Understanding Correlation of Peculiar Velocity Vectors in the Cosmicflows-3 Database

by

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Thanks to everyone!

Abstract

My abstract goes here.

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1 Introduction

In localized regions of space, smaller scale influences such as gravitational attraction dominate pulling matter towards regions of higher density. A simple example of this would be a galaxy being pulled towards a larger, more massive galaxy. While this behavior is most certainly still present at scales much larger than individual galaxies, the “motion” that dominates in this realm is due cosmological expansion which is often referred to more simply as the “stretching” of space between objects. While the driving factor for this large scale motion is still in principle due to gravity, what we see is that objects in space are being stretched away from each other often faster than they are moving toward regions of higher density. However, the motion towards regions of higher density at large scales can be seen as deviations from an idealized isotropic model of cosmological expansion. These aberrations in the model that indicate large scale flows in contrast to the linear growth of the universe can tell us more about the origin of the universe from the Big Bang and what the behavior of the universe may look like in the future. In this thesis we concern ourselves with the influence of the deviations from the model of cosmological expansion and propose a method to adequately model the large scale motions of the universe.

2 Background

2.1 Background: Observational Cosmology Primer

2.1.1 Classical Doppler Effect

Suppose you are standing on the sidewalk and along the road a parked ambulance turns on its siren. As the ambulance siren blares, it approaches you and you notice that the pitch of the siren seems to have increased such that it sounds higher than it did previously. Then, as the ambulance passes and recedes behind you another change occurs, but now the pitch seems to have decreased such that it sounds much deeper than before. This is a classic example of the phenomenon known as the *Doppler effect* where a stationary observer experiences a change in frequency due to a moving wave source. More precisely, the Doppler effect produces an observed shift in the original wave emitted such that an approaching wave source (i.e. the ambulance moving towards the observer) will seemingly appear to have an increase in frequency, meanwhile a receding wave source (i.e. the ambulance moving away from the observer) will appear to have a decrease in frequency. However, it is important to keep in mind that while a shift is observed, in actuality there are no changes to the emitted wave or the wave source itself. The relationship for the *Doppler shifted* frequency observed is given by the expression:

$$f_{\text{obs}} = f_{\text{em}} \left(\frac{v_{\text{wave}} + v_{\text{receiver}}}{v_{\text{wave}} + v_{\text{source}}} \right) \tag{2.1}$$

where f_{obs} describes the observed Doppler shifted frequency, f_{em} is the emitted frequency by the wave source, v_{wave} is the speed of the emitted wave and is defined by wave velocity equation as the wavelength λ_{em} times the frequency f_{em}, v_{receiver} is the velocity of the observer receiving the signal (positive when moving towards the source and negative in the opposite direction), v_{source} is the velocity of the moving wave source emitting the signal (positive when receding from the observer and negative in the opposite direction)[1].

From Eq. **??**, we can see that for an approaching ambulance for which the observer is at rest: $v_{\text{receiver}} = 0$, v_{source} is negative indicating the ambulance is moving towards the observer, and v_{wave} is the speed of sound at approximately 343 m/s. Given these conditions, we can write the Doppler effect for sound observed by a stationary observer (as in our ambulance example) as

$$f_{\text{obs}} = f_{\text{em}} \left(\frac{1}{1 - v_{\text{source}}/v_{\text{wave}}} \right) \tag{2.2}$$

where we have simplified the equation in terms of the ratio of the source velocity to the speed of the wave.

While the Doppler shifted sound waves experienced by an observer can accurately be described by Eq. **??** and Eq. **??**, this form of the equation is only true for wave sources that move much slower than the speed of light. For electromagnetic waves (such as visible light) special relativity dictates that the relationship must be altered such that Lorentz symmetry[2] is upheld. Therefore, the relativistic form of the Doppler effect becomes

$$\nu_{\text{obs}} = \nu_{em} \sqrt{\frac{1-\beta}{1+\beta}} \tag{2.3}$$

where ν is the relativistic notation for frequency (equivalent to f in the classical case) and the dimensionless $\beta = v/c$ is ratio between the relative velocity $v =$ v_source $-$ v_rec to the speed of light c. Note that β is positive when the observer and the source are receding away from each other and negative when the observer and the source are approaching each other—analogous to v_{source} in the classical case. The relativistic Doppler effect may seem quite different from our classical case in Eq. **??**; however, if we Taylor expand about the low-velocity limit (for $v \ll c$) where the relative velocity is much slower than the speed of light we find that $\nu\text{obs} = \nu_{\text{em}} \left(1 - \beta + \mathcal{O}(\beta 2)\right)$. Using the same low-velocity analysis on @eq:classicaldop, we find that $f_{\text{obs}} = f_{\text{em}} \left(1 - \beta\right)$ such that the relativistic Doppler effect reduces to the classical case for relative velocities much less than the speed of light.

[1] As you may have noticed, the relationship results in a constant Doppler shift in the frequency which is not what you would actually observe when an ambulance passes by. The increasing and decreasing in frequency is actually a product of the ambulance passing parallel to the observer which results in a changing angle between the moving wave source and the line of sight between the ambulance and the observer such that $v_{_{LOS}} = v_{\text{source}} \cdot \cos\theta$. However, if the ambulance were to drive directly at the observer, they would would hear a constant increase or decrease in the frequency as is the case in our example.

[2] Lorentz symmetry states simply that the laws of physics must uphold in any reference frame.

2.1.2 Redshift

The Doppler effect is particularly useful in astronomy because each element has a unique spectrum of emission and absorption lines that will appear Doppler shifted by a moving astronomical source. Astronomers may draw useful information from Doppler shifts in spectroscopic measurements, such as the distance, velocity, and composition of an astronomical object. This is accomplished by comparing the position of the spectral lines of a specified element against their rest frame wavelength. We will find it useful to define the relativistic Doppler shift in terms of the redshift z of the initial signal as

$$z \equiv \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} - 1 = \frac{\Delta\lambda}{\lambda_{\text{em}}} = \sqrt{\frac{1+\beta}{1-\beta}} - 1 \tag{2.4}$$

where we have used the wave velocity relation to rewrite the Doppler shift in Eq. **??** in terms of wavelength. The term redshift is used because it implies that the signal's wavelength has been lengthened, synonymous to longer wavelengths in the visible spectrum corresponds to red light. Likewise, a negative redshift is equivalent to a blueshift, a shortening of the signal's wavelength. From the limiting case for the low-velocity regime where $v \ll c$, using our previous result in Eq. **??** we find that

$$z \simeq \frac{v}{c} \tag{2.5}$$

where receding sources appear redshifted and approaching sources appear blueshifted relative to an observer.

2.1.3 Standard Candles

A Standard candle is a class of astronomical objects that possess an intrinsic quality shared amongst the class, a 'standard' so to speak, that provides a well known luminosity. Examples of standard candles include Cepheid variable stars, planetary nebula, Tully-Fisher relation for spiral galaxies, and supernovae amongst others. All of these contribute to the cosmic distance ladder, a chain of various techniques used to measure different length scales. The foundation of the distance ladder is the geometric effect of measuring nearby astronomical objects called parallax. The parallax uses the apparent displacement of a distant object as the observer changes their position to measure the distance and is described by $d = 1/p$ (for $p \ll 1\,\text{radian}$), where d is the distance in parsecs (1 parsec $\simeq$ 3.26 ly) and p is the parallax angle in arcseconds. While this method provides some of the most accurate distance measurements, it is limited by the apparent displacement which for large distances becomes difficult to resolve\footnote{Parallax measurements from ground-based telescopes are able to resolve to distances of about 50-100 , while space-based telescopes are able to extend this limitation to 200-300 within 10% accuracy.}.

Despite the limitations of the parallax, the accuracy of the method allows astronomers to calibrate other methods that can measure distances at much greater length scales on the cosmic distance ladder. Cepheid variables are stars that undergo a period of pulsation—a defined period of contraction and expansion—during which a change in brightness is observed[3]. Due to the change in physical size, the absolute brightness or *luminosity* of the Cepheid changes such that we observe a change in apparent brightness on Earth. What makes Cepheid variables an important standard candle is that not only does this intrinsic quality holds amongst the class, but more importantly, from empirical evidence we know that there is a direct relationship between the period of pulsation and the true luminosity of Cepheids: unsurprisingly, we call this the *period–luminosity relationship*. Distance measurements are normally a challenge to calculate due to the required luminosity as defined by the inverse square law for light:

$$\text{apparent brightness} = \frac{\text{Luminosity}}{4\pi \cdot d^2} \tag{2.6}$$

where d is the distance, L is the luminosity, and B is the apparent brightness as measured on Earth. But, by measuring the period of pulsation from changes in apparent brightness we are able to determine the luminosity and thus measure distance to Cepheids in galaxies far greater than the reach of parallax measurements. The accuracy of the period–luminosity relationship is dependent, however, on standard luminosity measurement to nearby Cepheids in which the parallax distance can be used to calibrate the method. For example, in 2002 the Hubble Space Telescope in partnership with the Hipparchus Satellite was able to use the parallax method to determine the most accurate distance measurement to the closest known Cepheid variable star, Delta Cephei, thus establishing a cosmic benchmark on the ladder for other distance methods. The period–luminosity relationship is often regarded as the stepping stone by which other methods are compared against to improve the accuracy of our distance measurements.

A more common astronomical method of measuring distance analogous to the inverse square law of light in Eq. **??** involves the magnitude system, a logarithmic magnitude scale of a stellar object's brightness[4]. Due to the logarithmic nature, the system is fairly counterintuitive such that an object brightness are given in 'reverse order' whereby brighter objects have a negative magnitude while dimmer

[3]It should be noted that when we mention brightness we are explicitly referring to the flux of light we measure in .

[4]invented by the Greek astronomer Hipparchus and Ptolemy. Eye sees logarithmically. Makes sense!

objects have a positive magnitude. We define the apparent magnitude (as seen from Earth) as

$$m \equiv m_a - m_b = -2.5 \log_{10} \left(\frac{B_a}{B_b} \right) \tag{2.7}$$

From Eq. **??** it is clear that a change of 1 magnitude corresponds to a 2.5 decrease in brightness between the objects. In order to standardize the relative comparison of magnitudes we compare stellar objects to the relative brightness to Vega such that objects that are brighter have a negative magnitude and objects that are dimmer have a positive magnitude[5]. Just as in Eq. **??**, in order to know the absolute magnitude we need to know the distance to the stellar object and we define this relationship as

$$M = m - 2.5 \log_{10} \left[\left(\frac{d}{10\,\mathrm{pc}} \right)^2 \right] \tag{2.8}$$

The convention used here is to calculate the absolute magnitude as would be seen from 10 away for all objects to standardize the method. If we simplify the equation we can rewrite Eq. **??** in terms of what we call the *distance modulus equation* such that

$$m - M = -5 + 5 \log_{10}(d) \tag{2.9}$$

Building off the stepping stone of the Cepheid' period–luminosity relationship, we can use the Tully-Fisher relation and Fundamental plane to extend the distance ladder to scales on the order of 1—the length scales of galaxy clusters. The Tully-Fisher relation is an empirically driven relationship between the rotational velocity of a galaxy and its absolute magnitude. In the context of the distance ladder it forms one of the most important distance estimators due to its measurement of the 21-centimeter hydrogen line, a far infrared line in the microwave spectrum, which is relatively unperturbed by objects that normally are opaque to shorter wavelengths[6]. Given the 21-centimeter hydrogen line, we can measure the redshift and blueshift due to the rotation of a galaxy and estimate the rotational velocity of the galaxy. However, given that we need distance estimates to measure the luminosity in terms of the absolute magnitude, again we can see the distance ladder in effect in that we use Cepheid distance measurements from both

[5]By this standard, it would naturally follow that Vega has a reference magnitude of 0

[6]This technique is especially important to physical cosmology for its accurate prediction of the ratio of luminous matter in the galaxy disk to the dark matter halo that surrounds it. For further reading refer to Desmond et al., 2015 on the Tully-Fisher Relation and mass-size relations to halo abundance. https://arxiv.org/abs/1506.00169

parallax and period-luminosity relationship within individual galaxies to calibrate the distance to the galaxy themselves. Subsequently, we can plot the rotational velocity and the absolute magnitude of individually calibrated galaxies to show the Tully-Fisher relation. Thus, given a rotational velocity from far redshift objects we can not only estimate the absolute magnitude, but more importantly a distance estimate that otherwise wouldn't be possible.

As we have seen, these techniques often overlap in the length scales they measure which is important for the calibration of the ladder through overlapping distance measurement within individual galaxies allowing allowing for multiple classes of distance estimators. Moreover, from Eq. **??** equation, we can see that the accuracy of our distance estimates depends on how well we know the object's true luminosity is known. Thus, determining the most suitable and accurate standard candles is the trickiest aspect of distance measurements as each method introduces some form of uncertainty. Regardless of the challenges in the method, distance measurements are one of the only methods that allow us to probe the cosmos and test our theories of the Universe. From this perspective, we see that distance measurement truly provide the foundation for understanding the structure and evolution of the universe—the *cosmology* of our universe.

2.1.4 Hubble's Law

In the late 1920s using distance and spectroscopic redshift measurements of Cepheid variables, Edwin Hubble found that a galaxy's relative velocity away from Earth is proportional to the distance between us. The relationship now known as Hubble's law can be expressed as

$$v \simeq cz = H_0 d \tag{2.10}$$

where we have expressed the relative velocity v (in km) in terms of the low-velocity limit from Eq. **??**, H_0 is the proportionality constant between the relative velocity and the distance to the object in , and d is the distance in Mpc. The proportionality constant H_0 is better known as the Hubble constant, but is most usefully written as $H_0 = 100\,h$ where h is the dimensionless Hubble parameter defined by the current accepted value.

While the redshift observed on Earth is often interpreted as a Doppler shift due to the method of measurement, this leads to a troubling interpretation of the Universe. That being that if the relative velocity is due to a Doppler redshift, than this would imply that everything in the night sky is receding away from us and that the Earth is located in a privileged place in the Universe. However, as Hubble amongst others had noticed, this statement is in obvious conflict with the Copernican principle which states that humans are not privileged observers of the weygaert/tim1publicpic/dtfe/2dFpanel.dtfe.lres.gif

Figure 2.1: img

Cosmos–i.e. that we are not located anywhere special with regards to the Universe around us. Another possible interpretation that does not require sacrificing the Copernican principle had been proposed independently as part of solutions to Einstein's field equation for general relativity by the Soviet mathematician Alexander Friedmann and the French astronomer Georges Lamaitre: that we reside in an expanding universe and Hubble's law provided the first empirical evidence supporting this theory. Under this interpretation, the relative velocity is not a result of intrinsic motion of galaxies away from us, but rather the result of the space between us stretching.

Moreover, Hubble's law not only implies that the Universe is expanding, but that it is doing so at an accelerating rate. The Hubble constant is currently measured in the range of 67-75 [7], meaning that every megaparsec of space is being stretched at a rate of approximately 70 km.

2.1.5 Peculiar Velocity

Hubble's law represents an empirical relationship for the expanding universe where each galaxy is static with respect to cosmological expansion. Given Eq. **??** we can estimate the cosmological redshift $z_{\rm cos}$ due to expansion from $z_{\rm cos} \equiv H_0 d/c$. However, an ideal Hubble's law neglects gravitational attraction to higher density regions of space which induce 'peculiar' motion that deviates from cosmological expansion velocity $H_0 d$ known as the Hubble flow. The consequence of a continuous inflow of matter towards these regions of space is a state of over-density known as gravitational collapse, which provides the means for the formation and growth of structure in the local Universe. Thus, measurements of peculiar velocity are a compelling cosmological probe of large scale structure due to being unbiased tracers for the underlying density field. We can estimate peculiar velocities as deviations within the observed redshift z_{obs} from Hubble's law as a result of Doppler redshift $z_{\rm doppler}$ due to local motion. Unfortunately we are observationally limited to receding and approaching wavelengths along the 1D line-of-sight component of a galaxy's full 3D peculiar velocity vector. Any transverse motion along the line-of-sight is far too small to be detected through spectroscopic measurements and thus we would also expect any contribution to the line-of-sight to be in the form of noise in the signal. Following Eq. **??**, at low-redshift where $v << c$, the line-of-sight component of the peculiar velocity v_p is given by the familiar form

[7] Add footnote on discrepancies between Planck Satellite and Supernovae Type Ia measurements

$$v_p \approx cz_{\mathrm{obs}} - H_0 d \tag{2.11}$$

where c is the speed of light, z_{obs} is the observed redshift, and $H_0 d$ is the cosmological expansion velocity at a given comoving distance d[8][9]. Therefore given a distance indicator, the comoving distance d can be measured independently from the observed redshift z_{obs} allowing us to estimate the peculiar velocity. The most common form of distance indicators are the various standard candles introduced in Sec 1.# which unfortunately suffer from large uncertainties. In contrast to the high accuracy redshift measurements available, the uncertainty on distance estimators can be upwards of $\simeq 20\%$ which can lead to uncertainties in the peculiar velocity measurement on the order of the velocity itself. Typical peculiar velocity measurement are expected to be of order 300 while the errors scale largely with the distance of the galaxy such that uncertainty δ_{v_p} can be approximated by $\delta_{v_p} \approx 0.20 H_0 d$. Given the large uncertaintites, pecular velocity measurements must be estimated through statistical means thus requiring large velocity surveys with both distance and redshift measurements[10].

[8]Comoving distance refers to the real distance between two objects at rest with respect to the Hubble flow. In simpler terms, comoving distance is the physical distance between two objects in the absence of cosmic expansion.

[9]Davis et al shows that due to redshift not being an additive property this crude approximation is only accurate for $z_{\mathrm{obs}} << 0.1$, but for the introductory purposes of this section we find this form more than sufficient. We will revisit this equation in Section 2.

[10]In Section 2. we will discuss an estimator method developed by Watkins Feldman that provides a more accurate method of statistically estimating peculiar velocities.

3 Theory: Method for Measuring Correlation in the Velocity Covariance Matrix (title is a work in progress)

In this section I will be writing about how we will be measuring correlation of peculiar velocities using a new method we are developing in this thesis. We begin with a given a set of distance and redshift measurements, from which we can calculate the peculiar velocity of each object as detailed in the background. We can then organize these measurements to represent a velocity field $v(r)$ where we can use the galaxies to trace the large-scale motion. Performing the Fourier transform of the peculiar velocity field allows us to measure the velocity power spectrum of the field. Since the velocity power spectrum is directly related to the density power spectrum, we can observe large-scale perturbations in the density spectrum and identify their origin. Since the density power spectrum relies on cosmological parameters, to estimate the bulk flow we must perform a likelihood analysis on the value of the cosmological parameters Ω_m, h and Ω_b based on on our peculiar velocity field data. The likelihood function involves a velocity covariance matrix for each peculiar velocity in the field. The method used in the past weighted the contribution of peculiar velocities to the bulk flow based on which scales they probe as measured in the velocity power spectrum. This method was useful in reducing the influence of small scale motions, but the bulk flow they measured disagreed with the value estimated from the standard cosmological model (ΛCDM model) using the cosmic microwave background (CMB) data. Thus, we will be reassessing the inclusion of the small-scale influences by using the velocity covariance matrix to probe the best cosmological parameters that match the data for measuring the bulk flow.

4 Data Analysis and Modeling

In this section I will be using the *Cosmicflows-3* (CF3) distance and redshift values to put our theory into effect! Below is a running list on how we will do this based on the theory in Chapter 3:

- We will calculate the peculiar velocity field
- Perform a Fourier transform on our data
- Measure the velocity and density power spectrum
- Calculate the velocity covariance matrix (i.e. the correlations)
- We will determine various maximum likelihood parameters based on the inclusion of the velocity correlations.

5 Results

In our results section we will determine which cosmological model best matches the velocity correlations present in the CF3 catalog using the velocity covariance matrix.

In our discussion/conclusion sections we will identify if disagreement is still found between our findings and the CMB data. If disagreement does exist and a plausible explanation for this disagreement can't be produced, then this would suggest a need to re-evaluate the current standard model. Personally, I hope that we find that our new method is consistent with the standard model as this would provide a valuable tool in understanding large scale motion. However, regardless of the results, we will either find a more suitable method for measuring the bulk flow or another method that despite taking small-scale influences into account still doesn't align with the standard model.