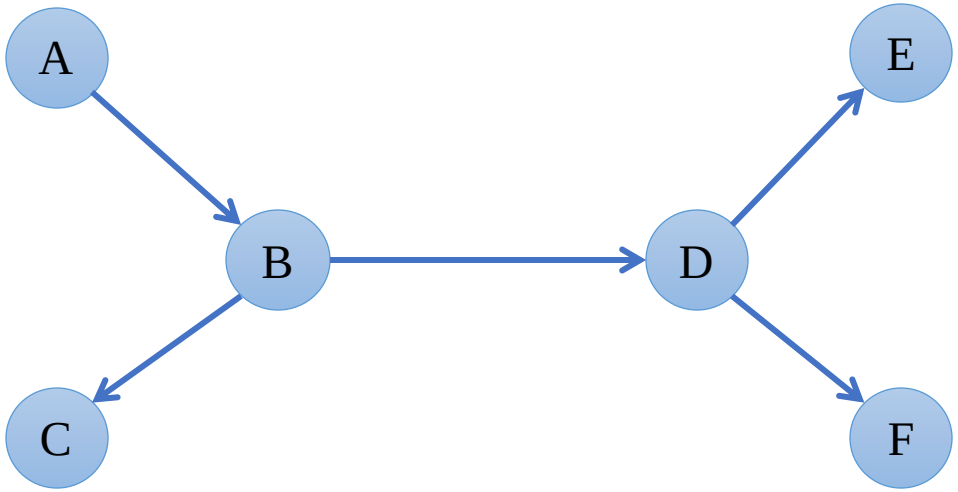
Constraint Satisfaction Problem

04/03/2025

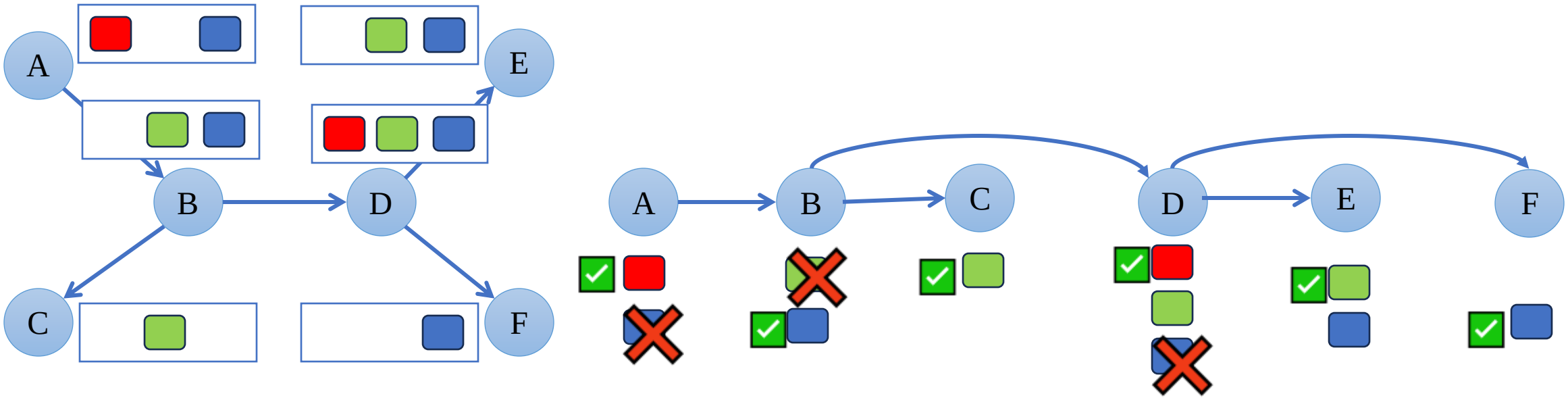
Koustav Rudra

Tree Structured CSP



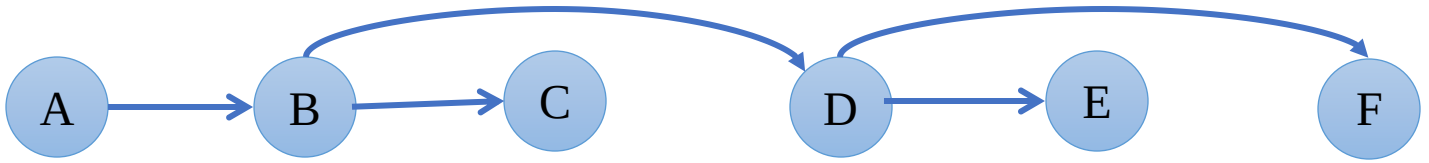
- No loop
- If constraint graph has no loop, the CSP could be solved in $O(nd^2)$ in worst case

Tree Structured CSP



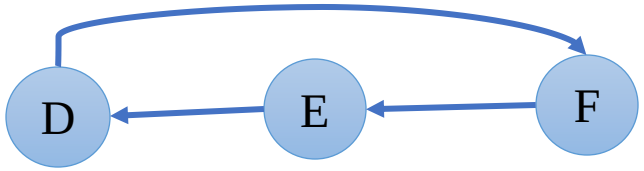
- Order:
  - Choose a Root variable
  - Order other variables in such a way that parents precede children
- Remove Backward:
  - For i=n to 2, REMOVEINCONSISTENT($Parent[x_i], x_i$)
- Assign Forward:
  - For i=1 to n, Assign x_i consistently with $Parent[x_i]$
- Runtime: $O(nd^2)$ Forward Pass: n, Backward Pass: n, Comparison: d^2

Tree Structured CSP



- After Backward pass all the root-to-leaf arcs are consistent
  - After a backward pass, each x→y has been made arc consistent
  - Y's children have been processed before y
  - Y's options can't be reduced

- Forward assignment will not backtrack

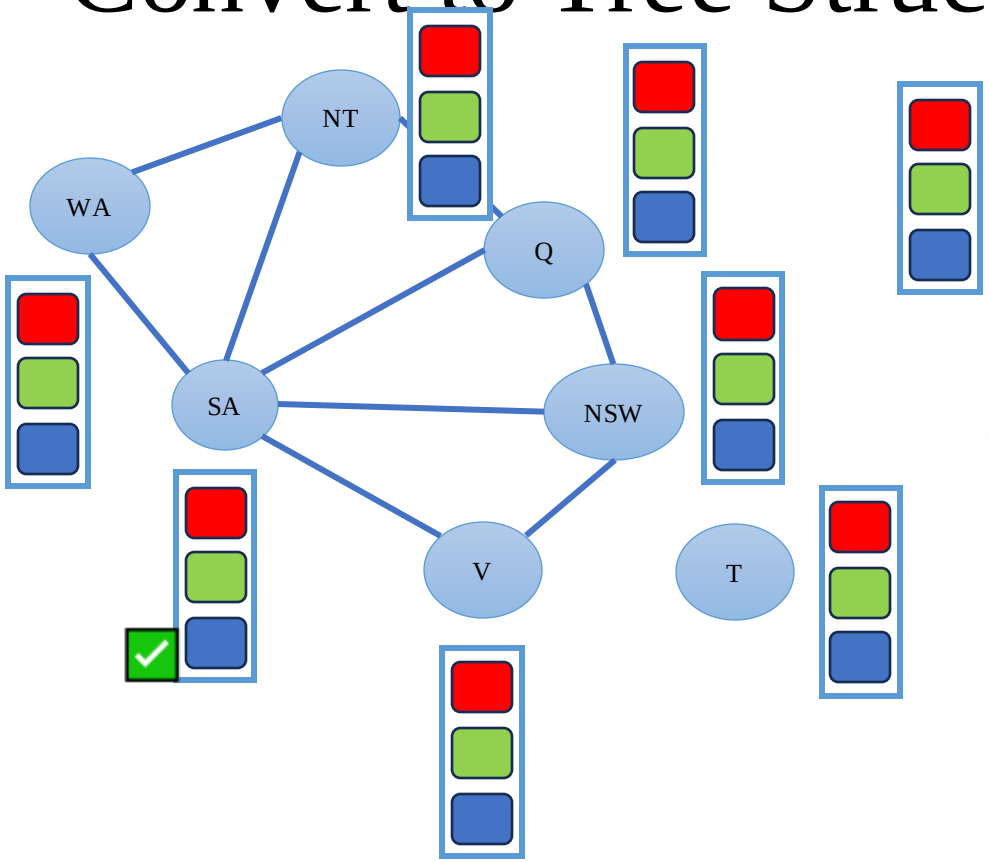


- Does not work on constraint graphs with cycles?

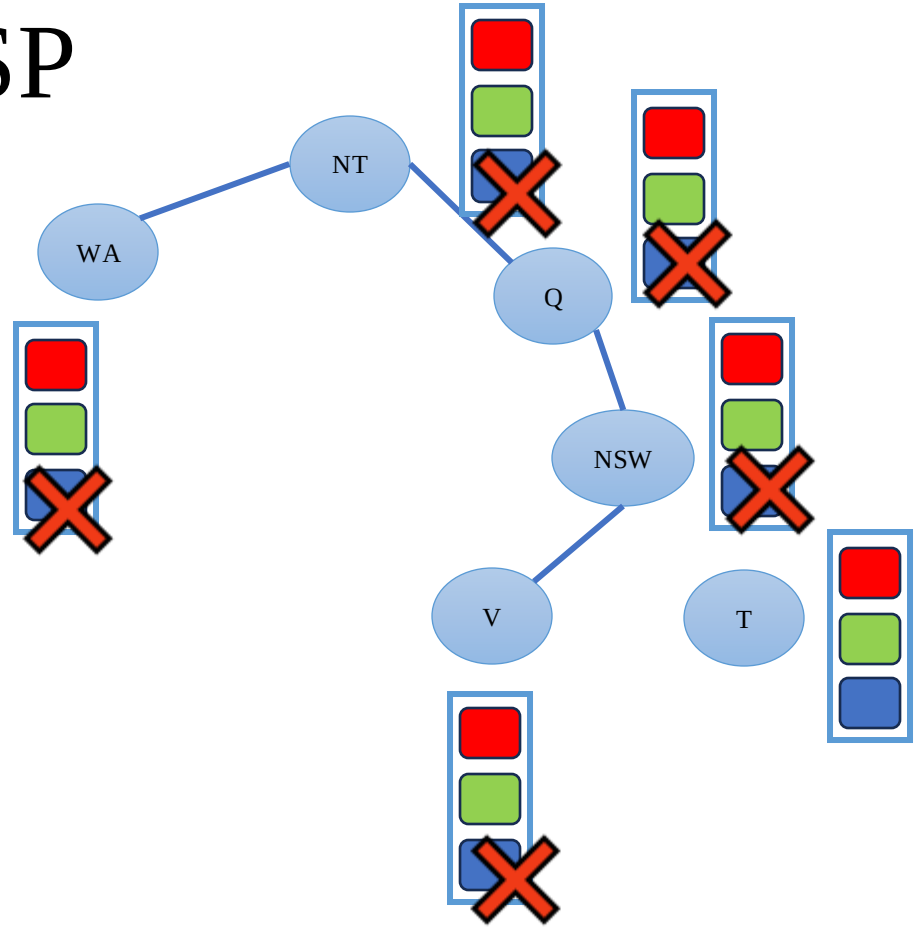
- Tree Structured CSP is not common

- How can we convert a CSP to Tree Structured CSP?

Convert to Tree Structured CSP



CUTSET={SA}



- Conditioning: Forcefully initiate a variable and prune the domains of the neighbours
- CUTSET-CONDITIONING:
  - Obtain a CUTSET of variables
  - Removing those will leave the constraint graph a tree
  - Instantiate (in all ways) the CUTSET

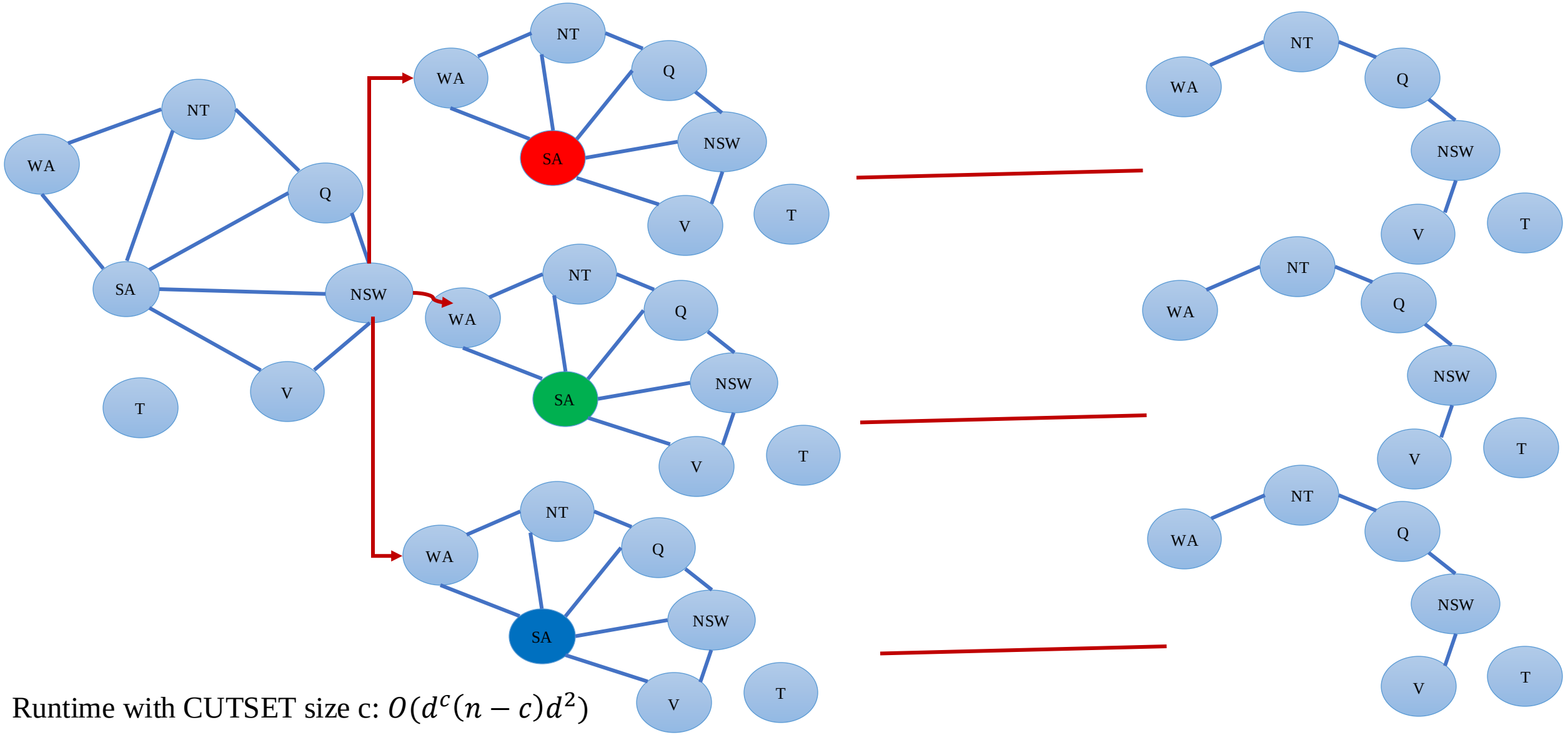
CUTSET Conditioning

Choose a CUTSET

Instantiate CUTSET

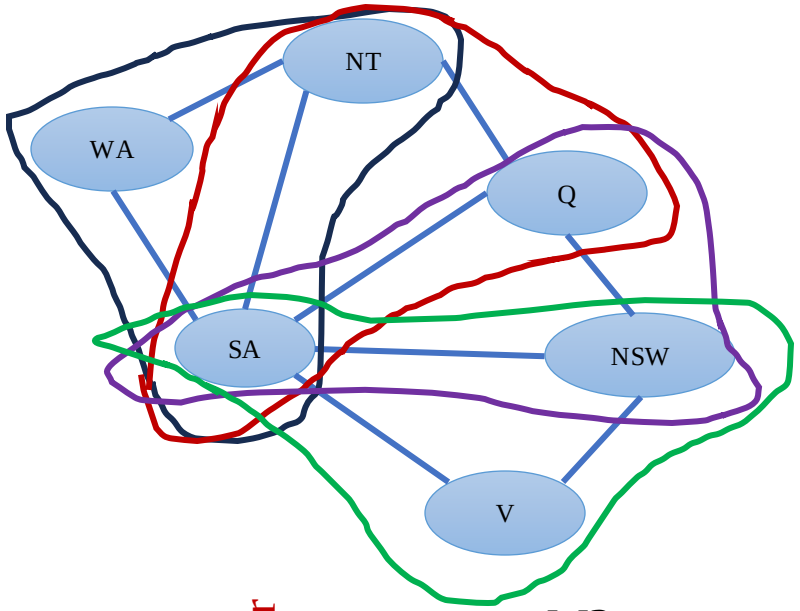
Compute Residual

Solve Residual TreeCSP

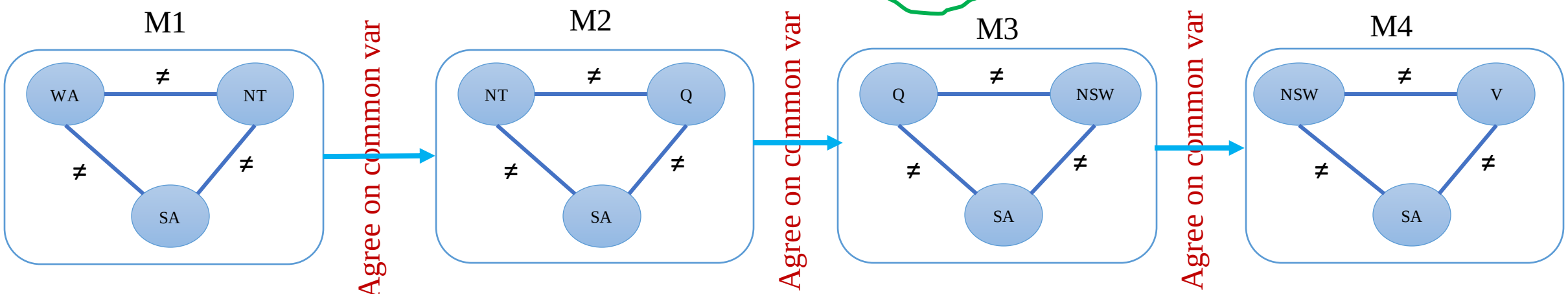


Runtime with CUTSET size c: $O(d^c(n-c)d^2)$

Tree Decomposing



Meta Constraint Graph



WA=R, NT=G, SA=B

Agree: $(M1, M2) \in \{\big((WA = G, NT = G, SA = G), (NT = G, Q = B, SA = G)\big), \dots\}$

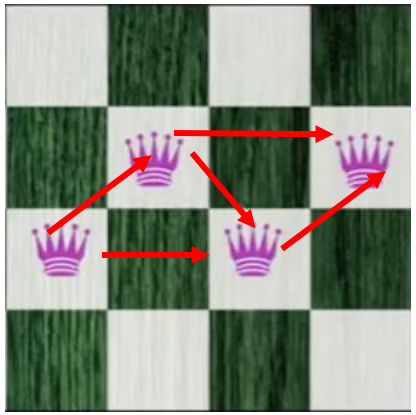
Iterative Improvement

- Start with a complete assignment with unsatisfied constraints

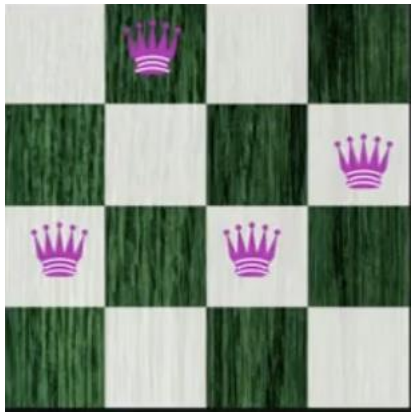
- Iteratively change solution
  - Reassign variable values
  - No data structure like stack maintained

- Algorithm
  - Variable selection: randomly select any conflicting variable
  - Value selection: min-conflict heuristics
    - Choose a value that violates the fewest constraints
    - (hill climb with h(n)=total number of constraints violated)

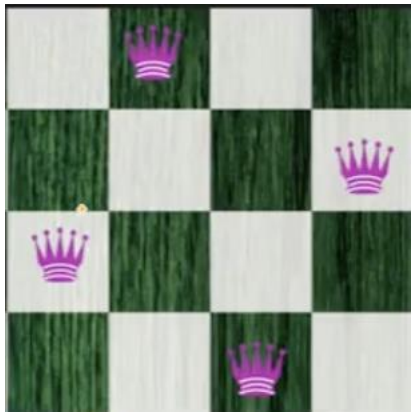
Iterative Improvement: 4 Queen Problem



h=5



h=2

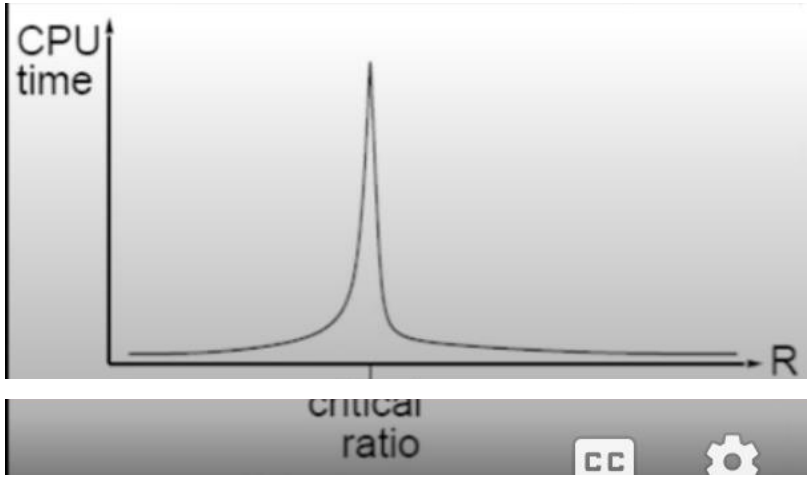


h=0

Min Conflict Heuristics

- Can solve N-queen problem for arbitrary n (~10M) with high probability in constant time
- Similar performance on random CSPs except for a narrow range

- $R = \frac{|Constraint\ Set|}{|Variable\ Set|}$



Comparison

Problem	Backtracking	BT+MRV	Forward Checking	FC + MRV	Min-Conflicts
N-Queens	>40,000K	13,500K	>40,000K	817K	4K

Summary

- CSP: Special instance of search problem
  - Generic (i.e., Problem Agnostic)
- Basic Algorithm: Backtracking
- Speedup: Ordering, Filtering, Problem Structure
- Iterative min-conflict (more practical)

Thank You