BU.230.730 Managing Financial Risk

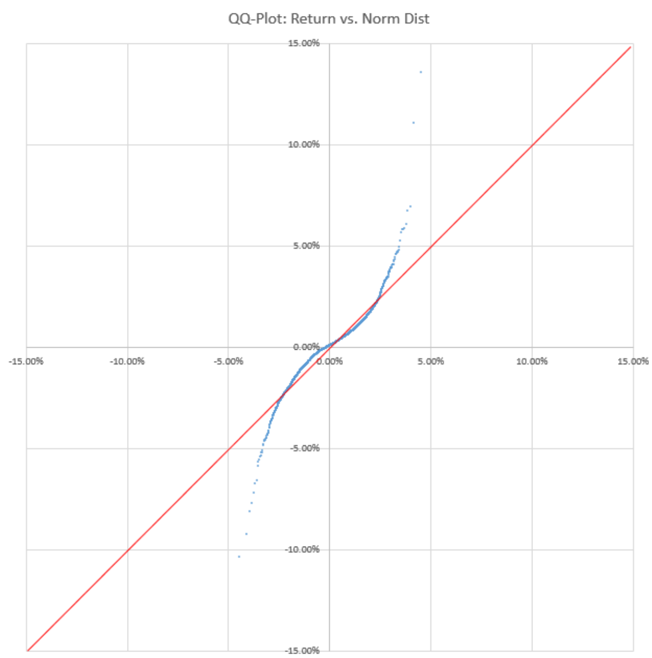
Lok Tin Kevin Chan

0 Goal:

- The end goal is to measure next week's tail risk by calculate the VaR and ES five days into the future. To do that, we need to construct the return distribution into the future. The methods of constructing the distribution include:
  - (i) Historical Simulation (HS)
  - (ii) GARCH + Monte Carlo Simulation (MCS) based on conditional normality assumptions
  - (iii) GARCH + Filtered Historical Simulation (FHS)

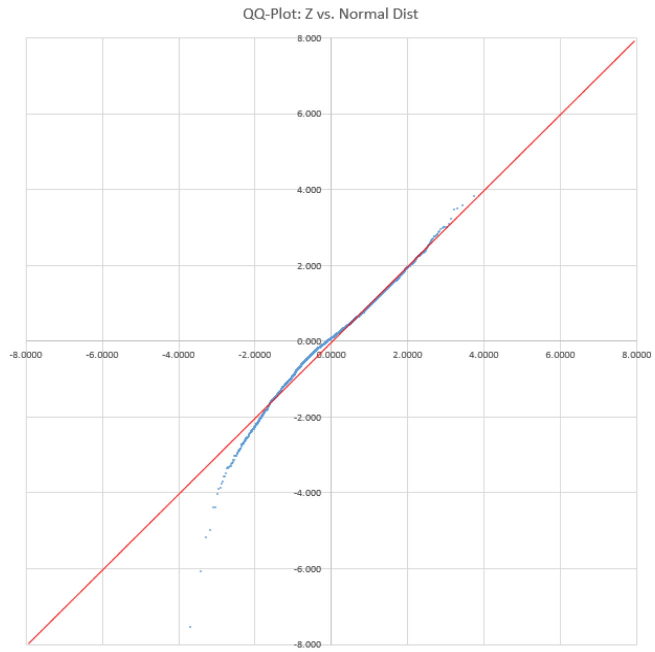
1 QQ-Plots

(i) QQ-Plot of Sample returns against a Standard Normal Distribution



Since we are only observing the pure sample returns, we are thus observing the <u>unconditional</u> distribution of returns.

Looking at the QQ-Plot, we can conclude that the <u>unconditional</u> distribution of the sample returns is not normally distributed. As the QQ-plot shows that while the sample return against a standard normal distribution is fairly symmetric, but it is leptokuritc. We observe evidence of "fat tails" of both right-hand and left-hand side relative to the normal distribution's tail (45^o line). Fatter tails means higher chance of getting extreme returns (both positives and negatives). It is interesting to note that there seem to be couple outliers, especially on the right tail (indicating a longer tail).



(ii) QQ-Plot of Sample z_t against Standard Normal Distribution

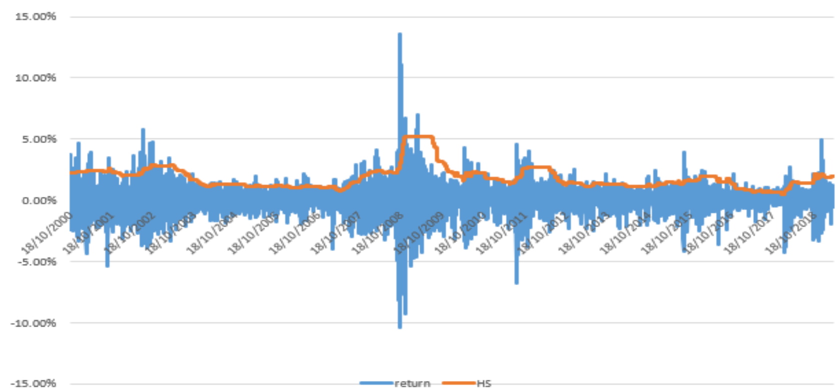
As sample z_t is the conditional standardized return (conditional on the predicted GARCH standard deviation), we are thus observing the conditional distribution of returns.

The above QQ-plot, shows that the z_t distribution is asymmetric, with fat tail to the left. It means that the standardized return (return adjusted for volatility) has much higher chance to fall into the negative regions than a normal distribution. On the other hand, the z_t distribution does not show significant fat tail on the positive side.

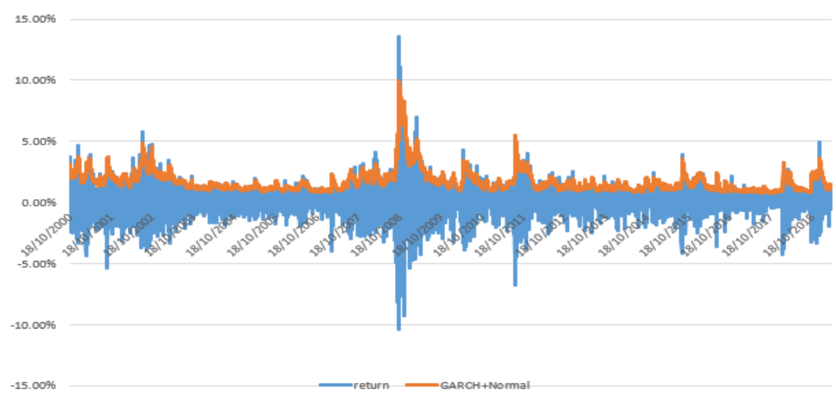
The results is important from a risk management perspective as GARCH(1,1) model is not able to fully explain the negative tail of return. If GARCH model is correct z_t should behave like a standard normal distribution.

2 $VaR_t^{0.05}$ from 18/10/2000 to 11/04/2019

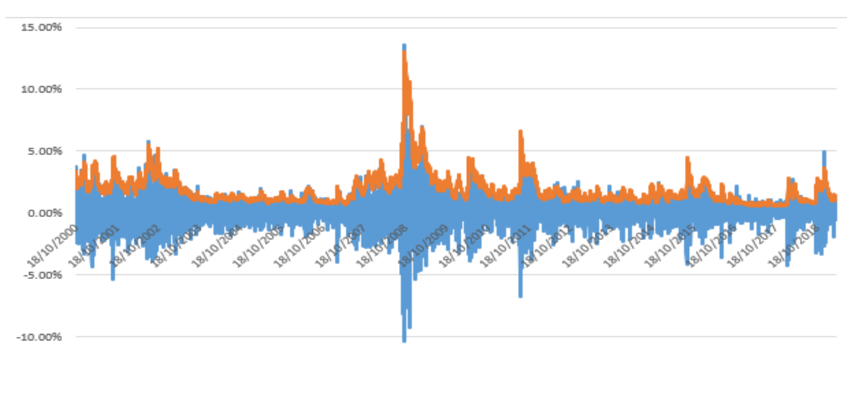
(i) Historical Simulation with m = 200
The results are given in `Cell (I230:I4877)` and imposed in Figure 3.



(ii) GARCH model + Conditional Normal assumption
The results are given in `Cell (J230:J4877)` and imposed in Figure 4.



(iii) GARCH model + Filtered Historical Simulation with m = 200
The results are given in `Cell (K230:K4877)` and imposed in Figure 5.



3 Unconditional Performances

VaR breach frequency in validation sample of the three methods is summarized in the table below:

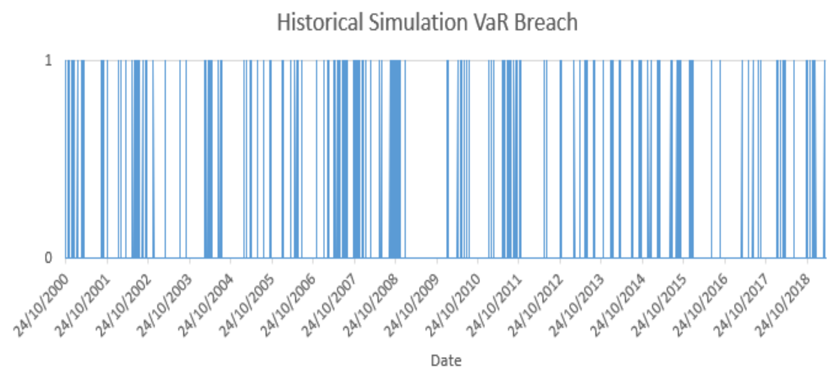
Method	Breach Frequency
Historical Simulation ($m = 200$)	5.25%
GARCH + Normal	5.36%
GARCH + FHS ($m = 200$)	4.93%

In order to compared with p, we consider the relative frequency instead of the absolute frequency of the breach. The theoretical value of the breach frequency for any given method with a $VaR^{0.05}$ should equal to 5%. From the above table, we can observe all three methods are somewhat close to the 5% theoretical value with GARCH+FHS having the closes breach frequency with only a 7 basis point lower from theoretical value which is mostly likely due to white noises. On the other hand, historical simulation and GARCH + Normal method's breach frequency resulted in 25 and 26 basis point higher than the expected relative frequency of 5% respectively.

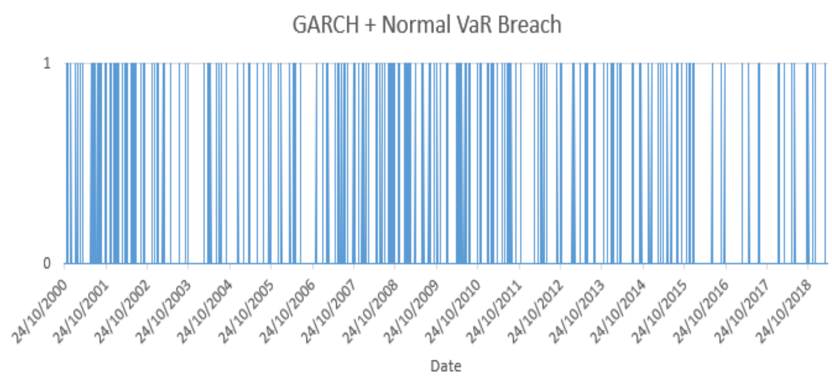
If a sample frequency is higher than the theoretical value, then VaR is too low in general.

4 Conditional Performance: VaR Breach Bar Chart

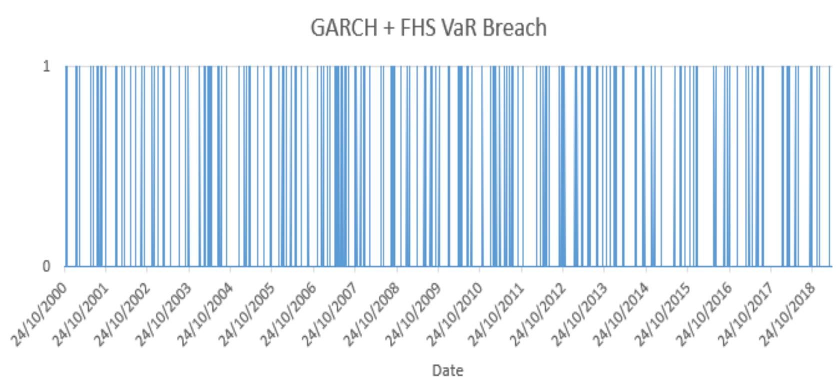
(i) Historical Simulation with m = 200



(ii) GARCH + Normal



(iii) GARCH + FHS



Eyeballing the VaR breach bar charts, we are able to observe pattern(non-randomness) of the breach in all three method:

(i) Historical Simulation:

We can clearly observe clustering effect of violations, for example during years of 2017 and 2018 there exist periods of continuous violations. While during periods of 2019 and 2010, on the other hand, there are consecutive void (no-breach) periods.

(ii) GARCH + Normal:

GARCH + Normal shows less clustering of violations compared to HS (m=200), but we are still able to observe some level of clustering during years 2001, 2002, 2008, 2009; while periods of void period during 2016, 2017, 2018.

(iii) GARCH + FHS:

GARCH + FHS shows the most randomness of violations out of all three models, but there still exist some clustering during period 2007 and sparsity of violations in 2017 and 2018.

Comparing all three methods, HS method has the highest level of clustering, followed by GARCH + Normal and then GARCH + FHS. If VaR is calculated perfectly, there should be no clustering and breaches should be even/randomly be spread throughout the period of interest, because breaches should be randomly independent events in theory.

In all three methods, we observe clustering periods and these correspond to Financial Crisis events:

(i) 2017-2018 period - Global Financial Crisis

(ii) 2001-2002 period - Internet bubble bursting

VaRs during these periods are <u>too low</u>, and of course during those periods it is a bad time to be underestimating risk because of the impact of crises. It indicates that investors (who rely on these VaR models) would be not prepared adequately for the crisis and thus suffer expected/unmanaged large losses.

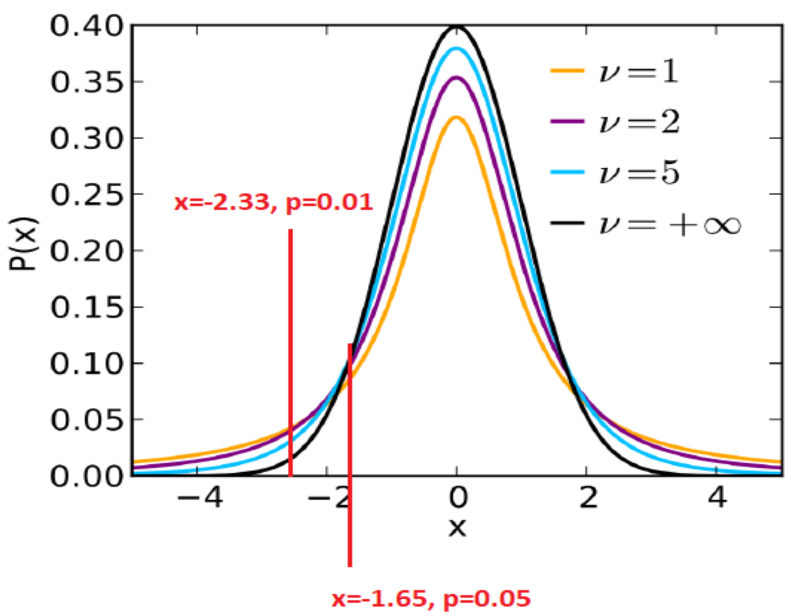
5 VaR from 0.05 to 0.01

Method	Breach Frequency
Historical Simulation ($m = 200$)	1.16%
GARCH + Normal	1.94%
GARCH + FHS ($m = 200$)	1.01%

GARCH + Normal's sample breach frequency is off the most at $VaR^{0.01}$, almost double the expected relative frequency of 1%. Reflecting a sever underestimation of VaR. As GARCH + Normal's VaR calculation is based on the assumption of normal distribution, but as the results shows the data is actually not normally distributed and should instead have a fatter and longer tail (especially on the left tail). Thus the distribution should be more leptokuritc, and thus a student t-distribution may be a better fit than normal distribution.

As demonstrate in both our QQ-Plots, we observe that the data follows a distribution with fatter and longer tail compared to normal distribution, thus compared to normal distribution, empirical distribution may be a better fit, as empirical distribution is non-parametric and includes observed extreme values in its distribution.

This observed problem is not as prominent when p = 0.05 as compared to p=0.01. The problem is originating from the long tail of the data as compared to the normal tail assumption in the GARCH + Normal model. The following graph (from lecture slides) compares the normal (black line) and fat and long tail Sudent-t distributions. It should be noted that VaR is a single-point tail measure of risk. For p=0.5, the VaR in the graph is x=0, and it is no difference between all distributions because we are not really looking at the tail. For p=0.05, x=-1.65 for normal distribution and the VaRs values are not off too much from normal since we are not looking at very far tail of the distribution. For p=0.01, x=-2.32, the problem will be prominent because we are comparing the farther tail of distribution and the 'fat-tail' effect will be more remarkable.



At p=0.05, I think that the problem will emerge again for the one-day ES since is an average of all the losses below VaR; not only the frequency, but also the severity of tail losses will be taken into account in ES. However, I think that the problem will still be hidden for the multi-day cumulative VaR since it is still a single-point tail risk measure, and as mentioned above, the lower 5% point for a fatter tail distribution may not be too different from that of a normal distribution.

6 VaR and ES with time horizon K = 5 days

(i) GARCH + MCS Normal (p=0.05)

Risk Measures	K=1	K=2	K=3	K=4	K=5
$VaR^{0.05}_{t,t+K}$	1.05%	1.54%	1.88%	2.12%	2.40%
$ES^{0.05}_{t,t+K}$	1.31%	1.91%	2.29%	2.60%	3.04%

(ii) GARCH + MCS FHS (p=0.05)

Risk Measures	K=1	K=2	K=3	K=4	K=5
$VaR^{0.05}_{t,t+K}$	1.04%	1.53%	1.81%	2.24%	2.43%
$ES^{0.05}_{t,t+K}$	1.42%	2.09%	2.50%	3.08%	3.50%

(iii) GARCH + MCS Normal (p=0.01)

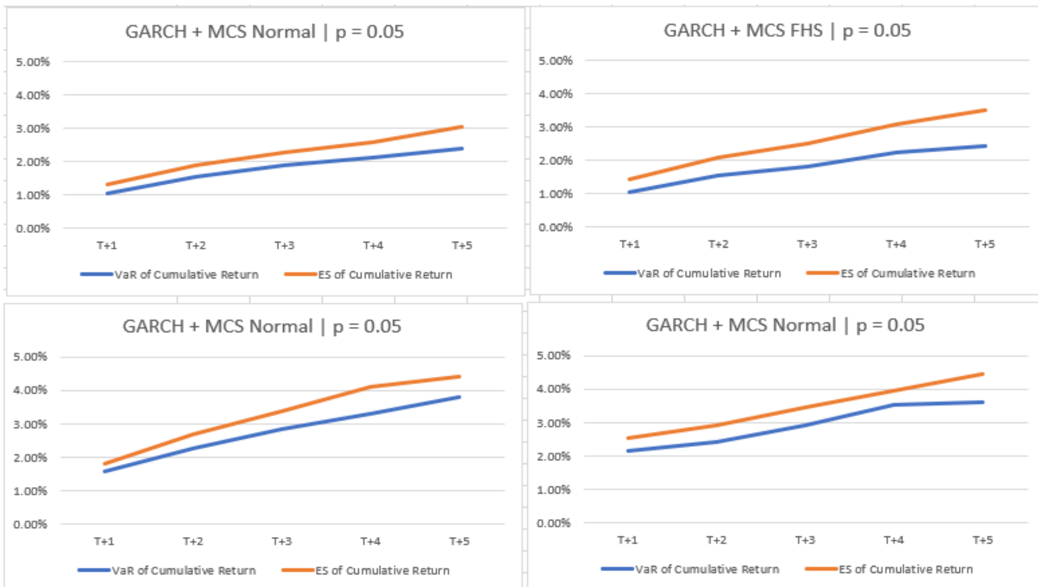
Risk Measures	K=1	K=2	K=3	K=4	K=5
$VaR^{0.05}_{t,t+K}$	1.59%	2.27%	2.83%	3.28%	3.81%
$ES^{0.05}_{t,t+K}$	1.79%	2.69%	3.36%	4.10%	4.41%

(iv) GARCH + MCS FHS (p=0.01)

Risk Measures	K=1	K=2	K=3	K=4	K=5
$VaR^{0.05}_{t,t+K}$	2.14%	2.41%	2.91%	3.51%	3.62%
$ES^{0.05}_{t,t+K}$	2.52%	2.90%	3.45%	3.96%	4.46%

Every time when the random number changes, the results change by a fairly amount. It is due to the simulated sample random fluctuations. To alleviate the problem of random fluctuations in the tails, one could

- increase the length of the simulation (now we simulate S=1000 paths, say we may increase it to S=10000 paths). By the law of large numbers, it would reduce the sample fluctuations.
- OR, we keep S=1000, but repeat the simulation experiment M (say M=500) times, and use the average results instead of one simulation result. By Central Limit Theorem, it would averaging out the sample fluctuations.



ES is greater than VaR in every horizon under BOTH models as seen in the above figure. Because ES not only look at the frequency of the lower bound losses, but also the severity (i.e., the size) of the losses. Especally for the fatter tail distributions, the extreme size losses will push up the ES.

Both VaR and ES are increasing for longer horizon. Because the prediction error for longer horizon should be higher than shorter horizon. For example the return uncertainty of a one year prediction from time t is much higher compared to a one day prediction from time t, thus equivalently the VaR and ES would be higher respectively for longer horizon.

At p=0.05, MCS gives similar VaR as compared with FHS method, but FHS gives larger ES as compared with MCS method. As explained in Question 5, the fat tail effect is not so prominent when p = 0.05 when computing VaR, but it does have considerable impact to the calculation of ES.

At p=0.01, MCS gives slightly lower VaR as compared with FHS method, but FHS gives much larger ES as compared with MCS method. But the difference at the larger K values (i.e., longer horizon) seems diminishing. The aggregation (i.e., culmulative) of returns may reduce the fat tailness of the distribution and move closer to a normal distribution.

My measures of next week's risk are:

$$VaR_{t,t+5}^{0.01} = 3.62\% \qquad\qquad ES_{t,t+5}^{0.01} = 4.46\%.$$

Finally, I will report the FHS method's results as they are, in general, larger values than the MCS, and lower the risk of under-reporting the the VaR and ES to the management. This non-parametric method matchs better the empirical distribution observed from the data.