Section 3.1

1. cofactor expansion along first row

$$\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix} = 3\begin{vmatrix} 3 & 2 \\ 5 & -1 \end{vmatrix} - 0\begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix} + 4\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix}$$

$$= 3(-13) - 0 + 40 = 1$$

cofactor expansion along second column

$$\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix} = -0 \cdot \begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix} + 3\begin{vmatrix} 3 & 4 \\ 0 & -1 \end{vmatrix} - 5\begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix}$$

$$= 0 - 9 + 10 = 1$$

2. cofactor expansion along first row

$$\begin{vmatrix} 0 & 4 & 1 \\ 5 & -3 & 0 \\ 2 & 3 & 1 \end{vmatrix} = 0 \cdot \begin{vmatrix} -3 & 0 \\ 3 & 1 \end{vmatrix} - 4\begin{vmatrix} 5 & 0 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & -3 \\ 2 & 3 \end{vmatrix}$$

$$= 0 - 20 + 21 = 1$$

cofactor expansion along second column

$$\begin{vmatrix} 0 & 4 & 1 \\ 5 & -3 & 0 \\ 2 & 3 & 1 \end{vmatrix} = -4 \cdot \begin{vmatrix} 5 & 0 \\ 2 & 1 \end{vmatrix} + (-3)\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 \\ 5 & 0 \end{vmatrix}$$

$$= -20 + 6 + 15 = 1$$

9.

$$\begin{vmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{vmatrix} = 3 \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 7 \end{vmatrix} - 0 + 0 - 0$$

cofactor expansion along 3rd row

$$3 \begin{vmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 7 \end{vmatrix} = 3\left(0 - 0 + 5 \begin{vmatrix} 7 & 2 \\ 3 & 1 \end{vmatrix}\right)$$

cofactor expansion along 1st row

$$= 3 \cdot 5 \cdot (7-6) = 15$$

12.

$$\begin{vmatrix} 3 & 0 & 0 & 0 \\ 7 & -2 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 3 & -8 & 4 & -3 \end{vmatrix} = 3 \begin{vmatrix} -2 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} - 0 + 0 - 0$$

cofactor expansion along 1st row

$$3 \begin{vmatrix} -2 & 0 & 0 \\ 6 & 3 & 0 \\ -8 & 4 & -3 \end{vmatrix} = 3\left(-2 \begin{vmatrix} 3 & 0 \\ 4 & -3 \end{vmatrix} - 0 + 0\right)$$

cofactor expansion along 1st row

$$= 3 \cdot -2 \cdot -9 = 54$$

15.

$$\begin{array}{ccc|cc} & & 0 & 10 & 0 \\ \hline 1 & 0 & 4 & 1 & 0 \\ 2 & 3 & 2 & 2 & 3 \\ 0 & 5 & -2 & 0 & 5 \\ \hline & & -6 & 0 & 40 \end{array}$$

$$-0-10-0+-6+0+40=24$$

19.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$ Swap row1 and row 2.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$$

$$\begin{vmatrix} c & d \\ a & b \end{vmatrix} = bc-ad = -(ad-bc)$$

Swapping a row negated the determinant.

20.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow \begin{bmatrix} a+kc & b+kd \\ c & d \end{bmatrix}$$

Replace row1 with row1 + k times row 2.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$$

$$\begin{vmatrix} a+kc & b+kd \\ c & d \end{vmatrix} = (a+kc)d - c(b+kd)$$

$$= ad + kcd - cb - ckd$$

$$= ad - bc$$

The determinants are the same.

21. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow \begin{bmatrix} a & b \\ kc & kd \end{bmatrix}$ Scale row 2 by k.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b \\ kc & kd \end{vmatrix} = akd - bkc$$

$$= k(ad - bc)$$

Scaling a row by k, scaled the determinant by k.

33. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $E = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\det E = \begin{vmatrix} 1 & k \\ 0 & 1 \end{vmatrix} = 1$$ since E is upper triangular

$$EA = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+kc & b+kd \\ c & d \end{bmatrix}$$

$$\det EA = \begin{vmatrix} a+kc & b+kd \\ c & d \end{vmatrix}$$

$$= (a+kc)d - c(b+kd)$$

$$= ad + kcd - cb - ckd$$

$$= ad - bc$$

Thus $\det EA = \det E \det A$.

37.

$$5A = \begin{bmatrix} 15 & 5 \\ 20 & 10 \end{bmatrix}$$

$$\det 5A = \begin{vmatrix} 15 & 5 \\ 20 & 10 \end{vmatrix} = 50$$

$$5 \det A = 5 \cdot \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = 5 \cdot 2 = 10$$

So $\det 5A \neq 5 \det A$.