Algorithms Fall 2016 Homework 12 Solution

December 8, 2016

Problem 1

We can design an algorithm as follows:

- Given $G = (V, E)$, returns NIL if G is nonhamiltonian.
- For each $e \in E$, we remove e from E and check if G is hamiltonian. If G is nonhamiltonian after removing e, that means e is on a hamiltonian cycle, we put e back to G.
- List the vertices on the hamiltonian cycle according to the edges left.

If HAM-CYCLE $\in$ P, we can run step 2 in polynomial time, that means this problem can be solved in polynomial time.

Problem 2

By the definition of reducibility, if $L_1 \leq_P L_2$ and $L_2 \leq_P L_3$, we have there exists two polynomial-time computable functions $f : \{0,1\}^* \to \{0,1\}^*$ and $g : \{0,1\}^* \to \{0,1\}^*$ such that:

- $x \in L_1$ if and only if $f(x) \in L_2$
- $y \in L_2$ if and only if $g(y) \in L_3$

which follows

- $x \in L_1$ if and only if $f(g(x)) \in L_3$

Since f and g are both polynomial-time computable, $f \cdot g$ is also a polynomial-time computable function. Thus according to the definition, we have $L_1 \leq_P L_3$.