(a_1 \vee a_2 \vee \neg a_3 \vee \neg a_4) \wedge$
$(a_1 \vee \neg a_2 \vee a_3 \vee \neg a_4) \wedge$
$(a_1 \vee \neg a_2 \vee \neg a_3 \vee a_4) \wedge$
$(a_1 \vee \neg a_2 \vee \neg a_3 \vee \neg a_4) \wedge$
$(\neg a_1 \vee a_2 \vee a_3 \vee \neg a_4) \wedge$
$(\neg a_1 \vee a_2 \vee \neg a_3 \vee a_4) \wedge$
$(\neg a_1 \vee a_2 \vee \neg a_3 \vee \neg a_4 \wedge$
$(\neg a_1 \vee \neg a_2 \vee a_3 \vee a_4) \wedge$
$(\neg a_1 \vee \neg a_2 \vee a_3 \vee \neg a_4) \wedge$
$(\neg a_1 \vee \neg a_2 \vee \neg a_3 \vee a_4) \wedge$
$(\neg a_1 \vee \neg a_2 \vee \neg a_3 \vee \neg a_4)$

a1	a2	a3	a4	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

can be simplified as:

$(\neg a_1 \vee \neg a_2) \wedge (\neg a_1 \vee \neg a_3) \wedge (\neg a_1 \vee \neg a_4) \wedge (\neg a_2 \vee \neg a_3) \wedge (\neg a_2 \vee \neg a_4) \wedge (\neg a_3 \vee \neg a_4)$

b) It is valid.

① subclaim. $\forall x \cdot \exists y \cdot P(x) \vee Q(y)) \Rightarrow (\forall x P(x)) \vee (\exists y\, Q(y))$.

Prove by contradiction

Assume LHS is true and RHS is false

LHS means for all x there exist some y makes P(x) or Q(y) is true.

RHS is false means for all x P(x) is false and there is no such y that makes Q(y) true.

suppose $\forall x P(x)$ is false then $\exists y \cdot Q(y)$ must be true to make LHS true.

Similarly, suppose $\exists y Q(y)$ is false then $\forall x P(x)$ must be true to make LHS true.

That is a contradiction to our assumption of RHS.

Therefore. we can prove this subclaim is true.

② Subclaim: $(\forall x P(x)) \vee (\exists y\, Q(y)) \Rightarrow \forall x \cdot \exists y\, P(x) \vee Q(y)$

Suppose LHS is true:

① $\forall x P(x)$ is true and $\exists y\, Q(y)$ is false, then P(x) is true for every x.

That means RHS is true regardless Q(y)

② $\exists y Q(y)$ is true and $\forall x P(x)$ is false. then for any x we can pick a y to make $P(x) \vee Q(y)$ true. RHS is also true.

Therefore, we can prove this FOL is valid.

c) It is invalid.

Proof: $(\forall x \cdot \exists y \cdot P(x,y) \vee Q(x,y)) \Rightarrow (\forall x \cdot \exists y \cdot P(x,y)) \vee (\forall x \cdot \exists y \cdot Q(x,y))$

Suppose $x = \{0,1\}$, $y = \{0,1\}$,

$P(1,1)$ is true, $P(1,0)$ is false, $P(0,1)$ is false, $P(0,0)$ is false

$Q(1,1)$ is true, $Q(0,1)$ is true, $Q(0,0)$ is false, $Q(1,0)$ is false

For LHS: when $x=0$, $y=1$, $P(x,y) \vee Q(x,y)$ is true

when $x=1$, $y=1$, $P(x,y) \vee Q(x,y)$ is true

$\therefore$ LHS is true

For RHS $\forall x \exists y P(x,y)$: when $x=0$, there is no y to make $P(x,y)$ true

so $\forall x \exists y P(x,y)$ is false.

For RHS $\forall x \exists y Q(x,y)$: when $x=0$, there is no y to make $Q(x,y)$ true.

so $\forall x \exists y Q(x,y)$ Therefore, RHS is false. we disprove this implication.

d) $\Phi = \exists x \exists y \exists z\ (P(x,y) \land P(z,y) \land P(x,z) \land \neg P(z,x))$

(a) $M_1 = \langle S_1, P_1 \rangle$, where $S_1 = N$ and $P_1 = \{(x,y) \mid x,y \in N \land x<y\}$.

Suppose $x=1$, $y=3$, $z=2$

Then $P(x,y)$ is true because $x<y$.

$P(z,y)$ is true because $z<y$.

$\neg P(z,x)$, $P(x,z)$ are true because $x<z$

$\therefore\ M_1 \models \Phi$

(b) $M_2 = \langle S_2, P_2 \rangle$ where $S_2 = N$ and $P_2 = \{(x,x+1) \mid x \in N\}$

Suppose $P(x,y)$ is true then $y=x+1$

$P(z,y)$ is true then $y=z+1$

Then $x=z$

Then $P(x,z)$ is false because $x=z$.

$\therefore\ M_2 \not\models \Phi$

c) $M_3 = \langle S_3, P_3 \rangle$, where $S_3 = \mathbb{P}(N)$, the powerset of natural number and $P_3 = \{(A,B) \mid A,B \in \mathbb{P}(N) \land A \subseteq B\}$

Suppose x contains 1 number

y contains 3 number

z contains 2 number.

Then $P(x,y)$ is true because $x \subseteq y$.

$P(z,y)$ is true because $z \subseteq y$

$\neg P(z,x)$, $P(x,z)$ are true because $x \subseteq z$.

$\therefore\ M_3 \models \Phi$

e) As Boolean variables a_n has disjunction compared to other a.

Then we have $\bigwedge_{1 \le i \le j \le n} (\neg a_i \lor \neg a_j)$.

Q3 a):

Assume n is the size of the square, M is the sum, r is row number and Val is the element value, c is column number.

① each cell element is distinct and its value is smaller than n^2:

$$\bigwedge_{1\le i,j,l,k\le n \,\wedge\, (i\ne l \vee j\ne k)} (Val_{ij} \ne Val_{lk})$$

$$\bigwedge_{1\le i,j\le n} (1\le Val_{ij} \le n^2)$$

② sum of each row, column, diagonal are equal.

$$\bigwedge_{1\le r\le n} \Sigma_{j=1}^{n} C_{rj} = M$$

$$\Sigma_{r=1}^{n} C_{rr} = M$$

$$\Sigma_{r=1}^{n} C_{r(n-r+1)} = M$$

$$\bigwedge_{1\le c\le n} \Sigma_{j=1}^{n} C_{ic} = M$$

Q4 e: The program is the last test function in my test_sym.py file. I took 172 seconds to run.

```
#Testing divergence
    def test_diverge(self):
        prg1 = "havoc x; c := 1; while c < 5 do {c:=c+1;if x < -100 then {f := x + 1;
x := x/100 + f}; if x < -1000 then {f := 2 * x; x := x/10 * f + x}; if x < 1000 then
{f := 3 / x;" \
               " x := x * f+ 10}; if x > 0 then {f := x /4; x := x*5/f}; if x >= 10 then {f :=
5+x; x := x-20*f}; if x = 100 then {f := 6-x; x := 100/f}}; print_state "
        ast1 = ast.parse_string(prg1)
        engine = sym.SymExec()
        st = sym.SymState()
        out = [s for s in engine.run(ast1, st)]
        self.assertEquals(len(out), 1)
```

```
pc: [10 < 21, 11 < 21, 12 < 21, 13 < 21, 14 < 21, 15 < 21, 16 < 21, 17 < 21, 18 < 21, 19 < 21]

..x: x!536
pc: [10 > x!536]

x: -1
pc: [Not(10 > x!536)]

.x: 10
y: 10
z: 0
a: 100
b: 1
pc: []

.x: x!543
pc: [10 < x!543, 9 < x!543]

...
----------------------------------------------------------------------
Ran 29 tests in 173.496s

OK
```