Algorithms Fall 2016 Homework 11 Solution

December 8, 2016

Problem 1

A simple polygon is defined as a flat shape consisting of straight, non-intersecting line segments or "sides" that are joined pair-wise to form a closed path. In other words, the polygon is simple if no sides intersect.
Assume that the input is a polygon which is represented by a set S of segments. Each segment only intersects with exactly another two different segments in S at the two endpoints respectively. Because we assume that the input set S is already a polygon, so the polygon is simple if and only if each segment in S exactly intersects with two other segments at its endpoints. We can apply a procedure SIMPLE-POLYGON which is very similar to $ANY-SEGMENTS-INTERSECT(S)$ on p. 1025 in the textbook. Here we only need to modify the intersection condition. In $SIMPLE-CHECK$ procedure, we use $MID-INTERSECT$ to check where "t $MID-INTERSECTs$ with s" means segment t intersects segment s at the point other than endpoints of s, which can be finished in $O(1)$ time.
The pseudo-code is as follows:

```
SIMPLE-POLYGON(S)
1  T = ∅
2  Sort the endpoints of the segments in S from left to right,
   breaking ties by putting left endpoints before right endpoints
   and breaking further ties by putting points with lower y-coordinates first.
3  for point p in S
4          let N be the set of segments that has endpoint p
5          if |N|! = 2
6                  return FALSE
7          for each s in N
8                  if p is the left endpoint of s
9                          INSERT(T, s)
10                         if Above (T, s) exists and MID − INTERSECTs with s
11                         or Below(T, s) exists and MID − INTERSECTs with s
12                                 return FALSE
13                 if p is the right endpoint of s
14                         if Above (T, s) exists and MID − INTERSECTs with s
15                         or Below(T, s) exists and MID − INTERSECTs with s
16                                 return FALSE
17                         DELETE(T, s)
18 return TRUE
```

If set S contains n segments, then the $SIMPLE-POLYGON$ runs in $O(n \log n)$ time. The sorting of end points takes $O(n \log n)$ time since the number of endpoints is also n. The for loop lines 3-17 iterates at most n times (once for a point) and each iteration takes $O(\log n)$ time due to the red-black-tree operation, same to that in the procedure $ANY-SEGMENTS-INTERSECTION$. Therefore the total running time is $O(n \log n)$.

Problem 2

There are two flaws:

- The problem can not be solved by divide-and-conquer algorithm if all points in P are on a same vertical line. Suppose all the points are on vertical line l, then P is divided into two point sets $P_L = P$ and $P_R = \emptyset$.

- The conclusion that at most 6 points can reside in the $\delta \times 2\delta$ rectangles is not always correct. The following figure is a counter-example where there are 7 points in the $\delta \times 2\delta$ rectangles, points $L_i \in P_L (i = 1,2,3,4)$ and points $R_i \in P_R (i = 1,2,3)$. Moreover, it is possible that $|R_1R_3| \geq \delta$ and $|R_2R_3| \geq \delta$, but $|L_3R_3| \leq \delta$.