

# Math 109—Polynomial Worksheet Name: key

1. Given  $h(x) = -x^5 + 4x^4 - 3x^3 - x^2 + 4$ , answer the following questions.

a. Describe the end behavior of  $h(x)$ .

degree = 5

leading coefficient is neg

$x \rightarrow \infty \quad y \rightarrow -\infty$

$x \rightarrow -\infty \quad y \rightarrow \infty$

b. State the maximum number of turns possible for the graph of  $h(x)$ .

4

c. What is the y-intercept of  $h(x)$ ?

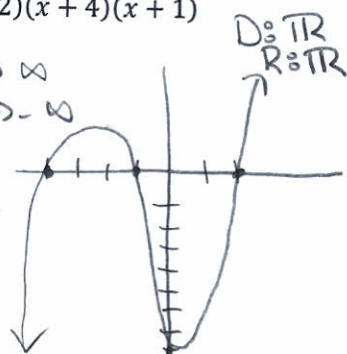
(0, 4)

For problems 2-7, use the end behavior (with limits), zeros, the y-intercept, and multiplicity to sketch the graph of the given polynomials.

2.  $y = (x-2)(x+4)(x+1)$

$x \rightarrow \infty \quad y \rightarrow \infty$   
 $x \rightarrow -\infty \quad y \rightarrow -\infty$

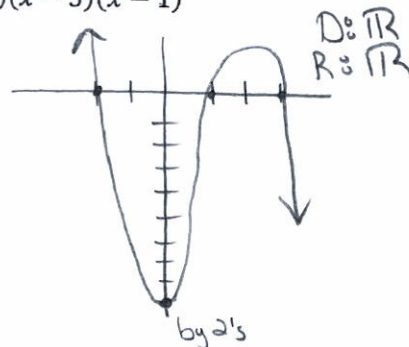
(2, 0)  
 (-4, 0)  
 (-1, 0)  
 (0, -8)



5.  $y = -3(x+2)(x-3)(x-1)$

$x \rightarrow \infty \quad y \rightarrow -\infty$   
 $x \rightarrow -\infty \quad y \rightarrow \infty$

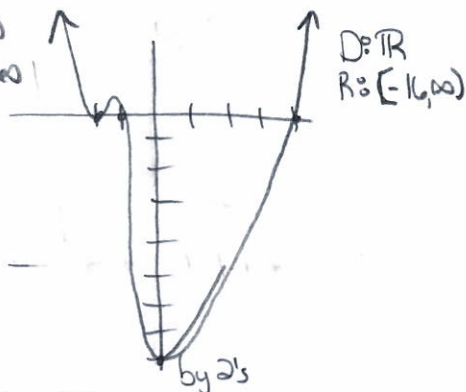
(-2, 0)  
 (3, 0)  
 (1, 0)  
 (0, -18)



3.  $f(x) = (x-4)(x+2)^2(x+1)$

$x \rightarrow \infty \quad y \rightarrow \infty$   
 $x \rightarrow -\infty \quad y \rightarrow -\infty$

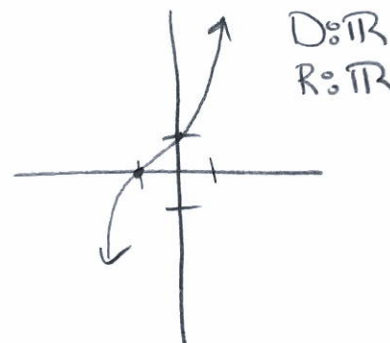
(4, 0)  
 (-2, 0) m2  
 (-1, 0)  
 (0, -16)



6.  $f(x) = (x+1)^3$

$x \rightarrow \infty \quad y \rightarrow \infty$   
 $x \rightarrow -\infty \quad y \rightarrow -\infty$

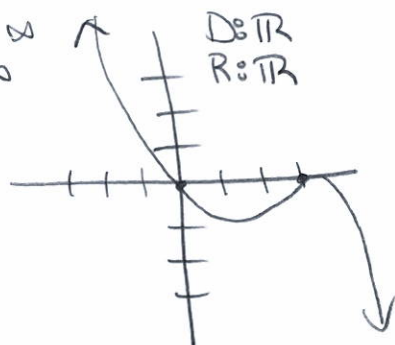
(-1, 0) m3  
 (0, 1)



4.  $g(x) = -x(x-3)^2$

$x \rightarrow \infty \quad y \rightarrow -\infty$   
 $x \rightarrow -\infty \quad y \rightarrow \infty$

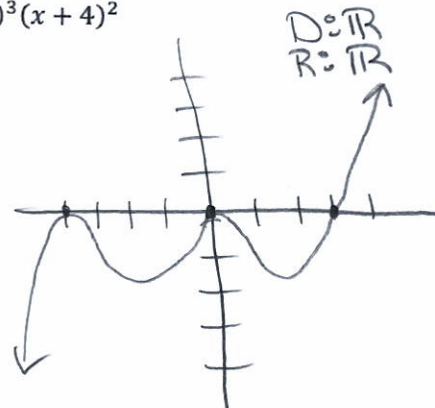
(0, 0)  
 (3, 0) m2  
 (0, 0) y-int



7.  $g(x) = x^2(x-3)^3(x+4)^2$

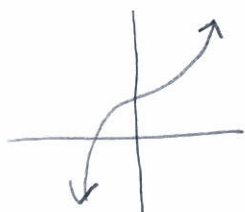
$x \rightarrow \infty \quad y \rightarrow \infty$   
 $x \rightarrow -\infty \quad y \rightarrow -\infty$

(0, 0) m2  
 (3, 0) m3  
 (-4, 0) m2  
 (0, 0) y-int



8. Sketch a third degree polynomial that satisfies the following given information.

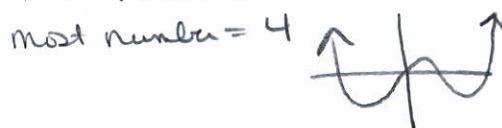
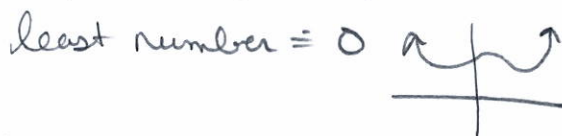
- a.  $g(x)$  has one x-intercept    b.  $g(x)$  has two x-intercepts    c.  $g(x)$  has three x-intercepts



9. Is it possible to draw a third degree polynomial with no x-intercept? Explain your answer.

No. Because of the end behavior any odd power polynomial will have as  $x \rightarrow \infty$   $y \rightarrow \infty$  or  $x \rightarrow \infty$   $y \rightarrow -\infty$   
 $x \rightarrow -\infty$   $y \rightarrow -\infty$      $x \rightarrow -\infty$   $y \rightarrow \infty$

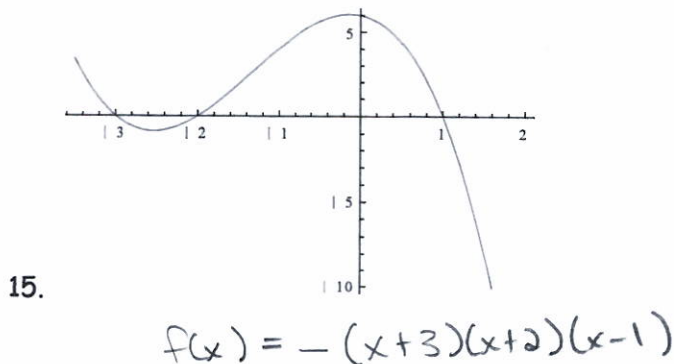
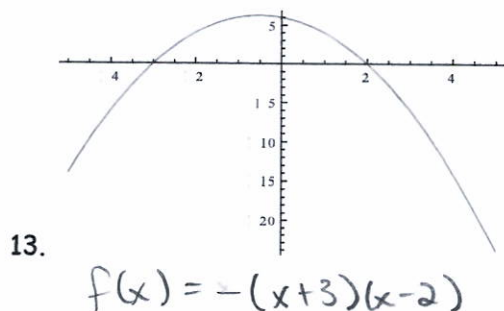
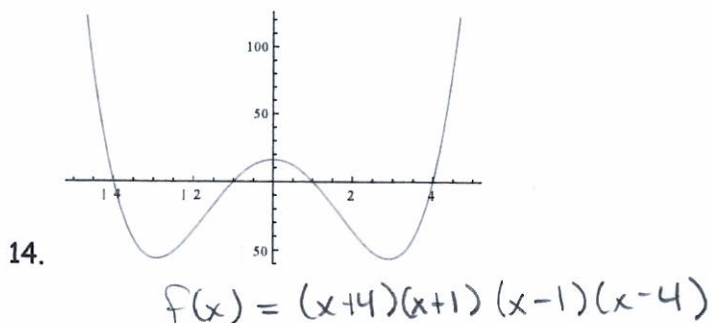
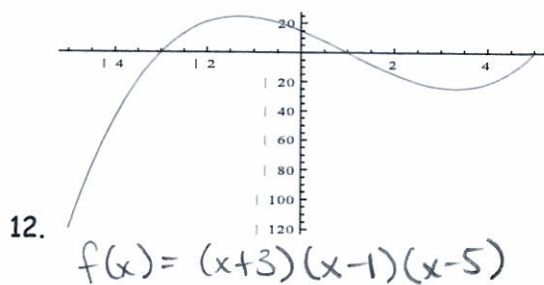
10. What is the least number of x-intercepts for a fourth degree polynomial? The most number of x-intercepts? (Please provide a picture for each as part of your explanation.)



11. What is the least number of x-intercepts for a seventh degree polynomial? The most number of x-intercepts?

least = 1    most 7

Write a polynomial equation for each of the following graphs. Leave your equation in factored form.



Find all of the zeros for the following polynomials and write the equation in factored form.

16.  $f(x) = x^3 - 3x^2 - 2x + 6$

$$x^3 - 3x^2 - 2x + 6 = 0$$

$$\frac{6}{1} = \pm 1, 2, 3, 6$$

$$\begin{array}{r|rrrr} & 1 & -3 & -2 & 6 \\ 3 & & 3 & 0 & -6 \end{array}$$

$$1 \quad 0 \quad -2 \quad 0$$

$$x^2 - 2 = 0$$

$$x = \pm\sqrt{2}$$

$$(3, 0)$$

$$(\pm\sqrt{2}, 0)$$

17.  $g(x) = 2x^3 + 9x^2 - 8x - 15$

$$2x^3 + 9x^2 - 8x - 15 = 0$$

$$\frac{15}{2} = \pm 1, 3, 5, 15$$

$$\begin{array}{r|rrrr} & 2 & 9 & -8 & -15 \\ -1 & & -2 & -7 & 15 \end{array}$$

$$2 \quad 7 \quad -15 \quad 0$$

$$2x^2 + 7x - 15$$

$$(2x - 3)(x + 5)$$

$$(-1, 0)$$

$$\left(\frac{3}{2}, 0\right)$$

$$(-5, 0)$$