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# ALGORITHMS

HW - 8

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Problem 1    29.1-6    (pg 858)

①

Show that the following linear program is infeasible:

$$\text{maximize: } 3x_1 - 2x_2$$

Subject to

$$x_1 + x_2 \leq 2$$

$$-2x_1 - 2x_2 \leq -10$$

$$x_1, x_2 \geq 0$$

By looking the problem above, firstly we try to divide the second constraint by '2'. And then we add it to the first constraint.

Then by doing this, the result would be  $0 \leq -3$  which is not valid.

Hence we can say that the given Program (linear) is infeasible.

Problem 2      29.3-7      (pg 879)

(2)

Firstly we convert the given problem into the format of equation.

$$\text{i.e. } Z = -x_1 - x_2 - x_3$$

$$x_4 = -10000 + 2x_1 + 7.5x_2 + 3x_3$$

$$x_5 = -30000 + 20x_1 + 5x_2 + 10x_3$$

Consider that  $x_1, x_2, x_3, x_4, x_5 \geq 0$

→ Now we will change to maximize the negative of the function:

We maximize the values of 'Z' from above:

$$\text{i.e. } -x_1 - x_2 - x_3 = -x_0$$

Then, we get:

$$Z = -x_0$$

$$x_4 = -10000 + 2x_1 + 7.5x_2 + 3x_3 + x_0$$

$$x_5 = -30000 + 20x_1 + 5x_2 + 10x_3 + x_0$$

Now,  $x_0, x_1, x_2, x_3, x_4, x_5 \geq 0$

→ Then  $x_0$  will enter &  $x_5$  variable will be the leaving one.

As it is the basic variable whose value in the basic solution is most negative.

(3)

After pivoting, we get

$$Z = -30000 + 20x_1 + 5x_2 + 10x_3 - x_5$$

$$x_0 = 30000 - 20x_1 - 5x_2 - 10x_3 + x_5$$

$$x_4 = 20000 - 18x_1 + 2.5x_2 - 7x_3 + x_5$$

$$\text{Then } x_0, x_1, x_2, x_3, x_4, x_5 \geq 0$$

→ The basic solution is feasible, so now we need to repeatedly call PIVOT until we obtain an optimal solution to  $L_{aux}$ .  
Now,  $x_2$  will be entering variable.

$$Z = -x_0$$

$$x_2 = 6000 - 4x_1 - 2x_3 + x_5/5 - x_0/5$$

$$x_4 = 35000 - 28x_1 - 12x_3 + (3/2)x_5 - x_0/2$$

$$x_0, x_1, x_2, x_3, x_4, x_5 \geq 0$$

→ Since the solution has  $x_0 = 0$ , we can tell that our initial problem was feasible.  
Hence we can remove it from the set of constraints. Now we use the original substitutions made initially.

And ~~we~~ the equation will be:

$$Z = -6000 + 3x_1 + x_3 - x_5/5$$

$$x_2 = 6000 - 4x_1 - 2x_3 + x_5/5$$

$$x_4 = 35000 - 28x_1 - 12x_3 + (3/2)x_5$$

$$\text{Then } x_1, x_2, x_3, x_4, x_5 \geq 0$$

→ Now we choose  $x_1$  as entering variable.

$$Z = -2250 - (2/7)x_3 - (3/28)x_4 - (11/280)x_5$$

$$x_1 = 1250 - (3/7)x_3 - (1/28)x_4 + (3/56)x_5$$

$$x_2 = 1000 - (2/7)x_3 + (1/7)x_4 - (1/70)x_5$$

$$\& x_1, x_2, x_3, x_4, x_5 \geq 0$$

Here all the co-efficients in the objective function are negative. Hence the basic solution is an optimal solution.

$$\text{i.e. } (x_1, x_2, x_3) = (1250, 1000, 0)$$