# Package 'CoFESWave'

January 14, 2020

2 CoFEScoherency

## **Description**

Computes the Wavelet Coherency matrix (wco) of two series x and y, using a Gabor Wavelet Transform. Different windows for smoothing can be used. Also computes the cross-Wavelet Transform (smoothed (scross)) or non-smoothed (cross)). It also gives the p-values for the wavelet coherency (pvCo) these are computed with n.sur surrogate series. The surrogates are constructed by fitting an ARMA(p,q) model to our series and building new samples by drawing errors from a Gaussian distribution. The major part of the code has been adopted from L. Aguiar-Conraria and M.J. Soares.

## Usage

```
CoFEScoherency(
  Х,
  у,
  dt = 1,
  dj = 0.25,
  low.period = 2 * dt,
  up.period = length(x) * dt,
  pad = 0,
  sigma = 1,
  wt.type = 0,
  wt.size = 5,
  ws.type = 0,
  ws.size = 5,
  n.sur = 0,
  p = 0,
  q = 0,
  Phase_diff = FALSE,
  low.fp = 32,
  up.fp = 128,
  date = NULL
```

# Arguments

```
Χ
                  one series
                  another serise (same length with x)
У
dt
                  sampling rate
                  Default: 1.
dj
                  frequency resolution (i.e. 1/dj = number of voices per octave)
                  Default: 0.25.
                  lower period of the decomposition
low.period
                  Default: 2*dt.
up.period
                  upper period of the decomposition
                  Default: length(x)*dt.
                  an integer (power of 2) defining the total length of the vectors x and x after zero
pad
                  padding; if pad is not a power of 2, pad with zeros to total length: 2^(next power
                  of 2 + 2)
sigma
                  the sigma parameter for the Gabor wavelet
                  Default: 1 - Morlet Wavelet.
wt.type
                  type of window for smoothing in time direction
```

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0 : Bartlett1 : Blackman2 : Rectangular3 : Hamming4 : Hanning5 : Triangular

Default: Bartlett.

wt.size (half) size of window in time

Default: 5.

ws.type type of window for smoothing in scale direction

0 : Bartlett
1 : Blackman
2 : Rectangular
3 : Hamming
4 : Hanning
5 : Triangular

Default: Bartlett.

ws.size (half) size of window in scale

Default: 5.

n.sur integer, number of surrogate series, if we want to compute p-values for the

Wavelet Coherency

Default: 0 - no computation.

p non-negative integers, orders of the ARMA(P,Q) model

Default: 0.

q non-negative integers, orders of the ARMA(P,Q) model

Default: 0.

Phase\_diff Calculate the phase-difference? Logical.

Default: TRUE.

low. fp lower periods used in the computation of phase-difference

Default: 32.

up.fp upper periods used in the computation of phase-difference

Default: 128.

date a date series

## **Details**

CoFEScoherency

#### Value

A list of class "CoFEScoherency" with elements of different dimensions.

Here is a detailed list of all elements: #'

wco complex wavelet coherency of x and y

periods the vector of Fourier periods (in time units) that correspond to the scales

used

scales the vector of scales, given by  $s0*2^{(j*dj)}$ ; j=0,...,11, where J1+1 is number of

scales and s0 is minimum scale

coi the "cone-of-influence", which is a vector of n\_x=length(x) points that contains

the limit of the region where the wavelet transforms are influenced by edge ef-

fects

pvCo p-values for wavelet coherency cross cross-wavelet transform of x and y

scross smoothed cross wavelet transform of x and y

rwco real coherency (absolute value of complex coheency)

phase.dif mean phase difference (in selected periods)

time.lag mean time-lag (in selected periods)
average.coer average coherency (average at all times)
average.cross average cross power (average at all times)

date a date series

#### Author(s)

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CoFES. Credits are also due to L. Aguiar-Conraria and M.J. Soares.

#### References

Aguiar-Conraria, L. and Soares, M.J. (2011) "The Continuous Wavelet Transform: A Primer", NIPE Working Paper 16/2011.

what? "Time-Dependent Spectral Analysis of Epidemiological Time-Series with Wavelets", Journal of the Royal Society Interface, 4, 625.36.

Torrence, C. and Compo, T.C., "A Prectical Guide to Wavelet Analysis" (1998), Bulletin of the American Meteorological Society, 79, 605.618.

## **Examples**

## more work

CoFESmpcoherency Multiple or Partial Wavelet Coherencies of several series

#### **Description**

Computes multiple(wmco) and/or partial wavelet coherencies (wpco) of columns of matrix X. These are computed using a Gabor Wavelet Transform (different Gabor wavelets can be used). Different windows for smoothing can be used. One can also compute the corresponding p\_values (if we take n.sur>0). The surrogates are constructed by fitting an ARMA(p,q) model to our series and building new samples by drawing errors from a Gaussian distribution.

The major part of the code has been adopted from L. Aguiar-Conraria and M.J. Soares.

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## Usage

```
CoFESmpcoherency(
  Χ,
  dt = 1,
  dj = 0.25,
  low.period = 2 * dt,
  up.period = nrow(X) * dt,
  pad = 0,
  sigma = 1,
  coher.type = 0,
  index.p = 2,
  wt.type = 0,
  wt.size = 5,
  ws.type = 0,
  ws.size = 5,
  n.sur = 0,
  p = 0,
  q = 0,
  Phase_diff = FALSE,
  low.fp = 32,
  up.fp = 128,
  date = NULL
```

## **Arguments**

Χ matrix with m columns (vectors corresponding to time-series) whose partial or multiple coherencies we want to compute. The first column X[,1] has a special role; e.g. in case of multiple coherency, we want to compte R1.(2...m) dt sampling rate Default: 1. dj frequency resolution (i.e. 1/dj = number of voices per octave)Default: 0.25. low.period lower period of the decomposition Default: 2\*dt. up.period upper period of the decomposition Default: length(x)\*dt. pad an integer (power of 2) defining the total length of the vectors x and x after zero padding; if pad is not a power of 2, pad with zeros to total length: 2^(next power of 2 + 2) sigma the sigma parameter for the Gabor wavelet Default: 1 - Morlet Wavelet. type of coherency we want to compute coher.type

> 0 or "part" : partial (default) 1 or "mult" : multiple

index.p index of series for partial coherency

Default: 2.

wt.type type of window for smoothing in time direction

0 : Bartlett1 : Blackman2 : Rectangular3 : Hamming4 : Hanning5 : Triangular

Default: Bartlett.

wt.size (half) size of window in time

Default: 5.

ws.type type of window for smoothing in scale direction

0 : Bartlett
1 : Blackman
2 : Rectangular
3 : Hamming
4 : Hanning
5 : Triangular

Default: Bartlett.

ws.size (half) size of window in scale

Default: 5.

n.sur integer, number of surrogate series, if we want to compute p-values for the

Wavelet Coherency

Default: 0 - no computation.

p non-negative integers, orders of the ARMA(P,Q) model

Default: 0.

q non-negative integers, orders of the ARMA(P,Q) model

Default: 0.

Phase\_diff Calculate the phase-difference? Logical.

Default: TRUE.

low.fp lower periods used in the computation of phase-difference

Default: 32.

up.fp upper periods used in the computation of phase-difference

Default: 128.

date a date series

## **Details**

CoFESmpcoherency

## Value

A list of class "CoFESmpcoherency" with elements of different dimensions.

Here is a detailed list of all elements: #'

wmpco Wavelet Multiple or Partial Coherency Matrix

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periods the vector of Fourier periods (in time units) that corresponds to the used scales scales the vector of scales, given by s0\*2^(j\*dj);j=0,...,J1, where J1+1 is number of

scales and s0 is minimum scale

the "cone-of-influence", which is a vector of n\_x=length(x) points that contains

the limit of the region where the wavelet transforms are influenced by edge ef-

fects

low.fp lower periods where phase-diff is computed up.fp upper periods where phase-diff is computed

pvMP matrix of p-values for the multiple/partial wavelet coherency

phase.dif mean phase difference (in the selected periods)

time.lag (in the selected periods)

average.wpco mean partial coherency (we take absolute value of partial coherency)

date a date series

## Author(s)

CoFES. Credits are also due to L. Aguiar-Conraria and M.J. Soares.

#### References

Aguiar-Conraria, L. and Soares, M.J. (2010)

Soares, M.J. (2010), "Multiple and Partial Wavelet Coherencies" (private notes) available at http:

Torrence, C. and Compo, T.C., "A Prectical Guide to Wavelet Analysis" (1998), Bulletin of the American Meteorological Society, 79, 605.618.

## **Examples**

## more work

CoFESWave WaveL2E: A package for ...

## **Description**

The WaveL2E package provides? categories of important functions: WaveL2E, plot.CoFESWaveWCO, etc.

## **CoFESWave functions**

WaveL2E

CoFESWave.image

Image plot of the wavelet power spectrum of a single time series

## **Description**

This function plots the wavelet power spectrum of a single time series, which is provided by an object of class "analyze.wavelet", or alternatively of class "analyze.coherency". (In the latter case, the series number or name must be specified.) The vertical axis shows the Fourier periods. The horizontal axis shows time step counts, but can be easily transformed into a calendar axis if dates are provided in either row names or as a variable named "date" in the data frame at hand. Both axes can be relabeled. In particular, an option is given to individualize the period and/or time axis by specifying tick marks and labels.

An option is given to raise wavelet power values to any (positive) exponent before plotting in order to accentuate the contrast of the image.

The color levels can be defined according to quantiles of values or according to equidistant breakpoints (covering the interval from 0 to maximum level), with the number of levels as a further parameter. A user-defined maximum level can be applied. In addition, there is an option to adopt an individual color palette.

Further plot design options concern: plot of the cone of influence, plot of wavelet power contour lines at a specified level of significance, plot of power ridges.

Finally, there is an option to insert and format a color legend (a right-hand vertical color bar) and to set the plot title. For further processing of the plot, graphical parameters of plot regions are provided as output.

The name and parts of the layout were inspired by a similar function developed by Huidong Tian and Bernard Cazelles (archived R package WaveletCo).

#### Usage

```
CoFESWave.image(
  WT,
  mv.series = 1,
  exponent = 1,
  plot.coi = TRUE,
  plot.contour = TRUE,
  siglvl = 0.1,
  col.contour = "white",
  plot.ridge = TRUE,
  1v1 = 0,
  col.ridge = "black",
  color.key = "quantile",
  n.levels = 100,
  color.palette = "rainbow(n.levels, start = 0, end = .7)",
  maximum.level = NULL,
  useRaster = TRUE,
  max.contour.segments = 250000,
  plot.legend = TRUE,
 legend.params = list(width = 1.2, shrink = 0.9, mar = 5.1, n.ticks = 6, label.digits =
    1, label.format = "f", lab = NULL, lab.line = 2.5),
  label.time.axis = TRUE,
```

```
show.date = FALSE,
  date.format = NULL,
 date.tz = NULL,
  timelab = NULL,
  timetck = 0.02,
  timetcl = 0.5,
 spec.time.axis = list(at = NULL, labels = TRUE, las = 1, hadj = NA, padj = NA),
  label.period.axis = TRUE,
  periodlab = NULL,
 periodtck = 0.02,
 periodtcl = 0.5,
 spec.period.axis = list(at = NULL, labels = TRUE, las = 1, hadj = NA, padj = NA),
 main = NULL,
 1wd = 2,
 lwd.axis = 1,
  graphics.reset = TRUE,
  verbose = FALSE
)
```

#### **Arguments**

WT an object of class "analyze.wavelet" or "analyze.coherency" In case class(WT) = "analyze.coherency": number (1 or 2) or name of the my.series series to be analyzed. Default: 1. exponent Exponent applied to values before plotting in order to accentuate the contrast of the image; the exponent should be positive. Default: 1. Plot cone of influence? Logical. plot.coi Default: TRUE. Plot contour lines to border the area of wavelet power significance? Logical. plot.contour Default: TRUE. level of wavelet power significance to be applied to the plot of contour lines. siglvl Default: 0.1. col.contour color of contour lines. Default: "white". Plot the wavelet power ridge? Logical. plot.ridge Default: TRUE. 1v1 minimum level of wavelet power for ridge to be plotted. Default: 0. ridge color. col.ridge Default: "black". color.key How to assign colors to power and coherence levels? Two options: "interval" or "i" equidistant breakpoints (from 0 through maximum value)

quantiles

Default: "quantile".

"quantile" or "q" :

n.levels Number of color levels.

Default: 100.

color palette Definition of color levels. (The color palette will be assigned to levels in reverse

order!)

Default: "rainbow(n.levels, start = 0, end = .7)".

maximum.level Maximum plot level of wavelet power considered; only effective in case of

equidistant breakpoints (color.key equaling "i").

Default: NULL (referring to maximum level observed).

useRaster Use a bitmap raster instead of polygons to plot the image? Logical.

Default: TRUE.

max.contour.segments

limit on the number of segments in a single contour line, positive integer.

Default: 250000 (options(...) default settings: 25000).

plot.legend Plot color legend (a vertical bar of colors and breakpoints)? Logical.

Default: TRUE.

legend.params a list of parameters for the plot of the color legend; parameter values can be set

selectively (style in parts adopted from image.plot in the R package fields

by Douglas Nychka):

width: width of legend bar.

Default: 1.2.

shrink: a vertical shrinkage factor.

Default: 0.9.

mar: right margin of legend bar.

Default: 5.1.

n.ticks: number of ticks for labels.

Default: 6.

label.digits: digits of labels.

Default: 1.

label.format: format of labels.

Default: "f".

lab: axis label.

Default: NULL.

lab.line: line (in user coordinate units) where to put the axis label.

Default: 2.5.

label.time.axis

Label the time axis? Logical.

Default: TRUE.

show date Show calendar dates? (Effective only if dates are available as row names or by

variable date in the data frame which is analyzed.) Logical.

Default: FALSE.

date.format the format of calendar date given as a character string, e.g. "%Y-%m-%d", or

equivalently "%F"; see strptime for a list of implemented date conversion specifications. Explicit information given here will overturn any specification stored in WT. If unspecified, date formatting is attempted according to as.Date.

Default: NULL.

date.tz a character string specifying the time zone of calendar date; see strptime. Ex-

plicit information given here will overturn any specification stored in WT. If un-

specified, "" (the local time zone) is used.

Default: NULL.

timelab Time axis label.

Default: "index"; in case of a calendar axis: "calendar date".

timetck

length of tick marks on the time axis as a fraction of the smaller of the width or height of the plotting region; see par. If timetck  $\geq 0.5$ , timetck is interpreted as a fraction of the length of the time axis, so if timetck = 1 (and timetcl = NULL), vertical grid lines will be drawn.

Setting timetck = NA is to use timetcl = -0.5 (which is the R default setting

of tck and tcl).
Default here: 0.02.

timetcl

length of tick marks on the time axis as a fraction of the height of a line of text; see par. With timetcl = -0.5 (which is the R default setting of tcl), ticks will be drawn outward.

Default here: 0.5.

spec.time.axis a list of tick mark and label specifications for individualized time axis labeling (only effective if label.time.axis = TRUE):

at: locations of tick marks (when NULL, default plotting will be applied). Valid tick marks can be provided as numerical values or as dates. Dates are used only in the case show.date = TRUE, however, and date formats should conform to as.Date or the format given in date.format.

Default: NULL.

labels: either a logical value specifying whether annotations at the tick marks are the tick marks themselves, or any vector of labels. If labels is non-logical, at should be of same length.

Default: TRUE.

las: the style of axis labels, see par.

Default: 1 (always horizontal).

hadj: adjustment of labels horizontal to the reading direction, see axis.

Default: NA (centering is used).

padj: adjustment of labels perpendicular to the reading direction (this can be a vector of adjustments for each label), see axis.

Default: NA (centering is used).

Mismatches will result in a reset to default plotting.

label.period.axis

Label the (Fourier) period axis? Logical.

Default: TRUE.

periodlab (Fourier) period axis label.

Default: "period".

periodtck

length of tick marks on the period axis as a fraction of the smaller of the width or height of the plotting region; see par. If periodtck >= 0.5, periodtck is interpreted as a fraction of the length of the period axis, so if periodtck = 1 (and periodtcl = NULL), horizontal grid lines will be drawn.

Setting periodtck = NA is to use periodtcl = -0.5 (which is the R default setting of tck and tcl).

Default here: 0.02.

periodtcl

length of tick marks on the period axis as a fraction of the height of a line of text; see par. With periodtcl = -0.5 (which is the R default setting of tcl) ticks will be drawn outward.

Default here: 0.5.

spec.period.axis

a list of tick mark and label specifications for individualized period axis labeling (only effective if label.period.axis = TRUE):

at: locations of tick marks (when NULL, default plotting will be applied). Valid tick marks can be provided as numerical and positive values only. Default: NULL.

labels: either a logical value specifying whether annotations at the tick marks are the tick marks themselves, or any vector of labels. If labels is non-logical, at should be of same length.

Default: TRUE.

las: the style of axis labels, see par. Default: 1 (always horizontal).

hadj: adjustment of labels horizontal to the reading direction, see axis. Default: NA (centering is used).

padj: adjustment of labels perpendicular to the reading direction (this can be a vector of adjustments for each label), see axis.

Default: NA (centering is used).

Mismatches will result in a reset to default plotting.

main an overall title for the plot.

Default: NULL.

lwd line width of contour lines and ridge.

Default: 2.

lwd.axis line width of axes (image and legend bar).

Default: 1.

graphics.reset Reset graphical parameters? Logical.

Default: TRUE.

verbose Print verbose output on the screen? Logical.

Default: FALSE.

## **Details**

CoFESWave.image

## Value

A list of class graphical parameters with the following elements:

op original graphical parameters

image.plt image plot region
legend.plt legend plot region

## Author(s)

CoFES. Credits are also due to Angi Roesch, Harald Schmidbauer, Huidong Tian, and Bernard Cazelles.

#### References

Aguiar-Conraria L., and Soares M.J., 2011. The Continuous Wavelet Transform: A Primer. NIPE Working Paper Series 16/2011.

Carmona R., Hwang W.-L., and Torresani B., 1998. Practical Time Frequency Analysis. Gabor and Wavelet Transforms with an Implementation in S. Academic Press, San Diego.

Cazelles B., Chavez M., Berteaux, D., Menard F., Vik J.O., Jenouvrier S., and Stenseth N.C., 2008. Wavelet analysis of ecological time series. Oecologia 156, 287–304.

Liu Y., Liang X.S., and Weisberg R.H., 2007. Rectification of the Bias in the Wavelet Power Spectrum. Journal of Atmospheric and Oceanic Technology 24, 2093–2102.

Tian, H., and Cazelles, B., 2012. WaveletCo. Available at https://cran.r-project.org/src/contrib/Archive/WaveletCo/, archived April 2013; accessed July 26, 2013.

Torrence C., and Compo G.P., 1998. A practical guide to wavelet analysis. Bulletin of the American Meteorological Society 79 (1), 61–78.

#### See Also

CoFESWave.Transform, CoFESWave.reconstruct

#### **Examples**

```
## Not run:
## The following example is adopted from Liu et al., 2007:
series.length <- 6*128*24
x1 <- periodic.series(start.period = 1*24, length = series.length)</pre>
x2 <- periodic.series(start.period = 8*24, length = series.length)</pre>
x3 <- periodic.series(start.period = 32*24, length = series.length)
x4 <- periodic.series(start.period = 128*24, length = series.length)</pre>
x < -x1 + x2 + x3 + x4
plot(x, type = "l", xlab = "index", ylab = "", xaxs = "i",
     main = "hourly series with periods of 1, 8, 32, 128 days")
## The following dates refer to the local time zone
## (possibly allowing for daylight saving time):
my.date <- seq(as.POSIXct("2014-10-14 00:00:00", format = "\</pre>
               by = "hour",
               length.out = series.length)
my.data <- data.frame(date = my.date, x = x)</pre>
## Computation of wavelet power:
## a natural choice of 'dt' in the case of hourly data is 'dt = 1/24',
## resulting in one time unit equaling one day.
## This is also the time unit in which periods are measured.
my.wt <- analyze.wavelet(my.data, "x",</pre>
                          loess.span = 0,
                          dt = 1/24, dj = 1/20,
                          lowerPeriod = 1/4,
                          make.pval = TRUE, n.sim = 10)
## Plot of wavelet power spectrum
## with color breakpoints referring to quantiles:
wt.image(my.wt, main = "wavelet power spectrum",
```

```
legend.params = list(lab = "wavelet power levels (quantiles)",
                               lab.line = 3.5,
                               label.digits = 2),
         periodlab = "period (days)")
## Note:
## The default time axis shows an index of given points in time,
## which is the count of hours in our example.
## The same plot, but with equidistant color breakpoints:
wt.image(my.wt, color.key = "i", main = "wavelet power spectrum",
         legend.params = list(lab = "wavelet power levels (equidistant)"),
         periodlab = "period (days)")
## Alternative styles of the time axis:
## The plot with time elapsed in days, starting from 0 and proceeding
## in steps of 50 days (50*24 hours),
## instead of the (default) time index:
index.ticks \leftarrow seq(1, series.length, by = 50*24)
index.labels <- (index.ticks-1)/24</pre>
## Insert your specification of the time axis:
wt.image(my.wt, color.key = "i", main = "wavelet power spectrum",
         legend.params = list(lab = "wavelet power levels (equidistant)"),
         periodlab = "period (days)", timelab = "time elapsed (days)",
spec.time.axis = list(at = index.ticks, labels = index.labels))
## The plot with (automatically produced) calendar axis:
wt.image(my.wt, color.key = "i", main = "wavelet power spectrum",
         legend.params = list(lab = "wavelet power levels (equidistant)"),
         periodlab = "period (days)",
         show.date = TRUE, date.format = "\
## Individualizing your calendar axis (works with 'show.date = TRUE')...
## How to obtain, for example, monthly date ticks and labels:
## The sequence of tick positions:
monthly.ticks <- seq(as.POSIXct("2014-11-01 00:00:00", format = "\
                      as.POSIXct("2016-11-01 00:00:00", format = "\
                      by = "month")
## Observe that the following specification may produce an error:
## 'seq(as.Date("2014-11-01"), as.Date("2016-11-01"), by = "month")'
## Time of the day is missing here!
## The sequence of labels (e.g. information on month and year only):
monthly.labels <- strftime(monthly.ticks, format = "\</pre>
## Insert your specification of the time axis:
wt.image(my.wt, color.key = "i", main = "wavelet power spectrum",
         legend.params = list(lab = "wavelet power levels (equidistant)"),
         periodlab = "period (days)",
         show.date = TRUE, date.format = "\
         spec.time.axis = list(at = monthly.ticks, labels = monthly.labels,
                                las = 2))
## The monthly ticks specify the midpoints of the colored cells and match
## the location of corresponding (default) time index ticks.
```

```
## Furthermore, the plot with an individualized period axis:
wt.image(my.wt, color.key = "i", main = "wavelet power spectrum",
         legend.params = list(lab = "wavelet power levels (equidistant)"),
         periodlab = "period (days)",
         show.date = TRUE, date.format = "\
         spec.time.axis = list(at = monthly.ticks, labels = monthly.labels,
                               las = 2),
         spec.period.axis = list(at = c(1,8,32,128)))
## Switching the time axis from index to time elapsed in hours
## (starting from 0, and proceeding in steps of 500 hours),
## and the period axis from days to hours:
index.ticks <- seq(1, series.length, by = 500)</pre>
index.labels <- index.ticks - 1</pre>
wt.image(my.wt, color.key = "i", main = "wavelet power spectrum",
         legend.params = list(lab = "wavelet power levels (equidistant)"),
         timelab = "time elapsed (hours)", periodlab = "period (hours)",
         spec.time.axis = list(at = index.ticks, labels = index.labels),
         spec.period.axis = list(at = c(1,8,32,128), labels = c(1,8,32,128)*24))
## A plot with different colors:
wt.image(my.wt, main = "wavelet power spectrum",
         legend.params = list(lab = "wavelet power levels (quantiles)",
                              lab.line = 3.5, label.digits = 2),
         color.palette = "gray((1:n.levels)/n.levels)", col.ridge = "yellow",
         periodlab = "period (days)")
## In the case of monthly (or quarterly) data, the time axis should be
## labeled at equally spaced time points. An example:
monthyear <- seq(as.Date("2014-01-01"), as.Date("2018-01-01"),
                 by = "month")
monthyear <- strftime(monthyear, format = "\</pre>
xx <- periodic.series(start.period = 6, length = length(monthyear))</pre>
xx <- xx + 0.2*rnorm(length(monthyear))</pre>
plot(xx, type = "l", xlab = "index", ylab = "", xaxs = "i",
     main = "monthly series with period of 6 months")
monthly.data <- data.frame(date = monthyear, xx = xx)</pre>
my.wt <- analyze.wavelet(monthly.data, "xx", loess.span = 0,</pre>
                          dt = 1, dj = 1/250,
                         make.pval = TRUE, n.sim = 250)
## Note:
## The natural choice of 'dt' in this example is 'dt = 1',
## resulting in periods measured in months.
## (Setting 'dt = 1/12' would result in periods measured in years.)
## The default wavelet power plot then shows the monthly:
wt.image(my.wt, main = "wavelet power spectrum",
         periodlab = "period (months)")
## The following plot shows the elapsed time, measured in months:
wt.image(my.wt, main = "wavelet power spectrum",
         periodlab = "period (months)", timelab = "time elapsed (months)",
```

```
spec.time.axis = list(at = 1:length(monthyear),
                               labels = (1:length(monthyear))-1))
## In case you prefer the monthyear labels themselves:
wt.image(my.wt, main = "wavelet power spectrum",
         periodlab = "period (months)", timelab = "month and year",
         spec.time.axis = list(at = 1:length(monthyear), labels = monthyear))
## You may sometimes wish to enhance your plot with additional information.
## There is an option to add further objects to the image plot region,
## by setting 'graphics.reset = FALSE'
## (but recall previous par settings after plotting):
op <- par(no.readonly = TRUE)</pre>
wt.image(my.wt, main = "wavelet power spectrum",
         periodlab = "period (months)",
         spec.period.axis = list(at = c(2,4,6,8,12)),
         spec.time.axis = list(at = 1:length(monthyear),
                               labels = substr(monthyear,1,3)),
         graphics.reset = FALSE)
abline(h = log2(6), lty = 3)
abline(v = seq(1, length(monthyear), by = 12), lty = 3)
mtext(2014:2018, side = 1,
      at = seq(1, length(monthyear), by = 12), line = 2)
par(op)
## For further axis plotting options:
## Please see the examples in our guide booklet,
## URL http://www.hs-stat.com/projects/WaveletComp/WaveletComp_guided_tour.pdf.
## End(Not run)
```

CoFESWave.reconstruct Reconstruction of a (detrended) time series from output provided by an object of class "analyze.wavelet" or "analyze.coherency"

## **Description**

This function reconstructs a (detrended) time series analyzed by wavelet transformation using either function analyze.wavelet or function analyze.coherency, subject to optional criteria concerning: minimum wavelet power, significance of wavelet power at a given significance level, specification of (Fourier) periods or period bands, exclusive use of the power ridge and/or the cone of influence. An option is provided to prevent the reconstructed series from final rescaling (applying the original (detrended) series' mean and standard deviation).

(If the object provided as input is of class "analyze.coherency", then the number or name of the time series must be specified.)

Optional: plot of wavelets used for reconstruction, plot of reconstructed series against original (detrended) series. An option is given to individualize the time axis by specifying tick marks and labels.

Output includes the original (detrended) and the reconstructed time series, along with reconstruction waves and parameters.

#### Usage

```
CoFESWave.reconstruct(
  WT,
  my.series = 1,
  1v1 = 0,
  only.ridge = FALSE,
  only.sig = TRUE,
  siglvl = 0.05,
  only.coi = FALSE,
  sel.period = NULL,
  sel.lower = NULL,
  sel.upper = NULL,
  rescale = TRUE,
  plot.waves = FALSE,
  plot.rec = TRUE,
  lty = 1,
  lwd = 1,
  col = 1:2,
  ylim = NULL,
  show.legend = TRUE,
  legend.coords = "topleft",
  legend.horiz = FALSE,
  legend.text = NULL,
  label.time.axis = TRUE,
  show.date = FALSE,
  date.format = NULL,
  date.tz = NULL,
  timelab = NULL,
  timetck = 0.02,
  timetcl = 0.5,
  spec.time.axis = list(at = NULL, labels = TRUE, las = 1, hadj = NA, padj = NA),
  main.waves = NULL,
  main.rec = NULL,
  main = NULL,
  lwd.axis = 1,
  verbose = TRUE
)
```

#### **Arguments**

WT	an object of class "analyze.wavelet" or "analyze.coherency"
my.series	In case class(WT) = "analyze.coherency": number (1 or 2) or name of the series to be analyzed.  Default: 1.
lvl	minimum level of wavelet power to be applied for the inclusion of reconstruction waves. Default: $\emptyset$ .
only.ridge	Select only the wavelet power ridge? Logical.  Default: FALSE.
only.sig	Use wavelet power significance in reconstruction? Logical.  Default: TRUE.

siglvl level of wavelet power significance to be applied for the inclusion of reconstruction waves. Default: 0.05. Exclude borders influenced by edge effects in reconstruction, i.e. include the only.coi cone of influence only? Logical. Default: FALSE. sel.period a vector of numbers to select Fourier periods (or closest available periods) and corresponding wavelets for the reconstruction. Default: NULL. sel.lower a number to define a lower Fourier period (or the closest available) for the selection of a band of wavelets for the reconstruction. (Only effective if sel.period = NULL.) Default: NULL. a number to define an upper Fourier period (or the closest available) for the sel.upper selection of a band of wavelets for the reconstruction. (Only effective if sel.period = NULL.) Default: NULL. Shall the reconstructed series finally be rescaled to attain the original (detrended) rescale series' mean and standard deviation? Logical. Default: TRUE. Shall reconstruction waves be plotted? Logical. plot.waves Default: FALSE. Shall the reconstructed series (together with the original (detrended) series) be plot.rec plotted? Logical. Default: TRUE. lty parameter for the plot of original vs. reconstructed series: line type, e.g. 1:2. Default: 1. lwd parameter for the plot of original vs. reconstructed series: line width, e.g. 1:2. Default: 1. col parameter for the plot of original vs. reconstructed series: color of lines. Default: 1:2. ylim numeric vector of length 2, providing the range of vertical coordinates for the plot. Default: NULL. show.legend Include legend into the plot of original vs. reconstructed series? Logical. Default: TRUE. coordinates to position the legend (as in function legend). legend.coords Default: "topleft". legend.horiz Set the legend horizontally rather than vertically? Logical. Default: FALSE. legend.text legend text. Default: c("original (detrended)", "reconstructed"). label.time.axis Label the time axis? Logical.

Default: TRUE.

show.date Show calendar dates? (Effective only if dates are available as row names or by

variable date in the data frame which is analyzed.) Logical.

Default: FALSE.

date.format the format of calendar date given as a character string, e.g. "%Y-%m-%d", or

equivalently "%F"; see strptime for a list of implemented date conversion specifications. Explicit information given here will overturn any specification stored in WT. If unspecified, date formatting is attempted according to as.Date.

Default: NULL.

date.tz a character string specifying the time zone of calendar date; see strptime. Ex-

plicit information given here will overturn any specification stored in WT. If un-

specified, "" (the local time zone) is used.

Default: NULL.

timelab Time axis label.

Default: "index"; in case of a calendar axis: "calendar date".

timetck length of tick marks on the time axis as a fraction of the smaller of the width

or height of the plotting region; see par. If timetck >= 0.5, timetck is interpreted as a fraction of the length of the time axis, so if timetck = 1 (and

timetcl = NULL), vertical grid lines will be drawn.

Setting timetck = NA is to use timetcl = -0.5 (which is the R default setting

of tck and tcl).

Default here: 0.02.

Belaut here. 0.02.

timetcl length of tick marks on the time axis as a fraction of the height of a line of text;

see par. With timetcl = -0.5 (which is the R default setting of tcl), ticks

will be drawn outward. Default here: 0.5.

spec.time.axis a list of tick mark and label specifications for individualized time axis labeling

(only effective if label.time.axis = TRUE):

at: locations of tick marks (when NULL, default plotting will be applied). Valid tick marks can be provided as numerical values or as dates. Dates are used only in the case show.date = TRUE, however, and date formats should conform to as.Date or the format given in date.format.

Default: NULL.

labels: either a logical value specifying whether annotations at the tick marks are the tick marks themselves, or any vector of labels. If labels is non-logical,

at should be of same length.

Default: TRUE.

las: the style of axis labels, see par.

Default: 1 (always horizontal).

hadj: adjustment of labels horizontal to the reading direction, see axis.

Default: NA (centering is used).

padj: adjustment of labels perpendicular to the reading direction (this can be a

vector of adjustments for each label), see axis.

Default: NA (centering is used).

Mismatches will result in a reset to default plotting.

main.waves an overall title for the plot of reconstruction waves.

Default: NULL.

main.rec an overall title for the plot of original vs. reconstructed series.

Default: NULL.

main an overall title for both plots.

Default: NULL.

lwd.axis line width of axes.

Default: 1.

verbose Print verbose output on the screen? Logical.

Default: TRUE.

#### Details

CoFESWave.reconstruct

#### Value

A list of class reconstruct with the following elements:

series a data frame building on WT\$series with the following columns:

date : the calendar date (if available as column

in WT\$series)

<x> : series <x>, with original name retained

(detrended, if loess.span != 0)

<x>.trend : the trend series (if loess.span != 0)
<x>.r : the reconstructed (detrended) series

Row names are taken over from WT\$series, and so are dates if given as row names. If WT is of class analyze.coherency, the second series in the coherency analysis is retained; if loess.span != 0, the second series is retained in the detrended version, and the trend is retained as well.

rec.waves data frame of scaled waves used for reconstruction

loess.span parameter alpha in loess controlling the degree of time series smoothing, if the

time series was detrended; no detrending if loess.span = 0.

lvl minimum level of wavelet power for waves (wave segments) to be included in

the reconstruction

only.coi Was the influence of edge effects excluded? I.e. was the cone of influence used

only?

only.sig Was wavelet power significance used in reconstruction?

siglvl level of wavelet power significance only.ridge Was the wavelet power ridge used only?

rnum.used the vector of Fourier period numbers used for reconstruction

rescale Was the reconstructed series rescaled according to the mean and standard devi-

ation taken from the original (detrended) series?

dt time resolution, i.e. sampling resolution in the time domain, 1/dt = number of

observations per time unit

dj frequency resolution, i.e. sampling resolution in the frequency domain, 1/dj =

number of suboctaves (voices per octave)

Period	the Fourier periods (measured in time units determined by dt, see the explanations concerning dt)
Scale	the scales (the Fourier periods divided by the Fourier factor)
nc	number of columns = number of observations = number of observation epochs; "epoch" meaning point in time
nr	number of rows = number of scales (Fourier periods)
axis.1	tick levels corresponding to the time steps used for (cross-)wavelet transformation: 1, 1+dt, 1+2dt, The default time axis in plot functions provided by WaveletComp is determined by observation epochs, however; "epoch" meaning point in time.
axis.2	tick levels corresponding to the log of Fourier periods: log2(Period). This determines the period axis in plot functions provided by WaveletComp.
date.format	the format of calendar date (if available)
date.tz	the time zone of calendar date (if available)

## Author(s)

CoFES. Credits are also due to Angi Roesch and Harald Schmidbauer.

#### References

Carmona R., Hwang W.-L., and Torresani B., 1998. Practical Time Frequency Analysis. Gabor and Wavelet Transforms with an Implementation in S. Academic Press, San Diego.

Liu Y., Liang X.S., and Weisberg R.H., 2007. Rectification of the Bias in the Wavelet Power Spectrum. Journal of Atmospheric and Oceanic Technology 24, 2093–2102.

Torrence C., and Compo G.P., 1998. A practical guide to wavelet analysis. Bulletin of the American Meteorological Society 79 (1), 61–78.

#### See Also

CoFESWave.Transform

# Examples

```
## Computation of wavelet power:
## a natural choice of 'dt' in the case of hourly data is 'dt = 1/24',
## resulting in one time unit equaling one day.
## This is also the time unit in which periods are measured.
my.w <- analyze.wavelet(my.data, "x",</pre>
                        loess.span = 0,
                        dt = 1/24, di = 1/20,
                        lowerPeriod = 1/4,
                        make.pval = TRUE, n.sim = 10)
## Plot of wavelet power spectrum (with equidistant color breakpoints):
wt.image(my.w, color.key = "interval",
         legend.params = list(lab = "wavelet power levels"),
         periodlab = "period (days)")
## Reconstruction of the time series,
## including significant components only:
reconstruct(my.w)
## The same reconstruction, but showing wave components first:
reconstruct(my.w, plot.waves = TRUE)
## Reconstruction, including all components whether significant or not:
reconstruct(my.w, only.sig = FALSE)
## Reconstruction, including significant components,
## but selected periods only (e.g. ignoring period 8):
reconstruct(my.w, sel.period = c(1,32,128))
## Reconstruction, including significant components,
## but the ridge only:
reconstruct(my.w, only.ridge = TRUE)
## Alternate styles of the time axis:
## The plot with time elapsed in days, starting from 0 and proceeding
## in steps of 50 days (50*24 hours),
## instead of the (default) time index:
index.ticks \leftarrow seq(1, series.length, by = 50*24)
index.labels <- (index.ticks-1)/24</pre>
## Insert your specification of time axis:
reconstruct(my.w, only.ridge = TRUE,
            timelab = "time elapsed (days)",
            spec.time.axis = list(at = index.ticks, labels = index.labels))
## See the periods involved:
my.rec <- reconstruct(my.w, only.ridge = TRUE)</pre>
print(my.rec$Period[my.rec$rnum.used])
## The original and reconstructed time series can be retrieved:
plot(my.rec$series$x, type = "1", xlab = "index", ylab = "")
lines(my.rec$series$x.r, col="red")
legend("topleft", legend = c("original", "reconstructed"),
       lty = 1, col = c("black", "red"))
## Please see also the examples in our guide booklet,
```

```
## URL http://www.hs-stat.com/projects/WaveletComp/WaveletComp_guided_tour.pdf.
## End(Not run)
```

CoFESWave.Transform

Computation of the wavelet power spectrum of a single time series

#### **Description**

The time series is selected from an input data frame by specifying either its name or its column number. Optionally, the time series is detrended, using loess with parameter loess.span. Internally, the series will be further standardized before it undergoes wavelet transformation.

The wavelet power spectrum is computed by applying the Morlet wavelet. P-values to test the null hypothesis that a period (within lowerPeriod and upperPeriod) is irrelevant at a certain time are calculated if desired; this is accomplished with the help of a simulation algorithm. There is a selection of models from which to choose the alternative hypothesis. The selected model will be fitted to the data and simulated according to estimated parameters in order to provide surrogate time series.

Wavelet transformation, as well as p-value computations, are carried out by calling subroutine wt.

The name and parts of the layout of subroutine wt were inspired by a similar function developed by Huidong Tian and Bernard Cazelles (archived R package WaveletCo). The basic concept of the simulation algorithm and of ridge determination build on ideas developed by these authors. The major part of the code for the computation of the cone of influence and the code for Fourier-randomized surrogate time series has been adopted from Huidong Tian.

Wavelet computation, the simulation algorithm and ridge determination build heavily on the use of matrices in order to minimize computation time in R.

This function provides a broad variety of final as well as intermediate results which can be further analyzed in detail.

#### Usage

```
CoFESWave.Transform(
 my.data,
 my.series = 1,
 loess.span = 0.75,
  dt = 1,
 dj = 1/20,
  lowerPeriod = 2 * dt,
  upperPeriod = floor(nrow(my.data)/3) * dt,
 make.pval = TRUE,
 method = "white.noise",
 params = NULL,
 n.sim = 100,
 date.format = NULL,
 date.tz = NULL,
  verbose = TRUE
)
```

#### **Arguments**

my . data data frame of time series (including header, and dates as row names or as sepa-

rate columnnamed "date" if available)

my. series name or column index indicating the series to be analyzed, e.g. 1, 2, "dji",

"ftse".

Default: 1.

loess. span parameter alpha in loess controlling the degree of time series smoothing, if the

time series is to be detrended; no detrending if loess. span = 0.

Default: 0.75.

dt time resolution, i.e. sampling resolution in the time domain, 1/dt = number of

observations per time unit. For example: a natural choice of dt in case of hourly data is dt = 1/24, resulting in one time unit equaling one day. This is also the time unit in which periods are measured. If dt = 1, the time interval between

two consecutive observations will equal one time unit.

Default: 1.

dj frequency resolution, i.e. sampling resolution in the frequency domain, 1/dj =

number of suboctaves (voices per octave).

Default: 1/20.

lowerPeriod lower Fourier period (measured in time units determined by dt, see the expla-

nations concerning dt) for wavelet decomposition.

If dt = 1, the minimum admissible value is 2.

Default: 2\*dt.

upperPeriod upper Fourier period (measured in time units determined by dt, see the expla-

nations concerning dt) for wavelet decomposition.

Default: (floor of one third of time series length)\*dt.

make.pval Compute p-values? Logical.

Default: TRUE.

method the method of generating surrogate time series; select from:

"white.noise" : white noise

"shuffle" : shuffling the given time series
"Fourier.rand" : time series with a similar spectrum

"AR" : AR(p)

"ARIMA" : ARIMA(p,0,q)

Default: "white.noise".

params a list of assignments between methods (AR, and ARIMA) and lists of parameter

values applying to surrogates. Default:  $\ensuremath{\mathsf{NULL}}$ 

Default includes two lists named AR and ARIMA:

• AR = list(...), a list containing one single element:

p : AR order. Default: 1.

• ARIMA = list(...), a list of six elements:

p : AR order.

Default: 1.

q : MA order.

Default: 1.

include.mean : Include a mean/intercept term?

Default: TRUE.

sd. fac : magnification factor to boost the

residual standard deviation.

Default: 1.

trim : Simulate trimmed data?

Default: FALSE.

trim.prop : high/low trimming proportion.

Default: 0.01.

n.sim number of simulations.

Default: 100.

date.format optional, and for later reference: the format of calendar date (if available in the

input data frame) given as a character string, e.g. "%Y-%m-%d", or equivalently "%F"; see strptime for a list of implemented date conversion specifications. Explicit information given here will be overwritten by any later specification given in e.g. wt.image. If unspecified, date formatting will be attempted ac-

cording to as.Date.

Default: NULL.

date.tz optional, and for later reference: a character string specifying the time zone

of calendar date (if available in the input data frame); see strptime. Explicit information given here will be overwritten by any specification given in e.g.

wt.image.

If unspecified, "" (the local time zone) will be used. Default: NULL.

verbose Print verbose output on the screen? Logical.

Default: TRUE.

#### Details

CoFESWave.Transform

Wavelet transformation, as well as p-value computations, are carried out by calling the internal function wt.

## Value

A list of class "analyze.wavelet" with elements of different dimensions.

The elements of matrix type (namely, Wave, Phase, Ampl, Power, Power.pval, Ridge) have the following structure:

columns correspond to observations (observation epochs; "epoch" meaning point in time), rows correspond to scales (Fourier periods) whose values are given in Scale (Period).

Here is a detailed list of all elements:

series a data frame with the following columns:

date : the calendar date

(if available as column in my.data)

<x> : the series which has been analyzed

(detrended, if loess.span != 0;

original name retained)

<x>.trend : the trend series (if loess.span != 0)

Row names are taken over from my. data, and so are dates if given as row names.

loess.span parameter alpha in loess controlling the degree of time series smoothing if the

time series was detrended; no detrending if loess.span = 0

dt time resolution, i.e. sampling resolution in the time domain, 1/dt = number of

observations per time unit

dj frequency resolution, i.e. sampling resolution in the frequency domain, 1/dj =

number of suboctaves (voices per octave)

Wave complex wavelet transform of the series

Phase phases
Ampl amplitudes

Power wavelet power in the time/frequency domain

Power . avg average wavelet power in the frequency domain (averages over time)

Power.pval p-values of wavelet power

Power.avg.pval p-values of average wavelet power

Ridge wavelet power ridge, in the form of a matrix of 0s and 1s

Period the Fourier periods (measured in time units determined by dt, see the explana-

tions concerning dt)

Scale the scales (the Fourier periods divided by the Fourier factor)

nc number of columns = number of observations = number of observation epochs;

"epoch" meaning point in time

nr number of rows = number of scales (Fourier periods)

coi.1, coi.2 borders of the region where the wavelet transforms are not influenced by edge

effects (cone of influence). The coordinates of the borders are expressed in terms

of internal axes axis.1 and axis.2.

axis.1 tick levels corresponding to the time steps used for (cross-)wavelet transforma-

tion: 1, 1+dt, 1+2dt, .... The default time axis in plot functions provided by WaveletComp is determined by observation epochs, however; "epoch" meaning

point in time.

axis.2 tick levels corresponding to the log of Fourier periods: log2(Period). This

determines the period axis in plot functions provided by WaveletComp.

date.format the format of calendar date (if available)
date.tz the time zone of calendar date (if available)

## Author(s)

CoFES. Credits are also due to Angi Roesch, Harald Schmidbauer, Huidong Tian, and Bernard Cazelles.

#### References

Aguiar-Conraria L., and Soares M.J., 2011. The Continuous Wavelet Transform: A Primer. NIPE Working Paper Series 16/2011.

Carmona R., Hwang W.-L., and Torresani B., 1998. Practical Time Frequency Analysis. Gabor and Wavelet Transforms with an Implementation in S.Academic Press, San Diego.

Cazelles B., Chavez M., Berteaux, D., Menard F., Vik J.O., Jenouvrier S., and Stenseth N.C., 2008. Wavelet analysis of ecological time series. Oecologia 156, 287–304.

Liu Y., Liang X.S., and Weisberg R.H., 2007. Rectification of the Bias in the Wavelet Power Spectrum. Journal of Atmospheric and Oceanic Technology 24, 2093–2102.

Tian, H., and Cazelles, B., 2012. WaveletCo. Available at https://cran.r-project.org/src/contrib/Archive/WaveletCo/, archived April 2013; accessed July 26, 2013.

Torrence C., and Compo G.P., 1998. A practical guide to wavelet analysis. Bulletin of the American Meteorological Society 79 (1), 61–78.

#### See Also

CoFESWave.reconstruct

#### **Examples**

```
## Not run:
## The following example is adopted from Liu et al., 2007:
series.length <- 6*128*24
x1 <- periodic.series(start.period = 1*24, length = series.length)</pre>
x2 <- periodic.series(start.period = 8*24, length = series.length)</pre>
x3 <- periodic.series(start.period = 32*24, length = series.length)
x4 <- periodic.series(start.period = 128*24, length = series.length)</pre>
x < -x1 + x2 + x3 + x4
plot(x, type = "l", xlab = "index", ylab = "", xaxs = "i",
     main = "hourly series with periods of 1, 8, 32, 128 days")
## The following dates refer to the local time zone
## (possibly allowing for daylight saving time):
my.date <- seq(as.POSIXct("2014-10-14 00:00:00", format = "\</pre>
               by = "hour",
               length.out = series.length)
my.data <- data.frame(date = my.date, x = x)</pre>
## Computation of wavelet power:
## a natural choice of 'dt' in the case of hourly data is 'dt = 1/24',
## resulting in one time unit equaling one day.
## This is also the time unit in which periods are measured.
## There is an option to store the date format and time zone as additional
## parameters within object 'my.wt' for later reference.
my.wt <- analyze.wavelet(my.data, "x",</pre>
                          loess.span = 0,
                          dt = 1/24, dj = 1/20,
                          lowerPeriod = 1/4,
                          make.pval = TRUE, n.sim = 10,
                          date.format = " \setminus
```

plot\_CoFESWaveWCO

plot\_CoFESWaveWCO

## **Description**

To plot (absolute value) of wavelet coherency or multiple or partial wavelet coherencies of several series, coi (and levels of sigificance)

## Usage

```
plot_CoFESWaveWCO(
   Coherency.Object,
   pE = 5,
   horizons.label = TRUE,
   low.FP = 32,
   up.FP = 128
)
```

## **Arguments**

Coherency.Object

output of function CoFEScoherency or CoFESmpcoherency

pE a constant to enhance the quality of picture

horizons.label Label the horizontal axis with labels (instead of numbers)? Logical.

Default: TRUE.

low.FP lower periods used in the frequency band up.FP upper periods used in the frequency band

## Author(s)

CoFES. Credits are also due to L. Aguiar-Conraria and M.J. Soares.

#### **Examples**

```
## more work
```

WaveL2E 29

WaveL2E

WaveL2E is a function

## Description

WaveL2E is a function

## Usage

```
WaveL2E(
  Х,
  date = NULL,
 block = 1,
  base_plot = TRUE,
  L2E = TRUE,
  Chi_square = TRUE
)
```

## Arguments

L2E

X	a series
date	a date series
block	number of observations within a block Default: 1.
base_plot	Plot wavelet power spectrum of x? Logical Default: TRUE.

Apply L2E thresholding method? Logical

Default: TRUE.

Apply L2E\_Chi\_square thresholding method? Logical Chi\_square

Default: TRUE.

## Value

A list of class "WaveL2E" with elements of different dimensions.

Here is a detailed list of all elements: #'

```
sig
                 tmp
                 tmp
dis0
                 tmp
thresh
                 tmp
qthresh
                 tmp
original
                 tmp
Ana_Wave
                 tmp
Emp_WaveL2E
                 tmp
{\tt Emp\_WaveL2E\_MAD}
                 tmp
```

30 WaveL2E

recon\_L2E tmp
PTV\_L2E tmp
PSL\_L2E tmp
recon\_Chi\_square tmp
PTV\_Chi\_square tmp
PSL\_Chi\_square tmp
date a date series

## Author(s)

CoFES.

# Examples

## WaveL2E(x)

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