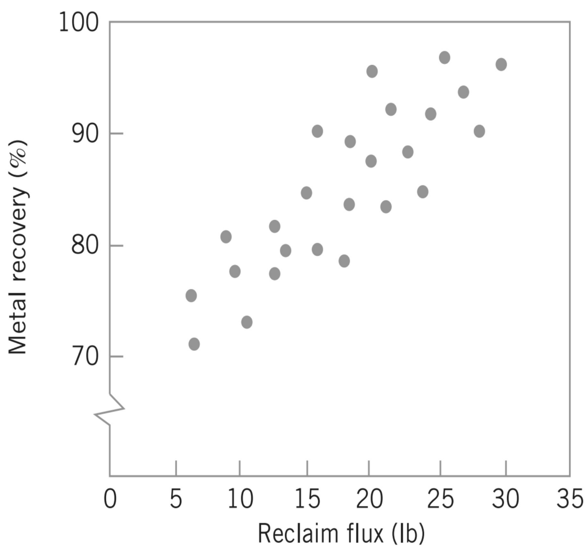
Scatter Diagram

The scatter diagram is used to identify a potential *relationship* between two variables.



The scatter diagram indicates a *strong positive correlation* between metal recovery and flux amount.

Note that *correlation does not necessarily imply causality!*

11 Control Charts

Types of Process Variability

Three types of process variability:

1. *Stationary and uncorrelated:* Data vary around a fixed mean in a stable or predictable manner.
2. *Stationary and autocorrelated:* Successive observations are dependent with tendency to move in long runs on either side of mean.
3. *Nonstationary process:* Drifts without any sense of a stable or fixed mean.

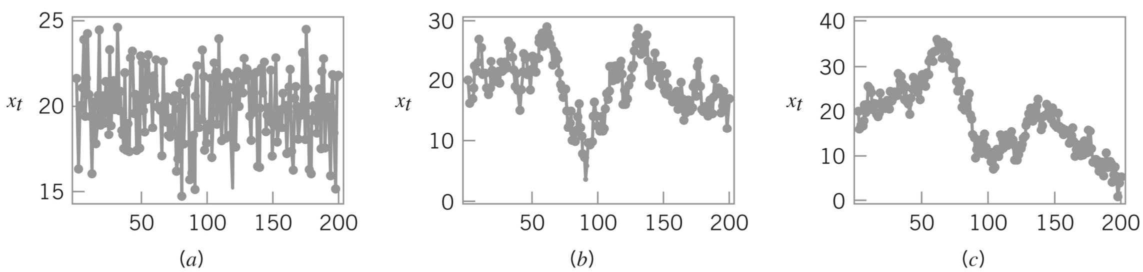
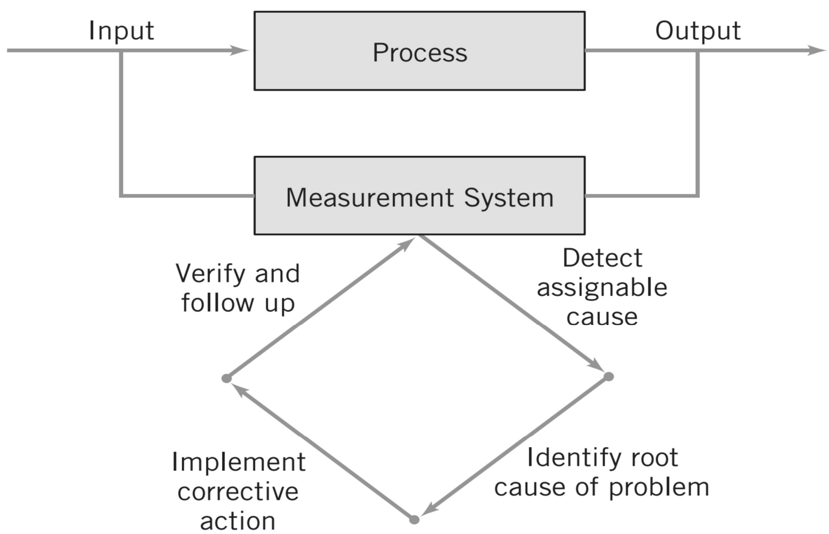


Figure: Data from three different processes: (a) stationary and uncorrelated, (b) stationary and autocorrelated, and (c) nonstationary.

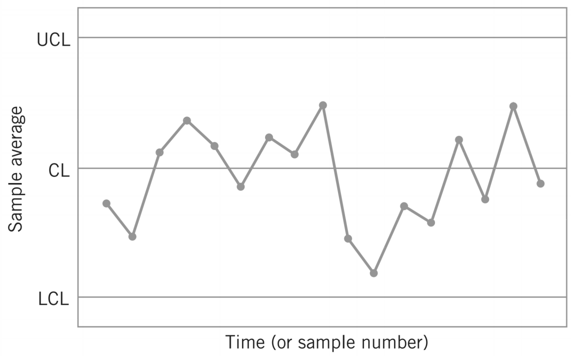
Process Improvement using Control Charts

The most important use of a control chart is to *improve* the process:

— Most processes *do not operate* in a state of statistical control.
— Routine use of control charts will assist in *identifying assignable causes*.
— Management, operator, and engineering action will usually be necessary to *eliminate the assignable causes*.



Elements of a Control Chart



A *control chart* contains:

- A *center line*
- An *upper control limit (UCL)*
- A *lower control limit (LCL)*

A point that plots within the control limits indicates the process *within control* – *no action* is necessary!

A point that plots outside the control limits is evidence that the process is *out of control* – *investigation* and *corrective action* are required!

A General Control Chart Model

Let w be a *sample statistic* that measures some quality characteristic of interest.

Suppose that the mean of w is μ_w and the standard deviation is σ_w.

$$\begin{aligned} \text{UCL} &= \mu_w + L\sigma_w \\ \text{Center line} &= \mu_w \\ \text{LCL} &= \mu_w - L\sigma_w \end{aligned}$$

where L is the *distance* of the control limits from the center line.

This model of control charts is also known as *Shewhart control charts*.

Control Chart Types and Design Elements

Control charts improve productivity, prevent defects, avoid unnecessary process adjustments, and offer diagnostic and capability insights.

Control charts are used to estimate process parameters to determine *process capability*.

Two general types of control charts:

1. *Control charts for variables:* Used when dealing with continuous, measurable data (e.g., weight, length, temperature).
2. *Control charts for attributes:* Used when dealing with discrete, countable data or categorical characteristics (e.g., defects, nonconformities).

Control chart *design* encompasses selection of *sample size, control limits,* and *sampling frequency*.

Type I & Type II Errors for Specifying Limits

Specifying the control limits is one of the critical decisions in designing a control chart.

By moving the control limits farther from the center line, we *decrease the risk* of a *type I error* – the risk of a point falling *beyond the control limits* – an out-of-control condition when no assignable cause is present.

However, widening the control limits will also *increase the risk* of a *type II error* – the risk of a point falling *between the control limits* when the process is really out of control.

If we move the control limits closer to the center line, the risk of type I error is *increased*, while the risk of type II error is *decreased*.

Example in Manufacturing

In *semiconductor manufacturing*, an important fabrication step is *photolithography*, in which a light-sensitive photoresist material is applied to the silicon wafer.

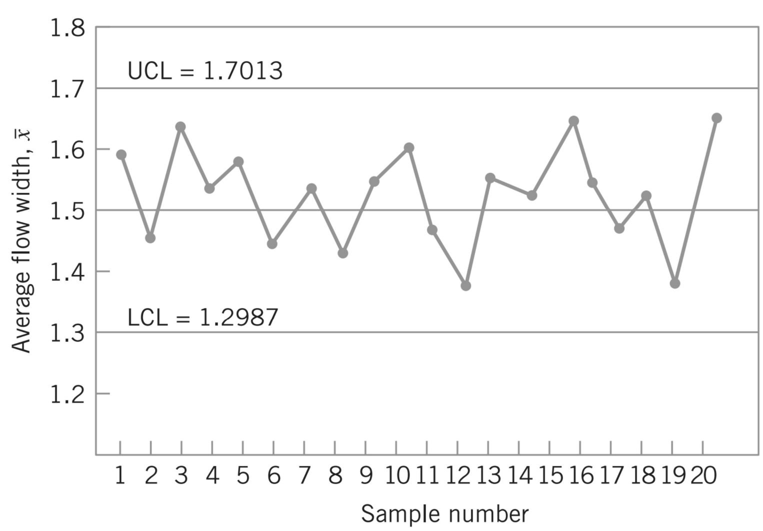
It is typical to follow development with a *hard-bake process* to increase resist adherence and etch resistance.

An important *quality characteristic* in hard bake is the *flow width* of the resist, a measure of how much it expands due to the baking process.

Suppose that flow width w can be controlled at a *mean* of 1.5 microns, and it is known that the *standard deviation* of flow width is 0.15 microns.

Example: Control Chart Construction

Every hour, a sample of 5 wafers is taken, the average flow width ($\bar{x}$) computed, and plotted on the chart. It is called an $\bar{x}$ control chart.



Note that all plotted points *fall inside* the *control limits*, so the process is considered to be in *statistical control.*

Example: Determination of Control Limits

Assuming σ to be the *process standard deviation*, standard deviation of sample average $\bar{x}$ can be computed as

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.15}{\sqrt{5}} = 0.0671$$

Assuming $\bar{x}$ is approximately normally distributed, the *control limits* are

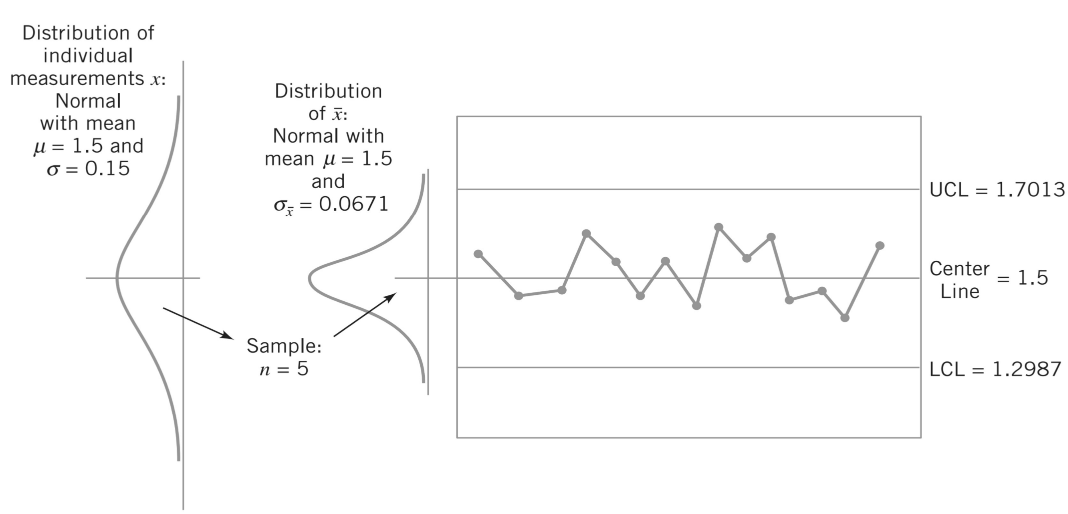
$$\text{UCL} = 1.5 + 3 \times 0.0671 = 1.7013$$
$$\text{LCL} = 1.5 - 3 \times 0.0671 = 1.2987$$

These are called *three-sigma control limits*. Here *sigma* refers to the standard deviation of the *statistic* $\bar{x}$, not the standard deviation of the quality characteristic.

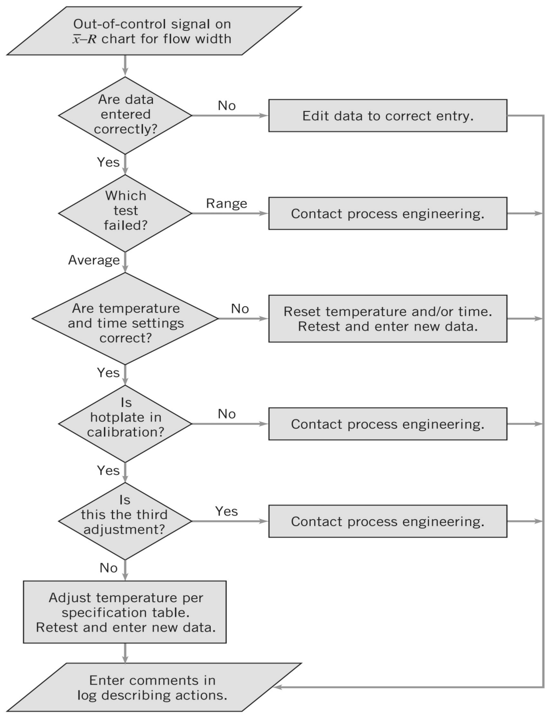
Example: Determination of Control Limits (cont'd)

The width of the control limits is *inversely proportional* to the sample size n for a given multiple of sigma.

Choosing the control limits is equivalent to setting up the *critical region* for testing the hypothesis: $H_0 : \mu_w = 1.5; H_1 : \mu_w \neq 1.5$



Example: Out-of-Control-Action Plans

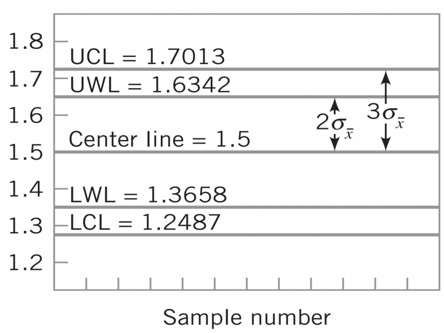


The *out-of-control action plan* (OCAP) is a flowchart of the sequence of activities that must take place following the occurrence of an *activating event*, i.e., *out-of-control signals* from the control chart.

The OCAP consists of *checkpoints*, i.e., potential assignable causes, and *terminators*, i.e., actions taken to resolve the out-of-control condition.

An OCAP is a *living document* in the sense that it will be modified over time.

Two Limits on Control Charts



The outer limits at *three-sigma* are called the *action limits*.

The inner limits at *two-sigma* are called *warning limits*.

The upper and lower warning limits can be calculated as

$$\text{UWL} = 1.5 + 2 \times 0.0671 = 1.6342$$
$$\text{LWL} = 1.5 - 2 \times 0.0671 = 1.3658$$

When *probability limits* are used, the *action limits* are 0.001 limits and the *warning limits* are 0.025 limits.

Sample Size and Frequency

Sample size and frequency can be determined using the *average run length (ARL)* of a control chart.

If the process observations are *uncorrelated*, the ARL can be calculated as

$$\text{ARL} = \frac{1}{p}$$

where p is the *probability* that any point exceeds the control limits.

For the chart with *three-sigma limits*, $p = 0.0027$ is the probability that a single point falls outside the limits when the process is in control.

Thus, the ARL of the chart is $\text{ARL}_0 = \frac{1}{0.0027} \approx 370$

Sample Size and Frequency (cont'd)

The performance of a control chart can be expressed using the *average time to signal (ATS)*.

If samples are taken at fixed intervals of time that are h hours apart, then

$$\text{ATS} = h \times \text{ARL} = \frac{h}{p}$$

The *larger sample size* given a fixed time interval h would allow process shift to be *detected more quickly* than with the smaller one.

Rational Subgroups

Rational subgrouping involves grouping together a set of individual measurements or observations taken from a process that share a *common source of variation*.

The goal is to create subgroups that represent a *homogeneous subset* of the process and *detect process shifts*.

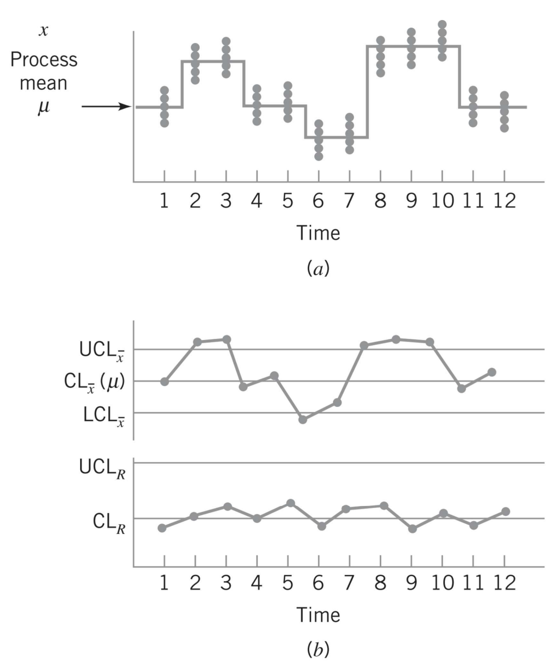
Subgroups could be based on *time, operators, production batches, machine types,* or any factor that may influence the process.

An R *chart (range chart)* is a type of control chart that displays the range *within* each rational subgroup, i.e., $R = x_{max} - x_{min}$.

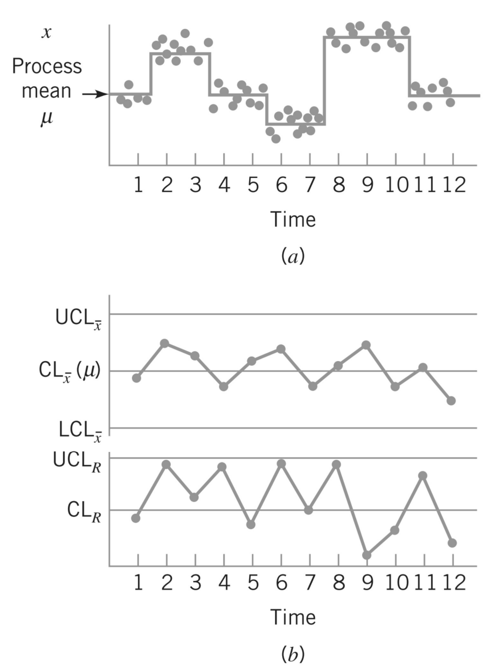
Thus, the R chart helps *monitor the variability* within each subgroup.

Rational Subgroups (cont'd)

The *snapshot* approach

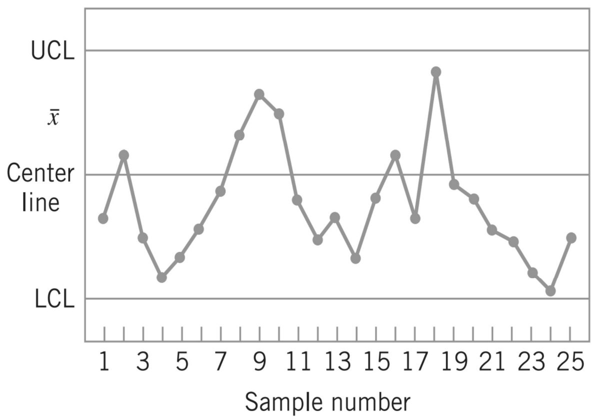


The *random sample* approach



Control Chart Patterns

A control chart may indicate an *out-of-control* condition when one or more points fall beyond the control limits or when the plotted points exhibit some *nonrandom pattern*.



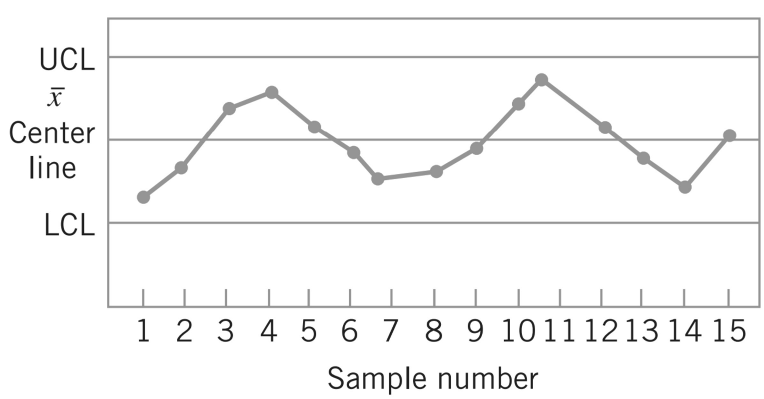
- Pattern is very *nonrandom* in appearance.
- 19 of out of 25 points plot below the center line – only 6 plot above.
- Following the 4th point, 5 points in a row increase in magnitude – a *run up*.
- An unusually long *run down* beginning with the 18th point.

Control Chart Patterns (cont'd)

The plotted sample averages exhibit a *cyclic behavior*, yet they all fall within the control limits!

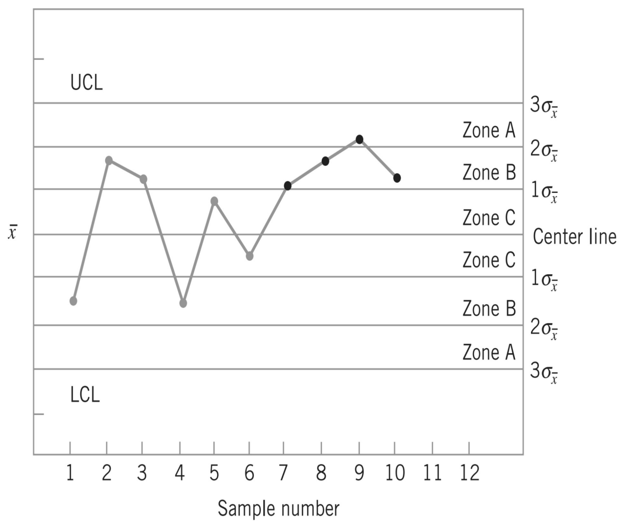
Such a pattern may indicate a problem with the process such as *operator fatigue, raw material deliveries, heat or stress buildup,* and so forth.

The yield may be improved by *elimination/reduction* of the sources of *variability* causing this cyclic behavior.



Detecting Nonrandom Patterns

The Western Electric *Statistical Quality Control Handbook* (1956) suggests a set of decision rules for detecting nonrandom patterns on control charts.



A process is out of control if either

1. *One* point plots outside the *three-sigma* control limits.
2. *Two out of three* consecutive points plot beyond the *two-sigma* warning limits.
3. *Four out of five* consecutive points plot at a distance of *one-sigma* or beyond from center line.
4. *Eight* consecutive points plot on *one side* of the center line.

Sensitizing Rules

Some of the sensitizing rules for control charts that are widely used in practice:

1. *One or more* points outside of the control limits.
2. *Two of three* consecutive points outside the two-sigma warning limits but still inside the control limits.
3. *Four of five* consecutive points beyond the one-sigma limits.
4. A run of *eight* consecutive points on one side of the center line.
5. *Six* points in a row steadily increasing or decreasing.
6. *Fifteen* points in a row in zone C.
7. *Fourteen* points in a row alternating up and down.
8. *Eight* points in a row on both sides of the center line with none in zone C.
9. An *unusual* or nonrandom pattern in the data.
10. One or more points near a warning or control limit.

Control Chart Application Phases

Phase I: Retrospective analysis of process data to *construct trial control* limits.

- Charts are effective at detecting large, sustained shifts in process parameters, outliers, measurement errors, data entry errors, etc.
- Facilitates identification and removal of assignable causes.

Phase II: Use of control charts to *monitor the process.*

- Process is assumed to be reasonably stable.
- Emphasis is on process monitoring, not on bringing an unruly process into control

Shewhart Constants

Table Shewhart constants

n	d2	d3	c4	A2	D3	D4	B3	B4
2	1.1284	0.8525	0.7979	1.8800	0.0000	3.2665	0.0000	3.2665
3	1.6926	0.8884	0.8862	1.0233	0.0000	2.5746	0.0000	2.5682
4	2.0588	0.8798	0.9213	0.7286	0.0000	2.2821	0.0000	2.2660
5	2.3259	0.8641	0.9400	0.5768	0.0000	2.1145	0.0000	2.0890
6	2.5344	0.8480	0.9515	0.4832	0.0000	2.0038	0.0304	1.9696
7	2.7044	0.8332	0.9594	0.4193	0.0757	1.9243	0.1177	1.8823
8	2.8472	0.8198	0.9650	0.3725	0.1362	1.8638	0.1851	1.8149
9	2.9700	0.8078	0.9693	0.3367	0.1840	1.8160	0.2391	1.7609
10	3.0775	0.7971	0.9727	0.3083	0.2230	1.7770	0.2837	1.7163
11	3.1729	0.7873	0.9754	0.2851	0.2556	1.7444	0.3213	1.6787
12	3.2585	0.7785	0.9776	0.2658	0.2833	1.7167	0.3535	1.6465
13	3.3360	0.7704	0.9794	0.2494	0.3072	1.6928	0.3816	1.6184
14	3.4068	0.7630	0.9810	0.2354	0.3281	1.6719	0.4062	1.5938
15	3.4718	0.7562	0.9823	0.2231	0.3466	1.6534	0.4282	1.5718
16	3.5320	0.7499	0.9835	0.2123	0.3630	1.6370	0.4479	1.5521
17	3.5879	0.7441	0.9845	0.2028	0.3779	1.6221	0.4657	1.5343
18	3.6401	0.7386	0.9854	0.1943	0.3913	1.6087	0.4818	1.5182
19	3.6890	0.7335	0.9862	0.1866	0.4035	1.5965	0.4966	1.5034
20	3.7349	0.7287	0.9869	0.1796	0.4147	1.5853	0.5102	1.4898
21	3.7783	0.7242	0.9876	0.1733	0.4250	1.5750	0.5228	1.4772
22	3.8194	0.7199	0.9882	0.1675	0.4345	1.5655	0.5344	1.4656
23	3.8583	0.7159	0.9887	0.1621	0.4434	1.5566	0.5452	1.4548
24	3.8953	0.7121	0.9892	0.1572	0.4516	1.5484	0.5553	1.4447
25	3.9306	0.7084	0.9896	0.1526	0.4593	1.5407	0.5648	1.4352

Example: Control Charts for Variables $\bar{x}$ & R

Flow Width Measurements (microns) for the Hard-Bake Process

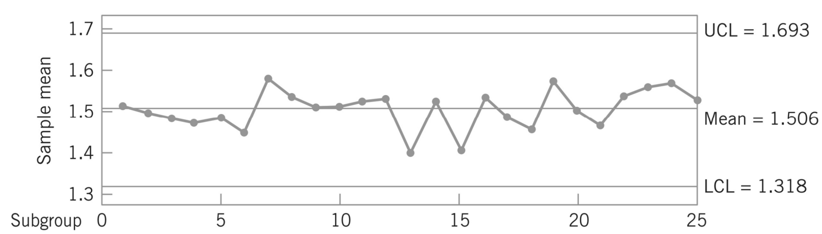
Sample Number	Wafers 1	2	3	4	5	$\bar{x}_i$	R_i
1	1.3235	1.4128	1.6744	1.4573	1.6914	1.5119	0.3679
2	1.4314	1.3592	1.6075	1.4666	1.6109	1.4951	0.2517
3	1.4284	1.4871	1.4932	1.4324	1.5674	1.4817	0.1390
4	1.5028	1.6352	1.3841	1.2831	1.5507	1.4712	0.3521
5	1.5604	1.2735	1.5265	1.4363	1.6441	1.4882	0.3706
6	1.5955	1.5451	1.3574	1.3281	1.4198	1.4492	0.2674
7	1.6274	1.5064	1.8366	1.4177	1.5144	1.5805	0.4189
8	1.4190	1.4303	1.6637	1.6067	1.5519	1.5343	0.2447
9	1.3884	1.7277	1.5355	1.5176	1.3688	1.5076	0.3589
10	1.4039	1.6697	1.5089	1.4627	1.5220	1.5134	0.2658
11	1.4158	1.7667	1.4278	1.5928	1.4181	1.5242	0.3509
12	1.5821	1.3355	1.5777	1.3908	1.7559	1.5284	0.4204
13	1.2856	1.4106	1.4447	1.6398	1.1928	1.3947	0.4470
14	1.4951	1.4036	1.5893	1.6458	1.4969	1.5261	0.2422
15	1.3589	1.2863	1.5996	1.2497	1.5471	1.4083	0.3499
16	1.5747	1.5301	1.5171	1.1839	1.8662	1.5344	0.6823
17	1.3680	1.7269	1.3957	1.5014	1.4449	1.4874	0.3589
18	1.4163	1.3864	1.3057	1.6210	1.5573	1.4573	0.3153
19	1.5796	1.4185	1.6541	1.5116	1.7247	1.5777	0.3062
20	1.7106	1.4412	1.2361	1.3820	1.7601	1.5060	0.5240
21	1.4371	1.5051	1.3485	1.5670	1.4880	1.4691	0.2185
22	1.4738	1.5936	1.6583	1.4973	1.4720	1.5390	0.1863
23	1.5917	1.4333	1.5551	1.5295	1.6866	1.5592	0.2533
24	1.6399	1.5243	1.5705	1.5563	1.5530	1.5688	0.1156
25	1.5797	1.3663	1.6240	1.3732	1.6887	1.5264	0.3224
						$\Sigma\bar{x}_i$ = 37.6400	ΣR_i = 8.1302
						$\bar{\bar{x}}$ = 1.5056	$\bar{R}$ = 0.32521

Example: Control Charts for Variables $\bar{x}$ & R

Phase I Application

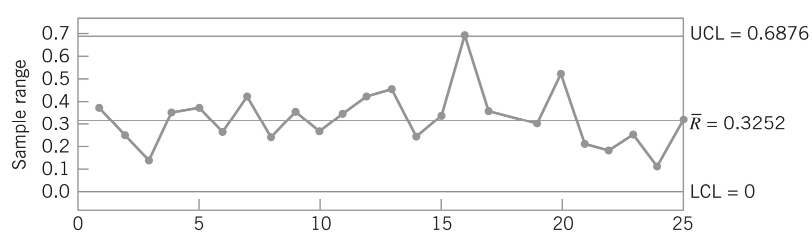
$\bar{x}$ Control Chart

$$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$$
$$\text{CL} = \bar{\bar{x}}$$
$$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$$



R Control Chart

$$\text{UCL} = \bar{R}D_4$$
$$\text{CL} = \bar{R}$$
$$\text{LCL} = \bar{R}D_3$$



Periodic revision of control limits and center lines is necessary for the effective use of any control chart. Practitioners often review *every week, every month, or every 25, 50, or 100 samples*.

Example: Control Charts for Variables $\bar{x}$ & R

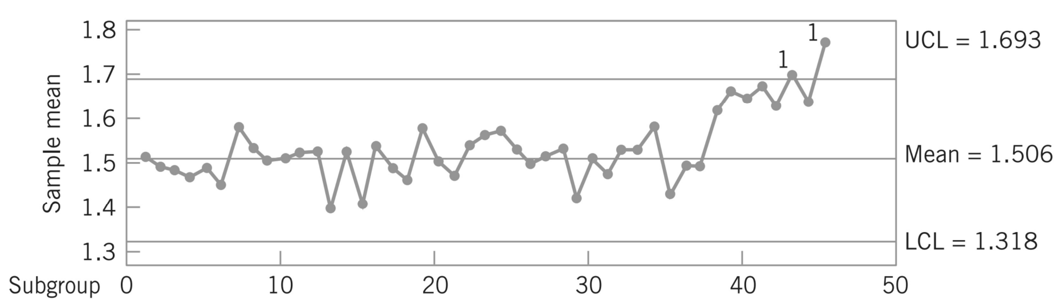
Phase II Application

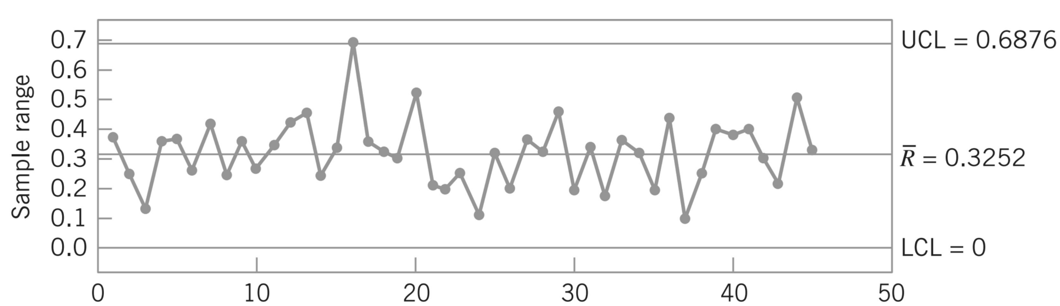
Additional Samples for Example

Sample Number	Wafers 1	2	3	4	5	$\bar{x}_i$	R_i
26	1.4483	1.5458	1.4538	1.4303	1.6206	1.4998	0.1903
27	1.5435	1.6899	1.5830	1.3358	1.4187	1.5142	0.3541
28	1.5175	1.3446	1.4723	1.6657	1.6661	1.5332	0.3215
29	1.5454	1.0931	1.4072	1.5039	1.5264	1.4152	0.4523
30	1.4418	1.5059	1.5124	1.4620	1.6263	1.5097	0.1845
31	1.4301	1.2725	1.5945	1.5397	1.5252	1.4724	0.3220
32	1.4981	1.4506	1.6174	1.5837	1.4962	1.5292	0.1668
33	1.3009	1.5060	1.6231	1.5831	1.6454	1.5317	0.3445
34	1.4132	1.4603	1.5808	1.7111	1.7313	1.5793	0.3181
35	1.3817	1.3135	1.4953	1.4894	1.4596	1.4279	0.1818
36	1.5765	1.7014	1.4026	1.2773	1.4541	1.4824	0.4241
37	1.4936	1.4373	1.5139	1.4808	1.5293	1.4910	0.0920
38	1.5729	1.6738	1.5048	1.5651	1.7473	1.6128	0.2425
39	1.8089	1.5513	1.8250	1.4389	1.6558	1.6560	0.3861
40	1.6236	1.5393	1.6738	1.8698	1.5036	1.6420	0.3662
41	1.4120	1.7931	1.7345	1.6391	1.7791	1.6716	0.3811
42	1.7372	1.5663	1.4910	1.7809	1.5504	1.6252	0.2899
43	1.5971	1.7394	1.6832	1.6677	1.7974	1.6970	0.2003
44	1.4295	1.6536	1.9134	1.7272	1.4370	1.6321	0.4839
45	1.6217	1.8220	1.7915	1.6744	1.9404	1.7700	0.3187

Example: Control Charts for Variables $\bar{x}$ & R

Phase II Application





Example: Control Charts for Variables $\bar{x}$ & R

Estimating Process Capability from Phase I:

What is the process capability if the specification limits on the flow width are 1.50 ± 0.50 microns?

Estimated standard deviation, $\hat{\sigma} = \frac{\bar{R}}{d_2} = 0.1398$ microns.

Process capability in terms of the *fraction of nonconforming units* produced:

$$
\begin{aligned}
p &= Pr(x < 1.0) + Pr(x > 2.0) \\
&= \Phi\left(\frac{1.0 - 1.5056}{0.1398}\right) + 1 - \Phi\left(\frac{2.0 - 1.5056}{0.1398}\right) \\
&= 0.00035 = 350 \text{ ppm or } 0.035\% \text{ nonconforming units}
\end{aligned}
$$

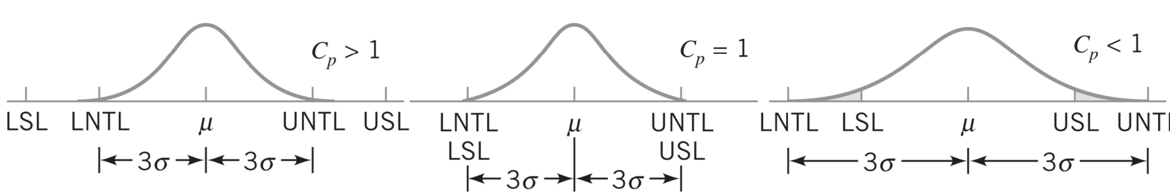
Example: Control Charts for Variables $\bar{x}$ & R

Estimating Process Capability from Phase I:

Process capability in terms of the *process capability ratio (PCR), C_p:*

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{2.0 - 1.0}{6 \times 0.1368} = 1.192$$

The percentage of the specification band that the process uses up, $P = (1/C_p) \times 100 = 100/1.192 = 83.89\%$



Example: Control Charts for Variables $\bar{x}$ & R

Operating-Characteristic (OC) Curves for Phase II Monitoring:

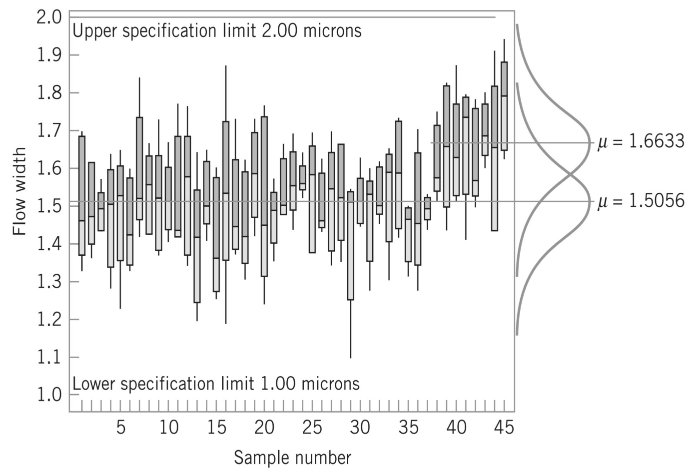
The *ability* of the $\bar{x}$ and R charts to *detect shifts in process quality* is described by their operating-characteristic (OC) curves.

For $L\sigma$ limits, i.e.,
$UCL = \mu + L\sigma$ & $LCL = \mu - L\sigma$

If the process mean shifts from μ to $\mu + k\sigma$, the *probability of not detecting* this shift (β-risk) is

$$\beta = \Phi(L - k\sqrt{n}) - \Phi(-L - k\sqrt{n})$$

where n is the sample/batch size.



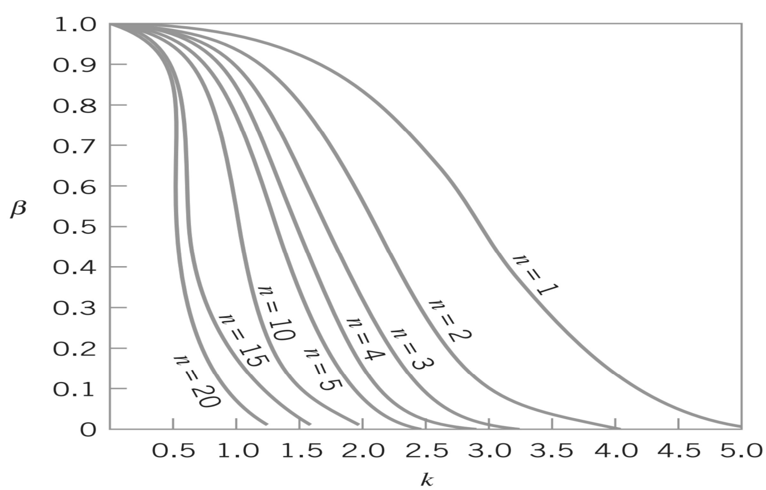
Example: Control Charts for Variables $\bar{x}$ & R

Operating-Characteristic (OC) Curves for Phase II Monitoring:

For the usual three-sigma limits ($L = 3$) and the sample size $n = 5$, the probability of detecting a shift from μ to $\mu + 2\sigma$ on the first sample following the shift is

$$\beta = \Phi(L - k\sqrt{n}) - \Phi(-L - k\sqrt{n}) = \Phi(3 - 2\sqrt{5}) - \Phi(-3 - 2\sqrt{5}) = 0.071$$

Figure: OC curves for the $\bar{x}$ chart with three-sigma limits:



Standard Normal CDF $\Phi(\cdot)$ Table

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Variables and Attributes Control Charts

Notation:

UCL	Upper Control Limit	$\bar{x}$	Average of Measurements
LCL	Lower Control Limit	$\bar{\bar{x}}$	Average of Averages
CL	Center Line	R	Range
n	Sample Size	$\bar{R}$	Average of Ranges
PCR	Process Capability Ratio	USL	Upper Specification Limit
$\hat{\sigma}$	Process Standard Deviation	LSL	Lower Specification Limit

Variables Data

$\bar{x}$ Control Chart	R Control Chart	s Control Chart	Capability Study
UCL = $\bar{\bar{x}} + A_2\bar{R}$	UCL = $\bar{R}\,D_4$	UCL = $B_4\bar{s}$	C_p = (USL − LSL)/(6$\hat{\sigma}$)
LCL = $\bar{\bar{x}} - A_2\bar{R}$	LCL = $\bar{R}\,D_3$	LCL = $B_3\bar{s}$	where $\hat{\sigma} = \bar{R}/d_2$
CL = $\bar{\bar{x}}$	CL = $\bar{R}$	CL = $\bar{s}$	

Attribute Data (p, np, c, and u Control Charts)

Control Chart Formulas

	p (fraction)	**np (number of nonconforming)**	**c (count of nonconformances)**	**u (count of nonconformances/unit)**
CL	$\bar{p}$	$n\bar{p}$	$\bar{c}$	$\bar{u}$
UCL	$\bar{p}+3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p}+3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c}+3\sqrt{\bar{c}}$	$\bar{u}+3\sqrt{\frac{\bar{u}}{n}}$
LCL	$\bar{p}-3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p}-3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c}-3\sqrt{\bar{c}}$	$\bar{u}-3\sqrt{\frac{\bar{u}}{n}}$
Notes	If n varies, use $\bar{n}$ or individual n_i	n must be a constant	n must be a constant	If n varies, use $\bar{n}$ or individual n_i

Factors for Constructing Variables Control Charts

Observations in Sample, n	Chart for Averages					Chart for Standard Deviations						Chart for Ranges				
	Factors for Control Limits			Factors for Center Line		Factors for Control Limits				Factors for Center Line		Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_3	B_4	B_5	B_6	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$

$$A = \frac{3}{\sqrt{n}} \qquad A_3 = \frac{3}{c_4\sqrt{n}}$$

$$c_4 \cong \frac{4(n-1)}{4n-3}$$

$$B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}} \qquad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}} \qquad B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}} \qquad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}$$