Section 3.3

3. $3x_1 - 2x_2 = 3$
$-4x_1 + 6x_2 = -5$

$$x_1 = \frac{\begin{vmatrix} 3 & -2 \\ -5 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -4 & 6 \end{vmatrix}} = \frac{8}{10} = \frac{4}{5} \qquad x_2 = \frac{\begin{vmatrix} 3 & 3 \\ -4 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -4 & 6 \end{vmatrix}} = \frac{-3}{10}$$

4. $-5x_1 + 2x_2 = 9$
$3x_1 - x_2 = -4$

$$x_1 = \frac{\begin{vmatrix} 9 & 2 \\ -4 & -1 \end{vmatrix}}{\begin{vmatrix} -5 & 2 \\ 3 & -1 \end{vmatrix}} = \frac{-1}{-1} = 1 \qquad x_2 = \frac{\begin{vmatrix} -5 & 9 \\ 3 & -4 \end{vmatrix}}{\begin{vmatrix} -5 & 2 \\ 3 & -1 \end{vmatrix}} = \frac{-7}{-1} = 7$$

11. $$\text{Cof } A = \begin{bmatrix} 0 & -5 & 5 \\ 1 & -1 & 2 \\ 0 & -5 & 10 \end{bmatrix} \qquad \text{adj } A = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -1 & -5 \\ 5 & 2 & 10 \end{bmatrix}$$

By Theorem 8,

$$A^{-1} = \frac{1}{\det A} \text{adj } A.$$

$$\det A = 0 \cdot 0 + -2 \cdot -5 + -1 \cdot 5 = 5$$
(using cofactor expansion along 1^{st} row.)

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 0 & 1 & 0 \\ -5 & -1 & -5 \\ 5 & 2 & 10 \end{bmatrix}$$

14.

$$\text{cof } A = \begin{bmatrix} 8 & 2 & -4 \\ 4 & 0 & -2 \\ -5 & -1 & 2 \end{bmatrix} \qquad \text{adj } A = \begin{bmatrix} 8 & 4 & -5 \\ 2 & 0 & -1 \\ -4 & -2 & 2 \end{bmatrix}$$

$$\det A = 1\cdot 8 + -1\cdot 2 + 2\cdot -4 = -2$$

(cofactor expansion along 1st row)

By Theorem 8,

$$A^{-1} = \frac{-1}{2}\begin{bmatrix} 8 & 4 & -5 \\ 2 & 0 & -1 \\ -4 & -2 & 2 \end{bmatrix}$$

18. Theorem 8 states that

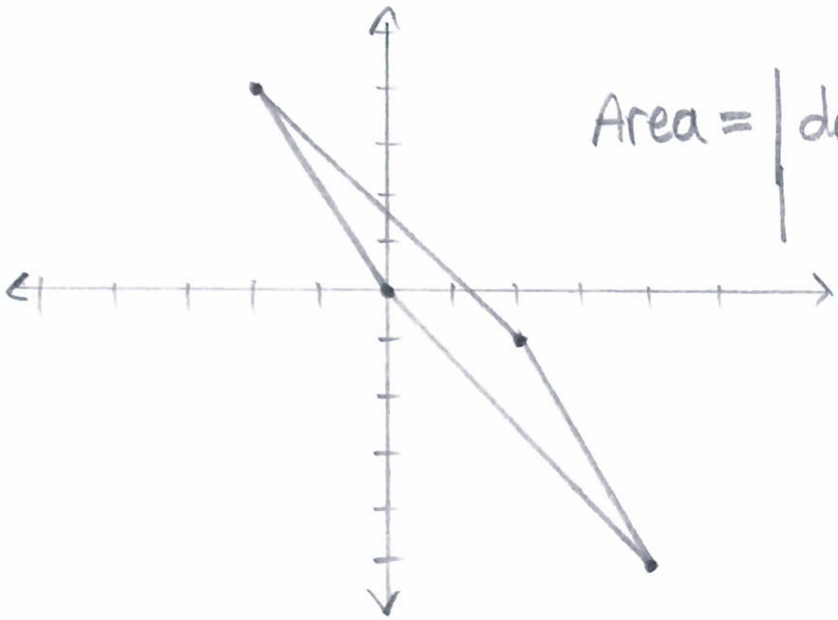
$$A^{-1} = \frac{1}{\det A}\text{adj } A.$$ If $\det A = 1$,

then $A^{-1} = \text{adj } A$.

Note that determinants of matrices with integer entries are integers.

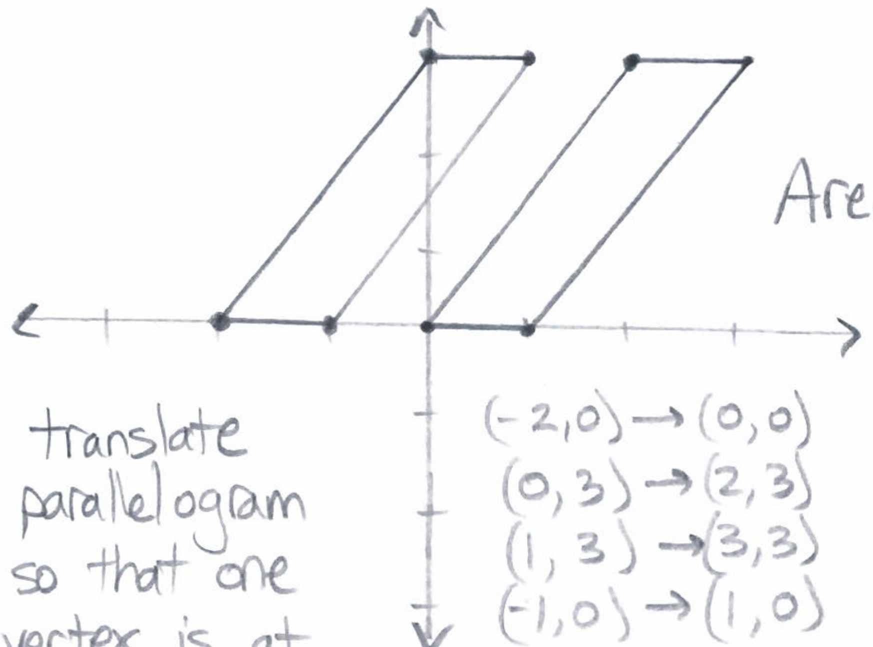
The adjugate matrix has cofactors as entries, and cofactors are determinants. Thus the adjugate matrix of A will be an integer matrix. Therefore A^{-1} will have integer entries.

20.



$$\text{Area} = \left| \det \begin{bmatrix} -2 & 4 \\ 4 & -5 \end{bmatrix} \right| = |-6| = 6$$

21.



$$\text{Area} = \left| \det \begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix} \right|$$

$$= |-3| = 3$$

translate parallelogram so that one vertex is at the origin

$(-2,0) \to (0,0)$
$(0,3) \to (2,3)$
$(1,3) \to (3,3)$
$(-1,0) \to (1,0)$

29. vertices of Δ are $\vec{o}, \vec{v}_1, \vec{v}_2$.

A triangle is half a parallelogram.

$$\text{Area} = \frac{1}{2} \left| \det [\vec{v}_1 \; \vec{v}_2] \right|$$

30. Vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

Translate to origin by subtracting (x_1, y_1) from each point.

$$(x_1, y_1) - (x_1, y_1) = (0, 0)$$
$$(x_2, y_2) - (x_1, y_1) = (x_2 - x_1, y_2 - y_1)$$
$$(x_3, y_3) - (x_1, y_1) = (x_3 - x_1, y_3 - y_1)$$

By problem 29, the area of the triangle is

$$\frac{1}{2} \left| \det \begin{bmatrix} x_2 - x_1 & x_3 - x_1 \\ y_2 - y_1 & y_3 - y_1 \end{bmatrix} \right|.$$

Now consider

$$\frac{1}{2} \left| \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \right| = \frac{1}{2} \left| \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 - x_1 & y_2 - y_1 & 0 \\ x_3 - x_1 & y_3 - y_1 & 0 \end{bmatrix} \right|$$

Using cofactor expansion along 3rd column we have

$$= \frac{1}{2} \left| \det \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{bmatrix} \right|$$

$$= \frac{1}{2} \left| \det \begin{bmatrix} x_2 - x_1 & x_3 - x_1 \\ y_2 - y_1 & y_3 - y_1 \end{bmatrix} \right| \quad \text{since } \det A = \det A^T$$

We have shown earlier that the above is equal to the area of the triangle.
(Note the problem should be the absolute value of the determinant.)