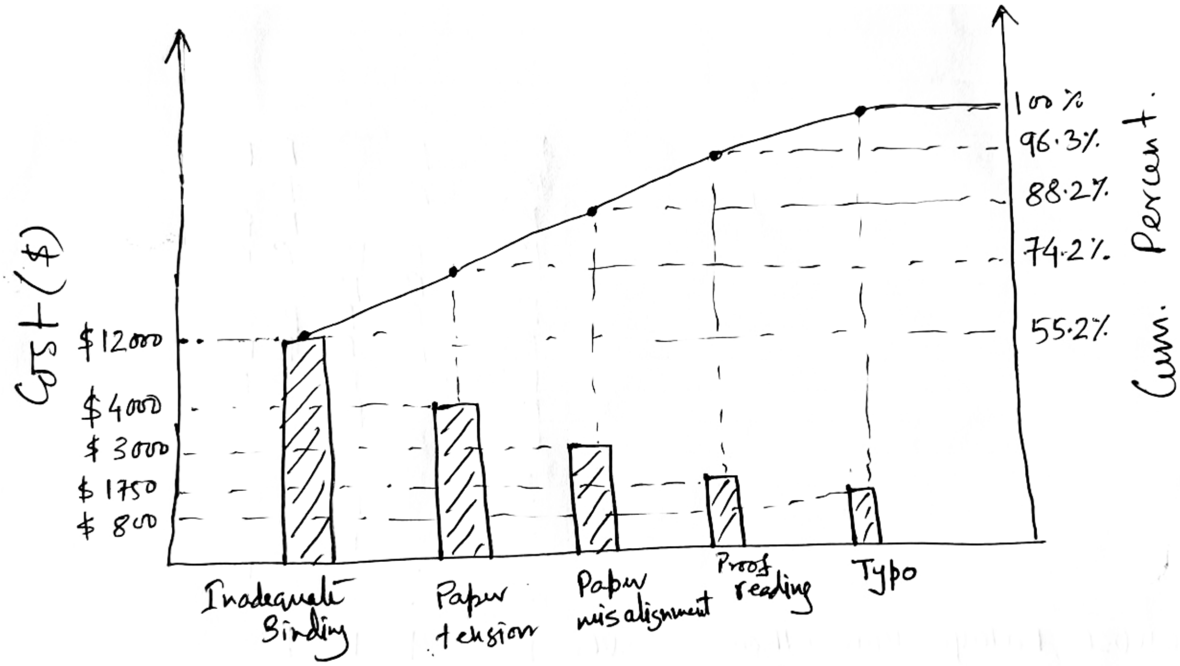
Q1



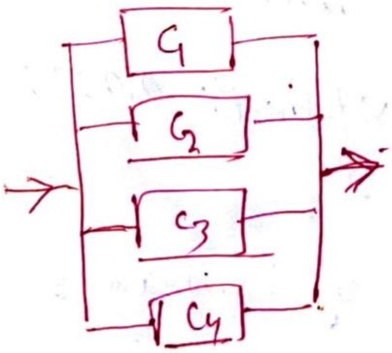
Costs	12000	4000	3000	1750	800	Total = $21550
Percent	55.7	18.6	13.9	8.1	3.7	
Cum%	55.7	74.2	88.2	96.3	100.0	

Pareto chart for error categories.

Budget = $18000 = $\frac{18000}{21550}$ = 83.53%

Areas to Tackle = Inadequate Binding,
Paper Tension,
Paper misalignment (66.66% rectification)

Q2



$n = 4. \quad p = 0.9$

$P(\text{System Functioning}) = P[X \geq 1]$

$= 1 - P[X = 0]$

$= 1 - {}^4C_0\, 0.9^0 \times 0.1^4$

$= 1 - 0.0001$

$= 0.9999$

$P(\text{System Failing})$

$= 1 - P(\text{Sys Func})$

$= 0.0001$

—x—

Q3

Outside diameter = quality issues. I X

$\mu = 40\text{m}, \quad \sigma = 2.5 \text{ mm}$

$\frac{x-\mu}{\sigma}$

Specification limits are: $(36, 45)$

$z_1 = \frac{36-40}{2.5} = -1.60$

$z_2 = \frac{45-40}{2.5} = \frac{5}{2.5} = 2.00$

Daily cost of scrap $= 2000 \times 0.0548 \times 0.50$

$= \$54.80$

$\Phi(z_1) = \Phi(-1.60) = 0.0548$

$\Phi(z_2) = \Phi(2.00) = 0.9772$

Daily cost of ~~repairwork~~ rework

$= 2000 \times 0.2 \times \$ 0.0228$

Total ~~daily cost~~ daily costs of rework & scrap $= \$63.92$

Q4

$\mu = 4000$ Kg

$\sigma = 25$ Kg.

$$P[X > 4050] = P\left[\frac{X-4000}{25} > \frac{4050-4000}{25}\right] = P[Z > 2] = 1 - P[Z \le 2] = 1 - \phi(Z)$$

$$Z = \frac{4050-4000}{25} \qquad \frac{x-\mu}{\sigma}$$

$$= 2$$

$$\boxed{1 - \phi(Z) = 0.0228}$$

Not meeting the reaurirement.

Only 2.28% of the products has a strength that exceeds 4050 Kg.

$$Z = \frac{x-\mu}{\sigma}$$

$$-1.645 = \frac{4050-\mu}{25}$$

$$\mu = 4050 + 1.645 \times 25 = 4091.125 \text{ kg.}$$

$$\boxed{\begin{aligned} &P[X>4050] = 0.95 \\ &\Rightarrow 1-\phi(z) = 0.95 \\ &\Rightarrow \phi(z) = 0.05 \\ &\Rightarrow z = -1.645 \end{aligned}}$$

$$Z = \frac{x-\mu}{\sigma}$$

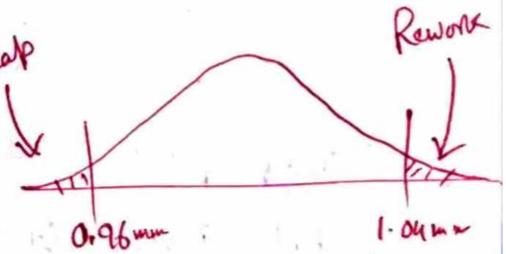
$$1 - P[X \le 4050]$$
$$= 1 - P[Z\sigma + \mu \le 4050]$$
$$= 1 - P\left[Z \le \frac{4050-\mu}{\sigma}\right]$$
$$= 1 - P[Z \le 2.00]$$
$$= 1 - \phi(2)$$

$\propto$

Q5

1.0 ± 0.04 mm

USL = 1.04 mm
LSL = 0.96 mm



(a) $X \sim N(0.98\,\text{mm}, 0.02\,\text{mm})$ $N, (\mu, \sigma)$

$\mu =$ $\sigma =$

$$Z_1 = \frac{USL - 0.98}{0.02} = \frac{1.04 - 0.98}{0.02} = 3$$

$$Z_2 = \frac{LSL - 0.98}{0.02} = \frac{0.96 - 0.98}{0.02} = -1$$

Proportion of conforming washers

$$= 1 - (\underbrace{0.1587}_{\text{Scrap.}} + \underbrace{0.0013}_{\text{Rework}}) = 0.84$$

Daily cost of rework $= 10000 \times 0.0013 \times 0.1$
$= \$1.30$

" " scrap $= 1000 \times 0.1587 \times 0.15$
$= \$238.05$

Daily total cost $= \$239.35$

(b) $\mu = 1.0, \quad \sigma = 0.02$

$$z_1 = \frac{0.96 - 1}{0.02} = -2.00 \qquad z_2 = \frac{1.04 - 1}{0.02} = 2.00$$

Proportion of rework $= 0.0228 = 1 - \Phi(z_2)$

" " (scrap $= 0.0228 = \Phi(z_1)$

Total Daily cost $+$) $= 10000 \times 0.0228\ (0.15 + 0.10)$

$= \$57$

(c) $\mu = 1.0, \quad \sigma = 0.015$

$$z_1 = \frac{0.96 - 1}{0.015} = -2.67$$

$$z_2 = \frac{1.04 - 1}{0.015} = +2.67$$

Scrap portion $= 0.0038$, Rework $= 0.0038$

Total Daily cost Scrap + Rework $= 10000 \times 0.0038 \times (0.15 + 0.1)$

$= \$9.5$

% decrease $= 98.03\%$

———— α ————

Q6

$f(x) = \lambda e^{-\lambda x}$ PDF

$f(T) = \lambda e^{-\lambda T}$

$\mu_T = \frac{1}{\lambda} = 10000$ hr.

$$\boxed{\int t f(t)\, dt = \int_0^{\infty} t \lambda e^{-\lambda t}\, dt = \frac{1}{\lambda}}$$

$$\boxed{CDF = \int_0^{t} f(t)\, dt = 1 - e^{-\lambda t}}$$

(a) $P(T > 8000)$

$= 1 - P(T \leq 8000)$

$= 1 - (1 - e^{-\lambda \times 8000})$

$= 0.449$

(b) $P(T > 15000 \mid T > 9000)$

$$= \frac{P(T > 15000)}{P(T > 9000)}$$

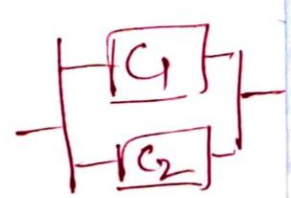
$$= \frac{e^{-\lambda 15000}}{e^{-\lambda 9000}}$$

$= e^{-\lambda 6000}$

$= P(T > 6000) = 0.549$

(c) $A \equiv$ Comp 1 operates 12000 hrs

$B \equiv$ Comp 2 " 12000 hrs.

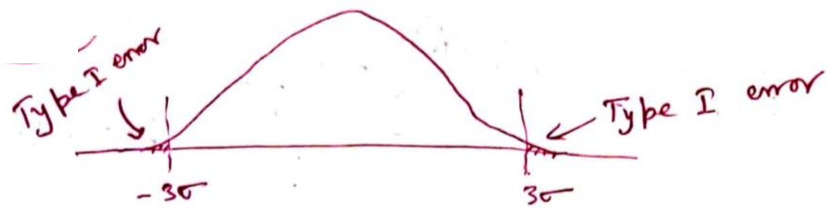


$P(A) = 1 - (1 - e^{-\lambda \times 12000}) = 0.30119$

$P(B) = 0.30119$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5117$

Q7



* Probability of Type I error $= P[X < \mu - 3\sigma] + P[X > \mu + 3\sigma]$

$$= P\left[\frac{X-\mu}{\sigma} < -3\right] + P\left[\frac{X-\mu}{\sigma} > 3\right]$$

$$= P[Z < -3] + P[Z > 3]$$

$$= \Phi(-3) + (1 - \Phi(3))$$

From Standard normal CDF / z table $\rightarrow$

$$= 0.00135 + (1 - 0.99865)$$

$$= 0.00135 + 0.00135$$

$$= 0.0027$$

* Probability of two out of three consecutive points falling outside the $2\text{-}\sigma$ limits $\Rightarrow$

$$= {}^3C_2\, p^2 \times (1-p) \quad \text{where } p = P[X < \mu - 2\sigma] + P[X > \mu + 2\sigma]$$

$$= P\left[\frac{X-\mu}{\sigma} < -2\right] + P\left[\frac{X-\mu}{\sigma} > 2\right]$$

$$= P[Z < -2] + P[Z > 2]$$

$$\cancel{= 0.02275 + (1-}$$

$$= \Phi(-2) + (1 - \Phi(2))$$

$$= 0.02275 + (1 - 0.97725)$$

$$= 0.02275 + 0.02275$$

$$= 0.0455$$

$$= {}^3C_2 \times 0.0455^2 \times (1 - 0.0455)$$

$$= 3 \times 0.0455^2 \times (1 - 0.0455)$$

$$= 0.005928$$

* Probability of 4 out of 5 consecutive points falling outside the one-sigma limits $\Rightarrow$

$$ {}^5C_4 \times p^4 \times (1-p) \quad \text{where } p = P[X < \mu - \sigma] + P[X > \mu + \sigma]$$

$$= P[Z < -1] + P[Z > 1]$$

$$= \Phi(-1) + [1 - \Phi(1)]$$

$$= 0.15866 + 0.15866 = 0.31732$$

$$= 5 \times 0.31732^4 \times (1 - 0.31732)$$

$$= 0.03461804$$

—x—

Q8

Average diameter = 15 mm = $\bar{X}$ = μ

Std. dev. of diameter = 0.8 mm = σ

$CL = \bar{X} = 15$ mm

Std. dev. of sample mean $= \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.8}{\sqrt{4}}$

$= 0.4$ mm

(a) One-sigma control limits: $\mu \pm 1\sigma_{\bar{x}} = 15 \pm 0.4 = (14.6, 15.4)$ mm

Two-sigma control limits: $\mu \pm 2\sigma_{\bar{x}}$

$= 15 \pm (2 \times 0.4)$

$= 15 \pm 0.8$

$= (14.2, 15.8)$ mm

(b) $3-\sigma$ control limits $= \mu \pm 3\sigma_{\bar{x}}$ mm

$= 15 \pm (3 \times 0.4)$ mm

$= 15 \pm 1.2$

$= (13.8, 16.2)$ mm

(c) Probability of false alarm = Prob. of Type I error

$= P[X < \mu - 3\sigma] + P[X > \mu + 3\sigma]$

$= P\left[\frac{X-\mu}{\sigma} < -3\right] + P\left[\frac{X-\mu}{\sigma} > 3\right]$

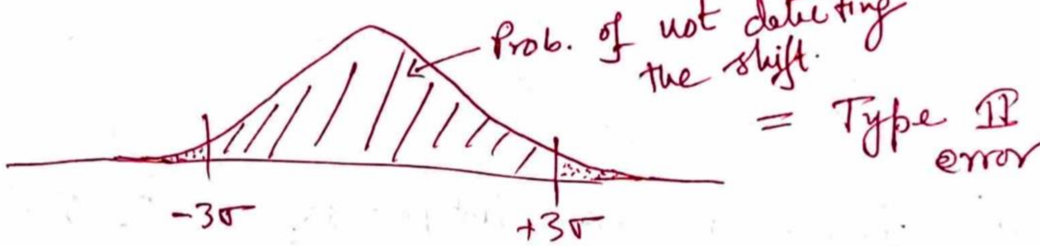
$= \Phi(-3) + (1 - \Phi(3))$

$= 0.0027$

(d)

Standardised normal values at the control limits after the shift:

$$z_1 = \frac{16.2 - 14.5}{0.4} = 4.25 \quad \Big| \quad z_2 = \frac{13.8 - 14.5}{0.4} = -1.75$$



$$\begin{aligned} \text{Prob.} &= \Phi(z_1) - \Phi(z_2) \\ &= \Phi(4.25) - \Phi(-1.75) \\ &= 1 - 0.04006 \\ &= 0.9599 \end{aligned}$$

$$ARL = \frac{1}{1 - 0.9599} = 24.9626$$

(e)

Prob. of detecting at second sample = Prob. of not detecting on first shift × Prob. of detecting on second shift

$$= 0.9599 \times (1 - 0.9599)$$

$$= 0.0385 \quad \text{Ans.}$$

** Prob. of failing to detect by the second sample $= 1 - 0.0786 = 0.9214$

Prob. of detecting by second sample $= (1 - 0.9599) + 0.0385$

$$= 0.0401 + 0.0385$$

$$= 0.0786$$

— x —

Q9

25 Samples $= m$ Sample / Batch Size $= 4 = n$

$\bar{\bar{x}} = \frac{1000}{25} = 40$ $\bar{R} = \frac{250}{25} = 10$

(a) $\bar{X}$ - Chart control limits,

$CL = \bar{\bar{X}} = 40$ minutes.

$UCL = \bar{\bar{X}} + A_2 \bar{R} = 40 + (0.7286) \times 10 = 47.286$ min.

$LCL = \bar{\bar{X}} - A_2 \bar{R} = 40 - (0.7286) \times 10 = 32.714$ min

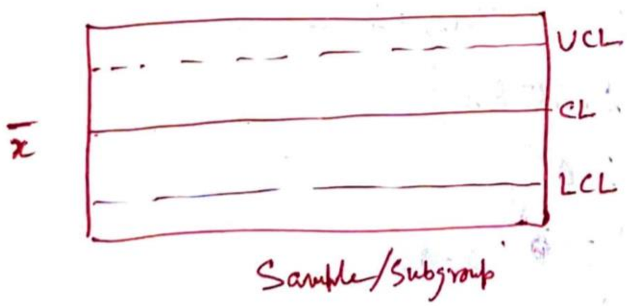
R - Chart control limits

$CL = \bar{R} = 10$ min $\boxed{R_i = x^i_{max} - x^i_{min}}$

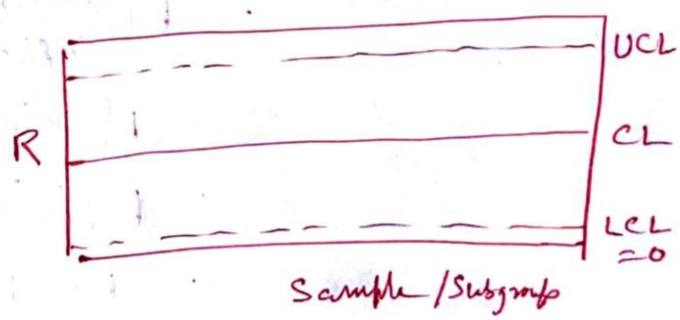
$UCL = \bar{R} D_4 = 10 \times 2.2821 = 22.821$ min

$LCL = \bar{R} D_3 = 10 \times 0.000 = 0$ min

$\bar{X}$ – Chart



R – Chart

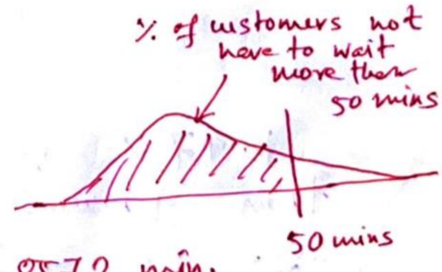


(b) Waiting time = X (Random variable)

Process mean = 40 min.

Process standard deviation $= \frac{\bar{R}}{d_2}$

$= \frac{10}{2.0588} = 4.8572$ min.



$P[X < 50] = P\left[\frac{x-40}{4.8572} < \frac{50-40}{4.8572}\right] = P[Z < 2.06]$

$= \Phi(2.06)$

$P[X<50] = \Phi(2.06) = 0.98030$

→ 98.03% of customers will not wait more than 50 mins.

(c) 2-σ limits for $\bar{X}$ - Chart

$$40 \pm 2 \times \frac{4.8572}{\sqrt{4}} = 40 \pm 4.8572$$

$$= (35.143, 44.857) \text{ min}$$

(d) New process average waiting time = 30 min.

$$P[X>40] = P\left[\frac{X-\mu}{\sigma} > \frac{40-\mu}{\sigma}\right]$$

$$= P\left[\frac{X-30}{4.8572} > \frac{40-30}{4.8572}\right]$$

$$= P[Z > 2.0588]$$

$$= P[Z > 2.06]$$

$$= 1 - P[Z \leq 2.06]$$

$$= 1 - \Phi(2.06)$$

$$= 1 - 0.9803$$

$$= 0.0197$$

→ 1.97% of customers will have to wait more than 40 mins.

$$P[X>50] = P\left[Z > \frac{50-30}{4.8572}\right] = P[Z>4.12] = 1-\Phi(4.12)$$

$$\approx 0.0000$$

Proportion of Customers have to wait more than 50 mins is negligible

—— X ——

Q10

Specifications on baking time = 10 ± 0.2 min.

USL = 10.2 min ; LSL = 9.8 min

20 Samples : $m = 20$; Sample / Batch Size = 6 ; $n = 6$

(a)

$\bar{x}$ – Chart

$CL = \bar{\bar{x}} = \frac{199.8}{20} = 9.99$ min

$UCL = \bar{\bar{x}} + A_3 \bar{s}$

$= 9.99 + 1.287 \times 0.07$

$= 10.080$ min

$LCL = \bar{\bar{x}} - A_3 \bar{s}$

$= 9.99 - 1.287 \times 0.07$

$= 9.90$ min

S – Chart

$CL = \bar{s} = \frac{1.40}{20} = 0.07$ min.

$UCL = B_4 \bar{s}$

$= 1.970 \times 0.07$

$= 0.1379$ min.

$LCL = B_3 \bar{s}$

$= 0.030 \times 0.07$

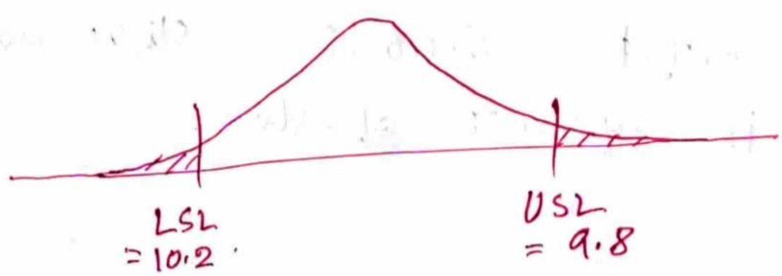
$= 0.0021$ min

(b) Process mean $= 9.99$ min $= \bar{\bar{x}}$

Process standard deviation $= \frac{\bar{s}}{c_4} = \frac{0.07}{0.9515} = 0.0736$

Shewhart constant

(c)



$P[x < LSL] + P[x > USL] = \Phi\left(\frac{LSL - 9.99}{0.0736}\right) + \left[1 - \Phi\left(\frac{USL - 9.99}{0.0736}\right)\right]$

$$= \phi\left(\frac{9.8-9.99}{0.0736}\right) + \left[1-\phi\left(\frac{10.2-9.99}{0.0736}\right)\right]$$

$$= 0.0049 + 0.0022$$

$$= 0.0071$$

Non conforming output = 0.71 %

The process is 99.29% capable.

(d) If the process mean shifts to 10 min, the standardized normal values at the specification limits are:

$$z_1 = \frac{9.8-10}{0.0736} = -2.72$$

$$z_2 = \frac{10.2-10}{0.0736} = 2.72$$

$$P[X < LSL] + P[X > USL]$$

$$= \phi(-2.72) + [1-\phi(2.72)]$$

$$= 0.0033 + 0.0033$$

$$= 0.0066$$

Non conforming output = 0.66 % (slight reduction)

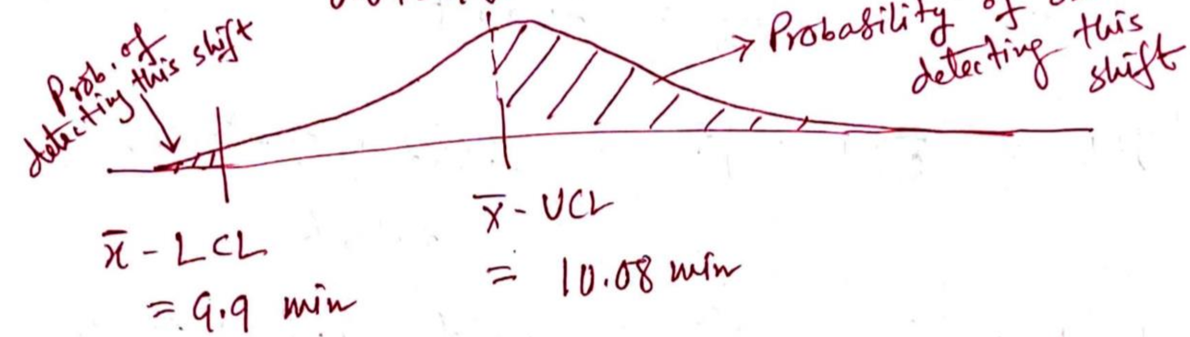
The process is 99.34% capable.

e)

Standardized normal values:

$$Z_1 = \frac{9.90 - 10.2}{(0.0736/\sqrt{6})} = -9.98$$

$$Z_2 = \frac{10.08 - 10.2}{0.0736/\sqrt{6}} = -3.99$$



Prob. of detecting this shift:

$$P[\bar{X} < LCL] + P[\bar{X} > UCL]$$

$$= \Phi(-9.98) + (1 - \Phi(-3.99))$$

$$\approx 1.0000$$

Probability of detection on the first sample taken $\approx 1.0000 \approx 100.00\%$

— ∝ —

Q11

Count of nonconformances chart

$\rightarrow$ c- chart

$$CL = \bar{c} = \frac{80}{30} = 2.667$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}} = 2.667 + 3\times\sqrt{2.667}$$
$$= 7.566$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}} = 2.667 - 4.899 \approx 0.0$$

For the specified ~~good~~ goal:

$$CL = 0.5 = \bar{c}$$
$$UCL = 0.5 + 3\sqrt{\bar{c}} = 0.5 + 3\times\sqrt{0.5}$$
$$= 0.5 + 2.121$$
$$= 2.621$$
$$LCL = 0.5 - 3\sqrt{0.5} = 0.5 - 2.121 \approx 0$$

[Count of nonconformances can not be negative]

Process mean $= \frac{2.667}{3} = 0.889$ blemishes per $100\ m^2$

" std. dev. $= \sqrt{0.889} = 0.943$

$X \equiv$ No. of blemishes per $100\,m^2$

$\hookrightarrow$ A Poisson random variable.

$$P[X < UCL] = P[X < 2.621] = P[X \leq 2]$$

$$= \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^{x}}{x!} \quad (\lambda = 0.889)$$

$$= e^{-0.889} \times \left(1 + 0.889 + \frac{0.889^2}{2!}\right)$$

$$= e^{-0.889} \times (1 + 0.889 + 0.3952)$$

$$= e^{-0.889} \times 2.2842$$

$$= 0.41107 \times 2.2842$$

$$= 0.93896$$

So, about 6.1% of the time, the process will be out of control.

— $\propto$ —

Q12

Natural Tolerance limits = 3-σ limits

$= 44 \pm 3(3) = (35, 53)$ ppm.

$z_1 = \frac{40-44}{3} = -1.33$

$z_2 = \frac{55-44}{3} = 3.67$

$P[X < LCL] + P[X > UCL] = \phi(z_1) + (1 - \phi(z_2))$

$= \phi(-1.33) + (1 - \phi(3.67))$

$= 0.0918$

—x—

Q13

$$C_p, PCR = \frac{USL - LSL}{6\sigma} = \frac{55-40}{6 \times 3}$$

$$= 0.833$$

$$C_p < 1$$

$$\frac{1}{C_p} = 1.2 = 120\%$$

The process uses up 120% of the specification range.

Process mean should be shifted to 47.5 ppm — mid point between the specification limits.

2)

$$\frac{55-47.5}{3} = +2.5 = z_1$$

$$\frac{40-47.5}{3} = -2.5 = z_2$$

$z_1 = z_2 = \pm 2.5$

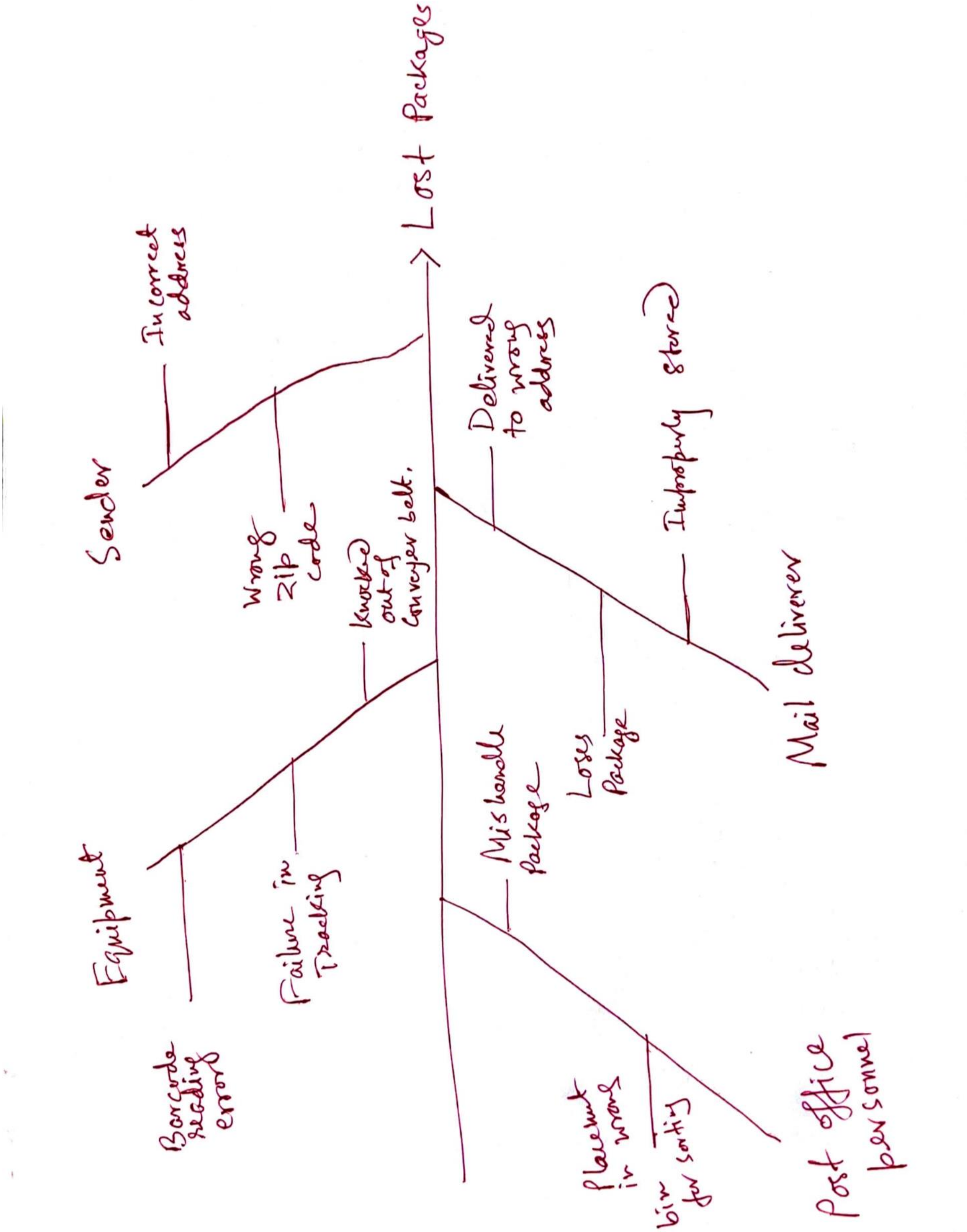
Proportion non conforming

$= 0.0062 \times 2$

$= 0.0124$

— $\propto$ —

Q14



Lost Packages
Sender
Incorrect address
Wrong zip code
Equipment
Knocked out of conveyer belt.
Failure in tracking
Barcode reading errors
Mail deliverer
Delivered to wrong address
Improperly stored
Loses package
Post office personnel
Mishandle package
Placement in wrong bin for sorting