Exam 4 Review

Covers Sec 5.1-5.6, 7.1, 7.3, 6.1, 6.2, ~~6.3,~~ 6.5

Sec 5.1: Estimating with Finite Sums

- ✓ Understand how rectangles are constructed to estimate the area underneath a curve using either the left hand endpoint, midpoint, or right hand endpoint of the intervals.
- ✓ Understand the derivation for the formulas for $R_n, L_n, \& M_n$ the different ways of estimating area using the sum of n rectangles
- ✓ Yellow worksheet

Problems to try:

1. From the book:

   Length of a road: You and a companion are about to drive a twisty stretch of dirt road in a car whose speedometer works but whose odometer (mileage counter) is broken. To find out how long this particular stretch of road is, you record the car's velocity at 10-sec intervals, with the results shown in the accompanying table. Estimate the length of the road using

   a. Left-endpoint values

   b. Right-endpoint values

2. Estimate the integral $\int_{-1}^{4}(x^2+2)dx$ using five intervals and the midpoints of the intervals. Draw in the rectangles on the graph. (Might also want to try using right and left endpoints and discussing which options is the best estimate.)

Sec 5.2: Sigma Notation and Limits of Finite Sums

- ✓ Know the different parts of sigma notation
- ✓ Know how to write a sum
- ✓ Know all of the common sums (i.e. the sum of 1, sum of i, etc.)
- ✓ Know how to find the sum with common sums
- ✓ Know how to find the limit of a sum

Problems to try:

3. Find the value of the sum $\sum_{i=1}^{n}(3+2i)^2$
4. Write the sum in sigma notation: $2+4+6+8+...+202$

Sec 5.3: The Definite Integral

- ✓ Know the definition of the definite integral
- ✓ Understand what the different parts of the definition of the definite integral mean
- ✓ Know the properties of definite integrals
- ✓ Know how to write the integral as a Riemann Sum
- ✓ Know how to evaluate an integral using different area properties (i.e. quarter circles, triangles, etc.)

Problems to try:

5. Write the Riemann Sum for an approximation of $\int_3^6 \ln x dx$ using n intervals and right endpts.
6. From the book:
   Suppose that f and h are integrable and that
   $\int_1^9 f(x)dx = -1$ $\int_7^9 f(x)dx = 5$ $\int_7^9 h(x)dx = 4$

Use the rules satisfied by definite integrals to find

- **a.** $\int_1^9 -2f(x)dx$
- **b.** $\int_7^9 [f(x) + h(x)]dx$
- **c.** $\int_7^9 [2f(x) - 3h(x)]dx$
- **d.** $\int_9^1 f(x)dx$
- **e.** $\int_1^7 f(x)dx$
- **f.** $\int_9^7 [h(x) - f(x)]dx$

Sec 5.4: The Fundamental Theorem of Calculus

- ✓ Know the two parts of the Fundamental Theorem of Calculus (FTC1 and FTC2)
- ✓ Remember that the function must be continuous in order to find the integral

Problems to try:

7. Problems from the book:
   Find $\frac{dy}{dx}$ in the following exercises:

   $y = \int_0^x \sqrt{(1+t^2)}dt$

   $y = \int_1^x \frac{1}{t} dt, x > 0$

   $y = \int_{\sqrt{x}}^0 \sin(t^2)\, dt$

Sec 5.5: Indefinite Integrals and the Substitution Rule

- ✓ Know the substitution rule for indefinite and definite integrals
- ✓ Know the properties of integrals involving symmetric functions
- ✓ Know all the formulas for integrals
- ✓ DO NOT FORGET THE C

Sec 5.6: Substitution and Area Between Curves

- ✓ Be able to use the substitution rule on a definite integral
- ✓ Find the area between two curves when one of them is not the x-axis. Remember to also think about doing horizontal rectangles vs. vertical rectangles.

Problems to try:

8. From the book: on pg 356 from 43-111 there are many different integrals that you can try. I would suggest doing some of each kind, and look for the harder ones.

Evaluating Indefinite Integrals

Evaluate the integrals in Exercises 43–72.

43. $\int 2(\cos x)^{-1/2}\sin x\,dx$ 44. $\int (\tan x)^{-3/2}\sec^2 x\,dx$

45. $\int (2\theta + 1 + 2\cos(2\theta + 1))\,d\theta$

46. $\int \left(\frac{1}{\sqrt{2\theta - \pi}} + 2\sec^2(2\theta - \pi)\right)d\theta$

47. $\int \left(t - \frac{2}{t}\right)\left(t + \frac{2}{t}\right)dt$ 48. $\int \frac{(t+1)^2 - 1}{t^4}\,dt$

49. $\int \sqrt{t}\sin(2t^{3/2})\,dt$ 50. $\int (\sec\theta\tan\theta)\sqrt{1 + \sec\theta}\,d\theta$

51. $\int e^x\sec^2(e^x - 7)\,dx$

52. $\int e^y\csc(e^y + 1)\cot(e^y + 1)\,dy$

53. $\int (\sec^2 x)\,e^{\tan x}\,dx$ 54. $\int (\csc^2 x)\,e^{\cot x}\,dx$

55. $\int_{-1}^{1}\frac{dx}{3x - 4}$ 56. $\int_{1}^{e}\frac{\sqrt{\ln x}}{x}\,dx$

57. $\int_0^4 \frac{2t}{t^2 - 25}\,dt$ 58. $\int \frac{\tan(\ln v)}{v}\,dv$

59. $\int \frac{(\ln x)^{-3}}{x}\,dx$ 60. $\int \frac{1}{r}\csc^2(1 + \ln r)\,dr$

61. $\int x3^{x^2}\,dx$ 62. $\int 2^{\tan x}\sec^2 x\,dx$

63. $\int \frac{3\,dr}{\sqrt{1 - 4(r-1)^2}}$ 64. $\int \frac{6\,dr}{\sqrt{4 - (r+1)^2}}$

65. $\int \frac{dx}{2 + (x-1)^2}$ 66. $\int \frac{dx}{1 + (3x+1)^2}$

67. $\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2 - 4}}$

68. $\int \frac{dx}{(x+3)\sqrt{(x+3)^2 - 25}}$

69. $\int \frac{e^{\sin^{-1}\sqrt{x}}\,dx}{2\sqrt{x - x^2}}$ 70. $\int \frac{\sqrt{\sin^{-1} x}\,dx}{\sqrt{1 - x^2}}$

71. $\int \frac{dy}{\sqrt{\tan^{-1} y}\,(1 + y^2)}$ 72. $\int \frac{(\tan^{-1} x)^2\,dx}{1 + x^2}$

Evaluating Definite Integrals

Evaluate the integrals in Exercises 73–112.

73. $\int_{-1}^{1}(3x^2 - 4x + 7)\,dx$ 74. $\int_0^1 (8s^3 - 12s^2 + 5)\,ds$

75. $\int_1^2 \frac{4}{v^2}\,dv$ 76. $\int_1^{27} x^{-4/3}\,dx$

77. $\int_1^4 \frac{dt}{t\sqrt{t}}$ 78. $\int_1^4 \frac{(1 + \sqrt{u})^{1/2}}{\sqrt{u}}\,du$

79. $\int_0^1 \frac{36\,dx}{(2x+1)^3}$ 80. $\int_0^1 \frac{dr}{\sqrt[3]{(7 - 5r)^2}}$

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81. $\int_{1/8}^{1} x^{-1/3}(1 - x^{2/3})^{3/2}\,dx$ 82. $\int_0^{1/2} x^3(1 + 9x^4)^{-3/2}\,dx$

83. $\int_0^{\pi}\sin^2 5r\,dr$ 84. $\int_0^{\pi/4}\cos^2\left(4t - \frac{\pi}{4}\right)dt$

85. $\int_0^{\pi/3}\sec^2\theta\,d\theta$ 86. $\int_{\pi/4}^{3\pi/4}\csc^2 x\,dx$

87. $\int_{\pi}^{3\pi}\cot^2\frac{x}{6}\,dx$ 88. $\int_0^{\pi}\tan^2\frac{\theta}{3}\,d\theta$

89. $\int_{-\pi/3}^{0}\sec x\tan x\,dx$ 90. $\int_{\pi/4}^{3\pi/4}\csc z\cot z\,dz$

91. $\int_0^{\pi/2} 5(\sin x)^{3/2}\cos x\,dx$ 92. $\int_{-\pi/2}^{\pi/2} 15\sin^4 3x\cos 3x\,dx$

93. $\int_0^{\pi/2}\frac{3\sin x\cos x}{\sqrt{1 + 3\sin^2 x}}\,dx$ 94. $\int_0^{\pi/4}\frac{\sec^2 x}{(1 + 7\tan x)^{2/3}}\,dx$

95. $\int_1^4\left(\frac{x}{8} + \frac{1}{2x}\right)dx$ 96. $\int_1^8\left(\frac{2}{3x} - \frac{8}{x^2}\right)dx$

97. $\int_{-2}^{-1} e^{-(x+1)}\,dx$ 98. $\int_{-\ln 2}^{0} e^{2w}\,dw$

99. $\int_0^{\ln 5} e^r(3e^r + 1)^{-3/2}\,dr$ 100. $\int_0^{\ln 9} e^{\theta}(e^{\theta} - 1)^{1/2}\,d\theta$

101. $\int_1^e \frac{1}{x}(1 + 7\ln x)^{-1/3}\,dx$ 102. $\int_1^3 \frac{(\ln(v+1))^2}{v+1}\,dv$

103. $\int_1^8 \frac{\log_4\theta}{\theta}\,d\theta$ 104. $\int_1^e \frac{8\ln 3\log_3\theta}{\theta}\,d\theta$

105. $\int_{-3/4}^{3/4}\frac{6\,dx}{\sqrt{9 - 4x^2}}$ 106. $\int_{-1/5}^{1/5}\frac{6\,dx}{\sqrt{4 - 25x^2}}$

107. $\int_{-2}^{2}\frac{3\,dt}{4 + 3t^2}$ 108. $\int_{\sqrt{3}}^{3}\frac{dt}{3 + t^2}$

109. $\int \frac{dy}{y\sqrt{4y^2 - 1}}$ 110. $\int \frac{24\,dy}{y\sqrt{y^2 - 16}}$

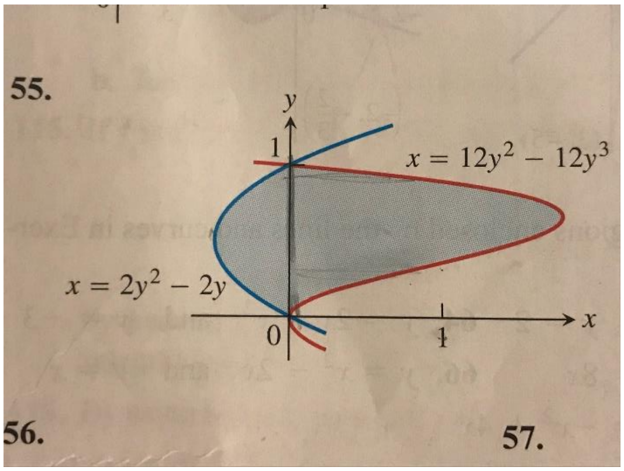
111. $\int_{\sqrt{2}/3}^{2/3}\frac{dy}{|y|\sqrt{9y^2 - 1}}$ 112. $\int_{-2/\sqrt{5}}^{-\sqrt{6}/\sqrt{5}}\frac{dy}{|y|\sqrt{5y^2 - 3}}$

Average Values

9. From the book: pg 351: 55, 63

Find the total area of the shaded region:

Find the area of the region enclosed by the line and/or curve.



$y = x^2 - 2$ and $y = 2$

10. Evaluate: $\int_{e}^{e^4} \frac{dx}{x\sqrt{\ln x}}$

11. Evaluate: $\int \frac{3x+4}{x^2+1}\,dx$

Sec 7.1: The Logarithm Defined as an Integral

- ✓ The only thing new in this section really is the integral for any exponential function, not just e^x, so remember it and remember all of the other rules for integrals.

Sec 7.4: Hyperbolic Functions

- ✓ Remember the definitions of hyperbolic functions and the identities for hyperbolic functions.
- ✓ Be able to find the derivative and integrals of the hyperbolic functions.

Problems to try:

12. Look back over your homework and quiz #12

Sec 6.1: Volumes by Slicing and Rotation About an Axis

- ✓ Know how to find the volume of a solid either using the disk method or the washer method. Remember that it is always top curve-bottom curve or right curve-left curve, even if the curve is the axis of rotation.
- ✓ Yellow worksheet

Problems to try:

13. From the book: pg 372: 44, 49

Find the volumes of the solids generated by revolving the regions bounded by the lines and curves about the x-axis

$$y = \sec(x)\,, \qquad y = \tan(x)\,, \qquad x = 0, \qquad x = 1$$

Find the volume of the solid generated by revolving each region about the given axis;

The region in the first quadrant bounded above by the curve $y = x^2$, below by the x-axis, and on the right by the line $x = 1$, about the line $x = -1$

~~**Sec 6.3**: Arc Length~~

- ✓ ~~Remember that you are just finding the sum of an infinite number of distances between points along the curve.~~
- ✓ ~~$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx$~~

~~*Problems to try:*~~

~~8. Review your homework. The focus should be on 1-10 on page 386.~~

Sec 6.5: Work

- ✓ Know how to find the work of a variable force or the work of pumping liquids from containers.
- ✓ I would suggest reviewing the two problems done in class and the ideas of how to set-up the work integrals and what each part represents.

Problems to try:

9. From the book: pg 398: 9, 18

Lifting an elevator cable: An electric elevator with a motor at the top has a multistrand cable weighing 4.5 lb/ft. When the car is at the first floor, 180 ft of cable are paid out, and effectively 0 ft are out when the car is at the top floor. How much work does the motor do just lifting the cable when it takes the car from the first floor to the top?

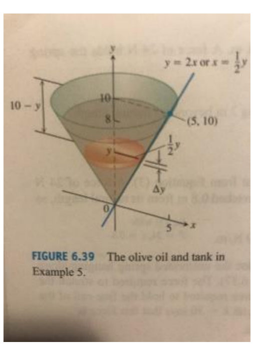


FIGURE 6.39 The olive oil and tank in Example 5.

Example 5: The conical tank in Figure 6.39 is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^2. How much work does it take to pump the oil to the rim of the tank?

a. **Pumping Milk:** Suppose that the conical container in Example 5 contains milk (weighing 64.5 lb/ft^2) instead of olive oil. How much work will it take to pump the contents to the rim?

b. **Pumping oil:** How much work will it take to pump the oil in Example 5 to a level 3 ft above the cone's rim?