Fuzzy Arithmetic Operation

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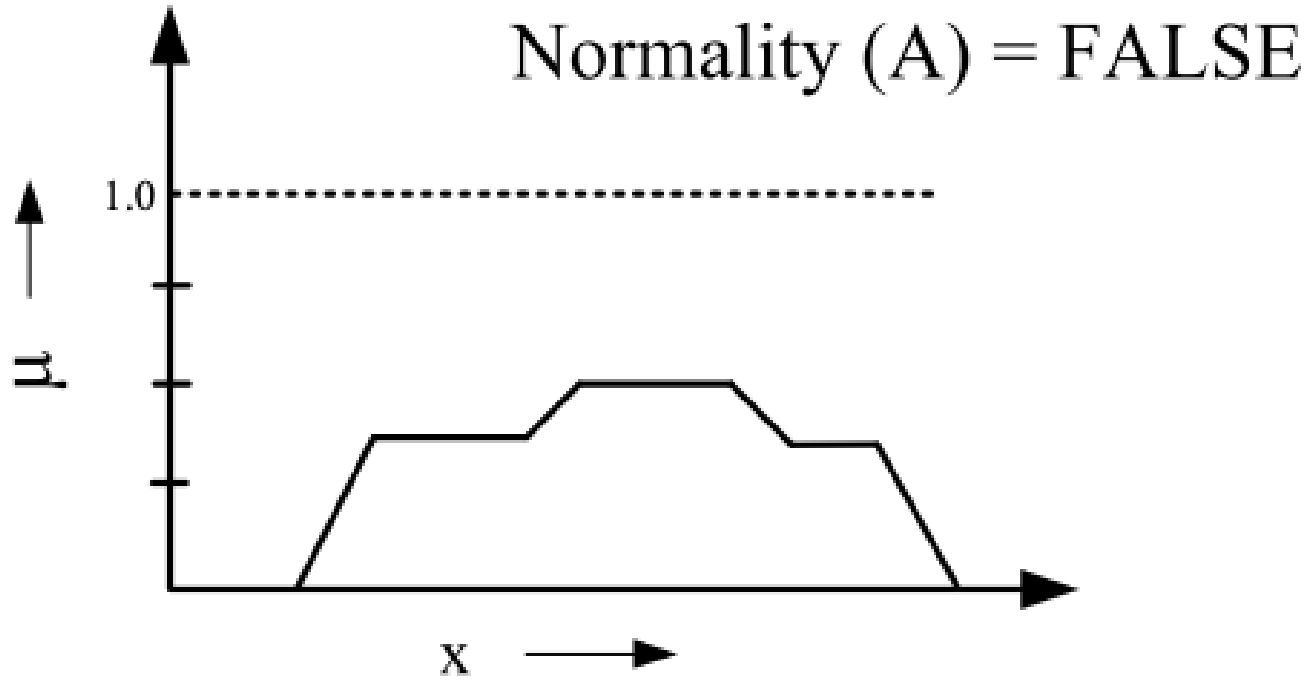
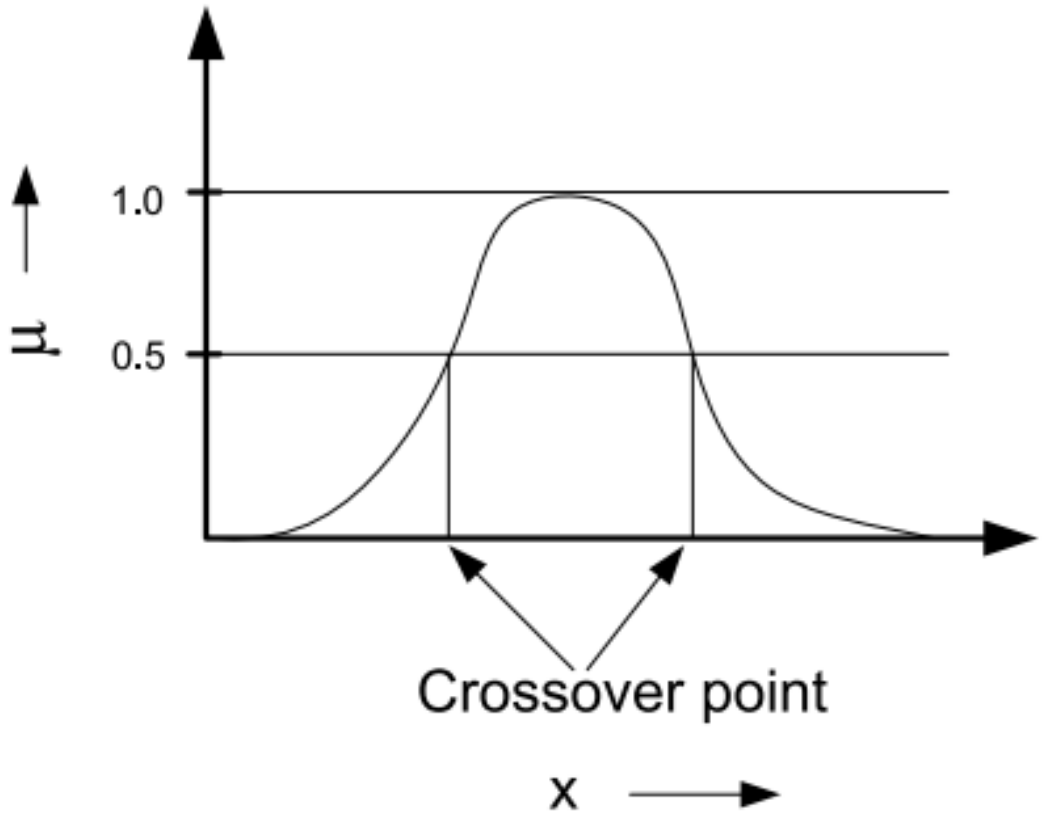
Normality

- A fuzzy set A is a normal if its core is non-empty. In other words, we can always find a point x ∈ X such that μA(x) = 1.



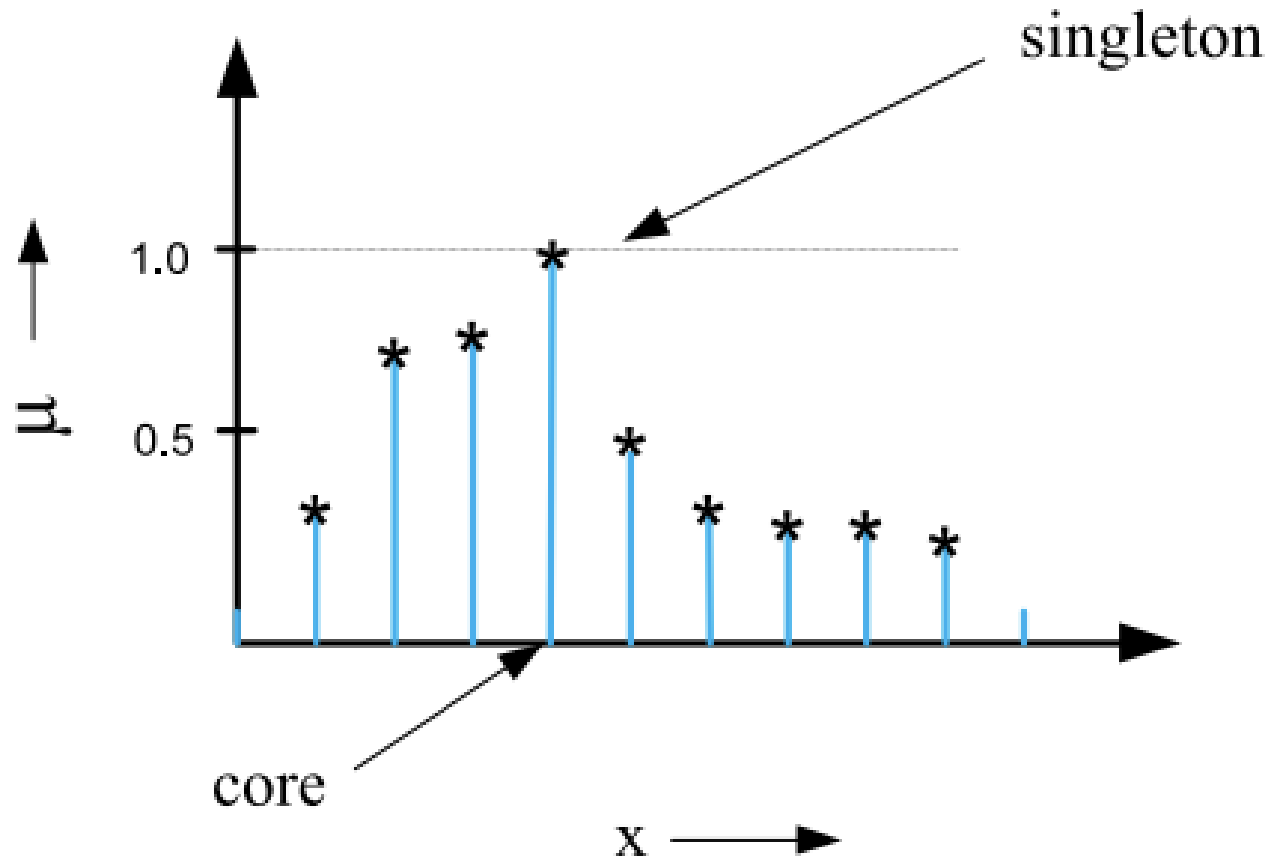
Crossover points

- A crossover point of a fuzzy set A is a point x ∈ X at which μA(x) = 0.5. That is Crossover (A) = {x|μA(x) = 0.5}



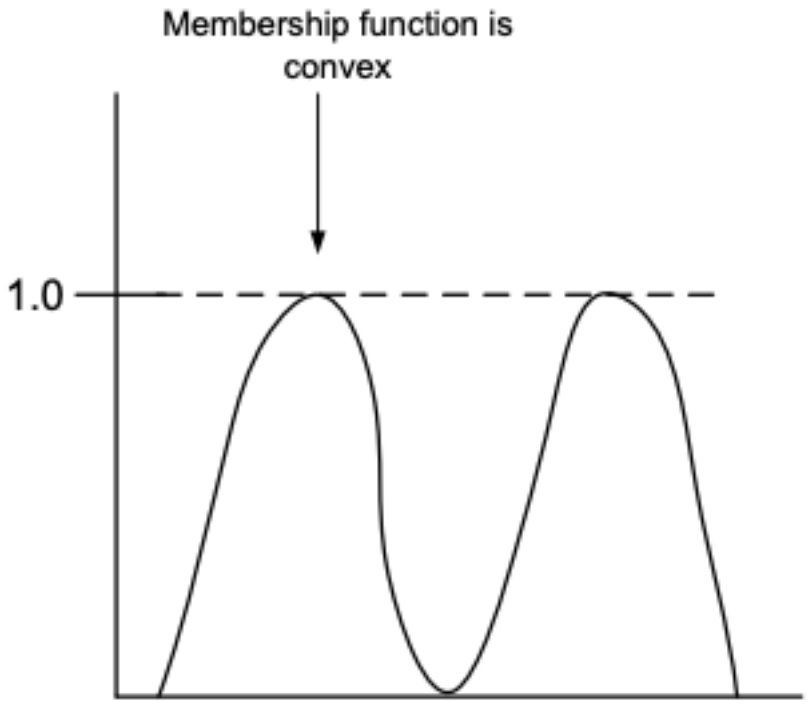
Fuzzy Singleton

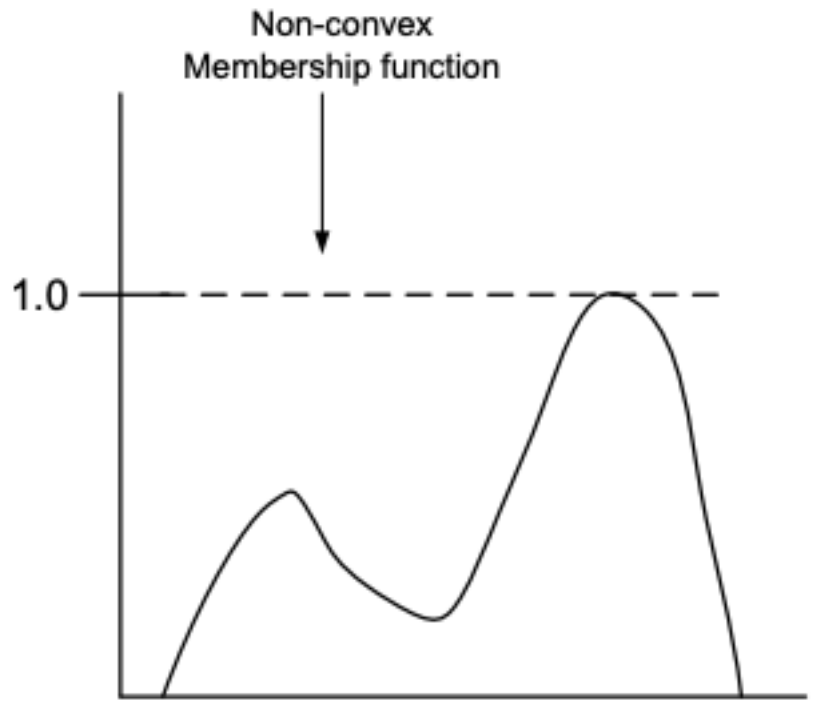
- A fuzzy set whose support is a single point in X with μA(x) = 1 is called a fuzzy singleton. That is |A| = { x | μA(x) = 1}.



Convexity

- Convexity : A fuzzy set A is convex if and only if for any x1 and x2 ∈ X and any λ ∈ [0, 1] μA (λx1 + (1 -λ)x2) ≥ min(μA(x1), μA(x2))

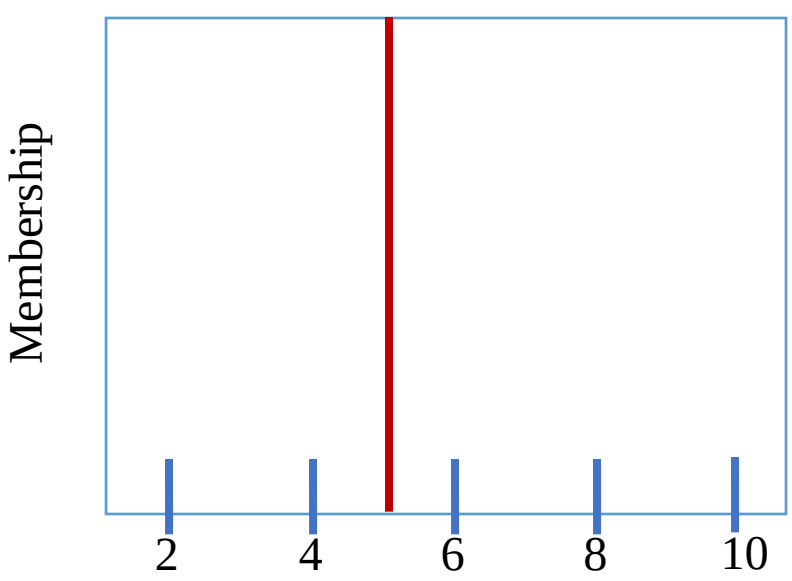




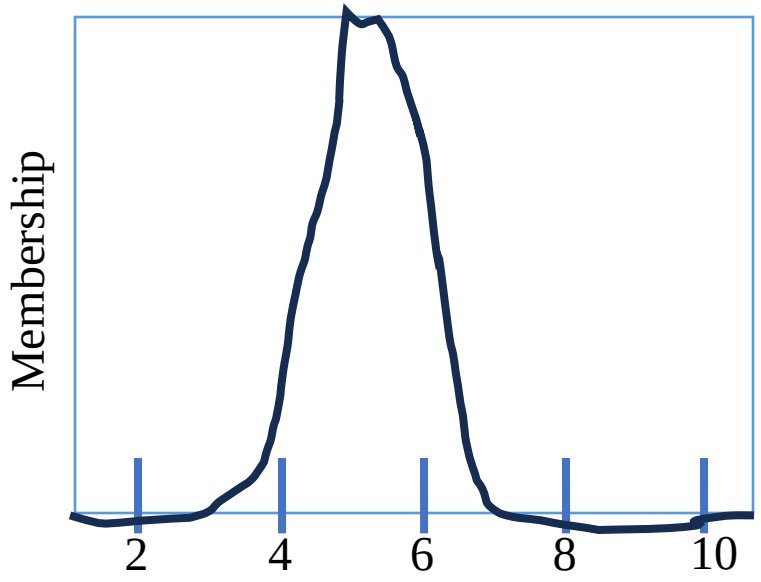
Fuzzy Number

- A Fuzzy number is a fuzzy set that holds the condition of normality and convexity
- Fuzzy numbers are the most basic types

A crisp number 5 or Fuzzy singleton 5



A Fuzzy number 5



Arithmetic Operations on Fuzzy Numbers

- There are four types of arithmetic operations that can be performed on fuzzy sets
  - Provided fuzzy sets are qualified for fuzzy numbers

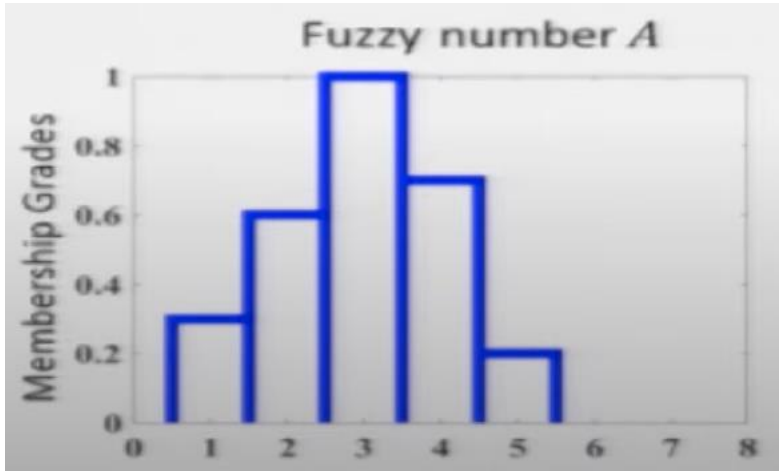
- These operations are
  - Addition on Fuzzy Numbers
  - Subtraction on Fuzzy Numbers
  - Multiplication on Fuzzy Numbers
  - Division on Fuzzy Numbers

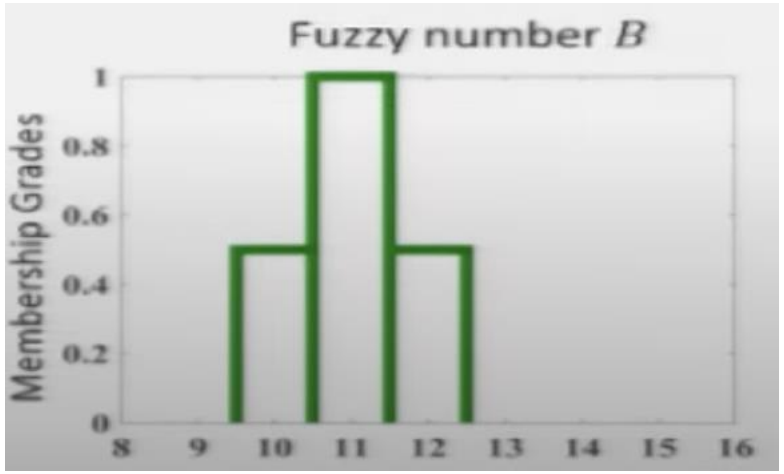
Addition of Fuzzy Numbers

- Let A and B are two Fuzzy numbers with the universe of discourse X
- If we perform the addition, it results in a new fuzzy number C as,
  - $C = A + B$
- The new fuzzy number C is defined as,
- For discrete: $C = \sum_x \mu_C(x^C)/x^C$
- For continuous: $C = \int \mu_C(x^C)/x^C$
- The membership function values of fuzzy number C are
  - $\mu_C(x^C) = \mu_{A+B}(x^C) = \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$
  - Where $x^C = x^A + x^B; \forall\, x^A, x^B, x^C \in X$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A+B}(x^C)/x^C = \sum_x \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]\,/x^C$

Addition of Fuzzy Numbers

- Example: Let us consider two fuzzy sets A and B with the universe of discourse $X \in [-20,20]$ as given below. Find the addition of fuzzy numbers A and B
- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$



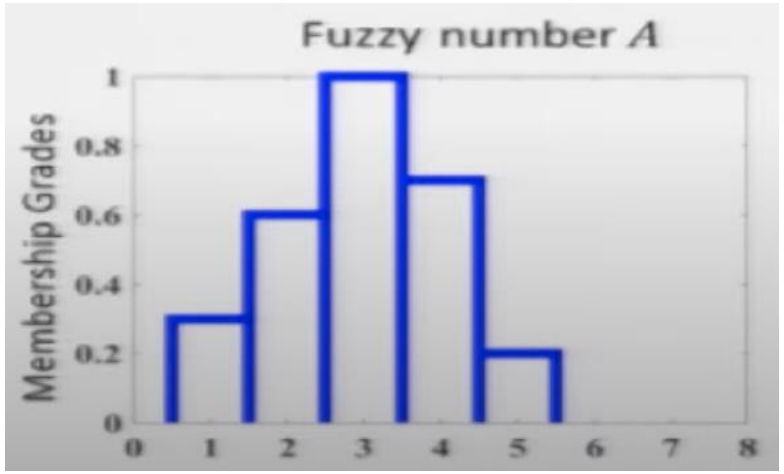


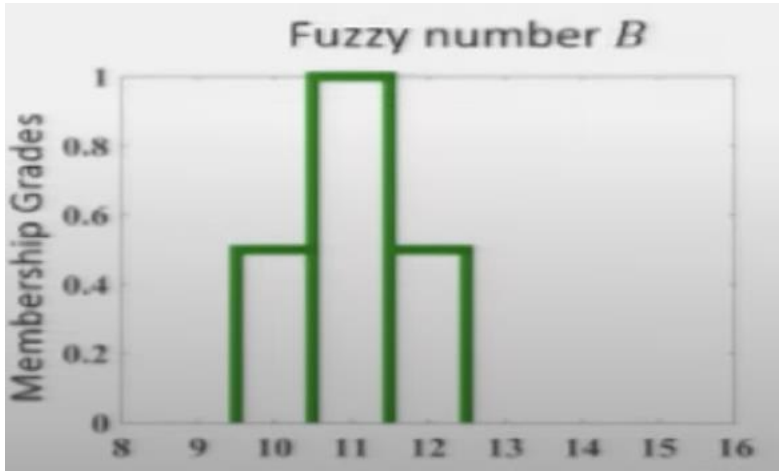
Subtraction of Fuzzy Numbers

- Let A and B are two Fuzzy numbers with the universe of discourse X
- If we perform the subtraction, it results in a new fuzzy number C as,
  - $C = A - B$
- The new fuzzy number C is defined as,
- For discrete: $C = \sum_x \mu_C(x^C)/x^C$
- For continuous: $C = \int \mu_C(x^C)/x^C$
- The membership function values of fuzzy number C are
  - $\mu_C(x^C) = \mu_{A-B}(x^C) = \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$
  - Where $x^C = x^A - x^B; \forall\, x^A, x^B, x^C \in X$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A-B}(x^C)/x^C = \sum_x \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]\,/x^C$

Subtraction of Fuzzy Numbers

- Example: Let us consider two fuzzy sets A and B with the universe of discourse $X \in [-20,20]$ as given below. Find the addition of fuzzy numbers A and B
- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$



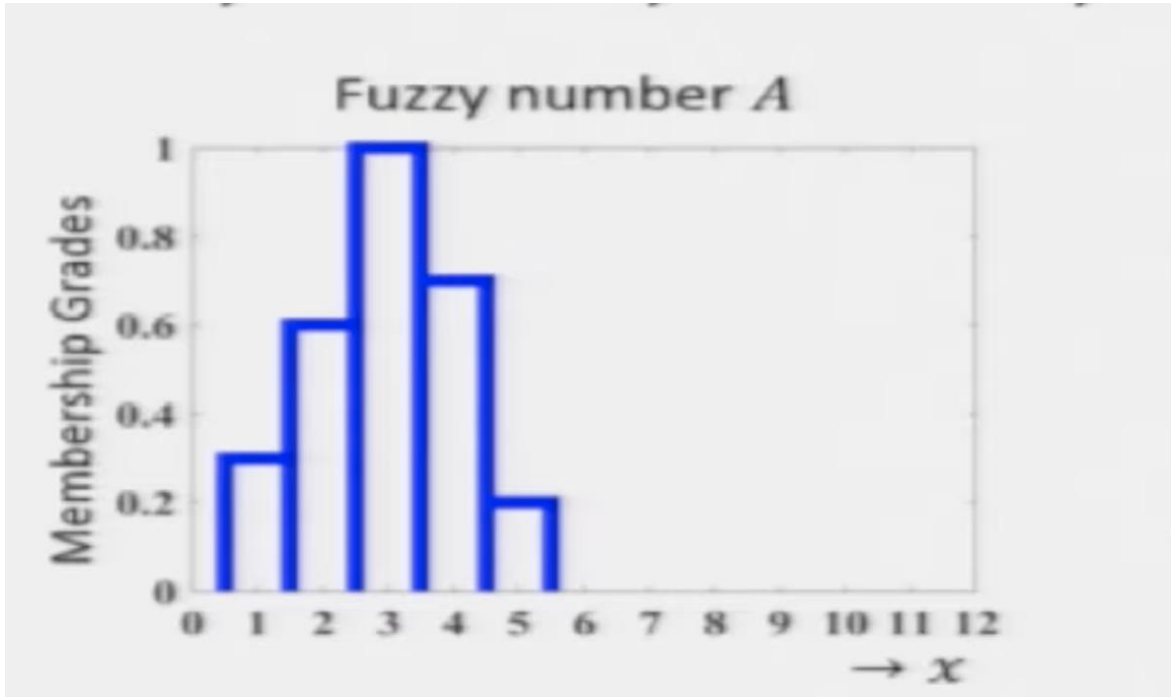


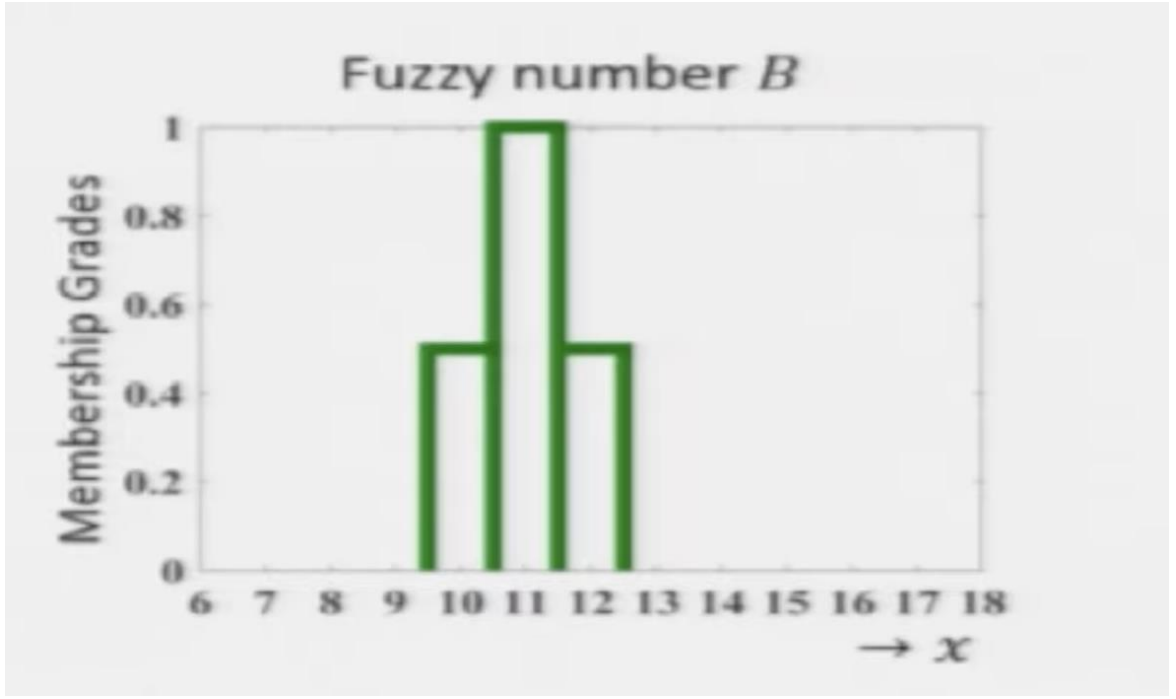
Multiplication of Fuzzy Numbers

- Let A and B are two Fuzzy numbers with the universe of discourse X
- If we perform the subtraction, it results in a new fuzzy number C as,
  - $C = A * B$
- The new fuzzy number C is defined as,
- For discrete: $C = \sum_x \mu_C(x^C)/x^C$
- For continuous: $C = \int \mu_C(x^C)/x^C$
- The membership function values of fuzzy number C are
  - $\mu_C(x^C) = \mu_{A*B}(x^C) = \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$
  - Where $x^C = x^A * x^B; \forall\, x^A, x^B, x^C \in X$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A*B}(x^C)/x^C = \sum_x \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]\,/x^C$

Multiplication of Fuzzy Numbers

- Example: Let us consider two fuzzy sets A and B with the universe of discourse $X \in [-15,15]$ as given below. Find the addition of fuzzy numbers A and B
- $A = \frac{0.3}{1} + \frac{0.6}{2} + \frac{1.0}{3} + \frac{0.7}{4} + \frac{0.2}{5}$
- $B = \frac{0.5}{10} + \frac{1.0}{11} + \frac{0.5}{12}$



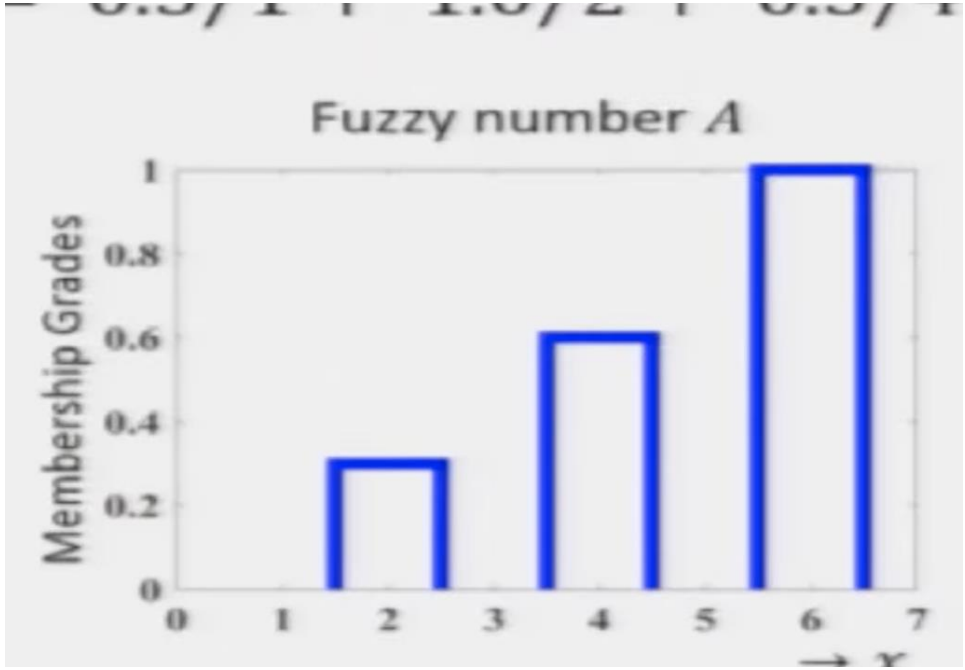


Division of Fuzzy Numbers

- Let A and B are two Fuzzy numbers with the universe of discourse X
- If we perform the subtraction, it results in a new fuzzy number C as,
  - $C = A \div B$
- The new fuzzy number C is defined as,
- For discrete: $C = \sum_x \mu_C(x^C)/x^C$
- For continuous: $C = \int \mu_C(x^C)/x^C$
- The membership function values of fuzzy number C are
  - $\mu_C(x^C) = \mu_{A\div B}(x^C) = \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]$
  - Where $x^C = x^A \div x^B; \forall\, x^A, x^B, x^C \in X$
- $C = \sum_x \mu_C(x^C)/x^C = \sum_x \mu_{A\div B}(x^C)/x^C = \sum_x \max_{x^A, x^B}[\mu_A(x^A) \wedge \mu_B(x^B)]\,/x^C$

Division of Fuzzy Numbers

- Example: Let us consider two fuzzy numbers A and B with the universe of discourse $X \in \mathbb{N}$ as given below. Find the addition of fuzzy numbers A and B
- $A = \frac{0.3}{2} + \frac{0.6}{4} + \frac{1.0}{6}$
- $B = \frac{0.5}{1} + \frac{1.0}{2} + \frac{0.5}{4}$



Thank You