`PHSX815_Project3`:
How to determine the weight of a coin

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April 2023

1 Introduction

We often assume a coin flip gives a fair 50% chance of heads or tails, but this assumption may not be correct. What if the coin instead gave a 60% chance of heads; how would we go about figuring this out? We need to test the coin, generating a data set by flipping it some number of times. How many times do we need to flip the coin to be satisfied with our data? If we want to test a number of coins, it would be best not to flip each coin more than necessary to avoid wasting time. To do this we will simulate coin tosses of a weighted coin and determine the weight and uncertainty through analysis of the data.

Sec. 2 provides a description of the code that simulates the coin toss experiments and predicts a weight of the coin based on the data. Sec. 3 performs an analysis on the simulated data, comparing the uncertainties for various numbers of flips. Sec. 4 provides a conclusion on how many flps are necessary to provide a desired uncertainty.

2 Code and Experimental Simulation

To create the simulated data, we take a Bernoulli distribution for a given weight parameter p. The coin toss is simulated by assigning a value of 1 whenever a random number between 0 and 1 is less than p and a value of 0 otherwise. This process is repeated n times to simulate a number of flips.

The process was performed for 50 different evenly distributed p values between 0 and 1 to create a near continuous distribution of true p values to compare to measured values. The measured values are obtained my minimizing a likelyhood function to the data from each simulated experiment. The experiment is run 1000 times at each true p value to create a Neyman construction. To compare the effect of changing the number of flips on the uncertainty of the measurement, we simulate 10, 100, and 1000 flips per experiment.

Figure 1 below plots a function of the parameter p proportional to likelyhood for a given simulated experiment with a true $p = 0.6$. In the plot of the left, there were 7 heads and 3 tails. In the plot on the right there were 59 heads and 41 tails. The function is generated using the simulated data set through the equation

$$f(p) = p^{n_H} \times (1-p)^{n_T} \tag{1}$$

where n_H is the number of heads and n_T is the number of tails in the data.

Figure 2 below shows the Neyman constructions at 10, 100, and 1000 flips per experiment. The p meas value is the maximum value of the functions shown in figure 1.

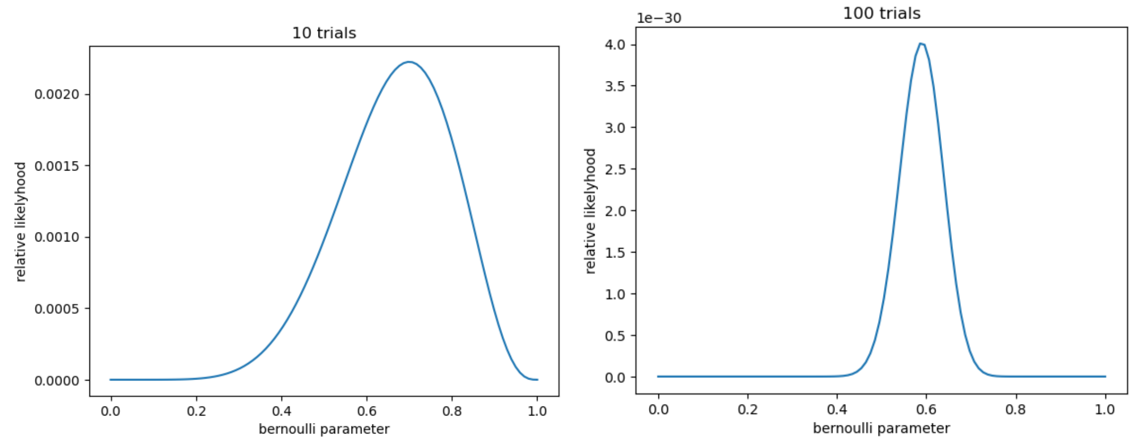

Figure 1: Relative likelyhood as a function of Bernoulli parameter for a coin with true $p = 0.6$. (Left) Likelyhood function for a data set of 10 coin tosses. (Right) Likelyhood function for a data set of 100 coin tosses. The likelyhood function tightens its peak for a higher number of flips.

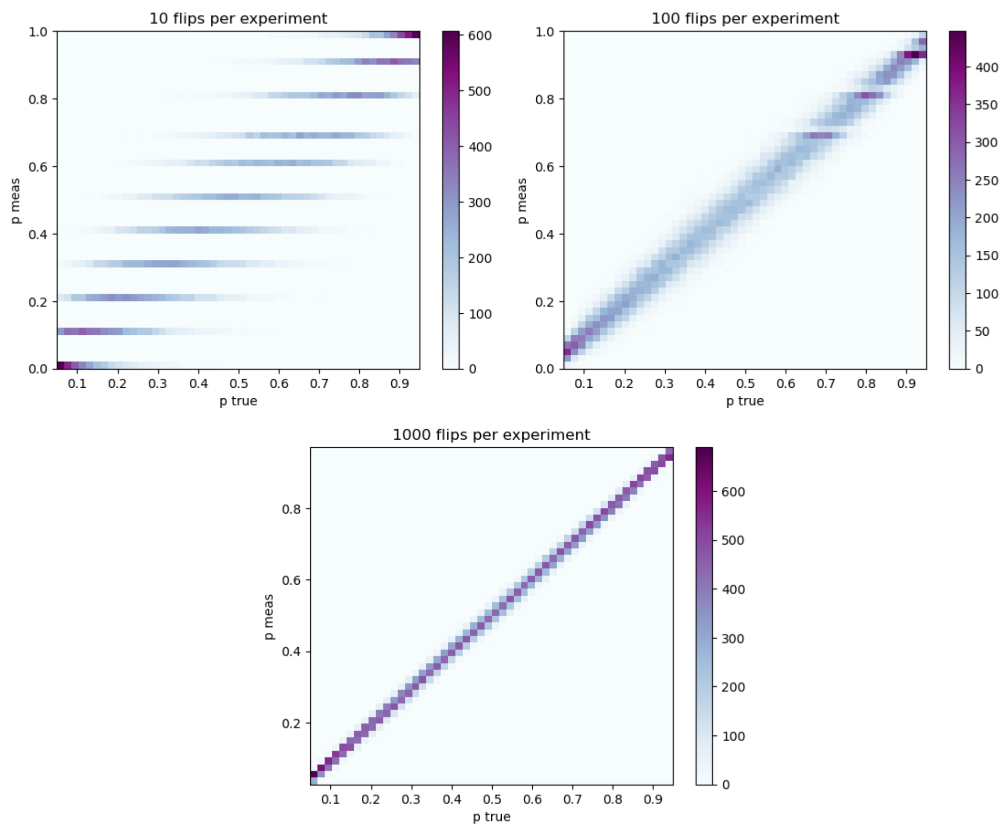

Figure 2: Neyman constructions plotting the measured p values as a function of true p value used to simulate experiments 50 p values used with 1000 experiments per p value at 10 flips (Left), 100 flips (Right), and 1000 flips (Bottom) per experiment

3 Analysis

The uncertainty for a given number of flips can be calculated using the p measured values for a given p true shown in Figure 2. Since the run time is too slow running a minimizer function on a large number of experiments, we calulate the maximum of equation (1) analytically by taking the derivative

$$\frac{\partial f}{\partial p} = n_H(p^{n_H-1})(1-p)^{n_T} + n_T(p^{n_H})(1-p)^{n_T-1} = 0 \tag{2}$$

which gives

$$p = \frac{n_H}{n_T + n_H} \tag{3}$$

when solving for p. Equation (3) is then used as the maximum p measured for each simulated experiment.

With 10 flips per experiment, a true value of 0.491 gave an average measurement of 0.501 with a standard deviation of 0.157 over the course of 1000 experiments. Taking two standard deviations as the 95% confidence interval, this leads to $\pm$ 0.314 as the uncertainty. 100 flips at the same 0.491 true value gives 0.492 $\pm$ 0.049 average measurement. 1000 flips gives 0.491 $\pm$ 0.016 average measurement.

4 Conclusion

Depending on the precision needed, determining the weight of a coin by flipping it over and over could be quite difficult. To 95% confidence, 10 flips is quite unreliable, leading to a measured value that could be off by as much as 30%. The uncertainty improves with more flips, but if one wanted to know the chance of flipping heads within 1%, this would take more than 1000 flips per coin. Perhaps accepting an uncertainty of 5% chance of heads is more reasonable, in which case testing 100 flips per coin .