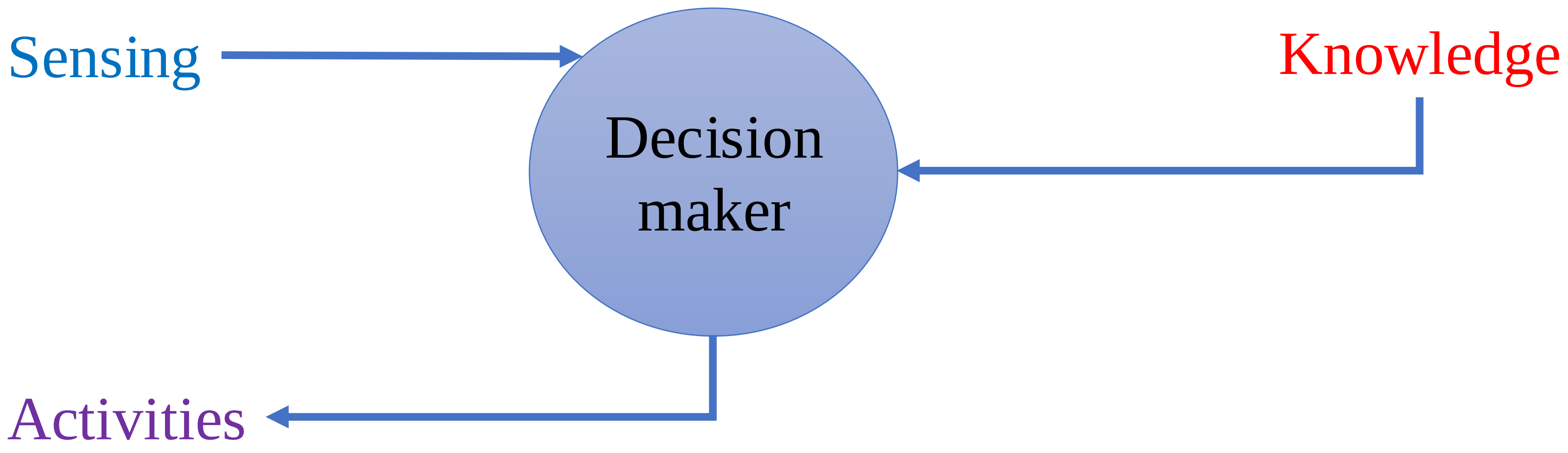
Knowledge Based System: Logic and Deduction

27/01/2025

Koustav Rudra

Knowledge and Intelligence



How to act given a particular scenario in the environment?

Machine: It is mandatory to have means of representing knowledge

How to represent knowledge in a way that machine can understand?

Represent knowledge in a machine

- We need a language to **represent** domain knowledge
  - Expect a machine to demonstrate an intelligent behaviour when that machine is left to work in a particular environment in a particular domain, provided we empower the machine with relevant knowledge from that domain

- There must be a method to use the knowledge
  - Understand the knowledge in which it is expressed

- **Inference**
  - Interpret knowledge in response to environmental fact that has been sensed

- **Syntax and semantics of language**
  - Grammar of a language
  - Laughs(Anil) == ?
  - Likes(Ashok, Akash) == ?

Logic is one such formal language

Logic

- A formal system for describing states of affairs, consisting of:
  - Syntax: describes how to make sentences, and

  - Semantics: describes the relation between the sentences and states of affairs

- Propositional Logic
- First Order Logic
- Temporal Logic
- Fuzzy Logic

Logical Deduction
Propositional Logic

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Objective

- How to represent simple facts in the language of propositional logic?

- How can we interpret propositional logic statement?
  - Understanding the meaning of propositional logic statement
  - Unless we understand the language we can’t act accordingly

- How to compute the meaning of compound proposition?
  - Collection of simple propositions and join them in some order
  - How to understand and integrate the meaning of individual propositions

Propositional Logic

- Anil is intelligent
- Anil is hardworking

Propositions

- If Anil is intelligent and Anil is hard... ...cores high

Compound proposition

combination

- Objects and Relations

Anil | Hardworking | Intelligent

- A Proposition (statement) can either be True or False

- Intelligent_Anil == Anil is intelligent
- Hardworking_Anil == Anil is hardworking

Towards the Syntax

- Let P stands for Intelligent_Anil
- Let Q stands for Hardworking_Anil

- What does $P \wedge Q$ (P and Q) mean?
- What does $P \vee Q$ (P or Q) mean?

- $P \wedge Q$ and $P \vee Q$ are compound propositions

Syntactic Elements of Propositional Logic

- Vocabulary
  - A set of propositional symbols (P, Q, R, etc.) each of which can be True or False

  - Set of **logical operators**
    - ∧ (AND), ∨ (OR), ~(NOT), → (implies)
    - Parenthesis () used for grouping

  - There are two special symbols
    - TRUE (T) and FALSE (F)
    - These are **logical constants**

How to form propositional sentences?

- Each symbol (a proposition or a constant) is a sentence

- If P is a sentence and Q is a sentence then
  - (P) is a sentence
  - P∧Q is a sentence
  - P∨Q is a sentence
  - ~P is a sentence
  - P→Q is a sentence
  - Nothing else is a sentence

Sentences are called well-formed formulae

Propositional Logic

- Given a set of atomic propositions AP

- Sentence → Atom | ComplexSentence

- Atom → True | False | AP

- ComplexSentence → (Sentence)
    - | Sentence Connective Sentence
    - | ~ Sentence

- Connective → $\wedge$ | $\vee$ | $\rightarrow$ | $\Leftrightarrow$

Implication →

- $P \rightarrow Q$

- If P is true then Q is true

- If it rains then the roads are wet

Equivalence ($\Leftrightarrow$)

- $P \Leftrightarrow Q$

- If P is True then Q is True and If Q is True then P is True

- If two sides of a triangle are equal then two base angles of the triangle are equal

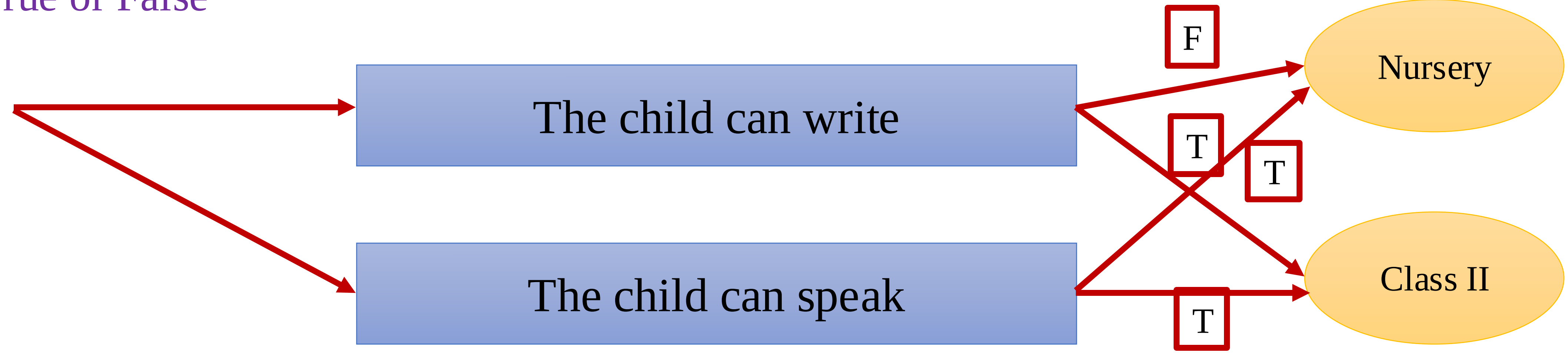
- $(P \rightarrow Q) \wedge (Q \rightarrow P)$

Example wffs

- P
- True
- $P \wedge Q$
- $(P \wedge Q) \rightarrow R$
- $(P \wedge Q) \vee R \rightarrow S$
- $\sim(P \vee Q)$
- $\sim(P \vee Q) \rightarrow R \wedge S$

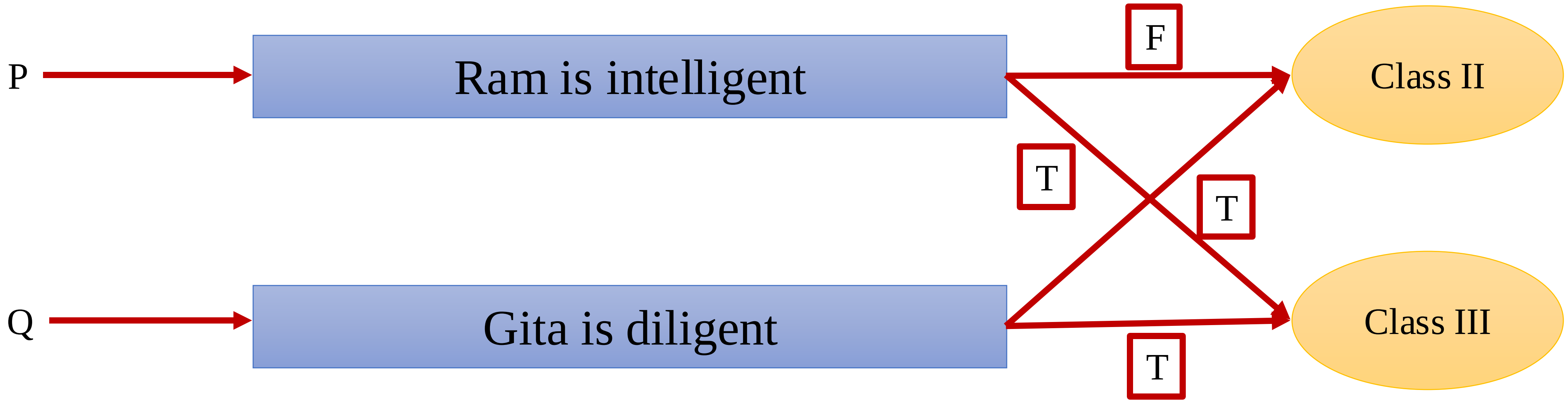
What does a wff mean --- Semantics?

- Interpretation in a world
- When we interpret a sentence in a world we assign meaning to it and it evaluates to either True or False
- P



- Same proposition could be interpreted in two different worlds in two different ways
- Interpretation attributes meaning or semantics to propositions

Semantics



- We deal with two symbols P and Q
- Truth values of P and Q depend on the way we interpret it in a particular world

How do we get a meaning?

- Sentences can be compound propositions

- **Steps:**

    - Interpret each atomic proposition in the **same world**

    - Assign Truth values to each interpretation

    - Compute the Truth value of compound proposition

Example

- P: likes(Akash, Aritra)
- Q: knows(Amit, Adway)

- **World:** Akash and Aritra are friends. Amit and Adway are known to each other.

- $P = T, Q = T$

- $P \wedge Q = T$
- $P \wedge \sim Q = F$

Validity of a sentence

- If a propositional sentence is true under all possible interpretation, it is VALID

- A sentence is VALID means it is True irrespective of the world in which we interpret it

- P∨~P is always True
  - Tautology

Satisfiability

- An interpretation is a mapping to a world

- A sentence is satisfiable by an interpretation **if**
  - Under that interpretation the sentence evaluates to **True**

- If NO interpretation makes a sentence True then
  - That sentence is called UNSATISFIABLE or INCONSISTENT
  - $P \wedge \sim P$

- If NO interpretation makes all the sentences in the set to be True then
  - The set of sentences is UNSATISFIABLE or INCONSISTENT

Inference in Propositional Logic

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Objective

- Infer the truth value of a proposition
- Reason towards new facts given a set of propositions
- Prove a proposition given a set of propositional facts

Truth Value Assignment

P	Q	P∧Q	P∨Q	~P	~Q	P→Q
T	T	T	T	F	F	T
T	F	F	T	F	T	F
F	T	F	T	T	F	T
F	F	F	T	T	T	T

De Morgan's Theorem

- $\sim(P \wedge Q) = \sim P \vee \sim Q$
- $\sim(P \vee Q) = \sim P \wedge \sim Q$

P	Q	P∨Q
T	T	T
T	F	T
F	T	T
F	F	F

~(P∨Q)
F
F
F
T

~P	~Q
F	F
F	T
T	F
T	T

~P∧~Q
F
F
F
T

Problem 2

- If P and Q are True, then what is the truth value of following statements?
  - S: $(\sim P \vee Q) \rightarrow P$

P	Q	$\sim P \vee Q$	S
T	T	T	T

Deduction using Propositional Logic: Steps

- Choice of Boolean variables a, b, c, d … which can take values True or False

- **Boolean Formulae developed** using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables

- **Codification of Sentences** of the argument into Boolean Formulae

- Developing the **Deduction Process** as obtaining truth of a Combined Formula expressing the complete argument

- Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Problem 1

- If I am the Director then I am well-known. I am the Director. So I am well-known.

Choice of Boolean variables a, b, c, d … which can take values True or False

- **Coding: Variables**
- a: I am the Director
- b: I am well-known

Boolean Formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables

- **Coding the sentences**

1. a→b
2. a
3. b

Codification of Sentences of the argument into Boolean Formulae

Developing the **Deduction Process** as obtaining truth of a Combined Formula expressing the complete argument

- **The final formula for deduction**
- ((a→b)Λa)→b

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Proof or Otherwise

a	b	$a \rightarrow b$	$((a \rightarrow b) \wedge a)$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Problem 2

- If I am the Director then I am well-known. I am not the Director. So I am not well-known.

Choice of Boolean variables a, b, c, d … which can take values True or False

Boolean Formulae developed using well defined connectors ~, Λ, V, →, etc, whose meaning (semantics) is given by their truth tables

Codification of Sentences of the argument into Boolean Formulae

Developing the **Deduction Process** as obtaining truth of a Combined Formula expressing the complete argument

- **Coding: Variables**
- a: I am the Director
- b: I am well-known

- **Coding the sentences**

1. $a \rightarrow b$
2. $\sim a$
3. $\sim b$

- **The final formula for deduction**
- $((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$

Determining the Truth or Validity of the formula and thereby proving or disproving the argument and Analyzing its truth under various Interpretations.

Proof or Otherwise

a	b	a→b	((a→b)∧~a	((a→b)∧~a)→~b
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Reasoning

- Using the given propositions which are assumed to be True
  - Trying to derive new facts which will also be True

- P: It is the month of July
- Q: It rains
- R: P→Q [If it is month of July then it rains]
- Premise: It is the month of July
- Conclude: It rains

Symbolic Deduction

Modus Ponens: One Inference Rule

- $P \rightarrow Q$
- P

- Q

- $P \rightarrow Q = \sim P \vee Q$
- $P \wedge \sim P \vee Q$
- $(P \wedge \sim P) \vee Q$
- $F \vee Q$
- Q

Allows us to deduce the truth of a consequent depending on the truth of the antecedents

Inference Rule: Importance

- We want to develop some mechanical procedures using which we can make the machine infer new facts
- Inference rules can be mechanically applied

- **Rules:**
  - If Not(Not(P)) then P
  - Chain Rule:
    - If P then Q
    - If Q then R
    - If P then R

Rules of Natural Deduction

- Modus Ponens: (a → b), a :- therefore b
- **Modus Tollens: (a → b), ~b :- therefore ~a**
- **Hypothetical Syllogism: (a → b), (b → c):- therefore (a → c)**
- **Disjunctive Syllogism: (a V b), ~a:- therefore b**
- **Constructive Dilemma: (a → b) Λ (c → d), (a V c) :- therefore (b V d)**
- **Destructive Dilemma: (a → b) Λ (c → d), (~b V ~d) :- therefore (~a V ~c)**
- **Simplification: a Λ b:- therefore a**
- **Conjunction: a, b:- therefore a Λ b**
- **Addition: a :- therefore a V b**

Inference Mechanisms

- Formal way of inferencing using propositional logic
- Truth Table Method
  - We can find out the truth of any compound proposition when we know the truth values of the individual propositions

- Deductive method
  - Inference rules which are not dependent on any interpretation
  - The propositions will evaluate to True or False based on some interpretation
  - Modus Ponen is one such inference rule

- Resolution
  - Propositions converted into clausal form
  - Negation of the goal, convert to clausal form
  - Iteratively apply propositions and prove NULL

Automated Reasoning

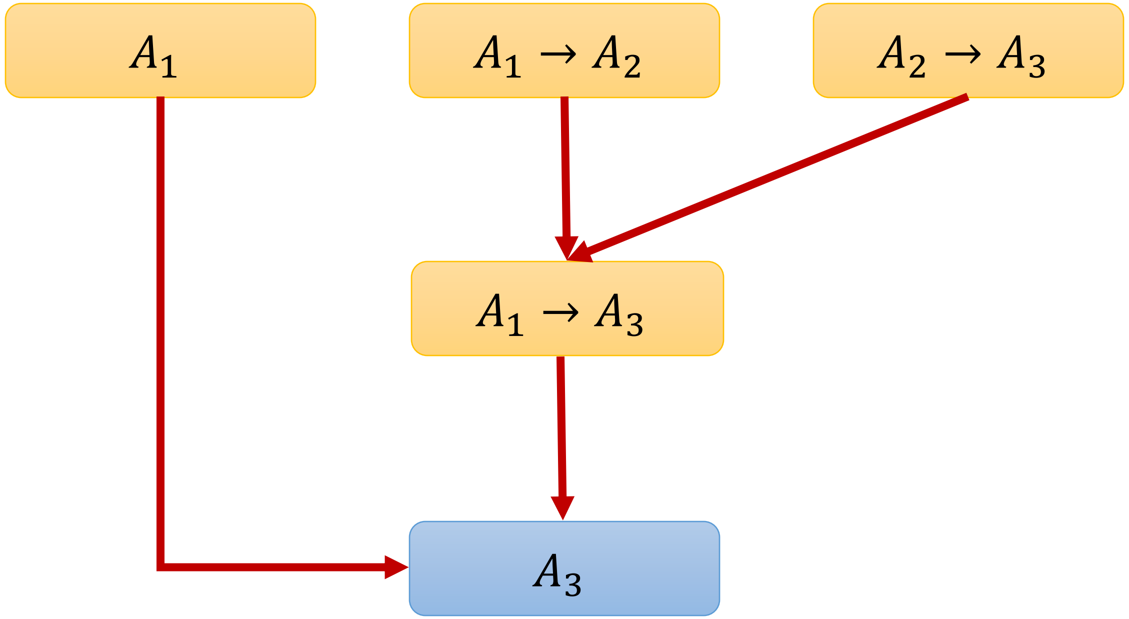
- In general, the inference problem is NP-complete [Cook's Theorem]

- If we restrict ourselves to Horn sentences , then repeated use of Modus Ponens gives us a polytime procedure.
  - Horn sentences are of the form:
    - $F1 \wedge F2 \wedge \ldots \wedge Fn \rightarrow G$

  - Forward chaining
  - Backward chaining

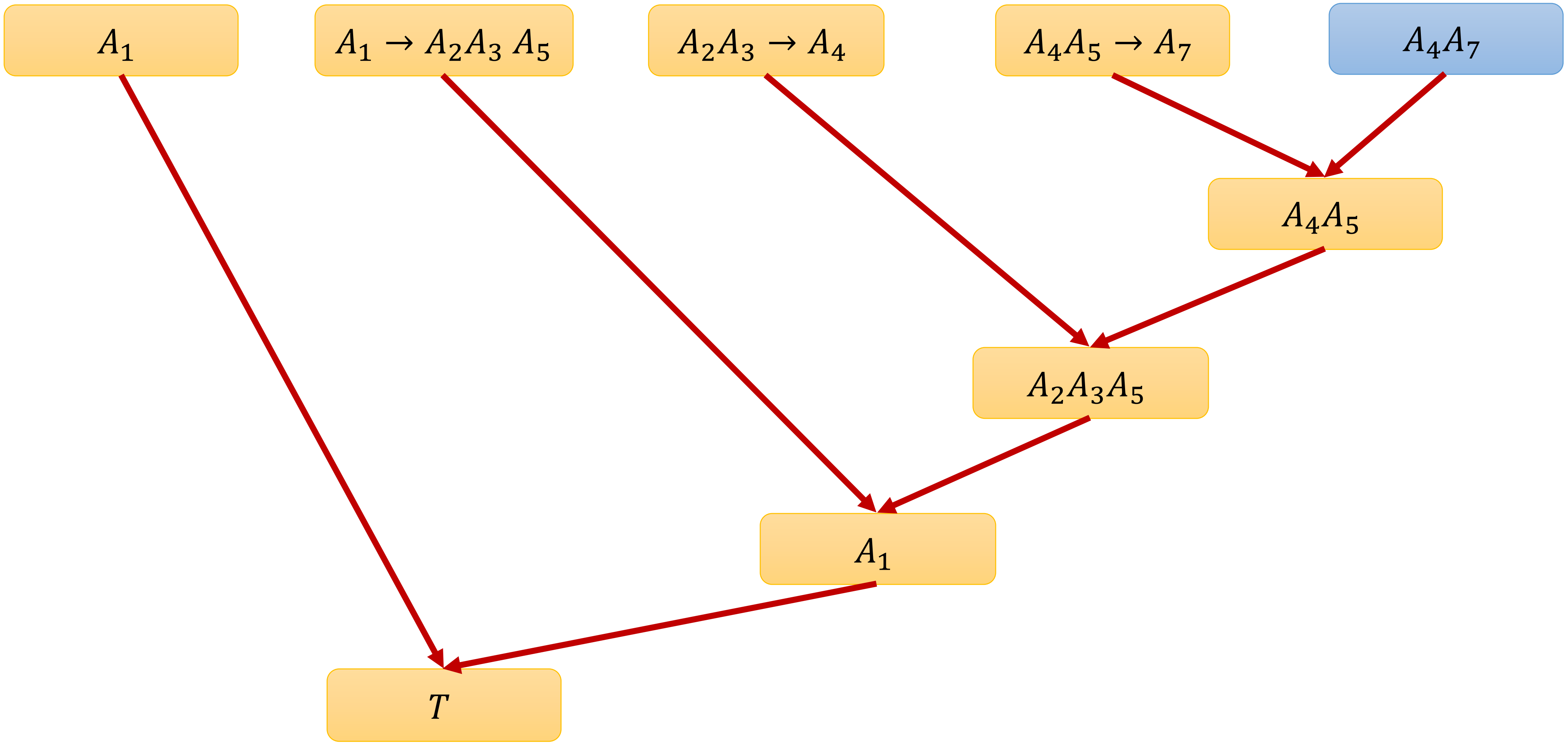
Automated Reasoning

- Forward Chaining



Automated Reasoning

- Backward chaining



Resolution

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Clause: A special form

- Literal – A single proposition or its negation
  - P, ~P

- A clause is a disjunction of literals
  - $P \vee Q \vee \sim R$

- Can we convert any proposition to a clausal form?

Converting compound proposition to clausal form

- Consider the sentence (wff)
  - $\sim(A \rightarrow B) \vee (C \rightarrow A)$
- Eliminate the implication sign
  - $\sim(\sim A \vee B) \vee (\sim C \vee A)$
- Eliminate double negation and reduce scope of “not” signs (De-Morgan Law)
  - $(A \wedge \sim B) \vee (\sim C \vee A)$
- Convert to conjunctive normal form by using distributive and associative laws
  - $(A \vee \sim C \vee A) \wedge (\sim B \vee \sim C \vee A)$
  - $(A \vee \sim C) \wedge (\sim B \vee \sim C \vee A)$
- Two clauses
  - $(A \vee \sim C)$
  - $(\sim B \vee \sim C \vee A)$

Why are we so interested in clausal form?

Helps us in applying interesting inference mechanism: **Resolution**

Resolution: Inference Mechanism

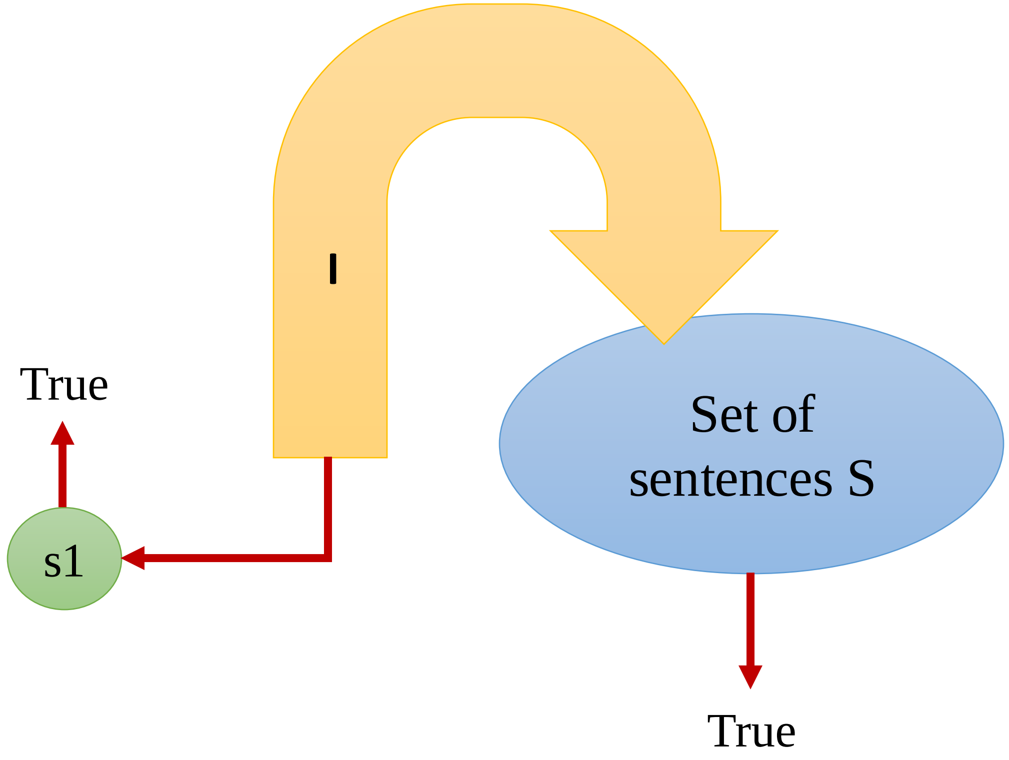
- Objective:
  - Learn to prove new facts given a set of facts
  - Given a set of facts proving a fact means proving the **logical entailment**
- A sound inference mechanism

Entailment

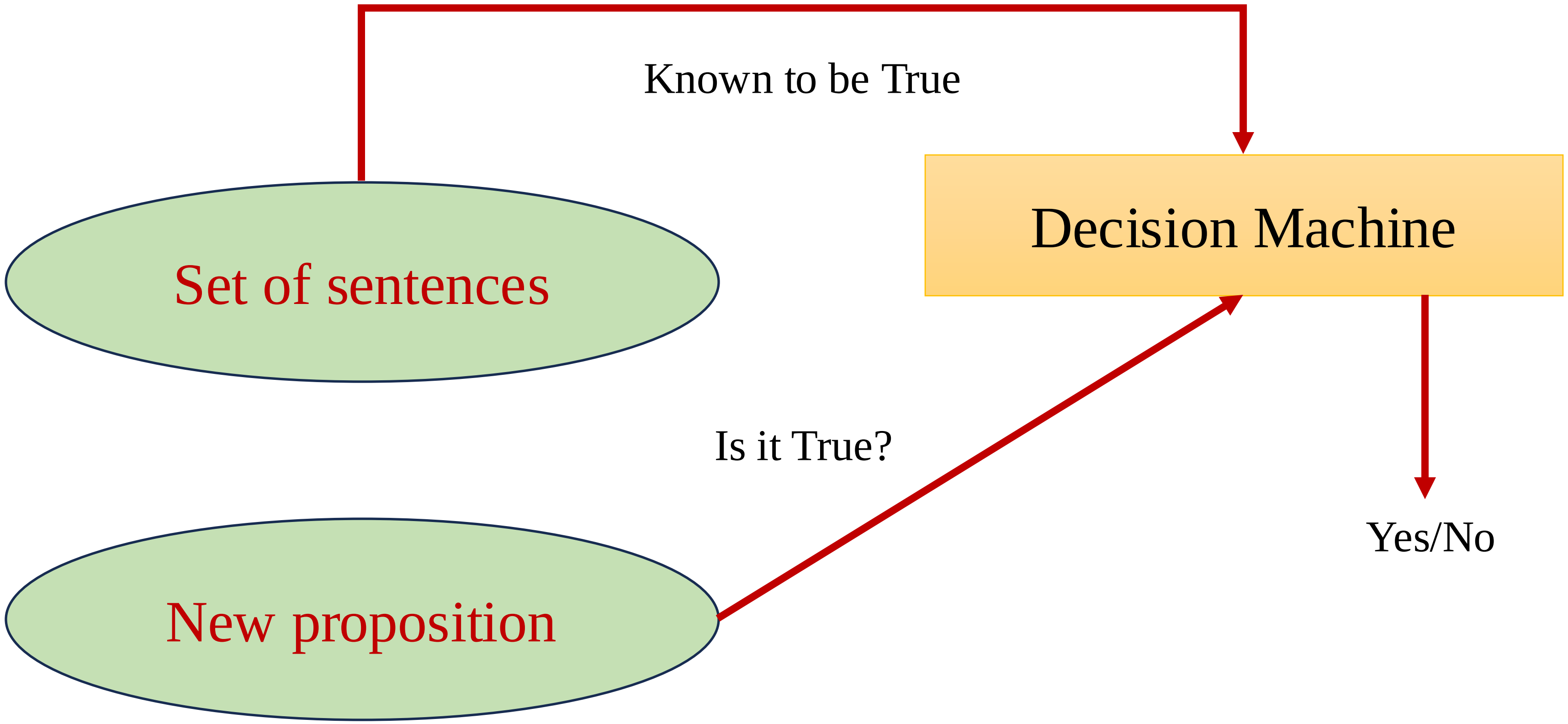
If a sentence s1 has a value True for all interpretations

that make all sentences in a set S True then

- S |- s1
- s1 logically follows from S
- s1 is a logical consequence of S
- S logically entails s1



Inference Mechanism



Resolution

- Suppose x is a literal
- S1 and S2 are two sets of propositional sentences represented in clausal form

- If we have $(x \lor S1) \land (\sim x \lor S2)$
  - Then we get $S1 \lor S2$
  - Here $S1 \lor S2$ is the resolvent
  - x is resolved upon

Problem 3

- If a triangle is equilateral then it is isosceles
- If a triangle is isosceles then two sides AB and AC are equal
- If AB and AC are equal then angle B and C are equal
- ABC is an equilateral triangle

- Prove angle B is equal to angle C

Problem 3: Proposition Form

- If a triangle is equilateral then it is isosceles
  - Equilateral(ABC)→Isosceles(ABC)

- If a triangle is isosceles then two sides AB and AC are equal
  - Isosceles(ABC)→Equal(AB,AC)

- If AB and AC are equal then angle B and C are equal
  - Equal(AB,AC)→Equal(B,C)

- ABC is an equilateral triangle
  - Equilateral(ABC)

Problem 3: Clausal Form

- Equilateral(ABC)→Isosceles(ABC)
  - ~ Equilateral(ABC)∨Isosceles(ABC)

- Isosceles(ABC)→Equal(AB,AC)
  - ~Isosceles(ABC)∨Equal(AB,AC)

- Equal(AB,AC)→Equal(B,C)
  - ~ Equal(AB,AC)∨Equal(B,C)

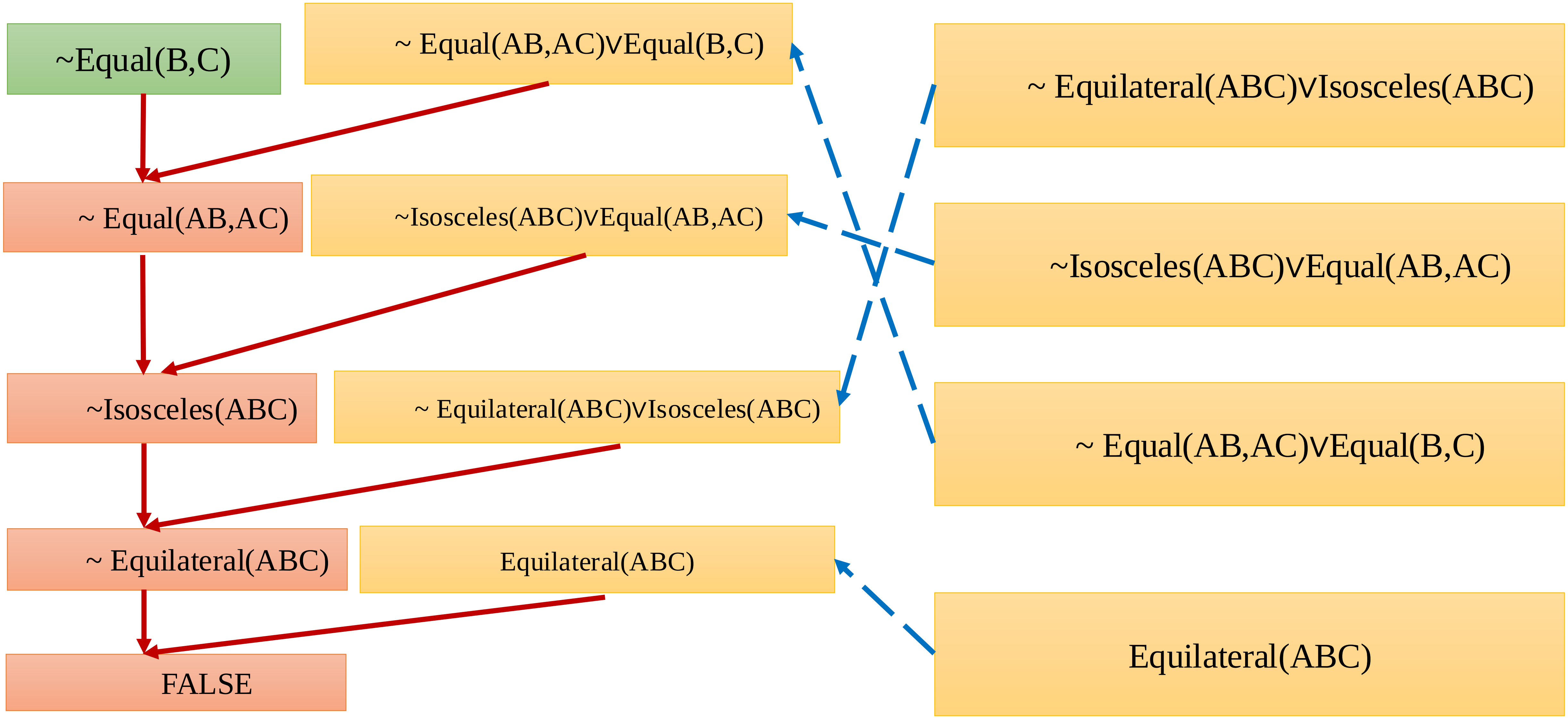
- Equilateral(ABC)

Proof by Refutation

- To Prove: Angle B is equal to Angle C: Equal(B,C)

- Let us disprove: NotEqual(B,C) = ~Equal(B,C)

- φ : $F1 \wedge F2 \wedge \ldots \wedge Fn \rightarrow G$
- φ : $\sim(F1 \wedge F2 \wedge \ldots \wedge Fn) \vee G$
- $\sim\varphi$: $F1 \wedge F2 \wedge \ldots \wedge Fn \wedge \sim G$



We have arrived in contradictory situation that is not supported by given set of facts

Thank You