

# Fully homomorphic encryption (FHE) schemes

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October 22, 2022

## 1 Preliminaries

Learning with error (LWE).

**Theorem 1** *content.*

**Lemma 1**  $x+y=z$

**Lemma 1.1**  $z-r+t=\mathcal{Z}$

Ring Learning with error (RLWE). Lemma.

## 2 FHE schemes

### 2.1 BV scheme

The BV scheme was introduced by Brakerski and Vinod Vaikuntanathan in [BV14] (LWE) and [BV11] (RLWE).

### 2.2 BGV scheme

[BGV14]

### 2.3 BFV scheme

In [Bra12], Brakerski proposed a FHE scheme based on the LWE problem. Then, Fan Junfeng and Vercauteren Frederik [FV12] introduced the RLWE version of Brakerski's scheme. The scheme is called BFV.

### BFV scheme

Let  $R = \mathbb{Z}$

- Key generation:
- Encryption:
- Decryption:
- Homomorphic operations
  - Add
  - MultConst
  - Mult

## References

- [BGV14] Zvika Brakerski, Craig Gentry, and Vinod Vaikuntanathan. (leveled) fully homomorphic encryption without bootstrapping. *ACM Transactions on Computation Theory (TOCT)*, 6(3):1–36, 2014.
- [Bra12] Zvika Brakerski. Fully homomorphic encryption without modulus switching from classical gapsvp. In *Annual Cryptology Conference*, pages 868–886. Springer, 2012.
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- [BV14] Zvika Brakerski and Vinod Vaikuntanathan. Efficient fully homomorphic encryption from (standard) lwe. *SIAM Journal on computing*, 43(2):831–871, 2014.
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