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HW-9

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Problem 30-2 (pg 921)

a) The sum of two Toeplitz matrices is Toeplitz, but the product of two Toeplitz matrices is not Toeplitz.

b) Let us consider 'A' as a Toeplitz matrix. We can use a vector of length $2n-1$ to represent it, given by: $(c_0, \ldots c_{2n-2})$
$= (a_{n,1}, a_{n-1,1}, \ldots, a_{2,1}, a_{1,1}, a_{1,2} \ldots a_{a,n})$

To add two Toeplitz matrices, we have to just add their associated vectors

c) We can say that, this is a multiplication of two polynomials. Specifically, let $P(x) = c_0 + c_1 x + \ldots + c_{2n-1} x^{2n-2}$. Let $(b_0, b_1, b_2, \ldots b_{n-1})$ be the vector of length 'n' by which we wish to multiply, and let y_k denote the k^{th} entry of the vector which results from the multiplication. Let $Q(x) = b_0 x^{n-1} + b_1 x^{n-2} + \ldots + b_{n-1}$. Then the co-efficient of $x^{n-k+n-1}$ in

$P(x)\, Q(x)$ is given by

$$\sum_{i=0}^{n-1} c_{n-k+i}\, b_i = \sum_{i=0}^{n-1} a_{k,i}\, b_i = y_k$$

Since we can multiply the polynomials in $O(n \lg n)$ & the needed results are just some of the co-efficients, we can multiply a Toeplitz matrix by an n-Vector in $O(n \lg n)$ & the needed results are just some of the co-efficients, we can multiply a Toeplitz matrix by an n-vector in $O(n \lg n)$.

d> We can view matrix multiplication as simply multiplication by an n vector carried out 'n' times. If b_j is the j^{th} column of the second resulting matrix. By part c, this can be done in $O(n^2 \lg n)$ time, which is asymtotically faster than even Strassen's algorithm.