INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

End-Autumn Semester Examination 2024-25

Date of Examination: 19/11/2024 **Session:** FN **Duration:** 3 Hrs **Full Marks:** 100

Subject No. AI61005 **Subject:** Artificial Intelligence: Foundations and Applications

Department/Center/School: Department of Artificial Intelligence

Specific charts, graph paper, log book etc., required NO

Special Instructions (if any) : Answer all the parts of a question in same place. Your answer should be precise and to the point for full credit. Detailed steps must be shown.

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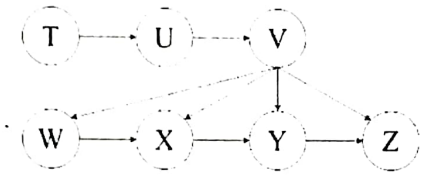
1. Consider the Bayes' net shown below. All variables have binary domains.



(a) Perform variable elimination to compute the query $P(U, W \mid Y = +y)$ and fill in the newly generated factors after eliminating each variable in the table given below. Consider the variable elimination ordering as T, V, X, Z. **[9]**

Factors Generated after Eliminating corresponding Variable (Left to right)			
T	V	X	Z

(b) Write down the heuristics of choosing the variable elimination ordering. Find a variable elimination ordering for the same query, i.e., for $P(U, W \mid Y = +y)$, for which the maximum size factor generated along the way is smallest. Fill in the variable elimination ordering and the factors generated in the table below. **[2+2 = 4]**

Eliminated Variable	**Factor Generated**

2. As part of a comprehensive study on people's happiness, important data were collected from graduating students. In an entirely optional survey that all students are required to complete, the following highly objective questions were asked:

- Do you party frequently [Party: Yes/No]?
- Are you wicked smart [Smart: Yes/No]?
- Are you creative [Creative: Yes/No]?
- Did you do well on all your homework assignments? [HW: Yes/No]
- Do you use a Mac? [Mac: Yes/No]
- Did your final project succeed? [Project: Yes/No]
- Did you succeed in your most important class? [Success: Yes/No]
- Are you currently Happy? [Happy: Yes/No]

All variables have binary domains. The values are either 0, corresponding to No response, or 1, corresponding to a Yes response. After consulting a behavioural psychologist, the following complete set of conditional relationships was obtained:

- HW depends only on Party and Smart
- Mac depends only on Smart and Creative
- Project depends only on Smart and Creative
- Success depends only on HW and Project
- Happy depends only on Party, Mac, and Success

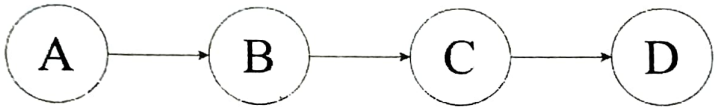
(a) Draw the Bayesian network and find the total number of independent parameters needed to construct the conditional probability tables. **[2]**

(b) Using only the Bayesian network structure from part (a), answer the following True/False questions and provide a brief explanation: **[2+2 = 4]**

(i) Party is dependent of Success given HW.

(ii) Party is independent of Smart given Success.

3. Consider the following Bayes' net and the corresponding distributions over the variables in the Bayes' net:



P(A)	
-a	3/4
+a	1/4

P(B|A)		
-a	-b	2/3
-a	+b	1/3
+a	-b	4/5
+a	+b	1/5

P(C|B)		
-b	-c	1/4
-b	+c	3/4
+b	-c	1/2
+b	+c	1/2

P(D|C)		
-c	-d	1/8
-c	+d	7/8
+c	-d	5/6
+c	+d	1/6

(a) You are given the following samples: **[1.5 * 2 = 3]**

$s_1: +a \ +b \ -c \ -d$ $\quad$ $s_5: +a \ -b \ -c \ +d$

$s_2: +a \ -b \ +c \ -d$ $\quad$ $s_6: +a \ +b \ +c \ -d$

$s_3: -a \ +b \ +c \ -d$ $\quad$ $s_7: -a \ +b \ -c \ +d$

$s_4: -a \ -b \ +c \ -d$ $\quad$ $s_8: -a \ -b \ +c \ +d$

(i) If these samples come from performing prior sampling, then calculate the sample estimate of $P\ (+c)$.

(ii) What will be the sample estimate of $P\ (+c|\ +\ a, -d)$?

(b) Estimate $P\ (-a|\ +\ b, -d)$ using Likelihood Weighting Sampling. Mention the weight of each sample, which is considered for estimation. **[2]**

(c) During Gibbs Sampling, samples are generated through an iterative process. Assume that the only evidence that is available is $A\ =\ +a$. Which of the following sequence(s) could have been generated by Gibbs Sampling? Justify your answer. **[1]**

i)
$s_1: +a \ -b \ -c \ +d$
$s_2: +a \ -b \ -c \ +d$
$s_3: +a \ -b \ +c \ +d$

ii)
$s_1: +a \ -b \ -c \ +d$
$s_2: +a \ -b \ -c \ -d$
$s_3: -a \ -b \ -c \ +d$

iii)
$s_1: +a \ -b \ -c \ +d$
$s_2: +a \ -b \ -c \ -d$
$s_3: +a \ +b \ -c \ -d$

iv)
$s_1: +a \ -b \ -c \ +d$
$s_2: +a \ -b \ -c \ -d$
$s_3: +a \ +b \ -c \ +d$

(d) Write the advantages/disadvantages of Likelihood Weighting Sampling over Rejection Sampling. Explain with an example. **[5]**

4. Consider a course registration system for a Master's program where students must take exactly 6 different courses in the available slots (S1, S2, S3, S4, S5, S6). There are 6 available courses (ML, AI, NLP, DB, CV, IR). The timings of the slots are:
S1: 9-10 a.m. S2: 9:30-10:30 a.m. S3: 10:00-11:00 a.m. S4: 10:00-11:00 a.m. S5: 10:30-11:30 a.m. S6: 10:30-11:30 a.m.
The courses are: (i). ML could be assigned to S3 or S4. (ii). AI could be assigned to S1 or S2. (iii). NLP for S1, S4, S5, S6. (iv). DB for S2, S4. (v). CV for S3, S4, S6. (vi). IR for S5

(a) Formulate this problem as a CSP problem in which there is one variable per slot, stating the domains, and constraints. Constraints should be specified formally and precisely, in an explicit way. **[6]**

(b) Draw the constraint graph associated with your CSP. **[4]**

(c) Search for a solution using basic backtracking. Only check whether any new assignment violates no constraint with previous assignments. As a tie breaker assign a course to a slot based on alphabetical order. **[5]**

Fill out this worksheet as you draw your search tree. There may be more rows than you need.

(i) Every time you assign a variable or remove a variable from propagation queue, fill out a new row in the table. (The same variable might appear in more than one row, especially if you have to backtrack.)

(ii) In that row indicate **which variable you assigned or de-queued**; write its **assigned** value if it has one (e.g. X=x), otherwise just write its name (X). In the second column, list the **values that were just eliminated from neighboring variables** as a result. If no values were just eliminated, write **NONE** instead.

(iii) If your search has to backtrack after assigning or de-queuing a variable: first, **finish listing** all values eliminated from neighboring variables in the current row. Next, check the backtrack box in that row. Then, continue with the next assignment in the following row as usual.

(iv) At some point, you might add several variables to your propagation queue at once. Break ties by adding variables to your propagation queue **in alphabetical order.**

	Var assigned or dequeued	List all values eliminated from neighboring variables	Backtrack?
Ex	X	Y≠B Z≠C (example)	√
1			

(d) Fill up the same table, but now implement backtracking with Forward checking and Minimum Value Ordering. Break the ties in alphabetic order. **[5]**

5. Consider the fuzzy production rule R: "If age is OLD, then experience is HIGH" with the following known membership functions:

- μ_{OLD}(age) : {0.2/Y, 0.5/A, 0.8/O}
- μ_{HIGH}(experience) : {0.4/L, 0.7/M, 0.9/H}

(a) Determine the relational matrix for the rule R. **[2]**

(b) Given the observed membership function $\mu_{MODERATELY\ OLD}$(age) = {0.3/Y, 0.6/A, 0.7/O}, determine the inferred membership function for $\mu_{MODERATELY\ HIGH}$(experience) using the max-min composition operator. **[3]**

6. Consider a Sugeno-type fuzzy inference system with the following rules;

- Rule 1: If x is C_1 and y is D_1, then $z = 3x + 2y + 7$
- Rule 2: If x is C_2 and y is D_2, then $z = 5x - y + 8$

Given the following membership functions:

- C_1 : Trapezoidal (2, 4, 6, 8)
- C_2 : Triangular (5, 7, 9)
- D_1 : Gaussian (mean = 5, sigma = 1.5)
- D_2 : Bell-shaped (a = 1.5, b = 0.7, c = 7) [membership function: $1 / (1 + ((x - c) / a)^{2 * b})$]

If x = 6 and y = 7, calculate the output value z using the system **[10]**

7. Let us consider two fuzzy numbers A and B with the universe of discourse $X \in [-1,2]$ as given below. Verify whether $(A-B)^2 = A^2 + B^2 - 2AB$ **[10]**

$$A = \frac{0.7}{(-1)} + \frac{1.0}{0} + \frac{0.5}{1} \qquad B = \frac{1.0}{(-1)} + \frac{0.5}{1} + \frac{0.7}{2}$$

8. (a) Consider lifestyle forecasting scenario in Figure 1. We have three states EAT(E), WORK(W), SLEEP (S). We begin in state E and MDP is shown. Immediate reward is 0. Answer the questions:

(i). Suppose you run a value iteration algorithm with discount factor 0.7. Fill up the given table for next time step. **[4]**

(ii). Assuming above values of states $V^2(E)$, $V^2(W), V^2(S)$ computed in (i), what is the implied policy π at this step. Write the values of $\pi(E), \pi(W), \pi(S)$. **[1]**

(iii). Suppose the weather adopts the policy a1 from all the states i.e., $\pi_1(E), \pi_1(W), \pi_1(S) = a2$. Evaluate the value of each state given this policy π_1. Calculate $V_{\pi_1}(E), V_{\pi_1}(W), V_{\pi_1}(S)$. **[1]**

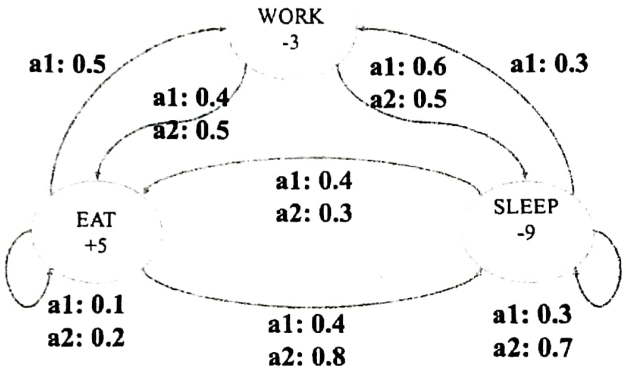


Figure 1

t	$V^t(E)$	$V^t(W)$	$V^t(S)$
1	+5	-3	-9
2			

(iv). Starting with π_1 as in (iii), do a policy update to find new policy π_2 (one step of policy iteration). Find the value of $\pi_2(E), \pi_2(W), \pi_2(S)$. **[4]**

(b). Write the steps in Genetic Algorithm (GA). **[5]**

(c). Write different reproduction types in GA. Explain the relation with local and global search? **[2+3]**

(d). Suppose a genetic algorithm uses chromosomes of the form x = abcdefgh with a fixed length of eight genes, where a, b, …, h are positions in chromosomes. The name of city is represented by capital letter in English alphabet {A to Z}. Weight of each city is given by their position in alphabetical order starting with 1 i.e., A = 1, B = 2, C = 3, …, Y = 25, Z = 26. The fitness of individual x be calculated as: $f(x) = (a + b) - (c + d) + (e + f) - (g + h)$,

Initial population is as follows: x1 = ABCDEFGH, x2 = STUVWXYZ, x3 = LKJIHGFE, x4 = TSRQPONM

(i). Evaluate the fitness of each individual, showing all your workings, and arrange them in order with the fittest first and the least fit last **[2]**

(ii). Perform the following crossover operations:

A. Cross the fittest two individuals using one–point crossover at the middle point. **[1.5]**

B. Cross the two least fittest individuals using a two–point crossover (after point b and after point f as mentioned in form x). **[1.5]**