Section 6.1

1. $\vec{u}\cdot\vec{u} = (-1)^2 + 2^2 = 5$ $\qquad \dfrac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}} = \dfrac{8}{5}$

$\vec{v}\cdot\vec{u} = 4\cdot -1 + 6\cdot 2 = 8$

5. $\left(\dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \left(\dfrac{-1\cdot 4 + 2\cdot 6}{4\cdot 4 + 6\cdot 6}\right)\begin{bmatrix}4\\6\end{bmatrix} = \dfrac{8}{52}\begin{bmatrix}4\\6\end{bmatrix} = \begin{bmatrix}8/13\\12/13\end{bmatrix}$

7. $\|\vec{w}\| = \sqrt{\vec{w}\cdot\vec{w}} = \sqrt{3^2+(-1)^2+(-5)^2} = \sqrt{35}$

9. $\begin{bmatrix}-30\\40\end{bmatrix}\cdot\dfrac{1}{\sqrt{(-30)^2+40^2}} = \begin{bmatrix}-30\\40\end{bmatrix}\cdot\dfrac{1}{50} = \begin{bmatrix}-3/5\\4/5\end{bmatrix}$

11. $\begin{bmatrix}7/4\\1/2\\1\end{bmatrix}\cdot\dfrac{1}{\sqrt{\left(\frac{7}{4}\right)^2+\left(\frac{1}{2}\right)^2+1^2}} = \begin{bmatrix}7/4\\1/2\\1\end{bmatrix}\cdot\dfrac{1}{\frac{\sqrt{69}}{4}} = \begin{bmatrix}7/\sqrt{69}\\2/\sqrt{69}\\4/\sqrt{69}\end{bmatrix}$

14. $\|\vec{u}-\vec{z}\| = \sqrt{(\vec{u}-\vec{z})\cdot(\vec{u}-\vec{z})} = \sqrt{4^2+(-4)^2+(-6)^2}$

$= \sqrt{68}$

15. $\begin{bmatrix} 8 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ -3 \end{bmatrix} = -16 + 15 = -1$ not orthogonal.

16. $\begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix} = 24 - 9 - 15 = 0$ orthogonal.

19. a. True - definition of length

b. True - Theorem 1 part c

c. True - see discussion of orthogonal vectors on page 335

d. False - see theorem 3, vectors orthogonal to the column space of A are in the null space of A^T.

e. True - see shaded box at bottom of pg 336.

28. Suppose $\vec{y}$ is orthogonal to $\vec{u}$ and $\vec{v}$.

Let $\vec{w}$ be a vector in Span $\{\vec{u}, \vec{v}\}$. Thus $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$. So

$\vec{w} = c_1\vec{u} + c_2\vec{v}$ for some constants

c_1 and c_2.

Consider $\vec{y} \cdot \vec{w} = \vec{y} \cdot (c_1 \vec{u} + c_2 \vec{v})$

$$= \vec{y} \cdot c_1 \vec{u} + \vec{y} \cdot c_2 \vec{v}$$

$$= c_1 (\vec{y} \cdot \vec{u}) + c_2 (\vec{y} \cdot \vec{v})$$

$$= c_1 \cdot 0 + c_2 \cdot 0$$

$$= 0 + 0 = 0$$

Thus $\vec{y}$ is orthogonal to $\vec{w}$, and since $\vec{w}$ is an arbitrary vector in Span $\{\vec{u}, \vec{v}\}$, $\vec{y}$ is orthogonal to every vector in Span $\{\vec{u}, \vec{v}\}$

30. Let W be a subspace and let $W^{\perp}$ be the set of vectors orthogonal to every vector in W.

a. Let $\vec{z}$ be in $W^{\perp}$, and let $\vec{u}$ be in W.

$$c\vec{z} \cdot \vec{u} = c(\vec{z} \cdot \vec{u}) = c \cdot 0 = 0$$

Thus $c\vec{z}$ is orthogonal to $\vec{u}$.

b. Let $\vec{z}_1, \vec{z}_2$ be in $W^{\perp}$ and let $\vec{u}$ be in W.

$$(\vec{z}_1 + \vec{z}_2) \cdot \vec{u} = \vec{z}_1 \cdot \vec{u} + \vec{z}_2 \cdot \vec{u}$$

$$= 0 + 0 = 0$$

Thus $\vec{z}_1 + \vec{z}_2$ is orthogonal to $\vec{u}$.

c. Parts a) and b) show that $W^{\perp}$ is closed under scalar multiplication and vector addition. Since the $\vec{0}$ is orthogonal to every vector, $\vec{0}$ is in $W^{\perp}$. Thus $W^{\perp}$ is a subspace by definition.