6 Circular Motion, Orbits, and Gravity

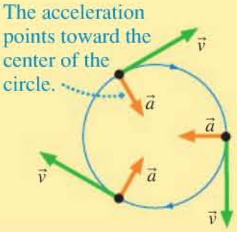
Motorcyclists in the "Globe of Death" ride their bikes on the inside of a spherical steel frame, seeming to defy gravity as they ride up the sides and upside down over the top. What prevents them from falling?

LOOKING AHEAD ▸▸

The goal of Chapter 6 is to learn about motion in a circle, including orbital motion under the influence of a gravitational force.

Uniform Circular Motion

A particle moving in a circle at a constant speed undergoes **uniform circular motion**. In this chapter you'll learn how to describe a particle's motion in terms of its *angular* position and *angular* velocity.



We'll also review the important idea that a particle moving in a circle has an acceleration directed toward the center of the circle.

Looking Back ◂◂
2.2 Uniform motion
3.8 Circular motion

Dynamics of Uniform Circular Motion

Because a particle moving in a circle has an acceleration that points toward the center of the circle, there must be a net force toward the center to cause this acceleration.

The net force on the girl is directed toward the center of the circle.

The normal force of the track and the car's weight combine to provide a net force toward the circle's center.

Looking Back ◂◂
5.2 Using Newton's second law

Apparent Forces in Circular Motion

An object moving in a circle appears to experience a force that "flings" it outward. You'll learn that these apparent forces are not real forces, but are in fact a consequence of Newton's first law.

What holds these riders in this carnival ride against the wall?

Looking Back ◂◂
5.3 Weight and apparent weight

Newton's Law of Gravity

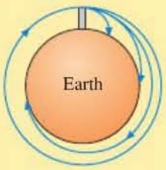
Newton discovered the law that governs gravity. You'll learn how it applies to an apple falling to earth or a rock falling on the moon, and how this law governs the motions of the moon, the planets, and even distant galaxies.

Newton's great insight was that the law of gravity described not only falling objects but also the orbits of the moon and planets.

Even the structure of distant galaxies is determined by Newton's law of gravity.

Gravity and Orbits

If an object moves fast enough, it can orbit the earth, the sun, or another planet.



An orbit can be thought of as projectile motion where the ground curves away just as fast as the object falls.

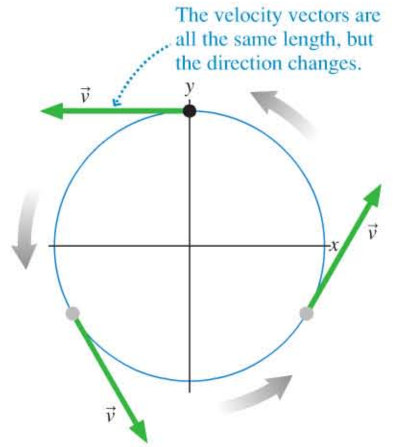
The space station appears weightless, but gravity still acts strongly on it; only its *apparent weight* is zero.

6.1 Uniform Circular Motion

We began our study of circular motion in Section 3.8. There, we learned how to describe the circular motion of a particle in terms of its period and frequency. We also learned that a particle moving in a circle has an acceleration—even if the particle's speed is constant—because the *direction* of its velocity is constantly changing.

In this chapter, we'll study the simplest kind of circular motion, in which a particle moves at a *constant* speed around its circular path. FIGURE 6.1 shows a particle undergoing this uniform circular motion. The particle might be a satellite moving in an orbit, a ball on the end of a string, or even just a dot painted on the side of a wheel. Regardless of what the particle represents, its velocity vector is always tangent to the circular path. The particle's speed v is constant, so the vector's length stays constant as the particle moves around the circle.

FIGURE 6.1 A particle in uniform circular motion.

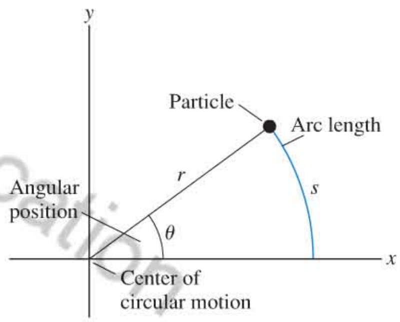


Angular Position

In order to describe the position of a particle as it moves around the circle, it is convenient to use the angle θ from the positive x-axis. This is shown in FIGURE 6.2. Because the particle travels in a circle with a fixed radius r, specifying θ completely locates the position of the particle. Thus we call angle θ the **angular position** of the particle.

We define θ to be positive when measured *counterclockwise* from the positive x-axis. An angle measured *clockwise* from the positive x-axis has a negative value. "Clockwise" and "counterclockwise" in circular motion are analogous, respectively, to "left of the origin" and "right of the origin" in linear motion, which we associated with negative and positive values of x.

FIGURE 6.2 A particle's angular position is described by angle θ.



Rather than measure angles in degrees, mathematicians and scientists usually measure angle θ in the angular unit of *radians.* In Figure 6.2, we also show the **arc length** s, the distance that the particle has traveled along its circular path. We define the particle's angle θ in **radians** in terms of this arc length and the radius of the circle:

$$\theta \text{ (radians)} = \frac{s}{r} \tag{6.1}$$

This is a sensible definition of an angle: The farther the particle has traveled around the circle (i.e., the greater s is), the larger the angle θ in radians. The radian, abbreviated rad, is the SI unit of angle. An angle of 1 rad has an arc length s exactly equal to the radius r. An important consequence of Equation 6.1 is that the arc length spanning the angle θ is

$$s = r\theta \tag{6.2}$$

NOTE ► Equation 6.2 is valid only if θ is measured in radians, not degrees. This very simple relationship between angle and arc length is one of the primary motivations for using radians. ◄

When a particle travels all the way around the circle—completing one *revolution,* abbreviated rev—the arc length it travels is the circle's circumference $2\pi r$. Thus the angle of a full circle is

$$\theta_{\text{full circle}} = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

We can use this fact to define conversion factors among revolutions, radians, and degrees:

$$1 \text{ rev} = 360° = 2\pi \text{ rad}$$

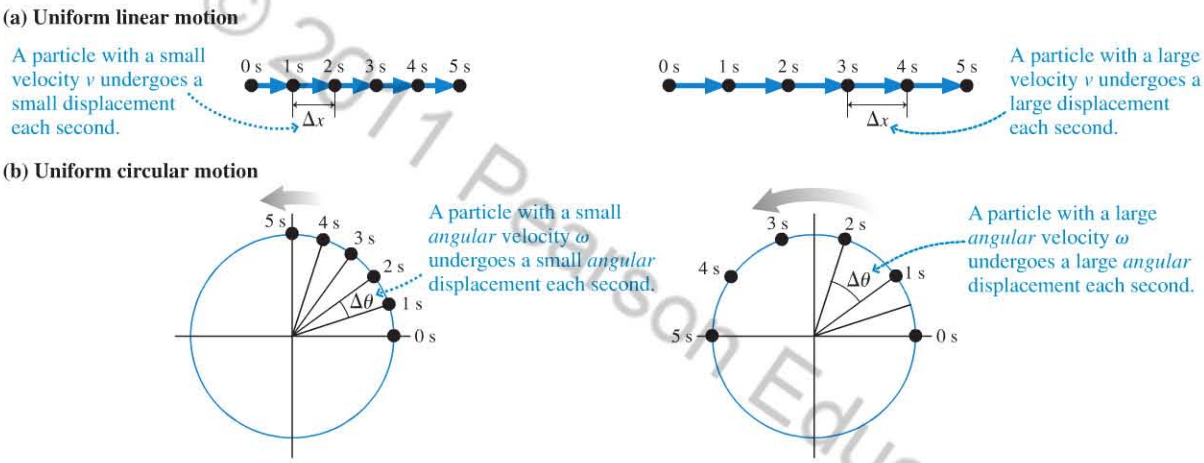
$$1 \text{ rad} = 1 \text{ rad} \times \frac{360°}{2\pi \text{ rad}} = 57.3°$$

We will often specify angles in degrees, but keep in mind that the SI unit is the radian. You can visualize angles in radians by remembering that 1 rad is just about 60°.

Angular Displacement and Angular Velocity

For the *linear* motion you studied in Chapters 1 and 2, a particle with a larger velocity undergoes a greater displacement in each second than one with a smaller velocity, as **FIGURE 6.3a** shows. **FIGURE 6.3b** shows two particles undergoing uniform *circular* motion. The particle on the left is moving slowly around the circle; it has gone only one-quarter of the way around after 5 seconds. The particle on the right is moving much faster around the circle, covering half of the circle in the same 5 seconds. You can see that the particle to the right undergoes twice the **angular displacement** $\Delta\theta$ during each interval as the particle to the left. Its **angular velocity,** the angular displacement through which the particle moves each second, is twice as large.

FIGURE 6.3 Comparing uniform linear and circular motion.



In analogy with linear motion, where $v_x = \Delta x/\Delta t$, we thus define the angular velocity as

$$\omega = \frac{\text{angular displacement}}{\text{time interval}} = \frac{\Delta\theta}{\Delta t} \qquad (6.3)$$

Angular velocity of a particle in uniform circular motion

The symbol ω is a lowercase Greek omega, *not* an ordinary w. The SI unit of angular velocity is rad/s.

Figure 6.3a shows that the displacement Δx of a particle in uniform linear motion changes by the same amount each second. Similarly, as Figure 6.3b shows, the *angular* displacement $\Delta\theta$ of a particle in uniform *circular* motion changes by the same amount each second. This means that **the angular velocity $\omega = \Delta\theta/\Delta t$ is constant for a particle moving with uniform circular motion.**

EXAMPLE 6.1 **Comparing angular velocities**

Find the angular velocities of the two particles in Figure 6.3b.

PREPARE For uniform circular motion, we can use any angular displacement $\Delta\theta$, as long as we use the corresponding time interval Δt. For each particle, we'll choose the angular displacement corresponding to the motion from $t = 0$ s to $t = 5$ s.

SOLVE The particle on the left travels one-quarter of a full circle during the 5 s time interval. We learned earlier that a full circle corresponds to an angle of 2π rad, so the angular displacement for this particle is $\Delta\theta = (2\pi \text{ rad})/4 = \pi/2$ rad. Thus its angular velocity is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2 \text{ rad}}{5 \text{ s}} = 0.31 \text{ rad/s}$$

The particle on the right travels halfway around the circle, or π rad, in the 5 s interval. Its angular velocity is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi \text{ rad}}{5 \text{ s}} = 0.63 \text{ rad/s}$$

ASSESS The angular velocity of the particle on the right is 0.63 rad/s, meaning that the particle travels through an angle of 0.63 rad each second. Because 1 rad $\approx 60°$, 0.63 rad is roughly 35°. In Figure 6.3b, the particle on the right appears to move through an angle of about this size during each 1 s time interval, so our answer is reasonable.

Angular velocity, like the velocity v_x of one-dimensional motion, can be positive or negative. The signs for ω noted in **FIGURE 6.4** are based on the convention that angles are positive when measured counterclockwise from the positive x-axis.

We've already noted how circular motion is analogous to linear motion, with angular variables replacing linear variables. Thus much of what you learned about linear kinematics and dynamics carries over to circular motion. For example, Equation 2.4 gave us a formula for computing a linear displacement during a time interval:

$$x_f - x_i = \Delta x = v_x \, \Delta t$$

You can see from Equation 6.3 that we can write a similar equation for the angular displacement:

$$\theta_f - \theta_i = \Delta\theta = \omega \, \Delta t \qquad (6.4)$$

Angular displacement for uniform circular motion

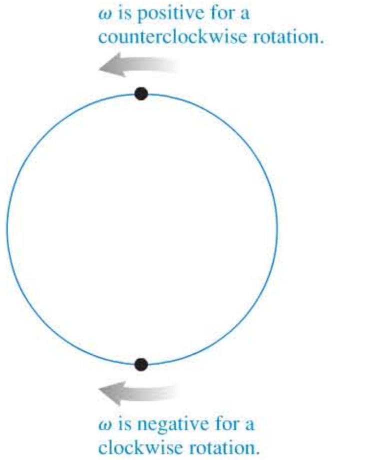


FIGURE 6.4 Positive and negative angular velocities.

For linear motion, we use the term *speed* v when we are not concerned with the direction of motion, *velocity* v_x when we are. For circular motion, we define the **angular speed** to be the absolute value of the angular velocity, so that it's a positive quantity irrespective of the particle's direction of rotation. Although potentially confusing, it is customary to use the symbol ω for angular speed *and* for angular velocity. If the direction of rotation is not important, we will interpret ω to mean angular speed. In kinematic equations, such as Equation 6.4, ω is always the angular velocity, and you need to use a negative value for clockwise rotation.

EXAMPLE 6.2 Kinematics at the roulette wheel

A small steel ball rolls counterclockwise around the inside of a 30.0-cm-diameter roulette wheel. The ball completes exactly 2 rev in 1.20 s.

a. What is the ball's angular velocity?
b. What is the ball's angular position at $t = 2.00$ s? Assume $\theta_i = 0$.

PREPARE Treat the ball as a particle in uniform circular motion.

SOLVE

a. The ball's angular velocity is $\omega = \Delta\theta/\Delta t$. We know that the ball completes 2 revolutions in 1.20 s, and that each revolution corresponds to an angular displacement $\Delta\theta = 2\pi$ rad. Thus

$$\omega = \frac{2(2\pi \text{ rad})}{1.20 \text{ s}} = 10.5 \text{ rad/s}$$

Because the rotation direction is counterclockwise, the angular velocity is positive.

b. The ball moves with constant angular velocity, so its angular position is given by Equation 6.4. Thus the ball's angular position at $t = 2.00$ s is

$$\theta_f = \theta_i + \omega \, \Delta t = 0 \text{ rad} + (10.5 \text{ rad/s})(2.00 \text{ s}) = 21.0 \text{ rad}$$

If we're interested in where the ball is in the wheel at $t = 2.00$ s, we can write its angular position as an integer multiple of 2π (representing the number of complete revolutions the ball has made) plus a remainder:

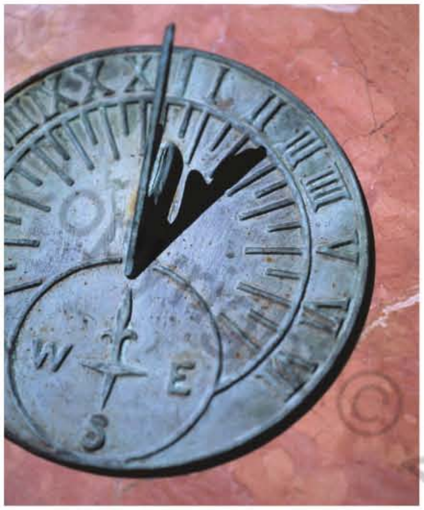
$$\theta_f = 21.0 \text{ rad} = 3.34 \times 2\pi \text{ rad}$$
$$= 3 \times 2\pi \text{ rad} + 0.34 \times 2\pi \text{ rad}$$
$$= 3 \times 2\pi \text{ rad} + 2.1 \text{ rad}$$

In other words, at $t = 2.00$ s, the ball has completed 3 rev and is 2.1 rad = 120° into its fourth revolution. An observer would say that the ball's angular position is $\theta = 120°$.

ASSESS Since the ball completes 2 revolutions in 1.20 s, it seems reasonable that it completes 3.34 revolutions in 2.00 s.

The angular speed ω is closely related to the period T and the frequency f of the motion. If a particle in uniform circular motion moves around a circle once, which by definition takes time T, its angular displacement is $\Delta\theta = 2\pi$ rad. The angular speed is thus

$$\omega = \frac{2\pi \text{ rad}}{T} \qquad (6.5)$$



◀ **Why do clocks go clockwise?** In the northern hemisphere, the rotation of the earth causes the sun to follow a circular arc through the southern sky, rising in the east and setting in the west. For millennia, humans have marked passing time by noting the position of shadows cast by the sun, which sweep in an arc from west to east—eventually leading to the development of the sundial, the first practical timekeeping device. In the northern hemisphere, sundials point north, and the shadow sweeps around the dial from left to right. Early clockmakers used the same convention, which is how it came to be clockwise.

We can also write the angular speed in terms of the frequency $f = 1/T$:

$$\omega = (2\pi \text{ rad})f \tag{6.6}$$

where f must be in rev/s. For example, a particle in circular motion with frequency 10 rev/s would have angular speed $\omega = 20\pi$ rad/s $= 62.8$ rad/s.

EXAMPLE 6.3 **Rotations in a car engine**

The crankshaft in your car engine is turning at 3000 rpm. What is the shaft's angular velocity?

PREPARE We'll need to convert rpm to rev/s; then we can use Equation 6.6.

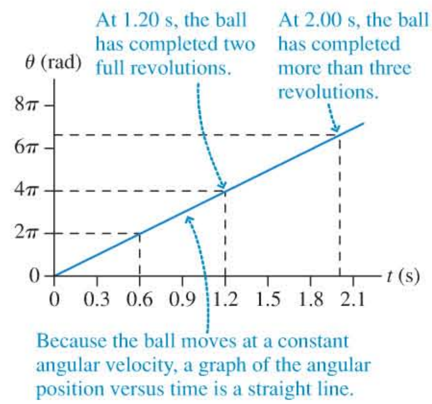
SOLVE We convert rpm to rev/s by

$$\left(3000 \frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 50 \text{ rev/s}$$

Thus the crankshaft's angular velocity is

$$\omega = (2\pi \text{ rad})f = (2\pi \text{ rad})(50 \text{ rev/s}) = 314 \text{ rad/s}$$

FIGURE 6.5 Angular position for the ball on the roulette wheel.



Angular-Position and Angular-Velocity Graphs

For the one-dimensional motion you studied in Chapter 3, we found that position- and velocity-versus-time graphs were important and useful representations of motion. We can use the same kinds of graphs to represent angular motion. Let's begin by considering the motion of the roulette ball of Example 6.2. We found that it had angular velocity $\omega = 10.5$ rad/s, meaning that its angular *position* changed by $+10.5$ rad every second. This is exactly analogous to the one-dimensional motion problem of a car driving in a straight line with a velocity of 10.5 m/s, so that its position increases by 10.5 m each second. Using this analogy, we can construct the **angular position-versus-time graph** for the roulette ball shown in FIGURE 6.5.

The angular velocity is given by $\omega = \Delta\theta/\Delta t$. Graphically, this is the *slope* of the angular position-versus-time graph, just as the ordinary velocity is the slope of the position-versus-time graph. Thus we can create an **angular velocity-versus-time graph** by finding the slope of the corresponding angular position-versus-time graph.

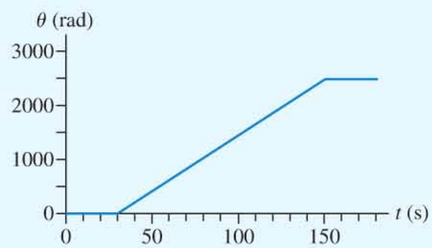
EXAMPLE 6.4 **Graphing a bike ride**

Jake rides his bicycle home from campus. FIGURE 6.6 is the angular position-versus-time graph for a small rock stuck in the tread of his tire. First, draw the rock's angular velocity-versus-position graph, using rpm on the vertical axis. Then interpret the graphs with a story about Jake's ride.

PREPARE Angular velocity ω is the slope of the angular position-versus-time graph.

SOLVE We can see that $\omega = 0$ rad/s during the first and last 30 s of Jake's ride because the horizontal segments of the graph have zero slope. Between $t = 30$ s and $t = 150$ s, an interval of 120 s,

FIGURE 6.6 Angular position-versus-time graph for Jake's bike ride.



the rock's angular velocity (the slope of the angular position-versus-time graph) is

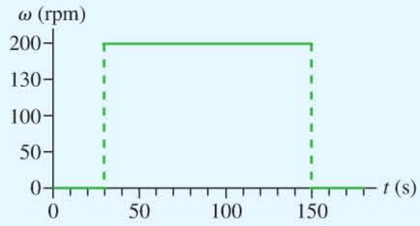
$$\omega = \text{slope} = \frac{2500\text{ rad} - 0\text{ rad}}{120\text{ s}} = 20.8\text{ rad/s}$$

We need to convert this to rpm:

$$\omega = \left(\frac{20.8\text{ rad}}{1\text{ s}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 200\text{ rpm}$$

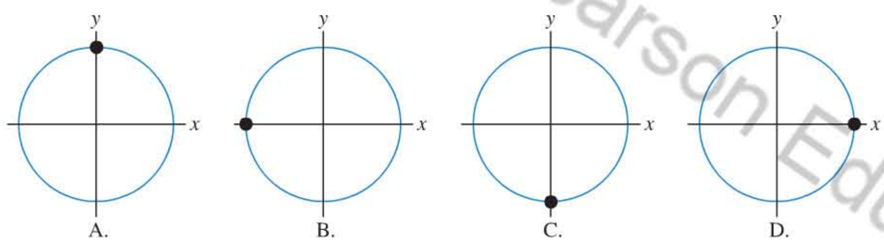
These values have been used to draw the angular velocity-versus-time graph of **FIGURE 6.7**. It looks like Jake waited 30 s for the light to change, then pedaled so that the bike wheel turned at a constant angular velocity of 200 rpm. 2.0 min later, he quickly braked to a stop for another 30-s-long red light.

FIGURE 6.7 Angular velocity-versus-time graph for Jake's bike ride.



ASSESS At 200 rpm for 2 minutes, the wheel would turn roughly 400 times or, at ≈6 rad/rev, through about 2400 rad. Our answer seems reasonable.

STOP TO THINK 6.1 Which particle has angular position $5\pi/2$?



6.2 Speed, Velocity, and Acceleration in Uniform Circular Motion

The preceding section described uniform circular motion in terms of angular variables. In Chapter 3, we introduced a description of uniform circular motion in terms of velocity and acceleration vectors. We will now unite these two different descriptions, which will enable us to consider a much wider range of problems.

Speed

In Chapter 3, we found that the speed of a particle moving with frequency f around a circular path of radius r is $v = 2\pi fr$. If we combine this result with Equation 6.5 for the angular speed, we find that speed v and angular speed ω are related by

$$v = \omega r \qquad (6.7)$$

Relationship between speed and angular speed

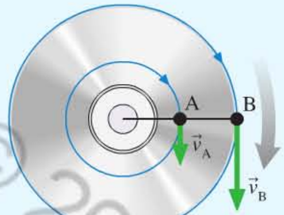
NOTE ► In Equation 6.7, **ω must be in units of rad/s.** If you are given a frequency in rev/s or rpm, you should convert it to an angular speed in rad/s. ◄

EXAMPLE 6.5 **Finding the speed at two points on a CD**

The diameter of an audio compact disc is 12.0 cm. When the disc is spinning at its maximum rate of 540 rpm, what is the speed of a point (a) at a distance 3.0 cm from the center and (b) at the outside edge of the disc, 6.0 cm from the center?

PREPARE Consider two points A and B on the rotating compact disc in FIGURE 6.8. During one period T, the disc rotates once, and both points rotate through the same angle, 2π rad. Thus the angular speed, $\omega = 2\pi/T$, is the same for these two points; in fact, it is the same for all points on the disc. But as they go around one time, the two points move different *distances;* the outer point B goes around a larger circle. The two points thus have different *speeds.* We can solve this problem by first finding the angular speed of the disc and then computing the speeds at the two points.

FIGURE 6.8 The rotation of an audio compact disc.



SOLVE We first convert the frequency of the disc from rpm to rev/s:

$$f = \left(540\ \frac{\text{rev}}{\text{min}}\right) \times \left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 9.00\ \text{rev/s}$$

We can compute the angular speed using Equation 6.6:

$$\omega = (2\pi\ \text{rad})(9.00\ \text{rev/s}) = 56.5\ \text{rad/s}$$

We can now use Equation 6.7 to compute the speeds of points on the disc. At point A, $r = 3.0\ \text{cm} = 0.030\ \text{m}$, so the speed is

$$v_B = \omega r = (56.5\ \text{rad/s})(0.030\ \text{m}) = 1.7\ \text{m/s}$$

At point B, $r = 6.0\ \text{cm} = 0.060\ \text{m}$, so the speed at the outside edge is

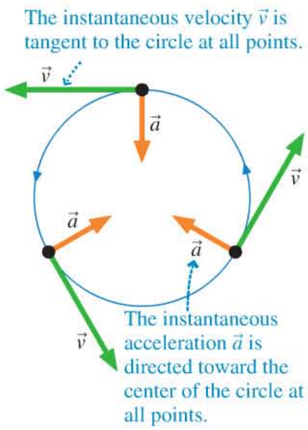
$$v_B = \omega r = (56.5\ \text{rad/s})(0.060\ \text{m}) = 3.4\ \text{m/s}$$

ASSESS The speeds are a few meters per second, which seems reasonable. The point farther from the center is moving at a higher speed, as we expected.

Velocity and Acceleration

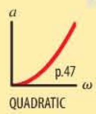
Although the *speed* of a particle in uniform circular motion is constant, its *velocity* is not constant because the *direction* of the motion is always changing. As you learned in Chapter 3, and as FIGURE 6.9 reminds you, there is an acceleration at every point in the motion, with the acceleration vector $\vec{a}$ pointing toward the center of the circle. We called this the *centripetal acceleration,* and we showed that for uniform circular motion the acceleration was given by $a = v^2/r$. Because $v = \omega r$, we can also write this relationship in terms of the angular speed:

FIGURE 6.9 Velocity and acceleration for uniform circular motion.



$$a = \frac{v^2}{r} = \omega^2 r \tag{6.8}$$

Centripetal acceleration for uniform circular motion

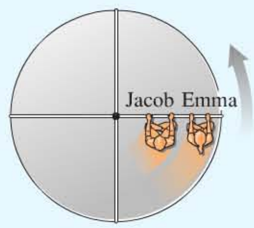


Acceleration depends on speed, but also distance from the center of the circle.

CONCEPTUAL EXAMPLE 6.6 **Who has the larger acceleration?**

Two children are riding in circles on a merry-go-round, as shown in FIGURE 6.10. Which child experiences the larger acceleration?

FIGURE 6.10 Top view of a merry-go-round.



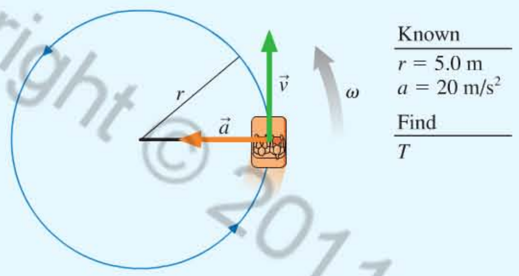
REASON As Example 6.2 showed, all points on the merry-go-round move at the same angular speed. The second expression for the acceleration in Equation 6.8 tells us that $a = \omega^2 r$. As the two children are moving with the same angular speed, Emma, with a larger value of r, experiences a larger acceleration.

ASSESS In Example 6.5, we saw that points farther from the center move at a higher speed. This would imply a higher acceleration as well, so our answer makes sense.

EXAMPLE 6.7 Finding the period of a carnival ride

In the Quasar carnival ride, passengers travel in a horizontal 5.0-m-radius circle. For safe operation, the maximum sustained acceleration that riders may experience is 20 m/s^2, approximately twice the free-fall acceleration. What is the period of the ride when it is being operated at the maximum acceleration?

FIGURE 6.11 Visual overview for the Quasar carnival ride.



PREPARE We will assume that the cars on the ride are in uniform circular motion. The visual overview of FIGURE 6.11 shows a top view of the motion of the ride.

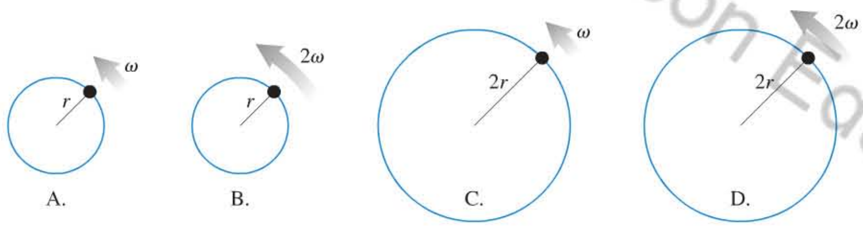
SOLVE The angular speed can be computed from the acceleration by rearranging Equation 6.8:

$$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{20\ m/s^2}{5.0\ m}} = 2.0\ rad/s$$

At this angular speed, the period is $T = 2\pi/\omega = 3.1$ s.

ASSESS One rotation in just over 3 seconds seems reasonable for a pretty zippy carnival ride. The period for this particular ride is actually 3.7 s, so it runs a bit slower than the maximum safe speed.

STOP TO THINK 6.2 Rank in order, from largest to smallest, the centripetal accelerations of particles A to D.



Hurling the heavy hammer This man is throwing a hammer that weighs over 30 pounds as far as he can by spinning the hammer around in a circle and then letting go. While he holds the handle, the hammer follows a circular path. He must provide a very large force directed toward the center of the circle to produce the centripetal acceleration, as you can see by how he is leaning away from the hammer. When he lets go, there is no longer a force directed toward the center, and the hammer will stop going in a circle and fly across the field.

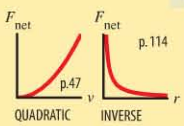
6.3 Dynamics of Uniform Circular Motion

Riders traveling around on a circular carnival ride are accelerating, as we have just seen. Consequently, according to Newton's second law, the riders must have a net *force* acting on them. In this section, we'll look at the forces that cause uniform circular motion.

We've already determined the acceleration of a particle in uniform circular motion—the centripetal acceleration of Equation 6.8. Newton's second law tells us what the net force must be to cause this acceleration:

$$\vec{F}_{net} = m\vec{a} = \left(\frac{mv^2}{r} = m\omega^2 r, \text{ toward center of circle}\right) \quad (6.9)$$

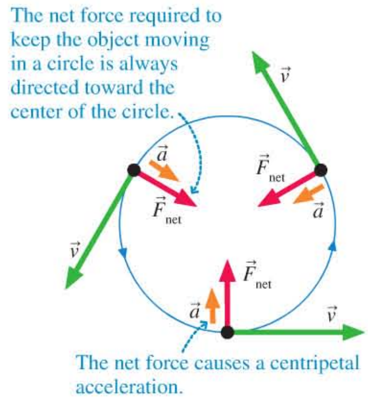
Net force producing the centripetal acceleration of uniform circular motion



In other words, **a particle of mass *m* moving at constant speed *v* around a circle of radius *r* must always have a net force of magnitude $mv^2/r = m\omega^2 r$ pointing toward the center of the circle,** as in FIGURE 6.12. It is this net force that causes the centripetal acceleration of circular motion. Without such a net force, the particle would move off in a straight line tangent to the circle.

The force described by Equation 6.9 is not a *new* kind of force. The net force will be due to one or more of our familiar forces, such as tension, friction, or the normal

FIGURE 6.12 Net force for circular motion.



force. Equation 6.9 simply tells us how the net force needs to act—how strongly and in which direction—to cause the particle to move with speed v in a circle of radius r.

In each example of circular motion that we will consider in this chapter, a physical force or a combination of forces directed toward the center produces the necessary acceleration. In some cases, the circular motion and the force are obvious, as in the hammer throw. Other cases are more subtle. For instance, for a car following a circular path on a level road, the necessary force is provided by the friction force between the tires and the road.

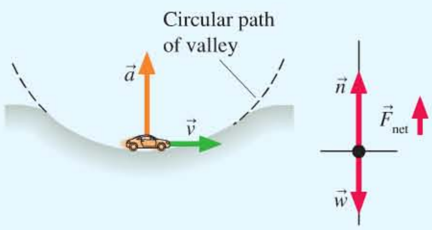
CONCEPTUAL EXAMPLE 6.8 Forces on a car

A car drives through a circularly shaped valley at a constant speed. At the very bottom of the valley, is the normal force of the road on the car greater than, less than, or equal to the car's weight?

REASON **FIGURE 6.13** shows a visual overview of the situation. The car is accelerating, even though it is moving at a constant speed, because its direction is changing. When the car is at the bottom of the valley, the center of its circular path is directly above it and so its acceleration vector points straight up. The free-body diagram of Figure 6.13 identifies the only two forces acting on the car as the normal force, pointing upward, and its weight, pointing downward. Which is larger: n or w?

Because $\vec{a}$ points upward, by Newton's second law there must be a net force on the car that also points upward. In order for this to be the case, the free-body diagram shows that the magnitude of the normal force must be *greater* than the weight.

FIGURE 6.13 Visual overview for the car in the valley.



ASSESS You have probably experienced this situation. As you drive through a dip in the road, you feel "heavier" than normal. As discussed in Section 5.3, this is because your apparent weight—the normal force that supports you—is greater than your true weight.

Solving Circular Dynamics Problems

4.2, 4.3, 4.4, 4.5 ActivPhysics

We have one basic equation for circular dynamics problems, Equation 6.9, which is just a version of Newton's second law. The techniques for solving circular dynamics problems are thus quite similar to those we have used for solving other Newton's second-law problems.

PROBLEM-SOLVING STRATEGY 6.1 Circular dynamics problems

PREPARE Begin your visual overview with a pictorial representation in which you sketch the motion, define symbols, define axes, and identify what the problem is trying to find. There are two common situations:

- If the motion is in a horizontal plane, like a tabletop, draw the free-body diagram with the circle viewed edge-on, the x-axis pointing toward the center of the circle, and the y-axis perpendicular to the plane of the circle.
- If the motion is in a vertical plane, like a Ferris wheel, draw the free-body diagram with the circle viewed face-on, the x-axis pointing toward the center of the circle, and the y-axis tangent to the circle.

SOLVE Newton's second law for uniform circular motion, $\vec{F}_{net} = (mv^2/r$, toward center of circle), is a vector equation. Some forces act in the plane of the circle, some act perpendicular to the circle, and some may have components in both directions. In the coordinate system described above, with the x-axis pointing toward the center of the circle, Newton's second law is

$$\sum F_x = \frac{mv^2}{r} = m\omega^2 r \qquad \text{and} \qquad \sum F_y = 0$$

Continued

That is, the net force toward the center of the circle has magnitude $mv^2/r = m\omega^2 r$ while the net force perpendicular to the circle is zero. The components of the forces are found directly from the free-body diagram. Depending on the problem, either:

- Use the net force to determine the speed v, then use circular kinematics to find frequencies or angular velocities.
- Use circular kinematics to determine the speed v, then solve for unknown forces.

ASSESS Make sure your net force points toward the center of the circle. Check that your result has the correct units, is reasonable, and answers the question.

Exercise 13

EXAMPLE 6.9 Analyzing the motion of a cart

An energetic father places his 20 kg child on a 5.0 kg cart to which a 2.0-m-long rope is attached. He then holds the end of the rope and spins the cart and child around in a circle, keeping the rope parallel to the ground. If the tension in the rope is 100 N, how much time does it take for the cart to make one rotation?

PREPARE We proceed according to the steps of Problem-Solving Strategy 6.1. **FIGURE 6.14** shows a visual overview of the problem. The main reason for the pictorial representation on the left is to illustrate the relevant geometry and to define the symbols that will be used. A circular dynamics problem usually does not have starting and ending points like a projectile problem, so subscripts such as x_i or y_f are usually not needed. Here we need to define the cart's speed v and the radius r of the circle.

The object moving in the circle is the cart plus the child, a total mass of 25 kg; the free-body diagram shows the forces. Because the motion is in a horizontal plane, Problem-Solving Strategy 6.1 tells us to draw the free-body diagram looking at the edge of the circle, with the x-axis pointing toward the center of the circle and the y-axis perpendicular to the plane of the circle. Three forces are acting on the cart: the weight force $\vec{w}$, the normal force of the ground $\vec{n}$, and the tension force of the rope $\vec{T}$.

Notice that there are two quantities for which we use the symbol T: the tension and the period. We will include additional information when necessary to distinguish the two.

SOLVE There is no net force in the y-direction, perpendicular to the circle, so $\vec{w}$ and $\vec{n}$ must be equal and opposite. There is a net force in the x-direction, toward the center of the circle, as there must be to cause the centripetal acceleration of circular motion. Only the tension force has an x-component, so Newton's second law is

$$\sum F_x = T = \frac{mv^2}{r}$$

We know the mass, the radius of the circle, and the tension, so we can solve for v:

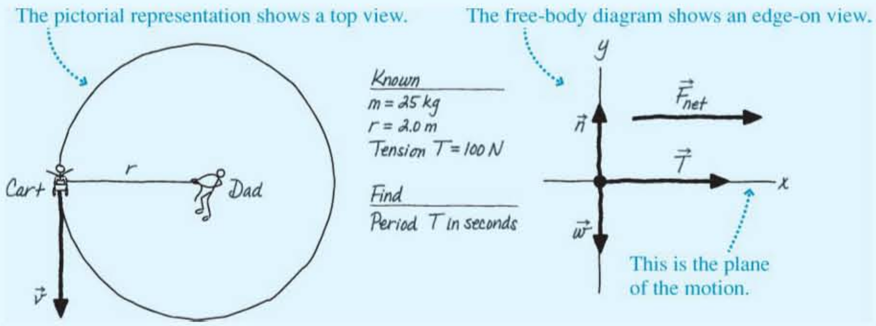
$$v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{(100\text{ N})(2.0\text{ m})}{25\text{ kg}}} = 2.83\text{ m/s}$$

From this, we can compute the period with a slight rearrangement of Equation 3.27:

$$T = \frac{2\pi r}{v} = \frac{(2\pi)(2.0\text{ m})}{2.83\text{ m/s}} = 4.4\text{ s}$$

ASSESS The speed is about 3 m/s. Because 1 m/s ≈ 2 mph, the child is going about 6 mph. A trip around the circle in just over 4 s at a speed of about 6 mph sounds reasonable; it's a fast ride, but not so fast as to be scary!

FIGURE 6.14 A visual overview of the cart spinning in a circle.



EXAMPLE 6.10 Finding the maximum speed to turn a corner

What is the maximum speed with which a 1500 kg car can make a turn around a curve of radius 20 m on a level (unbanked) road without sliding? (This radius turn is about what you might expect at a major intersection in a city.)

PREPARE We start with the visual overview in FIGURE 6.15. The car moves along a circular arc at a constant speed—uniform circular motion—for the quarter-circle necessary to complete the turn. The motion before and after the turn is not relevant to the problem. The more interesting issue is *how* a car turns a corner. What force or forces can we identify that cause the direction of the velocity vector to change? Imagine you are driving a car on a frictionless road, such as a very icy road. You would not be able to turn a corner. Turning the steering wheel would be of no use; the car would slide straight ahead, in accordance with both Newton's first law and the experience of anyone who has ever driven on ice! So it must be *friction* that causes the car to turn.

The top view of the tire in Figure 6.15 shows the force on one of the car's tires as it turns a corner. If the road surface were frictionless, the tire would slide straight ahead. The force that prevents an object from sliding across a surface is *static friction.* Static friction $\vec{f}_s$ pushes *sideways* on the tire, toward the center of the circle. How do we know the direction is sideways? If $\vec{f}_s$ had a component either parallel to $\vec{v}$ or opposite $\vec{v}$, it would cause the car to speed up or slow down. Because the car changes direction but not speed, static friction must be perpendicular to $\vec{v}$. Thus $\vec{f}_s$ causes the centripetal acceleration of circular motion around the curve. With this in mind, the free-body diagram, drawn from behind the car, shows the static friction force pointing toward the center of the circle. Because the motion is in a horizontal plane, we've again chosen an x-axis toward the center of the circle and a y-axis perpendicular to the plane of motion.

SOLVE The only force in the x-direction, toward the center of the circle, is static friction. Newton's second law along the x-axis is

$$\sum F_x = f_s = \frac{mv^2}{r}$$

The only difference between this example and the preceding one is that the tension force toward the center has been replaced by a static friction force toward the center.

Newton's second law in the y-direction is

$$\sum F_y = n - w = ma_y = 0$$

so that $n = w = mg$.

The net force toward the center of the circle is the force of static friction. Recall from Equation 5.10 in Chapter 5 that static friction has a maximum possible value:

$$f_{s\,max} = \mu_s n = \mu_s mg$$

Because the static friction force has a maximum value, there will be a maximum speed at which a car can turn without sliding. This speed is reached when the static friction force reaches its maximum value $f_{s\,max} = \mu_s mg$. If the car enters the curve at a speed higher than the maximum, static friction cannot provide the necessary centripetal acceleration and the car will slide.

Thus the maximum speed occurs at the maximum value of the force of static friction, or when

$$f_{s\,max} = \frac{m{v_{max}}^2}{r}$$

Using the known value of $f_{s\,max}$, we find

$$\frac{m{v_{max}}^2}{r} = f_{s\,max} = \mu_s mg$$

Rearranging, we get

$${v_{max}}^2 = \mu_s gr$$

For rubber tires on pavement, we find from Table 5.1 that $\mu_s = 1.0$. We then have

$$v_{max} = \sqrt{\mu_s gr} = \sqrt{(1.0)(9.8\text{ m/s}^2)(20\text{ m})} = 14\text{ m/s}$$

ASSESS 14 m/s ≈ 30 mph, which seems like a reasonable upper limit for the speed at which a car can go around a curve without sliding. There are two other things to note about the solution:

- The car's mass canceled out. The maximum speed *does not* depend on the mass of the vehicle, though this may seem surprising.
- The final expression for v_{max} *does* depend on the coefficient of friction and the radius of the turn. v_{max} decreases if μ is less (a slipperier road) or if r is smaller (a tighter turn). Both make sense.

FIGURE 6.15 Visual overview of a car turning a corner.

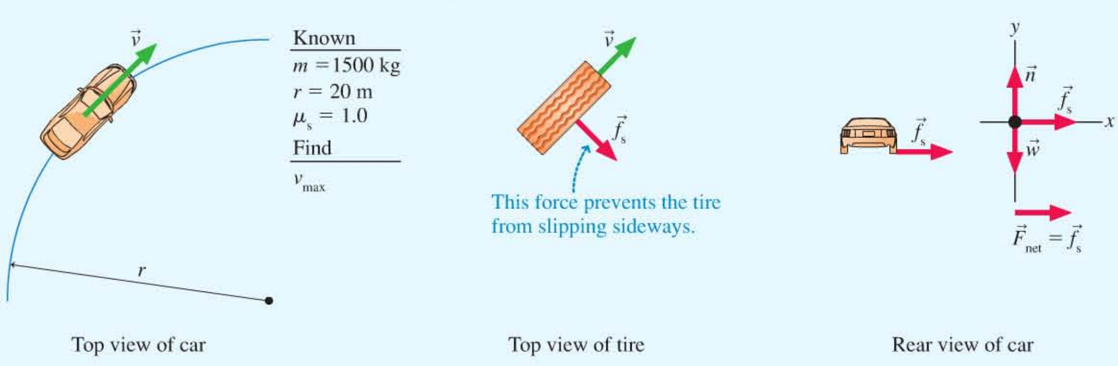


FIGURE 6.16 Wings on an Indy racer.

A banked turn on a racetrack.

Because v_{max} depends on μ_s and because μ_s depends on road conditions, the maximum safe speed through turns can vary dramatically. Wet or icy roads lower the value of μ_s and thus lower the maximum speed of turns. A car that easily handles a curve in dry weather can suddenly slide out of control when the pavement is wet. Icy conditions are even worse. If you lower the value of the coefficient of friction in Example 6.10 from 1.0 (dry pavement) to 0.1 (icy pavement), the maximum speed for the turn goes down to 4.4 m/s—about 10 mph!

Race cars turn corners at much higher speeds than normal passenger vehicles. One design modification of the *cars* to allow this is the addition of wings, as on the car in **FIGURE 6.16**. The wings provide an additional force pushing the car *down* onto the pavement by deflecting air upward. This extra downward force increases the normal force, thus increasing the maximum static friction force and making faster turns possible.

There are also design modifications of the *track* to allow race cars to take corners at high speeds. If the track is banked by raising the outside edge of curved sections, the normal force can provide some of the force necessary to produce the centripetal acceleration, as we will see in the next example. The curves on racetracks may be quite sharply banked. Curves on ordinary highways are often banked as well, though at more modest angles suiting the lower speeds.

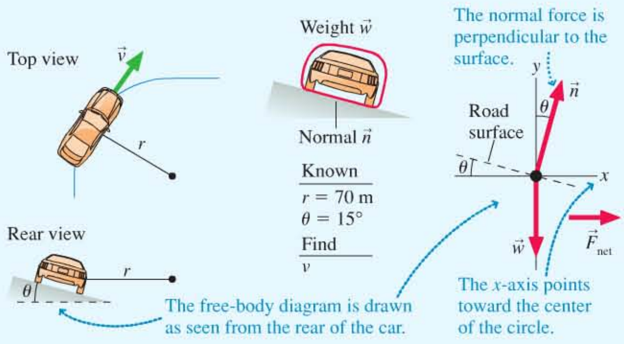
EXAMPLE 6.11 Finding speed on a banked turn

A curve on a racetrack of radius 70 m is banked at a 15° angle. At what speed can a car take this curve without assistance from friction?

PREPARE After drawing the pictorial representation in **FIGURE 6.17**, we use the force identification diagram to find that, given that there is no friction acting, the only two forces are the normal force and the car's weight. We can then construct the free-body diagram, making sure that we draw the normal force perpendicular to the road's surface.

Even though the car is tilted, it is still moving in a *horizontal* circle. Thus, following Problem-Solving Strategy 6.1, we choose the x-axis to be horizontal and pointing toward the center of the circle.

FIGURE 6.17 Visual overview for the car on a banked turn.



SOLVE Without friction, $n_x = n\sin\theta$ is the only component of force toward the center of the circle. It is this inward component of the normal force on the car that causes it to turn the corner. Newton's second law is

$$\sum F_x = n\sin\theta = \frac{mv^2}{r}$$

$$\sum F_y = n\cos\theta - w = 0$$

where θ is the angle at which the road is banked, and we've assumed that the car is traveling at the correct speed v. From the y-equation,

$$n = \frac{w}{\cos\theta} = \frac{mg}{\cos\theta}$$

Substituting this into the x-equation and solving for v give

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta = \frac{mv^2}{r}$$

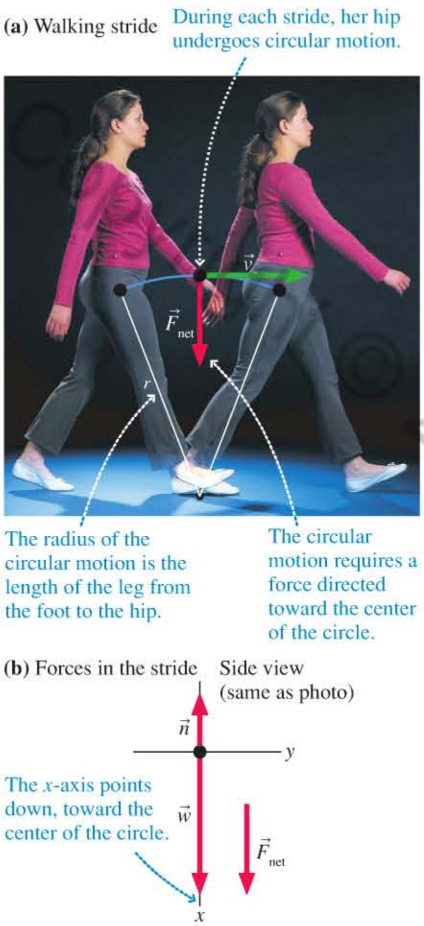
$$v = \sqrt{rg\tan\theta} = 14 \text{ m/s}$$

ASSESS This is $\approx$30 mph, a reasonable speed. Only at this exact speed can the turn be negotiated without reliance on friction forces.

Maximum Walking Speed

Humans and other two-legged animals have two basic gaits: walking and running. At slow speeds, you walk. When you need to go faster, you run. Why don't you just walk faster? There is an upper limit to the speed of walking, and this limit is set by the physics of circular motion.

FIGURE 6.18 Analysis of a walking stride.



Think about the motion of your body as you take a walking stride. You put one foot forward, then push off with your rear foot. Your body pivots over your front foot, and you bring your rear foot forward to take the next stride. As you can see in FIGURE 6.18a, the path that your body takes during this stride is the arc of a circle. **In a walking gait, your body is in circular motion as you pivot on your forward foot.**

A force toward the center of the circle is required for this circular motion, as shown in Figure 6.18. FIGURE 6.18b shows the forces acting on the woman's body during the midpoint of the stride: her weight, directed down, and the normal force of the ground, directed up. Newton's second law for the x-axis is

$$\sum F_x = w - n = \frac{mv^2}{r}$$

Because of her circular motion, the net force must point toward the center of the circle, or, in this case, down. In order for the net force to point down, the normal force must be *less* than her weight. Your body tries to "lift off" as it pivots over your foot, decreasing the normal force exerted on you by the ground. The normal force becomes smaller as you walk faster, but n cannot be less than zero. Thus the maximum possible walking speed v_{max} occurs when $n = 0$. Setting $n = 0$ in Newton's second law gives

$$w = mg = \frac{m{v_{max}}^2}{r}$$

Thus

$$v_{max} = \sqrt{gr} \tag{6.10}$$

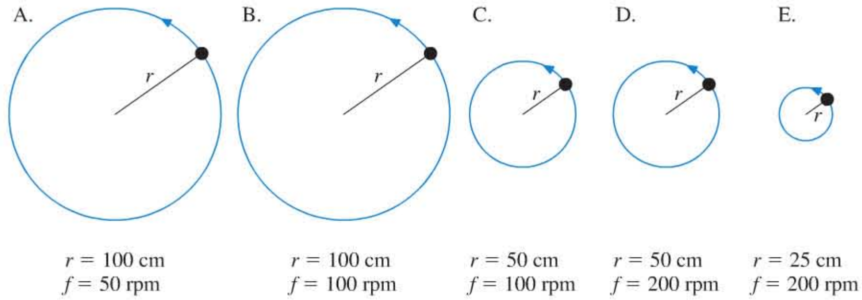
The maximum possible walking speed is limited by r, the length of the leg, and g, the free-fall acceleration. This formula is a good approximation of the maximum walking speed for humans and other animals. The maximum walking speed is higher for animals with longer legs. Giraffes, with their very long legs, can walk at high speeds. Animals such as mice with very short legs have such a low maximum walking speed that they rarely use this gait. Mice generally run to get from one place to another.

For humans, the length of the leg is approximately 0.7 m, so we calculate a maximum speed of

$$v_{max} \approx 2.6 \text{ m/s} \approx 6 \text{ mph}$$

You *can* walk this fast, though it becomes energetically unfavorable to do so at speeds above 4 mph. Most people make a transition to a running gait at about this speed. Children, with their shorter legs, must make a transition to a running gait at a much lower speed.

STOP TO THINK 6.3 A block on a string spins in a horizontal circle on a frictionless table. Rank in order, from largest to smallest, the tensions T_A to T_E acting on the blocks A to E.



6.4 Apparent Forces in Circular Motion

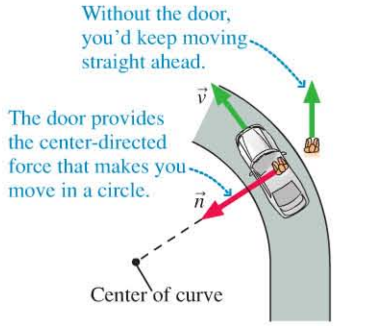
FIGURE 6.19 shows a carnival ride that spins the riders around inside a large cylinder. The people are "stuck" to the inside wall of the cylinder! As you probably know from experience, the riders *feel* that they are being pushed outward, into the wall. But our analysis has found that an object in circular motion must have an *inward* force to create the centripetal acceleration. How can we explain this apparent difference?

FIGURE 6.19 Inside the Gravitron, a rotating circular room.

Centrifugal Force?

If you are a passenger in a car that turns a corner quickly, you may feel "thrown" by some mysterious force against the door. But is there really such a force? FIGURE 6.20 shows a bird's-eye view of you riding in a car as it makes a left turn. You try to continue moving in a straight line, obeying Newton's first law, when—without having been provoked—the door starts to turn in toward you and so runs into you! You do then feel the force of the door because it is now the force of the door, pushing *inward* toward the center of the curve, that is causing you to turn the corner. But you were not "thrown" into the door; the door ran into you.

FIGURE 6.20 Bird's-eye view of a passenger in a car turning a corner.



A "force" that *seems* to push an object to the outside of a circle is called a *centrifugal force*. Despite having a name, there really is no such force. What you feel is your body trying to move ahead in a straight line (which would take you away from the center of the circle) as outside forces act to turn you in a circle. The only real forces, those that appear on free-body diagrams, are the ones pushing inward toward the center. **A centrifugal force will never appear on a free-body diagram and never be included in Newton's laws.**

With this in mind, let's revisit the rotating carnival ride. A person watching from above would see the riders in the cylinder moving in a circle with the walls providing the inward force that causes their centripetal acceleration. The riders *feel* as if they're being pushed outward because their natural tendency to move in a straight line is being resisted by the wall of the cylinder, which keeps getting in the way. But feelings aren't forces. The only actual force is the contact force of the cylinder wall pushing *inward*.

Apparent Weight in Circular Motion

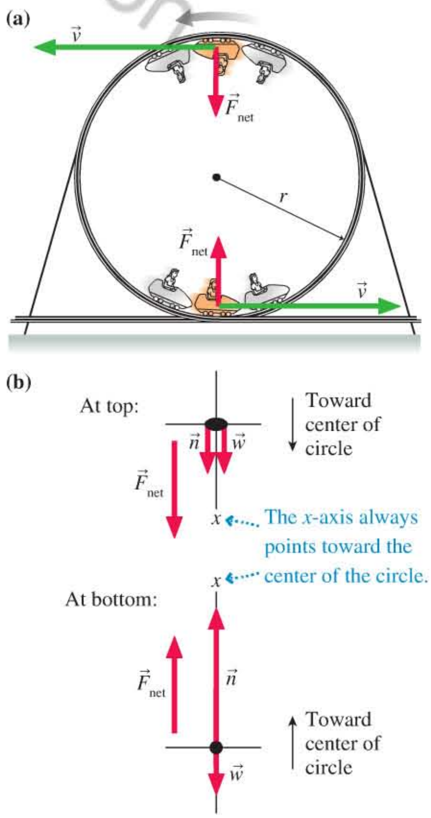
Imagine swinging a bucket of water over your head. If you swing the bucket quickly, the water stays in. But you'll get a shower if you swing too slowly. Why does the water stay in the bucket? Or think about a roller coaster that does a loop-the-loop. How does the car stay on the track when it's upside down? You might have said that there was a centrifugal force holding the water in the bucket and the car on the track, but we have seen that there really isn't a centrifugal force. Analyzing these questions will tell us a lot about forces in general and circular motion in particular.

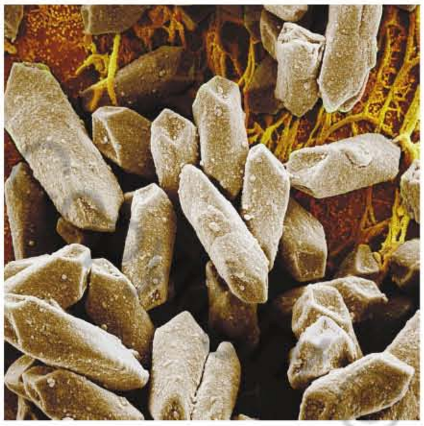
FIGURE 6.21a shows a roller coaster car going around a vertical loop-the-loop of radius r. If you've ever ridden a roller coaster, you know that your sensation of weight changes as you go over the crests and through the dips. To understand why, let's look at the forces on passengers going through the loop. To simplify our analysis, we will assume that the speed of the car stays constant as it moves through the loop.

FIGURE 6.21b shows a passenger's free-body diagram at the top and the bottom of the loop. Let's start by examining the forces on the passenger at the bottom of the loop. The only forces acting on her are her weight $\vec{w}$ and the normal force $\vec{n}$ of the seat pushing up on her. Recall from Chapter 5 that a person's apparent weight is the magnitude of the force that supports her. Here the seat is supporting the passenger with the normal force $\vec{n}$, so her apparent weight is $w_{app} = n$. Based on our understanding of circular motion, we can say:

- She's moving in a circle, so there must be a net force directed toward the center of the circle—currently directly above her head—to provide the centripetal acceleration.

FIGURE 6.21 A roller coaster car going around a loop-the-loop.





When "down" is up BIO You can tell, even with your eyes closed, what direction is down. This sense is due to small crystals of calcium carbonate, called *otoliths,* in your inner ears. Gravity pulls the otoliths down, so a normal force from a sensitive supporting membrane must push them up. Your brain interprets "down" as the opposite of this normal force. Normally, what your ears tell you is "down" is really down. But at the top of a loop in a roller coaster, the normal force is directed down, so your inner ear tells you that "down" is up! If your ears tell you one thing and your eyes another, it can be disorienting.

- The net force points *upward,* so it must be the case that $n > w$.
- Her apparent weight is $w_{app} = n$, so her apparent weight is greater than her true weight ($w_{app} > w$). Thus she "feels heavy" at the bottom of the circle.

This situation is the same as for the car driving through a valley in Conceptual Example 6.8. To analyze the situation quantitatively, we'll apply the steps of Problem-Solving Strategy 6.1. As always, we choose the x-axis to point toward the center of the circle or, in this case, vertically upward. Then Newton's second law is

$$\sum F_x = n_x + w_x = n - w = \frac{mv^2}{r}$$

From this equation, her apparent weight is

$$w_{app} = n = w + \frac{mv^2}{r} \tag{6.11}$$

The passenger's apparent weight at the bottom is *greater* than her true weight w, which agrees with your experience when you go through a dip or a valley.

Now let's look at the roller coaster car as it crosses the top of the loop. Things are a little trickier here. As Figure 6.21b shows, whereas the normal force of the seat pushes up when the passenger is at the bottom of the circle, it pushes *down* when she is at the top and the seat is above her. It's worth thinking carefully about this diagram to make sure you understand what it is showing.

The passenger is still moving in a circle, so there must be a net force *downward,* toward the center of the circle, to provide her centripetal acceleration. As always, we define the x-axis to be toward the center of the circle, so here the x-axis points vertically downward. Newton's second law gives

$$\sum F_x = n_x + w_x = n + w = \frac{mv^2}{r}$$

Note that w_x is now *positive* because the x-axis is directed downward. We can solve for her apparent weight:

$$w_{app} = n = \frac{mv^2}{r} - w \tag{6.12}$$

If v is sufficiently large, her apparent weight can exceed the true weight, just as it did at the bottom of the track.

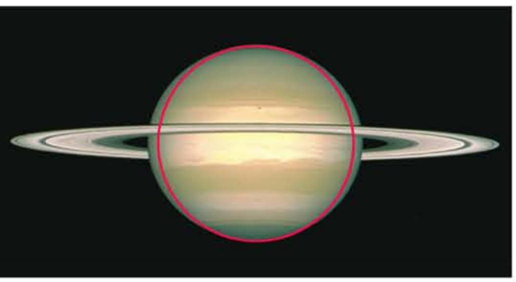
But let's look at what happens if the car goes slower. Notice from Equation 6.12 that, as v decreases, there comes a point when $mv^2/r = w$ and n becomes zero. At that point, the seat is *not* pushing against the passenger at all! Instead, she is able to complete the circle because her weight force alone provides sufficient centripetal acceleration.

The speed for which $n = 0$ is called the *critical speed* v_c. Because for n to be zero we must have $mv_c^2/r = w$, the critical speed is

$$v_c = \sqrt{\frac{rw}{m}} = \sqrt{\frac{rmg}{m}} = \sqrt{gr} \tag{6.13}$$

What happens if the speed is slower than the critical speed? In this case, Equation 6.12 gives a *negative* value for n if $v < v_c$. But that is physically impossible. The seat can push against the passenger ($n > 0$), but it can't *pull* on her, so the slowest possible speed is the speed for which $n = 0$ at the top. Thus, **the critical speed is the slowest speed at which the car can complete the circle.** If $v < v_c$, the passenger cannot turn the full loop but, instead, will fall from the car as a projectile! (This is why you're always strapped into a roller coaster.)

Water stays in a bucket swung over your head for the same reason. The bottom of the bucket pushes against the water to provide the inward force that causes circular motion. If you swing the bucket too slowly, the force of the bucket on the water drops to zero. At that point, the water leaves the bucket and becomes a projectile following a parabolic trajectory onto your head!



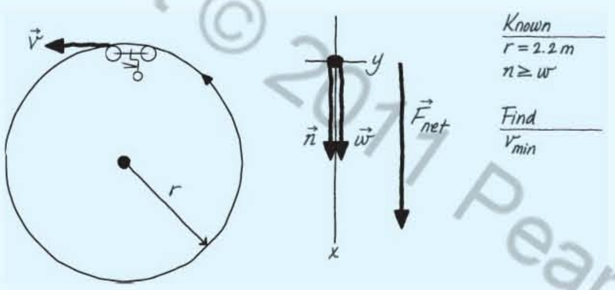
A fast-spinning world Saturn, a gas giant planet composed largely of fluid matter, is quite a bit larger than the earth. It also rotates much more quickly, completing one rotation in just under 11 hours. The rapid rotation decreases the apparent weight at the equator enough to distort the fluid surface; the planet is noticeably out of round, as the red circle shows. The diameter at the equator is 11% greater than the diameter at the poles.

EXAMPLE 6.12 **How slow can you go?**

A motorcyclist in the Globe of Death, pictured at the start of the chapter, rides in a 2.2-m-radius vertical loop. To keep control of the bike, the rider wants the normal force on his tires at the top of the loop to equal or exceed his and the bike's combined weight. What is the minimum speed at which the rider can take the loop?

PREPARE The visual overview for this problem is shown in **FIGURE 6.22**. At the top of the loop, the normal force of the cage on the tires is a *downward* force. In accordance with Problem-Solving Strategy 6.1, we've chosen the x-axis to point toward the center of the circle.

FIGURE 6.22 Riding in a vertical loop around the Globe of Death.



SOLVE We will consider the forces at the top point of the loop. Because the x-axis points downward, Newton's second law is

$$\sum F_x = w + n = \frac{mv^2}{r}$$

The minimum acceptable speed occurs when $n = w$; thus

$$2w = 2mg = \frac{m{v_{\min}}^2}{r}$$

Solving for the speed, we find

$$v_{\min} = \sqrt{2gr} = \sqrt{2(9.8\ \text{m/s}^2)(2.2\ \text{m})} = 6.6\ \text{m/s}$$

ASSESS The minimum speed is ≈ 15 mph, which isn't all that fast; the bikes can easily reach this speed. But normally several bikes are in the globe at one time. The big challenge is to keep all of the riders in the cage moving at this speed in synchrony. The period for the circular motion at this speed is $T = 2\pi r/v \approx 2$ s, leaving little room for error!

Centrifuges

The *centrifuge,* an important biological application of circular motion, is used to separate the components of a liquid with different densities. Typically these are different types of cells, or the components of cells, suspended in water. You probably know that small particles suspended in water will eventually settle to the bottom. However, the downward motion due to gravity for extremely small objects such as cells is so slow that it could take days or even months for the cells to settle out. It's not practical to wait for biological samples to separate due to gravity alone.

The separation would go faster if the force of gravity could be increased. Although we can't change gravity, we can increase the apparent weight of objects in the sample by spinning them very fast, and that is what the centrifuge in **FIGURE 6.23** does. By using very high angular velocities, the centrifuge produces centripetal accelerations that are thousands of times greater than free-fall acceleration. As the centrifuge effectively increases gravity to thousands of times its normal value, the cells or cell components settle out and separate by density in a matter of minutes or hours.

A centrifuge.

EXAMPLE 6.13 **Analyzing the ultracentrifuge**

An 18-cm-diameter ultracentrifuge produces an extraordinarily large centripetal acceleration of 250,000g, where g is the free-fall acceleration due to gravity. What is its frequency in rpm? What is the apparent weight of a sample with a mass of 0.0030 kg?

PREPARE The acceleration in SI units is

$$a = 250{,}000(9.80\ \text{m/s}^2) = 2.45 \times 10^6\ \text{m/s}^2$$

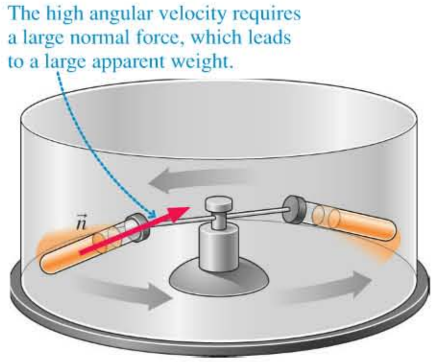
The radius is half the diameter, or $r = 9.0\ \text{cm} = 0.090\ \text{m}$.

SOLVE The centripetal acceleration is related to the angular speed by $a = \omega^2 r$. Thus

$$\omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{2.45 \times 10^6\ \text{m/s}^2}{0.090\ \text{m}}} = 5.22 \times 10^3\ \text{rad/s}$$

Continued

FIGURE 6.23 The operation of a centrifuge.



TRY IT YOURSELF

Human centrifuge BIO If you spin your arm rapidly in a vertical circle, the motion will produce an effect like that in a centrifuge. The motion will assist outbound blood flow in your arteries and retard inbound blood flow in your veins. There will be a buildup of fluid in your hand that you will be able to see (and feel!) quite easily.

By Equation 6.6, this corresponds to a frequency

$$f = \frac{\omega}{2\pi} = \frac{5.22 \times 10^3 \text{ rad/s}}{2\pi} = 830 \text{ rev/s}$$

Converting to rpm, we find

$$830 \frac{\text{rev}}{\text{s}} \times \frac{60 \text{ s}}{1 \text{ min}} = 50{,}000 \text{ rpm}$$

At this rotation rate, the 0.0030 kg mass has an apparent weight

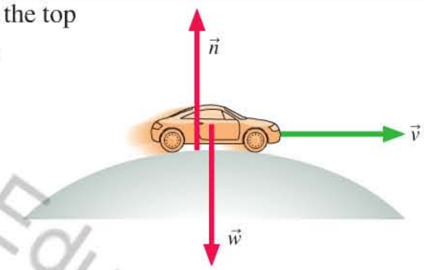
$$w_{\text{app}} = ma = (3.0 \times 10^{-3} \text{ kg})(2.45 \times 10^6 \text{ m/s}^2) = 7.4 \times 10^3 \text{ N}$$

The three gram sample has an effective weight of about 1700 pounds!

ASSESS Because the acceleration is 250,000g, the apparent weight is 250,000 times the actual weight. The forces in the ultracentrifuge are very large and can destroy the machine if it is not carefully balanced.

STOP TO THINK 6.4 A car is rolling over the top of a hill at constant speed v. At this instant,

A. $n > w$.
B. $n < w$.
C. $n = w$.
D. We can't tell about n without knowing v.



6.5 Circular Orbits and Weightlessness

The Space Shuttle orbits the earth in a circular path at a speed of over 15,000 miles per hour. What forces act on it? Why does it move in a circle? Before we start considering the physics of orbital motion, let's return, for a moment, to projectile motion. Projectile motion occurs when the only force on an object is gravity. Our analysis of projectiles made an implicit assumption that the earth is flat and that the free-fall acceleration, due to gravity, is everywhere straight down. This is an acceptable approximation for projectiles of limited range, such as baseballs or cannon balls, but there comes a point where we can no longer ignore the curvature of the earth.

Orbital Motion

FIGURE 6.24 Projectiles being launched at increasing speeds from height h on a smooth, airless planet.

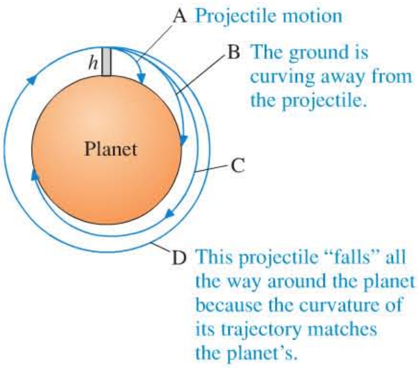


FIGURE 6.24 shows a perfectly smooth, spherical, airless planet with a vertical tower of height h. A projectile is launched from this tower with initial speed v_i parallel to the ground. If v_i is very small, as in trajectory A, the "flat-earth approximation" is valid and the problem is identical to Example 3.11 in which a car drove off a cliff. The projectile simply falls to the ground along a parabolic trajectory.

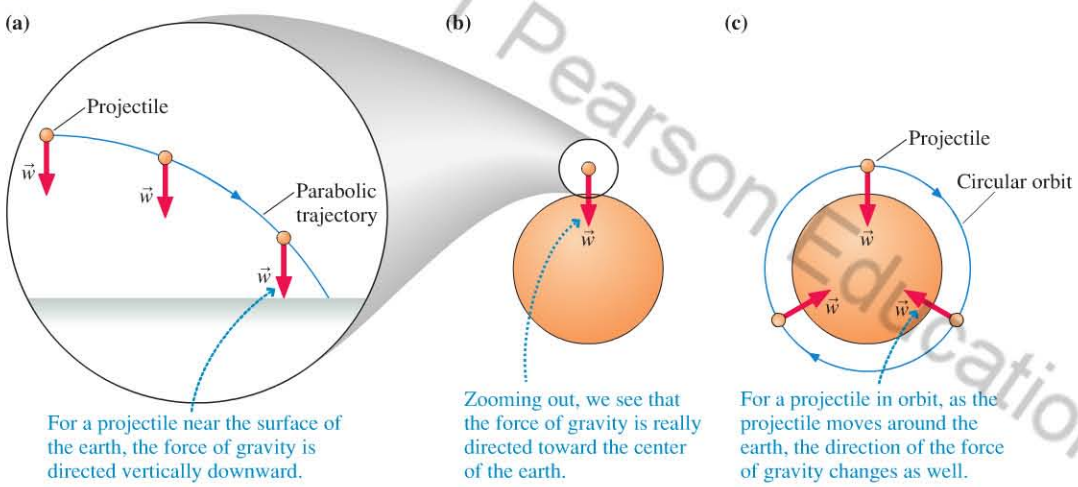
As the initial speed v_i is increased, it seems to the projectile that the ground is curving out from beneath it. It is still falling the entire time, always getting closer to the ground, but the distance that the projectile travels before finally reaching the ground—that is, its range—increases because the projectile must "catch up" with the ground that is curving away from it. Trajectories B and C are like this.

If the launch speed v_i is sufficiently large, there comes a point at which the curve of the trajectory and the curve of the earth are parallel. In this case, the projectile "falls" but it never gets any closer to the ground! This is the situation for trajectory D. The projectile returns to the point from which it was launched, at the same speed at which it was launched, making a closed trajectory. Such a closed trajectory around a planet or star is called an **orbit**.

The most important point of this qualitative analysis is that, in the absence of air resistance, **an orbiting projectile is in free fall.** This is, admittedly, a strange idea, but one worth careful thought. An orbiting projectile is really no different from a thrown baseball or a car driving off a cliff. The only force acting on it is gravity, but its tangential velocity is so great that the curvature of its trajectory matches the curvature of the earth. When this happens, the projectile "falls" under the influence of gravity but never gets any closer to the surface, which curves away beneath it.

When we first studied free fall in Chapter 2, we said that free-fall acceleration is always directed vertically downward. As we see in **FIGURE 6.25**, "downward" really means "toward the center of the earth." For a projectile in orbit, the direction of the force of gravity changes, always pointing toward the center of the earth.

FIGURE 6.25 The force of gravity is really directed toward the center of the earth.



As you have learned, a force of constant magnitude that always points toward the center of a circle causes the centripetal acceleration of uniform circular motion. Because the only force acting on the orbiting projectile in Figure 6.25 is gravity, and we're assuming the projectile is very near the surface of the earth, we can write

$$a = \frac{F_{\text{net}}}{m} = \frac{w}{m} = \frac{mg}{m} = g \tag{6.14}$$

An object moving in a circle of radius r at speed v_{orbit} will have this centripetal acceleration if

$$a = \frac{(v_{\text{orbit}})^2}{r} = g \tag{6.15}$$

That is, if an object moves parallel to the surface with the speed

$$v_{\text{orbit}} = \sqrt{gr} \tag{6.16}$$

then the free-fall acceleration provides exactly the centripetal acceleration needed for a circular orbit of radius r. An object with any other speed will not follow a circular orbit.

The earth's radius is $r = R_e = 6.37 \times 10^6$ m. The orbital speed of a projectile just skimming the surface of a smooth, airless earth is

$$v_{\text{orbit}} = \sqrt{gR_e} = \sqrt{(9.80\text{ m/s}^2)(6.37 \times 10^6\text{ m})} = 7900\text{ m/s} \approx 18{,}000\text{ mph}$$

We can use v_{orbit} to calculate the period of the satellite's orbit:

$$T = \frac{2\pi r}{v_{\text{orbit}}} = 2\pi\sqrt{\frac{r}{g}} \tag{6.17}$$

For this earth-skimming orbit, $T = 5065\ \text{s} = 84.4\ \text{min}$.

Of course, this orbit is unrealistic; even if there were no trees and mountains, a real projectile moving at this speed would burn up from the friction of air resistance. Suppose, however, that we launched the projectile from a tower of height $h = 200\ \text{mi} \approx 3.2 \times 10^5\ \text{m}$, above most of the earth's atmosphere. This is approximately the height of low-earth-orbit satellites, such as the Space Shuttle. Note that $h \ll R_e$, so the radius of the orbit $r = R_e + h = 6.69 \times 10^6\ \text{m}$ is only 5% larger than the earth's radius. Many people have a mental image that satellites orbit far above the earth, but in fact most satellites come pretty close to skimming the surface.

At this slightly larger value of r, Equation 6.17 gives $T = 87$ min. The actual period of the Space Shuttle at an altitude of 200 mi is about 91 minutes, so our calculation is very good—but not perfect. As we'll see in the next section, a correct calculation must take into account the fact that the force of gravity gradually gets weaker at higher elevations above the earth's surface.

Zero apparent weight in the Space Shuttle.

Weightlessness in Orbit

When we discussed *weightlessness* in Chapter 5, we saw that it occurs during free fall. We asked the question, at the end of Section 5.4, whether astronauts and their spacecraft are in free fall. We can now give an affirmative answer: They are, indeed, in free fall. They are falling continuously around the earth, under the influence of only the gravitational force, but never getting any closer to the ground because the earth's surface curves beneath them. Weightlessness in space is no different from the weightlessness in a free-falling elevator. **Weightlessness does *not* occur from an absence of weight or an absence of gravity.** Instead, the astronaut, the spacecraft, and everything in it are "weightless" (i.e., their *apparent* weight is zero) because they are all falling together.

Rotating space stations BIO The weightlessness astronauts experience in orbit has serious physiological consequences. Astronauts who spend time in weightless environments lose bone and muscle mass and suffer other adverse effects. One solution is to introduce "artificial gravity." On a space station, the easiest way to do this would be to make the station rotate, producing an apparent weight. The designers of this space station model for the movie *2001: A Space Odyssey* made it rotate for just that reason.

The Orbit of the Moon

If a satellite is simply "falling" around the earth, with the gravitational force causing a centripetal acceleration, then what about the moon? Is it obeying the same laws of physics? Or do celestial objects obey laws that we cannot discover by experiments here on earth?

The radius of the moon's orbit around the earth is 3.84×10^8 m. If we use Equation 6.17 to calculate the period of the moon's orbit, the time the moon takes to circle the earth once, we get

$$T = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{3.84 \times 10^8\ \text{m}}{9.80\ \text{m/s}^2}} = 655\ \text{min} \approx 11\ \text{h} \tag{6.18}$$

This is clearly wrong; the period of the moon's orbit is approximately one month.

Newton believed that the laws of motion he had discovered were *universal* and so should apply to the motion of the moon as well as to the motion of objects in the laboratory. But why should we assume that the free-fall acceleration g is the same at the distance of the moon as it is on or near the earth's surface? If gravity is the force of the earth pulling on an object, it seems plausible that the size of that force, and thus the size of g, should diminish with increasing distance from the earth.

Newton proposed the idea that the earth's force of gravity decreases with the square of the distance from the earth. This is the basis of *Newton's law of gravity,* a topic we will study in the next section. The force of gravity is less at the distance of the moon—exactly the strength needed to make the moon orbit at the observed rate. The moon, just like the Space Shuttle, is simply "falling" around the earth!

6.6 Newton's Law of Gravity

A popular image has Newton thinking of the idea of gravity after an apple fell on his head. This amusing story is at least close to the truth. Newton himself said that the "notion of gravitation" came to him as he "sat in a contemplative mood" and "was occasioned by the fall of an apple." It occurred to him that, perhaps, the apple was attracted to the center of the earth but was prevented from getting there by the earth's surface. And if the apple was so attracted, why not the moon? Newton's genius was his sudden realization that **the force that attracts the moon to the earth (and the planets to the sun) was identical to the force that attracts an apple to the earth.** In other words, gravitation is a *universal* force between all objects in the universe! This is not shocking today, but no one before Newton had ever thought that the mundane motion of objects on earth had any connection at all with the stately motion of the planets around the sun.

Isaac Newton was born to a poor farming family in 1642, the year of Galileo's death. He entered Trinity College at Cambridge University at age 19 as a "subsizar," a poor student who had to work his way through school. Newton graduated in 1665, at age 23, just as an outbreak of the plague in England forced the universities to close for two years. He returned to his family farm for that period, during which he made important experimental discoveries in optics, laid the foundations for his theories of mechanics and gravitation, and made major progress toward his invention of calculus as a whole new branch of mathematics.

Gravity Obeys an Inverse-Square Law

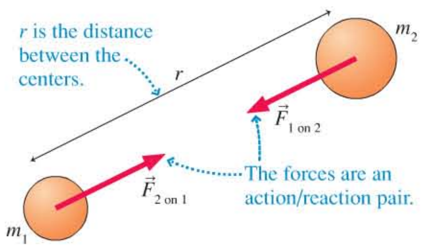
Newton also recognized that the strength of gravity must decrease with distance. These two notions about gravity—that it is universal and that it decreases with distance—form the basis for Newton's law of gravity.

Newton proposed that *every* object in the universe attracts *every other* object with a force that has the following properties:

1. The force is inversely proportional to the square of the distance between the objects.
2. The force is directly proportional to the product of the masses of the two objects.

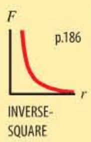
FIGURE 6.26 shows two spherical objects with masses m_1 and m_2 separated by distance r. Each object exerts an attractive force on the other, a force that we call the **gravitational force.** These two forces form an action/reaction pair, so $\vec{F}_{1\text{ on }2}$ is equal in magnitude and opposite in direction to $\vec{F}_{2\text{ on }1}$. The magnitude of the forces is given by Newton's law of gravity.

FIGURE 6.26 The gravitational forces on masses m_1 and m_2.



Newton's law of gravity If two objects with masses m_1 and m_2 are a distance r apart, the objects exert attractive forces on each other of magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2} \tag{6.19}$$



The forces are directed along the line joining the two objects.

The constant G is called the **gravitational constant.** In the SI system of units,

$$G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$$

NOTE ► Strictly speaking, Newton's law of gravity applies to *particles* with masses m_1 and m_2. However, it can be shown that the law also applies to the force between two spherical objects if r is the distance between their centers. ◄

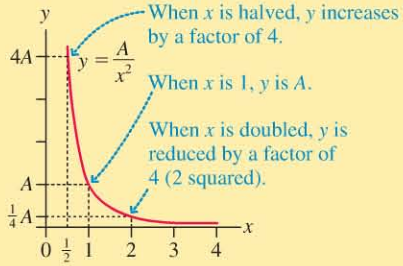
As the distance r between two objects increases, the gravitational force between them decreases. Because the distance appears squared in the denominator, Newton's law of gravity is what we call an **inverse-square** law. Doubling the distance between two masses causes the force between them to decrease by a factor of 4. This mathematical form is one we will see again, so it is worth our time to explore it in more detail.

Inverse-square relationships

Two quantities have an **inverse-square relationship** if y is inversely proportional to the *square* of x. We write the mathematical relationship as

$$y = \frac{A}{x^2}$$

y is inversely proportional to x^2



Here, A is a constant. This relationship is sometimes written as $y \propto 1/x^2$.

SCALING As the graph shows, inverse-square scaling means, for example:

- If you double x, you decrease y by a factor of 4.
- If you halve x, you increase y by a factor of 4.
- If you increase x by a factor of 3, you decrease y by a factor of 9.
- If you decrease x by a factor of 3, you increase y by a factor of 9.

Generally, **if x *increases* by a factor of C, y *decreases* by a factor of C^2.** If x *decreases* by a factor of C, y *increases* by a factor of C^2.

RATIOS For any two values of x—say, x_1 and x_2—we have

$$y_1 = \frac{A}{x_1^2} \quad \text{and} \quad y_2 = \frac{A}{x_2^2}$$

Dividing the y_1-equation by the y_2-equation, we find

$$\frac{y_1}{y_2} = \frac{A/x_1^2}{A/x_2^2} = \frac{A}{x_1^2}\frac{x_2^2}{A} = \frac{x_2^2}{x_1^2}$$

That is, the ratio of y-values is the inverse of the ratio of the squares of the corresponding values of x.

LIMITS As x becomes large, y becomes very small; as x becomes small, y becomes very large.

Exercises 23, 24

CONCEPTUAL EXAMPLE 6.14 Varying gravitational force

The gravitational force between two giant lead spheres is 0.010 N when the centers of the spheres are 20 m apart. What is the distance between their centers when the gravitational force between them is 0.160 N?

REASON We can solve this problem without knowing the masses of the two spheres. The key is to consider the ratios of forces and distances. Gravity is an inverse-square relationship; the force is related to the inverse square of the distance. The force *increases* by a factor of (0.160 N)/(0.010 N) = 16, so the distance must *decrease* by a factor of $\sqrt{16} = 4$. The distance is thus (20 m)/4 = 5.0 m.

ASSESS This type of ratio reasoning is a very good way to get a quick handle on the solution to a problem.

EXAMPLE 6.15 Gravitational force between two people

You are seated in your physics class next to another student 0.60 m away. Estimate the magnitude of the gravitational force between you. Assume that you each have a mass of 65 kg.

PREPARE We will model each of you as a sphere; this is not a particularly good model, but it will do for making an estimate. We will take the 0.60 m as the distance between your centers.

SOLVE The gravitational force is given by Equation 6.19:

$$F_{\text{(you) on (other student)}} = \frac{Gm_{\text{you}}m_{\text{other student}}}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(65\ \text{kg})(65\ \text{kg})}{(0.60\ \text{m})^2}$$

$$= 7.8 \times 10^{-7}\ \text{N}$$

ASSESS The force is quite small, roughly the weight of one hair on your head.

There is a gravitational force between all objects in the universe, but the gravitational force between two ordinary-sized objects is extremely small. Only when one (or both) of the masses is exceptionally large does the force of gravity become important. The downward force of the earth on you—your weight—is large because the earth has an enormous mass. And the attraction is mutual; by Newton's third law, you exert an upward force on the earth that is equal to your weight. However, the large mass of the earth makes the effect of this force on the earth negligible.

EXAMPLE 6.16 Gravitational force of the earth on a person

What is the magnitude of the gravitational force of the earth on a 60 kg person? The earth has mass 5.98×10^{24} kg and radius 6.37×10^6 m.

PREPARE We'll again model the person as a sphere. The distance r in Newton's law of gravity is the distance between the *centers* of the two spheres. The size of the person is negligible compared to the size of the earth, so we can use the earth's radius as r.

SOLVE The force of gravity on the person due to the earth can be computed using Equation 6.19:

$$F_{\text{earth on person}} = \frac{GM_e m}{R_e^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})(60\ \text{kg})}{(6.37 \times 10^6\ \text{m})^2}$$

$$= 590\ \text{N}$$

ASSESS This force is exactly the same as we would calculate using the formula for the weight force, $w = mg$. This isn't surprising, though. Chapter 5 introduced the weight of an object as simply the "force of gravity" acting on it. Newton's law of gravity is a more fundamental law for calculating the force of gravity, but it's still the same force that we earlier called "weight."

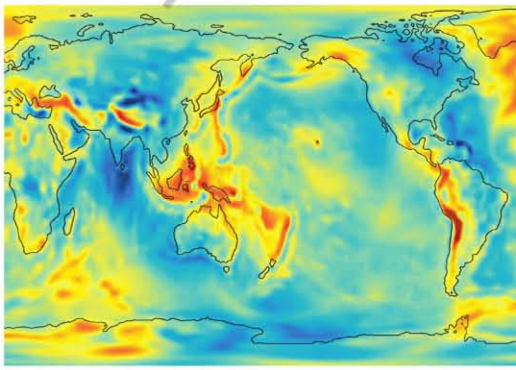
NOTE ► We will use uppercase R and M to represent the large mass and radius of a star or planet, as we did in Example 6.16. ◄

The force of gravitational attraction between the earth and you is responsible for your weight. If you were to venture to another planet, your *mass* would be the same but your *weight* would vary, as we discussed in Chapter 5. We will now explore this concept in more detail.

Gravity on Other Worlds

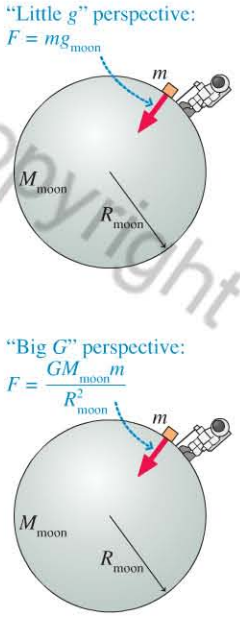
When astronauts ventured to the moon, television images showed them walking—and even jumping and skipping—with some ease, even though they were wearing life support systems with a mass of over 80 kg. This was a visible reminder that the weight of objects is less on the moon. Let's consider why this is so.

FIGURE 6.27 on the next page shows an astronaut on the moon weighing a rock of mass m. When we compute the weight of an object on the surface of the earth, we



Variable gravity When we calculated the force of the earth's gravity, we assumed that the earth's shape and composition are uniform. Neither is quite true, so there is a very small variation in gravity at the surface of the earth, as shown in this image. Red means slightly stronger surface gravity; blue, slightly weaker. These variations are caused by differing distances from the earth's center and by unevenness in the density of the earth's crust. Though these variations are important for scientists studying the earth, they are small enough that we can ignore them for the computations we'll do in this textbook.

FIGURE 6.27 An astronaut weighing a mass on the moon.



use the formula $w = mg$. We can do the same calculation for a mass on the moon, as long as we use the value of g on the moon:

$$w = mg_{moon} \tag{6.20}$$

This is the "little g" perspective. Falling-body experiments on the moon would give the value of g_{moon} as $1.62\ m/s^2$.

But we can also take a "big G" perspective. The weight of the rock comes from the gravitational attraction of the moon, and we can compute this weight using Equation 6.19. The distance r is the radius of the moon, which we'll call R_{moon}. Thus

$$F_{moon\,on\,m} = \frac{GM_{moon}m}{R^2_{moon}} \tag{6.21}$$

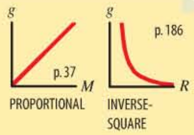
Because Equations 6.20 and 6.21 are two names and two expressions for the same force, we can equate the right-hand sides to find that

$$g_{moon} = \frac{GM_{moon}}{R^2_{moon}}$$

We have done this calculation for an object on the moon, but the result is completely general. At the surface of a planet (or a star), the free-fall acceleration g, a consequence of gravity, can be computed as

$$g_{planet} = \frac{GM_{planet}}{R^2_{planet}} \tag{6.22}$$

Free-fall acceleration on the surface of a planet



If we use values for the mass and the radius of the moon from the table inside the cover of the book, we can compute $g_{moon} = 1.62\ m/s^2$. This means that an object would weigh less on the moon than it would on the earth, where g is $9.80\ m/s^2$. A 70 kg astronaut wearing an 80 kg spacesuit would weigh over 330 lb on the earth but only 54 lb on the moon.

The low lunar gravity makes walking very easy, but a walking pace on the moon would be very slow. Earlier in the chapter we found that the maximum walking speed is $v_{max} = \sqrt{gr}$, where r is the length of the leg. For a typical leg length of 0.7 m and the gravity of the moon, the *maximum* walking speed would be about 1 m/s, just over 2 mph—a very gentle stroll!

Equation 6.22 gives g at the surface of a planet. More generally, imagine an object at distance $r > R$ from the center of a planet. Its free-fall acceleration at this distance is

$$g = \frac{GM}{r^2} \tag{6.23}$$

This more general result agrees with Equation 6.22 if $r = R$, but it allows us to determine the "local" free-fall acceleration at distances $r > R$. Equation 6.23 expresses Newton's idea that the size of g should decrease as you get farther from the earth.

As you're flying in a jet airplane at a height of about 10 km, the free-fall acceleration is about 0.3% less than on the ground. At the height of the Space Shuttle, about 300 km, Equation 6.23 gives $g = 8.9\ m/s^2$, about 10% less than the free-fall acceleration on the earth's surface. If you use this slightly smaller value of g in Equation 6.17 for the period of a satellite's orbit, you'll get the correct period of about 90 minutes. This value of g, only slightly less than the ground-level value, emphasizes the

◀ **Walking on the moon** BIO The low lunar gravity made walking at a reasonable pace difficult for the Apollo astronauts, but the reduced weight made jumping quite easy. Videos from the surface of the moon often show the astronauts getting from place to place by hopping or skipping—not for fun, but for speed and efficiency.

point that an object in orbit is not "weightless" due to the absence of gravity, but rather because it is in free fall.

EXAMPLE 6.17 Gravity on Saturn

Saturn, at 5.68×10^{26} kg, has nearly 100 times the mass of the earth. It is also much larger, with a radius of 5.85×10^7 m. What is the value of g on the surface of Saturn?

SOLVE We can use Equation 6.22 to compute the value of g_{Saturn}:

$$g_{\text{Saturn}} = \frac{GM_{\text{Saturn}}}{R_{\text{Saturn}}^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.68 \times 10^{26}\ \text{kg})}{(5.85 \times 10^7\ \text{m})^2} = 11.1\ \text{m/s}^2$$

ASSESS Even though Saturn is much more massive than the earth, its larger radius gives it a surface gravity that is not markedly different from that of the earth. If Saturn had a solid surface, you could walk and move around quite normally.

EXAMPLE 6.18 Finding the speed to orbit Deimos

Mars has two moons, each much smaller than the earth's moon. The smaller of these two bodies, Deimos, has an average radius of only 6.3 km and a mass of 1.8×10^{15} kg. At what speed would a projectile move in a very low orbit around Deimos?

SOLVE The free-fall acceleration at the surface of Deimos is quite small:

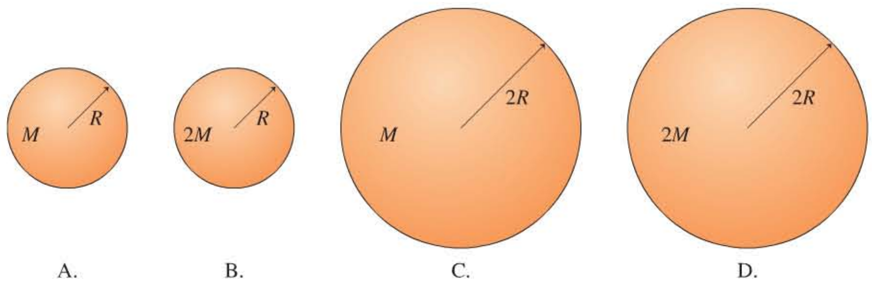
$$g_{\text{Deimos}} = \frac{GM_{\text{Deimos}}}{R_{\text{Deimos}}^2} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(1.8 \times 10^{15}\ \text{kg})}{(6.3 \times 10^3\ \text{m})^2}$$
$$= 0.0030\ \text{m/s}^2$$

Given this, we can use Equation 6.16 to calculate the orbital speed:

$$v_{\text{orbit}} = \sqrt{gr} = \sqrt{(0.0030\ \text{m/s}^2)(6.3 \times 10^3\ \text{m})} = 4.3\ \text{m/s} \approx 10\ \text{mph}$$

ASSESS This is quite slow. With a good jump, you could easily launch yourself into an orbit around Deimos!

STOP TO THINK 6.5 Rank in order, from largest to smallest, the free-fall accelerations on the surfaces of the following planets.



6.7 Gravity and Orbits

The planets of the solar system orbit the sun because the sun's gravitational pull, a force that points toward the center, causes the centripetal acceleration of circular motion. Mercury, the closest planet, experiences the largest acceleration, while Pluto, the most distant, has the smallest. ActivPhysics ONLINE 4.6

FIGURE 6.28 on the following page shows a large body of mass M, such as the earth or the sun, with a much smaller body of mass m orbiting it. The smaller body is called a **satellite**, even though it may be a planet orbiting the sun. Newton's second law tells us that $F_{M \text{ on } m} = ma$, where $F_{M \text{ on } m}$ is the gravitational force of the large body

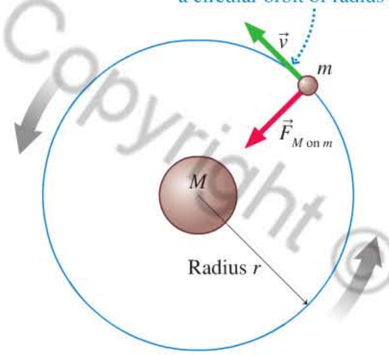


FIGURE 6.28 The orbital motion of a satellite is due to the force of gravity.

on the satellite and a is the satellite's acceleration. $F_{M \text{ on } m}$ is given by Equation 6.19, and, because it's moving in a circular orbit, the satellite's acceleration is its centripetal acceleration, mv^2/r. Thus Newton's second law gives

$$F_{M \text{ on } m} = \frac{GMm}{r^2} = ma = \frac{mv^2}{r} \tag{6.24}$$

Solving for v, we find that the speed of a satellite in a circular orbit is

$$v = \sqrt{\frac{GM}{r}} \tag{6.25}$$

Speed of a satellite in a circular orbit of radius r about a star or planet of mass M

A satellite must have this specific speed in order to maintain a circular orbit of radius r about the larger mass M. If the velocity differs from this value, the orbit will become elliptical rather than circular. Notice that the orbital speed does not depend on the satellite's mass m. This is consistent with our previous discoveries that free-fall motion and projectile motion due to gravity are independent of the mass.

For a planet orbiting the sun, the period T is the time to complete one full orbit. The relationship among speed, radius, and period is the same as for any circular motion, $v = 2\pi r/T$. Combining this with the value of v for a circular orbit from Equation 6.25 gives

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

If we square both sides and rearrange, we find the period of a satellite:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \tag{6.26}$$

Relationship between the orbital period T and radius r for a satellite in a circular orbit around an object of mass M

In other words, **the square of the period of the orbit is proportional to the cube of the radius of the orbit.**

NOTE ► The mass M in Equation 6.26 is the mass of the object at the center of the orbit. ◄

This relationship between radius and period had been deduced from naked-eye observations of planetary motions by the 17th-century astronomer Johannes Kepler. One of Newton's major scientific accomplishments was to use his law of gravity and his laws of motion to prove what Kepler had deduced from observations. Even today, Newton's law of gravity and equations such as Equation 6.26 are essential tools for the NASA engineers who launch probes to other planets in the solar system.

The table inside the back cover of this book contains astronomical information about the sun and the planets that will be useful for many of the end-of-chapter problems. Note that planets farther from the sun have longer periods, in agreement with Equation 6.26.

EXAMPLE 6.19 Locating a geostationary satellite

Communication satellites appear to "hover" over one point on the earth's equator. A satellite that appears to remain stationary as the earth rotates is said to be in a *geostationary orbit*. What is the radius of the orbit of such a satellite?

PREPARE For the satellite to remain stationary with respect to the earth, the satellite's orbital period must be 24 hours; in seconds this is $T = 8.64 \times 10^4$ s.

SOLVE We solve for the radius of the orbit by rearranging Equation 6.26. The mass at the center of the orbit is the earth:

$$r = \left(\frac{GM_e T^2}{4\pi^2}\right)^{\frac{1}{3}} = \left(\frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\ \mathrm{kg})(8.64 \times 10^4\ \mathrm{s})^2}{4\pi^2}\right)^{\frac{1}{3}}$$
$$= 4.22 \times 10^7\ \mathrm{m}$$

ASSESS This is a high orbit; the radius is about 7 times the radius of the earth. Recall that the radius of the Space Shuttle's orbit is only about 5% larger than that of the earth.

Gravity on a Grand Scale

Although relatively weak, gravity is a long-range force. No matter how far apart two objects may be, there is a gravitational attraction between them. Consequently, gravity is the most ubiquitous force in the universe. It not only keeps your feet on the ground, but also is at work on a much larger scale. The Milky Way galaxy, the collection of stars of which our sun is a part, is held together by gravity. But why doesn't the attractive force of gravity simply pull all of the stars together?

The reason is that all of the stars in the galaxy are in orbit around the center of the galaxy. The gravitational attraction keeps the stars moving in orbits around the center of the galaxy rather than falling inward, much as the planets orbit the sun rather than falling into the sun. In the nearly 5 billion years that our solar system has existed, it has orbited the center of the galaxy approximately 20 times.


A spiral galaxy, similar to our Milky Way galaxy.

The galaxy as a whole doesn't rotate at a fixed angular speed, though. All of the stars in the galaxy are different distances from the galaxy's center, and so orbit with different periods. Stars closer to the center complete their orbits in less time, as we would expect from Equation 6.26. As the stars orbit, their relative positions shift. Stars that are relatively near neighbors now could be on opposite sides of the galaxy at some later time.

The rotation of a *rigid body* like a wheel is much simpler. As a wheel rotates, all of the points keep the same relationship to each other; every point on the wheel moves with the same angular velocity. The rotational dynamics of such rigid bodies is a topic we will take up in the next chapter.

STOP TO THINK 6.6 If the mass of the moon were doubled but it stayed in its present orbit, how would its orbital period change?

A. The period would increase.
B. The period would decrease.
C. The period would stay the same.

INTEGRATED EXAMPLE 6.20 A hunter and his sling

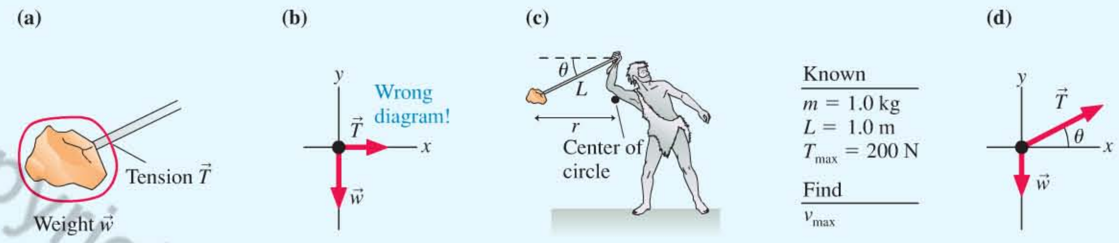
A Stone Age hunter stands on a cliff overlooking a flat plain. He places a 1.0 kg rock in a sling, ties the sling to a 1.0-m-long vine, then swings the rock in a horizontal circle around his head. The plane of the motion is 25 m above the plain below. The tension in the vine increases as the rock goes faster and faster. Suddenly, just as the tension reaches 200 N, the vine snaps. If the rock is moving toward the cliff at this instant, how far out on the plain (from the base of the cliff) will it land?

PREPARE We model the rock as a particle in uniform circular motion. We can use Problem-Solving Strategy 6.1 to analyze this part of the motion. Once the vine breaks, the rock undergoes projectile motion with an initial velocity that is horizontal.

The force identification diagram of **FIGURE 6.29a** on the next page shows that the only contact force acting on the rock is the tension in the vine. Because the rock moves in a horizontal circle, you may be tempted to draw a free-body diagram like **FIGURE 6.29b**, on the next page where $\vec{T}$ is directed along the x-axis. You will quickly run into trouble, however, because in this diagram the net force has a downward y-component that would cause the rock to rapidly accelerate downward. But we know that it moves in a horizontal circle and that the net force must point toward the center of the circle. In this free-body diagram, the weight force $\vec{w}$ points straight down and is certainly correct, so the difficulty must be with $\vec{T}$.

Continued

FIGURE 6.29 Visual overview of a hunter swinging a rock.



As an experiment, tie a small weight to a string, swing it over your head, and check the angle of the string. You will discover that the string is not horizontal but, instead, is angled downward. The sketch of **FIGURE 6.29c** labels this angle θ. Notice that the rock moves in a *horizontal* circle, so the center of the circle is not at his hand. The x-axis points horizontally, to the center of the circle, but the tension force is directed along the vine. Thus the correct free-body diagram is the one in **FIGURE 6.29d**.

Once the vine breaks, the visual overview of the situation is shown in **FIGURE 6.30**. The important thing to note here is that the initial x-component of velocity is the speed the rock had an instant before the vine broke.

SOLVE From the free-body diagram of Figure 6.29d, Newton's second law for circular motion is

$$\sum F_x = T\cos\theta = \frac{mv^2}{r}$$
$$\sum F_y = T\sin\theta - mg = 0$$

where θ is the angle of the vine below the horizontal. We can use the y-equation to find the angle of the vine:

$$\sin\theta = \frac{mg}{T}$$
$$\theta = \sin^{-1}\left(\frac{mg}{T}\right) = \sin^{-1}\left(\frac{(1.0\text{ kg})(9.8\text{ m/s}^2)}{200\text{ N}}\right) = 2.81°$$

where we've evaluated the angle at the maximum tension of 200 N. The vine's angle of inclination is small but not zero.

Turning now to the x-equation, we find the rock's speed around the circle is

$$v = \sqrt{\frac{rT\cos\theta}{m}}$$

Be careful! The radius r of the circle is not the length L of the vine. You can see in Figure 6.29c that $r = L\cos\theta$. Thus

$$v = \sqrt{\frac{LT\cos^2\theta}{m}} = \sqrt{\frac{(1.0\text{ m})(200\text{ N})(\cos 2.81°)^2}{1.0\text{ kg}}} = 14\text{ m/s}$$

Because this is the horizontal speed of the rock just when the vine breaks, the initial velocity $(v_x)_i$ in the visual overview of the projectile motion, Figure 6.30, must be $(v_x)_i = 14$ m/s. Recall that a projectile has no horizontal acceleration, so the rock's final position is

$$x_f = x_i + (v_x)_i\,\Delta t = 0\text{ m} + (14\text{ m/s})\Delta t$$

where Δt is the time the projectile is in the air. We're not given Δt, but we can find it from the vertical motion. For a projectile, the vertical motion is just free-fall motion, so we have

$$y_f = y_i + (v_y)_i\Delta t - \frac{1}{2}g(\Delta t)^2$$

The initial height is $y_i = 25$ m, the final height is $y_f = 0$ m, and the initial vertical velocity is $(v_y)_i = 0$ m/s. With these values, we have

$$0\text{ m} = 25\text{ m} + (0\text{ m/s})\Delta t - \frac{1}{2}(9.8\text{ m/s}^2)(\Delta t)^2$$

Solving this for Δt gives

$$\Delta t = \sqrt{\frac{2(25\text{ m})}{9.8\text{ m/s}^2}} = 2.3\text{ s}$$

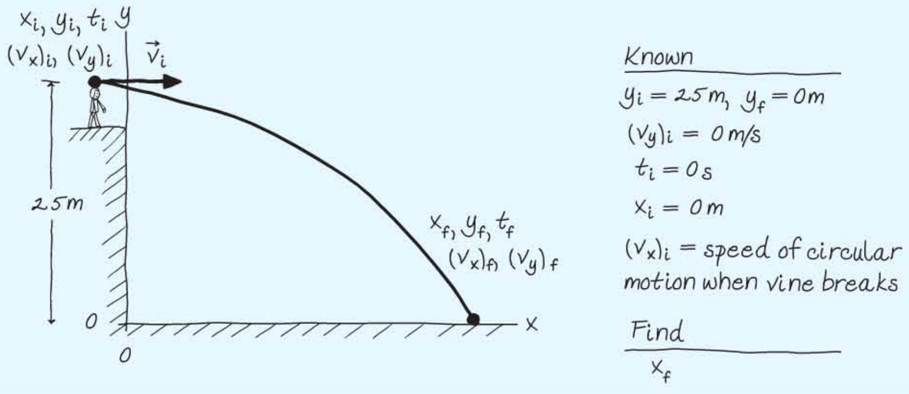
Now we can use this time to find

$$x_f = 0\text{ m} + (14\text{ m/s})(2.3\text{ s}) = 32\text{ m}$$

The rock lands 32 m from the base of the cliff.

ASSESS The circumference of the rock's circle is $2\pi r$, or about 6 m. At a speed of 14 m/s, the rock takes roughly half a second to go around once. This seems reasonable. The 32 m distance is about 100 ft, which seems easily attainable from a cliff over 75 feet high.

FIGURE 6.30 Visual overview of the rock in projectile motion.



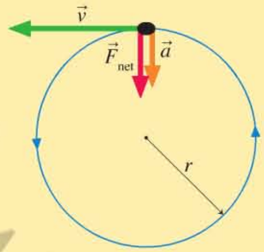
SUMMARY

The goal of Chapter 6 has been to learn about motion in a circle, including orbital motion under the influence of a gravitational force.

GENERAL PRINCIPLES

Uniform Circular Motion

An object moving in a circular path is in uniform circular motion if v is constant.



- The speed is constant, but the direction of motion is constantly changing.
- The **centripetal acceleration** is directed toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$

- This acceleration requires a net force directed toward the center of the circle. Newton's second law for circular motion is

$$\vec{F}_{net} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle}\right)$$

Universal Gravitation

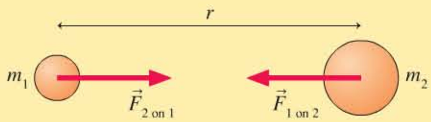
Two objects with masses m_1 and m_2 that are distance r apart exert attractive gravitational forces on each other of magnitude

$$F_{1\,on\,2} = F_{2\,on\,1} = \frac{Gm_1m_2}{r^2}$$

where the gravitational constant is

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

This is **Newton's law of gravity.** Gravity is an inverse-square law.



IMPORTANT CONCEPTS

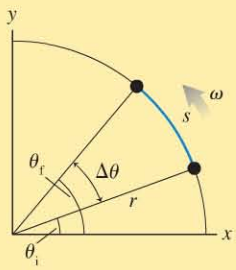
Describing circular motion

We define new variables for circular motion. By convention, counterclockwise is positive.

Angular position: θ

Angular displacement: $\Delta\theta = \theta_f - \theta_i$

Angular velocity: $\omega = \dfrac{\Delta\theta}{\Delta t}$



Angles are measured in radians, where 1 rev = 360° = 2π rad. The SI units of angular velocity are rad/s.

Period: T = time for one complete circle.

Frequency: $f = \dfrac{1}{T}$

Uniform circular motion kinematics

For uniform circular motion:

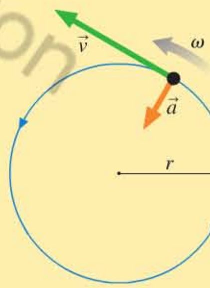
$$\omega = 2\pi f \qquad \theta_f - \theta_i = \Delta\theta = \omega\,\Delta t$$

The velocity, acceleration, and circular motion variables are related as follows:

$$v = \frac{2\pi r}{T}$$

$$v = \omega r$$

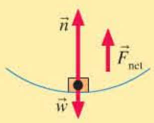
$$a = \frac{v^2}{r} = \omega^2 r$$



APPLICATIONS

Apparent weight and weightlessness

Circular motion requires a net force pointing to the center. The apparent weight $w_{app} = n$ is usually not the same as the true weight w. n must be > 0 for the object to be in contact with a surface.



In orbital motion, the net force is provided by gravity. An astronaut and his spacecraft are both in free fall, so he feels weightless.

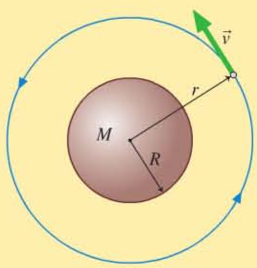
Planetary gravity and orbital motion

For a planet of mass M and radius R, the free-fall acceleration on the surface is

$$g = \frac{GM}{R^2}$$

The speed of a satellite in a low orbit is

$$v = \sqrt{gr}$$



A **satellite** in a circular orbit of radius r around an object of mass M moves at a speed v given by

$$v = \sqrt{\frac{GM}{r}}$$

The period and radius are related as follows:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

Problems labeled can be done on a Workbook Dynamics Worksheet; INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

QUESTIONS

Conceptual Questions

1. The batter in a baseball game hits a home run. As he circles the bases, is his angular velocity positive or negative?
2. Viewed from somewhere in space above the north pole, would a point on the earth's equator have a positive or negative angular velocity due to the earth's rotation?
3. A cyclist goes around a level, circular track at constant speed. Do you agree or disagree with the following statement? "Since the cyclist's speed is constant, her acceleration is zero." Explain.
4. In uniform circular motion, which of the following quantities are constant: speed, instantaneous velocity, angular velocity, centripetal acceleration, the magnitude of the net force?
5. A particle moving along a straight line can have nonzero acceleration even when its speed is zero (for instance, a ball in free fall at the top of its path). Can a particle moving in a circle have nonzero *centripetal* acceleration when its speed is zero? If so, give an example. If not, why not?
6. Would having four-wheel drive on a car make it possible to drive faster around corners on an icy road, without slipping, than the same car with two-wheel drive? Explain.
7. BIO Large birds like pheasants often walk short distances. Small birds like chickadees never walk. They either hop or fly. Why might this be?
8. When you drive fast on the highway with muddy tires, you can hear the mud flying off the tires into your wheel wells. Why does the mud fly off?
9. A ball on a string moves in a vertical circle as in Figure Q6.9. When the ball is at its lowest point, is the tension in the string greater than, less than, or equal to the ball's weight? Explain. (You may want to include a free-body diagram as part of your explanation.)

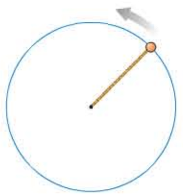


FIGURE Q6.9

10. Give an everyday example of circular motion for which the centripetal acceleration is mostly or completely due to a force of the type specified: (a) Static friction. (b) Tension.
11. Give an everyday example of circular motion for which the centripetal acceleration is mostly or completely due to a force of the type specified: (a) Gravity. (b) Normal force.
12. It's been proposed that future space stations create "artificial gravity" by rotating around an axis. (The space station would have to be much larger than the present space station for this to be feasible.)
    a. How would this work? Explain.
    b. Would the artificial gravity be equally effective throughout the space station? If not, where in the space station would the residents want to live and work?
13. A car coasts at a constant speed over a circular hill. Which of the free-body diagrams in Figure Q6.13 is correct? Explain.

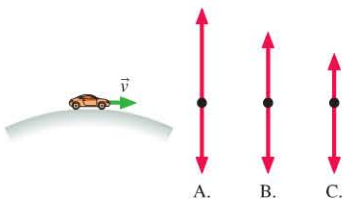


FIGURE Q6.13

14. Riding in the back of a pickup truck can be very dangerous. If the truck turns suddenly, the riders can be thrown from the truck bed. Why are the riders ejected from the bed?
15. Variation in your apparent weight is desirable when you ride a roller coaster; it makes the ride fun. However, too much variation over a short period of time can be painful. For this reason, the loops of real roller coasters are not simply circles like Figure 6.21a. A typical loop is shown in Figure Q6.15. The radius of the circle that matches the track at the top of the loop is much smaller than that of a matching circle at other places on the track. Explain why this shape gives a more comfortable ride than a circular loop.

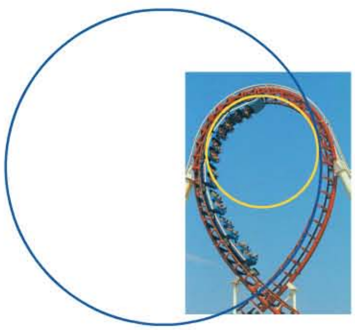


FIGURE Q6.15

16. A small projectile is launched parallel to the ground at height $h = 1$ m with sufficient speed to orbit a completely smooth, airless planet. A bug rides in a small hole inside the projectile. Is the bug weightless? Explain.
17. Why is it impossible for an astronaut inside an orbiting space shuttle to go from one end to the other by walking normally?
18. If every object in the universe feels an attractive gravitational force due to every other object, why don't you feel a pull from someone seated next to you?
19. A mountain climber's weight is less on the top of a tall mountain than at the base, though his mass is the same. Why?
20. Is the earth's gravitational force on the sun larger, smaller, or equal to the sun's gravitational force on the earth? Explain.

Multiple-Choice Questions

21. | A ball on a string moves around a complete circle, once a second, on a frictionless, horizontal table. The tension in the string is measured to be 6.0 N. What would the tension be if the ball went around in only half a second?
A. 1.5 N B. 3.0 N C. 12 N D. 24 N

22. | As seen from above, a car rounds the curved path shown in Figure Q6.22 at a constant speed. Which vector best represents the net force acting on the car?

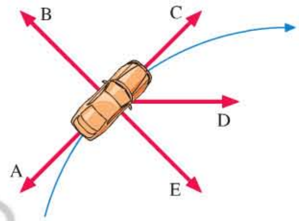


FIGURE Q6.22

23. | Suppose you and a friend, each of mass 60 kg, go to the park and get on a 4.0-m-diameter merry-go-round. You stand on the outside edge of the merry-go-round, while your friend pushes so that it rotates once every 6.0 s. What is the magnitude of the (apparent) outward force that you feel?
A. 7 N B. 63 N C. 130 N D. 260 N

24. | The cylindrical space station in Figure Q6.24, 200 m in diameter, rotates in order to provide artificial gravity of g for the occupants. How much time does the station take to complete one rotation?
A. 3 s B. 20 s C. 28 s D. 32 s

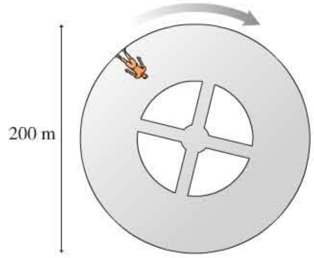


FIGURE Q6.24

25. || Two cylindrical space stations, the second four times the diameter of the first, rotate so as to provide the same amount of artificial gravity. If the first station makes one rotation in the time T, then the second station makes one rotation in time
A. $T/4$ B. $2T$ C. $4T$ D. $16T$

26. | A newly discovered planet has twice the mass and three times the radius of the earth. What is the free-fall acceleration at its surface, in terms of the free-fall acceleration g at the surface of the earth?
A. $\frac{2}{9}g$ B. $\frac{2}{3}g$ C. $\frac{3}{4}g$ D. $\frac{4}{3}g$

27. || Suppose one night the radius of the earth doubled but its mass stayed the same. What would be an approximate new value for the free-fall acceleration at the surface of the earth?
A. 2.5 m/s^2 B. 5.0 m/s^2 C. 10 m/s^2 D. 20 m/s^2

28. | Currently, the moon goes around the earth once every 27.3 days. If the moon could be brought into a new circular orbit with a smaller radius, its orbital period would be
A. More than 27.3 days.
B. 27.3 days.
C. Less than 27.3 days.

29. || Two planets orbit a star. Planet 1 has orbital radius r_1 and planet 2 has $r_2 = 4r_1$. Planet 1 orbits with period T_1. Planet 2 orbits with period
A. $T_2 = \frac{1}{2}T_1$ B. $T_2 = 2T_1$ C. $T_2 = 4T_1$ D. $T_2 = 8T_1$

30. | A particle undergoing circular motion in the xy-plane stops on the positive y-axis. Which of the following does *not* describe its angular position?
A. $\pi/2$ rad B. π rad C. $5\pi/2$ rad D. $-3\pi/2$ rad

Questions 31 through 33 concern a classic figure-skating jump called the axel. A skater starts the jump moving forward as shown in Figure Q6.31, leaps into the air, and turns one-and-a-half revolutions before landing. The typical skater is in the air for about 0.5 s, and the skater's hands are located about 0.8 m from the rotation axis.


FIGURE Q6.31

31. || What is the approximate angular speed of the skater during the leap?
A. 2 rad/s B. 6 rad/s C. 9 rad/s D. 20 rad/s

32. | The skater's arms are fully extended during the jump. What is the approximate centripetal acceleration of the skater's hand?
A. 10 m/s^2 B. 30 m/s^2 C. 300 m/s^2 D. 450 m/s^2

33. | What is the approximate speed of the skater's hand?
A. 1 m/s B. 3 m/s C. 9 m/s D. 15 m/s

PROBLEMS

Section 6.1 Uniform Circular Motion

1. || What is the angular position in radians of the minute hand of a clock at (a) 5:00, (b) 7:15, and (c) 3:35?
2. | A child on a merry-go-round takes 3.0 s to go around once. What is his angular displacement during a 1.0 s time interval?
3. ||| What is the angular speed of the tip of the minute hand on a clock, in rad/s?
4. || An old-fashioned vinyl record rotates on a turntable at 45 rpm. What are (a) the angular speed in rad/s and (b) the period of the motion?
5. ||| The earth's radius is about 4000 miles. Kampala, the capital of Uganda, and Singapore are both nearly on the equator. The distance between them is 5000 miles.
   a. Through what angle do you turn, relative to the earth, if you fly from Kampala to Singapore? Give your answer in both radians and degrees.
   b. The flight from Kampala to Singapore takes 9 hours. What is the plane's angular speed relative to the earth?

6. A Ferris wheel rotates at an angular velocity of 0.036 rad/s. At $t = 0$ min, your friend Seth is at the very top of the ride. What is Seth's angular position at $t = 3.0$ min, measured counterclockwise from the top? Give your answer as an angle in degrees between 0° and 360°.
7. A turntable rotates counterclockwise at 78 rpm. A speck of dust on the turntable is at $\theta = 0.45$ rad at $t = 0$ s. What is the angle of the speck at $t = 8.0$ s? Your answer should be between 0 and 2π rad.
8. A fast-moving superhero in a comic book runs around a circular, 70-m-diameter track five and a half times (ending up directly opposite her starting point) in 3.0 s. What is her angular speed, in rad/s?
9. Figure P6.9 shows the angular position of a potter's wheel.
   a. What is the angular displacement of the wheel between $t = 5$ s and $t = 15$ s?
   b. What is the angular velocity of the wheel at $t = 15$ s?

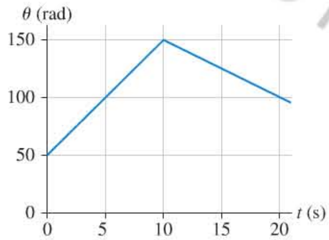


FIGURE P6.9

10. The angular velocity (in rpm) of the blade of a blender is given in Figure P6.10.
    a. If $\theta = 0$ rad at $t = 0$ s, what is the blade's angular position at $t = 20$ s?
    b. At what time has the blade completed 10 full revolutions?

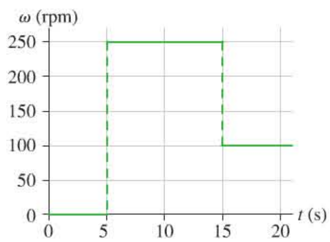


FIGURE P6.10

Section 6.2 Speed, Velocity, and Acceleration in Uniform Circular Motion

11. A 5.0-m-diameter merry-go-round is turning with a 4.0 s period. What is the speed of a child on the rim?
12. The blade on a table saw spins at 3450 rpm. Its diameter is 25.0 cm. What is the speed of a tooth on the edge of the blade, in both m/s and mph?
13. The horse on a carousel is 4.0 m from the central axis.
    a. If the carousel rotates at 0.10 rev/s, how long does it take the horse to go around twice?
    b. How fast is a child on the horse going (in m/s)?
14. The radius of the earth's very nearly circular orbit around the sun is 1.50×10^{11} m. Find the magnitude of the earth's (a) velocity, (b) angular velocity, and (c) centripetal acceleration as it travels around the sun. Assume a year of 365 days.
15. Your roommate is working on his bicycle and has the bike upside down. He spins the 60-cm-diameter wheel, and you notice that a pebble stuck in the tread goes by three times every second. What are the pebble's speed and acceleration?
16. To withstand "g-forces" of up to $10g$, caused by suddenly pulling out of a steep dive, fighter jet pilots train on a "human centrifuge." $10g$ is an acceleration of 98 m/s^2. If the length of the centrifuge arm is 12 m, at what speed is the rider moving when she experiences $10g$?

Section 6.3 Dynamics of Uniform Circular Motion

17. Figure P6.17 is a bird's-eye view of particles on a string moving in horizontal circles on a tabletop. All are moving at the same speed. Rank in order, from largest to smallest, the tensions T_1 to T_4.

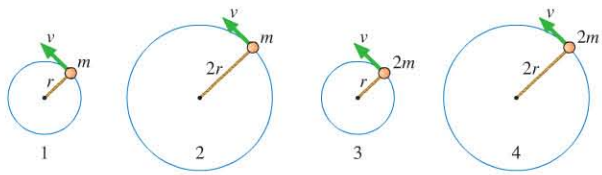


FIGURE P6.17

18. A 200 g block on a 50-cm-long string swings in a circle on a horizontal, frictionless table at 75 rpm.
    a. What is the speed of the block?
    b. What is the tension in the string?
19. A 1500 kg car drives around a flat 200-m-diameter circular track at 25 m/s. What are the magnitude and direction of the net force on the car? What causes this force?
20. A fast pitch softball player does a "windmill" pitch, illustrated in Figure P6.20, moving her hand through a circular arc to pitch a ball at 70 mph. The 0.19 kg ball is 50 cm from the pivot point at her shoulder. At the lowest point of the circle, the ball has reached its maximum speed.
    a. At the bottom of the circle, just before the ball leaves her hand, what is its centripetal acceleration?
    b. What are the magnitude and direction of the force her hand exerts on the ball at this point?


FIGURE P6.20

21. A baseball pitching machine works by rotating a light and stiff rigid rod about a horizontal axis until the ball is moving toward the target. Suppose a 144 g baseball is held 85 cm from the axis of rotation and released at the major league pitching speed of 85 mph.
    a. What is the ball's centripetal acceleration just before it is released?
    b. What is the magnitude of the net force that is acting on the ball just before it is released?
22. You're driving your pickup truck around a curve with a radius of 20 m. A box in the back of the truck is pressed up against the wall of the truck. How fast must you drive so that the force of the wall on the box equals the weight of the box?

Section 6.4 Apparent Forces in Circular Motion

23. The passengers in a roller coaster car feel 50% heavier than their true weight as the car goes through a dip with a 30 m radius of curvature. What is the car's speed at the bottom of the dip?
24. You hold a bucket in one hand. In the bucket is a 500 g rock. You swing the bucket so the rock moves in a vertical circle 2.2 m in diameter. What is the minimum speed the rock must have at the top of the circle if it is to always stay in contact with the bottom of the bucket?
25. As a roller coaster car crosses the top of a 40-m-diameter loop-the-loop, its apparent weight is the same as its true weight. What is the car's speed at the top?
26. A typical laboratory centrifuge rotates at 4000 rpm. Test tubes have to be placed into a centrifuge very carefully because of the very large accelerations.
    a. What is the acceleration at the end of a test tube that is 10 cm from the axis of rotation?
    b. For comparison, what is the magnitude of the acceleration a test tube would experience if stopped in a 1.0-ms-long encounter with a hard floor after falling from a height of 1.0 m?

Section 6.5 Circular Orbits and Weightlessness

27. A satellite orbiting the moon very near the surface has a period of 110 min. Use this information, together with the radius of the moon from the table on the inside of the back cover, to calculate the free-fall acceleration on the moon's surface.

Section 6.6 Newton's Law of Gravity

28. The centers of a 10 kg lead ball and a 100 g lead ball are separated by 10 cm.
    a. What gravitational force does each exert on the other?
    b. What is the ratio of this gravitational force to the weight of the 100 g ball?
29. The gravitational force of a star on an orbiting planet 1 is F_1. Planet 2, which is twice as massive as planet 1 and orbits at twice the distance from the star, experiences gravitational force F_2. What is the ratio F_2/F_1?
30. The free-fall acceleration at the surface of planet 1 is 20 m/s^2. The radius and the mass of planet 2 are twice those of planet 1. What is the free-fall acceleration on planet 2?
31. What is the ratio of the sun's gravitational force on you to the earth's gravitational force on you?
32. Suppose the free-fall acceleration at some location on earth was exactly 9.8000 m/s^2. What would it be at the top of a 1000-m-tall tower at this location? (Give your answer to five significant figures.)
33. a. What is the gravitational force of the sun on the earth?
    b. What is the gravitational force of the moon on the earth?
    c. The moon's force is what percent of the sun's force?
34. What is the free-fall acceleration at the surface of (a) Mars and (b) Jupiter?

Section 6.7 Gravity and Orbits

35. Planet X orbits the star Omega with a "year" that is 200 earth days long. Planet Y circles Omega at four times the distance of Planet X. How long is a year on Planet Y?
36. Satellite A orbits a planet with a speed of 10,000 m/s. Satellite B is twice as massive as satellite A and orbits at twice the distance from the center of the planet. What is the speed of satellite B?
37. The Space Shuttle is in a 250-mile-high orbit. What are the shuttle's orbital period, in minutes, and its speed?
38. The *asteroid belt* circles the sun between the orbits of Mars and Jupiter. One asteroid has a period of 5.0 earth years. What are the asteroid's orbital radius and speed?
39. An earth satellite moves in a circular orbit at a speed of 5500 m/s. What is its orbital period?

General Problems

40. How fast must a plane fly along the earth's equator so that the sun stands still relative to the passengers? In which direction must the plane fly, east to west or west to east? Give your answer in both km/h and mph. The radius of the earth is 6400 km.
41. The car in Figure P6.41 travels at a constant speed along the road shown. Draw vectors showing its acceleration at the three points A, B, and C, or write $\vec{a} = \vec{0}$. The lengths of your vectors should correspond to the magnitudes of the accelerations.

FIGURE P6.41

42. In the Bohr model of the hydrogen atom, an electron (mass $m = 9.1 \times 10^{-31}$ kg) orbits a proton at a distance of 5.3×10^{-11} m. The proton pulls on the electron with an electric force of 8.2×10^{-8} N. How many revolutions per second does the electron make?
43. A 75 kg man weighs himself at the north pole and at the equator. Which scale reading is higher? By how much? Assume the earth is a perfect sphere. Explain why the readings differ.
44. A 1500 kg car takes a 50-m-radius unbanked curve at 15 m/s. What is the size of the friction force on the car?
45. A 500 g ball swings in a vertical circle at the end of a 1.5-m-long string. When the ball is at the bottom of the circle, the tension in the string is 15 N. What is the speed of the ball at that point?
46. Suppose the moon were held in its orbit not by gravity but by a massless cable attached to the center of the earth. What would be the tension in the cable? See the inside of the back cover for astronomical data.
47. A 30 g ball rolls around a 40-cm-diameter L-shaped track, shown in Figure P6.47, at 60 rpm. Rolling friction can be neglected.

FIGURE P6.47

    a. How many different contact forces does the track exert on the ball? Name them.
    b. What is the magnitude of the net force on the ball?
48. A 5.0 g coin is placed 15 cm from the center of a turntable. The coin has static and kinetic coefficients of friction with the turntable surface of $\mu_s = 0.80$ and $\mu_k = 0.50$. The turntable very slowly speeds up to 60 rpm. Does the coin slide off?

49. A *conical pendulum* is formed by attaching a 500 g ball to a 1.0-m-long string, then allowing the mass to move in a horizontal circle of radius 20 cm. Figure P6.49 shows that the string traces out the surface of a cone, hence the name.
   a. What is the tension in the string?
   b. What is the ball's angular velocity, in rpm?

   Hint: Determine the horizontal and vertical components of the forces acting on the ball, and use the fact that the vertical component of acceleration is zero since there is no vertical motion.

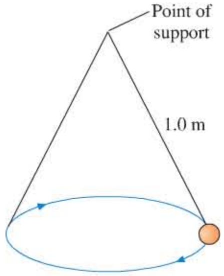


FIGURE P6.49

50. In an old-fashioned amusement park ride, passengers stand inside a 3.0-m-tall, 5.0-m-diameter hollow steel cylinder with their backs against the wall. The cylinder begins to rotate about a vertical axis. Then the floor on which the passengers are standing suddenly drops away! If all goes well, the passengers will "stick" to the wall and not slide. Clothing has a static coefficient of friction against steel in the range 0.60 to 1.0 and a kinetic coefficient in the range 0.40 to 0.70. What is the minimum rotational frequency, in rpm, for which the ride is safe?

51. INT The 0.20 kg puck on the frictionless, horizontal table in Figure P6.51 is connected by a string through a hole in the table to a hanging 1.20 kg block. With what speed must the puck rotate in a circle of radius 0.50 m if the block is to remain hanging at rest?

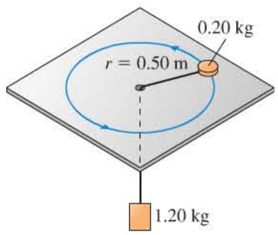


FIGURE P6.51

52. While at the county fair, you decide to ride the Ferris wheel. Having eaten too many candy apples and elephant ears, you find the motion somewhat unpleasant. To take your mind off your stomach, you wonder about the motion of the ride. You estimate the radius of the big wheel to be 15 m, and you use your watch to find that each loop around takes 25 s.
   a. What are your speed and magnitude of your acceleration?
   b. What is the ratio of your apparent weight to your true weight at the top of the ride?
   c. What is the ratio of your apparent weight to your true weight at the bottom?

53. A car drives over the top of a hill that has a radius of 50 m. What maximum speed can the car have without flying off the road at the top of the hill?

54. INT A 100 g ball on a 60-cm-long string is swung in a vertical circle whose center is 200 cm above the floor. The string suddenly breaks when it is parallel to the ground and the ball is moving upward. The ball reaches a height 600 cm above the floor. What was the tension in the string an instant before it broke?

55. BIO While a person is walking, his arms (each with typical length 70 cm measured from the shoulder joint) swing through approximately a 45° angle in 0.50 s. As a reasonable approximation, we can assume that the arm moves with constant speed during each swing.
   a. What is the acceleration of a 1.0 g drop of blood in the fingertips at the bottom of the swing?
   b. Draw a free-body diagram for the drop of blood in part a.
   c. Find the magnitude and direction of the force that the blood vessel must exert on the drop of blood.
   d. What force would the blood vessel exert if the arm were not swinging?

56. INT The two identical pucks in Figure P6.56 rotate together on a frictionless, horizontal table. They are tied together by strings 1 and 2, each of length l. If their common angular speed is ω, what are the tensions in the two strings?

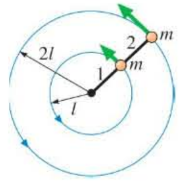


FIGURE P6.56

57. BIO INT The ultracentrifuge is an important tool for separating and analyzing proteins in biological research. Because of the enormous centripetal accelerations that can be achieved, the apparatus (see Figure 6.23) must be carefully balanced so that each sample is matched by another on the opposite side of the rotor shaft. Failure to do so is a costly mistake, as seen in Figure P6.57. Any difference in mass of the opposing samples will cause a net force in the horizontal plane on the shaft of the rotor. Suppose that a scientist makes a slight error in sample preparation, and one sample has a mass 10 mg greater than the opposing sample. If the samples are 10 cm from the axis of the rotor and the ultracentrifuge spins at 70,000 rpm, what is the magnitude of the net force on the rotor due to the unbalanced samples?


FIGURE P6.57

58. The Space Shuttle orbits 300 km above the surface of the earth.
   a. What is the force of gravity on a 1.0 kg sphere inside the Space Shuttle?
   b. The sphere floats around inside the Space Shuttle, apparently "weightless." How is this possible?

59. A sensitive gravimeter at a mountain observatory finds that the free-fall acceleration is 0.0075 m/s^2 less than that at sea level. What is the observatory's altitude?

60. Suppose we could shrink the earth without changing its mass. At what fraction of its current radius would the free-fall acceleration at the surface be three times its present value?

61. Planet Z is 10,000 km in diameter. The free-fall acceleration on Planet Z is 8.0 m/s^2.
   a. What is the mass of Planet Z?
   b. What is the free-fall acceleration 10,000 km above Planet Z's north pole?

62. What are the speed and altitude of a geostationary satellite (see Example 6.19) orbiting Mars? Mars rotates on its axis once every 24.8 hours.

63. BIO a. What is the free-fall acceleration on Mars?
   b. Estimate the maximum speed at which an astronaut can walk on the surface of Mars.

64. INT How long will it take a rock dropped from 2.0 m above the surface of Mars to reach the ground?

65. ||| A 20 kg sphere is at the origin and a 10 kg sphere is at $(x, y) = (20\text{ cm}, 0\text{ cm})$. At what point or points could you place a small mass such that the net gravitational force on it due to the spheres is zero?
INT

66. ||| a. At what height above the earth is the free-fall acceleration 10% of its value at the surface?
b. What is the speed of a satellite orbiting at that height?

67. | Mars has a small moon, Phobos, that orbits with a period of 7 h 39 min. The radius of Phobos' orbit is 9.4×10^6 m. Use only this information (and the value of G) to calculate the mass of Mars.

68. || You are the science officer on a visit to a distant solar system. Prior to landing on a planet you measure its diameter to be 1.80×10^7 m and its rotation period to be 22.3 h. You have previously determined that the planet orbits 2.20×10^{11} m from its star with a period of 402 earth days. Once on the surface you find that the free-fall acceleration is 12.2 m/s^2. What are the masses of (a) the planet and (b) the star?

69. ||||| Europa, a satellite of Jupiter, is believed to have a liquid ocean of water (with a possibility of life) beneath its icy surface. In planning a future mission to Europa, what is the fastest that an astronaut with legs of length 0.70 m could walk on the surface of Europa? Europa is 3100 km in diameter and has a mass of 4.8×10^{22} kg.
BIO

In Problems 70 through 73 you are given the equation (or equations) used to solve a problem. For each of these, you are to

a. Write a realistic problem for which this is the correct equation. The last two questions should involve real planets. Be sure that the answer your problem requests is consistent with the equation given.
b. Finish the solution of the problem.

70. || $60\text{ N} = (0.30\text{ kg})\omega^2(0.50\text{ m})$

71. || $(1500\text{ kg})(9.80\text{ m/s}^2) - 11{,}760\text{ N} = (1500\text{ kg})v^2/(200\text{ m})$

72. || $$\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.90 \times 10^{27}\text{ kg})}{r^2} = \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}{(6.37 \times 10^6\text{ m})^2}$$

73. || $$\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})(1000\text{ kg})}{r^2} = \frac{(1000\text{ kg})(1997\text{ m/s})^2}{r}$$

Passage Problems

Orbiting the Moon

Suppose a spacecraft orbits the moon in a very low, circular orbit, just a few hundred meters above the lunar surface. The moon has a diameter of 3500 km, and the free-fall acceleration at the surface is 1.6 m/s^2.

74. | The direction of the net force on the craft is
A. Away from the surface of the moon.
B. In the direction of motion.
C. Toward the center of the moon.
D. Nonexistent, because the net force is zero.

75. | How fast is this spacecraft moving?
A. 53 m/s B. 75 m/s C. 1700 m/s D. 2400 m/s

76. | How much time does it take for the spacecraft to complete one orbit?
A. 38 min B. 76 min C. 110 min D. 220 min

77. | The material that comprises the side of the moon facing the earth is actually slightly more dense than the material on the far side. When the spacecraft is above a more dense area of the surface, the moon's gravitational force on the craft is a bit stronger. In order to stay in a circular orbit of constant height and speed, the spacecraft could fire its rockets while passing over the denser area. The rockets should be fired so as to generate a force on the craft
A. Away from the surface of the moon.
B. In the direction of motion.
C. Toward the center of the moon.
D. Opposite the direction of motion.

STOP TO THINK ANSWERS

Stop to Think 6.1: A. Because $5\pi/2$ rad $= 2\pi$ rad $+ \pi/2$ rad, the particle's position is one complete revolution (2π rad) plus an extra $\pi/2$ rad. This extra $\pi/2$ rad puts the particle at position A.

Stop to Think 6.2: D > B > C > A. The centripetal acceleration is $\omega^2 r$. Changing r by a factor of 2 changes the centripetal acceleration by a factor of 2, but changing ω by a factor of 2 changes the centripetal acceleration by a factor of 4.

Stop to Think 6.3: $T_D > T_B = T_E > T_C > T_A$. The center-directed force is $m\omega^2 r$. Changing r by a factor of 2 changes the tension by a factor of 2, but changing f (and thus ω) by a factor of 2 changes the tension by a factor of 4.

Stop to Think 6.4: B. The car is moving in a circle, so there must be a net force toward the center of the circle. The center of the circle is below the car, so the net force must point downward. This can be true only if $w > n$.

Stop to Think 6.5: B > A > D > C. The free-fall acceleration is proportional to the mass, but inversely proportional to the square of the radius.

Stop to Think 6.6: C. The period of the orbit does not depend on the mass of the orbiting object.