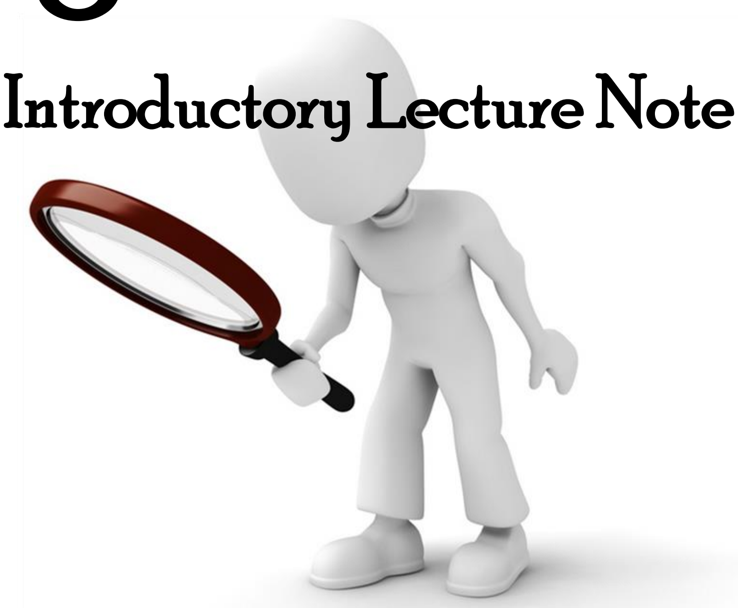
Reliability and Testing Methods:

An Introductory Lecture Note



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Chapter 1

Introduction to Reliability

Reliability is the performance against requirements over a period of time. Reliability measurements always have a time factor. IPC-SM-785 defines reliability as the ability of a product to function under given conditions and for a specified period of time without exceeding acceptable failure levels.

According IPC standard, J-STD-001B, which deals with solder joint reliability, electronic assemblies are categorized in three classes of products, with increasing reliability requirements.

Class 1, or general, electronic products, including consumer products. Reliability is desirable, but there is little physical threat if soldier joints fail.

Class 2, or dedicated service, electronics products, including industrial and commercial products (computers, telecommunications, etc.). Reliability is important, and solder joint failures may impede operations and increase service costs.

Class 3, or high-performance, electronics products, including automotive, avionics, space, medical, military, or any other applications where reliability is critical and solder joint can be life/mission threatening.

Class 1 products typically have a short design life, e.g. 3 to 5 years, and may not experience a large number of stress cycle. Class 2 and 3 products have longer design lives and may experience larger temperature swings. For example, commercial aircraft may have to sustain over 20,000 takeoffs and landings over a 20- year life, with cargo bay electronics undergoing thermal cycles from ground level temperatures (perhaps as high as 50 C at 35,000 feet).

The metrics used to measure reliability include:

Percent failure per thousand hours

MTBF: mean time between failure

MTTF: mean time to failure

FIT: failure in time, typically failures per billion hours of operation

Reliability is a hierarchical consideration at all levels of electronics, from materials to operating systems because materials are used to make components. Components compose subassemblies and subassemblies compose assemblies. Assemblies are combined into systems of ever-increasing complexity and sophistication.

Importance of Reliability

a. Increased demand for interconnectivity between sub-systems

Due to the increased inclination for module design concept, individual sub-systems may be constructed by different manufacturers and later assembled by yet another manufacturer. An example of such could be oscilloscope in which the power supply module, the tube, the control knob and the casing may be manufactured separately according to a given specification. Failure in any of these sub-systems could lead to a failure of the entire system. Hence, the demand for maximum reliability on the part of the manufacturer of sub-systems.

b. Demand from the Military, Communication and Industrial Sectors

While a relatively low reliability can still be tolerated in consumer electronic equipment (Radios, VCRs, Televisions etc.) a higher reliability is expected in industrial equipment (Control panels, temperature controllers, timers, motors, heaters etc.) and even higher degree is required in military, communication and aerospace equipment. A communication satellite is not expected to fail suddenly or unexpectedly, cutting off millions of intercontinental calls. Neither is the control circuitry of an atomic bomb expected to fail when it is to be triggered off.

c. Cost of Repair and Maintenance

If reliability studies are not embarked upon to determine and improve reliability of an item, an unfortunate end-consumer may end up spending more on repair and maintenance of the item than on the initial cost price. This is common for cheap imitations or household electronics equipment, like radios, VCRs, stereos, amplifiers and CD players.

d. Market Competition

Due to competition between manufacturers of similar products (e.g. radios -Trident, International, Sharp, Sony, etc.) no manufacturer can afford to develop the reputation of unreliability. Therefore, each endeavor to produce the most reliable goods at the least cost so as to have an edge over competitors. In addition, a manufacturer has to consider the cost of damages he has to pay for an item that fails during warranty period.

Definition of Terms

An Item: is defined as a part, (e.g.) resistor), sub-system (e.g. power supply module) or equipment (e.g. oscilloscope) which can be individually considered and separately tested.

Failure: is the termination of the ability of an item to perform its required functions.

A Fault: is a deviation of the unique operational characteristics of an item. A state of fault is regarded as a failure.

An Item: is considered to have failed under any of the following three conditions:

a. When it becomes completely inoperative,
b. When it is operative but unable to perform any longer the required function (for example, an electric motor now rotating at a speed lower than normal).
c. When it is still operative but is unsafe for continued us, (for example, an electric item that shocks when it is under use).

Reliability is the characteristic of an item expressed in probability that it will perform a function under stated conditions for a stated period of time. For example, the reliability of a television boaster might be given as 0.8 over a 10-hour period with an ambient temperature of 25°C.

Mean Time Between Failure (MTBF)

This term is applicable to only repairable items, MTBF can be defined as the mean value of the length of time which elapses between failures. It can be computed based on two main types of tests:

i. **The replacement method:**

In this method, an item is put on test until it fails, then it is repaired, and put back into service until it fails again, is repaired, put back etc.

If t_o is the starting time

t_1 = time of first failure

t_2 = time of second failure

t_n = time of nth failure

n = number of failures

$$m = \frac{(t_1 - t_0) + (t_2 - t_1) + \dots\dots\dots + (t_n - t_{n-1})}{n}$$

In the above example, the time taken for repair is counted as negligible. However, in practice, this downtime is taken into consideration.

Worked Example

Continuous tests were conducted on an electrical item and faults were repaired immediately it occurred at the following times.

Note that the downtime is to be neglected.

Failures	0	1	1	1	1	1	1
Time (x 100hrs)	$t_o = 0$	$t_1 = 2$	$t_2 = 4$	$t_3 = 8$	$t_4 = 10$	$t_5 = 14$	$t_6 = 15$

Calculate the MTBF.

Solution

MTBF,

$$m = \frac{(t_1 - t_0) + (t_2 - t_1) + (t_3 - t_2) + (t_4 - t_3) + (t_5 - t_4) + (t_6 - t_5)}{n}$$

$$m = \frac{(2 - 0) + (4 - 2) + (8 - 4) + (10 - 8) + (14 - 10) + (15 - 14)}{6}$$

$$m = \frac{15}{6} \times 10^2 = 250\ hrs$$

ii. The non-replaceable method

This method is used when the time between failures is so large that the item cannot be repeatedly put back into test. Instead, a large number of failures and survivors are noted. Under this method, the test time should not intrude into the wear out period of the item's life-span.

$$m = \frac{test\ hours\ for\ failures + test\ hours\ for\ survivors}{number\ of\ failures}$$

$$m = \frac{total\ componenet\ test\ (survivors) hours}{total\ number\ of\ failures}$$

Worked Example

In MTBF test, 50 components were tested for a period lasting 200 hours. The time of failures of the components are as shown in the table below. 35 components survived without failure, assuming that wear-out failure can be ignored.

Calculate:

i. total test hours before failure
ii. total test hours without failure
iii. total survival hours
iv. MTBF

Number of Components	Time of Failure (hours)
6	100
5	140
4	175

Solution

i. Total test hours before failures fore failures (for components which failed)

$$= (6 \times 100) + (5 \times 140) + (4 \times 175) = 2000\ hours$$

ii. Total test hours without failure (test hours for survivors)

$$= 35 \times 200 = 700\ hours$$

iii. Total survival hours

$$= 2000 + 7000 = 9000\ hours$$

iv. MTBF $m = \frac{9000}{15} = 600\ hours$

Note that if the replacement method had been used, it would have taken 600 hours before the item failed once, and hence, 600 hours for 10 failures. Using the non-replacement method, a large number of items (50) were used to obtain an MTBF (600hrs) which was greater than the test hours (200 hours).

Failure rate (λ)

This is the number of failure per unit time. For many electronic system λ is constant in the useful life period of the item and is equal in value to the reciprocal of the MTBF (m)

$$\lambda = \frac{1}{m} \quad \text{Or} \quad m = \frac{1}{\lambda}$$

For instance, if MTBF = 5000 hours

then $\lambda = \frac{1}{5000}$ failures per hour

= 20% per 1000 hours

= 200 faults per 10^6 hours

Note that a system with the highest MTBF will have lowest failure rate and hence be the most reliable.

Mean time to failure (MTTF)

While MTBF is applicable to items that can be repaired on failure, MTTF is used for items that are non-reparable (such as resistors, capacitors, electric bulbs, batteries etc.). It is the average time an item may be expected to function before failure. Supposing N items are tested in a specified way (by applying certain electrical, mechanical, heat or humidity conditions) until all have failed. If the times of each item failure is given by $t_1, t_2, t_3 \ldots\ldots\ldots\ldots\ldots t_n$, then the MTTF is given as:

$$MTTF = \frac{\sum_{i=1}^{N}(t_i - t_o)}{N}$$

$$M = \frac{(t_1 - t_0) + (t_2 - t_0) + \ldots\ldots\ldots\ldots + (t_N - t_0)}{N}$$

Where t_0 = starting (or reference) time

$(t_1 - t_0)$ = period to 1st failure

$(t_2 - t_0)$ = period to 2nd failure

$(t_N - t_0)$ = period to Nth failure

N= total number of failed items

Worked Example

Life testing is made on six (non-repairable) electrical lamps and the following results were obtained:

Failures	0	1	5	1	2	4
Time (100 hours)	$t_0 = 0.5$	$t_1 = 4$	$t_2 = 10$	$t_3 = 16$	$t_4 = 20$	$t_5 = 23$

Calculate the MTTF.

Solution:

$$MTTF = \frac{(t_1 - t_0) + 5(t_2 - t_0) + (t_3 - t_0) + 2(t_4 - t_0) + 4(t_5 - t_0)}{N}$$

$$MTTF = \frac{(4 - 0.5) + 5(10 - 0.5) + (16 - 0.5) + 2(20 - 0.5) + 4(23 - 0.5)}{1 + 5 + 1 + 2 + 4}$$

$$MTTF = \frac{195.5}{13} = 15.04$$

Classification of failures

Failures can be classified according to:

(i) Causes of failure
(ii) Timing of failure
(iii) Degree of failure
(iv) Combination of failures

(i) By Causes of failures

a) **Misuse Failure:** this is caused by using the item at stress beyond its stipulated capabilities e.g. applying 230V a.c. mains to an equipment specified for use with 110V a.c. mains. Only such failures are the fault of the consumer (user).
b) **Inherent Weakness Failure:** These are cause by a weakness which is inherent in the item itself. Such an item fails when it is used within its stated capabilities as specified by the manufacturer.

(ii) By Timing of failures

a) **Sudden Failures:** these could not have been anticipated by prior examination.
b) **Gradual Failures:** these could have been anticipated by prior examination.

(iii) By Degree of failures

a) **Partial Failure:** these are failures resulting from a deviation in the characteristics of item beyond special limit but the item can still perform part of the required functions (e.g. an electric fan rotating at a speed lower than normal)

b) **Complete Failure:** these are failures resulting from a total inability on the part of the item to perform its purchased function (e.g. an oscilloscope that refuses to show a trace when switched on).

(iv) By Combination of failures

a) **Catastrophic Failure:** failures which are both **sudden** and **complete** e.g. blowing of a fuse on short-circuiting or open-circuit on a rheostat.

b) **Degradation Failures:** these are failures which are both **gradual** and **partial** e.g. change in the value of a resistor due to change in climatic conditions.

Failure Rate

This can be defined mathematically as:

$$t = \frac{1}{NS} \quad \frac{dN_F}{dt}$$

where N_S = Number of surviving items

N_F = Number of failure items in time interval D_t as $D_t \rightarrow 0$

Worked Example

10 items have failed out of 1010 put on test during a period of 5000 hours. Calculate the failure rate.

Solution

$$\lambda = \frac{1}{1000} \times \frac{10}{5000} = 2 \times 10^{6}\ failures\ per\ hour$$

Or 0.0002%/hour Or 0.2%/10^3 hours

Basic failure rate is constant for most electronic components once they are in their useful life period. Below are some sample failure rates:

Failure rate in 1 hour%/10^3hours	
Capacitor	0.1
Resistor (carbon film type)	0.05
Diode (silicon 1 watt)	0.005
Thermistor	0.06
Triode value	1.8
Transformer (R.F)	0.03

In practice, the failure rate of components is affected by the environment, temperature and the fraction of its power rating at which it is operated.

The overall failure rate of each component in an equipment is given by

$$\lambda_o = n\,.\,\lambda\,.\,W_E\,.\,W_T\,.\,W_R$$

where n = quantity of the same component in the equipment

λ = basic failure rate

W_E = weighting faction due to environment (other than temperature)

W_R = weighting fraction due to rating

W_T = weighting factor due to temperature

Where λ_T = total failure rate for an equipment

$\lambda_{01}\,.\,\lambda_{02}\,.\,.\,.\,.\,.\,.\,.\,\lambda_{0n}$ = overall failure rate contributed by all the individual components e. g. transistor, resistors, diodes etc.

Reliability, $R = e^{-st\lambda(t)} dt$ if λ is not constant with time

$R = e^{-} \int_{o}^{\infty} \lambda(t)\, dt$

But if λ is a constant, then

$$R = e^{-\lambda t}$$

MTBF or MTTF $m = \int_{o}^{\alpha} R(t)\, dt$

If λ is constant, $m = \frac{1}{\lambda}$

Failure Density Function

$F(t) = \frac{-\, dR(t)}{dt}$

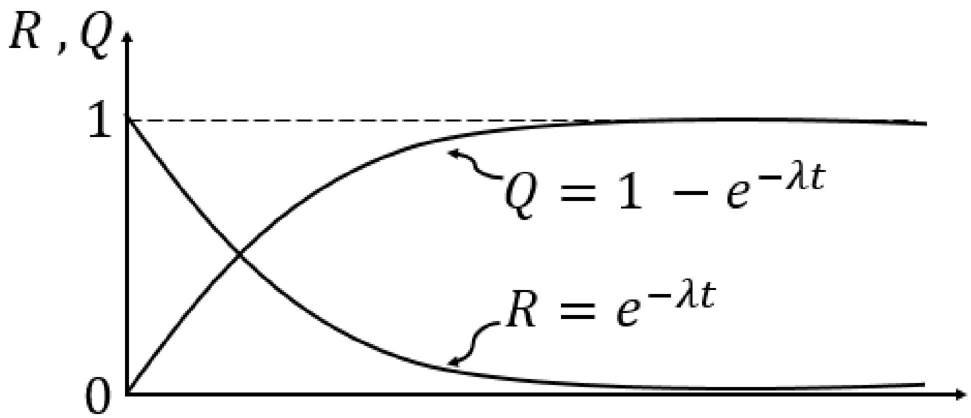
Reliability (R) and Unreliability (Q) Curves (λ = constant)

R = probability of no failure with time t

Q = probability of failure within time t

From statistical theory, $R + Q = 1$

$$Q = 1 - R = 1 - e^{-\lambda t}$$



At $\quad t = 1/\lambda$

$$R = e^{-\lambda t} = e^{-1} = 0.37$$

$$Q = 1 - e^{-1} = 0.63$$

If N_S = number of survivors after time t

N_F = number of failures after time

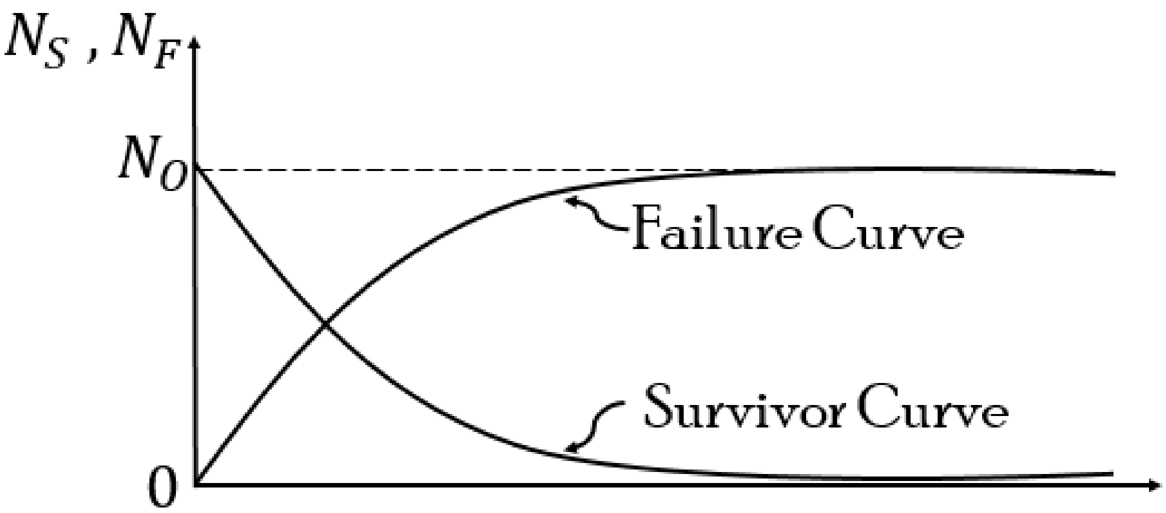
$$N_O = N_S + N_F$$

$$R = \frac{N_S}{N_O} = 1 - \frac{N_F}{N_O} = e^{-\lambda t} \qquad \text{where } N_O = N_S + N_F$$

$$N_O - N_F = N_O\, e^{-\lambda t} = N_S$$

$$Ns = N_O e^{-\lambda t}$$

$$N_F = N_O - Ns(1 - e^{-\lambda t})$$



Worked Example

1000 similar components, each with a constant failure rate of 5%/10^3hr are put into test together. Calculate the time before failure of the following number of components.

(i) 100
(ii) 500

Solution

We know that $$R = e^{-\lambda t} = 1 - \frac{N_F}{N_O}$$

$$\text{But } \lambda = \frac{5}{100} \times \frac{1}{1000} = 5 \times 10^{-5}$$

(i) $N_O = 1000$

$N_F = 100$

$$e^{-(5 \times 10^{-5} t_1)} = 1 - \frac{100}{1000} = 0.9$$

$$-5 \times 10^{-5} t_1 = log_e 0.9$$

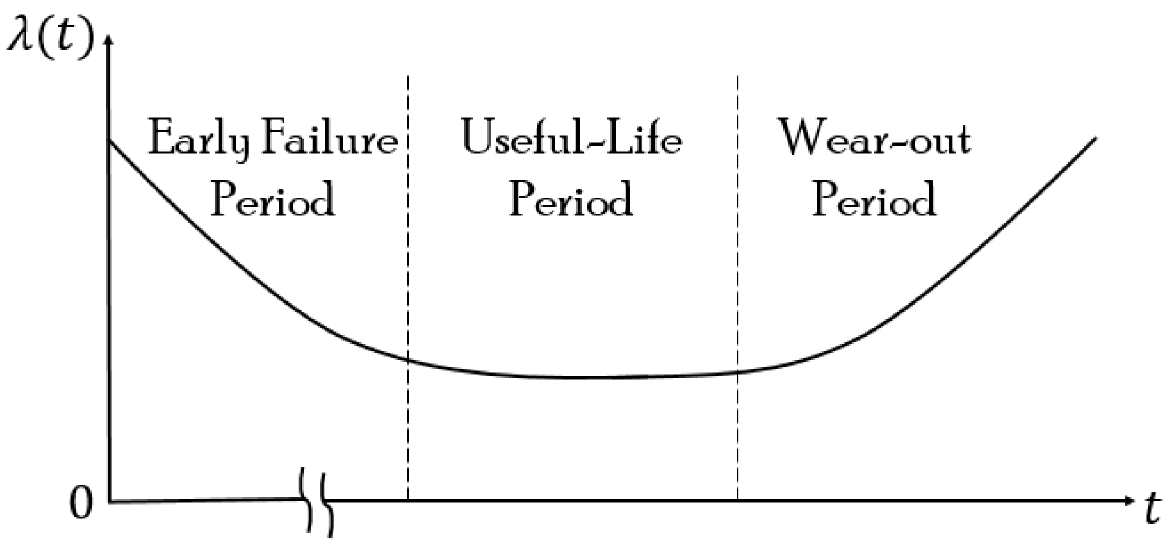
$$t_1 = 2107 \; hours$$

(ii) $$(-5 \times 10^{-5} t_2) = 1 - \frac{500}{1000} = 0.5$$

$$-5 \times 10^{-5} t_2 = log_e 0.5$$

$$t_2 = 13863 \; hours$$

Failure pattern (both – tub curve) of an equipment.



The bath-tub curve shows 3 distinct phases in the life of the equipment

(1) Early Failure or (Infant mortality) Period

These are as a result of weak parts that escape final screening before leaving the manufacturer

Causes of faults

a) **Manufacturing Faults:** e.g. faults in welds, joints, connections, insulations, etc. along with dirt, impurities, cracks that were not detected before dispatch from the manufacturer.

b) **Design fault:** are those caused by inaccurate design which may be undetected before leaving the manufacturer. It can be avoided by first manufacturing, testing and improving upon a porotype before mass production.

c) **Misuse Fault:** due to incompetent use by the customer, and also by hazardous environment for which it was not designed.

d) Packaging, Transportation and Storage faults

e) **Installation Faults:** due to incorrect installation and poor commissioning. These could combine to form a failure during the early life of the item.

The warranty period usually concedes with early failure period.

Constant Failure Period

The failure rate is constant in this period and it is also known as the useful life period. This is the period in which the item is expected to perform its purchased functions. Items have random or stress-related faults. Below are examples of systems and their approximate useful life period;

System	**Useful Life Period**
Telephone	> 15 years
Television	7-10 years based on 2000 hours per year

Electric Power System	> 15 years

Note that MTBF and MTTF are only relevant in the useful life period.

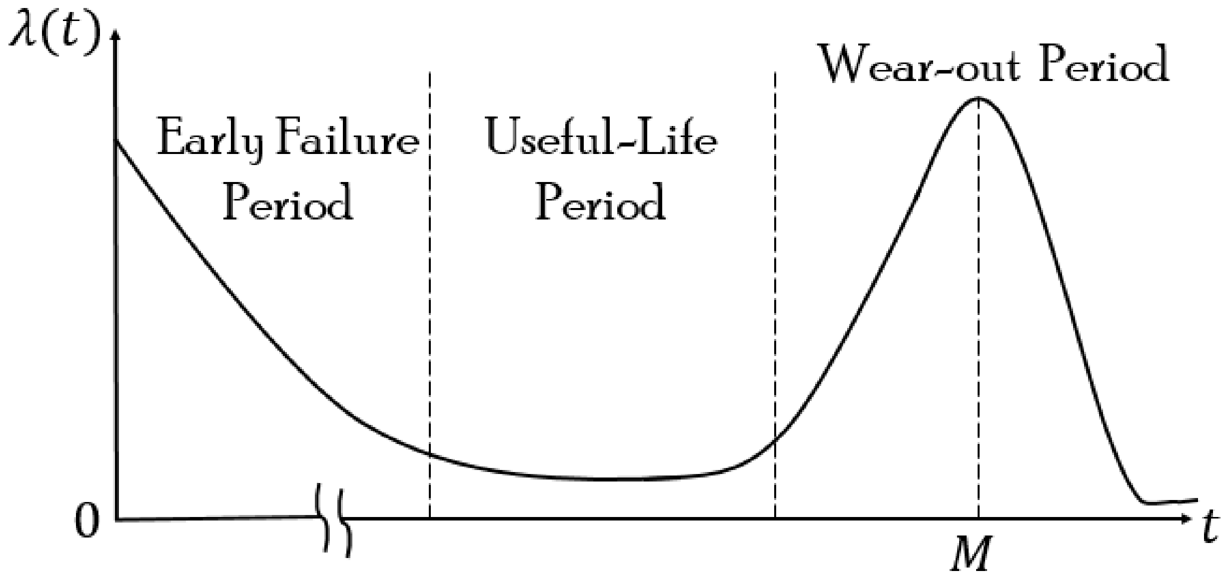
Wear out Failure Period

Towards the end of the useful life period on the item, its failure begins to increase (see the bathtub curve). This is due to deterioration of parts as a result of corrosion, oxidation, insulation breakdown, wear-out period, and the item becomes more of a liability than an asset due to the cost of frequent system downtimes, and, in most cases is withdrawn and replaced.

It is however possible to extend the useful life of an item by proper planned maintenance and repairs. Since the lifespan of an equipment/system is determined by that of the parts that it is comprised of, a lifespan can be significantly extended by replacement of parts that are approaching their wear-out phase.

It should be noted that semiconductors' components have a flat characteristic curve after early failure period. Thus, they are subjected to stress beyond their capabilities, they tend to have very long lifespans.

The early failure is represented by a gamma distribution, the useful life period by a Weibull distribution and the wear-out period by a normal (or gaussian) distribution.



M represents the mean wear-out period (not to be confused with MTBF or MTTF). The failures in the wear-out period tend to be clustered around M.

If the $t_{1,}$ $t_{2,}$ $t_{3,}$ . . . . , t_n represents the lifespan of n components during wear-out test, then the standard deviation is given by;

$$\sigma = \frac{\sqrt{(t_1 - M)^2 + (t_2 - M)^2 + - - - - - - - + (t_n - M)^2}}{n}$$

It is noted that 60% of the failures occurs within a period $M \pm \sigma$

i.e. between the time $(M - \sigma)$ to $(M + \sigma)$ approximately 95 percent occur within the period between $(M - 2\sigma)$ to $(M + 2\sigma)$ and 99.7 percent occur within a period between $(M - 3\sigma)$ to $(M + 3\sigma)$

it should be noted that if $t_{1,}$ $t_{2,}$ $t_{3,}$ . . . . , t_n represent the times to wear-out failure, then mean time wear-out

$$M = \frac{t_1 + t_2 + t_3 + \ldots\ldots + t_n}{n}$$

$$M = \frac{\text{Total testtime of all components which failed from wearout}}{Total\ number\ of\ components\ which\ failed\ from\ wearout}$$

Worked Example

30 components are put on accelerated life, test, and their failures occur at the following times from start of the test (inhouse)

18	186	470	760	802	828	959
40	230	533	788	812	831	
98	310	608	795	812	842	
118	317	714	800	814	850	

Calculate the mean time wear-out failure.

Solution

It can be safely assumed that the first two failures might have belonged to early failure period, followed by about 10 failures which are random in nature. Thus, we estimate the first twelve failures, up to714 hours because they are unlikely to be due to wear-out.

In addition, the very long life of 959 is not typical of the rest, and is hence not included.

Total hours components were on test

= 760 + 788 + 795 + 800 + 802 +812 + 812 + 814 + 828 + 831 + 842 + 860 = 9,744

Number of failures = 12

Mean time to wear-out $M = \frac{9744}{12}$ = 812 hours

Worked Example

An Electronic system works for 24 hours daily and continuous for 50 days. The following components listed below are used for its construction

Component	Number used	Failure rate (%per 10^3 hours)
Transistor	30	0.08
Diodes	4	0.05
Capacitors	100	0.01
Resistors	140	0.05
Connections (soldered)	700	0.001

Weighting Factors

Environment (all components) 2.0

Temperature (all components) 1.5

Rating

Capacitors 3.0

Resistors 2.0

Transistors 2.0

Diodes 1.5

Calculate, from the data given above, the system reliability for the 50-day operating period.

Solution

Components	$n_i\,\lambda$	Weighting Factors			$\lambda_o = n_i\,\lambda W_E\, W_r\, W_R$
	(%/10³)	W_E	W_T	W_R	(% /10³)
Transistor 30	0.08	2.0	1.5	2.0	14.4
Diodes 4	0.05	2.0	1.5	1.5	0.9
Capacitors 100	0.1	2.0	1.5	3.0	9.0
Resistors 140	0.5	2.0	1.5	2.0	42.0
Connections (soldered) 700	0.001	2.0	1.5	-	2.1

$$\lambda_T = \sum \lambda_O = (14.4 + 0.9 + 9.0 + 42.0 + 2.1)\%/10^3 hrs$$

$$\lambda_T = 68.4\%/10^3 hrs$$

Total operating time, $t = 24 \times 50\ hrs = 1200\ hrs$

$$R = e^{-\lambda\, t} = e^{-0.684 \times 10^{-3} \times 1200}$$

$$R = 0.44$$

Tutorial Questions

1a) A reliability test was conducted on a sample of 1000 similar component at a fixed temperature. The number of survivors at the end of the 100th hour was 950, and at the end of 200th hour was 920.

Calculate the average failure rate (1000hrs) over

i. The first 100-hour period (Answer 52.65%/10^3hr)

ii. The second 100-hour period (Answer 32.6%/10^3hr)

b. From the result obtained in 1(a), state with reason the region of the bathtub curve which can be said to follow this failure rate pattern.

2. In a test to determine the MTBF of a certain component, 100 were tested for period of 2000 hours. The times of failure of the component are shown in the Table 1 below, 55 components survived without failure. Assuming that wear-out failure can be ignored, calculate

1) Total survivor period (Answer 181010 hrs)
2) MTBF (Answer 12067 hrs)
3) Failure rate (Answer 8.28%/10^3hr)

Table 1

Number of Components	Time of Failure (hour)
1	250
1	300
4	415
5	800
4	1200

3a) Explain what you understand by the following statement;

i. Chance failures are distributed exponentially

ii. Approximately 63% of chance failures occur before a time equal to the MTBF

3b) Calculate the failure of a component having a reliability of 0.9 percent for a period of 500 hours. Assume exponential failure.

4. A radio receiver has sub-unit with failure rates shown in table 2 below. Calculate the reliability over a period of 5000 operating hours.

Table 2

Unit	Number in use	Failure rate (%/10^3 hr)
RF amplifier	1	0.4
Oscillator	2	0.08
IF amplifier	1	0.2
Detector	1	0.8
AF amplifier	3	0.04
Power supply	1	0.9

(Answer 0.873)

5a Define MTBF

b. State the mathematical expression for reliability in terms of MTBF and operating time, t.

c. It is desired to achieve a reliability of 0.85 for certain component over a period of 500 hours. Calculate the necessary MTBF.

6a Explain why weighting factors are important in the determination of MTBF of an equipment.

b. An electronic equipment contains the following component operating under the conditions stated against them:

20 silicon transistors (>1w) operating at 0.1 max-power rating.

10 germanium transistors (<1w) operating at 0.5 of max-power rating.

50 silicon diodes (>1w) operating at 0.1 of max-power rating.

100 resistors (composition) operating at 0.1 of max-power rating

50 resistors (carbon film) operating at 0.5 max-power rating

40 capacitors (ceramic) operating at 0.1 of max working voltage

20 capacitors (electrolytic/tantalum) operating at 0.5 of max working voltage

10 fuses, 10 relays, (sealed, each coil), 1 transformer (variable) 400 soldered connections.

Calculate the MTBF if the equipment is to operate in the following locations;

i. In an air-conditioned room (Answer 12780)
ii. In a room at a normal temperature (Answer 6390)

7. In a test lasting 100hours to determine the MTBF for a certain component, 8 out of 50 failed one after each of the following period (in hours) 4, 9, 15, 25, 40, 42, and 81. Assuming wear-out failure may be ignored, calculate

i. MTBF (Answer 554)
ii. Anticipated number of failures out of a batch of 1000 if the test is run over 2000 hours (Answer 973)

8a Explain the difference between the term MTBF and "Mean Wear-out life"

b. A wear-out test gave the life data shown in the table below. Calculate the upper confidence limit for the true mean wear-out life at 95% confidence level.

Number of Components	Life hours
5	390
8	450
10	450
7	550
6	600

(Answer 522 hours)

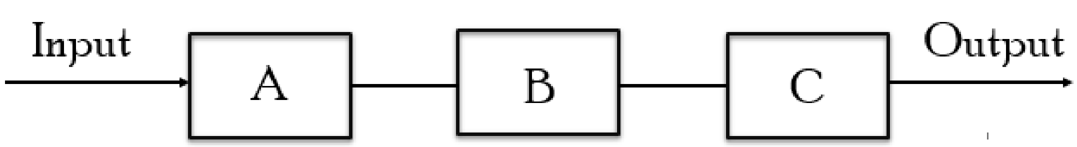
Chapter 2

Reliability Prediction

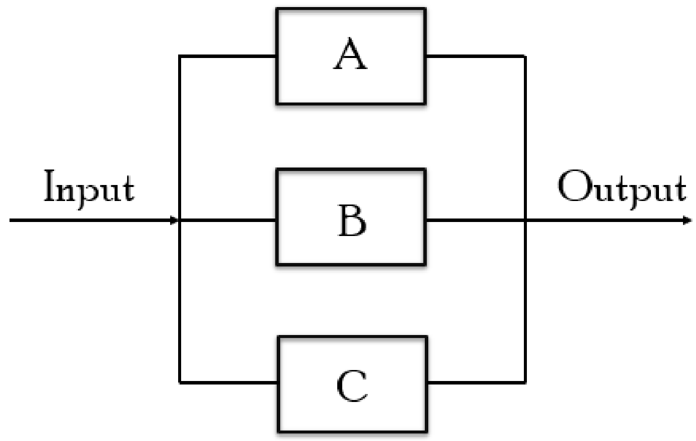
In practice, complex equipment and systems are made up of a number of sub-units each having their own reliability and failure rates. Thus, reliability prediction is the process of calculating the anticipated system reliability from assumed component failure rates. Such system reliability depends on the inter-relationship between the functional blocks, whether series, parallel, or series-parallel.

A Series System:

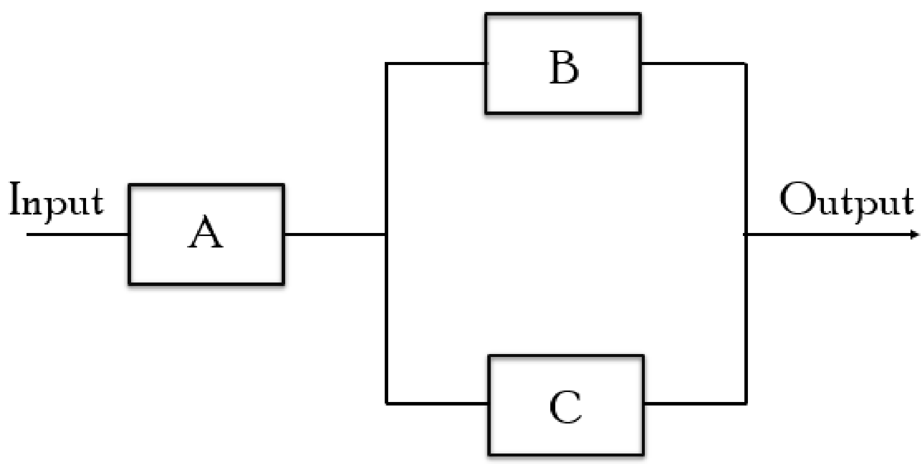
This is one in which the failure of any subunit is capable of causing a failure of the system. In the figure below, the system fails if A, B or C fail.



A Parallel System: is one in which does not fail until all sub-units have failed.



A Series-Parallel System: is one in which fails if the block in series connection (block A) fails od if both blocks (block B and C) in parallel connection fail. Thus some blocks are more crucial to system operation than others.



Reliability and MTBF of a Series System

If the series system comprises *n* sub-units of reliabilities

$$R_{1,}\ \ R_{2,}\ \ R_{3,}\ \ .\ .\ .\ .\ .\ .\ .\ .\ ,R_n$$

Failure rate s $\lambda_{1,}\ \ \lambda_{2,}\ \ \lambda_{3,}\ \ .\ .\ .\ .\ .\ .\ .\ .\ ,\lambda_n$

Then, the system reliability R_s is given by

$$R_s = R_{1,}\ \ R_{2,}\ \ R_{3,}\ \ .\ .\ .\ .\ .\ .\ .\ .\ ,R_n$$

If $R_1 = e^{-\lambda_1 t},\ R_2 = e^{-\lambda_2 t},\ .\ .\ .\ .\ .\ .\ .\ ,R_n = e^{-\lambda_n t}$

$$R_s = R_1\ .R_2 \ldots\ R_n = e^{-\lambda_1 t} \times e^{-\lambda_2 t} \times\ .\ .\ .\ .\ .\ .\ . \times e^{-\lambda_n t}$$

$$R_s = e^{-(\lambda_1 + \lambda_2 + \ldots\ldots + \lambda_n)t}$$

$R_s = e^{-\lambda_s t}$ (where $\lambda_s = \lambda_1 + \lambda_2 + .\ .\ .\ .\ .\ .\ . + \lambda_n$)

MBTF, $M_S = \dfrac{1}{\lambda_s} = \dfrac{1}{\lambda_1 + \lambda_2 + \ldots\ldots + \lambda_n}$

For *n* similar sub-units of equal reliability and failure rate

$$Rs = e^{-n\lambda t}$$

$$M_S = \frac{1}{n\lambda}$$

Reliability and MTBF of a Parallel Systems

$$R_p = 1 - [(1 - R_1)\,(1 - R_2)\ .\ .\ .\ .\ .\ .\ .\ (1 - R_n)]$$

For n similar subunits of equal reliability

$$R_p = 1 - (1 - R)^n$$

MBTF

Recall that $M = \int_o^\alpha R(t)\, dt$

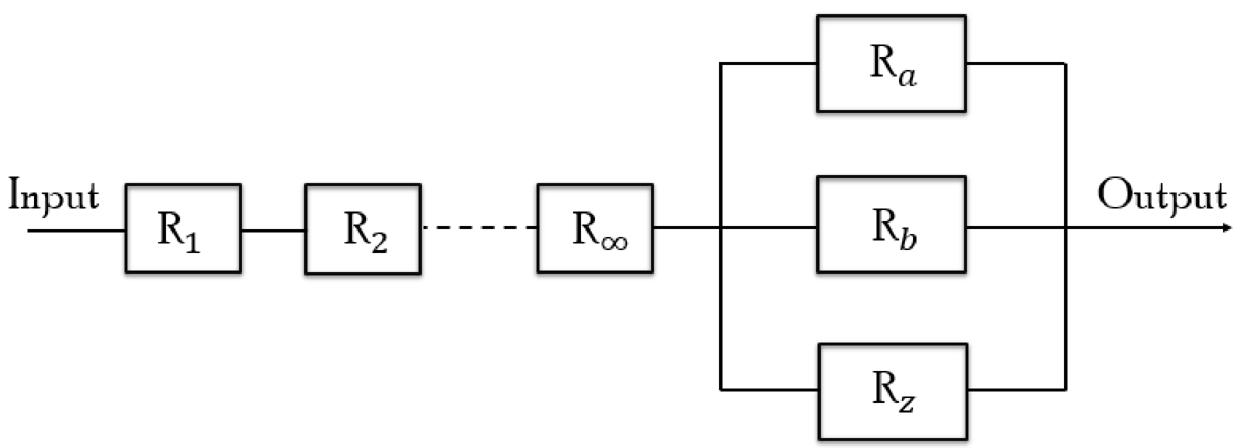
So, for parallel system

$$M = \int_o^\infty R_p(t)\, dt$$

For n similar subunits of equal failure rate X

$$M_P = \frac{1}{\lambda} + \frac{1}{2\lambda} + \frac{1}{3\lambda} + ------- \frac{1}{n\lambda}$$

Reliability and MTBF of a Series-Parallel System



$$R_s = R_1 \times R_1 \times ------- R_\infty$$

$$R = 1 - [(1 - R_a)\,(1 - R_b) ----(1 - R_z)]$$

$$R_{sp} = R_s \times R_P$$

$$= (R_1 \times R_1 \times . . . R_\infty)\,[1 - (1 - R_a)\,(1 - R_b). . .(1 - R_z)]$$

MTBF

$$Msp = \int_{o}^{\infty} R_{sp}(t)\, dt$$

Worked Example

Compare the reliability of a series system with a parallel system, if each system contains 3 sub-units having reliabilities 0.95, 0.85, and 0.75 respectively.

Solution

$$Rs = 0.95\ x\ 0.85\ x\ 0.75 = 0.605$$

$$R = \ [1 - (1 - 0.95)(1 - 0.85)(1 - 0.75)]$$

$$= 1 - (0.05\ \times\ 0.15\ \times 0.25$$

$$= 0.998$$

Worked Example

A complex communication satellite has an in-built microwave repeater unit, having a mean time to failure of 40,000 hours. the link is operative if one channel is working and the reliability of the switching unit is 0.95, calculate the reliability for one-year operating period using

i. A single channel
ii. Two parallel channels
iii. Three parallel channels

Solution

$$\lambda = \frac{1}{4\times10^4} = 0.25\times10^4\ failures/hour$$

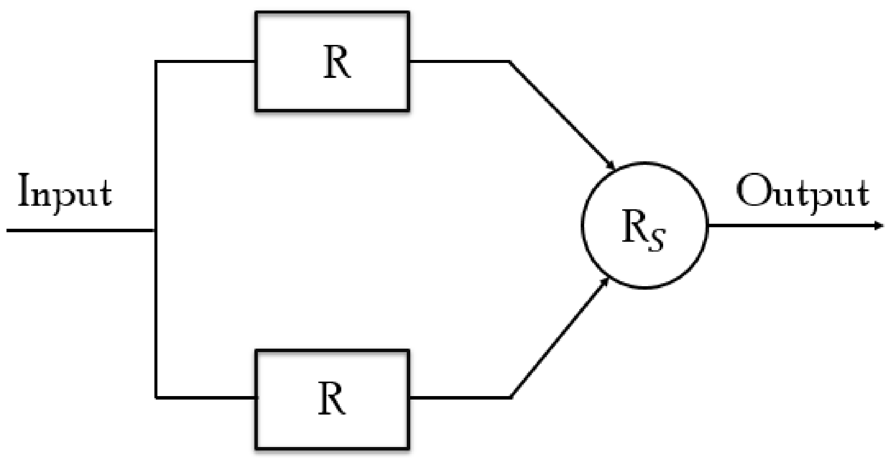
Operating period $t = 1yr = 8760hrs\ (24 \times 365)$

i. Using a single channel (no switching is necessary in this case)

$$R_1 = R = e^{-\lambda t} = e^{-0.25\times8760\times10^{-4}}$$

$$R_1 = 0.803$$

ii. 2 parallel channels switching unit



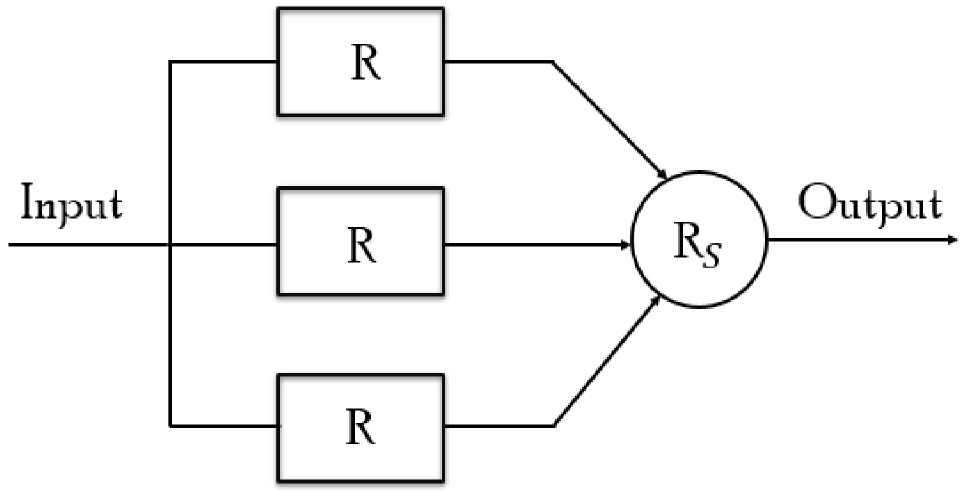
$$R_P = 1-[(1-R)(1-R)] = 1-(1-R)^2$$

$$= 1-(1-0.803)^2$$

$$= (1-0.197)^2 = 0.961$$

$$R_2 = R_P\times R_s = 0.961 \times 0.95 = 0.913$$

iii. 3 Parallel Channels + Switching Unit



$$R_2 = 1-(1-R)^3$$
$$= 1-(1-0.803)^3$$
$$= 1-0.197)^3 = 0.992$$
$$R_3 \;= R_P \times \; R_s = 0.992 \; \times \; 0.95 = 0.9424$$

Worked Example

An electric generating set has a probability of successful operation of 0.95 for a single day, and it is expected to operate for 5 days (not continuously)

a) Discuss the probability distribution for the two-possible outcome (success or failure)
b) Find the possibility that the generating set will operate successfully for (i) 3 days (ii) at least 4 days.

Solution

(a) Since we have two possible outcomes (success/failure) with fixed probabilities of occurring, we can employ binomial distribution methods.

If p = probability of success

q = probability of failure

then, p + q = 1

n = numbers of trails (i.e. number of days = 5)

Therefore, probability distribution of the two possible outcomes can be determined from the binomial expansion of $(p + q)^5$

using pascal's triangle

$$(p + q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$$

Where

p^5= probability that the generator will operate successfully for all 5 days.

$5p^4q$= probability that the generator set will operate successfully for 4 days and fail for a day.

q^5= probability that the generator will operate without success for the 5 days.

Since $p + q = 1 \; then \; (p + q)^5 = 1$

(b) (i) Probability of 3-day successful operation = $10p^3q^2$

$$since\; p = 0.95, q = 1 - p = 1 - 0.95 = 0.05$$

$$10p^3q^2 = 10 \times 0.95^3 \times 0.05^2 = 0.02$$

(ii) Probability that the generating set will operate successfully for at least 4 days = $p^5 + 5p^4q$

$$= 0.95^4 + (5 \times 0.95^4 \times 0.05) = 0.978$$

In general, probability of 'r' successes in the trails is given by

$$P_r = n(r \times p^r \times (1-p)^{n-r}$$

$$\text{Where,}^n C_r = \frac{n!}{r!\ (n-r)!}$$

Worked Example

An electric power system consists of 3 sections connected in series, the section has mean times between failure of 10,000hrs, 5,000hrs and 4,000hrs respectively. Calculate the MTBF of the system.

Solution

If $\lambda_{1,}\ \lambda_{2,}\ \lambda_{3,}$ represent the failure rates of each section respectively and the sections are connected in series.

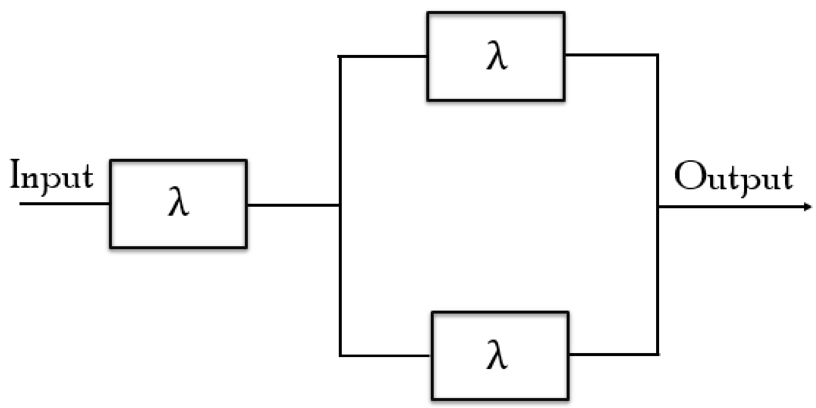
$$\lambda_{S=}\ \lambda_1 + \lambda_2 + \lambda_3\ ;\ MTBF = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$MTBF = \left[\frac{1}{10 \times 10^3} + \frac{1}{5 \times 10^3} + \frac{1}{4 \times 10^3}\right] \times 10^{-3}/hr$$

$$Ms = \left(\frac{1}{0.1 + 0.2 + 0.25}\right) \times 10^{-3} = 1818\ hrs$$

Worked Example

Calculate the MBTF of the system shown below if the failure rate of each unit is λ.



A series-parallel circuit

Solution

Reliability of the parallel blocks, $R_P = 1 - (1-R)(1-R) = 2R - R^2$

Reliability of the series block, $R = e^{-\lambda t}$

Reliability of the whole system $R_T = R(2R - R^2)$

$= 2R^2 - R^3$

MTBF of the system

$$M_T = \int_o^\infty (2R^2 - R^3)\, dt$$

$$= \int_o^\infty (2e^{-2\lambda t} - e^{-3\lambda t})\, dt = \frac{1}{\lambda} - \frac{1}{3\lambda} = \frac{2}{3\lambda}$$

$$M_T = \frac{2}{3\lambda}$$

Worked Example

A wear-out test gave the following results:

No of Components	2	1	4	6	3
Life per component hr	800	1000	1200	1300	1500

Calculate:

(i) The mean wear-out life of the components,

(ii) The upper confidence limit for the components mean wear-out life, at 95% confidence level.

Solution

(i) Sum of lives of components

$$(2 \times 800) + (1 \times 1000) + (4 \times 1200) + (6 \times 1300) + (3 \times 1500)$$
$$= 19700 \text{ hrs}$$

$$\text{Mean wear-out, } M = \frac{19{,}700}{16} = 1231\ hrs$$

(ii) Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} f_i(t_i - m)}{n}}$$

In order to determine σ, we embark on the following arrangement

f_i	$(t_i - m)$	$(t_i - m)^2$	$f_i(t_i - m)^2$
2	$(800 - 1231)$	185761	371522
1	$(1000 - 1231)$	53361	53361
4	$(1200 - 1231)$	961	3844
6	$(1300 - 1231)$	4761	28566
3	$(1500 - 1231)$	72361	217083
			674,376

$$\sigma = \sqrt{\frac{674{,}376}{16}} = 205hrs$$

Standard deviation for the mean distribution, σ_m is

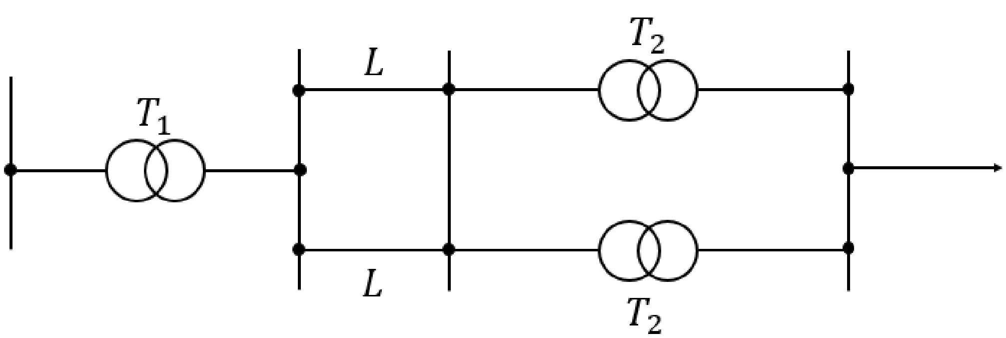
$$\sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{205}{\sqrt{16}} = 51hrs$$

The upper confidence limit for the time mean wear-out life at a 95% level of confidence is

$$(m + 2\sigma_m) = 1231 + (2 \times 51) = 1333hrs$$

Worked Example

A power transmission system shown below consists of a step-up transformer T_1, 2 circuits of power transmission lines L, & 2 step-down transformers T_2. Full power can be transmitted to a consumer through any of the 2-transmission line circuits. The step-down transformer can handle 50% power only. The unreliability of transformer T_1, each transmission line circuits and each step-down transformer are $QT_I = 0.005, QL = 0.03\ and\ QT_2 = 0.004\ respectively.$



A power transmission system

Failures in all these components are random and independent. Assuming that constant power is transmitted, determine the reliability of the system under the following conditions: (a) 100% power transmission (b) 50% power transmission.

Solution

(a) $$R\ (100\%) = RT_1 \times (1 - Q^2L) \times R^2\ T_2$$

$where\ RT_1 = (1 - QT_I)\ ;\ Q_L = 1 - R_L\ ;\ RT_2 = 1 - QT_2$

$$\therefore R(100\%) = (0.995 \times (1 - 0.03^2) \times 0.996^2 = 0.986$$

(b) $$(RT_2 + QT_2)^2 = R^2\ T_2 + 2RT_2Q_2 + Q^2\ T_2$$

We note that only the middle term $2RT_2QT_2$ gives the probability that one of the T_2 transformers is operational.

$\therefore$ Reliability of the system for 50% power transmission is

$$R\ (50\%) = RT_I \times (1 - Q^2L) \times 2RT_2QT_2$$

$$R\ (50\%) = 0.995 \times (-0.03^2) \times (2 \times 0.996 \times 0.004) = 0.008$$

Note

$1 - Q^2L$ is the reliability of a unit consisting of 2 parallel sub-units.

Worked Example

An electrical generating set designed for continuous operation fails twice in a period of 123 days. The total time for repairs during the period is 3 days. Determine the following parameters.

(i) MTBF (ii) MTTR (iii) AVAILABILITY

Solution

(i) Total operating time = $(123 - 3) = 120\ days$

Total number of failures = 2

$$\text{MTBF} = \frac{120}{2} = 60 days$$

(ii) $$\text{MTTR} = \frac{total\ time\ for\ repairs}{total\ no\ of\ failures} = \frac{3}{2}\text{ days or } 1\tfrac{1}{2}\text{ day}$$

$$\text{Availability} = \frac{MTBF}{MTBF+MTTR} = \frac{60}{60+3/2} \times 100\% = 97.6\%$$

Redundancy

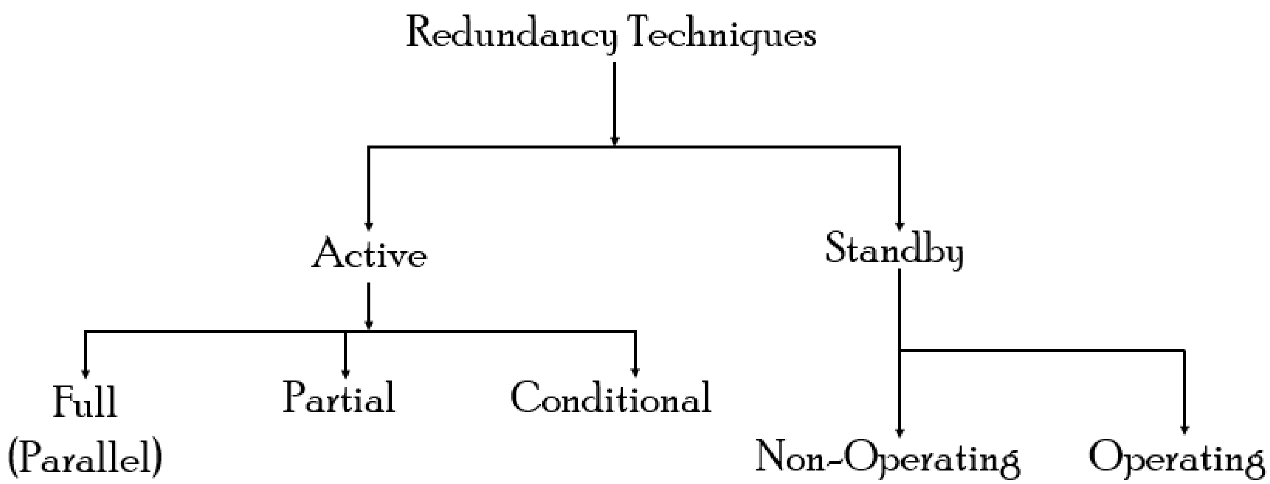
Redundancy is the method of improving system reliability. It is a technique of including one or more redundant units as a means of duplicating components, sub-units or equipment that are crucial to system operation, so that in event of failure, the redundant unit up the task and system operation continues without interruption.

An example is the altimeter that indicates to the pilot the height at which he is flying. Since that failure of the aircraft (i.e. the crash of the plane) must be ultimately avoided, a good design may incorporate 3 altimeters into the aircraft. If one goes faulty, any two which give the same reading can be sufficient for height measurement, the remaining two are the redundant ones included to reduce the probability of a crash (to improve system reliability)

Redundancy may be applied at the part, circuit or equipment level. Due to cost, weight, space, complexity, time to design and maintenance expenses, redundancy is used when all other techniques of improving reliability have been exhausted or when part improvement is costlier than duplication.

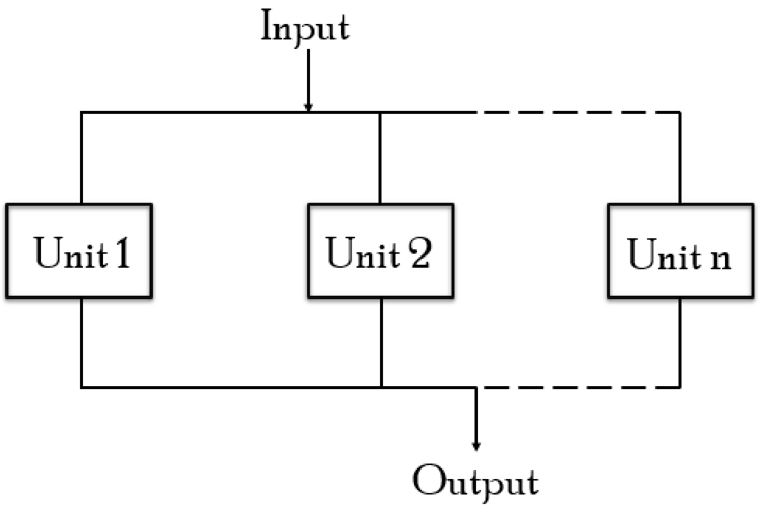
In addition, it may reduce system downtime in the event of preventive maintenance and repair. In some situations, due to inaccessibility (as in space-craft, satellite or underwater equipment, etc.) the equipment cannot be easily maintained, and redundancy is the only way of ensuring a reasonable useful life period.

Classification of Redundancy

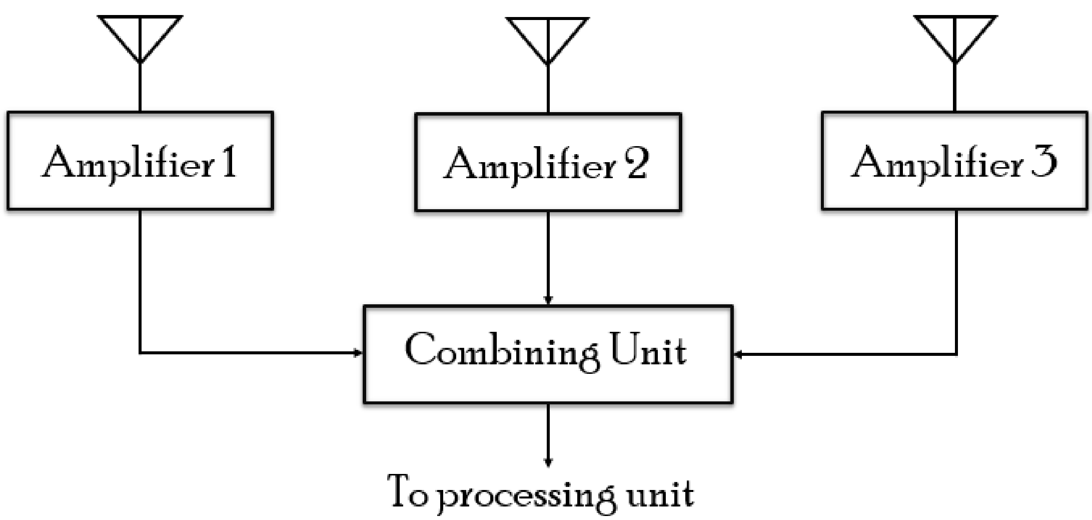


Active Redundancy

In this case, all the units are energized simultaneously to perform a given function.



An example of active redundancy is in the diversity reception of long distance radio transmission. Three or more well-spaced series are used to pick up the same signal. After amplification, a combining unit selects the largest signal for subsequent processing. Another example is that of the altimeters mentioned above.



Full (Parallel) Active Redundancy

Although all units are energized simultaneously, system operation is sustained as long as at least one unit remains operating.
In this case,

$$R_P = 1 - (1 - R_1)(1 - R_2) - - - -(1 - R_n) = 1 - (1 - R)^2$$

$$M = \frac{1}{\lambda} + \frac{1}{2\lambda} + \frac{1}{3\lambda} - - - - - - \frac{1}{n\lambda} +$$

Where R = reliability of each unit
λ = failure rate of each unit

Partial Active Redundancy

In this case, more than one of the identical units is needed to sustain system operation. An example is an aircraft with four engines. It can land safely if three are working, but will fail with only one functional engine. Binomial expressions can be used to solve this type of questions.

Worked Example

The unreliability of an aircraft during flight is 0.01. What is the reliability of successful flight if the aircraft can complete the flight on at least three out of its four engines?

Solution

For successful flight

a) All four engines must work
b) 3 out of four engines must work

From binomial expression,

$$(R + Q)^4 = R^4 + 4R^3Q + 6R^2Q^2 + 4RQ^3 + Q^4$$

The system reliability (probability of a successful flight)

$$R_{ap} = R^4 + 4R^3Q$$

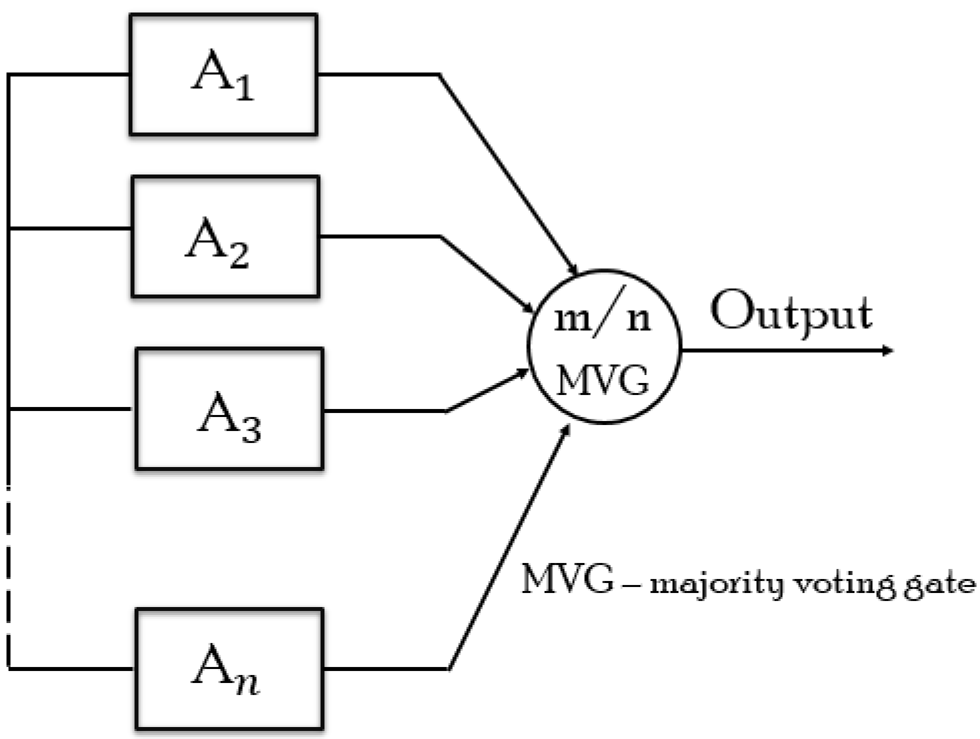
$$Q = 0.01; \qquad R = 0.99$$

$$R_{ap} = 0.99^4 + 4\,(0.99)^3 \times 0.01$$

$$= 0.0999$$

Conditional (Majority Voting) Active Redundancy

In this case, m out of n units are required to be working for the system to function. This is called m-out-of-n (or m/n) redundancy.



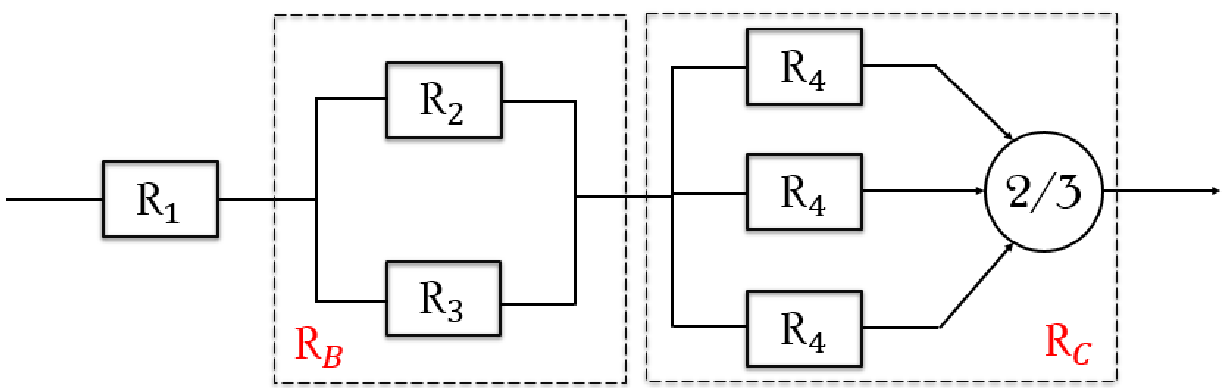
For instance, a 4/6 MVG will act as a decision maker, sampling the readings of 6 redundant units and presenting the option favoured by at least 4 of them. The output of the MVG can be made to trigger off various control processes.

An example of conditional redundancy is in aircraft flying controls where three units are employed in redundancy. A sensing system automatically switches off one unit if it does not agree with the rest.

$$R_{m-n} = 1 - \sum_{i=0}^{M-1} {}^{n}C_i \, R^i (1-R)^{n-1}$$

Worked Example

Determine the reliability of the system shown below:



Solution

Assuming the MVG is perfect (reliability = 1)

$$R_s = R_1 \times R_B \times R_C$$

$$R_B = 1 - [(1-R_2)(1-R_3)]$$

For R_C, note that $(R+Q)^3 = R^3 + 3R^2Q + 3RQ^2 + Q^3$

$$R_C = 1 - (\text{unreliability}) = 1 - (\text{probability of failure})$$

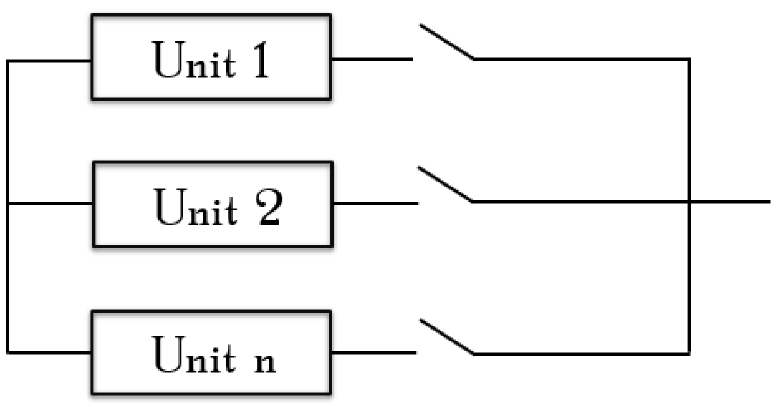
$$R_C = 1 - (3RQ^2 + Q^3)$$

$$R_C = 1 - 3R_4(1 - R_4)^2 + (1 - R_4)^3$$

Alternatively, $$R_C = R^3 + 3R^2Q$$

Passive (Standby) redundancy

In this case, only one unit is energized at a time, and a provision is made to switch automatically from a failed unit to one of the standby units. An example is two generators connected with automatic switching devices. If the first fails during operation, the other one is switched on to replace it so that there is no system failure. In effect, standby redundancy involves the use of additional units which are only energized when the operating unit fails,



The reliability for standby redundancy is given by the first n terms of the Poisson expression.

$$R_{sh} = R(t) = e^{-\lambda t}[\,1 + \lambda t + \frac{\lambda^2 t^2}{2!} + \ . \ . \ . + \frac{\lambda^{(n-1)} t^{(n-1)}}{(n-1)!}\,]$$

For two units (i.e. one in standby), $R_{sh} = e^{-\lambda t}(1 + \lambda t)$

It is assumed that the sensing and switching devices are perfect i.e. failure-free, and all units have identical failure rates.

If two units have failure rates λ_1 and λ_2 the above expression becomes

$$R_{sh} = \frac{\lambda_1 R_1 - \lambda_2 R_2}{\lambda_2 - \lambda_1}$$

Worked Example

A computer and two other similar computers in standby redundancy are available for control of a chemical process, such that if a computer fails, another is instantaneously switched on for operation. The failure rate of each computer is 0.01 failure per hour.

Calculate the system reliability if the operating period is 50 hours and switch is perfect, using:

i. A single computer
ii. One standby computer
iii. Two standby computers

Solution

i. For a single computer

$$R = e^{-\lambda t}$$

$$R = e^{-0.01 \times 50} = e^{-0.5} = 0.606$$

ii. For one standby computer

$$R = e^{-\lambda t}(1 + \lambda t)$$

$$R = e^{-0.01 \times 50}(1 + 0.01 \times 50) = e^{-0.5}(1{,}5) = 0.909$$

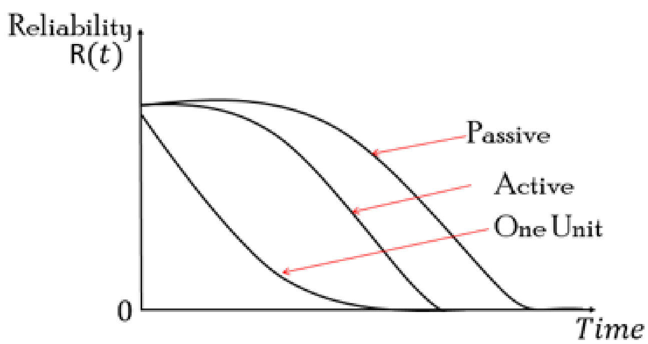
iii. For two standby computers

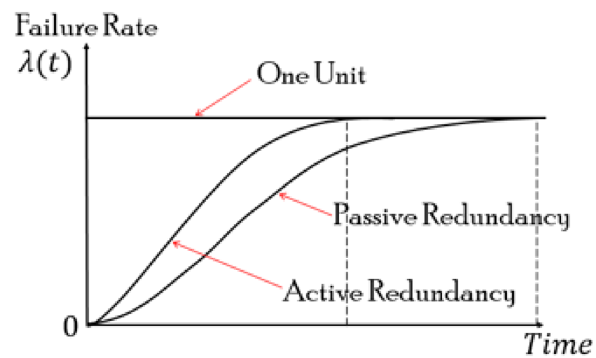
$$R = e^{-\lambda t}(1 + \lambda t + \frac{\lambda^2 t^2}{2!})$$

$$R = e^{-\lambda t}(1.5 + 0.125) = 0.985$$

Comparison of Reliability Improvement using Active and Passive Redundancy

Graphical Comparison





It is assumed that for passive redundancy, the switching is perfect. It is observed from the above graphs that reliability stays high for active or passive redundancy, compared to that of one unit, but that of active redundancy falls more sharply with time than with passive redundancy. It must be noted, however, that equal numbers of redundant units are assumed for both cases. A greater number of units generally means a longer period of higher reliability and a sharper decline.

If each unit has a constant failure rate, the failure rates for active and passive redundancy are shown above. Obviously, the failure rate for an active redundant system is higher than that of a standby system visiting at all times.

Practical Comparison

With active redundancy, there is no inherent ability to detect whenever one of the units fails. Thus, if two units are connected in active redundancy and one fails, the operator does not detect because the system continues its normal operation until the last unit fails. This is undesirable, as part of the advantages of redundancy is eroded.

To avoid this, a detecting device may be incorporated into the system design to detect and signify the failure of any unit to redundancy, there is inherent provision for detecting failure and switching over to another unit.

Mathematical Comparison

For one unit, $MBTF = \frac{1}{\lambda} = K$

For two similar units in active redundancy,

$$MBTF = \frac{1}{\lambda} + \frac{1}{2\lambda} = K + \frac{K}{2} = \frac{3}{2}K$$

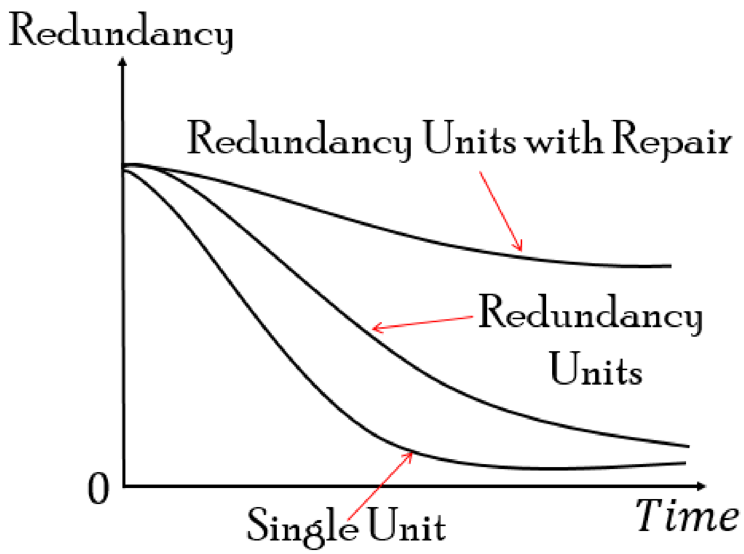
For the same units in standby redundancy,

$$MBTF = \frac{1}{\lambda} + \frac{1}{\lambda} = 2K$$

From the above, it is clear that standby redundancy has a lower system failure rate.

Redundancy with Periodic Repair

Redundancy with repair of failed units means that the system as a whole can be operated continuously while failed units are being repaired. Thus, overall system failures, reliability and MBTF can be defined.



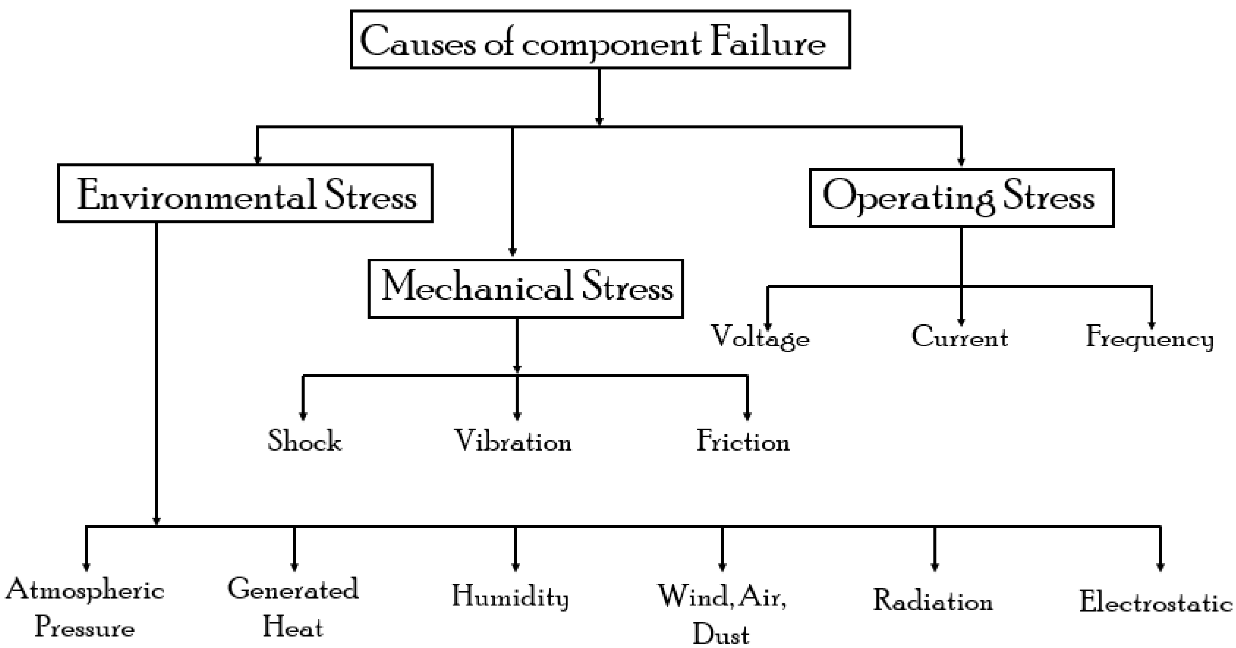
If a system with redundant units and Reliability $R(t)$ is repaired every T hour, then,

$$System\ MBTF = \frac{\int_0^{\infty} R(t)\, dt}{1 - R(t)}$$

where $R(t)$ is the reliability without repairs, $R(T)$ is the probability of no failure for T hours.

Chapter 3

Causes & Remedies of Component failure



Environmental Stress

These are due to factors such as weather, moisture, sea water etc. which are a function of the type of place where the equipment is being used. Equipment that are to be used outdoor are especially exposed to the full effects of weather, and are likely to experience environmental stress.

Effects of atmospheric temperature

It has been discovered that failure rate of an electronic component approximately doubles with every 10°C rise in temperature.

Fixed applied voltage

In general, extremes of temperature increase failure rate because of the following:

- **i.** The effects of expansion and contraction of materials with change in temperature.
- **ii.** Possible change in component value with temperature.
- **iii.** The increase in the rate of chemical reaction with temperature (corrosion etc.) between a component and a contaminant.
- **iv.** Melting, softening and freezing of some component material.

Remedy

For high temperature environments

- i. Forced ventilation by use of fans incorporated into the equipment
- ii. Use of components with low temperature coefficient of expansion.

For low temperature environment

- i. Indirect heating of the equipment
- ii. Correct choice of components

Effect of Generated Heat

Semiconductor devices that carry large currents are likely to generate a lot of heat under operation. This heat can cause failure of the component if care is not taken, leading to a system failure.

Remedy

i. Use of heat sinks
ii. Forced ventilation by fans
iii. Careful choice of components
iv. Reduction in generated heat by using derating concepts.

Effect of Humidity

Humidity can be very dangerous to component, especially when combined with high temperatures. This is because if a film of water vapor can be formed on a component as PCB surfaces it can become ionized to form a conducting path, hence creating a short circuit fault between the circuit. In addition, such a film or water can encourage the growth of fungi which is undesirable.

Remedy

i. Insulating materials such as silicon and polystyrene which do not absorb moisture can be used for equipment casing.
ii. Sensitive equipment or even entire circuit modules may be sealed up using varnish etc.

Effects of atmospheric pressure

Very low pressure (at high altitudes) can cause leaking of dielectric capacitors. This could be the fate of equipment transported by air in an unpressured casing.

Remedy

i. Use of pressured casing
ii. Dust free paths between conductors

Effects of air impurities (e.g. dust, dirt, sand, salt etc.)

i. Increased contact resistance for components, switches etc.
ii. The salinity of sea water may cause corrosion and promote degradation of insulation in an equipment.

Remedy

i. Periodic removal of dust by blowing.
ii. Sealing of component if necessary

Mechanical Stress

These are brought about by the effect of shocks, vibrations and friction on an equipment. This brings about weakening of equipment supports, loosening of wires and connections bent and possibly damaged components. However, such stresses have little effects on electronic components which are small in size. For additional security, such components can be protected by encasing them in epoxy resin etc. secure them down to the supports surface.

Heavy components such as transformers and electromechanical components are more liable to malfunction due to mechanical vibration shocks.

Remedy

i. Use of antivibration mountings, lucking nuts etc.
ii. Use of encapsulation for sensitive components.

Methods of reducing equipment/component failure

i. Careful part selection
ii. Use of improved technology (e.g. use of integrated circuits to replace discrete components)
iii. Use of redundancy
iv. Use of worst-case design
v. Use of derating concepts

   Derating is a process by which a component/equipment is operated below the rated (environmental, mechanical or operating) stress level of a parameter in order to reduce the component/equipment failure rate and consequently increase the reliability.

$$\boxed{\lambda_2 = \lambda_1 \left(\frac{V_2}{V_1}\right)^n \ K^{(t_2 - t_1)}} \longrightarrow \text{Arrhenius Law}$$

λ_1 = Failure rate at voltage V_1 and temperature t_1

λ_2 = Failure rate at voltage V_2 and temperature t_2

K and n are constants

To use the above law to solve problems

i. Let $V_2 = V_1 = V$. Keep the voltage constant. Vary the temperature from t_1 to t_2 and determine the new failure rate.

$$\lambda_2 = \lambda_1 (\frac{V_2}{V_1})^n \ K^{(t_2 - t_1)}$$

$$\lambda_2 = \lambda_1 K^{(t_2 - t_1)}$$

$$Log_e \ K = \frac{Log_e \ (\frac{\lambda_2}{\lambda_1})}{t_2 - t_1}$$

ii. For a constant temperature

$$\lambda_2 = \lambda_1 (\frac{V_2}{V_1})^n \ K^0$$

$$\lambda_2 = \lambda_1 (\frac{V_2}{V_1})^n$$

$$n = \frac{Log_e \ (\frac{\lambda_2}{\lambda_1})}{Log_e \ (\frac{V_2}{V_1})}$$

Worked Example

In a test to determine failure rate, three sets of observations under the following conditions

i. Failure rate for rated voltage and rated temperature 0.025
ii. Failure rate for rated voltage and twice rated voltage temperature 0.100
iii. Failure rate for half rated voltage and rated temperature 0.00125

If rated temperature is 30°C, calculate the probable failure rated at one-third rated voltage and two-third rated temperature.

Solution

Rated temperature = 30°C, twice rated temperature

=60°C, two-third rated temperature

$$\frac{2}{3} \times 30°C = 20°C$$

If $V_1 =$ rated voltage $= V$, $t_1 = rated\ temp. = t = 30°C$

$$\therefore V_2 = V, \qquad t_2 = 2t = 2 \times 30 = 60$$

$$\lambda_2 = \lambda_1 (\frac{V_2}{V_1})^n \ K^{(t_2 - t_1)}$$

$$0.100 = 0.025 (\frac{V_2}{V_1})^n \ K^{(60 - 30)}$$

$$0.100 = 0.025 \ K^{30}$$

$$K^{30} = 4$$

$$Log_e K = \frac{Log_e 4}{30}$$

$$K = 1.047$$

From test C, $V_3 = {}^{V}/_{2}$, $t_3 = t_1 = 30$

$$\lambda_3 = \lambda_1 (\frac{V_3}{V_1})^n \ K^{(t_3 - t_1)}$$

$$0.00125 = 0.025 (\frac{1}{2})^n \ K^{(30 - 30)}$$

$$\frac{0.00125}{0.025} = (\frac{1}{2})^n$$

$$Log_e(0.05) = n \ Log_e(0.5)$$

$$n = \frac{Log_e(0.05)}{Log_e(0.5)} = \frac{-2.996}{-0.693} = 4.323$$

$$\therefore\ \lambda_2 = 0.025(\frac{V_2}{V_1})^{4.323} \times 1.047^{(t_2 - t_1)}$$

For ${}^{1}/{}_{3}$ rated voltage and ${}^{2}/{}_{3}$ rated temperature

$$\lambda_2 = 0.025(\frac{1}{3})^{4.323} \times 1.047^{(20-30)}$$

$$\lambda_2 = 0.025 \times \frac{1}{3^{4.323}} \times \frac{1}{1.047^{10}}$$

$$\lambda_2 = 0.0001371$$

Note the decrease in failure rate from 0.025 (at rated temp. & voltage) to 0.0001371 (at $1/3$ rated voltage and $2/3$ rated temperature)

Chapter 4

Maintainability

Maintainability (m) is the probability that a unit or system will be restored to operational efficiency within a stated time provided that the prescribed procedures for maintenance are performed.

To achieve a high maintainability, the following factors among others ought to be considered:

i. Efficiency and skills of the maintenance crew
ii. Availability of spare parts, necessary tools and equipment
iii. The dependability of power supply etc.

Definition of terms

Maintenance: is a combination of any actions carried out to retain an item in normal operational standard, or in the event of failure, to restore it to the same.

Objectives of maintenance

i. To extend the useful life span of an item
ii. To increase the availability of equipment
iii. To ensure operational readiness of all equipment needed for emergency purposes
iv. To ensure the safety of the personnel involved

v. To avoid exorbitant costs which may be incurred on event of system failure

Mean time to repair (MTTR):

Is the mean of the time required to perform maintenance actions. Such maintenance actions may include fault localizations, fault isolation, fault correction and the removal, replacement or reassembly, alignment and adjustment and testing. The time for each of these operations is determined and the MTTR is given as:

$$MTTR = \frac{\sum n_i \lambda_i t_{mi}}{\sum n_i \lambda_i}$$

n_i = quantity of similar parts

λ_i = parts failure rate

t_{mi} = predicted maintenance operation time

Repair rate (µ):

This is the number of maintenance actions that can be carried out on a particular item per hour. It is the average number of items that can be restored to normal working condition per hour, assuming no time lag between repair jobs.

$$\mu = \frac{1}{MTTR}$$

Maintenance time constant (t_m):

It is the permissible repair time. The maintainability equation can be given as:

$$m = 1 - e^{-\mu t_m}$$

Worked Example

The average time to repair a computer keyboard is 2 hours. Determine the maintainability of the system for a time of $3\frac{1}{2}$ hours.

Solution

$$MTTR = 2\ hours$$

$$\mu = \frac{1}{MTTR} = \frac{1}{2} = 0.5$$

Maintainability, m

$$m = 1 - e^{-\mu t_m}$$

$$m = 1 - e^{-0.5 \times 3.5} = 0.826$$

Utilization Factor (U):

This is the ratio of the operating time (t_{op}) of system to the maintenance time (t_m , idle time t_{id}) (that is, the time lost due to administrative protocols etc.) and the operating time t_{op}.

$$U = \frac{t_{op}}{t_m + t_{id} + t_{op}}$$

Therefore, during a given time period e.g. a year, 10 years etc., the utilization factor is the fraction of that time for which the item was in operating condition.

Availability (A)

This is the probability that an item will perform its required function at a stated instance of time or over stated period of time. If the idle time is zero, then Availability is given as:

$$A = U_{max} = \frac{t_{op}}{t_m(min) + t_{op}}$$

$$But \quad t_{op} = MTBF$$

$$t_m(min) = MTTR$$

$$\therefore \; A = \frac{MTBF}{MTTR + MTBF}$$

Worked Example

A distribution transformer designed for continuous operation fails twice in a period of 200 days. The total time for repair is 7 days. Calculate:

i. MTBF (in days)
ii. MTTR (in days)
iii. Availability

Solution

Total operating time $= 200 - 7 = 193\ days$

Total failures $= \ 2$

$$MTBF = \frac{Total\ operating\ time}{number\ of\ failures} = \frac{193}{2} = 96.5$$

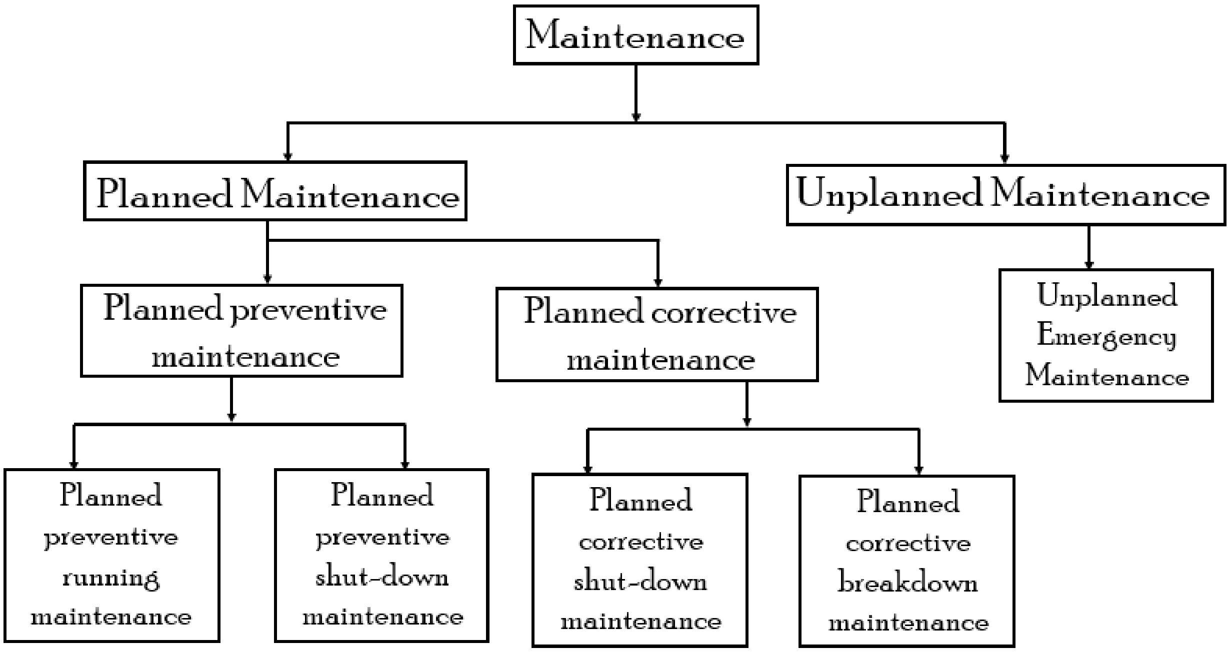
$$MTTR = \frac{Total\ time\ for\ repairs}{Total\ number\ of\ failures} = \frac{7}{2} = 3.5$$

$$A = \frac{MTBF}{MTTR\ +\ \ MTBF} = \frac{96.5}{3.5 + 96.5} = 0.965$$

Repairability

This is the probability that a unit or system will be restored to operational efficiency within a given "active repair time" if the recommended maintenance is carried out on it.

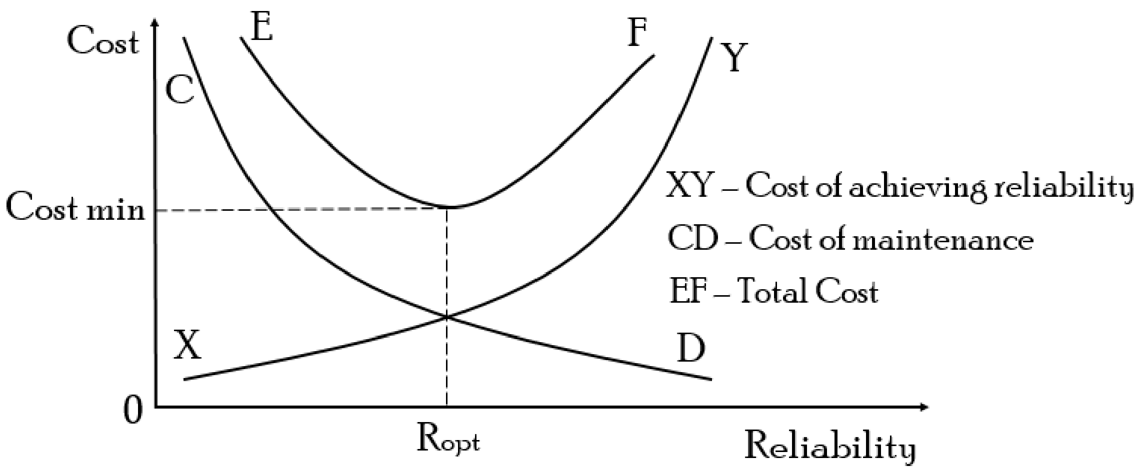
Classification of maintenance



Methods of improving maintainability

i. Provision of easily accessible maintenance check points
ii. Provision of suitable tools and test equipment that are easy to operate.
iii. Provision of adequate training for the maintenance personnel
iv. Provision of well written maintenance and repairs manual
v. Provision of adequate spare
vi. Provision of conducive environment for maintenance and repair

Relationship between reliability cost



Failure Reporting

This is the procedure by which an equipment designer is made to be aware of shortcomings in his design so that he can adopt a corrective measure in later designs.

Tutorial

1. A radio receiver has sub-units with their corresponding qualities, failure rates and average maintenance action times indicated against them as shown in Table 1 below:

Units	Quantity	Failure rate (%/10^3 hr)	Average Maintenance actual time (hr)
RF Amplifier	1	0.50	0.45
Oscillator	1	0.60	0.25
IF Amp	2	0.25	0.50
Detector	1	0.75	0.36
AF Amp	3	0.50	0.64
Power Supply	1	0.90	0.40

Using the data above, calculate for the radio receiver:

i. Mean time to repair (MTTR)

ii. The repair time.

Chapter 5

Specifications

Specification is a detailed description of the required characteristics of a device, component, equipment, system, product or process. Specifications are used by manufacturers in stating the characteristics of an end product which may be an equipment, a system etc.

Example of Specification of a function generator

RANGE: 0.001 Hz to 10Hz
STABILITY: 0.03% (after initial warm up)

OUTPUT:
Functions: Sine – distortion less than 1%
Square – rise time less than 3
Triangle – non-linearity less than 1 % of maximum amplitude
Amplitude: 4 mV to 20mV peak to peak
Impedance: 50 to 600

Uses of Specifications

i. They are used by manufacturers as an indication of the expected characteristics of a product.
ii. They are used as guidelines provided by the design department to control the manufacturing department of a plant.
iii. They are used to give buyers a basis for comparison between similar products, in the process of selecting that which is best suited for their needs.
iv. They act as a basis of contract agreement between the manufacturer and the buyer.

Typical items of information required in specifications

The example below shows item of information relevant to specification of measuring test instruments:

Range

i. Minimum and maximum measurable magnitudes

Accuracy

i. Provides information about maximum acceptable tolerances i.e. range of possible errors.

Input Characteristics

i. Specific value or range of values of information on sources or factors causing restriction on input impedance.

Output Characteristics

i. Type of display required e.g. digital or graphical
ii. Effect of frequency compensating probes, if used.

Stability

i. Maximum acceptable time between calibration
ii. Availability of built-in calibration facility

Power Input

i. Mains supply – r. m. s., amplitude value and frequency
ii. D. C. source stating

Reliability

i. MTBF
ii. Consequences of failure
iii. Special requirements on spares and maintenance

Frequency

i. Range
ii. Scale accuracy
iii. Frequency stability

Main Output

i. Waveforms
ii. Output impedance
iii. Minimum usable output

Chapter 6

Testing Methods

Categories of Testing

i. Prototype Testing
ii. Pre-production Testing
iii. Production Testing

i. Prototype Testing

Before the mass production of a design, a prototype is usually produced and thoroughly tested to prove a design will meet the specification. If such test reveals any inadequacies, the design is modified and tests repeated.

It is normal for a prototype to be tested by the reliability or the quality control department and the designer may be called to question for its performance and reliability. Three major errors may occur at this stage.

i. Design errors: prototype fails even though manufactured according to design.
ii. Manufacturing errors: the prototype was manufactured as designed, but some errors are incurred during manufacturing.
iii. Part faults: the prototype was manufactured as designed, but the parts are faulty

ii. Pre-production (Qualification) Testing

This is done after prototype testing before mass production to ensure the specifications can be met under normal manufacturing conditions. Such tests include environmental tests, reliability tests, maintainability tests, packaging and transport test, physical and electrical characteristics tests and ergonomics test.

iii. Production Testing

This is done on mass produced items to ensure they conform with the specifications.

Reliability demonstration test

This is a test which is normally carried out by a manufacturer to prove to a customer his item is as good as it claims to be. When a customer receives an untested item, he may decide to test it by himself to verify whether or not it satisfies its specifications. This test is called reliability acceptance test.