`PHSX815_Project2`: Which dice set was used?

Kenny Couberly

February 2023

1 Introduction

When working for a company that makes two types of dice sets, I forgot to label the boxes before sending them to shipping. The first dice set contains 10 standard 6 sided dice, while the second dice set contains a random distribution of 10 dice bewteen 1 and 20 sides. Each possibility is equally likely in this random distribution. The first dice set is the better seller, so we carry 75% standard dice sets and 25% randomized dice sets.

After shipping out one of these unlabled dice sets, it was used on 5 occasions where the customer rolled each of the 10 dice in the set and recorded the average result. They recorded average results of 3.2, 3.7, 4.2, 4.5, and 5.6. However, the customer did not share which dice set they received.

Sec. 2 presents the hypotheses we are testing to see which best explains the customer's results. Sec. 3 provides a description of how the simulated data used to compare the 2 hypotheses was created as well as the results of the simulation. Sec. 4 performs an analysis on the simulated data, comparing the hypotheses using a log-likelyhood ratio. Finally, a conclusion between the hypotheses is reached in Sec. 5.

2 Hypotheses to the customer's results

The first hypothesis is that a standard six sided die was used for every roll. This simple hypothesis is expected to produce a symmetric distribution that peaks between 3-4. The second hypothesis is that a random distribution of dice were used. This random distribution is of dice between 1 and 20 sides that are simulated using an evenly spaced categorical distribution with 20 divisions. We will compare the hypotheses by comparing the likelyhood in their final distributions that they produce an average roll result of at least 4.

3 Code and Experimental Simulation

To create the simulated data, we take a categorical distribution with n divisions to simulate the roll of an n sided die. To simulate the simple hypothesis, we used a categorical distribution $n = 6$ and repeated the roll 10 times, then recorded the average result. This would represent one experiment. This process was repeated 10000 times to simulate 10000 experiments.

To simulate the complex hypothesis, the die was first selected using a categorical distribution $n = 20$. The result from this first categorical distribution would then be used for the dice roll. This was repeated 10 times, each time repeating the simulated die selection using the categorical distribution

$n = 20$ before simulating the dice roll. After 10 rolls, the average was collected representing one experiment. Again, the process was repeated 10000 times. Figure 1 shows histograms of the results of these two simulations.

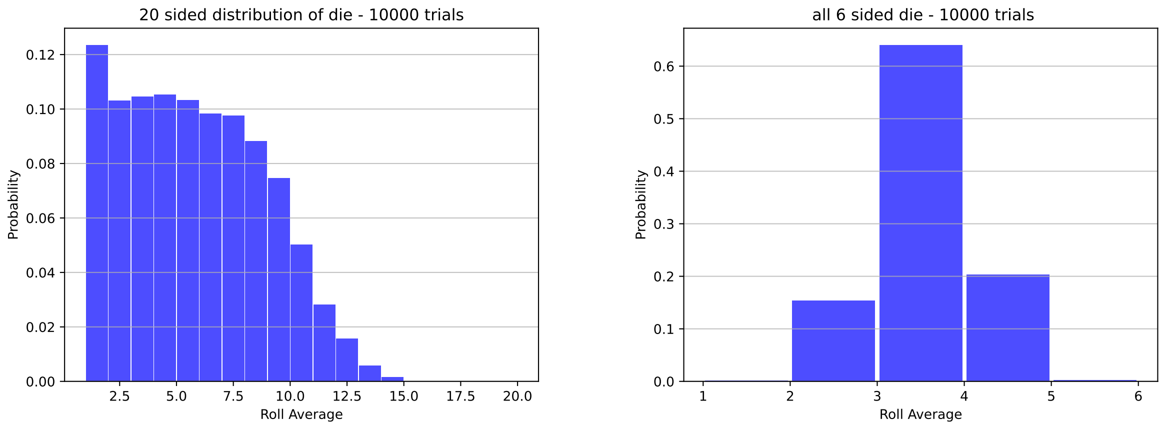

Figure 1: Distributions of the roll averages after 10 rolls for two different hypotheses (Left) Distribution using randomly selected dice over a categorical (n=20) distribution. (Right) Distribution using only standard 6-side dice. Each entry is a simulated experiment where 10 rolls are averaged. 10000 experiments are simulated for each hypothesis. Histograms are normalized to show bins as a portion of the total number of experiments.

4 Analysis

Looking at the two distributions, the simple hypothesis is a relatively symmetric plot that peaks with averages between 3 and 4, while the complex hypothesis peaks between 1 and 2 and trails off for higher roll averages. In comparing the hypothesis, we will consider bins of size one, so a roll average of 3.2 will use the probability of a roll average between 3 and 4 according to the distributions. By this method, an average of 3.2 will be given a probability of 0.6402 by the simple hypothesis, since this is the probability of the bin from 3 to 4.

There is also the probability of a randomly selected box belonging to each hypothesis to consider. Since 75% of the boxes correspond to the simple hypothesis, there is a 0.75 base probability to consider. By using Bayes theorem, let B be the hypothesis and A be the result.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{1}$$

means that for the average of 3.2, we have $0.6402 \times 0.75 = 0.48015$ as the likelyhood of the simple hypothesis, since $P(B) = 1$ where B is the result being between 3 and 4.

Following this process for each hypothesis, we can sum the 5 results and compare the hypotheses using their log-likelyhood ratio. Letting the simple hypothesis be A and the complex hypothesis be A', the log-likelyhood ratio is

$$\frac{\Sigma \log(P(A|B_i))}{\Sigma \log(P(A'|B_i))} \tag{2}$$

where B_i is each of the 5 averages the customer got. The simulated experiments gave probabilities of 0.6402 from 3 to 4, 0.2033 from 4 to 5, and 0.0022 from 5 to 6 for the simple hypothesis, and probabilities of 0.1045 from 3 to 4, 0.1053 from 4 to 5, and 0.1033 from 5 to 6 for the complex hypothesis. Using these results, the customer's data gives a log-likelyhood ratio of 0.6386.

5 Conclusion

The distribution of roll averages shows a higher probability of getting the customer's result with the simple hypothesis than with the complex hypothesis. Most of the customer's roll averages fell withing the 3 to 4 and 4 to 5 ranges, both of which had a higher probability using the simple hypothesis. In addition, the chance of randomly selecting a box of all 6 sided dice was higher than selecting a box of randomly selected dice given the higher number of standard dice sets. The log-likelyhood ratio of 0.6386 confirms this result. Since this log-likelyhood ratio is positive, it suggests that the simple hypothesis is the more likely explanation.