Proof that every planar graph can be colored with five colors:

1. Show that any simple planar graph has a vertex of degree 5 or less.

Proof by contradiction: Assume all vertices of a simple planar graph G have degree greater than or equal to 6.

Pick a component of G, and in this component $6V \leq$ the sum of the degrees which is equal to $2E$.

Every connected planar graph has degree 5 or less → every component has one too or each disconnected simple planar graph can be made connected by adding edges, so subtracting edges means the inequality still holds

Using the formula $V - E + F = 2$, $F = 2 - V + E$. Plugging this into the inequality $2F \leq 3E$, $3(2 - V + E) \leq 2E$.

This simplifies to $6 - 3V + 3E \leq 2E$, and $E \leq 3V - 6$.

So $6V \leq 2E \leq 6V - 12$, and $6V \leq 6V - 12$, so $0 \leq -12$, which is false.

Therefore any simple planar graph needs to have at least one vertex of degree 5 or less.

2. Use induction to show that every planar graph can be colored with 6 colors.

Assume the graph G is connected, and if it is not, apply this proof to each component of the graph since vertices in different components will never be connected, so the colors will never conflict with each other.

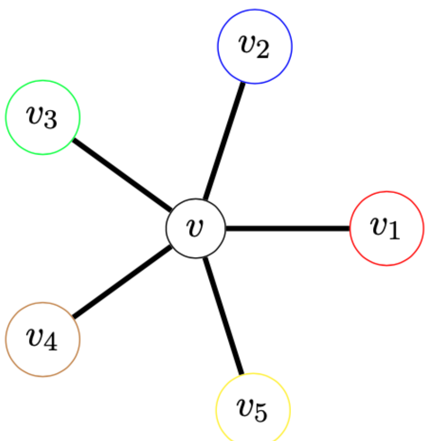
Base Case: Consider a simple planar graph with one vertex. Pick a color for this vertex, and the graph is colored with 6 or less colors, so the base case holds.

Inductive Step: For $1 \leq k$, assume that every planar graph with k or fewer vertices can be 6-colored.

Consider a planar graph with $k + 1$ vertices. We know that this graph has a vertex of degree 5 or fewer. Remove this vertex and the edges connected to it, and by the inductive hypothesis, we know that we can 6-color the remaining graph. Since this vertex is connected to at most 5 other vertices it is adjacent to at most 5 colors, so we can put it and its adjacent edges back into the graph, and color it one of the one or more available colors for the vertex. The planar graph with $k + 1$ vertices is now successfully 6-colored. Done by induction.

3. Use induction to show that every planar graph can be colored with 5 colors.

Base Case: Consider a simple planar graph with one vertex. Pick a color for this vertex, and the graph is colored with 5 or less colors, so the base case holds.
Inductive Step: For 1 ≤ k, assume that every planar graph with k or fewer vertices can be 5-colored.
Consider a planar graph with k + 1 vertices. We know that this graph has a vertex of degree 5 or fewer. Let v be the vertex in the graph with minimum degree. Remove v and the edges connected to it, and by the inductive hypothesis, we know that we can 5-color the remaining graph. If v has degree 4 or less or has degree 5 and its surrounding 5 vertices are colored with 4 or less colors, there is at least one available color to color v with, and we are done by induction. If v has degree 5 and its surrounding 5 vertices are colored with the 5 different colors, suppose the graph is arranged as in the image. If there is not a path from v1 to v3 that alternates between red and green show that you can change v1 to green (and swap a few other vertices between red and green- if the red was connected to greens then change those to red and etc) to get a valid 5 coloring with v1 and v3 both green, now you can color v red.



If there is a path from v1 to v3 the alternates between red and green, there is not a path from v2 to v5 alternating between blue and yellow because the two paths would have to cross at some point, making the graph non planar.
So every planar graph has a 5-coloring, and we are done by induction.

```
def fiveColor(G) :
	If G.number_of_nodes() == 1 :
		G.nodes[1] ['color'] = 'lightcoral'
	else :
		H = G.remove_node(1)
		fiveColor(H)
```

write pseudocode algorithm for the 5 color theory
implement a recursive algorithm that gives a 5 coloring of a planar graph with an input of an adjacency matrix of a graph
Graph with k vertices
For n = 1 - k → find a vertex v in the graph that appears in the matrix the least amount of times (smallest degree)
For all other k - 1 vertices find a 5 coloring

5 color
If vertex set of G is 1 element, color vertex color 1
recursion

If there exists a vertex of degree 4
G = G - v
5 color G
G = G w/ v
Color v remaining color

If smallest degree vertex is 5
G = G - v
5 color G
If adjacent vertices aren't all different
	Color v remaining color
Else
//Iterate over pairs of adjacent vertices and try to find a path b/w them that does not alternate colors
*might not have to check all 10
	For x in range (1, 11) *for 10 pairs of colors
		G = subgraph on vertices w/ color in x
		If there is a path b/w the two vertices you're looking at

If not → break
Change v2 to color v1
*iteration to change colors of vertices adjacent to v2

6 color pseudocode

```
6 color
    If vertex G is size 1
        Color v color 1
    Else
        Find vertex v of smallest degree
        G = G - v
        6 color G
        G = G with v
        Color v a color not used on vertices adjacent to it
```

python libraries

- Networkx