# Appendix B

# MATLAB Overview

This appendix provides a broad overview of a few choice features of MATLAB. It should be pointed out, however, that MATLAB is both a programming language as well as a software suite. Thus it is just as appropriate to refer to "MATLAB code" as to "open MATLAB."

The choice of which features to include here primarily results from their usefulness in the 351L laboratory. Deeper help on these topics as well as the myriad of other functions in MATLAB can be obtained from a number of outside references, including those in the bibliography.

## B.1 Basic MATLAB Interface

The MATLAB desktop consists of four basic windows:

- Command Window
- Command History
- Current Directory
- Workspace

A fifth important window—the Editor—can be opened by typing edit at the command prompt (>>). When running experiments with many windows open, it is sometimes helpful to dock the Editor into the MATLAB desktop. The Editor window has an icon similar to  $\P$  that can be used to dock it with the main MATLAB desktop.

The Command Window is useful for executing files or making quick calculations. Lengthy processes are better suited to be written in a script—more on that later. The Command Window may be cleared by issuing the command clc.

The Command History logs all of the actions executed in the Command Window as well as sign-in and exit times from MATLAB. It is most useful for finding commands issued previously if there is a lot of output cluttering the Command Window. Also, you may select several lines from the command history and drag them into the Editor if you wish to save them into a script.

The Current Directory is familiar to Windows users. It should be noted, however, that MATLAB can only execute scripts from the Current Directory (and subdirectories) and the MATLAB root directory. Thus if you wrote code for Lab 1 and wanted to run it in your Lab 2 directory, it would be easiest to copy-paste the file.

The Workspace lists all of the variables (and their type) that are currently active. To clear the Workspace, use the command clear all. To delete a certain variable (call it var), use clear var. Also note that clearing the Command Window does not clear the Workspace and vice-versa.

#### B.1.1 Statements

Statements are entered in the Command Window at the command prompt: >>. The comment symbol in MATLAB is the percent sign: %. When MATLAB encounters this symbol, it ignores the remaining text on that line. Comments are useful for creating "headers" for scripts. It is recommended that you include some authorship information in every script you write.

```
>> % Kane Warrior
>> % Lab 1, Section 2
>> % Data Analysis and Plotting Script
>> % 17 July 2008
```

Results from statements issued are by default stored in the single variable named ans. This variable will show up in the Workspace and appear in the Command Window:

Pressing the *Enter* key causes MATLAB to execute the statement in the command prompt.

If a statement is too long for a single line, one may issue a carriage return by typing three periods (...) and then the *Enter* key.

Note: From now on the explicit typing of **<Enter>** will be assumed to be understood and therefore omitted.

Multiple statements may be issued on one line by separating them with a comma. The statements are executed from left to right.

Variables are user-defined entities that allow the saving and recalling of information throughout a MATLAB session. Variables may be named according to certain conventions, the most important of which is that they must begin with a letter. All variables currently active appear in the Workspace. A quick list of all of the variable names can be produced in the Command Window with the who command. A more complete table with variable names, dimensions, sizes, and classes is obtained by using the plural form: whos.

Statements that are terminated with a semicolon (;) are executed, but the results are not shown in the Command Window. Any variables assigned are saved and may be displayed by typing that variable's name subsequently:

```
Example Code:

>> a = 25; b = sqrt(a) + 2.5;

>> a, b
a =

25
b =

7.5000
>>
```

### B.1.2 Help

MATLAB's biggest advantage over other programming languages is its library of functions. To aid users in finding a function, MATLAB has two *very* useful commands.

To see a list of functions that have a certain word in the title or description, use the lookfor command.

```
>> lookfor sinc
DIRIC Dirichlet, or periodic sinc function
SINC Sin(pi*x)/(pi*x) function.
INVSINC Desired amplitude and frequency response for invsinc filters
>>
```

To get explicit information on a single function whose name is known, the help function is extremely useful. It explains the syntax and purpose of every function in the MATLAB library.

```
>> help sqrt
SQRT Square root.
SQRT(X) is the square root of the elements of X.
Complex results are produced if X is not positive.

See also sqrtm.
>>
```

#### B.1.3 M-Files

Files with extension .m are executable in MATLAB. There are two flavors of m-files: Script and Function. All m-files contain plain text, and as such can be edited with any text editor. MATLAB provides a customized text editor, as discussed above. However, you may read, write, and edit m-files with any plain text editor (such as Notepad, WinEdt, emacs, etc.).

## Scripts

Scripts are helpful because they can lump dozens or even thousands of Command Line statements into a single action. The conventions for naming scripts are very similar to those for naming variables:

- Do Not begin filenames with numbers
- Do Not include punctuation anywhere in filenames

• **Do Not** use a filename that is already used for a MATLAB function or an existing variable

There are a few useful shortcut keys when writing and debugging scripts. The F5 key saves and executes the entire active script. When dealing with long files, it is often useful to execute only the code you have just written. The F9 key executes only the text that is currently highlighted.

#### **Functions**

There are thousands of functions in the MATLAB library. However, one may find it necessary to write a new function—often a collection of other functions—to suit one's specific needs. Just like MATLAB native functions, user-defined functions have a strict yet simple syntax. To define a new function, there are four essential components that must be the first executable line in the m-file:

- The MATLAB command function
- Output variables
- The user-defined function name
- Input variables

Output variables are contained in [square brackets] and separated by commas. Input variables are contained in (parentheses) and also separated by commas. Comments may appear above the line containing the function command, and these will appear when the help command is issued in relation to this new function.

```
- Example Code: -
% Kane Warrior
% 17 July 2008
% VECTOR function
% INPUTS:
% x
        A complex number
%
% OUTPUTS:
% m
        Magnitude of x
% theta Angle of x in degrees counterclockwise from
            positive Real axis
%
function [m,theta] = vector(x)
m=abs(x);
                            %Use the native function
                                 % abs.
theta=angle(x)*180/pi;
                            %Use the native function
```

```
% angle and convert
% from radians to
% degrees.
```

### B.1.4 Storage and Retrieval Commands

Data can be stored as variables, which can then be saved to memory. The syntax is:

```
save filename <variables>
```

This saves the variables listed in *<variables>*—separated by spaces—as a file called filename.mat in the current directory. If no variables are specified, all of the variables in the Workspace are saved.

Upon clearing the Workspace, the variables can be restored by issuing load *filename*. This will restore the variables saved in filename.mat to the workspace and overwrite any existing variables with the same name.

### **B.1.5** Numeric Format

There are several types of numbers recognized by MATLAB. So far we have only dealt with rationals. We will, however, need more general numbers (such as non-repeating decimals and complex numbers).

The default setting in MATLAB is for numbers to be truncated after four digits to the right of the decimal and nine to the left. After this, the number will be presented in scientific notation. If this is not convenient, the command format long will always give 15 digits to the right of the decimal. To revert, use format short.

#### Floating Point Numbers

Floating point numbers represent real numbers. They are how computers interpret what we know as 2, 1/2, and  $\log 2$ . Floating point numbers come in various precisions, the most common of which are single and double. The default for MATLAB is double precision. This should not become an issue in this course, so we will leave it at that.

As an example, let us issue the following commands:

```
>> s = 0.5555;

>> 1 = 0.55555

1 =

0.5555

>> format long

>> s - 1

ans =

-4.999999999999449e-005
```

### Complex Numbers

One feature that sets MATLAB apart from other languages is how it deals with complex numbers. Upon startup, the variables  $\mathbf{i}$  and  $\mathbf{j}$  are predefined to both be  $\sqrt{-1}$ . Using these variables, we can define complex numbers. MATLAB, in turn, saves them in memory as a single variable with a real and a complex part. For example, we create a complex variable below.

```
Example Code:

>> c = 3 + 1j
c =
3.0000 + 1.0000i
```

The Workspace resulting from the previous section and this action is shown in Figure B.1.

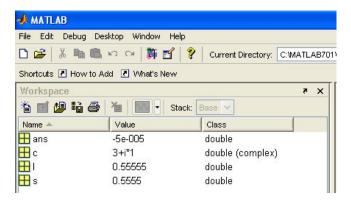


Figure B.1: Example workspace showing floating point real and complex numbers

The built-in functions listed in Table B.1 are particularly useful when dealing with complex numbers.

Function	Example
real(c)	3
imag(c)	1
abs(c)	3.1623
angle(c)	$0.3218^{a}$

Table B.1: Common built-in MATLAB functions dealing with complex arguments. Here the examples use c=3+i.

One must be careful because the variable names i and j can be reassigned by the user to be something other than  $\sqrt{-1}$ . Say, for example, that on

<sup>&</sup>lt;sup>a</sup>As with all built-in MATLAB angular functions, the default is always radians.

lines 10-20 in a script there is a for loop that uses i as its index and runs up to i = 100. Then on line 25, the script tries to execute complex variable assignment x = 4+5\*i;. Of course, unless the variable i was cleared between lines 20 and 25, the script will create x = 504 and not the desired x = 4.0000 + 5.0000\*i. The moral is to be very careful when using i and j as non-native variables.

#### Arrays

Arrays are the heart and soul of MATLAB (MATrix LABoratory). Arrays are groups of numbers that travel around together. They may be grouped so they stay in the right order, so that they perform some mathematical operation, or any other reason. We are all familiar with one- and two-dimensional arrays. We already know them as *vectors* and *matrices*, respectively. However, MATLAB can handle higher-dimensional arrays; for these it is easier to think of every two-dimensional object as being a matrix stacked among others.

Arrays are defined using square brackets: []. Entries in a row may be separated by either a comma or whitespace (space bar). A row is terminated and a new one begun with either a semicolon or a line break within the brackets.

In MATLAB, array dimensions are always listed with rows first and columns second. For example, if we wanted to call out the 6 from the variable TwoDimArray, we would enter

```
>> TwoDimArray(2,3)
ans =
```

6

This is because the 6 lies in the second row and the third column. Some helpful array creation and basic modification commands are listed in Table B.2.

Function	Description	Example
ones(m,n)	Create an $m$ by $n$ array	ones(2,3) = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$
	filled with ones	0 0
zeros(m,n)	Same as ones, but filled with zeros.	zeros(3,2)= 0 0
	micd with zeros.	$\begin{bmatrix} 0 & 0 \\ 1 & \end{bmatrix}$
transpose(A) or A.'	Transpose the array $A$	rvec.' = 2/3
size(A)	Return the dimensions of $A$ in an array	4   size(cvec) = 4 1
length(A)	Returns the maximal dimension of $A$	length(rvec) = 4
:	Used to span indices, "to"	TwoDimArray(1:2,2:3) $= \begin{array}{c} 2 & 3 \\ 5 & 6 \end{array}$

Table B.2: Common built-in MATLAB functions useful for defining and manipulating arrays. The examples use the variables created in Section B.1.5.

We can also make an array by concatenating smaller arrays. Special attention must be paid to the dimensions in order to make a valid concatenation.

```
Example Code:

>> h = [1 2 3], k = [4; 7], m = [5 6; 8 9]
h =

1 2 3
k =

4
7
m =

5 6
8 9
>> n = [h; k m]
n =

1 2 3
```

```
4 5 6
7 8 9
```

Vectors, as we have seen, are one-dimensional examples of arrays. To create vectors with equally spaced elements, the colon (:) notation is helpful. The syntax is  $start\ value:\ step\ size:\ end\ value.$  The default step size is 1.

```
Example Code:

>> c = 2:5, d = 9:-1:3

c =

2  3  4  5

d =

9  8  7  6  5  4  3
```

A useful trick is to use a vector to call indices from another vector. For example, using the above variable c, we can call the second through the fifth values from d in the following way:

```
Example Code:

>> d(c)
ans =
8 7 6 5
```

The following are more complex examples of arrays and indices.

```
— Example Code: —
>> m = [1.5 -2.4 3.5 0.7; -6.2 3.1 -5.5 4.1; ...
1.1 2.2 -0.1 0]
m =
    1.5
            -2.4
                    3.5
                            0.7
    -6.2
            3.1
                    -5.5
                            4.1
    1.1
            2.2
                    -0.1
>> n = m(1:2,2:4), o = m(:,1:2), p = m(2,:)
                    0.7
    -2.4
            3.5
    3.1
            -5.5
                    4.1
            -2.4
    -6.2
            3.1
    1.1
            2.2
    -6.2
            3.1
                    -5.5
                            4.1
```

#### Strings

Strings (or sometimes character strings) are very much like vectors. The difference is that the entries are characters instead of numbers. Strings are defined by single quotation marks.

```
Example Code:

>> str = 'Signal and System Analysis';
>> disp(str)
Signals and System Analysis
>> str(1:6)
Signal
```

Strings are useful when we wish to load several files that have been given sequential names (like *data1.dat*, *data2.dat*, etc.). In this case, a **for** loop may be created that executes commands on the string **data** concatenated with the index 1, 2, etc.

#### Logicals

When MATLAB performs a logical operation (see Section B.2.2), the output is always binary in nature. That is, the output is an array of 1's and 0's. This output is then dual-purpose: It contains numbers in indexed positions (like arrays) but it also contains "true/false" values in indexed positions. In this sense, logical arrays are very handy because they serve dual purposes and may be processed very quickly.

There is a key difference between arrays and logicals regarding indexing. When using an array to call the index, the value of the elements of the array is important. When using logicals, only the position of the "true" elements is important.

```
Example Code:

>> a = 2:1:5, b = a>3
a =

2  3  4  5
b =

0  0  1  1
>> a(b)
ans =

4  5
>> a([3:4])
ans =

4  5
>> notlogical = [0 0 1 1];
>> a(notlogical)
```

```
??? Subscript indices must either be real positive
integers or logicals.
```

The last line is an error because MATLAB thinks we are looking to create an array with the first two elements being the  $0^{th}$  element of **a** and the third and fourth elements as the  $1^{st}$  element of **a**. When MATLAB looks for the  $0^{th}$  element of **a**, it gets confused because indices can only be positive integers  $(1,2,3,\ldots)$ .

#### Cells

Cells are convenient ways to store objects of several different types. They are created and their indices are called with {curly braces} and act very much like arrays. However, a one-dimensional cell may have an entire matrix at position 1 but a single character at position 2. Cells do not have a clear analog like arrays do to matrices, so it may be more convenient to simply think of them as "multimedia" arrays or the mix-plate of MATLAB.

#### **Special Numbers**

In addition to the imaginary numbers i and j, MATLAB has two other special predefined variables. They are pi and Inf. The variable pi is MATLAB's closest estimate at the irrational number  $\pi$ . The variable Inf represents infinity and results from actions like dividing by zero or that result in a number larger than MATLAB can handle.

Furthermore, operations like  $\frac{0}{0}$  and  $\sin \infty$  produce undefined results and are represented by NaN, which stands for "not a number."

## B.2 Operations

The operations in the above sections have been minimal and intuitive. Here we look at some of the more advanced operations MATLAB can perform.

## **B.2.1** Arithmetic Operations

MATLAB defines all arithmetic operations in matrix terms. Note that a single number may be either a scalar or a one-by-one matrix, but its operations are all treated as matrix ones. Issuing help arith and help slash will list the available arithmetic operations.

As one would expect, the *order* of operations is often important. We have seen that MATLAB executes commands from left to right. However, within a command, MATLAB follows the familiar *PEMDAS* (parentheses, exponents, multiplication and division, then addition and subtraction) order.

It is also important to observe that the operators "slash" / and "backslash" \ represent two different matrix operations. For example, B/A is right division and represents  $BA^{-1}$  whereas A\B is left division and represents  $A^{-1}B$ . We know that these may be two *very* different quantities, so it is important to differentiate between them.

To use an operator on an element-by-element basis rather than its matrix math sense, use the modifier of a period (.) prior to the operator. This tells MATLAB to perform the operation among entries with the same location in their respective arrays.

```
3 2
21 32
n =
3.0000 0.5000
2.3333 2.0000
o =
3 1
343 4096
```

## **B.2.2** Logical Operators

The logical operations AND, OR, and NOT are specified by ampersand (&), pipe (|), and tilde (~) respectively. They can be used in conjunction with the relation operators listed in Table B.3 to construct logical arrays as in the examples below. As usual, "true" is represented by 1 and "false" by 0.

Relation	Symbol
Less than	<
Less than or equal to	<=
Greater than	>
Greater than or equal to	>=
Equals exactly	==
Does not equal	~=

Table B.3: Relation operator symbols

```
Example Code:

>> a = [1 3 2; 4 6 5], b = a>2 & a<=5
a =

1 3 2
4 6 5
b =

0 1 0
1 0 1
>> c = [1 5 3 4 7 8], d = c>4
c =

1 5 3 4 7 8
d =

0 1 0 0 1 1
```

Further, logical arithmetic operations may be mixed in a single command. Priority is given to the arithmetic operations first, however.

```
Example Code:

>> 5 + 2 > 6 * 8

ans =

0

>> 5 + 2 > 6

ans =

1

>> 5 + (2>6) * 8

ans =

5
```

## **B.2.3** Mathematical Operations

Another of MATLAB's strengths is its vast library of built-in mathematical functions. These range from utterly simple to quite complicated. A sampling of commonly useful math operations is listed in Tables B.4a and B.4b.

## **B.2.4** Data Analysis Operations

One may use MATLAB to perform analysis on sets of data collected elsewhere. There are several built-in MATLAB functions that calculate certain statistics or ease the process of data analysis. Table B.5 contains a sample of these.

## B.3 Flow Control

Like many other programming languages, MATLAB uses flow control statements to repetitively or selectively execute statements. Once activated, all statements are executed until the end command.

#### B.3.1 for Statement

The for statement executes commands a fixed number of times. It is equivalent to the do command found in other languages. For example, to evaluate the summation

$$x(t) = \sum_{k=1}^{3} t^{\sqrt{1.2k}} \sqrt{k}$$

for times between 0 and 0.8 seconds at intervals of 0.2 seconds, we would execute:

```
t = 0 : 0.2 : 0.8;
x = zeros(size(t));
for k = 1 : 3
x = x + sqrt(k)*t.^sqrt(1.2*k);
```

Command	Description
exp(x)	Base $e$ exponent of $x$
log(x)	Base $e$ logarithm of $x$
log10(x)	Base 10 logarithm of $x$
<pre>complex(a,b)</pre>	Construct the complex number $a + bi$
conj(x)	Complex conjugate of $x$
round(x)	Round $x$ to the nearest integer
floor(x)	Round $x$ toward $-\infty$
ceil(x)	Round $x$ toward $\infty$

(a) Commonly used built-in mathematical functions

Command	Description
sin(x)	$\sin x$
cos(x)	$\cos x$
tan(x)	$\tan x$
asin(x)	$\arcsin x$
acos(x)	$\arccos x$
atan(x)	$\arctan x$
atan2(y,x)	$ \arctan(yi + x) $ Restricted to $-\pi$ to $\pi$
(b) Commonly	used built-in trigonometry

(b) Commonly used built-in trigonometry functions

Table B.4

```
end
>> x
x =
0 0.3701 1.0130 1.8695 2.9182
```

In the above example, we *initialized* the variable  $\mathbf{x}$  to be all zeros before we executed the for loop. This is good programming practice because in larger or nested loops, it is faster for the system to assign values to already existing elements, rather than resize the variable on every iteration.

Nested for loops involve executing entire sub-loops on every iteration of outer loops. In the example below, m starts at 1, and n runs from 1 to 4 before m advances to 2; Then the process repeats.

Function	Description	Example
find(a)	Returns indices of nonzero elements	find(a) = 1 3 4 6
find(a>b)	Returns indices for which logical expression is true	find(a>2) = 4 6
max(a,[],d)	Returns the maximum elements along the direction specified	max(a) = 4
min(a,[],d)	Returns the minimum elements along the direction specified	$\min(a) = 0$
mean(a,d)	The average of the elements	mean(a) = 1.6667
<pre>sum(a,d)</pre>	The sum of the elements	sum(a) = 10

Table B.5: Commonly used data analysis functions. All relevant examples operate on the vector a = (1, 0, 2, 3, 0, 4). The optional arguments [], d can be used to specify the dimension on which the function is to act. In all cases d = 1 operates on columns and returns a row, and d = 2 is the opposite.

```
— Example Code: —
y = zeros(3,4);
for m = 1 : 3
    for n = 1 : 4
        y(m,n) = m + n;
    end
end
    2
        3
            4
                5
    3
                6
        5
            6
                7
```

It is often important to remember to place the semicolon after each line inside of the loops. If omitted, MATLAB will display that line every time it is iterated. This can significantly slow the execution time of a script.

#### B.3.2 while Statement

The while statement is similar to the for statement except that the loop terminates when a logical condition is satisfied. To find the largest power of 2 that is less than 5,000, we might use

```
n = 1;
while n*2 < 5000
    n = n*2;
end

>> n
n = 4096
```

Care should be taken when using while loops because there is a possibility that poorly stated logical conditions may never be satisfied; this results in an endless loop.

<u>Important</u>: Should you find yourself in an infinite loop, press Control-c in the <u>Command Window to terminate the current process</u>.

Another common use of while statements is to perform tasks while a certain variable counts down towards zero. Here it is appropriate to introduce a concept called "machine zero." MATLAB systems have a least counting element, it is the smallest quantity that the system can differentiate between numbers. On most modern installations, it is around  $10^{-16}$ . So if a while loop is set to run until the counter reaches zero, it will actually run until the counter reaches  $\pm 10^{-16}$ .

This may be a problem if the counter only asymptotically approaches zero. It may take many iterations to reach the machine zero. In this case, it is useful to define a number that is "close enough" to zero for the loop to terminate. This value, called the stopping point, represents a trade-off between absolute accuracy and execution speed.

#### B.3.3 if Statement

The if statement allows us to selectively execute statements based on the outcome of a logical expression. Further, they can be nested within other loops as well as other if statements.

```
for k = 1 : 4
   if k == 1
        x(k) = 3*k;
   else
        if k == 2 | k == 4
            x(k) = k/2;
        else
            x(k) = 2*k;
   end
```

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```
end
end
>> x
x =
3 1 6 2
```

In addition, if statements are often used to issue warnings or check for data errors before performing intense calculations.

```
c = 't'; n = 2;
if c == 'f'
    c = 'false'
    y = NaN;
end
d = 0.1 : 0.1 : 0.4;
if c == 't'
    if n == 2
        y = 100*d(n);
elseif n == 1
        y = 10*d(n);
else
        y = 0;
end
end
```

Which of course yields c = t and y = 20.

When used with if statements, the break command is very useful. If a loop is to be executed a variable number of times, if statements can be used to check the status of certain criteria, and then the break command can be issued to halt the loop.

## B.4 Plotting

Plotting—in two dimensions or three—is an extremely useful tool in the analysis and presentation of data. Even if they are never destined to be published, plots often give a "big picture" idea of a data set that is more valuable than numerical or statistical analysis alone.

As such, MATLAB has a wealth of graphical display functions from the urbane to the imaginative. The most basic is the plot(x,y) command. This places a connected plot of the data pairs x(k), y(k) for all k in the current figure window. The following three lines of code generate the plot in Figure B.2.

```
x = linspace(0, 2*pi);
y = sin(x);
plot(x,y);
```

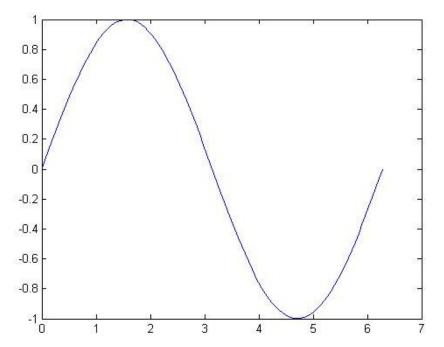


Figure B.2: Simple plot generated from three lines of code.

Of course, the plot in Fig. B.2 is presentable and the ease with which it was generated is nice. But it is lacking somewhat in presentation (what are we looking at?) and functionality (why are we looking at it?). As such, MATLAB provides many more commands to create detailed graphs and dress them up nicely.

In particular, the plot command itself has many optional arguments. Issuing help plot details all of the different line and marker styles available.

We now expand on the three-line example above to incorporate some of the commands detailed in Table B.6. The resulting graph is shown in Figure B.3.

```
x = linspace(0, 2*pi);
y1 = sin(x);
y2 = cos(x);
ax = [0 2*pi -1.5 1.5];
figure;
```

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Command	Description
figure(a)	Makes a the active figure window
hold on	Places subsequent plot commands on the same axes instead of replacing them
hold off	Releases hold on command
<pre>title('text')</pre>	Places text as a title above the plot
<pre>xlabel('text')</pre>	Labels x-axis of plot with text
<pre>ylabel('text')</pre>	Labels y-axis of plot with vertically rotated text.
<pre>text(x,y,'text')</pre>	Places text on the plot at point $(x, y)$
<pre>legend('First','Second')</pre>	Places a legend that labels two plots on the same axes
axis([x1 x2 y1 y2])	Scales the x- and y-axes to fit between $x_1$ , $x_2$ , $y_1$ , and $y_2$
<pre>subplot(m,n,id)</pre>	Splits the figure window into an $m$ by $n$ array of plots with plot number id as the active plot <sup><math>a</math></sup>

Table B.6: Common plot editing commands

<sup>&</sup>lt;sup>a</sup>In a subplot array, the plots are numbered starting at 1 in the upper left and count across the first line, then across the second line, etc.

```
subplot(2,1,1)
plot(x,y1,'o')
axis(ax)
title('Basic Sinusoids')
ylabel('y-axis')
legend('Sin(x)')
subplot(2,1,2)
plot(x,y2,'+-')
axis(ax)
xlabel('x-axis')
ylabel('y-axis')
legend('Cos(x)')
```

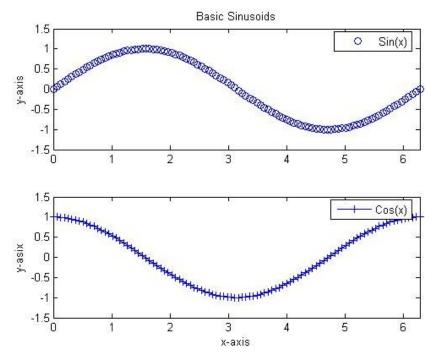


Figure B.3: Embellished plots using tools from Table B.6

## B.5 Signal Analysis

We know that signals can be represented in many ways. Two of the most common forms are the time and frequency domains. This section will discuss MATLAB's power to convert a signal between these two representations.

Consider a relatively simple time signal  $x(t) = \sin 2\pi 50t + \sin 2\pi 120t$ . We see that x(t) is simply the sum of two unit-amplitude sinusoids (one at 50 Hz and one at 120 Hz). To make things interesting, consider now passing x(t) through a system in which it experiences additive white Gaussian noise (AWGN) and emerges as y(t). A realization of this scenario is coded below with a sampling frequency of 1000 Hz and plotted in the time domain in Fig. B.4.

```
fs = 1000;
t = 0 : 1/fs : 1;
x = sin(2*pi*50*t) + sin(2*pi*120*t);
y = x + 0.5*randn(size(t));
plot(t,x,t,y);
title('Sinusoid Corrupted by AWGN');
xlabel('Time (s)');
ylabel('Volts')
```

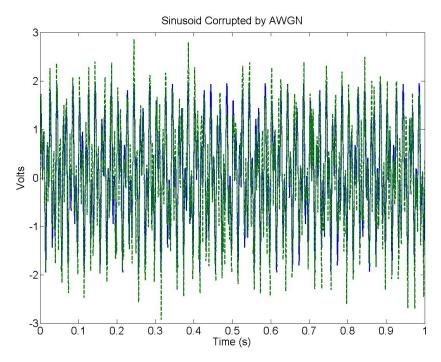


Figure B.4: Realization of sinusoidal signal (solid blue line) and corrupted output signal (dashed green line)

Looking at the original signal's trace, it is difficult to discern the two frequencies of which it is composed. If we were only given the corrupted signal, it would

be nigh impossible to report with any certainty the sinusoids that compose the original signal.

Thus, we would like to look at these signals in the frequency domain. MAT-LAB's fft command (fast Fouier transform) will come to our rescue. Before we cavalierly use fft, let us endeavor to understand it a little better.

Equation (2.92) in [4] gives the discrete exponential Fourier coefficients of signal g(t) as

$$D_n = \lim_{T_s \to 0} \frac{1}{N_0} \sum_{k=0}^{N_0 - 1} g(kT_s) e^{-jn\omega_0 T_s k} \quad \text{for } n = 0, 1, \dots, \frac{N_0}{2}$$
 (B.1)

where  $N_0$  is the number of samples,  $T_s$  is the interval between samples, and  $T_0 = 2\pi/\omega_0$  is the period of the signal.

The built-in fft uses approximation algorithms to produce the complex-valued coefficients for real sampled signals. A key observation is that the FFT of a (non-periodic) signal depends both on the period of observation  $(T_0)$  and the sampling frequency  $(f_s = 1/T_s)$ . It is also important to note that the Nyquist-Shannon sampling theorem (Section 6.1 in [4]) states that we may only measure frequency spectra up until one half of the sampling frequency.

Now we are ready to analyze the power spectra of our two example signals, x(t) and y(t).

```
— Example Code: ·
NO = length(x);
fnyq = 0.5*fs;

f = 0 : 1/(N0*Ts) : fnyq;

X = fft(x);

Y = fft(y);
Ts = 1/fs;
Xnyq = X(1:length(f));
Ynyq = Y(1:length(f));
Pxx = Xnyq .* conj(Xnyq) / NO;
Pyy = Ynyq .* conj(Ynyq) / NO;
figure;
subplot(2,1,1)
loglog(f,Pxx,'b')
ylabel('Spectral Power of x(t) [V^2/Hz]');
title('Power Spectra of Sinusoid Corrupted by AWGN')
subplot(2,1,2);
loglog(f,Pyy,'g--')
ylabel('Spectral Power of y(t) [V^2/Hz]');
xlabel('Frequency (Hz)');
```

The results are plotted in Figure B.5. We can see that the sampling rate was enough to capture the power peaks at the two component frequencies. (If

we had sampled less frequently, the Nyquist frequency would have been smaller, and we may have lost the peak at 120 Hz.) Further, we see that the corrupted signal also has strong power at the same frequencies as its originator.

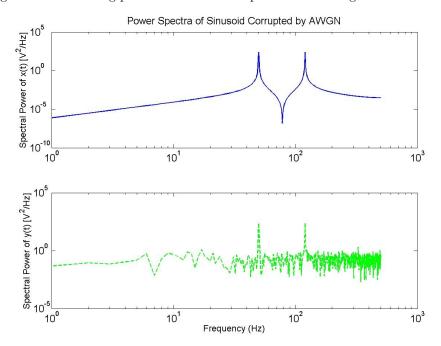


Figure B.5: PSD of the original uncorrupted signal compared with that of the corrupted signal.

## B.6 Symbolic MATLAB

MATLAB has evolved over the years and recently sprouted a toolbox called Symbolic MATLAB. If the foundation of MATLAB is to make engineering applications easier to code, then the purpose of Symbolic MATLAB is to make engineering concepts easier to learn. This toolbox allows us to manipulate the equations we find in textbooks directly, without having to translate them into pseudo-code first.

The basic building block of Symbolic MATLAB is the sym command. This command takes an argument and turns it into a "sym object." For creating a symbolic expression with several variables (such as a, b, and x) use syms a b x.

At this point, we are ready to start building symbolic expressions. The Symbolic Toolbox is useful when performing extensive analysis on equations without being burdened by discretization or roundoff errors. For example, if we needed to quickly solve a differential equation such as

$$\frac{df(t)}{dt} = f(t) + \sin t, \qquad f(0) = 1 \tag{B.2}$$

we could use Symbolic MATLAB as follows:

The last line illustrates the subs command where we may substitute numerical values and check their evaluations. You may note that if we had not provided the initial condition f(0) = 1, Symbolic MATLAB would have solved the indefinite ODE and provided us with the solution in terms of a constant of integration C1.

In addition to dsolve (the differential equation solver), Symbolic MATLAB has many other useful commands for building, manipulating, and evaluating analytic statements. A short list of these may be found in Table B.7.

Function	Description	Example
subs(f,a)	Substitutes $a$ as the independent variable in equation $f$	subs(f,pi) = -1
<pre>diff(f,'t')</pre>	Differentiates $\mathbf{f}$ with respect to $t$	$   diff(f,'t') = -\sin(t) $
<pre>int(f,'t')</pre>	Indefinite integral of $f$ with respect to $t$	$\inf(f,'t') = \sin(t)$
int(f,'t',a,b)	Integral of f over $[a, b]$	int(f,'t',a,b) = $= sin(b) - sin(a)$
<pre>simplify(f)</pre>	Analytically simplify	$simplify(f^2 + g^2) = 1$
<pre>compose(f,g)</pre>	Functional composition of $f$ with $g$	$\begin{array}{c} \texttt{compose(f,g) =} \\ &= f \circ g = \cos(\sin t) \end{array}$
fourier(f)	Fourier transform	fourier(h) = $\frac{2}{1+w^2}$

Table B.7: Useful Symbolic MATLAB functions. In the examples in this table, we have issued the commands syms t a b, f = cos(t), g = sin(t), and h = exp(-abs(t)).

As a final example, we present the classic calculus assignment of comparing a function with it's MacLaurin series. Here we choose  $f(t) = \sin(t + \pi/4)$ , and

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we wish to compare accuracy for  $t \in [-\pi, \pi]$  for MacLaurin series of degree 4 and 8. The results are plotted in Figure B.6.

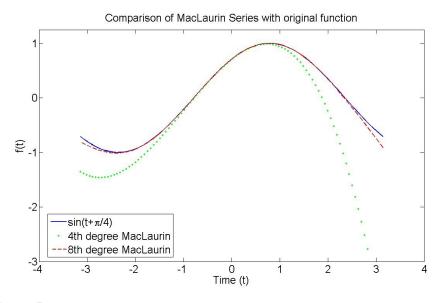


Figure B.6: Resulting plot from Symbolic MATLAB MacLaurin (Taylor) series example.

## Bibliography

[1] Stephen J. Chapman. MATLAB Programming for Engineers, 2nd edition. Thomson Engineering, 2001

- [2] Mathworks website. http://www.mathworks.com/
- [3] EE351L Laboratory Manual, 1st edition.
- [4] B. P. Lathi. Modern Digital and Analog Communication Systems, 3rd edition. Oxford UP, 1998.