

Symbolic Execution

Testing, Quality Assurance, and Maintenance
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based on slides by Prof. Johannes Kinder and others

How would you test this program?

Remember this little program Foo()?

Is it possible to **automatically** generate a test-suite that

- achieves full branch coverage
- discovers whether division by 0 is possible
- identifies all infeasible test requirements (i.e., dead code)

Remarkably, the answer is YES

```
def Foo(x, y):  
    """ requires: x and y are int  
        ensures: returns floor(max(x,y)/min(x, y)) """  
    if x > y:  
        return x / y  
    else  
        return y / x
```

Symbolic execution in a nutshell

- to identify division by 0, add explicit tests (i.e., oracles)
- **traverse** the program to compute each program path
 - path1: $x > y$, $y == 0$; path2: $x > y$, $y != 0$, return x / y ; ...
- **solve** constraints for each path using a constraint (or logic) solver
 - path1: $x=10$, $y=0$; path2: $x=10$, $y=1$; ...
- **run** the program on tests generated by previous step
- all testing is now automatic

```
def Foo(x, y):  
    """ requires: x and y are int  
        ensures: returns floor(max(x,y)/min(x, y)) """  
    if x > y:  
        assert y != 0  
        return x / y  
    else:  
        assert x != 0  
        return y / x
```

$\text{floor}(z)$ is the largest integer not greater than z

Symbolic Execution is a Combination of

Automatically explore program paths

- Execute program on “symbolic” input values
- “Fork” execution at each branch
- Record branching conditions

Constraint solver

- Decides path feasibility
- Generates test cases for paths and bugs

History

Int. Conference on Reliable Software 1975

James C. King:

A new approach to program testing

Robert S. Boyer, Bernard Elspas, Karl N. Levitt:

*SELECT—a formal system for testing and debugging programs
by symbolic execution*



Recent work on proving the correctness of programs by formal analysis [5] shows great promise and appears to be the ultimate technique for producing reliable programs. However, the practical accomplishments in this area fall short of a tool for routine use. Fundamental problems in reducing the theory to practice are not likely to be solved in the immediate future.

History (2)

SAT / SMT solvers lead to boom in 2000s

- Constraint solving becomes a commodity
- Makes classic algorithms viable in practice

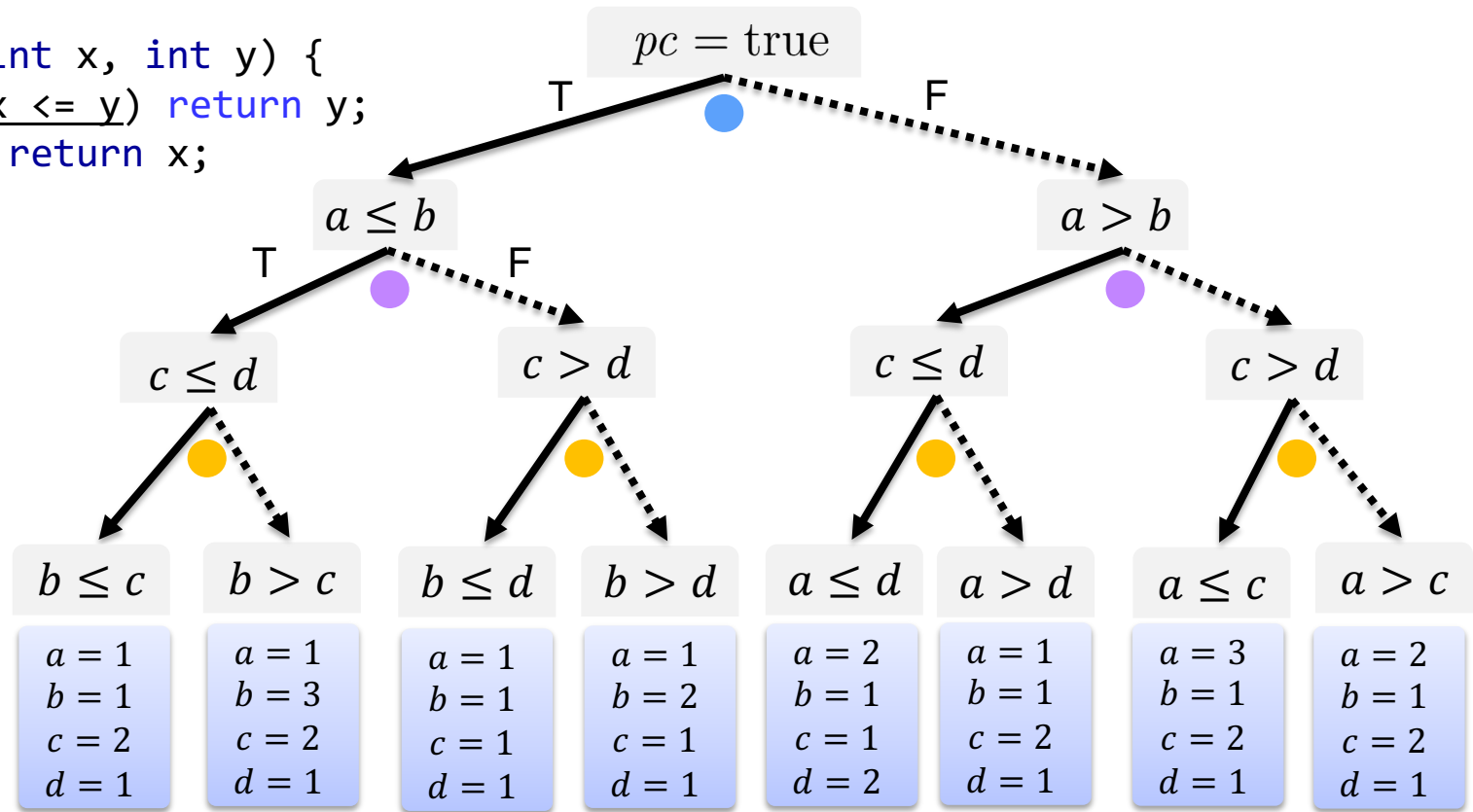
Conceptual breakthroughs (Dynamic Symbolic Execution)

- Patrice Godefroid, Nils Klarlund, Koushik Sen: *DART: directed automated random testing*. PLDI 2005
- Cristian Cadar, **Vijay Ganesh**, Peter M. Pawlowski, David L. Dill, Dawson R. Engler: *EXE: automatically generating inputs of death*. CCS 2006

Symbolic Execution Illustrated

```
int Max(int a, int b, int c, int d) {
    return Max(Max(a, b ●), Max(c, d ●) ●);
}
```

```
int Max(int x, int y) {
    if (x <= y) return y;
    else return x;
}
```



pc = true

x = *X*

r = 0

```
1 int proc(int x) {  
2  
3     int r = 0  
4  
5     if (x > 8) {  
6         r = x - 7  
7     }  
8  
9     if (x < 5) {  
10        r = x - 2  
11    }  
12  
13    return r  
    }
```


Symbolic
program state

$pc = \text{true}$

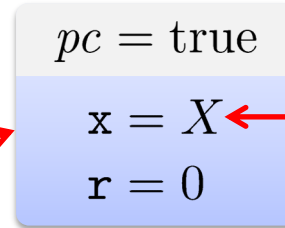
$x = X$

$r = 0$

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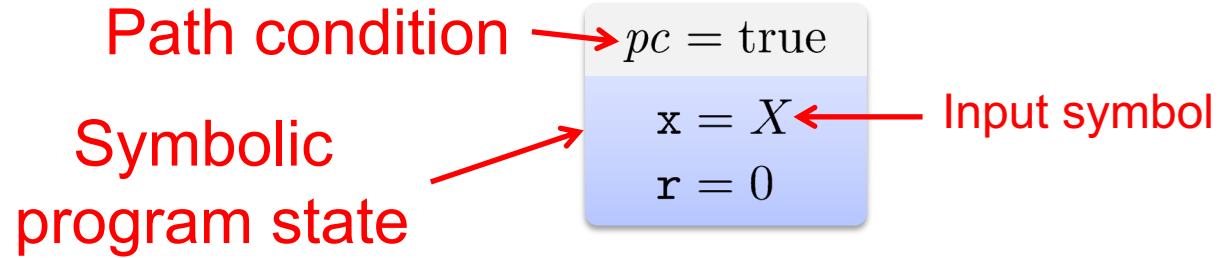
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Symbolic
program state



Input symbol

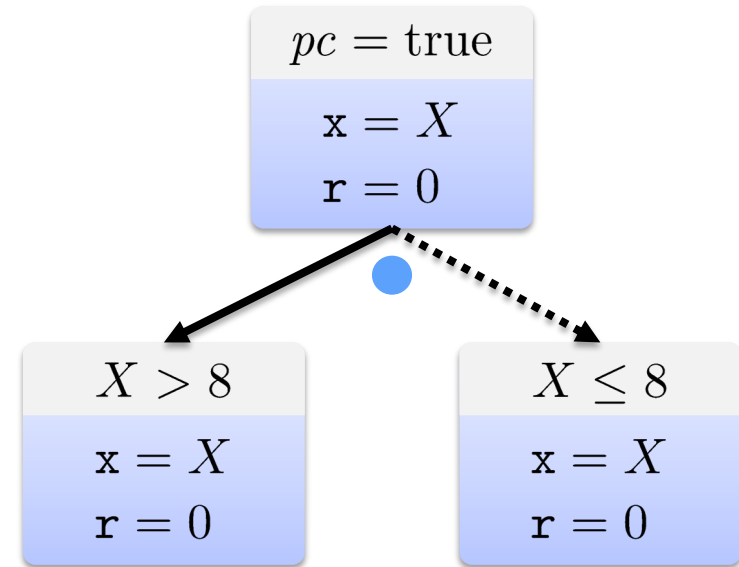
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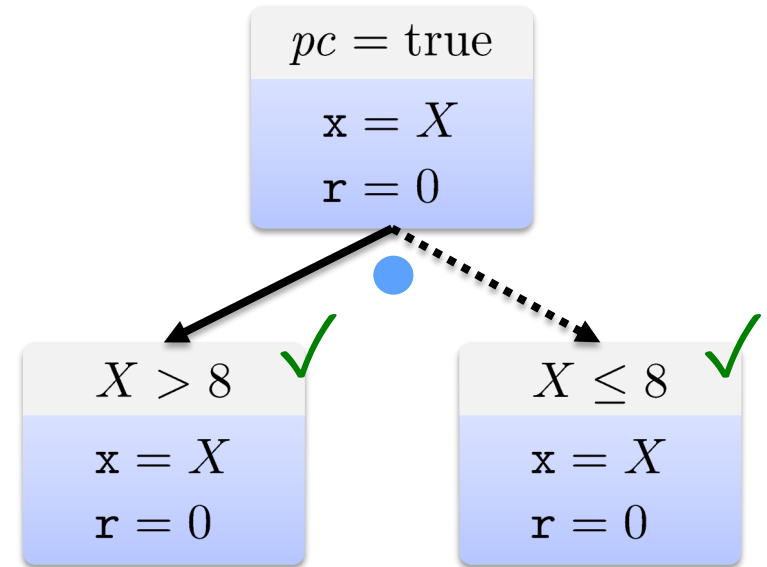
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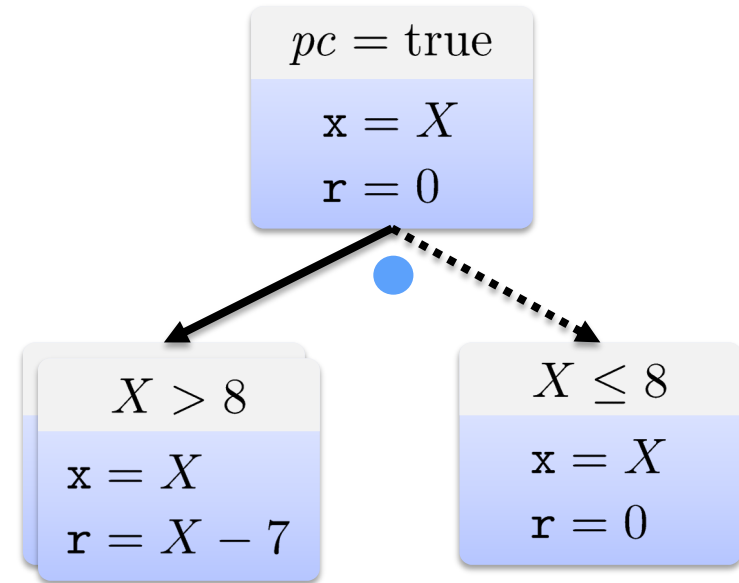
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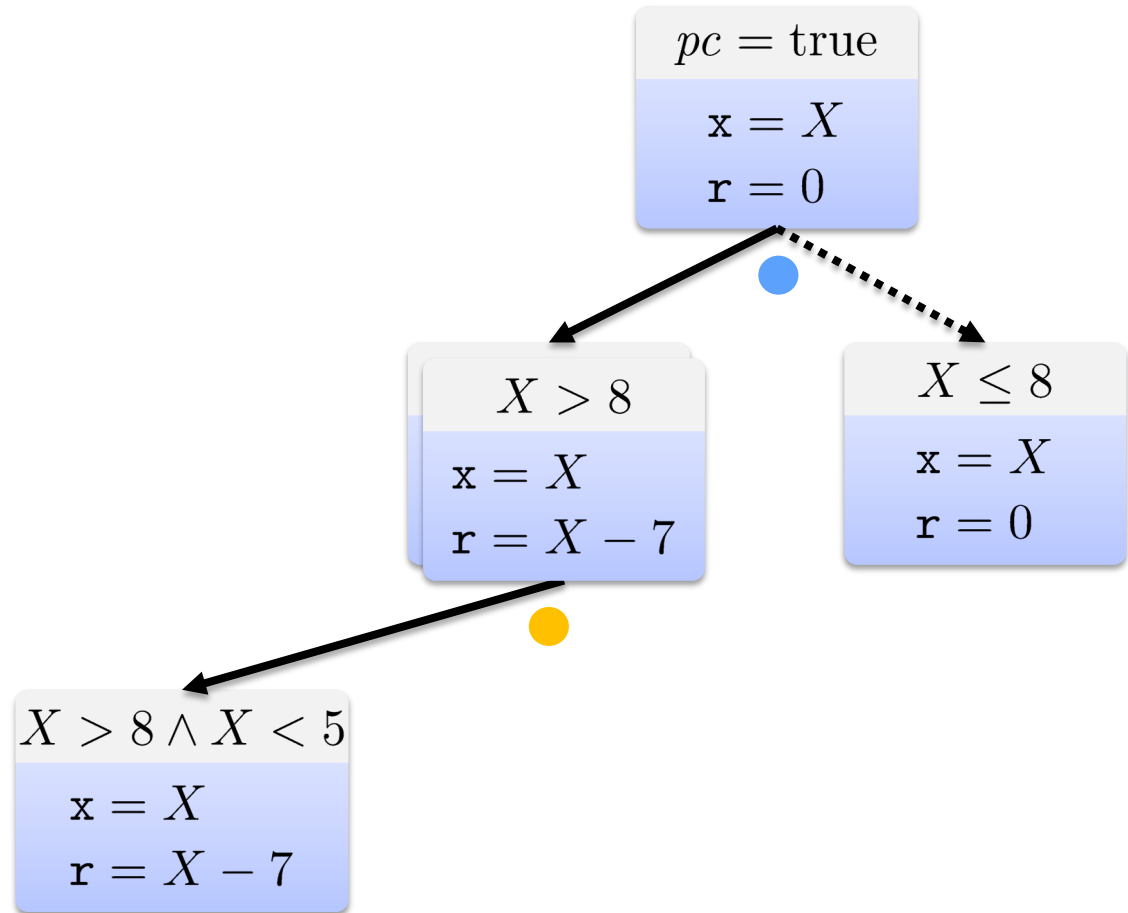
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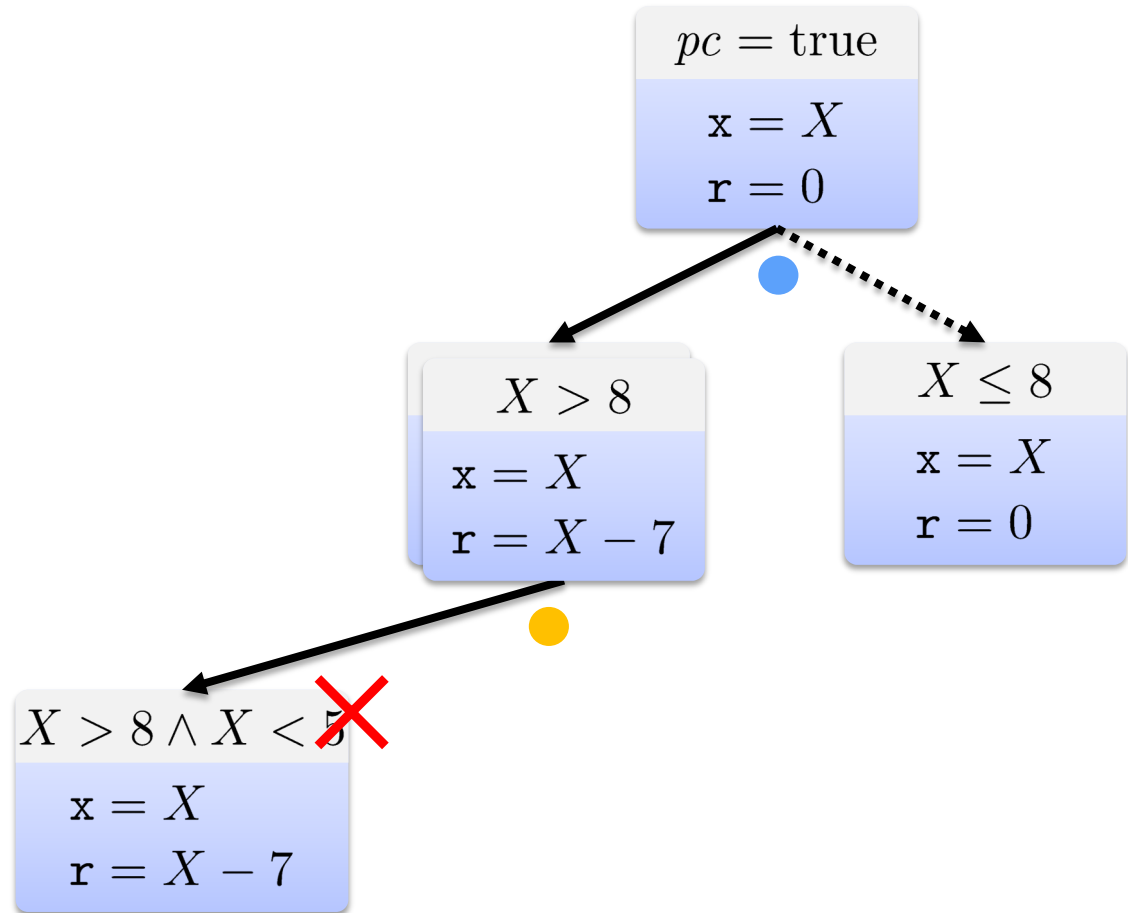
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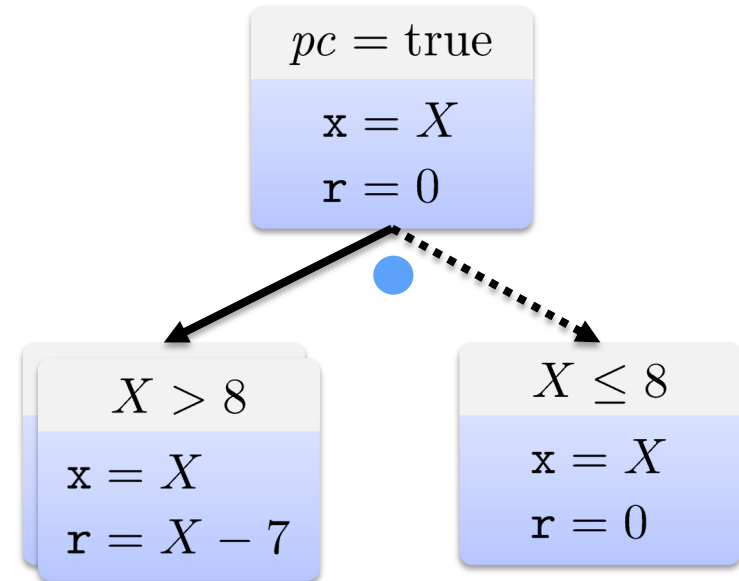
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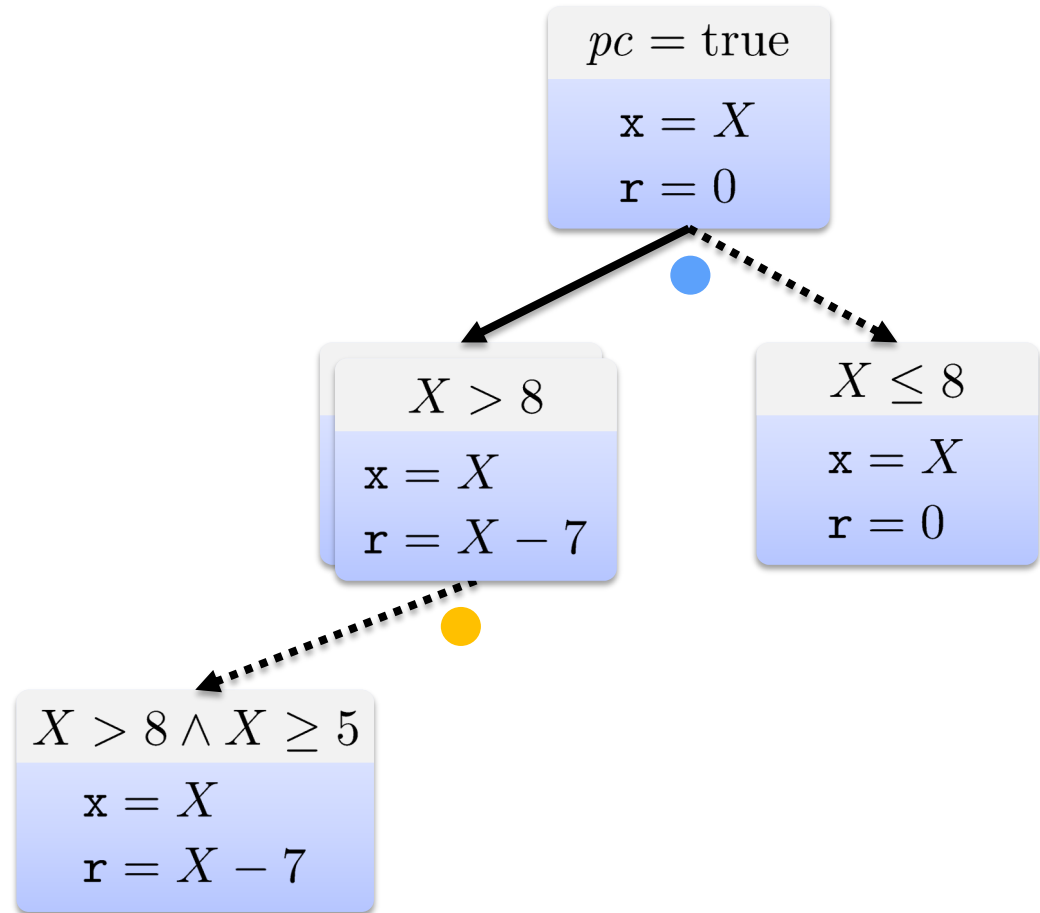
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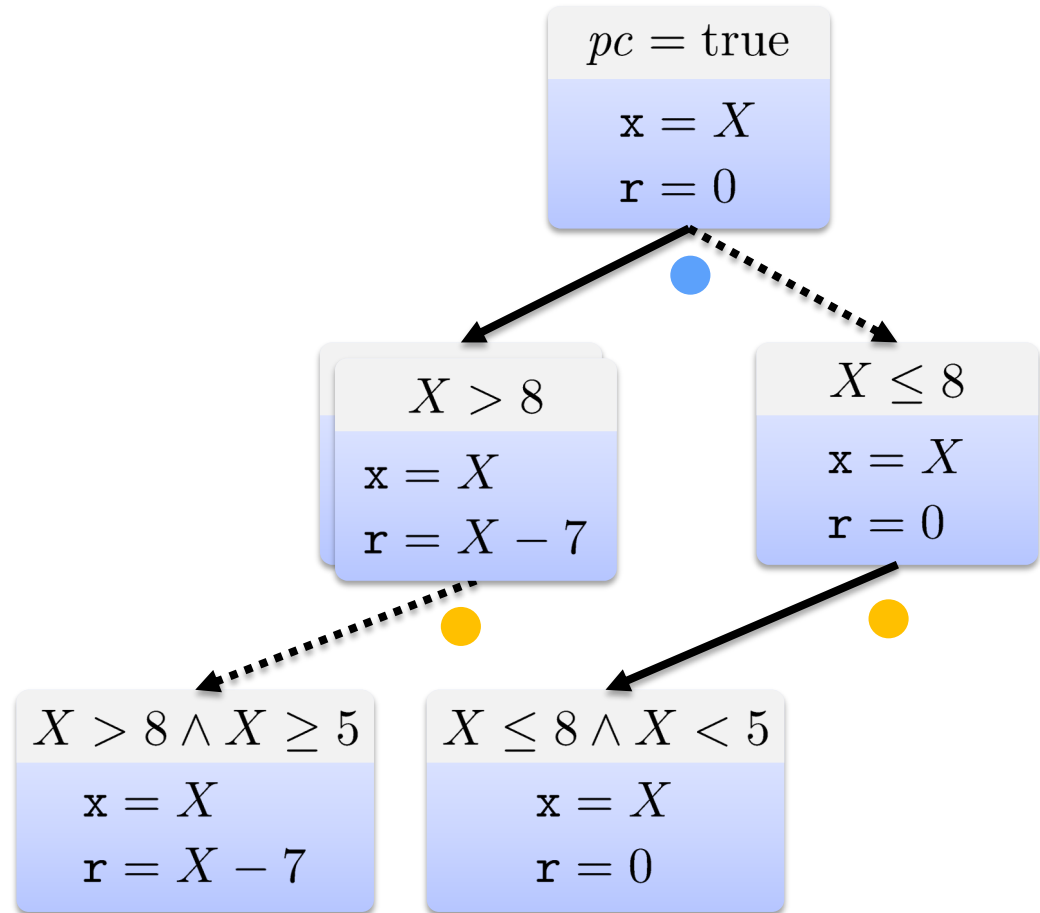
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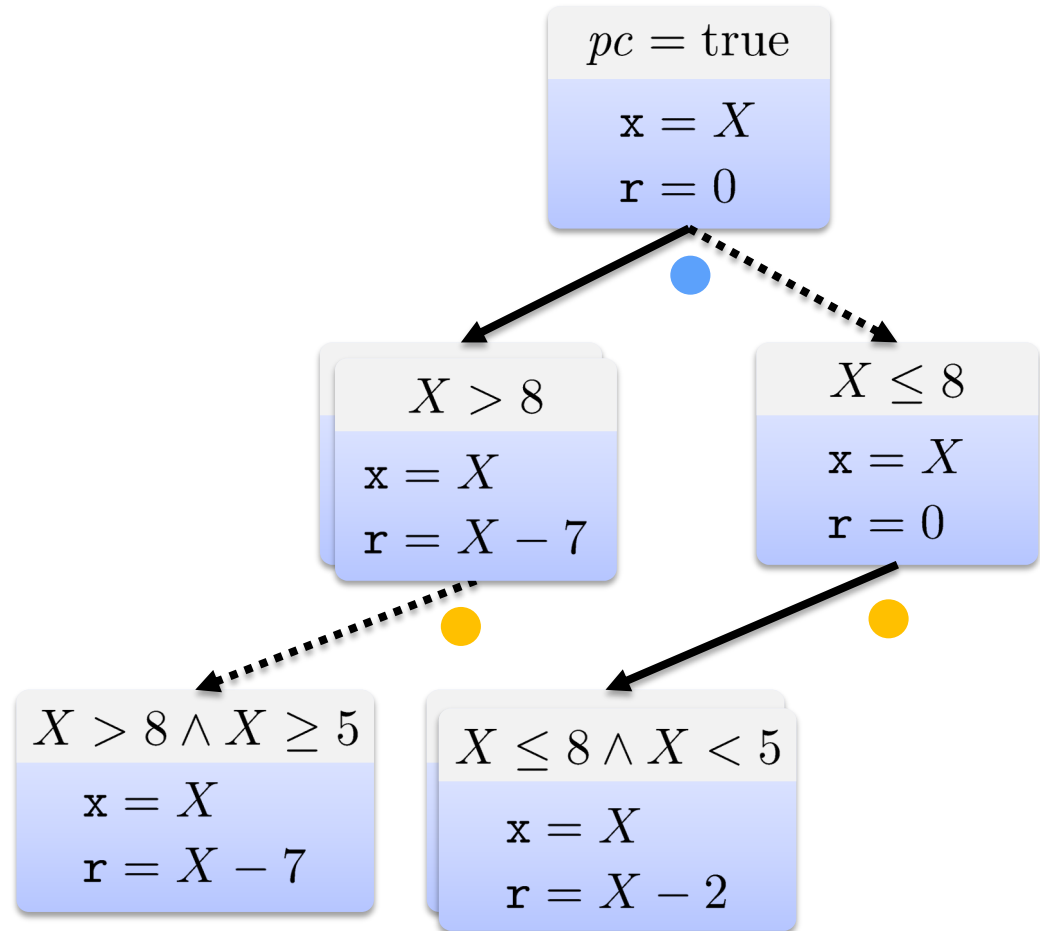
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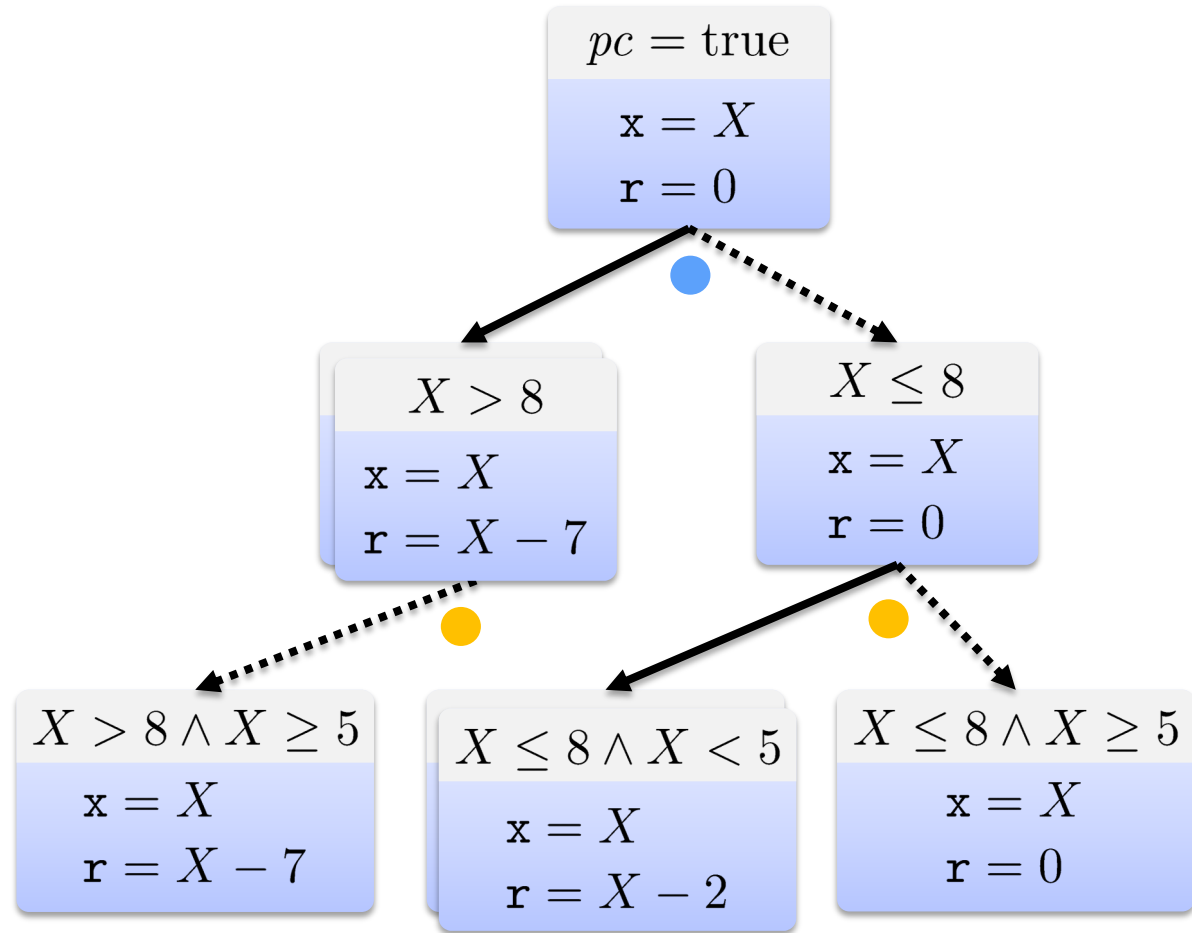
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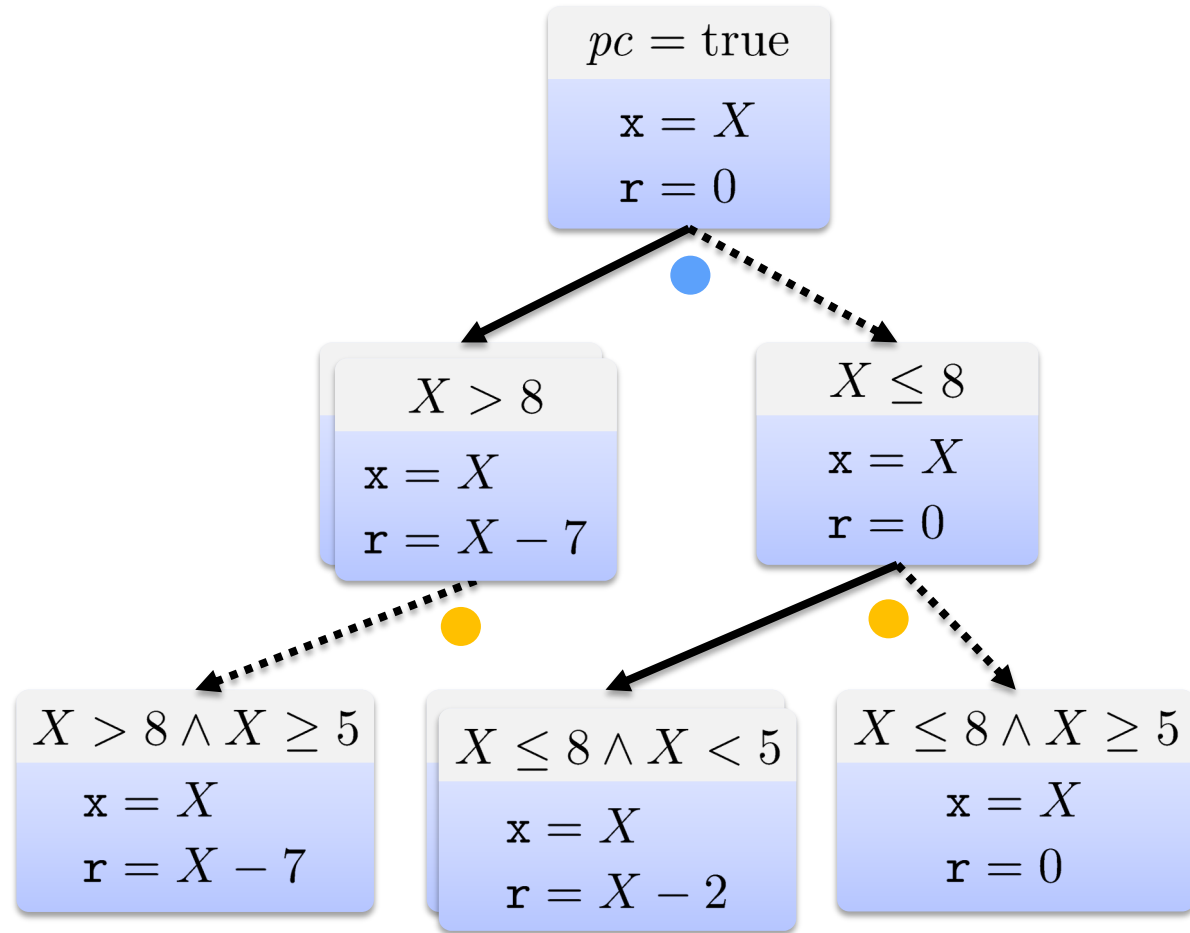
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```



Satisfying assignments:

$X = 9$

$X = 4$

$X = 7$

Test cases:

proc(9)

proc(4)

proc(7)

Symbolic Execution

Analysis of programs by tracking symbolic rather than actual values

- a form of **Static Analysis**

Symbolic reasoning is used to reason about *all* the inputs that take the same path through a program

Builds constraints that characterize

- conditions for executing paths
- effects of the execution on program state

Symbolic Execution

Uses symbolic values for input variables.

Builds constraints that characterize the conditions under which execution paths can be taken.

Collects **symbolic path conditions**

- a path condition for a path P is a formula PC such that PC is satisfiable if and only if P is executable

Uses theorem prover (**constraint solver**) to check if a path condition is satisfiable and the path can be taken.

Symbolic Execution: Alternative Explanation

Symbolic execution creates a **functional** representation of each **path** in a Control Flow Graph of a program

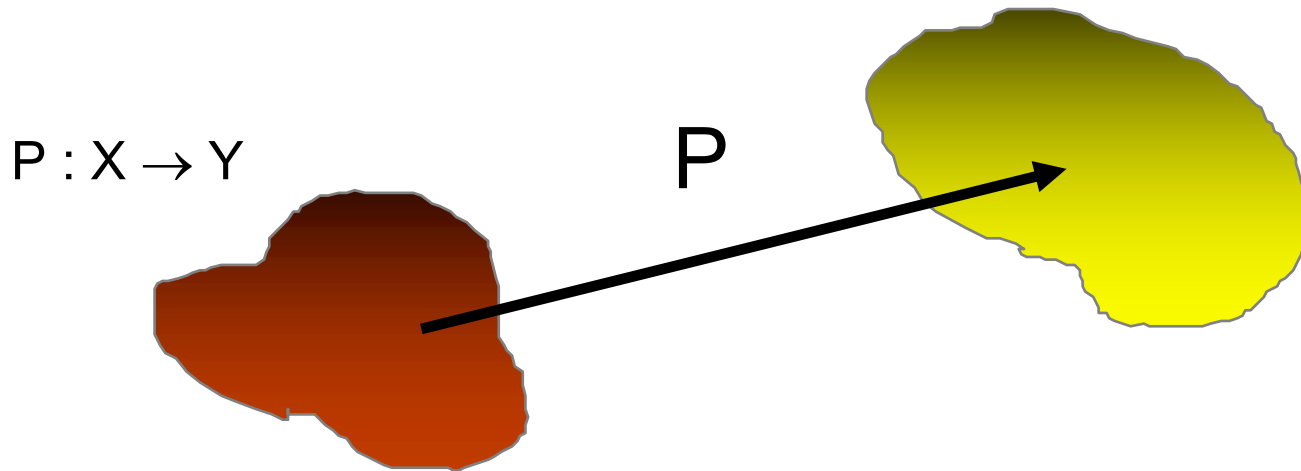
- i.e., each program path becomes a function from inputs to outputs

Notation:

For a program path P (i.e, a sequence of instructions)

- $D[P]$ is the domain for path P
 - the inputs that force the program to take path P_i
- $C[P]$ is the computation for path P
 - the result of executing the path

Program as a Function



Program P is composed of partial functions corresponding to the executable paths

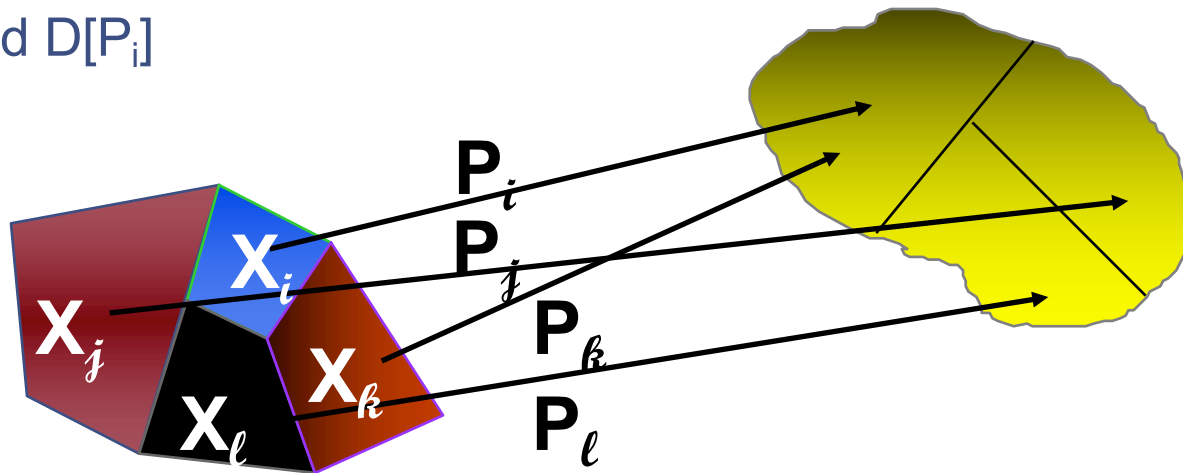
$$P = \{P_1, \dots, P_r\}$$

Each partial function $P_i : X_i \rightarrow Y$ maps some part of the input of the program to the output

Effect of Symbolic Execution

X_i is the domain of path P_i

Denoted $D[P_i]$



$$X = D[P_1] \cup \dots \cup D[P_r] = D[P]$$

$$D[P_i] \cap D[P_j] = \emptyset, i \neq j$$

Exercise: Find a Violation

```
int x=0, y=0, z=0;
if (a ●) {
    x = -2;
}
if (b < 5 ●) {
    if (!a && c ●)
        { y = 1; }
    z = 2;
}
assert(x+y+z != 3);
```

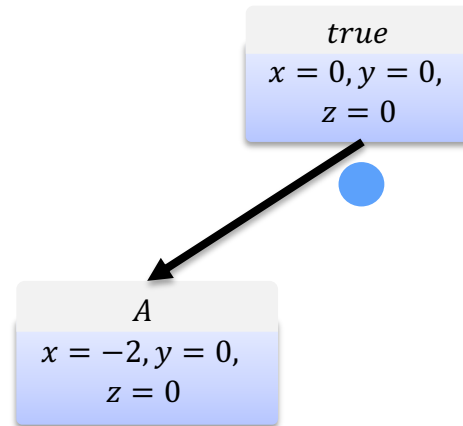
true
 $x = 0, y = 0,$
 $z = 0$

Common to all states:

$a = A, b = B, c = C$

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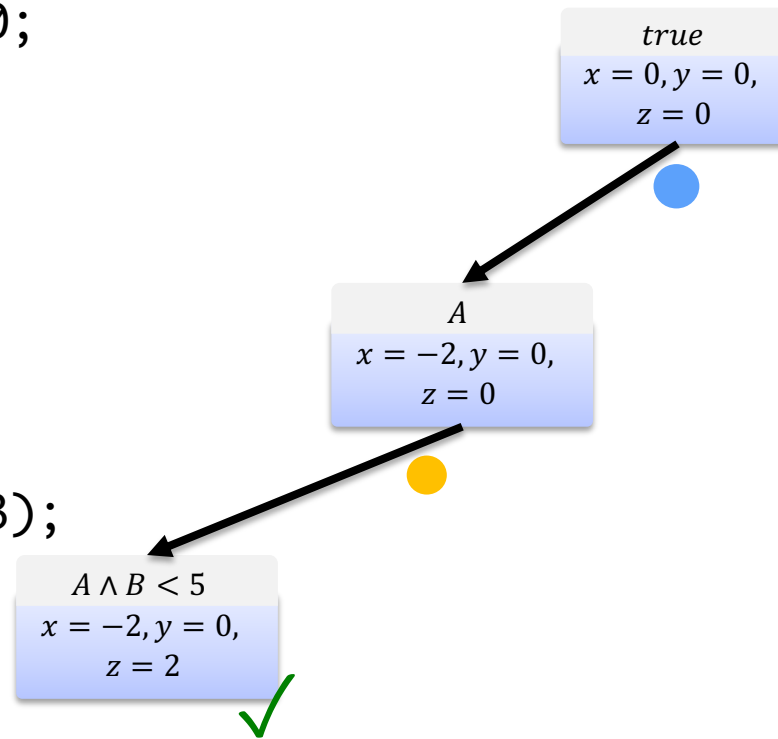


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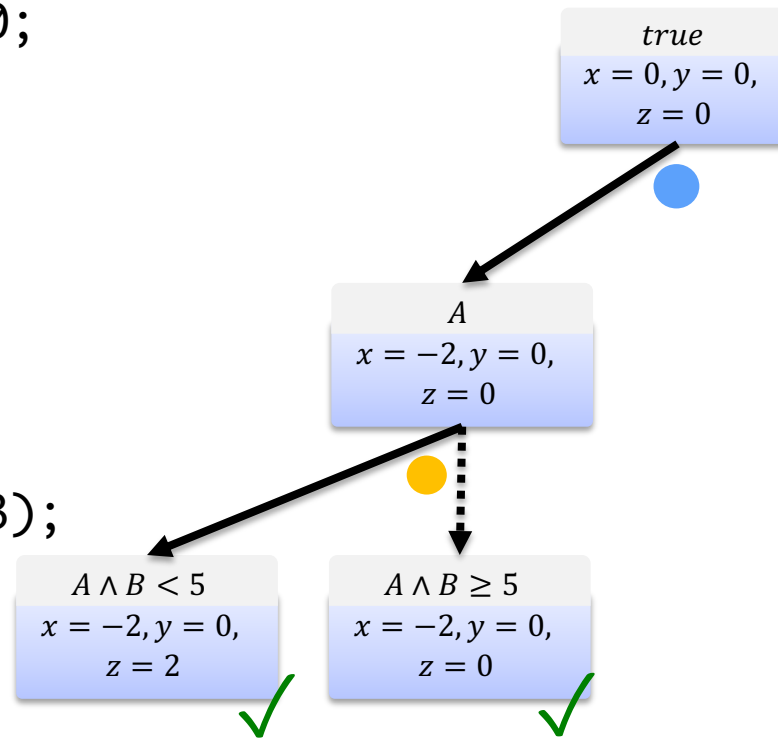


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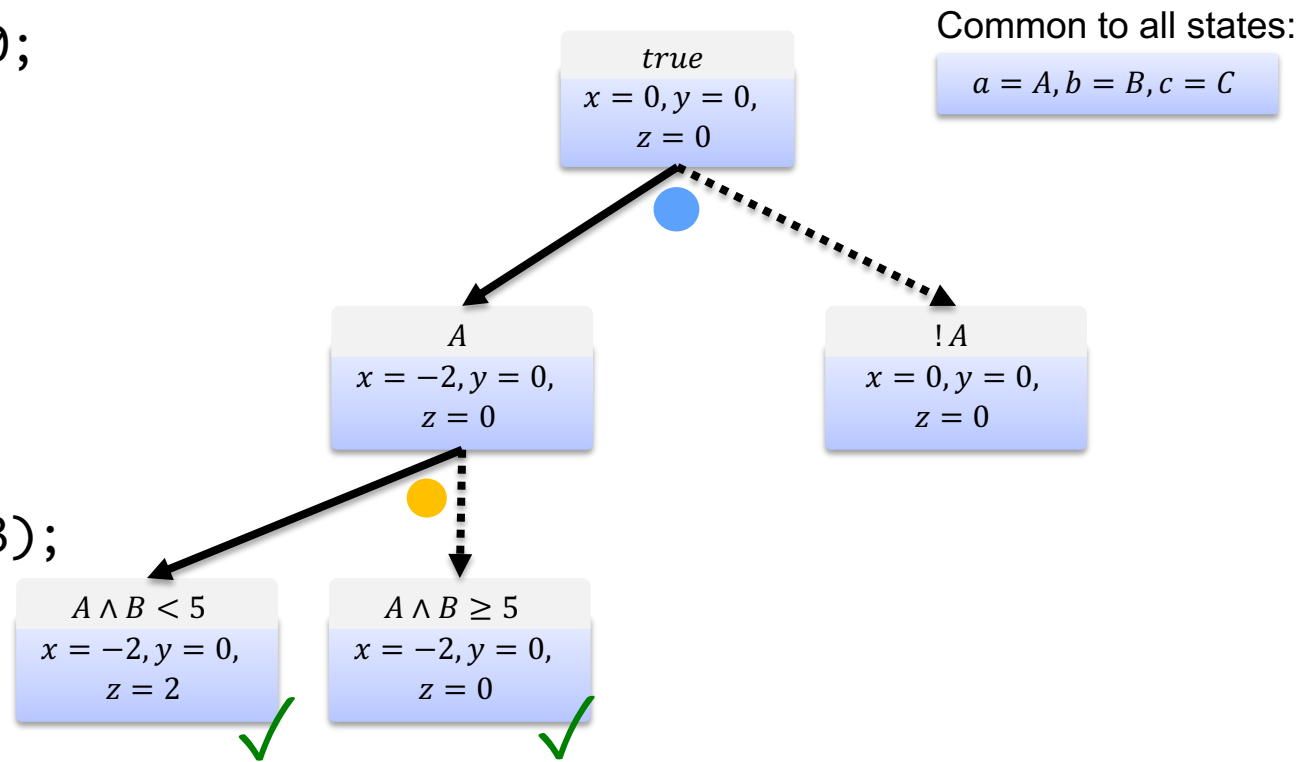


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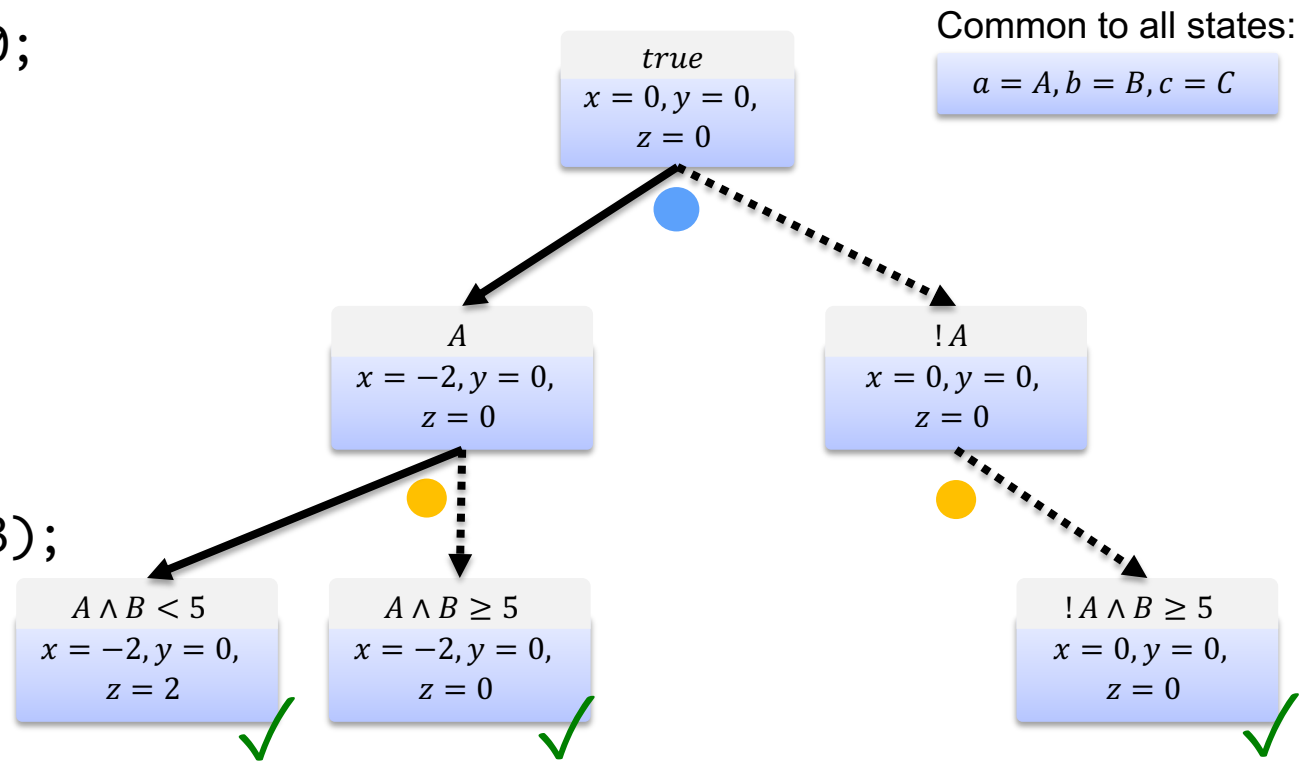
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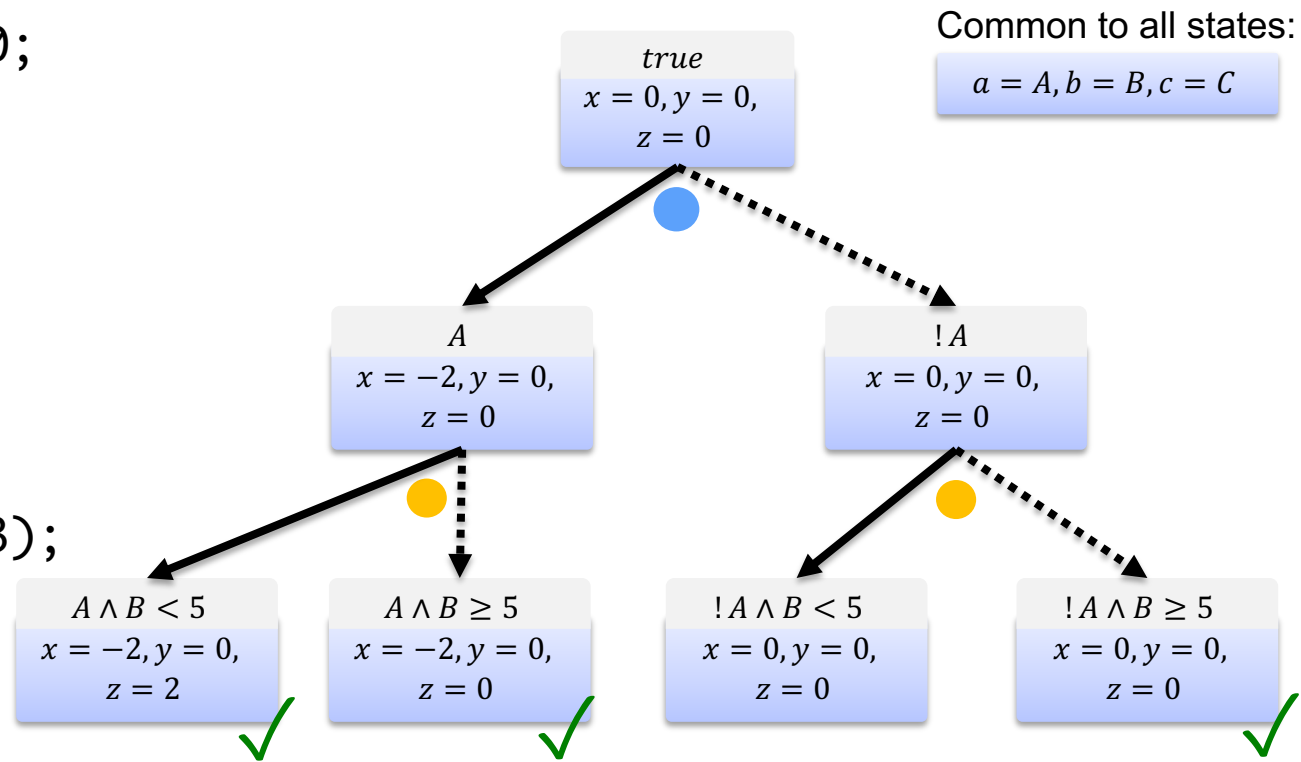
Exercise: Use SymExec to Find a Violation

```
int x=0, y=0, z=0;
if (a ●) {
    x = -2;
}
if (b < 5 ●) {
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    z = 2;
}
assert(x+y+z != 3);
```



Exercise: Find a Violation

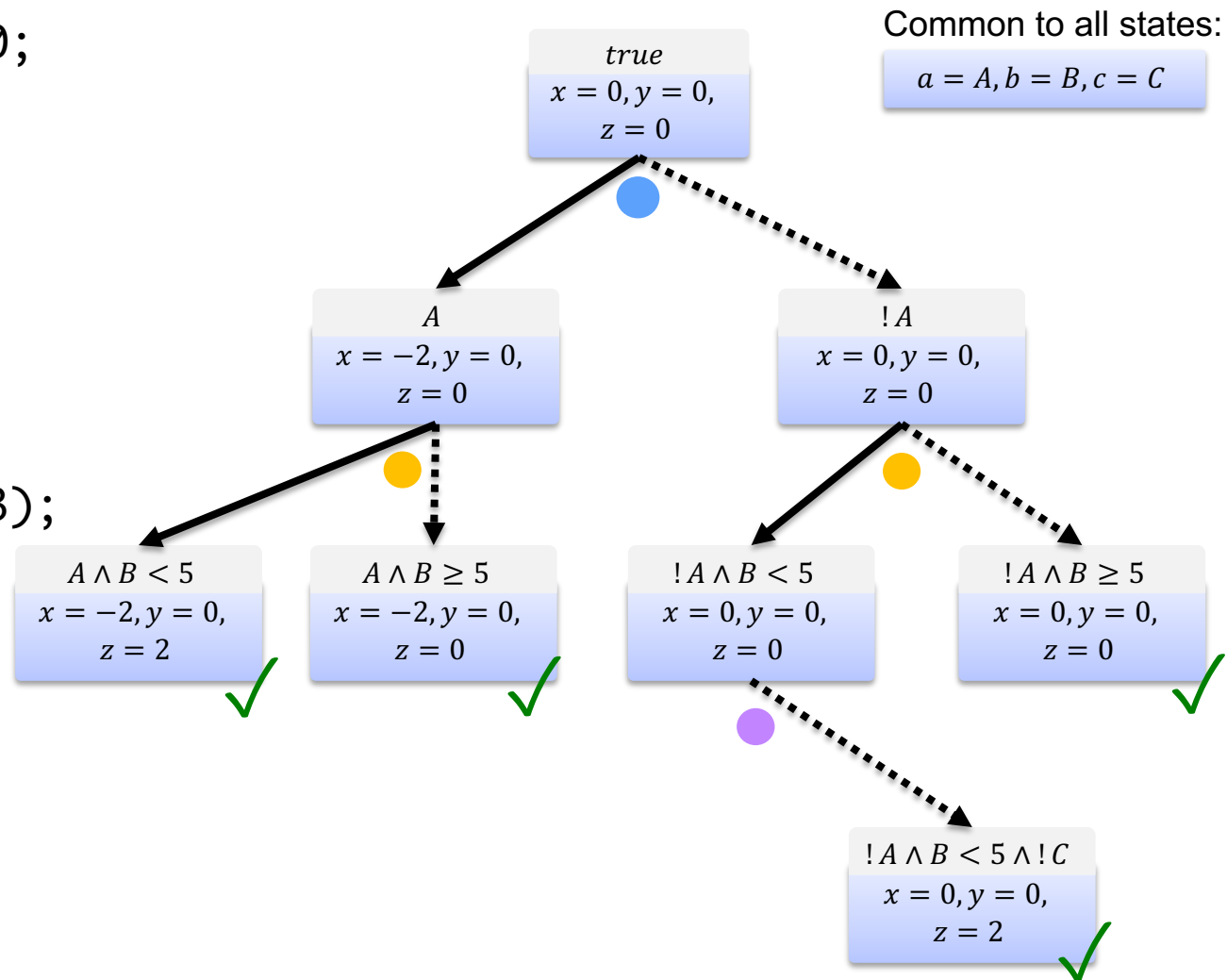
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Exercise: Find a Violation

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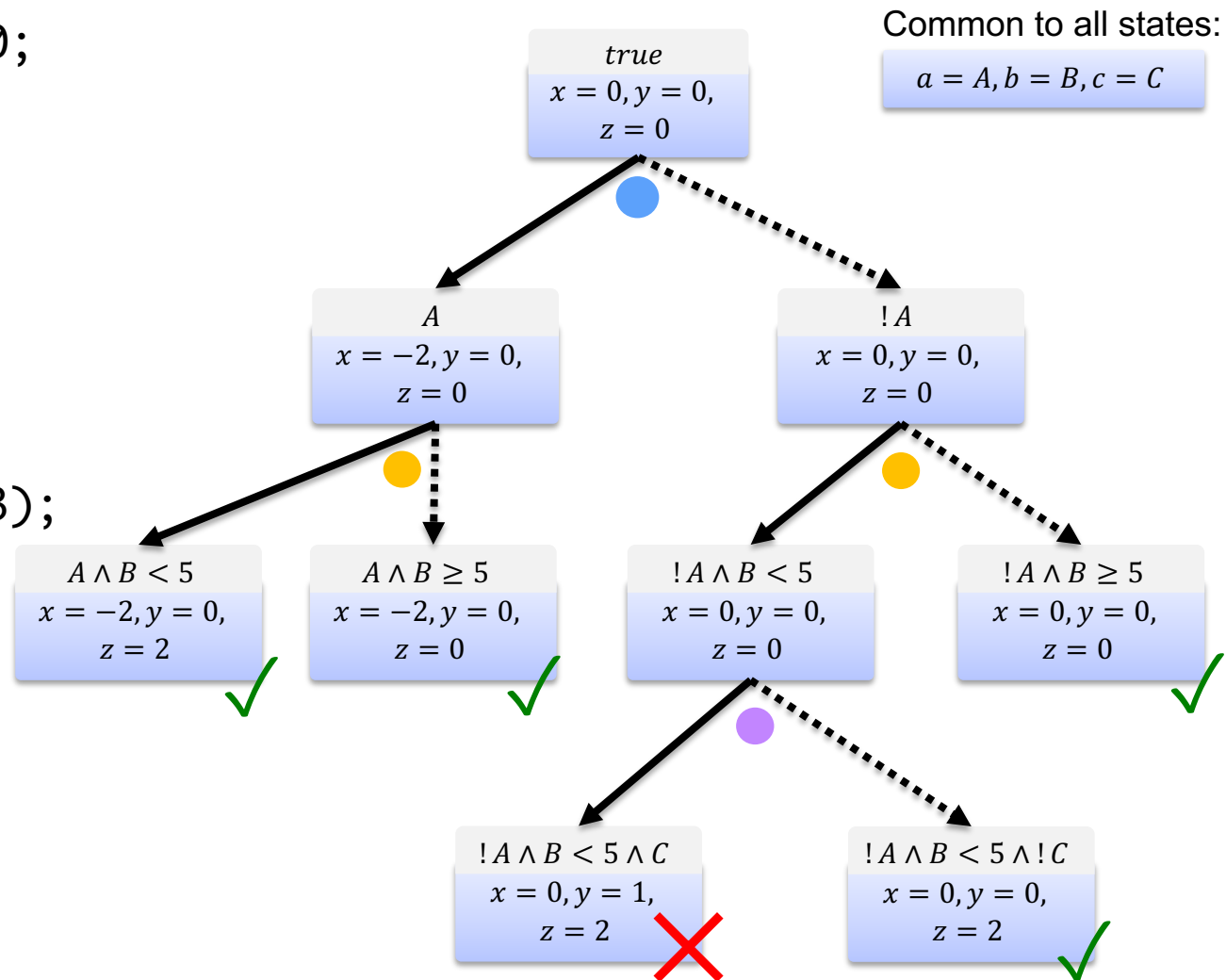
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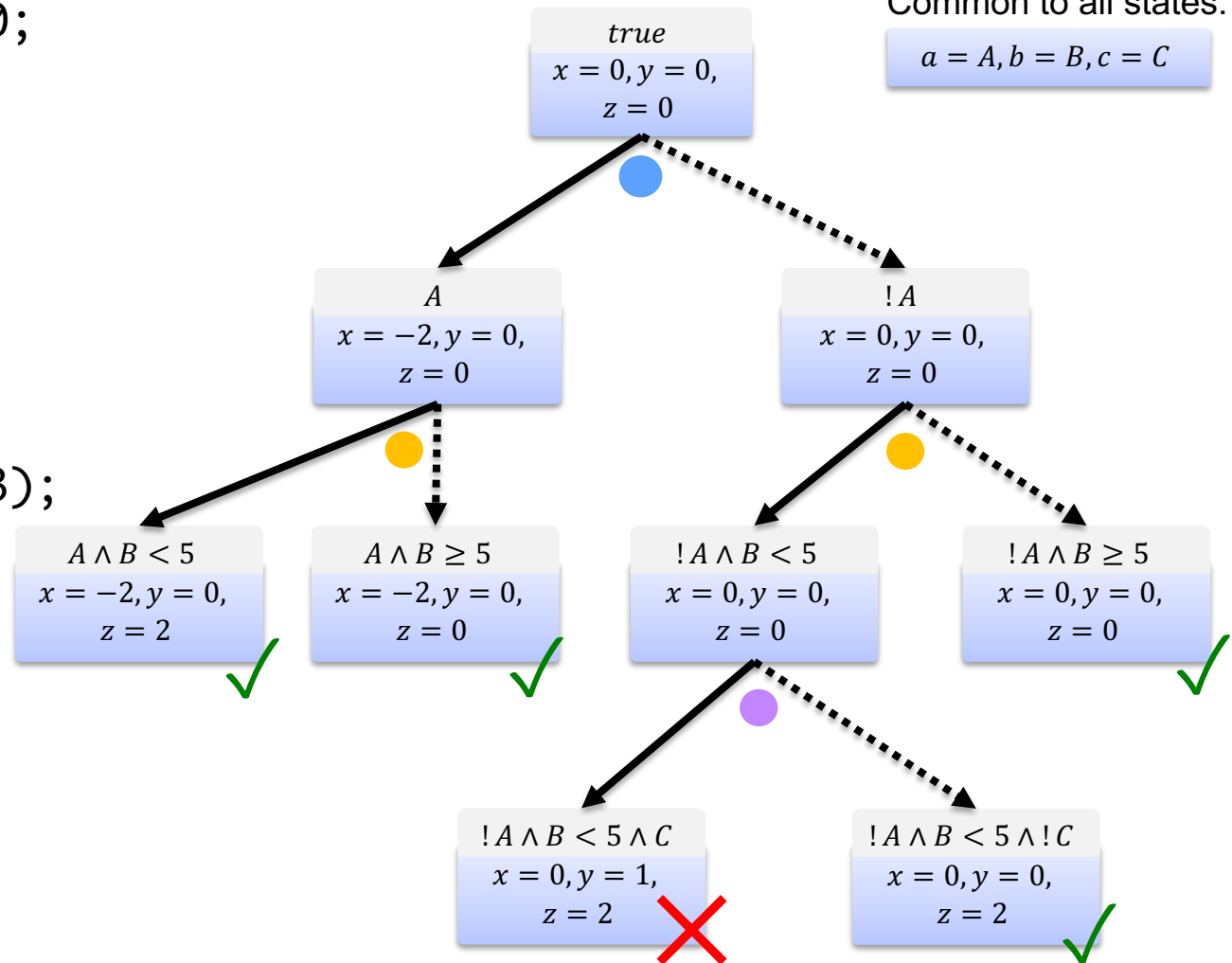


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}
assert(x+y+z != 3);
```

Common to all states:

$a = A, b = B, c = C$



$a = \text{false}, b = 2, c = \text{true} \rightarrow x = 0, y = 1, z = 2 : \text{assert}(0+1+2 \neq 3)$

Finding Bugs using Symbolic Execution

Symbolic execution enumerates paths

- Runs into bugs that trigger whenever path executes
- Assertions, buffer overflows, division by zero, etc., require specific conditions

Assertions (and other oracles) are compiled into conditionals

- Treat assertions as conditions
- Creates explicit error paths
- Bug exists if error() call is reachable

`assert x != NULL`




`if (x == NULL)
 error();`

Finding Bugs with Symbolic Execution

Instrument program with properties

- Translate any safety property to reachability
- Same as fuzzing

Division by zero

$y = 100 / x$  $\text{assert } x \neq 0$
 $y = 100 / x$

Buffer overflows

$a[x] = 10$  $\text{assert } x \geq 0 \ \&\& \ x < \text{len}(a)$

Implementation can be explicit or implicit

- explicit: like sanitizers, instrument the code with checks
- Implicit: symbolic execution engine injects extra checks at runtime

Problems of (Classical) Symbolic Execution

Some code is hard to analyze

- it is surprising how hard it can be to solve even very simple-looking constraints
- some code (e.g., crypto hash) is provably hard to invert

Path explosion

- Number of paths is exponential in the size of the program
- control flow, loops, procedures, concurrency, ...

Inputs: real code has more than just integers!

- pointers, data structures, ...
- files, databases, ...
- threads, thread schedules, ...
- sockets, ...