Solve

① $2\sin(2x)+\sqrt{3}=0$

$\sin(2x)=-\sqrt{3}/2$

$2x=\sin^{-1}(-\sqrt{3}/2)$

$2x=4\pi/3,\ 5\pi/3$

$[0,2\pi) \Rightarrow 2\pi/3,\ 5\pi/6$

All $\Rightarrow \{x \mid x=2\pi/3+\pi k\}$

$\{x \mid x=5\pi/6+\pi k\}$

② $(\sec x-\sqrt{2})(\sqrt{3}\sec x+2)=0$

$\sec x-\sqrt{2}=0 \qquad \sqrt{3}\sec x+2=0$

$\sec x=\sqrt{2} \qquad \sec x=-2/\sqrt{3}$

$[0,2\pi) \Rightarrow x=\pi/4,\ 7\pi/4 \qquad x=5\pi/6,\ 7\pi/6$

All $\Rightarrow \{x \mid x=\pi/4+2\pi k\}$ & $\{x \mid x=7\pi/4+2\pi k\}$

$\{x \mid x=5\pi/6+2\pi k\}$ & $\{x \mid x=7\pi/6+2\pi k\}$

③ $\cos(2x)+\sin^2x=0$

$1-2\sin^2x+\sin^2x=0$

$1=\sin^2x$

$\sin x=\pm 1$

$[0,2\pi) \Rightarrow x=\pi/2,\ 3\pi/2$

All $\Rightarrow \{x \mid x=\pi/2+\pi k\}$

④ $\tan(3x)=-\sqrt{3}$

$3x=\tan^{-1}(-\sqrt{3})$

$3x=2\pi/3,\ 5\pi/3$

$[0,2\pi) \Rightarrow 2\pi/9,\ 5\pi/9$

All $\Rightarrow \{x \mid x=2\pi/9+\pi/3\,k\}$

⑤ $3\sec^2x\tan x=4\tan x$

$3\sec^2x\tan x-4\tan x=0$

$\tan x(3\sec^2x-4)=0$

$\tan x=0 \qquad 3\sec^2x-4=0$

$\sec^2x=4/3$

$\sec x=\pm 2/\sqrt{3}$

$[0,2\pi) \Rightarrow 0,\ \pi,\ \pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6$

All $\Rightarrow \{x \mid x=\pi k\}$

$\{x \mid x=\pi/6+\pi k\}$

$\{x \mid x=5\pi/6+\pi k\}$

Verify

(1) $\cos(3x) = \cos x\,(1-4\sin^2 x)$

$\cos(x+2x) =$

$\cos x\cos(2x) - \sin x\sin(2x) =$

$\cos x(1-2\sin^2 x) - \sin x(2\sin x\cos x) =$

$\cos x - 2\cos x\sin^2 x - 2\cos x\sin^2 x =$

$\cos x(1-4\sin^2 x) =$

(2) $\tan^2 x = \dfrac{1-\cos(2x)}{1+\cos(2x)}$

$= \dfrac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)}$

$= \dfrac{\cancel{1}-\cancel{1}+2\sin^2 x}{\cancel{1}+2\cos^2 x-\cancel{1}}$

$= \dfrac{2\sin^2 x}{2\cos^2 x}$

$= \tan^2 x$

(3) $\cot(-x) = \dfrac{1-\sin^2 x}{\cos(-x)\sin(-x)}$

$= \dfrac{\cos^{\cancel{2}} x}{\cancel{\cos x}\sin(-x)}$

$= \dfrac{\cos x}{\sin(-x)}$

$= \dfrac{\cos(-x)}{\sin(-x)}$

$= \cot(-x)$

(4) $\tan^2 x = \dfrac{1-\sin^2 x\csc^2 x+\sin^2 x}{\cos^2 x}$

$= \dfrac{\cancel{1}-\cancel{1}+\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

(5) $\dfrac{\cos(x+y)}{\cos x\cos y} = 1-\tan x\tan y$

$\dfrac{\cos x\cos y + \sin x\sin y}{\cos x\cos y} =$

$\dfrac{\cos x\cos y}{\cos x\cos y} + \dfrac{\sin x\sin y}{\cos x\cos y} =$

$1+\tan x\tan y =$

(6) $\dfrac{\cos(2x)}{\sin^2 x} = \csc^2 x - 2$

$\dfrac{1-2\sin^2 x}{\sin^2 x} =$

$\dfrac{1}{\sin^2 x} - \dfrac{2\sin^2 x}{\sin^2 x} =$

$\csc^2 x - 2 =$

(7) $\dfrac{\cos(x+y)}{\sin(x-y)} = \dfrac{1-\tan x\tan y}{\tan x-\tan y}$

$\left(\dfrac{\cos(x)\cos y-\sin x\sin y}{\sin x\cos y-\cos x\sin y}\right)\left(\dfrac{\frac{1}{\cos x\cos y}}{\frac{1}{\cos x\cos y}}\right) =$

$\dfrac{1-\tan x\tan y}{\tan x-\tan y} =$

(8) $\dfrac{1}{\csc x-\cot x} = \dfrac{1+\cos x}{\sin x}$

$\dfrac{1}{\frac{1}{\sin x}-\frac{\cos x}{\sin x}} =$

$\dfrac{1}{\frac{1-\cos x}{\sin x}} =$

$\left(\dfrac{1+\cos x}{1+\cos x}\right)\dfrac{\sin x}{1-\cos x} =$

$\dfrac{\sin x(1+\cos x)}{1-\cos^2 x} =$

$\dfrac{\cancel{\sin x}(1+\cos x)}{\sin^{\cancel{2}} x}$

(9) $\sec^2 x = \dfrac{\csc x}{\csc x-\sin x}\left(\dfrac{\sin x}{\sin x}\right)$

$= \dfrac{1}{1-\sin^2 x}$

$= \dfrac{1}{\cos^2 x}$

$= \sec^2 x$

(10) $\left(\dfrac{\sec x}{\sec x}\right)\dfrac{\sec x}{\sec x\tan x} - \dfrac{\tan x}{\sec x}\left(\dfrac{\tan x}{\tan x}\right) = \cos x\cot x$

$\dfrac{\sec^2 x-\tan^2 x}{\sec x\tan x} =$

$\dfrac{1}{\sec x\tan x} =$

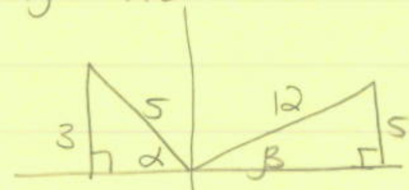
$\dfrac{1}{\sec x}\cdot\dfrac{1}{\tan x} =$

$\cos x\cot x =$

Simplify or Solve

1) $\sin(\alpha+\beta)$ given $\sin\alpha = 3/5$ II & $\sin\beta = 5/12$ I

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{3}{5}\right)\left(\frac{\sqrt{119}}{12}\right) + \left(\frac{-4}{5}\right)\left(\frac{5}{12}\right)$$
$$= \frac{3\sqrt{119}}{60} - \frac{20}{60}$$



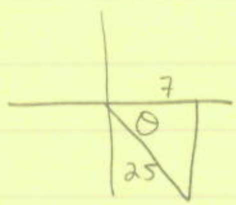
$3^2+b^2=5^2$
$b=4$

$5^2+a^2=12^2$
$a=\sqrt{119}$

$$\sin(\alpha+\beta) = \frac{3\sqrt{119}-20}{60}$$

2) $\sin\left(\frac{\theta}{2}\right) = \frac{3}{5}$ & $\frac{3\pi}{4} < \frac{\theta}{2} < \pi \Rightarrow \frac{3\pi}{2} < \theta < 2\pi$

$$\left(\pm\sqrt{\frac{1-\cos\theta}{2}} = \frac{3}{5}\right)^2$$
$$\frac{1-\cos\theta}{2} = \frac{9}{25}$$
$$1-\cos\theta = \frac{18}{25}$$
$$\cos\theta = \frac{7}{25}$$
$$\sec\theta = \frac{25}{7}$$



$7^2+b^2=25^2$
$b=24$

$\sin\theta = -24/25$
$\csc\theta = -25/24$
$\tan\theta = -25/7$
$\cot\theta = -7/25$

3) Factor $\cos^2x\tan x - 2\cos x\tan x - 3\tan x$
$$\tan x(\cos^2x - 2\cos x - 3)$$
$$\tan x(\cos x - 3)(\cos x + 1)$$

4) $f(x) = \sin x\cos x$
$$f(-x) = \sin(-x)\cos(-x)$$
$$= -\sin x\cos x$$
$$-f(x) = -\sin x\cos x$$

Does $f(x) = f(-x) \rightarrow$ No, then not even
Does $f(-x) = -f(x) \rightarrow$ Yes, then odd