Test #3 Review

1. pg 290 = #7, 8

7. $f(x) = x - 2\ln x \qquad 1 \le x \le 3$

$f'(x) = 1 - \frac{2}{x} = 0$

$1 = \frac{2}{x}$

$x = 2$

x	$f(x)$
1	$1 - 2\ln(1) = 1$
2	$2\ln(2) = -.386$
3	$1 - 2\ln(3) = -1.197$

endpts

c.v.

$\boxed{\text{global max of } 1 \text{ @ } x = 1 \quad \text{global min of } -1.197 \text{ @ } x = 3}$

8. $f(x) = \frac{\ }{x} + \ln(x^2) \qquad 1 \le x \le 4$

$f'(x) = \frac{-4}{x^2} + \frac{2x}{x^2} = 0$

$\frac{4}{x^2} = \frac{2x}{x^2}$

then $x = 2$ & $x = 0$ ~~is~~ undefined

not in domain

x	$f(x)$
1	$\frac{4}{1} + \ln(1) = 4$
2	$\frac{4}{2} + \ln(4) = 3.386$
4	$\frac{4}{4} + \ln(16) = 3.77$

$\boxed{\text{global max of } 4 \text{ @ } x = 1 \quad \text{global min of } 3.386 \text{ @ } x = 2}$

2. $g(0) = 0 \qquad [0, 2]$

$g(2) = 6$

then by the MVT $g'(c) =$ average slope

$g'(c) = \frac{g(2) - g(0)}{2 - 0}$

$g'(c) = \frac{6 - 0}{2 - 0}$

$\boxed{g'(c) = 3}$

the instanteous slope = the average slope of 3 @ a pt c in (0, 2)

3. pg 236 ⇒ 1, 2, 5, 7

1. $f(x) = x^2 + 2x - 1 \quad [0, 1]$

$f'(x) = 2x + 2$ by MVT $\quad 2x + 2 = \frac{f(1) - f(0)}{1 - 0}$

$2x + 2 = \frac{2 + 1}{1 - 0}$

$2x + 2 = 3 \Rightarrow \boxed{x = 1/2}$

2. $f(x) = x^{2/3}$ $[0,1]$

$f'(x) = \frac{2}{3x^{1/3}}$ by MVT $\frac{2}{3x^{1/3}} = \frac{f(1)-f(0)}{1-0}$

$\frac{2}{3x^{1/3}} = \frac{1-0}{1-0}$

$2 = 3x^{1/3}$

$(2/3)^3 = x$

$\boxed{8/27 = x}$

5. $f(x) = \sin^{-1}x$ $[-1,1]$

$f'(x) = \frac{1}{\sqrt{1-x^2}}$ by MVT $\frac{1}{\sqrt{1-x^2}} = \frac{f(1)-f(-1)}{1+1}$

$\frac{1}{\sqrt{1-x^2}} = \frac{\pi/2 + \pi/2}{2}$

$\frac{1}{\sqrt{1-x^2}} = \frac{\pi}{2}$

$\left(\frac{2}{\pi}\right)^2 = \left(\sqrt{1-x^2}\right)^2$

$4/\pi^2 = 1-x^2$

$\frac{4}{\pi^2} = 1-x^2$

$x^2 = 1 - 4/\pi^2$

$x = \pm\sqrt{1-4/\pi^2}$

$\boxed{x = \pm .77}$

7. $f(x) = x^3 - x^2$ $[-1,2]$

$f'(x) = 3x^2 - 2x$ by MVT $3x^2 - 2x = \frac{f(2)-f(-1)}{2+1}$

$3x^2 - 2x = \frac{4+2}{2+1}$

$3x^2 - 2x = 2$

$3x^2 - 2x - 2 = 0$

$\boxed{x = \frac{1 \pm \sqrt{7}}{3} = -.548, 1.215}$

4. Check notes from Thursday Oct 27th $\Rightarrow$ you can find these in an e-mail sent to you

5. pg 291 ⇒ #29, 39, 53

29. $y = -x^3 + 6x^2 - 9x + 3$

$y' = -3x^2 + 12x - 9 = 0$

$x = 1, 3$ possible extrema

$y'' = -6x + 12 = 0$

$x = 2$ possible inflection pt

intervals	$(-\infty, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
pt	0	$3/2$	$5/2$	4
sign of f'	−	+	+	−
Slope of f	↘	↗	↗	↘
sign of f"	+	+	−	−
Concavity of f	↑	↑	↓	↓
Shape of f	◡	◡	◠	◠

global max ⇒ none

global min ⇒ none

local max ⇒ $(3, 3)$

local min ⇒ $(1, -1)$

inflection pt ⇒ $(2, 1)$

inc ⇒ $(1, 3)$

dec ⇒ $(-\infty, 1) \cup (3, \infty)$

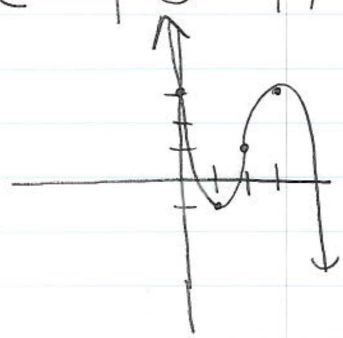
concave ↑ ⇒ $(-\infty, 2)$

concave ↓ ⇒ $(2, \infty)$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

no asymptotes



39. $y = \ln(x^2 - 4x + 3) \Rightarrow x^2 - 4x + 3 > 0$

$(x-3)(x-1) > 0$

$x > 3$ & $x < 1$

$y' = \dfrac{2x - 4}{x^2 - 4x + 3} = 0$

$x = 2$ c.v. doesn't work b/c not in domain

$x \neq 1, 3$

$$y'' = \frac{(x^2 - 4x + 3)(2) - \overbrace{(2x-4)(2x-4)}^{4x^2 - 16x + 16}}{(x^2 - 4x + 3)^2}$$

$$= \frac{2x^2 - 8x + 6 - 4x^2 + 16x + 16}{(x^2 - 4x + 3)^2}$$

$$= \frac{-2x^2 + 8x + 10}{(x^2 - 4x + 3)^2}$$

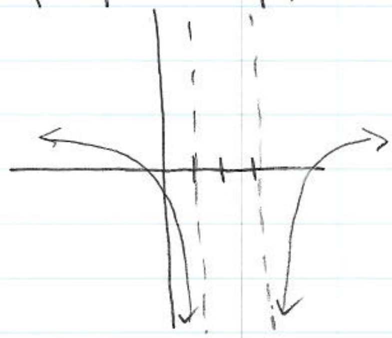
$x =$ none

$x \neq 1, 3$

→

intervals	$(-\infty, 1)$	$(1,3)$	$(3,\infty)$
pt	0		4
f' sign	$-$		$+$
f slope	↘	✓ domain	↗
f'' sign	$-$		$-$
f concavity	↓		↓
Shape	⌒		⌒

global max ⇒ none
global min ⇒ none
local max ⇒ none
local min ⇒ none
inflection pts ⇒ none
inc ⇒ $(3,\infty)$
dec ⇒ $(-\infty,1)$
Concave ↓ ⇒ $(-\infty,1) \cup (3,\infty)$
concave ↑ ⇒ none
D: $(-\infty,1)\,(3,\infty)$
R: $(-\infty,\infty)$
v.a. $x = 1, 3$



53. $y = \dfrac{x+1}{x-3} \Rightarrow$ h.a. $y = 1$
v.a. $x = 3$

$y' = \dfrac{(x-3)(1) - (x+1)(1)}{(x-3)^2}$

$= \dfrac{-4}{(x-3)^2} = 0$

$x \neq 3$

$y'' = \dfrac{(x-3)^{\cancel{2}}(0) - (-4)(2\cancel{(x-3)}(1))}{(x-3)^{\cancel{4}3}}$

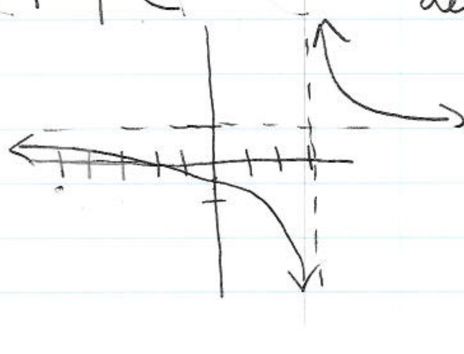
$y'' = \dfrac{8}{(x-3)^3}$

$x = 3$

intervals	$(-\infty, 3)$	$(3,\infty)$
pt	0	4
f' sign	$-$	$-$
f slope	↘	↘
f'' sign	$-$	$+$
f concavity	↓	↑
shape	⌒	◡

global max ⇒ none
global min ⇒ none
local max ⇒ none
local min ⇒ none
inflection pts ⇒ none
inc ⇒ none
dec ⇒ $(-\infty,3) \cup (3,\infty)$

concave ↑ ⇒ $(3,\infty)$
Concave ↓ ⇒ $(-\infty,3)$
D: $\{x \mid x \neq 3\}$
R: $\{y \mid y \neq 1\}$
v.a. $x = 3$
h.a. $y = 1$



6. T by the 2nd Derivative test for Extrema
7. T by the 2nd Derivative test for Concavity
8. $\lim_{x\to 1} \frac{\ln x}{\sin(\pi x)} \Rightarrow \frac{0}{0}$ yes b/c of indeterminate form

$\lim_{x\to\infty} x^{\cos(1/x)} \Rightarrow \infty^1 = \infty$ no b/c lim is ∞

$\lim_{x\to -\infty} [(e^x - 1)\ln|x|] \Rightarrow -1\cdot\infty = -\infty$ no b/c lim is $-\infty$

$\lim_{x\to\infty} x^3 e^{-x^2} \Rightarrow \lim_{x\to\infty} \frac{x^3}{e^{x^2}} \Rightarrow \frac{\infty}{\infty}$ yes b/c of indeterminate form

$\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\tan x}\right) \Rightarrow \infty - \infty$ yes w/ some re-arranching it can be of the form to use L' Hopital's

9. pg 291 ⇒ #73-83 odd

73. $\lim_{x\to 0} \frac{10^x - 1}{x} = \frac{0}{0}$

$\lim_{x\to 0} \frac{10^x \ln 10}{1} = \boxed{\ln 10}$

75. $\lim_{x\to 0} \frac{2^{\sin x} - 1}{e^x - 1} = \frac{0}{0}$

$\lim_{x\to 0} \frac{2^{\sin x}\ln(2)\cos x}{e^x} = \boxed{\ln 2}$

77. $\lim_{x\to 0} \frac{5 - 5\cos x}{e^x - x - 1} = \frac{0}{0}$

$\lim_{x\to 0} \frac{5\sin x}{e^x - 1} = \frac{0}{0}$

$\lim_{x\to 0} \frac{5\cos x}{e^x} = \boxed{5}$

79. $\lim_{x\to 0^+} \frac{x - \ln(1+2x)}{x^2} = \frac{0}{0}$

$\lim_{x\to 0^+} \frac{1 - \frac{2}{1+2x}}{2x} \Rightarrow \lim_{x\to 0^+} \frac{\frac{1+2x-2}{1+2x}}{2x}$

$\Rightarrow \lim_{x\to 0^+} \frac{2x-1}{2x(1+2x)} = \boxed{-\infty}$

81. $\lim_{x\to 0^+} \left(\frac{e^x}{x} - \frac{1}{x}\right) \Rightarrow \infty - \infty$

$\lim_{x\to 0^+} \left(\frac{e^x - 1}{x}\right) = \frac{0}{0}$

$\lim_{x\to 0^+} \frac{e^x}{1} = \boxed{1}$

83. $\lim_{x\to\infty} \left(1+\frac{b}{x}\right)^{kx} \Rightarrow (1^{\infty})$

$y = \lim_{x\to\infty} \left(1+\frac{b}{x}\right)^{kx} \Rightarrow \ln y = \ln \lim_{x\to\infty} \left(1+\frac{b}{x}\right)^{kx} \Rightarrow \ln y = \lim_{x\to\infty} kx \ln\left(1+\frac{b}{x}\right)$

$\Rightarrow \ln y = \lim_{x\to\infty} \frac{\ln(1+b/x)}{\frac{1}{kx}} \left(\frac{0}{0}\right) \Rightarrow \ln y = \lim_{x\to\infty} \frac{\frac{-b/x^2}{1+b/x}}{\frac{-1}{kx^2}} \cdot \frac{-kx^2}{1}$

$\Rightarrow \ln y = \lim_{x\to\infty} \frac{-b}{\cancel{x^2}} \cdot 1+b/x \cdot -k\cancel{x^2}$

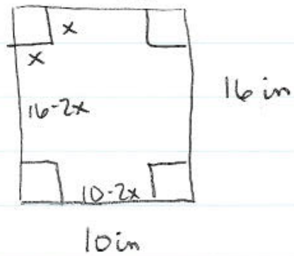
$\Rightarrow \ln y = \lim_{x\to\infty} bk(1+b/x)$

$\ln y = bk$

$y = e^{bk} \Rightarrow \boxed{\lim_{x\to\infty} \left(1+\frac{b}{x}\right)^{kx} = e^{bk}}$

10. pg 292 ⇒ #93 pg 268 ⇒ #8,9

93.



$V = l \cdot w \cdot h$

$V = x(10-2x)(16-2x)$

$V = x(160-52x+4x^2)$

$V = 4x^3-52x^2+160x$

$V' = 12x^2-104x+160 = 0$

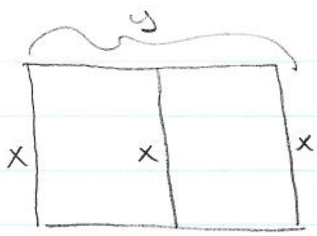
$x = 20/3, 2$

$x \Rightarrow [0,10]$

x	V
0	0
2	144
20/3	-17.77
10	0

when 2 in cut from corners a max of 144 in^3 is the Volume I know it is a max b/c

(8) $\nearrow$ 2 $\searrow$ (3)

8. 

$A = xy$

$216 = xy$

$y = 216/x$

$P = 3x + 2y$

$P = 3x + 2(216/x)$

$P = 3x + 432/x$

$P' = 3 - 432/x^2 = 0$

$3x^2 = 432$

$x^2 = 144$

$x = 12$ (not ± b/c of length)

$y = 18$

outer perimeter ⇒ 60 m

total fence ⇒ 72 m

9. 

$V = 500\text{ ft}^3$

$V = x^2 h$

$500 = x^2 h$

$500/x^2 = h$

$SA = 4xh + x^2$

$SA = 4x(500/x^2) + x^2$

$SA = 2000/x + x^2$

$SA' = -2000/x^2 + 2x = 0$

$2000/x^2 = 2x$

$2000 = 2x^3$

$1000 = x^3$

$10 = x$

$5 = h$

sq base 10 ft × 10 ft

ht 5 ft

by minimizing the SA the weight of the tank will also be minimized

12. pg 292 ⇒ 97, 101, 107, 109, 111

97. $\int (x^3 + 5x - 7)\,dx = \frac{x^4}{4} + \frac{5x^2}{2} - 7x + C$

101. $\int \frac{dr}{(r+5)^2} = \frac{-1}{r+5} + C$

107. $\int \sec^2\left(\frac{s}{10}\right) ds = 10\tan\left(\frac{s}{10}\right) + C$

109. $\int \csc\sqrt{2}\theta \cot\sqrt{2}\theta \, d\theta = -\frac{1}{\sqrt{2}} \csc\sqrt{2}\theta + C$

111. $\int \sin^2\left(\frac{x}{4}\right) dx = \int \frac{1 - \cos\left(2 \cdot \frac{x}{4}\right)}{2} dx = \int \frac{1}{2} - \frac{\cos\left(\frac{x}{2}\right)}{2} dx$

$= \frac{1}{2}x - \sin\left(\frac{x}{2}\right) + C$

13. $f(x)=\frac{-1}{1+x^2} \Rightarrow F(x)=\cot^{-1}x+C$

$f(x)=e^{-5x} \Rightarrow F(x)=-\frac{1}{5}e^{-5x}+C$

$f(x)=\sec x\tan x \Rightarrow F(x)=\sec x+C$

14. $a(t)=6t+4$

$v(t)=3t^2+4t+C$ @ $v(0)=-6$

$-6=3(0)^2+4(0)+C$

$\boxed{v(t)=3t^2+4t-6}$

$s(t)=t^3+2t^2-6t+C$ @ $s(0)=9$

$9=(0)^3+2(0)^2-6(0)+C$

$\boxed{s(t)=t^3+2t^2-6t+9}$

1. pg 305 ⇒ 11

11. left hand ⇒ total distance is the area under the velocity curve

$D=10(0+44+15+35+30+44+35+15+22+35+44+30)$

width ↑ sum of heights ↑

$D=3490$ ft

right hand

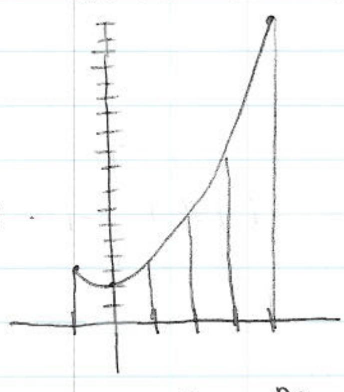
$D=10(44+15+35+30+44+35+15+22+35+44+30+35)$

$D=3840$ ft

2. $\int_{-1}^{4}(x^2+2)\,dx$

$\int_{-1}^{4}(x^2+2)\,dx=1(\text{ hts })$

$=31.25$



$\Delta x=1$

mdpts ⇒ $x=-.5, .5, 1.5, 2.5, 3.5$

hts ⇒ $2.25, 2.25, 4.25, 8.25, 14.25$

3. $\sum_{i=1}^{n}(3+2i)^2=\sum_{i=1}^{n}9+12i+4i^2=\sum_{i=1}^{n}9+12\sum_{i=1}^{n}i+4\sum_{i=1}^{n}i^2$

$2n^3+3n^2+n$

$n(2n^2+3n+1)$

$9n+\cancel{12}\,6\left(\frac{n(n+1)}{\cancel{2}}\right)_{n^2+n}+\cancel{4}\,2\left(\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right)$

use the formulas on pg 304-309

$=9n+6n^2+6n+\frac{4}{3}n^3+2n^2+\frac{2}{3}n=\boxed{\frac{4}{3}n^2+8n^2+\frac{47}{3}n}$

4. $2+4+6+8+\ldots+202$ arithmetic sum w/ a difference of 2

$a_n = a + (n-1)d$ $\quad a_n = 2 + 2n - 2$

$202 = 2 + (n-1)(2)$ $\quad a_n = 2n$

$101 = n$

$$\boxed{\sum_{n=1}^{101} 2n}$$

5. $\int_3^6 \ln x \, dx \Rightarrow \lim_{n\to\infty} \sum_{k=1}^{n} \ln x \, \Delta x$

6. pg 322 ⇒ #10

10. $\int_1^9 f(x)\,dx = -1$ $\quad \int_7^9 f(x)\,dx = 5$ $\quad \int_7^9 h(x)\,dx = 4$

a. $\int_1^9 -2f(x)\,dx \Rightarrow -2(-1) = 2$

b. $\int_7^9 [f(x) + h(x)]\,dx = 5 + 4 = 9$

c. $\int_7^9 [2f(x) - 3(h(x))]\,dx = 2(5) - 3(4) = -2$

d. $\int_9^1 f(x)\,dx = -\int_1^9 f(x)\,dx = -(-1) = 1$

e. $\int_1^7 f(x)\,dx = \int_1^9 f(x)\,dx - \int_7^9 f(x)\,dx = -1 - 5 = -6$

f. $\int_9^7 [h(x) - f(x)]\,dx = -\int_7^9 [(h(x) - f(x)]\,dx = -(4-5) = 1$