Sample Questions

Q2.1 What do you understand by dimensions of quality? Name any three quality dimensions and briefly discuss what they represent and what customer concerns they address.

Q2.2 Explain the two main types of quality definitions. What do you understand by quality improvement? Explain nonconforming and defective products.

Q2.3 Deming's philosophy was summarized in 14 points. Discuss any three of these points and provide your opinion on the same.

Q2.4 Discuss any three of the Deming's seven deadly diseases of management. Plot the Shewhart cycle and discuss the four steps briefly.

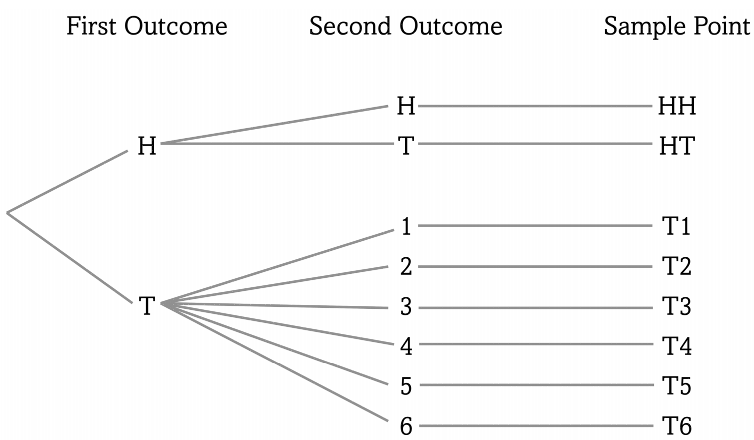
Q2.5 Name four main types of quality costs and briefly explain each type.

$\boxed{3}$ Probability Theory

Sample Space

The *set of all possible outcomes* of an experiment is called the sample space S.

- The *experiment* term is used to describe a process that generates data.
- Each outcome is called an *element* of the sample space or a *sample point*.



Events

An event is a *subset of a sample space* or a group of outcomes of the sample space whose members have some *common characteristics*.

An *event* A with respect to a particular *sample space* S is a set of possible outcomes. *Examples* include:

- Event A that is outcome of tossing a dice divisible by 3.
- Operational states of components in which system is considered functioning.

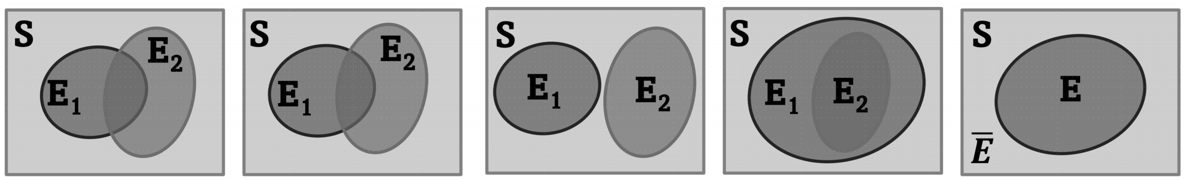
Types of Events:

- *Null or Empty*: Event that contain no outcomes of the sample space.
- *Union*: Consists of all possible outcomes that are either in E_1 or in E_2 or in both E_1 and E_2.
- *Intersection*: Consists of all outcomes that are both in E_1 or in E_2.

Events (cont'd)

- *Mutually Exclusive*: Event in which E_1 and E_2 cannot occur both.
- *Containment of an event by another*: Intersection of E_1 and E_2 consists of all possible outcomes of E_2.
- *Complement*: Consists of all outcomes not contained in E.

Can you identify the event type?



(1) (2) (3) (4) (5)

Permutations & Combinations

A *permutation* is an arrangement of objects *in a specific order*. It is the number of ways to arrange a set of items without repetition.

The number of permutations of n distinct objects taken r at a time:

$$^nP_r = \frac{n!}{(n-r)!}$$

A *combination* is a selection of objects *without considering the order*. It is the number of ways to choose a subset from a larger set, without the order.

The number of combinations of n distinct objects taken r at a time:

$$^nC_r = \frac{n!}{r!(n-r)!} = \frac{^nP_r}{r!} = {^nC_{n-r}}$$

Probability Laws

If an experiment can result in any one of N different equally likely outcome, and if exactly n of these outcomes correspond to event A, then the *probability of event* A can be written as

$$P(A) = \frac{\text{No. of favorable outcomes}}{\text{Total number of outcomes}} = \lim_{n \to \infty} \frac{n}{N}$$

The *probability of an event* A obeys following postulates:

- $P(A)$ is non-negative, $0 \leq P(A) \leq 1$.
- Probability of a certain event equals 1.
- Probability of complement, i.e., A not occurring is $P(\bar{A}) = 1 - P(A)$.

Probability Laws (cont'd)

Law of Idempotence:

- $P(A \cup A) = P(A \cap A) = P(A)$
- $P(A \cup A \cup A \cup A) = P(A \cap A \cap A \cap A) = P(A)$
- $P(A \cup B \cup A \cup B) = P(A \cup B)$
- $P(A \cap B \cap A \cap B) = P(A \cap B)$

Independent and *mutually exclusive* events:

- Two events A and B are said to be independent if the occurrence of event A does not depend on the occurrence of event B and vice versa, then

$$P(A|B) = P(A); \; P(B|A) = P(B)$$

- Two events A and B are said to be mutually exclusive if both can not occur simultaneously or alternatively have no elements in common, i.e., $A \cap B = \phi$:

$$P(A \cap B) = 0$$

Probability Laws (cont'd)

Law of Intersection:

- Joint probability that events A and B will occur is $P(A \cap B)$
- If the events are dependent, then

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

- If the events are independent, then

$$P(A \cap B) = P(A)P(B) = P(B)P(A)$$

Union Law:

- Probability of any one of the two events A or B occurring is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If A and B are independent:

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$

Probability Laws (cont'd)

- If A and B are dependent:

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A)P(B|A) \\ &= P(A) + P(B) - P(B)P(A|B) \end{aligned}$$

- If A and B are mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

- Probability of any one of the three events A, B or C occurring is:

$$\begin{aligned} P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\ - P(B \cap C) + P(A \cap B \cap C) \end{aligned}$$

Conditional Probability

Conditional probability of obtaining *outcome A given that B has occurred* can be denoted as

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(B)}$$

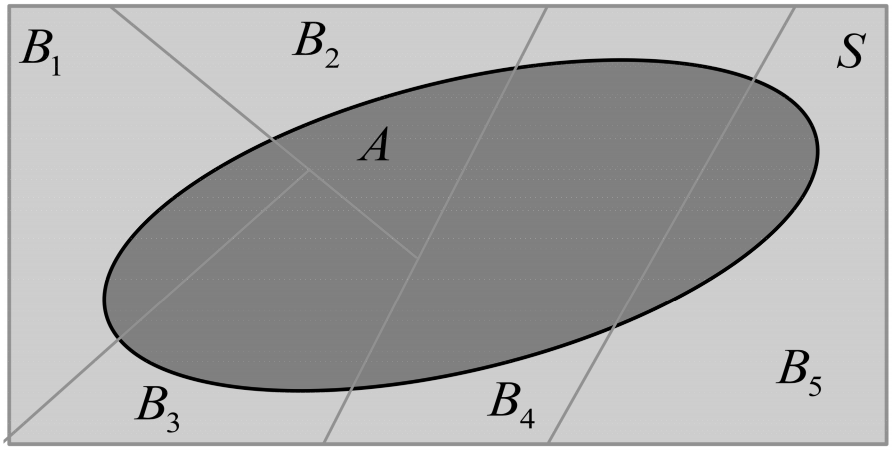
Conditional probability of obtaining *outcome B given that A has occurred* can be denoted as

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

Total Probability Theorem

If the events $B_1, B_2, \cdots, B_k$ partition the sample space S completely then for any event A in S:

$$P(A) = \sum_{i=1}^{k} P(A \cap B_i) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$



Bayes' Rule

Bayes' rule allows us to *update our belief* in the probability of an event based on new evidence.

The formula for Bayes' rule is expressed as:

LIKELIHOOD
The probability of "B" being true, given "A" is true

PRIOR
The probability of "A" being true. This is the knowledge.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

POSTERIOR
The probability of "A" being true, given "B" is true

MARGINAL
The probability of "B" being true

Random Variables

A *random variable* is a mathematical concept used in probability theory and statistics to describe an uncertain or random quantity.

- It represents a numerical outcome or value that results from a random experiment, process, or event.

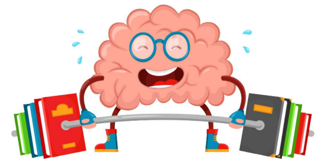
There are *two main types* of random variables:

- *Discrete* random variable
- *Continuous* random variable

A *discrete random variable* can only take on specific, distinct values, often integers or a countable set of values.

A *continuous random variable* can take on any value within a specified range or interval.

Exercise Problems



Q3.1 An automated quality control system in a factory is designed with 5 identical inspection units. The system is considered to fail when 3 or more of these inspection units fail. *(a)* How many are total system states? *(b)* How many of them belong to failure states?

Q3.2 In a factory producing light bulbs, each bulb has a probability of 0.02 of being defective. To ensure quality, a random sample of 10 bulbs is tested. Calculate the probability that exactly 2 bulbs out of the 10 are defective.

Q3.3 A software testing team needs to test a new application with 4 different modules. Each module can pass or fail independently. Assuming the probability of passing for each module to be 0.8, calculate the probability of at least 2 modules passing out of 4.

Exercise Problems

Q3.4 In a manufacturing facility, the probability that a product meets the initial quality standards after the first inspection is $P(I) = 0.83$. The probability that it passes the final quality check is $P(F) = 0.82$. The probability that a product meets both the initial and final quality standards is $P(I \cap F) = 0.78$. *(a)* What is the probability that the product passes the final quality check, given that it met the initial quality standards? *(b)* What is the probability that the product met the initial quality standards, given that it passes the final quality check?

Q3.5 In a pharmaceutical manufacturing process, the probability that a batch of medication passes the initial quality control (QC) inspection is $P(QC_1) = 0.90$. If it passes the initial QC inspection, the probability that it passes the secondary QC inspection is $P(QC_2 \mid QC_1) = 0.85$. If it passes the secondary QC inspection, the probability that it passes the final QC inspection is $P(QC_3 \mid QC_2) = 0.80$. *(a)* What is the overall probability that a batch of medication passes all three QC inspections? *(b)* Given that a batch has passed the final QC inspection, what is the probability that it passed the initial QC inspection?

Exercise Problems

Q3.6 In a certain assembly plant, three machines, B_1, B_2, and B_3 make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

Q3.7 A bicycle company produces bicycles in three of their manufacturing plants at different locations. Three plants, P_1, P_2, and P_3 manufactures 20%, 35%, and 45%, respectively, of the bicycles. It is known from past experience that 4%, 1%, and 2% of the bicycles made by each plant, respectively, are defective. Use the total probability theorem and the Bayes' rule to the answer the following questions. *(a)* Suppose that a finished bicycle is randomly selected. What is the probability that it is defective? *(b)* What is the probability that a defective bicycle found belongs to plant P_3? *(c)* What will be the change in probability that a defective bicycle belongs to plant P_3 if the proportion of manufacturing bicycles are changed from $(20\%, 35\%, 45\%)$ to $(45\%, 35\%, 20\%)$, respectively, for the three plants.

Answers to Exercise Problems

Q3.1 [Ans: (a) 32; (b) 16]

Q3.2 [Ans: 0.0153]

Q3.3 [Ans: 0.9728]

Q3.4 [Ans: (a) 0.94; (b) 0.95]

Q3.5 [Ans: (a) 0.612; (b) 1.0]?

Q3.6 [Ans: 0.0245]

Q3.7 [Ans: (a) 0.0205; (b) 0.439; (c) 0.282 $\downarrow$]