Fuzzy Rule

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Fuzzy Rule

- A Fuzzy implication of the form:
  - If X is A then Y is B
  - A and B are two linguistic variables defined by Fuzzy sets A and B
  - On the universe of discourses X and Y, respectively

- Antecedent: X is A
- Consequence: Y is B

If-Then Rules

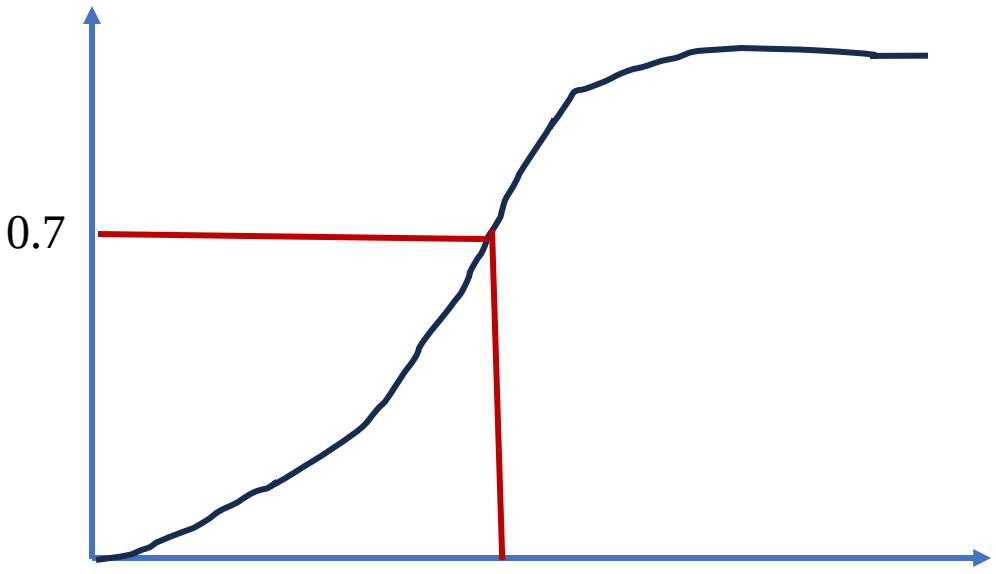
- Use Fuzzy sets and Fuzzy operators as the subjects and verbs of fuzzy logic to form rules
  - If x is A then Y is B
  - Where A and B are linguistic terms defined by fuzzy sets on the sets X and Y respectively

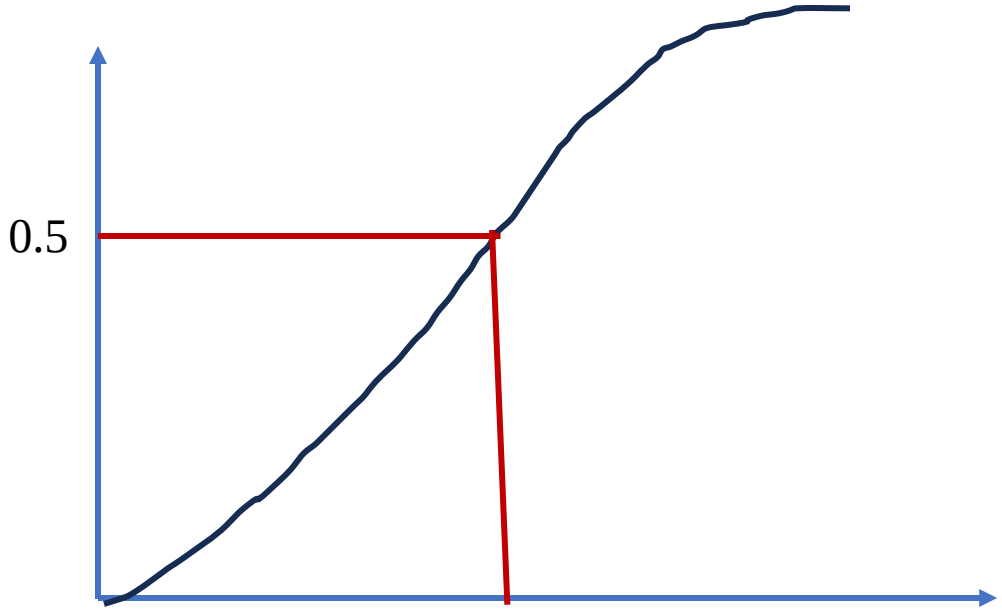
  - If velocity is small, then current needed is small

  - If temp is high, put cooler as moderate

If-Then Rules

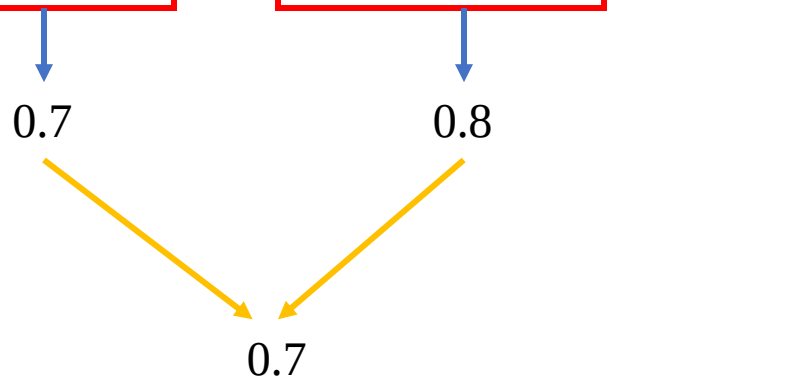
- If x is A is true then …
- If 70 is high is true then …





If-Then Rules

- If x is A is and y is B then …
- If 70 is high and 5 is medium then …



Max-min rule of composition

- Given N observations E_i over X and hypothesis H_i over Y we have N rules:
- If $E_1\ then\ H_1$
- If $E_2\ then\ H_2$

- If $E_N\ then\ H_N$

- $\mu_H = \max\{\min(\mu_{E_1}), \min(\mu_{E_2}), \dots, \min(\mu_{E_N})\}$

Fuzzy Cartesian Product

- A is a fuzzy set on the universe of discourse X with $\mu_A(x)|x \in X$
- B is a fuzzy set on the universe of discourse Y with $\mu_B(y)|y \in Y$
- $R = A \times B \subset X \times Y$
- $\mu_R(X,Y) = \mu_{A\times B}(X,Y) = \min\{\mu_A(x), \mu_B(y)\}$
- $A = \{(a_1, 0.2), (a_2, 0.7), (a_3, 0.4)\}$, $\mathrm{B} = \{(b_1, 0.5), (b_2, 0.6)\}$

$R = A \times B$

	b_1	b_2
a_1	0.2	0.2
a_2	0.5	0.6
a_3	0.4	0.4

Fuzzy Implication: Example 2

- Suppose P and T represent Pressure and Temperature
- P = {1,2,3,4}
- T = {10, 15, 20, 25, 30, 35, 40, 45, 50}
- $T_{HIGH} = \{(20,0.2), (25,0.4), (30,0.6), (35,0.6), (40,0.7), (45,0.8), (50,0.8)\}$
- $P_{LOW} = \{(1,0.8), (2,0.8), (3,0.6), (4,0.4)\}$
- if temperature is HIGH, then pressure is LOW, $R: T_{HIGH} \rightarrow P_{LOW}$

	1	2	3	4
20	0.2	0.2	0.2	0.2
25	0.4	0.4	0.4	0.4
30	0.6	0.6	0.6	0.4
35	0.6	0.6	0.6	0.4
40	0.7	0.7	0.6	0.4
45	0.8	0.8	0.6	0.4
50	0.8	0.8	0.6	0.4

Set of Support

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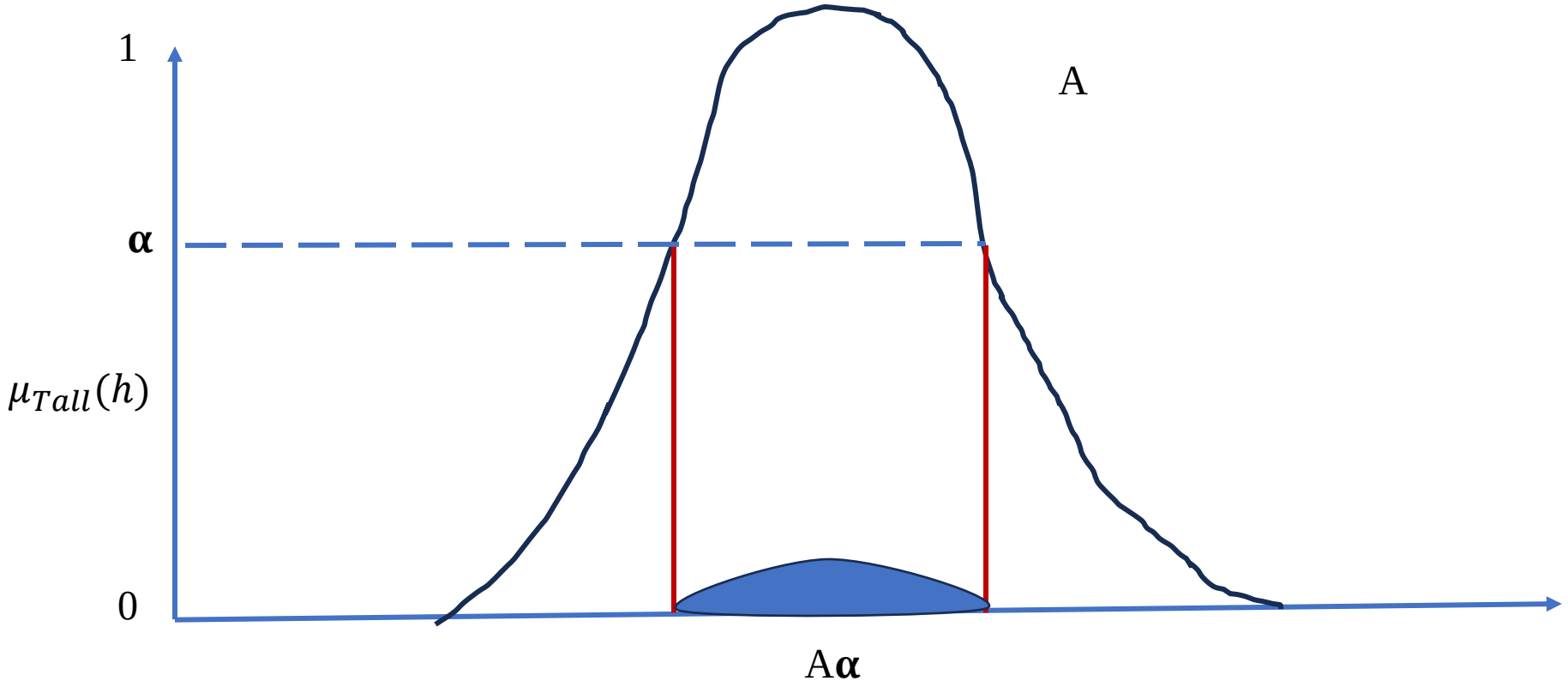
Set of Support

- It is a Crisp set
- Consist of elements whose membership values in the corresponding Fuzzy Set is greater than zero

α Cut / Horizontal Cut

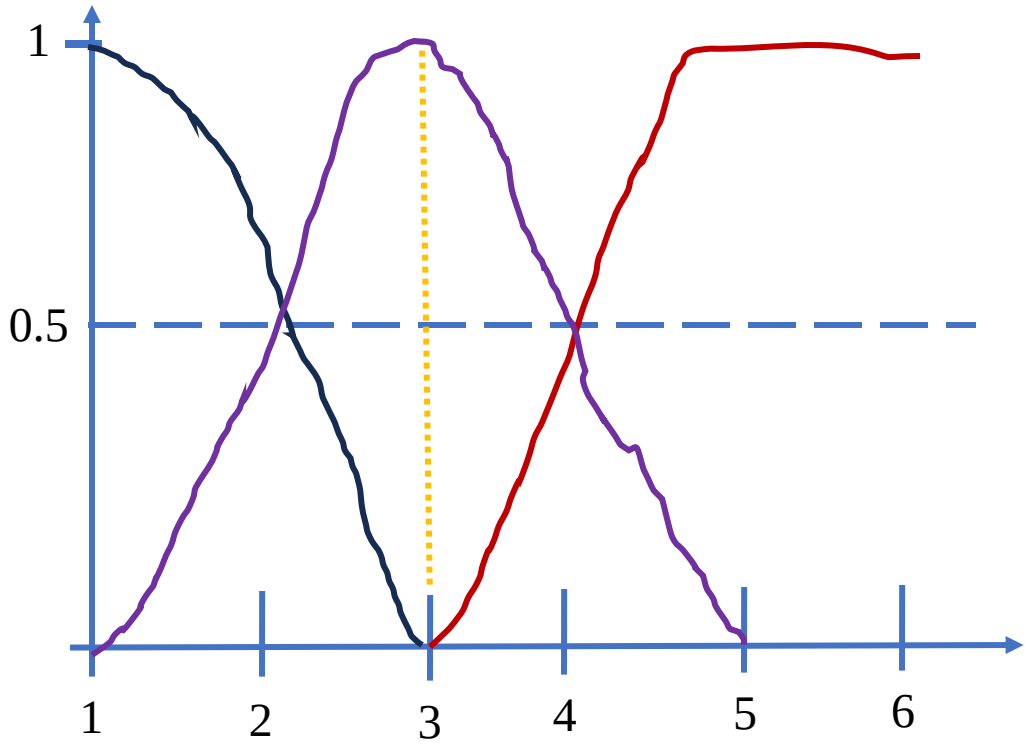
- It is a crisp set
- Consist of elements whose membership values in the corresponding Fuzzy Set is greater than **α**



Fuzziness and Probability

PDF: Probability Density Function for throwing a dice
"Small Number": Membership Function of the concept small

How to do Fuzzy Inferencing?



Thank You