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Tax Competition for Heterogeneous Firms with Endogenous Entry†

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This paper models tax competition for mobile firms that are differentiated by their productivities. Because taxes affect the distribution of firms, they affect wages, prices, and the number of firms. From the social planner's perspective, optimal taxes efficiently distribute income between private and public consumption and are harmonized, providing the optimal number of firms. This is not a Nash equilibrium. As is common in such models, equilibrium taxes are inefficiently low. Furthermore, there is no pure strategy equilibrium with equal taxes resulting in too many firms. This illustrates a new distortion from tax competition and a new benefit from harmonization. (*JEL* H21, H25, H87)

Recent innovations to the literature on international trade and foreign direct investment (FDI) have incorporated heterogeneous firms into models of imperfect competition that previously assumed identical firms.[1] We build upon these by incorporating imperfectly competitive heterogeneous firms and endogenous entry into a general equilibrium model of tax competition for mobile investment. Although a handful of tax competition models with imperfect competition exist, firms in these models are identical, and their number is exogenous.[2] In our model, firms differ in their labor productivity. Because of this, low productivity firms prefer a low-cost location even if this has a higher tax. Since taxes affect location decisions, they affect relative wages, prices, and the number of firms in

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[1] Examples in the trade literature include Fabio Ghironi and Mark J. Melitz (2005) and Melitz (2003). Examples in the FDI literature include Volker Nocke and Stephen Yeaple (2008), Yeaple (2003b), and Elhanan Helpman, Melitz, and Yeaple (2004).

[2] Examples include Ben Ferrett and Ian Wooton (2005), Richard E. Baldwin and Paul Krugman (2004), and Eckhardt Janeba (1998).

equilibrium. World welfare maximizing taxes are harmonized, inducing the optimal degree of entry, and efficiently allocate income between the private and public sectors. Individual governments, however, have an incentive to lower their own tax to attract more profitable firms. This race to the bottom in taxes leads to the well-known result of public good underprovision. Furthermore, there is no pure strategy Nash equilibrium with equal taxes, potentially resulting in too many firms compared to the (unconstrained) social optimum. This is because unequal taxes lower wages in one country, exacerbating the excessive entry (and the underproduction) found in monopolistic competition style models. This provides a new motivation for tax harmonization.

Our use of discrete yet endogenously entering firms is a departure from the standard models of tax competition.[3] Typically, one of two approaches is used. The first assumes that investors divide their activities across locations. Since each location has diminishing marginal returns, even with tax differentials, each location receives positive investment levels.[4] We call this the "continuous investment" approach. The second approach assumes a given number of firms (typically one) for which governments compete. Here, firms choose a single location, meaning that countries that do not host have no investment. We call this the "discrete investment" approach.[5]

These modeling assumptions have important implications for strategic behavior and the ability to include public goods. Under discrete investment, competition for the firm amounts to a second-price auction in which governments bid their own value for the firm, and the winner pays the second highest value.[6] This leads to a best response in which governments marginally undercut one another's taxes until all gains from hosting are exhausted. In contrast, when investment is continuous, optimal taxes trade off against the size of the tax base and the share of profits collected by the host government. Thus, there is not the weakly dominant strategy found in the discrete investment case. A key implication of this difference is that it is often impossible to include a necessary public good in the discrete investment case because typically there exist equilibria in which some countries do not host FDI and therefore collect no tax revenue. Thus, while both approaches predict a race to the bottom in taxes, only the continuous investment model can be used to describe the inefficiencies tax competition causes for public goods provision. In our model, due to the heterogeneity in their productivities, not all firms flock to the low-tax location.[7] Thus, even though a single firm's location is discrete, each government collects positive tax revenues from the firms in equilibrium. Therefore, we are able to discuss public goods provision in a model with discrete investment.

To maximize world welfare, optimal taxes must meet two conditions. First, they should efficiently allocate income between the private and public sectors. Second, they should be equal across countries, resulting in equal wages across countries and the optimal number of firms (subject to the distortions created by imperfect

[3] John Douglas Wilson (1999), Thomas A. Gresik (2001), and Clemens Fuest, Bernd Huber, and Jack Mintz (2005) provide overviews of the existing literature.

[4] Wilson (1986) and George R. Zodrow and Peter Mieszkowski (1984) are seminal papers in this vein.

[5] Recent examples include Andreas Haufler and Ian Wooton (1999, 2006) and Horst Raff (2004).

[6] See Dan A. Black and William H. Hoyt (1989) or Ronald B. Davies and Christopher J. Ellis (2007) for details on this second-price auction result.

[7] This is in contrast to Baldwin and Krugman (2004) in which mobile firms all agglomerate in one location or the other depending on relative tax rates.

competition). However, there are no equilibrium outcomes that satisfy these conditions. Beginning from optimal taxes, each country will find it strictly beneficial to undercut the overseas tax. Doing so creates two benefits for the low-tax country. First, it guarantees the high productivity (and high profit) firms. Second, attracting additional firms drives up local wages, increasing real income. These discrete income gains outweigh the marginal loss to tax revenues, a result comparable to that found in the discrete investment case. As a result, equilibrium taxes will be too low. However, as in the continuous investment case, the need to provide a public good tempers the race to the bottom since for low overseas taxes, rather than matching taxes, it is preferable to have a strictly higher tax to increase tax revenues. Nevertheless, matching the other nation's tax remains a dominant strategy. This implies that there are equilibrium outcomes in which countries' taxes are unequal. This encourages entry, drawing resources away from productive firms and lowering world welfare. This is, then a new inefficiency from tax competition that cannot be found in the existing literature due to its assumption of an exogenous number of firms.

The closest antecedent of our model is that of Wilson (1987). In a model of continuous investment, he considers strategic taxation of mobile capital by small countries. As is standard in continuous investment models, tax differences encourage capital to flow to the low-tax location. An unusual feature of his model is that this leads to complete specialization (i.e., the low-tax country produces only the capital-intensive good and the high-tax country produces only the labor-intensive good). One might compare our differences across firms to Wilson's differences across sectors. However, several features differentiate our results from his. First, we consider large countries in which governments anticipate the terms of trade effects that their taxes create. This introduces strategic effects not found in his model. Second, his sectors exhibit constant returns to scale and perfect competition. We, however, allow for increasing returns to scale and assume a monopolist in each variety. Finally, the number of sectors in Wilson's model is exogenous whereas the number of varieties in our model is endogenous. Thus, his model would be incapable of revealing the inefficiency from excessive entry.

The paper proceeds as follows. Section I lays out the baseline model. Section II derives the world-welfare maximizing level of taxes. Section III describes the properties of the Nash equilibria. Section IV considers the sensitivity of the results to alternative assumptions, including the presence of vertical FDI. Section V concludes.

I. The Model

In this section, we present the basic framework of our model. Consider a world with two countries labeled A and B. Each country k has a fixed labor endowment given by $\overline{L_k}$, which is the sole factor of production. Without loss of generality, let $\overline{L_A} \geq \overline{L_B}$. The sequence of moves is the following. First, tax rates are set, either jointly by a social planner or simultaneously in the case of competition. Second, firms simultaneously choose which of the countries to locate in and how much to produce. Finally, consumption occurs and payoffs accrue. As is standard, we apply subgame perfection.

A. *Consumers*

Utility from private consumption of the representative consumer in country k is of the Dixit-Stiglitz form:

$$U_k = \left(\int_0^N x_k(i)^{(\sigma-1)/\sigma} \, di \right)^{\sigma/(\sigma-1)}, \tag{1}$$

where N is the number of varieties and $\sigma > 1$. Pre-tax private income in k is I_k, which is the sum of wage income and pre-tax profits of firms that are located in k.[8] This is taxed at a rate t_k, thus, the same tax rate is applied to wage income and profits.[9] Consumers maximize utility subject to their budget constraint:

$$\int_0^N p(i)\, x_k(i) \, di \le (1 - t_k) I_k.$$

As is well known, this yields a demand function for private consumers for each firm of

$$x_k(i) = P^{1-\sigma} p(i)^{-\sigma} (1 - t_k) I_k,$$

where P is a price index of the form

$$P = \left(\int_0^N p(i)^{1-\sigma} \, di \right)^{1/(1-\sigma)}. \tag{2}$$

Tax revenues are used by the government to fund public consumption of these same N goods, where the relative valuation of is the same as those given by the consumer's utility function, i.e., the government maximizes a function:

$$G_k = \left(\int_0^N g_k(i)^{(\sigma-1)/\sigma} \, di \right)^{\sigma/(\sigma-1)}. \tag{3}$$

The government of each country must run a balanced budget, i.e.:

$$\int_0^N p(i) g_k(i) \, di \le t_k I_k.$$

[8] We consider generalizations of the distribution of profits in Section IV. Also, we do not consider firms coming from a third country as is done in Davies (2005) and Kjetil Bjorvatn and Carsten Eckel (2006).
[9] The case of different taxes on labor and profits is considered in Section IV.

One interpretation of G_k is that the tax revenues support consumption by individuals without income (such as the unemployed or the elderly). Alternatively, G_k can be the consumption of a corrupt government official (as in the so-called Leviathan models).[10] A third interpretation is that it represents the government's transformation of the N products into a publicly provided good. Regardless of the interpretation, government consumption in k results in public demand in k for each firm of

$$g_k(i) = P^{1-\sigma} p(i)^{-\sigma} t_k I_k,$$

implying that total demand for each firm from country k is

$$X_k(i) = P^{1-\sigma} p(i)^{-\sigma} I_k.$$

We assume that there is no price discrimination between countries (which is guaranteed by free resale and zero trade costs).[11] Thus, where $I = I_A + I_B$, the firm's worldwide demand is $X(i) = P^{1-\sigma} p(i)^{-\sigma} I$.

B. *Firms*

A given firm i makes two choices. First, it decides in which country to locate (or to not enter at all).[12] Second, given entry, it decides on a production level. Firms make these decisions simultaneously, i.e., taking choices of other firms as given. As is standard, firms take wages as given. Firm i's after-tax profits, when based in country k, are given by

$$\pi_k(i) = (1 - t_k)[p(i)q(i) - w_k a(i)q(i) - w_k F], \tag{4}$$

where t_k is the tax in country k, w_k is the wage rate in country k, $q(i)$ is firm i's output, $a(i)$ is firm i's exogenously endowed unit labor requirement, and F is the amount of labor firm i requires to cover fixed costs.[13] As in other models of firm

[10] Examples of this include Geoffrey Brennan and James M. Buchanan (1980) and Jeremy Edwards and Michael Keen (1996).

[11] If trade costs are positive, this increases the desirability of locating in the country with the greater income, as this is where more of the firm's output will be sold. As shown by Haufler and Wooten (1999) in a model with a single firm, this gives an advantage to this larger economy in the tax competition game. A similar advantage would be found in our model.

[12] A useful conceptualization of the model is to attribute a given firm's productivity to an internationally mobile entrepreneur. This entrepreneur earns rents from this firm-specific asset, i.e., entrepreneurial income amounts to firm's profits. Given a country of residence, an entrepreneur becomes a part of that country's representative consumer which, under this preference structure, implies that entrepreneurial income, demand, and welfare enter the model consistent with the above formulation. Note that since an entrepreneur would consider public good provision as well as after tax income when deciding where to reside, government expenditures are best interpreted as income transfers among people or corruption. However, as discussed in footnote 27, variants of the model exist allowing for entrepreneurs and public goods that yield comparable results.

[13] For simplicity, we assume that these productivity parameters are known to the firm owners. Alternatively, we could endow each of the entrepreneurs in footnote 12 with an additional factor of production that can be used to obtain a draw from the distribution of productivity parameters, but has no other use. Since there is no opportunity cost to obtaining a draw, this would simply result in a more complicated version of the present model.

heterogeneity, this unit labor requirement drives the differences across firms.[14] We assume that this is increasing in the index, i.e., $a(0) < a(i) < a(j)$ for $0 < i < j$, implying that firm 0 is the most productive firm. We assume that $F < min\,\{\overline{L_A}, \overline{L_B}\}$, ensuring that either country is capable of hosting at least one firm with labor remaining for positive production.[15] With a continuum of firms, this is easily satisfied. Given a location, profit maximization yields

$$q_k(i) = \left(\frac{\sigma}{(\sigma-1)} a(i) w_k\right)^{-\sigma} P^{(\sigma-1)}\, I, \tag{5}$$

resulting in a markup over marginal cost:

$$p(i) = \frac{\sigma}{(\sigma-1)}\, a(i) w_k. \tag{6}$$

This condition allows us to rewrite profits so that, at their maximum,

$$\pi_k(i) = (1 - t_k)\, \frac{1}{(\sigma-1)}\, w_k[a(i)\, q(i) - (\sigma-1)F], \tag{7}$$

where, for this firm's market to clear, its quantity $q_k(i)$ equals its worldwide demand $X_k(i)$. For future use, it is important to note that by equation (5) , we can show that a firm's labor demand is falling in its index.

In order to more easily describe the distribution of firms and derive best response tax rates, it is useful to make a distinction between the high-tax country and the low-tax country. We will label our countries such that $t_1 \leq t_2$, and refer to country 1 as the low-tax country.[16] If $t_1 < t_2$ and $w_1 \leq w_2$, then all firms will locate in country 1 since this location offers lower taxes and lower costs. As described in detail below, this is incompatible with endogenous wages since it implies an excess labor supply in country 2. Thus, $w_1 \geq w_2$ with strict equality only if $t_1 = t_2$. Additional implications are that $p_1(i) \geq p_2(i)$ and $q_1(i) \leq q_2(i)$ for each firm i. For notational convenience define $\theta \equiv w_2/w_1 \leq 1$, which holds with equality only when taxes are equal.[17]

With free entry, firms enter until the last firm earns zero profits by doing so. This last firm is firm N. Whenever taxes are unequal, we find that

$$a(N)q_1 - (\sigma-1)\,F < a(N)q_2 - (\sigma-1)F,$$

[14] In an earlier version of this paper (Davies and Eckel 2007), productivity is the same across firms, but firms differ in the labor requirement for the fixed cost. The results are qualitatively identical to those here.

[15] It is important to recognize that our firms are not multinationals according to the standard definition because they have their headquarters (where the fixed cost takes place) and their production in the same country. We consider the case of multinationals in Section V.

[16] Note that whether country A or B corresponds to country 1 depends on its relative taxes. The purpose behind this distinction is to ease the derivation of the impacts of unequal taxes.

[17] The potential for different wages does not exist in models with an additional, freely traded good produced under constant returns to scale and perfect competition, as this equalizes wages across borders.

implying that pre-tax profits are greater for this firm in the low-cost country 2. Although country 2 has a higher tax rate, since firm N earns no profits, this is not a deterrent. Note that since this firm earns zero profits, in equilibrium

$$q_2(N) = a(N)^{-1}(\sigma - 1)F, \tag{8}$$

which is decreasing in N, which itself depends on relative taxes. Using equation (5) for a generic firm i producing in country 2 and equation (8), we see that

$$q_2(i) = a(i)^{-\sigma} a(N)^{\sigma-1}(\sigma - 1)F. \tag{9}$$

Since all other firms have strictly lower production costs for the same quantity, if they were to enter country 2, they would earn positive profits (and potentially even higher profits by entering country 1). Taking the ratio of the quantity solutions of a given firm in countries 1 and 2 and using (9), allows us to derive that for a given firm i:

$$q_1(i) = \theta^{\sigma} q_2(i) = \theta^{\sigma} a(i)^{-\sigma} a(N)^{\sigma-1}(\sigma - 1)F. \tag{10}$$

An important aspect of this solution is that, unlike monopolistic competition with identical technologies across firms, in our setting, only the last firm to enter has zero profits. Therefore, because of the heterogeneous productivities, profit taxes have the ability to influence firm location even with free entry. In addition, when tax rates differ, there exists a firm λ that is indifferent between the two countries. Specifically, defining the relative after-tax rate $\tau \equiv (1 - t_2)/(1 - t_1) \leq 1$ and using equations (7) and (10) to evaluate profits in each country for firm λ:

$$\theta^{\sigma} a(N)^{\sigma-1} - a(\lambda)^{\sigma-1} = \tau\theta\left[a(N)^{\sigma-1} - a(\lambda)^{\sigma-1}\right]. \tag{11}$$

This implies that whenever tax rates differ there is a distribution of firms such that firms 0 to λ locate in country 1 and firms λ to N locate in country 2. See Figure 1. Intuitively, since the high productivity firms have high profits but use little labor, they are attracted by country 1's low-tax rate but are not overly put off by its high wage. Low productivity firms, on the other hand, use more labor and earn smaller profits. Therefore, the cost savings from using country 2's low-cost workers outweighs the additional tax burden. Firms with indexes higher than N do not enter.

C. *Labor Markets and Income*

In order to clear the labor market in each country, wages must adjust so that labor supply equals labor demand. As noted above, if country 1 has lower taxes and (weakly) lower wages, all firms will locate there. This would lead to an excess supply of labor in country 2, pushing w_2 down. As a result, if $t_1 \leq t_2$, labor markets in the two countries will clear only if $w_2 \leq w_1$, with strict equality only when

Figure 1. Distribution of Firms

taxes are equal.[18] For notational simplicity, define $\mu_1(\lambda) \equiv \lambda^{-1} \int_0^{\lambda} a(j)^{1-\sigma}\, dj$. Thus, dropping the argument of μ_1 and summing across the firms that country 1 hosts, in equilibrium,

$$\overline{L_1} = \lambda \theta^{\sigma} (\sigma - 1)\, Fa(N)^{\sigma - 1}\, \mu_1 + \lambda F. \tag{12}$$

Similarly, defining $\mu_2(\lambda, N) = (N - \lambda)^{-1} \int_{\lambda}^{N} a(j)^{1-\sigma}\, dj$, in country 2,

$$\overline{L_2} = (N - \lambda) a(N)^{\sigma-1} (\sigma - 1) F \mu_2 + (N - \lambda) F. \tag{13}$$

This also allows us to write equilibrium values of income (and the price index):

$$I_1 = w_1 \lambda F \theta^{\sigma} a(N)^{\sigma-1}\, \sigma \mu_1, \tag{14}$$

$$I_2 = w_2 (N - \lambda)\, a(N)^{\sigma-1}\, F \sigma \mu_2, \tag{15}$$

and

$$P = \frac{\sigma}{(\sigma - 1)} w_1 (\lambda \mu_1 + (N - \lambda) \theta^{1-\sigma}\, \mu_2)^{1/(1-\sigma)}. \tag{16}$$

D. *Comparative Statics*

We now have a system of three equilibrium equations, one describing the location-indifferent firm λ (equation 11), and two describing labor market equilibria (equations 12 and 13). We also have three endogenous variables, the index of the indifferent firm λ, the number of firms N, and relative wages θ. Using this system of equations, we can derive how these variables move with the relative after-tax variable $\tau = (1 - t_2)/(1 - t_1)$.

LEMMA 1: *The index of the indifferent firm and the number of firms are decreasing in τ. Relative wages are increasing in τ.*

[18] This highlights the role heterogeneity plays in our model. If firms are identical, they will all flock to the one country or the other. Furthermore, since with free entry and homogeneity profits equal zero for all firms, profit taxes would have no impact on the total number of firms or their distribution across countries. If this were the case, there would be no strategic interaction between countries.

PROOF:

By direct calculation:

$$(17)\quad \frac{d\lambda}{d\tau} = -\frac{[\Psi^{\sigma-1}-1]}{\Delta}\sigma(\sigma-1)\lambda\theta^{\sigma}a(N)^{\sigma-1}\mu_1 \times \{\sigma+(\sigma-1)^2(N-\lambda)a(N)^{\sigma-2}a'(N)\mu_2\}<0$$

$$(18)\quad \frac{dN}{d\tau} = -\frac{[\Psi^{\sigma-1}-1]}{\Delta}\{1+\Psi^{\sigma-1}(\sigma-1)\}\,\sigma(\sigma-1)\lambda\theta^{\sigma}a(N)^{\sigma-1}\mu_1<0,$$

and

$$(19)\quad \frac{d\theta}{d\tau} = \frac{[\Psi^{\sigma-1}-1]}{\Delta}\theta\Big\{\big(a(\lambda)^{(\sigma-1)}+(\sigma-1)\theta^{\sigma}\Psi^{\sigma-1}\big) \times \big(\sigma+(\sigma-1)^2(N-\lambda)a(N)^{\sigma-2}a'(N)\mu_2\big) + \{1+\Psi^{\sigma-1}(\sigma-1)\}(\sigma-1)^2\lambda\theta^{\sigma}a(N)^{\sigma-2}a'(N)\mu_1\Big\}>0,$$

where $\Psi \equiv a(N)/a(\lambda) \geq 1$, and

$$\Delta = \{1+\Psi^{\sigma-1}(\sigma-1)\}(\sigma-1)^2\theta^{\sigma-1}\lambda a(N)^{\sigma-2}a'(N)\mu_1\{(\sigma-1)\tau\sigma\theta\Psi^{\sigma-1}+\tau\theta\} + \{\sigma+(\sigma-1)^2(N-\lambda)a(N)^{\sigma-2}a'(N)\mu_2\} \times \Big\{(\sigma-1)\Psi^{\sigma}a(N)^{-1}a'(\lambda)(\theta^{\sigma}-\tau\theta)\sigma(\sigma-1)\lambda\theta^{\sigma-1}a(N)^{\sigma-1}\mu_1 + \big((\sigma-1)\theta^{\sigma}\Psi^{\sigma-1}+1\big)^2\theta^{-1}\Big\}>0.$$

The intuition behind these comparative statics is straightforward. Suppose that t_1 rises, implying an increase in the relative after-tax rate τ. This rise will lead the initially indifferent firm to strictly prefer country 2. As a result, the indifferent firm's index falls. This shift in firms toward country 2 increases labor demand there, thereby increasing country 2's relative wage θ. This rise in costs in country 2 means that the firm that previously just covered its costs now has negative profits, leading it to exit and N to fall.

Lemma 1 states that changes in the relative after-tax rates between countries affect not only the allocation of firms between countries (and thus relative labor demand), but they also have an impact on the absolute number of firms. As equation (18) shows, the relationship between the relative after-tax rate and the number of firms is negative, implying that tax harmonization (an increase of τ to its maximum of one) results in the *minimum* level of variety. This result will play a central role in the welfare analysis.

E. *The Distribution of Firms under Equal Taxes*

One difficulty with the above analysis is that when taxes are equal, wages must also be equal, otherwise all firms will flock to one location or the other. The difficulty this presents is that at this point, all firms are indifferent between locations. As such, there exist many distributions of the firms across the two locations that are consistent with this equilibrium besides those in which firms agglomerate according to their productivity. Furthermore, even if we use a distribution such that firms zero to λ locate in one country and the remainder locate in the other, there is no obvious way to assign the high productivity firms to one country or the other. Therefore, we make the assumption that when tax rates are equal, country A hosts firms zero to λ with probability β and firms λ to N with probability $1 - \beta$. It will be useful to establish certain results regarding the distribution of firms in the equal tax case.

PROPOSITION 1: *Assume equal taxes. Then:*

(i) *Regardless of which country hosts the high productivity firms, N remains the same.*

(ii) *If A has more labor than B, then when A hosts the high- (low-) productivity firms, it hosts more firms in equilibrium than when B hosts the high- (low-) productivity firms.*

(iii) *A country hosts more firms when hosting the low-productivity firms than when it hosts the high-productivity firms.*

(iv) *If A hosts the low-productivity firms, then it hosts more firms in equilibrium than B does. The reverse also holds when endowments are equal.*

(v) *When a country hosts the high-productivity firms, more labor is devoted to production than when it hosts the low-productivity firms.*

PROOF:

(i) When tax rates are equal (which implies equal wages),

$$\overline{L_A} + \overline{L_B} = \overline{L_1} + \overline{L_2} = (\sigma - 1)\, a\,(N)^{\sigma-1}\, F \int_0^N a(j)^{1-\sigma}\, dj + NF,$$

i.e., the total number of firms is the same regardless of whether the relatively large A hosts the low- or the high-productivity firms. Denote this number of firms N^*.

(ii) Although N^* is independent of who hosts the high-productivity firms, λ is not.

When country A hosts the high-productivity firms, the number it hosts is λ_A^*, where

$$\overline{L_A} = (\sigma - 1)a(N^*)^{\sigma-1} F \int_0^{\lambda_A^*} a(j)^{1-\sigma}\, dj + \lambda_A^* F.$$

Similarly, when country B hosts the high-productivity firms, the number it hosts is λ_B^*:

$$\overline{L_B} = (\sigma - 1)a(N^*)^{\sigma-1} F \int_0^{\lambda_B^*} a(j)^{1-\sigma}\, dj + \lambda_B^* F.$$

Since $\overline{L_A} \geq \overline{L_B}$ and $a(i)$ is increasing in i, this implies that

$$\lambda_A^* \geq \lambda_B^*, \tag{20}$$

with strict equality only when endowments are equal. This means that in equilibrium A would host at least as many high-productivity firms as B would. This, in turn, implies that

$$N^* - \lambda_B^* \geq N^* - \lambda_A^*, \tag{21}$$

with strict equality only when endowments are equal. This implies that in equilibrium A would host at least as many low-productivity firms as B would.

(iii) Since a firm's total labor demand is decreasing in its index, the average firm in country 2 uses less labor than the average firm in country 1 does. As a result, when a given country k hosts the low-productivity firms, it must host more firms than when it hosts the high-productivity firms in order to exhaust its labor supply, i.e.,

$$\lambda_k^* < N^* - \lambda_{-k}^*. \tag{22}$$

(iv) Combining equations (20)–(22) implies that because B is no larger than A, when B hosts the high-productivity firms, it hosts fewer firms than A does. Furthermore, when $\overline{L_A} = \overline{L_B}$, this implies that where $\lambda_A^* = \lambda_B^* = \lambda^*$,

$$\lambda^* - (N^* - \lambda^*) = a(N^*)^{\sigma-1}(\sigma - 1)[(N^* - \lambda^*)\mu_2 - \lambda^*\mu_1] < 0. \tag{23}$$

(v) Using this result, the amount of labor devoted to production is greater when a country hosts the high-productivity firms:

$$\lambda_k^*\mu_1(\lambda_k^*) > (N^* - \lambda_{-k}^*)\,\mu_2(\lambda_k^*, N^*). \tag{24}$$

F. *National Welfare*

The national welfare in country k depends on private and public consumption:

$$v_k(t_k, t_{-k}) = U_k^{\alpha}\, G_k^{(1-\alpha)}. \tag{25}$$

One interpretation of this function is that of a representative consumer who derives utility from his own private consumption and a publicly provided good created by a production function given by equation (3). Alternatively, this can represent a function that weights the utility of income earners relative to that of those consuming out of tax revenues (be they the unemployed or Leviathan government officials). Defining $T_k = t_k^{(1-\alpha)}(1 - t_k)^{\alpha}$, and using the results above for quantities and prices, when tax rates differ, we can write country 1's indirect national welfare as

$$v_1 = T_1(\sigma - 1)(\lambda\mu_1 + \theta^{1-\sigma}\,(N - \lambda)\mu_2)^{1/(\sigma-1)}\,\lambda\theta^{\sigma}\,a(N)^{\sigma-1}\,F\mu_1, \tag{26}$$

while that for country 2 is

$$v_2 = T_2(\sigma - 1)(\lambda\mu_1 + \theta^{1-\sigma}\,(N - \lambda)\mu_2)^{1/(\sigma-1)}\,(N - \lambda)\theta a\,(N)^{\sigma-1}\,F\mu_2. \tag{27}$$

Inspection of these shows that if a country's tax rate equals one or zero, regardless of the other country's tax rate, national welfare is zero because all income is allocated to the public or private sector. For a given pair of harmonized tax rates $t_1 = t_2 = t_k$, recall that country k has a probability β of receiving the high-productivity firm, and its expected welfare is

$$v_k^*(t_k, t_k) = T_k(\sigma - 1)\left(\int_0^{N^*} a(j)^{1-\sigma}\,dj\right)^{1/(\sigma-1)} a(N^*)^{\sigma-1}F \tag{28}$$
$$\times\,[\beta\lambda_k^*\mu_1(\lambda_k^*) + (1 - \beta)(N^* - \lambda_{-k}^*)\mu_2(\lambda_{-k}^*, N)],$$

where the term in brackets is proportional to the expected amount of labor used in production in country k. By Proposition 1(v), $v_2(t, t) < v_k^*(t, t) < v_1(t, t)$, i.e., with equal taxes, national income and welfare are greater when hosting the high-productivity firms.

II. Optimal Taxes

To derive optimal taxes, consider a social planner that maximizes the sum of the two countries' welfares, which is given by

$$W = v_A\,(t_A, t_B) + v_B\,(t_A, t_B) \tag{29}$$

by choosing the two countries' tax rates.[19] Using equations (26) and (27), this reduces to

$$(30)\quad W = (\sigma - 1)\theta^{\sigma} a(N)^{\sigma-1} F(\lambda\mu_1 + \theta^{1-\sigma}(N-\lambda)\mu_2)^{1/(\sigma-1)}(T_1\lambda\mu_1 + T_2(N-\lambda)\mu_2),$$

where λ and N depend on relative taxes or, if taxes are equal, λ depends on the random assignment of the low-cost firms (making this an expected world welfare). The solution to this is found in our next proposition.

PROPOSITION 2: *The social planner's optimum is to set* $t_A = t_B = 1 - \alpha$.

PROOF:

First, we examine how W's two components behave in τ, i.e., treating T_1 and T_2 as fixed. From equation (26), and using the comparative statics (17), (18), and (19), we see that

$$(31)\quad \begin{aligned}\frac{dv_1}{d\tau} = \Gamma^{-1} T_1\theta^{\sigma}\big\{&(\theta^{1-\sigma} - 1)\sigma\theta^{\sigma} a(\lambda)^{1-\sigma} a(N)^{\sigma-1}\lambda\mu_1\\ &+ \big[\sigma - \big(1 + a(\lambda)^{1-\sigma}a(N)^{\sigma-1}(\sigma - 1)\big)\big]\theta\lambda\mu_1\\ &+ \big[(\sigma - 1)\theta^{1-\sigma} - \theta(\sigma - 1)a(\lambda)^{1-\sigma}a(N)^{\sigma-1}\big](N-\lambda)\mu_2\big\},\end{aligned}$$

where

$$\Gamma \equiv \frac{\Delta a(\lambda)^{\sigma-1}(\lambda\mu_1 + \theta^{1-\sigma}(N-\lambda)\mu_2)^{1-(1/(\sigma-1))}}{\{\sigma + (\sigma-1)^2(N-\lambda)a(N)^{\sigma-2}a'(N)\mu_2\}\big[a(N)^{\sigma-1} - a(\lambda)^{\sigma-1}\big]\lambda\mu_1 F(\sigma-1)a(N)^{\sigma-1}} > 0.$$

Evaluating this at $t_1 = t_2$ yields

$$(32)\quad \frac{dv_1}{d\tau} = \Gamma^{-1} T_1(\sigma-1)a(\lambda)^{1-\sigma}\big[a(\lambda)^{\sigma-1} - a(N)^{\sigma-1}\big](\lambda\mu_1 + (N-\lambda)\mu_2) < 0.$$

Repeating this for country 2, we see that

$$(33)\quad \begin{aligned}\frac{dv_2}{d\tau} = T_2\theta a(\lambda)^{1-\sigma}\Gamma^{-1}\big\{&(\theta^{1-\sigma} - 1)(N-\lambda)\mu_2\sigma\theta^{\sigma} a(N)^{\sigma-1}\\ &+ (\sigma-1)\theta^{\sigma}\big(a(N)^{\sigma-1} - a(\lambda)^{(\sigma-1)}\big)\lambda\mu_1\\ &+ \big(\theta^{\sigma}(\sigma-1)a(N)^{\sigma-1} - (\theta\sigma - 1)a(\lambda)^{(\sigma-1)}\big)(N-\lambda)\mu_2\big\},\end{aligned}$$

[19] Note that when taxes are equal and the interpretation of G is a publicly provided good, this amounts to maximizing the indirect utility of the world representative consumer, making this a natural counterpart to the individual governments' objectives.

which, evaluated at $t_1 = t_2$, reduces to

$$(34)\quad \frac{dv_2}{d\tau} = T_2 a(\lambda)^{1-\sigma}\Gamma^{-1}\,(\sigma - 1)\big(a(N)^{\sigma-1} - a(\lambda)^{(\sigma-1)}\big)[\lambda\mu_1 + (N-\lambda)\mu_2] > 0.$$

Combining equations (32) and (34) (which at $t_1 = t_2$ implies that $T_1 = T_2$):

$$(35)\quad \left.\frac{\partial W}{\partial \tau}\right|_{\tau=1} = 0.$$

Thus, to maximize the real value of world welfare, the social planner harmonizes taxes. Note that this does not specify the level of taxes, merely their relative values.

Taking τ as given, the impact of country k's tax on its distribution of income between the private and public sectors is $(1-\alpha)t_k^{-1} - \alpha(1-t_k)^{-1}$, which equals zero when $t_k = 1 - \alpha$. Since the total impact of a country's tax rate on worldwide welfare is the combination of its effect on τ and its effect on its distribution of income, the social planner will set $t_A = t_B = 1 - \alpha$ in order to reach an optimum. Furthermore, note that at this solution, λ falls out of equation (30), implying that the social planner is indifferent as to which country hosts the high-productivity firms.

Thus, the social planner harmonizes taxes and sets them so that the marginal value of income is equalized between the public and private sectors. Furthermore, once taxes are equal, the social planner is indifferent between which country hosts the high-productivity firms. As such, this result is not dependent on the β assignment scheme under equal taxes (more on this below). Note that the social planner's solution is still only a constrained optimum since, as described by Avinash K. Dixit and Joseph E. Stiglitz (1977), the market is characterized by imperfect competition among firms, none of which can earn negative equilibrium profits.

While the optimal distribution of income between sectors is standard, it is perhaps less expected that the world welfare maximum involves tax harmonization. This is because the number of firms in our model is lowest when taxes are equal. With the love for variety Dixit-Stiglitz preferences represent, one might expect that the social planner would implement unequal taxes, thereby creating a low-cost location and encouraging entry, as discussed by Douglas Holtz-Eakin and Mary E. Lovely (1996). However, due to the resource constraint, optimality in the Dixit-Stiglitz class of models represents a tradeoff between the scale of firms (i.e., quantity of a given firm) and the scope of variety (N). Often, increasing output per firm comes at a cost of reduced factor availability, which reduces the number of firms. They show that the effect that dominates depends on the specifics of the model. In the special case of Dixit and Stiglitz (1977), the presence of a numeraire good creates an elastic labor supply in the differentiated goods industry. This results in the same scale of firms in both the market and socially optimal equilibria. In our model, we do not have such a numeraire. Thus, since total labor used in production equals $\overline{L_1} + \overline{L_2} - NF$, tax harmonization reduces N and increases the average amount of labor used in production. However, due to firm heterogeneity, this does not imply that production by each remaining firm rises. From Lemma 1, λ falls from harmonization. Thus, in country 1, there are fewer firms using $\overline{L_1}$, implying that production per firm there rises. In country 2, however, inspection of equation (9) shows that production per firm falls. This is because the labor freed by exiting firms is more than used up by entry of firms relocating from country 1.[20] Thus, not only does average labor used in production rise, there is a shift in labor allocation away from the relatively unproductive firms in country 2 toward the more productive firms in country 1. This added productivity gain induced by harmonization ensures that the welfare gain from increased output more than offsets the loss created by the reduction in varieties.

III. Nash Equilibrium Taxes

With the optimal taxes in hand, we ask whether it is possible to achieve them through uncoordinated tax competition, and find that this is not the case.

PROPOSITION 3: *The social planner's optimum is not achievable as a Nash equilibrium outcome.*

PROOF:

Let $t_A = t_B = 1 - \alpha$. Recalling that $v_2 < v^* < v_1$, we know that a country is strictly better off hosting the high-productivity firms with certainty rather than only having a chance at doing so. This is seen by comparing the income of country 1 with that of country 2 when taxes are equal. The ratio of equations (14) and (15) at this point is

$$\frac{I_1}{I_2} = \frac{\lambda_k^* \, \mu_1(\lambda_k^*)}{(N^* - \lambda_{-k}^*)\mu_2(\lambda_k^*, N^*)} > 1 \tag{36}$$

by equation (24). This discrete gain outweighs the marginal welfare loss from reallocating a share of income from the public to the private sector through an infinitesimal tax cut. Therefore, for each country k, its best response tax rate, $t_k(t_{-k})$, is such that $t_k(1 - \alpha) < 1 - \alpha$, implying that optimal taxes are not a Nash equilibrium.

Thus, optimal taxes are not an equilibrium. Furthermore, it is possible to show that even inefficiently low-harmonized taxes are not an equilibrium outcome in pure strategies.

PROPOSITION 4: *There are no pure strategy equilibria with harmonized taxes.*

PROOF:

To reach our result, we begin by examining a country's behavior given that it is the low-tax country 1, then examine behavior given that it is the high-tax country 2, and then combine them to finish the proof.

We begin with country 1. Recalling equation (32), and noting that $d\tau/dt_1 = \tau/(1 - t_1) > 0$, we see that ignoring the distribution of income between private and public consumption, country 1 has a dominant strategy of undercutting country 2's tax rate.

[20] An additional way to see this is that with harmonization, the relative wage in country 1 falls. This increases labor demand of remaining firms and increases their output. This is simultaneously an increase in the relative wage of country 2, reducing labor demand by firms there, and thus their output.

The full effect of t_1 on v_1, however, must also take into account the effect of t_1 on the distribution of income. Therefore, the actual first-order condition for country 1 is

$$(37)\quad \frac{dv_1}{dt_1} = \left[(1-\alpha)t_1^{-1} - \alpha(1-t_1)^{-1}\right] T_1(\sigma-1)(\lambda\mu_1 + \theta^{1-\sigma}(N-\lambda)\mu_2)^{1/(\sigma-1)}\lambda F\theta^{\sigma} a(N)^{\sigma-1}\mu_1 + \frac{\tau\Gamma^{-1}T_1\theta^{\sigma}}{1-t_1}\Big\{(\theta^{1-\sigma}-1)\sigma\theta^{\sigma}\Psi^{\sigma-1}\lambda\mu_1 + \left[\sigma - \left(1+\Psi^{\sigma-1}(\sigma-1)\right)\right]\theta\lambda\mu_1 + \left[(\sigma-1)\theta^{1-\sigma} - \theta(\sigma-1)\Psi^{\sigma-1}\right](N-\lambda)\mu_2\Big\}.$$

In order to understand the best response function $t_1(t_2)$ this implies, it is useful to consider two values of t_2. First, when $t_2 = 1$, $t_1(1) = 1-\alpha$, i.e., it will efficiently allocate income between the public and private sectors. The intuition is that when $t_2 = 1$, country 1 has no incentive to use its tax to affect firm location. The second value of t_2, denoted $\overline{t_2}$, is such that if country 1 knew that it would host the high- productivity firms, equation (37) equals zero by setting $t_1 = \overline{t_2}$. For country k this is

$$(38)\quad \overline{t_2}_k = \frac{(1-\alpha)}{\left(1 + \dfrac{\Gamma^{-1}[\Psi^{\sigma-1}-1]}{(\lambda_k^*\mu_1(\lambda_k^*) + (N^*-\lambda_k^*)\mu_2(\lambda_k^*,N^*))^{(1/(\sigma-1))-1}\lambda_k^* Fa(N^*)^{\sigma-1}\mu_1(\lambda_k^*)}\right)},$$

where Γ is a function of λ_k^* and N^*. Note that $\overline{t_2}_k < 1-\alpha$. At this $\overline{t_2}_k$, although it would be beneficial for 1 to reduce t_1 below $\overline{t_2}_k$ in terms of affecting τ, this would cause too great a distortion to the distribution of income. However, if this country actually set its tax rate equal to that of the other, it would not know for certain that it would receive the high-productivity firms. Thus, the best response is $t_1(\overline{t_2}) < \overline{t_2}$. Graphically, this looks as in Figure 2, where, for values of $t_2 \leq \overline{t_2}$, the best response lies just to the left of the 45 degree line.[21] However, it is straightforward to show that given the strict convexity of the preferences, for each value of t_2, there is a unique value of $t_1(t_2)$ corresponding to it implying that the best response does not bend backward.

This desire to undercut the overseas tax is comparable to that in discrete investment models in which there is a discrete change in welfare generated by undercutting the overseas tax, as this guarantees winning of the firm. Unlike those models, there is also a desire to strictly undercut the other country in order to increase wages and attract more firms.[22] Thus, even though the endogenous variables in our model

[21] Note that we have not proven the exact shape of the portion above this point and that these are just for illustrative purposes. However, it is straightforward to show that given the strict convexity of the preferences, for each value of t_2, there is a unique value of $t_1(t_2)$ corresponding to it implying that the best response does not bend backward. As shown below, the critical feature of the best response is that it does not lie on the 45 degree line.

[22] Since higher wages imply higher prices for goods produced in this country, this also acts as a terms of trade argument for lowering taxes. This is akin to the export subsidy terms of trade argument put forth by Motoshige

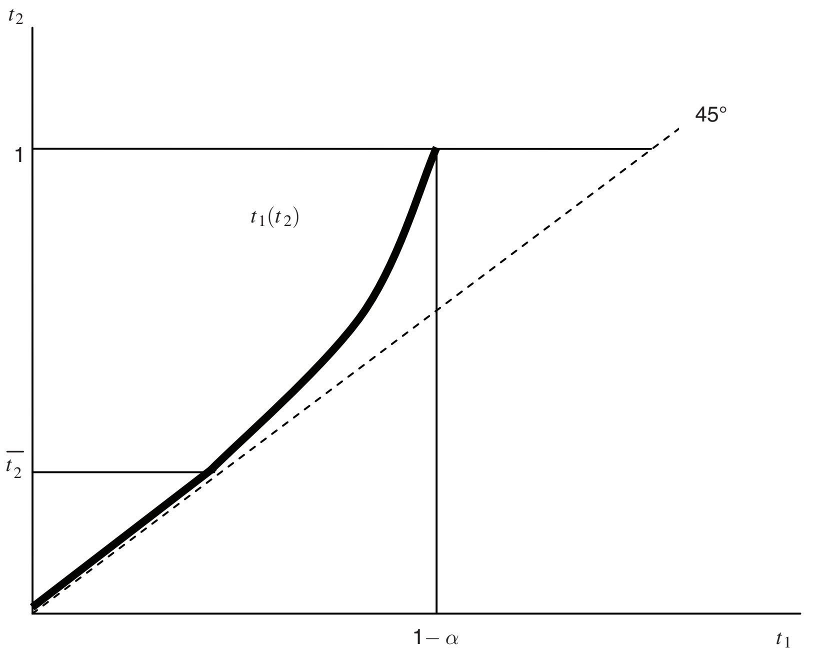

FIGURE 2. BEST RESPONSE OF COUNTRY 1

move continuously, this leads to a dominant strategy not found in continuous investment models.

Turning to country 2, we recall equation (34) and note that $d\tau/dt_2 = -1/(1 - t_1) < 0$. Thus, ignoring the distribution of income, country 2 wants to increase τ to its highest value by setting $t_2 = t_1$. Looking at the total impact of t_2 on v_2, we find that

$$(39)\quad \frac{dv_2}{dt_2} = \left[(1-\alpha)t_2^{-1} - \alpha(1-t_2)^{-1}\right]$$
$$\times\; T_2(\sigma-1)(\lambda\mu_1 + \theta^{1-\sigma}(N-\lambda)\mu_2)^{1/(\sigma-1)}(N-\lambda)\theta a(N)^{\sigma-1} F\mu_2$$
$$- \frac{T_2\theta\Gamma^{-1}}{(1-t_1)}\Big\{(\theta^{1-\sigma}-1)(N-\lambda)\mu_2\sigma\theta^{\sigma}\Psi^{\sigma-1} + (\sigma-1)\theta^{\sigma}(\Psi^{\sigma-1}-1)\lambda\mu_1$$
$$+ (\theta^{\sigma}(\sigma-1)\Psi^{\sigma-1} - (\theta\sigma-1))\, a(\lambda)^{(\sigma-1)}(N-\lambda)\mu_2\Big\}.$$

As before, it is instructive to consider the best response $t_2(t_1)$ at two key values of t_1. First, when $t_1 = 0$, if country 2 matches this tax it devotes no income to the public sector, minimizing its welfare. However, if country 2 allocates income efficiently by setting $t_2 = 1 - \alpha$, the first term goes to zero while the second is negative, implying that this is not a best response. Thus $0 < t_2(0) < 1 - \alpha$.

Itoh and Kazuharu Kiyono (1987), since, in their model, subsidizing exports of marginally exported products drives up wages, prices, and the welfare generated by other exported goods.

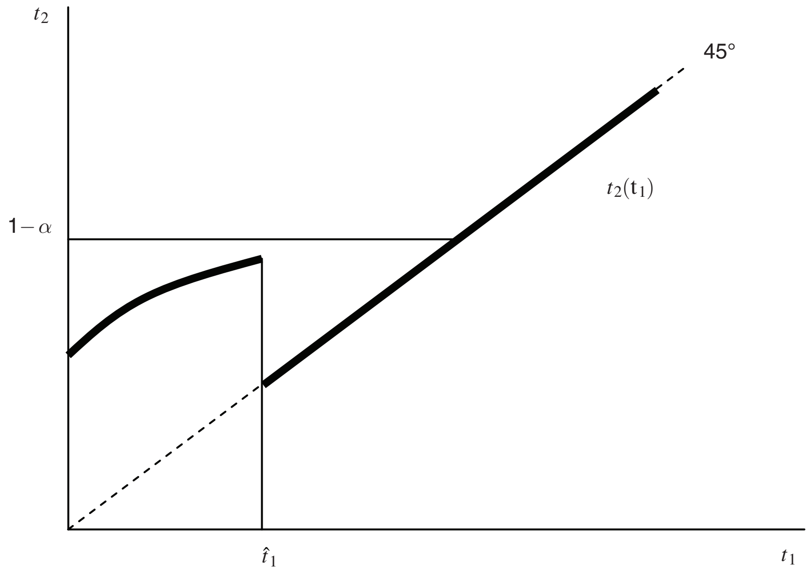

Figure 3. Best Response of Country 2

Next, similar to country 1's $\overline{t_2}$, there exists a value of t_1 denoted by $\tilde{t}_1$, such that if country 2 knew for sure it would receive the low-productivity firms, its first order condition equals zero by setting $t_2(\tilde{t}_1) = \tilde{t}_1$. Specifically, for country k:

$$(40)\quad \tilde{t}_{1k} = \frac{(1-\alpha)}{1+\left(\dfrac{\Gamma^{-1}(\Psi^{\sigma-1}-1)}{(\lambda^*_{-k}\mu_1(\lambda^*_{-k})+(N^*-\lambda^*_{-k})\mu_2(\lambda^*_{-k},N^*))^{(1/(\sigma-1))-1}(N^*-\lambda^*_{-k})a(N^*)^{\sigma-1}F\mu_2(\lambda^*_{-k},N^*)}\right)},$$

where Γ is a function of N^* and λ^*_{-k}. Note that this is less than $1-\alpha$.

In practice, however, once country 2 matches its tax rate to that of country 1, it receives a discrete boost in expected income, since it now has a positive probability of receiving the high productivity firms. Because of this, there is a strict income advantage to matching tax rates at this point. Therefore, there will exist a tax rate $\hat{t}_{1k}$ by country 1 for which country 2 is indifferent between having a higher tax rate with its superior allocation of income and an equal tax rate with a higher expected income level. Given the above discussion, it follows that $0 < \hat{t}_{1k} < \tilde{t}_{1k}$. Thus, country 2's best response is characterized by $t_2(t_1) \geq t_1$ with strict inequality only when $t_1 < \hat{t}_{1k}$. See Figure 3, where, again, strict concavity of preferences rules out a backward-bending best response.[23]

[23] Note that, as in Figure 2, we have not proven the slope of this best response above the 45 degree line. Nevertheless, as shown below, the critical feature of the best response is that it is never optimal to match tax rates.

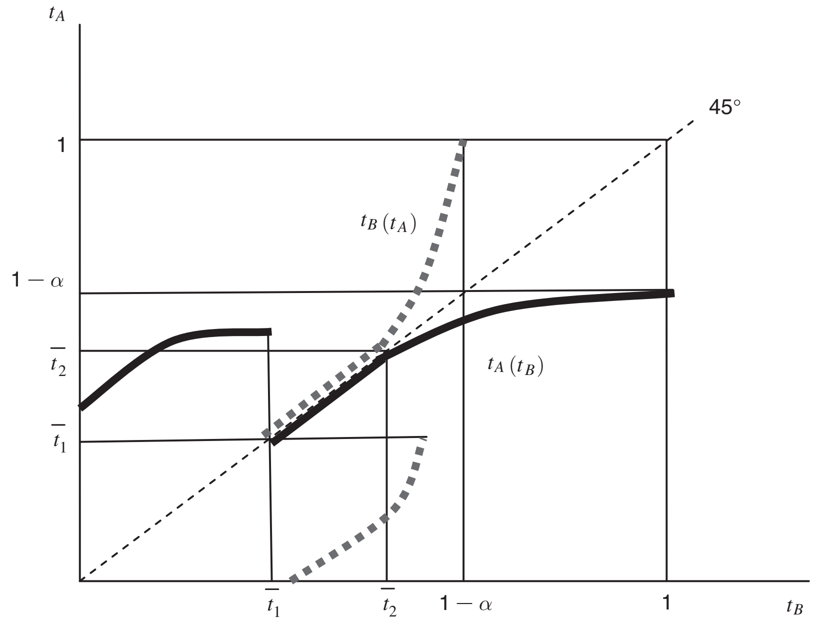

Figure 4. Best Responses When $L_A = L_B$

Combining these to form the best response for country k, three facts come to the forefront. First, when $t_{-k} = 0$, country k will find it desirable to set a strictly higher tax rate and allocate some income to the public sector. Second, when $t_{-k} = 1$, country k will choose to set a strictly smaller tax rate (in fact, one equal to $1 - \alpha$). Thus, for low values of t_{-k} country k will choose to be the high-tax country 2, whereas for high values of t_{-k}, it will choose to be the low-tax country 1. Third, when $t_{-k} = \hat{t}_{1k}$ for country k, we know that k is indifferent between maintaining a tax rate higher than $\hat{t}_{1k}$ and matching tax rates *if it is unable to lower its tax further.* However, it can lower its tax further and, given the discussion for country 1, will find it desirable to do so. Thus, country k's indifference is, in fact, between having a strictly higher tax rate or a strictly smaller tax rate. Therefore, the jump in country k's best response will happen when $-k$'s tax rate is $t_{-k} = \overline{t_{1k}} < \hat{t}_{1k}$, implying that there exist two optimal tax rates at this point, i.e., $t_k(\overline{t_{1k}}) = \{\underline{t_k}, \overline{t_k}\}$, where $\underline{t_k} < \overline{t_{1k}} < \overline{t_k}$. The key implication of this is that matching is never a best response. Therefore, there do not exist pure strategy equilibria with harmonized taxes.

Thus, neither the efficient distribution of income nor the efficient number of firms will occur as part of a pure strategy equilibrium. Furthermore, since country k only assigns positive probability to setting $t_k = 1 - \alpha$ when the other country places positive probability to setting $t_{-k} = 1$, which $-k$ will never do, there is no possibility of efficient income distribution arising in either country as an equilibrium outcome.

What, then, can be said about the Nash equilibria? For the case of identical countries, we can show that there are no pure strategy Nash equilibria.

PROPOSITION 5: *When countries are identical, there are no pure strategy Nash equilibria.*

PROOF:

With identical countries, $\lambda_A^* = \lambda_B^*$. Thus, comparing equations (38) and (40), we see that the difference between $\overline{t_1}_k$ and $\tilde{t}_{1k}$ is that equation (38) has $\lambda_k^* \mu_1(\lambda_k^*)$ in its denominator, whereas equation (40) has $(N^* - \lambda_k^*)\mu_2(\lambda_k^*, N^*)$ in its denomintor. Using equation (24), this implies that $\overline{t_2}_k > \tilde{t}_{1k}$. Therefore, $\overline{t_1}_A = \overline{t_1}_B < \overline{t_2}_A = \overline{t_2}_B$, i.e., that point at which the jump in country 2's best response occurs (i.e., $\overline{t_1}_A$) is before country 1's best response moves discretely away from the 45 degree line. This implies that best responses have discrete jumps in them as shown in Figure 4.[24] Given the symmetry between countries, this rules out the possibility of pure strategy Nash equilibria.

These best responses combine features of those found in both the continuous and discrete investment models. When *B*'s tax rate is moderate (in the middle of Figure 4), the dominant factor in *A*'s decision making is the effect of its tax rate on the distribution of firms. As a result, as in the discrete investment models it chooses to undercut *B*'s tax. However, when *B*'s tax rate is very high or very low (the extremes of Figure 4), *A* becomes far more cognizant of the tradeoff between the tax base and the allocation of income between sectors. This then leads to behavior comparable to that found in the continuous investment models.

What can be said when *A* has strictly more labor than *B*? Given the above discussion, it is still clear that no pure strategy equilibrium exists with equal taxes. If a pure strategy equilibrium does exist, then we can place the following restrictions on it.

PROPOSITION 6: *Suppose that country A is strictly larger than B. Any pure strategy Nash Equilibrium must be such that* $t_B < t_A < 1 - \alpha$. *The existence of such a pure strategy equilibrium requires that A's labor endowment be sufficiently large relative to B's.*

PROOF:

For there to exist a pure strategy Nash equilibrium in which *B* has the higher tax, then it must be that $\overline{t_2}_A$, the lowest tax by *B* for which *A* is willing to be the low-tax country, is greater than $\tilde{t}_{1_B}$, the highest tax by *A* for which *B* is willing to be the high tax country, i.e., that $\tilde{t}_{1_B} - \overline{t_2}_A > 0$. For this to be true, it must be that

$$\lambda_A^* \mu_1(\lambda_A^*) < (N^* - \lambda_A^*)\mu_2(\lambda_A^*, N^*). \tag{41}$$

Using equation (24) for country *B*, this, in turn, requires that

$$\lambda_A^* \mu_1(\lambda_A^*) < (N^* - \lambda_A^*)\mu_2(\lambda_A^*, N^*) < \lambda_B^* \mu_1(\lambda_B^*) \tag{42}$$

which, since $\lambda_A^* > \lambda_B^*$ and $\lambda\mu_1(\lambda)$ is increasing in λ, cannot be true.

[24] Discontinuous best-responses are found in a number of papers considering tax competition between parent and host countries (e.g., Roger H. Gordon 1992, Davies and Gresik 2003, and Davies 2003).

Thus, any pure strategy Nash equilibria must be such that the larger country sets the higher tax. Note that if such a pure strategy equilibrium exists, it must be such that both taxes are less than the socially optimal tax of $1 - \alpha$. To see this, recall that by Proposition 3, the best response to a tax of $1 - \alpha$ is less than this level. Thus, $t_B < 1 - \alpha$. Using this in equation (39), we see that evaluating $dv_A/dt_A|_{t_A=1-\alpha} < 0$, indicating that $t_A < 1 - \alpha$ as well.

For such a pure strategy equilibrium to occur, a necessary condition is that $\tilde{t}_{1_A} - \overline{t_2}_B > 0$, since only if this is true will there exist tax rates for which A is willing to have a discretely higher tax, and B is willing to have a discretely lower tax (i.e., this is necessary for $\overline{t_1}_A - \overline{t_2}_B > 0$). For $\tilde{t}_{1_A} - \overline{t_2}_B > 0$, it must be that

$$\lambda_B^* \mu_1(\lambda_B^*) < (N^* - \lambda_B^*)\mu_2(\lambda_B^*, N^*). \tag{43}$$

When countries are the same in size, this condition fails. Keeping total labor supply constant but lowering $\overline{L}_B$ also lowers λ_B^*, reducing the left-hand side of equation (43) and increasing the right-hand side of equation (43). Furthermore, as $\overline{L}_B$ approaches zero, λ_B^* does as well. This implies that there must be a sufficiently large degree of asymmetry to have $\overline{t_1}_A - \overline{t_2}_B > 0$. As a result, only when asymmetries are sufficiently large is there a possibility of a pure strategy Nash equilibrium.

This then predicts that if there is a pure strategy Nash equilibrium, the larger country will set the higher tax since its large labor supply makes the marginal firm less elastic to its tax. To see this, note that as country B becomes small, $d\lambda/d\tau$ and $dN/d\tau$ approach zero, i.e., the distribution of firms becomes unresponsive to the tax, thereby reducing the incentive for the high-tax country to lower its tax.[25] It is worth noting that this matches the empirical results of Michael P. Devereux, Ben Lockwood, and Michela Redoano (2008) who find that within the Organisation for Economic Co-operation and Development (OECD) relatively large countries set higher statutory corporate taxes. M. Peter van der Hoek (2003) finds a similar pattern in European Union taxes. Note that we are not claiming that a pure strategy Nash equilibrium exists. This is a necessary but not a sufficient condition. The only point at which pure strategy equilibria definitely exist is in the limit when all labor is in A. In the limit, since B cannot support any firms, the distribution of firms no longer depends on taxes. Thus, A's dominant strategy is to set its tax equal to $1 - \alpha$ and any tax rate by B is a best response. Thus, there exists a continuum of pure strategy Nash equilibria. However, as long as both countries have positive labor supplies, regardless of whether the Nash equilibria are in pure or mixed strategies, all equilibrium outcomes are inefficient.

IV. Alternative Assumptions

The previous results are robust to several alternative assumptions. In this section, we consider several permutations of the baseline model. First, consider alternative firm distributions with equal taxes. For example, assume that the large country A

[25] A similar result is found in Wilson (1991).

hosts the high productivity firms when $\tau = 1$. In this case, B would be willing to match A's tax for intermediate tax levels. Nevertheless, A benefits by marginally undercutting B's tax. Thus A's best response eliminates pure strategy equilibria with equal taxes. Alternatively, we could distribute firms so that average profits, average productivities, or the number of firms are the same across countries. In each case, by marginally undercutting the overseas tax, a country creates a discrete shift in its income by attracting only the most productive firms, eliminating pure strategy Nash equilibria with equal taxes. Furthermore, it is still a best response to set a tax rate of $1 - \alpha$ only when the other country sets its tax equal to 1. Since the social planner is indifferent to how firms are distributed when $t_A = t_B = 1 - \alpha$, these alternatives do not change the socially optimal taxes.

Second, we can change the assumption that wage income and profits are taxed equally. If we allow for different tax rates, then because labor is exogenously endowed, a wage tax is a nondistortionary lump sum tax. If wage income is sufficiently large so that enough tax revenues can be generated for public use, then it is optimal for governments to use the wage tax to transfer income between private and public sectors, and use its profit tax to attract the desired number of firms. This separates the need to balance τ against income allocation. Given the results above for a country's preferred τ, it is clear that this leads to a race to the bottom which, unless taxes are bounded from below, implies that profit taxes shoot toward negative infinity.[26] It is worth noting that in this case, if both profit tax rates are bounded at the same point, this equilibrium is efficient relative to the social planner's problem. This is because the minimum tax rate effectively harmonizes profit taxes, and wage taxes distribute income optimally.

Third, we assumed that firm profits accrue entirely to local income. However, this strong of an assumption is not necessary for our results. If we replace it with one assuming that the majority of a firm's profits go to local income, all of our results hold. This is because, when taxes are equal, it still strictly benefits a country to undercut the other's tax because of the boost to income this provides. The primary difference is that the discrete gain from doing so is smaller than before because the country only keeps a majority of the profit earned by these high profit firms. As another alternative, suppose that an exogenous portion of after-tax profits accrues to each country regardless of its location (as might occur with diversified ownership). With equal taxes (and equal wages), private income is the same regardless of whether or not a country receives the high-productivity firms. However, since profits are higher in the country hosting these firms, tax revenues are higher in these countries as well. Therefore there is, again, the discrete welfare gain from hosting the high-productivity firms, indicating that countries will seek to undercut one another's tax rate resulting in comparable findings.[27]

[26] Note that what is important is that governments compete in average effective tax rates. With additional tax instruments applied to firms, location will still be driven by the average effective tax rate. When this includes both marginal rates, such as τ, and lump sum taxes, even for a given average effective tax, the combination of the two has the potential to create interesting effects on the distribution of firms. We leave this to future work.

[27] Suppose one takes the entrepreneur interpretation of the genesis of firms (as in footnote 12) in which the entrepreneur i, herself, is immobile, but that she can set up the firm wherever she chooses and with some portion (include 100 percent) of after-tax profits returning to her. Now, public good provision does not enter into the firm location decision. This would yield results the same as with an immobile owner but a mobile production facility.

Fourth, we can consider best responses when firms are able to geographically fragment their activities, i.e., become multinationals. In the literature on FDI, there are two broad classes of multinational firms: vertical firms that engage in headquarters activity in one country and production in another (Helpman 1984), and horizontal firms that have their headquarters in one country but produce in multiple countries (James R. Markusen 1984). Consistent with this literature, let the location of the fixed cost represent the headquarter activity. In our model, as noted by Markusen (1984), the absence of trade costs and constant returns to scale in production eliminate the need for multiple production facilities.[28] Thus, if multinationals exist in our model, they are of the vertical type.[29] An important difference between our setting and the standard one is that in the typical model of vertical FDI, multinationals arise due to factor price differences across countries. Typically, headquarter services are skilled-labor intensive relative to production. Therefore, if countries differ in their relative endowments and factor prices are not equalized through trade, then the skilled-labor abundant country hosts the headquarter activity, and the other country hosts production. In our model, however, there is only one factor of production. Nevertheless, as discussed above, when taxes differ there can still exist wage differences across countries, providing a reason for vertical FDI.

In order to explore the implications of vertical FDI, it is necessary to make some assumptions regarding tax jurisdictions. Specifically, we assume that countries only levy taxes on firms headquartered within their borders. We also assume that the parent part of the multinational (i.e., where the fixed cost occurs) pays its subsidiary (where production occurs) w_k per unit of output, where k is the country hosting production. This amounts to assuming that there is no ability to transfer price.[30] As a result, the only tax base for a given firm is found in the country hosting its headquarters.[31]

When taxes are equal, as before, wages will be equal. Therefore, there is no need for firms to fragment their activities and all of the properties above hold. Now suppose that $t_1 < t_2$, which, for labor markets to clear, implies that $w_1 > w_2$. In this case, all firms will seek to locate their production in the low-cost country 2, since there are no tax advantages to locating production in country 1. We assume that the labor supply in 2 is large enough to handle this. Unlike production, there are advantages to locating the headquarters in the low-tax country 1. The primary difference this causes is that the indifferent firm λ now pays w_2 on production costs regardless of where it locates its headquarters. Thus, equation (11) becomes

$$\left[\theta a(N)^{\sigma-1} - a(\lambda)^{\sigma-1}\right] = \tau\theta\left[a(N)^{\sigma-1} - a(\lambda)^{\sigma-1}\right]. \tag{44}$$

[28] In fact, in this setting, if there are costs to building each production facility, only single production-location firms will exist.

[29] Evidence of vertical FDI is found by Henrik Braconier, Pehr-Johan Norback, and Dieter Urban (2005); Gordon H. Hanson, Raymond J. Mataloni, Jr., and Matthew J. Slaughter (2005); Yeaple (2003); and Susan E. Feinberg and Michael P. Keane (2001).

[30] We discuss relaxing this momentarily.

[31] An advantage of this is that it eliminates the need to consider double tax issues, the strategic aspects of which are considered by Eric W. Bond and Larry Samuelson (1989), Janeba (1995), and Davies (2003). We leave a more realistic treatment of this issue to future work.

Once again, this implies that low-productivity firms will find it advantageous to locate their headquarters in country 2 because the tax savings are small compared to the wage savings on the fixed cost. Furthermore, this yields the same equation determining the last firm to enter equation (8). Now, however, since the cost advantages to locating in country 2 are smaller, the indifferent firm has a higher index than in the baseline model.

Despite this change in the equilibrium λ, there is still a discrete income benefit to undercutting the other nation's tax rate for the same reasons described above. In fact, since FDI increases the profits of high-productivity firms, doing so attracts even more profitable firms than it did before. This, then, provides a greater income boost than in the baseline model, therefore the introduction of FDI only increases the severity of the race to the bottom tax competition. Thus, as in the baseline model, there are no pure strategy Nash equilibria with equal taxes, and all equilibrium outcomes will have taxes less than $1 - \alpha$. Since the equilibrium with vertical FDI is the same as the baseline case when taxes are equal, allowing vertical FDI does not change the solution to the social planner's problem, implying that all Nash equilibrium outcomes are again inefficient.

Finally, it is important to recognize that alternative assumptions on the social welfare function can result in tax harmonization being undesirable from the social planner's perspective. First, if countries have different α's then harmonization creates distortions for at least one of them vis-à-vis its income distribution. As such, the social planner may choose to set differing tax rates in the two locations. Second, if countries' welfares are unequally weighted in the social welfare function, the social planner has two reasons to maintain different tax rates. The first of these is that the social planner is no longer indifferent over which country hosts the high-productivity firms. By setting its favored nation's tax just under the other, this ensures that its favored nation will host these firms, providing a boost to the social welfare function. Furthermore, by lowering the favored nation's tax relative to the others, this benefits that country (at the expense of the other) by sending even more firms to it. Thus, using unequal tax rates is a method of shifting income from the high-tax to the low-tax country. Nevertheless, this is an inefficient way of doing so since lump sum transfers could be used to shift this income without the distortions unequal taxes create.

V. Conclusion

This paper has incorporated recent innovations from the trade literature on mobile firms into a tax competition model by modeling competition between governments for heterogeneous, imperfectly competitive firms with endogenous entry. These new features highlight a heretofore unrecognized aspect of tax competition—that it can encourage excessive firm entry. This adds to the typical woe of tax competition, the underprovision of a public good. Furthermore, our framework allows us to study the extent of this problem, even in a model where firms choose a single location, something that cannot be done in other models of discrete investment.

An implication of our results is that tax coordination, or at least de facto coordination by imposing a minimum tax rate across countries, can improve welfare relative to the Nash equilibrium. This then lends some support to the drive for such coordination by the OECD (1998, 2000) or the European Union (see van der Hoek 2003, for a discussion). While there are certainly reasons to caution against harmonization (such as varying preferences over public versus private consumption), we hope that our results add further depth to this lively and important debate.

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