Exam 1 Review

① $(-4,9)$ and $(5,6)$

Slope

$M = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{6-9}{5+4}$

$= -\dfrac{3}{9} = -\dfrac{1}{3}$

$M = -1/3$

Distance

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(5+4)^2 + (6-9)^2}$

$= \sqrt{9^2 + 3^2}$

$= \sqrt{81+9}$

$= \sqrt{90}$

$d = 3\sqrt{10}$

Midpoint

$M.P. = \left(\dfrac{x_2+x_1}{2}, \dfrac{y_2+y_1}{2}\right)$

$= \left(\dfrac{5-4}{2}, \dfrac{6+9}{2}\right)$

$(1/2, 15/2)$

or

$(.5, 7.5)$

② $x^2+6x+y^2+8y=0$ Center? radius? Graph it?

$(x^2+6x+\_\_\_) + (y^2+8y+\_\_\_) = 0$

$(x^2+6x+\underline{9}) + (y^2+8y+\underline{16}) = 9+16$

$(x+3)^2 + (y+4)^2 = 25$

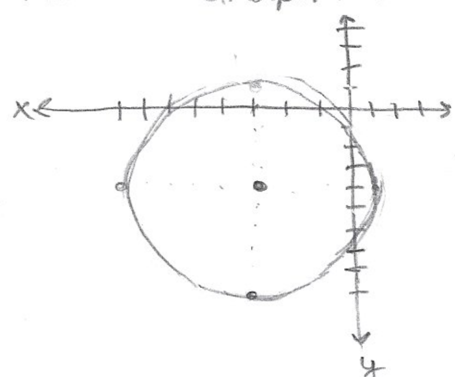
center

$(-3,-4)$

radius

$r^2 = 25$

$r = 5$



③ equation: $x-2y=-8$ through pt $(6,-1)$

$x+8 = 2y$

$\frac{1}{2}x+4 = y$

part 1 **parallel**

$m = 1/2$ * has same slope

$y = 1/2x + b$

$-1 = 1/2(6) + b$

$-1 = 3 + b$

$-4 = b$

$\boxed{y = 1/2x - 4}$

perpendicular

$m = -2$ * negative reciprocal of original

$y = -2x + b$

$-1 = -2(6) + b$

$-1 = -12 + b$

$11 = b$

$\boxed{y = -2x + 11}$

④ $(2, 2/3)$ and $(-1/2, -2)$

Slope

$$m = \frac{y^1 - y^2}{x^1 - x^2} = \frac{2/3 - (-2)}{2 - (-1/2)}$$

$$m = \frac{\frac{8}{3}}{\frac{5}{2}} = \frac{8}{3} \cdot \frac{2}{5} = \frac{16}{15}$$

$$y = \frac{16}{15}x + b$$

$$-2 = \frac{16}{15}\left(-\frac{1}{2}\right) + b$$

$$-2 = -\frac{8}{15} + b$$

$$-\frac{22}{15} = b$$

$$\boxed{y = \frac{16}{15}x - \frac{22}{15}}$$

⑤ a. $y - 3x = 5$

$3(x+1) = y - 2 \Rightarrow 3x + 3 = y - 2 \Rightarrow y = 3x + 5$

$(3x+5) - 3x = 5$

$5 \overset{\checkmark}{=} 5$ $\boxed{\infty \text{ solutions}}$ Dependent

b. $\left(\frac{x}{2} + \frac{y}{2} = 5\right) \cdot 3 = \frac{3x}{2} + \frac{3y}{2} = 15$

$\frac{3x}{2} - \frac{2y}{3} = 2$

$$-\left(\frac{3x}{2} - \frac{2y}{3} = 2\right)$$

$$0x + \frac{13}{6}y = 13$$

$$y = 6$$

$\frac{x}{2} + \frac{(6)}{2} = 5$

$\frac{x}{2} + 3 = 5$

$\frac{x}{2} = 2$

$x = 4$

$\boxed{\text{solution } (4, 6)}$ independent

c. $x - y = 7$

$y - x = 5 \Rightarrow y = 5 + x$

$x - (5 + x) = 7$

$x - 5 - x = 7$

$-5 \neq 7$ $\boxed{\text{No solution.}}$ Inconsistent

6 a. ① $x+y=5$
② $y-z=2$
③ $x+z=3$

$x+y=5$
$\underline{y-z=2} \Rightarrow y=2+z$
$x+2+z=5$
④ $x+z=3$
$z=3-x$

③ $x+z=3$
$x+3-x=3$
$0=0$
$\boxed{\infty \text{ solutions}}$
dependent

b. $x+y+2z=7.5$ ①
$3x+4y+z=12$ ②
$5x+2y+5z=21$ ③

$-3(x+y+2z=7.5) \Rightarrow -3x-3y-6z=-22.5$ ①
$\underline{3x+4y+z=12}$ ②
★ ④ $y-5z=-10.5$

② $5(3x+4y+z=12) \Rightarrow 15x+20y+5z=60$
③ $-3(5x+2y+5z=21) \Rightarrow \underline{-15x-6y-15z=-63}$
★ ⑤ $12y-10z=-3$

④ $(y-5z=-10.5)-2 \Rightarrow -2y+10z=21$
⑤ $12y-10z=-3$ $\quad \underline{12y-10z=-3}$
$10y=18$
$y=9/5$

④ $y-5z=-10.5$
$\left(\frac{9}{5}\right)-5z=-10.5$ $\quad -\frac{105}{10}-\frac{18}{10}$
$-5z=-\frac{123}{10}$
$z=\frac{123}{2}$

① $x+\frac{9}{5}+2\left(\frac{123}{2}\right)=7.5$
$x+\frac{9}{5}+123=7.5$
$x+\frac{9}{5}=115.5$ $\quad \frac{1155}{10}$
$x=\frac{1137}{10}$

solution: $\left(\frac{1137}{10}, 9/5, 123/2\right)$
independent

c. $x-2y+3z=5$ ①
$2x-4y+6z=3$ ②
$2x-3y+z=9$ ③

① $-2(x-2y+3z=5) \Rightarrow -2x+4y-6z=-10$
② $\quad \underline{2x-4y+6z=3}$
$0\neq-7$

$\boxed{\text{no solution}}$
inconsistent

⑦ $f(x) = x^2 + 3$ $\qquad f(-3),\ f(a+1),\ \dfrac{f(x+h)-f(x)}{h}$

$f(-3) = (-3)^2 + 3$
$= 9 + 3$
$f(-3) = 12$

$f(a+1) = (a+1)^2 + 3$
$= a^2 + 2a + 1 + 3$
$f(a+1) = a^2 + 2a + 4$

$\dfrac{f(x+h)-f(x)}{h}$ $\qquad f(x+h) = (x+h)^2 + 3$
$= x^2 + 2xh + h^2 + 3$

$= \dfrac{x^2+2xh+h^2+3-(x^2+3)}{h}$

$= \dfrac{2xh+h^2}{h} \Rightarrow \dfrac{h(2x+h)}{h} \Rightarrow 2x+h$

$\dfrac{f(x+h)-f(x)}{h} = 2x+h$

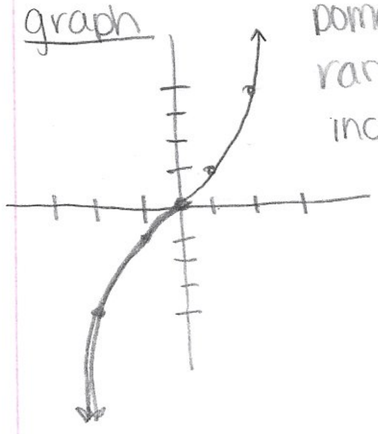
⑧ Difference quotient $= \dfrac{f(x+h)-f(x)}{h}$ $\qquad f(x) = x^2 - 2x + 1$

$f(x+h) = (x+h)^2 - 2(x+h) + 1$
$= x^2 + 2xh + h^2 - 2x - 2h + 1$
$f(x+h) - f(x) = \cancel{x^2} + 2xh + h^2 \cancel{-2x} - 2h \cancel{+1} - (\cancel{x^2} \cancel{-2x} \cancel{+1})$
$= 2xh + h^2 - 2h$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(2x+h-2)}{h} = \boxed{2x+h-2}$

⑨ a. $-x^2$ for $x \le 0$
x^2 for $x > 0$

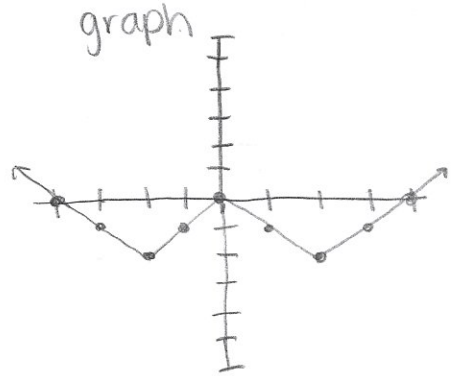
graph



Domain: $(-\infty, \infty)$
range: $(-\infty, \infty)$
increasing: $(-\infty, \infty)$

b. $\begin{cases} -x-4 & \text{for } x \le -2 \\ -|x| & \text{for } -2 < x < 2 \\ x-4 & \text{for } x > 2 \end{cases}$

graph



range: $(-\infty, \infty)$
domain: $(-2, \infty)$
decrease: $(-\infty, -2) \cap (0, 2)$
increase: $(-2, 0) \cap (2, \infty)$
constant @ pts $(-2, -2), (0, 0)$ & $(2, -2)$

(10) $y = -\frac{1}{2}|x-4|+3$

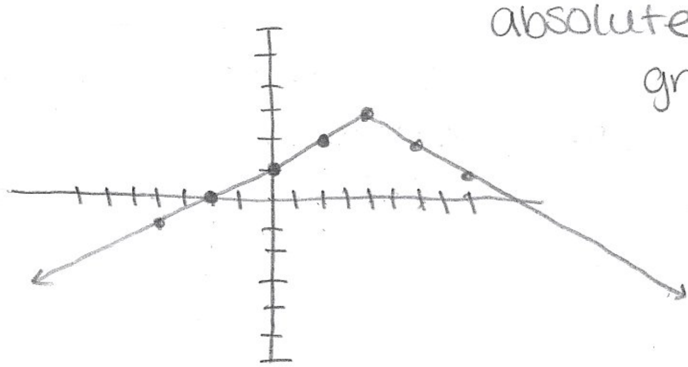
<u>graph</u>

domain: $(-\infty, 3)$

range: $(-\infty, \infty)$

increase: $(-\infty, 4)$

decrease: $(4, \infty)$



absolute function graph

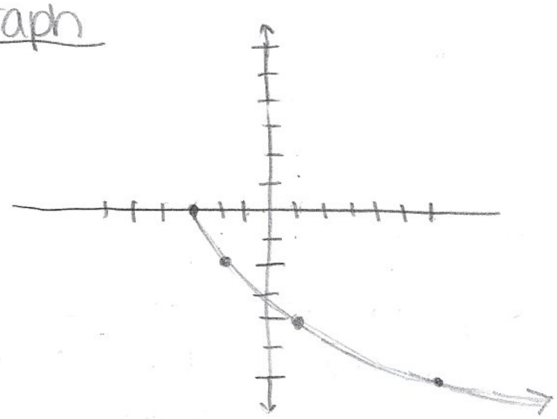
transformations:

- reflect over x-axis
- stretched by a factor of 2
- moved right 4 and up 3

symmetry on the line $x=4$

(11) $y = -2\sqrt{x+3}$

<u>graph</u>



square root function

domain: $y \leq 0$

range : $x \geq -3$

decrease: $(-3, \infty)$

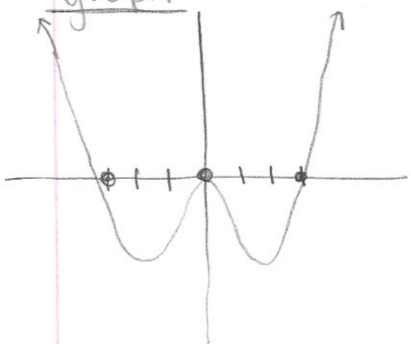
transformations:

- reflect across x-axis
- stretched by a factor of 2
- moved left 3

NO symmetry

(12) a. $f(x) = x^4 - 9x^2 \Rightarrow x^2(x^2-9)$

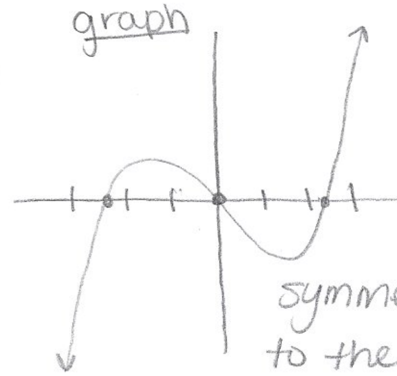
graph



symmetry w/ respect to the y-axis

b. $f(x) = -x^3 - 5x \Rightarrow -x(x^2-5)$

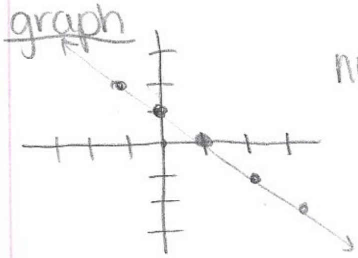
graph



symmetry w/ respect to the origin

c. $f(x) = -x+1$

graph



no symmetry

2.4

(13) $f(x) = x^2+3 \quad g(x) = 2x-7$

a. $g(f(x))$

$= 2(x^2+3) - 7$

$= 2x^2+6-7$

$\boxed{= 2x^2 - 1}$

b. $f(g(2)) = \boxed{12}$

$g(2) = 2(2) - 7$

$g(2) = 4-7 = -3$

$f(g(2)) = (-3)^2 + 3$

$= 9+3$

$= 12$

c. $\dfrac{f(x)}{g(x)} = \dfrac{x^2+3}{2x+7}$

d. $(f+g)(2) = 4$

$(f+g) = x^2+3+2x-7$

$= x^2+2x-4$

$(f+g)(2) = (2)^2 + 2(2) - 4$

$= 4+4-4$

$= 4$

2.5 13 a. invertible

$f(x) = 3x - 21$

$x = 3y - 21$

$x + 21 = 3y$

$\frac{1}{3}x + 7 = y$

Domain & range:

all real numbers

b. invertible

$y = \sqrt{9 - x^2}$

$x = \sqrt{9 - y^2}$

$x^2 = 9 - y^2$

$x^2 - 9 = -y^2$

$9 - x^2 = y^2$

$y = \sqrt{9 - x^2}$

range: $(-3, 3)$

domain: $(0, 3)$

c. Not invertible