**Test #3 Review**
**(Covers 4.1-4.8, 5.1-5.4)**

**Sec 4.1:** Extreme Values of Functions
- ✓ Know the difference between the global extrema and local extrema.
- ✓ Know how to find extrema
- ✓ Don't forget to check the endpoints (if there are any) for possible extrema
- ✓ Make sure the critical values are in the interval and domain of f(x)
- ✓ It is possible to have multiple max/mins if they are the same value
- ✓ Yellow worksheet
- ✓ The Extreme Value Theorem
- ✓ Definition of a critical number

*Problems to try:*

1. Find the absolute maximum and absolute minimum values of $f$ over the interval.
   a. $f(x) = x - 2\ln(x), 1 \le x \le 3$
   b. $f(x) = \frac{4}{x} + \ln(x^2), 1 \le x \le 4$

**Sec 4.2:** The Mean Value Theorem
- ✓ Know the Mean Value Theorem
- ✓ Know the Intermediate Value Theorem from Sec 2.6
- ✓ Find c values that satisfy the Mean Value Theorem

*Problems to Try:*

2. If g(x) is differentiable for all x, and g(0) = 0 and g(2) = 6, then there exists a c on the interval (0,2) where g'(c) =
3. From the book:
   a. Find the value or values of c that satisfy the equation
   $$\frac{f(b) - f(a)}{b - a} = f'(c)$$
   in the conclusion of the Mean Value Theorem for the functions and intervals in the following problems
      a. $f(x) = x^2 + 2x - 1. [0,1]$
      b. $f(x) = x^{\frac{2}{3}}, [0,1]$
      c. $f(x) = \sin^{-1} x. [-1,1]$
      d. $f(x) = x^3 - x^2, [-1,2]$
4. Be able to define the three theorems (Extreme Value, Mean Value, Intermediate Value THM)

**Sec 4.3:** Monotonic Functions and the First Derivative Test
**Sec 4.4:** Concavity and Curve Sketching
- ✓ Know the first and second derivative tests for extrema. Remember that both tests determine local max/min just using different derivatives. KNOW THEM WELL!!!!
- ✓ Use the table and find all the parts: increasing/decreasing intervals, concave up/down intervals, local max, local min, inflection points, vertical and horizontal asymptotes

- ✓ Definitions of concavity and inflection point

*Problems to Try:*

5. Problems from the book:
   a. Graph the curves for the following problems:
      a. $y = -x^3 + 6x^2 - 9x + 3$
      b. $y = \ln(x^2 - 4x + 3)$
      c. $y = \frac{x+1}{x-3}$
6. T or F: If f'(c) = 0 and f''(c) = 0, then f(c) is neither a maximum or minimum.
7. T or F: If f''(x) < 0 when x < c and f''(x) > 0 when x > c then f has an inflection point at c.

**Sec 4.5:** Indeterminate Forms and L'Hopital's Rule

- ✓ Know all the different indeterminate forms and how to get the multiplication, subtraction, and power forms into division form so that you can use L'Hopitals Rule
- ✓ Remember that you always need to show the indeterminate form!
- ✓ Know L'Hopitals Rule

*Problems to Try:*

8. Indicate whether or not the following limits could be evaluated with L'Hopital's Rule, give reasons for your conclusions.

$$\lim_{x\to 1}\frac{\ln x}{\sin(\pi x)}$$

$$\lim_{x\to -\infty}[(e^x - 1)\ln|x|]$$

$$\lim_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{\tan x}\right)$$

$$\lim_{x\to\infty} x^{\cos(1/x)}$$

$$\lim_{x\to\infty} x^3 e^{-x^2}$$

9. From the book:
   a. Find the limits:
      1. $\lim_{x\to 0}\frac{10^x - 1}{x}$
      2. $\lim_{x\to 0}\frac{2^{\sin(x)} - 1}{e^x - 1}$
      3. $\lim_{x\to 0}\frac{5 - 5\cos(x)}{e^x - x - 1}$
      4. $\lim_{x\to 0^+}\frac{t - \ln(1+2t)}{t^2}$
      5. $\lim_{x\to 0^+}\left(\frac{e^t}{t} - \frac{1}{t}\right)$
      6. $\lim_{x\to\infty}\left(1 + \frac{b}{x}\right)^{kx}$

**Sec 4.6:** Applied Optimization Problems

- ✓ Find a function for the quantity that you are trying to optimize
- ✓ Convert the function to a single variable function using other information
- ✓ Find critical points and determine if max/min
- ✓ Use points to find optimal value
- ✓ I would try redoing some of the homework problems to refresh your memory of these types of problems

*Problems to Try:*

10. Problems from the book:
    a. Open top box: An open-top rectangle box is constructed from a 10 in. -by-16 in. piece of cardboard by cutting squares of equal side length from the corners and folding up the sides. Find analytically the dimensions of the box of largest volume and the maximum volume. Support your answers graphically.
    b. The shortest fence: A 216 $m^2$ rectangle plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions.
    c. Designing a tank: Your iron works as contracted to design and build a 500 $ft^3$, square-based, open-top, rectangular steel holding tank for a paper company. The tank is to be made by welding thin stainless-steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weigh as little as possible.
        a. What dimensions do you tell the shop to use?
        b. Briefly describe how you took weight into account.

11. The problems from class, see I-learn site.

**Sec 4.8:** Antiderivatives

- ✓ Finding the original function knowing the rate of change
- ✓ DO NOT FOGET THE C!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
- ✓ Know how to find the general antiderivative and a specific antiderivative given some specific points.
- ✓ Know the general form of the power rule, ln, and how to go back from the basic derivatives.
- ✓ Remember that you can always check yourself by taking the derivative of the antiderivative to get what you started with.

*Problems to Try:*

12. Problems from the book:
    a. Find the indefinite integrals (most general antiderivative). Check your answers by differentiation.
        a. $\int (x^3 + 5x - 7)dx$
        b. $\int \left(\frac{1}{(r+5)^2}\right) dr$
        c. $\int \sec^2 \frac{s}{10} ds$
        d. $\int \csc(\sqrt{2\theta} \cot(\sqrt{2\theta}\, d\boldsymbol{\theta}$
        e. $\int \sin^2 \frac{x}{4} dx$ (Hint: $\sin^2 \theta = \frac{1-\cos(2\boldsymbol{\theta})}{2}$)
13. Find the antiderivatives (these are just a few that could represent what you need to know for the test):$f(x) = -\frac{1}{1+x^2}$, $f(x) = e^{-5x}$, $f(x) = \sec x \tan x$

**Sec 5.1:** Estimating with Finite Sums

- ✓ Understand how rectangles are constructed to estimate the area underneath a curve using either the left hand endpoint, midpoint, or right hand endpoint of the intervals.
- ✓ Understand the derivation for the formulas for $R_n, L_n, \& M_n$ the different ways of estimating area using the sum of n rectangles
- ✓ Yellow worksheet

*Problems to try*:

1. From the book:

   **Length of a road:** You and a companion are about to drive a twisty stretch of dirt road in a car whose speedometer works but whose odometer (mileage counter) is broken. To find out how long this particular stretch of road is, you record the car's velocity at 10-sec intervals, with the results shown in the accompanying table. Estimate the length of the road using

   **a.** Left-endpoint values

   **b.** Right-endpoint values

2. Estimate the integral $\int_{-1}^{4}(x^2+2)dx$ using five intervals and the midpoints of the intervals. Draw in the rectangles on the graph. (Might also want to try using right and left endpoints and discussing which options is the best estimate.)

**Sec 5.2:** Sigma Notation and Limits of Finite Sums

- ✓ Know the different parts of sigma notation
- ✓ Know how to write a sum
- ✓ Know all of the common sums (i.e. the sum of 1, sum of i, etc.)
- ✓ Know how to find the sum with common sums
- ✓ Know how to find the limit of a sum

*Problems to try:*

3. Find the value of the sum $\sum_{i=1}^{n}(3+2i)^2$
4. Write the sum in sigma notation: $2+4+6+8+...+202$

**Sec 5.3:** The Definite Integral

- ✓ Know the definition of the definite integral
- ✓ Understand what the different parts of the definition of the definite integral mean
- ✓ Know the properties of definite integrals
- ✓ Know how to write the integral as a Riemann Sum

- ✓ Know how to evaluate an integral using different area properties (i.e. quarter circles, triangles, etc.)

*Problems to try:*

5. Write the Riemann Sum for an approximation of $\int_3^6 \ln x dx$ using n intervals and right endpts.
6. From the book:
   Suppose that $f$ and $h$ are integrable and that
   $\int_1^9 f(x)dx = -1$ $\int_7^9 f(x)dx = 5$ $\int_7^9 h(x)dx = 4$

**Use the rules satisfied by definite integrals to find**

   a. $\int_1^9 -2f(x)dx$
   b. $\int_7^9 [f(x) + h(x)]dx$
   c. $\int_7^9 [2f(x) - 3h(x)]dx$
   d. $\int_9^1 f(x)dx$
   e. $\int_1^7 f(x)dx$
   f. $\int_9^7 [h(x) - f(x)]dx$

**Sec 5.4:** The Fundamental Theorem of Calculus

- ✓ Know the two parts of the Fundamental Theorem of Calculus (FTC1 and FTC2)
- ✓ Remember that the function must be continuous in order to find the integral

*Problems to try:*

7. Problems from the book:
   Find $\frac{dy}{dx}$ in the following exercises:
   $y = \int_0^x \sqrt{(1+t^2)}dt$

   $y = \int_1^x \frac{1}{t} dt, x > 0$

   $y = \int_{\sqrt{x}}^0 \sin(t^2)\, dt$

14. A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6\ cm/sec$ and its initial displacement is $s(0) = 9\ cm$. Find the position function $s(t)$.