Math 109: Arithmetic Review

Precalculus focuses on college algebra and trigonometry. As you learn different concepts in algebra and trigonometry you will use arithmetic skills. This document helps you review some key arithmetic concepts that you must know. Complete the practice problems to ensure that you will be ready to use these skills as they are needed during the semester.

Example 1—Finding a common denominator

In order to add and subtract fractions, all fractions must have a common denominator. To find a common denominator (but not necessarily the least common denominator) you can multiply the denominators together.

$$\frac{5}{8}-\frac{4}{7}$$

The common denominator is $8*7=56$. Now you need to multiply by a "form of one" so that the fractions have the common denominator. A "form of one" is any fraction that is equal to 1 (below we use $\frac{7}{7}$ and $\frac{8}{8}$).

$$\left(\frac{7}{7}\right)\left(\frac{5}{8}\right)-\left(\frac{4}{7}\right)\left(\frac{8}{8}\right)$$
$$=\frac{35}{56}-\frac{32}{56}$$

Now that the fractions have a common denominator we can subtract the fractions, by subtracting the numerators, and simplify the result.

$$\frac{3}{56}$$

This idea can be extended to fractions with algebraic expressions in the numerator and/or the denominator.

In order to multiply and divide fractions you do not need a common denominator.
To multiply fractions: multiply the numerators and multiply the denominators.
To divide fractions: multiply the 1st fraction by the reciprocal of the 2nd fraction.

Example 2—Finding a common denominator with algebraic expressions

$$\frac{x-2}{x+1}-\frac{x-3}{x+4}$$

Once again, the common denominator can be found by multiplying the denominators together:
$(x+1)(x+4)=x^2+5x+4$

We now multiply each factor by a "form of one" (below we use $\left(\frac{x+4}{x+4}\right)$ and $\left(\frac{x+1}{x+1}\right)$) so that the fraction will have the common denominator:

$$\left(\frac{x+4}{x+4}\right)\left(\frac{x-2}{x+1}\right)-\left(\frac{x-3}{x+4}\right)\left(\frac{x+1}{x+1}\right)$$

Notice that we multiplied each fraction by a different "form of one" so that the denominators are the same. We multiply the numerators and denominators:

$$\frac{x^2+2x-8}{x^2+5x+4}-\frac{x^2-2x-3}{x^2+5x+4}$$

We now can subtract the fractions. Remember that the minus must be distributed to each term in the numerator of the second fraction:

$$\frac{(x^2 + 2x - 8) - (x^2 - 2x - 3)}{x^2 + 5x + 4}$$
$$= \frac{4x - 5}{x^2 + 5x + 4}$$

Example 3—Expanding Polynomials

When we expand polynomials we perform multiplication through distribution or the FOIL method. Remember that x^3 really means $x * x * x$. So in expanding polynomials we apply this rule:

$$(x + 2)^2$$
$$= (x + 2)(x + 2)$$
$$= x^2 + 2x + 2x + 4$$
$$= x^2 + 4x + 4$$

Example 4—Simplifying through the Order of Operations

When we simplify an algebraic expression it is important to remember the order of operations. First we compute anything inside parentheses. Next we simplify anything with an exponent. Third in the order of mathematical operation is multiplication and division, which ever operation comes first from left to right in the equation. Finally we complete any addition or subtraction, again completing the operations from left to right in the equation.

$$5 - 2 * 3^2 + 7 - 18 \div 3 - 2^2$$
$$5 - 2 * 9 + 7 - 18 \div 3 - 4$$
$$5 - 18 + 7 - 6 - 4$$
$$-16$$

Example 5—Quadratic Equations

This is not an arithmetic concept, but I would like you to review the quadratic equation in preparation for our first class. The quadratic equation is an equation with a squared variable:

$$y = ax^2 + bx + c$$

There are a few different ways that we can solve this equation. We set y equal to 0 (always!) and then we can factor, use the quadratic formula, the square root property, or complete the square (which we will learn more about later in the semester). The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving a quadratic equation using factoring:

$$x^2 - 3x = -4$$
$$x^2 - 3x + 4 = 0$$
$$(x - 4)(x + 1) = 0$$
$$x = 4, -1$$

Solving a quadratic equation using the quadratic formula:

$$3x^2 + x - 5 = 0$$

I do not see the factors as easily, so I will use the quadratic formula:

$$x = \frac{-1 \pm \sqrt{1 - 4(3)(-5)}}{2 * 3}$$
$$x = \frac{-1 \pm \sqrt{61}}{6}$$

This does not simplify, but I can find decimal approximation values:

$$x = 1.135, -1.468$$

Homework Problems

Please try the following problems before our first class meeting.

1. Solve: $4x^2 + 12x = -9$
2. Solve: $x^2 - 3x - 10 = 0$
3. Expand: $(x - 1)^3$
4. Add: $\frac{x+3}{x-1} + \frac{x}{x+2}$
5. Subtract: $\frac{x^2+3}{x+2} - \frac{3x^2-2x+1}{x+3}$