Diffusion Processes on Complex Networks

Assignment 1.

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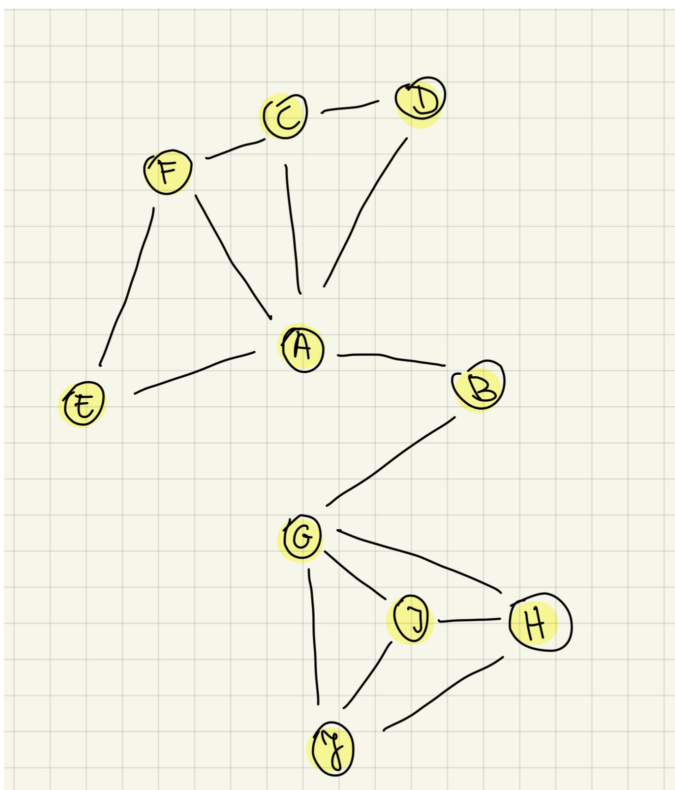
Date: 17.03.2021

Task 1.

We consider the undirected network defined by the following set of links:

Alice	Bob	Bob	Gail	Irene	Gail
Carl	Alice	Gail	Harry	Irene	Jen
Alice	David	Harry	Jen	Ernst	Frank
Alice	Ernst	Jen	Gail	David	Carl
Alice	Frank	Harry	Irene	Carl	Frank

(a) Draw the network by hand.



(b) How many nodes are there?

There are 10 nodes.

(c) What is the density of the network?

Density is a number of actual connections divided by a number of potential connections. We can calculate potential connections using below formula

$$PC = \frac{n(n-1)}{2},$$

where n is a number of nodes.

In this case, the density is equal to

$$D = \frac{AC}{PC} = \frac{15}{\frac{10(10-2)}{2}} = \frac{1}{3}$$

(d) Calculate the degree of each node. Who is the most central node according to this measure?

Degree of a node is a number of edges connected to that node. In this case:

- Alice – 5
- Bob – 2
- Carl – 3
- David – 2
- Ernst – 2
- Frank – 3
- Gail – 4
- Harry – 3
- Irene – 3
- Jen – 3

Then, Alice is the most central node, according to this measure.

(e) Calculate the clustering of each node and the average clustering of the network.

The clustering coefficient is given by

$$c_i = \frac{2L_i}{d_i(d_i-1)},$$

where L_i is a number of links between neighbours of node i and d_i is a degree of node i. In this case:

- Alice – $\frac{3}{10}$
- Bob – 0
- Carl – $\frac{2}{3}$
- David – 1
- Ernst – 1
- Frank – $\frac{2}{3}$
- Gail – $\frac{1}{2}$
- Harry – 1
- Irene – 1
- Jen – 1

And the average clustering of the network is equal to

$$c_n = 0,713.$$

(f) Calculate the closeness centrality for each node. Who is the most central node according to this measure?

The closeness can by given by

$$C_i = \frac{n-1}{\sum_j d(i,j)},$$

where $d(i,j)$ can be defined as the number of edges in a shortest path between nodes i and j in the network.

In this case:

- Alice – $\frac{9}{16}$
- Bob – $\frac{9}{16}$
- Carl – $\frac{9}{22}$
- David – $\frac{9}{23}$
- Ernst – $\frac{9}{23}$
- Frank – $\frac{9}{22}$
- Gail – $\frac{9}{18}$
- Harry – $\frac{9}{24}$
- Irene – $\frac{9}{24}$
- Jen – $\frac{9}{24}$

Then, Alice and Bob are the most central nodes, according to this measure.

(g) Calculate the betweenness centrality for each node. Who is the most central node according to this measure?

Betweenness is calculated by given formula

$$B_i = \sum_{j \neq i \neq k} \frac{\sigma_{jk}(i)}{\sigma_{jk}},$$

where $\sigma_{jk}(i)$ is a number of a shortest paths between nodes j and k passing through node i and σ_{jk} is a number of a shortest paths between nodes j and k.

In this case:

- Alice – 22
- Bob – 20
- Carl – $\frac{1}{2}$
- David – 0
- Ernst – 0
- Frank – $\frac{1}{2}$
- Gail – 18
- Harry – 0
- Irene – 0
- Jen – 0

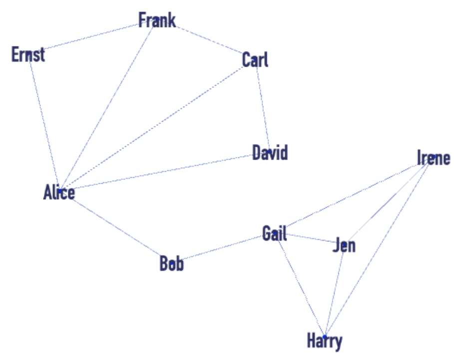
Then, Alice is the most central node, according to this measure.

Task 2.

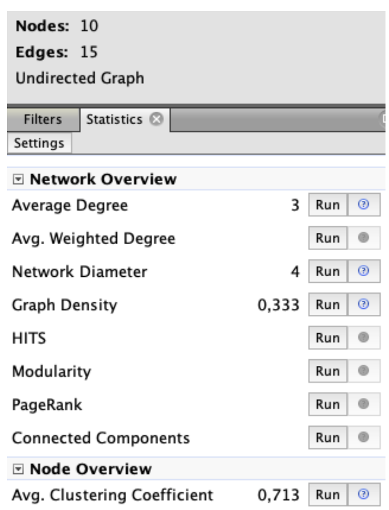
(a) Prepare a CSV file with the edge list.

Target	Source
Alice	Bob
Carl	Alice
Alice	David
Alice	Ernst
Alice	Frank
Bob	Gail
Gail	Harry
Harry	Jen
Jen	Gail
Harry	Irene
Irene	Gail
Irene	Jen
Ernst	Frank
David	Carl
Carl	Frank

(b)Visualize the network by making use of Gephi software.



(c) Calculate the basic network measures within Gephi.



Id	Degree	Clustering Coefficient	Closeness Centrality	Betweenness Centrality
Bob	2	0.0	0.5625	20.0
Alice	5	0.3	0.5625	22.0
Carl	3	0.666667	0.409091	0.5
David	2	1.0	0.391304	0.0
Ernst	2	1.0	0.391304	0.0
Frank	3	0.666667	0.409091	0.5
Gail	4	0.5	0.5	18.0
Harry	3	1.0	0.375	0.0
Jen	3	1.0	0.375	0.0
Irene	3	1.0	0.375	0.0

Task 3.

An undirected unweighted network of size N may be represented through a symmetric adjacency matrix $A \in R^{N \times N}$, which has $a_{ij} = 1$, if nodes i and j are connected, and $a_{ij} = 0$ otherwise. We assume that $a_{ii} = 0$, so there are no self-loops in the network.

Let e be a column vector of N elements all equal to 1, i.e. $e = (1, 1, \ldots, 1)^T$, where the superscript T indicates the transposition.

Write expressions for or answer each of the following by making use of the above quantities and the matrix formalism (no sum symbol allowed!):

(a) the vector k whose elements are degrees k_i of the nodes $i = 1, 2, 3, \ldots, N$.

Answer:

$$k = (A \cdot c)^T.$$

(b) the total number L of links in the network

Answer:

$$L = \frac{(A \cdot c)^T \cdot e}{2}.$$

(c) the matrix N whose element n_{ij} is equal to the number of common neighbors of nodes i and j

Answer:

$$N = A^2.$$

(d) the number T of triangles present in the network. A triangle is three vertices, each connected by edges to both of the others (hint: trace of a matrix)

Answer:

$$T = \frac{tr(A^3)}{6}.$$