Constraint Satisfaction Problem: Filtering

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Filtering

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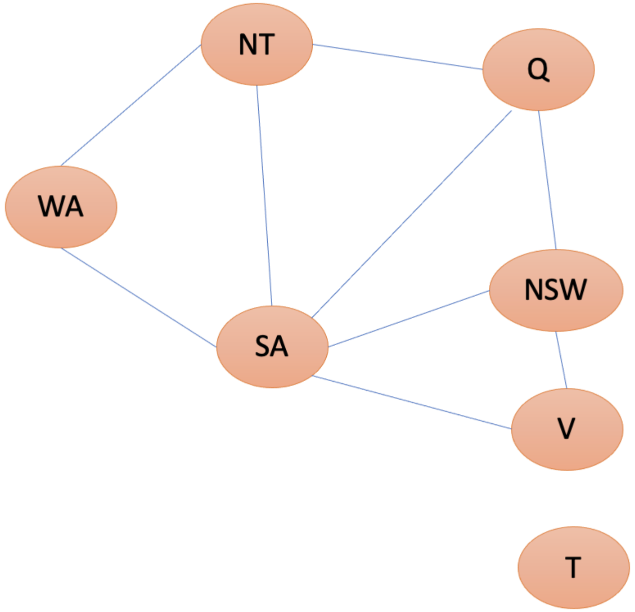
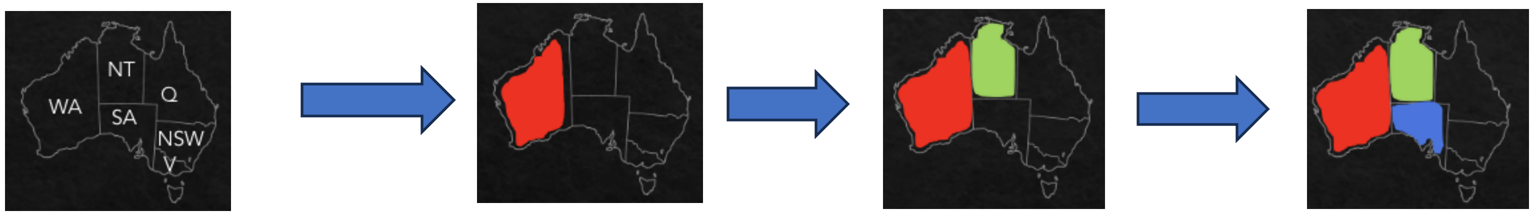
Ordering: Minimum Remaining Values



- Choose the variable to expand that has fewest legal values left in its domain
  - Most constrained variable
  - X1 = [R, G, B] X2 = [B]

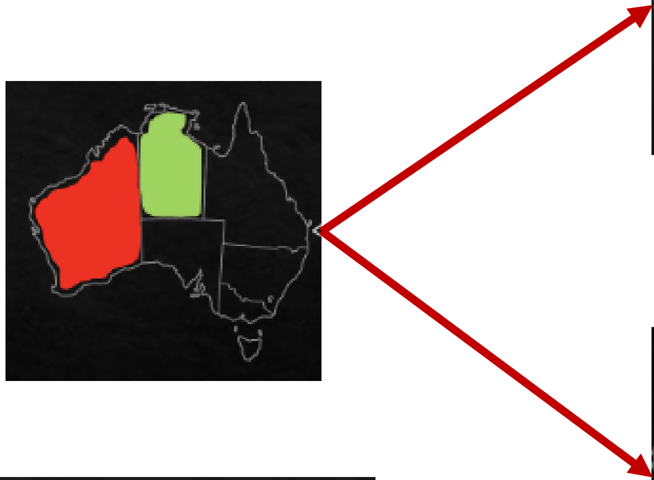


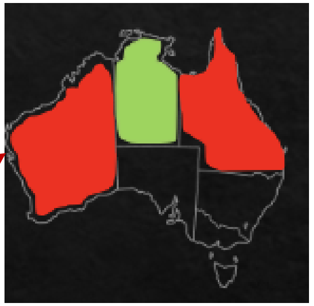
WA	NT	Q	NSW	V	SA
🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦
🟥	🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟥 🟩 🟦	🟩 🟦

Most Constraint Variable or Fail-Fast ordering

Ordering: Least Constraining Values (LCV)

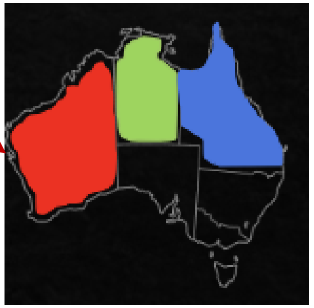
- Choose the value of a variable that rules out the fewest values in the remaining variables



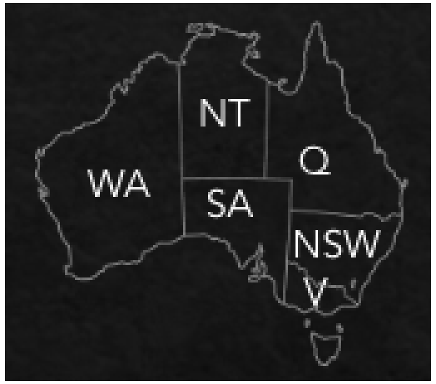


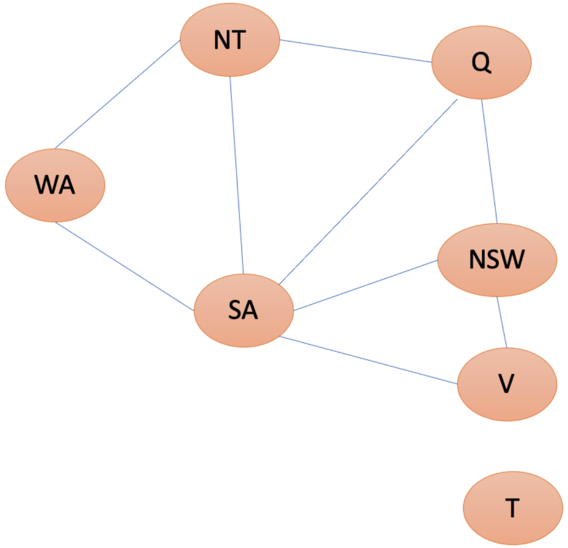
Q = {Red}
SA = {Blue}

Least constraining variable



Q = {Blue}
SA = {}





CSPs: Recap

- CSP Structure
  - Variables
  - Domains
  - Constraints
    - Implicit (code to compute)
    - Explicit (list of legal tuples)
    - Unary / Binary / n-ary
  - Goals
    - Find any solution
    - Find optimal solution

CSP Solver

- Backtracking give huge gain in speed

- Ordering
  - Which variable should be processed next (MRV)?
  - In what order values of the chosen variable be tried (LCV)?

- Filtering
  - Can we detect eventual failure early?
  - Arc consistency

NP-hard

- Structure
  - Can we exploit the problem structure?

CSP: Efficient Solver

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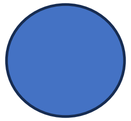
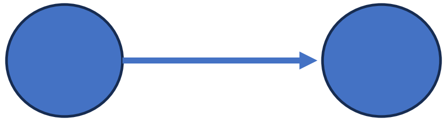
K-Consistency

- Consistency ➔ Non-violence of constraints

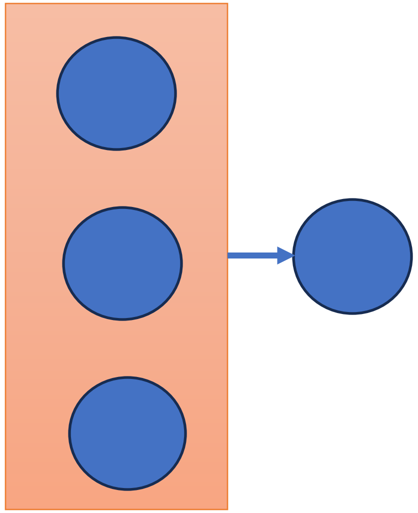
- Degrees of consistency
  - 1-consistency (node consistency)
    - Unary constraints



  - 2-consistency (Arc consistency)
    - Any consistent assignment to one can be extended to other
    - Binary



  - K-consistency
    - Any consistent assignment to K-1 nodes can be extended to the Kth node

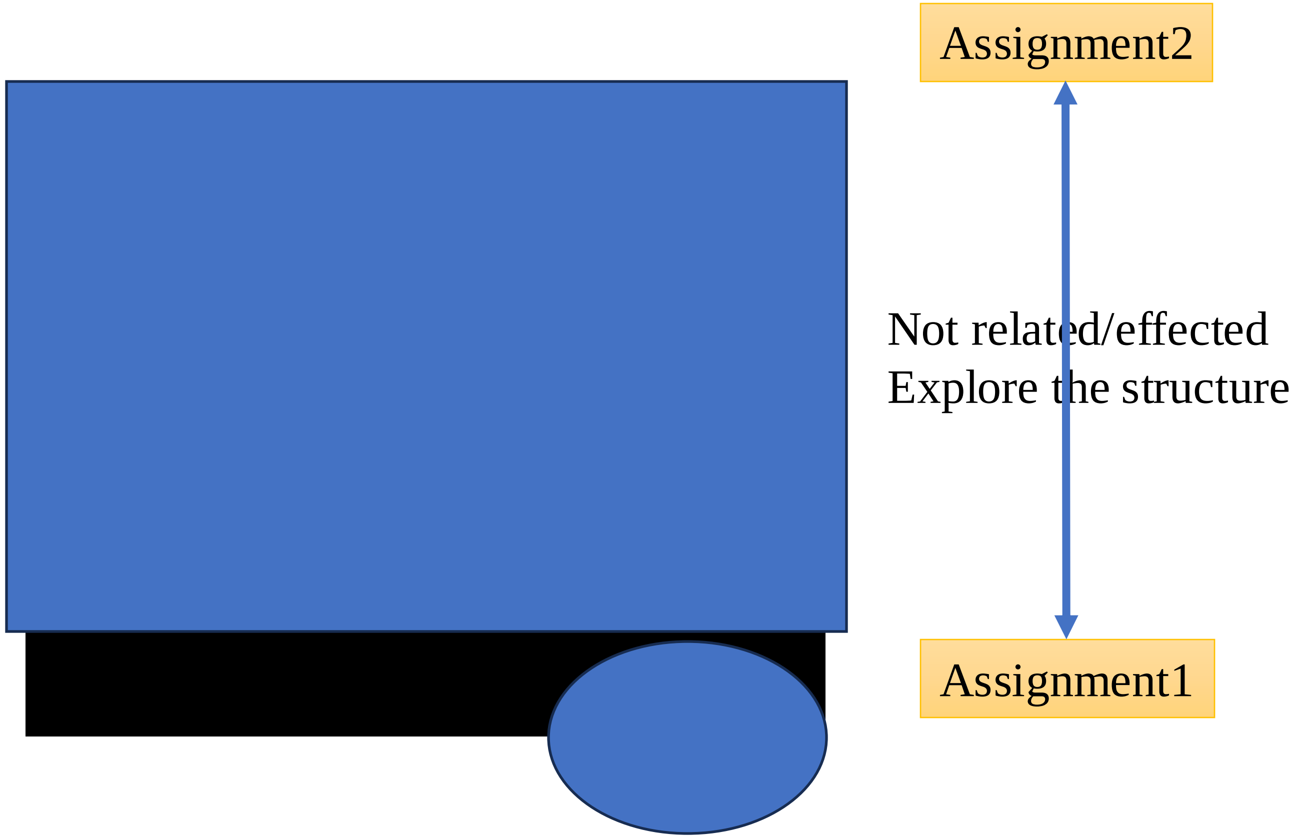


Strong K-Consistency

- Strong K-consistent ➔ K-1, K-2,…,1 consistent
- Strong N-consistency ensures solution without backtracking [N variable CSP]
    - Choose random assignment of any variable
    - Choose a new variable
    - 2-consistency ➔ there is a choice consistent with the first
    - Choose another variable
    - 3-consistency ➔ there is a choice consistent with the first two
    - …
- What is the limitation?
    - Enforcing strong N-consistency as hard as having the solution
- Trade-off between arc-consistency and K-consistency
    - E.g., 3-consistency aka Path-consistency

Problem Structure

- Independent Subproblems
  - Mainland and Tasmania do not interact
- How to identify independent subproblems?
  - Connected components of constraint graph
- What is the benefit?
  - Without decomposition running time:$O(d^n)$
  - Let n variables broken into subproblem of c variables
  - Worst case: $O(\frac{n}{c}d^c)$ ➔ Linear in n
- Let n=100, c=20, d=2
  - Without decomposition: 2^{100}
  - With decomposition: $5 * 2^{20}$



Assignment2

Not related/effected
Explore the structure

Assignment1

Very uncommon to find a problem structure

Thank You