

Sequential location among firms with foresight

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Existing theory poorly describes the product diversity in a modern market economy largely because such theory is founded on an inadequate concept of equilibrium. Standard analysis regards decision makers as naive in their anticipations of the response of rivals to their decisions and neglects the substantial costs of relocating in the product characteristic space. In this paper, we construct an equilibrium model of firms in which each firm locates in sequence with correct expectations of the way its decisions influence the decisions of firms yet to locate. The nature of the equilibrium is explored in a series of familiar examples taken from the literature.

1. Introduction

■ The modern theory of the firm seems ill-equipped to cope with some aspect of firms' market decisions. The bulk of this theory is designed to describe the nature of market equilibrium in a well-defined industry that produces a well-defined product. Existing theory has difficulty with the diversity of product characteristics offered in a market where products are differentiated. When nonconvexities are unimportant, of course, a result much like perfect competition—with the product space filled by a complete spectrum of product varieties and the particular tastes of each consumer satisfied—can be expected (Rosen, 1974; Visscher, 1975). However, there exist cases in which imperfect competition is of interest. Most product differentiation occurs in industries with some economies of scale that limit the extent of product variety. The perfectly competitive model modified to permit product variety is thus only a benchmark. The significance of deviations from the perfect competition benchmark cannot be judged until an adequate model is found to describe the firm's product characteristic choice in *imperfectly* competitive markets.

Chamberlin's model (1933) posited imperfectly competitive mar-

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kets in which products are differentiated, but he made no predictions about the variety of products one should expect to see in market equilibrium. Discussion of product differentiation in oligopoly, the other market structure marked by nonhomogeneous products, is limited essentially to work stemming from the plant location model of Lösch (1954) and the famous duopoly location model of Hotelling (1929).

The Hotelling model has been an obvious foundation on which to build a theory of location in product characteristic space. Hotelling analyzed the behavior of duopolists locating a single store on a finite one-dimensional geographical market. The concept of location extends easily to any choice of a product characteristic. For example, firms often “locate” along a single dimension in choosing product durability and quality (sudsiness, softness, cleaning power, absorbency, etc.). Hotelling asserted that in his problem the duopolists, able to make costless relocations, attain a noncooperative equilibrium back-to-back in the center of the market.

The Hotelling model has been adopted with some success by Downs (1964) to explain centralist tendencies in political platforms and by Steiner (1961) to explain similarities in television programming on different channels. But a fundamental assumption of the Hotelling solution—that location, like price, can be altered costlessly—makes it inappropriate for the study of much actual market product differentiation. There may well be firms like the proverbial ice cream vendors on the beach that face small costs of relocating as Hotelling assumed. But it commonly seems that the costs of relocation are quite substantial. Relocation in product characteristic space may mean retooling dies, changing consumer images via advertising, or moving a plant site.

Given these not inconsequential relocation costs, it may be more reasonable to model firms as making location decisions once-and-for-all, one firm at a time, with firms being aware of the relative permanence of their decisions and thus taking some care to anticipate the decision rules firms entering later in the sequence will follow.¹ We offer such a model here and argue that theory must go in this direction to advance further the usefulness of models of the firm.

Firms in our model locate one at a time. It can be argued that firms enter in sequence because some entrants become aware of a profitable market before others or require longer periods of time in which to “tool up.” Further, each firm’s location decision is once-and-for-all; relocation is assumed prohibitively expensive. Moving a store after witnessing the location choices of others or retooling product dies as new products enter is a very costly procedure. Each firm is assumed to choose the profit maximizing market position based on the observed choices of firms already located and the location *rules* that subsequent, equally rational entrants and potential entrants will use. Thus, each firm takes into consideration the effect of its location decision upon the ultimate configuration of the industry.

Each firm in our solution recognizes that other potential entrants into the market are not unlike itself; no firm mistakenly considers itself a profit-maximizer in a world of fools. The expectations of the firm

¹ Hay (1976) has independently derived the infinite line equilibrium for our example 1 below by using the foresighted sequential solution concept.

about the response of other firms to its own decisions are rational in the sense that the expectations are consistent with the predictions of the model. In this way we are able to avoid incompatible incentives among firms that mark most other oligopoly models. Other oligopoly models offer a wide variety of *ad hoc* reaction functions, each with the property that in equilibrium each firm has incorrect expectations about the reaction of rival firms to changes in its own decision.

Further advantages of such a model are several. First, entry into the market can be endogenous in this model. Unlike many oligopoly theories, we need not assume that a specific number of firms enter, assign reaction functions to each firm, and solve for equilibrium locations. Instead, we can allow firms to enter sequentially and then determine simultaneously the equilibrium locations *and* the equilibrium number of firms. The resulting equilibrium may have all successful entrants making positive profits and yet further entry is deterred because the ensuing price competition would leave the further entrants with negative profits. Thus, the foresighted sequential entry solution makes formal the notion introduced in the limit-pricing literature (cf. Sherman, 1974, Ch. 14) of barriers to entry due to the threat of potential competition. Second, this model should provide a better description of behavior for some problems than other models provide. The test of any model is the accuracy of its predictions. We do not test our model rigorously, but we believe that it is likely to predict behavior better than other models in many cases. In part, this hunch is due to the assumed sophistication of the economic agents in our theory, together with the observation that when much is at stake and bad decisions are not easily corrected, it pays agents to be sophisticated in their decision making. Armchair perceptions of the world are, at least, not inconsistent with our model. Firms do not try to imitate an existing product when introducing a new product line, but rather aim for the "gaps" in the existing product spectrum.² Similarly, we find in our model that firms space themselves and do not cluster in the Hotelling fashion.

To introduce our proposed solution concept, in Section 2 we examine a series of examples that display important aspects of market equilibrium. The first example is the so-called "Hotelling problem" of a fixed number of firms competing on a finite linear market by location alone, but we employ our solution concept rather than the conventional one. That is, the firms enter sequentially and once-and-for-all with each entering firm correctly anticipating the decision of the remaining firms in the sequence of entrants.

The second example embellishes the first by making entry endogenous and unrestricted. Firms are spaced evenly in equilibrium at a distance determined by the fixed setup cost required for entry, with no tendency for firms to pair up back-to-back as in the Hotelling solution. The solution is unique in further contrast to the Hotelling solution where either no equilibrium exists (with three firms) or multiple equilibria can exist.³

The third example is the true Hotelling problem in which firms

² A plethora of marketing models are now available to determine the "locations" of such gaps and the size of the market the gap represents. See Kotler (1971, Chapter 17).

³ As Gerard Butters observed at the March 1976 NBER-NSF Conference on Theoretical Industrial Organization, there exist multiple equilibria in the "Hotelling" solution when the number of firms exceeds five.

compete noncooperatively on the basis of both location and price. There is no noncooperative solution to the original Hotelling problem because of a discontinuity in the reaction function of each firm. After modifying the Hotelling structure to ensure continuity and using numerical methods, we find that duopolists seek maximum separation rather than the minimum separation claimed by Hotelling. With three firms, no noncooperative equilibrium in pure (nonrandomized) strategies was obtained even for this modified Hotelling structure.

In the fourth example, however, we assume sequential entry and employ our recursive solution concept for the modified Hotelling structure to obtain a unique equilibrium without reliance on randomized strategies. Endogenous entry by firms is easily admitted to this model.

The final example studies an industry in which entrants choose in sequence only production capacities. In this example with restricted entry, complete monopoly results.

Section 3 notes the tendency to monopoly in the sequential entry equilibrium when the number of locations a firm can occupy is not arbitrarily constrained. Section 3 also presents potential reasons for the apparent limit to the number of positions a firm can occupy.

■ The following examples help illustrate the solution concept we offer. The examples represent familiar structures in the literature. The object is to point out the nature of the equilibrium when firms locate sequentially and to contrast the result with that from other well-known solutions.

2. Some examples

□ **Example 1.** The first example will be what is commonly called the Hotelling (1929) spatial location problem but with sequential location. The space of potential locations is the unit interval, and the uniform density of customers is N . Customers patronize the nearest store and purchase one unit (price is set exogenously and is equal at each store). Initially we follow Hotelling (or, for more than two firms, Eaton and Lipsey, 1975) and limit entry to a fixed number of firms, n . In sequence firms each locate one store on the unit interval to maximize profits, given the location of firms already located, knowledge that firms yet to enter will attempt to maximize profit, and knowledge of the fixed number of firms that will be permitted to enter.

The nature of the solution, assuming that the fixed cost of entry is sufficiently small that n firms can locate profitably, is obtained by backward inductive reasoning from the location decision of firm n to that of firm 1. The problem of firm n is simply to maximize its profits (proportional to the length of the interval that it serves) by choice of its location given the locations of the $n - 1$ firms that have already located. Since firm n is a profit maximizer, there is a decision rule it should follow in deciding where to locate. Because firm $n - 1$ knows firm n is a profit maximizer, firm $n - 1$ can use the rule firm n will apply to predict how firm n will locate as it varies its own position. Given that it can predict the response by firm n to its choice of position, firm $n - 1$ can determine a profit maximizing decision rule for itself as a function of the $n - 2$ occupied locations it observes when it locates. Firm $n - 2$ can use the decision rules it knows firms n and $n - 1$ will use to predict how their locational choices will

respond as it varies its own position. Given that it can predict the response to its choice by n and $n - 1$, firm $n - 2$ obtains a decision rule for itself as a function of the $n - 3$ occupied positions it sees. Working backward this way, the decision of the first firm can be found, and from that, the ultimate structure of the industry is determined.

For example, suppose the number of firms is two. After the first firm has located, the second firm will have a choice of locating in one of two disjoint intervals on either side of the first firm. The profit maximizing decision rule of the second firm is clearly to choose the larger interval and to locate as closely as possible to the position of firm 1. As Hotelling first noted, the second firm captures the trade of all consumers in the "hinterland" from it to the end of the market and must split the consumers between it and the other firm, so the best position is that which maximizes the size of the hinterland. Recognizing that this is the decision rule firm 2 will apply, the first firm can do no better than to locate in the market center and face an equal probability of firm 2's locating on either side an epsilon distance away (like Hotelling we assume no firm can locate on top of another).

Consider now the three-firm case. No Hotelling equilibrium exists (firms continue to leapfrog to obtain an outside position), but a well-defined equilibrium exists assuming foresighted sequential entry. Given the symmetry of the problem, we assume without loss of generality that x_1 , the location of the first firm, is less than $1/2$. The optimal decision rule for the third firm conditional upon the decisions of firms 1 and 2 is as follows:

- (i) If $x_2 \leq 1/2$, locate just to the right of $\max(x_1, x_2)$.
- (ii) If $x_2 > 1/2$ and $x_1 > \max[1 - x_2, (x_2 - x_1)/2]$, locate just to the left of x_1 .
- (iii) If $x_2 > 1/2$ and $1 - x_2 > \max[x_1, (x_2 - x_1)/2]$, locate just to the right of x_2 .
- (iv) If $x_2 > 1/2$ and $(x_2 - x_1)/2 \geq \max[x_1, 1 - x_2]$, locate at $(x_1 + x_2)/2$.
[Actually the third firm is indifferent among all points between x_1 and x_2 so our selection of the midpoint is somewhat arbitrary.]

Given this rule for firm 3 and the location of firm 1, the best location rule for the second firm is

$$x_2 = 2/3 + 1/2 x_1 \quad \text{if } x_1 \leq 1/4$$

or

$$x_2 = 1 - x_1 \quad \text{if } 1/4 < x_1 \leq 1/2.$$

Given these rules for firms 2 and 3, the optimal location for firm 1 is $1/4$. Therefore, the equilibrium locations for firms 1 and 2 are at $1/4$ and $3/4$ and the third firm locates between them. Clustering is thus avoided for all intents and purposes.⁴

□ **Example 2.** The method used to analyze the previous example becomes impractical when there are more than a few firms. However, we are able to characterize the solution when there are an infinite

⁴ It is perhaps worth noting that this solution invites mutually beneficial side payments, if they can be arranged. Observe that each of the first two entrants gains in market share $1/2$ of any further distance it can persuade the third firm to move away from it, and the third firm's market share is the same at all locations between the two initial entrants.

number of potential entrants and a fixed cost of locating. The infinite number of potential entrants assumption is admittedly unrealistic, but we need not apologize because the assumption imposes the same restrictions upon observed market structure as would be obtained for any finite n -firm problem for which n exceeds the number of firms which the market can profitably support. Letting α be market share needed to cover fixed costs, $[1/\alpha]$ is a bound on the number of firms which this market can profitably support.

To evaluate the profitability of a location decision, a firm must have expectations as to the way the industry structure will develop conditional upon its decision and the existing structure. We conjecture (and later verify that the conjecture is consistent with maximizing behavior on the part of subsequent potential entrants) that the firm locating on the unit interval expects the following to occur:

- (i) If two firms are located at x_A and x_B , where $x_A < x_B$, and there are no firms located between them, then $x_B - x_A \leq 2\alpha$ implies that no firms will ever locate in the $[x_A, x_B]$ interval, $2\alpha < x_B - x_A \leq 4\alpha$ implies that a subsequent firm will locate at $(x_A + x_B)/2$, and $x_B - x_A > 4\alpha$ implies that the next firm to locate in the interval will locate with equal probability at $x_A + 2\alpha$ or $x_B - 2\alpha$.
- (ii) If a firm is located at x and no other firm is located to its left, then $x < \alpha$ implies that no firm will ever locate in the $[0, x]$ interval, and $x > \alpha$ implies that the next firm to locate in the $[0, x]$ interval will choose the point α .
- (iii) If a firm is located at x and no other firm is located to its right, then $1 - x \leq \alpha$ implies that no firm will ever locate in the $[x, 1]$ interval, and $1 - x > \alpha$ implies that the next firm to locate in the $[x, 1]$ interval will choose point $1 - \alpha$.

Given this expectations conjecture, we now characterize the optimal location rule for a firm conditional upon industry structure at the time of its decision.

Given the conjecture above, the firm can predict its ultimate market share conditional upon its location decision and the set of locations already occupied. Its (expected) market share will be divided into sales to customers located to its left and sales to customers located to its right. The left- (right-) hand share depends only upon the distance z to its nearest existing left- (right-) hand competitor or to the end point if there is no such competitor. The left- (right-) hand market share is denoted by $w(z, e)$, where $e = 0$ if there is a competitor to the left (right) and $e = 1$ if there is not.

The left-hand value function $w(z, 0)$ must satisfy the functional equation

$$w(z, 0) = \begin{cases} \frac{1}{2} w(z - 2\alpha, 0) + \frac{1}{2}\alpha & \text{if } z > 4\alpha \\ \frac{1}{4} z & \text{if } 2\alpha < z \leq 4\alpha \\ \frac{1}{2} z & \text{if } 0 < z \leq 2\alpha, \end{cases} \quad (1)$$

where z is the distance to the nearest existing left-hand competitor. If this distance exceeds 4α , the conjecture implies that the next firm to locate between them will choose with equal probability a point 2α from either the nearest left-hand rival or the firm in question. In the former case the distance to the nearest left-hand competitor is reduced to $z - 2\alpha$. In the latter case the distance becomes 2α , and no

further reduction to the nearest left-hand competitor occurs. Consequently, its realized left-hand market share would be α under the latter contingency. This explains the first relationship. When $2\alpha < z \leq 4\alpha$, the conjecture implies that one additional firm will locate at the midpoint between the firm and its nearest existing left-hand rival. If $z \leq 2\alpha$, no subsequent firm will locate in that interval. This explains the remaining relations in (1). By an identical argument, the right-hand market share must also satisfy the functional equation (1) when z is defined to be the distance to the nearest right-hand competitor.

We can now write the explicit function satisfying the functional equation (1). Let l be the integer part of $z/2\alpha$ and r the remainder. Then

$$w(z, 0) = (1 - 2^{-l})\alpha + 2^{-l-1}r. \quad (2)$$

Another function $w(z, 1)$ is needed to specify the expected market share conditional upon selecting a point for which there is no existing nearest competitor to the left (right). By the conjecture, if $x > l$, the first firm to locate between 0 and x will choose location α . If on the other hand $x \leq \alpha$, no subsequent firm will locate to its left. Thus

$$w(z, 1) = \begin{cases} w(z - \alpha, 0) & \text{if } z > \alpha \\ z & \text{if } z \leq \alpha. \end{cases} \quad (3)$$

The same function specifies right-hand market share when there is no existing right-hand competitor, with z denoting the distance from x to the right-hand end point 1.

Let x_1, x_2, \dots, x_k be the locations on $[0, 1]$ that k previous firms have already occupied, where $x_i > x_j$ if $i > j$ (the size of the subscript here does *not* refer to the order of entry over time). The end points of the interval are $x_0 \equiv 0$ and $x_{k+1} \equiv 1$. Let $e(j) = 0$ for $j = 1, \dots, k$ and $e(j) = 1$ for $j = 0, k + 1$. This function indicates whether the point x_j is or is not an end point.

The expected value of the firm if it locates at point x between x_j and x_{j+1} is then the sum of left- and right-hand market share less the fixed cost of locating α ; that is,

$$v_j(x) = w[|x - x_j|, e(j)] + w[|x_{j+1} - x|, e(j + 1)] - \alpha.$$

The optimal location x conditional upon entry between x_j and x_{j+1} is (see Figure 1)

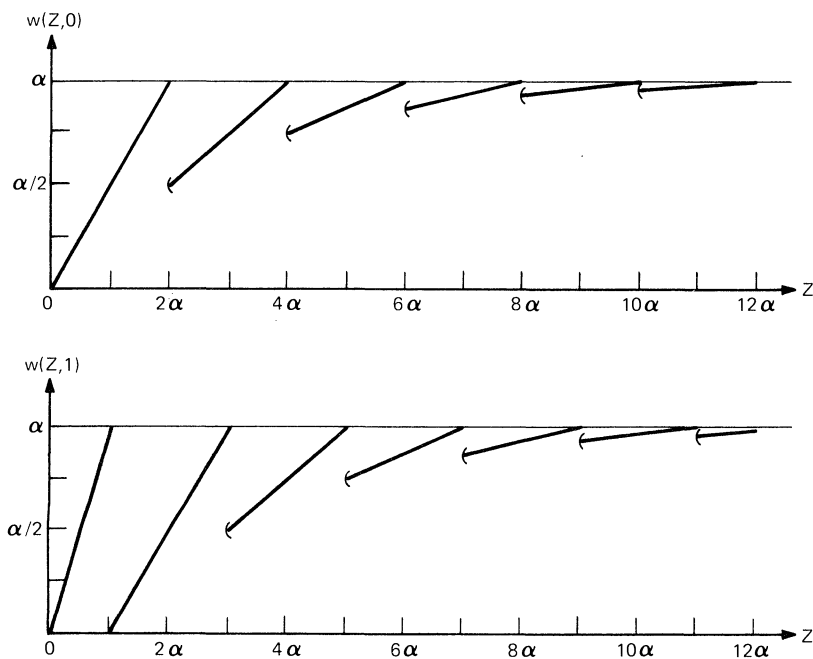
$$\begin{aligned} x &= \alpha && \text{if } e(j) = 1 \text{ and } x_{j+1} > \alpha; \\ x &= 1 - \alpha && \text{if } e(j + 1) = 1 \text{ and } x_j < 1 - \alpha; \\ x &= x_j + 2\alpha \text{ or } x = x_{j+1} - 2\alpha && (4) \\ x &= (x_j + x_{j+1})/2 && \text{if } x_{j+1} - x_j > 4\alpha \text{ and } e(j) = 0; \\ &&& \text{if } 2\alpha < x_{j+1} < x_j \leq 4\alpha. \end{aligned}$$

The optimum decision under any other circumstance is not to locate.

Subsequent firms to enter face essentially the same decision problem as that outlined above (there may be additional points occupied and, therefore, a different k , of course). Given that all firms share the conjecture, maximizing behavior on their part implies that their location decisions will satisfy (4). As (4) implies the expectations conjecture, the conjecture and maximizing behavior on the part of subsequent firms are consistent. Therefore, our conjecture is an equilibrium conjecture.

The equilibrium rule is not completely determined until we specify

FIGURE 1
THE w VALUE FUNCTION



how to choose among the $k + 1$ intervals. Let

$$v_j = \text{maximum } v_j(x) \\ x_j < x < x_{j+1}.$$

If the maximum v_j is 0, the firm does not locate. If the maximum v_j is positive, the firm chooses in any way among the j for which v_j is greatest. Letting the chosen j be j^* , the firm locates optimally in the $[x_{j^*}, x_{j^*+1}]$ interval. This decision rule maximizes expected profits.

With this equilibrium rule the first firm will locate α from one end (assuming $\alpha \leq 1/2$) and the second firm will locate α from the other end (assuming $\alpha < 1/2$). Subsequent firms locate 2α from the nearest competitor until this is no longer possible. If $1/\alpha$ happens to be an even integer or $1/\alpha < 4$, no subsequent firms enter. Otherwise, a final firm locates at the midpoint of that remaining unoccupied interval with length exceeding 2α . Except for the last firm, the spacing is uniform. Firms locate as far away from their nearest competitor as is possible without inviting entry in between.⁵

□ **Example 3.** Although a duopoly model with competition by location only (and not by price) on a finite, one-dimensional market is sometimes referred to as “the Hotelling problem,” the true Hotelling structure admits noncooperative price determination as well. Indeed, there is a bit of a sequential element in the manner in which the duopolists’ locations and prices are determined in the original Hotelling problem. *Given* any duopolists’ locations, Nash equilibrium prices are determined. *Given* that prices will be determined in this noncooperative manner after locations are chosen, profits can be

⁵ This location principle is appropriate, but spacing is not uniform, if the density of customers is not uniform.

written as a function of locations alone, and Nash equilibrium locations are then sought. The difficulty with this solution concept, as others have noted (Smithies, 1941; Eaton, 1976; and Salop, 1976), is that when locations in Nash are sufficiently close, Nash equilibrium prices will not exist. The nonexistence of equilibrium is a problem that frequently arises when reaction functions are, as in this case, discontinuous. The source of the discontinuity in the price reaction function here is that a lower price by one of the firms does not always gain the firm market share in a smooth continuous fashion. A price sufficiently low can capture the entire market, whereas a price slightly higher loses the rival firm's entire "hinterland."

In this example, we drop the foresighted sequential solution concept temporarily to look more carefully at the Hotelling problem and its original solution concept. We modify the Hotelling structure to avoid discontinuities in firms' price reaction functions, yet maintain the spirit of the Hotelling problem. Rather than locating stores in geographical space, consider firms that choose, in addition to price, a level of product quality (or rather defectiveness) to be thought of as customer waiting time at a retail outlet. Given the manner in which this product quality characteristic is admitted to our model, the price setting game is concave so price reaction functions are continuous; this is sufficient to insure the existence of equilibrium prices, given locations.

Each firm is constrained arbitrarily to offering a single waiting time. The profits of firm i are

$$\pi_i = [p_i - C(x_i)]q_i - F,$$

where p_i is price at outlet i , $C(x_i)$ is cost per unit sold when waiting time is x_i , F is the fixed cost of entry, and q_i is the quantity sold at outlet i . Consumers are indexed by their valuation of the (dis)amenity, to be thought of in this example as value of waiting time, $0 \leq v \leq V_m$, and are distributed uniformly over this interval with density N .

Consumer of type v purchases one unit from the outlet offering the smallest total price

$$p_i + vx_i.$$

If firm h offers the next shorter waiting time, x_h , and higher price, p_h , and firm j offers the next longer waiting time, x_j , and lower price, p_j , then sales q_i to firm i offering x_i and p_i are

$$q_i = N \left[\frac{p_h - p_i}{x_i - x_h} - \frac{p_i - p_j}{x_j - x_i} \right],$$

or the length of the value of time interval served by firm i multiplied by N . If no firm offers a longer wait, then

$$q_i = N \left[\frac{p_h - p_i}{x_i - x_h} \right].$$

If no firm offers a shorter wait, then

$$q_i = N \left[V_m - \frac{p_i - p_j}{x_j - x_i} \right].$$

In this revised Hotelling problem, each firm always gains or loses market share with small changes in price in a continuous fashion, thus eliminating the troublesome aspect of the original Hotelling structure.

Equilibrium prices, given location, are now assured. Therefore, equilibrium overall is more likely than in Hotelling's formulation. Using numerical methods, we obtain equilibrium prices and waiting time "locations" of duopolists employing Hotelling's adjustment mechanism in a sample problem. The cost function employed was of the form

$$C(x_i) = A/(A + Bx_i),$$

where A and B are constants.⁶ The equilibrium found in the sample problem examined is unique. However, more startling than the existence of a unique equilibrium where Hotelling's problem offers none is the nature of the equilibrium obtained. In contrast to the minimum differentiation Hotelling claimed, duopolists in the revised Hotelling problem have equilibrium locations far apart! The results of the exercise are displayed in Table 1. An intuitive explanation for the sig-

TABLE 1
HOTELLING EQUILIBRIUM

NUMBER OF FIRMS	x_i	$C(x_i)$	P_i	q_i	π_i
2	1.000	0.500	3.854	3.500	11.717
	0.040	0.962	7.208	6.500	40.640

nificant equilibrium spread between the duopolists' locations can be given. Locations too close together (i.e., products that are too close a substitute for one another) make price cutting appear too lucrative, given the naive expectations of Hotelling-like competitors. When the duopolists are close together, small decreases in one firm's price yield large increases in the market share captured from its competitors. Intense price competition, then, when the firms are in close proximity makes such locations undesirable from the point of view of either firm, even though if price competition could be avoided, those locations would be chosen as Hotelling asserted.⁷

Despite the modification in the structure of the Hotelling problem that makes equilibrium more likely, existence of an equilibrium still cannot be guaranteed. An example with three firms and the same parameters as the previous example produced no equilibrium. In the sample problem, when two firms begin far apart as in the two-firm equilibrium and a third firm is between the two widely separated firms, then the two firms with the longest waiting times proceed to jump over each other, attempting to offer a slightly shorter wait until one competitor gets too close to the firm offering the shortest wait. At that point the remaining firm lengthens his wait time dramatically to capture the portion of the market that has now been vacated. This permits the middle firm to increase its wait time somewhat (and avoid stiff price

⁶ In the sample problem, $N = 1$, $V_m = 10$, $A = 2$, $B = 2$, and $0.040 \leq x_i \leq 1.000$.

⁷ McGuire and Staelin (1976) show that wholesale dealers may use distributorships to buffer themselves from extreme price competition in much the same way as the duopolists in our example seek a buffer zone of waiting time locations between them to avoid aggressive price cutting.

competition with the shortest wait firm), whereupon the leapfrog process back to the shortest waiting time begins again.

This nonexistence problem with the Hotelling solution concept in pure (nonrandomized) strategies⁸ does not enhance the appeal of the theory. An added benefit of considering firms to locate in sequence once-and-for-all is the virtual guarantee of a well-defined equilibrium.

□ **Example 4.** The modified Hotelling structure (waiting time example) introduced in the previous section is used here as a vehicle for examining the nature of the equilibrium when firms with foresight enter sequentially and once-and-for-all.

If both waiting time “location” and price were chosen once-and-for-all, there would be difficulties with the sequential choice solution concept as severe as the nonexistence of an equilibrium using the Hotelling solution concept. For, if the first entrant must make an irreversible choice of both “location” and price, then the second entrant can always do at least as well as the first entrant, thus making entry later in the sequence preferred to entry earlier. The second entrant can always do at least as well as the first in this case, because the second firm always has the option available of choosing the same “location” as the first at an arbitrarily smaller price, leaving the first firm no *market share at all*. Therefore, if firm 2 considers a location at which its profits would be more than a very small amount less than firm 1, then firm 2 can do better by choosing the firm 1 location and a price slightly less than that of firm 1, in which case firm 2 will then have profits greater than firm 1 with certainty. The sequential “equilibrium” in this case would involve the second entrant’s being more profitable than the first. As it now stands, the model has no means of predicting when or whether the first of the potential entrants will enter rather than wait when, as in this case, each potential entrant would prefer entering second to entering first. Thus, the sequential solution concept appears inappropriate when later entry is preferred to earlier entry.

Fortunately, the choice of price is generally not so inflexible as the choice of product quality characteristics. Consequently, while it may be reasonable to assume that product quality is chosen sequentially and once-and-for-all, it may be better to assume that price can be costlessly varied by each firm after witnessing the price choice of all other firms. Therefore, in the example of this section, we compute the equilibrium in which each firm is assumed to locate by offering one waiting time, given the observed waiting times chosen by firms already in, the correct expectations of the way the waiting times of firms yet to enter will be chosen, and the knowledge that prices will be determined non-cooperatively in Nash fashion once waiting time offerings are chosen. The assumption that prices are costlessly changed by each firm in response to the decisions of other firms eliminates much (not all) of the indeterminacy in achieving equilibrium that was noted above when each firm’s price was assumed permanent.

The equilibrium waiting times in our example can be obtained by recursive methods. The procedure is as follows. Solve for the decision rule of firm $n + 1$, the first firm failing to enter successfully. This rule

⁸ Eaton (1976) documents similar difficulties with the original Hotelling structure when nonexistence due to discontinuities in the reaction functions of each firm is assumed away. Salop (1976) analyzes the Hotelling structure when the linear market is a circle and finds nonexistence is still pervasive.

defines the choice (including the choice not to enter) of waiting time by firm $n + 1$, given the previous choices of other firms. Substitute that decision rule for the expected choice of firm $n + 1$ in the maximization problem of firm n , the last firm to enter successfully. Solve for the profit maximizing decision rule of firm n as a function only of the choices of the $n - 1$ firms already entered. Proceed until the decision of the first firm is obtained. Use the decision rules derived to deduce the industry structure and confirm that firm $n + 1$ indeed fails to enter.

Table 2 describes the equilibria computed for $n \leq 3$ for specific assumptions about the size of the fixed costs of entry and demand.⁹

TABLE 2
SEQUENTIAL EQUILIBRIUM

FIXED COST OF ENTRY F	EQUILIBRIUM NUMBER OF FIRMS	ORDER OF ENTRY	x_i	$C(x_i)$	P_i	q_i	π_i
0.300	2	1	0.500	1.000	1.548	6.101	3.041
		2	0.590	0.848	1.198	3.899	1.064
0.200	2	1	0.500	1.000	1.311	6.062	1.685
		2	0.551	0.907	1.109	3.938	0.595
0.160	3	1	0.500	1.000	1.356	5.555	1.818
		2	0.564	0.886	1.071	3.569	0.500
		3	0.833	0.600	0.836	0.875	0.040

Several aspects of the sequential equilibrium are evident in the results of this example. As one might expect, the size of the fixed setup cost acts as a barrier to entry; the greater is the fixed cost, the smaller is the number of firms that enter. However, a change in the size of the fixed cost alters the "location" choices of the entrants, even when the change is not sufficient to influence the number of successful entrants. For example, the affect of an increase in the fixed cost from 0.20 to 0.30 does not merely decrease the profits of the two successful entrants without affecting location. Indeed profits of both firms actually increase! Closer scrutiny of the results produces the reason. When the fixed cost is at the lower level, the second entrant must locate closer to the first to forestall further entry by a third firm that would lower profits even more. The second entrant would prefer to locate farther from the first if entry could be artificially restricted to two firms, because price competition between rivals near one another is intense, and prices then get cut to less profitable levels. A higher fixed cost of entry, therefore, permits the second entrant to move farther from the first, where it can enjoy more of a monopoly position and higher equilibrium prices without attracting further entry. Thus, paradoxically, a higher fixed cost within certain limits actually increases a firm's profitability because of its value as a barrier to entry.

Table 2 also shows that when fixed costs are reduced to a certain level (0.16 in the example computed), a third firm successfully enters. At that level of fixed cost, firm 1 and firm 2 "location" choices which are close enough together to limit further entry are no longer consistent with profit maximizing behavior. The equilibrium sequence is

⁹ A listing of the FORTRAN computer program used to compute these equilibria is available upon request from the authors. For the results in the tables, $N = 1$, $V_m = 10$, $A = 40$, $B = 1$, $0.5 \leq x_i \leq 1.0$, all i .

characterized by successive entrants' choosing successively longer waiting times and no firm's choosing a waiting time arbitrarily close to the choice of any other firm. Profits and market share are larger the earlier in the sequence that a firm enters; i.e., it pays to be first in the market. It is worth noting that the three-firm equilibrium in Table 2 is the unique equilibrium in the example computed for any number of potential entrants greater than or equal to three. Since the fourth firm in the sequence does not enter profitably, neither does any other firm later in the sequence. Therefore, for the equilibrium described in Table 2, no specific number of potential entrants need be assumed initially as long as the number exceeds three. Without loss of generality, the number of potential entrants can be assumed infinite.

It is conceivable that there are cases in which it pays a firm to enter later rather than earlier. In such an instance, a firm may delay entry in the hope that some other firm will go first. A similar kind of indeterminacy is sometimes mentioned in discussions of the incentives to innovate in the absence of a patent law. Rather than devote resources in new product innovation, firms may wait for some other firm to innovate first in the hope of appropriating costlessly the value of the innovation by using the information thus revealed about the innovation by the first firm (cf. Sherman, 1974, pp. 180–185). The sequential solution concept is inappropriate in this case unless the gaming aspects of firms strategically delaying entry are treated explicitly.

In practice, the indeterminacy in such a situation might be resolved by a single firm's obtaining sufficient venture capital to locate at multiple positions such that no remaining potential position offers profits. The result in this case is complete monopoly. Indeed, sequential foresighted entry results in monopoly anytime the number of locations any one firm can choose is not restricted because all equilibrium locations are profitable, and we expect the first firm in the sequence to choose all profitable locations if possible.

□ **Example 5.** This example emphasizes our contention that making entry into the industry endogenous is crucial and further demonstrates the proposition asserted in the previous example that without a constraint on the number of "locations" any one firm may occupy, the resulting industry structure is monopoly. "Location" by a firm in this example will correspond to a choice of physical plant capacity. Firms produce a homogeneous product, and market price is determined by the total amount of plant capacity in the industry. Assume that industry marginal revenue is a decreasing function of industry capacity, $F > 0$ is the fixed cost of entering the industry, and C is the constant marginal cost of additional capacity.

With a large number of potential entrants, the equilibrium capacity selection rule is to build just sufficient capacity to insure that no subsequent firm can enter profitably. That is, the first firm should be a "limit capacity" monopolist. Suppose the first firm stops short and thus invites further successful entry. The market price is the demand price corresponding to the total industry capacity. The first firm captures sales equal to the size of its capacity choice. Had the first firm chosen the entire industry capacity, however, market price would be no different, further entry would still be forestalled, yet the first firm would sell more than had it chosen smaller capacity; firm 1 clearly profits more by

extending capacity to the ultimate industry size. This is the equivalent of the first firm in example 4 opting for as many locations as necessary to preclude subsequent entry unless otherwise constrained.

A very different solution may result if, rather than unfettered endogenous entry, a predetermined, fixed, finite number of entrants is assumed and the fixed cost of entry is then assumed small enough to permit that number to enter successfully. This is the conventional (and we think unsatisfactory) procedure for modeling a market equilibrium. The resulting capacity choices, given foresighted sequential entry, may differ significantly from the case above. If the capacity choices of earlier firms permit firm n to enter with positive profit, then firm n will not drive market price down to the limit entry level as in the previous case. The demand curve faced by firm n is that portion of the market demand remaining after the first $n - 1$ firms make their capacity choices. Firm n can induce by its capacity choice any demand price less than that corresponding to industry capacity after $n - 1$ firms have “located”; the market price it *elects* to induce is that which maximizes its profits. Thus, earlier firms in the sequence choose capacity small enough so that profit maximizing firms “locating” later in the sequence elicit the market price most favorable to the earlier firms.¹⁰

■ The “foresighted sequential entry” equilibrium we obtain in each of the previous examples guarantees nonnegative wealth to all successful entrants and positive wealth to some. This raises the following interesting question: since firms move sequentially, why does the first firm to enter not locate at all positions in the equilibrium industry structure and obtain all potential profits in the market rather than be content with only the first location and less profits? In other words, why should the first firm not be a monopoly?

Foresighted sequential entry would appear to imply more monopoly market structure than is common in the economy. It is worth considering amendments that would bring the predictions of the model closer to reality. One possibility is that there are costs to occupying multiple locations not yet in the model. Alternatively, perhaps firms *do* occupy all profitable locations initially, but as time passes, additional locations become desirable, yet the first entrant into the market has no special incentive to expand into these locations. We shall discuss briefly how each of these possibilities might be incorporated into our model.

There are several potential constraints on the number of different locations a firm might want to occupy. A constraint may be financial. It is well known that firms exhibit preference for financing new ventures from internal retained earnings rather than external equity or debt capital. The reason for this preference may not be an internal cost of capital that is significantly lower than the market rate of interest. It could be merely that confidentiality is worth a premium. Borrowing by the firm or interesting investors in new equity shares involves revealing investment plans. Therefore, obtaining the financ-

3. An observation and concluding remarks

¹⁰ If the demand is linear, the number of potential entrants is limited to n , and the fixed cost of entry is sufficiently small, then the sequential entry solutions can be determined analytically. Letting a be the intercept of the inverse industry demand curve and c be the constant marginal cost of production, capacity of the i th firm is $(a - c)/2^i$. Thus the first firm to enter is twice the size of the second, which is twice the size of the third, etc.

ing necessary to locate in enough positions to exhaust all profitable market opportunities through external means may in itself eliminate those market opportunities by forcing the firm to divulge too much information. This could be an argument for a model in which the number of potential locations a firm can occupy is limited by cash flow. In this spirit, Spence (1977) imposes financial constraints in a dynamic setting and shows that monopoly does not result.

Alternatively, a constraint on the potential locations available to any individual firm may ultimately result in diseconomies of scale. The question is, why can a big firm not do everything that a small firm can do? Williamson (1975) explores the hypothesis that there are limits to the effective reach of the managerial hierarchy. The burgeoning literature on the internal organizations of the firm may soon indicate the pervasiveness of such scale diseconomies. Yet to be resolved if we are to rely on span of control explanations of multiple firm markets is why we observe such widely disparate firm sizes and why, as suggested by Williamson's research, the use of relatively autonomous divisions cannot reduce such diseconomies.

There are other potential costs to monopolizing when the market is "discovered." The first firm may be uncertain about the extent of the market. If the firm is risk averse, it may sample the market initially to obtain information even though to do so provides information to other potential entrants and relinquishes its monopoly position. Another possibility is that rapid adjustment of firm size is more costly than slow adjustment.¹¹ The first firm might then find it more profitable to add capacity slowly, even though such a policy invites loss of market share. Similar results were found by Flaherty (1976) and Kydland (1976) in dynamic equilibrium analyses with costly adjustment. Even though the assumption of increasing cost of adjustment is standard in investment demand theory (see Lucas, 1967), the origins of these costs have not been explored. One appealing justification is that adding productive capacity requires new personnel that must be screened. Conceivably, the likelihood of mistakenly accepting the wrong job candidate increases the larger is the ratio of screening rate to the current size of the organization. The argument here is not that there is a limit to the span of control within the firm, but rather that the effectiveness of screening job applicants to find a good "match" to existing tasks and personnel depends on the time horizon over which the screening proceeds. This hypothesis warrants further study.

Thus, elaborations of the sequential location model developed can add explanatory power to the theory. As the above discussion indicates, much useful work remains in the development of the theory of sequential location. Yet, as an alternative to other *solution concepts*, the nature of the market equilibrium described here has something substantial to offer. Market agents in this model make choices based on the way they think choices of all other agents will be made conditional on their own decision, rather than based simply on the observed current choices of their present rivals. It is only when firms consider the potential influence of their decisions on choices of competitors that in equilibrium no firm can improve its situation by adopting a different decision rule. Equilibrium defined in this fashion is

¹¹ We thank Oliver Williamson for this suggestion.

appealing if firms realize that competitors have motivations similar to their own. And if one argues that firms are unaware of rivals' motives and can be modeled most successfully with that assumption, then we must explain why more economists are not managing firms.

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