
Machine learning-aided modeling of fixed income instruments

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Abstract

The fixed income market is very important to the economy. Sovereign issues are influenced by central bank policy, and corporate issues are viewed relative to these sovereign issues. Compared to equities, bonds have lower liquidity and transparency; hence, there is less public data available. We demonstrate the applicability of machine learning models in forecasting interest rates for U.S. Treasuries of varying maturities, as well as clean prices of corporate issues.

1 The Fixed Income Market

Fixed income markets have strategic importance to the global economy. The global bond market is valued over \$100 trillion, as compared to S&P's \$64 trillion valuation of the global equity markets. [1] Although most bonds are available to individual investors, the majority are purchased by large institutional funds and insurance companies for the purpose of hedging risk.

A bond is issued with a defined term (lifespan) and coupon (interest) rate. A credit rating agency rates bonds based on the issuer's creditworthiness. As there is a greater variety of bonds than stocks, secondary trades of a bond do not take place in a market, but through direct trader-to-trader negotiations. This system reduces liquidity and transparency for trades, which increases risk for both the buyer and the seller. Having an improved ability to forecast prices would mitigate some pricing risk, and, given the size of institutional trades, avoid large losses.

Sovereign Bonds. Sovereign bonds are fixed income debt issued by a sovereign government. These bonds are usually considered the lowest risk available in their respective countries, and thus offer the lowest rates in the country. The U.S. government currently holds a top AAA rating from major credit agencies. The interest rates are influenced by the monetary policies of the country's central bank.

Corporate Bonds. Corporate bonds are issued by companies, and are usually considered higher risk than sovereign bonds. The perceived additional risk of investing in a particular company leads to higher interest rates for corporate bonds as compared to sovereign bonds. Prior to 2002, U.S. corporate bond trade prices were not publicly available except through compulsory reporting of large trades by insurance companies. [2] As buyers had limited information, this caused bond prices to be relatively volatile. In response to regulation, the Financial Industry Regulatory Authority (FINRA), a trade group, set up a public database (TRACE) where licensed security dealers would be required to report trade information. It officially launched in 2006.

Matrix Pricing. Many bonds trade rarely. This complicates the task of determining a suitable purchase price. The most popular technique used in this scenario is called "matrix pricing" [3]. Matrix pricing involves adding an arbitrary spread to a roughly-comparable bond: a more-actively

Table 1: RF prediction metrics

Term	Test R^2	Test MSE
3	0.9943	0.0016
10	0.9876	0.0021
30	0.9618	0.0026

Table 2: SVR prediction metrics

Term	Test R^2	Test MSE
3	0.9907	0.0026
10	0.9754	0.0042
30	0.796	0.0139

traded bond from the same company, a similar bond from a different company, or a U.S. Treasury bond [4]. Dealers will usually hold this quote barring large shifts in the price of the comparable [5]. The arbitrary nature of this technique can lead to discrepancies in valuation. Using machine learning on existing data, our goal is to forecast the prices of such bonds without finding comparables.

Summary. The rest of our paper will be organized as follows. In Section 2 we introduce the machine learning models used in forecasting. We will continue with our bond experiments in Section 3. Finally, we discuss these results in Section 4, and delve into further avenues of exploration.

2 Forecasting Models

Our problem is to be able to forecast a future series of interest rates (for Treasuries) and a future series of trade prices (for corporate bonds). As more work has been done with stock forecasting, we will choose two models that have been used successfully, and adapt them for our experiments.

Random Forests (RF). The random forest [6][7] model is considered an ensemble-based learning method. The training process entails constructing multiple decision trees (hence, the "forest"). The output is the mean output of the decision trees. In our tests, we set the number of decision trees to 100.

Support Vector Regression (SVR). By contrast, the support vector regression [8][9][10] model seeks to fit a curve to the data while minimizing the distance from the points to be within a certain margin. The SVR model will find a higher-dimensional space where the data is linear, and return the line of best fit in that space. For our SVR model, we used a radius basis function (RBF) kernel with $C = 0.8$ and $\epsilon = 0.2$.

3 Experiments

We use a lag method of forecasting: given the previous n prices, can we predict the next price? For our experiments, we used a lag of 5 points.

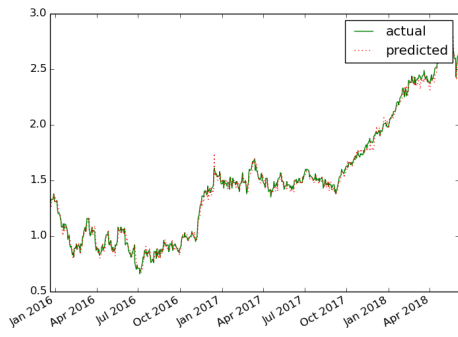
3.1 Treasuries

Data. We considered U.S. Treasuries of 3 year, 10 year, and 30 year terms. The daily nominal interest rates were obtained from CRSP for the period of October 27, 1993 to June 5, 2018. The training-test data ratio used for our experiments was 90:10. We had conducted preliminary tests using smaller proportions of training data (70:10, 40:10, and so on). While some of these trials had good results, the trained models were much more vulnerable to regime changes in price. If the the testing or training intervals contained extreme rises and falls, the model would give distorted predictions. To minimize these disruptions, we decided to use 90:10 by default.

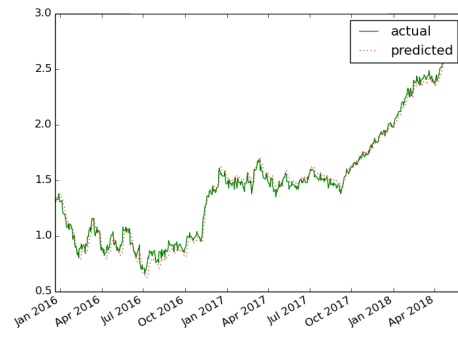
Results. In Figures (1), (2), and (3), we see the results of the RF and SVR models. We use the R^2 coefficient and the mean squared error (MSE) to measure our accuracy. Tables 1 and 2 show these results on our test data.

3.2 Corporate Bonds

We considered three bonds: a 30-year Exxon bond issued in 1991, a 30-year Johnson&Johnson bond issued in 1993, and a 15-year Alcoa bond issued in 2007. The first two bonds are rated Aaa by Moody's. The Alcoa bond is rated Ba2 (below investment grade) by Moody's. Figures (4), (5), and (6) show the prediction results.

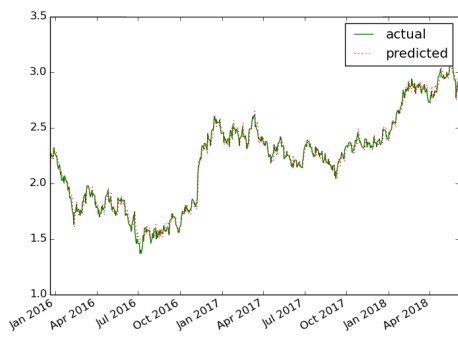


(a) RF

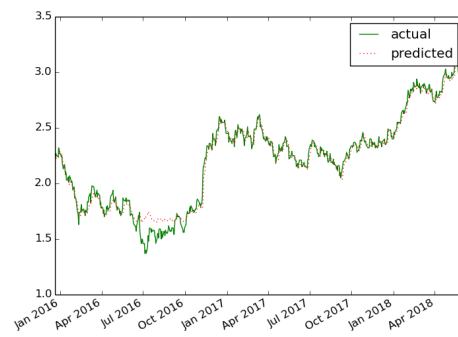


(b) SVR

Figure 1: Predictive performance for a 3-year Treasury bond

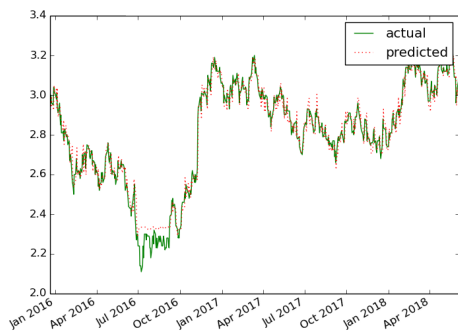


(a) RF



(b) SVR

Figure 2: Predictive performance for a 10-year Treasury bond

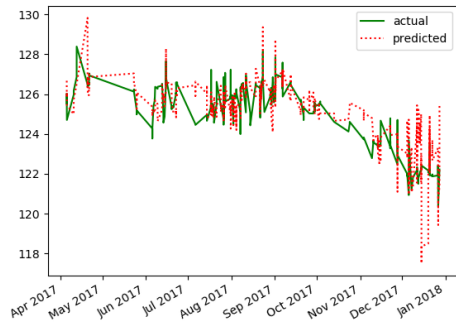


(a) RF

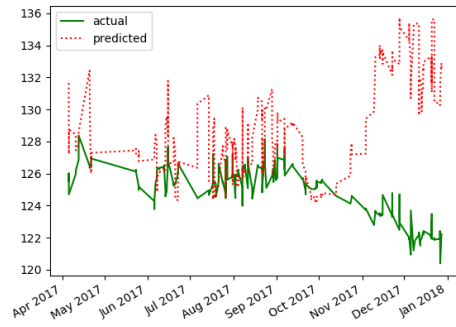


(b) SVR

Figure 3: Predictive performance for a 30-year Treasury bond

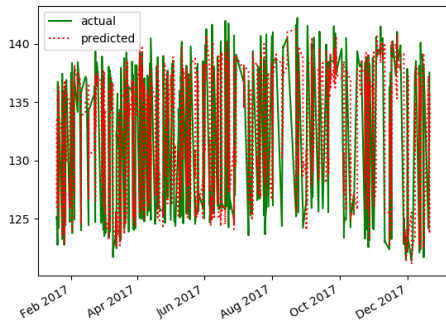


(a) RF

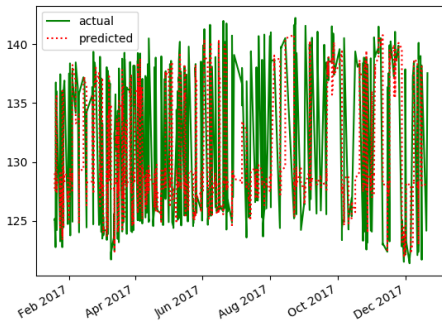


(b) SVR

Figure 4: Predictive performance for a 30-year Exxon bond issued in 1991



(a) RF

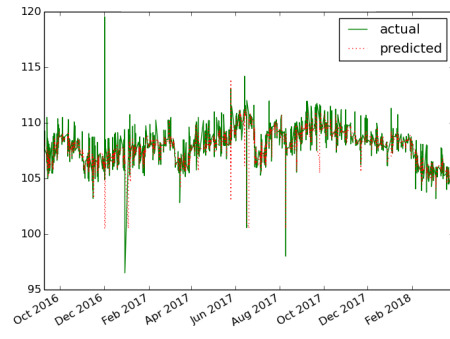


(b) SVR

Figure 5: Predictive performance for a 30-year J&J bond issued in 1993



(a) RF



(b) SVR

Figure 6: Predictive performance for a 15-year Alcoa bond issued in 2007

Table 3: RF corporate metrics

Bond	Exxon		J&J		Alcoa	
Winsorized?	No	Yes	No	Yes	No	Yes
Test R^2	0.446	0.779	0.32	0.594	0.649	0.942
Test MSE	1.815	0.711	28.902	15.895	0.96	0.128

Table 4: SVR corporate metrics

Bond	Exxon		J&J		Alcoa	
Winsorized?	No	Yes	No	Yes	No	Yes
Test R^2	-10.682	-1.289	0.083	0.347	0.379	0.727
Test MSE	38.883	7.373	38.953	25.587	1.7	0.602

3.3 Winsorizing

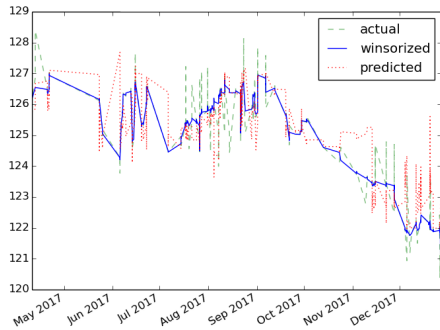
As the corporate bond data consists of manual reports from dealers, there is the possibility of data input errors. There also may be individual trades that diverge from the overall trend. In this situation, winsorizing the data should minimize disruption caused by the degree of divergence, simplifying the task of fitting the trend. This process entails finding outliers, and replacing them with a chosen high or low margin point.

For our margins, we calculated the moving median standard deviation. First, we obtained the moving median using a median filter with a kernel size of 5 points. We then found the absolute deviation of each of the points from the median, and standardized it to the 75th percentile. We winsorized the data to lie within ± 2 deviations. Subsequently, we trained and tested our models as before.

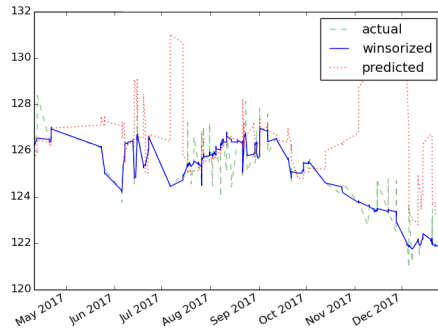
In tables 3 and 4, we see how both our metrics are greatly improved when using the winsorized data. We confirm this using Figures (7), (8), and (9). The winsorized data preserves most of the peaks and troughs of the original line, while scaling down the more extreme points.

4 Discussion

Model Efficacy. In our tests, the RF model performs distinctly better than the SVR. The RF achieved a better fit to the data when it was close and had fewer divergent areas. Furthermore, we note that training models with a 5 point lag achieves good results in many cases, providing a credible data-agnostic alternative to matrix pricing.

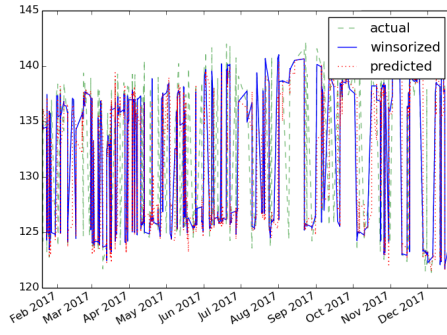


(a) RF

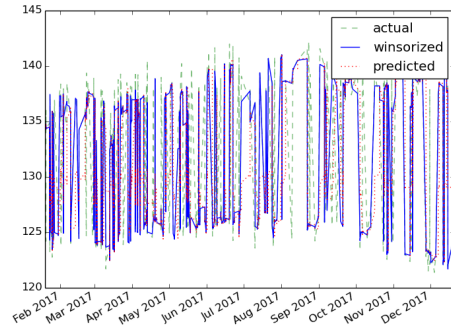


(b) SVR

Figure 7: Winsorized performance for a 30-year Exxon bond issued in 1991

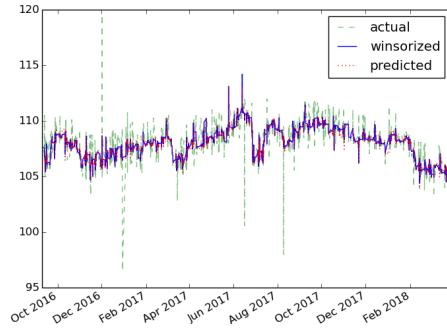


(a) RF

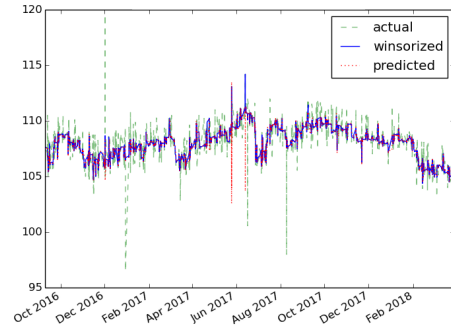


(b) SVR

Figure 8: Winsorized performance for a 30-year J&J bond issued in 1993



(a) RF



(b) SVR

Figure 9: Winsorized performance for a 15-year Alcoa bond issued in 2007

Treasuries vs. Corporates. Treasury bond rates were not very divergent, and hence easier to fit for both models. This can be attributed to central bank policies to avoid sudden large shifts in rates, resulting in a sort of data filter.

On the other hand, the individual corporate transactions had a higher degree of divergence, as there are fewer filtering influences. Taking this variation into account, we note that the quantity of datapoints is key for maintaining accuracy. We had about 5x as many data points for Alcoa as for Exxon, so the models for Alcoa were more accurate. Alcoa is a junk-rated bond and has a shorter lifespan than the other two bonds, so holders have less of an incentive to hold.

5 Further Directions

There are several further directions in which this work can be expanded upon.

Machine Learning. While we covered the RF and SVR models, there are other forecasting models than can be used for this problem of forecasting. Long short-term memory (LSTM) [11], a type of deep learning model, can be used when larger datasets are available. Using hyperparameter optimization, one can arrive at a model better adjusted to the particular dataset. It is also possible to use cascading classifiers to reduce the overall forecasting error.

Data Preprocessing. Further preprocessing of the corporate bond data can be useful in prediction. As in other instruments, there may be seasonality and other recurring patterns. We can also alter the lag length of input data in case longer-range patterns are needed.

Effects of the Market. Our model can be further refined by incorporating the effects of factors external to the issuer. Many stock models take into account the effects of company announcements and investor sentiment when making price predictions, as well as company fundamentals [12]. Bond prices may also be affected by these and other factors.

Asset Variations. The models in this paper could be extended to measure the predictive quality of bond covenant provisions. For example, what effects do issuer call options have on the market trade price? What about conversion clauses, or insurance?

Sovereign Issuer Relationships. Besides the U.S., many other countries release nominal rates for their outstanding sovereign debt. This model could be applied to forecast their rates as well; one can even take into account currency and trade relationships for these models. Such a model can be useful when forecasting in foreign corporate bond markets.

6 Conclusion

In this paper, we have demonstrated two types of fixed-income forecasting problems that can be handled using machine learning models. Treasury interest rates can be easily forecasted from past data given a long enough history. Corporate bond transaction prices, although noisier, can also be predicted to a degree, depending on the amount and quality of data available. From this work, we can test more complex models to better explain corporate bond prices. Overall, having an effective forecasting tool can help increase market liquidity at accurate prices, aiding investors and issuers alike.

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