Lab 6

Frequency Modulation

6.1 Pre-Lab

0. Read Section 6.2.

1. Use MATLAB to generate a narrow-band cosine-modulated FM signal with modulation index $\beta = 0.2$, and a wide-band cosine-modulated FM signal with $\beta = 5$. Print these waveforms. See Equation (6.6) in Section 6.2.1.

2. For the circuit in Figure 6.1 and the given V_{in} (DC) values, fill in Table 6.1 with the voltage at input pin 5.
   Hint: Assume the input impedance at pin 5 is infinite.

3. For the circuit in Figure 6.1 and assuming the output at pin 4 is a 2 kHz triangular wave, discuss what the output signal looks like.
   Hint: Consider the three cases

   (a) $V_4 > 0.7$ Volts

   (b) $0.7 \text{ Volts} > V_4 > -0.7$ Volts

   (c) $V_4 < -0.7$ Volts

6.2 Overview

This experiment covers the topics of voltage controlled oscillator (VCO) frequency modulators and phase-locked loop (PLL) frequency demodulators.

6.2.1 VCO-FM Modulator

For linear frequency modulation (FM) we vary the derivative of the angle linearly with the input signal. Let the transmitted signal be represented as

$$y(t) = A \cos \theta(t) \tag{6.1}$$

V_{in} (Volts)	V_5 (Volts)
$+3$	
$+1$	
0	
-1	
-3	

Table 6.1: Recorded voltages at pin 5

where

$$\frac{d\theta(t)}{dt} = \omega_c + k_f x(t) \tag{6.2}$$

Note that $x(t)$ is the input message and the instantaneous angular frequency is $\omega(t) = d\theta/dt$.

From Equation (6.2), we know that the instantaneous phase of the signal is

$$\theta(t) = \int [\omega_c + k_f x(t)]\, dt = \omega_c t + k_f \int x(t) dt \tag{6.3}$$

So we can write the corresponding expression for the transmitted signal. For example, if we let $x(t) = A \cos \omega_m t$, then the transmitted signal can be expressed as

$$y(t) = \cos \left[\omega_c t + \frac{k_f A}{\omega_m} \sin \omega_m t + \theta_0 \right] \tag{6.4}$$

where θ_0 is of course the constant of integration. For convenience we can choose $\theta_0 = 0$.

If we define $\Delta\omega = k_f A$, then

$$y(t) = \cos \left[\omega_c t + \frac{\Delta\omega}{\omega_m} \sin \omega_m t \right] \tag{6.5}$$

and we define the *modulation index* (for tone modulation) as

$$\beta = \frac{\Delta\omega}{\omega_m} \tag{6.6}$$

For this same input signal $x(t)$, we know that the instantaneous angular frequency is

$$\omega(t) = \omega_c + \Delta\omega \cos \omega_m t \tag{6.7}$$

Note that this value varies as a sinusoid between $\omega_c + k_f A$ and $\omega_c - k_f A$.

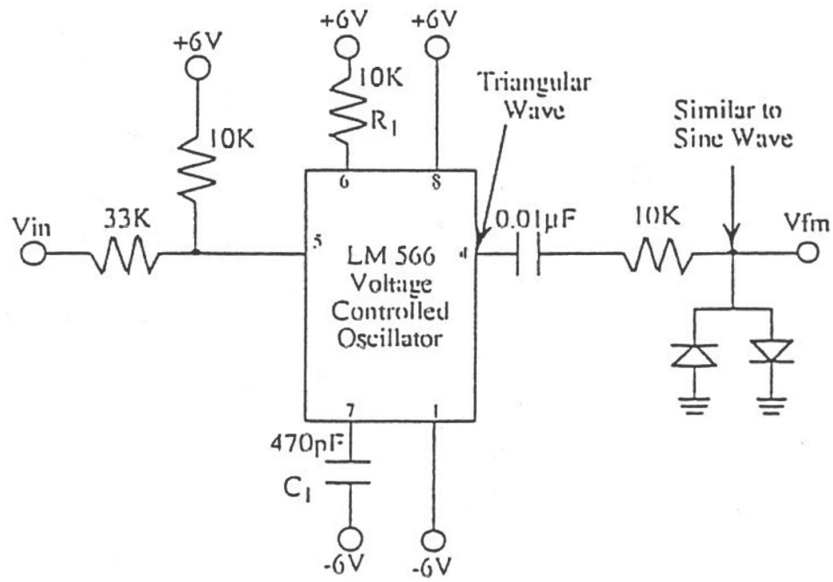

Figure 6.1: Voltage controlled oscillator (VCO) used to generate FM signals

In this lab, a VCO converts an input voltage to an output frequency. We will use the LM566 VCO in the configuration shown in Fig. 6.1. Several requirements for the LM566 are listed below:

- The input voltage at pin 5 must be kept in the range from $0.75V_{cc}$ to V_{cc}.
- The resistor at pin 6, R_1, is the timing resistor. The capacitor at pin 7, C_1, is the timing capacitor.
- The approximate output frequency to input voltage relationship is

  $$f_0 = \frac{2.4(V^+ - V_5^+)}{R_1 C_1 V_{cc}}$$

  where

  $$\begin{aligned} V^+ &= V_{cc} - V_1 \\ V_5^+ &= V_5 - V_1 \\ V_{cc} &= 6 \text{ Volts} = -V_1 \end{aligned}$$

- The linear relationship between input voltage and output frequency holds for a frequency up to about 500 kHz (i.e., the VCO has a limited bandwidth). This will limit the values of R_1 and C_1.

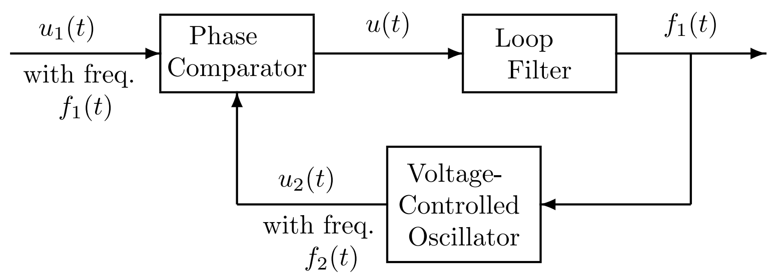

Figure 6.2: Phase-locked loop block diagram

6.2.2 PLL-FM Demodulator

The phase-locked loop (PLL) is used to demodulate a frequency modulated signal. This circuit synchronizes a signal from an internal oscillator that keeps track with the frequency and phase of the input signal. The PLL consists of the three basic functional blocks in Figure 6.2:

- Voltage controlled oscillator
- Phase comparator
- Loop filter

The phase-locked loop works as follows:

- The VCO oscillates at a frequency $f_2(t)$ which is determined by the output signal, $f(t)$. The angular frequency of the VCO is given by

$$\omega_2(t) = 2\pi f_2(t) = \omega_0 k_0 f(t)$$

  where ω_0 is the center angular frequency of the VCO and k_0 is the gain of VCO.

- The phase comparator compares the phase of $u_2(t)$, which is the output signal from the VCO, with the phase of $u_1(t)$, which is the input signal, to create an output signal $u(t)$. This output signal is approximately proportional to (or at least linear in) the phase error. The expression is

$$u(t) = k_d \theta_e(t) = k_d(\theta_1(t) - \theta_2(t))$$

  where k_d represents the gain of the phase comparator. The output signal of the phase comparator consists of the desired low frequency components and undesirable high frequency components.

- The loop filter cancels the undesired high frequency components from the comparator. In many cases, this is a first-order lowpass filter.

To see how the three building blocks work together, consider the case that the phase error $\theta_e(t) \equiv 0$. When this is true, the output of the phase comparator $u(t)$ and output of the loop filter, $f(t)$, are also zero. This condition allows the VCO to operate at its center frequency.

Now assume that the phase error is nonzero. The phase comparator creates a nonzero output signal $u(t)$ and after a small delay the loop filter will create a signal $f(t)$. This causes the VCO to change its operating frequency in such a way that the phase error will finally vanish. (Students of EE 351 will recognize this as a closed-loop error-rejecting control system.)

For the particular PLL that we use in this lab, the Free Running Frequency (f_0) and Loop Gain (LG) are important parameters.

$$f_0 = \frac{0.3}{R_0 C_0} \tag{6.8}$$

$$LG = k_o k_d \tag{6.9}$$

The Loop Gain relates the amount of phase change between the input signal and the VCO signal for a shift in input signal frequency (assuming the loop remains in lock). In Eq. (6.9), k_o is the oscillator sensitivity in rad/sec/Volt, and k_d is the phase detector sensitivity in Volts/rad.

The phases of the input and feedback signals are compared and the PLL tends to make the phase difference between the two signals equal to zero. In Figure 6.2, the VCO in the feedback is an FM modulator, and the center frequency can be set equal to the center frequency of the input signal, $u_1(t)$.

In our PLL-FM demodulator, $u_1(t)$ is the output of the FM modulator, thus is expressed the same as $y(t)$ from Eq. (6.1). Rewriting this relationship [and using Eq. (6.3)], we have

$$u_1(t) = y(t) = \cos\left[\omega_c t + k_f \int x(t)dt\right] \tag{6.10}$$

For ease of reference, let $\theta_1(t) = k_f \int x(t)dt$.

The equation for $u_2(t)$ is

$$u_2(t) = \cos\left[\omega_c t + k_0 \int f(t)dt\right] \tag{6.11}$$

Again, let $\theta_2(t) = k_0 \int f(t)dt$.

Since each signal has the same center frequency ω_c, the phase comparator compares the instantaneous values of $\theta_1(t)$ and $\theta_2(t)$. The difference in phases is transformed into a voltage level proportional to the phase difference and then lowpass filtered to yield $f(t)$. This voltage is then input to the VCO for the feedback. As a result, the difference between $\theta_1(t)$ and $\theta_2(t)$ will be made smaller. This process occurs continually, such that after transients are discarded, $\theta_1(t) = \theta_2(t)$ for all t. Thus,

$$k_0 \int f(t)dt = k_f \int x(t)dt$$
$$\Rightarrow f(t) = \frac{k_f}{k_0} x(t)$$

where $x(t)$ is the original message sent and $f(t)$ is the demodulated output.

6.3 Procedure

This experiment proceeds in two parts: One for the modulator and another for the demodulator.

6.3.1 Modulator

1. Build the modulator circuit as shown in Fig. 6.1.
2. Find k_f.
   [Recall: The instantaneous angular frequency is $\omega(t) = \omega_c + k_f x(t)$.]
   (a) Apply the input voltages given in the Pre-Lab and record the output angular frequency (at pin 4) for each input voltage. (Find f_0 first, then calculate $\omega_0 = 2\pi f_0$.)
   (b) Plot ω_0 versus V_{in}. Make a best-fit line through all the data points. The slope of this line is k_f. Note that k_f has units of rad/sec/Volt.
3. Now apply the sinusoidal input signal $x(t) = A\cos(2\pi \cdot 2000t)$. Note that for the DC input, we observed one frequency at the output. With this sinusoidal input, we will observe a range of frequencies corresponding to each instantaneous input voltage value. This signal looks like a ribbon on the oscilloscope.
   Since the value of A determines the range of frequencies at the output, we must first calculate the desired A for different modulation indices. To do this, complete Table 6.2 using Equations (6.12) to calculate the values.
   $$\Delta\omega = \beta\omega_m = Ak_f \tag{6.12a}$$
   $$\Rightarrow A = \frac{\Delta\omega}{k_f} \tag{6.12b}$$
   In Eqs. (6.12), $k_m = 2$ kHz and k_f is the value calculated from Step 2 in this section. The value of $\Delta\omega$ can be calculated by observing
   $$\omega_{max} = \frac{2\pi}{T_{min}} \quad \text{and} \quad \omega_{min} = \frac{2\pi}{T_{max}}$$
   $$\Rightarrow \Delta\omega = \frac{\omega_{max} - \omega_{min}}{2} \tag{6.13}$$
4. Observe and record the corresponding spectrum in step 3 for different β.

β	$\Delta\omega^c$	A^c	$\Delta\omega^o$
0.20			
0.50			
0.80			
1.10			
1.40			

Table 6.2: Parameters for modulations. Columns 2–3 are calculated values. Column 4, $\Delta\omega^o$, should be used for recording observed values.

6.3.2 Demodulator

1. Build the FM demodulator using the LM565 phase-locked loop as shown in Fig. 6.3.
2. Apply the output of the modulation circuit whose message is sinusoidal to the FM demodulator and observe the demodulated signal. Compare this with the input message signal of the modulator. Sketch both waveforms. How do they compare?
3. Repeat step 2 using a square wave as the message signal. Which signal is more easily demodulated, the sinusoid or the square wave? Why?

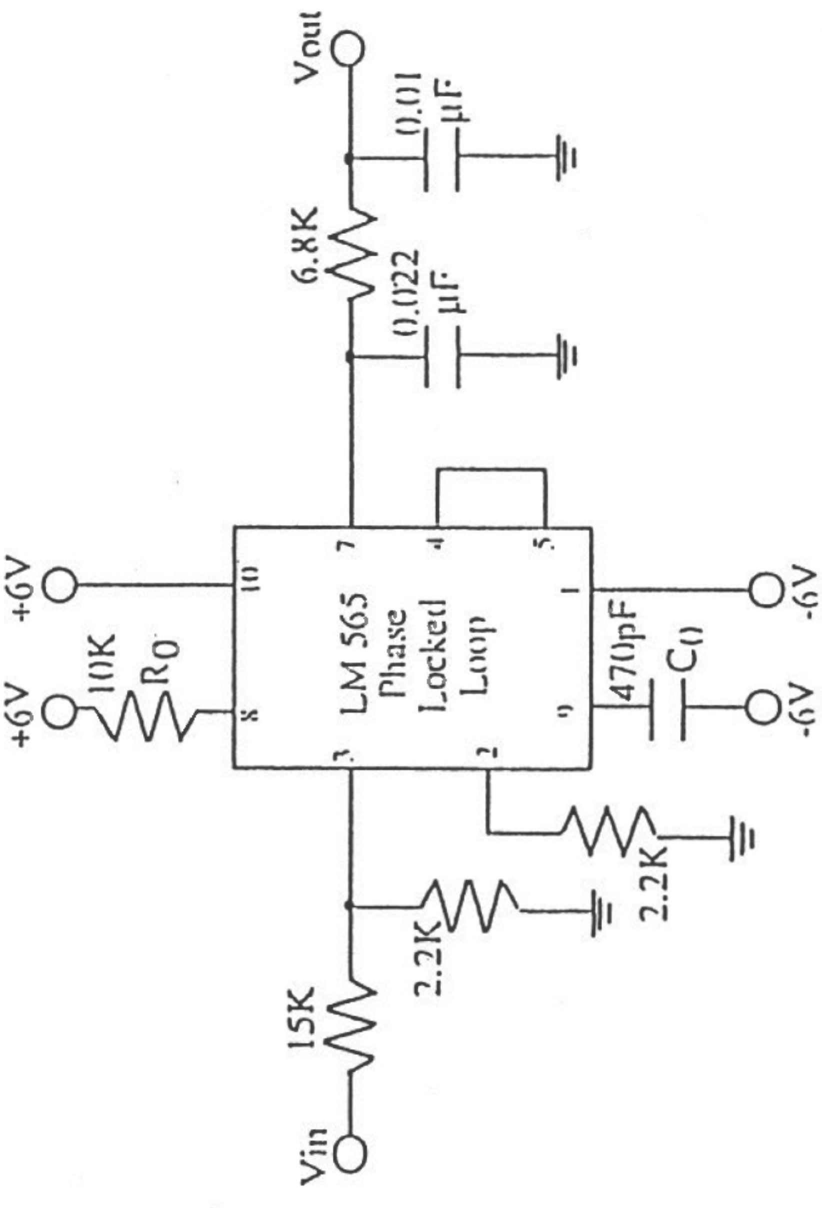

Figure 6.3: Phase-locked loop for FM demodulation