(3) (a) $\int e^y \csc(e^y+1)\cot(e^y+1)\,dy$ $\quad u = e^y+1$, $du = e^y\,dy$

$= \int \csc u \cot u\,du$

$= -\csc u + C$

$= -\csc(e^y+1) + C$

(b) $\int 3^{-2x}\,dx$ $\quad u = -2x$, $du = -2dx$

$= -\frac{1}{2}\int 3^u\,du$

$= -\frac{1}{2}\left(\frac{3^u}{\ln 3}\right) + C$

$= \frac{-3^{-2x}}{2\ln 3} + C$

(c) $\int_1^8 \frac{2}{3x} - \frac{8}{x^2}\,dx$

$= \frac{2}{3}\ln|x| + \frac{8}{x}\Big|_1^8$

$= \left(\frac{2}{3}\ln 8 + 1\right) - \left(8\right)$

$= \frac{2}{3}\ln 8 - 7$

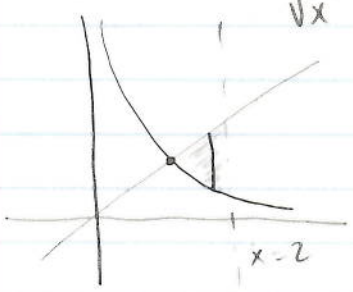
(d) $\int 4\cosh(3x - \ln 2)\,dx$ $\quad u = 3x - \ln 2$, $du = 3dx$

$= \frac{4}{3}\int \cosh u\,du$

$= \frac{4}{3}\sinh u + C$

$= \frac{4}{3}\sinh(3x - \ln 2)$

(4) (a) $y = x$ $y = \frac{1}{\sqrt{x}}$ $x = 2$



$x = \frac{1}{\sqrt{x}}$

$x^{3/2} = 1$

$x = 1^{2/3}$

$x = 1$

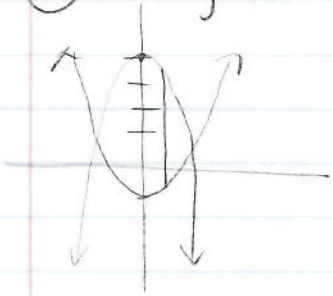
intersect at $(1, 1)$

$$A = \int_1^2 x - \frac{1}{\sqrt{x}}\, dx = \frac{x^2}{2} + 2\sqrt{x}\,\Big|_1^2$$

$$= \left(2 + 2\sqrt{2}\right) - \left(\frac{1}{2} + 2\right)$$

$$A = -\frac{1}{2} + 2\sqrt{2}$$

(b) $4x^2 + y = 4$ $x^4 \cdot y = 1 \Rightarrow y = 4 - 4x^2$ & $y = x^4 - 1$



$x^4 - 1 = 4 - 4x^2$

$x^4 + 4x^2 - 5 = 0$

$(x^2 + 5)(x^2 - 1)$

$x^2 = -5$ $x = \pm 1$

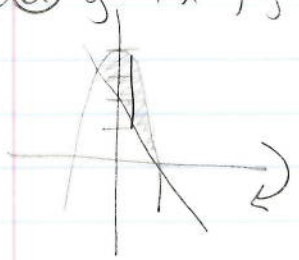
~~$x = \pm i\sqrt{5}$~~

$$A = \int_{-1}^{1} (4 - 4x^2) - (x^4 - 1)\, dx = 5x - \frac{4x^3}{3} - \frac{x^5}{5}\,\Big|_{-1}^{1}$$

$$= \left(5 - \frac{4}{3} - \frac{1}{5}\right) - \left(-5 + \frac{4}{3} + \frac{1}{5}\right)$$

$$A = \frac{104}{15}$$

(5) (a) $y = 4 - x^2$, $y = 2 - x$



$4 - x^2 = 2 - x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

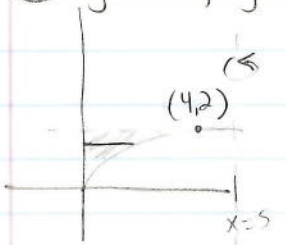
$x = 2, -1$

$$V = \int_{-1}^{2} \pi(4-x^2)^2 - \pi(2-x)^2\,dx = \pi\int_{-1}^{2} 16 - 8x^2 + x^4 - 4 + 4x - x^2\,dx$$

$$= \pi\left(12x - 3x^3 + x^2 + \frac{x^5}{5}\right)\Big|_{-1}^{2}$$

$$= \frac{93}{5}\pi$$

(b) $y = \sqrt{x}$, $y = 2$, $x = 0$



$2 = \sqrt{x}$

$4 = x$

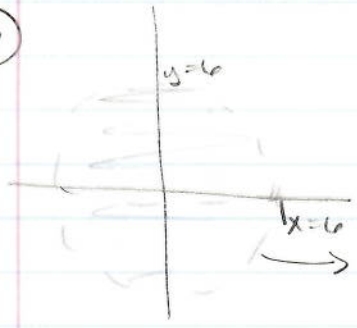
$$V = \int_{0}^{2} \pi(5-0)^2 - \pi(y^2-0)^2\,dy$$

$$= \pi\int_{0}^{2} 25 - y^4\,dy$$

$$= \pi\left(25y - \frac{y^5}{5}\right)\Big|_{0}^{2}$$

$$= \frac{218}{5}\pi$$

(6)



$x = \sqrt{36-y^2}$

$\omega = 62.4 \text{ lb/ft}^3$

$$W = \int_{-6}^{6} 62.4\left(\pi\left(\sqrt{36-y^2}\right)^2\right)(6-y)\,dy$$

$$= 62.4\pi \int_{-6}^{6} (6-y)(36-y^2)\,dy$$

$$= 62.4\pi \int_{-6}^{6} 216 - 6y^2 - 36y + y^3\,dy$$

$$= 62.4\pi \left(216y - 2y^3 - 18y^2 + \frac{y^4}{4}\right)\Big|_{-6}^{6}$$

$$= 62.4\pi\,(1728)$$

$$= 107827.2\pi \text{ ft-lbs}$$

$$= 338749.1394 \text{ ft-lbs}$$