Laws of Probability Relevant to Reliability Prediction

Review the following laws of probability theory:

- Multiplication rule [Applied in series systems]
- Addition Rule [Applied in parallel systems]

Binomial Distribution

To explain this type of distribution, we shall consider an example.

Suppose we have an electric generating set which has a probability of successful operation of 0.95 for a single day, and it is expected to operate for 5 days [not continuously]. Suppose we are required to do the following:

(1) Discuss the probability distribution of the two possible outcomes

(2) Find the possibility that the generator set will operate successfully for

(i) 3 days

(ii) at least 4 days.

Solution

In answering this question, we should note the following points.

1) There are two possible outcomes, namely, ‘success’ or ‘failure’.
2) The possibility of success may be represented by p and of failure by q, such that $p + q = 1$. In other words, the probability of failure may be represented by $(1-p)$.
3) The number of trials is equal to the number of days the generating set is put on operation ie $n = 5$

Solution

4) The probability distribution of the two possible outcomes, can be determined through binomial expansion $(p + q)^5$

5) Using the idea of Pascal's triangle the coefficients of expansion of $(p + q)^5$ can be obtained as 1,5,10,10,5,1

Now, the expansion of $(p + q)^5$ gives

$(p + q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$

In discussing the probability distribution of the two possible outcomes, we can say that:

Solution contd.

P^5 = probability that the generating set will operate successfully for all the 5 days.

$5p^4q$ = probability that the generating set will operate successfully for 4 days and fail for a day.

$10p^3q^2$ = probability that the genenerating set will operate successfully for 3 days and fails 2 days'

$10p^2q^3$ = probability that the generating set will operate successfully for 2 days and fail 3 days.

$5pq^4$ = probability that the generating set will operate successfully for 1 day and fail 4 days.

Q^5 = probability that the generating set will operate without success for the 5 days.

Solution contd.

The sum of terms in the RHS of the expansion $(p+q)^5$ is equal to one 1.

i. Probability that the set will operate successfully for 3 days = $10p^3q^2$

since $p = 0.95$, $q = 1 – 0.95 = 0.05$

therefore $10p^3q^2 = 10 \times 0.95^3 \times 0.05^2$

$= 0.02$

Solution contd.

ii. Probability that the set will operate successfully for at least 4 days is the sum of the probabilities that it will operate for 4 days and for the 5 – day period, ie

$$p\text{ (at least 4 days)} = p^5 + 5p^4q$$

$$= 0.95^5 + 5 \times (0.95)^4 \times 0.05$$

$$= 0.978$$

Solution contd.

In more general form, the expansion of $(p + q)^n$ is given as

$(p + q)^n = p^n$ x $np^{n-1}q + n(n-1)/2!p^{n-2}q^2 +$ ……… + ${}^nC_r p^{n-r} q^r +$ …… q^n

$Pr = {}^nC_r$ x p^r x $(1 - p)^{n-r}$ i.e probability of r successes in n trials.

Note

a) Since $p + q = 1$, therefore $(p + q)^n = 1$

Consequently, the sum of all the terms in the RHS of the expansion $(p + q)^n$ is always equal to 1.

b) This idea of binomial distribution will be useful in finding the reliability of systems with partial redundancy and conditional (majority voting) redundancy.

Redundancy

The parallel arrangement of system modules is often called redundancy and is used in critical applications (e.g. nuclear reactors, petrochemical plants, aircrafts, etc) where very high reliability is required. If three parallel paths are used with each path having a reliability of 0.85, for example, a system reliability of 0.997 is achieved. Redundancy can be defined as the provision of more than one means of getting an item to perform a given function. Use of redundancy is meant to ensure that a system continues to function satisfactorily in spite of failure of some of the items from which it is built-up.

Redundancy

It is important for systems employing redundancy that protection against common failure routes (called common mode failures) is included. Separate and independent power supplies, for example are essential.

Advantages

1) It aid planned preventive maintenance
2) The existence of a redundant element can allow for repair, in some cases, with no system downtime.
3) In situation where equipment cannot be maintained e.g. space – craft, redundant elements may prolong operating time significantly.

Limitation

The applications of redundancy is not without limitations.

It increases weight, space, complexity, Cost,

time to design and maintenance cost.

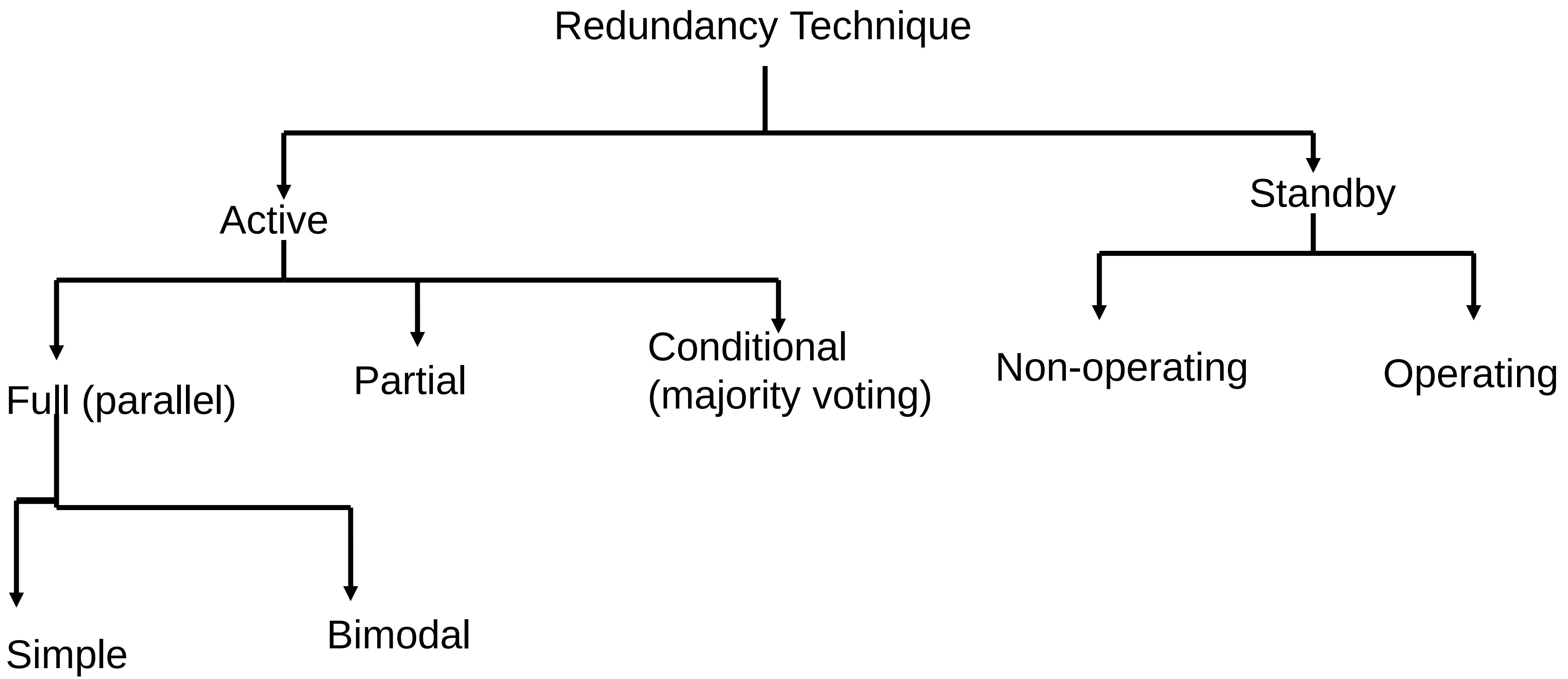
Classification of Redundancy

There are two main types of redundancy in common use namely:

- Active redundancy and
- Standby (passive) redundancy.

However, redundancy may further be subdivided under different headings as summarised in figure.

Classification of Redundancy



Active Redundancy

In this case, all the units are energized simultaneously to perform a given function.

In the context of active redundancy, the expected function can be performed even if only one out the several units is working.

The example of active redundancy is the connection of three altimeters to measure how high the plane is flying from the ground level in an aircraft.

Passive or Standby Redundancy

In this case one of the alternative units or systems is energised at a time, and there is provision to switch in another unit if one fails.

For example, in many establishments where continuous supply of electricity is necessary (e.g. operating theatre of a hospital) provision is made to switch over to a standby generator if mains electricity supply fails.

Partial Active Redundancy

In the case of partial active redundancy, the number of units permitted to fail is less than in full active redundancy. Unlike the full redundancy, more than one path must continue to function if the system as a whole is to work.

An example is an aircraft with four engines. It could almost certainly land safely if three were still working, but safe landing might be difficult, if not impossible, with only one engine.

Calculation of reliability involving partial redundancy is based on the use of binomial expression.

Practice problems

1) The unreliability of an aircraft engine during flight is 0.01. what is the reliability of successful flight if the aircraft can complete the flight on at least three of its four engines?
2) A power transmission system shown in figure Q2 consists of a step-up transformer T_1, two circuits of power transmission lines L, and two step-down transformers T_2. Full power can be transmitted to a consumer through any of the two transmission lines circuits. The step-down transformers can handle 50% power only. The unreliabilities of transformer T_1, each transmission lines circuit and each step-down transformer are Q_{T1}=0.005, Q_L = 0.03 and Q_{T2} =0.004 respectively.

Examples contd.

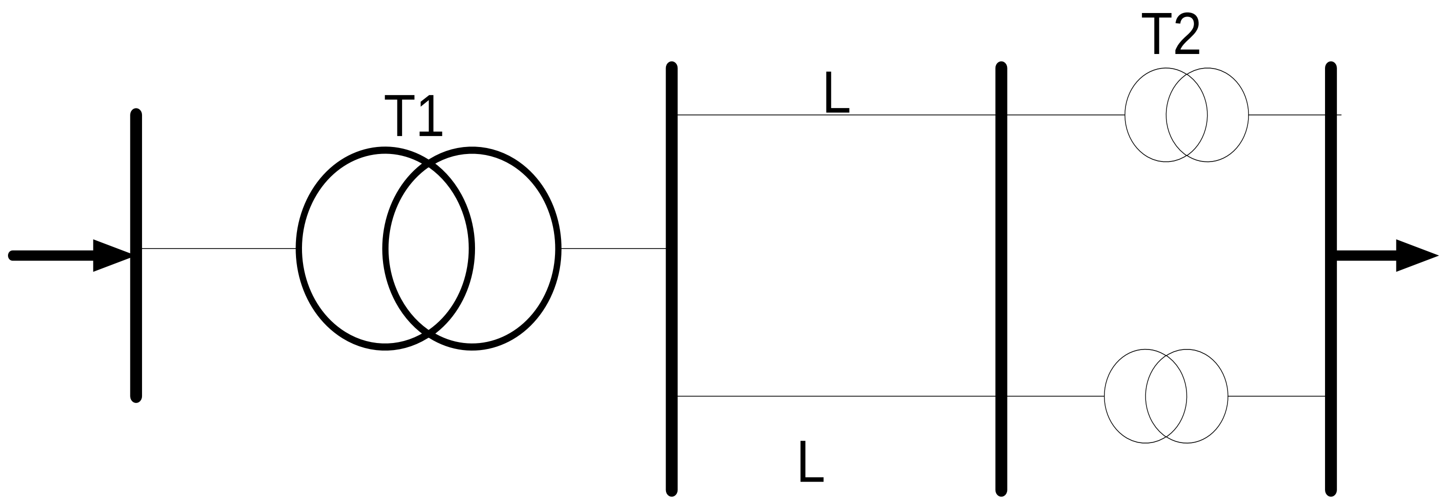


Fig Q2

Examples contd.

Failures in all these components are random and independent. Assuming that constant power is transmitted, determine the reliability of the system under the following conditions.

(a) 100% power transmission

(b) 50% power transmission.

Majority Voting Redundancy

In majority voting redundancy (sometimes called conditional active redundancy) m-out-of-n-units of a system are required to be functioning for the system to function.

This concept is illustrated in fig8

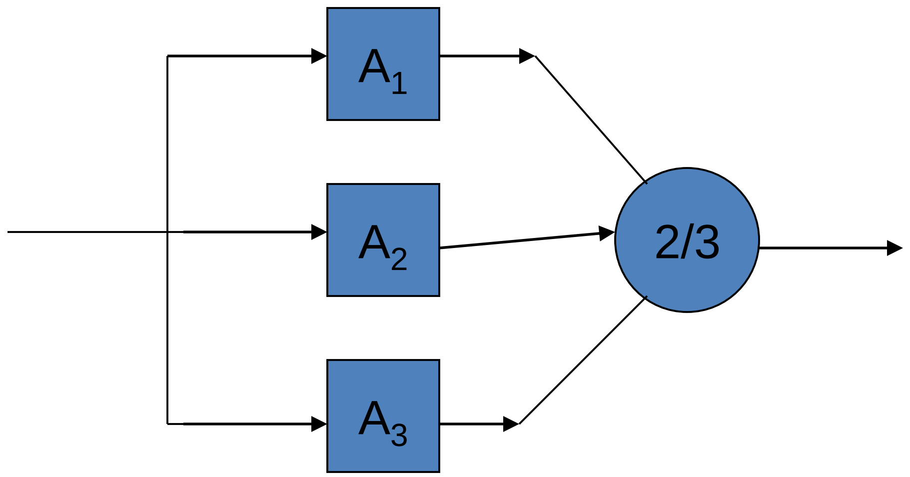


Fig. 7 Majority Voting Redundancy.

MVG contd.

In Fig.7, there are three processing units whose outputs are sent to a 2-out –of -3 or 2/3 majority voting gate (MVG), which is a decision maker.

The output of the MVG will be the same as that recorded by the majority of the processing units.

The reliability of an m/n system where all n units have the same reliability value ,R, is given by the binomial expression.

$$R_{m/n} = 1 - \sum_{i=0}^{m-1} nC_i R^i (1-R)^{n-i} \qquad 10$$

MVG contd.

This assumes that the reliability of the MVG is $R_v = 1$. If $R_v \pm 1$, then the system reliability is given by $R_{sym} = R_{m/n} \times R_v$ 11

The reliability of the system can also be found via binomial distribution approach. Consider the case of a ¾ - system in which all four units have reliability R. The unreliability of each units is Q with R + Q = 1.

For all four units, we have

$(R + Q)^4 = R^4 + 4R^3Q + 6R^2Q^2 + 4RQ^3 + Q^4 = 1$

MVG contd.

N ow

R^4 = Probability that all 4 units are Reliable.

$4R^3Q$ = Probability that 3 units are reliable
and 1 unit is unreliable.

$6R^2Q^2$ = Probability that 2 units are reliable and
2 units are unreliable.

$4RQ^3$ = Probability that 1 unit is reliable and 3 units
are unreliable.

Q^4 = Probability that 4 units are unreliable.

MVG contd.

The probability that at least 3 out of 4 units are functioning is $R_{3/4} = R^4 + 4R^3Q$.

For each unit, $Q = 1 – R$, hence

$R_{3/4} = R^4 + 4R^3(1 – R)$

Alternatively,

$$R_{3/4} = R^4 + 4R^3Q = 1 – (6R^2Q^2 + 4RQ^3 + Q^4)$$
$$= 1-[6R^2(1-R)^2+4R(1-R)^3+(1-R)^4]$$

System Reliability can also be evaluated in cases where the units have different reliability values. Consider a 2/3 system with units having reliabilities R_1, R_2 and R_3 respectively.

MVG contd.

We have

$(R_1 + Q_1)(R_2 + Q_2)(R_3 + Q_3) = 1$ Which when expanded yields:

$R_1R_2R_3+R_1R_2Q_3+R_1R_3Q_2+R_2R_3Q_1+R_1Q_2Q_3$

$+R_2Q_1Q_3+R_3Q_1Q_2+Q_1Q_2Q_3=1$

Hence

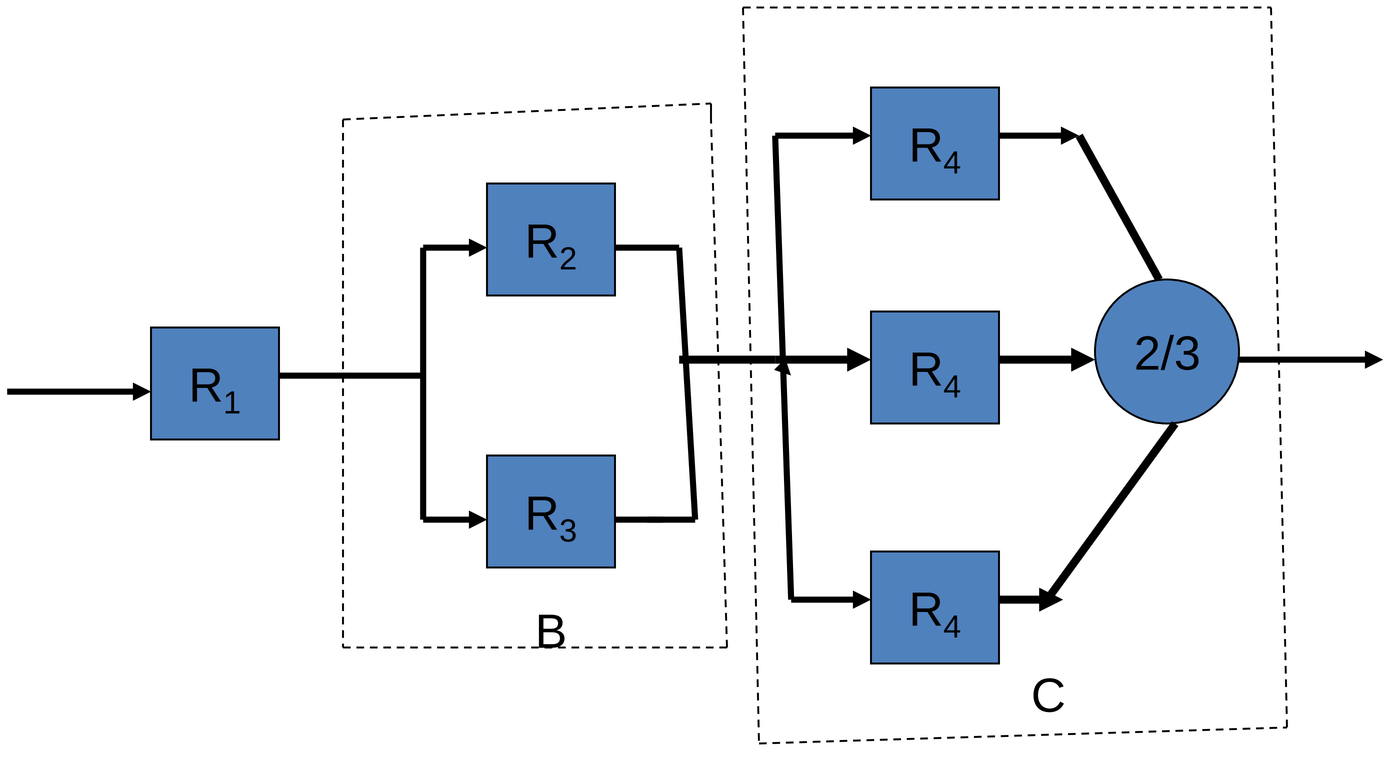
$R_{2/3} = R_1R_2R_3 + R_1R_2Q_3 + R_1R_3Q_2 + R_2R_3Q_1$

NOTE

Other terms in the LHS Of the expression above containing more than one unreliability (i.e. $R_1Q_2Q_3$, $R_2Q_1Q_3$ and $Q_1Q_2Q_3$) are not useful in finding $R_{2/3}$ therefore they are absent in the value of R_{2-3} above.

Example1

Determine the reliability of the system of fig Q1. Assume that the 2/3 MVG is perfect.



FigQ1

Passive or Standby Redundancy

In this case the system is provided with additional units, which are only energized when the active operating unit fails. Provision is also made for detecting the state of the active unit and switching to the passive standby unit in the event that the active unit fails.

The reliability of a system with passive redundancy is given by the first n terms of the poisson distribution.

Passive or Standby Redundancy cotd.

$$R = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2 t^2}{2!} + \bullet\bullet\bullet + \frac{\lambda^{n-1} t^{n-1}}{(n-1)!}\right.$$

For two units, one active and one standby, the reliability is given by the first two terms i.e.

$$R_{ab} = e^{-\lambda t}(1 + \lambda t)$$

Passive or Standby Redundancy cotd.

It is assumed that the device that senses failure of the active unit and then switches over to the standby unit is perfectly reliable. Moreover, all units are assumed to be identical with the same failure rate, λ

Example 2

A process control system has three process computers, two of which are on standby redundancy. Each computer has a failure rate, λ = 0.01 per hour. Calculate the system reliability if the operating period is 50 hours and the switch is perfect, using

i. A single computer

ii. One standby computer

iii. Two standby computers

Practice problem 1

Three temperature sensing elements X, Y, and Z, having reliabilities of 0.9, 0.8 and 0.7 respectively are connected to the same points of a process control system. An alarm is designed to be given if any two or more of these temperature sensors record a temperature above a certain prescribed level. Determine the reliability of the system if:

(i) The elements are connected in a 2-out-of-3 configuration,

(ii) All the elements must function for system success.

Practice problem 2

An electric supply system has three protective devices which are incorporated into it and installed in order to cut off electrical supply in the event of any untoward incident. The three protective devices are designated A, B, C, and it has been found that the unreliability of each of these devices is $Q_a = 0.005$, $Q_b = 0.01$ and $Q_c = 0.001$. It is given that if any two or more of the three devices fail to operate, then the system will fail.

Determine the unreliability of the system. Assume that other component parts of the system are perfectly reliable.

Hints: since there are three protective devices in the system, then $R_a+Q_a)(R_b+Q_b)(R_c+Q_c) =1$. On expansion of the LHS, terms containing more than one Q are those needed.