Modeling Ethnic Conflict with Altruism

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***Abstract*—We present a game theoretic model for ethnic conflict that explicitly accounts for endogenous altruism and spite between ethnic groups. We treat ethnic groups as expected utility maximizers whose available actions are the amount of resources invested toward conflict with each of their opponents. We prove existence and uniqueness of pure Nash equilibria in the two-player case and demonstrate why, even though extensive simulations suggest that equilibria exist, traditional proof methods fail when considering more than two players.**

I. INTRODUCTION

At first blush, controlling ethnic conflict might seem to be a fool's errand. However, modeling conflict between players—in both descriptive and proscriptive capacities—is at the heart of game theory. We propose a novel model for conflict between ethnic groups motivated by sociological research and developed with mathematical precision in mind.

Sociologists have studied ethnic conflict in a game theoretic framework since Rabushka and Shepsle's seminal work in 1972 [1] that delineated numerous examples worldwide of conflicting political interests of groups drawn along ethnic lines. Their model included finite action sets and was developed primarily to support the authors' thesis that democracy often does not serve plural societies well. Since then, many authors have tried their hand at modeling human conflict. For example, Fearon and Laitin [2] identified equilibrium policing behaviors when individuals from two ethnic groups are randomly and repeatedly selected to play a prisoner's dilemma game. Additionally, Alj and Haurie [3] found equilibrium strategies for games played by families who are endowed with feelings of altruism toward their successive generations.

Particularly relevant to accurately modeling ethnic groups is the incorporation of non-strictly selfish motivations. The sociology literature records the presence of altruism as a salient cultural characteristic [4], and there are theories from sociobiology that propose that altruism may be rooted in evolution [5]. However, the analytical models of conflict—particularly ethnic—tend to incorporate any effects of observed altruism by modifying each group's selfish preferences [6]. This practice inherently introduces a coupling between a group's strictly selfish preferences and its sentiments towards its opponents that we feel need not be obfuscated within utility functions.

Decoupling selfish preferences from sentiments via linear combinations has been proposed by economists [7] and engineers [8]. In their investigation of the effects of altruism in traffic routing games, the authors of [8] benefit from equilibrium existence results for games with nonatomic players. Also, their implementation of altruism lumps players' sentiments about others into a single weighting coefficient whereas our model allows for individualized altruism sentiments among each of the atomic players. This allows for more fine-tuned control in multiplayer games where there is no reason to believe that a group's sentiments toward all other ethnicities will be uniform.

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II. MODELING ETHNIC CONFLICT

We endeavor to model the interactions between $L \geq 2$ ethnic groups as players in a noncooperative game. Each player i is endowed with a budget of $\Pi_i > 0$ resources that they may draw from and invest toward conflict with other players. We denote player i's action as a vector $\theta_i = (\theta_{i1}, \ldots, \theta_{iL})$ where $\theta_{ij} \geq 0, i \neq j$ represents the amount of resources player i decides to invest toward conflict with player j, and $\theta_{ii} \geq 0$ is the amount that player i chooses to save. Each player's action is subject to a budget constraint

$$\sum_{j=1}^{L} \theta_{ij} = \Pi_i$$

for all $i = 1, \ldots, L$.

The success of institutional oversight is modeled by the factor $I \in [0,1]$. This quantity is the probability that a supervisory institution will be successful at preventing all conflict. With probability $1 - I$, however, the institution will fail and two things will happen as a result.

First, a certain fraction $c \in [0,1]$ of each player's resources invested toward conflict will be spent. For each i, we denote player i's remaining resources after subtracting expenditures by

$$\bar{\Pi}_i(\theta_i) = \Pi_i - c \sum_{j \neq i} \theta_{ij} = (1-c)\Pi_i + c\theta_{ii}.$$

Hereafter, the dependence on θ will be implied, and we will simply use $\bar{\Pi}_i$.

Second, there will be a conflict between each pair of players i and j whenever $\theta_{ij} + \theta_{ji} > 0$. We assume that the likelihood of victory is proportional to investment, so player i will win a conflict with player j with probability

$$\frac{\theta_{ij}}{\epsilon + \theta_{ij} + \theta_{ji}},$$

where $\epsilon > 0$ is an arbitrarily small number introduced to maintain continuity.

At stake in a conflict is a fraction of a player's unspent resources. When player i wins a conflict with player j, player i receives a payout of

$$\frac{\lambda \bar{\Pi}_j}{L-1}$$

from player j where $\lambda \in [0, 1]$ represents the fraction of players' unspent resources that may be lost to opponents.

We measure players' preferences over outcomes with the expected value of their resources after conflict. Player i's strictly selfish utility is then

$$\bar{U}_i(\theta) = I\Pi_i + (1 - I)\left(\bar{\Pi}_i + \frac{\lambda}{L-1}\sum_{j\neq i}\frac{\theta_{ij}\bar{\Pi}_j - \theta_{ji}\bar{\Pi}_i}{\epsilon + \theta_{ij} + \theta_{ji}}\right).$$

Players' sentiments toward each other are an integral feature of the model. We generally dub these sentiments as altruism, however they may encompass both negative feelings (spite) or positive ones (traditional altruism). We account for altruism in a vector β_i for each player i so that its true utility is the inner product

$$U_i(\theta) = \beta_i^T \bar{U}(\theta),$$

where $\bar{U}(\theta) = \left(\bar{U}_1(\theta), \ldots, \bar{U}_L(\theta)\right)^T$ is the column vector of strictly selfish utilities and $\beta_{ij} \in [-1, 1], i \neq j$ is player i's altruism coefficient towards player j. To normalize, we restrict each player i to have $\beta_{ii} = 1$.

III. Multiplayer Analysis

Analysis of this model is interesting for a number of reasons. Because of the all-to-all nature of players' actions, the complexity of the action spaces grows with the square of the number of players. Additionally, for more than two players, the game loses many of its amenable characteristics.

As an infinite game—one in which the strategy spaces are continuous—it must have at least one mixed Nash equilibrium [9]. However, the mixed Nash solution concept is not appealing because the introduction of probability measures over actions does not have an implementable realization in a one-shot game. We would like to determine if the game possesses any pure Nash equilibria—that is—if there are any joint actions such that no player has an incentive to unilaterally deviate.

Using gradient play ([10]), we have performed extensive Monte Carlo simulations of this game with up to 10 players and have observed convergence to an equilibrium in every run. This leads us to conjecture that the game has pure Nash equilibria for an arbitrary number of players. In general, however, our model does not fit into any of the common classes of games known to have pure Nash equilibria.

It is not a supermodular game [11] because the action set for each player i,

$$B_i = \{x \in \mathbb{R}^L : \sum_{j=1}^{L} x_j = \Pi_i, x_j \geq 0, j = 1, \ldots, L\}, \quad (1)$$

is not a lattice [12], nor have we been able to discern a potential function that would indicate that the model is any kind of potential game [13].

Perhaps most surprisingly, however, is that for three or more players, the game lacks even the quasiconcavity property that would allow us to apply the existence results of [14] and [15].

For example, with three strictly selfish players and $I = 0.6813$, $c = 0.9930$, $\lambda = 0.8952$, with endowment

$$\Pi = \begin{bmatrix} 1 \\ 45 \\ 1 \end{bmatrix}$$

there are two points in the joint action space that defy the quasiconcavity definition [16]. For fixed actions by players 1 and 2:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0.0475 & 0.5837 & 0.3689 \\ 1.3871 & 14.1044 & 29.5085 \end{bmatrix}, \quad (2)$$

we plot the utility of player 3 for all convex combinations of the two actions

$$\theta_3^{(1)} = \begin{bmatrix} 0.4385 & 0.1806 & 0.3809 \end{bmatrix}$$
$$\theta_3^{(2)} = \begin{bmatrix} 0.1377 & 0.7823 & 0 \end{bmatrix}$$

in Fig. 1.

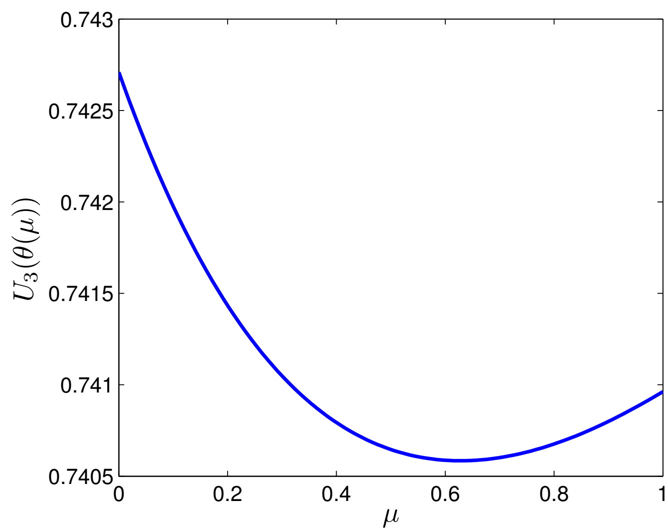

Fig. 1. Example of non-quasiconcavity in player 3's own actions. Here $\theta_1(\mu)$ and $\theta_2(\mu)$ are constant as given in (2), and $\theta_3(\mu) = \mu\theta_3^{(1)} + (1-\mu)\theta_3^{(2)}$. The utility of player 3 is not quasiconcave because for $0.4 \leq \mu < 1$ we have that $U_3(\theta(\mu)) < \min\{U_3(\theta^{(1)}), U_3(\theta^{(2)})\}$.

Without an explicit existence result for games with three or more players, we restrict further analysis to the two-player case where sharper results can be obtained.

IV. Two-Player Analysis

When considering the model with $L = 2$ players, the game can be shown to have the desirable properties of a unique pure Nash equilibrium.

A. Existence of Pure Nash Equilibria

With two players, the game possesses the concavity property which endows it—via the Kakutani fixed point theorem—with at least one pure Nash equilibrium [14], [15].

Proposition 1: The two-player game has at least one Nash equilibrium.

Proof: We proceed by checking the conditions required in [15] are satisfied. First, action sets (1) are closed subsets of the real line $[0, \Pi_i]$ for $i = 1, 2$ and are therefore compact and convex.

Each utility function, $U_i(\theta)$, $i = 1, 2$, is the sum of affine and continuous fractional terms and is therefore continuous.

Although only quasiconcavity in one's own action is required, we show that for a fixed opponent's action, the utility function of each player is strictly concave. Consider player 1's utility (the analysis for player 2 is symmetric):

$$U_1(\theta) = I(\Pi_1 + \beta_{12}\Pi_2) + (1 - I) \Big[\Pi_1 + \beta_{12}\Pi_2 - c(\theta_{12} + \beta_{12}\theta_{21}) + \lambda(1 - \beta_{12}) \frac{\theta_{12}\Pi_2 - \theta_{21}\Pi_1}{\epsilon + \theta_{12} + \theta_{21}}\Big].$$

By fixing the action of player 2, θ_{21}, we see that the second derivative with respect to θ_{12} is strictly negative,

$$\frac{\mathrm{d}^2 U_1}{\mathrm{d}\theta_{12}^2} = -\lambda(1 - I)(1 - \beta_{12}) \frac{\epsilon\Pi_2 + \theta_{21}(\Pi_1 + \Pi_2)}{(\epsilon + \theta_{12} + \theta_{21})^2} < 0,$$

barring certain boundary cases for the game parameters. Thus the game has at least one Nash equilibrium in pure strategies. ■

Some special attention ought to be paid to the corner cases in the existence proof. When $\lambda = 0$, both players' utilities become linear in their own actions. Similarly, whenever $\beta_{ij} = 1, i \neq j$, player i's utility becomes linear in its own action. Both of these cases preserve the strictly concave conclusion. Lastly, if $I = 1$, then the game is null and players receive identical utilities for all actions. These boundary cases are of little importance if we aim to model real-world scenarios where the equilibrium actions are not trivial.

B. Uniqueness of Nash Equilibrium

Games that posses a pure Nash equilibrium are attractive for several reasons. Principally, they are considered "solvable" in that rational players, when presented with a pure Nash equilibrium, will not elect to deviate. Even more prized, however, are games with a unique pure Nash equilibrium. These are games in which there is only one "solution" and for which convergent numerical methods do not depend on initial conditions.

Proposition 2: The two-player game has exactly one pure Nash equilibrium.

Proof: We shall show that the game satisfies the sufficient condition for diagonal strict concavity in Theorem 6 in [17]. For all actions, we need to show that there exists a real vector $r > 0$ such that the symmetric version of Rosen's pseudo-gradient is negative definite:

$$D(\theta, r) = G(\theta, r) + G^T(\theta, r) < 0,$$

where for the two-player game, the pseudo-gradient looks like

$$G(\theta, r) = \begin{bmatrix} r_1 \frac{\partial^2 U_1}{\partial \theta_{12}^2} & r_1 \frac{\partial^2 U_1}{\partial \theta_{12}\partial \theta_{21}} \\ r_2 \frac{\partial^2 U_2}{\partial \theta_{12}\partial \theta_{21}} & r_2 \frac{\partial^2 U_2}{\partial \theta_{21}^2} \end{bmatrix}.$$

The first leading principal minor of $D(\theta, r)$ is negative for all actions and all $r_1 > 0$:

$$[D(\theta, r)]_{1,1} = -4r_1(1 - \beta_{12}) [\epsilon\Pi_2 + \theta_{21}(\Pi_1 + \Pi_2)] \frac{\lambda(1 - I)}{(\epsilon + \theta_{12} + \theta_{21})^3} < 0.$$

Furthermore, the determinant of the second leading principal minor,

$$\left(\frac{\lambda(1 - I)}{(\epsilon + \theta_{12} + \theta_{21})^3}\right)^2 \Big\{16 r_1 r_2 (1 - \beta_{12})(1 - \beta_{21}) [\epsilon\Pi_2 + \theta_{21}(\Pi_1 + \Pi_2)] [\epsilon\Pi_1 + \theta_{12}(\Pi_1 + \Pi_2)] - [\epsilon(\Pi_1 + \Pi_2) + (\theta_{21} - \theta_{12})(\Pi_1 + \Pi_2)]^2 [r_2(1 - \beta_{21}) - r_1(1 - \beta_{12})]^2\Big\},$$

is positive for any r that satisfies

$$r_2 = \frac{1 - \beta_{12}}{1 - \beta_{21}} r_1 > 0.$$

■

A careful treatment of the corner cases is given in Section V with the help of the best response functions.

With uniqueness established, we are able to describe exactly how the model parameters affect equilibrium actions.

V. PARAMETER ANALYSIS

The amount of resources invested toward conflict at equilibrium is highly dependent on the values of the various parameters of the model. For two players, we can express the best response of player 1 in closed form

$$BR_1(\theta_2) = \begin{cases} -\epsilon + \sqrt{(1 - \beta_{12}) \frac{\lambda}{c} \Pi_2 \epsilon}, & \theta_{21} = 0 \\ \min\left\{\Pi_1, -\theta_{21} + \sqrt{\tau_1}\right\}, & 0 < \theta_{21} \leq \tau_1 \\ 0, & \theta_{21} > \tau_1 \end{cases}$$

where $\tau_1 = (1 - \beta_{12}) \frac{\lambda}{c} (\Pi_1 + \Pi_2)$ is a threshold value. The symmetric result holds for $BR_2(\theta_1)$.

Using the closed form of the best response functions, we can look for fixed points in the set of joint actions for the corner cases mentioned in Section IV-B. In the cases where $\lambda = 0$ or $\beta_{12} = \beta_{21} = 1$, both players' best response to any action of their opponent is to invest nothing toward conflict, so $\theta_{12} = \theta_{21} = 0$ is the unique equilibrium.

If $I = 1$, then players are indifferent across all their actions, and any joint action is an equilibrium. However, as discussed in Section IV-A, this case is utterly uninteresting.

In the case where one player is perfectly altruistic toward its opponent ($\beta_{12} = 1$, say), but the opponent does not exactly reciprocate ($\beta_{21} < 1$), then we see that it is always in player 1's best interest to invest nothing in conflict. However, the less altruistic player then has an incentive to invest a tiny amount towards conflict resulting in a unique equilibrium at

$$\theta_{12} = 0,$$
$$\theta_{21} = -\epsilon + \sqrt{(1 - \beta_{21})\frac{\lambda}{c}\Pi_1 \epsilon}.$$

The perfectly altruistic player does not mind guaranteed loss in battle because it derives just as much utility from its opponent success as it would from its own.

Fig. 2 shows the best response curves for the symmetric game where $\Pi_1 = \Pi_2 = 10$, $\frac{\lambda}{c} = 1$, and no altruism (β is the identity matrix). The unique equilibrium in this case is for both players to invest half of their budget towards conflict and save the other half.

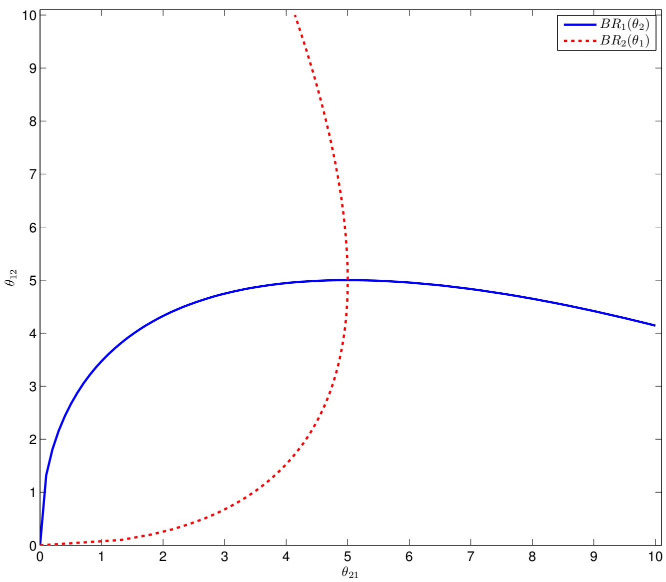

Fig. 2. Best response curves for the symmetric two-player game with $\Pi_1 = \Pi_2 = 10$, $\frac{\lambda}{c} = 1$, and $\beta_{12} = \beta_{21} = 0$.

As one might expect, unleveling the playing field by endowing one player with a larger budget will skew the actions considerably. Letting all other parameters remain the same, Fig. 3 shows the best response curves for the game where $\Pi_1 = 100$ and $\Pi_2 = 10$. The weaker player must commit all of its resources to conflict in equilibrium whereas the stronger player commits only a fraction—albeit still greater than when the players were evenly matched.

The ratio of the parameters $\frac{\lambda}{c}$ is an important factor in determining the equilibrium investments. Conceptually, it represents the relative weights behind risking unspent resources (larger λ) and disincentives to invest in conflict (smaller c). Fig. 4 illustrates how the crossing of the best response curves shifts closer to the origin as $\frac{\lambda}{c}$ decreases, and away (toward higher levels of investment) as $\frac{\lambda}{c}$ increases.

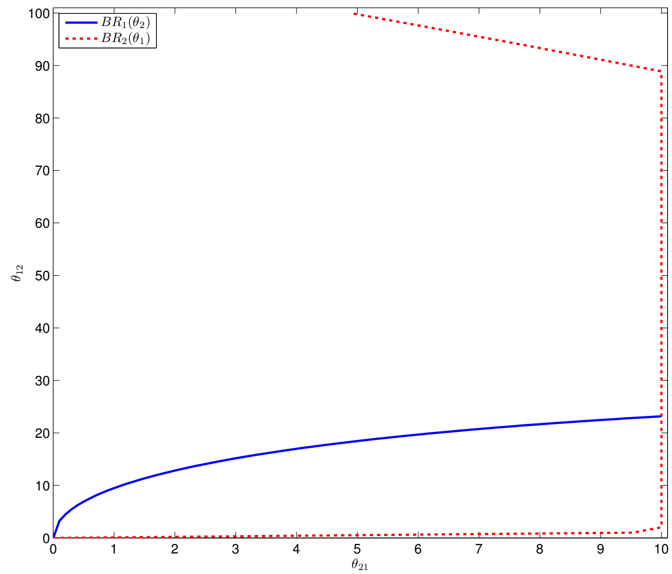

Fig. 3. Best response curves for the asymmetric two-player game with $\Pi_1 = 100$, $\Pi_2 = 10$, $\frac{\lambda}{c} = 1$, and $\beta_{12} = \beta_{21} = 0$.

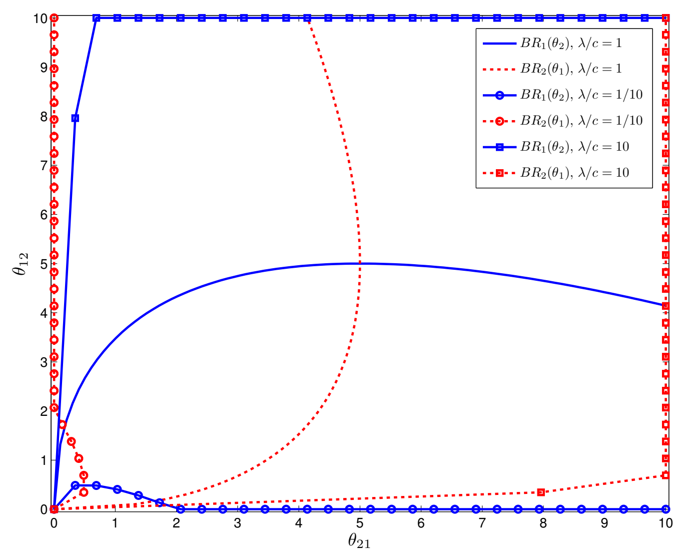

Fig. 4. Best response curves for three risk ratios, $\frac{\lambda}{c}$, in symmetric two-player games.

The most salient feature of this model is its explicit incorporation of altruism. By considering various combinations of altruism between two players, we see the model predict likely equilibria in Fig. 5. When both players hold positive altruism towards one another, equilibrium investment in conflict is less than in the symmetric non-altruistic case. Analogously, both players feeling spiteful leads to an increase in equilibrium investments.

In the case where one player is altruistic and the other spiteful, we see in Fig. 5 that the altruistic player (player 1) "turns the other cheek" at equilibrium and endures a much higher investment from its opponent.

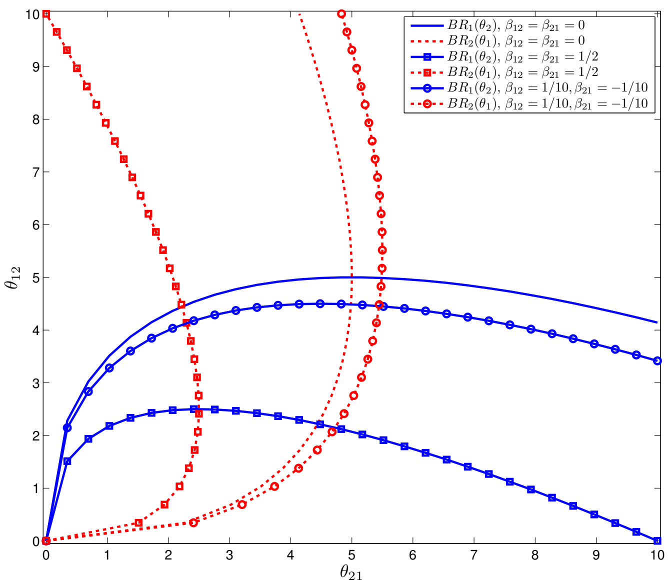

Fig. 5. Best response curves for the two-player game with $\beta_{12} = 0.5$, $\beta_{21} = 0.5$, equal budgets, and $\frac{\lambda}{c} = 1$.

VI. CONCLUSIONS

We have constructed a sociologically motivated model for ethnic conflict that explicitly incorporates endogenous quantities of altruism and spite. We provide sharp Nash equilibrium existence and uniqueness results for cases with two players and demonstrate the non-triviality of the model by showing that existence results for the multiplayer cases that rely on traditional proof methods like supermodularity, potential functions, and quasiconcavity are not applicable. By adjusting the exogenous model parameters, we observe expected shifts in equilibrium behavior in the two-player game.

The prospects for controlling the level of conflict between ethnic groups are limited to one's ability to affect the model parameters. While not surprising, the clearest method for reducing investment in conflict would be to encourage higher levels of altruism amongst the players.

Future topics of research include investigation of the existence of equilibria in the multiplayer case. The authors believe strongly that equilibria must be guaranteed to exist as we have observed ubiquitous convergence of gradient play in extensive simulations. Furthermore, the accuracy of the model might benefit from analyzing the equilibrium policies of repeating the one-shot game presented here. This method might also improve appreciation for mixed equilibria in the one-shot game.

An unfortunate but plausible common thread can be observed in nearly all the equilibria of the two-player game: Investment toward conflict is nonzero. While this does not strictly imply that violence is inevitable, it does lend credence to Bueno de Mesquita's observation that "[even] those cherishing peace and preferring diplomatic solutions over violent solutions to international disputes, however, must at least act as if they are prepared to fight" [18].

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