



Euler and the 336 Million Dollar Software Patent

A revised version of the talk given at Facebook on February 13, 2015

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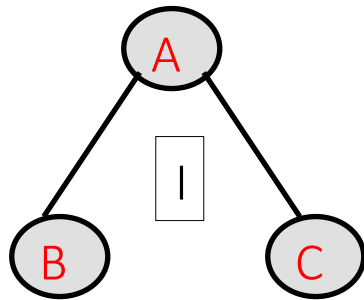
Ben Holland, Iowa State University

Acknowledgement: Team members at Iowa State University and EnSoft, DARPA contracts FA8750-12-2-0126 & FA8750-15-2-0080

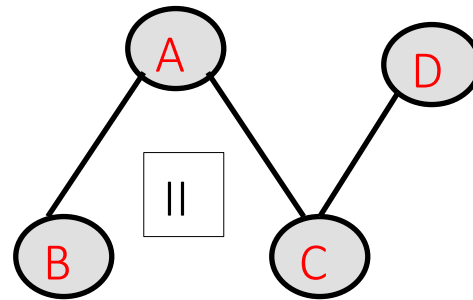
Vertex Rank problem

Think of rank as a measure of importance of the vertex.

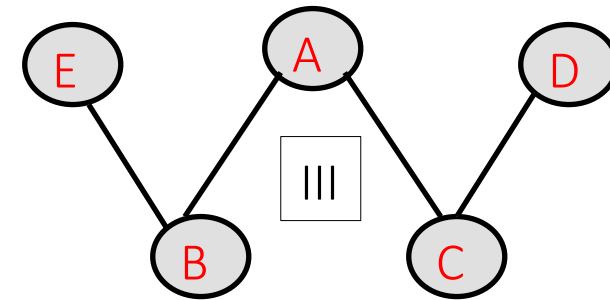
Problem: Develop a quantitative measure for rank



A has higher rank, B and C have the same rank

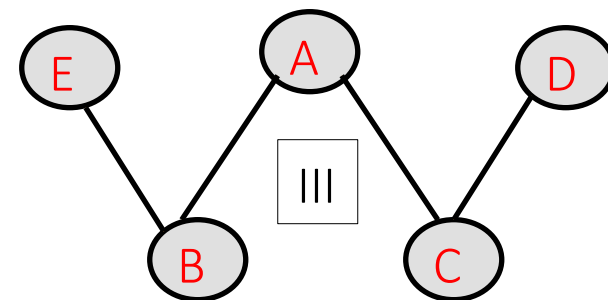
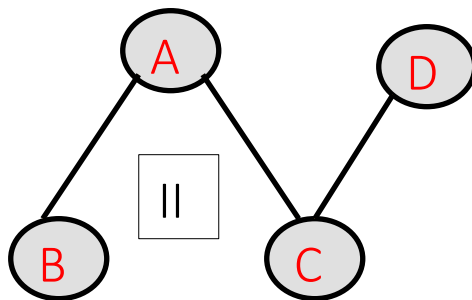
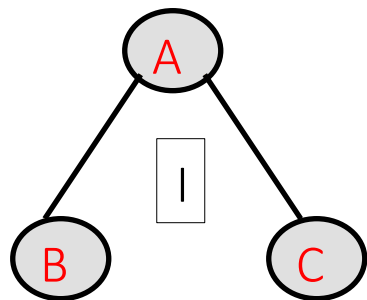


A, C have higher rank than B and D. A and C have the same rank. B and D have the same rank.



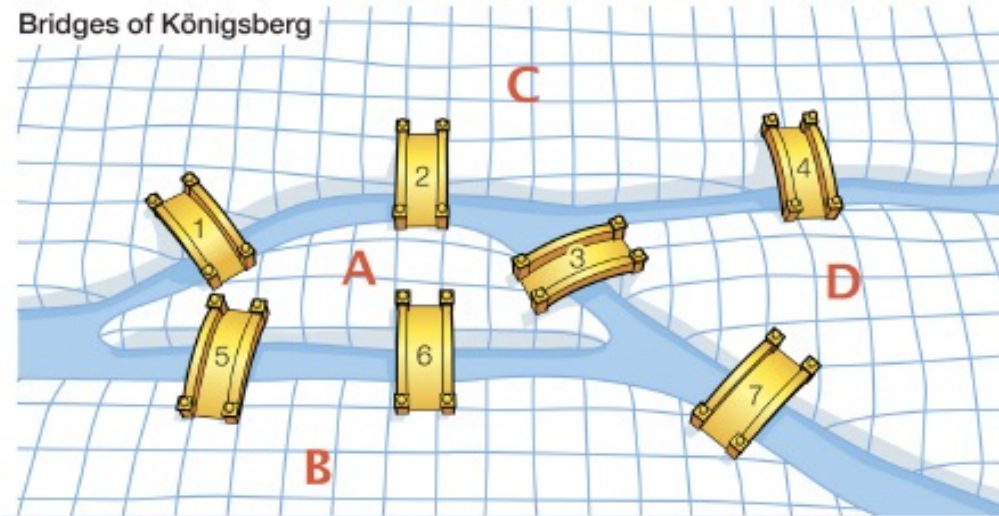
D, E have the same rank. B, C have the same but higher rank. A has the highest rank

A quantitative measure of vertex rank



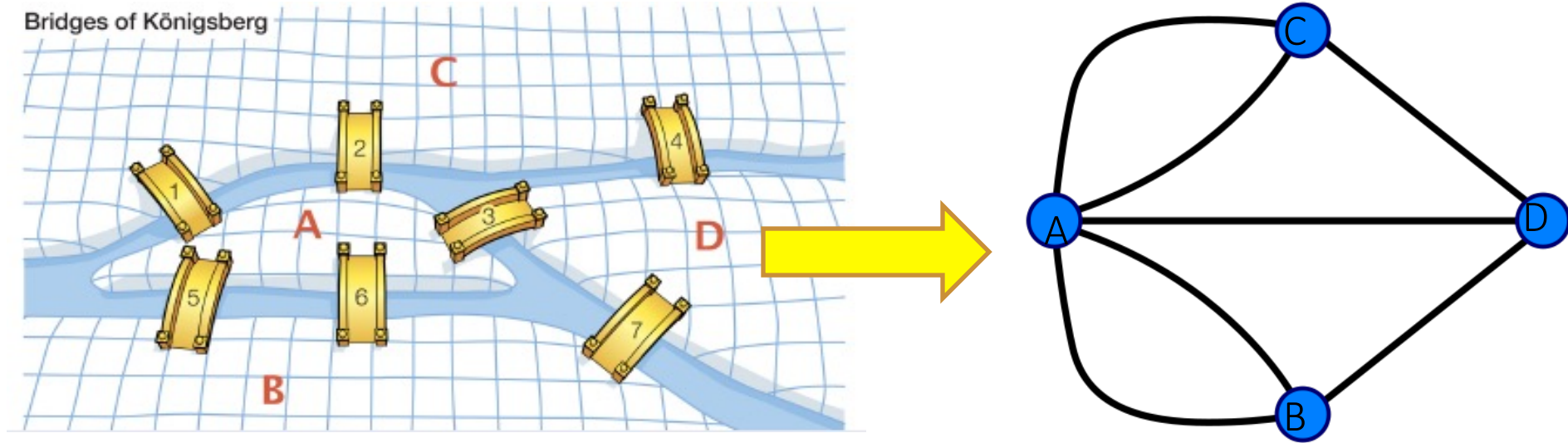
	A	B	C	D	E	Total
I	1.459	0.7702	0.7702			2.9994
II	1.298	0.7017	1.298	0.7017		3.9994
III	1.2297	1.1952	1.1927	0.6726	0.6727	4.9998

The *seven bridge* problem



Problem: Is it possible to make a loop in which every bridge is crossed exactly once?

Euler's solution (1735)



A loop that goes through all edges exactly once is called an *Euler loop*

Theorem: A Euler loop exists if and only if each vertex has even degree

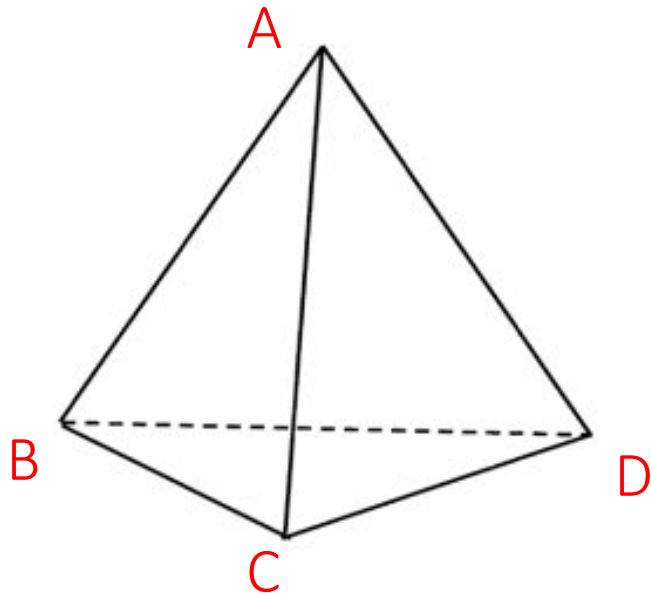
William Rowan Hamilton's Icosian Game (1857)

Dodecahedron – a polyhedron with 12 faces



Problem: Find a loop through the edge graph of the dodecahedron visiting every vertex exactly once.

Let us do a simpler case.



Loop: ABCDA

A brief history

1. **Euler (1735)**: Find a loop through a graph visiting every edge exactly once.
2. **Hamilton (1857)**: Find a loop, visiting every vertex exactly once.
3. **Traveling Salesman Problem (1930)**: Find a loop with the shortest distance, visiting every vertex exactly once.
4. **PageRank (1996)**: the backbone of Google search engine, U. S. patent 6,285,999 – Google paid Stanford University received 336 million dollars.

We live in the age of large graphs

Two broad categories of Mathematics: (A)Continuous mathematics, (B)Discrete mathematics

With the advent of powerful computers, many applications of of Discrete Mathematics have evolved: genetics, computer networks, internet search engines, social networks, the so called big data, ... and many yet to evolve

Calculus of variables (Leibnitz and Newton) came hundreds of years after introduction of “variable” (Diophantus, Brahmagupta, Khwarizmi)

Calculus of graphs in 21-th century three hundred years after Euler introduced graphs



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<http://www.ensoftcorp.com/atlas/>

Atlas: a graph database platform with interactive visualization for managing the complexity software systems.

A loop iteration count problem

```
For J = 1 to n
  for K= 1 to J
    count = count + 1;
```

How many times will the innermost loop iterate?

<i>J</i>	1	2		3						
<i>K</i>	1	1	2	1	2	3				

The answer is $1 + 2 + 3 + \dots + (n-2) + (n-1) + n$

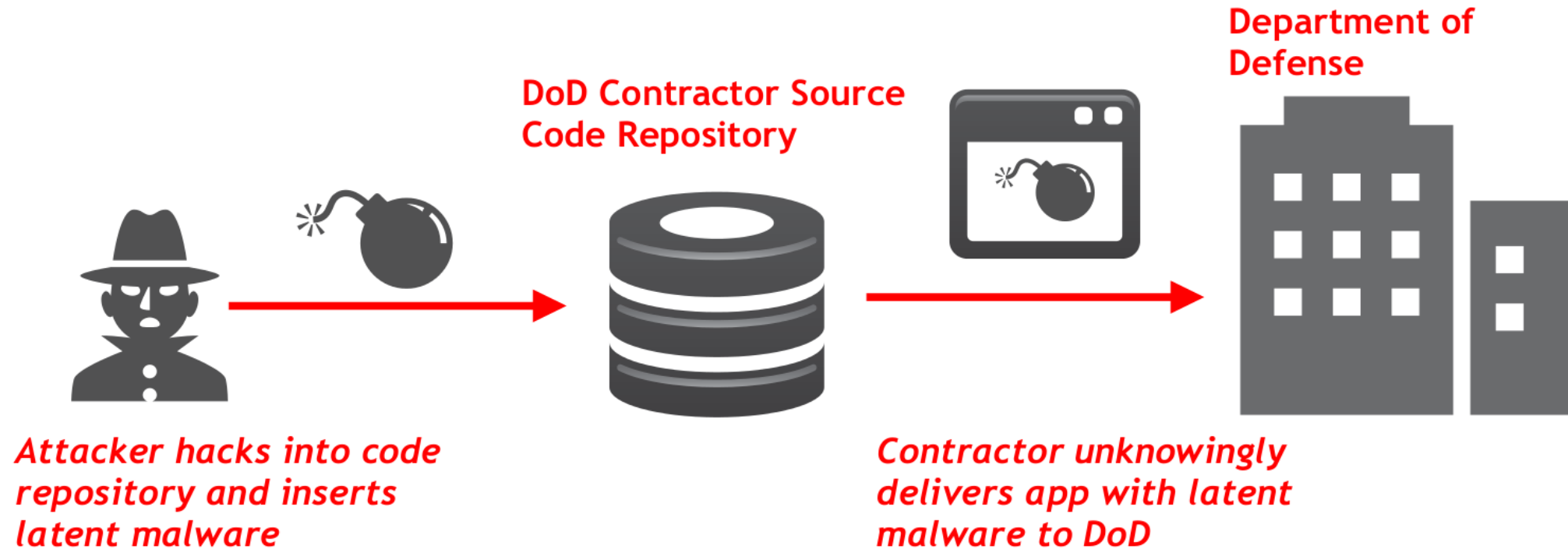
The formula $\frac{1}{2} n(n+1)$ works for counting the number of iterations of a 2-level loop.

Consider a t -level nested loop. For example, a loop with $t = 3$

```
count = 0;  
  for J = 1 to  $n$   
    for K = 1 to J  
      for L = 1 to K  
        count = count + 1;
```

How many times will the innermost loop iterate?

DARPA: We created the Internet, we will create the technology to defend it



What have we learnt?

