Section 1.1

1. $x_1 + 5x_2 = 7$
$-2x_1 - 7x_2 = -5$

The corresponding augmented matrix is $\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right]$ Replace Row 2 with the sum of Row 2 and twice Row 1.

$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right]$ Scale Row 2 by multiplying it by $\frac{1}{3}$.

$\left[\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right]$ Replace Row 1 with the sum of Row 1 and $-5 \cdot$ Row 2.

$\left[\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right]$

Since we used row operations the system that corresponds to this augmented matrix has the same solution set as the original system.

$x_1 + 0x_2 = -8$
$0x_1 + x_2 = 3$

The solution is $\boxed{x_1 = -8,\ x_2 = 3}$

2. $2x_1 + 4x_2 = -4$
$5x_1 + 7x_2 = 11$

Augmented matrix

$$\left[\begin{array}{cc|c} 2 & 4 & -4 \\ 5 & 7 & 11 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 2 & 4 & -4 \\ 5 & 7 & 11 \end{array}\right]$$ Scale Row 1 by $\frac{1}{2}$

$$\left[\begin{array}{cc|c} 1 & 2 & -2 \\ 5 & 7 & 11 \end{array}\right]$$ Replace Row 2 by the sum of Row 2 and -5 Row 1.

$$\left[\begin{array}{cc|c} 1 & 2 & -2 \\ 0 & -3 & 21 \end{array}\right]$$ Scale Row 2 by $-\frac{1}{3}$

$$\left[\begin{array}{cc|c} 1 & 2 & -2 \\ 0 & 1 & -7 \end{array}\right]$$ Replace Row 1 by the sum of Row 1 and -2 Row 2.

$$\left[\begin{array}{cc|c} 1 & 0 & 12 \\ 0 & 1 & -7 \end{array}\right]$$

Solution: $x_1 = 12$, $x_2 = -7$

5. $\begin{bmatrix} 1 & -4 & \boxed{5} & 0 & 7 \\ 0 & 1 & \boxed{-3} & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{bmatrix}$

The next two row operations should make the boxed entries zero.

① Replace Row 2 with the sum of Row 2 and 3 times Row 3.

② Replace Row 1 with the sum of Row 1 and -5 times Row 3.

7. $\begin{bmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

I would stop row operations at this point. Since this matrix is an augmented matrix, row 3 corresponds to the equation $0x_1 + 0x_2 + 0x_3 = 1$. This equation says $0 = 1$. Thus the system has no solution.

9. $\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix}$

Scale Row 4 by $\frac{1}{2}$

$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

Replace Row 3 by the sum of Row 3 and 3 times Row 4

$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

Replace Row 2 by the sum of Row 2 and 3 times Row 3.

$\begin{bmatrix} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

Replace Row 1 by the sum of Row 1 and Row 2

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Solution set:

$$\begin{aligned} x_1 &= 4 \\ x_2 &= 8 \\ x_3 &= 5 \\ x_4 &= 2 \end{aligned}$$

13. $$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{bmatrix}$$

Swap Row 2 and Row 3

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 2 & 2 & 9 & 7 \end{bmatrix}$$

Replace Row 3 with the sum of Row 3 and -2 times Row 1

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{bmatrix}$$

Replace Row 3 with the sum of Row 3 and -2 times Row 2

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{bmatrix}$$

Scale Row 3 by $\frac{1}{5}$

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Replace Row 2 with the sum of Row 2 and -5 times Row 3.

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Replace Row 1 with the sum of Row 1 and 3 times Row 3.

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Solution:

$$x_1 = 5$$
$$x_2 = 3$$
$$x_3 = -1$$

17. If the three lines have a common point of intersection, then the system of equations has a solution.

$$\begin{aligned} x_1 - 4x_2 &= 1 \\ 2x_1 - x_2 &= -3 \\ -x_1 - 3x_2 &= 4 \end{aligned} \longrightarrow \begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{bmatrix}$$

Replace Row 2 with the sum of Row 2 and -2 times Row 1

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ -1 & -3 & 4 \end{bmatrix}$$

Replace Row 3 with the sum of Row 3 and Row 1

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{bmatrix}$$

Replace Row 3 with the sum of Row 3 and Row 2.

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

Scale Row 2 by $1/7$

$$\begin{bmatrix} 1 & -4 & 1 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{bmatrix}$$

Replace Row 1 by the sum of Row 1 and 4 times Row 2

$$\begin{bmatrix} 1 & 0 & -13/7 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = -13/7$$
$$x_2 = -5/7$$

Yes the three lines have a common point of intersection $(-13/7, -5/7)$.

19. $\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$

Replace Row 2 with the sum of Row 2 and -3 times Row 1.

$\begin{bmatrix} 1 & h & 4 \\ 0 & 6-3h & -4 \end{bmatrix}$

This system will be consistent as long as $6-3h \neq 0$. If $6-3h = 0$, then the second row would read $[0\ 0\ -4]$. The equation corresponding to this row would be $0x_1 + 0x_2 = -4$. There is no solution to that equation.

So $6-3h=0$ implies $h=2$. As long as $h \neq 2$, the system corresponding to the augmented matrix will be consistent.

23. a. TRUE see commentary on page 6 after blue box with elementary row operations.

b. FALSE A 5×6 matrix has 5 rows and 6 columns

c. TRUE see definition on page 3

d. TRUE see blue box on page 7

24. a. TRUE see highlighted box on page 7

b. FALSE see definition on bottom of page 6. Two matrices are row equivalent if there is a sequence of elementary row operations that transforms one matrix into the other.

c. FALSE an inconsistent system has no solution.

d. TRUE see definition of equivalent on page 3

31.

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}$$

Replace Row 3 with the sum of Row 3 and -4 times Row 1

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$$

Replace Row 3 with the sum of Row 3 and 4 times Row 1.

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}$$