# Business Statistics: A Decision-Making Approach

6th Edition

# Chapter 18

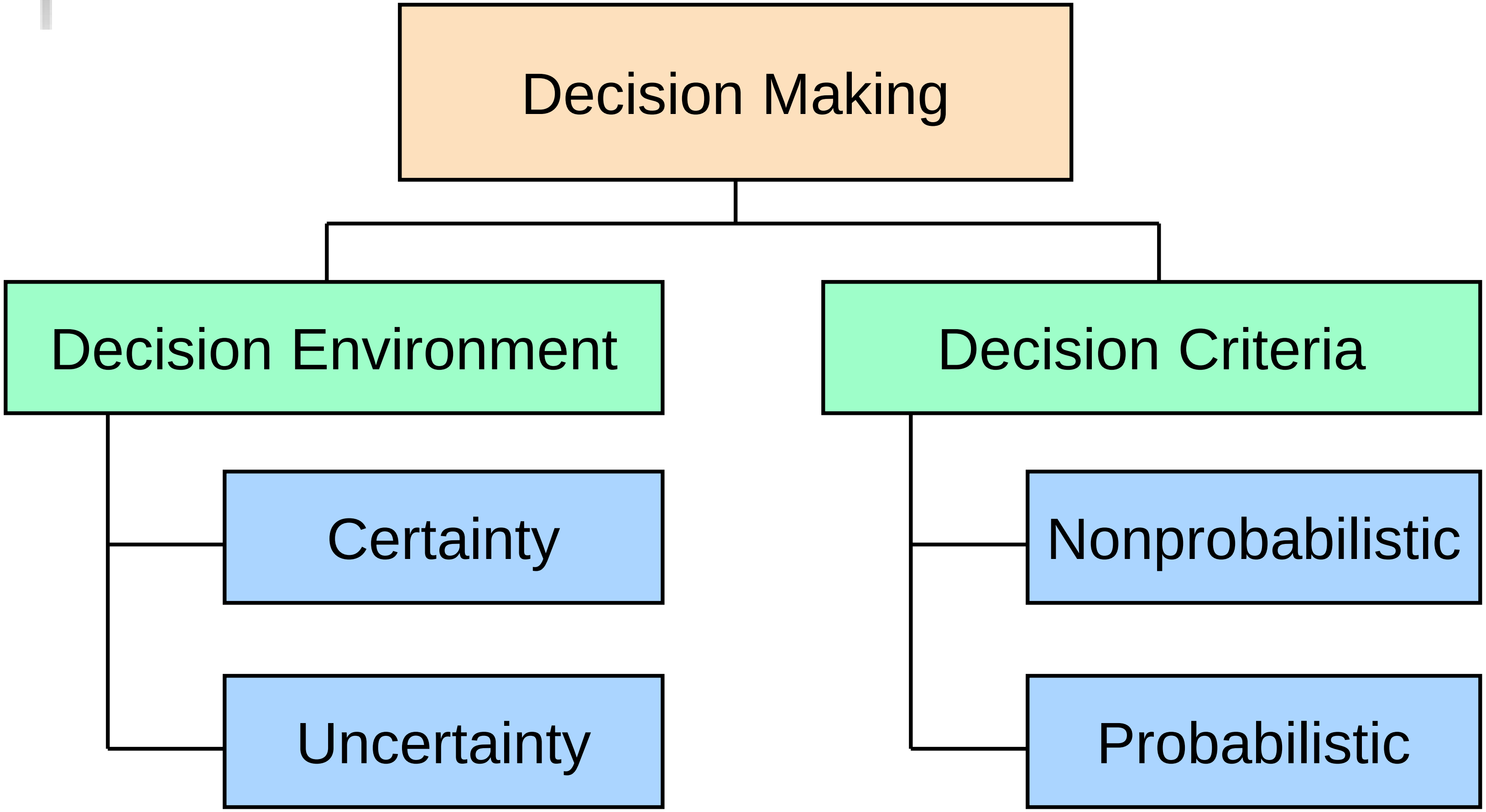
## Introduction to Decision Analysis

# Chapter Goals

**After completing this chapter, you should be able to:**

- Describe the decision environments of certainty and uncertainty
- Construct a payoff table and an opportunity-loss table
- Define and apply the expected value criterion for decision making
- Compute the value of perfect information
- Develop and use decision trees for decision making

# Decision Making Overview



# The Decision Environment

- Decision Environment
  - Certainty *
  - Uncertainty

Certainty: The results of decision alternatives are known

Example:

Must print 10,000 color brochures

Offset press A: $2,000 fixed cost
+ $.24 per page

Offset press B: $3,000 fixed cost
+ $.12 per page

# The Decision Environment

*(continued)*

- Decision Environment
  - Certainty
  - Uncertainty *

Uncertainty: The outcome that will occur after a choice is unknown

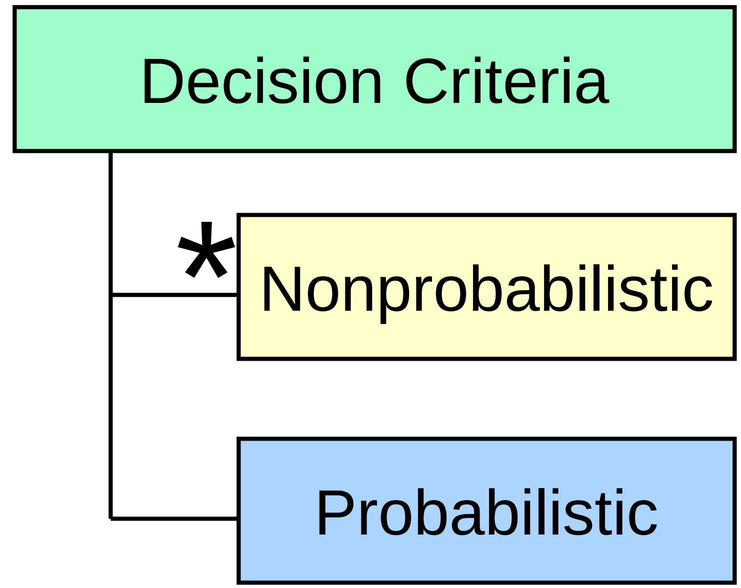
Example:

You must decide to buy an item now or wait. If you buy now the price is $2,000. If you wait the price may drop to $1,500 or rise to $2,200. There also may be a new model available later with better features.

# Decision Criteria

**Nonprobabilistic Decision Criteria:** Decision rules that can be applied if the probabilities of uncertain events are not known.

- maximax criterion
- maximin criterion
- minimax regret criterion



# Decision Criteria

*(continued)*

Probabilistic Decision Criteria:
Consider the probabilities of uncertain events and select an alternative to maximize the expected payoff of minimize the expected loss

- Decision Criteria
  - Nonprobabilistic
  - * Probabilistic

- maximize expected value
- minimize expected opportunity loss

# A Payoff Table

A payoff table shows alternatives, states of nature, and payoffs

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | |
|---|---|---|---|
| | Strong Economy | Stable Economy | Weak Economy |
| Large factory | 200 | 50 | -120 |
| Average factory | 90 | 120 | -30 |
| Small factory | 40 | 30 | 20 |

# Maximax Solution

The maximax criterion (an optimistic approach):

1. For each option, find the maximum payoff

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | |
|---|---|---|---|
| | Strong Economy | Stable Economy | Weak Economy |
| Large factory | 200 | 50 | -120 |
| Average factory | 90 | 120 | -30 |
| Small factory | 40 | 30 | 20 |

| 1. Maximum Profit |
|---|
| 200 |
| 120 |
| 40 |

# Maximax Solution

*(continued)*

The maximax criterion (an optimistic approach):

1. For each option, find the maximum payoff
2. Choose the option with the greatest maximum payoff

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | | 1. Maximum Profit |
|---|---|---|---|---|
| | Strong Economy | Stable Economy | Weak Economy | |
| Large factory | 200 | 50 | -120 | 200 |
| Average factory | 90 | 120 | -30 | 120 |
| Small factory | 40 | 30 | 20 | 40 |

2. Greatest maximum is to choose **Large factory**

# Maximin Solution

The maximin criterion (a pessimistic approach):

1. For each option, find the minimum payoff

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | | 1. Minimum Profit |
|---|---|---|---|---|
| | Strong Economy | Stable Economy | Weak Economy | |
| Large factory | 200 | 50 | -120 | -120 |
| Average factory | 90 | 120 | -30 | -30 |
| Small factory | 40 | 30 | 20 | 20 |

# Maximin Solution

*(continued)*

The maximin criterion (a pessimistic approach):

1. For each option, find the minimum payoff
2. Choose the option with the greatest minimum payoff

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | | 1. Minimum Profit |
|---|---|---|---|---|
| | Strong Economy | Stable Economy | Weak Economy | |
| Large factory | 200 | 50 | -120 | -120 |
| Average factory | 90 | 120 | -30 | -30 |
| Small factory | 40 | 30 | 20 | 20 |

2. Greatest minimum is to choose **Small factory**

# Opportunity Loss

Opportunity loss is the difference between an actual payoff for a decision and the optimal payoff for that state of nature

| **Investment Choice** (Alternatives) | **Profit in $1,000's** (States of Nature) | | |
|---|---|---|---|
| | Strong Economy | Stable Economy | Weak Economy |
| Large factory | 200 | 50 | -120 |
| Average factory | 90 | 120 | -30 |
| Small factory | 40 | 30 | 20 |

Payoff Table

The choice "Average factory" has payoff 90 for "Strong Economy". Given "Strong Economy", **the choice of "Large factory" would have given a payoff of 200, or 110 higher**. Opportunity loss = 110 for this cell.

# Opportunity Loss

*(continued)*

Payoff Table

| **Investment Choice** (Alternatives) | **Profit in $1,000's** (States of Nature) Strong Economy | Stable Economy | Weak Economy |
|---|---|---|---|
| Large factory | 200 | 50 | -120 |
| Average factory | 90 | 120 | -30 |
| Small factory | 40 | 30 | 20 |

Opportunity Loss Table

| **Investment Choice** (Alternatives) | **Opportunity Loss in $1,000's** (States of Nature) Strong Economy | Stable Economy | Weak Economy |
|---|---|---|---|
| Large factory | 0 | 70 | 140 |
| Average factory | 110 | 0 | 50 |
| Small factory | 160 | 90 | 0 |

# Minimax Regret Solution

## The minimax regret criterion:

1. For each alternative, find the maximum opportunity loss (or “regret”)

### Opportunity Loss Table

| Investment Choice (Alternatives) | Opportunity Loss in $1,000’s (States of Nature) | | | 1. Maximum Op. Loss |
|---|---|---|---|---|
| | Strong Economy | Stable Economy | Weak Economy | |
| Large factory | 0 | 70 | 140 | 140 |
| Average factory | 110 | 0 | 50 | 110 |
| Small factory | 160 | 90 | 0 | 160 |

# Minimax Regret Solution

*(continued)*

## The minimax regret criterion:

1. For each alternative, find the maximum opportunity loss (or "regret")
2. Choose the option with the smallest maximum loss

### Opportunity Loss Table

| Investment Choice (Alternatives) | **Opportunity Loss in $1,000's** (States of Nature) | | | 1. Maximum Op. Loss |
|---|---|---|---|---|
| | Strong Economy | Stable Economy | Weak Economy | |
| Large factory | 0 | 70 | 140 | 140 |
| Average factory | 110 | 0 | 50 | 110 |
| Small factory | 160 | 90 | 0 | 160 |

2. Smallest maximum loss is to choose **Average factory**

# Expected Value Solution

- The expected value is the weighted average payoff, given specified probabilities for each state of nature

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | |
|---|---|---|---|
| | Strong Economy (.3) | Stable Economy (.5) | Weak Economy (.2) |
| Large factory | 200 | 50 | -120 |
| Average factory | 90 | 120 | -30 |
| Small factory | 40 | 30 | 20 |

Suppose these probabilities have been assessed for these states of nature

# Expected Value Solution

*(continued)*

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | |
|---|---|---|---|
| | Strong Economy (.3) | Stable Economy (.5) | Weak Economy (.2) |
| Large factory | 200 | 50 | -120 |
| Average factory | 90 | 120 | -30 |
| Small factory | 40 | 30 | 20 |

Expected Values

→ 61
→ 81
→ 31

Maximize expected value by choosing **Average factory**

Example: EV (Average factory) = 90(.3) + 120(.5) + (-30)(.2)
= 81

# Expected Opportunity Loss Solution

## Opportunity Loss Table

| Investment Choice (Alternatives) | Opportunity Loss in $1,000's (States of Nature) | | | Expected Op. Loss (EOL) |
|---|---|---|---|---|
| | Strong Economy (.3) | Stable Economy (.5) | Weak Economy (.2) | |
| Large factory | 0 | 70 | 140 | 63 |
| Average factory | 110 | 0 | 50 | 43 |
| Small factory | 160 | 90 | 0 | 93 |

Minimize expected op. loss by choosing **Average factory**

Example: EOL (Large factory) = 0(.3) + 70(.5) + (140)(.2) = 63

# Cost of Uncertainty

- Cost of Uncertainty (also called Expected Value of Perfect Information, or EVPI)

- Cost of Uncertainty

  = Expected Value Under Certainty (EVUC)

  – Expected Value without information (EV)

so: $\boxed{\text{EVPI} = \text{EVUC} - \text{EV}}$

# Expected Value Under Certainty

- Expected Value Under Certainty (EVUC):

EVUC = expected value of the best decision, given perfect information

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | |
|---|---|---|---|
| | Strong Economy (.3) | Stable Economy (.5) | Weak Economy (.2) |
| Large factory | 200 | 50 | -120 |
| Average factory | 90 | 120 | -30 |
| Small factory | 40 | 30 | 20 |

| | | |
|---|---|---|
| 200 | 120 | 20 |

Example: Best decision given "Strong Economy" is "Large factory"

# Expected Value Under Certainty

*(continued)*

| Investment Choice (Alternatives) | Profit in $1,000's (States of Nature) | | |
|---|---|---|---|
| | Strong Economy (.3) | Stable Economy (.5) | Weak Economy (.2) |
| Large factory | 200 | 50 | -120 |
| Average factory | 90 | 120 | -30 |
| Small factory | 40 | 30 | 20 |

| | | |
|---|---|---|
| 200 | 120 | 20 |

- Now weight these outcomes with their probabilities to find EVUC:

$$EVUC = 200(.3)+120(.5)+20(.2)$$
$$= \boxed{124}$$

# Cost of Uncertainty Solution

- **Cost of Uncertainty (EVPI)**

  = Expected Value Under Certainty (EVUC)

  – Expected Value without information (EV)

Recall: EVUC = 124

EV is maximized by choosing "Average factory", where EV = 81

so: **EVPI = EVUC – EV**

**= 124 – 81**

**= 43**

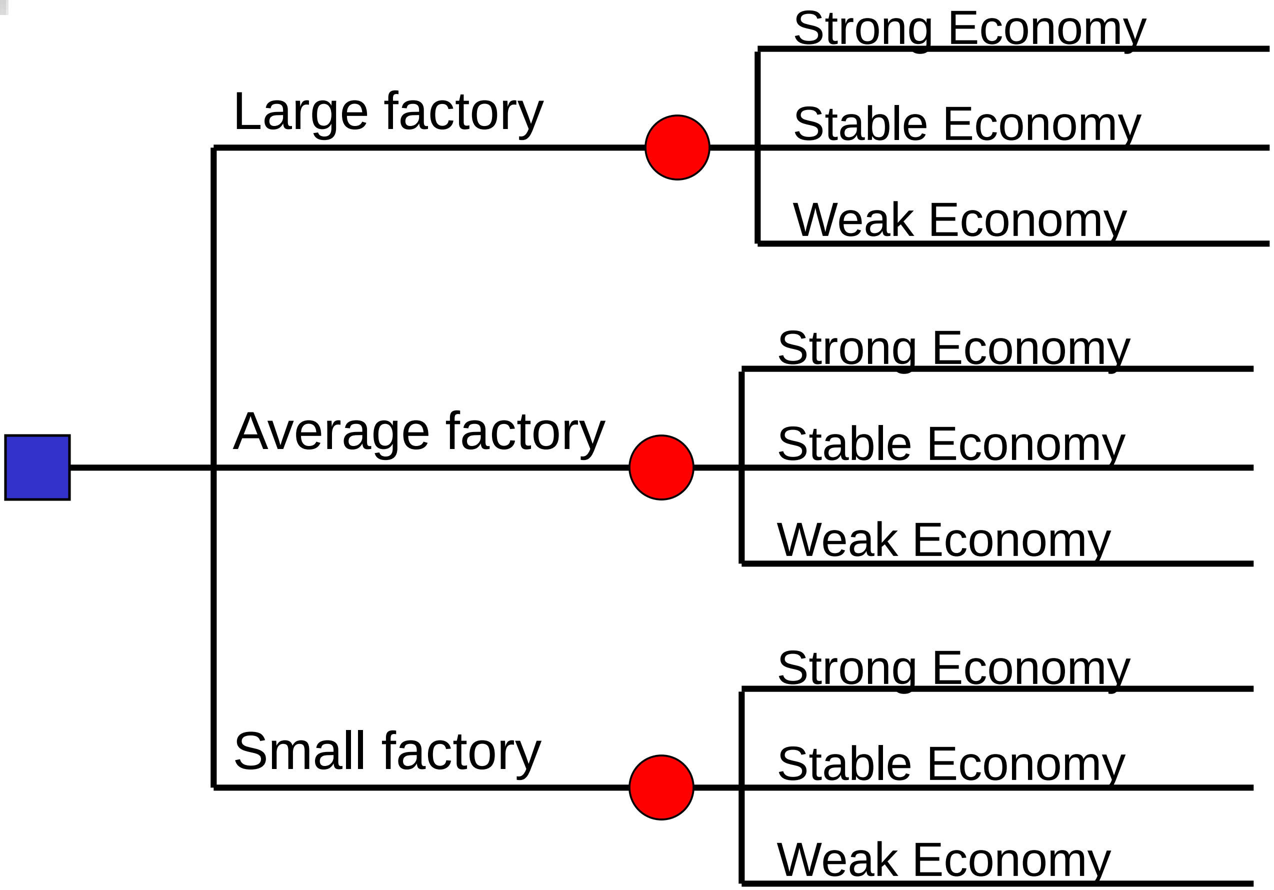
# Decision Tree Analysis

- A Decision tree shows a decision problem, beginning with the initial decision and ending will all possible outcomes and payoffs.

■ Use a square to denote decision nodes

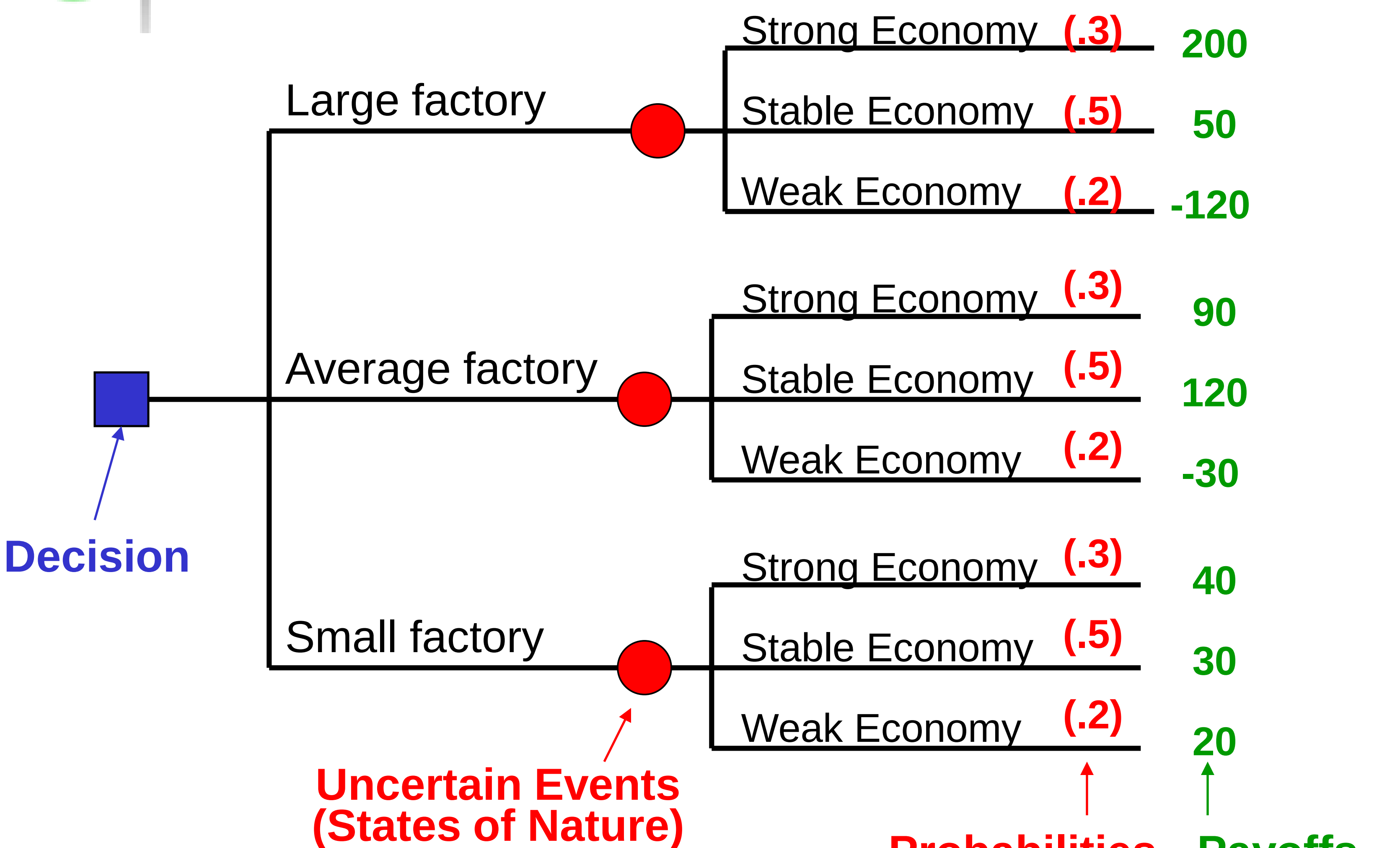
● Use a circle to denote uncertain events

# Sample Decision Tree

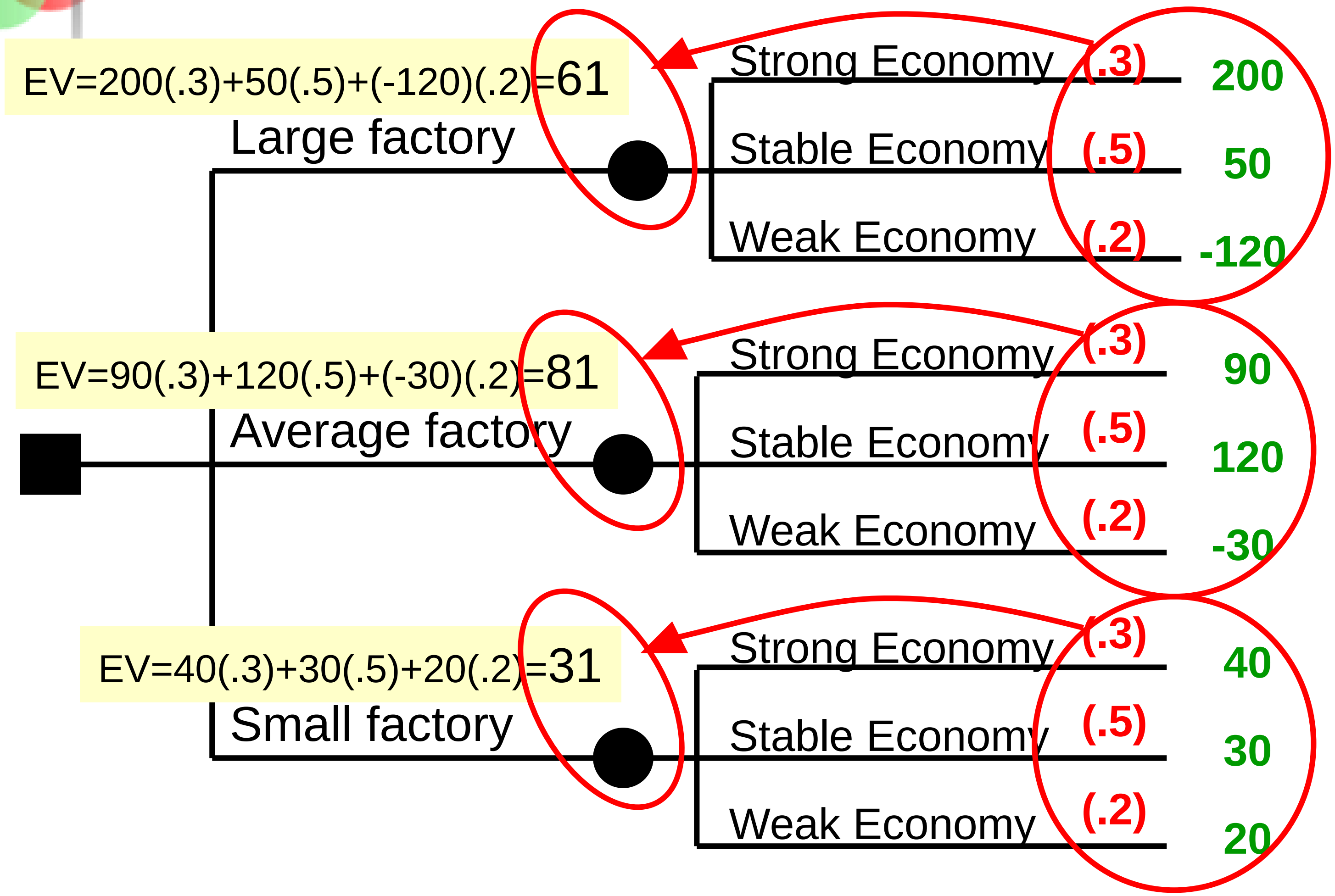


# Add Probabilities and Payoffs

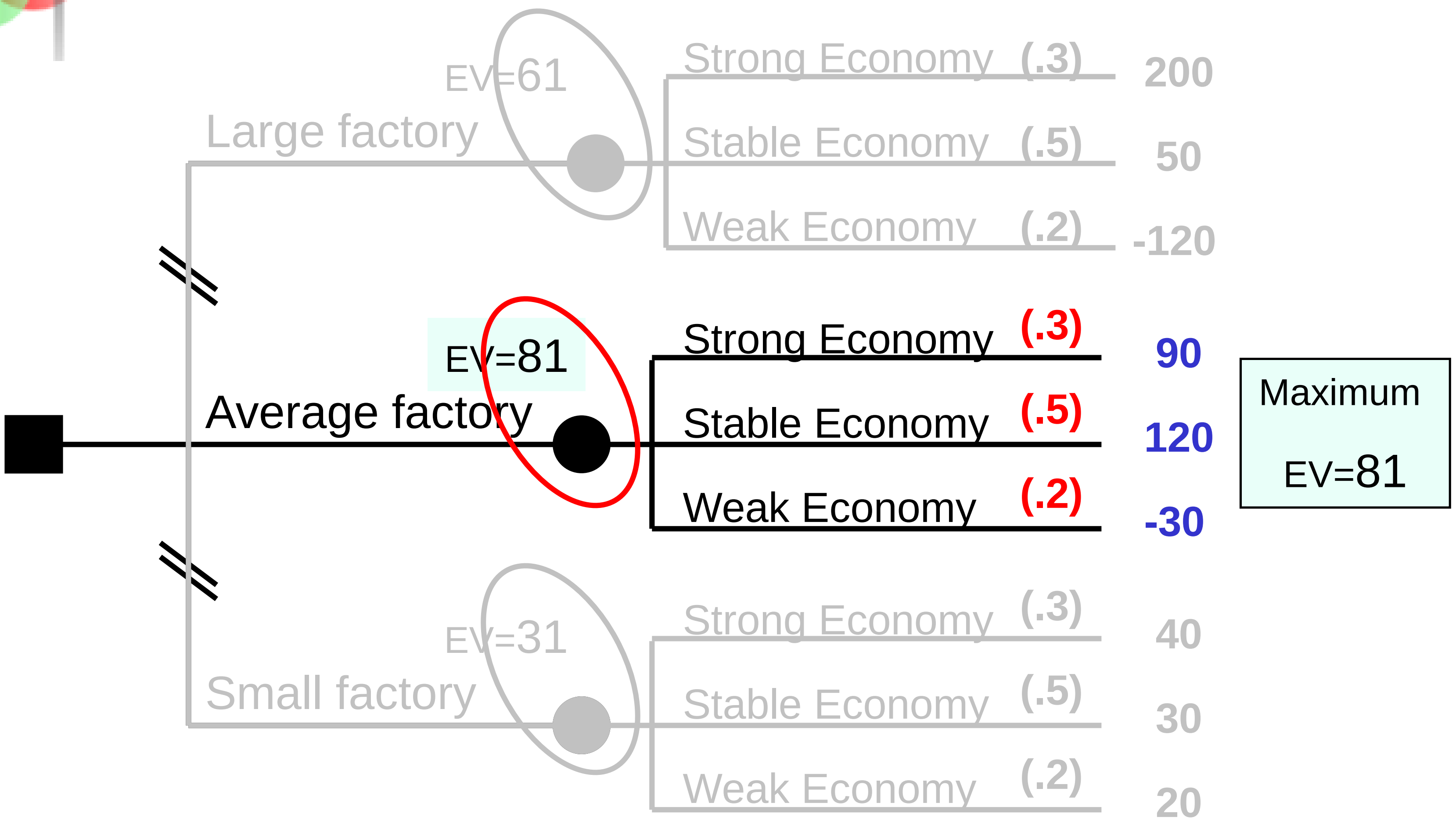
*(continued)*



# Fold Back the Tree



# Make the Decision



# Chapter Summary

- Examined decision making environments
  - certainty and uncertainty
- Reviewed decision making criteria
  - nonprobabilistic:  maximax, maximin, minimax regret
  - probabilistic: expected value, expected opp. loss
- Computed the Cost of Uncertainty (EVPI)
- Developed decision trees and applied them to decision problems