Section 2.7

1. $\begin{bmatrix} 1.25 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Any matrix of the form $\begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$ will have the same effect on homogeneous coordinates for $\mathbb{R}^2$ as A has on the usual coordinates for $\mathbb{R}^2$.

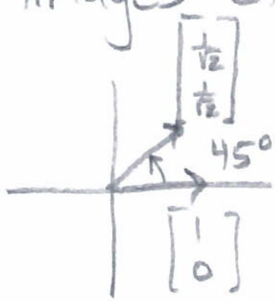
3. Translate by $(3,1)$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

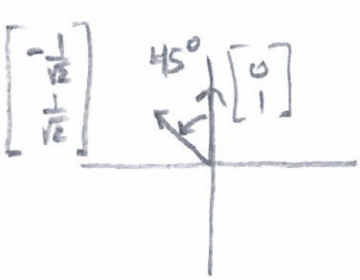
Rotate $45°$ about the origin

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

To find the rotation matrix we can find the images of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ under the rotation.



Since the vector has length 1, the x-coordinate will be $\frac{1}{\sqrt{2}}$ and the y-coordinate will be $\frac{1}{\sqrt{2}}$ as well. (think pythagorean theorem $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1^2$)



Similarly, when $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is rotated $45°$, we come to the point $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

In general a rotation by ϕ degrees counter clockwise can be done by the matrix

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}.$$

To translate first, then rotate, our matrix would be equal to the product of

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \sqrt{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 2\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}.$$

4. Translate by $(-2,3)$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

Scale x by .8 and y by 1.2

$$\begin{bmatrix} .8 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translate first, then scale, yields the product

$$\begin{bmatrix} .8 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .8 & 0 & -1.6 \\ 0 & 1.2 & 3.6 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Reflect through x-axis

$$\begin{bmatrix}1\\0\end{bmatrix} \longrightarrow \begin{bmatrix}1\\0\end{bmatrix}$$

$$\begin{bmatrix}0\\1\end{bmatrix} \longrightarrow \begin{bmatrix}0\\-1\end{bmatrix}$$

Rotate 30° about origin

$$\begin{bmatrix}1\\0\end{bmatrix} \longrightarrow \begin{bmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{bmatrix}$$

$$\begin{bmatrix}0\\1\end{bmatrix} \longrightarrow \begin{bmatrix}-\frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{bmatrix}$$

(use 30°-60°-90° triangles)
(or matrix given in problem 3)

Reflect through x-axis

$$\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Rotate 30° about origin

$$\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0\\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0\\ 0 & 0 & 1\end{bmatrix}$$

To reflect, then rotate, the product is

$$\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0\\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2} & 0\\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0\\ 0 & 0 & 1\end{bmatrix}$$

9. ABD where A is 2×2, B is 2×2, D is 2×200.

Multiplying BD there are 4 multiplications per column of D. Thus BD requires 800 multiplications. $A(BD)$ will require the same number of multiplications since A is 2×2 and BD is 2×200. Thus $A(BD)$ requires a total of 1600 multiplications.

Multiplying AB there are 8 multiplications.

i.e. $$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{bmatrix}$$

Since AB is 2×2 and D is 2×200 $(AB)D$ requires 800 multiplications.

Thus $(AB)D$ requires a total of 808 multiplications.

It seems that the second method would be about twice as fast as the first.