0.1 Monopolistic Competition Theory

Let there be two counties, A and B, that uniquely make up a market. We assume for simplicitly that each market is unique, and independent of each other. Firms first make a decision among markets, and then on the margin make a choice between the two counties. Each county has a locatioon specific representative consumer, with a strictly concave utility function $U_i(G, H, (\sum_{i=1}^{n} q_i^{\rho})^{\frac{1}{\rho}})$, where for concavity $\rho < 1$, and all goods are assumed to be substitues. Let G denote consumption of a government public good, and H an amount of housing, and the didfferentiated goods q_i are a consumers consumption, with $y = (\sum_{i=1}^{n} q_i^{\rho})^{\frac{1}{\rho}}$. Individuals are each endowed with their own capital stock K_i, which is tradeable across counties and an external market at rate r, as a result individual face the budget constraint, $H + \sum_{i=1}^{n} p_i q_i \leq rK_i$, which we have normalized by the price of housing.

There are two firm sectors. The first is a competitive housing market, where firms use a strictly concave production function $F(K)$. Then, governments can impose a tax on capital τ_r, leading to the production function;

$$\pi_c = \max_K F(K) - (r + \tau_r)K$$

Of importance here is how firm size and number of firm entries respond to taxes. The solution to this subproblem is well understood in the Tiebout literature. First note from the firms first order conditions we get that the optimal level of K^* is tacitly defined by,

$$\frac{d\pi_c}{dK} = F_K(K^*) - (r + \tau_r) = 0$$

Then, taking a full differential and holding $dr = 0$, we get

$$F_{KK}(K^*)dK - d\tau_r = 0$$

$$\frac{dK}{d\tau_r} = \frac{1}{F_{KK}} < 0$$

Further, since the production function is strictly concave, if we let there be a positive change in taxes the optimal firm size shrinks and more firms enter the market. Thus $\frac{dn_c^*}{d\tau_k} > 0$. This result follows from the traditional theory of the firm in a competitive market.

The second industry is a monopolistic competition sector defined by a linear cost function. Then the firms problem becomes, subject to a tax on prices,

$$\pi = \max_{p_i}(p_i - \tau_p - c)q_i - f$$

Again, in this case we want to look at how prices, quantities, and how many product types enter the market subject to changes in tax rates. First we show that as taxes increase, $d\tau_r > 0$, that optimal prices for firms increaes to compensate. From Dixit and Stiglitz (1977) and Tirole (1988) we note that using a two-stage budgeting process for the government,

$$\epsilon_i = \frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i} = \frac{1}{1-\rho}$$

Which, in our model implies that optimal price is

$$p_i^* = \frac{c+\tau}{\rho}$$

$$\frac{dp_i^*}{d\tau} = \frac{1}{\rho} > 0$$

Then, the optimal production point is where firms are just breaking even, or

$$(p_i^* - \tau_p - c)q_i = f$$

$$(\frac{c+\tau_p}{\rho} - \tau_p - c)q_i = f$$

$$q_i^* = \frac{f\rho}{(c+\tau_p)(1-\rho)}$$

Then, as a result, we further get that

$$\frac{dq_i}{d\tau_p} < 0$$

Here we get the usual monopolistic competition solution from Dixit and Stiglitz and Tirole

$$U(G, rK_i - nqp, n^{\frac{1}{\rho}}q)$$

Finally, the optimal number of firms, given government spending G is defined by,

$$U_H(G, rK_i - npq, n^{\frac{1}{\rho}}q)(c+\tau) = n^{\frac{1}{\rho}-1}\rho U_y(G, rK_i - npq, n^{\frac{1}{\rho}}q)$$

Let the utility function now take the form $U = G^{\psi}H^{\sigma}Y^{1-\sigma}$, for $\psi, \sigma < 1$. Then we get

$$\sigma(rK_i - npq)^{\sigma-1}(n^{\frac{1}{\rho}}q)^{1-\sigma}(c+\tau) = (1-\sigma)\rho^{\frac{1}{\rho}-1}(rK_i - npq)^{\sigma}(n^{\frac{1}{\rho}}q)^{-\sigma}$$

$$\frac{\sigma}{(1-\sigma)\rho}n^{\frac{1}{\rho}}q(c+\tau) = rK_i - npq$$

$$\frac{\sigma(c+\tau)}{(1-\sigma)\rho}n^{\frac{1}{\rho}} = \frac{rK_i}{q} - np$$

Differentiating with respect to τ we get

$$\frac{\sigma(c+\tau)}{(1-\sigma)\rho}\frac{1}{\rho}n^{\frac{1}{\rho}-1}\frac{\partial n}{\partial \tau} = \frac{rK_i}{\frac{\partial q}{\partial \tau}} - \frac{\partial n}{\partial \tau}p - n\frac{\partial p}{\partial \tau}$$

$$(\frac{\sigma(c+\tau)}{(1-\sigma)\rho}\frac{1}{\rho}n^{\frac{1}{\rho}-1} + p)\frac{\partial n}{\partial \tau} = \frac{rK_i}{\frac{\partial q}{\partial \tau}} - n\frac{\partial p}{\partial \tau}$$

Since the left hand side term is positive everywhere, and the left hand term is negative everywhere, we get that

$$\frac{\partial n}{\partial \tau} < 0$$

Thus, the number of varieties in the market decreases as taxes increase.

——

Then the social planners problem is the second best in this case. The social planner cannot dictate the consumption bundles of consumers, but can try to find a non-zero bundle of taxes that improves their social welfare through the provision of government services. The consumers problem then becomes, given G,

$$\max_{H,q_i} U(G, H, \left(\sum_{i=1}^{n} q_i^{\rho}\right)^{\frac{1}{\rho}})$$

$$s.t. \quad H + \sum_{i=1}^{n} q_i p_i \leq rK_i$$

Now note that for the competitive firms, we have

$$D(p) = rK_i - nqp = n_c^* q_c^*$$

$$\implies \frac{\partial D(p)}{d\tau_p} = 0 = \frac{\partial n_c^*}{\partial \tau_p} q + n_c^* \frac{\partial q_c^*}{\partial \tau}$$

$$\implies \frac{\partial n_c^*}{\partial \tau_p} = \frac{n_c^*}{q_c^*} \frac{\partial q_c^*}{\partial \tau}$$

Further from knowing the general solutions to this problem, we can restructure the government's problem to be

$$U\left(\tau_r n_c^* K_c^* + \tau_p n_m^* \frac{f\rho}{(c+\tau_p)(1-\rho)}, rK_i - n_m^* \frac{f(c+\tau)}{(c+\tau_p)(1-\rho)}, n^{\frac{1}{\rho}} \frac{f\rho}{(c+\tau_p)(1-\rho)}\right)$$