Review key for ~~test~~ 4a

1) $(1-\sin\alpha)(1+\sin\alpha)$

$1-\sin^2\alpha \Rightarrow \boxed{\cos^2\alpha}$

2) $\csc x \tan x + \sec(-x)$

$\cancel{\frac{1}{\sin x}}\left(\frac{\cancel{\sin x}}{\cos x}\right) + \frac{1}{\cos(x)} = \frac{2}{\cos x} = \boxed{2\csc x}$

3) $(1-\csc x)(1-\csc(-x))$

$(1-\csc(x))(1+\csc(x))$

$1-\csc^2 x \Rightarrow -\cot^2 x$

4) $\frac{\cos^2 x - \sin^2 x}{\sin 2x} \Rightarrow \frac{\cos 2x}{\sin 2x} \Rightarrow \boxed{\cot 2x}$

5) $\frac{1}{1+\sin\alpha} + \frac{\sin\alpha}{\cos^2\alpha}$

$\frac{1}{1+\sin\alpha} + \frac{\sin\alpha}{1-\sin^2\alpha} \Rightarrow \frac{1-\sin\alpha}{(1+\sin\alpha)(1-\sin\alpha)} + \frac{\sin\alpha}{1-\sin^2\alpha}$

$= \frac{1-\cancel{\sin\alpha}+\cancel{\sin\alpha}}{1-\sin^2\alpha}$

$= \frac{1}{\cos^2\alpha} \Rightarrow \boxed{\sec^2\alpha}$

6) $2\sin\left(\frac{\pi}{2}-x\right)\cos\left(\frac{\pi}{2}-x\right)$

$2\cos x\sin x \Rightarrow \sin 2\theta$

7) $\frac{2\tan 2\cancel{x}}{1-\tan^2 2x} \Rightarrow u=2x$

$\frac{2\tan u}{1-\tan^2 u} \Rightarrow \frac{\frac{2\sin u}{\cos u}}{1-\frac{\sin^2 u}{\cos^2 u}} \Rightarrow \frac{\frac{2\sin u}{\cos u}}{\frac{\cos^2 u-\sin^2 u}{\cos^2 u}} \Rightarrow \frac{2\sin u}{\cancel{\cos u}}\left(\frac{\cos^{\cancel{2}} u}{\cos^2 u-\sin^2 u}\right)$

$= \frac{2\sin u\cos u}{\cos^2 u-\sin^2 u} \Rightarrow \frac{2\sin 2u}{\cos 2u}$

$2\tan 2u$

$\boxed{2\tan 4x}$

$$\tan(2w+2w)$$

8) $$\frac{\tan 2w - \tan 4w}{1+\tan 2w \tan 4w} = \frac{\tan 2w - \dfrac{\tan(2w)+\tan(2w)}{1-\tan^2(2w)}}{1+\tan 2w \tan 4w}$$

$$= \frac{\tan 2w - \dfrac{2\tan(2w)}{1-\tan^2(2w)}}{1+\tan(2w)\left(\dfrac{2\tan(2w)}{1-\tan^2(2w)}\right)}$$

$$= \frac{\tan 2w - \dfrac{2\tan(2w)}{1-\tan^2(2w)}}{1+\dfrac{2\tan^2(2w)}{1-\tan^2(2w)}}$$

$$= \frac{\dfrac{(\tan(2w))(1-\tan^2(2w))}{1-\tan^2(2w)} - \dfrac{2\tan(2w)}{1-\tan^2(2w)}}{\dfrac{1-\tan^2(2w)+2\tan^2(2w)}{1-\tan^2(2w)}}$$

$$= \frac{\dfrac{\tan(2w)-\tan^3(2w)-2\tan(2w)}{\cancel{1-\tan^2(2w)}}}{\dfrac{1-\tan^2(2w)+2\tan^2(2w)}{\cancel{1-\tan^2(2w)}}}$$

$$= \frac{-(\tan(2w))\cancel{(\tan^2(2w)+1)}}{\cancel{1+\tan^2(2w)}}$$

$$= -\tan(2w)$$

9) $\sin(3\theta)\cos(6\theta) - \cos(3\theta)\sin(6\theta)$ $\quad u = 3\theta$

$\sin(u)\cos(2u) - \cos(u)\sin(2u)$

$\sin(u)(\cos^2 u - \sin^2 u) - \cos u(2\sin u\cos u)$

$\sin(u)\cos^2 u - \sin^3 u - 2\sin u\cos^2 u$

$-\sin^3(u) - \sin u\cos^2 u$

$-\sin^3(u) - \sin u(1 - \sin^2 u)$

$\cancel{-\sin^3(u)} - \sin u + \cancel{\sin^3 u}$

$-\sin u \Rightarrow \boxed{-\sin(3\theta)}$

10) $\dfrac{\sin 2y}{1+\cos 2y} \Rightarrow \dfrac{2\sin y\cancel{\cos y}}{\cancel{1}+2\cos^{\cancel{2}} y\cancel{-1}} \Rightarrow \dfrac{\cancel{2}\sin y}{\cancel{2}\cos y}$

$= \boxed{\tan y}$

11) $\dfrac{1-\cos 2z}{\sin 2z} \Rightarrow \dfrac{\cancel{1-1}+2\sin^2 z}{2\sin z\cos z} \Rightarrow \dfrac{\cancel{2\sin^{2}} z}{\cancel{2\sin z}\cos z}$

$= \dfrac{\sin z}{\cos z} \Rightarrow \boxed{\tan z}$

12) $\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$ $\quad u = \frac{x}{2}$

$\cos^2(u) - \sin^2(u)$

$\cos(2u) \Rightarrow \cos\left(\cancel{2}\left(\frac{x}{\cancel{2}}\right)\right) \Rightarrow \boxed{\cos(x)}$

47) $\sec 2\theta = \dfrac{1+\tan^2\theta}{1-\tan^2\theta}$

$$= \frac{\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}} \Rightarrow \frac{\frac{\cos^2\theta+\sin^2\theta}{\cancel{\cos^2\theta}}}{\frac{\cos^2\theta-\sin^2\theta}{\cancel{\cos^2\theta}}} = \frac{1}{\cos^2\theta - \sin^2\theta}$$

$= \dfrac{1}{\cos(2\theta)}$

$\boxed{= \sec(2\theta)}$

Note $\frac{1}{\tan\theta} = \cot\theta$

51) $\cot(\alpha - 45^\circ) = \frac{1+\tan\alpha}{\tan\alpha - 1}$

$\frac{1}{\tan(\alpha - 45^\circ)} =$

$\frac{1+\tan\alpha\tan(45^\circ)}{\tan\alpha - \tan(45^\circ)} =$

$\frac{1+\tan\alpha}{\tan\alpha - 1} =$

53) $\frac{\sin 2x}{2\csc x} = \sin^2 x\cos x$

$\frac{\cancel{2}\sin x\cos x}{\cancel{2}\left(\frac{1}{\sin x}\right)}$

$\sin^2 x\cos x = \sin^2 x\cos x$

57) $\cos(4x) = 8\sin^4 x - 8\sin^2 x + 1$

$\cos(2x)\cos(2x) - \sin(2x)\sin(2x) =$

$(1-2\sin^2 x)(1-2\sin^2 x) - (2\sin x\cos x)(2\sin x\cos x) =$

$1 - 4\sin^2 x + 4\sin^4 x - 4\sin^2 x\cos^2 x =$

$1 - 4\sin^2 x + 4\sin^4 x - 4\sin^2 x(1-\sin^2 x) =$

$1 - 4\sin^2 x + 4\sin^4 x - 4\sin^2 x + 4\sin^4 x =$

$1 - 8\sin^2 x + 8\sin^4 x =$

69) $2\cos(2x) + 1 = 0$

$\cos(2x) = -\frac{1}{2}$

$2\cos^2(x) - 1 = -1/2$

$2\cos^2(x) = 1/2$

$\cos^2(x) = 1/4$

$\cos(x) = \frac{1}{2}$ $\quad \{x \mid \frac{\pi}{3} + k\pi \text{ or } \frac{2\pi}{3} + k\pi\}$

71) $(\sqrt{3}\csc x - 2)(\csc x - 2) = 0$

$\sqrt{3}\csc x - 2 = 0$ $\qquad \csc x - 2 = 0$

$\csc x = \frac{2}{\sqrt{3}}$ $\qquad \csc x = 2$

$x = \frac{\pi}{6} + \pi k$

$x = \frac{\pi}{3} + \pi k$

$\{x \mid x = \frac{\pi}{3} + k\pi \text{ or } x = \frac{\pi}{6} + k\pi\}$

73) $2\sin^2 x + 1 = 3\sin x$

$2\sin^2 x + 1 - 3\sin x = 0$

$2\sin^2 x - 3\sin x + 1 = 0$ $\qquad 2x^2 - 3x + 1 \qquad 2, 1$

$(2\sin x - 1)(\sin x - 1) = 0$ $\qquad 2x^2 - 2x - x + 1$

$2x(x-1) - 1(x-1)$

$\sin x = 1/2$ $\qquad \sin x = 1$ $\qquad (2x-1)(x-1)$

$x = \frac{\pi}{6} + \pi k$ $\qquad x = \frac{\pi}{2} + 2\pi k$

$\{x \mid x = \frac{\pi}{6} + k\pi \text{ or } \frac{\pi}{2} + 2k\pi\}$

83) $\sin\alpha\cos\alpha = \frac{1}{2}$ $\qquad 0$

$\frac{1}{2}(\sin(\alpha+\alpha) + \sin(\alpha-\alpha)) = 1/2$

$\sin(2\alpha) = 1$ $\qquad u = 2\alpha$

$\sin(u) = 1$ $\qquad u = \frac{\pi}{2}$

$2\alpha = \frac{\pi}{2} \Rightarrow \boxed{\alpha = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}}$

87) $\sin^2\alpha + \cos^2\alpha = \frac{1}{2}$

$1 \neq \frac{1}{2}$ None