Math 109—Rational Function

I. Domain & Vertical Asymptotes

A. State the domain of each rational function.

B. Give the equation of the vertical asymptote(s) of the rational function.

1. $f(x)=\frac{3}{2x-1}$

$2x-1=0$

$x=\frac{1}{2}$

$D: \{x \mid x \neq \frac{1}{2}\}$

2. $f(x)=\frac{x}{(x+1)(x-3)}$

$(x+1)(x-3)=0$

$x=-1, 3$

$D: \{x \mid x \neq -1, 3\}$

3. $f(x)=\frac{x^2+1}{x^2-4}$

$x^2-4=0$

$x=\pm 2$

$D: \{x \mid x \neq \pm 2\}$

4. $f(x)=\frac{3x^2}{x^2-4x-12}$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=6, -2$

$D: \{x \mid x \neq 6, -2\}$

II. Graphs & Range

A. Match each rational function with its graph.

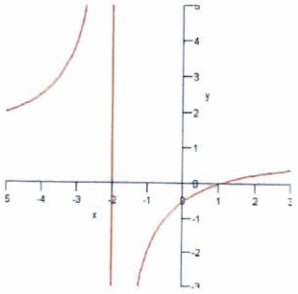
B. State the range for each rational function.

C. Find each of the horizontal asymptotes.

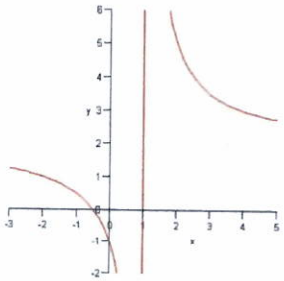
5. $f(x)=\frac{2x+1}{x-1}$ b

6. $f(x)=\frac{x+1}{x-2}$ c

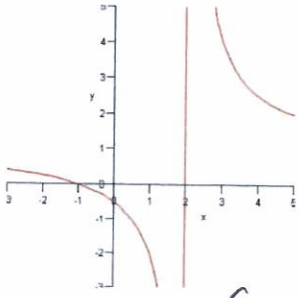
7. $f(x)=\frac{x-1}{x+2}$ a



a



b



c

III. **Graph and find all of the important parts**

A. Find the horizontal and vertical asymptote for each rational function (using limits as appropriate).

B. Sketch the graph of each rational function.

C. Find the domain and range.

8. $f(x) = \frac{1}{x+6}$

$x+6=0$

V.a. $x=-6$

$1 \neq 0$

no x-int

$(0, \frac{1}{6})$

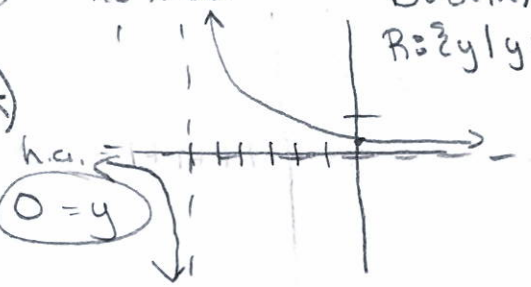
$D: \{x \mid x \neq -6\}$

$R: \{y \mid y \neq 0\}$

$\lim_{x\to\infty} \left(\frac{1}{x+6}\right)\left(\frac{\frac{1}{x}}{\frac{1}{x}}\right)$

$\lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{6}{x}} = 0 = y$

h.a.



9. $f(x) = \frac{2x}{x-3}$

$x-3=0$

V.a $x=3$

$2x=0$

$x=0$

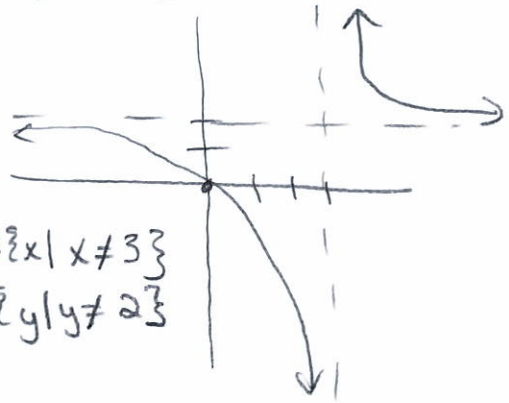
$(0,0)$ x-int

y-int $(0,0)$

$\lim_{x\to\infty} \left(\frac{2x}{x-3}\right)\left(\frac{\frac{1}{x}}{\frac{1}{x}}\right)$

$\lim_{x\to\infty} \frac{2}{1-\frac{3}{x}} = 2 = y$

h.a.



$D: \{x \mid x \neq 3\}$

$R: \{y \mid y \neq 2\}$

10. $f(x) = -\frac{x}{x^2-4}$

$x^2-4=0$

V.a $x=\pm 2$

$-x=0$

$x=0$

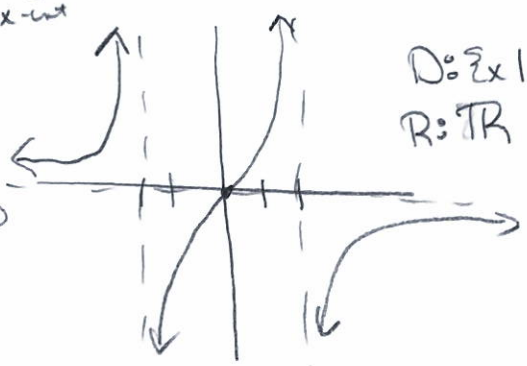
$(0,0)$ x-int

$(0,0)$ y-int

$\lim_{x\to\infty} \left(\frac{-x}{x^2-4}\right)\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)$

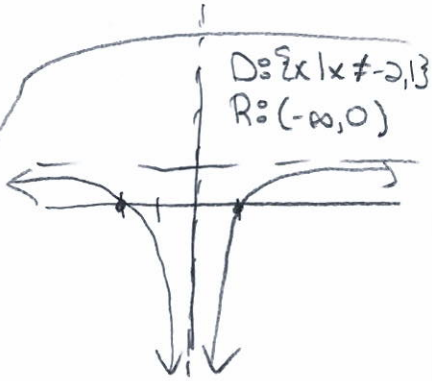
$\lim_{x\to\infty} \frac{-\frac{1}{x}}{1-\frac{4}{x^2}} = 0 = y$

h.a.



$D: \{x \mid x \neq \pm 2\}$

$R: \mathbb{R}$



$D: \{x \mid x \neq -2, 1\}$

$R: (-\infty, 0)$

11. $f(x) = \frac{x^2+5}{2x^2-x-1}$

$2x^2-x-1=0$

$(2x+1)(x-1)=0$

$x = -\frac{1}{2}, 1$ V.a

$x^2+5=0$

$x \neq$

no x-int

y-int

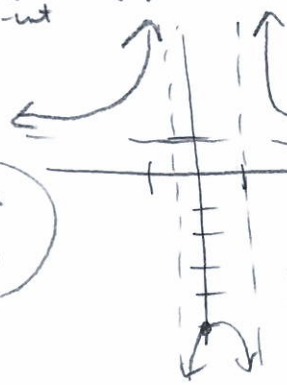
$(0,-5)$

$\lim_{x\to\infty} \left(\frac{x^2+5}{2x^2-x-1}\right)\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)$

$\lim_{x\to\infty} \frac{1+\frac{5}{x^2}}{2-\frac{1}{x}-\frac{1}{x^2}} = \frac{1}{2} = y$

h.a

$y=\frac{1}{2}$



$D: \{x \mid x \neq -\frac{1}{2}, 1\}$

$R: (-\infty, -5] \cup (\frac{1}{2}, \infty)$

↳ based on your graph!

12. $f(x) = \frac{(x+2)(x-1)}{x^2}$

$x^2=0$

$x=0$ V.a

$(x+2)(x-1)=0$

$x=-2, 1$

$(-2,0)(1,0)$ x-int

no y-int

$\lim_{x\to\infty} \left(\frac{x^2+x-2}{x^2}\right)\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)$

$\lim_{x\to\infty} \frac{1+\frac{1}{x}-\frac{2}{x^2}}{1} = 1 = y$ h.a