

MLE Example (cont'd)

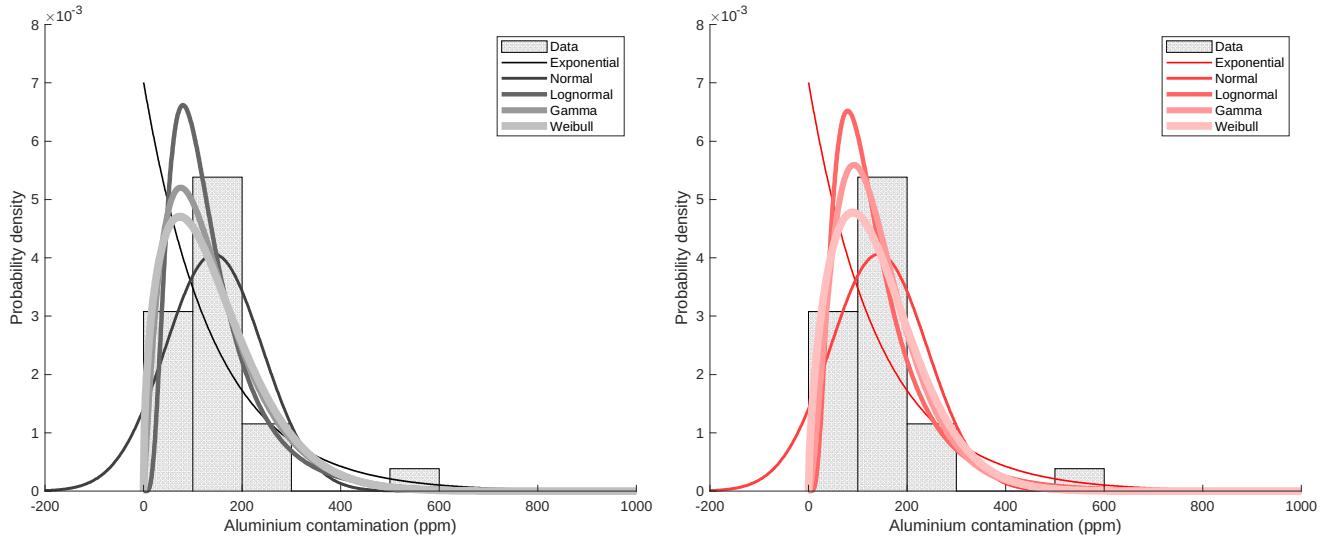


Figure: Various distributions fitted to the aluminium contamination data using **method of moments** (*left*) and **method of maximum likelihood** (*right*).

8 Six-Sigma Quality

Six-Sigma & Other Tools

Use of statistical and other analytical tools for process improvement has grown steadily over the years.

- *Statistical quality control*: Origins in 1920, explosive growth during WW II, 1950s.
- *Operations Research (OR)* (1940s)
- *FDA, EPA* in the 1970's
- *Total Quality Management (TQM)* movement in the 1980's
- *Re-engineering* of business processes (late 1980's)
- *Six-Sigma*: Origins at Motorola in 1987, expanded impact during 1990s to present.

What is Six Sigma (6σ)?

Sigma (σ) is a statistical concept that represents how much *variation* there is in a process relative to *customer specifications*.

Sigma value is based on *defects per million opportunities (DPMO)*.

Six Sigma (6σ) is equivalent to *3.4 DPMO*.

The variation in the process is so small that the resulting products and services are *99.9996% defect free*.

Focus of Six Sigma is on *process improvement*.

A process is an *organized sequence of activities* that produces an output that adds value to the organization. All work is performed in (interconnected) processes.

Amount of Variation vs. Sigma Value

Variation	Effect	Sigma Value
Too much	Hard to produce output within customer specifications	Low (0 – 2)
Moderate	Most output meets customer specifications	Middle (3 – 5)
Very little	Virtually all output meets customer specifications	High (6)

Origin & Popularization of Six Sigma

Started at *Motorola* Corporation in the mid-1980's, when the company discovered that products with a high first-pass yield (i.e., those that made it through the production process defect-free) rarely failed in actual use, resulting in higher customer satisfaction.

Popularized by former *General Electric* CEO Jack Welch's commitment to achieving Six Sigma capability (realized \$12 Billion savings over 5 years).

"Six Sigma is a quality program that improves your customers' experience, lowers your costs and builds better leaders." – *Jack Welch, the former CEO of General Electric*.

Why use σ as a metric?

- Focuses on defects
- Establishes a common metric to make comparisons easier
- A more sensitive indicator than other metrics

Origin & Popularization of Six Sigma (cont'd)

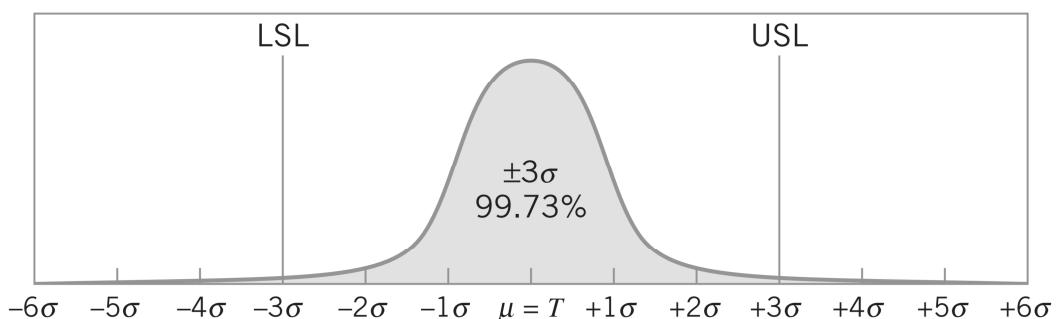
Six Sigma is good for any business, not just for manufacturing!

Six Sigma has been successful in improving processes throughout

- Operations
- Sales
- Marketing
- Information technology
- Finance
- Customer services
- Human resources.

Every business suffers from the two key problems that Six Sigma can solve: **defects** and **delay**.

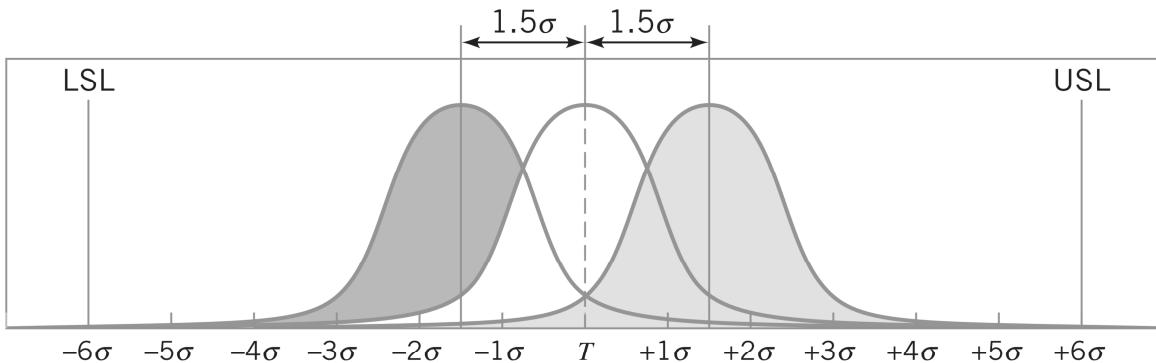
Six Sigma Concept



Spec. Limit	Percentage Inside Specs	ppm Defective
±1 Sigma	68.27	317300
±2 Sigma	95.45	45500
±3 Sigma	99.73	2700
±4 Sigma	99.9937	63
±5 Sigma	99.999943	0.57
±6 Sigma	99.999998	0.002

(a) Normal distribution centered at the target (T)

Six Sigma Concept (cont'd)



Spec. Limit	Percentage inside specs	ppm Defective
± 1 Sigma	30.23	697700
± 2 Sigma	69.13	608700
± 3 Sigma	93.32	66810
± 4 Sigma	99.3790	6210
± 5 Sigma	99.97670	233
± 6 Sigma	99.999660	3.4

(b) Normal distribution with the mean shifted by $\pm 1.5\sigma$ from the target

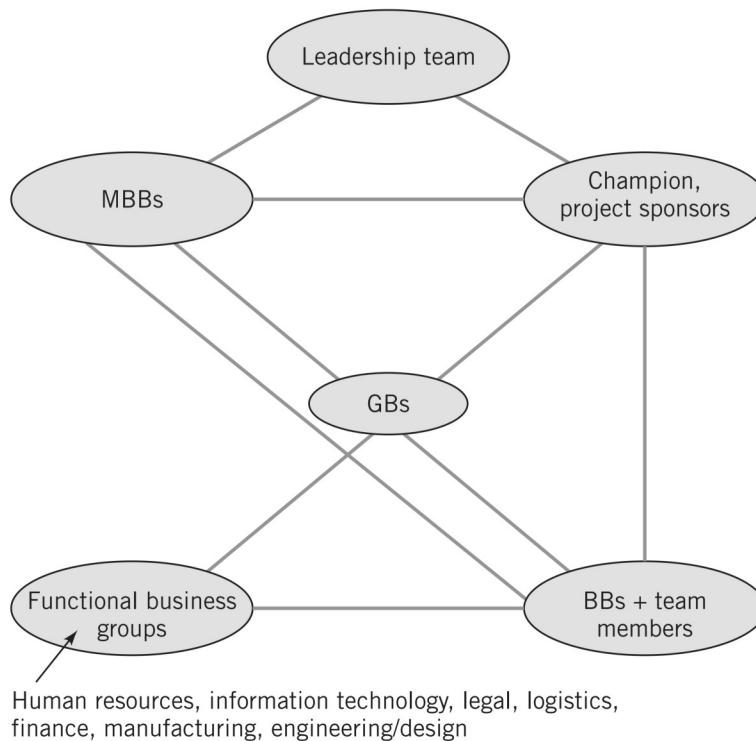
Structure of a Six Sigma Organization

Companies involved in a Six Sigma effort utilize specially trained individuals, called *Green Belts (GBs)*, *Black Belts (BBs)*, and *Master Black Belts (MBBs)* to lead teams focused on projects.

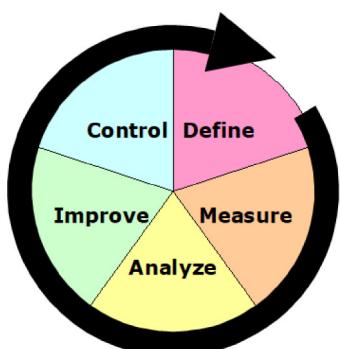
The Six Sigma organization's structure highlights key connections between *functional units*:

- The *leadership team* holds ultimate responsibility for approving improvement projects and overseeing progress.
- *Project champions* facilitate identification, team assembly, resource allocation, project advancement, often managing multiple projects.
- *Black Belts* lead project completion, while *Green Belts*, with less experience, either lead smaller projects or support Black Belt-led teams.
- *Master Black Belts* serve as technical leaders, aiding in project identification, consulting on technical matters, and *training Green and Black Belts*.

Structure of a Six Sigma Organization (cont'd)

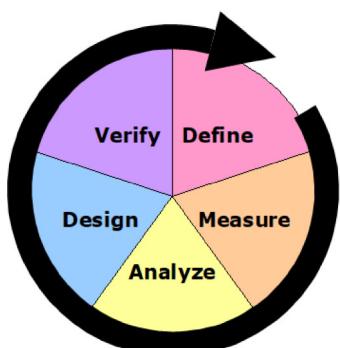


Six Sigma Methodology



DMAIC: Define - Measure - Analyze - Improve - Control

This method is used to improve the current capabilities of an existing process. This is by far the most commonly used methodology of sigma improvement teams.



DMADV: Define - Measure - Analyze - Design - Verify

This method is used when you need to create or completely redesign a process, product, or service to meet customer requirements. DMADV teams are usually staffed by senior managers and Six Sigma experts.



Exercise Problems (cont'd)

Q8.1 A manufacturing company produces 10,000 units of a product each day. The current process operates at a defect rate of 3.4 defects per million opportunities (DPMO), corresponding to Six Sigma quality. (a) Calculate the number of defective units expected per day. (b) If the company wants to improve to a defect rate of 2.0 DPMO, how many defective units will they have after achieving this new quality level?



Exercise Problems (cont'd)

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Q8.2 A company produces metal rods with a target diameter of 10 mm, with an upper specification limit (USL) of 10.05 mm and a lower specification limit (LSL) of 9.95 mm. The standard deviation of the process is 0.01 mm. Determine the sigma level at which the process is operating. Suggest improvements needed to bring the process to Six Sigma quality if it is not already operating at that level.

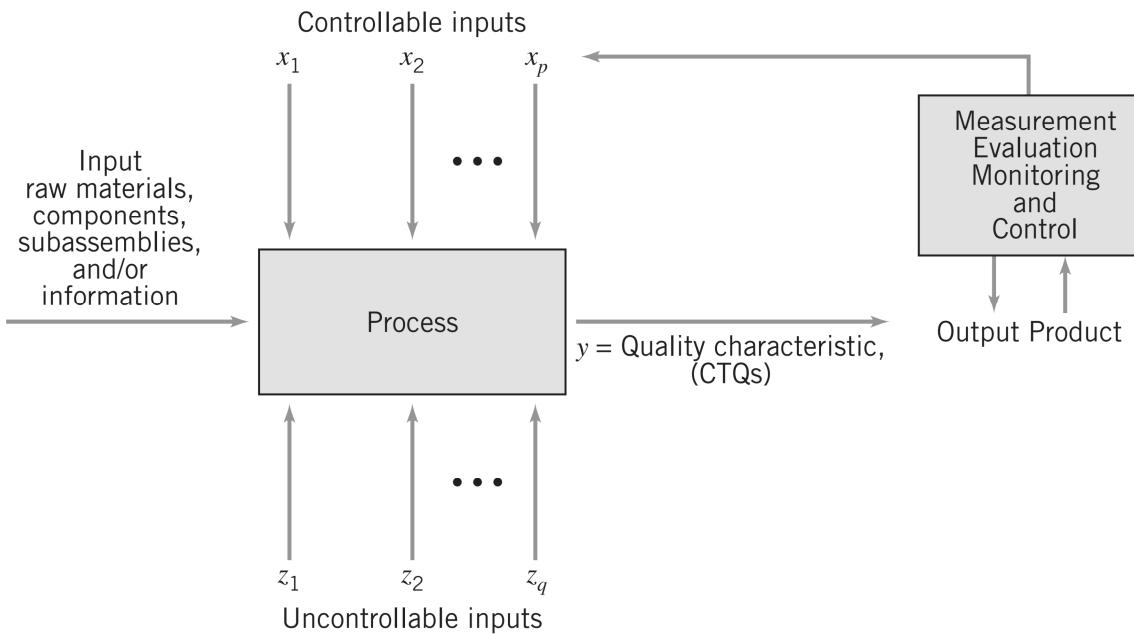
Answers to Exercise Problems

Q8.1 [Ans: 0.034; 0.02]

Q8.2 [Ans: 5σ ; $SD \approx 0.0083mm$]

9 Statistical Methods for Quality Improvement

Process Inputs & Outputs



Statistical Methods

Quality improvement programs focus specifically on *three* major areas.

1 *Statistical Process Control (SPC)*: Used to monitor, control, and improve processes by employing statistical techniques to understand and manage variations in manufacturing and production.

- Control charts and other problem-solving tools
- Reducing variability through elimination of assignable causes
- On-line technique

2 *Design of Experiments (DOE)*: Used to determine the effect of the controllable input factors on the output product parameters.

- Discovering the key factors that influence process performance
- Process optimization
- Off-line technique

3 *Acceptance Sampling*: Used to inspect and classify a randomly selected sample lot for acceptance or disposition.

Statistical Process Control (SPC)

One of the primary techniques of SPC is the application of *control charts*.

A *control chart* plots the averages of measurements of a quality characteristic in samples taken from the process.

It has a *center line (CL)* and *upper* and *lower control limits (UCL & LCL)*

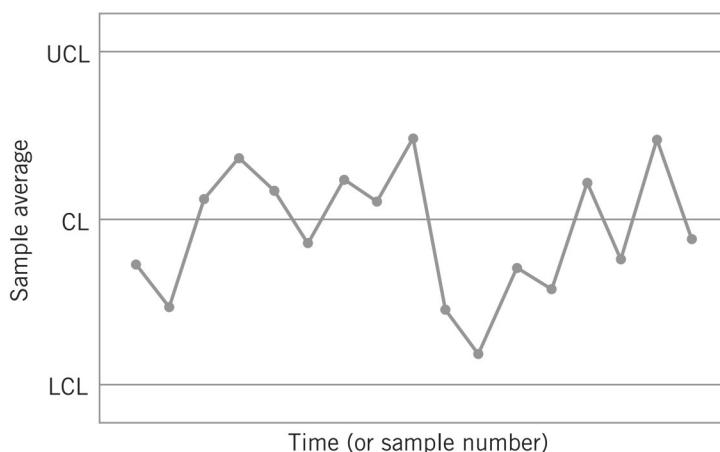


Figure: Walter A. Shewhart (1891-1967) developed the first control chart in about 1924.

- The *center line* represents where the process characteristic should fall if there are no unusual sources of variability present.
- The *control limits* are determined from various statistical considerations.

A Typical Control Chart

Systematic use of control charts is an excellent way to reduce variability!



When *unusual sources of variability* are present, sample averages will plot outside the control limits

- This is a signal that some investigation of the process should be made and corrective action taken to remove any variability.

Design of Experiments (DOX)

Design of experiments is helpful in discovering the *key variables influencing the quality characteristics* of interest in the process.

A *designed experiment* is an approach to systematically varying the controllable input factors in the process and determining the effect these factors have on the output product parameters.

- Statistically designed experiments are invaluable in reducing the variability in the quality characteristics.
- Determines the levels of the controllable variables that optimize process performance.

Design of Experiments (DOX) (cont'd)

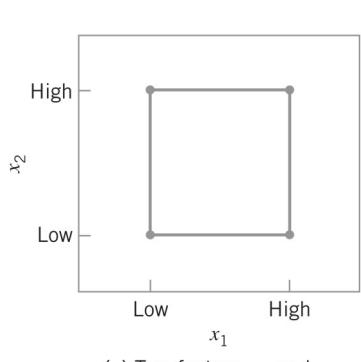
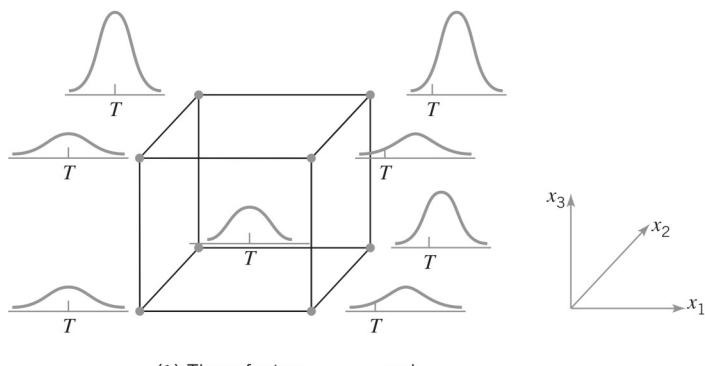
(a) Two factors, x_1 and x_2 (b) Three factors, x_1 , x_2 , and x_3

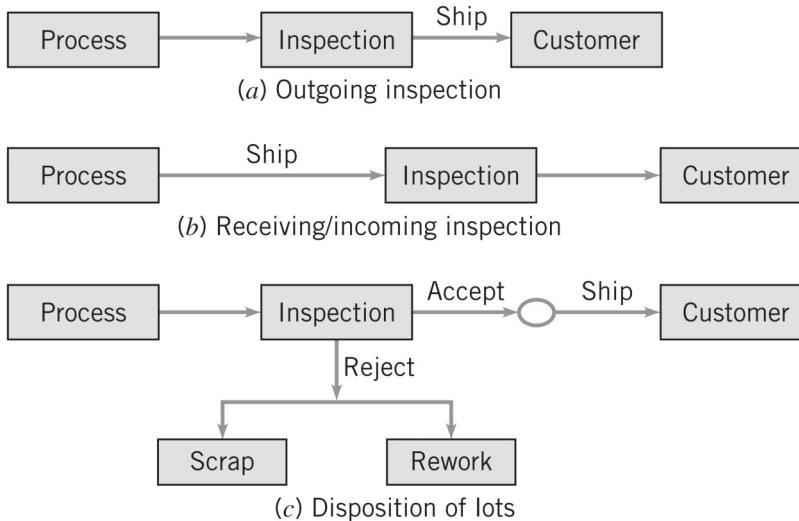
Figure: Factorial designed experiment for a process, where factors are varied together in such a way that all possible combinations of factor levels are tested. In (a), two factors with two levels (low & high) resulted in four possible test combinations. In (b), three factors with two levels resulted in eight test combinations.

Increasing x_1 from low to high increases the average level of the process output and could shift it off the target value (T). The process variability gets substantially reduced when x_2 and x_3 are at their high levels.

Acceptance Sampling

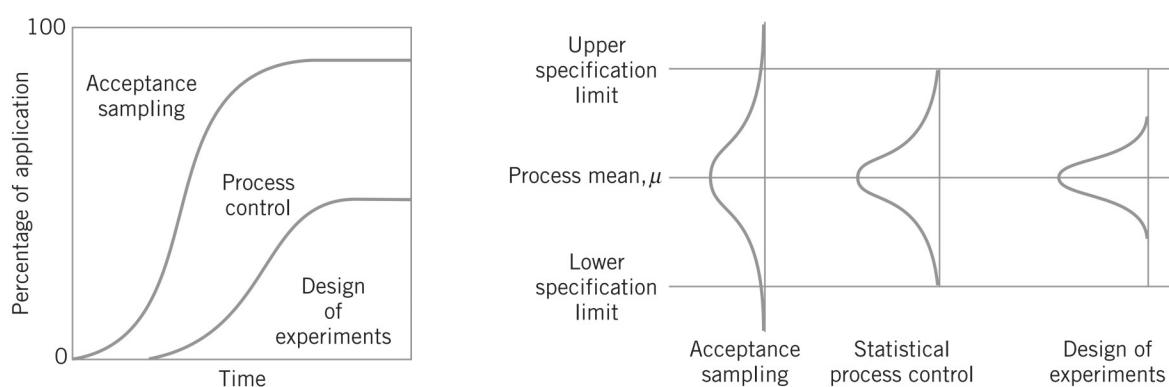
Acceptance sampling is defined as the *inspection* and *classification* of a sample of units selected at random from a larger batch or lot and the ultimate decision about disposition of the lot.

- *Outgoing inspection* is performed immediately following production.
- *Incoming inspection* is performed after products are received from supplier.



When to Use These Statistical Methods?

Phase diagram of the *use of quality-engineering methods* (left) and the corresponding *systematic reduction of process variability* (right).



This figure illustrates how companies start by not recognizing quality problems, then use basic acceptance sampling-based inspection methods. Over time, they realize that quality can't just be checked; they need to improve processes. They then use tools like SPC and DOX to make better products.

10 Quality Control Tools

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Basic Tools

If a product is to meet or exceed customer expectations, generally it should be produced by a process that is *stable* or *repeatable*.

The process must be capable of operating with little variability around the *target* or *nominal* dimensions of the product's *quality characteristics*.

Quality control (QC) tools are useful in achieving process stability and improving capability through the reduction of variability.

QC tools - the *magnificent seven*:

- 1 Histogram or stem-and-leaf plot
- 2 Check sheet
- 3 Pareto chart
- 4 Cause-and-effect diagram
- 5 Defect concentration diagram
- 6 Scatter diagram
- 7 Control chart

Chance & Assignable Causes of Variation

In any production process, a certain amount of *inherent* or *natural variability* will always exist.

This variability results from *chance (common) causes* of variation.

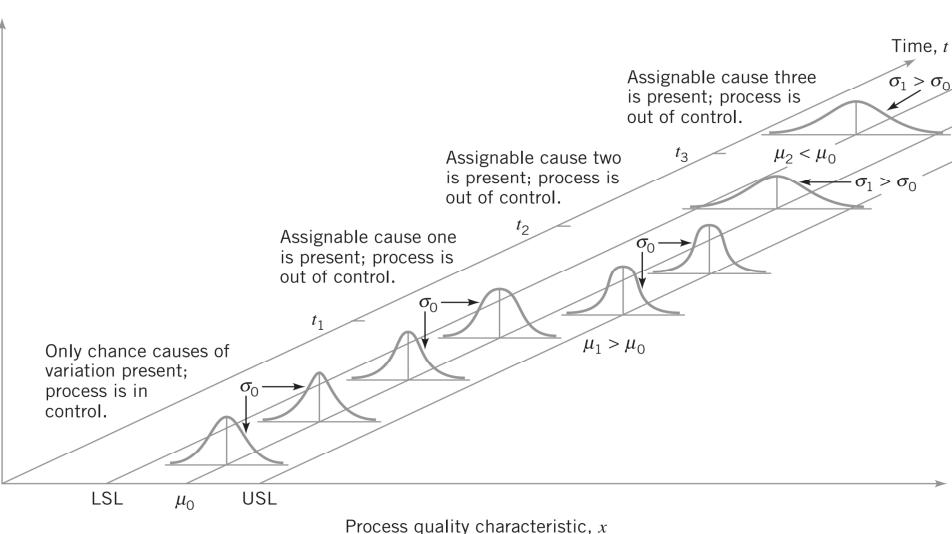
Another type of variation is caused by specific factors, known as *assignable (special) causes* of variation, that are not inherent in the process under normal conditions.

Assignable causes are *specific, identifiable*, and often *controllable*. When an assignable cause is present, it leads to non-random, significant changes in the process output.

Chance & Assignable Causes of Variation (cont'd)

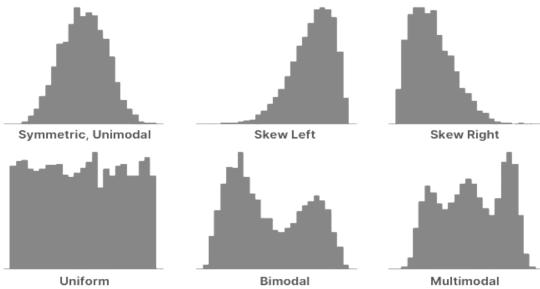
A process that is operating with only chance causes of variation present is *in statistical control*.

A process that is operating in the presence of assignable causes is an *out-of-control* process.



Histogram

A histogram displays the *frequency of data points* within specified intervals, providing a visual summary of the *variation* and *shape* of the data.

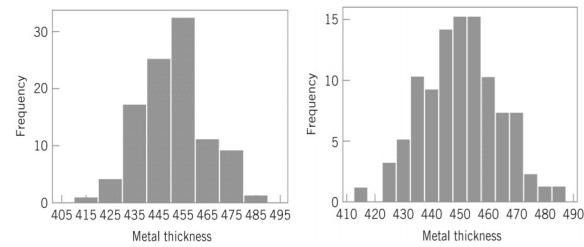


The *choice of bin width* can influence the appearance of histogram.

Bin width adjustment allows for a balance between *capturing details* and *smoothing* out noise in data.

In practice, the *no. of bins* are selected $\approx \sqrt{n}$ for n observations.

Sturges's rule can also be employed, where $\text{no. of bins} = 1 + \log_2 n$.



Stem-and-Leaf Plot

A *stem-and-leaf plot* is a visual representation of a dataset where each data point is split into a *stem (leading digits)* and a *leaf (trailing digit)*.

The *stems* are typically arranged *vertically in ascending order*, and the *leaves* are listed *horizontally* next to their respective stems.

Clusters of leaves near a stem indicate data concentrations that help identifying the *outliers*.

Stem	Leaf
5	
6	2 7
7	5 6 9
8	2 7
9	2
10	

Key: 6|2 = 6.2 score

Sorted quiz score (out of 10) data:
6.2, 6.7, 7.5, 7.6, 7.9, 8.2, 8.7, 9.2

Check Sheet

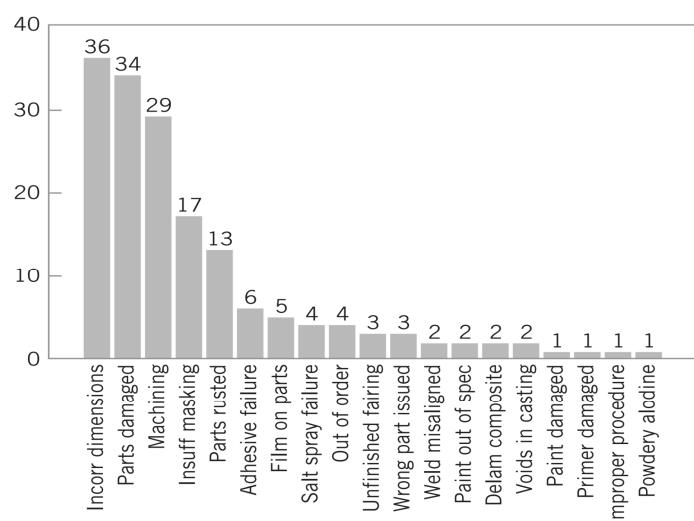
The check sheet is a *time-oriented summary* of historical or current operating data that helps identifying trends or other patterns.

CHECK SHEET DEFECT DATA FOR 2002–2003 YTD													
Defect	2002												Total
	1	2	3	4	5	6	7	8	9	10	11	12	
Parts damaged	1	3	1	2	1	10	3	2	2	7	2	34	
Machining problems	3	3			1	8	3	8	3			29	
Supplied parts rusted	1	1	2	9								13	
Masking insufficient	3	6	4	3	1							17	
Misaligned weld	2											2	
Processing out of order	2										2	4	
Wrong part issued	1			2								3	
Unfinished fairing	3											3	
Adhesive failure		1					1		2	1	1	6	
Powdery alodine		1										1	
Paint out of limits			1							1		2	
Paint damaged by etching	1											1	
Film on parts		3	1	1								5	
Primer cans damaged			1									1	
Voids in casting				1	1							2	
Delaminated composite					2							2	
Incorrect dimensions					13	7	13	1	1	1	1	36	
Improper test procedure						1						1	
Salt-spray failure								4		2		4	
TOTAL	4	5	14	12	5	9	9	6	10	14	20	7	166

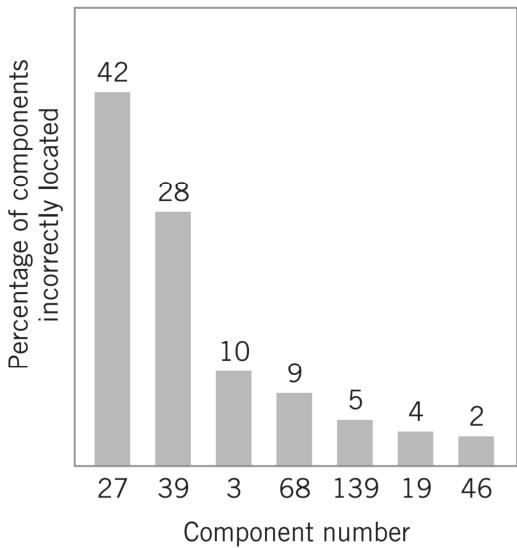
Pareto Chart

The Pareto chart is a *frequency distribution or histogram* of attribute data arranged by category.

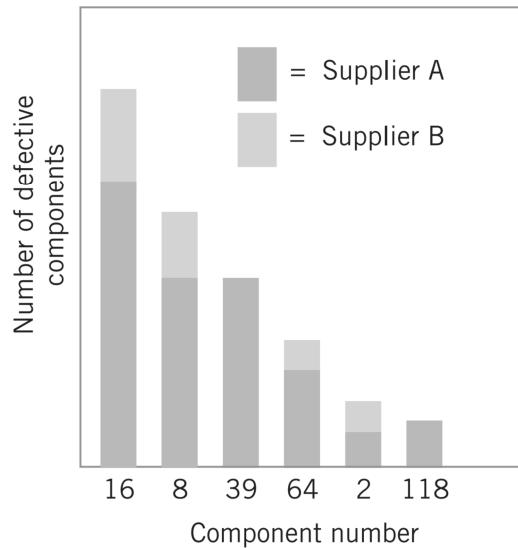
The Pareto chart does not automatically identify the *most important* defects, but only the *most frequent* ones.



Pareto Chart Examples



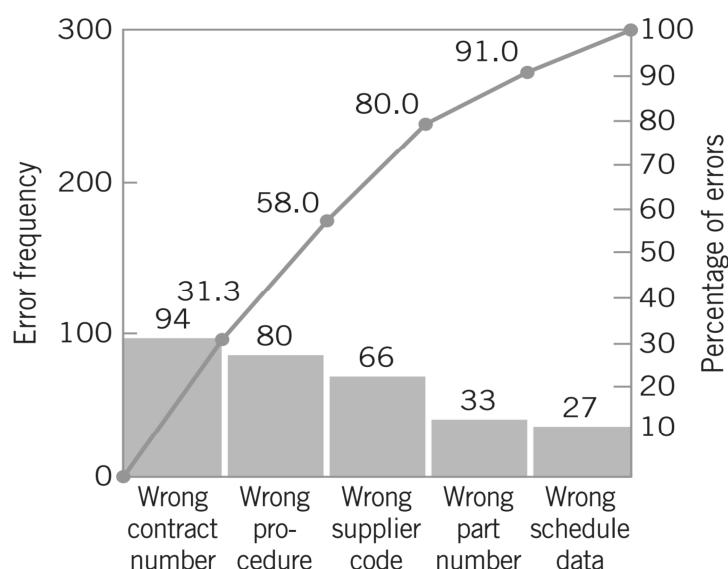
A Pareto chart for an electronics assembly process.



A *stacked* Pareto chart for electronics manufacturing.

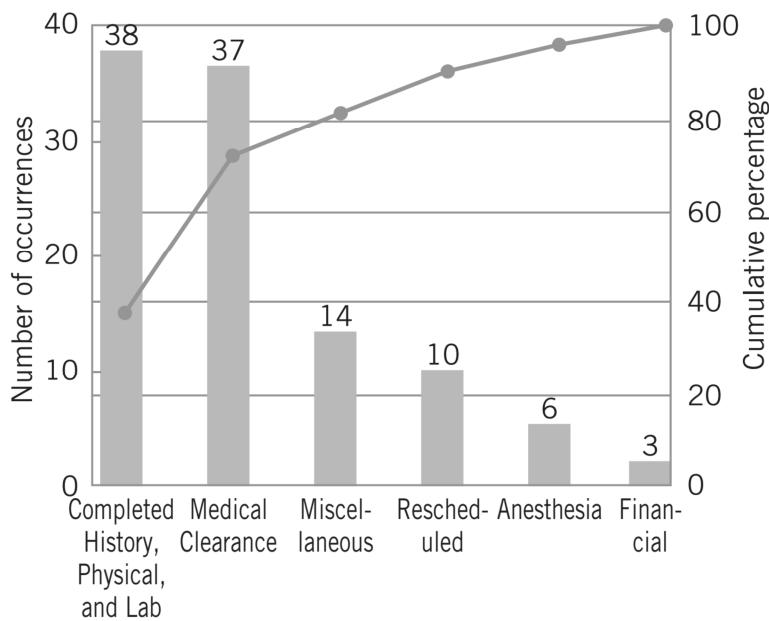
Pareto Chart Examples ^(cont'd)

Pareto charts are widely used in *nonmanufacturing applications*. A quality improvement team was investigating errors on purchase orders to reduce the organization's number of purchase order changes.



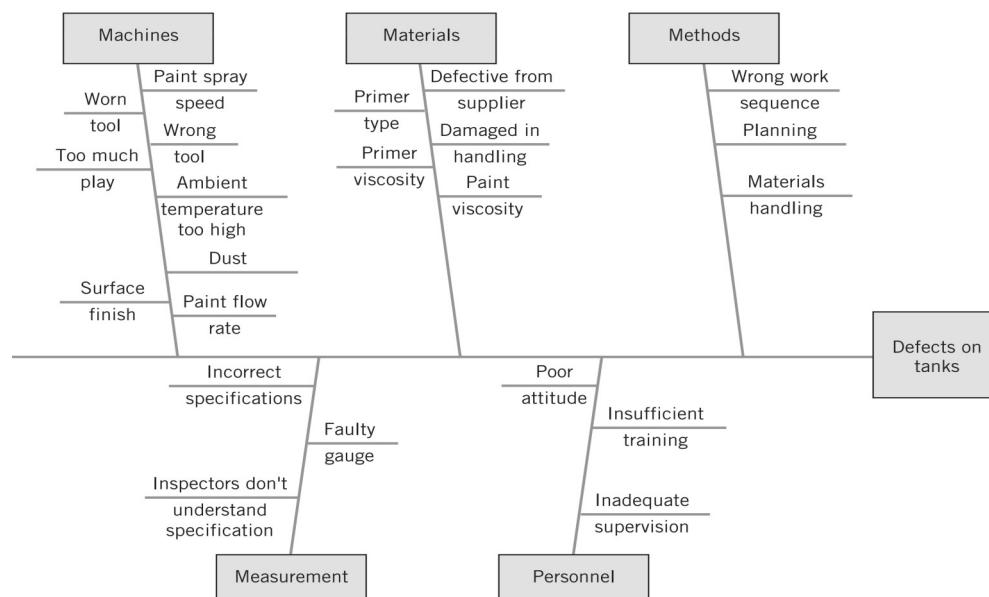
Pareto Chart Examples (cont'd)

A Pareto chart for a *hospital* to reflect the reasons for cancellation of scheduled outpatient surgery.



Cause-and-Effect Diagram

In situations where *causes are not obvious*, the cause-and-effect diagram is a formal tool useful in identifying potential causes.



An example from the *tank manufacturing* process.

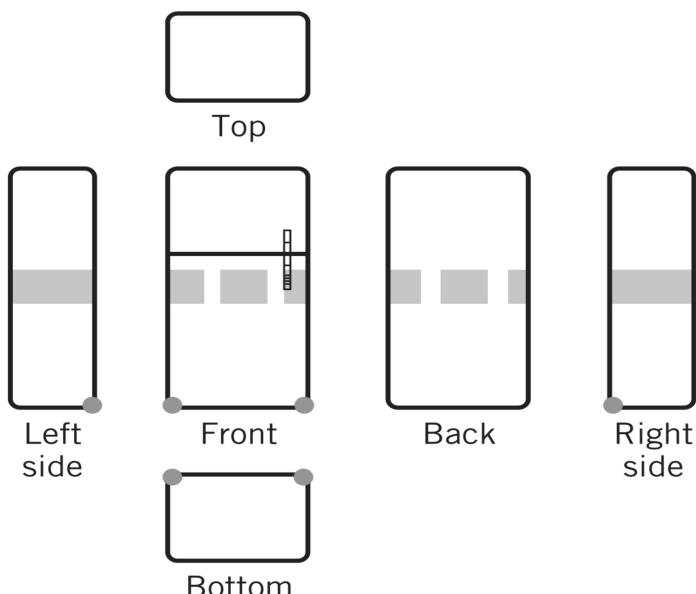
Cause-and-Effect Diagram (cont'd)

Steps for *constructing* a cause-and-effect diagram:

- 1 Define the problem or effect to be analyzed.
- 2 Form the team to perform the analysis.
- 3 Draw the effect box and the center line.
- 4 Specify the *major potential cause categories* and join them as boxes connected to the center line.
- 5 Identify the *possible causes and classify* them into the categories. Create new categories, if necessary.
- 6 Rank order the causes to identify those that seem most likely to impact the problem.
- 7 Take corrective action.

Defect Concentration Diagram

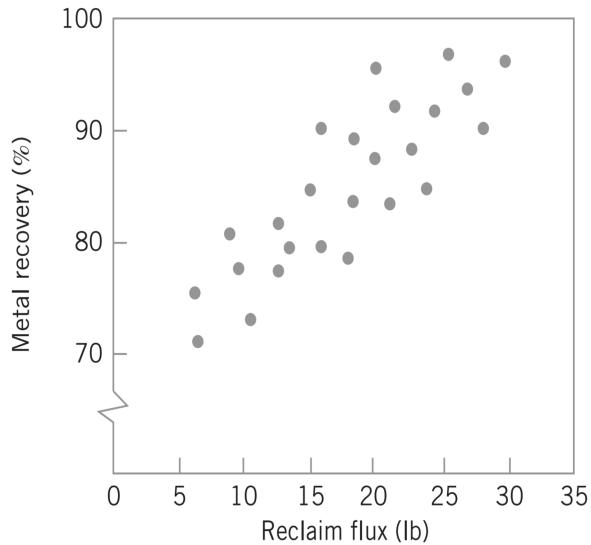
A defect concentration diagram is a picture of the unit, showing all relevant views with various *defects* and their *locations* drawn on it.



Defect concentration diagram for the final assembly stage of a *refrigerator* manufacturing process.

Scatter Diagram

The scatter diagram is used to identify a potential *relationship* between two variables.



The scatter diagram indicates a *strong positive correlation* between metal recovery and flux amount.

Note that *correlation does not necessarily imply causality!*