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ALGORITHMS

HW - 12

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Problem 1 : (34.2-3) pg 1065

(1)

Show that if HAM-CYCLE ∈ P then the problem of listing the vertices of a hamiltonian cycle, in order, is polynomial time solvable.

Suppose that 'G' is hamiltonian. which means that there is a hamiltonian cycle. Pick any one vertex 'v' in the graph, and consider all the possibilities of deleting all but two of the edges passing through that vertex. For some pair of edges to save, the resulting graph must still be hamiltonian because the hamiltonian graph that existed originally only used two edges. Since the degree of the vertex is bounded by the number of vertices minus one we are only less than squaring that number by looking at all pairs $(\binom{n-1}{2} \in O(n^2))$. This means that we are only running the polynomial tester polynomially many independent times, so the runtime is polynomial. Once we have all the pair of vertices where deleting all the others coming off 'v'.

still results in a hamiltonian graph, we will remember those as special, and once that we will never again try to delete. We repeat the processes with both of the vertices that are now adjacent to 'v', testing hamiltonicity of each way of picking a new vertex to save. We continue in this process until we are left with only $|V|$ edge, and so, we have ~~to~~ just constructed a hamiltonian cycle.

Problem-2: (34.3-2) pg 1077

Show that the $\leq_p$ relation is a transitive relation on languages. That is, show that if:

$$L_1 \leq_p L_2 \quad \text{and} \quad L_2 \leq_p L_3$$

Then,

$$L_1 \leq_p L_3.$$

Now, let $L_1 \leq_p L_2$ be (i)

&

$L_2 \leq_p L_3$ be (ii)

③

Let f_1 be the polynomial time reduction function such that $x \in L_1$ if & only if $f_1(x) \in L_2$. Similarly let (ii) and f_2 be the polynomial time reduction function such that $x \in L_2$ if and only if $f_2(x) \in L_3$. Then we can compute ~~for~~ f_2 & f_1 in polynomial time, and $x \in L_1$ if & only if $f_2(f_1(x)) \in L_3$.

$\therefore L_1 \leq_p L_3$. Hence $\leq_p$ relation is transitive. And proved.

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