INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Mid Spring Semester Examination 2023-24

Date of Examination: 19-02-2024 **Session:(FN/AN)** AN **Duration:** 2 Hrs **Full Marks:** 60

Subject No. : RE30003 **Subject :** INTRODUCTION TO QUALITY

Department/Center/School: Subir Chowdhury School of Quality and Reliability

Specific charts, graph paper, log book etc., required: No

Special Instructions: Answers to all problems should be given in the same order as they appear in the question paper. Non-programmable digital calculator can be used for calculations. Clearly mention reasonable assumption(s), if any, while answering the questions. Answer all questions.

1) **[25]**

a) What do you understand by the dimensions of quality? Name any three quality dimensions and briefly discuss what they represent and the customer concerns they address. **(3)**

b) Explain the two main types of quality definitions. What is your understanding of quality improvement? Define nonconforming and defective products, and elaborate on the distinctions between them. **(3)**

c) Name three statistical methods for quality improvement, briefly explaining each. Additionally, draw a visual representation of a control chart, highlighting its main elements. Discuss the stages where different quality engineering or improvement methods are typically applied within an organization. **(5)**

d) Explain the four steps (the PDCA cycle) of the Shewhart cycle. **(3)**

e) Deming's philosophy was summarized in 14 points. Discuss any three of these points and share your opinion on each. **(3)**

f) Explain the concept of Six Sigma and outline the steps of the DMAIC framework. **(4)**

g) Name four main types of quality costs and provide a brief explanation for each. **(4)**

2) A bicycle company produces bicycles in three of their manufacturing plants at different locations: P1, P2, and P3. These plants manufacture 20%, 35%, and 45%, respectively, of the bicycles. Past experience indicates that 4%, 1%, and 2% of the bicycles made by each plant, respectively, are defective. Use the total probability theorem and Bayes' rule to answer the following questions. **[7]**

a) Suppose a finished bicycle is randomly selected. What is the probability that it is defective? **(2)**

b) What is the probability that a defective bicycle belongs to Plant P3? **(3)**

c) What will be the change in this probability if the proportions of manufacturing bicycles are switched from 20%, 35%, and 45% to 45%, 35%, and 20%, respectively, for the three plants? **(2)**

3) An examination was conducted within a class consisting of 12 students. The scores of these students are 69, 77, 76, 82, 75, 67, 62, 79, 87, 92, 72, and 88 out of 100. **[8]**

a) What is the average score of the students? Create a box-and-whisker plot for the examination scores. Determine the interquartile range (IQR) of the scores. Did you identify any outliers? If not, explain why. **(4)**
b) Create a stem-and-leaf plot for the examination scores and explain the steps involved. Can you identify the stem with the highest concentration of data points? **(4)**

4) Consider a process that consists of a sequence of n independent trials, i.e., the outcome of each trial does not depend in any way on the outcome of previous trials. When the outcome of each trial is either a "success" or a "failure," the trials are called Bernoulli trials. If the probability of "success" on any trial is p, then the number of "successes" X in n Bernoulli trials follows the binomial distribution: **[10]**

$$P(X=k) = {}^{n}C_k\, p^k (1-p)^{n-k}, \quad k = 0, 1, 2, \cdots, n$$

where

$${}^{n}C_k = \frac{n!}{k!(n-k)!}$$

The mean (μ) and variance (σ^2) of any discrete distribution can be determined using the following formulas:

$$\mu = \sum_{k=0}^{n} kP(X=k), \quad \sigma^2 = \sum_{k=0}^{n} (k-\mu)^2 P(X=k)$$

a) Using the above expressions, show that the expectation or mean of a binomial random variable is *np*. **(3)**
b) Using the above expressions, show that the variance of a binomial random variable is *np(1-p)*. **(3)**
c) Suppose the probability of defective or nonconforming items in a population is 0.05. In a sample of 25 items, determine the expected or mean number of items that are defective or nonconforming. **(1)**
d) What are the mean and standard deviation for the number of defective or nonconforming items in another sample of 35 items? **(1)**
e) What are the probabilities of obtaining one or fewer defective or nonconforming items in both 25 and 35 samples? **(2)**

5) A battery manufacturer is supposed to deliver 1000 lithium-ion batteries to an automobile manufacturer for their electric vehicles (EVs) platform. As a quality characteristic, the automobile manufacturer is very critical about the reliability of the batteries. However, the battery manufacturer has provided information only on the time-to-failure (TTF) distribution of the batteries. **[10]**

The TTF can be represented as a random variable T, the TTF distribution can be represented as $f(t)$, and the corresponding reliability function can be represented as $R(t)$, which is the probability of the TTF being more than a particular operating time t, i.e., $R(t) = P(T>t)$. The reliability can be calculated as

$$R(t) = P(T>t) = \int_{t}^{\infty} f(t)dt$$

The relation between the reliability function *R(t)* and the TTF distribution *f(t)* can be written as

$$f(t) = -\frac{dR(t)}{dt}$$

The mean time to failure (MTTF) is also an important quality characteristic for the battery. The MTTF can be computed using the following mathematical expression:

$$MTTF = \int_0^\infty t f(t)dt$$

a) Using the above expressions, show that the MTTF can also be written as **(3)**

$$MTTF = \int_0^\infty R(t)dt$$

b) The battery manufacturer claimed that the TTF of batteries follows an exponential distribution given below:

$$f(t) = 0.00019e^{-0.00019t}, \quad t \geq 0$$

where t is in *hours*. Using the above expressions, derive the reliability function for the batteries. **(3)**

c) What is the MTTF of the batteries? Report the decrease in the battery reliability after using them until their MTTF. **(2)**

d) Determine the operational duration of these batteries for a drop in 90% of the reliability. How many supplied batteries out of 1000 will survive until this operational duration? **(2)**