ISO 14298 : 2021

WEEK 1

19/09/2022

LECTURE 1

INTRODUCTION

* Note the recommended texts

- Concept: Design and maintenance of what was designed; achieving little or no failure, making predictions.
  - Quality control
  - Reliability || Maintainability || Availability
  - Calculations involved

26/09/2022

WEEK 2

LECTURE 2

BASIC RELIABILITY CONCEPTS

* RELIABILITY
  - A design engineering discipline
  - Scientific knowledge to assure a product will perform its intended function.
  - Required duration; Given environment.
  - Measure of Product performance over time; Time Component involved
  - Other design principles are involved [System archi; Processes; Material
    - Series, Parallel
    - Components

* ITS DEFINITIONS

⇒ - Perform a required function
- In a particular environment, under certain operational conditions
- For a specific period of time

⇒ A probability for performance without failure; where there're certain expectations.

* IMPORTANCE TO AN ENGINEER

- To maintain Reputation (for an organization)
- Customer satisfaction.
- Warranty Costs
- Repeat business
- Cost analysis
- Customer reliability.
- Competitive advantage

* BENEFICIARIES OF GOOD RELIABILITY

⇒ Manufacturer ; Customer
- Reliability related costs

* To achieve reliable products:

- Acknowledge its importance
- Quantitative assessments.

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- Communicate with the customers to capture their needs.
- Perform failure analysis, and document it.
- Project management

* GOAL

- To evaluate inherent reliability, and improve it.

* TERMINOLOGIES

- COMPONENTS (Parts of a system)
- MEAN LIFE

- Average life of the total number (n_0) of components under test, measured over the entire life curve

- FAILURE

- Inability of a component/system to perform its intended function, for a time, in a specific environment; under these conditions.

⇒ Inoperative

⇒ Operative but unable to perform the required function.

⇒ Unsafe for continuous use (causing a shock)

- RELIABILITY

Frozen of 1

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- FAILURE RATE (λ) [Percentage/10^3 hour]

- No of failures occuring per unit time.

$$\lambda_i = \lim_{\Delta t \to 0} \frac{1}{N_s} \times \frac{\Delta N_f}{\Delta t} = \frac{1}{N_s} \times \frac{dN_f}{dt}$$

N_s = Number of surving items after a life test

N_f = Number of failed items during the time interval

* EXAMPLE 1

$\frac{1}{96.0} \times \frac{50}{100}$

- 10 items have failed out of 1000 put on test during a period of 5000 hours. Calculate the failure rate

Solution

$$\lambda = \frac{1}{1000} \times \frac{10}{5000}$$

$= 2 \times 10^{-6}$ failures per hour

$$\Rightarrow \% \lambda = \frac{10}{1000} \times \frac{1}{5000} \times 100$$

$= 2 \times 10^{-4}$ percent per hour

$= 0.2$ percent / 10^3 hours

$$= \frac{1}{N_s} \times \frac{dN_f}{dt}$$

$$= \frac{1}{1000} \times \frac{10}{5000} \times 100$$

$$= \frac{1}{500000} \times 100$$

$= 2 \times 10^{-5} \times 100$

$= 0.002$ percent/hour

* FAILURE RATE OF COMPONENTS

- Capacitor :

- Diode

* Conditions that affect failure rate

- w_E, w_T, w_A [Weighting factors]

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- Overall failure rate of each component :

$$\lambda_o = \lambda_n w_E w_T w_R$$

- Total failure rate

$$\Rightarrow \lambda_T = \lambda_{o1} + \lambda_{o2} + \lambda_{o3} + \cdots + \lambda_{on}$$

* EXAMPLE 2

- Electronic system in question
- Works for 24 hours daily and continuously for 50 days

⇒ Components	Nos. used	λ_f % per 10^3 hours
Transistors	30	0.05
Diodes	4	0.05
Capacitors	100	0.01
Resistors	140	0.05
Connections	700	0.001

- w_E for all components = 2.0

- $w_E = 2.0$

$w_T = 1.5$

RATING	
Cap.	3.0
Res.	2.0
Trans.	2.0
Diodes	1.5

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- Construct a table showing all the data given, calculate:
  - λ_o
  - System failure rate
  - System reliability

SOLUTION

- THE TABLE

$\lambda_{oi} = n_i \lambda_i W_E W_T W_R$ (% /1000 hrs)

Components	Nos. used	λ_i % per 10^3 hrs	W_E	W_T	W_R	$\lambda_{oi} = n_i \lambda_i W_E W_R W_T$
Transistors	30	0.08	2.0	1.5	2.0	14.4
Diodes	4	0.05	2.0	1.5	1.5	0.9
Capacitors	100	0.01	2.0	1.5	3.0	9.0
Resistors	140	0.05	2.0	1.5	2.0	42
Connection	700	0.001	2.0	1.5		2.1

$\lambda_{oi} = n \lambda_i W_E W_T W_R$

⇒ Trans $(\lambda_{oi}) = 30 \times 0.08 \times 2 \times 1.5 \times 2 = 14.4$

⇒ Diodes $= 4 \times 0.05 \times 2 \times 1.5 \times 1.5 = 0.9$

$\lambda_{oi} = n_i \lambda_i W_E W_T W_R$

⇒ Capacitors $= 100 \times 0.01 \times 2 \times 1.5 \times 3 = 9$

⇒ Resistors $= 140 \times 0.05 \times 2 \times 1.5 \times 2 = 42$

⇒ Connection $= 700 \times 0.01 \times 2 \times 1.5 = 2.1$

System failure rate

⇒ $\lambda_T = 14.4 + 0.9 + 9 + 42 + 2.1$

$= 68.4\% / 10^3$ hrs or $0.534 / 10^3$ hrs

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For unrepairable parts

* STILL ON TERMINOLOGIES

- MEAN TIME TO FAILURE (MTTF): Survival time for all of the components divided by the no. of failures.
- MEAN TIME BETWEEN FAILURES (MTBF): Basic measure of system reliability. (In hours) ⇒ For repairable items [- Replacement - Non-replacement]

* EXAMPLE 5

(i) Total test hours before failure:

$= (6 \times 100) + (5 \times 140) + (4 \times 175)$

$= 2000$ hours

$MTBF = m = \frac{1}{\lambda}$ ⇒ Reciprocal of failure rate

(ii) Total test hours without failure

$= 35 \times 2000$

$= 7000$ hours

(iii) Total survival hours

$= 2000 + 7000$

$= 9{,}000$ hours

(iv) $MTBF = m = \frac{2000 + 7000}{50 - 35}$

$= \frac{9000}{15} = 600$ Hours (Faults per hour)

⇒ A test lasting only 200 hours was used to obtain an estimate of 600 hours as the MTBF.

- Let the seven units of a particular product be assumed to have been tested for a period of 8 hours

If three of the units fail after 15, 35 and 60 hours respectively, calculate the product failure rate.

* MTTF

$$MTTF = \frac{\sum_{1}^{N} t_i - t_0}{N}$$ ← starting (Reference time)

$$= \frac{(t_1 - t_0) + (t_2 - t_0) + \dots + (t_n - t_0)}{N}$$

Period to 1st failure; Period to 2nd failure; ← Period to Nth failure

Total N = No of failed components

* EXAMPLE 3

- Life testing is made on six (non-repairable) electrical lamps, and the following results were obtained as shown below; MTTF = ?

Failure	0	1	1	1	1	2
Times (100 Hrs)	$t_0 = 0$	$t_1 = 4$	$t_2 = 10$	$t_3 = 16$	$t_4 = 20$	$t_5 = 23$

Solution

$$\Rightarrow MTTF = \frac{(t_1 - t_0) + (t_2 - t_0) + \dots + (t_n - t_0)}{N}$$

$$= \frac{(4-0) + (10-0) + (16-0) + (20-0) + (2 \times 23 - 0)}{6}$$

$$= \frac{4+10+16+20+46}{6} = \frac{96}{6} \times \text{times} = \frac{96}{6} \times 100 = 1600 \text{ H}$$

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WEEK 4

LECTURE 3

10/10/2022

MATHEMATICAL RELATIONSHIP OF SOME BASIC TERMS

* RELIABILITY AND FAILURE RATE

$$N_0 = N_S + N_f \quad ; \quad N_S = N_0 - N_f$$

$$R_{(t)} = \frac{N_S}{N_0} \quad ; \quad \lambda_{(t)} = \frac{1}{N_S} \times \frac{dN_f}{dt}$$

$$\lambda_{(t)}\, dt = \frac{1}{N_0 - N_f} \times dN_f$$

2 $\quad \int x^n dt = \frac{x^{n+1}}{n+1} \quad ; \quad \int \frac{1}{1+x} dx = \ln x + c$

a $\quad \int \frac{1}{2x+5} dx = \ln 2x+5$

$ax+b$

$$-\int_0^t \lambda_{(t)} dt = \int_0^{N_f} \frac{1}{N_0 - N_f} dN_f$$

Negative

$$\Rightarrow -\lambda_{(t)} = \left[\log_e N_0 - N_f\right]_0^{N_f}$$

$$-\lambda_{(t)} = \left[\log_e (N_0 - N_f) - \log_e (N_0 - 0)\right]$$

$$\Rightarrow -\lambda t = \log_e \left[\frac{N_0 - N_f}{N_0}\right]$$

$$e^{-\lambda t} = \frac{N_0}{N_0} - \frac{N_f}{N_0}$$

$$= 1 - \frac{N_f}{N_0} \quad \text{--- (i)}$$

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$$R(t) = \frac{N_s}{N_0}$$

$$= \frac{N_0 - N_f}{N_0}$$

$$= \frac{N_0}{N_0} - \frac{N_f}{N_0}$$

$$= 1 - \frac{N_f}{N_0} \quad \text{--- (2)}$$

Comparing equ (1) and (2)

$$R(t) = e^{-\lambda t}$$

when $\lambda(t) = \text{Constant}$

⇒ If failure rate is not constant;

$$R(t) = e^{-\int_0^t \lambda(t)\,dt}$$

⇒ Bathtub curve

$$\lambda_t = -\frac{\partial R_t}{\partial t}$$

$$f(t) = \lambda e^{-\lambda t}$$

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- Derivative of reliability $(R(t))$ wrt time $\Rightarrow$ FAILURE DENSITY FUNCTION, $f(t)$

$$\Rightarrow f(t) = -\frac{dR_t}{dt} \quad ; \quad R(t) = e^{-\lambda t}$$

$$\Rightarrow f(t) = \lambda e^{-\lambda t}$$

- Poisson distribution

$$P_r = \frac{(\lambda_t)^r}{r!} e^{-\lambda t}$$

Our target is zero failure

$$\Rightarrow P_0 = \frac{(\lambda_t)^0}{0!} e^{-\lambda t}$$

$$= 1(e^{-\lambda t})$$

$$= e^{-\lambda t} = R(t)$$

Probability of zero failure

* RELATIONSHIP BETWEEN MTBF, MTTF, AND FAILURE RATE

$\Rightarrow$ MTBF (M) is

$$M = \int_0^\infty R(t)\, dt$$

$$\Rightarrow M = \int_0^\infty e^{-\lambda t}\, dt$$

$$M = -\frac{1}{\lambda}\left[e^{-\lambda t}\right]_0^\infty$$

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$$= \frac{1}{\lambda}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{-1}{\lambda}\left[0 - 1\right] \quad ; M = \frac{1}{\lambda} = MTBF$$

Mean Time To Failure $= \frac{1}{\lambda}$

⇒ For components that are repairable

$$\therefore \lambda = \frac{1}{MTBF} = \frac{1}{M}$$

$$R(t) = e^{-\lambda t} = e^{-t/M} \quad ; \lambda = \frac{1}{M}$$

- After one MTBF ; $t = M$, the probability of survival,

$$R(t) = e^{-1} = 0.37$$

* EXAMPLE

① A device has $\lambda(t) = 2 \times 10^{-6}$ failure/hr, what is the reliability for an operating period of 500 hrs?

Solution

$$\lambda = 2 \times 10^{-6}$$

$$t = 500 \text{ hrs}$$

$$R(t) = e^{-\lambda t}$$

$$= e^{-(2 \times 10^{-6})(500)}$$

$$= 0.999 \text{ [Highly reliable]}$$

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A CONTINUATION

2. There are 2000 items in the test, how many failures are expected in 500 hrs?

Assume that strict quality control has eliminated premature failures, so we can assume a constant failure rate.

Solution

$$R(t) = \frac{1 - N_f}{N_0} \; ; \; R(t) = 0.999$$

$\frac{1}{2}$

$$\Rightarrow 0.99 = \frac{1 - N_f}{2000}$$

$$\Rightarrow 0.999 = \frac{2000 - N_f}{2000}$$

$$1998 = 2000 - N_f$$

$$N_f = 2000 - 1998$$

$$N_f = 2 \text{ failures}$$

* RELIABILITY EQUATIONS AND CURVES WHEN FAILURE RATE IS CONSTANT.

- In this case ; $R = e^{-\lambda t}$

Probability of No failures occuring at a time

$$\Rightarrow R + Q = 1$$

$$Q = 1 - R = 1 - e^{-\lambda t}$$; Q is unreliability

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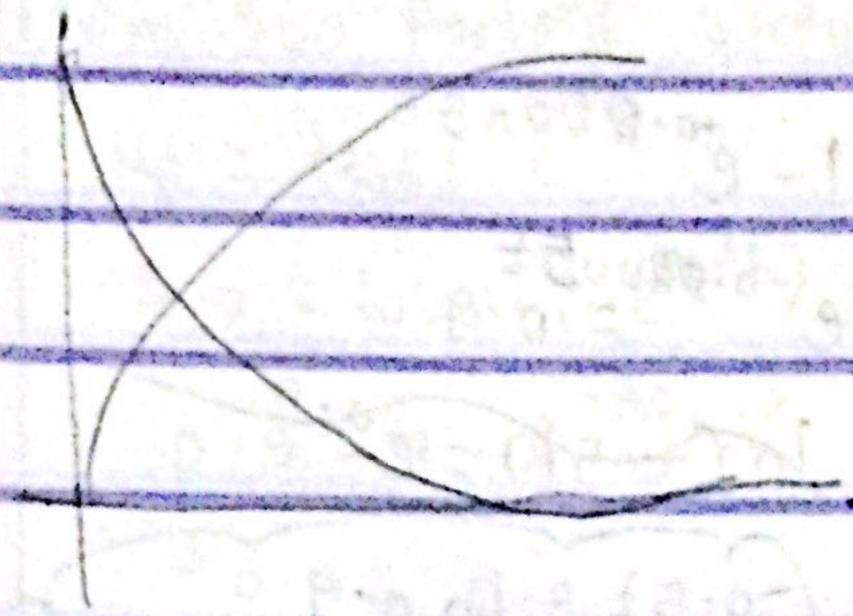
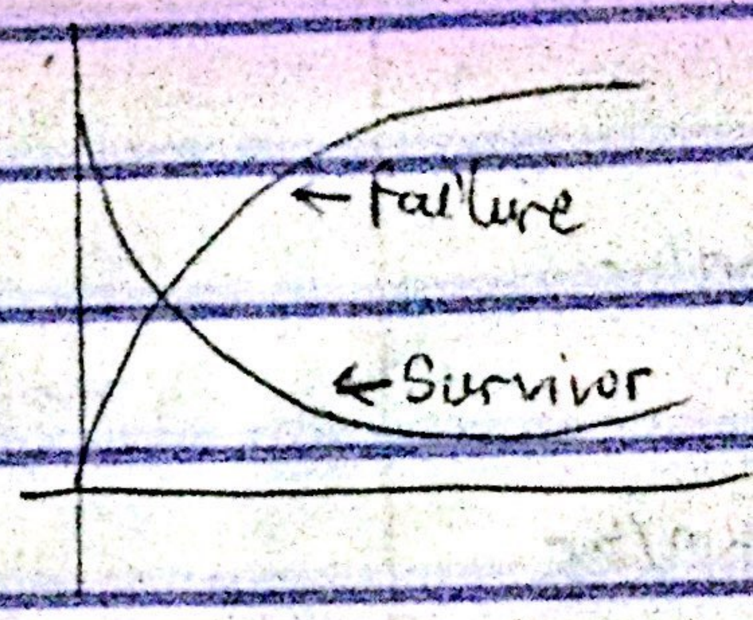


$t \to 0$, $R \to 1$ and $Q \to 0$

$t = \frac{1}{\lambda}$, $R = 0.37$ and $Q \to 0.63$

* SURVIVOR AND FAILURE EQUATION AND CURVES

$$R = 1 - N_f = e^{-\lambda t}$$



N_0

$N_s = N_0 - N_f$

$$\Rightarrow N_s = N_0 e^{-\lambda t}$$

$$N_f = N_0 - N_0 e^{-\lambda t}$$

$$N_f = N_0 (1 - e^{-\lambda t})$$

$$R = \frac{N_s}{N_0} = e^{-\lambda t}$$

$$Q = 1 - e^{-\lambda t} = \frac{N_f}{N_0}$$

* EXAMPLE

One thousand similar components each of constant failure of $5\% / 10^3$ hrs are put into test together. Calculate the time lag before failure of the following number of components.

(i) 100 (ii) 500

Solution

$$\lambda = 5 \times \frac{1}{1000}$$

~~= 0.5 failures/hour~~ 5×10^{-5} failures/hr

$$= Q = 1 - e^{-0.5t} = \frac{N_f}{N_0} \qquad Q = 1 - e^{-0.00005t} = \frac{N_f}{N_0}$$

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$\frac{N_f}{N_0} = 1 - e$ $\qquad 0.9 = e^{-0.00005t}$

$\frac{100}{1000} = 0.1$ $\qquad 0.1 = 1 - e^{-0.00005t}$

$e^{-0.00005t} = 0.9$

$-0.00005t = \ln(0.9)$ $\qquad \ln(-0.5t) = e^{0.9}$

$\Rightarrow -0.00005t = -0.1054 \Rightarrow$ $\quad -0.5t = \ln 0.9$

$t = \frac{-0.1054}{-0.00005}$ $\qquad -0.5t = -0.1054$

$= 2{,}108$ seconds $\qquad t = 0.21$ seconds

$= 0.590$ hrs $\qquad N_0 = 1000$

$N_f = 100$

$N_f = N_0 - N_0 e^{-\lambda t}$

$N_s = 1000 - 100$

$= 900$

$\Rightarrow 900 = 1000 e^{-\lambda t}$

$\Rightarrow 0.9 = e^{-\lambda t}$

$\ln(0.9) = -\lambda t$

$\ln(0.9) = -0.00005t$

$t = 0.56$ hrs

$\frac{5}{100} \times 1000$

$= \frac{5}{100} \times \frac{1000}{1000}$

$e^{-\lambda t}$

km/hr

$\frac{50 \times 1000}{3600}$

$N_f = N_0(1 - e^{-\lambda t})$

$100 = 1000(1 - e^{-\lambda t})$

$\Rightarrow 0.1 = 1 - e^{-\lambda t}$

$\Rightarrow 0.9 = e^{-\lambda t}$

$\ln(0.9) = -\lambda t$

$\Rightarrow \ln(0.9) = -0.00005t$

$\Rightarrow -0.10536 = -0.00005t$

$t = \frac{-0.10536}{-0.00005} = 2{,}107$ s

$= 0.58$ hrs

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⇒ For 500 failed components;

$N_f = N_0(1-e^{-\lambda t})$

$500 = 1000(1-e^{-\lambda t})$

$0.5 = 1 - e^{-\lambda t}$

$\Rightarrow 0.5 = e^{-\lambda t}$

$0.5 = -0.00005t$

$\ln(0.5) = -0.00005t$

$\therefore t = \dfrac{\ln(0.5)}{0.00005}$

$= 13802.94$ hrs

$N_s = 1000 - 500$

$= 500$

$N_s = N_0 e^{-\lambda t}$

$500 = 1000 e^{-0.00005t}$

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* FAILURE PATTERN OF EQUIPMENT

Ms - Products are designed to fit into Constant failure rate part of the BATHTUB curve

- The early failure curve.
- Random failure curve
- Wear out failure curve

⇒ Take note of the diagram of the bathtub curve.

* FAILURE REASONS

- Design faults
- Wrong usage
- Installation fault

* CONSTANT FAILURE PERIOD

- Gamma distribution
- Poisson distribution
- Normal distribution

* DIFFERENCE BETWEEN RELIABILITY AND QUALITY

- R(t) i Function of time and environment
  Quality isn't
- R(t) i Product of good design , Quality i Product of good manufacturi

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* RELIABILIY OF COMPUTERS

- Financial implications of reliability-

  Costs & R(t) $\propto$ Production costs

  " $\propto$ Design & Development costs

* FAILURE

- Causes

  - Misuse

- Inherent weakness failure
- Time of failure i Degree of failure
- Combination of failure.

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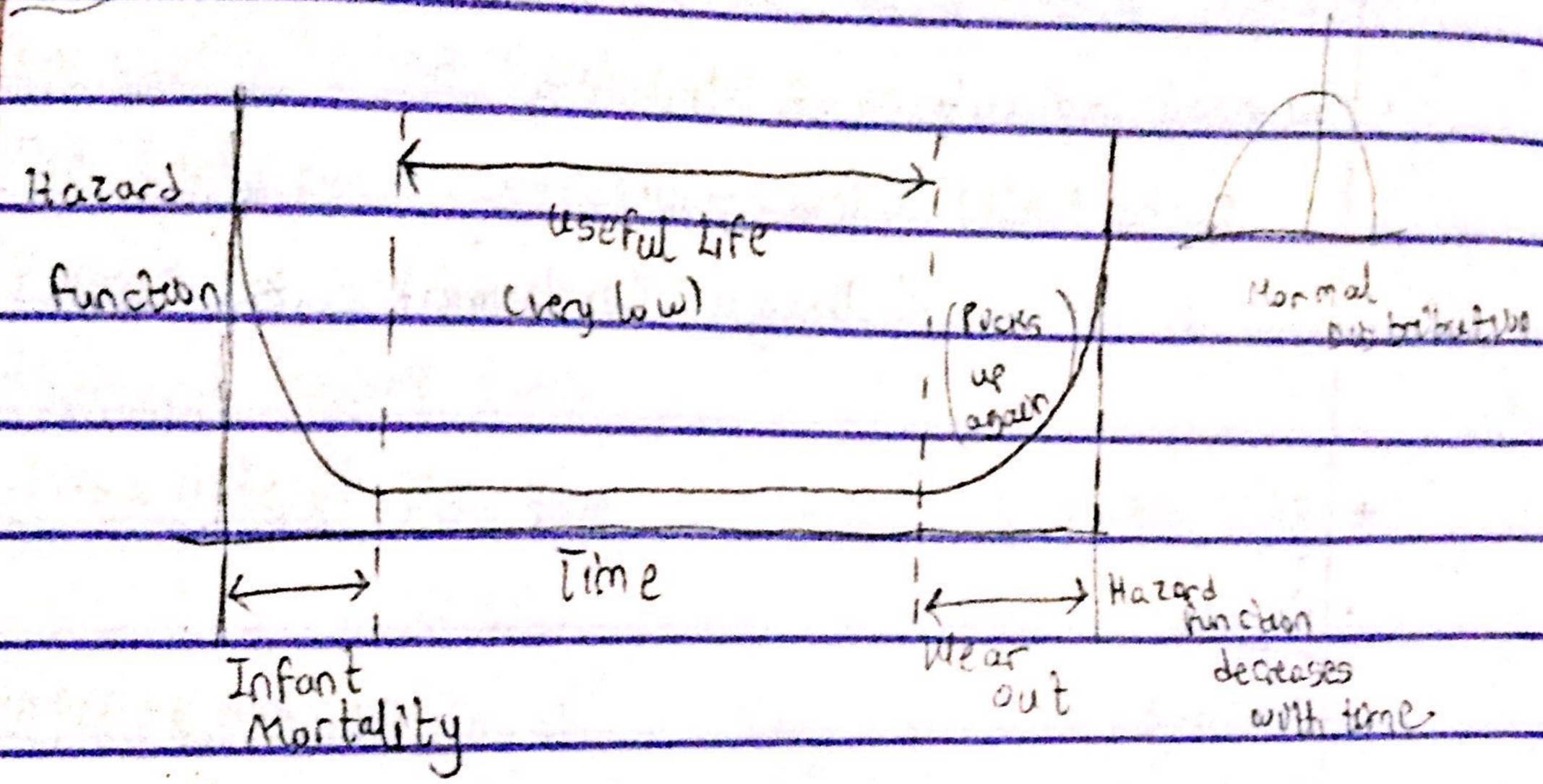
17/10/2022

LECTURE 4

* BATHTUB CURVE



$h = e^{-\lambda t}$

- Representation of reliability performance
- High hazard function ⇒ Failure rate
- Hazard function is constant at a point

* LIFE DISTRIBUTIONS

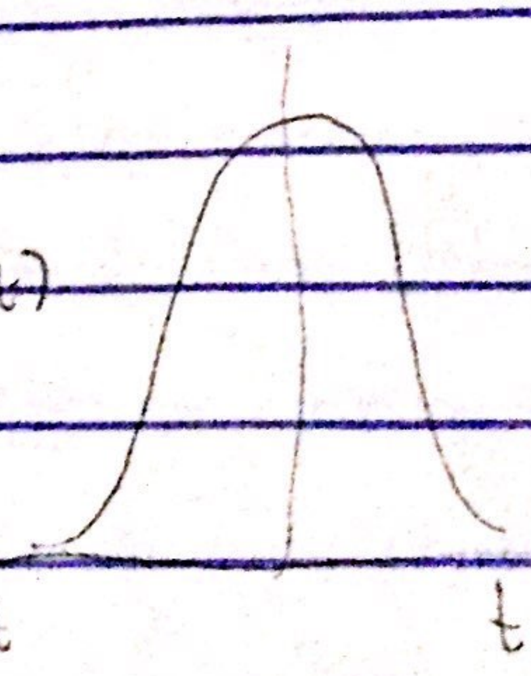
- At 5 hours, ? At 6 hours, ? At 22 hours, ?
- Practical examples of devices being tested before deploying.
- No of failures with time.
- The distribution is PDF

⇒ PDF ; CDF ; Hazard function

↓

Failure that can happen at any time

PDF ⇒ f(t)



$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} f(x)\,dx$$

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- x changes to t (No of failures wrt time)

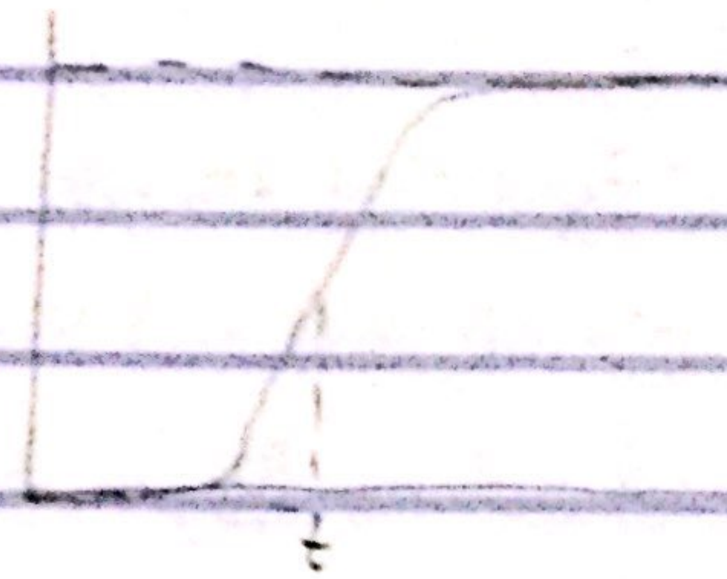
$f(x) \Rightarrow f(t)$

$\Rightarrow \lambda e^{-\lambda t}$

$\Rightarrow$ CDF, $F(t)$: Probability tht the measured value will fail bet $-\infty$ to t

$$F(t) = \int_{-\infty}^{t} f(t)\,dt$$

$\Rightarrow 1 - e^{-\lambda t}$

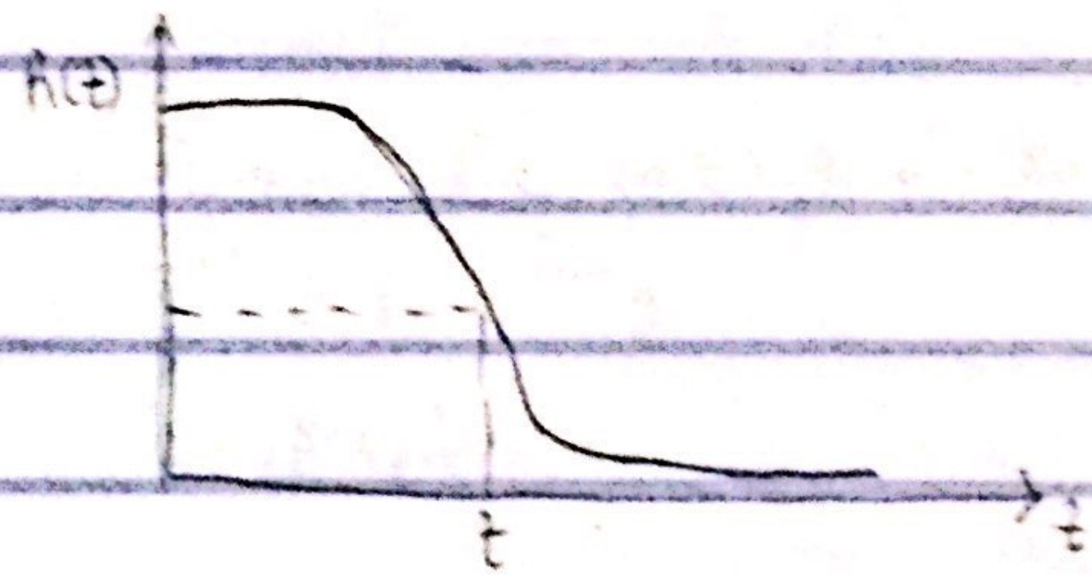


- Survival function

$$R(t) = 1 - F(t) = \int_{t}^{\infty} f(t)\,dt = 1 - \int_{-\infty}^{t} f(t)\,dt$$

$e^{-\lambda t}$

$\Rightarrow$ Thus, CDF can be termed as unreliability



$(x > Y)$

$\Rightarrow 1 - P(x < Y)$

$\Rightarrow$ HAZARD FUNCTION [h(t)]

$f(t) = \lambda e^{-\lambda t}$ [PDF] $-\infty$ to ∞

$F(t) = 1 - e^{-\lambda t}$ [CDF] $-\infty$ to t

$R(t) = e^{-\lambda t}$ [Reliability]

$$h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)}$$

Conditional probability of failure in the interval t to $(t + dt)$, given that no failure has occured by t

$$h(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

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* PARTICULAR LIFE DISTRIBUTIONS

- Wear out questions are based on normal distribution

~~Weibull Distribution~~

- The exponential ; Weibull ; Lognormal distributions

• Exponential models the useful life portion of the bath-tub curve

- Used when $h(t)$ is constant $[h(t)=\lambda]$ $\beta=1$

$$\frac{f(t)}{R(t)} \qquad h(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

- For repaired items, λ is failure rate.
- $1/\lambda = MTBF$, θ

- $t=0$; 63.2% of items will have failed

$$e^{-\lambda t} \; ; \; e^{-\lambda \times \frac{1}{\lambda}} = e^{-1} = 0.3679$$

$$Q(t) \Rightarrow 1-e^{-1} = 0.632$$

* Calculate the reliability when $t = MTBF$

$$R = e^{-\lambda t} \; ; \; t = \frac{1}{\lambda}$$

$$e^{-\lambda \times \frac{1}{\lambda}} = e^{-1}$$

$$= 0.3678$$

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- **Weibull** $\beta < 1$ $e^{-(t/\eta)^{\beta}}$

- Used for non-constant $h(t)$.
- Survival function

$$R(t) = \underset{(Exp)}{e}^{-(t/\eta)^{\beta}}$$

⇒ Can work for a constant function only when $\beta = 1$, and $\eta = \frac{1}{\lambda}$

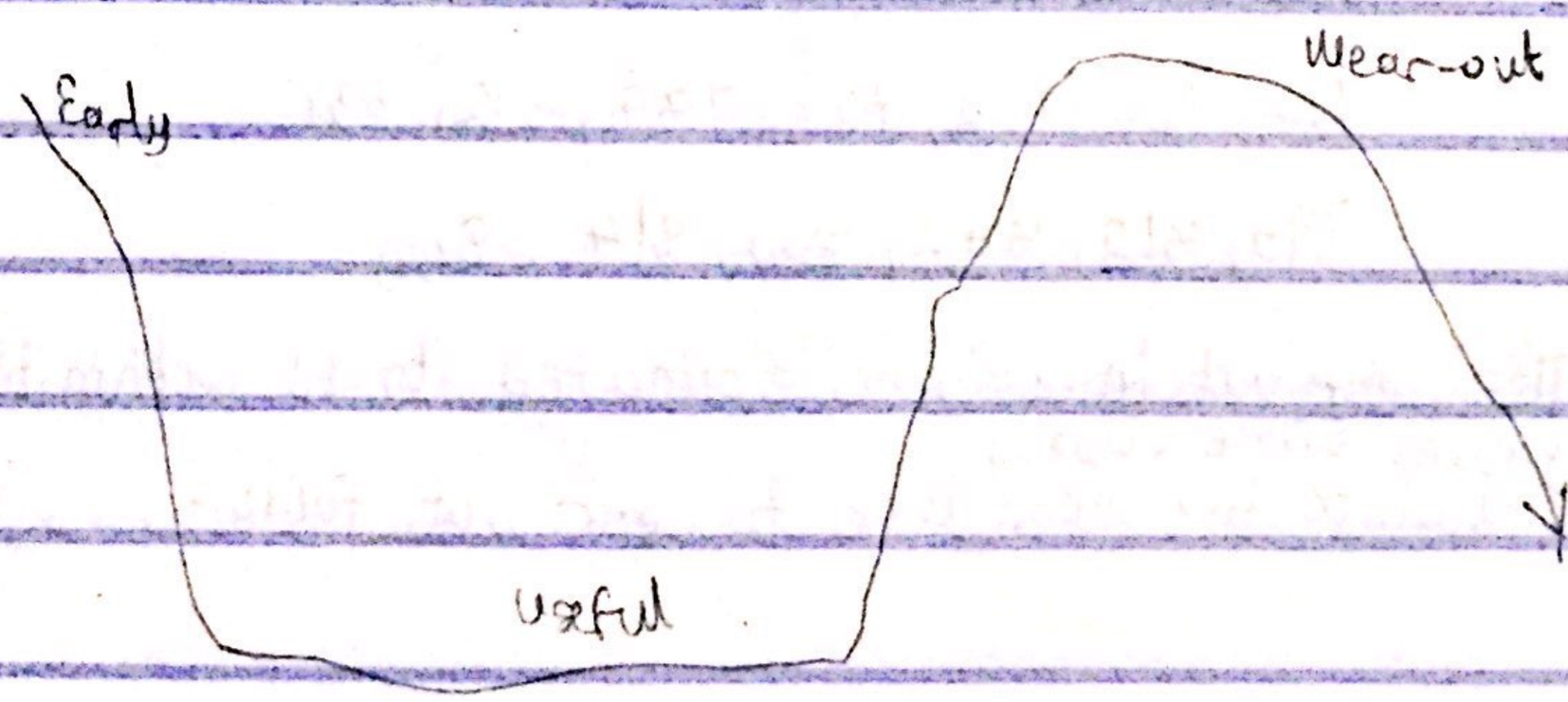
$$R(t) = e^{-(t \times \lambda)^{1}}$$
$$= e^{-\lambda t}$$

- Else, you can use it for early life ($\beta < 1$) and wear out period ($\beta > 1$)
- When $\beta > 3.5$, the Weibull distribution is an approximation for the Normal distribution.
- 3 parameters ⇒ t, η & β

- **Lognormal**

- A point where you can predict wear out of a component.
- Hazard function is increasing.

* MATH RELATION DUE TO WEAR OUT



- Failures due to wear out are clustred around the Mean, μ.

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$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} f_i (t_i - M)^2}{n}}$$

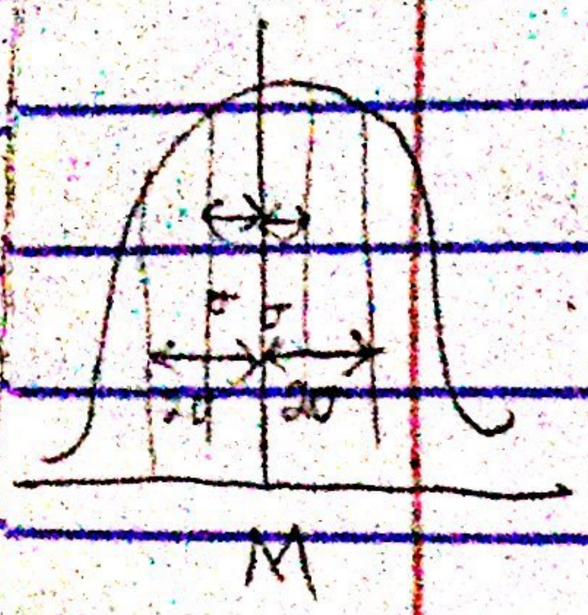
$$M = \frac{t_1 + t_2 + t_3 + \dots + t_s}{n}$$

(wear-out life)

$t_1, t_2, t_3 \dots$ → time to each wear out failure

t_s = Total time on test of all components which failed from wear out

n = Total no of components which failed from wear out



68% occurs when M is $\pm \sigma_m$

δ_m

* CONFIDENCE LIMITS

- σ ⇒ Normal SD
- σ_m = SD for means distribution

$$\Rightarrow \delta_m = \frac{\delta}{\sqrt{n}}$$

* EXAMPLE

- 30 components are put on accelerated life test.
- Their failures occur at the times:

714, 760, 802, 828, 788, 812, 831

795, 812, 842, 800, 814, 860

- The wear out time was estimated to be within this range of values. [close values]
- Calculate the mean time to wear out failure.

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$$M = \frac{t_1 + t_2 + t_3 + \cdots + t_{13}}{13}$$

$$= \frac{10454}{13}$$

$$= 804.46$$

If 714 is removed

$$\Rightarrow \frac{9744}{13}$$

$$= 749.54$$

$M + 2\sigma_m$

95% ↑ PRACTICE

No of failed components	2	1	4	6	3
Life per components (hrs)	800	1000	1200	1300	1500

$M + \sigma_m$

68%

(i) Mean wear out life:

$$M = \frac{t_1 + t_2 + t_3 + \cdots t_n}{n}$$

$$= \frac{(800 \times 2) + (1000 \times 1) + (4 \times 1200) + (6 \times 1300) + (3 \times 1500)}{2 + 1 + 4 + 6 + 3}$$

$$= \frac{1600 + 1000 + 4800 + 7800 + 4500}{16} = \frac{19700}{16}$$

$$= \cancel{1456.25} \quad = 1231.25$$

$n = 91$ 12th

$N_0 = 1000$

$N_s = 950$

$N_f = 50$

$\lambda = ?$

$$\frac{N_s}{N_0}$$

$$e^{-\lambda t} = 1 - \frac{N_f}{N_0}$$

$$e^{-\lambda(100)} = 1 - \frac{50}{1000}$$

$$-100\lambda = \ln(0.95)$$

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$\bar{x} = 1231.25$

f	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
2	-431.25	185976.5625	371953.125
1	-231.25	53476.5625	53476.5625
4	-31.25	976.5625	3906.25
6	68.75	4726.5625	28359.375
3	268.75	72226.5625	216679.6875

51.325 674375

$$\sigma = \sqrt{\frac{674375}{16}}$$

205

$$= 205.3$$

$$\Rightarrow \delta_m = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{205.3}{\sqrt{16}}$$

$$= 51.325$$

$$\Rightarrow M \pm 2\sigma_m$$

$\Rightarrow$ upper limit

$$1231.25 + 2(51.325)$$

$$= 1333.9$$

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ASSIGNMENT

* A transmission line fails on average 2 times per year. If the number of failures can be described by Poisson's distribution;

(i) What is the probability of having 2 failures in 5 years?

(ii) What is the probability of having 3 failures in 10 years?

$$P(K) = \frac{(\lambda t)^K e^{-\lambda t}}{K!}$$

$\int_{-\infty}^{t}$ 24/10/2022

WEEK 6

68% $\mu \pm \sigma$

LECTURE 5

CONCEPT OF RELIABILITY PREDICTION

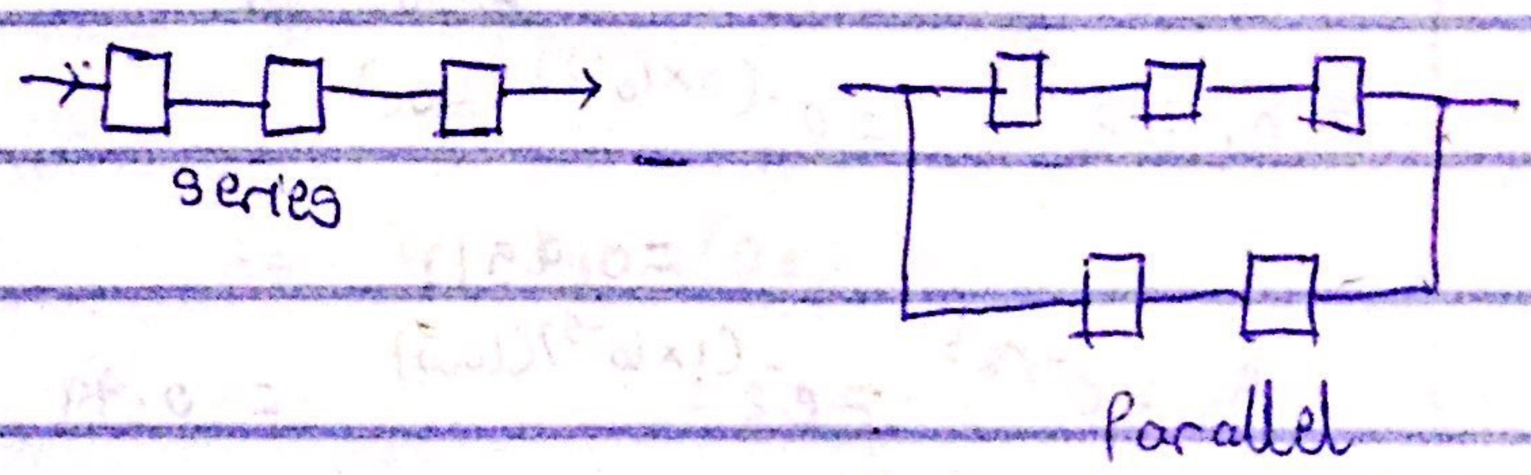
95% $\mu \pm 2\sigma$ To calculate reliability of a system that's made up of sub-units.

99.7% * SYSTEM TYPES

$\mu \pm 3\sigma$ - Series ~~or~~ i The failure of one affects the rest

- Parallel

- Series-parallel system.



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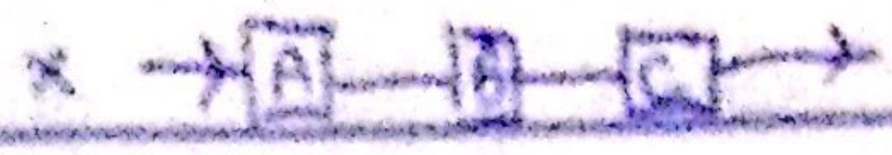
...ERED RELIABILITY SYSTEM

$R_s = R_A \cdot R_B \cdot R_C$; Each subunit has its own reliability



$R_A = e^{-\lambda_1 t}$, $R_B = e^{-\lambda_2 t}$, $R_C = e^{-\lambda_3 t}$, ..., $R_N = e^{-\lambda_N t}$

$\Rightarrow R_s = e^{-\lambda_1 t} \times e^{-\lambda_2 t} \times e^{-\lambda_3 t} \times \dots \times e^{-\lambda_N t}$

$e^{-(\lambda_1 + \lambda_2 + \lambda_3 \dots \lambda_N)t}$

$R_s = e^{-\lambda_s t}$

system failure rate.

EXAMPLE

3 subunits

R, $R_s = ?$, $t = 1000$ hours

$MTBF_1 = 20000$ hours, $MTBF_2 = 100,000$ hours

$MTBF_3 = 50,000$ hours

$\Rightarrow m = \frac{1}{\lambda}$; $\lambda = \frac{1}{m}$

$\lambda_1 = \frac{1}{m_1} = \frac{1}{20000} = 5 \times 10^{-5}/hr$

$\lambda_2 = \frac{1}{60,000} = 1 \times 10^{-5}/hr$

$\lambda_3 = \frac{1}{50,000} = 2 \times 10^{-5}/hr$

$\Rightarrow R_1 = e^{-\lambda_1 t} = e^{-(5 \times 10^{-5})(1000)}$

$= 0.9512$

$R_2 = e^{-\lambda_2 t} = e^{-(1 \times 10^{-5})(1000)} = 0.99$

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$R_3 = e^{-\lambda_2 t} = e^{-(2\times10^{-5})(1000)}$

$= 0.9802$

$\Rightarrow R_s = R_1 \cdot R_2 \cdot R_3$; $R_s = e^{-\lambda_s t}$ $= 0.9230$

$\lambda_s = \lambda_1 + \lambda_2 + \lambda_3$

$= 0.9512 + 0.99 + 0.9802$

$= 2.9214$

$R_s = e^{-(2.9214)(1000)}$

$=$

② 2 sub units in series

- $MTBF_1 = 5000$ hours
- $MTBF_2 = 8000$ hours

(a) $\lambda_1 = \frac{1}{m_1} = \frac{1}{5000} = 2\times10^{-4}/hr$

$\lambda_2 = \frac{1}{m_2} = \frac{1}{8000} = 1.25\times10^{-4}/hr$

(b) MTBF for the complete system ;

$m_s = \frac{1}{\lambda_s}$

$\lambda_s = \lambda_1 + \lambda_2 = (2+1.25)10^{-4}$

$= 3.25\times10^{-4}/hr$ $= 32.5\times10^{3}$ hrs

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$$m_s = \frac{1}{3.25\times10^{-4}} = 3076.923 \text{ hours}$$

(c) $R(t)$ for 14 days of continuous operation of the system

$$\Rightarrow R(t) = e^{-\lambda_s t}$$

$$= e^{-(3.25\times10^{-4})(14\times24 \text{ hours})}$$

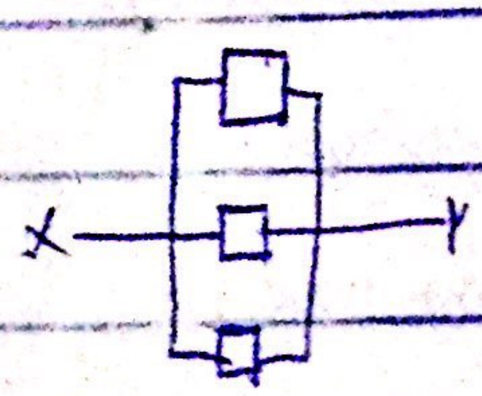
$$= 0.8966$$

* PARALLEL RELIABILITY SYSTEM

- Much more reliable, as the system doesn't fail until all the components have failed.

$$Q_p = Q_A \cdot Q_B \cdot Q_C$$

$$\Rightarrow 1 - R_p = (1-R_A)\cdot(1-R_B)\cdot(1-R_C)$$



$\Rightarrow R_p = 1 -$ (Probability that all modules have failed)

$$\Rightarrow R_p = 1 - [(1-R_A)\cdot(1-R_B)\cdot(1-R_C)]$$

* EXAMPLES

- Compare the reliability of a series system with a parallel system
- 3 sub-units with ; $R_1 = 0.95$, $R_2 = 0.85$, $R_3 = 0.75$

$$\Rightarrow R_s = R_1 \cdot R_2 \cdot R_3$$

$$= (0.95)\cdot(0.85)(0.75) = 0.6056$$

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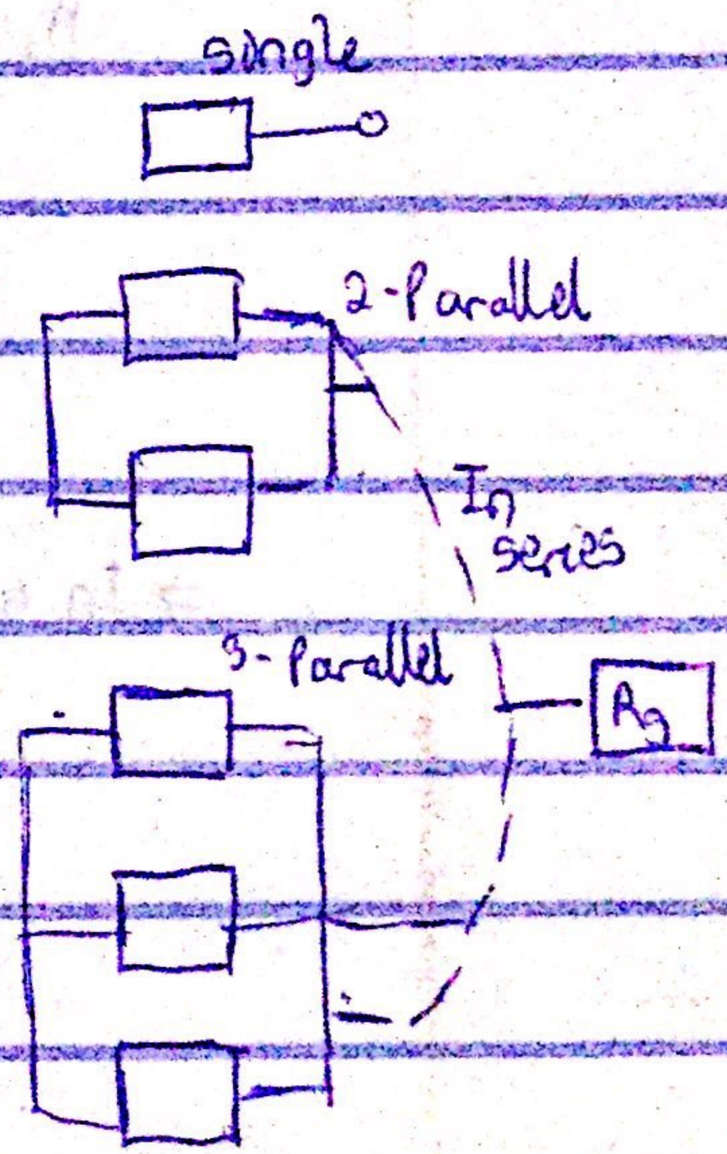
$$R_p = 1-[(1-R_1)\cdot(1-R_2)\cdot(1-R_3)]$$
$$= 1-[(1-0.95)\cdot(1-0.85)\cdot(1-0.75)]$$
$$= 1-[(0.05)(0.15)(0.25)]$$
$$= 1-0.001875$$
$$R_p = 0.998$$

⇒ The parallel system has a higher reliability than the series system ; $0.998 > 0.6056$

(2). Complex comm. satellite

- MTTF = 40,000 hours
- The link is operative if one channel is working
- $R_{switching\ unit} = 0.95$
- Reliability for one year = ?



(a) For a single channel

⇒ $t = 1$ year

$= 365 \times 24$

$= 8760$ hours

$R_1 = e^{-\lambda t}$; $\lambda = \frac{1}{40000} = 2.5\times10^{-5}$/hrs

$R_1 = e^{-(2.5\times10^{-5})(8760)}$

$= 0.8033$

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(b) Two parallel channels

$$R_p = 1-[(1-R_1)\cdot(1-R_2)]$$

$$\Rightarrow R_1 = R_2 = R$$

$$R_p = 1-[(1-0.8033)(1-0.8033)]$$

$$R_p = 1-[0.1967]^2$$

$$= 0.9613$$

The two channels are in series with R_s

$$\Rightarrow R_p \cdot R_s \quad , R_s = 0.95$$

$$= 0.9613 \times 0.95$$

$$= 0.9132$$

(c) Three parallel channels

$$R_1 = R_2 = R_3 = R$$

$$\Rightarrow R_p = 1-[(1-R)^3]$$

$$= 1-(0.1967)^3$$

$$= 0.9924$$

$\Rightarrow$ In series with R_s

$$\Rightarrow R_p \cdot R_s$$

$$= 0.9924 \times 0.95$$

$$= 0.9428$$

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* SERIES-PARALLEL

- 3 reliable states:

⇒ A not reliable, B & C are reliable

$$(1-R_A)R_BR_C$$

⇒ B not reliable, A & C are reliable

$$R_A(1-R_B)R_C$$

⇒ All modules are reliable

$$R_AR_BR_C$$

$$\Rightarrow R_{SP} = (1-R_A)R_BR_C + R_A(1-R_B)R_C + R_AR_BR_C$$

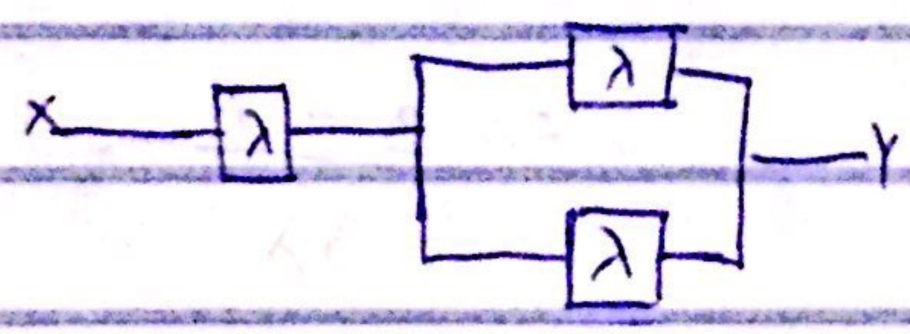
$$= R_BR_C - R_AR_BR_C + R_AR_C - R_AR_BR_C + R_AR_BR_C$$

$$R_BR_C - R_AR_BR_C + R_AR_C$$

$$= R_C(R_A + R_B - R_AR_B)$$

$$R_{SP} = R(2R - R^2) = 2R^2 - R^3$$

* EXAMPLE



- MTBF = ?

Solution

- Reliability of the parallel blocks

$$\Rightarrow R_P = 1 - [(1-R)(1-R)] = 2R - R^2$$

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- Reliability of the series block, $R = e^{-\lambda t}$

$\Rightarrow$ Reliability of the whole system

$$\Rightarrow R_T = R(2R - R^2)$$

$$= 2R^2 - R^3$$

MTBF of the system

$$M_T = \int_0^\infty [2R^2 - R^3]\,dt$$

$$= \int_0^\infty [2e^{-2\lambda t} - e^{-3\lambda t}]\,dt$$

$$= \left[-\frac{2}{2\lambda} e^{-2\lambda t}\right]_0^\infty - \left[-\frac{1}{3\lambda} e^{-3\lambda t}\right]_0^\infty$$

$$= \frac{-2}{2\lambda}[e^\infty - e^0] - \left(\frac{-1}{3\lambda}(e^\infty - e^0)\right)$$

$$= \frac{-1}{\lambda}(-1) - \left(\frac{-1}{3\lambda} \times -1\right)$$

$$= \frac{1}{\lambda} - \frac{1}{3\lambda}$$

$$= \frac{3-1}{3\lambda}$$

$$= \frac{2}{3\lambda}$$

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WEEK 7

31/10/2022

LECTURE 6

LAWS OF PROBABILITY RELEVANT TO RELIABILITY PREDICTION

- Multiplication rule (Series systems) ⇒ $P_{ab} = P_a \times P_b$
- Addition rule (Parallel systems)

* BINOMIAL DISTRIBUTION

- Electric generating set with a probability of 0.95 for a single day.
- Expected to operate for 5 days (Not continuously)

⇒ Two possible outcomes: Success [operating] or Failure [Not operating]

- Success = p ; Failure = q

$$p+q=1 \quad ; \quad q=1-p$$

⇒ No of trials = No of days in operation. ; $n=5$

- $(p+q)^n = 1 \Rightarrow (p+q)^5 = 1$

⇒ Using Pascal's triangle

```
                1                0
              1   1              1
            1   2   1            2
          1   3   3   1          3
        1   4   6   4   1        4
      1   5  10  10   5   1      5 ←
```

$$(p+q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$$

successful operation for all 5 days ↗ ↖ No success for all 5 days

- $5p^4q$ = 4 successful days of operation, 1 day without sucess

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* Possibility of successful operation for:

(i) 3 days

$\Rightarrow 10p^3q^2$; $p=0.95$

$q=1-0.95 = 0.05$

$\Rightarrow 10(0.95)^3(0.05)^2 = 0.0214$

(ii) For at least 4 days (i.e., 4 & 5 days)

$\Rightarrow p^5 + 5p^4q$

$= (0.95)^5 + 5(0.95)^4(0.05)$

$= 0.977$

$\Rightarrow (p+q)^n = p^n + np^{n-1}q + \frac{n(n-1)}{2!}p^{n-2}q^2$

more than
part must
if the who
must wo

$\Rightarrow p+q=1, (p+q)^n = 1$

- useful for finding the reliability of systems with partial redundancy

* REDUNDANCY

- Parallel arrangement of system modules, and is used in critical applications where very high reliability is required.
- Definition: The provision of more than one means of getting an item to perform a given function.

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- Advantages

- Aids planned preventive maintenance.
- Redundant element allows for repair, with no down time.

- Limitation

- Higher weight, space, complexity, cost, maintenance, design time.

* CLASSIFICATION

Redundancy Technique

- Active ← All units are powered at the same time
  - Full → more than one part must function if the whole system must work.
  - Partial ← Binomial → Less units have to function than in full
  - Conditional
- One unit powered at a time → Passive Standby
  - Non-operating
  - Operating

- Active

- Eg. Connection of 3 altimeters to measure plane height from ground.

- Standby

- Eg. Power system in an operating theatre.

* EXERCISES

(1) Unreliability of an aircraft engine during flight is 0.01 = q

- Reliability of successful flight if the aircraft can complete the flight

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on at least 3 of its 4 engines = q

Solution

- No. of engines, n = 4
- $q = 0.01$; $p = 1 - 0.01 = 0.99$

$(p+q)^n = 1$ | $(p+q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$

⇒ At least 3 engines are needed for a successful flight

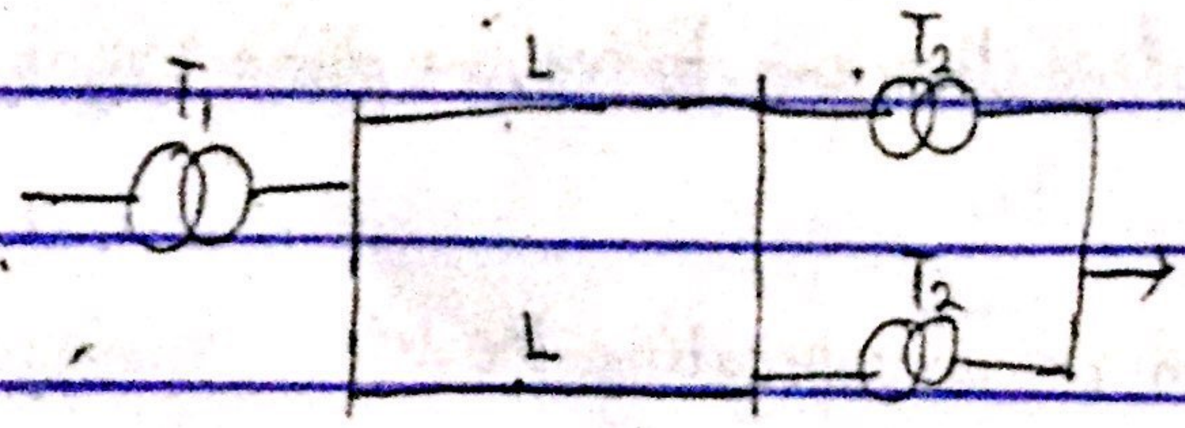
$$\Rightarrow p^4 + 4p^3q = (0.99)^4 + 4(0.99)^3(0.01)$$
$$= 0.999$$

⇒ Conditions for a successful flight:

- All the four engines are working $[p^4]$
- Only 1 engine has failed $[4p^3q]$

(2) Power transmission system

- 1 step up transformer, T_1 ; 2 T-lines, L ; 2 step down transformers T_2.



- The step-down transformers can handle 50% of power only.
- $Q_{T1} = 0.005$, $Q_L = 0.03$, $Q_2 = 0.004$
- Full power can be transmitted to a consumer through any of the two t-line circuits

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- Assuming constant power is transmitted...

(i) 100% power transmission.

⇒ Conditions for 100% Power transmission:

- T_1 at work
- At least one L at work
- The two T_2's at work

⇒ $Q_{T1} = 0.005$; $P_{T1} = 1 - 0.005$

$= 0.995$

⇒ $Q_L = 0.03$; $P_L = 0.97$

⇒ $Q_{T2} = 0.004$; $P_{T2} = 1 - 0.004$

$= 0.996$

~~⇒ $P_{T1}^2 + R_{100\%} =$~~

- At least one t-line at work: $(P_L + Q_L)^2 = 1$

$(1-q^2)(1-q)$ $\quad P_L^2 + 2P_L Q_L = (1 - Q^2)$

$1 - q - q + q^2$ $\quad R_{100\%} = R_{T1} \times (1 - Q^2)$

$1 - 2q + q^2$ $\quad \Rightarrow (P_{T2} + Q_{T2})^2 = P_{T2}^2 + 2P_{T2}Q_{T2} + Q_{T2}$

⇒ T_1 at work : $(P_{T1} + Q_{T1}) = 1$

$1 - Q_{T1} \Rightarrow P_{T1} = 0.995$

At least one of the 2

⇒ ~~The two~~ lines at work : $(P_L + Q_L)^2 = 1$

$\Rightarrow P_L^2 + 2P_L Q_L + Q_L^2 = 1$ $\quad [P_L^2 + 2P_L Q_L]$

$\Rightarrow$

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07/11/202

⇒ Both of the step down transformers, T_2

$$\Rightarrow (P_{T_2} + Q_{T_2})^2 = P_{T_2}^2 + 2P_{T_2}Q_{T_2} + Q_{T_2}^2 = 1 \quad \Rightarrow \text{only } P_{T_2}^2 \text{ req}$$

$$= 1 - Q_{T_2}^2$$

$$\Rightarrow P_{100\%} = P_{T_1} \times [1 - Q_L^2] \times [(P_{T_2} + Q_{T_2})^2]$$

$$= 0.995 \times [1 - (0.03)^2] \times [0.996^2 + 2(0.996)(0.004) + 0.004^2]$$

$$= 0.995 \times 0.9991 \times$$

$$\Rightarrow P_{100\%} = P_{T_1} \times [1 - Q_L^2] \times [P_{T_2}^2]$$

$$= (0.995) \times [1 - 0.03^2] \times (0.996^2)$$

$$= 0.986$$

(b) 50% power transmission

$$P_{T_1} \times [1 - Q_L^2] \times [2P_{T_2}Q_{T_2}]$$

$$(0.995) \times (1 - 0.03^2) \times (2 \times 0.996 \times 0.004)$$

$$= 0.00792$$

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* PRACTICE PROBLEMS

07/11/2022

WEEK 8

LECTURE 7

* TEST CORRECTION

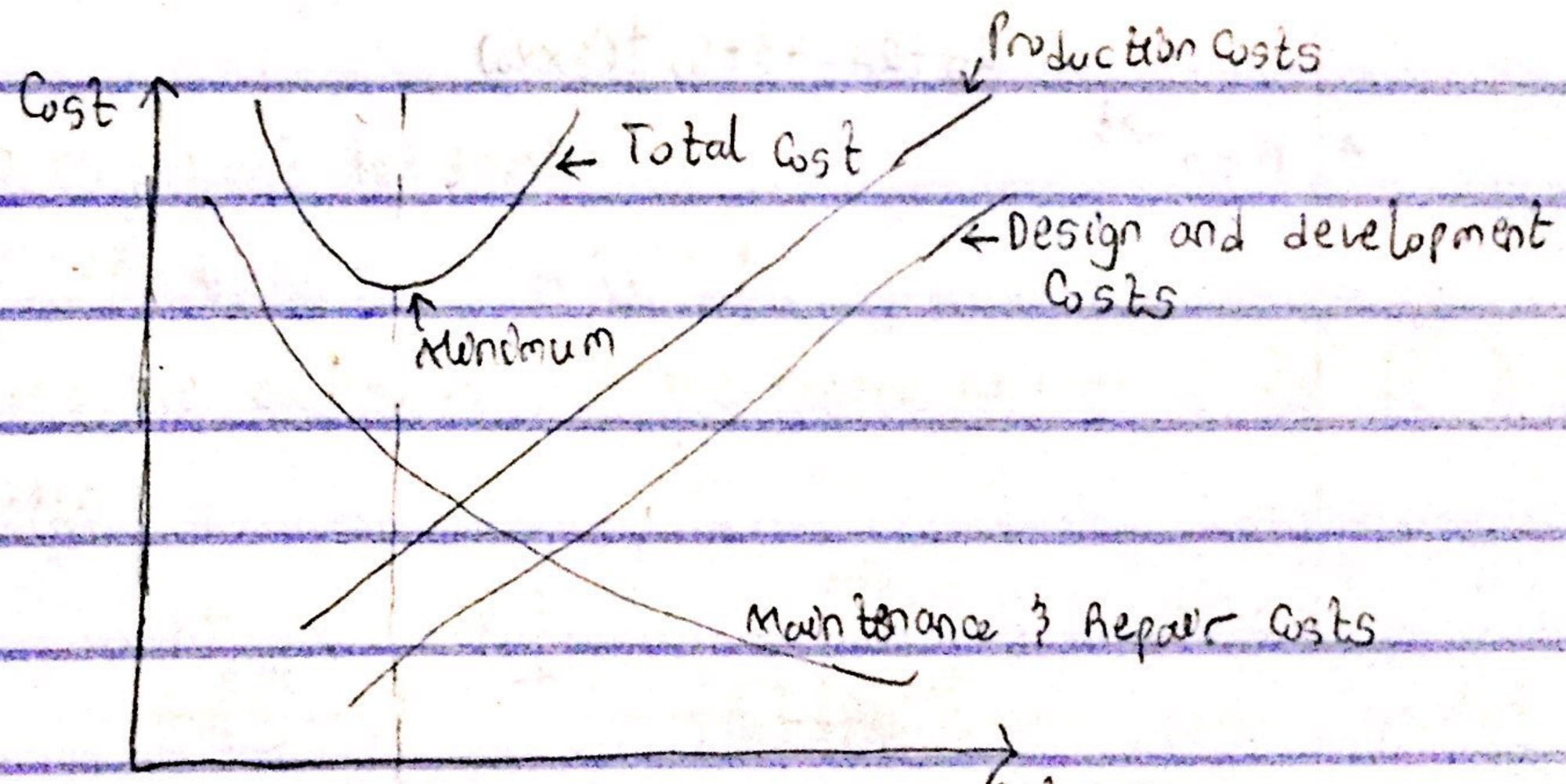
- Spellings
- Formulas
- Correctly labelled diagrams.

⇒ $P_r = \frac{(\lambda t)^r e^{-\lambda t}}{r!}$

⇒ Cost factors

- Design and development
- Maintenance and repair
- Production ~~and main~~

$\frac{MTBF}{MTBF + DT}$ [Availability]



Financial implications of Reliability

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$$\sigma = \sqrt{\frac{\Sigma f(x-M)^2}{n}} \quad , \quad M = \frac{fx}{f}$$

f	x-m	f	x-M	$(x-M)^2$
5	390	1950	-96.11	
8	450	3600	-36.11	
10	450	4500	-36.11	
7	550	3850	63.89	
6	600	3600	113.89	
36		17500		

$\sigma = 69.93$

$$e = \frac{\text{No. of failures}}{20+24+45+60+(3\times 90)} = \lambda$$

$\Rightarrow R = e^{-\lambda t}$

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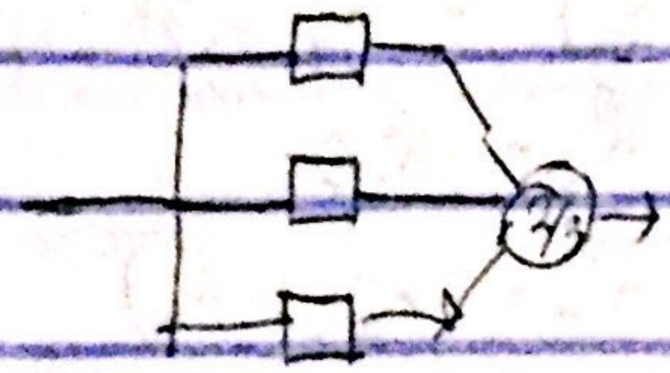
MAJORITY VOTING REDUNDANCY

- Conditional active redundancy
- m out of n units required for a system to function



- R for an m/n system where all n units have the same R

$$\Rightarrow R_{m/n} = 1 - \sum_{i=0}^{m-1} {}^{n}C_i R^i (1-R)^{n-i}$$

- MVG

$R_v = 1$; $R_{sys} = R_{m/n} \times R_v$

- Binomial distribution can also be used.

- Eg. for a 3/4 system

$$(R+Q)^4 = 1$$

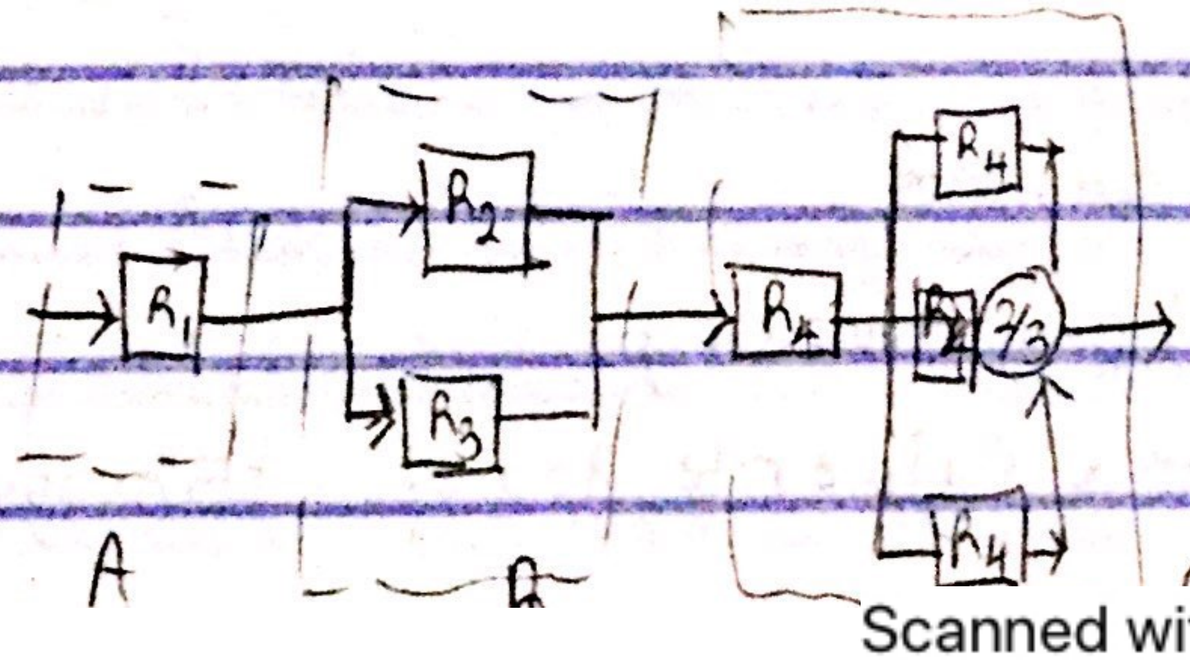
⇒ used when the R for the subunits are the same.

- When Rs are not the same,

$$(R_1+Q_1)(R_2+Q_2)(R_3+Q_3) = 1$$

- Choose the expression with the needed amount of Rs [eg. 2/3]

* EXAMPLE



$R_{sys} = ?$

MVG is perfect

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$\Rightarrow R_S = R_A \times R_B \times R_C$

$R_A = R_1 \quad ; R_B = 1-[(1-R_2)(1-R_3)]$

$(R_4+Q_4)^3 = R_4^3 + 3R_4^2 Q_4 + 3R_4 Q_4^2 + Q_4^3$

$\Rightarrow R_C = R_4^3 + 3R_4^2 Q_4$

$= R_4^3 + 3R_4^2(1-R_4)$

$Q_4 = 1-R_4$

$\Rightarrow R_S = R_1 \times 1-[(1-R_2)(1-R_3)] \times R_4^3 + 3R_4^2 Q_4$

$= R_1 \times [1-(1-R_2)(1-R_3)] \times (R_4^3 + 3R_4^2(1-R_4))$

* PASSIVE OR STANDBY REDUNDANCY

- Based on system energized when the active unit fails

$$R = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2 t^2}{2!} + \dots + \frac{\lambda^{n-1} t^{n-1}}{(n-1)!}\right]$$

- 2 units, one active, one standby;

$$R_{ab} = e^{-\lambda t}(1+\lambda t)$$

- Senses failure to the active unit, switches over to the stand unit.

- EXAMPLE

- 3 process computers
- Two on standby redundancy
- Each with a failure rate, $\lambda = 0.01$, $t = 50$ hours

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(i) For a single computer:

$$R_t = e^{-\lambda t} \quad , \lambda = 0.01, \; t = 50$$

$$= e^{-(0.01)(50)}$$

$$= 0.6065$$

(ii) For one standby computer $[n=2]$

$$R(t) = e^{-\lambda t}[1 + \lambda t]$$

$$= e^{-(0.01)(50)}[1 + (0.01 \times 50)]$$

$$= e^{-0.5}[1 + 0.5]$$

$$= 0.9098$$

(iii) For two standby computers $(n=3)$

$$R_{3t} = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2 t^2}{2!}\right]$$

$$= e^{-0.5}\left[1 + 0.5 + \frac{(0.01)^2 (50)^2}{2}\right]$$

$$e^{-0.5}(1.625)$$

$$= 0.9856$$

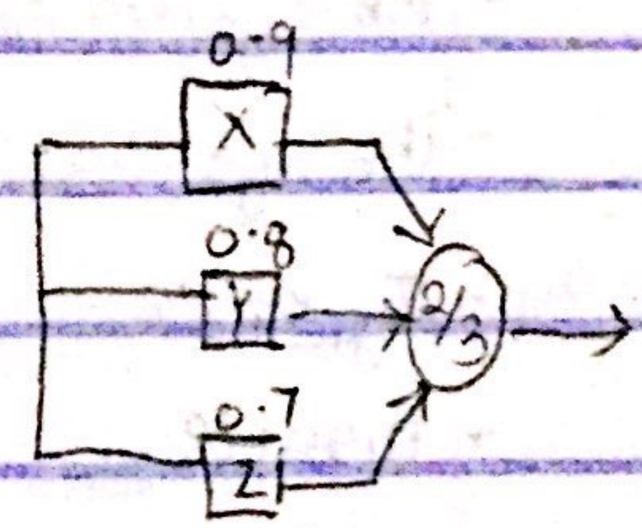
* PRACTICE PROBLEMS

(1) $R_X = 0.9$, $R_Y = 0.8$, $R_Z = 0.7$

$Q_X = 0.1$, $Q_Y = 0.2$, $Q_Z = 0.3$

$(R_X + Q_X)(R_Y + Q_Y)(R_Z + Q_Z) = 1$

$(R_X + Q_X)(R_Y R_Z + R_Y Q_Z + R_Z Q_Y + Q_Y Q_Z)$



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$R_X R_Y R_Z + R_X R_Y Q_Z + R_X R_Z Q_Y + R_X Q_Y Q_Z + R_Y R_Z Q_X + R_Y Q_X Q_Z + R_Z Q_X Q_Y$
$+ Q_X Q_Y Q_Z$

⇒ For 2/3, choose the expressions with at least 2 Rs;

$R_X R_Y R_Z + R_X R_Y Q_Z + R_X R_Z Q_Y + R_Y R_Z Q_X$

$= (0.9 \times 0.8 \times 0.7) + (0.9 \times 0.8 \times 0.3) + (0.9 \times 0.7 \times 0.2) + (0.8 \times 0.7 \times 0.1)$

$= 0.504 + 0.216 + 0.126 + 0.056$

$= 0.902$

14/11/2022

WEEK 9

LECTURE 8

MARKOV MODELS

- Assign various states to a system and determine the probabil of being in these states

- **Assumptions**
  - The system is memoryless (only the existing state)
  - The system is stationary

- **Types**
  - Discrete
  - Continuous Markov modelling.

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* DISCRETE

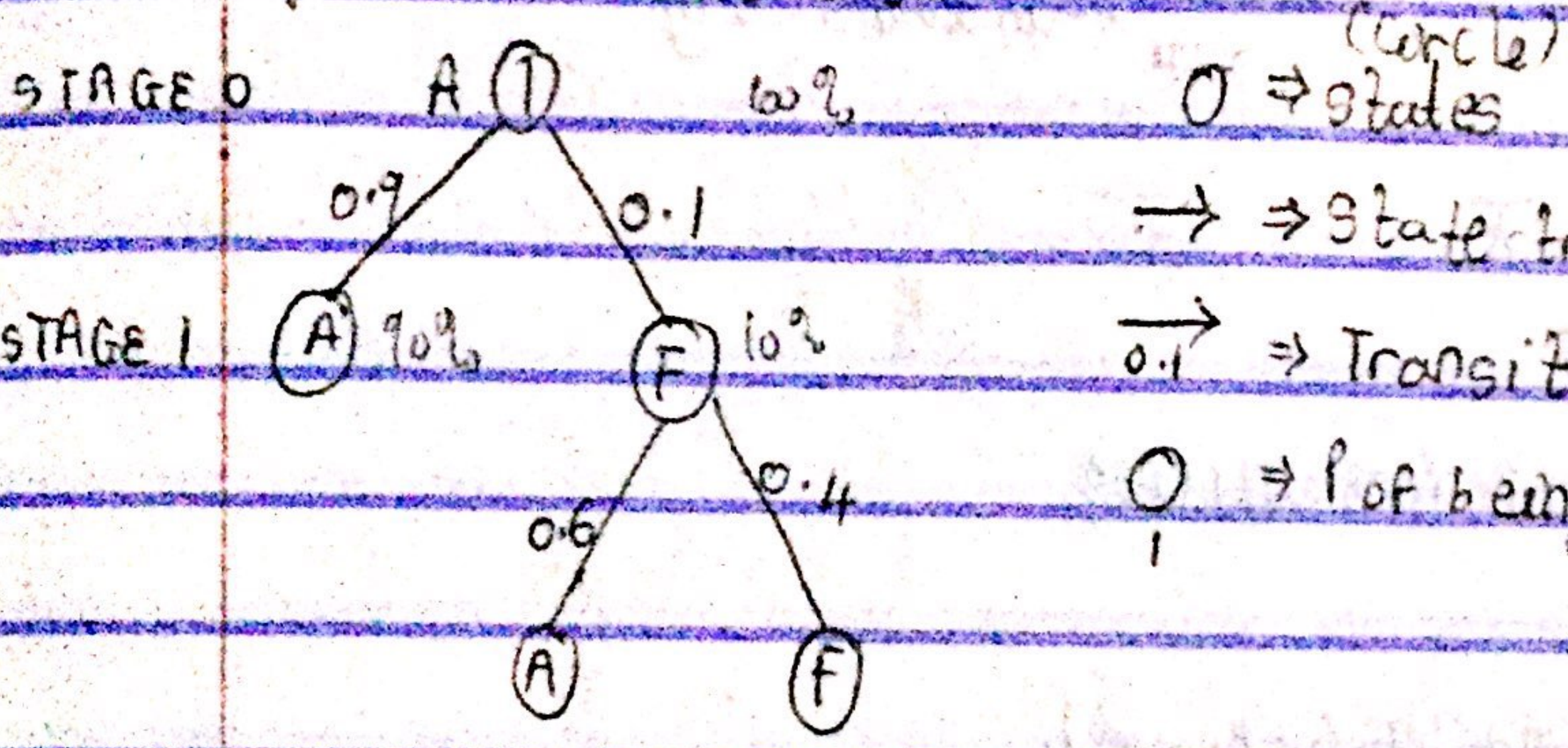
- Easier to understand
- A system that consists of a single component and 2 possible states:

⇒ Available state [A]

⇒ Failed state [F]

⇒ If component is available, A ; It has 10% chance of failing in the next time interval.

⇒ If component has failed, F ; 60% chance of being repaired in the next time interval



- Stage 2 can be reached from a transition from either state A or state F.

• Diagram of a discrete Markov Chain.

NOTE: The sum of all transitions out of any state must equal unity

* CONTINUOUS MARKOV MODELING

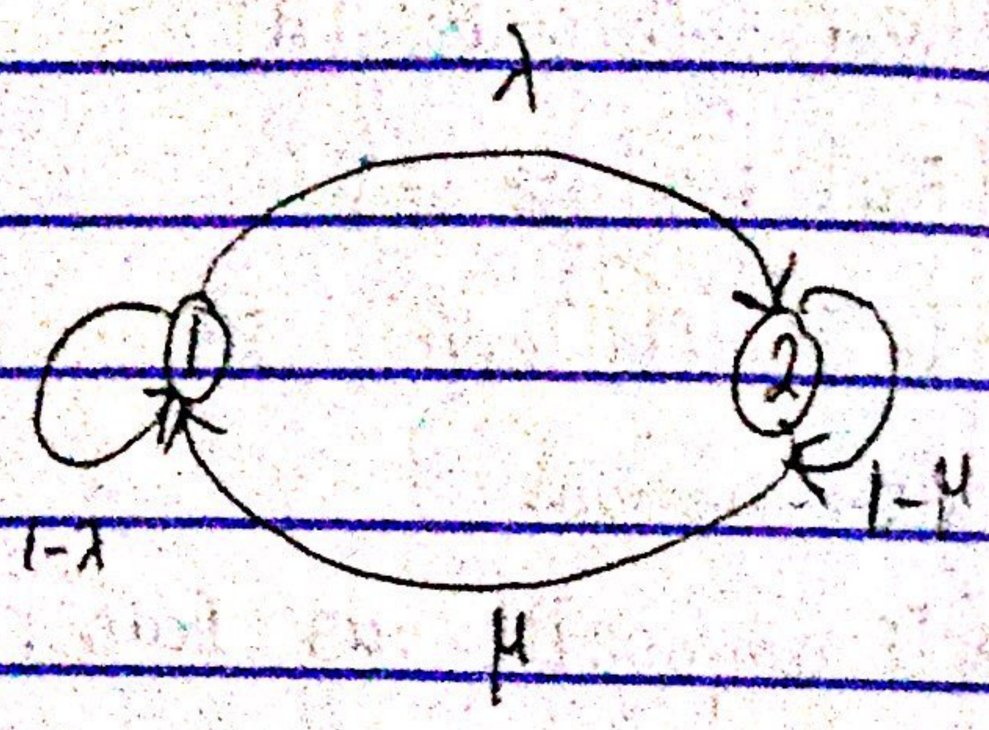
- Constant state transitions.
- P(state transitions) are replaced by transition rates.
- Considering a two state system:

⇒ Working state

⇒ A failed state

⇒ Failure rate : λ

⇒ Repair rate : μ



$$P_1 = \frac{\mu}{\lambda+\mu} \leftarrow \text{availability}, \quad P_2 = \frac{\lambda}{\lambda+\mu} \leftarrow \text{unavailability}$$

1-P

- $MTTF = \frac{1}{\lambda}$
- $MTTR = \frac{1}{\mu}$

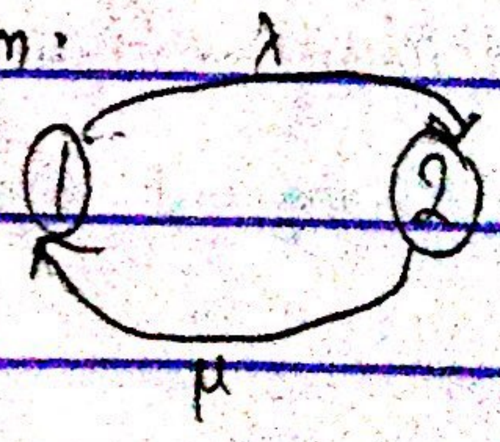
• STEADY-STATE PROBABILITIES

- $P.Q = 0$

↓

system availability (unknown)

- $\Sigma P_i = 1$
- Q ⇒ Transition intensity matrix, generated from the state transition diagram:



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$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} = \begin{bmatrix} -\lambda & \lambda \\ +\mu & -\mu \end{bmatrix}$$

* EXAMPLE

- Solve for ss probabilities and derive an expression for availability and unavailability in one component system with two states

solution

- To find the ss probabilities or availabilities, the following equations are used:

$$P \cdot Q = 0 \quad \& \quad \Sigma P_i = 1$$

- But $P_i = P_1, P_2$ & Q

Hence, $[P_1, P_2] \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix} = 0$

$$\Rightarrow -\lambda P_1 + \mu P_2 = 0 \quad \text{--(i)}$$

$$\lambda P_1 + (-\mu P_2) = 0$$

$$\lambda P_1 - \mu P_2 = 0 \quad \text{--(ii)}$$

$$\lambda P_1 = \mu P_2$$

$$P_1 = \frac{\mu P_2}{\lambda} \quad \text{OR} \quad P_2 = \frac{\lambda P_1}{\mu}$$

$$\Rightarrow P_1 + P_2 = 1 \quad [\Sigma P_i = 1]$$

$$P_1 + \frac{\lambda}{\mu} P_1 = 1 \quad \Rightarrow P_1 \left[1 + \frac{\lambda}{\mu}\right] = 1$$

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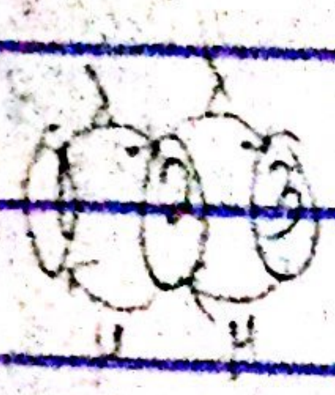
$$P_1\left[\frac{\mu+\lambda}{\mu}\right]=1$$

$$P_1=\frac{1}{\frac{\mu+\lambda}{\mu}}=\frac{\mu}{\mu+\lambda}$$ ⟨Availability⟩

⟨Buried⟩



$$\Rightarrow P_1=\frac{\mu P_2}{\lambda}\cancel{+P_2} \qquad P_1+P_2=1$$

$$\Rightarrow \frac{\mu}{\lambda}P_2+P_2=1$$

$$\begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{23} & Q_{33} \end{bmatrix}$$

$$\Rightarrow P_2\left[\frac{\mu}{\lambda}+1\right]=1$$

$$\Rightarrow P_2\left[\frac{\mu+\lambda}{\lambda}\right]=1$$

$$P_2=\frac{1}{\frac{\mu+\lambda}{\lambda}}=\frac{\lambda}{\mu+\lambda}$$ ⟨Unavailability⟩

$$\Rightarrow P_i=\left[\frac{\mu}{\mu+\lambda},\frac{\lambda}{\mu+\lambda}\right]$$

* EXERCISE

(a) Complete the transition rate matrix;

$$Q=\begin{bmatrix} -6 & - & 2 \\ 1 & -3 & - \\ 0 & 1 & - \end{bmatrix}$$

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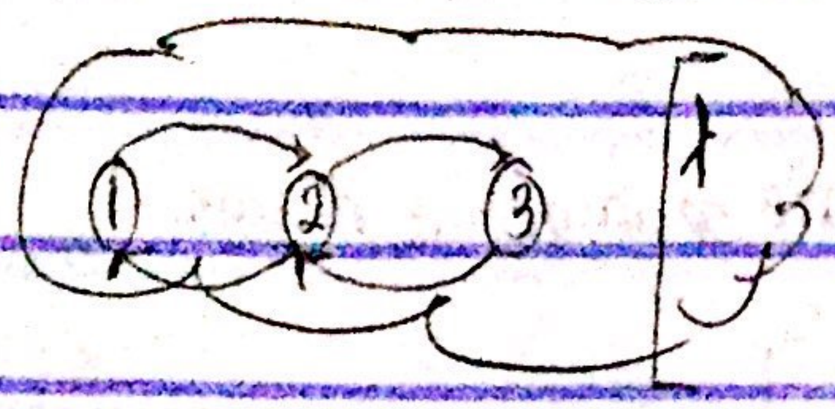
(b) Write the equation system for the ss distribution and solve it.

Solution



$-6 + x + 2 = 0$

$x = 4$

$1 + (-3) + z = 0$

$z = 2$

$0 + 1 + w = 0$

$w = -1$

(a) Every row is $= 0$

$P.Q = 0 \Rightarrow \begin{bmatrix} -6 & 4 & 2 \\ 1 & -3 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

(b) For the equation system

$$[P_1, P_2 P_3] \begin{bmatrix} -6 & +4 & 2 \\ 1 & -3 & 2 \\ 0 & 1 & -1 \end{bmatrix} = 0$$

$P_1 = \frac{1}{21}$ $\quad \cancel{\Rightarrow -6P_1 - 4P_2 + 2P_3 = 0}$ $\quad -6P_1 + P_2 = 0 \quad --(i)$

$+4P_1 + (-3P_2) + P_3 = 0 \quad --(ii)$

$P_2 = \frac{6}{21}$ $\quad P_2 = 6P_1$ $\quad 2P_1 + 2P_2 - P_3 = 0 \quad --(iii)$

$\Rightarrow P_1 + P_2 + P_3 = 1 \quad --(iv)$

$P_3 = \frac{14}{21}$ $\quad$ Solving simultaneously <Calculator>

$2P_1 + 12P_1 - P_3 = 0 \Rightarrow 14P_1 - P_3 = 0$

$+4P_1 - 18P_1 + P_3 = 0 \Rightarrow -22P_1 + P_3 = 0$

$P_1 = \frac{1}{21}$; $P_2 = \frac{6}{21}$, $P_3 = \frac{14}{21}$

$= \frac{2}{7}$ $\quad = \frac{2}{3}$

$P_1 + 6P_1 + P_3 = 0$ $\quad \frac{1}{21} + \frac{6}{21} + P_3 = 1$

$-6(\frac{1}{21}) + P_2 = 0$ $\quad 2P_1 + 12P_1 - P_3 = 1$ $\quad 21P_1 = 1$ $\quad 1 - \frac{1}{21} = \frac{21-1}{21} = \frac{20}{21}$

$P_2 = \frac{6}{\ldots}$ $\quad 7P_1 + P_3 = 0$ $\quad P_1 = \frac{1}{21}$

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* MAINTENANCE AND MAINTAINABILITY

- Basic differences

* Maintenance: Performed on unfailed systems

⇒ IEEE TASK FORCE ; Activity of restoration

- Primary objectives

- To extend the useful life span of an item.
- To increase availability
- To ensure operational readiness
- Safety
- To avoid exorbitant expenses

* Maintainability

- Ease and speed with which a failed item can be repaired.
- Quantifiable, Dimensionless.

- Factors for high maintainability

- System location
- Availability of spare parts
- Efficiency & skill of maintenance crew

* Aim

- Least time ; Least cost ; Least expenditure

* Eg.

- Fault localization ; Isolation ; Correction.

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$$MTTR = \frac{\sum n_i \lambda_i t_{m_i}}{n_i \lambda_i}$$

⇒ Repair rate, $\mu = \frac{1}{MTTR}$ (Maintainance action rate)

MTTR (Mean time to repair)

- Maintenance time constraint, t_m
- $M = 1 - e^{-\mu t_m} = 1 - \exp\left[\frac{-t_m}{MTTR}\right]$ [Maintainability equation]

* example

Average $t = \frac{1}{2}$ hour Average time to repair any fault

- $M = ?$, for $t_m = 2\frac{1}{2}$ hours [maintenance time]

Solution

$M = 1 - e^{-\mu t_m}$; $\mu = \frac{1}{MTTR}$, $MTTR = \frac{1}{2}$

$\mu = 1/\frac{1}{2} = 2$

$t_m = 2\frac{1}{2}$

$M = 1 - e^{-(2.5)(2)}$

$= 0.993$

* Utilisation Factor

$$U = \frac{t_{op}}{t_m + t_{id} + t_{op}}$$

* Availability

$$A = \frac{t_{op}}{t_{op} + t_m \text{ (min)}}$$

t_m becomes as small as possible.

$t_{op} = MTBF$ $t_m = MTTA$

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* example

- System fails twice in a period of 123 days
- Total time for repairs = 3 days
- MTTR, MTBF (days), Availability = ?

$t_{op} = 123 - 3 = 120$ days , no of failures = 2

$\Rightarrow MTBF = \frac{120}{2} = 60$ days

$\Rightarrow MTTR = \frac{TTR}{\text{no of failures}} = \frac{3}{2}$ days ; $A = \frac{MTBF}{MTTR + MTBF} \times 100$

* Unavailability, $\bar{A}$

$$\bar{A} = 1 - A = \frac{MTTR}{MTBF + MTTR}$$

* REPAIRABILITY : (Unit will be restored to operational efficiency)
* Maintenance action time.

* 99 Availability (A_{99})

$$A_{99} = \frac{MTBF}{MTBF + MTTR} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda} + \frac{1}{\mu}}$$

$$= \frac{\mu}{\mu + \lambda}$$

$$\Rightarrow \bar{A}_{99} = 1 - A_{99} = \frac{\lambda}{\lambda + \mu}$$

* Availability of items in series and parallel combinations.

$\Rightarrow A_S = A_1 A_2 \ldots A_n$

$\Rightarrow A_P = 1 - [(1-A_1)(1-A_2)\ldots(1-A_n)]$

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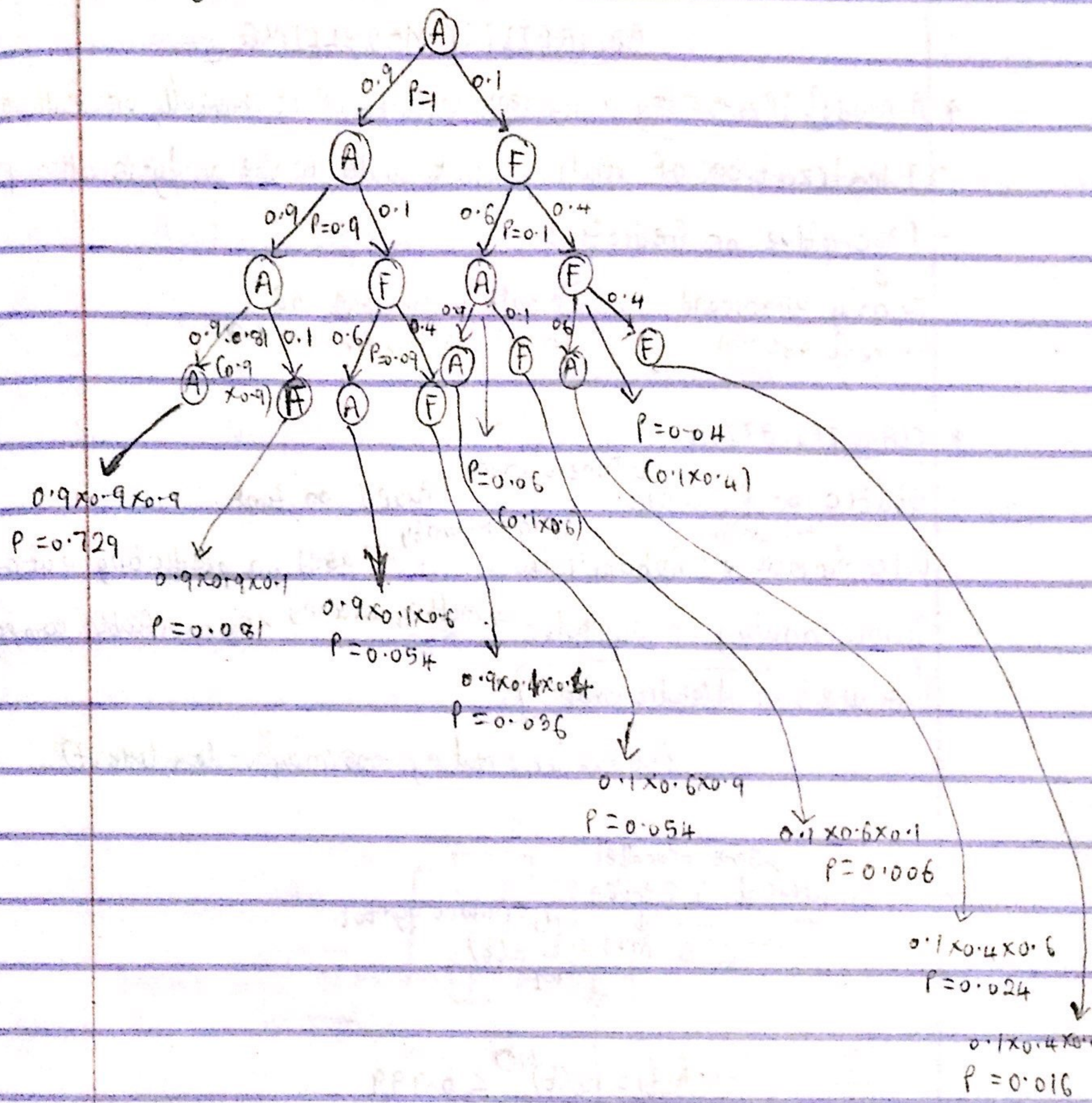
P(Being in each state) = sum of each path



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WEEK 10

21/11/2022

LECTURE 9

RELIABILITY MODELLING

* A Model; Representing a system or parts of it Physically or mathematic
  - Idealization of reality that aids in the analysis of a prob
  - Descriptive or Predictive.
    - ↓ To only understand a real system
    - ↘ To both understand and Predict system's Performance.

* CLASSIFICATION
  - Static or Dynamic ↙ Time varying {Based on time}
  - Deterministic ↙ certainty or Probabilistic ↙ uncertainty {Based on predicting outcome
  - Iconic, analog or symbolic → Math equations {Quantifiable componen
    - ↓ Look like the real (static char.)
    - Behave like real systems (may not look like it)

⇒ Combinational ; series ↗ Series + Parallel

$$R(t) = \prod_{i=1}^{N} R(t)$$ ↙ Product symbol

$$R(t) = R_c(t)^{100} \geq 0.999$$

$$\Rightarrow R_c(t) = \sqrt[100]{0.999}$$

$$= 0.99999$$

⇒ Parallel systems

$$1 - \prod_{i=1}^{N} [1 - R_i(t)]$$

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E.g

- Minimum R of 0.999
- Has a reliability of only 0.85

⇒ Min. number of modules needed to achieve the required reliability =

$$R(t) = 1 - [1 - R_m(t)]^N$$

$$\Rightarrow 1 - [1 - 0.85]^N \geq 0.999$$

$$1 - 0.999 \geq (1 - 0.850)^N \Rightarrow 0.001 \geq 0.15^N$$

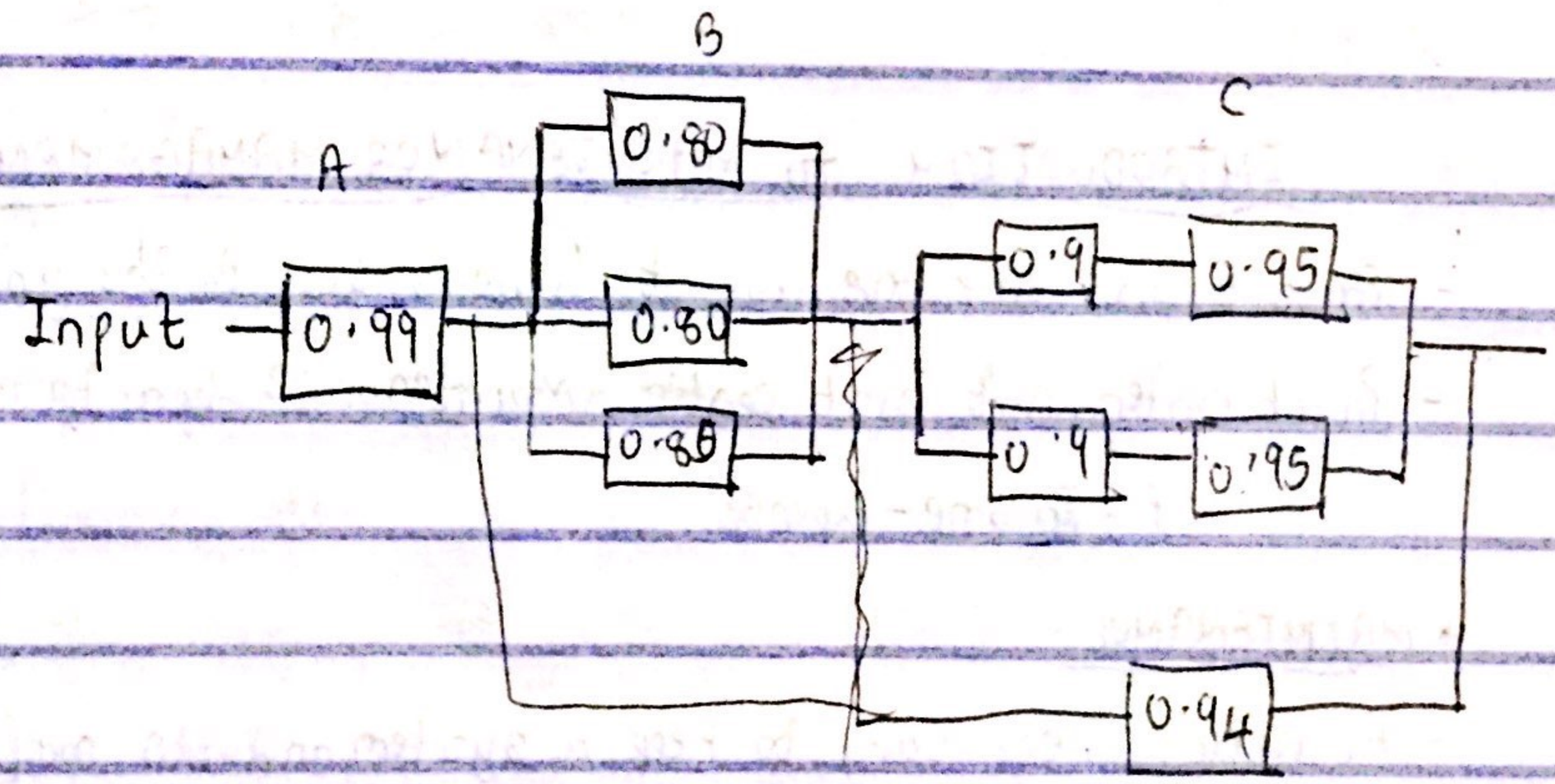
$$\log 0.001 = \log 0.15^n$$ ~~0.15~~

$$\Rightarrow \log 0.001 = N \log 0.15$$

$$N = \frac{\log 0.001}{\log 0.15} = 3.64$$

$N \approx 4$ <Has to be an integer>

⇒ Series parallel combination.



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$0.95 \times 0.9 = 0.855$

$= 1 - [(1-0.855)(1-0.855)]$

$= 0.97898$ (2 decimal places)

$1 - [(1-0.8)(1-0.8)(1-0.6)]$

$= 0.992$

$\Rightarrow 0.992 \times 0.97898$

$= 0.971148$

$\Rightarrow 1 - [(1-0.971148)(1-0.94)]$

$= 0.99827$

$\Rightarrow R_T = 0.99827 \times 0.99$

$= 0.988$

INTRODUCTION TO MAINTENANCE MANAGEMENT

- Efficient and effective use at lowest cost is the goal.
- Profit center, not cost center (Maintenance department)

$P = \text{Income} - \text{Expense}$

* **MAINTENANCE**

- Activities necessary to keep a system and its components in working order.
- Maintaining capability, controlling cost.

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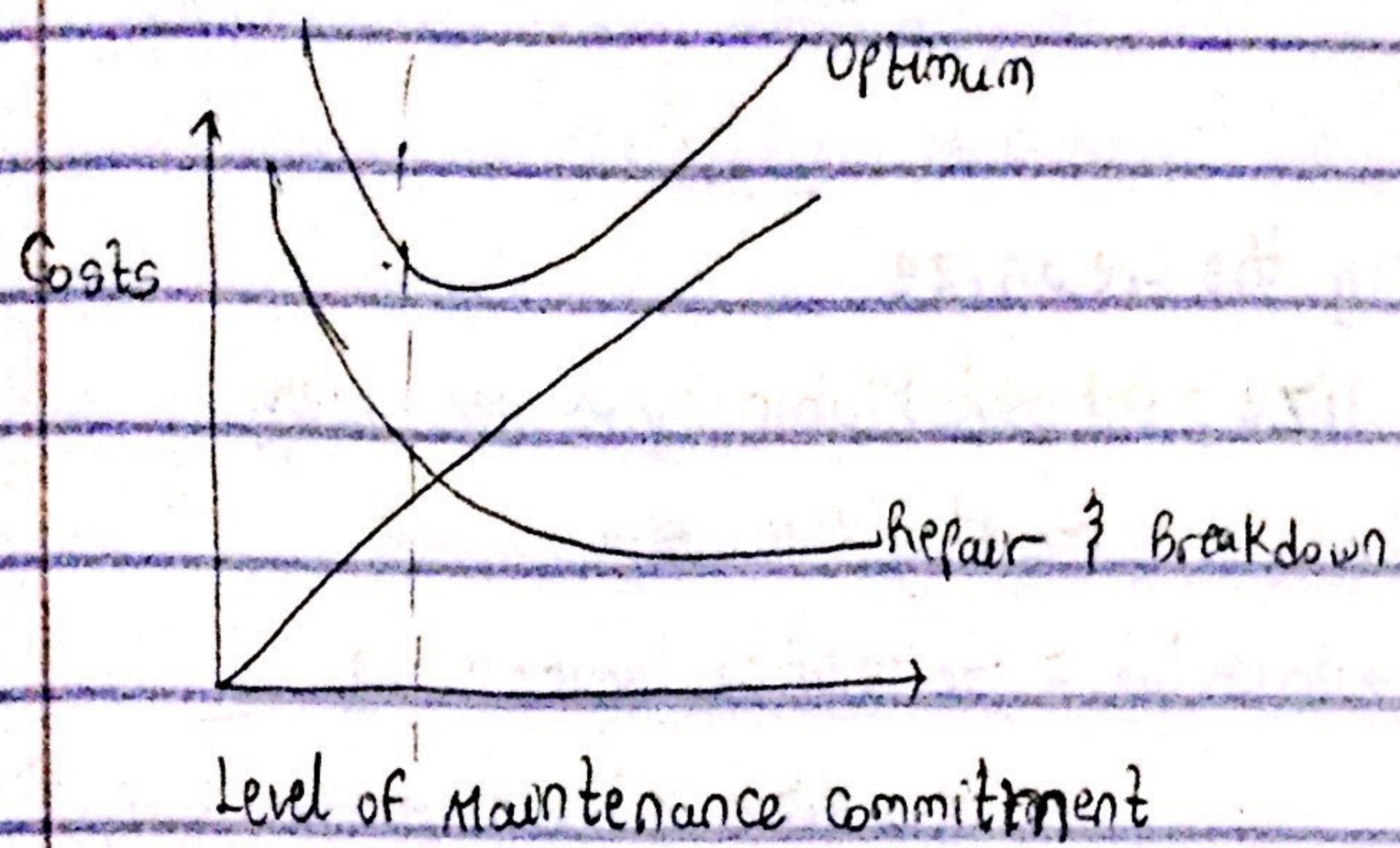

e^{-ux}

$x \geq 0$

- Comparison of maintenance costs with other graphs.

* MAINTENANCE OBJECTIVES

- Primary and Secondary → Pollution and noise control; Security
  - Compliance

$\mu = \frac{1}{w}$

- Maintaining existing equipment
- Inspection, cleaning, lubrication
- Modifications
- Installations
- Alterations

$P(S > a) = e^{-\frac{1}{30}t}$

$P(S < x) = 1 - e^{-\mu}$

* THE MAINTENANCE FUNCTION

- Planning
  - Necessary to set goals and objectives.
  - Procedures
- Scheduling
  - The execution of activities (Planned or unplanned) based on priorit

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⇒ Backlog

- Determining the crewsize

⇒ Crew size = $\frac{\text{Scheduled labor-hours per week}}{\text{Backlog} \times \text{Hours per week}}$

⇒ Backlog = $\frac{\text{Scheduled labor-hours}}{\text{Crew size} \times \text{Hours per week}}$

- Queing theory is implemented.

* Examples

① Calculate the crew size for a maintenance department in order to maintain a backlog of four weeks if 1,500 hours of maintenance activities are scheduled per week.
Assume a 35-hour work week.

② Assume that the equipment is operated 300 work days for 8 hours per day and with a known MBTF of 200 hours.
Failure costs are estimated at $ ~~$~~5,000 per occurence.
A preventive maintenance option would cost $450 per scheduled routine.
Could you justify the PM program if the PM routine every 120 hours reduces the probability of failure to 35%.

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WEEK 11

28/11/2022

LECTURE 10

QUEUING MODELS

* ASSIGNMENT REVISION

• Crew size = $\frac{1500}{4 \times 35}$ = 10.71 ≈ 11 people

* Queing theory

- To understand the behaviour of waiting lines
- Enable the operation managers to make better decisions regarding the operation of these lines.

⇒ Input into the system (Those arriving at the queue, awaiting service)

- Finite or Infinite (size)
- Arrival pattern (Poisson distribution) ; Rate of equipment failure

⇒ Waiting line

- Line length ; Can be unlimited. In maintenance, it will be limited.
- Line discipline ; order in which service is offered. [FIFO]
-

⇒ Service

- Pattern of service time
- Configuration of service facility.

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$P(\text{Service time} > x) = e^{-ux}$

$x \geq 0$; $u = 1/\text{Average service time}$

EXAMPLE

$x = 1$ hour

$P(\text{service time} > 60 \text{ minutes}) = 2.718^{-(\frac{1}{30})(60)}$

$= 0.135$

$= 13.53\%$

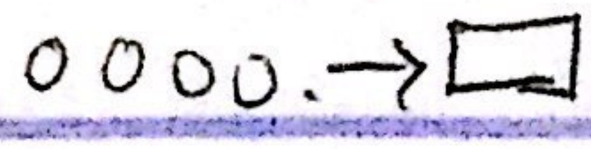
$\Rightarrow$ 13.5% likelihood that an arriving unit will require more than 1 hour of service

$\Rightarrow P(\text{Service time} \leq x) = 1 - e^{-ux}$

Graph for average service time

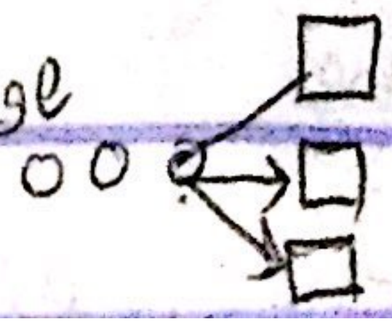
Service configuration

Single channel, single phase (One server for all customers ; simplest queue configuration)



Single channel, Multiple phase (Different servers for different tasks)

Multiple channel, single phase



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$1 - \frac{\lambda}{\mu}$ $\left(\frac{\lambda}{\mu}\right)^n P_0$

- Multiple channel, multiple phase.

- Focus on the maintenance department as the waiting lin

* NEEDED

- Avg. no of customers waiting
- Waiting & being served <system>
- Time on queue

* QM CALCULATIONS

(1) Avg. number of units waiting for service

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

(2) $L_s = \frac{\lambda}{\mu-\lambda}$

$$L_s = \frac{\lambda}{\mu-\lambda}$$

(3) $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$$

(4) $W_s = \frac{1}{\mu-\lambda}$

$$W_s = \frac{1}{\mu-\lambda}$$

(5) $P_0 = 1 - \frac{\lambda}{\mu}$ (Idle)

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(6) $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \Rightarrow P$ that a certain number of units are in the system.

* EXAMPLE A

- Let's assume that the equipment either because of breakdown or expected planned performance enters the waiting line at rate of 3 hours per day [Arrival rate $= \lambda$].
- The Maintenance department services the equipment on a FIFO basis.
- The average repair time is 2 hours, Given an 8 hour day, the service rate, $\mu = \frac{8}{2} = 4$ per day

(1) What is the average wait time in the line.

Solution

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{3}{4(4-3)}$$

$$= \frac{3}{4} = 0.75 \text{ day} \times 8 \text{ hours}$$

$$= 6 \text{ hours}$$

(2) What is the average total time in the repair shop?

Solution

$$\Rightarrow W_s = \frac{1}{\mu-\lambda} = \frac{1}{4-3} = 1 \text{ day}$$

$$= 8 \text{ hours.}$$

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* EXAMPLE B

- Assuming a constant failure rate of 0.025, calculate the preventive maintenance interval if only a 25% failure risk can be tolerated.

Solution

$\lambda = 0.025$, tolerated failure risk = 25%

$= 0.25$

$1 - 0.25 = 0.75$

$R = 1 - Q = e^{-\lambda t}$; $Q = 0.25$

$\Rightarrow 0.75 = e^{-0.025t}$

~~$\log 0.75 = \log e^{-0.025t}$~~

~~$\ln 0.75 = e^{-0.025t}$~~

$\ln 0.75 = \ln e^{-0.025t}$

$\ln 0.75 = -0.025t$

$\Rightarrow -0.2877 = -0.025t$

$t = \frac{-0.2877}{-0.025}$

$= 11.508$

≈ 12 hours

* EXAMPLE C (Previous assignment ; No 2)

- Assume that the equipment is operated 300 work days for 8 hours per day and with known MTBF of 200 hours. Failure cost are estimated at \$5,000 per occurrence. A PM maintenance option would cost \$450 per scheduled outline.

$1 - \frac{1}{\mu}$

$= \frac{\mu - \lambda}{\mu}$

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- Justify the PM program if performing the PM routine every 120 hours reduces the probability of failure to 35%?

Solution

300 days × 8 hours/day = 2400 hrs

No of failures = $\frac{2400}{MTBF}$ = $\frac{2400}{200}$

= 12 failures

⇒ Cost of failures = 12 × \$5000 (without PM)

= \$60,000 per year

⇒ No of scheduled PM = $\frac{2400}{120}$ = 20 schedules

⇒ Cost of scheduled PM = 20 × \$450

= \$9,000

⇒ No of failures before scheduled maintenance = 20 × Scheduled PM × 0.35
(In between scheduled maintenance)

= 7 failures

⇒ Cost of failures = 7 × \$5000

= \$35,000

⇒ Total cost of PM = \$9000 + \$35,000

= \$44,000

- From the calculation, \$44,000 is less than \$60,000. Thus, this PM schedule is recommended.

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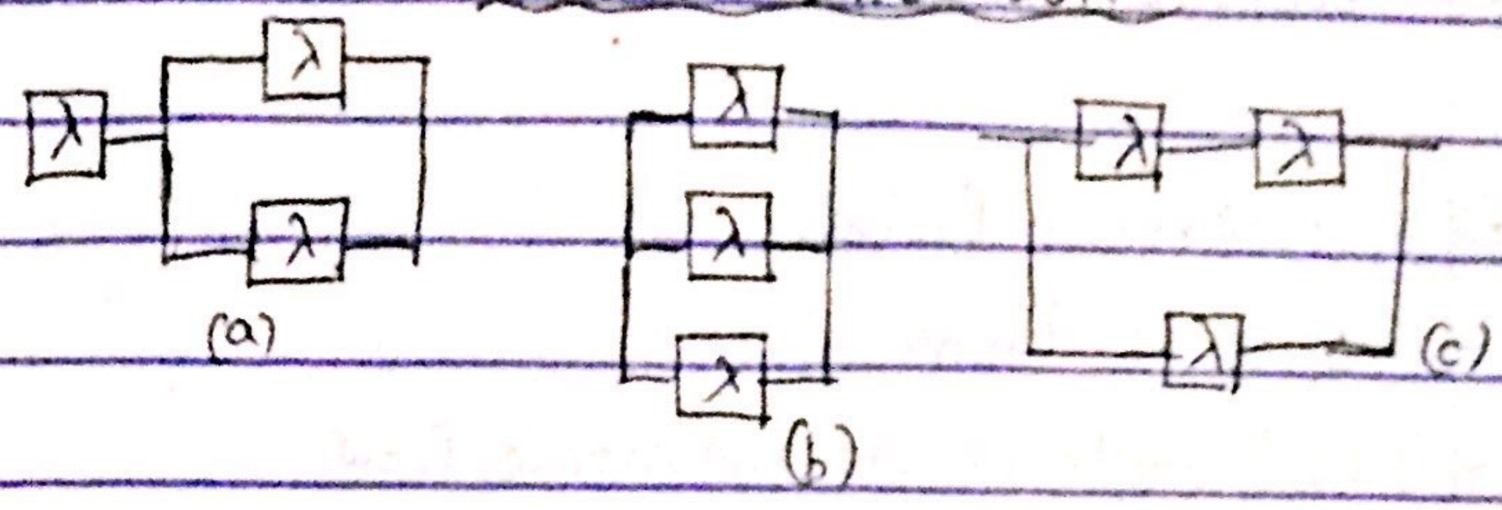
09/01/2022

WEEK 12

LECTURE 11

TEST 2 CORRECTION

① 

(a) Reliability in parallel

$\Rightarrow R_p = 1 - [(1-R)(1-R)]$

$= 1 - [1 - 2R + R^2] = 1 - 1 + 2R - R^2$

$= 2R - R^2$

$\Rightarrow$ In series : $R \times (2R - R^2)$

$= 2R^2 - R^3$

But $MTBF = \int_0^\infty 2R^2 - R^3$

$= 2\int_0^\infty e^{-2\lambda} dt - \int_0^\infty e^{-3t} dt = 2\left[\frac{e^{-2\lambda}}{-2\lambda}\right]_0^\infty - \left[\frac{e^{-3\lambda}}{-3\lambda}\right]_0^\infty$

$= \frac{2}{-2\lambda}[e^{-2\lambda\infty} - e^0] + \frac{1}{3\lambda}[e^{-3\lambda\infty} - e^0]$

$= \frac{2}{-2\lambda}[0-1] + \frac{1}{3\lambda}[0-1]$

$= \frac{1}{\lambda} - \frac{1}{3\lambda} = \frac{3-1}{3\lambda}$

$= 2/3\lambda$

$7/6\lambda$ (b) $1 - [(1-R)(1-R)(1-R)]$

$\frac{11}{6\lambda}$ (c) $1 - [(1-R^2)(1-R)]$

Highest

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(b) Preventive, Predictive, Corrective maintenance.

(c) 6 Scheduling factors

- Priority of tasks
- Size & skills of the maintenance crew
- Amount of time required to complete the tasks
- Equipment availability
- Labour needed for the task.
- Type of tools to be used

(d) MAINTENANCE

⇒ How much is left to be done

⇒ Backlog should NOT be kept at or near zero level, because the maintenance personnel will be idle when there are no repairs to be made.

⇒ 2-3 weeks of backlog is recommended.

(e) Backlog = 4 weeks

35 hour work week

1500 hours of maintenance

⇒ Crew size = $\frac{\text{Scheduled labour (hrs per week)}}{\text{Backlog}}$

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$$= \frac{1500}{4 \times 35} = 10.7$$

$$\approx 11$$

(f) Constant failure rate, $\lambda = 0.002$

- 20% failure rate $= Q$
- Maintenance interval = ?

$$\Rightarrow R = 1 - Q = 80\% = 0.8$$

$$\Rightarrow R = e^{-\lambda t}$$

$$0.8 = e^{-0.002t}$$

$$\ln 0.8 = -0.002t$$

$$t = \frac{\ln 0.8}{-0.002} = 111.57 \text{ hrs}$$

(g) PM PROGRAM

$\Rightarrow$ Time of operation $= 260 \times 16$

$= 4160$ hrs

$\Rightarrow$ No of failures per year $= \frac{4160}{66} = 63.03$ [MTBF = 66 hrs]

$\therefore$ Cost without maintenance

$= 63.03 \times 7{,}500 = ₦472{,}500$ per year

$\Rightarrow$ No of scheduled PM $= \frac{\text{Time of operation}}{\text{scheduled time}}$

$$= \frac{4160}{6} = 693.33$$

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⇒ Cost of PM = 693.33 × 250

₦173,250

⇒ Failure per year calculated = 63.03

⇒ 20% of 63.03 = 12.6

⇒ Cost of failure at 20% = 12.6 × 7500

= 94,500

∴ Total cost using PM = 94,500 + 173,250

= 267,750

∴ Since ₦267,750 < ₦472,500, the PM is justified.

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