AIFA
Searching Game Trees

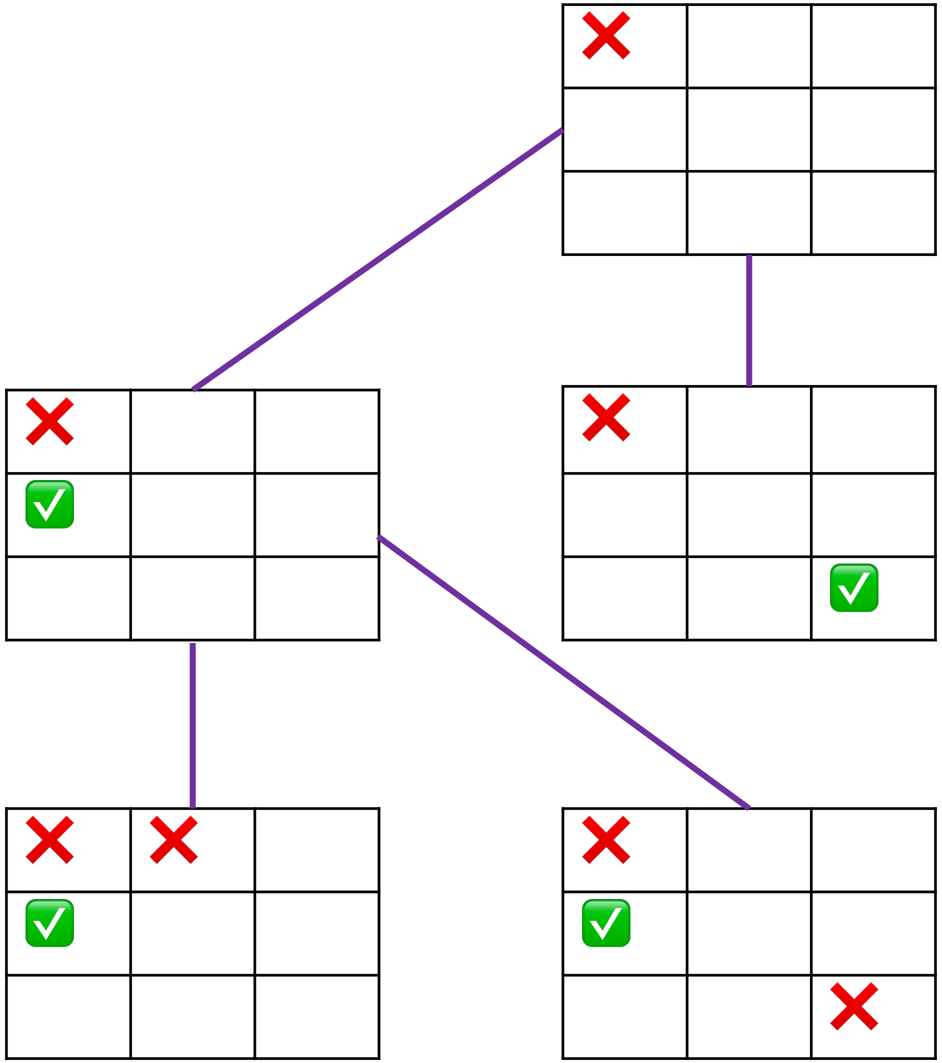
21/01/2025

Koustav Rudra

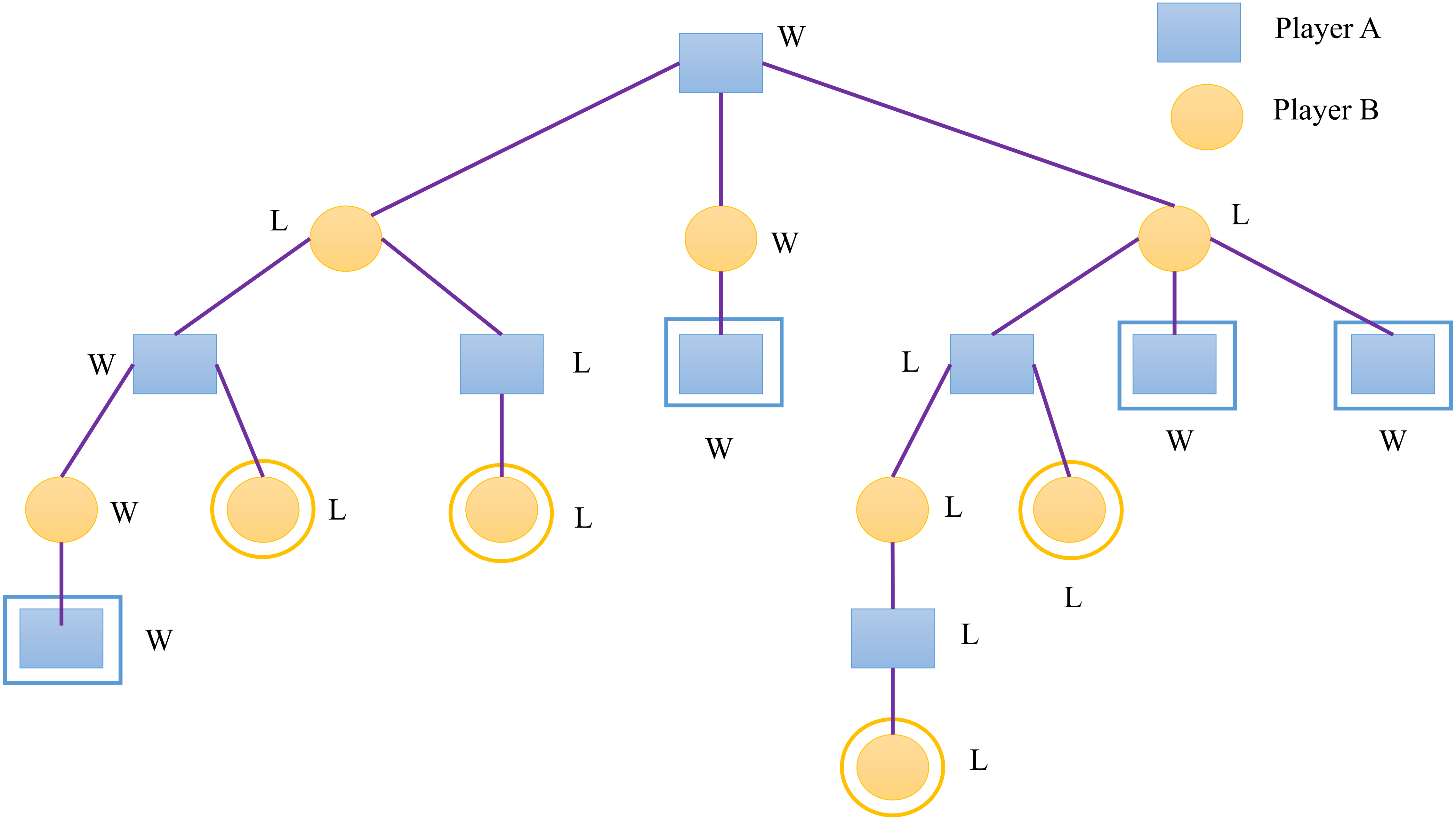
Searching Game Trees

- Consider an OR tree with two types of OR nodes, namely Min nodes and Max nodes
- In Min nodes, select the min cost successor
- In Max nodes, select the max cost successor

- Terminal nodes are winning or losing states
  - It is often infeasible to search up to the terminal nodes
  - We use heuristic costs to compare non-terminal nodes



Player A


Player A
Player B
W
L
W
L
W
L
W
L
L
W
W
W
W
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L
L
L
W
L
L
L

Searching Game Trees

- We will expand these moves upto a certain depth

- We will have some heuristic functions to evaluate the position of the game after that many lookaheads

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MinMax Trees

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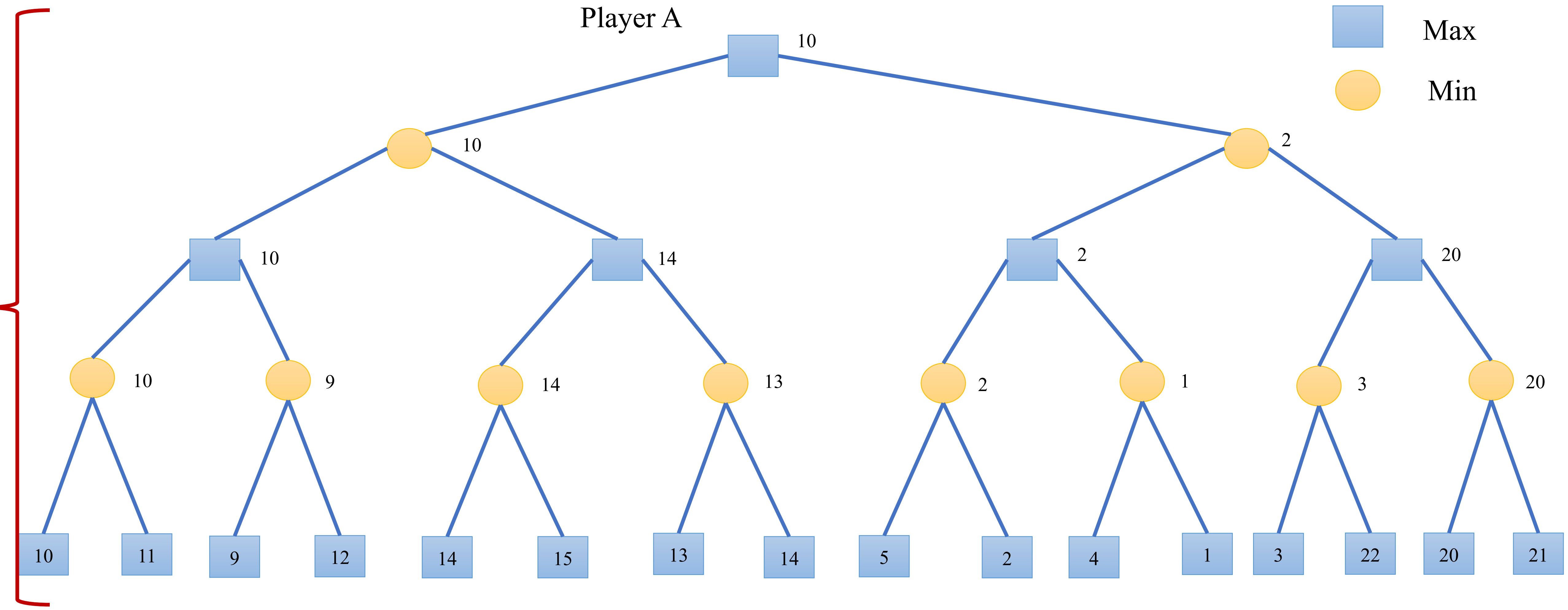
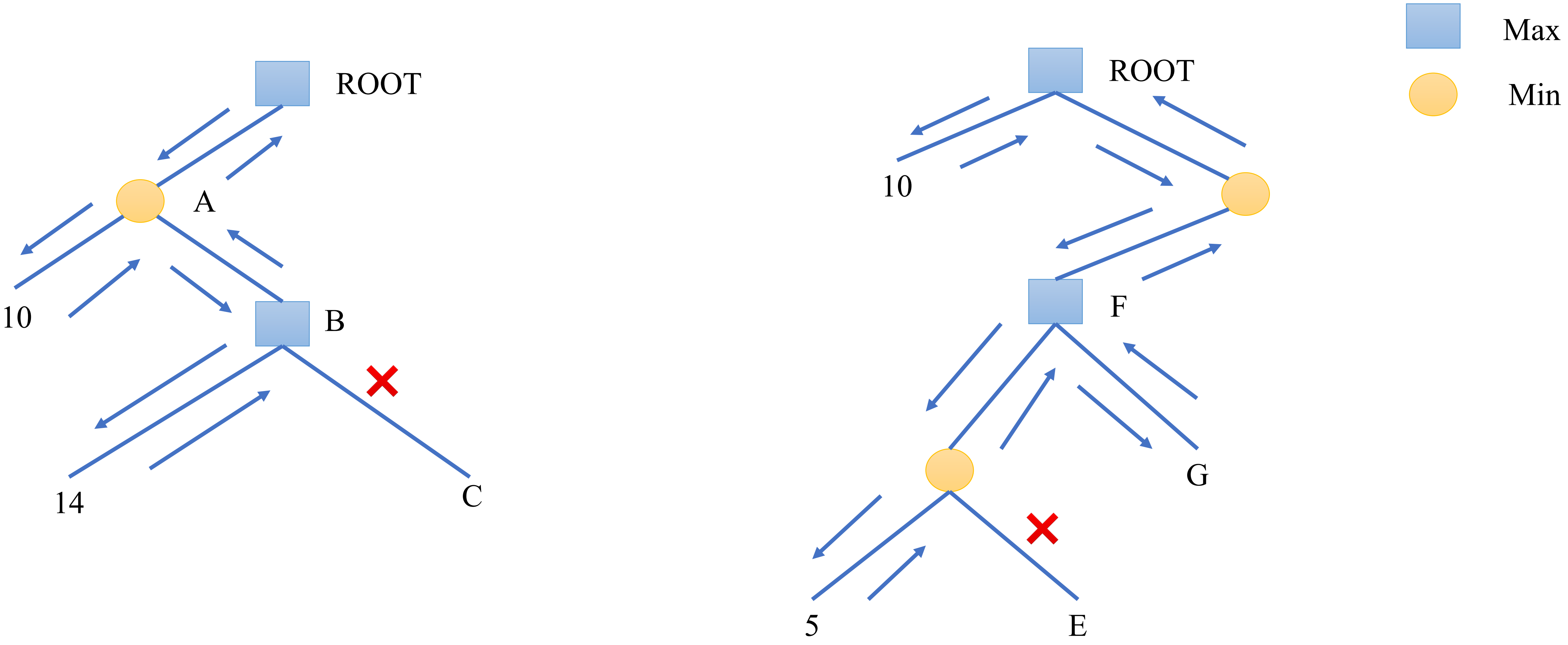


- Looked ahead up to this many number of moves
- Found out the cost value
    - How much cost I have to incur to win the game

Shallow and Deep Pruning



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AlphaBeta Pruning

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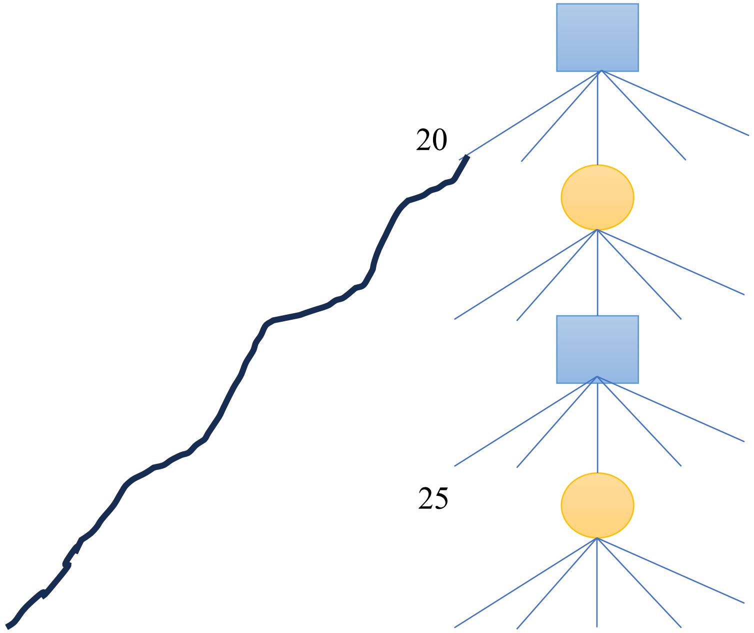
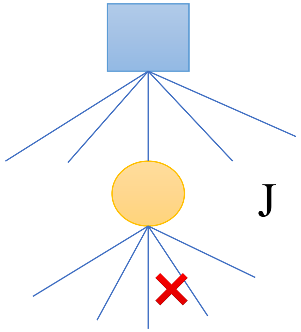
Alpha-Beta Pruning

- Alpha bound of J
    - The max current val of all MAX ancestors of J
    - Exploration of a min node, J, is stopped when its value equals or falls below alpha
    - In a min node, we update beta



What are we looking in MIN node?

Whether its current value fallen below the value backed up in the max ancestor of the node



$\alpha(J) = Current \max val\ of\ all\ MAX$ ancestors of J

Alpha-Beta Pruning

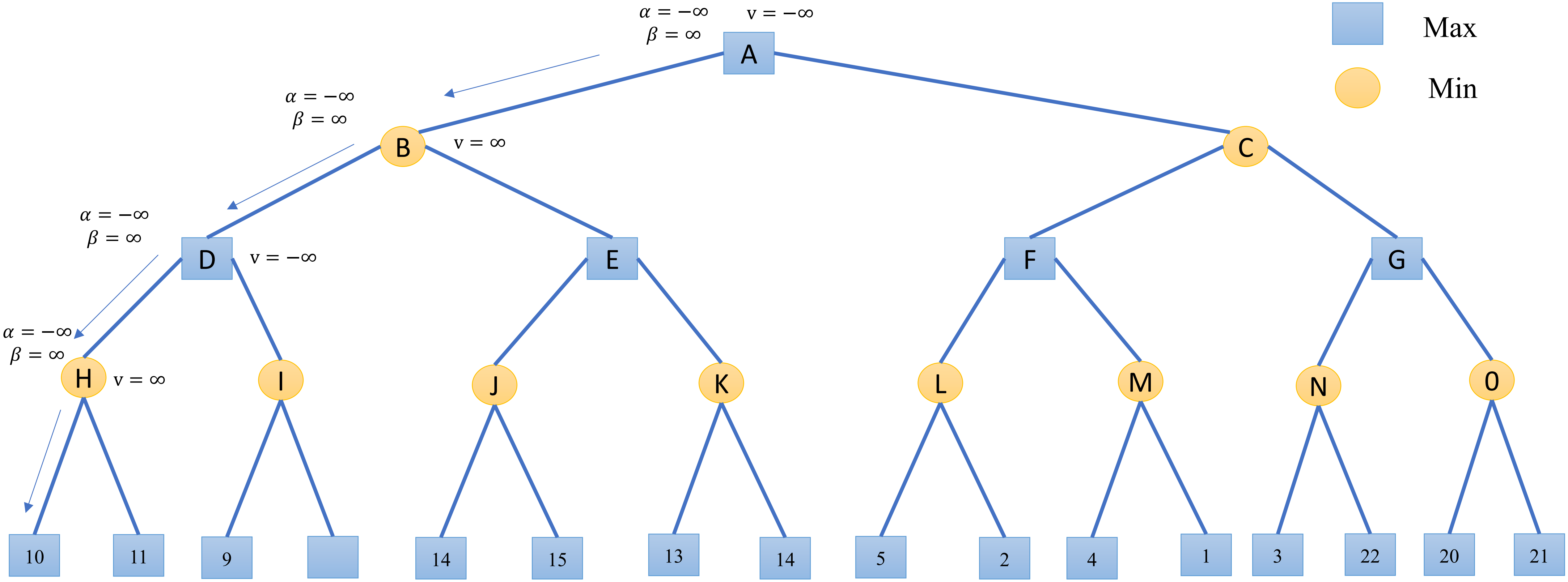
- Alpha bound of J
  - The max current val of all MAX ancestors of J
  - Exploration of a min node, J, is stopped when its value equals or falls below alpha
  - In a min node, we update beta

- Beta bound of J
  - The min current val of all MIN ancestors of J
  - Exploration of a max node, J, is stopped when its value equals or exceeds beta
  - In a max node, we update alpha

- In both min and max nodes, we return when $\alpha \geq \beta$

Alpha-Beta Pruning

- Alpha = best already explored option along path to the root for maximizer
- Beta = best already explored option along path to the root for minimizer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$\beta = \infty$
$v = -\infty$
A
Max
Min
$\alpha = -\infty$
$\beta = \infty$
B
$v = \infty$
C
$\alpha = -\infty$
$\beta = \infty$
D
$v = -\infty$
E
F
G
$\alpha = -\infty$
$\beta = \infty$
H
$v = \infty$
I
J
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14
15
13
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5
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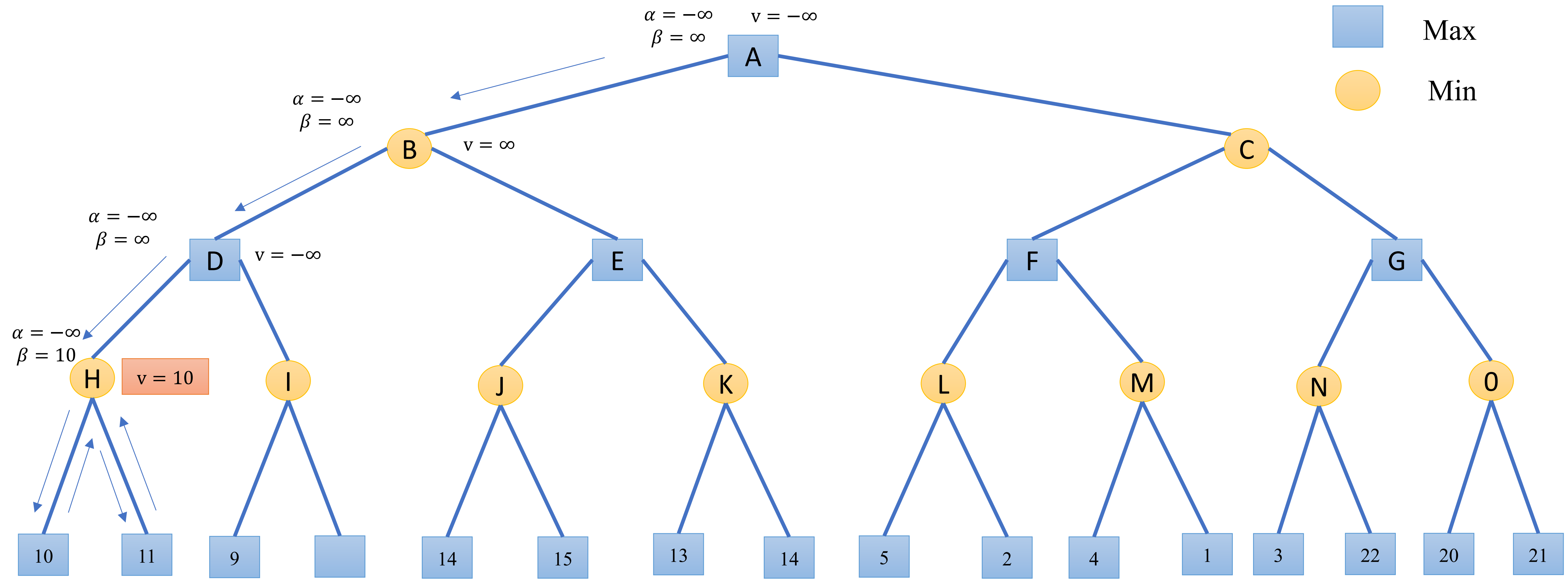
4
1
3
22
20
21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$\beta = \infty$
v = $-\infty$
A
Max
Min
$\alpha = -\infty$
$\beta = \infty$
B
v = ∞
C
$\alpha = -\infty$
$\beta = \infty$
D
v = $-\infty$
E
F
G
$\alpha = -\infty$
$\beta = 10$
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v = 10
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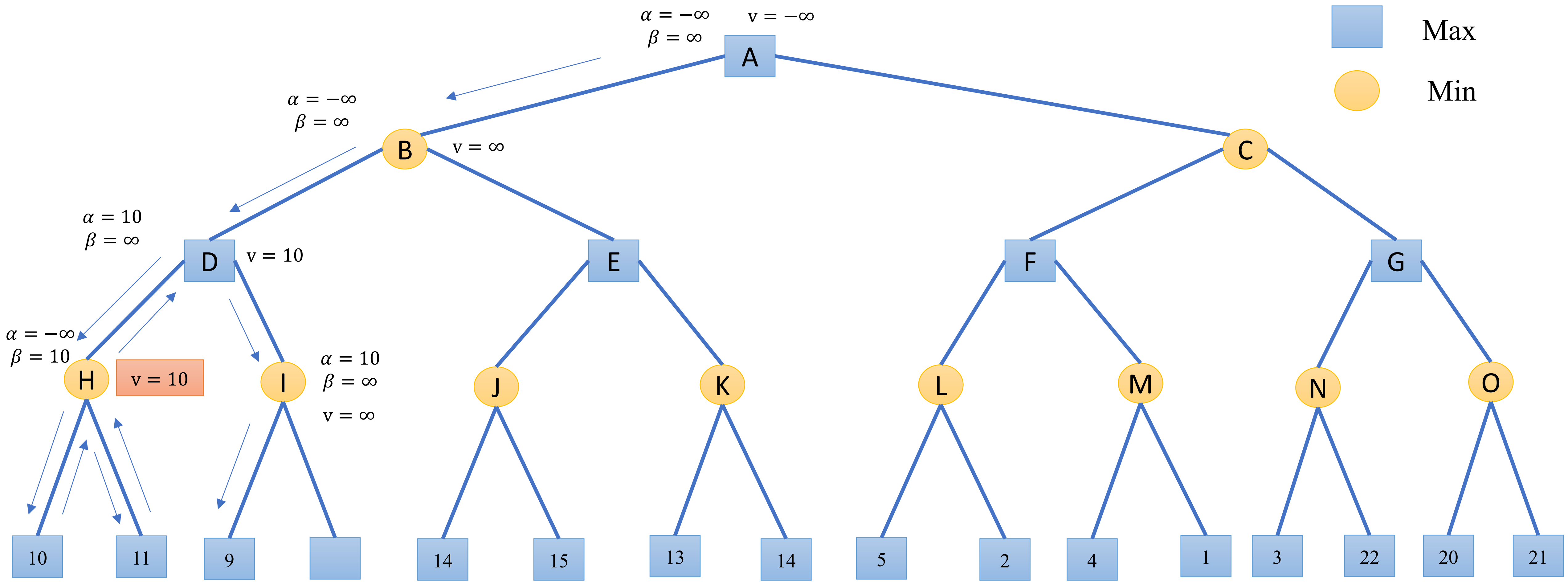
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$\beta = \infty$
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A
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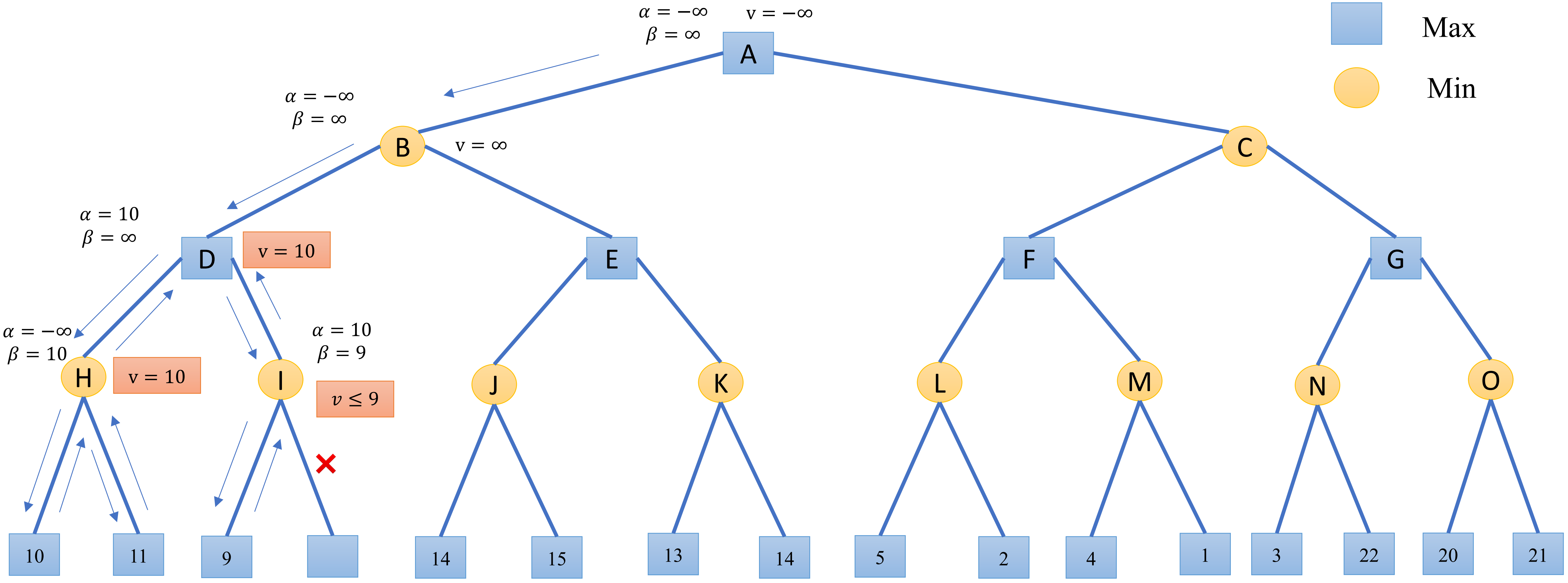

$\alpha = -\infty$
$\beta = \infty$
v = $-\infty$
A
Max
Min
$\alpha = -\infty$
$\beta = \infty$
B
v = ∞
C
$\alpha = 10$
$\beta = \infty$
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v = 10
E
F
G
$\alpha = -\infty$
$\beta = 10$
H
v = 10
$\alpha = 10$
$\beta = 9$
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$v \leq 9$
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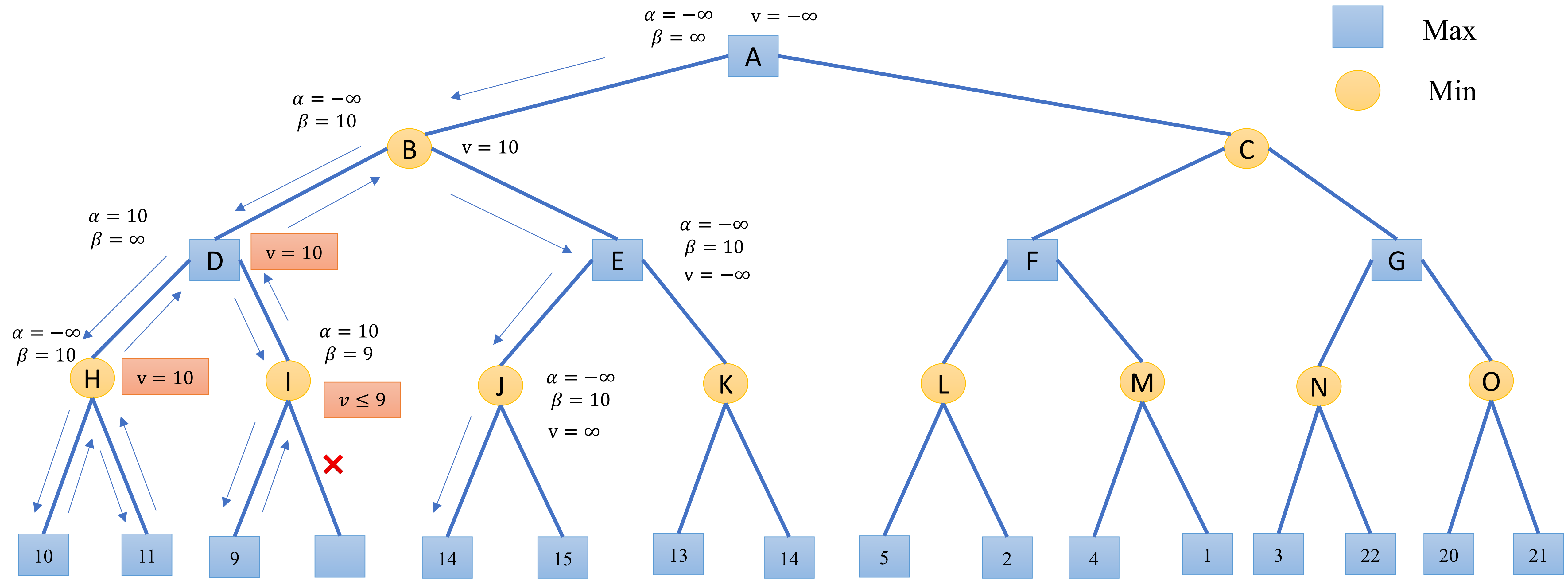
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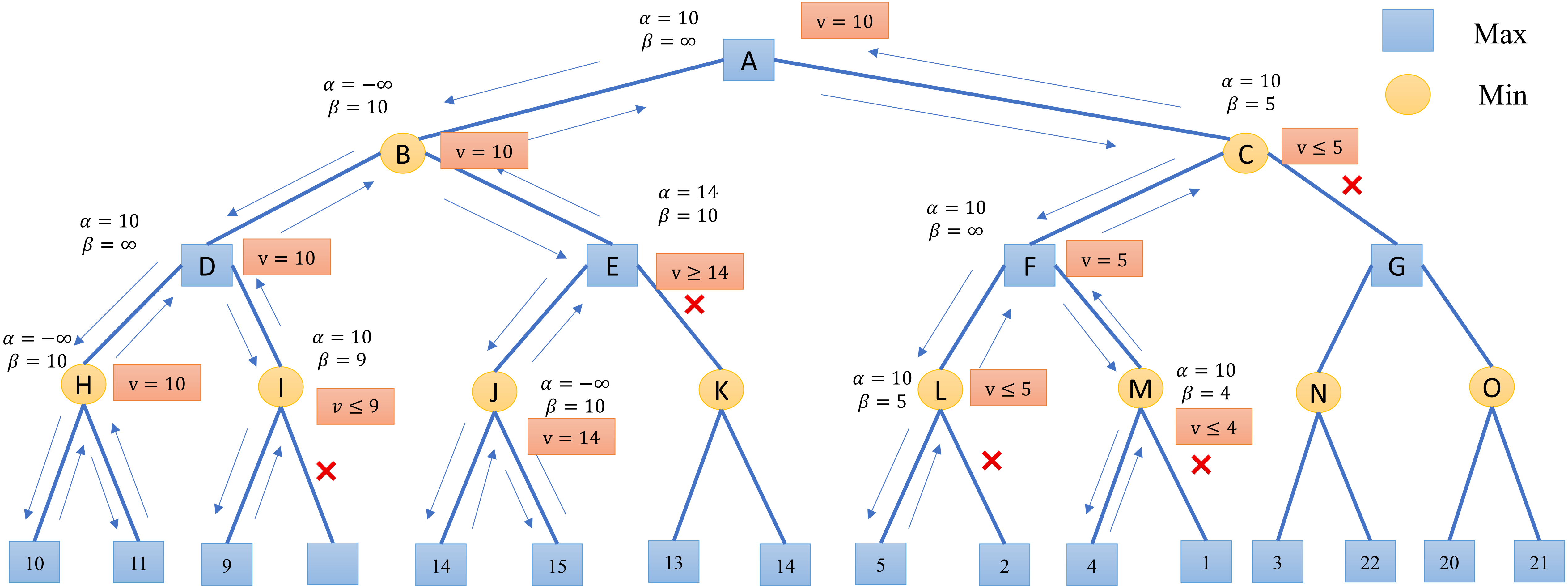

$\alpha = 10$
$\beta = \infty$
v = 10
A
Max
Min
$\alpha = -\infty$
$\beta = 10$
B
v = 10
$\alpha = 10$
$\beta = 5$
C
v ≤ 5
$\alpha = 10$
$\beta = \infty$
D
v = 10
$\alpha = 14$
$\beta = 10$
E
v ≥ 14
$\alpha = 10$
$\beta = \infty$
F
v = 5
G
$\alpha = -\infty$
$\beta = 10$
H
v = 10
I
$\alpha = 10$
$\beta = 9$
$v \leq 9$
J
$\alpha = -\infty$
$\beta = 10$
v = 14
K
$\alpha = 10$
$\beta = 5$
L
v ≤ 5
M
$\alpha = 10$
$\beta = 4$
v ≤ 4
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21

Alpha-Beta Procedure: V(J; α, β)

```
1. If J is a terminal, return V(J) = h(J)

2. If J is a max node:
    1. For each successor $J_k$ of J in succession:
        1. Set $\alpha = max\begin{cases} \alpha \\ V(J_k; \alpha, \beta) \end{cases}$
        2. If $\alpha \geq \beta$, then return $\beta$, else continue
    2. Return $\alpha$
3. If J is a min node:
    1. For each successor $J_k$ of J in succession:
        1. Set $\beta = min\begin{cases} \beta \\ V(J_k; \alpha, \beta) \end{cases}$
        2. If $\alpha \geq \beta$, then return $\alpha$, else continue
    2. Return $\beta$
```

Thank You