Math 109—Thursday June 21, 2018—Section 6.1

- ✓ Colin please say prayer to begin class.
- ✓ Put yourself in a group of 4 or 5. You will complete this worksheet in your notes. These three pages must be completed for Monday's class.
- ✓ Begin by sharing your answers to the prep and discussing any problems you were not able to figure out. Ask your group members for assistance. (Spend no more than 10-15 minutes on this.)
  - Answers to the Prep Assignment are:

1. $\frac{99}{70}$	2. $\frac{5x+6}{x(x+2)}$	3. $\frac{-2x^3+21x^2-9x-80}{(x-3)(x+4)(x-2)}$	4. $(x-5)(x+2)$	5. $y(x-2)^2$
6. $(3x+1)(x-2)$	7. $x+1$	8. $2x+x^2$	9. x	10. $2x^2-1$

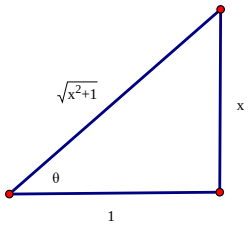
- ✓ The following are the objectives for Section 6.1:
  - Know the identities for section 6.1 found on the Trigonometric Identities sheet. Most of them you already know, but you will also go through the derivations of the new ones so that you understand why they work the way they do.
  - Simplify trigonometric expressions using the different identities and algebraic techniques (e.g., common denominator, factor).
  - Rewriting trigonometric expressions to find equivalent expressions.
  - Determine if a trigonometric function is even or odd. This is a review of previous topics (learned in the first unit and there is a video in the resources on I-Learn) but seen with the trigonometric functions.
- ✓ Review the Definition Identities, Reciprocal Identities, Pythagorean Identities, and Even and Odd Identities on the identity sheet.
  - Please note that the identities that have the symbol ▪ next to it must be memorized. For this section you need to have 15 identities memorized, but you already know 11 of them!
  - Use the powerpoint (posted on I-Learn under resources) to learn about the derivation of the Pythagorean Identities and the Even and Odd Identities. (You do not need to go through the derivations of the Double Angle Identities at this time.) Go through the powerpoint in presentation mode and answer the questions as you go.
- ✓ Simplify the following trigonometric expressions. Remember that you are not solving, just using identities to simplify. Use the algebraic simplification techniques that you used in your prep assignment if needed. Be sure to show all of your work. (Please do in your notebook.)

Questions			
$\frac{\tan(x)}{\sec(x)}$	$\sin(x)+\cot(x)\cos(x)$	$\sin(-x)\cot(-x)$	$\frac{1}{1+\cos(-x)}+\frac{1}{1-\cos x}$
Answers			
$\sin(x)$	$\csc(x)$	$\cos(x)$	$2\csc^2(x)$

- ✓ A big part of this course is to recognize equivalent forms of equations or expressions and determine which is best for the given situation. You did this previously when asked to work with the quadratic function in the standard form versus the vertex form. All though you could use either form, sometimes one form was better than the other. The concept of an identity is that no matter what the input is, you will get the same output using equivalent forms. In terms of the Trigonometry

Identities, this means that given $\sin^2(x) + \cos^2(x) = 1$, I can put any angle in for x and the answer will always be 1. But, sometimes this identity is more useful if it is written as $\sin^2(x) = 1 - \cos^2(x)$ (or some other form). It is the same identity but written in a more useful manner. This next part begins to address the concept of equivalent forms.

- Example 1: Given $\sin(\tan^{-1}(x))$ find an equivalent expression given $x > 0$. If you look at a similar example in the book or help me solve on MML you will find one way to do this problem using identities. But, this question is used in Calculus in a different way. I am going to walk you through this alternative way, but it will be your choice as to which way you do the problem.
  - Begin by focusing on $\tan^{-1}(x)$. In this expression what do you input? What is the output? (input a side length and output an angle)
  - This means that you can write $\tan^{-1}(x) = \theta$ which means $\tan(\theta) = x$. Draw a right triangle and label the given information based on this equation. Remember that $\tan(\theta) = \frac{opp}{hyp}$ from $S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$. It may help to see $\tan(\theta) = x$ as $\tan(\theta) = \frac{x}{1}$
    - Draw your triangle and label
  - Find the 3rd side of the triangle using Pythagorean's Theorem.
  - Now go back to the original problem of $\sin(\tan^{-1}(x))$. If $\tan^{-1}(x)$ inputs a side length, x, and outputs an angle, θ, and $\tan^{-1}(x) = \theta$ then $\sin(\tan^{-1}(x)) = \sin(\theta)$. Using your triangle find $\sin(\theta)$ and thus you have found $\sin(\tan^{-1}(x)) = \frac{x}{\sqrt{x^2+1}}$. Because $x > 0$ this means that we could be looking in the 1st or 4th quadrant. But, $\tan^{-1}(x)$ when $x > 0$ puts you in the 1st quadrant. Thus $\sin(\tan^{-1}(x))$ is a positive value and not a negative value.
    - Your picture should look similar to this:



- Example 2: Given that $\tan(x) = \frac{2}{3}$ and x is in Quadrant III, find the other 5 trigonometric ratios. Again, you can use different identities to determine this (for example you could use the Reciprocal and Pythagorean Identities). Or, you can draw a picture, in the right quadrant, and use what you know about right triangles to determine the other trig ratios.
  - Answers: $\cot(x) = \frac{3}{2}, \sec(x) = -\frac{\sqrt{13}}{3}, \cos(x) = -\frac{3}{\sqrt{13}},\ \sin(x) = -\frac{2}{\sqrt{13}},\ \csc(x) = -\frac{\sqrt{13}}{2}$

✓ Determine if a function is even or odd. In the powerpoint you reviewed how to determine if a function is even or odd and the derivation of the Even and Odd Identities. Now we can use those identities to determine if a function is even, odd or neither. Remember that a function can have symmetry, but not be an even function (symmetry over the y-axis) or an odd function (rotational symmetry about the origin).

- Determine if the following functions are even, odd, or neither.

$f(x) = \frac{\cos(2x)}{x}$	$f(x) = \sin(x) + \cos(x)$

  - The answers are: a. $f(x)$ is an odd function and b. $f(x)$ is neither odd or even

Math 109—Friday June 22, 2018—Section 6.2

- ✓ Adam please say prayer to start class.
- ✓ Put yourselves back into your groups from yesterday.
- ✓ Finish up any questions from Section 6.1, and ask Brooke any questions that need to be clarified. Spend about 30 minutes working on this.
- ✓ The following are the objectives for Section 6.2
  - Do algebraic simplification of trigonometric expressions.
  - Verify trigonometric identities.
- ✓ Answer the following questions. Once again think about the algebraic simplification techniques that you reviewed and discussed in your prep assignment from yesterday.

Multiply:

1. $(1+\tan x)(1-\tan x)$
2. $(2\sin\alpha - 1)^2$

Factor:

1. $\sin^2\beta + \sin\beta - 2$
2. $\sec^2 x - \cot^2 x$
3. $\cos^2\theta\tan\theta - 2\cos\theta\tan\theta - 3\tan\theta$
4. $\sin^4\alpha - \cos^4\alpha$
5. $1 - \csc^2 x$

- ✓ On the Trigonometric Identities read and discuss the strategies for verifying identities. The idea of verifying identities is not solving. You are working towards equivalent expressions—you just happen to know what equivalent expression you are trying to get to! Please note that you are only allowed to work one side because you are striving to write one side of the equation to look like the other.
- ✓ As a group, complete the following verification problems. Do your best and bring your results to share in class on Monday.

$1+\sec x\sin x\tan x = \sec^2 x$	$\cot(x)\sin(x) - \cos^2(x)\sec(x) = 0$	$\tan^2 x = \dfrac{1-\sin^2 x\csc^2 x + \sin^2 x}{\cos^2 x}$

- ✓ Please work on your Section 6.1 and 6.2 homework in MML.
- ✓ Thank you for working hard while I have been gone. On Monday we will spend a few minutes reviewing/clarifying before we move on with more verification problems.