ECSE 4480
ROBOTICS I

MiniProject 3: Spatial Robot Arm Motion

I, Chuizheng Kong, certify that the following work is my own and completed in accordance with the academic integrity policy as described in the Robotics I course syllabus.

Chuizheng Kong

Robotics I Mini-Project 3: Spactial Robot Arm Motion

```
clearvars
cd('C:\Users\kongc\Dropbox\RPI_2021FALL\RoboticsI\FALL21Robotics\MiniProject3')
load('S_sphere_path.mat')
addpath(genpath('C:\Users\kongc\Dropbox\RPI_2021FALL\RoboticsI\MATLABrelated\'))
addpath(genpath('C:\Users\kongc\Dropbox\RPI_2021FALL\RoboticsI\FALL21Robotics\MiniProject3'))
```

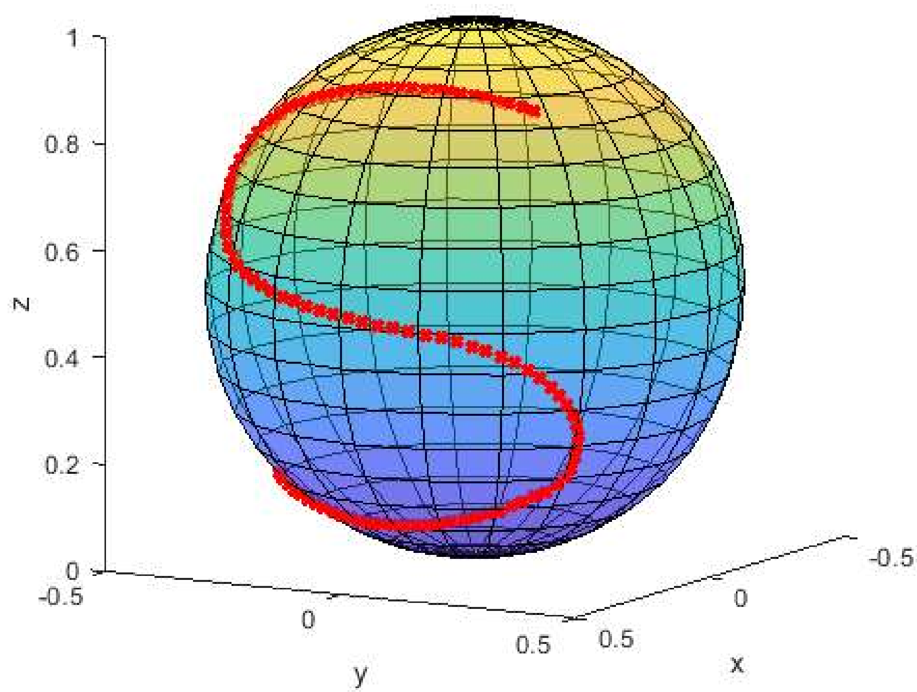
Part 1: Representations of Orientation

Conventional Cartesian positions (x,y,z) is an intuitive way to represent position and orientation; however, when dealing with robotic arms with constraints between links, end-effector might run into singularities. With other representation of rotations that accounts for the spherical path of robot arm.

```
% loading s-curve data
```

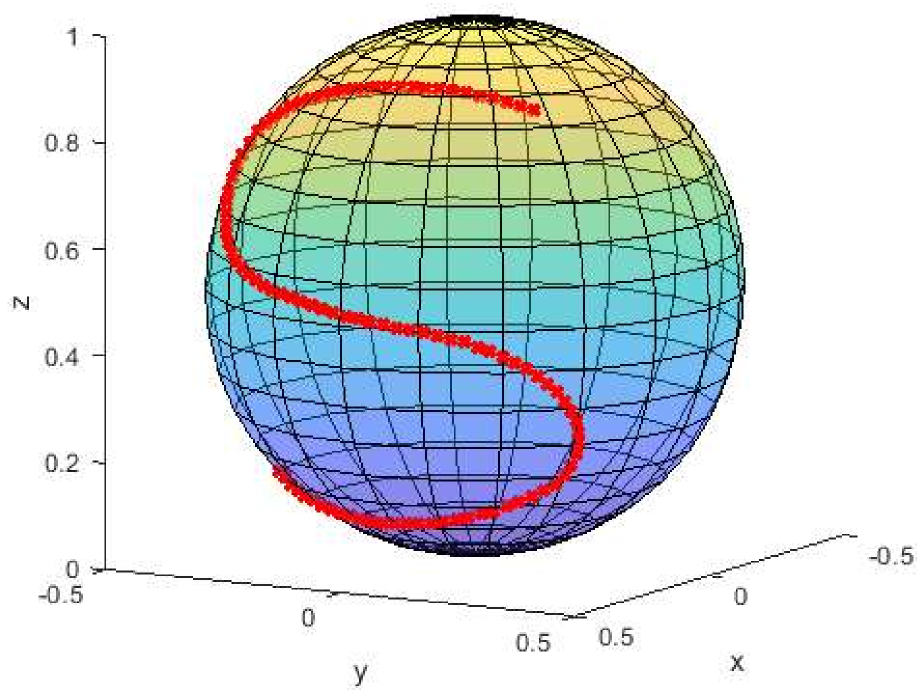
First we discretize the path into equal segments and store them in lambda

```
% plot the spherical S
figure(1);plot3(p_S(1,:),p_S(2,:),p_S(3,:),'rx','linewidth',3);
xlabel('x');ylabel('y');zlabel('z');
hold on;
% add a 3d sphere
surf(X,Y,Z)
% make it transparent
alpha .5
axis(r*[-1 1 -1 1 0 2]);axis('square');
view(120,10);
```

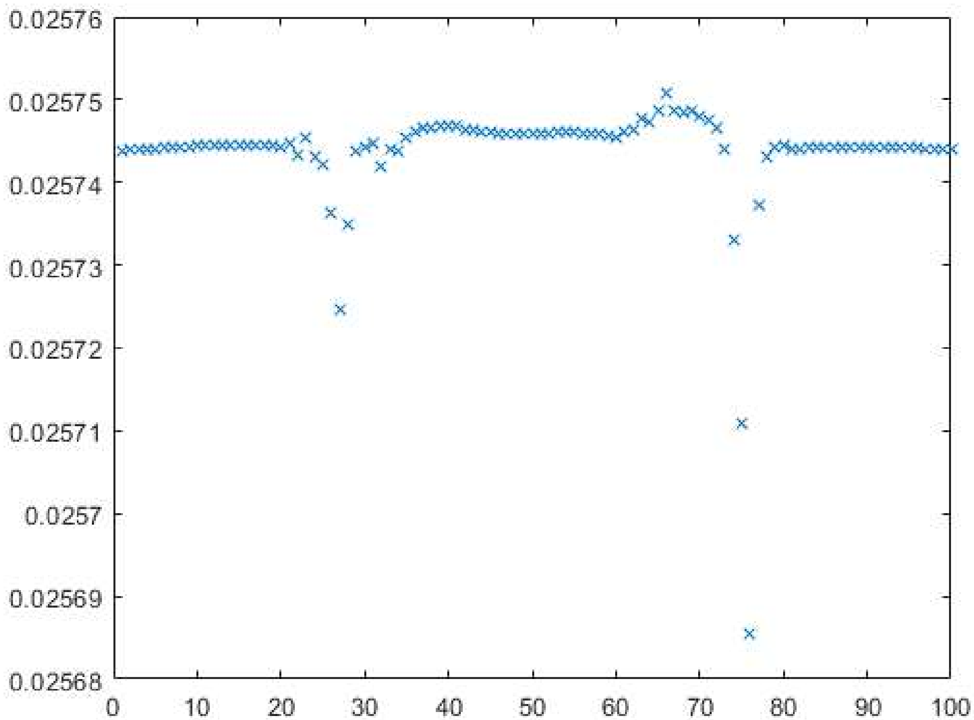



```
% convert to equal path length grid
```

```
diffS=vecnorm(diff(p_S')');
ls=[0 cumsum(diffS)];
lf=sum(diffS);
N=100;
l=(0:lf/N:lf);
pS=interp1(ls,p_S',l,'spline')';  % x = ls, f(x) = p_S, l = x_1, pS = f1(x_1)
% plot it out again with equal path length
figure(2);plot3(pS(1,:),pS(2,:),pS(3,:),'rx','linewidth',3);
xlabel('x');ylabel('y');zlabel('z');
hold on;
% 3d sphere
surf(X,Y,Z)
% make it transparent
alpha 0.4
axis(r*[-1 1 -1 1 0 2]);axis('square');
view(120,10);
```




```
% check the path length is indeed equal
dlvec=vecnorm(diff(pS')');
figure(3);plot(dlvec,'x')
```




```
dl=mean(dlvec);
disp(max(abs(dlvec-dl)));
```

```
5.8157e-05
```

Here we are allocating variables for Unit quaternions, eular angles, and angle-axis product.

```
%-----------------------------------
% next we find xT, yT, zT
N=length(pS);
xT=zeros(3,N);zT=zeros(3,N);yT=zeros(3,N);
quat=zeros(4,N); % allocate space for unit quaternion representation of R_{0T}
euler_rpy = zeros(3,N);
angle_axis = zeros(4,N);
```

We can see that the bottom of the sphere lies on (0,0,0) together with the base of the robot. For the end effector to be perpendicular to the sphere, "xT" should be the extention of a radius passing throught any points in "pS".

```
pc = r*[0;0;1];  % the distance from (0,0,0) to the center of the sphere.
% r is the radius of the sphere.
r = 0;p=0;q=0;
ak = 0;theta = 0;
for i=1:N
    xT(:,i)=(pS(:,i)-pc)/norm(pS(:,i)-pc);
    if i<N
        yT(:,i)=(pS(:,i+1)-pS(:,i));
    else
        yT(:,i)=yT(:,i-1);
    end
    yT(:,i)=yT(:,i)-yT(:,i)'*xT(:,i)*xT(:,i);
```

```
    yT(:,i)=yT(:,i)/norm(yT(:,i));
    zT(:,i)=cross(xT(:,i),yT(:,i));
    R=[xT(:,i) yT(:,i) zT(:,i)];
    quat(:,i)=R2q(R);
    [r,p,y] = R2rpy(R);
    euler_rpy(:,i) = [r,p,y]';
    [ak,theta] = R2kth(R);
    angle_axis(:,i) = [ak;theta];
end
```

Next we will look at three different representation of the path.

(a) Unit Quaternions

In this representation, three angles are represented in four DOF.

$$q_0 = \frac{1}{2}\sqrt{1 + r_{11} + r_{22} + r_{33}}$$

$$q_1 = \frac{1}{4q_0}(r_{32} - r_{23})$$

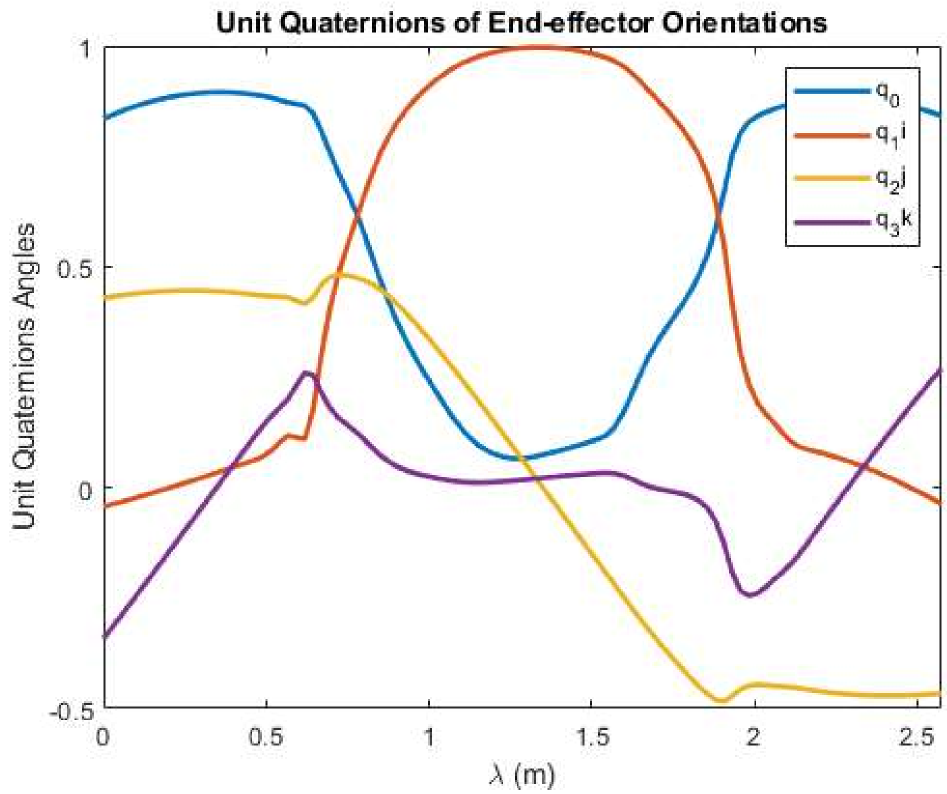
$$q_2 = \frac{1}{4q_0}(r_{13} - r_{31})$$

$$q_3 = \frac{1}{4q_0}(r_{21} - r_{12})$$

where rotation matrix:

$$R_{0T} \in \mathbb{R}^{3\times3}$$

```
figure;
plot(l,quat,'LineWidth',2)
legend('q_0','q_1i','q_2j','q_3k')
xlim([0,l(end)])
xlabel('\lambda (m)')
ylabel('Unit Quaternions Angles')
title('Unit Quaternions of End-effector Orientations')
```



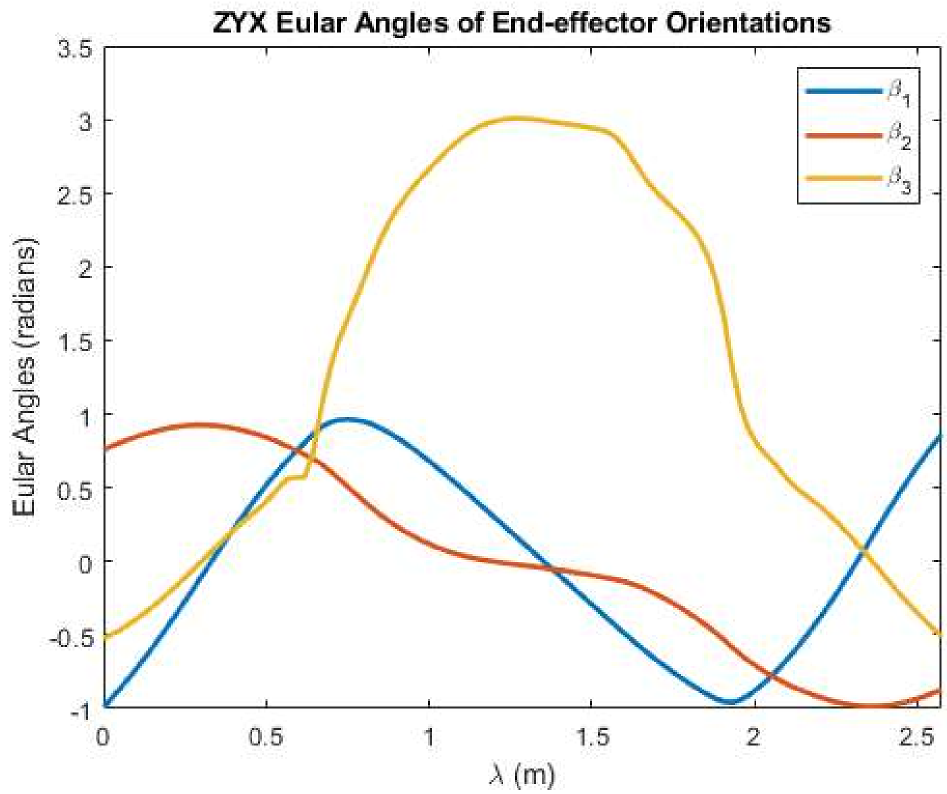
(b) Euler Angles

Any cartisian rotation can be represented by 1 of the 12 Euler Angles in either fixed frame or body frame.

In this problem we need to solve ZYX (body frame or rpy) Eular angle for each orientation of the end-effector frame.

$$R_{0T} = R_z(\beta_1)R_y(\beta_2)Rx(\beta_3)$$

```
figure;
plot(l,euler_rpy,'LineWidth',2)
legend('\beta_1','\beta_2','\beta_3')
xlim([0,l(end)])
xlabel('\lambda (m)')
ylabel('Eular Angles (radians)')
title('ZYX Eular Angles of End-effector Orientations')
```

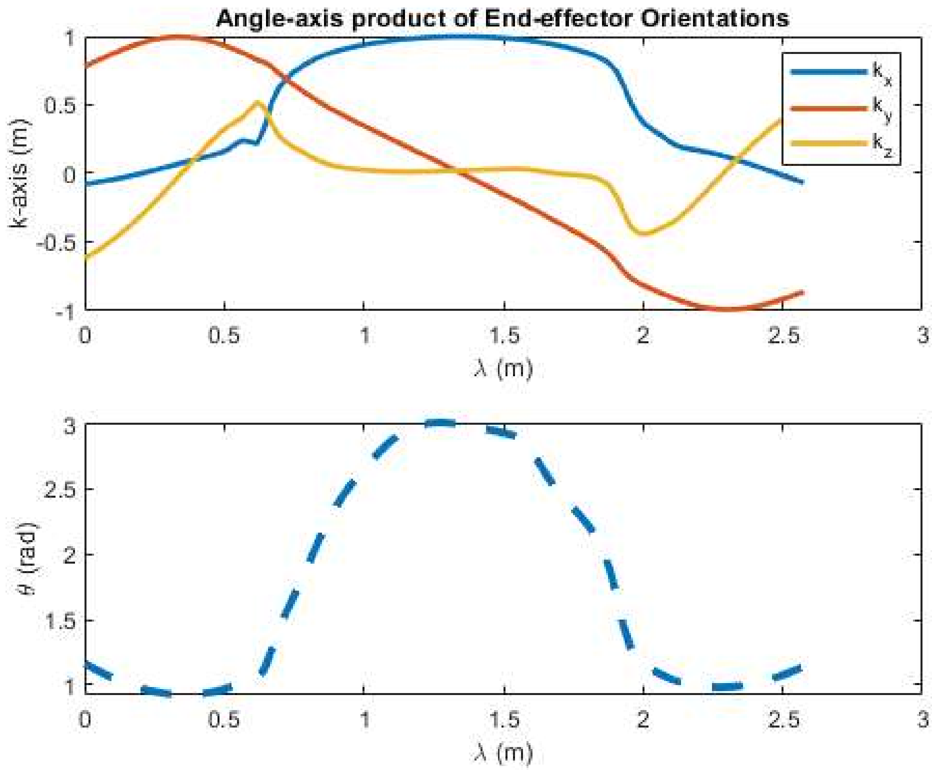


(C) Angle-axis product

Another way of thinking about rotation is that any combinations of SO(3) rotation can be represented as rotation of certain degree about one axis:

$$R_{0T} = e^{\beta^{\times}}, \text{ where } \beta = k\theta$$

```
figure;
subplot(2,1,1)
plot(l,angle_axis(1:3,:),'LineWidth',2)
legend('k_x','k_y','k_z')
xlabel('\lambda (m)')
ylabel('k-axis (m)')
title('Angle-axis product of End-effector Orientations')
subplot(2,1,2)
plot(l,angle_axis(4,:),'LineWidth',3,'LineStyle',"--")
xlabel('\lambda (m)')
ylabel('\theta (rad)')
```



Part 2: ABB IRB 1200-5/0.9 robot

The purpose of this part is to understand the motion control of IRB 1200 and to derive relativd equations to model its end-effector motion.

(a)

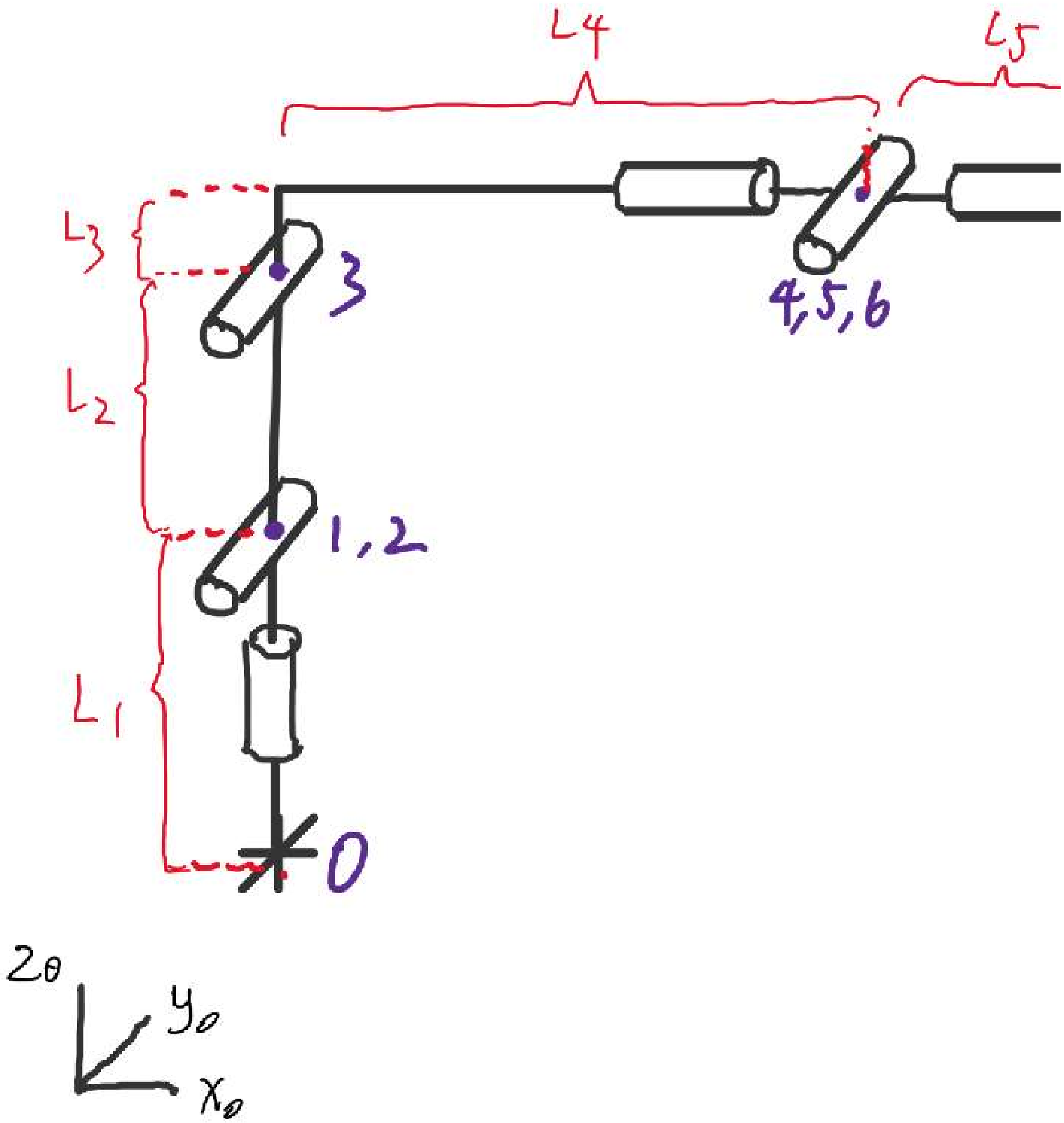
Given physical parameters of IRB 1200, we want to be able to represent it in POE, SDH, and MDH.

```
syms L1 L2 L3 L4 L5 real
syms q1 q2 q3 q4 q5 q6 real
```

1. First in Product of Exponential

```
figure;
imshow('IRB1200drawing.png')
```

IRB 1200




```
% P: ositon vector of each link
zz=zeros(3,1); ex = [1;0;0]; ey = [0;1;0]; ez = [0;0;1];
p01=0*ex+L1*ez;
p12=zz;
p23=L2*ez;
p34=L3*ez+L4*ex;
p45=zz;
p56=zz;
p6T=L5*ex;
% H: rotation axis
```

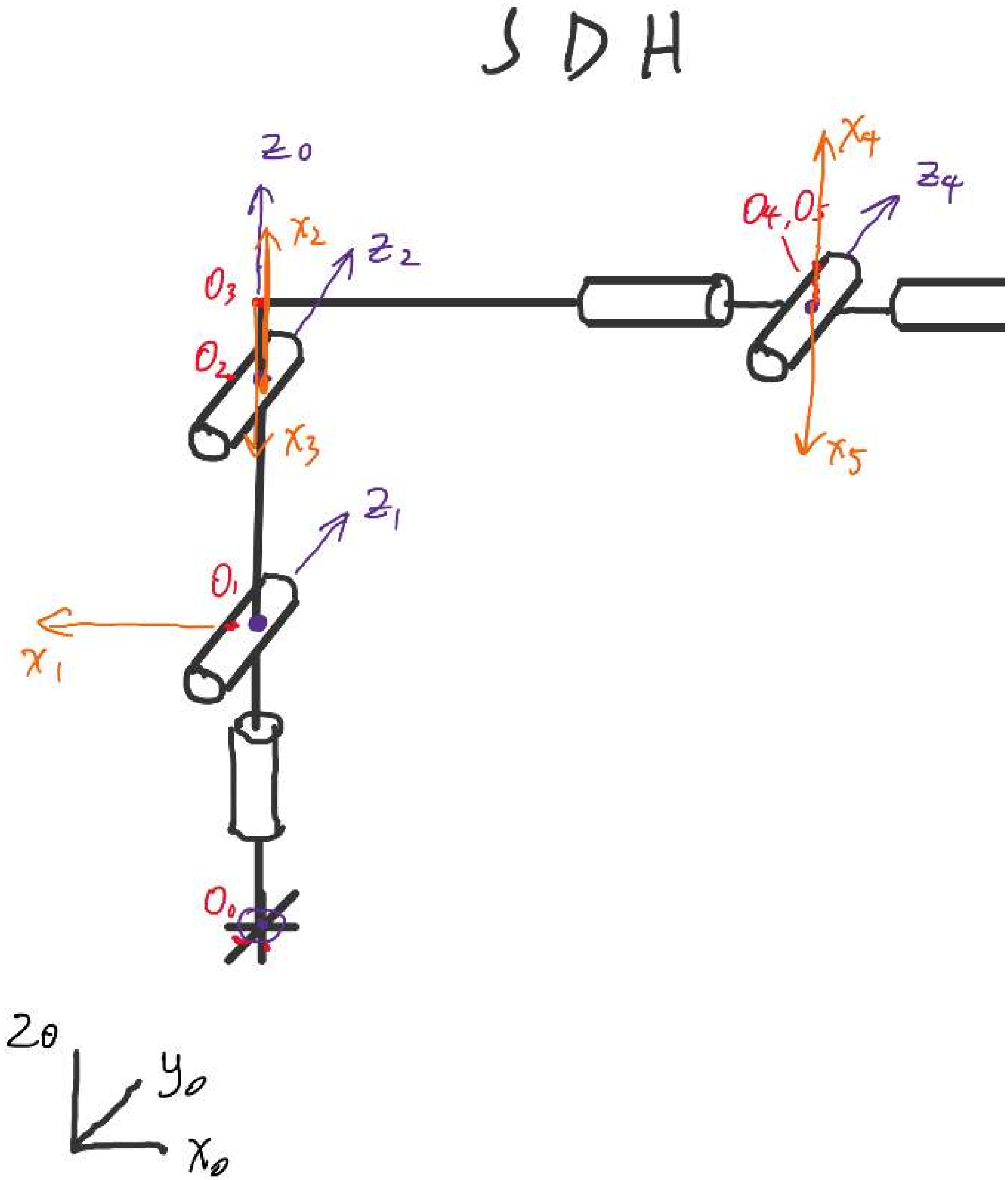
```
h1=ez;
h2=ey;
h3=ey;
h4=ex;
h5=ey;
h6=ex;
% Show in table
POE_i = {'1';'2';'3';'4';'5';'6';'T'};
POE_p = {'L1*ez';'0';'L2*ez';'L3*ez+L4*ex';'0';'0';'L5*ex'};
POE_h = {'ez';'ey';'ey';'ex';'ey';'ex';'ex'};
POE_config = table(POE_i,POE_h,POE_p)
```

POE_config = 7×3 table

	POE_i	POE_h	POE_p
1	'1'	'ez'	'L1*ez'
2	'2'	'ey'	'0'
3	'3'	'ey'	'L2*ez'
4	'4'	'ex'	'L3*ez+L4*ex'
5	'5'	'ey'	'0'
6	'6'	'ex'	'0'
7	'T'	'ex'	'L5*ex'

2. Representing IRB1200 in Standard Denavit-Hartenberg frame

```
imshow("SDH.png")
```




```
% Show in table
SDH_i = {'1';'2';'3';'4';'5';'6'};
SDH_d = string([L1,0,0,L4,0,L5]');
SDH_a = string([0,L2,-L3,0,0,0]');
SDH_alpha = string(sym([pi/2,0,pi/2,pi/2,pi/2,0]'));
SDH_theta = string((sym([pi,pi/2,pi,pi,pi,pi])+[q1,q2,q3,q4,q5,q6])');
SDH_config = table(SDH_i,SDH_d,SDH_a,SDH_alpha,SDH_theta)
```

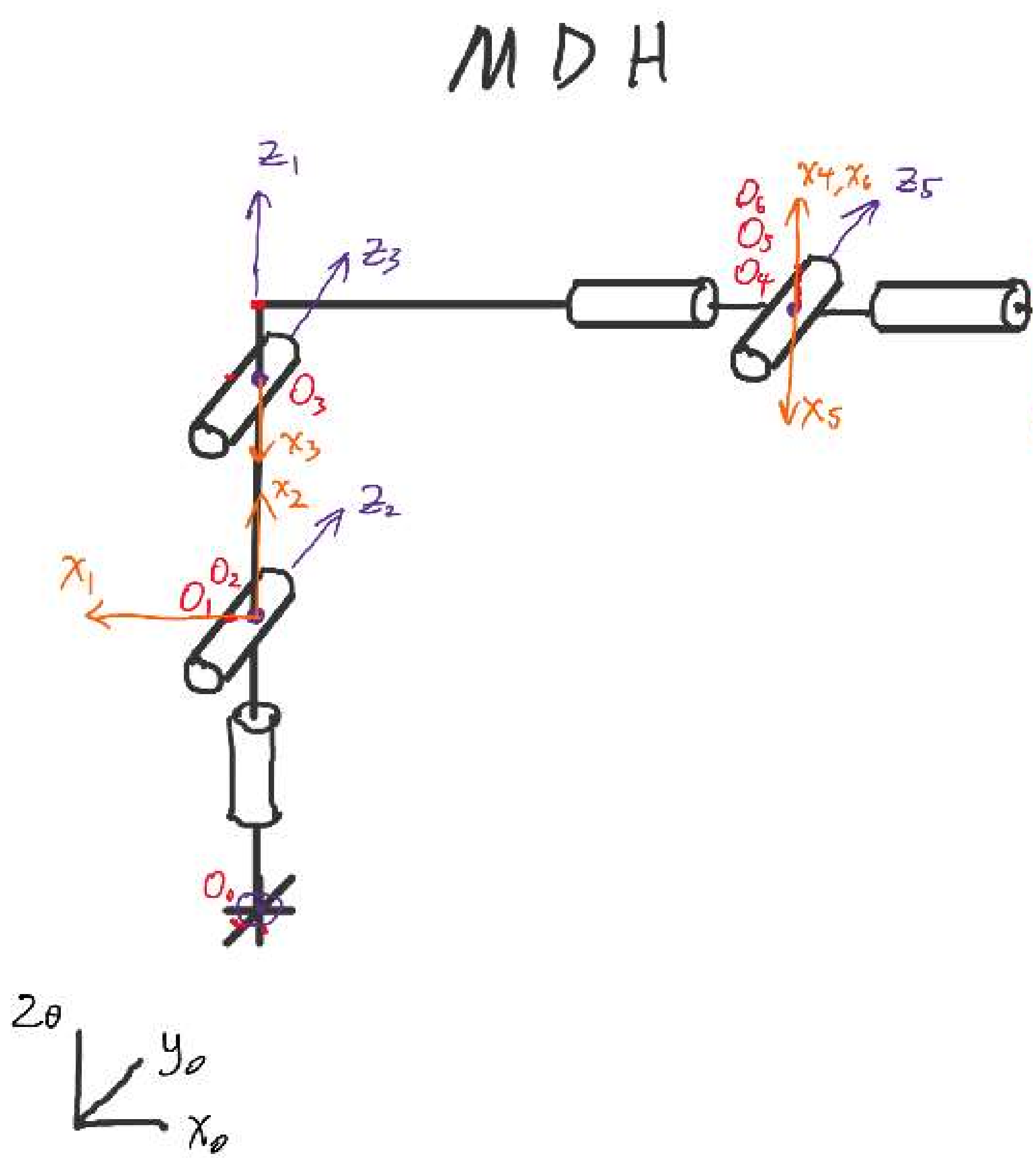
SDH_config = 6×5 table

	SDH_i	SDH_d	SDH_a	SDH_alpha	SDH_theta

	SDH_i	SDH_d	SDH_a	SDH_alpha	SDH_theta
1	'1'	"L1"	"0"	"pi/2"	"q1 + pi"
2	'2'	"0"	"L2"	"0"	"q2 + pi/2"
3	'3'	"0"	"-L3"	"pi/2"	"q3 + pi"
4	'4'	"L4"	"0"	"pi/2"	"q4 + pi"
5	'5'	"0"	"0"	"pi/2"	"q5 + pi"
6	'6'	"L5"	"0"	"0"	"q6 + pi"

3. Representing IRB1200 in Modified Denavit-Hartenberg frame

```
imshow("MDH.png")
```




```
% Show in table
MDH_i = {'1';'2';'3';'4';'5';'6';'7'};
```

```
MDH_d = string([L1,0,0,L4,0,0,L5]');
MDH_ai_1 = string([0,0,L2,-L3,0,0,0]');
MDH_alpha = string(sym([0,pi/2,0,pi/2,pi/2,pi/2,0]'));
MDH_theta = string((sym([pi,pi/2,pi,pi,pi,pi,pi])+[q1,q2,q3,q4,q5,q6,0])');
MDH_config = table(MDH_i,MDH_d,MDH_ai_1,MDH_alpha,MDH_theta)
```

```
MDH_config = 7×5 table
```

	MDH_i	MDH_d	MDH_ai_1	MDH_alpha	MDH_theta
1	'1'	"L1"	"0"	"0"	"q1 + pi"
2	'2'	"0"	"0"	"pi/2"	"q2 + pi/2"
3	'3'	"0"	"L2"	"0"	"q3 + pi"
4	'4'	"L4"	"-L3"	"pi/2"	"q4 + pi"
5	'5'	"0"	"0"	"pi/2"	"q5 + pi"
6	'6'	"0"	"0"	"pi/2"	"q6 + pi"
7	'7'	"L5"	"0"	"0"	"pi"

(b)

With POE, SDH, and MDH defined, calculate and compare the forward kinematics of all three representations. We do that by first define a sets of angles and then measure the difference of the end-effector frames between all three methods.

```
q = [0,0,0,0,0,pi/2]; %  end-effector pointing up ward
```

1. In POE convention, calculate the forward kinematics

```
% IRB 1200 robot using POE convention
irb1200_poe.q = q;
irb1200_poe.P=[p01 p12 p23 p34 p45 p56 p6T]; % unit in meter
irb1200_poe.H=[h1 h2 h3 h4 h5 h6];
irb1200_poe.joint_type=[0 0 0 0 0 0]; % all rotational joint
irb1200_poe = nlinkfwdkin(irb1200_poe);
disp('T_{0T} from POE')
```

```
T_{0T} from POE
```

```
disp(irb1200_poe.T)
```

$$\begin{pmatrix} 1 & 0 & 0 & L_4+L_5 \\ 0 & \frac{1}{9007199254740992} & -1 & 0 \\ 0 & 1 & \frac{1}{9007199254740992} & L_1+L_2+L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. In SDH convention, calculate the forward kinematics

```
irb1200_sdh.d = [L1,0,0,L4,0,L5];
irb1200_sdh.a = [0,L2,-L3,0,0,0];
irb1200_sdh.alpha = sym([pi/2,0,pi/2,pi/2,pi/2,0]);
irb1200_sdh.theta = sym([pi,pi/2,pi,pi,pi,pi])+q;
% Match then end-effector frame to POE
T6T=[[[0 0 1 ; 0 -1 0; 1 0 0] zeros(3,1)];[zeros(1,3) 1]];
```

```
irb1200_sdh = fwdkinsdh(irb1200_sdh);
T0T_sdh = simplify(irb1200_sdh.T*T6T)
```

T0T_sdh =

$$\begin{pmatrix} 1 & 0 & 0 & L_4+L_5 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_1+L_2+L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
disp('difference between POE and SDH')
```

difference between POE and SDH

```
disp(double(irb1200_poe.T-T0T_sdh));
```

```
   1.0e-15 *

         0         0         0         0
         0    0.1110         0         0
         0         0    0.1110         0
         0         0         0         0
```

With very small discrepancy (10^{-15}), we assured that POE and SDH has the same transformation.

3. In MDH convention, calculate the forward kinematics

```
irb1200_mdh.d = [L1,0,0,L4,0,0,L5];
irb1200_mdh.a = [0,0,L2,-L3,0,0,0];
irb1200_mdh.alpha = sym([0,pi/2,0,pi/2,pi/2,pi/2,0]);
irb1200_mdh.theta = sym([pi,pi/2,pi,pi,pi,pi,pi])+[q,0];
% Match then end-effector frame to POE
T7T=[[[0 0 -1 ; 0 1 0; 1 0 0] zeros(3,1)];[zeros(1,3) 1]];
irb1200_mdh = fwdkinmdh(irb1200_mdh);
T0T_mdh = simplify(irb1200_mdh.T*T7T)
```

T0T_mdh =

$$\begin{pmatrix} 1 & 0 & 0 & L_4+L_5 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_1+L_2+L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
disp('difference between POE and MDH')
```

difference between POE and MDH

```
disp(double(irb1200_poe.T-T0T_mdh));
```

```
   1.0e-15 *

         0         0         0         0
         0    0.1110         0         0
         0         0    0.1110         0
         0         0         0         0
```

With very small discrepancy (10^{-15}), we assured that POE and MDH has the same transformation.

Therefore, POE, SDH, and MDH are the same transformation.

(c)

Using subproblems, calculate inverse kinematics for IRB1200 using POE

1. we include numerical parameters for IRB1200

```
% ABB IRB 1200 parameters
L1=399.1;
L2=448;
L3=42;
L4=451;
L5=82;
% P: positon vector of each link
p01=0*ex+L1*ez;
p12=zz;
p23=L2*ez;
p34=L3*ez+L4*ex;
p45=zz;
p56=zz;
p6T=L5*ex;
% H: rotation axis
h1=ez;
h2=ey;
h3=ey;
h4=ex;
h5=ey;
h6=ex;
% IRB 1200 robot using POE convention
irb1200.P=[p01 p12 p23 p34 p45 p56 p6T]/1000; % unit in meter
irb1200.H=[h1 h2 h3 h4 h5 h6];
irb1200.joint_type=[0 0 0 0 0 0]; % all rotational joint
```

2. we then perform forward kinematics with a known set of joint angles.

```
% Create the Robot object
radius=.01;
[irb1200_rbt,colLink]=defineRobot(irb1200,radius);
irb1200.q = [0.1,0.2,-0.3,0.4,0.3,-0.2];  % known angles
disp('Preset angles');
```

```
Preset angles
```

```
disp(irb1200.q)
```

```
    0.1000    0.2000   -0.3000    0.4000    0.3000   -0.2000
```

```
irb1200 = nlinkfwdkin(irb1200);
disp('T_{0T} from POE method')
```

```
T_{0T} from POE method
```

```
disp(irb1200.T);
```

```
    0.9614   -0.1762    0.2115    0.6097
    0.2121    0.9638   -0.1613    0.0707
   -0.1755    0.1999    0.9640    0.9106
         0         0         0    1.0000
```

```
robot = irb1200;
```

3. preparing variables for inverse kinematics calculation

```
p0T = robot.T(1:3,4);
r0T = robot.T(1:3,1:3);
links = robot.P; %  3xn joint length in its own frame
% postion vectors of the arm
p01 = links(:,1); p12 = links(:,2); p23 = links(:,3); p34 = links(:,4);
p45 = links(:,5); p56 = links(:,6); p6T = links(:,7);
```

$$p_{0T} = p_{01} + R_{01}p_{12} + R_{02}p_{23} + R_{03}p_{34} + R_{04}p_{45} + R_{05}p_{56} + R_{06}p_{6T}$$

$$p_{0T} - p_{01} - R_{06}p_{6T} = R_{02}p_{23} + R_{03}p_{34}$$

known on the left, unknown on the right, we can rewrite this equation to the follwoing:

$$||(p_{16})_0|| = ||R_{02}(p_{23} + R_{23}p_{34})||$$

$$||p_{16}|| = ||p_{23} - (-e^{h3\times q3}p_{34})||$$

we can solve for $q3$ in R_{23} using subproblem3:

```
% a sphere with r = norm(p16)
% a p23 offseted cone of p34 waist height
% intersecting the sphere
% max two sol (maybe infinity?)
p16 = p0T - p01 - r0T*p6T;
q3 = subprob3(robot.H(:,3),-p34,p23,norm(p16));
```

as q3(2) = -0.3 matches with the previously generated angle, procede to solve for $q1$ and $q2$ using subproblem2 with equation:

$$e^{-h1^{\times}q1}(p_{16})_0 = e^{h2^{\times}q2}(p_{23} + e^{h3^{\times}q3}p_{34})$$

```
% two cone with equal waist height intersecting
% max two sol
q1 = zeros(length(q3)*2,1);
q2 = zeros(length(q3)*2,1);
q3_holder = q2;  % to expand q3 to 4 by 1
for i = 1:length(q3)
    p24 = p23 + expm(hat(robot.H(:,3))*q3(i))*p34;
    [q1(2*i-1:2*i,:),q2(2*i-1:2*i,:)] = subprob2(-robot.H(:,1),robot.H(:,2),p16,p24);
    q3_holder(2*i-1:2*i,:) = ones(2,1)*q3(i);
end
q3 = q3_holder;
```

For the rest angles: $q4, q5, q6$, we can solve them from the following equations:

$$R_{0T} = R_{01}R_{12}R_{23}R_{34}R_{45}R_{56}R_{6T}$$

$$R_{32}R_{21}R_{10}R_{0T} = R_{34}R_{45}R_{56}$$

rewrite in matrix exponential form to:

$$e^{-h4^{\times}q4}\left(e^{-h3^{\times}q3-h2^{\times}q2-h1^{\times}q1}R_{0T} \cdot h6\right) = e^{h5^{\times}q5} \cdot h6$$

solve for $q4$ and $q5$ using subproblem2:

```
% next, obtain q4 q5 with subproblem2
q4 = zeros(length(q3)*2,1);
```

```
q5 = q4;
q123_holder = zeros(length(q3)*2,3);
for i = 1:length(q3)
    r3T = expm(-hat(robot.H(:,3))*q3(i))*expm(-hat(robot.H(:,2))*q2(i))...
        *expm(-hat(robot.H(:,1))*q1(i))*r0T;
    [q4(2*i-1:2*i,:),q5(2*i-1:2*i,:)] = subprob2(-robot.H(:,4),robot.H(:,5),...
        r3T*robot.H(:,6),robot.H(:,6));
    q123_holder(2*i-1:2*i,:) = ones(2,3).*[q1(i),q2(i),q3(i)];
end
q1 = q123_holder(:,1); q2 = q123_holder(:,2); q3 = q123_holder(:,3);

% lastly obtain q6 with subproblem1
q6 = zeros(length(q3),1);
for i = 1:length(q3)
    r5T = expm(-hat(robot.H(:,5))*q5(i))*expm(-hat(robot.H(:,4))*q4(i))...
        *expm(-hat(robot.H(:,3))*q3(i))*expm(-hat(robot.H(:,2))*q2(i))...
        *expm(-hat(robot.H(:,1))*q1(i))*r0T;
    q6(i) = subprob1(robot.H(:,6),robot.H(:,3),r5T*robot.H(:,3));
end
qout = wrapToPi([q1,q2,q3,q4,q5,q6])
```

```
qout = 8×6

    0.1000    1.3853   -2.6559    0.1162    1.4490    0.1697
    0.1000    1.3853   -2.6559   -3.0254   -1.4490   -2.9719
   -3.0416   -0.2000   -2.6559   -2.8879    0.4764   -0.0426
   -3.0416   -0.2000   -2.6559    0.2537   -0.4764    3.0990
    0.1000    0.2000   -0.3000    0.4000    0.3000   -0.2000
    0.1000    0.2000   -0.3000   -2.7416   -0.3000    2.9416
   -3.0416   -1.3853   -0.3000   -3.0260    1.6335    0.1911
   -3.0416   -1.3853   -0.3000    0.1156   -1.6335   -2.9505
```

```
robot.q
```

```
ans = 1×6

    0.1000    0.2000   -0.3000    0.4000    0.3000   -0.2000
```

Clearly, row 5 was the input angles before forward kinematics.

Part 3: S-shaped Path Tracking with IRB1200

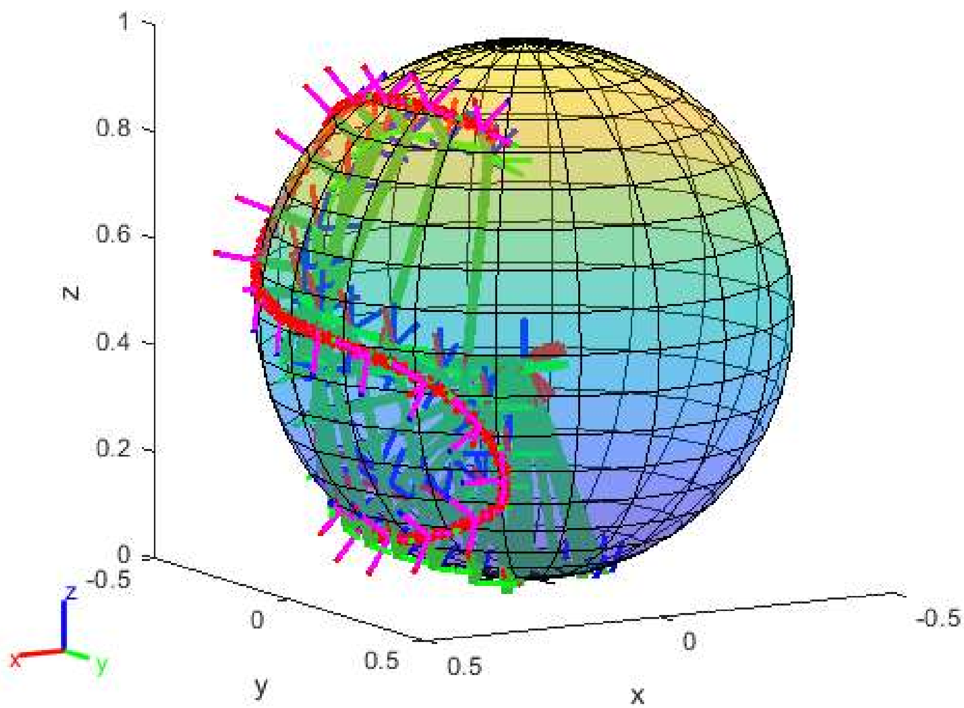
Now that we have inverse kinematics to configure all six angles of the robot at specific end-effector postions, we can have IRB1200 to trace out the s-shaped path. At the same time, we can study which pose out of all eight poses gives us the fastest path speed.

1. With end-effector orientation [xT,yT,zT] and postion pS calcuated from Part 1, we can track the S-shape using file inkinABB.m

```
solnNumber = 1;
q = zeros(6,N);
for i=1:N
    % specify end effector SE(3) frame
    %Td{i}=[[xT(:,i) yT(:,i) zT(:,i)]*R6T' pS(:,i);[0 0 0 1]];
    Td{i}=[[xT(:,i) yT(:,i) zT(:,i)] pS(:,i);[0 0 0 1]];
    irb1200.T=Td{i};
    %
    qsolns=invkinABB(irb1200); % output eight solutions
    q(:,i) = qsolns(solnNumber,:)';  % first solution (q = 1 x 6)
```

```
    % check forward kinematics to make sure the IK solution is correct
    irb1200.q=q(:,i)';
    irb1200=nlinkfwdkin(irb1200);
    T{i}=irb1200.T;
    % show robot pose (ever 10 frames)
    if mod(i,5)==0
        disp(norm(T{i}-Td{i}));
        figure(2);show(irb1200_rbt,q(:,i),'collision','on');
        view(150,10);
    end
end
```

```
3.7049e-16

2.6442e-16

3.3870e-16

2.5821e-16

2.6702e-16

3.5370e-16

5.2458e-16

3.2182e-16
```



2. Now if we pose an angular velocity constraints $\frac{dq}{dt} = 2$rad/sec, we would like to know which pose out of all eight poses has the fastest constant path velocity $\frac{d\lambda}{dt}$:

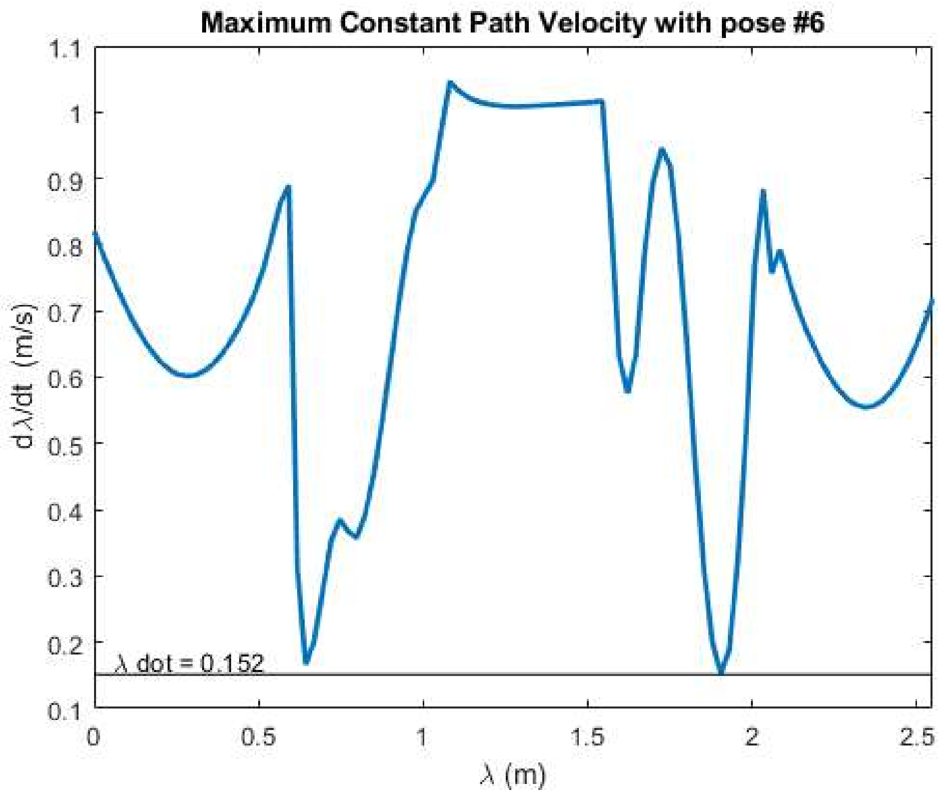
$$\frac{dq}{dt} = \frac{dq}{d\lambda} \cdot \frac{d\lambda}{dt}$$

as $\frac{d\lambda}{dt}$ need to satisfy all angles at this instance, the maximum $\frac{dq}{d\lambda}$ would give us a safe $\frac{d\lambda}{dt}$.

```
ldot = zeros(8,100);
q3s = zeros(8,101);
dq3 = zeros(8,1);
for sol = 1:8
    for i = 1:N
        Td{i}=[[xT(:,i) yT(:,i) zT(:,i)] pS(:,i);[0 0 0 1]];
        irb1200.T=Td{i};
        qsolns=invkinABB(irb1200); % output eight solutions
        q(:,i) = qsolns(sol,:)';
    end
    dqdl = max(abs(diff(q')'))./diff(l')';
    ldot(sol,:) = 2./dqdl;
    %  elbow joint q3 variation
    q3s(sol,:) = q(3,:);
    dq3(sol) = max(q(3,:)') - min(q(3,:)');
end
[pose_min,pose_min_index] = min(ldot,[],2);
[ldot_max,pose] = max(pose_min)
```

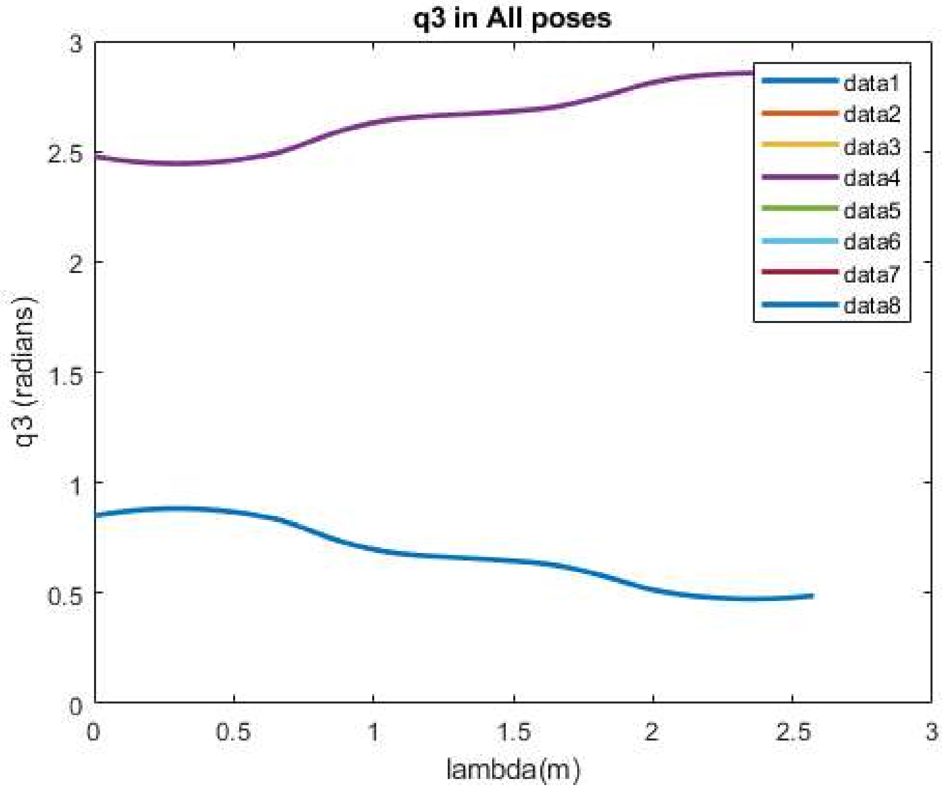
```
ldot_max = 0.1518
pose = 6
```

```
figure;plot(l(1:end-1),ldot(pose,:),'linewidth',2);
hold on;
hline(ldot_max,'k',['\lambda dot = ',num2str(round(ldot_max,3))]);
hold off
xlabel('\lambda (m)')
ylabel('d\lambda/dt  (m/s)')
xlim([0,l(end-1)])
title('Maximum Constant Path Velocity with pose #6')
```



while there are 8 ways for IRB1200 to track the path, most other pose has intense discontinuities across the path. pose 6, however, has overall smooth path. It's speed was constrained by near singlerity position near the turning point of the "S" shape.

```
figure;plot(l,q3s,'LineWidth',2);
xlabel('lambda(m)')
ylabel('q3 (radians)')
title('q3 in All poses')
legend
```




```
dq3
```

```
dq3 = 8×1
    0.4096
    0.4096
    0.4096
    0.4096
    0.4096
    0.4096
    0.4096
    0.4096
```

It is quite counterintuitive that the variation of q3 are the same for all poses; however, thinking more indepthly, there're only two choices from the result of subproblem3.

Part 4: End-effector Spatial Velocity

Knowing pose #6 to be the fastest pose with $\frac{d\lambda}{dt} = 0.152m/s$, the end-effector angular velocity can be calculated using the follwoing equation:

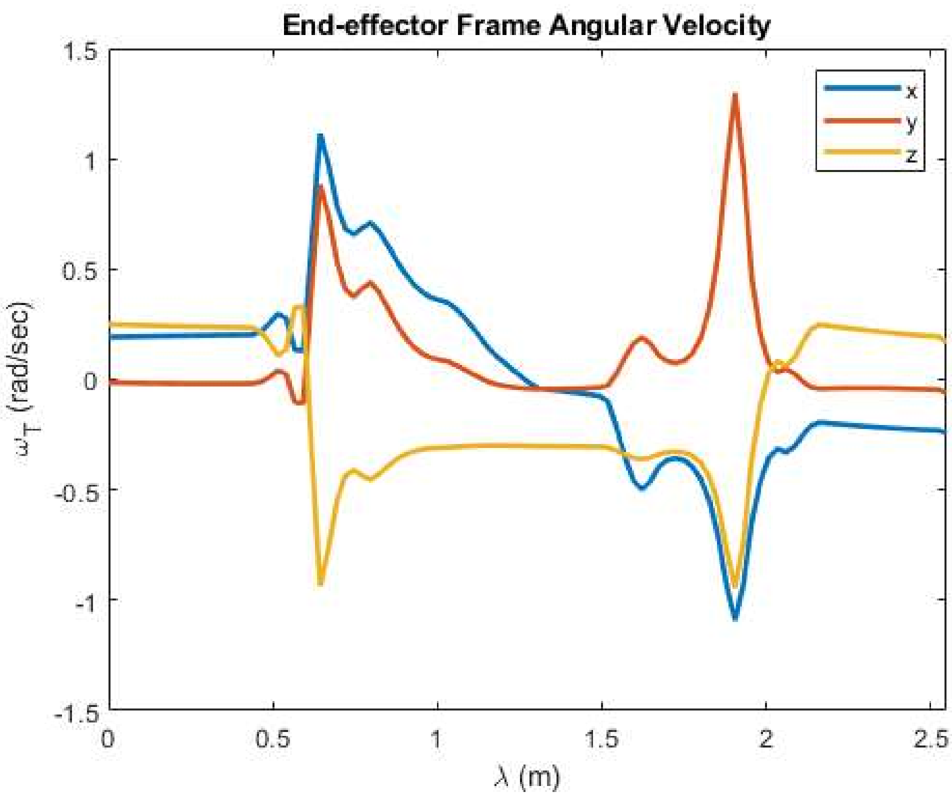
$$\dot{R}_{0T}(k) = \frac{R_{0T}(k+1) - R_{0T}(k)}{\lambda_{k+1} - \lambda_k} \cdot \frac{d\lambda}{dt}$$

$$(\omega_T(k))_0 = \left(\dot{R_{0T}}(k) R_{0T}^T(k)\right)^V$$

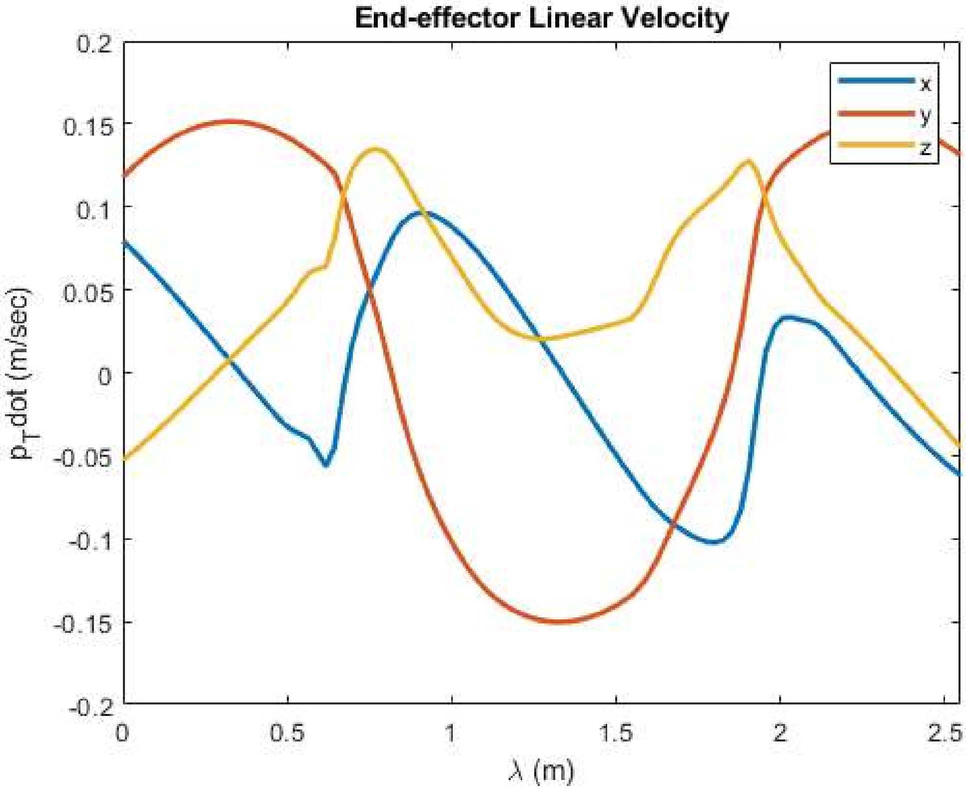
$$(\dot{P_T}(k))_0 = \frac{p_{0T}(k+1) - p_{0T}(k)}{\lambda(k+1) - \lambda(k)} \cdot \frac{d\lambda}{dt}, \text{ where } k \in \{1, 2, \cdots, N-1\}$$

```
wT = zeros(3,N-1);
pTdot = zeros(3,N-1);
for k = 1:N-1
    dT0T = Td{k+1} - Td{k};
```

```
    % calculating omegaT
    dr0T = dT0T(1:3,1:3);
    rdot = dr0T/(l(k+1)-l(k))*ldot_max;
    r0T = Td{k}(1:3,1:3);
    wT(:,k) = vee(rdot*r0T');
    % calculating pTdot
    dp0T = dT0T(1:3,4);
    p0T = Td{k}(1:3,4);
    pTdot(:,k) = dp0T/(l(k+1)-l(k))*ldot_max;
end

figure;
plot(l(1:N-1),wT,'linewidth',2);
xlabel('\lambda (m)')
ylabel('\omega_{T} (rad/sec)')
legend('x','y','z')
xlim([0,l(N-1)])
title('End-effector Frame Angular Velocity')
```




```
figure;
plot(l(1:N-1),pTdot,'linewidth',2);
xlabel('\lambda (m)')
ylabel('p_Tdot (m/sec)')
legend('x','y','z')
xlim([0,l(N-1)])
title('End-effector Linear Velocity')
```



By visually inspecting the result, both plots are quite reasonable. The kink at $\lambda = 0.64$ was also shown at the $\dot{\lambda}$ plot from Part3 4). As arm pos gets closer to singularity, the rotation speed of joints increase and cause the two spikes in the first plot.