PHSX815_Project4:
Determining surface temperature based on spectral distribution

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1 Introduction

Wien's displacement law is an important tool in using spectra to determine surface temperatures of radiating objects in our universe. In this project, I will simulate spectral measurement of black-body radiation used to determine the surface temperature of stars. I will plot histograms of energies as a function of wavelength to simulate experimental measurements. After finding the peak wavelength, I can predict the surface temperature using Wien's displacement law

$$\lambda_{peak} = \frac{b}{T} \tag{1}$$

where b is Wien's displacement constant. The temperature will be altered as a parameter in simulating measurements and compared to the temperature predicted in equation (1). The figure below shows example distributions for a number of surface temperatures.

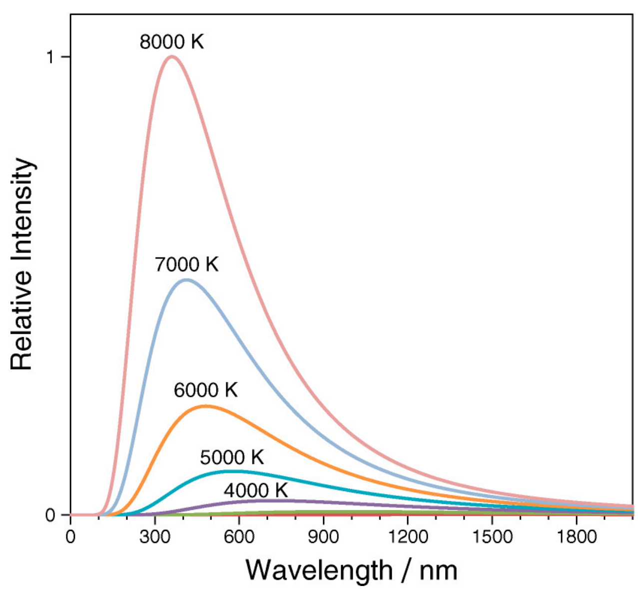

Figure 1: Sample distributions of spectral intensity as a function of wavelength corresponding to different surface temperatures [1]

The distributions shown in the figure above result from Planck's law

$$\mu(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{exp(hc/\lambda kT) - 1} \tag{2}$$

with a constant temperature T.

2 Code and Experimental Simulation

The experimental data is simulated by assigning a probability distribution to a range of wavelengths from 1 nm to 2000 nm and taking 30,000 samples from the distribution. The probability distribution is obtained by normalizing equation (2) to 1 over the wavelength interval [1nm,2000nm]. The results were then plotted into a histogram with 25 bins. The maximum bin is considered to be the measured maximum wavelength. This is meant to simulate an experiment where 25 wavelengths over the range 0 to 2000 nm are sampled and the maximum wavelength measure is taken to be λ_{max}. Data was simulated for stars of 4 different temperatures shown in the figure below.

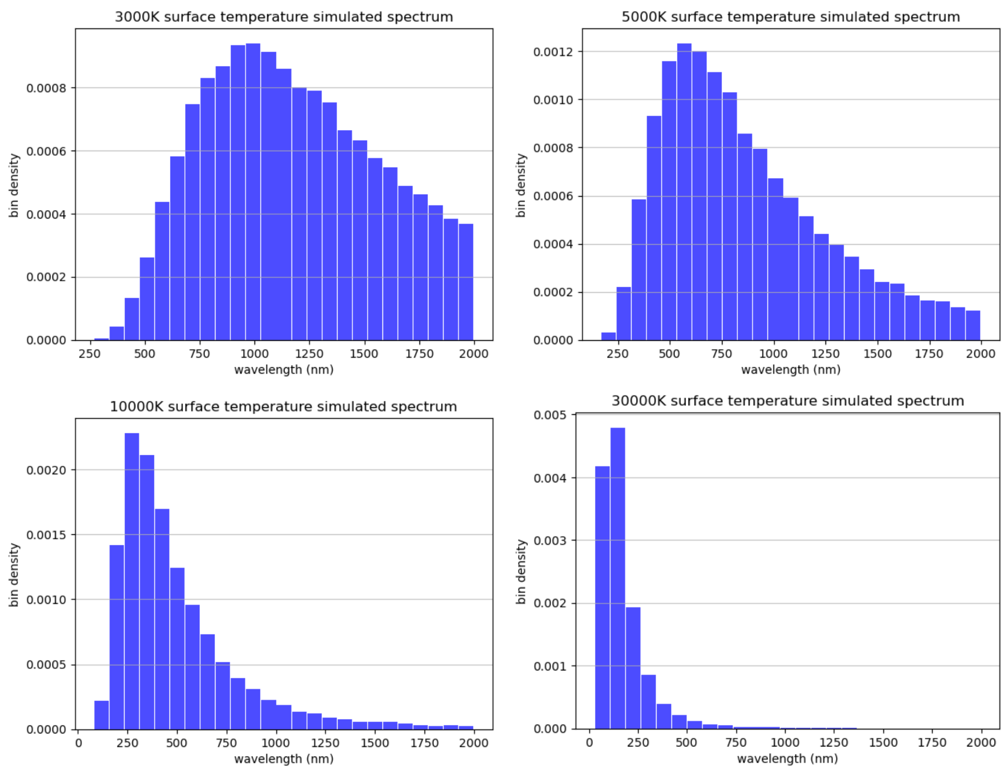

Figure 2: Histograms of simulated data used to determine peak wavelength for a number of temperatures. λ_{max} for each histogram: 925 nm (Top Left), 533 nm (Top Right), 234 nm (Bottom Left), 108 nm (Bottom right).

3 Analysis

Using the literature constant of $b = 2898\mu mK$ in equation (1), we can calculate the predicted temperature based on our histograms. [1] Doing so, we calculate a surface temperature

$$T_{surface} = \frac{2898\mu mK}{\lambda_{max} * 1\mu m/1000nm} \tag{3}$$

for each peak wavelength. This gives surface temperatures of 3132, 5433, 12384, and 26883 K for the stars simulated at 3000, 5000, 10000, and 30000 K respectively. In using the histograms to find the maximum wavelength, a range from 1 to 2000 nm is collected into 25 bins, resulting in a bin size of 80 nm. This results in an uncertainty of $\pm$ 40 nm when taking the center of the largest bin as the peak wavelength. Given a random sampling of 30,000 samples from the distribution, let the sampling uncertainty be negligable compared to this $\pm$ 40 nm. Then we use propagation of uncertainty to to calculate the uncertainty in surface temperature for each star.

$$\delta T = \frac{\partial T}{\partial \lambda_{max}} \delta \lambda_{max} \tag{4}$$

to get 3135 ± 136 K, 5433 ± 408 K, 12384 ± 1963 K, and 26883 ± 9938 K as our predicted surface temperatures.

4 Conclusion

I simulated experimental spectra via planck's law using star surface temperatures of 3000, 5000, 10000, and 30000 K, and took histograms of the resulting distributions. By taking the peak wavelengths and applying Wien's displacement law, the simulated measurements predicted surface temperatures of 3135 ± 136 K, 5433 ± 408 K, 12384 ± 1963 K, and 26883 ± 9938 K. Although two of the predicted surface temperatures fell within the expected range of uncertainty based on the construction of the histograms, the other two predicted temperatures fell outside projected uncertainty compared to the actual temperatures used to construct the simulation. This suggests another source of uncertainty in addition to the construction of the histogram. Perhaps the consideration of sampling uncertainty to be negligable was inaccurate or the sampling method was not truly random.

References

[1] Ball, D.W. (2013) 'Wien's displacement law as a function of frequency', Journal of Chemical Education, 90(9), pp. 1250–1252. doi:10.1021/ed400113z.