Section 2.2

1. $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}^{-1} = \frac{1}{32-30}\begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -\frac{5}{2} & 4 \end{bmatrix}$

Check: $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -5/2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 8 & 5 \\ -7 & -5 \end{bmatrix}^{-1} = \frac{1}{-40-(-35)}\begin{bmatrix} -5 & -5 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -7/5 & -8/5 \end{bmatrix}$

Check: $\begin{bmatrix} 8 & 5 \\ -7 & -5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -7/5 & -8/5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

6. The system is the same as the matrix equation

$\begin{bmatrix} 8 & 5 \\ -7 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -9 \\ 11 \end{bmatrix}$ or $A\vec{x} = \vec{b} \Rightarrow \vec{x} = A^{-1}\vec{b}$

Using Theorem 5,

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -7/5 & -8/5 \end{bmatrix}\begin{bmatrix} -9 \\ 11 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

8. A is invertible and $AD = I$.

$$AD = I$$
$$A^{-1}(AD) = A^{-1}I \qquad \text{(multiply by } A^{-1} \text{ on the left)}$$
$$(A^{-1}A)D = A^{-1}$$
$$ID = A^{-1}$$
$$D = A^{-1}$$

9. a. True, see definition of invertible on page 105

b. False, $(AB)^{-1} = B^{-1}A^{-1}$ see Theorem 6 part b.

c. False, $ad - bc \neq 0$ is necessary see Thm 4.

d. True, see Theorem 5, the unique solution to $A\vec{x} = \vec{b}$ is $\vec{x} = A^{-1}\vec{b}$.

e. True, see shaded box on page 109.

13. Given $AB = AC$, and A is invertible.

Since $AB = AC$, $A^{-1}(AB) = A^{-1}(AC)$.

Simplifying, $(A^{-1}A)B = (A^{-1}A)C$.

This implies $IB = IC$, which means $B = C$.

In general, $AB = AC$ does not imply $B = C$.

For example if A is the zero matrix, then

$$\underset{n\times n}{A}\,\underset{n\times p}{B} = \underset{n\times n}{A}\,\underset{n\times p}{C} = \underset{n\times p}{\text{the zero matrix}}$$

14. Given $(B-C)D=0$, where B and C are $m\times n$ matrices and D is invertible.

Since D is invertible, we can multiply the equation above by D^{-1} on the right.

$$(B-C)DD^{-1} = 0\cdot D^{-1}$$

$$(B-C)I = 0$$

$$B-C=0$$

$$B=C.$$

18. P is invertible, $A=PBP^{-1}$. Solve for B.

Since $A=PBP^{-1}$ and P is invertible, we may multiply on the left by P^{-1} and on the right by P.

$$P^{-1}AP = P^{-1}(PBP^{-1})P = (P^{-1}P)B(P^{-1}P)$$

$$= IBI = B, \quad \text{so } B=P^{-1}AP.$$

23. A is $n \times n$ and $A\vec{x} = \vec{0}$ has only the trivial solution.

Since $A\vec{x} = \vec{0}$ has only the trivial solution, each column of A is a pivot column. Since A is $n \times n$, A has n pivot columns, and A is row equivalent to I_n.

29. $\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 4 & 7 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -4 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 4 & -1 \end{array}\right]$

$\sim \left[\begin{array}{cc|cc} 1 & 0 & -7 & 2 \\ 0 & 1 & 4 & -1 \end{array}\right]$ Thus the inverse is $\begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix}$

Check: $\begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

32. $\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & -4 & 0 & 0 & 1 \end{array}\right] \sim$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 2 & -2 & 2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -7 & 2 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 10 & -2 & 1 \end{array}\right]$$

Since the last row has 0 0 0, we cannot row reduce to the identity matrix.

By Theorem 7, $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$ is not invertible.