

① Definition $y = 3x^2 - 2x + 4$

$$y' = \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 2(x+h) + 4 - 3x^2 + 2x - 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 4 - 3x^2 + 2x - 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6xh + 3h^2 - 2h}{h}$$

$$\boxed{y' = 6x - 2}$$

② $y = \frac{1}{\sqrt{x} - 1}$

$$y' = \frac{(\sqrt{x} - 1)(0) - 1\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x} - 1)^2}$$

$$= \frac{-\frac{1}{2\sqrt{x}}}{(\sqrt{x} - 1)^2}$$

$$\boxed{y' = \frac{-1}{2\sqrt{x}(\sqrt{x} - 1)^2}}$$

③ $y = x^7 + \sqrt{7}x - \frac{1}{1+\pi}$

$$\boxed{y' = 7x^6 + \sqrt{7}}$$

④ $y = x^2 \cot(5x)$

$$y' = 2x \cot(5x) + x^2(-\csc^2(5x) \cdot 5)$$

$$\boxed{y' = 2x \cot(5x) - 5x^2 \csc^2(5x)}$$

⑤ $y = 2^{3x^2}$

$$\boxed{y' = 2^{3x^2} (\ln 2)(6x)}$$

⑥ $y = \ln(\sec(x))$

$$y' = \frac{\sec x \tan x}{\sec x}$$

$$\boxed{y' = \tan x}$$

$$\begin{aligned} 7) \quad x^2 y^2 &= e^{2x} + y \\ 2xy^2 + x^2(2y \frac{dy}{dx}) &= 2e^{2x} + \frac{dy}{dx} \\ 2x^2 y \frac{dy}{dx} - \frac{dy}{dx} &= 2e^{2x} - 2xy^2 \\ \frac{dy}{dx} &= \frac{2e^{2x} - 2xy^2}{2x^2 y - 1} \end{aligned}$$

$$\begin{aligned} 8) \quad y &= (1+x^2)e^{\tan^{-1}x} \\ \ln y &= \ln[(1+x^2)e^{\tan^{-1}x}] \\ \ln y &= \ln(1+x^2) + \tan^{-1}x \end{aligned}$$

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \frac{2x}{1+x^2} + \frac{1}{1+x^2} \\ \frac{dy}{dx} &= \left(\frac{2x+1}{1+x^2} \right) (1+x^2)e^{\tan^{-1}x} \\ y' &= (2x+1)e^{\tan^{-1}x} \end{aligned}$$

$$\begin{aligned} y &= (1+x^2)e^{\tan^{-1}x} \\ y' &= (1+x^2)e^{\tan^{-1}x} \left(\frac{1}{1+x^2} \right) + 2xe^{\tan^{-1}x} \\ \text{OR } y' &= (1+2x)e^{\tan^{-1}x} \end{aligned}$$

$$\begin{aligned} 9) \quad y &= \log_2 \left(\frac{x^2}{2} \right) \\ y' &= \frac{x}{\left(\frac{x^2}{2} \right) \ln 2} \end{aligned}$$

$$y' = \frac{2}{x \ln 2}$$

$$\begin{aligned} 10) \quad y &= \left(\frac{1+\sin x}{1-\cos x} \right)^2 \\ y' &= 2 \left(\frac{1+\sin x}{1-\cos x} \right) \left[\frac{(1-\cos x)(\cos x) - (1+\sin x)(-\sin x)}{(1-\cos x)^2} \right] \\ y' &= 2(1+\sin x) \frac{(\cos x - \cos^2 x + \sin x + \sin^2 x)}{(1-\cos x)^3} \end{aligned}$$

[OR]

$$\ln y = 2 [\ln(1+\sin x) - \ln(1-\cos x)]$$

$$\frac{1}{y} \frac{dy}{dx} = 2 \left[\frac{\cos x}{1+\sin x} - \frac{\sin x}{1-\cos x} \right]$$

$$\frac{dy}{dx} = 2 \left[\frac{\cos x}{1+\sin x} - \frac{\sin x}{1-\cos x} \right] \left(\frac{1+\sin x}{1-\cos x} \right)^2$$

$$(11) \log_4(3x^2+3) + 4xy = \cos(3y^2) - 7^x$$

$$\frac{6x}{(3x^2+3)\ln 4} + 4y + 4x \frac{dy}{dx} = -\sin(3y^2) - 6y \frac{dy}{dx} - 7^x \ln 7$$

$$4x \frac{dy}{dx} + 6y \sin(3y^2) \frac{dy}{dx} = -7^x \ln 7 - 4y - \frac{6x}{(3x^2+3)\ln 4}$$

$$\boxed{\frac{dy}{dx} = \frac{-7^x \ln 7 - 4y - \frac{6x}{(3x^2+3)\ln 4}}{4x + 6y \sin(3y^2)}}$$

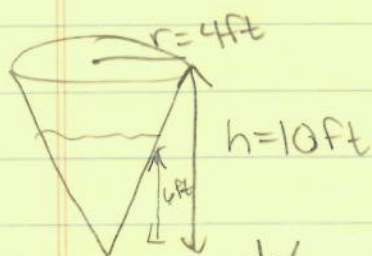
$$(12) y = 3 \sqrt{\frac{x(x+1)^2(x-2)}{5(x^2+1)(2x+3)^3}}$$

$$\ln y = \frac{1}{3} [\ln x + 2\ln(x+1) + \ln(x-2) - \ln 5 - \ln(x^2+1) - 3\ln(2x+3)]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{3} \left[\frac{1}{x} + \frac{2}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{6}{2x+3} \right]$$

$$\boxed{\frac{dy}{dx} = \frac{1}{3} \left[\frac{1}{x} + \frac{2}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{6}{2x+3} \right] \cdot 3 \sqrt{\frac{x(x+1)^2(x-2)}{5(x^2+1)(2x+3)^3}}}$$

(13)



$$\frac{dh}{dt} = ?$$

$$\frac{dV}{dt} = -5 \text{ ft}^3/\text{min}$$

$$\frac{r}{h} = \frac{4}{10}$$

$$r = \frac{2}{5} h$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{2}{5} h\right)^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{4}{25} h^2\right) h$$

$$V = \frac{4}{75} \pi h^3$$

$$\frac{dV}{dt} = \frac{4}{25} \pi h^2 \frac{dh}{dt}$$

$$-5 = \frac{4}{25} \pi (6)^2 \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = -27.63 \text{ ft}/\text{min}}$$

(14) Eqn of tangent line

$$y = 1 + \cos x$$

$$\frac{dy}{dx} = -\sin x$$

$$\left. \frac{dy}{dx} \right|_{\pi/2} = -1$$

$$y - 1 = -1(x - \pi/2)$$

$$y = -x + \pi/2 + 1$$