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# Biological reference points and projections

## Black sea bass Research Track

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# Outline

- General description of reference points in multi-stock extension of WHAM (TOR 5)
- BRPs for black sea bass application (TOR 5)
- Description of projections
- Example 3 year projection for black sea bass

# Equilibrium SSB per recruit calculations

The  $n_r \times n_r$  matrix of equilibrium spawning stock biomass per recruit for stock  $s$  in region  $r'$  (columns) given recruiting in region  $r$  (rows) as a function of a fully-selected  $F$ ,  $\tilde{F}$ , is defined as

$$\widetilde{\mathbf{O}}_s(\tilde{F}) = \sum_{a=1}^A \mathbf{O}_{s,a}(\tilde{F}) \mathbf{O}_{s,a}(\tilde{F}, \delta_s) \text{diag}(\mathbf{f}_{s,a})$$

where  $\mathbf{O}_{s,a}(\tilde{F}, \delta_s)$  is  $n_r \times n_r$  is the matrix of probabilities of surviving and occurring in region  $r'$  at age  $a + \delta_s$  given starting in region  $r$  at age  $a$ . It is the upper left sub-matrix of the probability transition matrix defined over the same interval. Similarly,

$$\mathbf{O}_{s,a}(\tilde{F}) = \begin{cases} \prod_{i=0}^{a-1} \mathbf{O}_{s,i}(\tilde{F}) & 1 \leq a < A \\ \left[ \prod_{i=0}^{a-1} \mathbf{O}_{s,i}(\tilde{F}) \right] \mathbf{O}_{s,+}(\tilde{F}) & a = A \end{cases}$$

is the upper left sub-matrix of the probability transition matrix of states at age  $a$  (columns) given being in each state (rows) at recruitment (age 1) where  $\mathbf{O}_{s,0} = \mathbf{I}$ . For the plus group  $a = A$ ,  $\mathbf{O}_{s,+} = (\mathbf{I} - \mathbf{O}_{s,A})^{-1}$ .  $\mathbf{f}_{s,a}$  is the vector of fecundities (product of weight and maturity) at age  $a$  in each region.

# Equilibrium Yield per recruit calculations

The matrix of equilibrium yield per recruit in each fleet (columns) as a function of  $\tilde{F}$  given recruiting in a given region (rows) is calculated as

$$\tilde{\mathbf{Y}}_s(\tilde{F}) = \sum_{a=1}^A \mathbf{O}_{s,a}(\tilde{F}) \mathbf{H}_{s,a}(\tilde{F}) \text{diag}(\mathbf{c}_{s,a})$$

where  $\mathbf{c}_a$  is the vector of catch weight at age for each fleet, and  $\mathbf{H}_{s,a}$  is the submatrix of  $\mathbf{P}_{s,a}$  with the probabilities of being captured in each fleet over the interval from  $a$  to  $a + 1$ .

- Currently, the model cannot distinguish catch weight at age by stock.
  - An empirical catch weight at age for the combined capture of all stocks is used.

# Combining SSB/R across stocks to determine $F_{40}$

- To find  $F_{40}$ , we use a weighted average of stock-specific components of spawning biomass per recruit as a function of fully-selected  $F$ :

$$\tilde{\mathcal{S}}(\tilde{F}) = \sum_{i=1}^{n_S} \omega_i \tilde{\mathcal{O}}_i(\tilde{F}, r = r(s_i), r' = r(s_i))$$

- weights can be user-specified or as a function of average recruitment for the different stocks:

$$\omega_i = \frac{\overline{R}_i}{\sum_{i=1}^{n_S} \overline{R}_i}$$

- $F_{40}$  is the value  $\tilde{F}^*$  such that

$$\tilde{\mathcal{S}}(\tilde{F}^*) = 0.4\tilde{\mathcal{S}}(\tilde{F} = 0)$$

- Same Newton methods as the standard WHAM package are used to solve for  $F_{40}$  internally
  - Internal calculations allow propagation of parameter uncertainty into that of the reference points.

# SSB and Yield at $F_{40}$

- An average recruitment is multiplied with SSB/R and Y/R for stock  $i$

$$\widetilde{\text{SSB}}_i = \overline{R}_i \widetilde{\mathbf{O}}_i \left( \widetilde{F}, r = r(s_i), r' = r(s_i) \right)$$

- For yield from stock  $i$  in fleet  $f$ :

$$\widetilde{Y}_{i,f} = \overline{R}_i \widetilde{\mathbf{Y}}_i \left( \widetilde{F}, r = r(s_i), f \right)$$

- The total yield for fleet  $f$  is just the sum across stocks:

$$\widetilde{Y}_f = \sum_{i=1}^{n_S} \widetilde{Y}_{i,f}$$

- User specifies the years to include for  $\overline{R}_i$ 
  - We used years 2000-2021 for black sea bass as in previous assessments.

# Average FAA for black sea bass BRPs

- All inputs to SSB/R and YPR/R are averaged over last five years
- FAA is averaged by fleet

	1	2	3	4	5	6	7	8+
North_Commercial	0.01	0.04	0.09	0.17	0.17	0.17	0.17	0.17
North_Recreational	0.01	0.09	0.10	0.15	0.26	0.40	0.56	0.56
South_Commercial	0.00	0.02	0.06	0.07	0.07	0.07	0.07	0.07
South_Recreational	0.03	0.08	0.15	0.21	0.25	0.26	0.26	0.26

# Selectivity for black sea bass BRPs

- Average fishing mortality at age and fleet is divided by the maximum of the total average FAA.

1	2	3	4	5	6	7	8+
0.05	0.23	0.4	0.61	0.76	0.91	1.07	1.07

	1	2	3	4	5	6	7	8+
North_Commercial	0.01	0.04	0.08	0.16	0.16	0.16	0.16	0.16
North_Recreational	0.01	0.08	0.09	0.14	0.25	0.38	0.52	0.52
South_Commercial	0.00	0.02	0.06	0.07	0.07	0.07	0.07	0.07
South_Recreational	0.03	0.07	0.14	0.20	0.23	0.24	0.25	0.25



# Weight at age for black sea bass BRPs

- Weight at age for SSB and fleets are averaged over last 5 years

	1	2	3	4	5	6	7	8+
BSB_North	0.11	0.20	0.37	0.54	0.75	0.99	1.19	1.53
BSB_South	0.09	0.19	0.35	0.48	0.63	0.81	0.93	1.50

	1	2	3	4	5	6	7	8+
North_Commercial	0.07	0.17	0.37	0.52	0.71	0.91	1.11	1.46
North_Recreational	0.11	0.20	0.37	0.54	0.75	0.99	1.19	1.53
South_Commercial	0.10	0.16	0.35	0.48	0.66	0.85	1.05	1.36
South_Recreational	0.09	0.19	0.35	0.48	0.63	0.81	0.93	1.50

# Maturity, M, movement at age for black sea bass BRPs

	1	2	3	4	5	6	7	8+
BSB_North	0	0.48	0.98	1.00	1.00	1	1	1
BSB_South	0	0.34	0.82	0.98	0.97	1	1	1

	1	2	3	4	5	6	7	8+
BSB_North	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
BSB_South	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4

	1	2	3	4	5	6	7	8+
North to South	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
South to North	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33

# Unfished equilibrium survival for black sea bass BRPs (North)

$$\mathbf{O}_{s,a} \left( \tilde{F} = 0 \right)$$

Recruits

	North	South
North	1	0
South	0	1

Age 2

	North	South
North	0.64	0.03
South	0.64	0.03

Age 3

	North	South
North	0.43	0.02
South	0.43	0.02

Age 4

	North	South
North	0.29	0.01
South	0.29	0.01

# Unfished equilibrium survival for black sea bass BRPs (North)

$$\mathbf{O}_{s,a} \left( \tilde{F} = 0 \right)$$

Age 5

	North	South
North	0.19	0.01
South	0.19	0.01

Age 6

	North	South
North	0.13	0.01
South	0.13	0.01

Age 7

	North	South
North	0.09	0
South	0.09	0

Age 8+

	North	South
North	0.18	0.01
South	0.18	0.01

# Equilibrium survival at $F_{40}$ for black sea bass BRPs (North)

Recruits

	North	South
North	1	0
South	0	1

Age 2

	North	South
North	0.63	0.03
South	0.63	0.03

Age 3

	North	South
North	0.37	0.02
South	0.37	0.02

Age 4

	North	South
North	0.21	0.01
South	0.21	0.01

# Equilibrium survival at $F_{40}$ for black sea bass BRPs (North)

$$\mathbf{O}_{s,a}(\tilde{F} = 0)$$

Age 5

	North	South
North	0.1	0
South	0.1	0

Age 6

	North	South
North	0.04	0
South	0.04	0

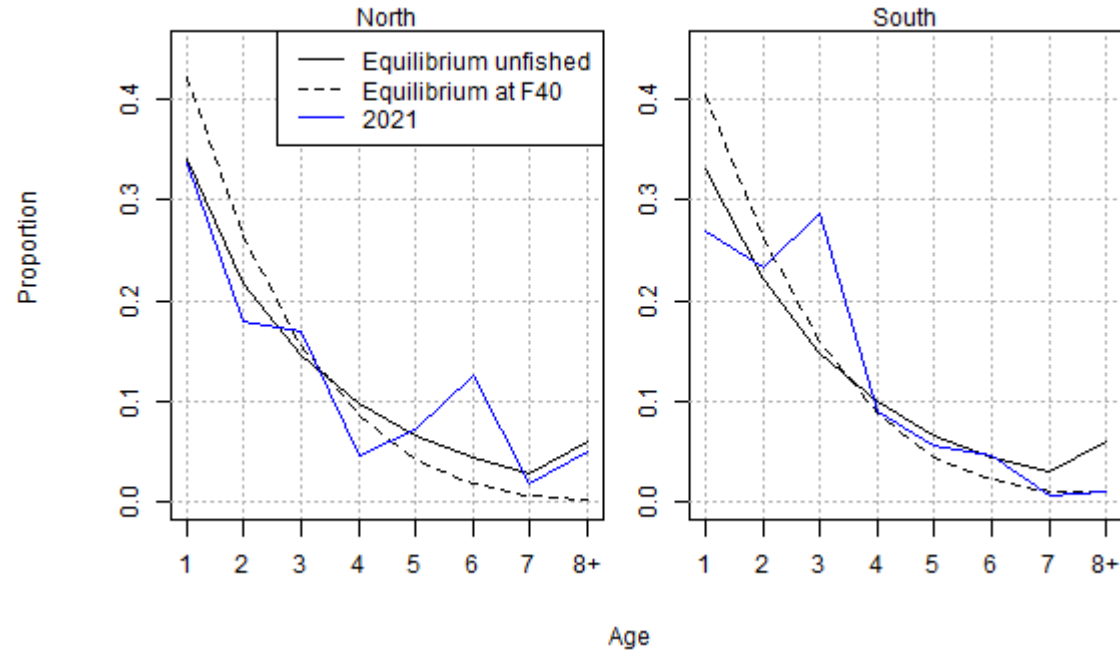
Age 7

	North	South
North	0.02	0
South	0.02	0

Age 8+

	North	South
North	0.01	0
South	0.01	0

# January 1 proportions at age in spawning region



$F_{40}$

- Total FAA (across all 4 fleets) :

1	2	3	4	5	6	7	8+
0.05	0.22	0.39	0.59	0.73	0.88	1.03	1.03

- FAA by region

1	2	3	4	5	6	7	8+
0.02	0.13	0.18	0.32	0.42	0.56	0.71	0.71
0.03	0.09	0.20	0.28	0.31	0.32	0.32	0.32



# $F_{40}$

## ■ FAA by region

1	2	3	4	5	6	7	8+
0.02	0.13	0.18	0.32	0.42	0.56	0.71	0.71
0.03	0.09	0.20	0.28	0.31	0.32	0.32	0.32

## ■ FAA by fleet

1	2	3	4	5	6	7	8+
0.01	0.04	0.09	0.17	0.17	0.17	0.17	0.17
0.01	0.09	0.10	0.15	0.26	0.39	0.54	0.54
0.00	0.02	0.06	0.07	0.07	0.07	0.07	0.07
0.03	0.08	0.15	0.21	0.24	0.25	0.25	0.25

# $F_{40}$

- FAA by fleet

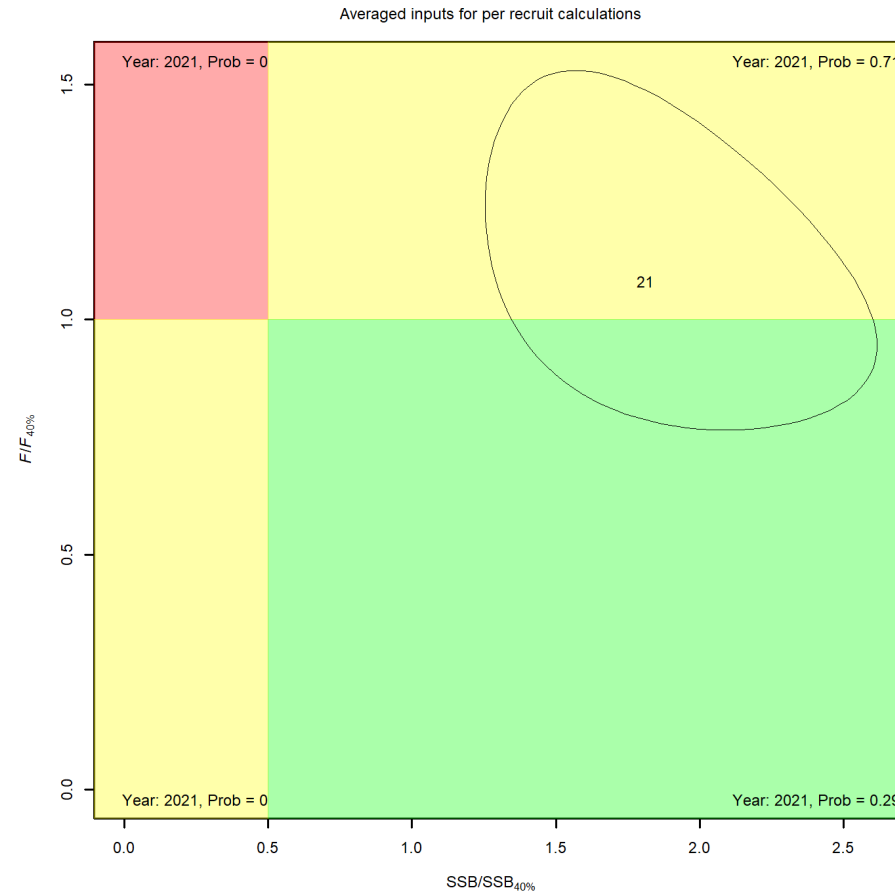
1	2	3	4	5	6	7	8+
0.01	0.04	0.09	0.17	0.17	0.17	0.17	0.17
0.01	0.09	0.10	0.15	0.26	0.39	0.54	0.54
0.00	0.02	0.06	0.07	0.07	0.07	0.07	0.07
0.03	0.08	0.15	0.21	0.24	0.25	0.25	0.25

- $FAA/(F_{40} = 1.03)$  = selectivity

1	2	3	4	5	6	7	8+
0.01	0.04	0.08	0.16	0.16	0.16	0.16	0.16
0.01	0.08	0.09	0.14	0.25	0.38	0.52	0.52
0.00	0.02	0.06	0.07	0.07	0.07	0.07	0.07
0.03	0.07	0.14	0.20	0.23	0.24	0.25	0.25

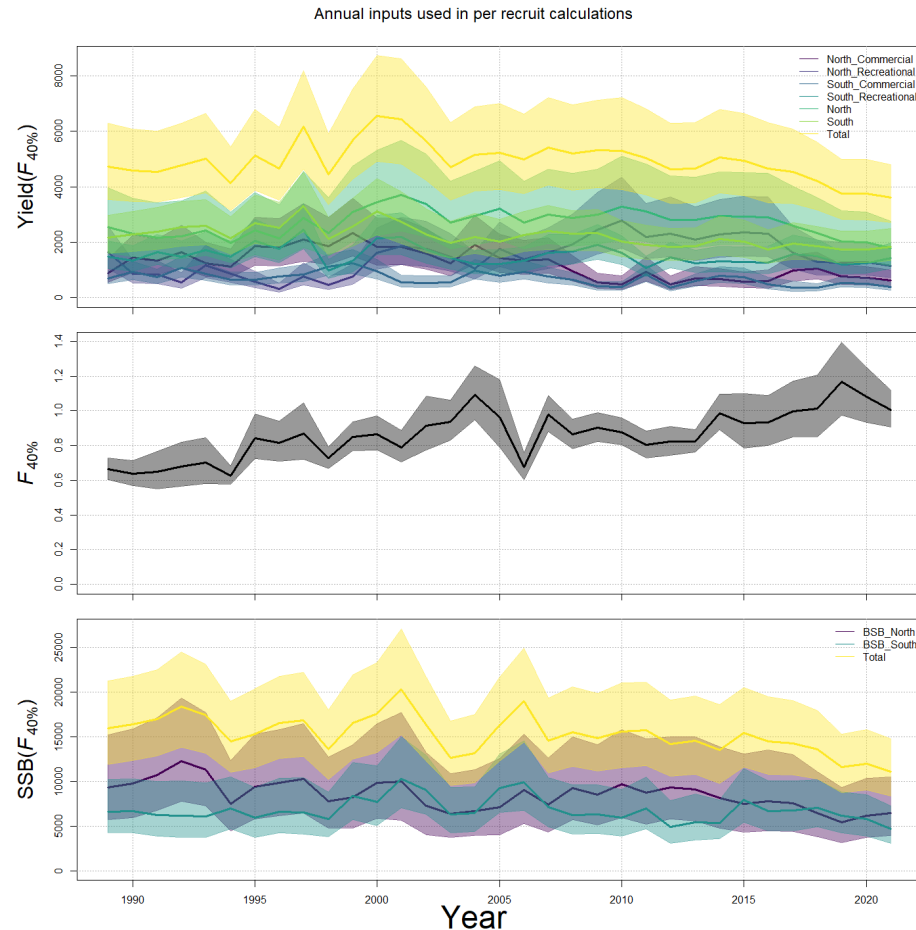
# Status relative to reference points

- WHAM and Mult-WHAM can calculate joint uncertainty of status of current F and SSB relative to reference points

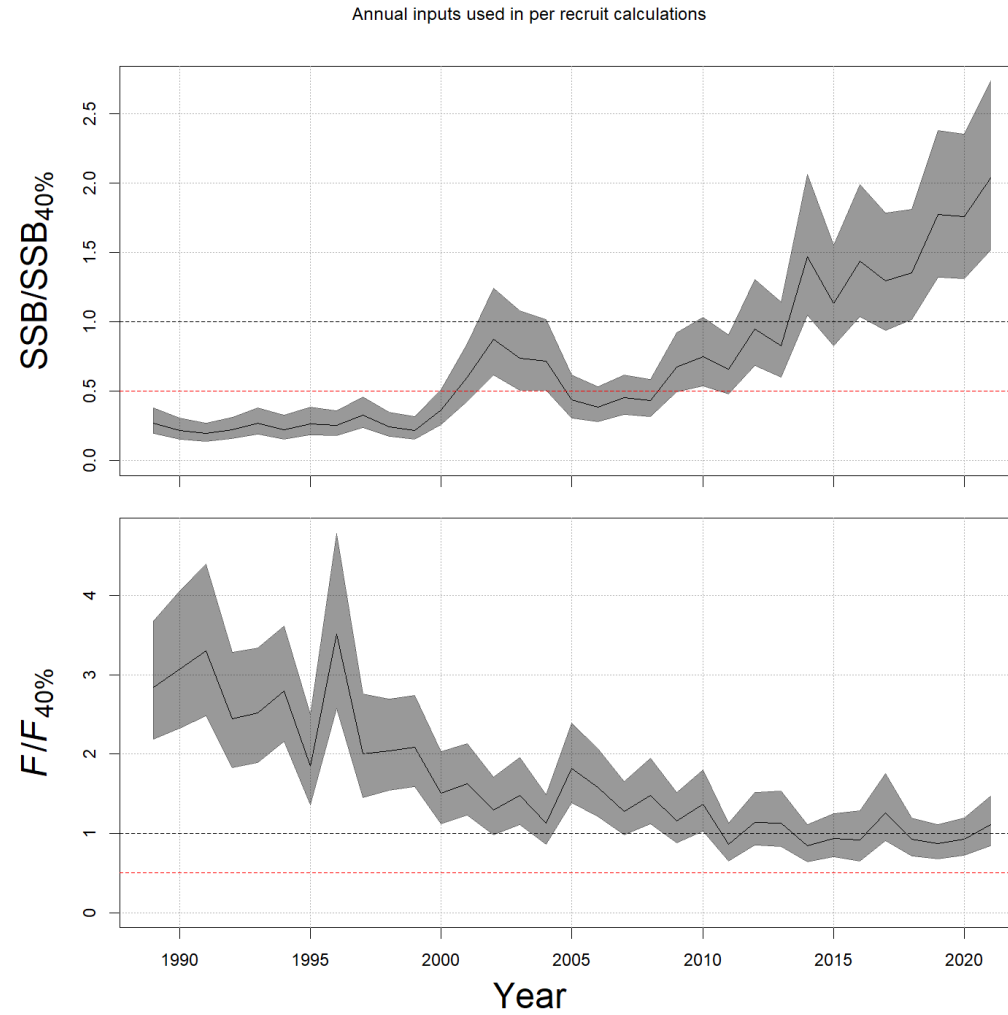


# Annual reference points

- Can calculate reference points every year using annual inputs instead of last 5 years.



# Annual status



## Projections (TOR 6)

# Projection options

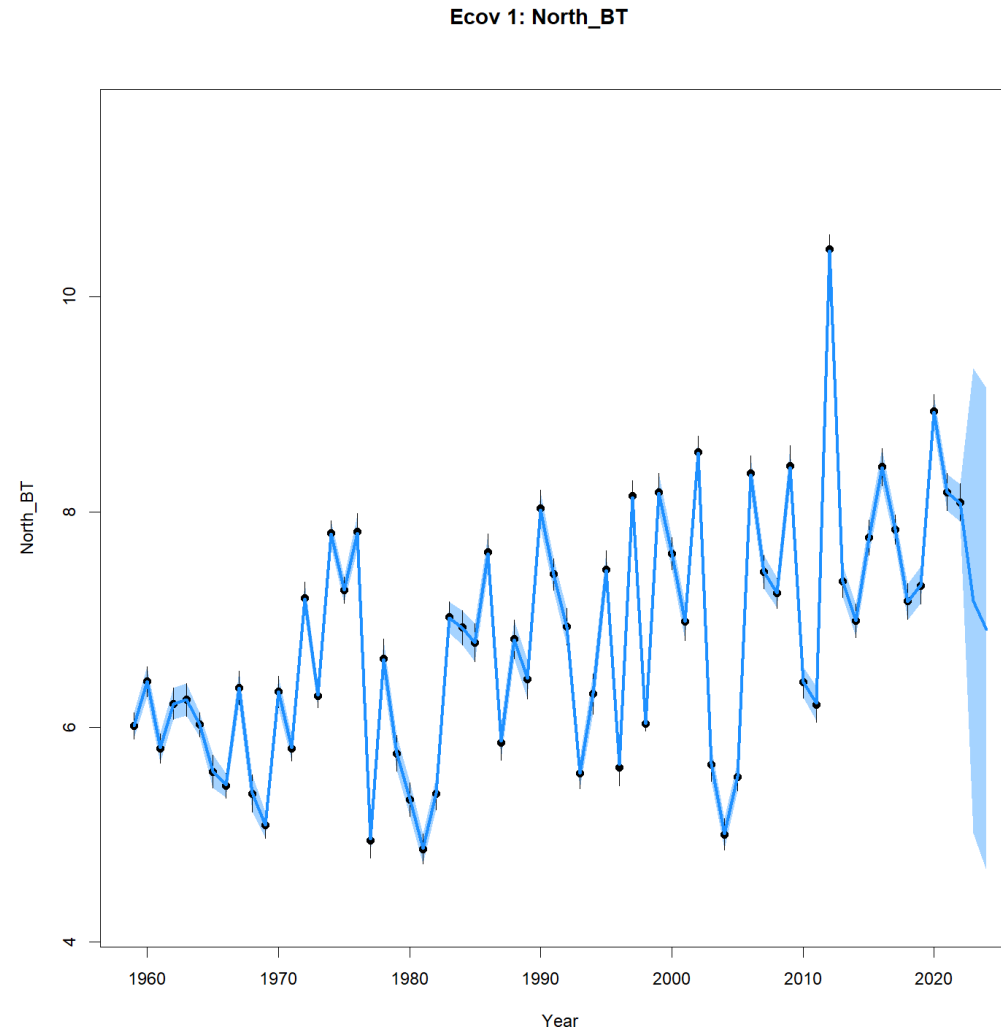
- Multi-WHAM has the same options as the standard WHAM package
- Continues time series models for some random effects:
  - Recruitment
  - Survival/movement transitions
  - Environmental covariates
  - Optional for M, and movement
  - Uncertainty in these estimates grows in projection years moving away from observations
  - **Under AR1 model projected random effects converge to the mean of the process**
- For other dynamics user specifies years to average (default is the same as that for prevailing BRPs).
- Various options for projected (year-specific) fully-selected F or FAA:
  - Status quo
  - $F_{40}$  (or some other percentage)
  - $F_{MSY}$  (if a S-R function is used)
  - user-specified fully-selected F
  - user-specified total Catch (appropriate FAA is calculated internally)

# 3-year projection example:

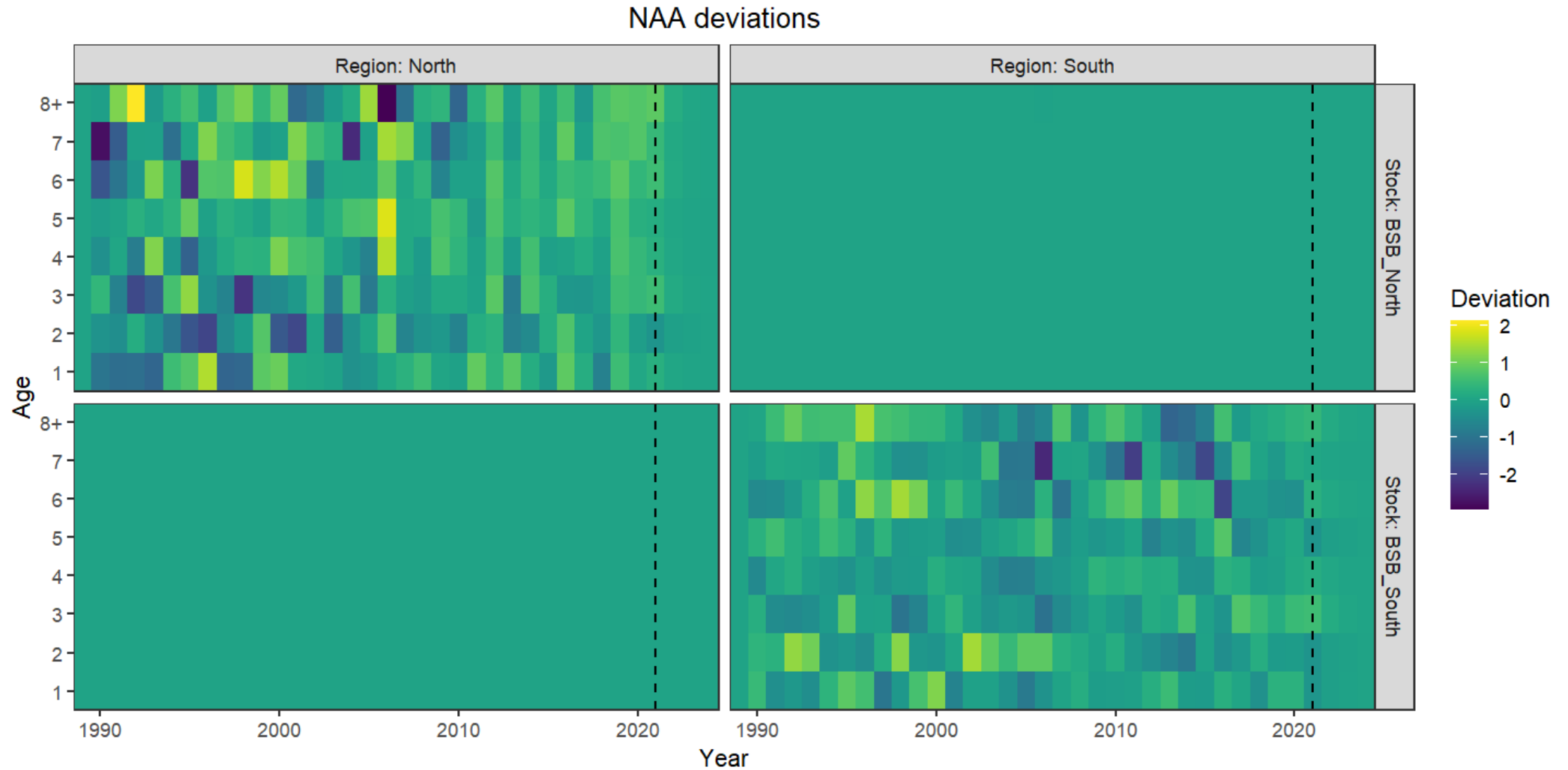
- Total Catch = 10 kmt in 2022
- Fully-selected total  $F = F_{40}$  in 2023, 2024
- Bottom temperature:
  - AR1 process random effects are forecast for 2022 and 2024 (observations end after 2022)
- 2DAR1 process random effects for recruitment and survival are forecast



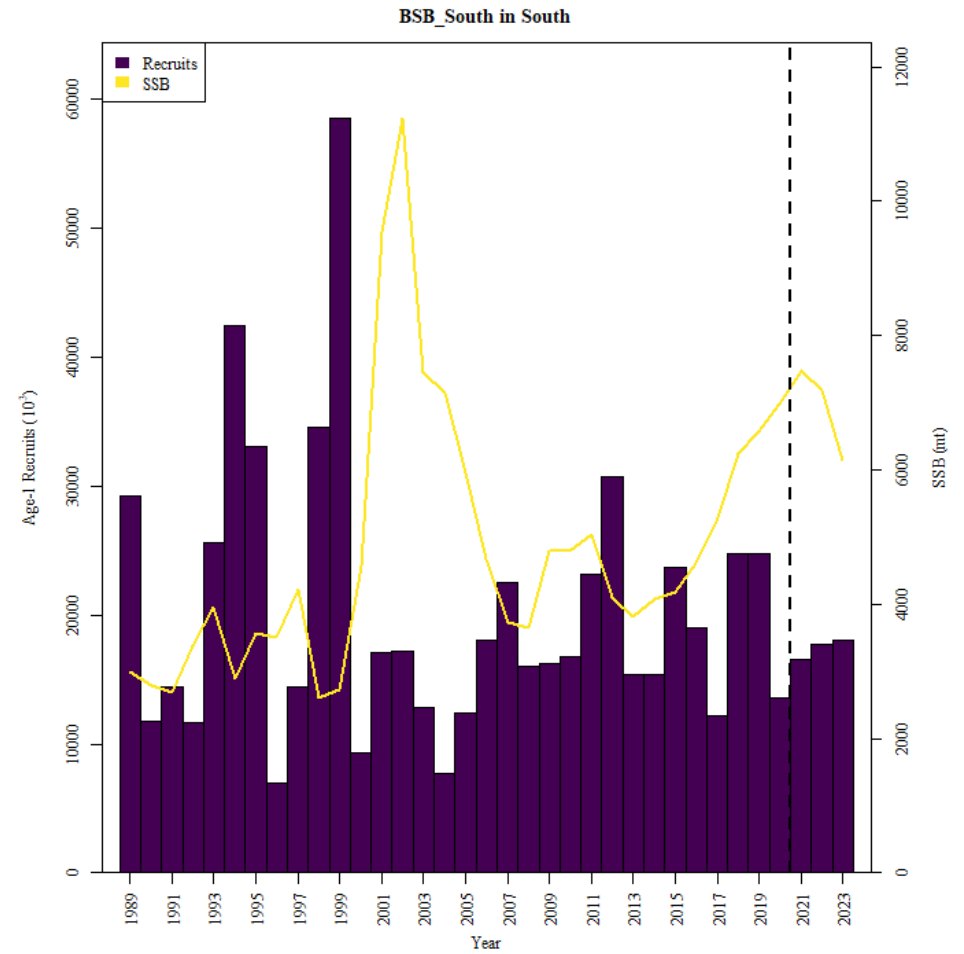
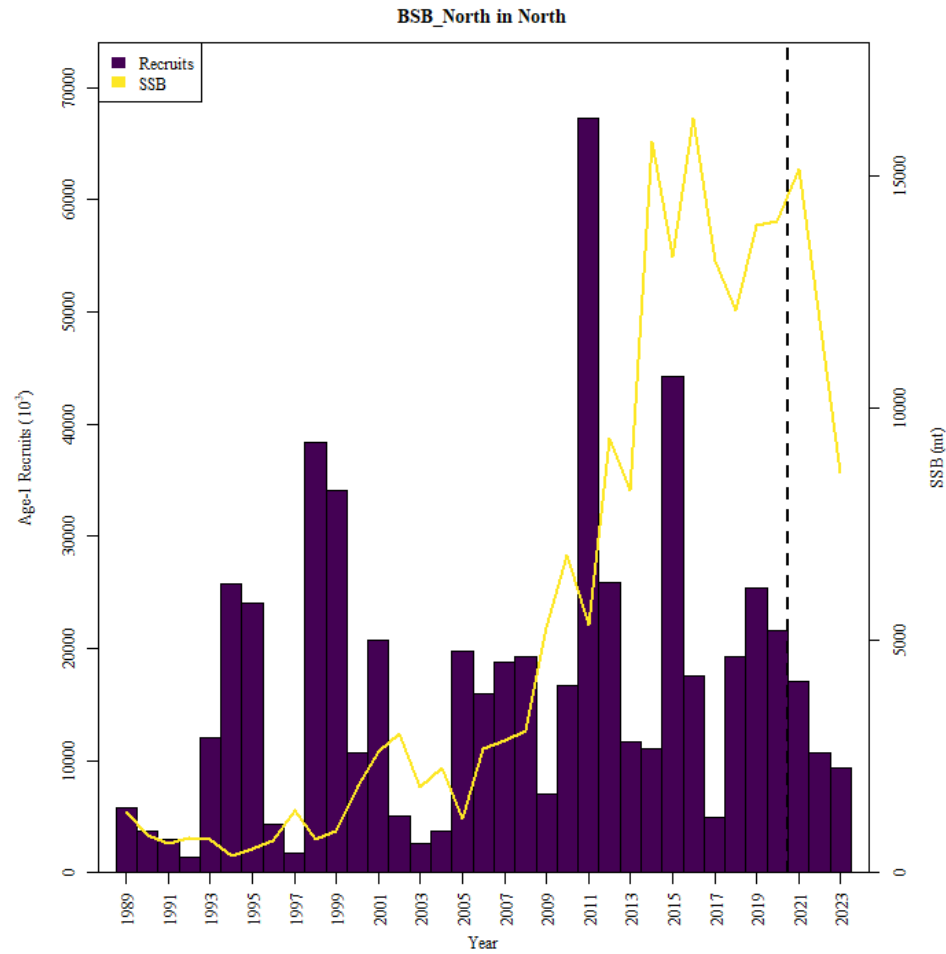
# Bottom temperature in the North



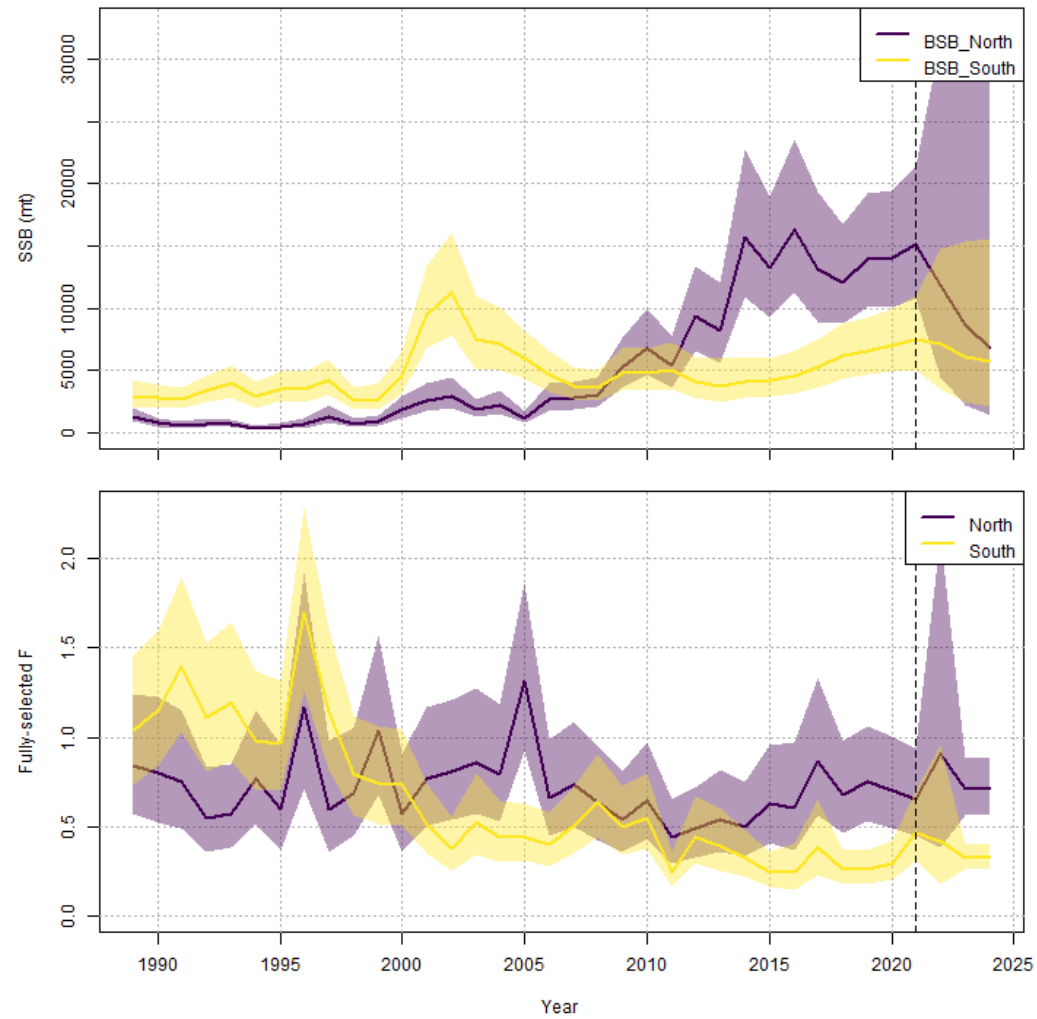
# Survival/movement deviation random effects



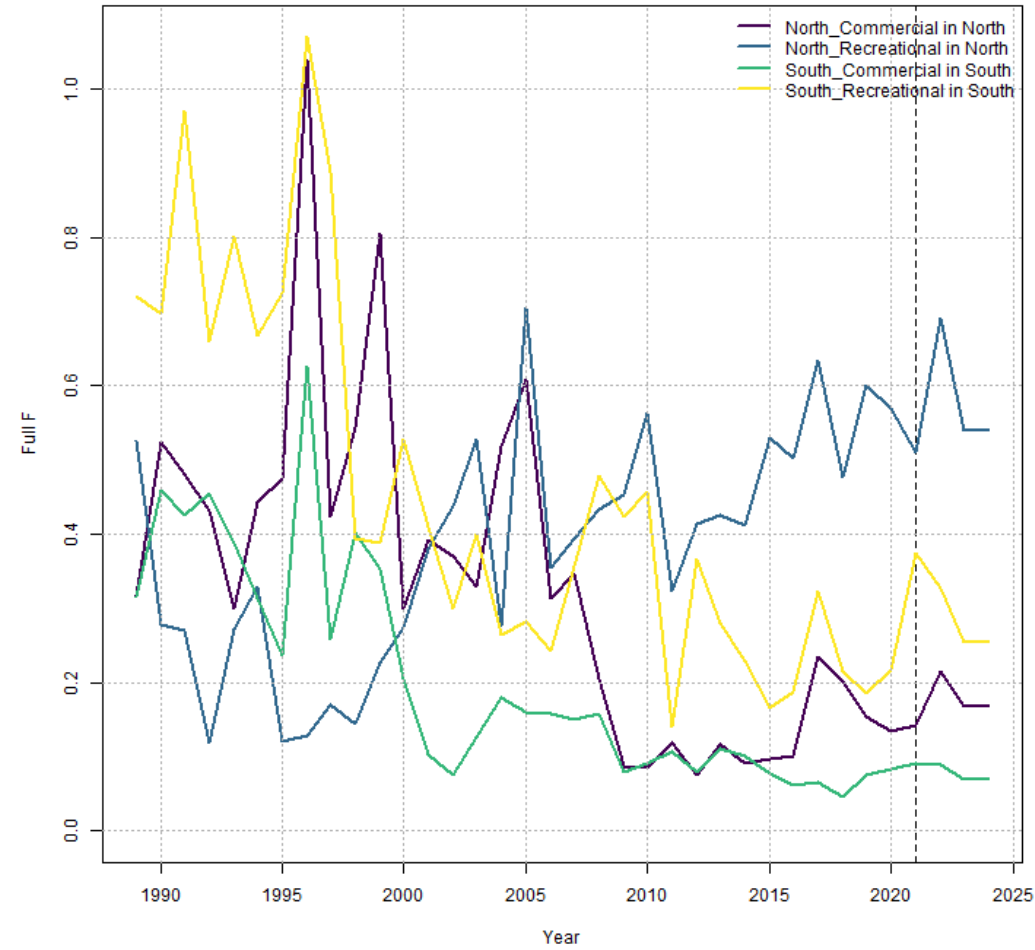
# SSB and recruitment



# SSB and F



# fleet-specific F



# Projected status

