State Space Search

13/01/2025

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Search Frameworks

- State space search
  - Uninformed/Blind search
  - Informed/Heuristic search

- Problem reduction search
  - Decompose the problem into parts
  - Solve parts best way
  - Integration by parts

- Game tree search

- Advances
  - Memory bound Search
  - Multi-objective Search
  - Learning how to search

State Space Search

- Basic Search Problem:

  - Given: [S,s,O,G] where
    - S is the [implicitly specified] set of states
    - s is the start state
    - O is the set of state transition operators
    - G is the set of goal states

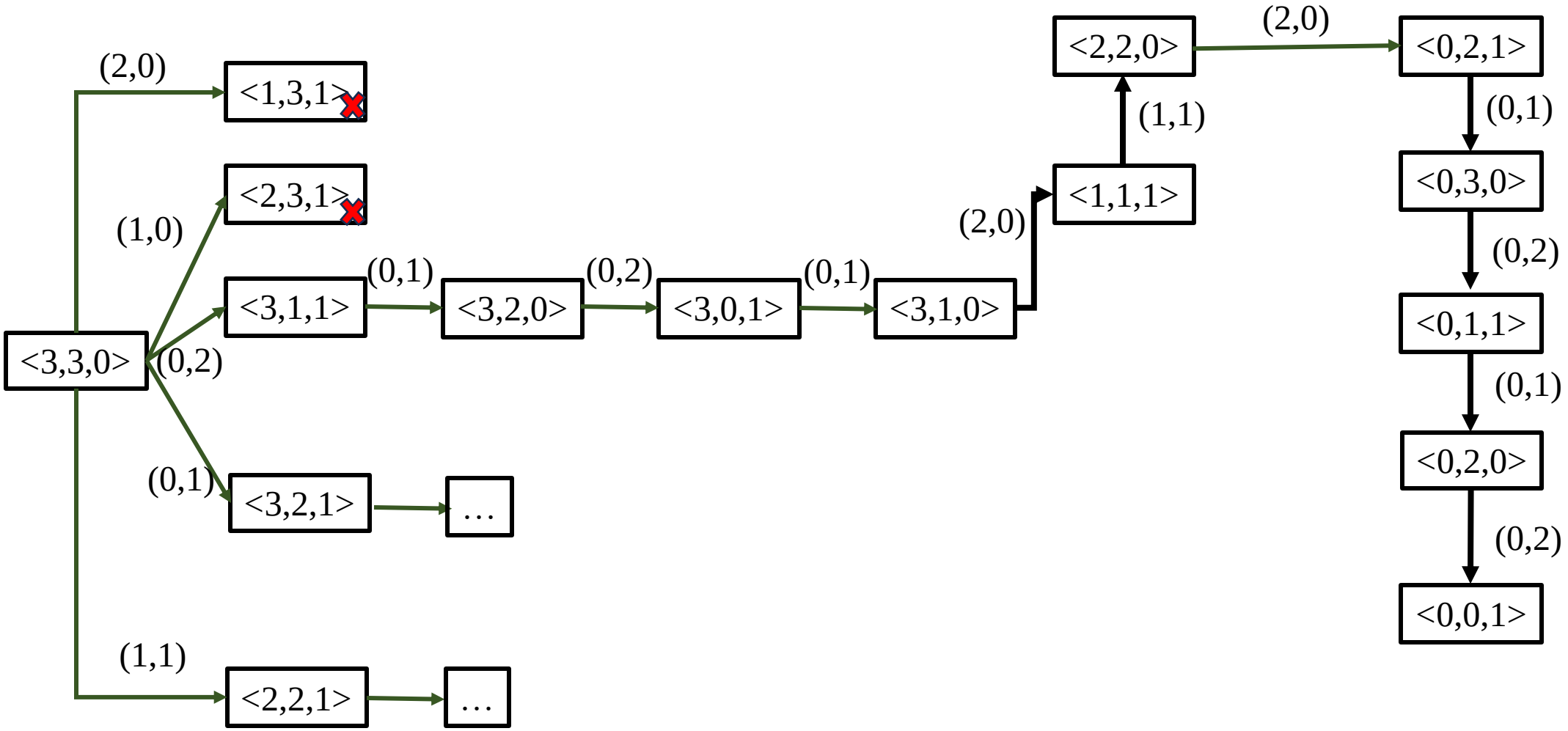
  - To find a sequence of state transitions leading from s to a goal state

Missionaries and Cannibals

- **State:** (#m, #c, 1/0)
- #m: Number of missionaries in the first bank
- #c: Number of cannibals in the first bank
- The last bit indicates whether the boat is in the first bank

- Start state: (3,3,0)
- Goal state: (0,0,1)
- Operators: Boat carries (#missionaries, #cannibals)
- (1,0), (0,1), (1,1)
- (2,0), (0,2)

Missionaries and Cannibals: Search



Search: Challenges

- Is the search space a graph or tree?
  - While exploring the states same state may appear multiple times

- Is it important to identify such repetitive states?
  - Yes
  - It may lead to infinite loop

Basic State Space Search

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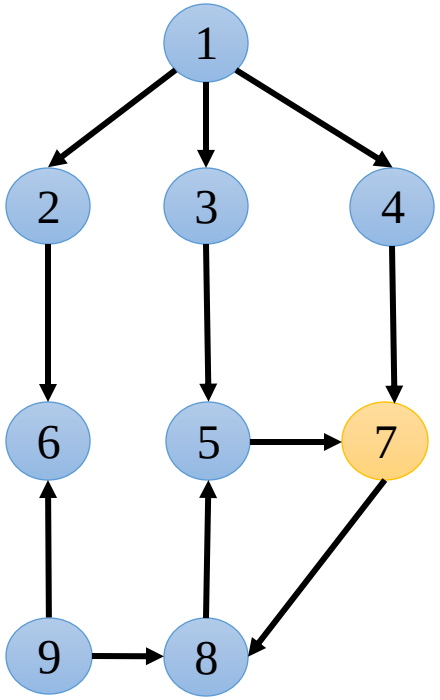
Basic Search Algorithm

- Initialize: Set OPEN={s}
- Fail:
  - If OPEN={}, Terminate with failure
- Select: Select a state, n, from OPEN
- Terminate:
  - If n∈G, terminate with success
- Expand:
  - Generate the successors of n using O and insert them in OPEN
- Loop:
  - Go to step 2

Which data structure should we use for OPEN?

Basic Search Algorithm

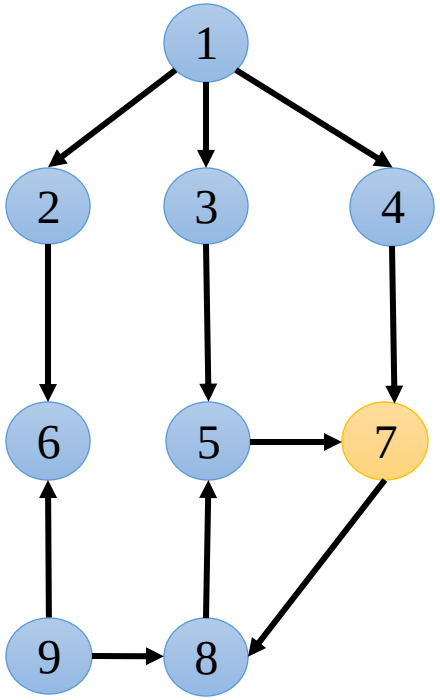
Open Set	Select State	Goal State	Terminate	Expanded Set
[1]	1	N	N	[4,3,2]
[4,3,2]	2	N	N	[4,3,6]
[4,3,6]	6	N	N	[4,3]
[4,3]	3	N	N	[4,5]
[4,5]	5	N	N	[4,7]
[4,7]	7	Y	Y	



Stack
Tie: Descending

Basic Search Algorithm

Open Set	Select State	Goal State	Terminate	Expanded Set
[1]	1	N	N	[4,3,2]
[4,3,2]	4	N	N	[3,2,7]
[3,2,7]	3	N	N	[2,7,5]
[2,7,5]	2	N	N	[7,5,6]
[7,5,6]	7	Y	Y	



Queue
Tie: Descending

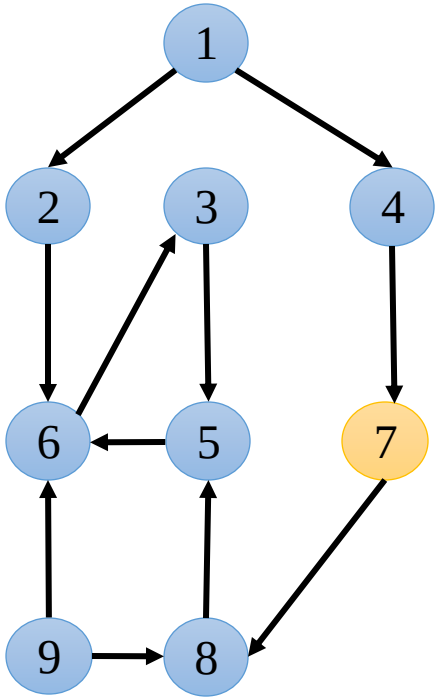
Basic Search Algorithm

- We don’t want to make whole state space explicit

- We only want to unfold that portion of the state space which is necessary to find out the goal

Basic Search Algorithm

Open Set	Select State	Goal State	Terminate	Expanded Set
[1]	1	N	N	[4,2]
[4,2]	2	N	N	[4,6]
[4,6]	6	N	N	[4,3]
[4,3]	3	N	N	[4,5]
[4,5]	5	N	N	[4,6]
[4,6]	6	N	N	[4,3]
[4,3]	3	N	N	[4,5]



Stack
Tie: Descending

How to maintain part of the state space that are already visited?

Basic State Space Search

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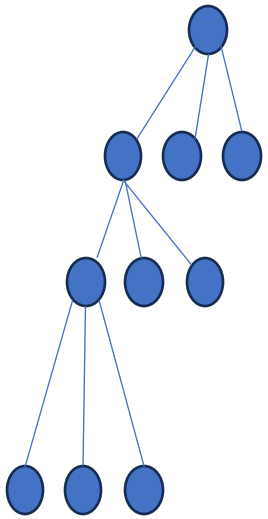
Basics Search Algorithm

- OPEN is a queue (FIFO) vs a stack (LIFO)
- Is this algorithm guaranteed to terminate?
- Under what circumstances will it terminate?

Complexity

- b: branching factor d: depth of the goal



- Breadth first search:
  - Time: $1 + b + b^2 + b^3 + \cdots + b^d = O(b^d)$
  - Space: $O(b^d)$

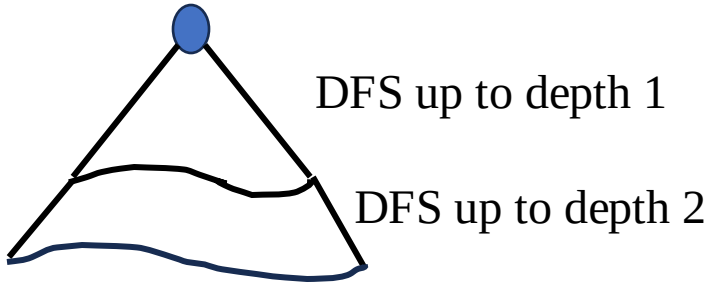
- Depth first search:
  - Time: $O(b^m)$
    - m: depth of state space tree
  - Space: $O(bm)$
  - State space tree is the tree that is obtained by applying the state transition operators repeatedly on the set of states

Trade-off between Space and Time

- BFS works well when the goal state is near the start state
- BFS uses too much space

- Nice trade-off between space and time
  - Depth first search to do breadth first search
  - Iterative deepening

Trade-off between Space and Time



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Space: $b + 2b + 3b + \ldots db$

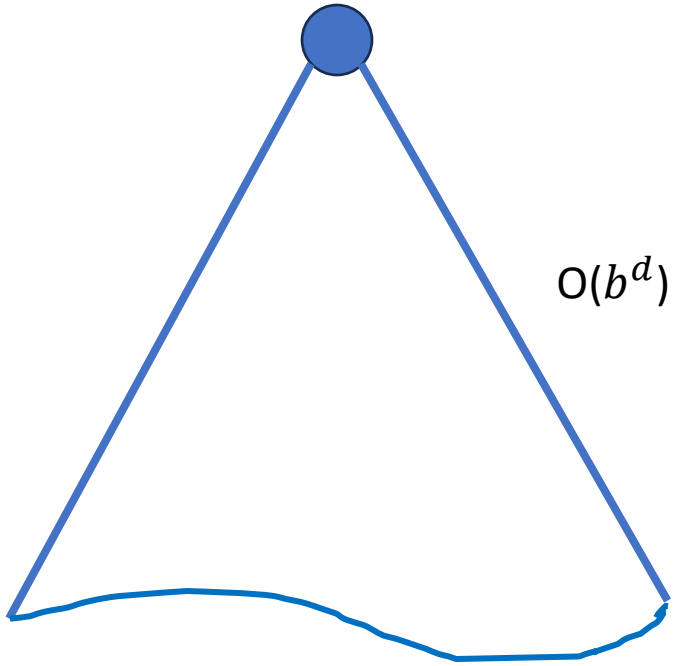
Time: $1 + b + b^2 + b^3 + \cdots + b^d$

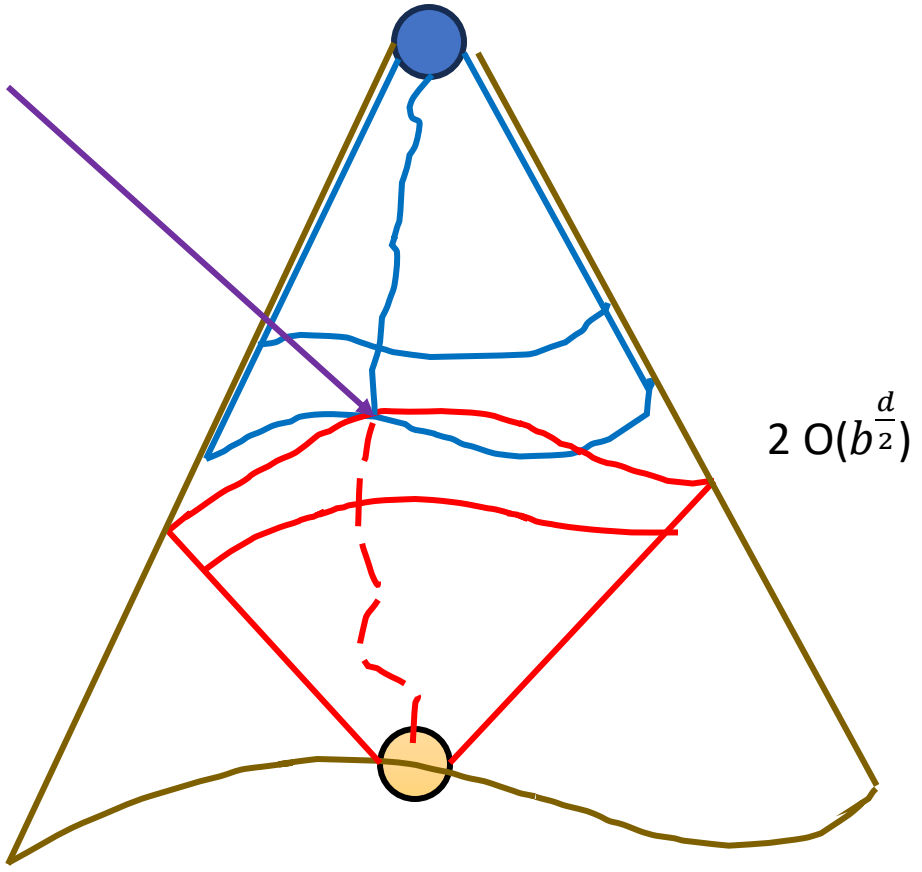
Trade-off between Space and Time

- Iterative deepening
  - Perform DFS repeatedly using increasing depth bounds
  - Works in O(b^d) time and O(bd) space

- Can we do something with time complexity?

Trade-off between Space and Time





Trade-off between Space and Time

- Iterative deepening
  - Perform DFS repeatedly using increasing depth bounds
  - Works in O(b^d) time and O(bd) space

- Bi-directional search
  - Possible only if the operators are reversible
  - Works in O($b^{\frac{d}{2}}$) time and O($b^{\frac{d}{2}}$) space

Complexity Comparison

Criterion	BFS	DFS	Depth Limited	Iterative Deepening	Bidirectional
Time	b^d	b^m	b^l	b^d	$b^{\frac{d}{2}}$
Space	b^d	bm	bl	bd	$b^{\frac{d}{2}}$
Optimal?	Yes	No	No	Yes	Yes
Complete?	Yes	No	Yes, if l>=d	Yes	Yes

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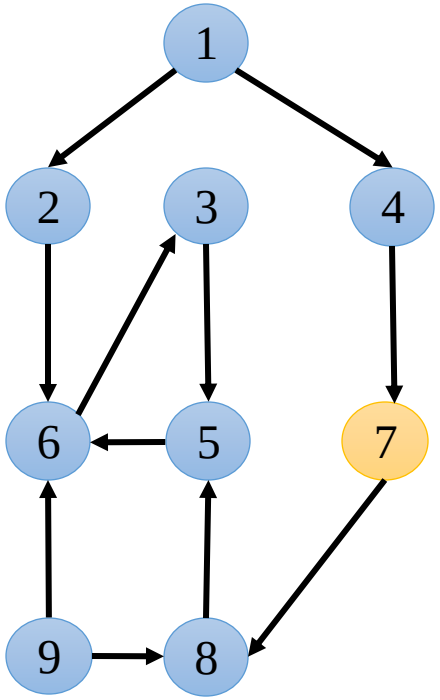

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How to maintain part of the state space that are already visited?

Search Algorithm: Saving Explicit Space

- Initialize: Set OPEN={s}, CLOSED = {}
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  - Save n is CLOSED
- Terminate:
  - If n∈G, terminate with success
- Expand:
  - Generate the successors of n using O
  - For each successor, m, insert m in OPEN,
    - Only if $m \notin [OPEN \cup CLOSED]$
- Loop:
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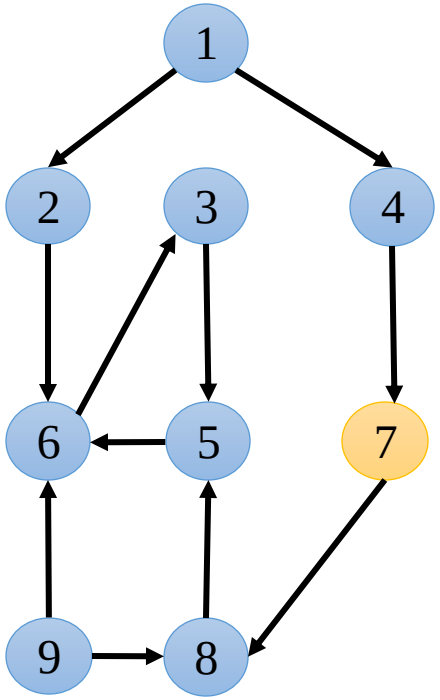
[4,5]	5	N	N	[4,~~6~~]
[4]	4	N	N	[7]
[7]	7	Y	Y	



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How to maintain data structure to trace the path from start to goal?

Search Algorithm: Saving Explicit Space

- How to maintain data structure to trace the path from start to goal?
- Whenever a new node m is added to the OPEN
  - Add a pointer from m to its parent n

Open Issues

- What will happen if state transition operators have associated cost?

- What will happen if the goal state have associated cost?

Thank You