Section 1.6

5. $B_2S_3 + H_2O \longrightarrow H_3BO_3 + H_2S$

Boron
Sulfur
Hydrogen
Oxygen

$$x_1\begin{bmatrix}2\\3\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\\1\end{bmatrix} = x_3\begin{bmatrix}1\\0\\3\\3\end{bmatrix} + x_4\begin{bmatrix}0\\1\\2\\0\end{bmatrix}$$

$$x_1\begin{bmatrix}2\\3\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\\1\end{bmatrix} + x_3\begin{bmatrix}-1\\0\\-3\\-3\end{bmatrix} + x_4\begin{bmatrix}0\\-1\\-2\\0\end{bmatrix} = \vec{0}$$

$$\begin{bmatrix}2&0&-1&0\\3&0&0&-1\\0&2&-3&-2\\0&1&-3&0\end{bmatrix} \sim \begin{bmatrix}2&0&-1&0\\0&0&3/2&-1\\0&0&3&-2\\0&1&-3&0\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&-1/2&0\\0&1&-3&0\\0&0&1&-\frac{2}{3}\\0&0&3&-2\end{bmatrix} \sim \begin{bmatrix}1&0&0&-1/3\\0&1&0&-2\\0&0&1&-2/3\\0&0&0&0\end{bmatrix}$$

$$\vec{x} = \begin{bmatrix}1/3\,x_4\\2\,x_4\\2/3\,x_4\\x_4\end{bmatrix} = x_4\begin{bmatrix}1/3\\2\\2/3\\1\end{bmatrix} = s\begin{bmatrix}1\\6\\2\\3\end{bmatrix} \text{ for } s \in \mathbb{R}$$

The balanced equation is

$$B_2S_3 + 6H_2O \rightarrow 2H_3BO_3 + 3H_2S$$

11. For each intersection, write an equation.

A: $x_1 + x_3 = 20$

B: $x_2 = x_3 + x_4$

C: $x_1 + x_2 + 80 = 0$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 60 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 60 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{array}\right]$$

$$x_1 = -x_3 + 20$$
$$x_2 = x_3 + 60$$
$$x_3 = x_3$$
$$x_4 = 60$$

$$\vec{x} = \begin{bmatrix} 20 \\ 60 \\ 0 \\ 60 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

If all the flows are nonnegative, then the largest x_3 can be is 20.

Otherwise, $x_1 < 0$.