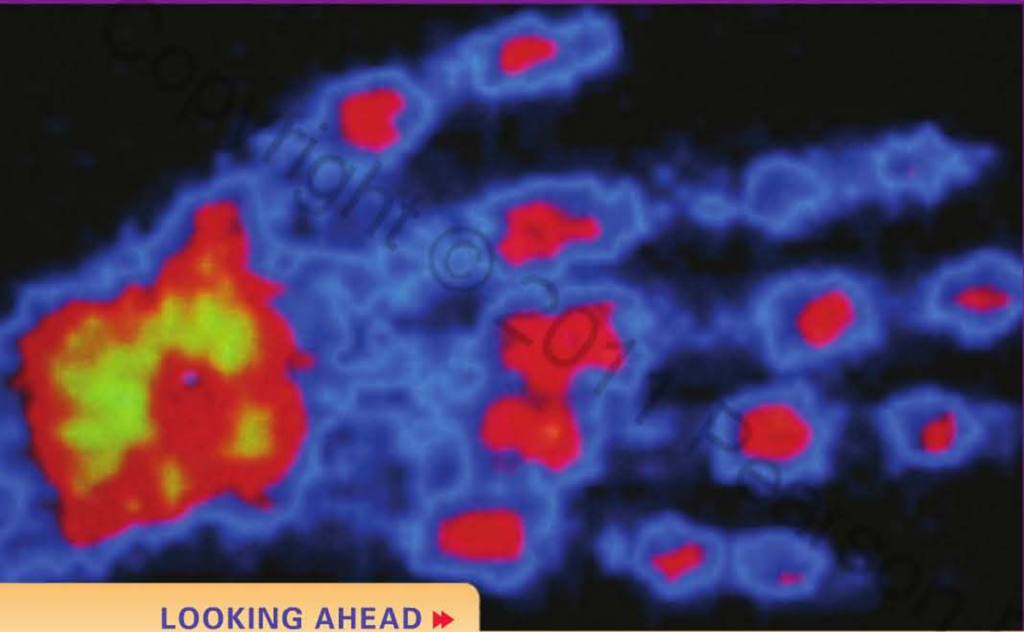
30 Nuclear Physics



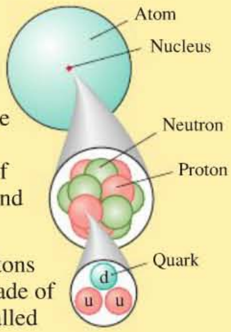
This is a *bone scan,* not an x ray. It was created using the radioactive decay of a particular type of nucleus inside the body. How can a process that occurs in the nucleus of an atom allow us to create an image of tissues of the human body?

LOOKING AHEAD

The goals of Chapter 30 are to understand the physics of the nucleus and some of the applications of nuclear physics.

Nuclear Structure

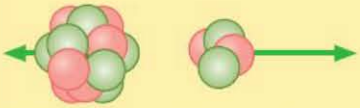
In this chapter, we'll look at the structure of the nucleus, the small positive core of the atom. The nucleus is made of positive protons and neutral neutrons.



Neutrons and protons are themselves made of smaller entities called **quarks.**

Nuclear Decay

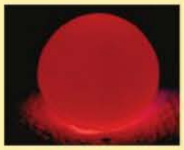
Certain nuclei **decay**, meaning they break into smaller pieces or otherwise change their structure.



We'll develop models to explain nuclear decays. The models are similar to those that explain electron transitions in atoms.

Looking Back
29.2 Models of the atom and the nucleus
29.6 Multielectron atoms

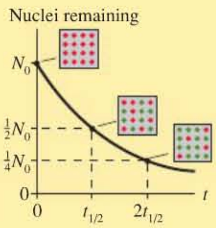
Nuclear transformations involve enormous energies.



Nuclear decay will keep this plutonium sphere glowing for decades.

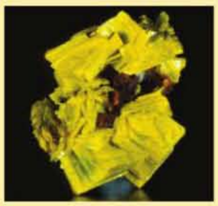
Looking Back
27.10 Relativistic energy

Nuclear decay is governed by a **half-life,** the time for half of the atoms in sample to decay.



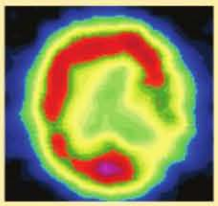
Nuclear Radiation

Nuclei decay by emitting high-energy charged particles or photons. This **nuclear radiation** comes in three main forms, corresponding to the three main types of decay.



The uranium in this mineral sample emits **alpha particles**—helium nuclei—as it decays.

The ghostly blue glow in the water around the reactor core is produced by **beta particles**—high-energy electrons—emitted by decaying nuclei.

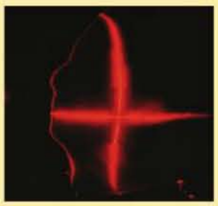


This image of the brain of a stroke patient was made with nuclei that decay by emitting **gamma rays**—high-energy photons. The area of reduced activity is clearly visible.

Applications

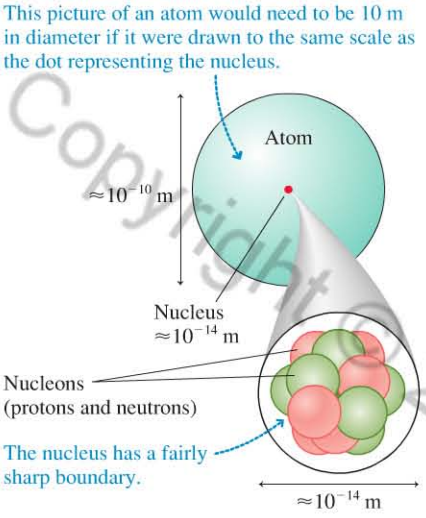
Nuclear physics is a practical subject with applications ranging from measuring ages to curing diseases.

Researchers measured the fraction of different carbon isotopes in charcoal from these cave paintings to determine that they were painted 30,000 years ago.



Radiation damages tissue; tumors are especially susceptible. Precisely targeted radiation can be used to treat cancer.

FIGURE 30.1 The nucleus is a tiny speck within an atom.



30.1 Nuclear Structure

For 29 chapters, we've made frequent references to properties of atoms that are due to the negative electrons surrounding the nucleus. In this final chapter it's time to dig deeper, to talk about the nucleus itself. In particular:

- What is nuclear matter? What are its properties?
- What holds the nucleus together? Why doesn't the repulsive electrostatic force blow it apart?
- What is the connection between the nucleus and radioactivity?

These were the questions asked by the pioneers of **nuclear physics,** the study of the properties of the atomic nucleus.

Let's review some information about the atom and the nucleus. The nucleus is a tiny speck in the center of a vastly larger atom. As FIGURE 30.1 shows, the nuclear diameter of roughly 10^{-14} m is only about 1/10,000 the diameter of the atom. What we call *matter* is overwhelmingly empty space!

The nucleus is composed of two types of particles: protons and neutrons. Together, these are referred to as **nucleons.** The role of the neutrons, which have nothing to do with keeping electrons in orbit, is an important issue that we'll address in this chapter. The number of protons Z is the element's atomic number. An element is identified by the number of protons in the nucleus, not by the number of orbiting electrons. Electrons are easily added and removed, forming negative and positive ions, but doing so doesn't change the element. The mass number A is defined to be $A = Z + N$, where N is the neutron number. The mass number is the total number of nucleons in a nucleus.

NOTE ► The mass number, which is dimensionless, is *not* the same thing as the atomic mass m. We'll look at actual atomic masses later. ◄

TABLE 30.1 Protons and neutrons

	Proton	Neutron
Number	Z	N
Charge q	$+e$	0
Spin	$\frac{1}{2}$	$\frac{1}{2}$
Mass, in u	1.00728	1.00866

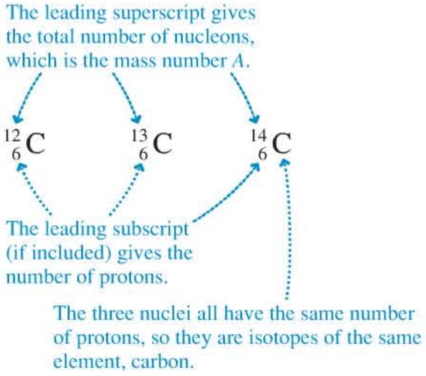
Protons and neutrons are virtually identical other than the fact that the proton has one unit of the fundamental charge e whereas the neutron is electrically neutral. The neutron is slightly more massive than the proton, but the difference is very small, only about 0.1%. Notice that the proton and neutron, like the electron, have an *inherent angular momentum* and magnetic moment with spin quantum number $s = \frac{1}{2}$. As a consequence, protons and neutrons obey the Pauli exclusion principle. Table 30.1 summarizes the basic properties of protons and neutrons.

Isotopes

As we learned in Chapter 29, not all atoms of the same element (and thus the same Z) have the same mass. There is a *range* of neutron numbers that happily form a nucleus with Z protons, creating a series of nuclei having the same Z-value (i.e., they are all the same chemical element) but different A-values. Each A-value in a series of nuclei with the same Z-value is called an *isotope.* Isotopes for some of the elements are given in a table in Appendix D.

The notation used to label isotopes uses the mass number A as a *leading* superscript, as shown in FIGURE 30.2. Hence ordinary carbon, which has six protons and six neutrons in the nucleus (and thus has $A = 12$), is written ^{12}C and pronounced "carbon twelve." The radioactive form of carbon used in carbon dating is ^{14}C. It has six protons, making it carbon, and eight neutrons, for a total of 14 nucleons. The isotope ^{2}H is a hydrogen atom in which the nucleus is not simply a proton but a proton and a neutron. Although the isotope is a form of hydrogen, it is called **deuterium.** Sometimes, for clarity, we will find it useful to include the atomic number as a leading *subscript.* Ordinary carbon is then written as $^{12}_{6}C$; deuterium as $^{2}_{1}H$.

FIGURE 30.2 Three isotopes of carbon.



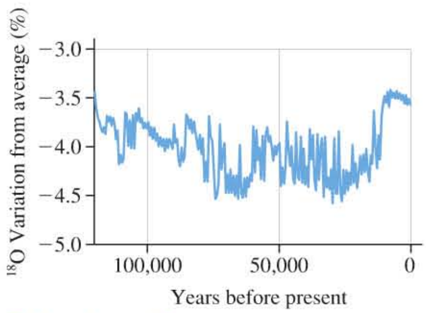
NOTE ► Adding the leading subscript doesn't provide any additional information. If the element is carbon, we know that it has 6 protons. But when we "balance" nuclear equations, we will find that the subscript is a useful tool and is worth including. ◄

The chemical behavior of an atom is largely determined by the orbiting electrons. Different isotopes of the same element have very similar *chemical* properties. ^{14}C will form the same chemical compounds as ^{12}C and will generally be treated the same by the body, a fact that permits the use of ^{14}C to determine the age of a sample. But the *nuclear* properties of these two isotopes are quite different, as we will see.

Most elements have multiple naturally occurring isotopes. For each element, the fraction of naturally occurring nuclei represented by one particular isotope is called the **natural abundance** of that isotope. For instance, oxygen has two primary isotopes, ^{16}O and ^{18}O. The data in Appendix D show that the natural abundance of ^{16}O is 99.76%, meaning that 9976 out of every 10,000 naturally occurring oxygen atoms are the isotope ^{16}O. Most of the remaining 0.24% of naturally occurring oxygen is the isotope ^{18}O, which has two extra neutrons.

The different masses of isotopes of an element can lead to some subtle differences in macroscopic behaviors. A water molecule made with the heavier ^{18}O isotope is slightly heavier as well, and will behave slightly differently than water made with the predominant ^{16}O isotope. Atmospheric water vapor is always deficient in ^{18}O compared to water in the ocean because the lighter molecules containing ^{16}O evaporate slightly more readily. Different atmospheric conditions lead to rain and snow (and thus lakes, rivers, and glaciers) with slightly different fractions of ^{18}O.

More than 3000 isotopes are known. The majority of these are **radioactive,** meaning that the nucleus is not stable but, after some period of time, will either fragment or emit some kind of subatomic particle in an effort to reach a more stable state. Many of these radioactive isotopes are created by nuclear reactions in the laboratory and have only a fleeting existence. Only 266 isotopes are **stable** (i.e., nonradioactive) and occur in nature. In addition, there are a handful of radioactive isotopes with such long decay times, measured in billions of years, that they also occur naturally.



Taking the earth's temperature When water vapor condenses to make snow crystals, the fraction of molecules containing ^{18}O is greater for snow that forms at higher atmospheric temperatures. Snow accumulating over tens of thousands of years has built up a thick ice sheet in Greenland. A core sample of this ice gives a record of the isotopic composition of the snow that fell over this time period. The above graph shows the isotopic composition of a single long core taken from the ice sheet, averaged over 50-year intervals; higher numbers correspond to higher temperatures. Broad trends, such as the increase in temperature at the end of the last ice age, are clearly seen.

Atomic Mass

You learned in Chapter 12 that atomic masses are specified in terms of the *atomic mass unit* u, defined such that the atomic mass of the isotope ^{12}C is exactly 12 u. The conversion to SI units is

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}$$

Alternatively, as we saw in Chapter 27, we can use Einstein's $E_0 = mc^2$ to express masses in terms of their energy equivalent. The energy equivalent of 1 u of mass is

$$\begin{aligned} E_0 &= (1.6605 \times 10^{-27} \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2 \\ &= 1.4924 \times 10^{-10} \text{ J} = 931.49 \text{ MeV} \end{aligned} \tag{30.1}$$

By noting that Einstein's formula implies $m = E_0/c^2$, we can write 1 u in the following form:

$$1 \text{ u} = \frac{E_0}{c^2} = 931.49 \left(\frac{\text{MeV}}{c^2}\right) \tag{30.2}$$

It may seem unusual, but the units MeV/c^2 are units of mass. This will be a useful unit for us when we need to compute energy equivalents. The energy equivalent of mass 1 MeV/c^2 is simply 1 MeV.

NOTE ► In the above equations, we have included more significant figures than usual. Many nuclear calculations look for the small difference between two masses that are almost the same. The two masses must be calculated or specified to four or five significant figures if their difference is to be meaningful. In calculations of nuclear energies, you should use the accurate values for nuclear masses given in Appendix D. ◄

Table 30.2 shows some important atomic mass values. Notice that the mass of a hydrogen atom is equal to the sum of the masses of a proton and an electron.

TABLE 30.2 Some atomic masses

Particle	Symbol	Mass (u)	Mass (MeV/c^2)
Electron	e	0.000549	0.51
Proton	p	1.007276	938.28
Neutron	n	1.008665	939.57
Hydrogen	^1H	1.007825	938.79
Helium	^4He	4.002602	3728.40

But a quick calculation shows that the mass of a helium atom (2 protons, 2 neutrons, and 2 electrons) is 0.03038 u *less* than the sum of the masses of its constituents. The difference is due to the *binding energy* of the nucleus, a topic we'll look at in Section 30.2.

The *chemical* atomic mass shown on the periodic table of the elements is the *weighted average* of the atomic masses of all naturally occurring isotopes. For example, chlorine has two stable isotopes: ^{35}Cl, with atomic mass $m = 34.97$ u, has an abundance of 75.8% and ^{37}Cl, at 36.97 u, has an abundance of 24.2%. The average, weighted by abundance, is $(0.758 \times 34.97\text{ u}) + (0.242 \times 36.97\text{ u}) = 35.45$ u. This is the value shown on the periodic table and is the correct value for most chemical calculations, but it is not the mass of any particular isotope of chlorine. Nuclear physics calculations involve the masses of specific isotopes, so you'll need to use these masses from Appendix D, not the chemical atomic masses given in the periodic table.

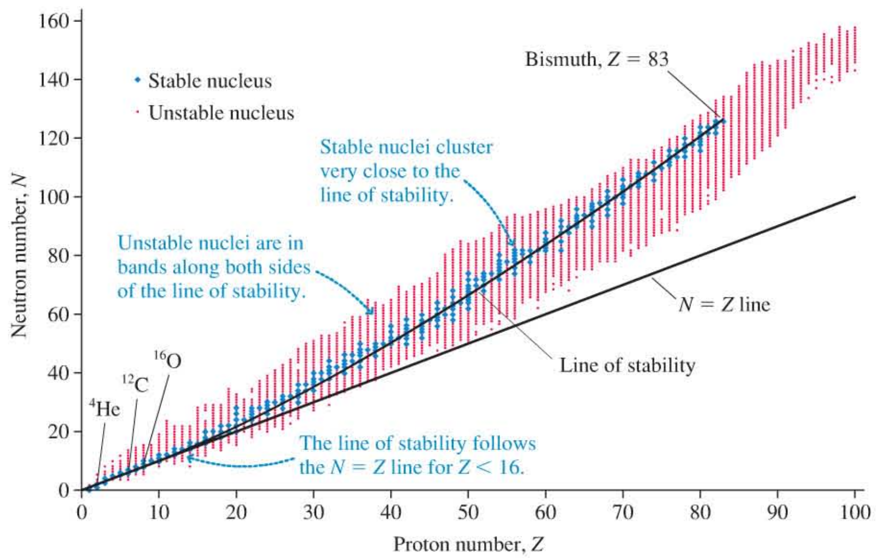
NOTE ► The atomic masses of the proton and the neutron are both ≈ 1 u. Consequently, the value of the mass number A is *approximately* the atomic mass in u. The approximation $m \approx A$ u is sufficient in many contexts, such as when we calculate speeds of gas molecules in Chapter 12. But in nuclear physics calculations we will need the more accurate mass values in Appendix D. ◄

STOP TO THINK 30.1 Three electrons orbit a neutral ^{6}Li atom. How many electrons orbit a neutral ^{7}Li atom?

30.2 Nuclear Stability

Because nuclei are characterized by two independent numbers, N and Z, it is useful to show the known nuclei on a plot of neutron number N versus proton number Z. **FIGURE 30.3** shows such a plot. Stable nuclei are represented by blue diamonds and unstable, radioactive nuclei by red dots.

FIGURE 30.3 Stable and unstable nuclei shown on a plot of neutron number N versus proton number Z.

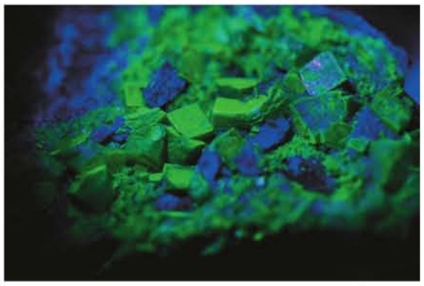


We can make several observations from this graph:

- The stable nuclei cluster very close to the curve called the **line of stability.**
- There are no stable nuclei with $Z > 83$ (bismuth). Heavier elements (up to $Z = 92$ (uranium)) are found in nature, but they are radioactive.

- Unstable nuclei are in bands along both sides of the line of stability.
- The lightest elements, with $Z < 16$, are stable when $N \approx Z$. The familiar isotopes ^{4}He, ^{12}C, and ^{16}O all have equal numbers of protons and neutrons.
- As Z increases, the number of neutrons needed for stability grows increasingly larger than the number of protons. The N/Z ratio is ≈ 1.2 at $Z = 40$ but has grown to ≈ 1.5 at $Z = 80$.

These observations—especially the fact that $N \approx Z$ for small Z but $N > Z$ for large Z—will be explained by the model of the nucleus that we'll explore in Section 30.3.



▶ **Unstable but ubiquitous uranium** All of the isotopes of uranium are unstable, but some have very long lives. Approximately half of the ^{238}U that was present at the formation of the earth 4.5 billion years ago is still around—and is all around you. Uranium is present at low concentrations in nearly all of the rocks and soil and water on the earth's surface. Much of the radiation that you are exposed to comes from this naturally occurring and widely distributed unstable element. Minerals with high concentrations of uranium are often fluorescent, as this sample of autunite ore photographed under ultraviolet light clearly shows.

Binding Energy

A nucleus is a *bound system.* That is, you would need to supply energy to disperse the nucleons by breaking the nuclear bonds between them. **FIGURE 30.4** shows this idea schematically.

You learned a similar idea in atomic physics. The energy levels of the hydrogen atom are negative numbers because the bound system has less energy than a free proton and electron. The energy you must supply to an atom to remove an electron is called the *ionization energy.*

In much the same way, the energy you would need to supply to a nucleus to disassemble it into individual protons and neutrons is called the **binding energy.** Whereas ionization energies of atoms are only a few eV, the binding energies of nuclei are tens or hundreds of MeV, energies large enough that their mass equivalent is not negligible.

We noted earlier that the mass of a helium atom is less than the mass of its constituents. Suppose we break a helium atom into two hydrogen atoms (taking account of the two protons and the two electrons) and two free neutrons as shown in **FIGURE 30.5**. The mass of the separated components is more than that of the helium atom. The difference in mass Δm arises from the energy that was put into the system to separate the tightly bound nucleons. The difference in masses is thus a measure of the binding energy:

$$\text{binding energy of helium nucleus} = \Delta m \cdot c^2$$

We can evaluate this energy B by converting the mass to MeV/c^2:

$$\begin{aligned} B &= \Delta m \cdot c^2 \\ &= \left((0.03038\ \text{u})\left(931.49\ \frac{\text{MeV}/c^2}{\text{u}}\right)\right)c^2 = (0.03038\ \text{u})(931.49\ \text{MeV/u}) \\ &= 28.30\ \text{MeV} \end{aligned}$$

Generally, the nuclear binding energy is computed by considering the mass difference between the atom and its separated components, Z hydrogen atoms and N neutrons:

$$B = (Zm_{\text{H}} + Nm_{\text{n}} - m_{\text{atom}}) \times (931.49\ \text{MeV/u}) \qquad (30.3)$$

Nuclear binding energy for an atom of mass m_{atom} with Z protons and N neutrons

FIGURE 30.4 The nuclear binding energy.

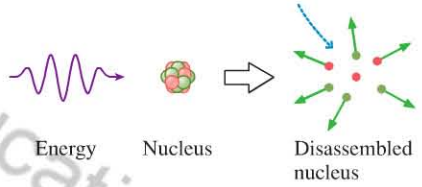
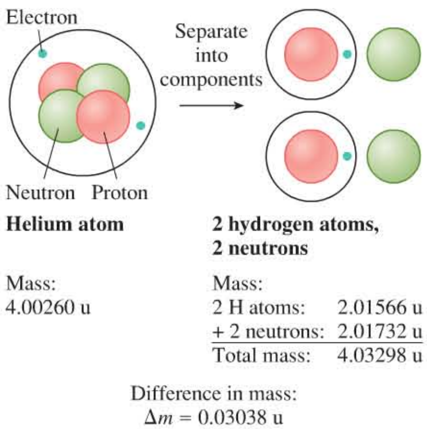


FIGURE 30.5 The binding energy of the helium nucleus.



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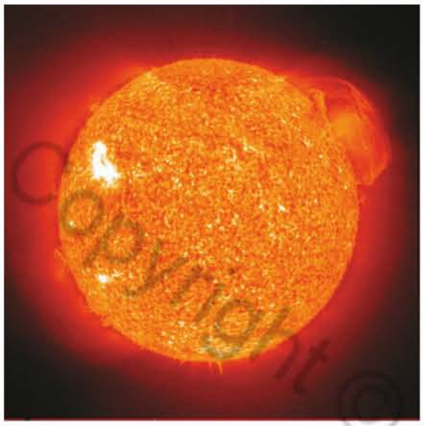


A nuclear fusion weight-loss plan The sun's energy comes from reactions that combine four hydrogen atoms to create a single atom of helium—a process called **nuclear fusion.** Because energy is released, the mass of the helium atom is less than that of the four hydrogen atoms. As the fusion reactions continue, the mass of the sun decreases—by 130 trillion tons per year! That's a lot of mass, but given the sun's enormous size, this change will amount to only a few hundredths of a percent of the sun's mass over its 10-billion-year lifetime.

19.3 ActivPhysics ONLINE

EXAMPLE 30.1 Finding the binding energy of iron

What is the nuclear binding energy of ^{56}Fe to the nearest MeV?

PREPARE Appendix D gives the atomic mass of ^{56}Fe as 55.934940 u. Iron has atomic number 26, so an atom of ^{56}Fe could be separated into 26 hydrogen atoms and 30 neutrons. The mass of the separated components is more than that of the iron nucleus; the difference gives us the binding energy.

SOLVE We solve for the binding energy using Equation 30.3. The masses of the hydrogen atom and the neutron are given in Table 30.2. We find

$$B = (26(1.007825\text{ u}) + 30(1.008665\text{ u}) - 55.934940\text{ u})(931.49\text{ MeV/u})$$
$$= (0.52846\text{ u})(931.49\text{ MeV/u}) = 492.26\text{ MeV} \simeq 492\text{ MeV}$$

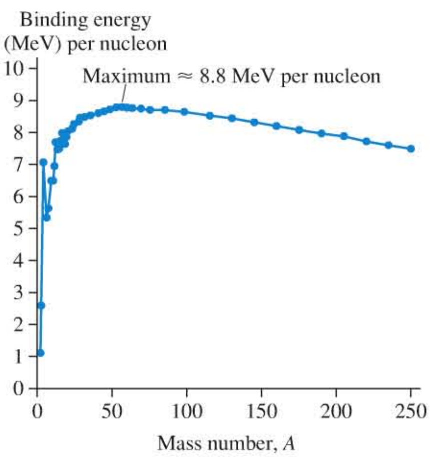
ASSESS The difference in mass between the nucleus and its components is a small fraction of the mass of the nucleus, so we must use several significant figures in our mass values. The mass difference is small—about half that of a proton—but the energy equivalent, the binding energy, is enormous.

How much energy is 492 MeV? To make a comparison with another energy value we have seen, the binding energy of a single iron nucleus is equivalent to the energy released in the metabolism of nearly *2 billion* molecules of ATP! The energy scale of nuclear processes is clearly quite different from that of chemical processes. In nuclear reactors, a small amount of nuclear "fuel" can produce a tremendous amount of energy.

As A increases, the nuclear binding energy increases, simply because there are more nuclear bonds. A more useful measure for comparing one nucleus to another is the quantity B/A, called the *binding energy per nucleon.* Iron, with $B = 492$ MeV and $A = 56$, has 8.79 MeV per nucleon. This is the amount of energy, on average, you would need to supply in order to remove *one* nucleon from the nucleus. Nuclei with larger values of B/A are more tightly held together than nuclei with smaller values of B/A.

FIGURE 30.6 is a graph of the binding energy per nucleon versus mass number A. The line connecting the points is often called the **curve of binding energy.**

FIGURE 30.6 The curve of binding energy for the stable nuclei.



For small values of A, adding more nucleons increases the binding energy per nucleon. If two light nuclei can be joined together to make a single, larger nucleus, the final nucleus will have a higher binding energy per nucleon. Because the final nucleus is more tightly bound, energy will be released in this *nuclear fusion* process. Nuclear fusion of hydrogen to helium is the basic reaction that powers the sun, as we have seen.

The curve of binding energy has a broad maximum at $A \approx 60$. Nuclei with $A > 60$ become less stable as their mass increases because adding nucleons *decreases* the binding energy per nucleon. *Alpha decay*, one of the three basic types of radioactive decay that we'll examine later in this chapter, occurs when a heavy nucleus becomes more stable by ejecting a small group of nucleons in order to decrease its mass, releasing energy in the process. The decrease in binding energy per nucleon as mass increases also explains why there are no stable nuclei beyond $Z = 83$.

A few very heavy nuclei, especially some isotopes of uranium and plutonium, are so unstable that they can be induced to fragment into two lighter nuclei in the process known as *nuclear fission*. For example, the collision of a slow-moving neutron with a ^{235}U nucleus causes the reaction

$$n + {}^{235}\text{U} \rightarrow {}^{236}\text{U} \rightarrow {}^{90}\text{Sr} + {}^{144}\text{Xe} + 2n \tag{30.4}$$

The ^{235}U nucleus absorbs the neutron to become ^{236}U, but ^{236}U is so unstable that it immediately fragments—in this case into a ^{90}Sr nucleus, a ^{144}Xe nucleus, and two neutrons. The less massive ^{90}Sr and ^{144}Xe nuclei are more tightly bound than the original ^{235}U nucleus, so a great deal of energy is released in this reaction. But the

^{90}Sr and ^{144}Xe nuclei *aren't* stable. As we've seen, nuclei with lower values of Z have relatively smaller numbers of neutrons, meaning there will be neutrons "left over" after the reaction. Equation 30.4 shows some free neutrons among the reaction products, but the two nuclear fragments have "extra" neutrons as well—they have too many neutrons and will be unstable. This is generally true for the products of a fission reaction. The fact that the waste products of nuclear fission are radioactive has important consequences for the use of nuclear fission as a source of energy.

▶ **A formidable chain reaction** For certain isotopes of uranium or plutonium, if the nucleus is struck by a neutron, it can split into two more tightly bound fragments, releasing tremendous energy. Extra neutrons are left over after the split, each of which can cause the fission of another nucleus—releasing more neutrons, which can produce further reactions. The net result is a nuclear fission **chain reaction.** A controlled reaction is the energy source of a nuclear power plant. An uncontrolled reaction is responsible for the terrible destructive power of a nuclear explosion, shown here in an above-ground test in Nevada in 1957.

30.3 Forces and Energy in the Nucleus

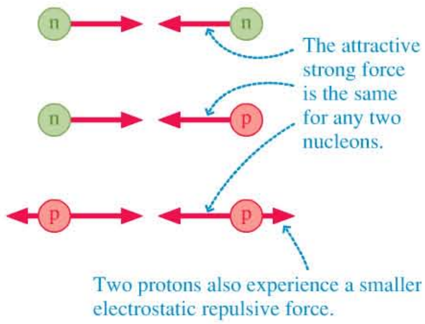
The nucleus of the atom is made of protons, which are positively charged, and neutrons, which have no charge. Why doesn't the repulsive force of the protons simply cause the nucleus to fly apart? This was a puzzle faced by early investigators. It soon became clear that a previously unknown force of nature operates within the nucleus to hold the nucleons together. This new force has to be stronger than the repulsive electrostatic force; hence it was named the **strong nuclear force,** or just the strong force.

The strong force has four important properties:

1. It is an *attractive* force between any two nucleons.
2. It does not act on electrons.
3. It is a *short-range* force, acting only over nuclear distances. We see no evidence for nuclear forces outside the nucleus.
4. Over the range where it acts, it is *stronger* than the electrostatic force that tries to push two protons apart.

FIGURE 30.7 summarizes the three types of interactions that take place within the nucleus. Many decades of research have shown that the strong force between two nucleons is independent of whether they are protons or neutrons. Charge is the basis for electromagnetic interactions, but it is of no relevance to the strong force. Protons and neutrons are identical as far as nuclear forces are concerned.

FIGURE 30.7 Forces between pairs of particles in the nucleus.



Protons throughout the nucleus exert repulsive electrostatic forces on each other, but, because of the short range of the strong force, a proton feels an attractive force only from the very few other protons with which it is in close contact. A nucleus with too many protons will be unstable because the repulsive electrostatic forces will overcome the attractive strong forces. Because neutrons participate in the strong force but exert no repulsive forces, **the neutrons provide the extra "glue" that holds the nucleus together.** In small nuclei, one neutron per proton is sufficient for stability. Hence small nuclei have $N \approx Z$. But as the nucleus grows, the repulsive force increases faster than the binding energy. More neutrons are needed for stability, so heavy nuclei have $N > Z$.

In Chapter 28 we learned that confining a particle to a region of space results in the quantization of energy. In Chapter 29 you saw the consequences of this quantization for electrons bound in atoms; in this chapter we will look at the quantized energy levels for protons and neutrons bound in the nucleus. Just as we did with atoms, we can "build up" the nuclear state by placing all the nucleons in the lowest energy levels consistent with the Pauli principle, which applies to nucleons just as it did to electrons. Each energy level can hold only a certain number of spin-up particles and spin-down particles, depending on the quantum numbers; additional nucleons must go into higher energy levels.

Although there is a great deal of similarity between the descriptions of nucleon energy levels in the nucleus and electron energy levels in the atom, there is one important difference: the energy scale. The electron energy levels are typically separated by a few eV, but the proton and neutron energy levels are separated by a few MeV—a million times as much.

Low-*Z* Nuclei

As our first example of an energy-level description of the nucleus, we'll consider the energy levels of low-*Z* nuclei ($Z < 8$). Because these nuclei have so few protons, we can neglect the electrostatic potential energy due to proton-proton repulsion and consider only the much larger nuclear potential energy. In that case, the energy levels of the protons and neutrons are essentially identical.

FIGURE 30.8 shows the three lowest allowed energy levels for protons and neutrons and the maximum number of protons and neutrons the Pauli principle allows in each. Energy values vary from nucleus to nucleus, but the spacing between these levels is several MeV.

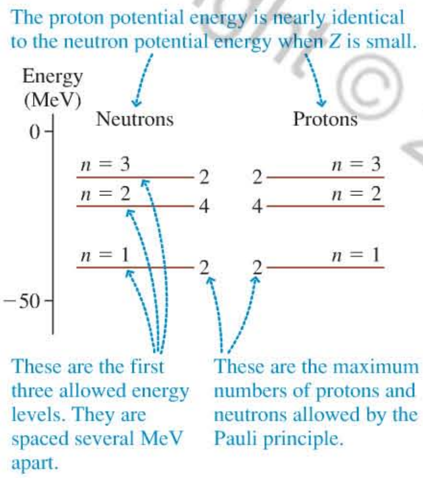


FIGURE 30.8 The three lowest energy levels of a low-*Z* nucleus. The neutron energy levels are on the left, the proton energy levels on the right.

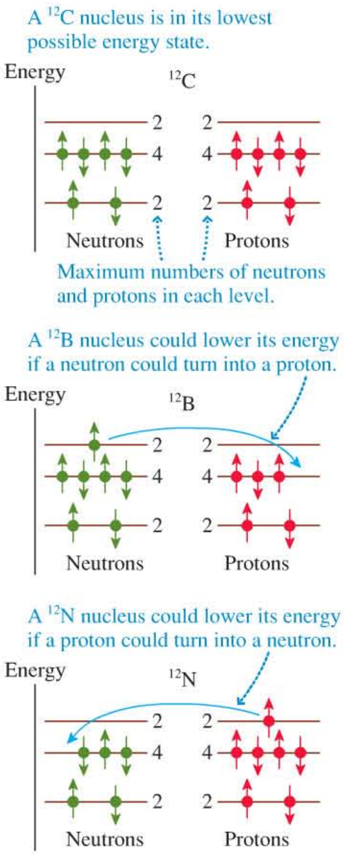
Suppose we look at a series of nuclei, all with $A = 12$ but with different numbers of protons and neutrons: ^{12}B, ^{12}C, ^{12}N. All have 12 nucleons, but only ^{12}C is a stable isotope; the other two are not. Why do we see this difference in stability?

FIGURE 30.9 shows the energy-level diagrams of ^{12}B, ^{12}C, and ^{12}N. Look first at ^{12}C, a nucleus with six protons and six neutrons. You can see that exactly six protons are allowed in the $n = 1$ and $n = 2$ proton energy levels. The same is true for the six neutrons. No other arrangement of the nucleons would lower the total energy, so this nucleus is stable.

^{12}B has five protons and seven neutrons. The sixth neutron fills the $n = 2$ neutron energy level, so the seventh neutron has to go into the $n = 3$ energy level. (This is just like the third electron in Li having to go into the $n = 2$ energy level because the first two electrons have filled the $n = 1$ energy level.) The $n = 2$ proton energy level has one vacancy because there are only five protons.

In atoms, electrons in higher energy levels move to lower energy levels by emitting a photon as the electron undergoes a quantum jump. That can't happen here because the higher-energy nucleon in ^{12}B is a neutron whereas the vacant lower energy level is that of a proton. But an analogous process could occur *if* a neutron could somehow turn into a proton, allowing it to move to a lower energy level. ^{12}N is just the opposite, with the seventh proton in the $n = 3$ energy level. If a proton could somehow turn into a neutron, it could move to a lower energy level, lowering the energy of the nucleus.

FIGURE 30.9 Nuclear energy-level diagrams of ^{12}B, ^{12}C, and ^{12}N.



You can see from the diagrams that the ^{12}B and ^{12}N nuclei have significantly more energy—by several MeV—than ^{12}C. If a neutron could turn into a proton, and vice versa, these nuclei could move to a lower-energy state—that of ^{12}C. In fact, that's exactly what happens! These nuclei are not stable. We'll explore the details in Section 30.4. Both ^{12}B and ^{12}N decay into the more stable ^{12}C in the process known as *beta decay*.

High-*Z* Nuclei

We can use the energy levels of protons and neutrons in the nucleus to give a qualitative explanation for one more observation. **FIGURE 30.10** shows the neutron and proton energy levels of a high-*Z* nucleus. In a nucleus with many protons, the increasing electrostatic potential energy raises the proton energy levels but not the neutron energy levels. Protons and neutrons now have a different set of energy levels.

As a nucleus is "built," by the adding of protons and neutrons, the proton energy levels and the neutron energy levels must fill to just about the same height. If there were neutrons in energy levels above vacant proton levels, the nucleus would lower its energy by changing neutrons into protons. Similarly, a proton would turn into a neutron if there were a vacant neutron energy level beneath a filled proton level. **The**

net result is that the filled levels for protons and neutrons are at just about the same height.

Because the neutron energy levels start at a lower energy, *more neutron states* are available than proton states. Consequently, a high-Z nucleus will have more neutrons than protons. This conclusion is consistent with our observation in Figure 30.3 that $N > Z$ for heavy nuclei.

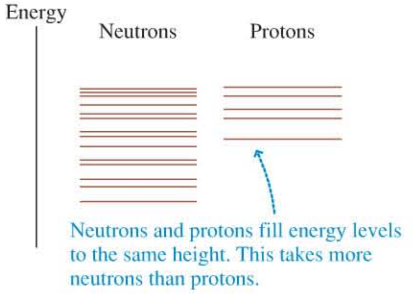


FIGURE 30.10 The proton energy levels are displaced upward in a high-Z nucleus because of the electric potential energy.

STOP TO THINK 30.2 Based on the model of nuclear energy levels and transitions you have seen, would you expect ^{13}C to be stable?

30.4 Radiation and Radioactivity

Some nuclei are unstable—they can decay—but what exactly does this mean? How do we know that they decay? What is left over when a nucleus decays?

The existence of nuclear decay was discovered in the 1890s by investigators who noticed high-energy emissions from certain atoms. Different nuclei emitted three different types of particle or ray, differentiated by their behavior in a magnetic field, as shown in **FIGURE 30.11**. Experiments to determine the nature of the emissions showed that **alpha particles** are helium nuclei and **beta particles** are electrons. **Gamma rays** were found to be similar to x rays, high-energy electromagnetic waves best described as photons. Table 30.3 summarizes the properties of alpha and beta particles and gamma rays.

TABLE 30.3 Three types of radiation

Radiation	Identification	Charge
Alpha, α	^{4}He nucleus	$+2e$
Beta, β	Electron	$-e$
Gamma, γ	High-energy photon	0

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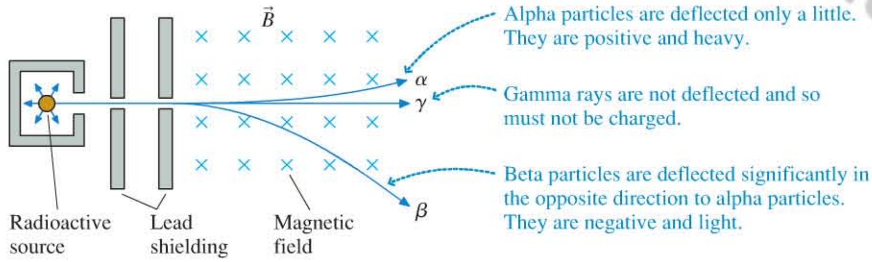
FIGURE 30.11 Identifying radiation by its deflection in a magnetic field.



We now define *radioactivity* or *radioactive decay* to be the spontaneous emission of particles or high-energy photons from unstable nuclei as they decay from higher-energy to lower-energy states. In this section, we will explore the decay mechanisms that result in the three different types of radiation, considering the changes in the nuclei that produce these decays.

Alpha Decay

We know that many large nuclei are unstable; it is energetically favorable for them to spontaneously break apart into smaller fragments. A combination of two neutrons and two protons, a ^{4}He nucleus, is an especially stable nuclear combination. When a large nucleus spontaneously decays by breaking into two smaller fragments, one of these fragments is almost always a helium nucleus—an alpha particle, symbolized by α.

An unstable nucleus that ejects an alpha particle loses two protons and two neutrons, so we can write this decay as

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + \alpha + \text{energy} \qquad (30.5)$$

Alpha decay of a nucleus

FIGURE 30.12 Alpha decay.

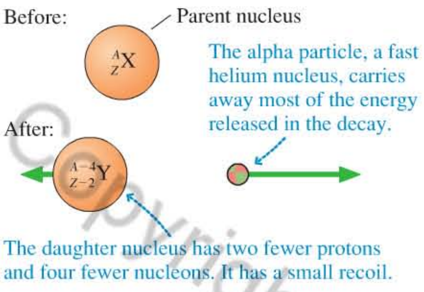


FIGURE 30.12 shows the alpha-decay process. The original nucleus X is called the **parent nucleus,** and the decay-product nucleus Y is the **daughter nucleus.**

Energy conservation tells us that an alpha decay can occur only when the mass of the parent nucleus is greater than the mass of the daughter nucleus plus the mass of the alpha particle. This requirement is often met for heavy, high-Z nuclei beyond the maximum of the curve of binding energy of Figure 30.6. It is energetically favorable for these nuclei to eject an alpha particle because the daughter nucleus is more tightly bound than the parent nucleus.

The daughter nucleus, which is much more massive than an alpha particle, undergoes only a slight recoil, as we see in Figure 30.12. Consequently, **the energy released in an alpha decay ends up mostly as the kinetic energy of the alpha particle.** We can compute this energy by looking at the mass energy difference between the initial and final states:

$$K_\alpha \approx \Delta E = (m_X - m_Y - m_{He})c^2 \tag{30.6}$$

EXAMPLE 30.2 **Analyzing alpha decay in a smoke detector**

Americium, atomic number 95, doesn't exist in nature; it is produced in nuclear reactors. An isotope of americium, ^{241}Am, is part of the sensing circuit in most smoke detectors. ^{241}Am decays by emitting an alpha particle. What is the daughter nucleus?

SOLVE Equation 30.5 shows that an alpha decay causes the atomic number to decrease by 2 and the atomic weight by 4. Let's write an equation for the decay showing the alpha particle as a helium nucleus, including the atomic weight superscript and the atomic number subscript for each element. There is no change in the total number of neutrons or protons, so the subscripts and superscripts must "balance" in the reaction:

$$^{241}_{95}\text{Am} \rightarrow {}^{237}_{93}? + {}^{4}_{2}\text{He} + \text{energy}$$

A quick glance at the periodic table reveals the unknown element in this equation, the daughter nucleus, to be an isotope of neptunium, $^{237}_{93}$Np.

ASSESS Balancing the two sides of the above reaction is similar to balancing the equation for a chemical reaction.

EXAMPLE 30.3 **Finding the energy of an emitted alpha particle**

The uranium isotope ^{238}U undergoes alpha decay to an isotope of thorium, ^{234}Th. What is the kinetic energy, in MeV, of the alpha particle?

PREPARE The decay products have less mass than the initial nucleus. This difference in mass is released as energy, most of which goes to the kinetic energy of the alpha particle. Because the energy of the alpha particle is only approximately equal to the reaction energy, we needn't use the full accuracy that the values in Appendix D provides. To 4 decimal places, the atomic mass of ^{238}U is 238.0508 u, that of ^{234}Th is 234.0436 u and that of ^{4}He—the alpha particle—is 4.0026 u.

SOLVE We can calculate the kinetic energy of the alpha particle using Equation 30.6:

$$K_\alpha = (238.0508\ \text{u} - 234.0436\ \text{u} - 4.0026\ \text{u})c^2$$
$$= (0.0046\ \text{u})c^2$$

If we convert from u to MeV/c^2, using the conversion factor 1 u = 931.49 MeV/c^2, the c^2 cancels, and we end up with

$$K_\alpha = \left(0.0046\ \text{u} \times \frac{931.49\ \text{MeV}/c^2}{1\ \text{u}}\right)c^2 = 4.3\ \text{MeV}$$

ASSESS This is a typical alpha-particle energy, corresponding to a speed of about 5% of the speed of light. Notice that with a careful use of conversion factors we never had to evaluate c^2.

Beta Decay

We identified beta decay as the emission of an electron e^-, the beta particle. A typical example of beta decay occurs in the carbon isotope ^{14}C, which undergoes the beta-decay process

$$^{14}\text{C} \rightarrow {}^{14}\text{N} + e^-$$

Carbon has $Z = 6$ and nitrogen has $Z = 7$. Because Z increases by 1 but A doesn't change, it appears that a neutron within the nucleus has changed itself into a proton by emitting an electron. That is, the basic decay process appears to be

$$\text{n} \rightarrow p + e^- \tag{30.7}$$

The electron is ejected from the nucleus but the proton is not. Thus the beta-decay process, shown in **FIGURE 30.13a**, is

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + \mathrm{e}^{-} + \text{energy} \tag{30.8}$$

Beta-minus decay of a nucleus

Do neutrons *really* turn into protons? It turns out that a free neutron—one not bound in a nucleus—is *not* a stable particle. It decays into a proton and an electron, with a half-life of approximately 10 minutes. This decay conserves energy because $m_n > m_p + m_e$. Furthermore, it conserves charge.

Whether a neutron *within* a nucleus can decay depends not only on the masses of the neutron and proton but also on the masses of the parent and daughter nuclei, because energy has to be conserved for the entire nuclear system. **Beta decay occurs only if** $m_X > m_Y$. ^{14}C can undergo beta decay to ^{14}N because $m(^{14}\mathrm{C}) > m(^{14}\mathrm{N})$. But $m(^{12}\mathrm{C}) < m(^{12}\mathrm{N})$, so ^{12}C is stable and its neutrons will not decay.

A few nuclei undergo a slightly different form of beta decay by emitting a *positron*. A positron, for which we use the symbol e^+, is identical to an electron except that it has a positive charge. As we saw in Chapter 27, the positron is the *antiparticle* of the electron. To distinguish between these two forms of decay, we call the emission of an electron *beta-minus decay* and the emission of a positron *beta-plus decay*.

Inside a nucleus undergoing beta-plus decay, a proton changes into a neutron and a positron:

$$ \mathrm{p}^{+} \rightarrow \mathrm{n} + \mathrm{e}^{+} \tag{30.9}$$

The full decay process, shown in **FIGURE 30.13b**, is

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + \mathrm{e}^{+} + \text{energy} \tag{30.10}$$

Beta-plus decay of a nucleus

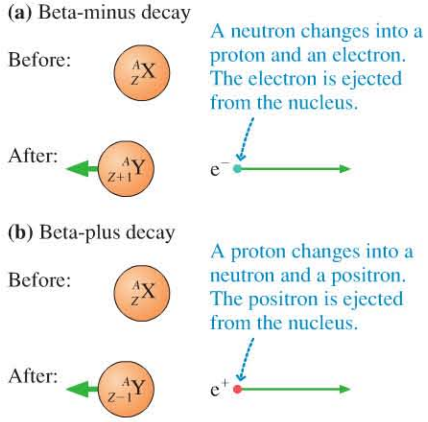
Beta-plus decay does *not* happen for a free proton because $m_p < m_n$. It *can* happen within a nucleus as long as energy is conserved for the entire nuclear system, but it is far less common than beta-minus decay.

In our earlier discussion of Section 30.3 we noted that the ^{12}B and ^{12}N nuclei could reach a lower-energy state if a proton could change into a neutron, and vice versa. Now we see that such changes can occur if the energy conditions are favorable. And, indeed, ^{12}B undergoes beta-minus decay to ^{12}C, while ^{12}N undergoes beta-plus decay to ^{12}C.

In general, beta decay is a process of nuclei with too many neutrons or too many protons that moves them closer to the line of stability in Figure 30.3.

NOTE ► The electron emitted in beta decay has nothing to do with the atom's valence electrons. The beta particle is created in the nucleus and ejected directly from the nucleus when a neutron is transformed into a proton and an electron. ◄

FIGURE 30.13 Beta decay.



EXAMPLE 30.4 Analyzing beta decay in the human body

Your body contains several radioactive isotopes. Approximately 20% of the radiation dose you receive each year comes from the radioactive decay of these atoms. Most of this dose comes from one potassium isotope, ^{40}K, which decays by beta-minus emission. What is the daughter nucleus?

SOLVE Rewriting Equation 30.8 as

$$ {}^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{20}? + \mathrm{e}^{-} + \text{energy} $$

we see that the daughter nucleus must be the calcium isotope ${}^{40}_{20}$Ca.

FIGURE 30.14 Gamma decay.

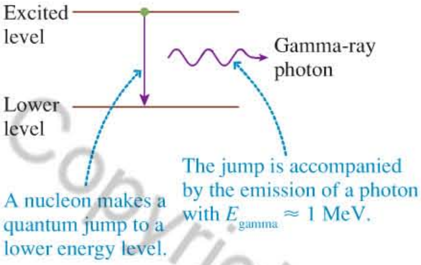
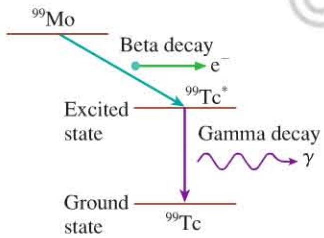


FIGURE 30.15 $^{99}Tc^*$, a gamma emitter, is produced in the beta decay of ^{99}Mo.



Gamma Decay

Gamma decay is similar to quantum processes you saw in earlier chapters. In Chapter 28, you learned that an atomic system can emit a photon with $E_{photon} = \Delta E_{atom}$ when an electron undergoes a quantum jump from an excited energy level to a lower energy level. Nuclei are no different. A proton or a neutron in an excited nuclear state, such as the one shown in FIGURE 30.14, can undergo a quantum jump to a lower-energy state by emitting a high-energy photon. This is the gamma-decay process.

The spacing between atomic energy levels is only a few eV. Nuclear energy levels, by contrast, are on the order of 1 MeV apart, meaning gamma-ray photons will have energies $E_{gamma} \approx 1$ MeV. Photons with this much energy are quite penetrating and deposit an extremely large amount of energy at the point where they are finally absorbed.

Nuclei left to themselves are usually in their ground states and thus cannot emit gamma-ray photons. However, alpha and beta decay often leave the daughter nucleus in an excited nuclear state, so gamma emission is often found to accompany alpha and beta emission.

Let's look at an example. One of the most important isotopes for medical imaging is ^{99}Tc, an isotope of the element technetium. An excited state of ^{99}Tc is produced in the beta decay of the molybdenum isotope ^{99}Mo:

$$^{99}\text{Mo} \rightarrow {}^{99}\text{Tc}^* + \text{e}^- + \text{energy}$$

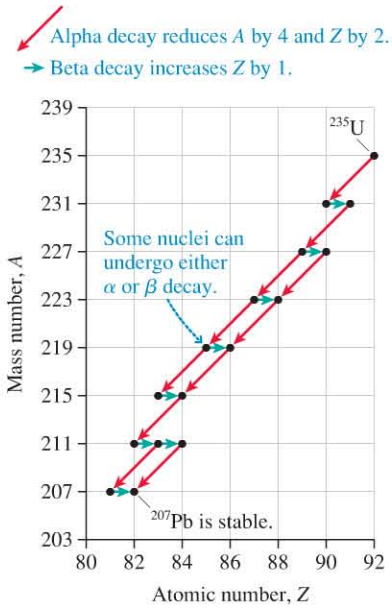
The asterisk signifies that the technetium nucleus is in an excited state. The excited nucleus then makes a transition to a lower-energy state via the emission of a 140 keV gamma ray:

$$^{99}\text{Tc}^* \rightarrow {}^{99}\text{Tc} + \gamma$$

The full decay process is shown in FIGURE 30.15. The final state of the technetium nucleus is much more stable than the excited state.

◀ **Isotopes on demand** BIO Many hospitals keep a radioactive molybdenum ^{99}Mo source on hand. The ^{99}Mo undergoes beta decay, but it's not the energy or the electron released in the decay that the hospital wants—it's the technetium daughter nucleus, ^{99}Tc, which is used in medical imaging procedures. ^{99}Tc decays very rapidly, so hospitals that use it must produce it on site. This happens in a radioactive "cow," a column with chemically bound ^{99}Mo. The column is inside a shielded enclosure, which protects the technician from radiation as she extracts the ^{99}Tc produced in the decay of the ^{99}Mo source.

FIGURE 30.16 ^{235}U decay series.



Decay Series

A radioactive nucleus decays into a daughter nucleus. In many cases, the daughter nucleus is also radioactive and decays to produce its own daughter nucleus. The process continues until reaching a daughter nucleus that is stable. The sequence of isotopes, starting with the original unstable isotope and ending with the stable isotope, is called a **decay series.**

The elements with $Z > 83$ present in the earth's crust are part of the decay series of a few long-lived isotopes of uranium and thorium. As an example, FIGURE 30.16 shows the decay series of ^{235}U, an isotope of uranium with a 700-million-year half-life. This is a very long time, but it is only about 15% the age of the earth, so most—but not all—of the ^{235}U nuclei present when the earth was formed have now decayed. There are many unstable nuclei in the decay series. Ultimately, all ^{235}U nuclei end as the ^{207}Pb isotope of lead, a stable nucleus.

Notice that some nuclei can decay by either alpha *or* beta decay. Thus there are a variety of paths that a decay can follow, but they all end at the same point.

Nuclear Radiation Is a Form of Ionizing Radiation

The energies of the alpha and beta particles and the gamma-ray photons of nuclear decay are typically in the range 0.1–10 MeV. These energies are much higher than the ionization energies of atoms and molecules, which, as we saw in Chapter 25, are

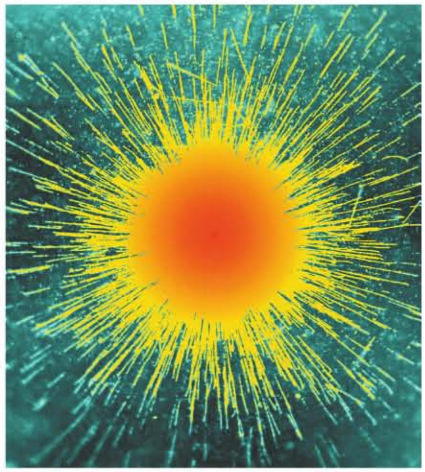
$\approx$10 eV. When the sun shines on your skin, it warms it; the low-energy photons are absorbed, their energy converted to thermal energy. The much higher energies of alpha and beta particles and gamma rays cause them to interact very differently with matter, *ionizing* atoms and *breaking* molecular bonds, leaving long trails of ionized atoms and molecules behind them before finally stopping. A particle with 1 MeV of kinetic energy can ionize $\approx$100,000 atoms or molecules. Alpha and beta particles and gamma rays are, like x rays, examples of ionizing radiation.

Ionization is the basis for the **Geiger counter,** the most well-known detector of nuclear radiation. **FIGURE 30.17** shows how a Geiger counter detects the passage of a beta particle. A Geiger counter, like other nuclear radiation detectors, measures only *ionizing radiation*.

When ionizing radiation enters the body, it causes damage in two ways. First, the ions drive chemical reactions that wouldn't otherwise occur. These reactions may damage the machinery of cells. Very large doses of ionizing radiation upset the delicate ionic balance that drives cellular transport, and can rapidly lead to cell death. For this reason, large doses of penetrating gamma rays are sometimes used to sterilize medical equipment inside and out.

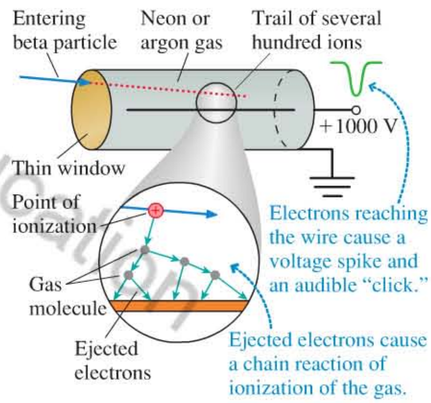
Second, ionizing radiation can damage DNA molecules by ionizing them and breaking bonds. If the damage is extensive, cellular repair mechanisms will not be able to cope, and the DNA will be permanently damaged, possibly creating a mutation or a tumor. Tissues with rapidly proliferating cells, such as bone marrow, are quite sensitive to ionizing radiation. Those with less-active cell reproduction, such as the nervous system, are much less sensitive.

NOTE ► Ionizing radiation causes damage to materials and tissues, but **objects irradiated with alpha, beta, or gamma radiation do not become radioactive.** Ionization drives chemical processes involving the electrons. An object could become radioactive only if its nuclei were somehow changed, and that does not happen. ◄



A speck of radium placed on a photographic plate emits alpha particles that leave clearly visible ionization trails.

FIGURE 30.17 The operation of a Geiger counter.



STOP TO THINK 30.3 The cobalt isotope ^{60}Co ($Z = 27$) decays to the nickel isotope ^{60}Ni ($Z = 28$). The decay process is

A. Alpha decay.
B. Beta-minus decay.
C. Beta-plus decay.
D. Gamma decay.

30.5 Nuclear Decay and Half-Lives

The decay of nuclei is different from other types of decay you are familiar with. A tree branch that falls to the forest floor decays. It darkens, becomes soft, and crumbles. You might be able to tell, just by looking at it, about how long it had been decaying. Nuclear decay is different. The nucleus doesn't "age" in any sense. Instead, a nucleus has a certain probability that, within the next second, it will spontaneously turn into a different nucleus and, in the process, eject an alpha or beta particle or a gamma ray.

We can use an analogy here: If you toss a coin, it always has a 50% probability of showing tails, no matter what previous tosses might have been. You might "expect" heads if you've tossed 10 tails in a row, but the 11th toss still has a 50% chance of coming up tails. Likewise with nuclei. If a nucleus doesn't decay in this second, it is no more or less likely to decay in the next. The nucleus remains just as it was, without any change, until the decay finally occurs. An "old" nucleus is identical to a "young" one.

In fact, the mathematics of radioactive decay is the same as that of tossing coins. Suppose you have a large number N_0 of coins. You toss them all and then keep those that come up heads while setting aside those that come up tails. Probability dictates that about half the coins will show tails and be set aside. Now you repeat the process over and over. With each subsequent toss, about half the coins are set aside—they "decay."

After the first toss, the number of coins you have left is about $(1/2)N_0$ because you set aside about half the coins. After the second toss, when you set aside about half of that half, the number of remaining coins is about $(1/2) \times (1/2)N_0$, or $(1/2)^2N_0 = N_0/4$. Half of these coins will be set aside in the third toss, leaving you with $(1/2)^3N_0$, or 1/8 of what you started with. After m tosses—assuming you started with a very large number of coins—the number of coins left is $N = (1/2)^m N_0$.

Similarly, if you start with N_0 unstable nuclei, after an interval of time we call one *half-life*, you'll find that you have $N = (1/2)N_0$ nuclei remaining. The **half-life** $t_{1/2}$ is the average time required for one-half the nuclei to decay. This process continues, with one-half the remaining nuclei decaying in each successive half-life. The number N nuclei remaining at time t is

$$N = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}} \tag{30.11}$$

Decay of nuclei in a radioactive sample in terms of half-life

Thus $N = N_0/2$ at $t = t_{1/2}$, $N = N_0/4$ at $t = 2t_{1/2}$, $N = N_0/8$ at $t = 3t_{1/2}$, and so on, with the ratio $t/t_{1/2}$ playing the role of the "number of tosses." **No matter how many nuclei there are at any point in time, the number decays by half during the next half-life.**

NOTE ► Each isotope that is unstable and decays has a characteristic half-life, which can range from a fraction of a second to billions of years. Appendix D provides nuclear data and half-lives for the isotopes referred to in this textbook. ◄

FIGURE 30.18 Half the nuclei decay during each half-life.

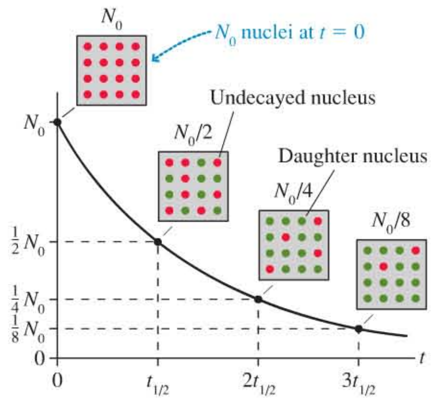
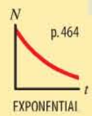


FIGURE 30.18 is a graph of the number of nuclei remaining after time t. This figure conveys two important ideas:

1. Nuclei don't vanish when they decay. The decayed nuclei have merely become some other kind of nuclei.
2. The decay process is random. We can predict that half the nuclei will decay in one half-life, but we can't predict which ones.

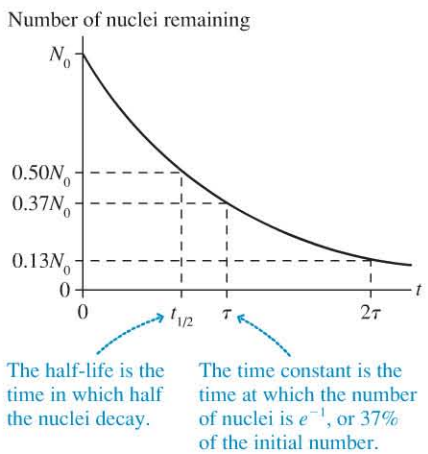
The graph has a form we have seen before: exponential decay. Mathematically, the number of nuclei remaining in a radioactive sample at time t is

$$N = N_0 e^{-t/\tau} \tag{30.12}$$



Exponential decay of nuclei in a radioactive sample

FIGURE 30.19 The number of radioactive atoms decreases exponentially with time.



where τ is the *time constant* for the decay. The exponential decay of the number of nuclei is analogous to the exponential decay of the capacitor voltage in an *RC* circuit or the amplitude of a damped harmonic oscillator.

FIGURE 30.19 is a graphical representation of Equation 30.12; it is the same graph as that of Figure 30.18, simply written in a different mathematical form. The number of radioactive nuclei decreases from N_0 at $t = 0$ to $N = N_0e^{-1} = 0.37N_0$ at time $t = \tau$. In practical terms, the number decreases by roughly two-thirds during one time constant.

NOTE ► There is no natural "starting time" for an exponential decay; you can choose any instant you wish to be $t = 0$. The number of radioactive nuclei present at that instant is N_0. If at one instant you have 10,000 radioactive nuclei whose time constant is $\tau = 10$ min, you'll have roughly 3700 nuclei 10 min later. The fact that you may have had more than 10,000 nuclei at an earlier time isn't relevant. ◄

We can relate the half-life to the time constant τ because we know, by definition, that $N = N_0/2$ at $t = t_{1/2}$. Thus, according to Equation 30.12,

$$\frac{N_0}{2} = N_0 e^{-t_{1/2}/\tau}$$

The N_0 cancels, and we can then take the natural logarithm of both sides to find

$$\ln\left(\frac{1}{2}\right) = -\ln 2 = -\frac{t_{1/2}}{\tau}$$

With one final rearrangement we have

$$t_{1/2} = \tau \ln 2 = 0.693\tau \tag{30.13}$$

That is, the half-life of a nuclear decay is 69.3% of the time constant for the decay. Whether we use Equation 30.11 and the half-life or Equation 30.12 and the time constant is a matter of convenience; both equations describe the same decay.

EXAMPLE 30.5 Determining the decay of radioactive iodine

Patients with Graves disease have an overactive thyroid gland. A common treatment uses radioactive iodine, which is taken up by the thyroid. The radiation emitted in its decay will damage the tissues of the gland. A single pill is produced with 4.0×10^{14} atoms of the isotope ^{131}I, which has a half-life of 8.0 days.

a. How many atoms remain 24 hours after the pill's creation, when the pill is delivered to a hospital?

b. Although the iodine in the pill is constantly decaying, it is still usable as long as it contains at least 1.1×10^{14} ^{131}I atoms. What is the maximum delay before the pill is no longer usable?

PREPARE The atoms in the sample undergo exponential decay, decreasing steadily in number.

SOLVE a. The half-life is $t_{1/2} = 8.0 \text{ days} = 192 \text{ hr}$. Using Equation 30.11, we can find the number of atoms remaining after 24 hr have elapsed:

$$N = (4.0 \times 10^{14})\left(\frac{1}{2}\right)^{24/192} = 3.7 \times 10^{14} \text{ atoms}$$

b. The time after which 1.1×10^{14} atoms remain is given by

$$1.1 \times 10^{14} = (4.0 \times 10^{14})\left(\frac{1}{2}\right)^{t/192}$$

To solve for t, we write this as

$$\frac{1.1 \times 10^{14}}{4.0 \times 10^{14}} = \left(\frac{1}{2}\right)^{t/192}$$

or

$$0.275 = \left(\frac{1}{2}\right)^{t/192}$$

Now, we take the natural logarithm of both sides:

$$\ln(0.275) = \ln\left(\left(\frac{1}{2}\right)^{t/192}\right)$$

We can solve for t by using the fact that $\ln(a^x) = x\ln(a)$. This allows us to "pull out" the $t/192$ exponent to find

$$\ln(0.275) = \left(\frac{t}{192}\right)\ln\left(\frac{1}{2}\right)$$

Solving for t, we find that the pill ceases to be useful after

$$t = 192\frac{\ln(0.275)}{\ln(1/2)} = 360 \text{ hr} = 15 \text{ days}$$

ASSESS The weakest usable concentration of iodine is approximately one-fourth of the initial concentration. This means that the decay time should be approximately equal to two half-lives, which is what we found.

Activity

The **activity** R of a radioactive sample is the number of decays per second. Each decay corresponds to an alpha, beta, or gamma emission, so the activity is a measure of how much radiation is being given off. A detailed treatment of the mathematics of decay shows that the activity of a sample of N nuclei having time constant τ (and half-life $t_{1/2}$) is

$$R = \frac{N}{\tau} = \frac{0.693N}{t_{1/2}} \tag{30.14}$$

A sample with $N = 1.0 \times 10^{10}$ nuclei decaying with time constant $\tau = 100$ s would, at that instant, have activity $R = 1.0 \times 10^8$ decays/s—in each second, 1.0×10^8 atoms would decay.

We see from Equation 30.14 that **activity is inversely proportional to the half-life.** If two samples have the same number of nuclei, the sample with the shorter half-life has the larger activity. We can combine Equation 30.14 with Equations 30.11 and 30.12 to obtain an expression for the variation of activity with time:

$$R = \frac{N}{\tau} = \frac{N_0 e^{-t/\tau}}{\tau} = R_0 e^{-t/\tau} = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}} \tag{30.15}$$

where $R_0 = N_0/\tau$ is the activity at $t = 0$. This equation has the same form as that for the decay of the sample. **The activity of a sample decreases exponentially** along with the number of remaining nuclei.

The SI unit of activity is the **becquerel,** defined as

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ decay/s or } 1 \text{ s}^{-1}$$

An older unit of activity, but one that continues in widespread use, is the **curie.** The conversion factor is

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

1 Ci is a substantial amount of radiation. The radioactive samples used in laboratory experiments are typically $\approx 1\ \mu$Ci or, equivalently, $\approx$40,000 Bq. These samples can be handled with only minor precautions. Larger sources of activity require thick shielding and other special precautions to prevent exposure to high levels of radiation.

Powered by decay This pellet is made of a short-lived isotope of plutonium. The short half-life means that this isotope has a very large activity. The rate of decay is such that it warms the pellet enough to make it glow. This pellet is intended for a thermoelectric generator, which uses the heat of the pellet to produce electricity. The radioactive decay of short-lived isotopes of plutonium has been used to provide power for spacecraft on voyages far from the sun. For many years, plutonium "batteries" were used to power heart pacemakers as well.

CONCEPTUAL EXAMPLE 30.6 **Relative activities of isotopes in the body**

^{40}K ($t_{1/2} = 1.3 \times 10^9$ yr) and ^{14}C ($t_{1/2} = 5.7 \times 10^3$ yr) are two radioactive isotopes found in measurable quantities in your body. Suppose you have 1 mole of each. Which is more radioactive—that is, which has a greater activity?

REASON Equation 30.14 shows that the activity of a sample is proportional to the number of atoms and inversely proportional to the half-life. Because both samples have the same number of atoms, the sample of ^{14}C, with its much shorter half-life, has a much greater activity.

EXAMPLE 30.7 **Determining the decay of activity**

A ^{60}Co (half-life 5.3 yr) source used to provide gamma rays to irradiate tumors has an activity of 0.43 Ci.

a. How many ^{60}Co atoms are in the source?
b. What will be the activity of the source after 10 yr?

PREPARE The activity of the source depends on the half-life and the number of atoms. If we know the activity and the half-life, we can compute the number of atoms. The number of atoms will undergo exponential decay, and so will the activity.

SOLVE a. Equation 30.14 relates the activity and the number of atoms. Rewriting the equation, we can relate the initial activity and the initial number of atoms as $N_0 = t_{1/2}R_0/0.693$. To use this equation, we need numbers in SI units. In Bq, the initial activity of the source is

$$R_0 = (0.43 \text{ Ci})\left(\frac{3.7 \times 10^{10} \text{ Bq}}{1 \text{ Ci}}\right) = 1.6 \times 10^{10} \text{ Bq}$$

The half-life $t_{1/2}$ in s is

$$t_{1/2} = (5.3 \text{ yr})\left(\frac{3.15 \times 10^7 \text{ s}}{1 \text{ yr}}\right) = 1.7 \times 10^8 \text{ s}$$

Thus the initial number of ^{60}Co atoms in the source is

$$N_0 = \frac{t_{1/2}R_0}{0.693} = \frac{(1.7 \times 10^8 \text{ s})(1.6 \times 10^{10} \text{ Bq})}{0.693}$$
$$= 3.9 \times 10^{18} \text{ atoms}$$

b. The variation of activity with time is given by Equation 30.15. After 10 yr, the activity is

$$R = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}} = (1.6 \times 10^{10} \text{ Bq})\left(\frac{1}{2}\right)^{10/5.3}$$
$$= 4.3 \times 10^9 \text{ Bq} = 0.12 \text{ Ci}$$

ASSESS Although N_0 is a very large number, it is a very small fraction of a mole. The sample contains only about 400 μg of ^{60}Co. In the first part of the question we needed to convert the half-life to s. The second part of the question used a ratio of two times, so we can use any units we like as long as the units of both times are the same.

Radioactive Dating

Many geological and archeological samples can be dated by measuring the decays of naturally occurring radioactive isotopes.

The most well-known dating technique uses the radioactive carbon isotope ^{14}C and is known as carbon dating or **radiocarbon dating.** ^{14}C has a half-life of 5730 years, so any ^{14}C present when the earth formed 4.5 billion years ago has long since decayed away. Nonetheless, ^{14}C is present in atmospheric carbon dioxide because high-energy cosmic rays collide with gas molecules high in the atmosphere to create ^{14}C nuclei from nuclear reactions with nitrogen and oxygen nuclei. The creation and decay of ^{14}C have reached a steady state in which the $^{14}C/^{12}C$ ratio is relatively stable at 1.3×10^{-12}.

All living organisms constantly exchange carbon dioxide with the environment, so the $^{14}C/^{12}C$ ratio in living organisms is also 1.3×10^{-12}. As soon as an organism dies, the ^{14}C in its tissue begins to decay and no new ^{14}C is added. As time goes on, the ^{14}C decays at a well-known rate. Thus, a measurement of the activity of an ancient organic sample permits a determination of the age. "New" samples have a higher fraction of ^{14}C than "old" samples.

The first step in radiocarbon dating is to extract and purify carbon from the sample. The carbon is then placed in a shielded chamber and its activity measured. This activity is then compared to the activity of an identical modern sample. Equation 30.15 relates the activity of a sample at a time t to its initial activity. If we assume that the original activity R_0 was the same as the activity of the modern sample, we can determine the time since the decay began—and thus the age of the sample.

A researcher is extracting a small sample of an ancient bone. By measuring the ratio of carbon isotopes present in the sample she will determine the age of the bone.

EXAMPLE 30.8 **Carbon dating a tooth**

A rear molar from a mammoth skeleton is dated using a measurement of its ^{14}C content. Carbon from the tooth is chemically extracted and formed into benzene. The benzene sample is placed in a shielded chamber. Decays from the sample come at an average rate of 11.5 counts per minute. A modern benzene sample of the exact same size gives 54.9 counts per minute. What is the age of the skeleton?

PREPARE We can assume that, thousands of years ago, the sample had an initial activity of 54.9 counts per minute—the activity of a modern sample. The present activity is lower due to the decay of the ^{14}C since the death of the mammoth.

SOLVE Equation 30.15 gives the decrease of the activity as a function of time as $R = R_0(1/2)^{t/t_{1/2}}$. The current activity is $R = 11.5$ counts per minute, and we assume that the initial activity was $R_0 = 54.9$. t is the time since the mammoth stopped growing—the age of the skeleton. We solve for t by rearranging terms and computing a natural logarithm, as in Example 30.5:

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$\ln\left(\frac{R}{R_0}\right) = \left(\frac{t}{t_{1/2}}\right)\ln\left(\frac{1}{2}\right)$$

We then solve for the time t:

$$t = \frac{t_{1/2}}{\ln(1/2)}\ln\left(\frac{R}{R_0}\right) = \frac{5730 \text{ yr}}{\ln(1/2)}\ln\left(\frac{11.5}{54.9}\right) = 12{,}900 \text{ yr}$$

ASSESS The final time is in years, the same unit we used for the half-life. This is a realistic example of how such radiocarbon dating is done; the numbers and details used in this example come from an actual experimental measurement.

CONCEPTUAL EXAMPLE 30.9 **Source contamination**

One possible problem with carbon dating is contamination with modern carbon sources. Suppose an archaeologist has unearthed and carbon-dated a fragment of wood that has absorbed carbon of recent vintage from organic molecules in groundwater. Does he underestimate or overestimate the age of the wood?

REASON Because the wood has absorbed modern carbon, it will have more ^{14}C than it would had it decayed undisturbed. The present activity is higher than it would otherwise be. This will lead to an underestimate of the age of the wood.

▶ **Responsible dating** BIO Measuring the activity of the carbon in a sample to determine the fraction of ^{14}C can require a significant amount of organic material—perhaps 25 g. For an artifact of great historical importance, such as this parchment fragment from the Dead Sea scrolls, this would be unacceptable. Instead, dates are obtained by using a mass spectrometer to directly measure the ratio of carbon isotopes, from which the age can be determined. This can be done with excellent accuracy on as little as 0.1 g of material.

Carbon dating can be used to date skeletons, wood, paper, fur, food material, and anything else made of organic matter. It is quite accurate for ages to about 15,000 years, about three half-lives. Beyond that, the difficulty of measuring the small remaining fraction of ^{14}C and some uncertainties about the cosmic ray flux in the past combine to decrease the accuracy. Even so, items are dated to about 50,000 years with a fair degree of reliability.

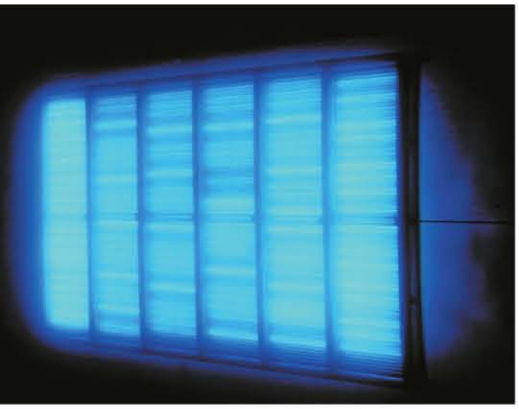
Isotopes with longer half-lives are used to date geological samples. Potassium-argon dating, using ^{40}K with a half-life of 1.25 billion years, is especially useful for dating rocks of volcanic origin.

STOP TO THINK 30.4 A sample starts with 1000 radioactive atoms. How many half-lives have elapsed when 750 atoms have decayed?

A. 0.25 B. 1.5 C. 2.0 D. 2.5

30.6 Medical Applications of Nuclear Physics

Nuclear physics has brought both peril and promise to society. Radioactivity can cause tumors. At the same time, radiation can be used to diagnose and cure some cancers. This section is a brief survey of medical applications of nuclear physics.



Ionizing radiation damages cells of the body, but it also damages bacteria and other pathogens. This gamma source is used for sterilizing medical equipment. The blue glow is due to the ionization of the air around the source.

Radiation Dose

Nuclear radiation disrupts a cell's machinery by altering and damaging biological molecules, as we saw in Section 30.4. The biological effects of radiation depend on two factors. The first is the physical factor of how much energy is absorbed by the body. The second is the biological factor of how tissue reacts to different forms of radiation.

Suppose a beta particle travels through tissue, losing kinetic energy as it ionizes atoms it passes. The energy lost by the beta particle is a good measure of the number of ions produced and thus the amount of damage done. In a certain volume of tissue, more ionization means more damage. For this reason, we define the radiation **dose** as the energy from ionizing radiation absorbed by 1 kg of tissue. The SI unit for the dose is the **gray,** abbreviated Gy. The Gy is defined as

$$1 \text{ Gy} = 1.00 \text{ J/kg of absorbed energy}$$

The number of Gy depends only on the energy absorbed, not on the type of radiation or on what the absorbing material is. Another common unit for dose is the *rad;* 1 rad = 0.01 Gy.

A 1 Gy dose of gamma rays and a 1 Gy dose of alpha particles have different biological consequences. To account for such differences, the **relative biological effectiveness** (RBE) is defined as the biological effect of a given dose relative to the biological effect of an equal dose of x rays. Table 30.4 shows the relative biological effectiveness of different forms of radiation. Larger values correspond to larger biological effects.

TABLE 30.4 Relative biological effectiveness of radiation

Radiation type	RBE
X rays	1
Gamma rays	1
Beta particles	1
Protons	5
Neutrons	5–20
Alpha particles	20

The radiation **dose equivalent** is the product of the energy dose in Gy and the relative biological effectiveness. Dose equivalent is measured in **sieverts,** abbreviated Sv. To be precise,

$$\text{dose equivalent in Sv} = \text{dose in Gy} \times \text{RBE}$$

One Sv of radiation produces the same biological damage regardless of the type of radiation. Another common unit of dose equivalent (also called biologically equivalent dose) is the *rem;* 1 rem = 0.01 Sv.

NOTE ► In practice, the term "dose" is often used for both dose and dose equivalent. Use the units as a guide. If the unit is Sv or rem, it is a dose equivalent; if Gy or rad, a dose. ◄

EXAMPLE 30.10 Finding energy deposited in radiation exposure

A 75 kg patient is given a bone scan. A phosphorus compound containing the gamma-emitter ^{99}Tc is injected into the patient. It is taken up by the bones, and the emitted gamma rays are measured. The procedure exposes the patient to 3.6 mSv (360 mrem) of radiation. What is the total energy deposited in the patient's body, in J and in eV?

PREPARE The exposure is given in Sv, so it is a dose equivalent, a combination of deposited energy and biological effectiveness. The RBE for gamma rays is 1. Gamma rays are penetrating, and the source is distributed throughout the body, so this is a whole-body exposure. Each kg of the patient's body will receive approximately the same energy.

SOLVE The dose in Gy is the dose equivalent in Sv divided by the RBE. In this case, because RBE = 1, the dose in Gy is numerically equal to the equivalent dose in Sv. The dose is thus 3.6 mGy = 3.6×10^{-3} J/kg. The radiation energy absorbed in the patient's body is

$$\text{absorbed energy} = (3.6 \times 10^{-3}\ \text{J/kg})(75\ \text{kg}) = 0.27\ \text{J}$$

In eV, this is

$$\text{absorbed energy} = (0.27\ \text{J})(1\ \text{eV}/1.6 \times 10^{-19}\ \text{J}) = 1.7 \times 10^{18}\ \text{eV}$$

ASSESS The total energy deposited, 0.27 J, is quite small; there will be negligible heating of tissue. But radiation produces its effects in other ways, as we have seen. Because it takes only ≈ 10 eV to ionize an atom, this dose is enough energy to ionize over 10^{17} atoms, meaning it can cause significant disruption to the cells of the body.

The question inevitably arises: What is a safe dose? Unfortunately, there is no simple or clear definition of a safe dose. A prudent policy is to avoid unnecessary exposure, and to weigh the significance of an exposure in relation to the *natural background*. We are all exposed to continual radiation from cosmic rays and from naturally occurring radioactive atoms in the ground, the atmosphere, and even the food we eat. This background averages about 3 mSv (300 mrem) per year, although there are wide regional variations depending on the soil type and the elevation. (Higher elevations have less atmospheric shielding, and thus have a larger exposure to cosmic rays. On the moon, with no protective atmosphere, the yearly dose would be 50 mSv.)

Table 30.5 lists the expected exposure from several different sources. A dental x ray subjects a person to approximately 1% of the yearly natural background that he or she would normally receive and is likely not a cause for significant worry. Mammograms involve a much larger dose, concentrated in a small region of the body. A nuclear medicine procedure, like a PET scan (which is discussed below), may involve an exposure that is much larger than the typical yearly background dose. This significant dose must be weighed against the medical benefits of the procedure.

TABLE 30.5 Radiation exposure

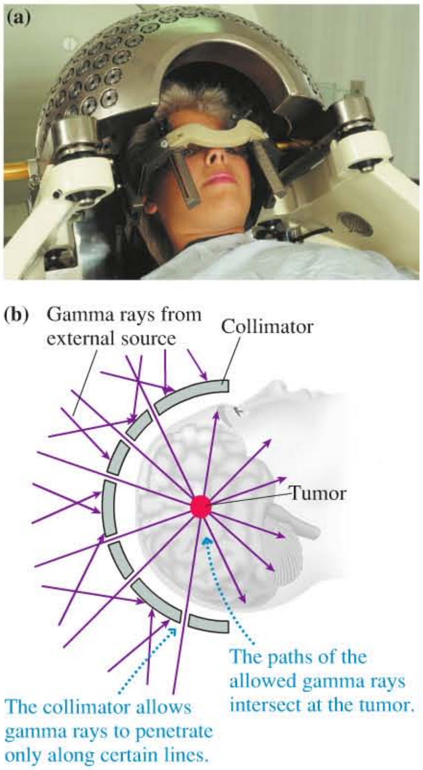
Radiation source	Typical exposure (mSv)
PET scan	7.0
Natural background (1 year)	3.0
Mammogram	0.70
Chest x ray	0.30
Transatlantic airplane flight	0.050
Dental x ray	0.030

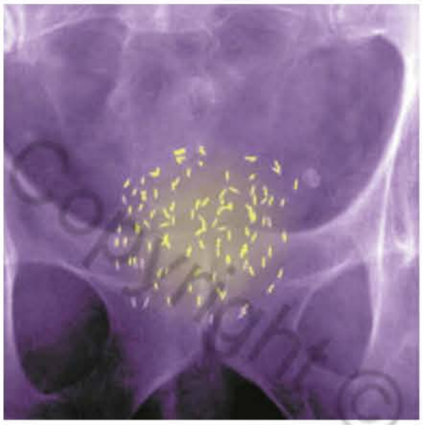
Nuclear Medicine

The tissues in the body most susceptible to radiation are those that are rapidly proliferating—including tumors. The goal in *radiation therapy* is to apply a large enough dose of radiation to destroy or shrink a tumor while producing minimal damage to surrounding healthy tissue.

In **FIGURE 30.20a**, a patient with a brain tumor is fitted with a metal *collimator* that absorbs gamma rays except for those traveling along desired paths. The collimator is fashioned so that gamma rays from an external source will be concentrated on the tumor, as shown in **FIGURE 30.20b**. Because the rapidly dividing cells of a tumor are much more sensitive to radiation than the tissues of the brain, and because surgical options carry risks of significant complications, radiation is a common means of treating tumors of the brain.

FIGURE 30.20 The use of gamma rays to treat a tumor in the brain.





Metal "seeds" containing radioactive ^{125}I in the prostate gland can shrink a tumor.

The detector is measuring gamma radiation emitted by isotopes taken up by tissues in the woman's head and neck.

Other tumors are treated by surgically implanting radioactive "seeds" within the tumor. One common type of seed contains ^{125}I, which undergoes a nuclear decay followed by emission of a 27.5 keV photon. The photon has a very short range so that it will damage only tissue close to the seed. Careful placement of the seeds permits a significant dose to a tumor with only minimal exposure to surrounding tissue.

Some tissues in the body will preferentially take up certain isotopes, allowing for treatment by isotope ingestion. A common treatment for hyperthyroidism, in which the thyroid gland is overactive, is to damage the gland with the isotope ^{131}I, a beta-emitter with a half-life of 8.0 days. A patient is given a tablet containing ^{131}I. The iodine in the blood is taken up and retained by the thyroid gland, resulting in a reduction of the gland's activity with minimal disruption of surrounding tissue.

Nuclear Imaging

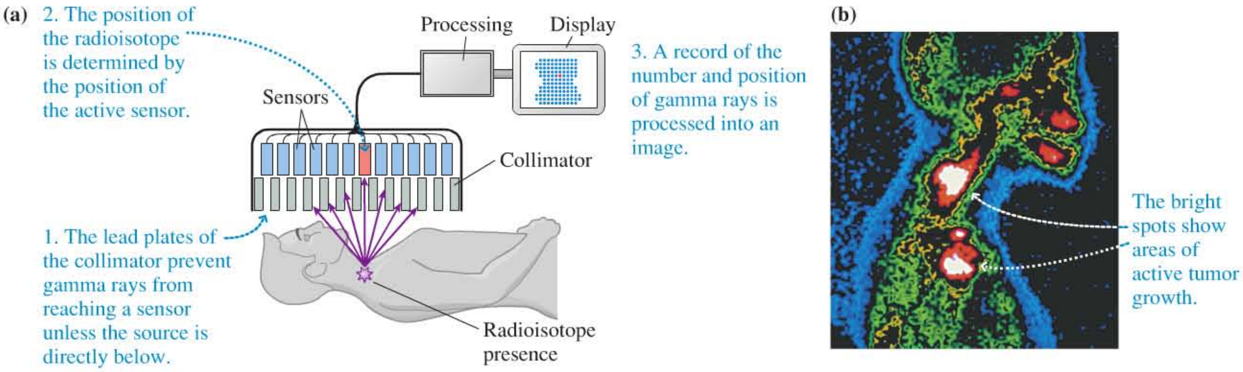
X rays from an external source may be used to make an image of the body, as described in Chapter 28. *Nuclear imaging* uses an internal source—radiation from isotopes in the body—to produce an image of tissues in the body. The bone-scan image that opened the chapter is an example of nuclear imaging.

There is a key difference between x rays and nuclear imaging procedures. **An x ray is an image of anatomical structure;** it is excellent for identifying structural problems like broken bones. **Nuclear imaging creates an image of the biological activity of tissues in the body.** For example, nuclear imaging can detect reduced metabolic activity of brain tissue after a stroke.

Let's look at an example that illustrates the difference between a conventional x-ray image or CAT scan and an image made with a nuclear imaging technique. Suppose a doctor suspects a patient has cancerous tissue in the bones. An x ray does not show anything out of the ordinary; the tumors may be too small or may appear similar to normal bone. The doctor then orders a scan with a **gamma camera,** a device that can measure and produce an image from gamma rays emitted within the body.

The patient is given a dose of a phosphorus compound labeled with the gamma-emitter ^{99}Tc. This compound is taken up and retained in bone tissue where active growth is occurring. The ^{99}Tc will be concentrated in the bones where there has been recent injury or inflammation—or where a tumor is growing. The patient is then scanned with a gamma camera. **FIGURE 30.21a** shows how the gamma camera can pinpoint the location of the gamma-emitting isotopes in the body and produce an image that reveals their location and intensity. A typical image is shown in **FIGURE 30.21b**. The bright spots show high concentrations of ^{99}Tc, revealing areas of tumor growth. The tumors may be too small to show up on an x ray, but their activity is easily detected with the gamma camera. With such early detection, the patient's chance of a cure is greatly improved.

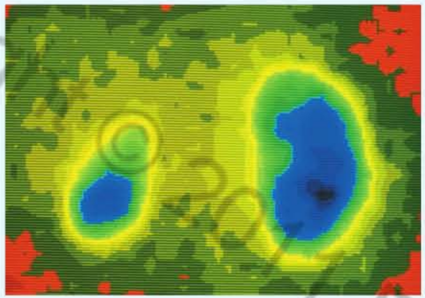
FIGURE 30.21 The operation of a gamma camera.



CONCEPTUAL EXAMPLE 30.11 **Using radiation to diagnose disease**

A patient suspected of having kidney disease is injected with a solution containing molecules that are taken up by healthy kidney tissue. The molecules have been "tagged" with radioactive ^{99}Tc. A gamma camera scan of the patient's abdomen gives the image in **FIGURE 30.22**. In this image, blue corresponds to the areas of highest activity. Which of the patient's kidneys has reduced function?

FIGURE 30.22 A gamma scan of a patient's kidneys.



REASON Healthy tissue should show up in blue on the scan because healthy tissue will absorb molecules with the ^{99}Tc attached and will thus emit gamma rays. The kidney imaged on the right shows normal activity throughout; the kidney imaged on the left appears smaller, so it has a smaller volume of healthy tissue. The patient is ill; the problem is with the kidney imaged on the left.

ASSESS Depending on the isotope and how it is taken up by the body, either healthy tissue or damaged tissue could show up on a gamma camera scan.

Positron-Emission Tomography

We have seen that a small number of radioactive isotopes decay by the emission of a positron. Such isotopes can be used for an imaging technique known as *positron-emission tomography,* or *PET.* PET is particularly important for imaging the brain.

The imaging process relies on the mass–energy conversion resulting from the combination of an electron and a positron. Suppose an electron and a positron are at rest, or nearly so; the combined momentum is nearly zero. The electron and positron have opposite charges and so will attract each other. When they meet, as we saw in Chapter 27, they completely annihilate—but energy and momentum are still conserved. The conservation of energy means that the annihilation will produce one or more high-energy photons—gamma rays. We learned in Chapter 27 that photons have momentum, so the annihilation can't produce a single photon because that photon would leave the scene of the annihilation with momentum. Instead the most likely result is a pair of photons directed exactly opposite each other, as shown in **FIGURE 30.23a**.

Most PET scans use the fluorine isotope ^{18}F, which emits a positron as it undergoes beta-plus decay to ^{18}O with a half-life of 110 minutes. ^{18}F is used to create an analog of glucose called fluorine-18 fluoro-deoxy-glucose (F-18 FDG). This compound is taken up by tissues in the brain. Areas that are more active are using more glucose, so the F-18 FDG is concentrated in active brain regions. When a fluorine atom in the F-18 FDG decays, the emitted positron immediately collides with a regular electron. The two annihilate to produce two gamma rays that travel out of the brain in opposite directions, as shown in Figure 30.23a.

FIGURE 30.23 Positron-emission tomography.

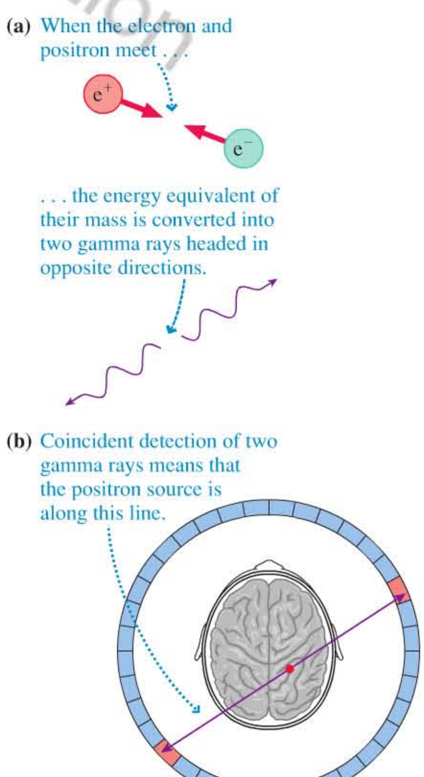
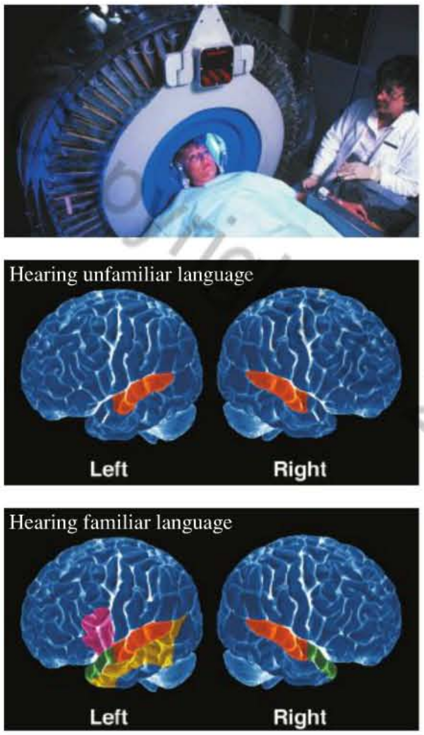


FIGURE 30.23b shows a patient's head surrounded by a ring of gamma-ray detectors. Because the gamma rays from the positron's annihilation are emitted back to back, simultaneous detection of two gamma rays on opposite sides of the subject indicates that the annihilation occurred somewhere along the line between those detectors. Recording many such pairs of gamma rays shows with great accuracy where the decays are occurring. A full scan will show more activity in regions of the brain where metabolic activity is enhanced, less activity in regions where metabolic activity is depressed. An analysis of these scans can provide a conclusive diagnosis of stroke, injury, or Alzheimer's disease.



STOP TO THINK 30.5 A patient ingests a radioactive isotope to treat a tumor. The isotope provides a dose of 0.10 Gy. Which type of radiation will give the highest dose equivalent in Sv?

A. Alpha particles B. Beta particles C. Gamma rays

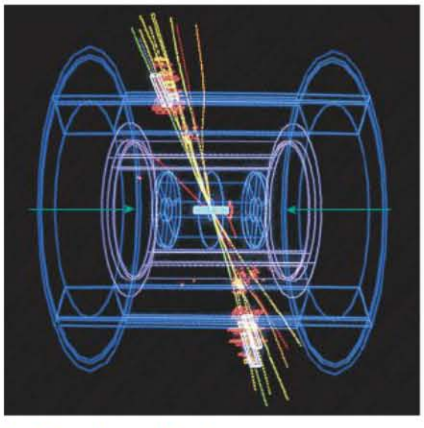
◀ **This is your brain on PET** BIO The woman in the top photo is undergoing a PET scan not to diagnose disease but to probe the workings of the brain. While undergoing a PET scan, a subject is asked to perform different mental tasks. The lower panels show functional images from a PET scan super-imposed on anatomical images from a CAT scan. The subject first listened to speech in an unfamiliar language; the active areas of the brain were those responsible for hearing. Next, she listened to speech in a familiar language, resulting in activity in the parts of the brain responsible for speech and comprehension.

30.7 The Ultimate Building Blocks of Matter

19.5 ActivPhysics

As we've seen, modeling the nucleus as being made of protons and neutrons allows a description of all of the elements in the periodic table in terms of just three basic particles—protons, neutrons, and electrons. But are protons and neutrons *really* basic building blocks? Molecules are made of atoms. Atoms are made of a cloud of electrons surrounding a positively charged nucleus. The nucleus is composed of protons and neutrons. Where does this process end? Are electrons, protons, and neutrons the basic building blocks of matter, or are they made of still smaller subunits?

This question takes us into the domain of what is known as **particle physics**—the branch of physics that deals with the basic constituents of matter and the relationships among them. Particle physics starts with the constituents of the atom, the proton, neutron, and electron, but there are many other particles below the scale of the atom. We call these particles **subatomic particles.**



Subatomic crash tests The above picture is the record of a collision between an electron and a positron (paths represented by the green arrows) brought to high speeds in a collider. The particles annihilated in the center of a detector that measured the paths of the particles produced in the collision. In this case, the annihilation of the electron and positron created a particle known as a Z boson. The Z boson then quickly decayed into the two jets of particles seen coming out of the detector.

Antiparticles

We've described the positron as the *antiparticle* to the electron. In what sense is a positron an *anti*electron? As we've seen, when a positron and an electron meet, they annihilate each other, turning the energy equivalent of their masses into the pure energy of two photons. Mass disappears and light appears in one of the most spectacular confirmations of Einstein's relativity.

Every subatomic particle that has been discovered has an antiparticle twin that has the same mass and the same spin but opposite charge. In addition to positrons, there are antiprotons (with $q = -e$), antineutrons (also neutral, but not the same as regular neutrons), and antimatter versions of all the various subatomic particles we will see. The notation to represent an antiparticle is a bar over the top of the symbol. A proton is represented as p, an antiproton as $\overline{\mathrm{p}}$.

Antiparticles provide interesting opportunities for creating "exotic" subatomic particles. When a particle meets its associated antiparticle, the two annihilate, leaving nothing but their energy behind. This energy must go somewhere. Sometimes it is emitted as gamma-ray photons, but this energy can also be used to create other particles.

The major tool for creating and studying subatomic particles is the *particle collider.* These machines use electric and magnetic fields to accelerate particles and their antiparticles, such as e and $\bar{e}$, or p and $\bar{p}$, to speeds very close to the speed of light. These particles then collide head-on. As they collide and annihilate, their mass-energy and kinetic energy combine to produce exotic particles that are not part of ordinary matter. These particles come in a dizzying variety—pions, kaons, lambda particles, sigma particles, and dozens of others—each with its own antiparticle. Most live no more than a trillionth of a second.

EXAMPLE 30.12 Determining a possible outcome of a proton-antiproton collision

When a proton and an antiproton annihilate, the resulting energy can be used to create new particles. One possibility is the creation of electrically neutral particles called *neutral pions.* A neutral pion has a rest mass of 135 MeV/c^2. How many neutral pions could be produced in the annihilation of a proton and an antiproton? Assume the proton and antiproton are moving very slowly as they collide.

PREPARE The mass of a proton is given in Table 30.2 as 938 MeV/c^2. The mass of an antiproton is the same. Because the proton and antiproton are moving slowly, with essentially no kinetic energy, the total energy available for creating new particles is the energy equivalent of the masses of the proton and the antiproton.

SOLVE The total energy from the annihilation of a proton and an antiproton is the energy equivalent of their masses:

$$E = (m_{\text{proton}} + m_{\text{antiproton}})c^2 = (938\text{ MeV}/c^2 + 938\text{ MeV}/c^2)c^2 = 1876\text{ MeV}$$

It takes 135 MeV to create a neutral pion. The ratio

$$\frac{\text{energy available}}{\text{energy required to create a pion}} = \frac{1876\text{ MeV}}{135\text{ MeV}} = 13.9$$

tells us that we have enough energy to produce 13 neutral pions from this process, but not quite enough to produce 14.

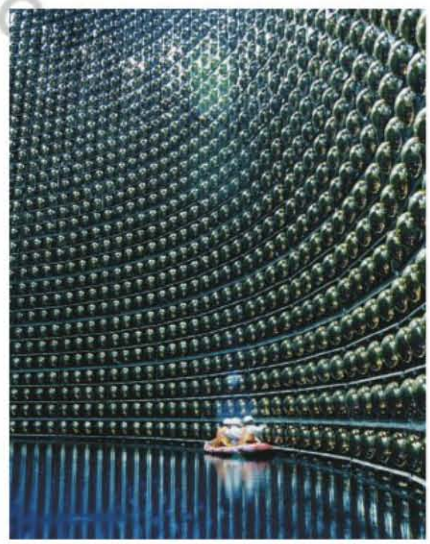
ASSESS Because the mass of a pion is much less than that of a proton or an antiproton, the annihilation of a proton and antiproton can produce many more pions than the number of particles at the start. Though the production of 13 neutral pions is a possible outcome of a proton-antiproton interaction, it is not a likely one. In addition to the conservation of energy, there are many other physical laws that determine what types of particles, and in what quantities, are likely to be produced.

Neutrinos

The most abundant particle in the universe is not the electron, proton, or neutron; it is a particle you may have never heard of. Vast numbers of these particles pass through your body every day, only in extremely rare cases leaving any trace of their passage. This elusive particle is the *neutrino,* a neutral, nearly massless particle that interacts only weakly with matter.

Some of the first studies of beta-minus decay, in the 1930s, found that neither energy nor momentum seemed to be conserved in the process. The physicist Wolfgang Pauli correctly suggested that, in addition to the electron, the beta-decay process emits a particle that, at that time, had not been detected. This new particle had to be electrically neutral, in order to conserve charge, and it had to be much less massive than an electron. It was later named the **neutrino,** meaning "little neutral one."

The neutrino is represented by the symbol ν, a lowercase Greek nu. There are three types of neutrinos. The neutrino involved in beta decay is the *electron neutrino* ν_e; it shows up in processes involving electrons and positrons. The electron neutrino



A big detector for a small particle The rubber raft in the photo is floating inside a particle detector designed to measure neutrinos. Neutrinos are so weakly interacting that a neutrino produced in a nuclear reaction in the center of the sun will likely pass through the entire mass of the sun and escape. Of course, the neutrino's weakly interacting nature also means that it is likely to pass right through a detector. The Super Kamiokande experiment in Japan monitors interactions in an enormous volume of water in order to spot a very small number of neutrino interactions.

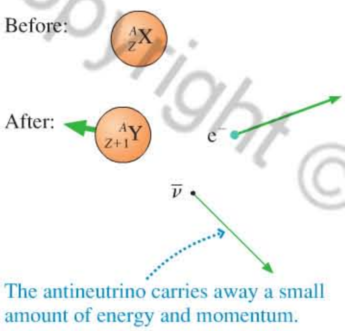


FIGURE 30.24 An accurate picture of beta decay must include the antineutrino.

of course has an antiparticle, the antineutrino $\overline{\nu}_e$. The full descriptions of beta-minus and beta-plus decays, including the neutrinos, are

$$n \rightarrow p^+ + e^- + \overline{\nu}_e$$
$$p^+ \rightarrow n + e^+ + \nu_e$$

FIGURE 30.24 shows that the electron and antineutrino (or positron and neutrino) share the energy released in the decay.

NOTE ► If we are concerned with the accurate balance of energy and momentum in a beta decay, we must include the antineutrino, but we can generally ignore the presence of this weakly interacting particle for the problems we solve in this chapter. ◄

It was initially thought that the neutrino, like the photon, was a *massless* particle. However, experiments in the last few years have shown that the neutrino mass, while tiny, is not zero. The best current evidence suggests a mass about one-millionth the mass of an electron. Experiments now under way will determine a more precise value. Because the neutrino is so abundant in the universe, this small mass is of great cosmological significance.

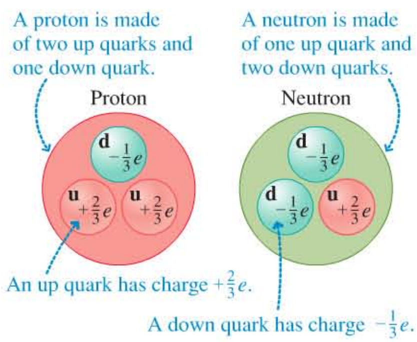
Quarks

The process of beta decay, in which a neutron can change into a proton, and vice versa, gives a hint that the neutron and the proton are *not* fundamental units but are made of smaller subunits.

There is another reason to imagine such subunits: the existence of dozens of subatomic particles—muons, pions, kaons, omega particles, and so on. Just as the periodic table explains the many different atomic elements in terms of three basic particles, perhaps it is possible to do something similar for this "subatomic zoo." In the 1960s, the American physicist Murray Gell-Mann and the Japanese physicist Kazuhiko Nishijima independently postulated the existence of particles that could be combined to make protons, neutrons, and other subatomic particles. Their ideas were soon confirmed in experiments showing that neutrons and protons seem to have internal structure—they appear to be made of three distinct subparticles.

We now understand protons and neutrons to be composed of smaller charged particles whimsically named **quarks.** The quarks that form protons and neutrons are called **up quarks** and **down quarks,** symbolized as u and d, respectively. The nature of these quarks and the composition of the neutron and the proton are shown in **FIGURE 30.25**.

FIGURE 30.25 The quark content of the proton and neutron.



NOTE ► It seems surprising that the charges of quarks are *fractions* of e. Don't charges have to be integer multiples of e? It's true that atoms, molecules, and all macroscopic matter must have $q = Ne$ because these entities are constructed from electrons and protons. But no law of nature prevents other types of matter from having other amounts of charge. ◄

A neutron and a proton differ by one quark. Beta decay can now be understood as a process in which a down quark changes to an up quark, or vice versa. Beta-minus decay of a neutron can be written as

$$\mathrm{d} \rightarrow \mathrm{u} + e^- + \bar{\nu}_\mathrm{e}$$

The existence of quarks thus provides an explanation of how a neutron can turn into a proton.

CONCEPTUAL EXAMPLE 30.13 Quarks and beta-plus decay

What is the quark description of beta-plus decay?

REASON In beta-plus decay, a proton turns into a neutron, with the emission of a positron and an electron neutrino. To turn a proton into a neutron requires the conversion of an up quark into a down quark; the total reaction is thus

$$\mathrm{u} \rightarrow \mathrm{d} + e^+ + \nu_\mathrm{e}$$

The quark model is very successful, but no one has ever seen a solitary quark. They don't exist alone, only inside other particles. In fact, the theory that describes quarks and their interactions specifies that we will *never* see a single quark all by itself. The force between quarks increases as they move apart, so an infinite amount of energy would be required to separate two quarks. Quarks must always be bound with other quarks, a principle known as **quark confinement.**

Fundamental Particles

Our current understanding of the truly *fundamental* particles—the ones that cannot be broken down into smaller subunits—is that they come in two basic types: **leptons** (particles like the electron and the neutrino) and quarks (which combine to form particles like the proton and the neutron). The leptons and quarks are described in Table 30.6. A few points are worthy of note:

- Each particle has an associated antiparticle.
- There are three *families* of leptons. The first is the electron and its associated neutrino, and their antiparticles. The other families are based on the muon and the tau, heavier siblings to the electron. Only the electron and positron are stable particles.
- There are also three families of quarks. The first is the up-down family that makes all "normal" matter. The other families are pairs of heavier quarks that form more exotic particles.

As far as we know, this is where the trail ends. Matter is made of molecules; molecules of atoms; atoms of protons, neutrons, and electrons; protons and neutrons of quarks. Quarks and electrons seem to be truly fundamental. But scientists of the early 20th century thought they were at a stopping point as well—they thought that they knew all of the physics that there was to know. As we've seen over the past few chapters, this was far from true. New tools such as the next generation of particle colliders will certainly provide new discoveries and new surprises.

The early chapters of this book, in which you learned about forces and motion, had very obvious applications to things in your daily life. But in these past few chapters we see that even modern discoveries—discoveries such as antimatter that may seem like science fiction—can be put to very practical use. As we come to the close of this book, we hope that you have gained an appreciation not only for what physics tells us about the world, but also for the wide range of problems it can be used to solve.

TABLE 30.6 Leptons and quarks

Leptons		Antileptons	
Electron	e^-	Positron	e^+
Electron neutrino	ν_e	Electron antineutrino	$\bar{\nu}_\mathrm{e}$
Muon	μ^-	Antimuon	μ^+
Muon neutrino	ν_μ	Muon antineutrino	$\bar{\nu}_\mu$
Tau	τ^-	Antitau	τ^+
Tau neutrino	ν_τ	Tau antineutrino	$\bar{\nu}_\tau$
Quarks		**Antiquarks**	
Up	u	Antiup	$\bar{\mathrm{u}}$
Down	d	Antidown	$\bar{\mathrm{d}}$
Strange	s	Antistrange	$\bar{\mathrm{s}}$
Charm	c	Anticharm	$\bar{\mathrm{c}}$
Bottom	b	Antibottom	$\bar{\mathrm{b}}$
Top	t	Antitop	$\bar{\mathrm{t}}$

INTEGRATED EXAMPLE 30.14 Čerenkov radiation

We know that nothing can travel faster than c, the speed of light in a vacuum. But we also know that light itself goes slower as it travels through a medium. Consequently particles moving at high speeds through a medium can be traveling faster than a light wave in the medium.

FIGURE 30.26 is a photo of the core of a nuclear reactor. The core is immersed in water, which carries away the thermal energy produced in the reactor. As high-energy electrons emerge with a speed very close to c from nuclear reactions in the core, they move through the water faster than the speed of light in water.

FIGURE 30.26 Čerenkov light illuminates the water surrounding a nuclear reactor core.

Recall that a shock wave—a sonic boom—is produced when an airplane moves faster than the speed of sound. An electron moving faster than the speed of light in water makes an electromagnetic shock wave—a light pulse analogous to a sonic boom. This particular type of electromagnetic radiation is known as **Čerenkov radiation,** or Čerenkov light, and it is responsible for the blue glow around the reactor core in Figure 30.26.

One source of high-speed electrons is the beta-minus decay of ^{133}Xe, a radioactive isotope of xenon that is produced in the fission of uranium and accumulates in the reactor core.

a. What is the daughter nucleus of this decay?
b. Assume that all of the energy released in the decay goes to kinetic energy of the emitted electron. What is the electron's kinetic energy?
c. Use the equations of special relativity to determine the electron's speed.
d. Would the emitted electron be moving at a speed high enough to cause Čerenkov light in water?
e. Based on the color of the Čerenkov light you can see in Figure 30.26, which of the following describes the spectrum of Čerenkov light?
   - The intensity of Čerenkov light is uniform at all frequencies.
   - The intensity of Čerenkov light is proportional to frequency.
   - The intensity of Čerenkov light is proportional to wavelength.

   Explain.

PREPARE ^{133}Xe undergoes beta decay, so the mass of this nucleus must be greater than that of the daughter nucleus. The "lost" mass is converted to energy, which we assume goes to the kinetic energy of the electron, so the kinetic energy of the electron will be $K = \Delta m \cdot c^2$. Once we know the electron's kinetic energy, we can determine the speed to see if it exceeds the speed of light in water. The speed of light in a medium is given by $v = c/n$, where n is the index of refraction. Water has $n = 1.33$.

SOLVE a. Equation 30.18 tells us that beta-minus decay increases Z by 1 while leaving A unchanged. Xe has $Z = 54$; $Z = 55$ is Cs (cesium), so the daughter nucleus, still with $A = 133$, is ^{133}Cs.

b. We use the data in Appendix D to find the mass difference between the parent and daughter nuclei:

$$\Delta m = m(^{133}\text{Xe}) - m(^{133}\text{Cs})$$
$$= 132.905906 \text{ u} - 132.905436 \text{ u} = 0.00047 \text{ u}$$

We assume that the energy corresponding to this mass difference is the kinetic energy of the emitted electron, so the electron's kinetic energy is

$$K = \Delta m \cdot c^2 = (0.00047 \text{ u})(931.49 \text{ MeV}/c^2)c^2 = 0.44 \text{ MeV}$$

c. The kinetic energy of the electron is large enough that we'll need to consider relativity—a classical treatment won't be sufficient. Equation 27.23 gives the relationship between an object's kinetic energy and its rest energy E_0 as $K = (\gamma - 1)E_0$. We can rearrange this to give γ in terms of a ratio of two energies that we know: the electron's rest energy, 0.51 MeV, and the electron's kinetic energy, 0.44 MeV. Because it's a ratio, we need not convert units:

$$\gamma = 1 + \frac{K}{E_0} = 1 + \frac{0.44 \text{ MeV}}{0.51 \text{ MeV}} = 1.9$$

We can now use the definition of γ to solve for the electron's speed:

$$\gamma = \frac{1}{\sqrt{1 - (v/c)^2}}$$
$$v = c\sqrt{1 - 1/\gamma^2} = 2.5 \times 10^8 \text{ m/s}$$

d. Čerenkov light will be emitted if the speed of the emitted electron is greater than the speed of a light wave in the water. The speed of light in water is

$$v = \frac{3.00 \times 10^8 \text{ m/s}}{1.33} = 2.3 \times 10^8 \text{ m/s}$$

The electron is moving faster than this, and so it will emit Čerenkov light.

e. The photo reveals that Čerenkov light appears blue, so more high-frequency blue light is emitted than low-frequency red light. The intensity is greater at higher frequencies, so the intensity is proportional to frequency—at least for visible light. The index of refraction decreases for very high frequencies, returning to $n = 1$ for x rays. Light speed at these very high frequencies is no longer slower than the particle speed, so Čerenkov light "cuts off" at very high frequencies.

ASSESS The energy of the beta particle is reasonably typical for particles emitted by nuclear decays, and we know that Čerenkov light is observed around reactor cores, so it's reasonable to expect the beta particle to be moving fast enough to emit Čerenkov light.

SUMMARY

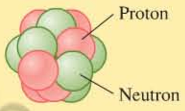
The goals of Chapter 30 have been to understand the physics of the nucleus and some of the applications of nuclear physics.

GENERAL PRINCIPLES

The Nucleus

The nucleus is a small, dense, positive core at the center of an atom.

Z protons, charge $+e$, spin $\frac{1}{2}$
N neutrons, charge 0, spin $\frac{1}{2}$



The **mass number** is

$$A = Z + N$$

Isotopes of an element have the same value of Z but different values of N.

The strong force holds nuclei together:

- It acts between any two nucleons.
- It is short range.

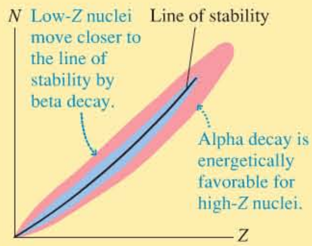
Adding neutrons to a nucleus allows the strong force to overcome the repulsive Coulomb force between protons.

The **binding energy** B of a nucleus depends on the mass difference between an atom and its constituents:

$$B = (Zm_{\mathrm{H}} + Nm_{\mathrm{n}} - m_{\mathrm{atom}}) \times (931.49\ \mathrm{MeV/u})$$

Nuclear Stability

Most nuclei are not **stable.** Unstable nuclei undergo **radioactive decay.** Stable nuclei cluster along the **line of stability** in a plot of the isotopes.



Mechanisms by which unstable nuclei decay:

Decay	Particle	Penetration
alpha	^{4}He nucleus	low
beta-minus	e^-	medium
beta-plus	e^+	medium
gamma	photon	high

Alpha and beta decays change the nucleus; the daughter nucleus is a different element.

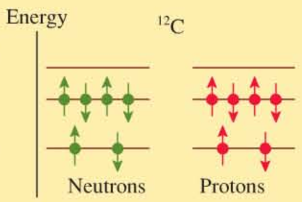
Alpha decay:
$${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + \alpha + \text{energy}$$

Beta-minus decay:
$${}^{A}_{Z}\mathrm{X} \rightarrow {}_{Z+1}^{\ \ A}\mathrm{Y} + \beta + \text{energy}$$

IMPORTANT CONCEPTS

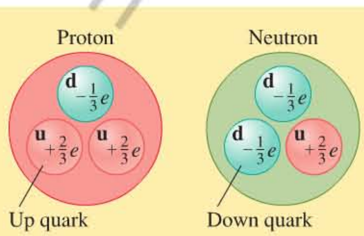
Energy levels

Nucleons fill nuclear energy levels, similar to filling electron energy levels in atoms. Nucleons can often jump to lower energy levels by emitting beta particles or gamma photons.



The quark model

Nucleons (and other particles) are made of quarks. Quarks and leptons are fundamental particles.



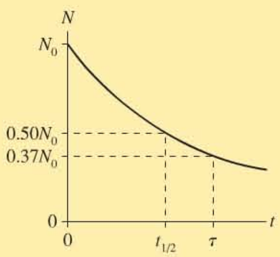
APPLICATIONS

Radioactive decay

The number of undecayed nuclei decreases exponentially with time t:

$$N = N_0 e^{-t/\tau}$$

$$N = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$$



The **half-life**

$$t_{1/2} = \tau \ln 2 = 0.693\tau$$

is the time in which half of any sample decays.

Measuring radiation

The **activity** of a radioactive sample is the number of decays per second. Activity is related to the half-life as

$$R = \frac{0.693N}{t_{1/2}} = \frac{N}{\tau}$$

The radiation **dose** is measured in grays, where

$$1\ \mathrm{Gy} = 1.00\ \mathrm{J/kg}\ \text{of absorbed energy}$$

The **relative biological effectiveness** (RBE) is the biological effect of a dose relative to the biological effects of x rays. The **dose equivalent** is measured in sieverts, where

$$\text{dose equivalent in Sv} = \text{dose in Gy} \times \text{RBE}$$

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

VIEW ALL SOLUTIONS

QUESTIONS

Conceptual Questions

1. Atom A has a larger atomic mass than atom B. Does this mean that atom A also has a larger atomic number? Explain.
2. Given that $m_H = 1.007825$ u, is the mass of a hydrogen atom ^{1}H greater than, less than, or equal to 1/12 the mass of a ^{12}C atom? Explain.
3. a. Is there a stable $^{30}_{3}$Li nucleus? Explain how you made your determination.
   b. Is there a stable $^{184}_{92}$U nucleus? Explain how you made your determination.
4. Rounding slightly, the nucleus ^{3}He has a binding energy of 2.5 MeV/nucleon and the nucleus ^{6}Li has a binding energy of 5 MeV/nucleon.
   a. What is the binding energy of ^{3}He?
   b. What is the binding energy of ^{6}Li?
   c. Is it energetically possible for two ^{3}He nuclei to join or fuse together into a ^{6}Li nucleus? Explain.
   d. Is it energetically possible for a ^{6}Li nucleus to split or fission into two ^{3}He nuclei? Explain.
5. A sample contains a mix of isotopes of an element. Using a spectrometer to measure the spectrum of emitted light will not reveal the mix of isotopes; analyzing the sample with a mass spectrometer will. Explain.
6. For each nuclear energy-level diagram in Figure Q30.6, state whether it represents a nuclear ground state, an excited nuclear state, or an impossible nucleus.

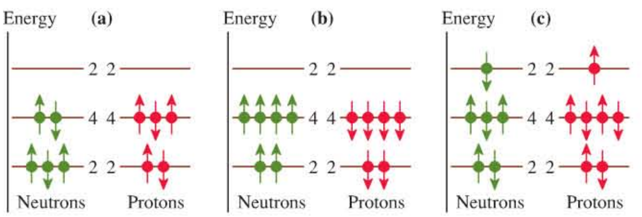


FIGURE Q30.6

7. Figure Q30.7 shows how the number of nuclei of one particular isotope varies with time. What is the half-life of the nucleus?

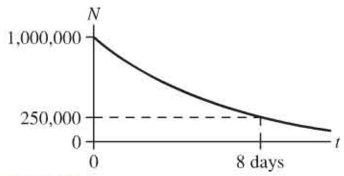


FIGURE Q30.7

8. A radioactive sample has a half-life of 10 s. 10,000 nuclei are present at $t = 20$ s.
   a. How many nuclei were there at $t = 0$ s?
   b. How many nuclei will there be at $t = 40$ s?
9. Nucleus A decays into the stable nucleus B with a half-life of 10 s. At $t = 0$ s there are 1000 A nuclei and no B nuclei. At what time will there be 750 B nuclei?
10. A radioactive sample's half-life is 1.0 min, so each nucleus in the sample has a 50% chance of undergoing a decay sometime between $t = 0$ and $t = 1$ min. One particular nucleus has not decayed at $t = 15$ min. What is the probability this nucleus will decay between $t = 15$ and $t = 16$ min?
11. Four samples each contain a single radioactive isotope. Sample A has 1 mol of matter and an activity of 100 Bq. Sample B has 10 mol and 100 Bq, sample C has 100 mol and 100 Bq, and sample D has 100 mol and 1000 Bq. Rank in order, from largest to smallest, the half-lives of these four isotopes. Explain.
12. Oil and coal generally contain no measurable ^{14}C. What does this tell us about how long they have been buried?
13. Radiocarbon dating assumes that the abundance of ^{14}C in the environment has been constant. Suppose ^{14}C was less abundant 10,000 years ago than it is today. Would this cause a lab using radiocarbon dating to overestimate or underestimate the age of a 10,000-year-old artifact? (In fact, the abundance of ^{14}C in the environment does vary slightly with time. But the issue has been well studied, and the ages of artifacts are adjusted to compensate for this variation.)
14. Identify the unknown X in the following decays:
    a. $^{222}_{86}\text{Rn} \rightarrow {}^{218}_{84}\text{Po} + X$ b. $^{228}_{88}\text{Ra} \rightarrow {}^{228}_{89}\text{Ac} + X$
    c. $^{140}_{54}\text{Xe} \rightarrow {}^{140}_{55}\text{Cs} + X$ d. $^{64}_{29}\text{Cu} \rightarrow {}^{64}_{28}\text{Ni} + X$
15. Are the following decays possible? If not, why not?
    a. $^{232}_{90}\text{Th} \rightarrow {}^{236}_{92}\text{U} + \alpha$ b. $^{238}_{94}\text{Pu} \rightarrow {}^{236}_{92}\text{U} + \alpha$
    c. $^{33}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + e^-$
16. What kind of decay, if any, would you expect for the nuclei with the energy-level diagrams shown in Figure Q30.16?

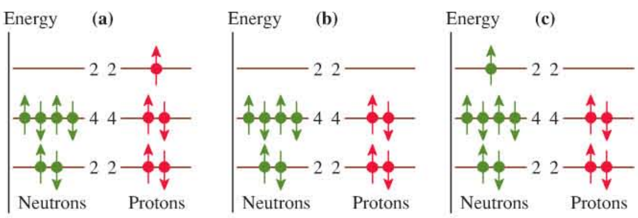


FIGURE Q30.16

17. The nuclei of ^{4}He and ^{16}O are very stable and are often referred to as "doubly magic" nuclei. Use what you know about energy levels to explain what is special about these particular nuclei.
18. BIO A and B are fresh apples. Apple A is strongly irradiated by nuclear radiation for 1 hour. Apple B is not irradiated. Afterward, in what ways are apples A and B different?
19. BIO A patient's tumor is irradiated with gamma rays from an external source. Afterward, is his body radioactive? Explain.

20. It's possible that a bone tumor will not show up on an x-ray
BIO image but will show up in a gamma scan. Explain why this is so.

21. Four radiation doses are as follows: Dose A is 10 rad with an
BIO RBE of 1, dose B is 20 rad with an RBE of 1, dose C is 10 rad with an RBE of 2, and dose D is 20 rad with an RBE of 2.
   a. Rank in order, from largest to smallest, the amount of energy delivered by these four doses.
   b. Rank in order, from largest to smallest, the biological damage caused by these four doses.

22. Two different sources of radiation give the same dose equiva-
BIO lent in Sv. Does this mean that the radiation from each source has the same RBE? Explain.

23. Some types of MRI can produce images of resolution and detail
BIO similar to PET. Though the images are similar, MRI is generally preferred over PET for studies of brain function involving healthy subjects. Why?

24. Sulfur colloid particles tagged with ^{99}Tc are taken up and
BIO retained by cells in the liver and spleen. A patient is suspected of having a liver tumor that would destroy these cells. Explain how a gamma camera scan could be used to confirm or rule out the existence of a tumor.

25. The first two letters in the acronym SPECT, which describes a
BIO nuclear imaging technique, stand for "single photon." Is a SPECT done with a gamma emitter or a positron emitter?

The following two questions concern an uncommon nuclear decay mode known as *electron capture*. Certain nuclei that are proton-rich but energetically prohibited from undergoing beta-plus decay can capture an electron from the 1*s* shell, which then combines with a proton to make a neutron. The basic reaction is

$$p + e^- \rightarrow n + \nu_e$$

26. Give a description of the electron capture process in terms of quarks.

27. Electron capture is usually followed by the emission of an x ray. Why?

Multiple-Choice Questions

28. | A significant fraction of the radiation dose you will receive
BIO during your life comes from radioactive materials in your body. The most important source of this radiation is the potassium isotope ^{40}K, which decays to the stable calcium isotope ^{40}Ca. What particle is emitted in the decay?
   A. A helium nucleus B. A neutron
   C. An electron D. A positron

29. | A certain watch's luminous glow is due to zinc sulfide paint that is energized by beta particles given off by *tritium*, the radioactive hydrogen isotope ^{3}H, which has a half-life of 12.3 years. This glow has about 1/10 of its initial brightness. How many years old is the watch?

   A. 20 yr B. 30 yr C. 40 yr D. 50 yr

30. | What is the unknown isotope in the following fission reaction: $n + {}^{235}U \rightarrow {}^{131}I + ? + 3n$
   A. ^{86}Rb B. ^{102}Rb C. ^{89}Y D. ^{102}Y

31. || The uranium in the earth's crust is 0.7% ^{235}U and 99.3% ^{238}U. Two billion years ago, ^{235}U comprised approximately 3% of the uranium in the earth's crust. This tells you something about the relative half-lives of the two isotopes. Suppose you have a sample of ^{235}U and a sample of ^{238}U, each with exactly the same number of atoms.
   A. The sample of ^{235}U has a higher activity.
   B. The sample of ^{238}U has a higher activity.
   C. The two samples have the same activity.

32. | Suppose you have a 1 g sample of ^{226}Ra, half-life 1600 years. How long will it be until only 0.1 g of radium is left?
   A. 1600 yr B. 3200 yr C. 5300 yr D. 16,000 yr

33. | A sample of ^{131}I, half-life 8.0 days, is registering 100 counts per second on a Geiger counter. How long will it be before the sample registers only 1 count per second?
   A. 8 days B. 53 days C. 80 days D. 800 days

34. || The complete expression for the decay of the radioactive hydrogen isotope *tritium* may be written as ${}^3H \rightarrow {}^3He + X + Y$. The symbols X and Y represent
   A. $X = e^+, Y = \bar{\nu}_e$ B. $X = e^-, Y = \nu_e$
   C. $X = e^+, Y = \nu_e$ D. $X = e^-, Y = \bar{\nu}_e$

35. | The quark composition of the proton and neutron are, respectively, uud and udd, where u is an up quark (charge $+\frac{2}{3}e$) and d is a down quark (charge $-\frac{1}{3}e$). There are also anti-up $\bar{u}$ (charge $-\frac{2}{3}e$) and anti-down $\bar{d}$ (charge $+\frac{1}{3}e$) quarks. The combination of a quark and an antiquark is called a *meson*. The mesons known as *pions* have the composition $\pi^+ = u\bar{d}$ and $\pi^- = \bar{u}d$. Suppose a proton collides with an antineutron. During such collisions, the various quarks and antiquarks annihilate whenever possible. When the remaining quarks combine to form a single particle, it is a
   A. Proton B. Neutron
   C. π^+ D. π^-

VIEW ALL SOLUTIONS

PROBLEMS

Section 30.1 Nuclear Structure

1. | How many protons and how many neutrons are in (a) ^{3}H, (b) ^{40}Ar, (c) ^{40}Ca, and (d) ^{239}Pu?

2. | How many protons and how many neutrons are in (a) ^{3}He, (b) ^{20}Ne, (c) ^{60}Co, and (d) ^{226}Ra?

3. || Use the data in Appendix D to calculate the chemical atomic mass of lithium, to two decimal places.

4. | Use the data in Appendix D to calculate the chemical atomic mass of neon, to two decimal places.

Section 30.2 Nuclear Stability

5. | Calculate (in MeV) the total binding energy and the binding energy per nucleon (a) for ^{3}H and (b) for ^{3}He.

6. | Calculate (in MeV) the total binding energy and the binding energy per nucleon (a) for ^{40}Ar and (b) for ^{40}K.

7. | Calculate (in MeV) the binding energy per nucleon for ^{3}He and ^{4}He. Which is more tightly bound?

8. || Calculate (in MeV) the binding energy per nucleon for ^{12}C and ^{13}C. Which is more tightly bound?

9. | Calculate (in MeV) the binding energy per nucleon for (a) ^{14}N, (b) ^{56}Fe, and (c) ^{207}Pb.

10. || When a nucleus of ^{235}U undergoes fission, it breaks into two smaller, more tightly bound fragments. Calculate the binding energy per nucleon for ^{235}U and for the fission product ^{137}Cs.

11. || When a nucleus of ^{240}Pu undergoes fission, it breaks into two smaller, more tightly bound fragments. Calculate the binding energy per nucleon for ^{240}Pu and for the fission product ^{133}Xe.

Section 30.3 Forces and Energy in the Nucleus

12. || Draw an energy-level diagram, similar to Figure 30.9 for the protons and neutrons in ^{11}Be. Do you expect this nucleus to be stable?

13. || Draw energy-level diagrams, similar to Figure 30.9, for all $A = 10$ nuclei listed in Appendix D. Show all the occupied neutron and proton levels. Which of these nuclei do you expect to be stable?

14. || Draw energy-level diagrams, similar to Figure 30.9, for all $A = 14$ nuclei listed in Appendix D. Show all the occupied neutron and proton levels. Which of these nuclei do you expect to be stable?

15. ||| You have seen that filled electron energy levels correspond to chemically stable nuclei. A similar principle holds for nuclear energy levels; nuclei with equally filled proton and neutron energy levels are especially stable. What are the three lightest isotopes whose proton and neutron energy levels are both filled, and filled equally?

Section 30.4 Radiation and Radioactivity

16. | ^{15}O and ^{131}I are isotopes used in medical imaging. ^{15}O is a
BIO beta-plus emitter, ^{131}I a beta-minus emitter. What are the daughter nuclei of the two decays?

17. | Spacecraft have been powered with energy from the alpha decay of ^{238}Pu. What is the daughter nucleus?

18. | Identify the unknown isotope X in the following decays.
   a. $^{234}\text{U} \rightarrow X + \alpha$
   b. $^{32}\text{P} \rightarrow X + e^-$
   c. $X \rightarrow {}^{30}\text{Si} + e^+$
   d. $^{24}\text{Mg} \rightarrow X + \gamma$

19. | Identify the unknown isotope X in the following decays.
   a. $X \rightarrow {}^{224}\text{Ra} + \alpha$
   b. $X \rightarrow {}^{207}\text{Pb} + e^-$
   c. $^{7}\text{Be} + e^- \rightarrow X$
   d. $X \rightarrow {}^{60}\text{Ni} + \gamma$

20. | What is the energy (in MeV) released in the alpha decay of ^{239}Pu?

21. | What is the energy (in MeV) released in the alpha decay of ^{228}Th?

22. || What is the total energy (in MeV) released in the beta decay of a neutron?

23. | Medical gamma imaging is generally done with the tech-
BIO netium isotope $^{99}\text{Tc}^*$, which decays by emitting a gamma-ray photon with energy 140 keV. What is the mass loss of the nucleus, in u, upon emission of this gamma ray?

Section 30.5 Nuclear Decay and Half-Lives

24. | The radioactive hydrogen isotope ^{3}H is called tritium. It decays by beta-minus decay with a half-life of 12.3 years.
   a. What is the daughter nucleus of tritium?
   b. A watch uses the decay of tritium to energize its glowing dial. What fraction of the tritium remains 20 years after the watch was created?

25. | The barium isotope ^{133}Ba has a half-life of 10.5 years. A sample begins with 1.0×10^{10} ^{133}Ba atoms. How many are left after (a) 2 years, (b) 20 years, and (c) 200 years?

26. | The cadmium isotope ^{109}Cd has a half-life of 462 days. A sample begins with 1.0×10^{12} ^{109}Cd atoms. How many are left after (a) 50 days, (b) 500 days, and (c) 5000 days?

27. || How many half-lives must elapse until (a) 90% and (b) 99% of a radioactive sample of atoms has decayed?

28. || The Chernobyl reactor accident in what is now Ukraine was
BIO the worst nuclear disaster of all time. Fission products from the reactor core spread over a wide area. The primary radiation exposure to people in western Europe was due to the short-lived (half-life 8.0 days) isotope ^{131}I, which fell across the landscape and was ingested by grazing cows that concentrated the isotope in their milk. Farmers couldn't sell the contaminated milk, so many opted to use the milk to make cheese, aging it until the radioactivity decayed to acceptable levels. How much time must elapse for the activity of a block of cheese containing ^{131}I to drop to 1.0% of its initial value?

29. |||| What is the age in years of a bone in which the $^{14}\text{C}/^{12}\text{C}$ ratio is measured to be 1.65×10^{-13}?

30. || ^{85}Sr is a short-lived (half-life 65 days) isotope used in bone
BIO scans. A typical patient receives a dose of ^{85}Sr with an activity of 0.10 mCi. If all of the ^{85}Sr is retained by the body, what will be its activity in the patient's body after one year has passed?

31. || What is the half-life in days of a radioactive sample with 5.0×10^{15} atoms and an activity of 5.0×10^{8} Bq?

32. ||| What is the activity, in Bq and Ci, of 1.0 g of ^{226}Ra? Marie
INT Curie was the discoverer of radium; can you see where the unit of activity named after her came from?

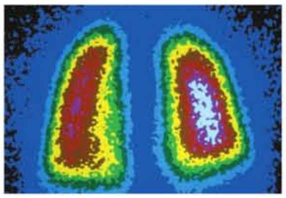
33. || Many medical PET scans use the isotope ^{18}F, which has a
BIO half-life of 1.8 hr. A sample prepared at 10:00 A.M. has an activity of 20 mCi. What is the activity at 1:00 P.M., when the patient is injected?

34. ||| An investigator collects a sample of a radioactive isotope with an activity of 370,000 Bq. 48 hours later, the activity is 120,000 Bq. What is the half-life of the sample?

Section 30.6 Medical Applications of Nuclear Physics

35. || A 50 kg nuclear plant worker is exposed to 20 mJ of neutron
BIO radiation with an RBE of 10. What is the dose in mSv?

36. || A gamma scan showing the
BIO active volume of a patient's lungs can be created by having a patient breathe the radioactive isotope ^{133}Xe, which undergoes beta-minus decay with a subsequent gamma emission from the daughter nucleus. A typical procedure gives a dose of 0.30 rem to the lungs. How much energy is deposited in the 1.2 kg mass of a patient's lungs?

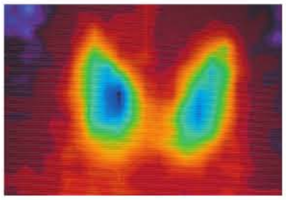


37. || How many rad of gamma-ray photons cause the same biological damage as 30 rad of alpha radiation?

38. | 150 rad of gamma radiation are directed into a 150 g tumor.
BIO How much energy does the tumor absorb?

39. || During the 1950s, nuclear bombs were tested on islands in the South Pacific. In one test, personnel on a nearby island received 10 mGy per hour of beta and gamma radiation. At this rate, how long would it take to receive a potentially lethal dose equivalent of 4.5 Sv?

40. ||| BIO ^{131}I undergoes beta-minus decay with a subsequent gamma emission from the daughter nucleus. Iodine in the body is almost entirely taken up by the thyroid gland, so a gamma scan using this isotope will show a bright area corresponding to the thyroid gland with the surrounding tissue appearing dark. Because the isotope is concentrated in the gland, so is the radiation dose, most of which results from the beta emission. In a typical procedure, a patient receives 0.050 mCi of ^{131}I. Assume that all of the iodine is absorbed by the 0.15 kg thyroid gland. Each ^{131}I decay produces a 0.97 MeV beta particle. Assume that half the energy of each beta particle is deposited in the gland. What dose equivalent in Sv will the gland receive in the first hour?



41. | BIO The doctors planning a radiation therapy treatment have determined that a 100 g tumor needs to receive 0.20 J of gamma radiation. What is the dose in Gy?

42. ||| BIO ^{90}Sr decays with the emission of a 2.8 MeV beta particle. Strontium is chemically similar to calcium and is taken up by bone. A 75 kg person exposed to waste from a nuclear accident absorbs ^{90}Sr with an activity of 370,000 Bq. Assume that all of this ^{90}Sr ends up in the skeleton. The skeleton forms 17% of the person's body mass. If 50% of the decay energy is absorbed by the skeleton, what dose equivalent in Sv will be received by the person's skeleton in the first month?

Section 30.7 The Ultimate Building Blocks of Matter

43. || BIO What are the minimum energies of the two oppositely directed gamma rays in a PET procedure?

44. |||| Positive and negative pions, denoted π^+ and π^-, are antiparticles of each other. Each has a rest mass of 140 MeV/c^2. Suppose a collision between an electron and positron, each with kinetic energy K, produces a π^+, π^- pair. What is the smallest possible value for K?

45. || In a particular beta-minus decay of a free neutron (that is, one not part of an atomic nucleus), the emitted electron has exactly the same kinetic energy as the emitted electron antineutrino. What is the value, in MeV, of that kinetic energy? Assume that the recoiling proton has negligible kinetic energy.

46. |||| INT The masses of the neutrinos are still not precisely determined, but let us assume for the purpose of this problem that the mass of an electron neutrino is one millionth the mass of an electron. What is the kinetic energy, in eV, of an electron neutrino moving at $0.999c$?

General Problems

47. | The chemical atomic mass of hydrogen, with the two stable isotopes ^{1}H and ^{2}H (deuterium), is 1.00798 u. Use this value to determine the natural abundance of these two isotopes.

48. || You learned in Chapter 29 that the binding energy of the electron in a hydrogen atom is 13.6 eV.
    a. By how much does the mass decrease when a hydrogen atom is formed from a proton and an electron? Give your answer both in atomic mass units and as a percentage of the mass of the hydrogen atom.
    b. By how much does the mass decrease when a helium nucleus is formed from two protons and two neutrons? Give your answer both in atomic mass units and as a percentage of the mass of the helium nucleus.
    c. Compare your answers to parts a and b. Why do you hear it said that mass is "lost" in nuclear reactions but not in chemical reactions?

49. ||| Use the graph of binding energy of Figure 30.6 to estimate the total energy released if a nucleus with mass number 240 fissions into two nuclei with mass number 120.

50. ||| Could a ^{56}Fe nucleus fission into two ^{28}Al nuclei? Your answer, which should include some calculations, should be based on the curve of binding energy of Figure 30.6.

51. ||| a. What are the isotopic symbols of all isotopes in Appendix D with $A = 17$?
    b. Which of these are stable nuclei?
    c. For those that are not stable, identify both the decay mode and the daughter nucleus.

52. ||| What is the activity in Bq and in Ci of a 2.0 mg sample of ^{3}H?

53. || The activity of a sample of the cesium isotope ^{137}Cs is 2.0×10^8 Bq. Many years later, after the sample has fully decayed, how many beta particles will have been emitted?

54. ||| A 115 mCi sample of a radioactive isotope is made in a reactor. When delivered to a hospital 16 hours later, its activity is 95 mCi. The lowest usable activity level is 10 mCi.
    a. What is the isotope's half-life?
    b. For how long after delivery is the sample usable?

55. ||| BIO You are assisting in an anthropology lab over the summer by carrying out ^{14}C dating. A graduate student found a bone he believes to be 20,000 years old. You extract the carbon from the bone and prepare an equal-mass sample of carbon from modern organic material. To determine the activity of a sample with the accuracy your supervisor demands, you need to measure the time it takes for 10,000 decays to occur.
    a. The activity of the modern sample is 1.06 Bq. How long does that measurement take?
    b. It turns out that the graduate student's estimate of the bone's age was accurate. How long does it take to measure the activity of the ancient carbon?

56. |||| BIO A sample of wood from an archaeological excavation is dated by using a mass spectrometer to measure the fraction of ^{14}C atoms. Suppose 100 atoms of ^{14}C are found for every 1.0×10^{15} atoms of ^{12}C in the sample. What is the wood's age?

57. || A sample of 1.0×10^{10} atoms that decay by alpha emission has a half-life of 100 min. How many alpha particles are emitted between $t = 50$ min and $t = 200$ min?

58. || A sample contains radioactive atoms of two types, A and B. Initially there are five times as many A atoms as there are B atoms. Two hours later, the numbers of the two atoms are equal. The half-life of A is 0.50 hours. What is the half-life of B?

59. |||| The technique known as potassium-argon dating is used to date old lava flows and thus any fossilized skeletons found in them, like this 1.8-million-year old hominid skull. The potassium isotope ^{40}K has a 1.28-billion-year half-life and is naturally present at very low levels. ^{40}K decays by beta emission into the stable isotope ^{40}Ar. Argon is a gas, and there is no argon in flowing lava because the gas escapes. Once the lava solidifies, any argon produced in the decay of ^{40}K is trapped inside and cannot escape. What is the age of a piece of solidified lava with a ^{40}Ar/^{40}K ratio of 0.12?

60. |||| Corals take up certain elements from seawater, including uranium but not thorium. After the corals die, the uranium isotopes slowly decay into thorium isotopes. A measurement of the relative fraction of certain isotopes therefore provides a determination of the coral's age. A complicating factor is that the thorium isotopes decay as well. One scheme uses the alpha decay of ^{234}U to ^{230}Th. After a long time, the two species reach an equilibrium in which the number of ^{234}U decays per second (each producing an atom of ^{230}Th) is exactly equal to the number of ^{230}Th decays per second. What is the relative concentration of the two isotopes—the ratio of ^{234}U to ^{230}Th—when this equilibrium is reached?
BIO

61. ||| All the very heavy atoms found in the earth were created long ago by nuclear fusion reactions in a supernova, an exploding star. The debris spewed out by the supernova later coalesced to form the sun and the planets of our solar system. Nuclear physics suggests that the uranium isotopes ^{235}U ($t_{1/2} = 7.04 \times 10^8$ yr) and ^{238}U ($t_{1/2} = 4.47 \times 10^9$ yr) should have been created in roughly equal amounts. Today, 99.28% of uranium is ^{238}U and 0.72% is ^{235}U. How long ago did the supernova occur?

62. ||| ^{235}U decays to ^{207}Pb via the decay series shown in Figure 30.16. The first decay in the chain, that of ^{235}U, has a half-life of 7.0×10^8 years. The subsequent decays are much more rapid, so we can take this as the half-life for the decay of ^{235}U to ^{207}Pb. ^{238}U decays, with a half-life of 4.5×10^9 years, via a similar decay series that ends in a different lead isotope, ^{206}Pb. Again, the subsequent decays are much more rapid, so we can take this as the half-life for the decay of ^{238}U to ^{206}Pb. The two uranium decay chains can be used to precisely determine the age of certain minerals that exclude lead from their crystal structure but easily incorporate uranium. When these minerals form, they contain both ^{238}U and ^{235}U, but no lead. As time goes on, the isotopes of uranium decay, producing isotopes of lead. A measurement of the ^{238}U/^{206}Pb ratio allows a determination of the age—which can be checked using a measurement of the ^{235}U/^{207}Pb ratio. If 75% of the ^{235}U present in a particular rock has decayed to lead, what percent of the ^{238}U has decayed to lead?

63. || A 75 kg patient swallows a 30 μCi beta emitter with a half-life of 5.0 days. The beta particles are emitted with an average energy of 0.35 MeV. Ninety percent of the beta particles are absorbed within the patient's body and 10% escape. What dose equivalent does the patient receive?
BIO

64. ||| About 12% of your body mass is carbon; some of this is radioactive ^{14}C, a beta-emitter. If you absorb 100% of the 49 keV energy of each ^{14}C decay, what dose equivalent in Sv do you receive each year from the ^{14}C in your body?
BIO INT

65. ||||| Ground beef may be irradiated with high-energy electrons from a linear accelerator to kill pathogens. In a standard treatment, 1.0 kg of beef receives 4.5 kGy of radiation in 40 s.
BIO INT
   a. How much energy is deposited in the beef?
   b. What is the average rate (in W) of energy deposition?
   c. Estimate the temperature increase of the beef due to this procedure. The specific heat of beef is approximately 3/4 of that of water.

66. ||| A 70 kg human body typically contains 140 g of potassium. Potassium has a chemical atomic mass of 39.1 u and has three naturally occurring isotopes. One of those isotopes, ^{40}K, is radioactive with a half-life of 1.3 billion years and a natural abundance of 0.012%. Each ^{40}K decay deposits, on average, 1.0 MeV of energy into the body. What yearly dose in Gy does the typical person receive from the decay of ^{40}K in the body?
BIO

67. |||| What dose in rads of gamma radiation must be absorbed by a block of ice at 0°C to transform the entire block to liquid water at 0°C?
INT

68. || A chest x ray uses 10 keV photons. A 60 kg person receives a 30 mrem dose from one x ray that exposes 25% of the patient's body. How many x-ray photons are absorbed in the patient's body?
BIO

69. |||| The plutonium isotope ^{239}Pu has a half-life of 24,000 years and decays by the emission of a 5.2 MeV alpha particle. Plutonium is not especially dangerous if handled because the activity is low and the alpha radiation doesn't penetrate the skin. But the tiniest speck of plutonium can cause problems if it is inhaled and lodges deep in the lungs. Let's see why.
BIO
   a. Soot particles are roughly 1 μm in diameter, and it is known that these particles can go deep into the lungs. How many ^{239}Pu atoms are in a 1.0-μm-diameter particle of ^{239}Pu? The density of plutonium is 19,800 kg/m^3.
   b. What is the activity, in Bq, of this 1.0-μm-diameter particle?
   c. The activity of the particle is very small, but the penetrating power of alpha particles is also very small, so the damage is concentrated. The alpha particles deposit their energy in a 50-μm-diameter sphere around the plutonium particle. In one year, what is the dose equivalent in mSv to this small sphere of tissue in the lungs? Assume that the tissue density is that of water.
   d. How does the exposure to this tissue compare to the natural background exposure?

70. ||| Uranium is naturally present at low levels in many soils and rocks. The ^{238}U decay series includes the short-lived radon isotope ^{222}Rn, with $t_{1/2} = 3.82$ days. Radon is a gas, and it can seep into basements. The Environmental Protection Agency recommends that homeowners take steps to remove radon if the radon activity exceeds 4 pCi per liter of air. The daughter nuclei from radon decay are of significant concern, but the radon itself does provide some exposure.
BIO
   a. How many ^{222}Rn atoms are there in 1.0 m^3 of air if the activity is 4.0 pCi/L?
   b. The range of alpha particles in air is 3 cm. Let's model a person as a 180-cm-tall, 25-cm-diameter cylinder with a mass of 65 kg. Only decays within 3 cm of the cylinder cause exposure, and only 50% of the decays direct the alpha particle toward the person. What is the dose equivalent (in mSv) for a person who spends an entire year in a room where the activity is 4 pCi/L?

Passage Problems

Nuclear Fission

The uranium isotope ^{235}U can *fission*—break into two smaller-mass components and free neutrons—if it is struck by a free neutron. A typical reaction is

$$^{1}_{0}\text{n} + ^{235}_{92}\text{U} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3^{1}_{0}\text{n}$$

As you can see, the subscripts (the number of protons) and the superscripts (the number of nucleons) "balance" before and after the fission event; there is no change in the number of protons or neutrons. Significant energy is released in this reaction. If a fission event happens in a large chunk of ^{235}U, the neutrons released may induce the fission of other ^{235}U atoms, resulting in a chain reaction. This is how a nuclear reactor works.

The number of neutrons required to create a stable nucleus increases with atomic number. When the heavy ^{235}U nucleus fissions, the lighter reaction products are thus neutron rich and are likely unstable. Many of the short-lived radioactive nuclei used in medicine are produced in fission reactions in nuclear reactors.

71. | What statement can be made about the masses of atoms in the above reaction?
    A. $m(^{235}_{92}\text{U}) > m(^{141}_{56}\text{Ba}) + m(^{92}_{36}\text{Kr}) + 2m(^{1}_{0}\text{n})$
    B. $m(^{235}_{92}\text{U}) < m(^{141}_{56}\text{Ba}) + m(^{92}_{36}\text{Kr}) + 2m(^{1}_{0}\text{n})$
    C. $m(^{235}_{92}\text{U}) = m(^{141}_{56}\text{Ba}) + m(^{92}_{36}\text{Kr}) + 2m(^{1}_{0}\text{n})$
    D. $m(^{235}_{92}\text{U}) = m(^{141}_{56}\text{Ba}) + m(^{92}_{36}\text{Kr}) + 3m(^{1}_{0}\text{n})$
72. | Because the decay products in the above fission reaction are neutron rich, they will likely decay by what process?
    A. Alpha decay
    B. Beta decay
    C. Gamma decay
73. | ^{235}U is radioactive, with a long half-life of 704 million years. The decay products of a ^{235}U fission reaction typically have half-lives of a few minutes. This means that the decay products of a fission reaction have
    A. Much higher activity than the original uranium.
    B. Much lower activity than the original uranium.
    C. The same activity as the original uranium.
74. | If a $^{238}_{92}$U nucleus is struck by a neutron, it may absorb the neutron. The resulting nucleus then rapidly undergoes beta-minus decay. The daughter nucleus of that decay is
    A. $^{239}_{91}$Pa B. $^{239}_{92}$U C. $^{239}_{93}$Np D. $^{239}_{94}$Pu

STOP TO THINK ANSWERS

Stop to Think 30.1: Three. Different isotopes of an element have different numbers of neutrons but the same number of protons. The number of electrons in a neutral atom matches the number of protons.

Stop to Think 30.2: Yes. ^{12}C has filled levels of protons and neutrons; the neutron we add to make ^{13}C will be in a higher energy level, but there is no "hole" in a lower level for it to move to, so we expect this nucleus to be stable.

Stop to Think 30.3: B. An increase of Z with no change in A occurs when a neutron changes to a proton and an electron, ejecting the electron.

Stop to Think 30.4: C. One-quarter of the atoms are left. This is one-half of one-half, or $(1/2)^2$, so two half-lives have elapsed.

Stop to Think 30.5: A. Dose equivalent is the product of dose in Gy (the same for each) and RBE (highest for alpha particles).

PART VII SUMMARY

Modern Physics

A common theme runs through the final chapters of this book: Nature is stranger than we thought it was. From the bizarre paradoxes of special relativity to the dizzying array of subatomic particles, the physical description of the world around us has taken on an almost science-fiction air in Part VII. But this material isn't science fiction; it's real, the product of decades of careful experiments.

Relativity requires us to stretch our notions of space and time. Time really does slow down for particles moving at high speeds, as decades of experiments have shown. Classical Newtonian physics has a comforting predictability, but this is left behind in quantum theory. We simply can't know an electron's position and velocity at the same time. Many decades of clever experiments have shown conclusively that *no* underlying laws can restore the predictability of classical physics at the atomic scale. And nuclear physics has shown that the alchemist's dream is true—you *can* turn one element into another!

This new physics is surprisingly practical. Relativistic corrections allow GPS systems to give extraordinarily accurate measurements of your position anywhere on the earth. Quantum mechanics is the theory underlying the development of computer chips and other modern electronics. And your smoke detector probably contains a small amount of an element not found in nature, an element created in a nuclear reactor.

As we conclude our journey, the knowledge structure for Part VII summarizes the key ideas of relativity, quantum physics, and nuclear physics. These are the theories behind the emerging technologies of the 21st century.

KNOWLEDGE STRUCTURE V Modern Physics

BASIC GOALS

What are the properties and characteristics of space and time?
What do we know about the nature of light and atoms?
How are atomic and nuclear phenomena explained by energy levels, wave functions, and photons?

GENERAL PRINCIPLES

Principle of relativity	All the laws of physics are the same in all inertial reference frames.
Quantization of energy	Particles of matter and photons of light have only certain allowed energies.
Uncertainty principle	$\Delta x \, \Delta p \geq h/2\pi$
Pauli exclusion principle	No more than one electron can occupy the same quantum state.

Relativity

- The speed of light c is the same in all inertial reference frames.
- No particle or causal influence can travel faster than c.
- Length contraction: The length of an object in a reference frame in which the object moves with speed v is

$$L = \sqrt{1-\beta^2}\,\ell \leq \ell$$

  where ℓ is the proper length.
- Time dilation: The proper time interval $\Delta\tau$ between two events is measured in a reference frame in which the two events occur at the same position. The time interval Δt in a frame moving with relative speed v is

$$\Delta t = \Delta\tau/\sqrt{1-\beta^2} \geq \Delta\tau$$

- Particles have energy even when at rest. Mass can be transformed into energy and vice versa: $E_0 = mc^2$.

Quantum physics

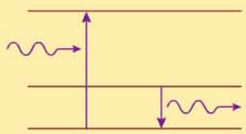
- Matter has wave-like properties. A particle has a de Broglie wavelength:

$$\lambda = \frac{h}{mv}$$

- Light has particle-like properties. A photon of light of frequency f has energy:

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

- The wave nature of matter leads to quantized energy levels in atoms and nuclei. A transition between quantized energy levels involves the emission or absorption of a photon.

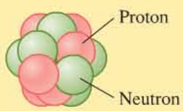


Properties of atoms

- Quantized energy levels depend on quantum numbers n and l.
- An atom can jump from one state to another by emitting or absorbing a photon of energy $E_{\text{photon}} = \Delta E_{\text{atom}}$.
- The ground-state electron configuration is the lowest-energy configuration consistent with the Pauli principle.

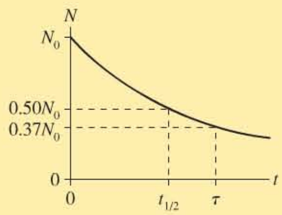
Properties of nuclei

- The nucleus is the small, dense, positive core at the center of an atom. The nucleus is held together by the strong force, an attractive short-range force between any two nucleons.



- Unstable nuclei decay by alpha, beta, or gamma decay. The number of nuclei decreases exponentially with time:

$$N = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$$



The Physics of Very Cold Atoms

Modern physics is a study of extremes. Relativity deals with the physics of objects traveling at near-light speeds. Quantum mechanics is about the physics of matter and energy at very small scales. Nuclear physics involves energies that dwarf anything dreamed of in previous centuries.

Some of the most remarkable discoveries of recent years are at another extreme—very low temperatures, mere billionths of a degree above absolute zero. Let's look at how such temperatures are achieved and some new physics that emerges.

You learned in Part III that the temperature of a gas depends on the speeds of the atoms in the gas. Suppose we start with atoms at or above room temperature. Cooling the gas means slowing the atoms down. How can we drastically reduce their speeds, bringing them nearly to a halt? The trick is to slow them, thus cooling them, using the interactions between light and atoms that we explored in Chapters 28 and 29.

Photons have momentum, and that momentum is transferred to an atom when a photon is absorbed. Part a of the figure shows an atom moving "upstream" against a laser beam tuned to an atomic transition. Photon absorptions transfer momentum, slowing the atom. Subsequent photon emissions give the atom a "kick," but in random directions, so on average the emissions won't speed up the atom in the same way that the absorptions slow it. A beam of atoms moving "upstream" against a correctly tuned laser beam is slowed down—the "hot" beam of atoms is cooled.

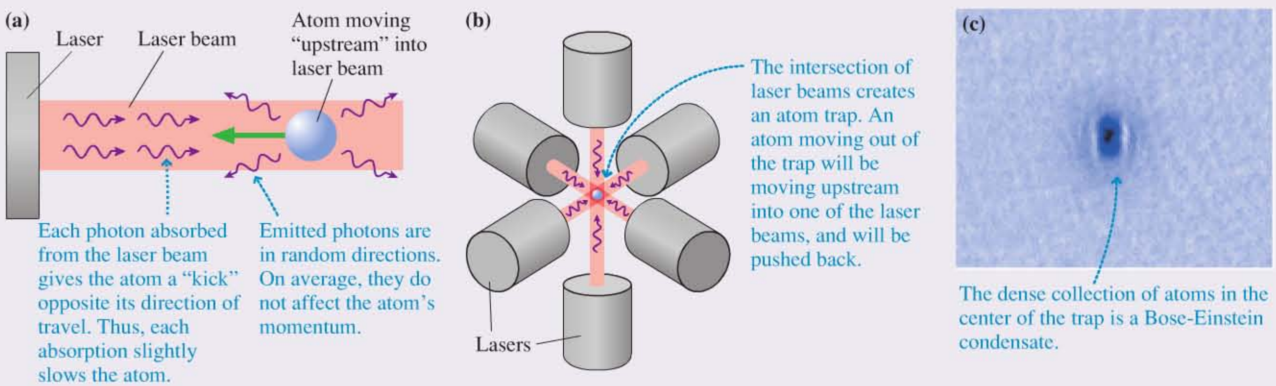
Once a laser cools the atoms, a different configuration of laser beams can trap them. Part b of the figure shows six overlapped laser beams, each tuned slightly *below* the frequency of an atomic absorption line. If an atom tries to leave the overlap region, it will be moving "upstream" against one of the laser beams. The atom will see that laser beam Doppler-shifted to a higher frequency, matching the transition frequency. The atom will then absorb photons from this laser beam, and the resulting kick will nudge it back into the overlap region. The atoms are trapped in what is known as *optical molasses* or, more generally, an *atom trap.* More effective traps can be made by adding magnetic fields. The final cooling of the atoms is by evaporation—letting the more energetic atoms leave the trap.

Ultimately, these techniques produce a diffuse gas of atoms moving at only $\approx$1 mm/s. This corresponds to a nearly unbelievable temperature of just a few nanokelvin—billionths of a degree above absolute zero. This is colder than outer space. The coldest spot in the universe is inside an atom trap in a physics lab.

Once the atoms are cooled, some very remarkable things happen. As we saw in Part VII, all particles, including atoms, have a wave nature. As the atoms slow, their wavelengths increase. In a correctly prepared gas at a low enough temperature, the de Broglie wavelength of an individual atom is larger than the spacing between atoms, and the wave functions of multiple atoms overlap. When this happens, some atoms undergo *Bose-Einstein condensation,* coalescing into one "super atom," with thousands of atoms occupying the same quantum state. An example of the resulting Bose-Einstein condensate is shown in part c of the figure. In the condensate, the atoms—that is, their wave functions—are all in the same place at the same time! This truly bizarre state of matter is a remarkable example of the counterintuitive nature of the quantum world.

Are there applications for Bose-Einstein condensation? Current talk of atom lasers and other futuristic concepts aside, no one really knows, just as the early architects of quantum mechanics didn't know that their theory would be used to design the chips that power personal computers.

At the start of the 20th century, there was a worry that everything in physics had been discovered. There is no such worry at the start of the 21st century, which promises to be full of wonderful discoveries and remarkable applications. What do you imagine the final chapter of a physics textbook will look like 100 years from now?



Laser cooling and trapping.

PART VII PROBLEMS

The following questions are related to the passage "The Physics of Very Cold Atoms" on the preceding page.

1. Why is it useful to create a very slow-moving assembly of cold atoms?
   A. The atoms can be more easily observed at slow speeds.
   B. Lowering the temperature this way permits isotopes that normally decay in very short times to persist long enough to be studied.
   C. At low speeds the quantum nature of the atoms becomes more apparent, and new forms of matter emerge.
   D. At low speeds the quantum nature of the atoms becomes less important, and they appear more like classical particles.
2. The momentum of a photon is given by $p = h/\lambda$. Suppose an atom emits a photon. Which of the following photons will give the atom the biggest "kick"—the highest recoil speed?
   A. An infrared photon
   B. A red-light photon
   C. A blue-light photon
   D. An ultraviolet photon
3. When an atom moves "upstream" against the photons in a laser beam, the energy of the photons appears to be ________ if the atom were at rest.
   A. Greater than
   B. Less than
   C. The same as
4. A gas of cold atoms strongly absorbs light of a specific wavelength. Warming the gas causes the absorption to decrease. Which of the following is the best explanation for this reduction?
   A. Warming the gas changes the atomic energy levels.
   B. Warming the gas causes the atoms to move at higher speeds, so the atoms "see" the photons at larger Doppler shifts.
   C. Warming the gas causes more collisions between the atoms, which affects the absorption of photons.
   D. Warming the gas makes it more opaque to the photons, so fewer enter the gas.
5. A gas of cold atoms starts at a temperature of 100 nK. The average speed of the atoms is then reduced by half. What is the new temperature?
   A. 71 nK B. 50 nK
   C. 37 nK D. 25 nK
6. Rubidium is often used for the type of experiments noted in the passage. At a speed of 1.0 mm/s, what is the approximate de Broglie wavelength of an atom of ^{87}Rb?
   A. 5 nm B. 50 nm
   C. 500 nm D. 5000 nm
7. A gas of rubidium atoms and a gas of sodium atoms have been cooled to the same very low temperature. What can we say about the de Broglie wavelengths of typical atoms in the two gases?
   A. The sodium atoms have the longer wavelength.
   B. The wavelengths are the same.
   C. The rubidium atoms have the longer wavelength.

The following passages and associated questions are based on the material of Part VII.

Splitting the Atom

"Splitting" an atom in the process of nuclear fission releases a great deal of energy. If all the atoms in 1 kg of ^{235}U undergo nuclear fission, 8.0×10^{13} J will be released, equal to the energy from burning 2.3×10^{6} kg of coal. What is the source of this energy? Surprisingly, the energy from this nuclear disintegration ultimately comes from the electric potential of the positive charges that make up the nucleus.

The protons in a nucleus exert repulsive forces on each other, but this force is less than the short-range attractive nuclear force. If a nucleus breaks into two smaller nuclei, the nuclear force will hold each of the fragments together, but it won't bind the two positively charged fragments to each other. This is illustrated in Figure VII.1. The two fragments feel a strong repulsive electrostatic force. The charges are large and the distance is small (roughly equal to the sum of the radius of each of the fragments), so the force—and thus the potential energy—is quite large.

In a fission reaction, a neutron causes a nucleus of ^{235}U to split into two smaller nuclei; a typical reaction is

$$n + {}^{235}\text{U} \rightarrow {}^{87}\text{Br} + {}^{147}\text{La} + \text{neutrons} + \approx 200 \text{ MeV}$$

Right after the nucleus splits, with only the electric force now acting on the two fragments, the electrostatic potential energy of the two fragments is

$$U = \frac{kq_1q_2}{r_1 + r_2}$$

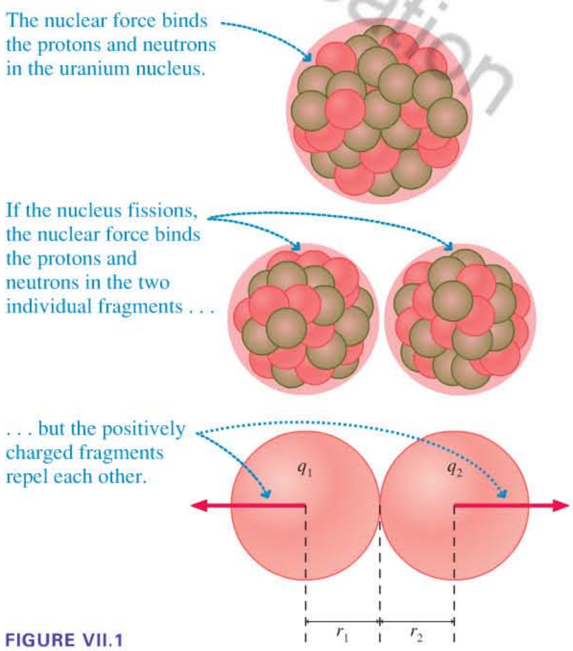


FIGURE VII.1

This is the energy that will be released, transformed into kinetic energy, when the fragments fly apart. If we use reasonable estimates for the radii of the two fragments, we compute a value for the energy that is close to the experimentally observed value of 200 MeV for the energy released in the fission reaction. The energy released in this *nuclear* reaction is actually *electric* potential energy.

8. How many neutrons are "left over" in the noted fission reaction?
   A. 1 B. 2 C. 3 D. 4
9. After a fission event, most of the energy released is in the form of
   A. Emitted beta particles and gamma rays.
   B. Kinetic energy of the emitted neutrons.
   C. Nuclear energy of the two fragments.
   D. Kinetic energy of the two fragments.
10. Suppose the original nucleus is at rest in the fission reaction noted above. If we neglect the momentum of the neutrons, after the two fragments fly apart,
   A. The Br nucleus has more momentum.
   B. The La nucleus has more momentum.
   C. The momentum of the Br nucleus equals that of the La nucleus.
11. Suppose the original nucleus is at rest in the fission reaction noted above. If we neglect the kinetic energy of the neutrons, after the two fragments fly apart,
   A. The Br nucleus has more kinetic energy.
   B. The La nucleus has more kinetic energy.
   C. The kinetic energy of the Br nucleus equals that of the La nucleus.
12. 200 MeV is a typical energy released in a fission reaction. To get a sense for the scale of the energy, if we were to use this energy to create electron-positron pairs, approximately how many pairs could we create?
   A. 50 B. 100 C. 200 D. 400
13. The two fragments of a fission reaction are isotopes that are neutron-rich; each has more neutrons than the stable isotopes for their nuclear species. They will quickly decay to more stable isotopes. What is the most likely decay mode?
   A. Alpha decay
   B. Beta decay
   C. Gamma decay

Additional Integrated Problems

14. The glow-in-the-dark dials on some watches and some keychain lights shine with energy provided by the decay of radioactive tritium, ^3_1H. Tritium is a radioactive isotope of hydrogen with a half-life of 12 years. Each decay emits an electron with an energy of 19 keV. A typical new watch has tritium with a total activity of 15 MBq.


A keychain light powered by the decay of tritium.

   a. What is the speed of the emitted electron? (This speed is high enough that you'll need to do a relativistic calculation.)
   b. What is the power, in watts, provided by the radioactive decay process?
   c. What will be the activity of the tritium in a watch after 5 years, assuming none escapes?
15. An x-ray tube is powered by a high-voltage supply that delivers 700 W to the tube. The tube converts 1% of this power into x rays of wavelength 0.030 nm.
BIO
   a. Approximately how many x-ray photons are emitted per second?
   b. If a 75 kg technician is accidentally exposed to the full power of the x-ray beam for 1.0 s, what dose equivalent in Sv does he receive? Assume that the x-ray energy is distributed over the body, and that 80% of the energy is absorbed.
16. Many speculative plans for spaceships capable of interstellar travel have been developed over the years. Nearly all are powered by the fusion of light nuclei, one of a very few power sources capable of providing the incredibly large energies required. A typical design for a fusion-powered craft has a 1.7×10^6 kg ship brought up to a speed of $0.12c$ using the energy from the fusion of ^2_1H and ^3_2He. Each fusion reaction produces a daughter nucleus and one free proton with a combined kinetic energy of 18 MeV; these high-speed particles are directed backward to create thrust.
   a. What is the kinetic energy of the ship at the noted top speed? For the purposes of this problem you can do a nonrelativistic calculation.
   b. If we assume that 50% of the energy of the fusion reactions goes into the kinetic energy of the ship (a very generous assumption), how many fusion reactions are required to get the ship up to speed?
   c. How many kilograms of ^2_1H and of ^3_2He are required to produce the required number of reactions?
17. A muon is a lepton that is a higher-mass (rest mass 105 MeV/c^2) sibling to the electron. Muons are produced in the upper atmosphere when incoming cosmic rays collide with the nuclei of gas molecules. As the muons travel toward the surface of the earth, they lose energy. A muon that travels from the upper atmosphere to the surface of the earth typically begins with kinetic energy 6.0 GeV and reaches the surface of the earth with kinetic energy 4.0 GeV. The energy decreases by one-third of its initial value. By what fraction does the speed of the muon decrease?
18. A muon is a lepton that is a higher-mass (rest mass 105 MeV/c^2) sibling to the electron. Muons are produced in the upper atmosphere when incoming cosmic rays collide with the nuclei of gas molecules. The muon half-life is 1.5 μs, but atmospheric muons typically live much longer than this because of time dilation, as we saw in Chapter 27. Suppose 100,000 muons are created 120 km above the surface of the earth, each with kinetic energy 10 GeV. Assume that the muons don't lose energy but move at a constant velocity directed straight down toward the surface of the earth. How many muons survive to reach the surface?