2 Motion in One Dimension

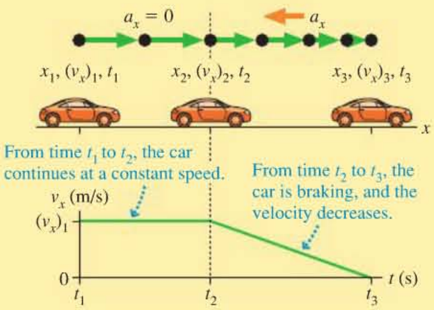
A horse can run at 35 mph, much faster than a human. And yet, surprisingly, a man can win a race against a horse if the length of the course is right. When, and how, can a man outrun a horse?

LOOKING AHEAD ▸▸

The goal of Chapter 2 is to describe and analyze linear motion.

Describing Motion

We began discussing motion in Chapter 1. In Chapter 2, you'll learn more ways to represent motion. You will also learn general strategies for solving problems.



Motion diagrams and graphs are key parts of the problem-solving strategies that we will develop in this chapter.

Looking Back ◂◂
1.5 Velocity vectors and motion diagrams

Position

Position is defined in terms of a coordinate system and units of our choosing.

A game of football is really about motion in one dimension with a well-defined coordinate system.

Looking Back ◂◂
1.2 Position and displacement

Velocity

Velocity is the rate of change of position.

A small change in position during an interval of time means a small velocity; a larger change means a larger velocity.

Looking Back ◂◂
1.3 Velocity

Acceleration

Acceleration is the rate of change of velocity.

A cheetah is capable of a rapid change in velocity—that is a large acceleration. We'll see how to solve problems of changing velocity by using the concept of acceleration.

Analyzing Motion

Once you know how to describe motion, you'll be ready to do some analysis.

The main engines of a Saturn V fire for $2^1/_2$ minutes. How high will the rocket be and how fast will it be going when the engines shut off?

Constant Velocity

One important case we will consider is motion in a straight line at a constant velocity— uniform motion.

Each minute, the ship moves the same distance in the same direction.

Constant Acceleration

Another special case is motion with constant acceleration.

We think of acceleration as "speeding up," but braking to a stop involves a change in velocity—an acceleration—as well.

Free Fall

Free fall is a special case of constant acceleration.

Once the coin leaves your hand, it's in free fall, and its motion is similar to that of a falling ball or a jumping gazelle.

2.1 Describing Motion

The modern name for the mathematical description of motion, without regard to causes, is **kinematics.** The term comes from the Greek word *kinema,* meaning "movement." You know this word through its English variation *cinema*—motion pictures! This chapter will focus on the kinematics of motion in one dimension, motion along a straight line.

Representing Position

As we saw in Chapter 1, kinematic variables such as position and velocity are measured with respect to a coordinate system, an axis that *you* impose on a system. We will use an x-axis to analyze both horizontal motion and motion on a ramp; a y-axis will be used for vertical motion. We will adopt the convention that the positive end of an x-axis is to the right and the positive end of a y-axis is up. This convention is illustrated in **FIGURE 2.1**.

Now, let's look at a practical problem of the sort that we first saw in Chapter 1. **FIGURE 2.2** is a motion diagram, made at 1 frame per minute, of a straightforward situation: A student walking to school. She is moving horizontally, so we use the variable x to describe her motion. We have set the origin of the coordinate system, $x = 0$, at her starting position, and we measure her position in meters. We have included velocity vectors connecting successive positions on the motion diagram, as we saw we could do in Chapter 1. The motion diagram shows that she leaves home at a time we choose to call $t = 0$ min, and then makes steady progress for a while. Beginning at $t = 3$ min there is a period in which the distance traveled during each time interval becomes shorter—perhaps she slowed down to speak with a friend. Then, at $t = 6$ min, the distances traveled within each interval are longer—perhaps, realizing she is running late, she begins walking more quickly.

FIGURE 2.1 Sign conventions for position.

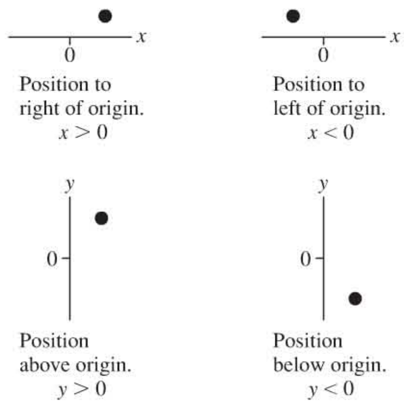


FIGURE 2.2 The motion diagram of a student walking to school and a coordinate axis for making measurements.

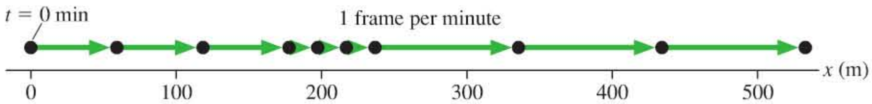


TABLE 2.1 Measured positions of a student walking to school

Time t (min)	Position x (m)	Time t (min)	Position x (m)
0	0	5	220
1	60	6	240
2	120	7	340
3	180	8	440
4	200	9	540

Every dot in the motion diagram of Figure 2.2 represents the student's position at a particular time. For example, the student is at position $x = 120$ m at $t = 2$ min. Table 2.1 lists her position for every point in the motion diagram.

The motion diagram of Figure 2.2 is one way to represent the student's motion. Presenting the data as in Table 2.1 is a second way to represent this motion. A third way to represent the motion is to make a graph. **FIGURE 2.3** is a graph of the positions of the student at different times; we say it is a graph of x versus t for the student. We have merely taken the data from the table and plotted these particular points on the graph.

NOTE ► A graph of "a versus b" means that a is graphed on the vertical axis and b on the horizontal axis. ◄

We can flesh out the graph of Figure 2.3, though. Common sense tells us that the student was *somewhere specific* at all times: There was never a time when she failed to have a well-defined position, nor could she occupy two positions at one time. (As reasonable as this belief appears to be, we'll see that it's not entirely accurate when we get to quantum physics!) We also can assume that, from the start to the end of her motion, the student moved *continuously* through all intervening points of space, so

FIGURE 2.3 A graph of the student's motion.

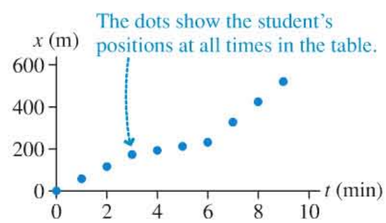
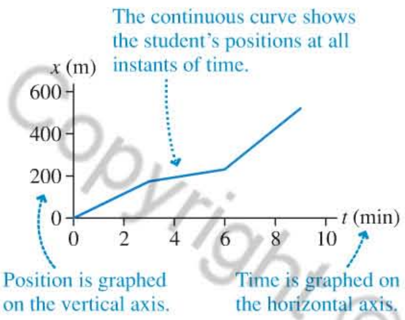


FIGURE 2.4 Extending the graph of Figure 2.3 to a position-versus-time graph.



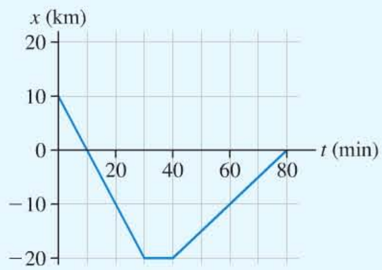
we can represent her motion as a continuous curve that passes through the measured points, as shown in FIGURE 2.4. Such a continuous curve that shows an object's position as a function of time is called a **position-versus-time graph** or, sometimes, just a *position graph.*

NOTE ► A graph is *not* a "picture" of the motion. The student is walking along a straight line, but the graph itself is not a straight line. Further, we've graphed her position on the vertical axis even though her motion is horizontal. A graph is an *abstract representation* of motion. We will place significant emphasis on the process of interpreting graphs, and many of the exercises and problems will give you a chance to practice these skills. ◄

CONCEPTUAL EXAMPLE 2.1 Interpreting a car's position-versus-time graph

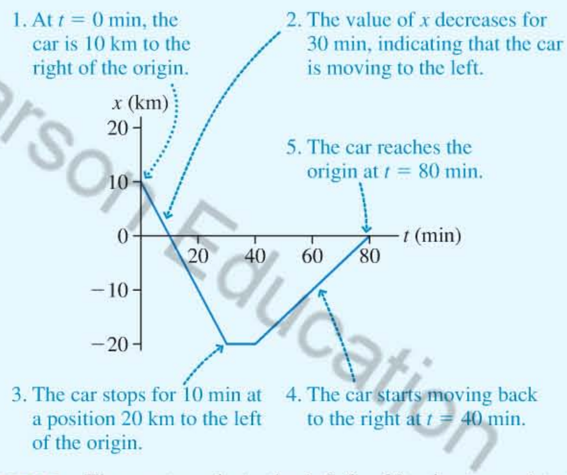
The graph in FIGURE 2.5 represents the motion of a car along a straight road. Describe (in words) the motion of the car.

FIGURE 2.5 Position-versus-time graph for the car.



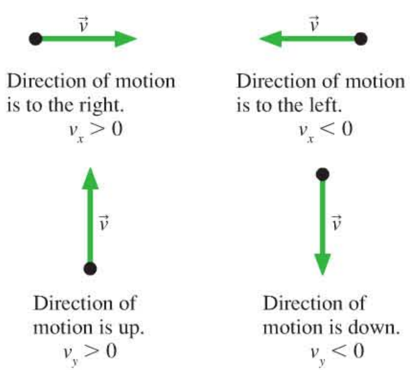
REASON The vertical axis in Figure 2.5 is labeled "x (km)"; position is measured in kilometers. Our convention for motion along the x-axis given in Figure 2.1 tells us that x increases as the car moves to the right and x decreases as the car moves to the left. The graph thus shows that the car travels to the left for 30 minutes, stops for 10 minutes, then travels to the right for 40 minutes. It ends up 10 km to the left of where it began. FIGURE 2.6 gives a full explanation of the reasoning.

FIGURE 2.6 Looking at the position-versus-time graph in detail.



ASSESS The car travels to the left for 30 minutes and to the right for 40 minutes. Nonetheless, it ends up to the left of where it started. This means that the car was moving faster when it was moving to the left than when it was moving to the right. We can deduce this fact from the graph as well, as we will see in the next section.

FIGURE 2.7 Sign conventions for velocity.



Representing Velocity

Velocity is a vector; it has both a magnitude and a direction. When we draw a velocity vector on a diagram, we use an arrow labeled with the symbol $\vec{v}$ to represent the magnitude and the direction. For motion in one dimension, vectors are restricted to point only "forward" or "backward" for horizontal motion (or "up" or "down" for vertical motion). This restriction lets us simplify our notation for vectors in one dimension. When we solve problems for motion along an x-axis, we will represent velocity with the symbol v_x. v_x will be positive or negative, corresponding to motion to the right or the left, as shown in FIGURE 2.7. For motion along a y-axis, we will use the symbol v_y to represent the velocity; the sign conventions are also illustrated in Figure 2.7. We will use the symbol v, with no subscript, to represent the speed of an object. **Speed is the *magnitude* of the velocity vector** and is always positive.

For motion along a line, the definition of velocity from Chapter 1 can be written as

$$v_x = \frac{\Delta x}{\Delta t} \tag{2.1}$$

This agrees with the sign conventions in Figure 2.7. If Δx is positive, x is increasing, the object is moving to the right, and Equation 2.1 gives a positive value for velocity. If Δx is negative, x is decreasing, the object is moving to the left, and Equation 2.1 gives a negative value for velocity.

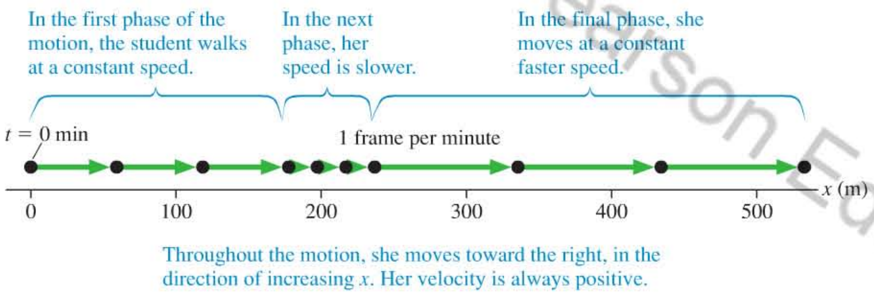
Equation 2.1 is the first of many kinematic equations we'll see in this chapter. We'll often specify equations in terms of the coordinate x, but if the motion is vertical, in which case we use the coordinate y, the equations can be easily adapted. For example, Equation 2.1 for motion along a vertical axis becomes

$$v_y = \frac{\Delta y}{\Delta t} \tag{2.2}$$

From Position to Velocity

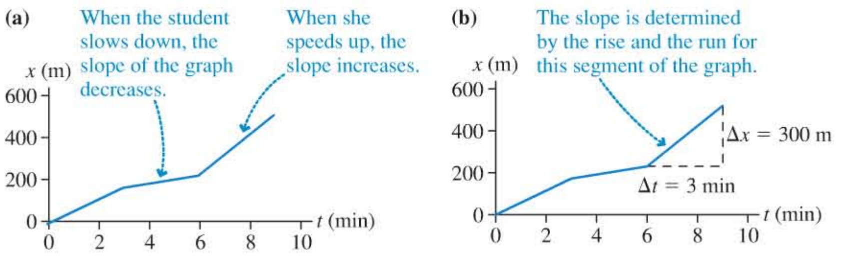
Let's take another look at the motion diagram of the student walking to school. As we see in FIGURE 2.8, where we have repeated the motion diagram of Figure 2.2, her motion has three clearly defined phases. In each phase her speed is constant (because the velocity vectors have the same length) but the speed varies from phase to phase.

FIGURE 2.8 Revisiting the motion diagram of the student walking to school.



Her motion has three different phases; similarly, the position-versus-time graph redrawn in FIGURE 2.9a has three clearly defined segments with three different slopes. Looking at the different segments of the graph, we can see that there's a relationship between her speed and the slope of the graph: **A faster speed corresponds to a steeper slope.**

FIGURE 2.9 Revisiting the graph of the motion of the student walking to school.



The correspondence is actually deeper than this. Let's look at the slope of the third segment of the position-versus-time graph, as shown in FIGURE 2.9b. The slope of a graph is defined as the ratio of the "rise," the vertical change, to the "run," the horizontal change. For the segment of the graph shown, the slope is:

$$\text{slope of graph} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$$

This ratio has a physical meaning—it's the velocity, exactly as we defined it in Equation 2.1. We've shown this correspondence for one particular graph, but it is a

Time lines BIO This section of the trunk of a pine tree shows the light bands of spring growth and the dark bands of summer and fall growth in successive years. If you focus on the spacing of successive dark bands, you can think of this picture as a motion diagram for the tree, representing its growth in diameter. The years of rapid growth (large distance between dark bands) during wet years and slow growth (small distance between dark bands) during years of drought are readily apparent.

general principle: **The slope of an object's position-versus-time graph is the object's velocity at that point in the motion.** This principle also holds for negative slopes, which correspond to negative velocities. We can associate the slope of a position-versus-time graph, a *geometrical* quantity, with velocity, a *physical* quantity. This is an important aspect of interpreting position-versus-time graphs, as outlined in Tactics Box 2.1.

TACTICS BOX 2.1 Interpreting position-versus-time graphs MP™

Information about motion can be obtained from position-versus-time graphs as follows:

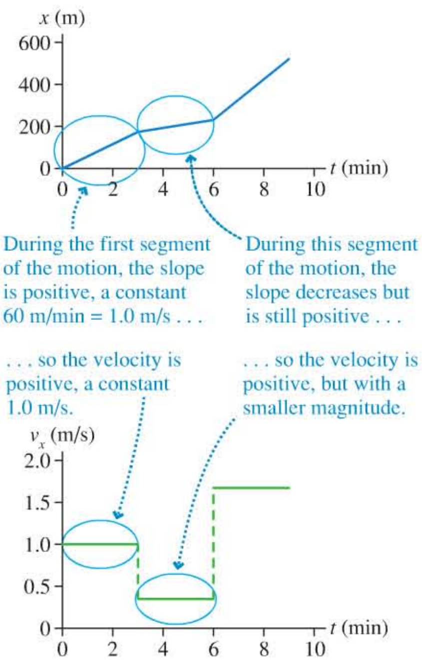
1. Determine an object's *position* at time t by reading the graph at that instant of time.
2. Determine the object's *velocity* at time t by finding the slope of the position graph at that point. Steeper slopes correspond to faster speeds.
3. Determine the *direction of motion* by noting the sign of the slope. Positive slopes correspond to positive velocities and, hence, to motion to the right (or up). Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).

Exercises 2,3

NOTE ► The slope is a ratio of intervals, $\Delta x/\Delta t$, not a ratio of coordinates; that is, the slope is *not* simply x/t. ◄

NOTE ► We are distinguishing between the actual slope and the *physically meaningful* slope. If you were to use a ruler to measure the rise and the run of the graph, you could compute the actual slope of the line as drawn on the page. That is not the slope we are referring to when we equate the velocity with the slope of the line. Instead, we find the *physically meaningful* slope by measuring the rise and run using the scales along the axes. The "rise" Δx is some number of meters; the "run" Δt is some number of seconds. The physically meaningful rise and run include units, and the ratio of these units gives the units of the slope. ◄

FIGURE 2.10 Deducing the velocity-versus-time graph from the position-versus-time graph.



We can now use the approach of Tactics Box 2.1 to analyze the student's position-versus-time graph that we saw in Figure 2.4. We can determine her velocity during the first phase of her motion by measuring the slope of the line:

$$v_x = \text{slope} = \frac{\Delta x}{\Delta t} = \frac{180\ \text{m}}{3\ \text{min}} = 60\frac{\text{m}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} = 1.0\ \text{m/s}$$

In completing this calculation, we've converted to more usual units for speed, m/s. During this phase of the motion, her velocity is constant, so a graph of velocity versus time appears as a horizontal line at 1.0 m/s, as shown in **FIGURE 2.10**. We can do similar calculations to show that her velocity during the second phase of her motion (i.e., the slope of the position graph) is +0.33 m/s, and then increases to +1.7 m/s during the final phase. We combine all of this information to create the **velocity-versus-time graph** shown in Figure 2.10.

An inspection of the velocity-versus-time graph shows that it matches our understanding of the student's motion: There are three phases of the motion, each with constant speed. In each phase, the velocity is positive because she is always moving to the right. The second phase is slow (low velocity) and the third phase fast (high velocity.) All of this can be clearly seen on the velocity-versus-time graph, which is yet another way to represent her motion.

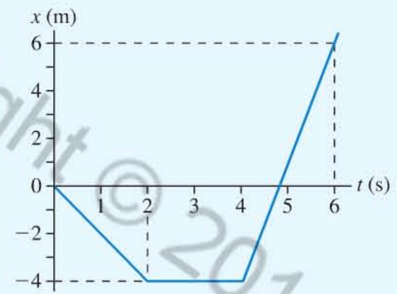
NOTE ► The velocity-versus-time graph in Figure 2.10 includes vertical segments in which the velocity changes instantaneously. Such rapid changes are an idealization; it actually takes a small amount of time to change velocity. ◄

EXAMPLE 2.2 **Analyzing a car's position graph**

FIGURE 2.11 gives the position-versus-time graph of a car.

a. Draw the car's velocity-versus-time graph.
b. Describe the car's motion in words.

FIGURE 2.11 The position-versus-time graph of a car.



PREPARE Figure 2.11 is a graphical representation of the motion. The car's position-versus-time graph is a sequence of three straight lines. Each of these straight lines represents uniform motion at a constant velocity. We can determine the car's velocity during each interval of time by measuring the slope of the line.

SOLVE

a. From $t = 0$ s to $t = 2$ s ($\Delta t = 2$ s) the car's displacement is $\Delta x = -4\text{ m} - 0\text{ m} = -4$ m. The velocity during this interval is

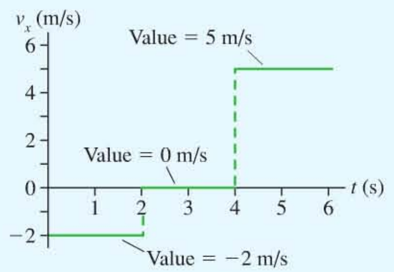
$$v_x = \frac{\Delta x}{\Delta t} = \frac{-4\text{ m}}{2\text{ s}} = -2\text{ m/s}$$

The car's position does not change from $t = 2$ s to $t = 4$ s ($\Delta x = 0$ m), so $v_x = 0$ m/s. Finally, the displacement between $t = 4$ s and $t = 6$ s ($\Delta t = 2$ s) is $\Delta x = 10$ m. Thus the velocity during this interval is

$$v_x = \frac{10\text{ m}}{2\text{ s}} = 5\text{ m/s}$$

These velocities are represented graphically in FIGURE 2.12.

FIGURE 2.12 The velocity-versus-time graph for the car.



b. The velocity-versus-time graph of Figure 2.12 shows the motion in a way that we can describe in a straightforward manner: The car backs up for 2 s at 2 m/s, sits at rest for 2 s, then drives forward at 5 m/s for 2 s.

ASSESS Notice that the velocity graph and the position graph look completely different. They should! The value of the velocity graph at any instant of time equals the *slope* of the position graph. Since the position graph is made up of segments of constant slope, the velocity graph should be made up of segments of constant *value*, as it is. This gives us confidence that the graph we have drawn is correct.

From Velocity to Position

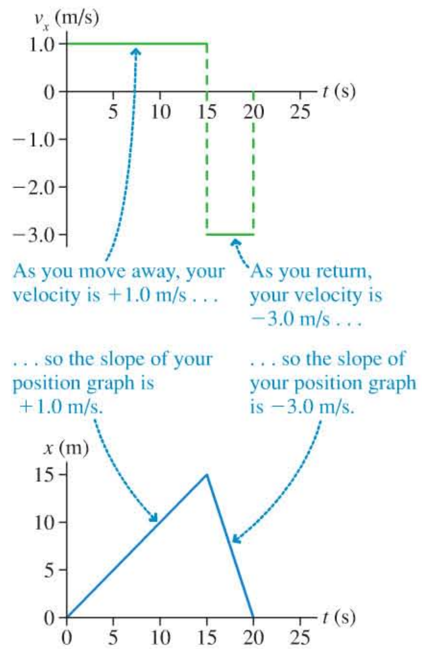
We've now seen how to move between different representations of uniform motion. There's one last issue to address: If you have a graph of velocity versus time, how can you determine the position graph?

Suppose you leave a lecture hall and begin walking toward your next class, which is down the hall to the west. You then realize that you left your textbook (which you always bring to class with you!) at your seat. You turn around and run back to the lecture hall to retrieve it. A velocity-versus-time graph for this motion appears as the top graph in FIGURE 2.13. There are two clear phases to the motion: walking away from class (velocity +1.0 m/s) and running back (velocity −3.0 m/s.) How can we deduce your position-versus-time graph?

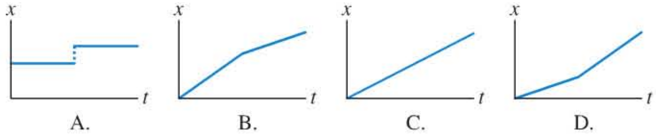
As before, we can analyze the graph segment by segment. This process is shown in Figure 2.13, in which the upper velocity-versus-time graph is used to deduce the lower position-versus-time graph. For each of the two segments of the motion, the sign of the velocity tells us whether the slope of the graph is positive or negative; the magnitude of the velocity tells how steep the slope is. The final result makes sense; it shows 15 seconds of slowly increasing position (walking away) and then 5 seconds of rapidly decreasing position (running back.) And you end up back where you started.

There's one important detail that we didn't talk about in the preceding paragraph: How did we know that the position graph started at $x = 0$ m? The velocity graph tells us the *slope* of the position graph, but it doesn't tell us where the position graph should start. Although you're free to select any point you choose as the origin of the coordinate system, here it seems reasonable to set $x = 0$ m at your starting point in the lecture hall; as you walk away, your position increases.

FIGURE 2.13 Deducing a position graph from a velocity-versus-time graph.



STOP TO THINK 2.1 Which position-versus-time graph best describes the motion diagram at left?



A ship on a constant heading at a steady speed is a practical example of uniform motion.

2.2 Uniform Motion

If you drive your car on a straight road at a perfectly steady 60 miles per hour (mph), you will cover 60 mi during the first hour, another 60 mi during the second hour, yet another 60 mi during the third hour, and so on. This is an example of what we call *uniform motion.* **Straight-line motion in which equal displacements occur during any successive equal-time intervals is called uniform motion or constant-velocity motion.**

NOTE ▶ The qualifier "any" is important. If during each hour you drive 120 mph for 30 min and stop for 30 min, you will cover 60 mi during each successive 1 hour interval. But you will *not* have equal displacements during successive 30 min intervals, so this motion is not uniform. Your constant 60 mph driving is uniform motion because you will find equal displacements no matter how you choose your successive time intervals. ◀

FIGURE 2.14 Motion diagram and position-versus-time graph for uniform motion.

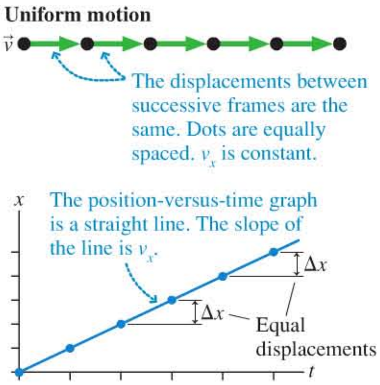


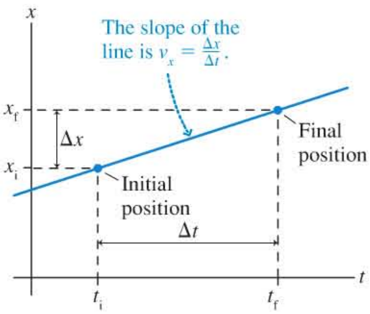
FIGURE 2.14 shows a motion diagram and a graph for an object in uniform motion. Notice that the position-versus-time graph for uniform motion is a straight line. This follows from the requirement that all values of Δx corresponding to the same value of Δt be equal. In fact, an alternative definition of uniform motion is: **An object's motion is uniform if and only if its position-versus-time graph is a straight line.**

Equations of Uniform Motion

An object is in uniform motion along the x-axis with the linear position-versus-time graph shown in FIGURE 2.15. Recall from Chapter 1 that we denote the object's initial position as x_i at time t_i. The term "initial" refers to the starting point of our analysis or the starting point in a problem. The object may or may not have been in motion prior to t_i. We use the term "final" for the ending point of our analysis or the ending point of a problem, and denote the object's final position x_f at the time t_f. As we've seen, the object's velocity v_x along the x-axis can be determined by finding the slope of the graph:

$$v_x = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \tag{2.3}$$

FIGURE 2.15 Position-versus-time graph for an object in uniform motion.



Equation 2.3 can be rearranged to give

$$x_f = x_i + v_x \,\Delta t \tag{2.4}$$

Position equation for an object in uniform motion (v_x is constant)

where $\Delta t = t_f - t_i$ is the interval of time in which the object moves from position x_i to position x_f. Equation 2.4 applies to any time interval Δt during which the velocity is constant. We can also write this in terms of the object's displacement, $\Delta x = x_f - x_i$:

$$\Delta x = v_x \,\Delta t \tag{2.5}$$

The velocity of an object in uniform motion tells us the amount by which its position changes during each second. An object with a velocity of 20 m/s *changes* its position by 20 m during every second of motion: by 20 m during the first second of its motion, by another 20 m during the next second, and so on. We say that position is changing at the *rate* of 20 m/s. If the object starts at $x_i = 10$ m, it will be at $x = 30$ m after 1 s of motion and at $x = 50$ m after 2 s of motion. Thinking of velocity like this will help you develop an intuitive understanding of the connection between velocity and position.

Physics may seem densely populated with equations, but most equations follow a few basic forms. The mathematical form of Equation 2.5 is a type that we will see again: The displacement Δx is *proportional* to the time interval Δt.

NOTE ► The important features of a proportional relationship are described below. In this text, the first time we use a particular mathematical form we will provide such an overview. In future chapters, when we see other examples of this type of relationship, we will refer back to this overview. ◄

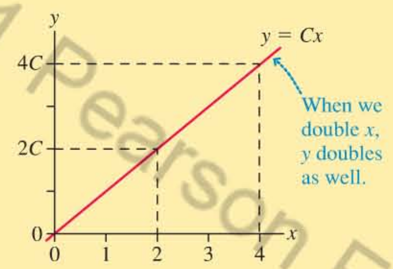
Proportional relationships

We say that y is **proportional** to x if they are related by an equation of this form:

$$y = Cx$$

y is proportional to x

We call C the **proportionality constant.** A graph of y versus x is a straight line that passes through the origin.



SCALING If x has the initial value x_1, then y has the initial value $y_1 = Cx_1$. Changing x from x_1 to x_2 changes y from y_1 to y_2. The ratio of y_2 to y_1 is

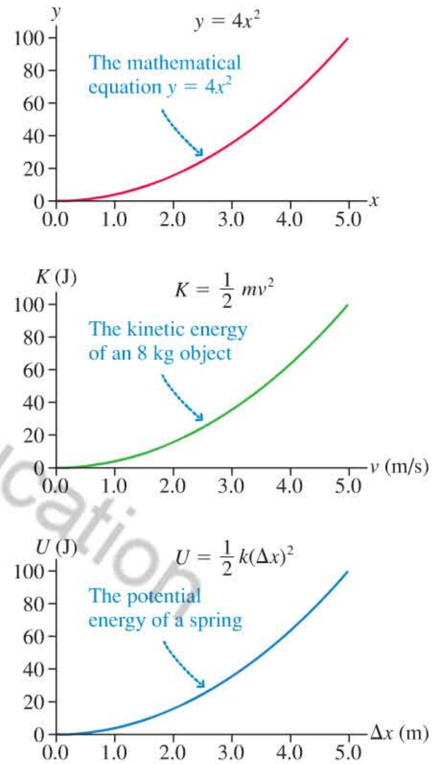
$$\frac{y_2}{y_1} = \frac{Cx_2}{Cx_1} = \frac{x_2}{x_1}$$

The ratio of y_2 to y_1 is exactly the same as the ratio of x_2 to x_1. If y is proportional to x, which is often written $y \propto x$, then x and y change by the same factor:

- If you double x, you double y.
- If you decrease x by a factor of 3, you decrease y by a factor of 3.

If two variables have a proportional relationship, we can draw important conclusions from ratios without knowing the value of the proportionality constant C. We can often solve problems in a very straightforward manner by looking at such ratios. This is an important skill called *ratio reasoning*.

Exercise 11



Mathematical Forms These three figures show graphs of a mathematical equation, the kinetic energy of a moving object versus its speed, and the potential energy of a spring versus the displacement of the end of the spring. All three graphs have the same overall appearance. The three expressions differ in their variables, but all three equations have the same **mathematical form.** There are only a handful of different mathematical forms that we'll use in this text. As we meet each form for the first time, we will give an overview. When you see it again, we'll insert an icon that refers back to the overview so that you can remind yourself of the key details.

EXAMPLE 2.3 If a train leaves Cleveland at 2:00 . . .

A train is moving due west at a constant speed. A passenger notes that it takes 10 minutes to travel 12 km. How long will it take the train to travel 60 km?

PREPARE For an object in uniform motion, Equation 2.5 shows that the distance traveled Δx is proportional to the time interval Δt, so this is a good problem to solve using ratio reasoning.

SOLVE We are comparing two cases: the time to travel 12 km and the time to travel 60 km. Because Δx is proportional to Δt, the ratio of the times will be equal to the ratio of the distances. The ratio of the distances is

$$\frac{\Delta x_2}{\Delta x_1} = \frac{60\text{ km}}{12\text{ km}} = 5$$

Continued

This is equal to the ratio of the times:

$$\frac{\Delta t_2}{\Delta t_1} = 5$$

$$\Delta t_2 = \text{ time to travel 60 km} = 5\Delta t_1 = 5 \times (10 \text{ min}) = 50 \text{ min}$$

It takes 10 minutes to travel 12 km; it will take 50 minutes—5 times as long—to travel 60 km.

ASSESS For an object in steady motion, it makes sense that 5 times the distance requires 5 times the time. We can see that using ratio reasoning is a straightforward way to solve this problem. We don't need to know the proportionality constant (in this case, the velocity); we just used ratios of distances and times.

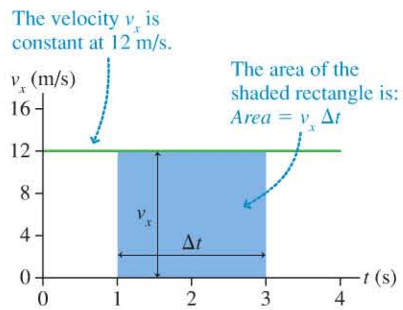
From Velocity to Position, One More Time

We've seen that we can deduce an object's velocity by measuring the slope of its position graph. Conversely, if we have a velocity graph, we can say something about position—not by looking at the slope of the graph, but by looking at what we call the *area under the graph*. Let's look at an example.

Suppose a car is in uniform motion at 12 m/s. How far does it travel—that is, what is its displacement—during the time interval between $t = 1.0$ s and $t = 3.0$ s?

Equation 2.5, $\Delta x = v_x \Delta t$, describes the displacement mathematically; for a graphical interpretation, consider the graph of velocity versus time in FIGURE 2.16. In the figure, we've shaded a rectangle whose height is the velocity v_x and whose base is the time interval Δt. The area of this rectangle is $v_x \Delta t$. Looking at Equation 2.5, we see that this quantity is also equal to the displacement of the car. The area of this rectangle is the area between the axis and the line representing the velocity; we call it the "area under the graph." We see that the **displacement Δx is equal to the area under the velocity graph during interval Δt.**

FIGURE 2.16 Displacement is the area under a velocity-versus-time graph.



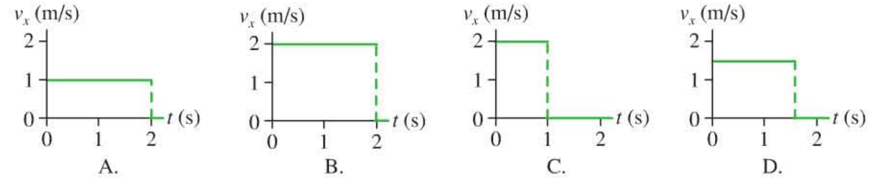
Whether we use Equation 2.5 or the area under the graph to compute the displacement, we get the same result:

$$\Delta x = v_x \Delta t = (12 \text{ m/s})(2.0 \text{ s}) = 24 \text{ m}$$

Although we've shown that the displacement is the area under the graph only for uniform motion, where the velocity is constant, we'll soon see that this result applies to any one-dimensional motion.

NOTE ► Wait a minute! The displacement $\Delta x = x_f - x_i$ is a length. How can a length equal an area? Recall that earlier, when we found that the velocity is the slope of the position graph, we made a distinction between the *actual* slope and the *physically meaningful* slope? The same distinction applies here. The velocity graph does indeed bound a certain area on the page. That is the actual area, but it is *not* the area to which we are referring. Once again, we need to measure the quantities we are using, v_x and Δt, by referring to the scales on the axes. Δt is some number of seconds, while v_x is some number of meters per second. When these are multiplied together, the *physically meaningful* area has units of meters, appropriate for a displacement. ◄

STOP TO THINK 2.2 Four objects move with the velocity-versus-time graphs shown. Which object has the largest displacement between $t = 0$ s and $t = 2$ s?



2.3 Instantaneous Velocity

The objects we've studied so far have moved with a constant, unchanging velocity or, like the car in Example 2.1, changed abruptly from one constant velocity to another. This is not very realistic. Real moving objects speed up and slow down, *changing* their velocity. As an extreme example, think about a drag racer. In a typical race, the car begins at rest but, 1 second later, is moving at over 25 miles per hour!

A drag racer moves with rapidly changing velocity.

For one-dimensional motion, an object changing its velocity is either speeding up or slowing down. When you drive your car, as you speed up or slow down—changing your velocity—a glance at your speedometer tells you how fast you're going *at that instant.* An object's velocity—a speed *and* a direction—at a specific *instant* of time t is called the object's **instantaneous velocity.**

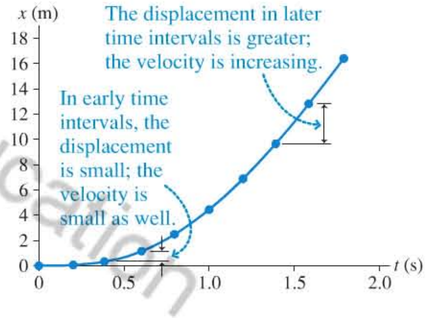
But what does it mean to have a velocity "at an instant"? An instantaneous velocity of magnitude 60 mph means that the rate at which your car's position is changing—at that exact instant—is such that it would travel a distance of 60 miles in 1 hour *if* it continued at that rate without change. Said another way, if *just for an instant* your car matches the velocity of another car driving at a steady 60 mph, then your instantaneous velocity is 60 mph. **From now on, the word "velocity" will always mean instantaneous velocity.**

For uniform motion, we found that an object's position-versus-time graph is a straight line and the object's velocity is the slope of that line. In contrast, **FIGURE 2.17** shows that the position-versus-time graph for a drag racer is a *curved* line. The displacement Δx during equal intervals of time gets greater as the car speeds up. Even so, we can use the slope of the position graph to measure the car's velocity. We can say that

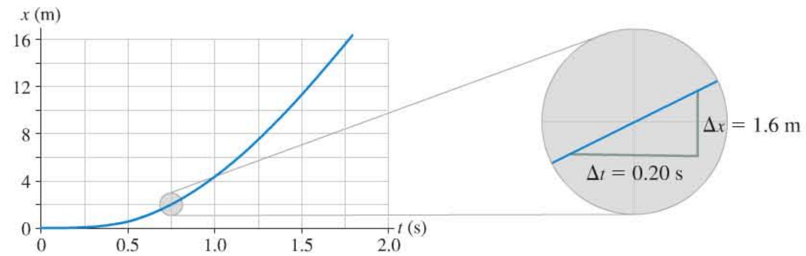
$$\text{instantaneous velocity } v_x \text{ at time } t = \text{slope of position graph at time } t \qquad (2.6)$$

But how do we determine the slope of a curved line at a particular point? The following table gives the necessary details.

FIGURE 2.17 Position-versus-time graph for a drag racer.



Finding the instantaneous velocity

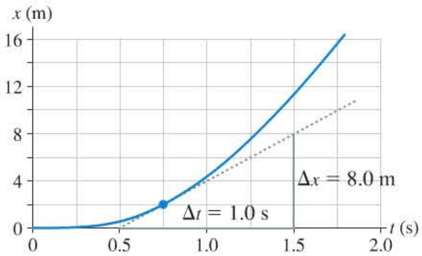


If the velocity changes, the position graph is a curved line. But we can still compute a slope by considering a small segment of the graph. Let's look at the motion in a very small time interval right around $t = 0.75$ s. This is highlighted with a circle, and we show a close-up in the next graph, at right.

Now that we have magnified a small part of the position graph, we see that the graph in this small part appears to have a constant slope. It is always possible to make the graph appear as a straight line by choosing a small enough time interval. We can find the slope of the line by calculating the rise over run, just as before:

$$v_x = \frac{1.6\text{ m}}{0.20\text{ s}} = 8.0\text{ m/s}$$

This is the slope of the graph at $t = 0.75$ s and thus the velocity at this instant of time.



Graphically, the slope of the curve at a particular point is the same as the slope of a straight line drawn *tangent* to the curve at that point. **The slope of the tangent line is the instantaneous velocity at that instant of time.**

Calculating rise over run for the tangent line, we get

$$v_x = \frac{8.0\text{ m}}{1.0\text{ s}} = 8.0\text{ m/s}$$

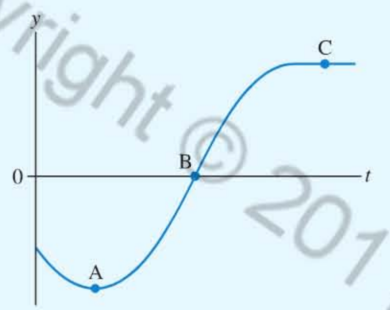
This is the same value we obtained from considering the close-up view.

CONCEPTUAL EXAMPLE 2.4 **Analyzing an elevator's position graph**

FIGURE 2.18 shows the position-versus-time graph of an elevator.

a. Sketch an approximate velocity-versus-time graph.
b. At which point or points is the elevator moving the fastest?
c. Is the elevator ever at rest? If so, at which point or points?

FIGURE 2.18 The position-versus-time graph for an elevator.



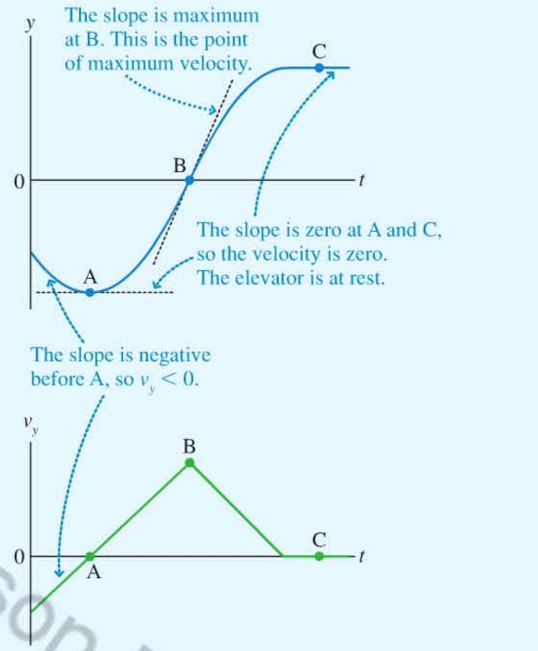
REASON a. Notice that the position graph shows y versus t, rather than x versus t, indicating that the motion is vertical rather than horizontal. Our analysis of one-dimensional motion has made no assumptions about the direction of motion, so it applies equally well to both horizontal and vertical motion. As we just found, the velocity at a particular instant of time is the slope of a tangent line to the position-versus-time graph at that time. We can move point-by-point along the position-versus-time graph, noting the slope of the tangent at each point. This will give us the velocity at that point.

Initially, to the left of point A, the slope is negative and thus the velocity is negative (i.e., the elevator is moving downward). But the slope decreases as the curve flattens out, and by the time the graph gets to point A, the slope is zero. The slope then increases to a maximum value at point B, decreases back to zero a little before point C, and remains at zero thereafter. This reasoning process is outlined in **FIGURE 2.19a**, and **FIGURE 2.19b** shows the approximate velocity-versus-time graph that results.

The other questions were answered during the construction of the graph:

b. The elevator moves the fastest at point B where the slope of the position graph is the steepest.

FIGURE 2.19 Finding a velocity graph from a position graph.



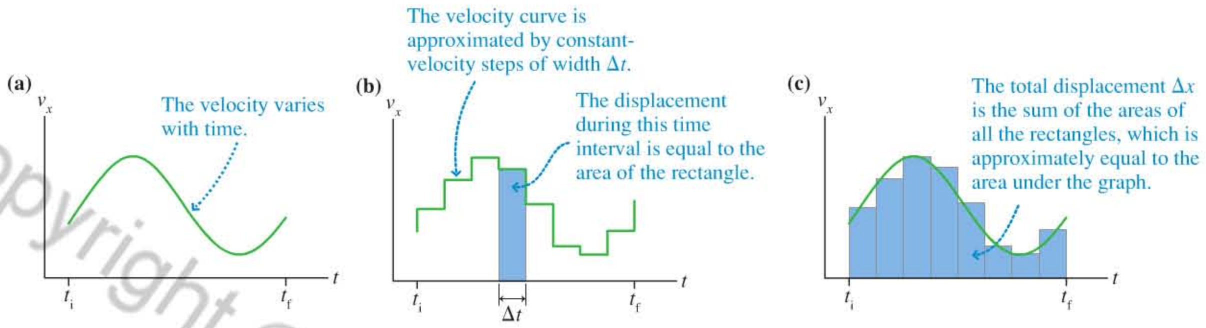
c. A particle at rest has $v_y = 0$. Graphically, this occurs at points where the tangent line to the position-versus-time graph is horizontal and thus has zero slope. Figure 2.19 shows that the slope is zero at points A and C. At point A, the velocity is only instantaneously zero as the particle reverses direction from downward motion (negative velocity) to upward motion (positive velocity). At point C, the elevator has actually stopped and remains at rest.

ASSESS The best way to check our work is to look at different segments of the motion and see if the velocity and position graphs match. Until point A, y is decreasing. The elevator is going down, so the velocity should be negative, which our graph shows. Between points A and C, y is increasing, so the velocity should be positive, which is also a feature of our graph. The steepest slope is at point B, so this should be the high point of our velocity graph, as it is.

For uniform motion we showed that the displacement Δx is the area under the velocity-versus-time graph during time interval Δt. We can generalize this idea to the case of an object whose velocity varies. **FIGURE 2.20a** on the next page is the velocity-versus-time graph for an object whose velocity changes with time. Suppose we know the object's position to be x_i at an initial time t_i. Our goal is to find its position x_f at a later time t_f.

Because we know how to handle constant velocities, let's *approximate* the velocity function of Figure 2.20a as a series of constant-velocity steps of width Δt as shown in **FIGURE 2.21b**. The velocity during each step is constant (uniform motion), so we can calculate the displacement during each step as the area of the rectangle under the curve. The total displacement of the object between t_i and t_f can be found as the sum of all the individual displacements during each of the constant-velocity steps. We can see in Figure 2.20b that the total displacement is approximately equal to the area under the graph, even in the case where the velocity varies. Although the approximation shown in the figure is rather rough, with only nine steps, we can imagine that it could be made as accurate as desired by having more and more ever-narrower steps.

FIGURE 2.20 Approximating a velocity-versus-time graph with a series of constant-velocity steps.



Consequently, an object's displacement is related to its velocity by

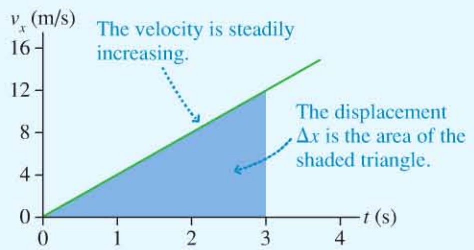
$$x_f - x_i = \Delta x = \text{area under the velocity graph } v_x \text{ between } t_i \text{ and } t_f \quad (2.7)$$

EXAMPLE 2.5 The displacement during a rapid start

FIGURE 2.21 shows the velocity-versus-time graph of a car pulling away from a stop. How far does the car move during the first 3.0 s?

PREPARE Figure 2.21 is a graphical representation of the motion. The question How far? indicates that we need to find a displacement Δx rather than a position x. According to Equation 2.7, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s.

FIGURE 2.21 Velocity-versus-time graph for the car of Example 2.5.



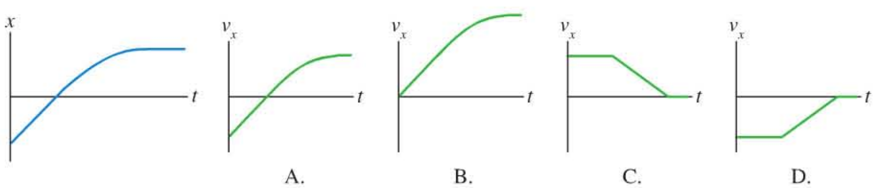
SOLVE The curve in this case is an angled line, so the area is that of a triangle:

$$\Delta x = \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s}$$
$$= \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}$$

The car moves 18 m during the first 3 seconds as its velocity changes from 0 to 12 m/s.

ASSESS The physically meaningful area is a product of s and m/s, so Δx has the proper units of m. Let's check the numbers to see if they make physical sense. The final velocity, 12 m/s, is about 25 mph. Pulling away from a stop, you might expect to reach this speed in about 3 s—at least if you have a reasonably sporty vehicle! If the car had moved at a constant 12 m/s (the final velocity) during these 3 s, the distance would be 36 m. The actual distance traveled during the 3 s is 18 m—half of 36 m. This makes sense, as the velocity was 0 m/s at the start of the problem and increased steadily to 12 m/s.

STOP TO THINK 2.3 Which velocity-versus-time graph goes with the position-versus-time graph on the left?



2.4 Acceleration

The goal of this chapter is to describe motion. We've seen that velocity describes the rate at which an object changes position. We need one more motion concept to complete the description, one that will describe an object whose velocity is changing.

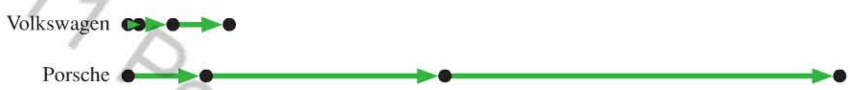
As an example, let's look at a frequently quoted measurement of car performance, the time it takes the car to go from 0 to 60 mph. Table 2.2 shows this time for two different cars.

TABLE 2.2 Performance data for vehicles

Vehicle	Time to go from 0 to 60 mph
1997 Porsche 911 Turbo S	3.6 s
1973 Volkswagen Super Beetle Convertible	24 s

Let's look at motion diagrams for the Porsche and the Volkswagen in FIGURE 2.22. We can see two important facts about the motion. First, the lengths of the velocity vectors are increasing, showing that the speeds are increasing. Second, the velocity vectors for the Porsche are increasing in length more rapidly than those of the VW. The quantity we seek is one that measures how rapidly an object's velocity vectors change in length.

FIGURE 2.22 Motion diagrams for the Porsche and Volkswagen.



When we wanted to measure changes in position, the ratio $\Delta x/\Delta t$ was useful. This ratio, which we defined as the velocity, is the *rate of change of position.* Similarly, we can measure how rapidly an object's velocity changes with the ratio $\Delta v_x/\Delta t$. Given our experience with velocity, we can say a couple of things about this new ratio:

- The ratio $\Delta v_x/\Delta t$ is the *rate of change of velocity.*
- The ratio $\Delta v_x/\Delta t$ is the *slope of a velocity-versus-time graph.*

We will define this ratio as the **acceleration,** for which we use the symbol a_x:

$$a_x = \frac{\Delta v_x}{\Delta t} \tag{2.8}$$

Definition of acceleration as the rate of change of velocity

Similarly, $a_y = \Delta v_y/\Delta t$ for vertical motion.

As an example, let's calculate the acceleration for the Porsche and the Volkswagen. For both, the initial velocity $(v_x)_i$ is zero and the final velocity $(v_x)_f$ is 60 mph. Thus the *change* in velocity is $\Delta v_x = 60$ mph. In m/s, our SI unit of velocity, $\Delta v_x = 27$ m/s.

Now we can use Equation 2.8 to compute acceleration. Let's start with the Porsche, which speeds up to 27 m/s in $\Delta t = 3.6$ s:

$$a_{\text{Porsche}\,x} = \frac{\Delta v_x}{\Delta t} = \frac{27\text{ m/s}}{3.6\text{ s}} = 7.5\,\frac{\text{m/s}}{\text{s}}$$

Here's the meaning of this final figure: Every second, the Porsche's velocity changes by 7.5 m/s. In the first second of motion, the Porsche's velocity increases by 7.5 m/s; in the next second, it increases by another 7.5 m/s, and so on. After 1 second, the velocity is 7.5 m/s; after 2 seconds, it is 15 m/s. This increase continues as long as the Porsche has this acceleration. We thus interpret the units as 7.5 meters per second, per second—7.5 (m/s)/s.

The Volkswagen's acceleration is

$$a_{\text{VW}\,x} = \frac{\Delta v_x}{\Delta t} = \frac{27\text{ m/s}}{24\text{ s}} = 1.1\,\frac{\text{m/s}}{\text{s}}$$

Cushion kinematics When a car hits an obstacle head-on, the damage to the car and its occupants can be reduced by making the acceleration as small as possible. As we can see from Equation 2.8, acceleration can be reduced by making the *time* for a change in velocity as long as possible. This is the purpose of the yellow crash cushion barrels you may have seen in work zones on highways—to lengthen the time of a collision with a barrier.

In each second, the Volkswagen changes its speed by 1.1 m/s. This is only 1/7 the acceleration of the Porsche! The reasons the Porsche is capable of greater acceleration has to do with what *causes* the motion. We will explore the reasons for acceleration in Chapter 4. For now, we will simply note that the Porsche is capable of much greater acceleration, something you would have suspected.

NOTE ► It is customary to abbreviate the acceleration units (m/s)/s as m/s^2. For example, we'll write that the Volkswagen has an acceleration of 1.1 m/s^2. When you use this notation, keep in mind the *meaning* of the notation as "(meters per second) per second." ◄

TABLE 2.3 Velocity data for the Volkswagen and the Porsche

Time (s)	Velocity of VW (m/s)	Velocity of Porsche (m/s)
0	0	0
1	1.1	7.5
2	2.2	15.0
3	3.3	22.5
4	4.4	30.0

Representing Acceleration

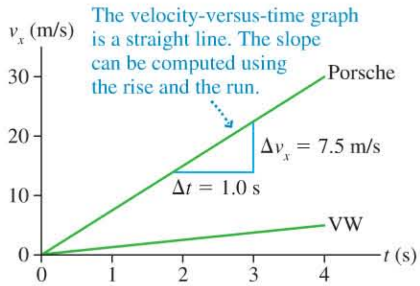
Let's use the values we have computed for acceleration to make a table of velocities for the Porsche and the Volkswagen we considered earlier. Table 2.3 uses the idea that the VW's velocity increases by 1.1 m/s every second while the Porsche's velocity increases by 7.5 m/s every second. The data in Table 2.3 are the basis for the velocity-versus-time graphs in **FIGURE 2.23**. As you can see, an object undergoing constant acceleration has a straight–line velocity graph.

The slope of either of these lines—the rise over run—is $\Delta v_x/\Delta t$. Comparing this with Equation 2.8, we see that the equation for the slope is the same as that for the acceleration. That is, **an object's acceleration is the slope of its velocity-versus-time graph:**

$$\text{acceleration } a_x \text{ at time } t = \text{slope of velocity graph at time } t \qquad (2.9)$$

The VW has a smaller acceleration, so its velocity graph has a smaller slope.

FIGURE 2.23 Velocity-versus-time graphs for the two cars.



CONCEPTUAL EXAMPLE 2.6 Analyzing a car's velocity graph

FIGURE 2.24a is a graph of velocity versus time for a car. Sketch a graph of the car's acceleration versus time.

REASON The graph can be divided into three sections:

- An initial segment, in which the velocity increases at a steady rate.
- A middle segment, in which the velocity is constant.
- A final segment, in which the velocity decreases at a steady rate.

In each section, the acceleration is the slope of the velocity-versus-time graph. Thus the initial segment has constant, positive acceleration, the middle segment has zero acceleration, and the final segment has constant, *negative* acceleration. The acceleration graph appears in **FIGURE 2.24b**.

ASSESS This process is analogous to finding a velocity graph from the slope of a position graph. The middle segment having zero acceleration does *not* mean that the velocity is zero. The velocity is constant, which means it is *not changing* and thus the car is not accelerating. The car does accelerate during the initial and final segments. The magnitude of the acceleration is a measure of how quickly the velocity is changing. How about the sign? This is an issue we will address in the next section.

FIGURE 2.24 Finding an acceleration graph from a velocity graph.

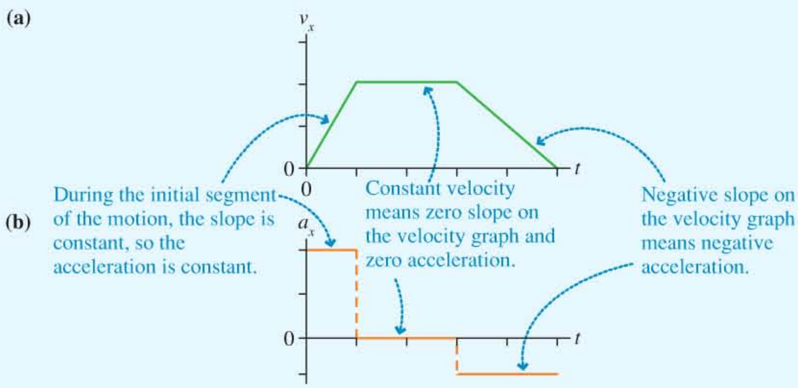
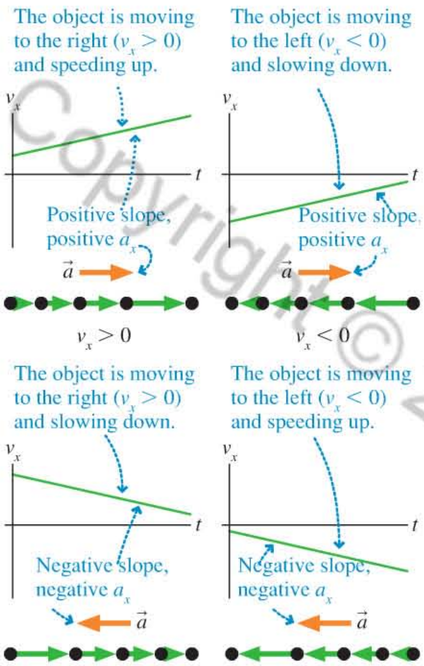


FIGURE 2.25 Determining the sign of the acceleration.



The Sign of the Acceleration

It's a natural tendency to think that a positive value of a_x or a_y describes an object that is speeding up while a negative value describes an object that is slowing down (decelerating). Unfortunately, this simple interpretation *does not work.*

Because an object can move right or left (or, equivalently, up and down) while either speeding up or slowing down, there are four situations to consider. FIGURE 2.25 shows a motion diagram and a velocity graph for each of these situations. As we've seen, an object's acceleration is the slope of its velocity graph, so a positive slope implies a positive acceleration and a negative slope implies a negative acceleration.

Acceleration, like velocity, is really a vector quantity, a concept that we will explore more fully in Chapter 3. Figure 2.25 shows the acceleration vectors for the four situations. The acceleration vector points in the same direction as the velocity vector $\vec{v}$ for an object that is speeding up and opposite to $\vec{v}$ for an object that is slowing down.

An object that speeds up as it moves to the right (positive v_x) has a positive acceleration, but an object that speeds up as it moves to the left (negative v_x) has a negative acceleration. Whether or not an object that is slowing down has a negative acceleration depends on whether the object is moving to the right or to the left. This is admittedly a bit more complex than thinking that negative acceleration always means slowing down, but our definition of acceleration as the slope of the velocity graph forces us to pay careful attention to the sign of the acceleration.

STOP TO THINK 2.4 A particle moves with the velocity-versus-time graph shown here. At which labeled point is the magnitude of the acceleration the greatest?

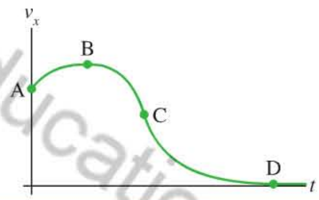


FIGURE 2.26 The red dots show the positions of the top of the Saturn V rocket at equally spaced intervals of time during liftoff.

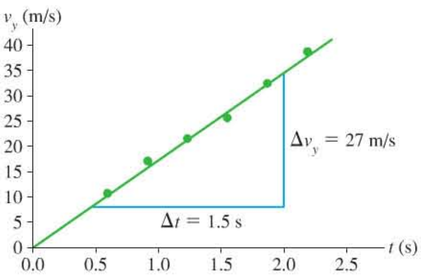
2.5 Motion with Constant Acceleration

For uniform motion—motion with constant velocity—we found in Equation 2.3 a simple relationship between position and time. It's no surprise that there are also simple relationships that connect the various kinematic variables in constant-acceleration motion. We will start with a concrete example, the launch of a Saturn V rocket like the one that carried the Apollo astronauts to the moon in the 1960s and 1970s. FIGURE 2.26 shows one frame from a video of a rocket lifting off the launch pad. The red dots show the positions of the top of the rocket at equally spaced intervals of time in earlier frames of the video. This is a motion diagram for the rocket, and we can see that the velocity is increasing. The graph of velocity versus time in FIGURE 2.27 shows that the velocity is increasing at a fairly constant rate. We can approximate the rocket's motion as constant acceleration.

We can use the slope of the graph in Figure 2.27 to determine the acceleration of the rocket:

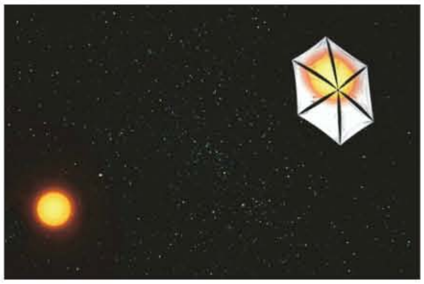
$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{27\ \text{m/s}}{1.5\ \text{s}} = 18\ \text{m/s}^2$$

FIGURE 2.27 A graph of the rocket's velocity versus time.



This acceleration is more than double the acceleration of the Porsche, and it goes on for quite a long time—the first phase of the launch lasts over 2 minutes! How fast is the rocket moving at the end of this acceleration, and how far has it traveled? To answer questions like these, we first need to work out some basic kinematic formulas for motion with constant acceleration.

▶ **Solar sailing** A rocket achieves a high speed by having a very high acceleration. A different approach is represented by a solar sail. A spacecraft with a solar sail accelerates due to the pressure of sunlight from the sun on a large, mirrored surface. The acceleration is minuscule, but it can continue for a long, long time. After an acceleration period of a few *years,* the spacecraft will reach a respectable speed!



Constant-Acceleration Equations

Consider an object whose acceleration a_x remains constant during the time interval $\Delta t = t_f - t_i$. At the beginning of this interval, the object has initial velocity $(v_x)_i$ and initial position x_i. Note that t_i is often zero, but it need not be. FIGURE 2.28a shows the acceleration-versus-time graph. It is a horizontal line between t_i and t_f, indicating a *constant* acceleration.

The object's velocity is changing because the object is accelerating. We can use the acceleration to find $(v_x)_f$ at a later time t_f. We defined acceleration as

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t} \tag{2.10}$$

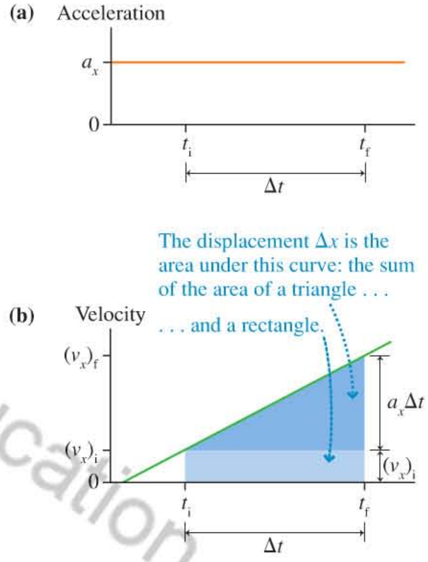
which is rearranged to give

$$(v_x)_f = (v_x)_i + a_x\,\Delta t \tag{2.11}$$

Velocity equation for an object with constant acceleration

NOTE ▶ We have expressed this equation for motion along the x-axis, but it is a general result that will apply to any axis. ◀

FIGURE 2.28 Acceleration and velocity graphs for motion with constant acceleration.



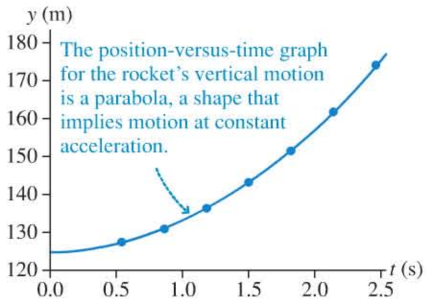
The velocity-versus-time graph for this constant-acceleration motion, shown in FIGURE 2.28b, is a straight line with value $(v_x)_i$ at time t_i and with slope a_x.

We would also like to know the object's position x_f at time t_f. As you learned earlier, the displacement Δx during a time interval Δt is the area under the velocity-versus-time graph. The shaded area in Figure 2.28b can be subdivided into a rectangle of area $(v_x)_i\,\Delta t$ and a triangle of area $\frac{1}{2}(a_x\,\Delta t)(\Delta t) = \frac{1}{2}a_x(\Delta t)^2$. Adding these gives

$$x_f = x_i + (v_x)_i\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \tag{2.12}$$

Position equation for an object with constant acceleration

FIGURE 2.29 Position-versus-time graph for the Saturn V rocket launch.



where $\Delta t = t_f - t_i$ is the elapsed time. The fact that the time interval Δt appears in the equation as $(\Delta t)^2$ causes the position-versus-time graph for constant-acceleration motion to have a parabolic shape. For the rocket launch of Figure 2.26, a graph of the position of the top of the rocket versus time appears as in FIGURE 2.29.

Equations 2.11 and 2.12 are two of the basic kinematic equations for motion with constant acceleration. They allow us to predict an object's position and velocity at a future instant of time. We need one more equation to complete our set, a direct relationship between displacement and velocity. To derive this relationship, we first use Equation 2.11 to write $\Delta t = ((v_x)_f - (v_x)_i)/a_x$. We can substitute this into Equation 2.12 to obtain

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x\,\Delta x \tag{2.13}$$

Relating velocity and displacement for constant-acceleration motion

In Equation 2.13 $\Delta x = x_f - x_i$ is the *displacement* (not the distance!). Notice that Equation 2.13 does not require knowing the time interval Δt. This is an important equation in problems where you're not given information about times. Equations 2.11, 2.12, and 2.13 are the key equations for motion with constant acceleration. These results are summarized in Table 2.4.

TABLE 2.4 Kinematic equations for motion with constant acceleration

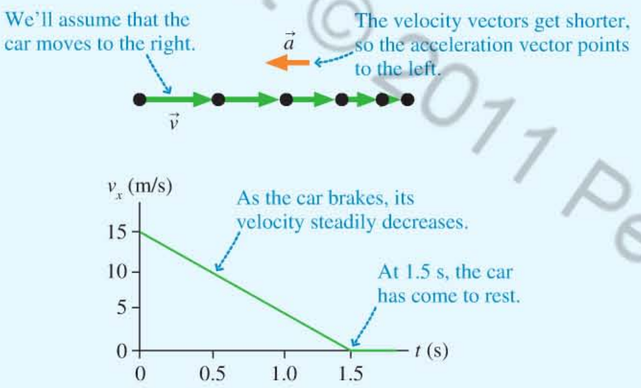
$(v_x)_f = (v_x)_i + a_x\,\Delta t$
$x_f = x_i + (v_x)_i\,\Delta t + \frac{1}{2}a_x(\Delta t)^2$
$(v_x)_f^2 = (v_x)_i^2 + 2a_x\,\Delta x$

EXAMPLE 2.7 Coming to a stop

As you drive in your car at 15 m/s (just a bit under 35 mph), you see a child's ball roll into the street ahead of you. You hit the brakes and stop as quickly as you can. In this case, you come to rest in 1.5 s. How far does your car travel as you brake to a stop?

PREPARE The problem statement gives us a description of motion in words. To help us visualize the situation, FIGURE 2.30 illustrates the key features of the motion with a motion diagram and a velocity graph. The graph is based on the car slowing from 15 m/s to 0 m/s in 1.5 s.

FIGURE 2.30 Motion diagram and velocity graph for a car coming to a stop.



SOLVE We've assumed that your car is moving to the right, so its initial velocity is $(v_x)_i = +15$ m/s. After you come to rest, your final velocity is $(v_x)_f = 0$ m/s. The acceleration is given by Equation 2.10:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t} = \frac{0 \text{ m/s} - 15 \text{ m/s}}{1.5 \text{ s}} = -10 \text{ m/s}^2$$

An acceleration of -10 m/s^2 (really -10 m/s per second) means the car slows by 10 m/s every second.

Now that we know the acceleration, we can compute the distance that the car moves as it comes to rest using Equation 2.12:

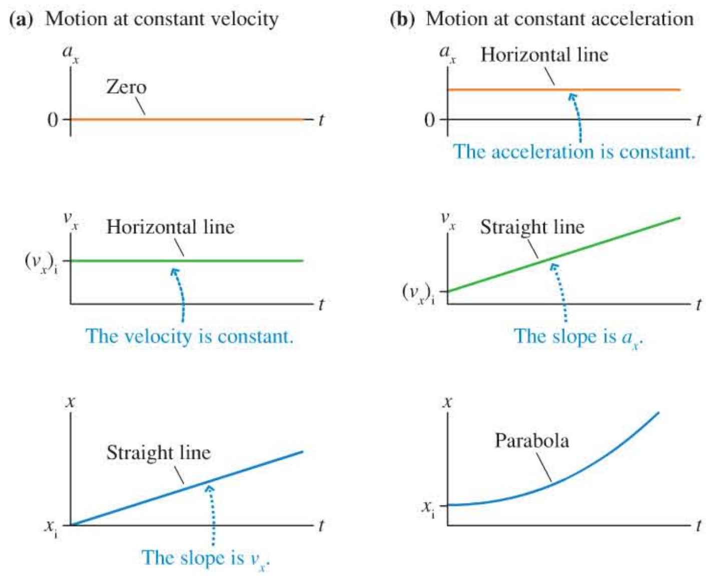
$$\begin{aligned} x_f - x_i &= (v_x)_i \Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \\ &= (15 \text{ m/s})(1.5 \text{ s}) + \tfrac{1}{2}(-10 \text{ m/s}^2)(1.5 \text{ s})^2 = 11 \text{ m} \end{aligned}$$

ASSESS 11 m is a little over 35 feet. That's a reasonable distance for a quick stop while traveling at about 35 mph. The purpose of the Assess step is not to prove that your solution is correct but to use common sense to recognize answers that are clearly wrong. Had you made a calculation error and ended up with an answer of 1.1 m—less than 4 feet—a moment's reflection should indicate that this couldn't possibly be correct.

1.1, 1.2, 1.3 ActivPhysics Online

Graphs will be an important component of our problem solutions, so we want to consider the types of graphs we are likely to encounter in more detail. FIGURE 2.31 is a graphical comparison of motion with constant velocity (uniform motion) and motion with constant acceleration. Notice that uniform motion is really a special case of constant-acceleration motion in which the acceleration happens to be zero.

FIGURE 2.31 Motion with constant velocity and constant acceleration. These graphs assume $x_i = 0$, $(v_x)_i > 0$, and (for constant acceleration) $a_x > 0$.



For motion at constant acceleration, a graph of position versus time is a *parabola*. This is a new mathematical form, one that we will see again. If $(v_x)_i = 0$, the second equation in Table 2.4 is simply

$$\Delta x = \tfrac{1}{2} a_x (\Delta t)^2 \tag{2.14}$$

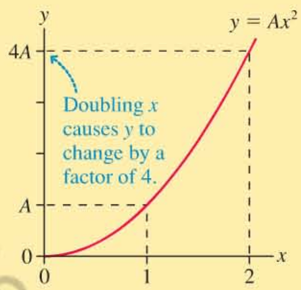
Δx depends on the *square* of Δt; we call this a *quadratic relationship*.

Quadratic relationships

Two quantities are said to have a **quadratic relationship** if y is proportional to the square of x. We write the mathematical relationship as

$$y = Ax^2$$

y is proportional to x^2



The graph of a quadratic relationship is a parabola.

SCALING If x has the initial value x_1, then y has the initial value $y_1 = A(x_1)^2$. Changing x from x_1 to x_2 changes y from y_1 to y_2. The ratio of y_2 to y_1 is

$$\frac{y_2}{y_1} = \frac{A(x_2)^2}{A(x_1)^2} = \left(\frac{x_2}{x_1}\right)^2$$

The ratio of y_2 to y_1 is the square of the ratio of x_2 to x_1. If y is a quadratic function of x, a change in x by some factor changes y by the square of that factor:

- If you increase x by a factor of 2, you increase y by a factor of $2^2 = 4$.
- If you decrease x by a factor of 3, you decrease y by a factor of $3^2 = 9$.

Generally, we can say that:

Changing x by a factor of c changes y by a factor of c^2.

Exercise 19

Getting up to speed BIO A bird must have a minimum speed to fly. Generally, the larger the bird, the faster the takeoff speed. Small birds can get moving fast enough to fly with a vigorous jump, but larger birds may need a running start. This swan must accelerate for a long distance in order to achieve the high speed it needs to fly, so it makes a frenzied dash across the frozen surface of a pond. Swans require a long, clear stretch of water or land to become airborne. Airplanes require an even faster takeoff speed and thus an even longer runway, as we will see.

EXAMPLE 2.8 Displacement of a drag racer

A drag racer, starting from rest, travels 6.0 m in 1.0 s. Suppose the car continues this acceleration for an additional 4.0 s. How far from the starting line will the car be?

PREPARE We assume that the acceleration is constant, so the displacement will follow Equation 2.14. This is a *quadratic relationship,* so the displacement will scale as the square of the time.

SOLVE After 1.0 s, the car has traveled 6.0 m; after another 4.0 s, a total of 5.0 s will have elapsed. The time has increased by a factor of 5, so the displacement will increase by a factor of 5^2, or 25. The total displacement is

$$\Delta x = 25(6.0 \text{ m}) = 150 \text{ m}$$

ASSESS This is a big distance in a short time, but drag racing is a fast sport, so our answer makes sense.

STOP TO THINK 2.5 A cyclist is at rest at a traffic light. When the light turns green, he begins accelerating at 1.2 m/s^2. How many seconds after the light turns green does he reach his cruising speed of 6.0 m/s?

A. 1.0 s B. 2.0 s C. 3.0 s D. 4.0 s E. 5.0 s

Dinner at a distance BIO A chameleon's tongue is a powerful tool for catching prey. Certain species can extend the tongue to a distance of over 1 ft in less than 0.1 s! A study of the kinematics of the motion of the chameleon tongue, using techniques like those in this chapter, reveals that the tongue has a period of rapid acceleration followed by a period of constant velocity. This knowledge is a very valuable clue in the analysis of the evolutionary relationships between chameleons and other animals.

Building a complex structure requires careful planning. The architect's visualization and drawings have to be complete before the detailed procedures of construction get under way. The same is true for solving problems in physics.

2.6 Solving One-Dimensional Motion Problems

The big challenge when solving a physics problem is to translate the words into symbols that can be manipulated, calculated, and graphed. This translation from words to symbols is the heart of problem solving in physics. Ambiguous words and phrases must be clarified, the imprecise must be made precise, and you must arrive at an understanding of exactly what the question is asking.

In this section we will explore some general problem-solving strategies that we will use throughout the text, applying them to problems of motion along a line.

A Problem-Solving Strategy

The first step in solving a seemingly complicated problem is to break it down into a series of smaller steps. In worked examples in the text, we use a problem-solving strategy that consists of three steps: *prepare, solve,* and *assess.* Each of these steps has important elements that you should follow when you solve problems on your own.

MP™ Problem-Solving Strategy

PREPARE The Prepare step of a solution is where you identify important elements of the problem and collect information you will need to solve it. It's tempting to jump right to the Solve step, but a skilled problem solver will spend the most time on this step, the preparation. Preparation includes:

- **Drawing a picture.** In many cases, this is the most important part of a problem. The picture lets you model the problem and identify the important elements. As you add information to your picture, the outline of the solution will take shape. For the problems in this chapter, a picture could be a motion diagram or a graph—or perhaps both.
- **Collecting necessary information.** The problem's statement may give you some values of variables. Other important information may be implied or must be looked up in a table. Gather everything you need to solve the problem and compile it in a list.
- **Doing preliminary calculations.** There are a few calculations, such as unit conversions, that are best done in advance of the main part of the solution.

SOLVE The Solve step of a solution is where you actually do the mathematics or reasoning necessary to arrive at the answer needed. This is the part of the problem-solving strategy that you likely think of when you think of "solving problems." But don't make the mistake of starting here! The Prepare step will help you be certain you understand the problem before you start putting numbers in equations.

ASSESS The Assess step of your solution is very important. When you have an answer, you should check to see whether it makes sense. Ask yourself:

- **Does my solution answer the question that was asked?** Make sure you have addressed all parts of the question and clearly written down your solutions.
- **Does my answer have the correct units and number of significant figures?**
- **Does the value I computed make physical sense?** In this book all calculations use physically reasonable numbers. You will not be given a problem to solve in which the final velocity of a bicycle is 100 miles per hour! If your answer seems unreasonable, go back and check your work.
- **Can I estimate what the answer should be to check my solution?**
- **Does my final solution make sense in the context of the material I am learning?**

The Pictorial Representation

Many physics problems, including 1-D motion problems, often have several variables and other pieces of information to keep track of. The best way to tackle such problems is to draw a picture, as we noted when we introduced a general problem-solving strategy. But what kind of picture should you draw?

In this section, we will begin to draw **pictorial representations** as an aid to solving problems. A pictorial representation shows all of the important details that we need to keep track of and will be very important in solving motion problems.

TACTICS BOX 2.2 Drawing a pictorial representation MP™

1. **Sketch the situation.** Not just any sketch: Show the object at the *beginning* of the motion, at the *end*, and at any point where the character of the motion changes. Very simple drawings are adequate.
2. **Establish a coordinate system.** Select your axes and origin to match the motion.
3. **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. *Every* variable used later in the mathematical solution should be defined on the sketch.

We will generally combine the pictorial representation with a **list of values,** which will include:

- *Known information.* Make a table of the quantities whose values you can determine from the problem statement or that you can find quickly with simple geometry or unit conversions.
- *Desired unknowns.* What quantity or quantities will allow you to answer the question?

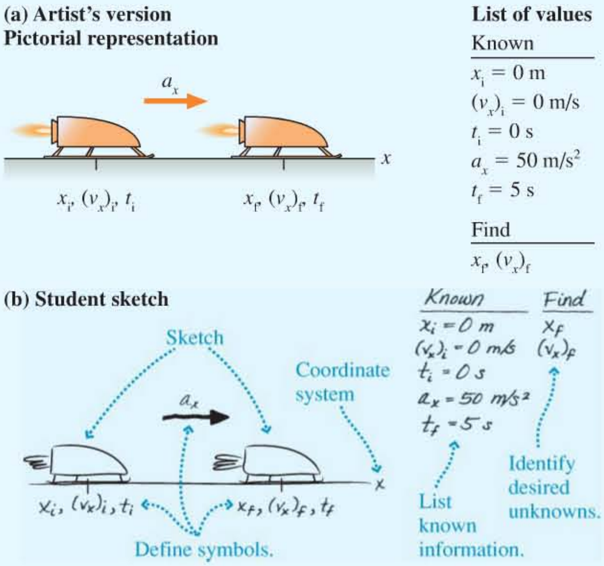
Exercise 21

EXAMPLE 2.9 Drawing a pictorial representation

Complete a pictorial representation and a list of values for the following problem: A rocket sled accelerates at 50 m/s^2 for 5 s. What are the total distance traveled and the final velocity?

PREPARE FIGURE 2.32a shows a pictorial representation as drawn by an artist in the style of the figures in this book. This is

FIGURE 2.32 Constructing a pictorial representation and a list of values.



certainly neater and more artistic than the sketches you will make when solving problems yourself! FIGURE 2.32b shows a sketch like one you might actually do. It's less formal, but it contains all of the important information you need to solve the problem.

NOTE ► Throughout this book we will illustrate select examples with actual hand-drawn figures so that you have them to refer to as you work on your own pictures for homework and practice. ◄

Let's look at how these pictures were constructed. The motion has a clear beginning and end; these are the points sketched. A coordinate system has been chosen with the origin at the starting point. The quantities x, v_x, and t are needed at both points, so these have been defined on the sketch and distinguished by subscripts. The acceleration is associated with an interval between these points. Values for two of these quantities are given in the problem statement. Others, such as $x_i = 0$ m and $t_i = 0$ s, are inferred from our choice of coordinate system. The value $(v_x)_i = 0$ m/s is part of our *interpretation* of the problem. Finally, we identify x_f and $(v_x)_f$ as the quantities that will answer the question. We now understand quite a bit about the problem and would be ready to start a quantitative analysis.

ASSESS We didn't *solve* the problem; that was not our purpose. Constructing a pictorial representation and a list of values is part of a systematic approach to interpreting a problem and getting ready for a mathematical solution.

The Visual Overview

The pictorial representation and the list of values are a very good complement to the motion diagram and other ways of looking at a problem that we have seen. As we translate a problem into a form we can solve, we will combine these elements into what we will term a **visual overview.** The visual overview will consist of some or all of the following elements:

- A *motion diagram.* A good strategy for solving a motion problem is to start by drawing a motion diagram.
- A *pictorial representation*, as defined above.
- A *graphical representation.* For motion problems, it is often quite useful to include a graph of position and/or velocity.
- A *list of values.* This list should sum up all of the important values in the problem.

Future chapters will add other elements to this visual overview of the physics.

EXAMPLE 2.10 Kinematics of a rocket launch

A Saturn V rocket is launched straight up with a constant acceleration of 18 m/s^2. After 150 s, how fast is the rocket moving and how far has it traveled?

PREPARE **FIGURE 2.33** shows a visual overview of the rocket launch that includes a motion diagram, a pictorial representation, and a list of values. The visual overview shows the whole problem in a nutshell. The motion diagram illustrates the motion of the rocket. The pictorial representation (produced according to Tactics Box 2.2) shows axes, identifies the important points of the motion, and defines variables. Finally, we have included a list of values that gives the known and unknown quantities. In the visual overview we have taken the statement of the problem in words and made it much more precise; it contains everything you need to know about the problem.

SOLVE Our first task is to find the final velocity. Our list of values includes the initial velocity, the acceleration, and the time interval, so we can use the first kinematic equation of Table 2.4 to find the final velocity:

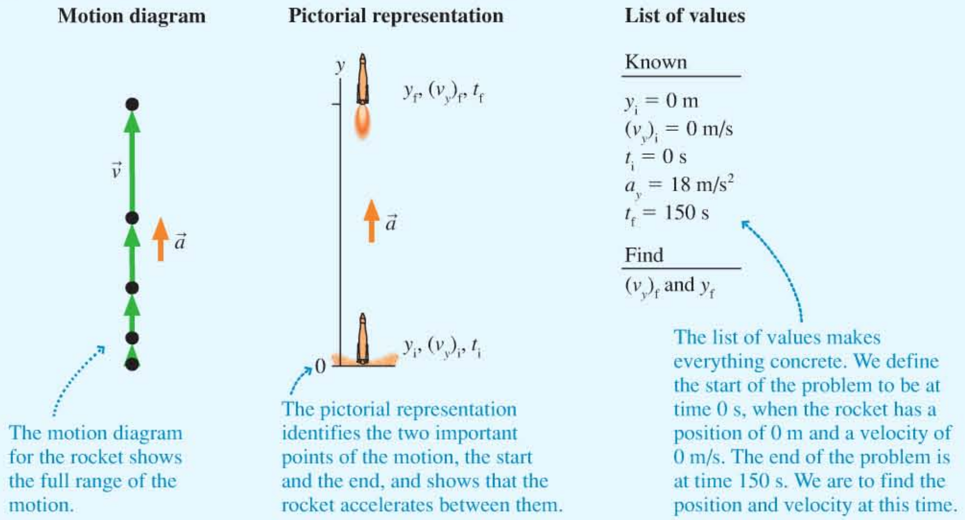
$$(v_y)_f = (v_y)_i + a_y\,\Delta t = 0\text{ m/s} + (18\text{ m/s}^2)(150\text{ s})$$
$$= 2700\text{ m/s}$$

The distance traveled is found using the second equation in Table 2.4:

$$y_f = y_i + (v_y)_i\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$= 0\text{ m} + (0\text{ m/s})(150\text{ s}) + \tfrac{1}{2}(18\text{ m/s}^2)(150\text{ s})^2$$
$$= 2.0 \times 10^5\text{ m} = 200\text{ km}$$

ASSESS The acceleration is very large, and it goes on for a long time, so the large final velocity and large distance traveled seem reasonable.

FIGURE 2.33 Visual overview of the rocket launch.



Problem-Solving Strategy for Motion with Constant Acceleration

1.4–1.6, 1.8, 1.9, 1.11–1.14 ActivPhysics

Earlier in this section, we introduced a general problem-solving strategy. Now we will adapt this general strategy to solving problems of motion with constant acceleration. We will introduce such specific problem-solving strategies in future chapters as well.

PROBLEM-SOLVING STRATEGY 2.1 **Motion with constant acceleration** MP™

PREPARE Draw a visual overview of the problem. This should include a motion diagram, a pictorial representation, and a list of values; a graphical representation may be useful for certain problems.

SOLVE The mathematical solution is based on the three equations in Table 2.4.

- Though the equations are phrased in terms of the variable x, it's customary to use y for motion in the vertical direction.
- Use the equation that best matches what you know and what you need to find. For example, if you know acceleration and time and are looking for a change in velocity, the first equation is the best one to use.
- Uniform motion with constant velocity has $a = 0$.

ASSESS Is your result believable? Does it have proper units? Does it make sense?

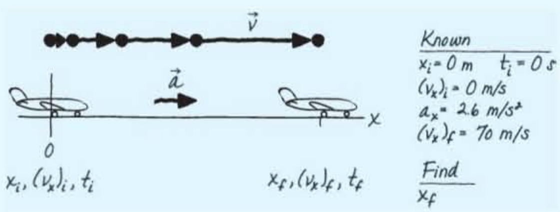
Exercise 25

EXAMPLE 2.11 Calculating the minimum length of a runway

A fully loaded Boeing 747 with all engines at full thrust accelerates at 2.6 m/s^2. Its minimum takeoff speed is 70 m/s. How much time will the plane take to reach its takeoff speed? What minimum length of runway does the plane require for takeoff?

PREPARE The visual overview of **FIGURE 2.34** summarizes the important details of the problem. We set x_i and t_i equal to zero at the starting point of the motion, when the plane is at rest and the acceleration begins. The final point of the motion is when the plane achieves the necessary takeoff speed of 70 m/s. The plane is accelerating to the right, so we will compute the time for the plane to reach a velocity of 70 m/s and the position of the plane at this time, giving us the minimum length of the runway.

FIGURE 2.34 Visual overview for an accelerating plane.



SOLVE First we solve for the time required for the plane to reach takeoff speed. We can use the first equation in Table 2.4 to compute this time:

$$(v_x)_f = (v_x)_i + a_x\,\Delta t$$

$$70\text{ m/s} = 0\text{ m/s} + (2.6\text{ m/s}^2)\,\Delta t$$

$$\Delta t = \frac{70\text{ m/s}}{2.6\text{ m/s}^2} = 26.9\text{ s}$$

We keep an extra significant figure here because we will use this result in the next step of the calculation.

Given the time that the plane takes to reach takeoff speed, we can compute the position of the plane when it reaches this speed using the second equation in Table 2.4:

$$x_f = x_i + (v_x)_i\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$= 0\text{ m} + (0\text{ m/s})(26.9\text{ s}) + \tfrac{1}{2}(2.6\text{ m/s}^2)(26.9\text{ s})^2$$

$$= 940\text{ m}$$

Our final answers are thus that the plane will take 27 s to reach takeoff speed, with a minimum runway length of 940 m.

ASSESS Think about the last time you flew; 27 s seems like a reasonable time for a plane to accelerate on takeoff. Actual runway lengths at major airports are 3000 m or more, a few times greater than the minimum length, because they have to allow for emergency stops during an aborted takeoff. (If we had calculated a distance far greater than 3000 m, we would know we had done something wrong!)

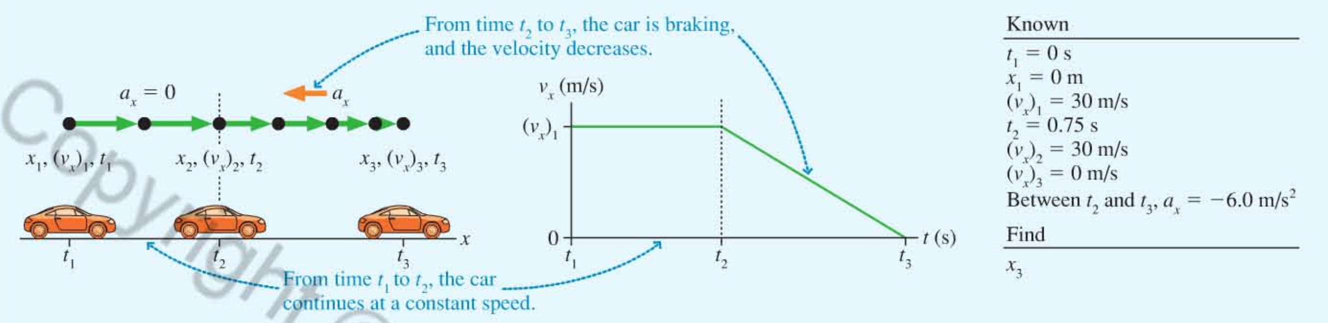
EXAMPLE 2.12 Finding the braking distance

A car is traveling at a speed of 30 m/s, a typical highway speed, on wet pavement. The driver sees an obstacle ahead and decides to stop. From this instant, it takes him 0.75 s to begin applying the brakes. Once the brakes are applied, the car experiences an acceleration of -6.0 m/s^2. How far does the car travel from the instant the driver notices the obstacle until stopping?

PREPARE This problem is more involved than previous problems we have solved, so we will take more care with the visual overview in **FIGURE 2.35**. In addition to a motion diagram and a pictorial representation, we include a graphical representation. Notice that there are two different phases of the motion: a constant-velocity phase before braking begins, and a steady slowing

Continued

FIGURE 2.35 Visual overview for a car braking to a stop.



down once the brakes are applied. We will need to do two different calculations, one for each phase. Consequently, we've used numerical subscripts rather than a simple i and f.

SOLVE From t_1 to t_2 the velocity stays constant at 30 m/s. This is uniform motion, so the position at time t_2 is computed using Equation 2.4:

$$x_2 = x_1 + (v_x)_1(t_2 - t_1) = 0 \text{ m} + (30 \text{ m/s})(0.75 \text{ s})$$
$$= 22.5 \text{ m}$$

At t_2, the velocity begins to decrease at a steady -6.0 m/s^2 until the car comes to rest at t_3. This time interval can be computed using the first equation in Table 2.4, $(v_x)_3 = (v_x)_2 + a_x \Delta t$:

$$\Delta t = t_3 - t_2 = \frac{(v_x)_3 - (v_x)_2}{a_x} = \frac{0 \text{ m/s} - 30 \text{ m/s}}{-6.0 \text{ m/s}^2} = 5.0 \text{ s}$$

The position at time t_3 is computed using the second equation in Table 2.4; we take point 2 as the initial point and point 3 as the final point for this phase of the motion and use $\Delta t = t_3 - t_2$:

$$x_3 = x_2 + (v_x)_2 \,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$
$$= 22.5 \text{ m} + (30 \text{ m/s})(5.0 \text{ s}) + \tfrac{1}{2}(-6.0 \text{ m/s}^2)(5.0 \text{ s})^2$$
$$= 98 \text{ m}$$

x_3 is the position of the car at the end of the problem—and so the car travels 98 m before coming to rest.

ASSESS The numbers for the reaction time and the acceleration on wet pavement are reasonable ones for an alert driver in a car with good tires. The final distance is quite large—it is more than the length of a football field.

1.7, 1.10 ActivPhysics ONLINE

2.7 Free Fall

A particularly important example of constant acceleration is the motion of an object moving under the influence of gravity only, and no other forces. This motion is called **free fall.** Strictly speaking, free fall occurs only in a vacuum, where there is no air resistance. But if you drop a hammer, air resistance is nearly negligible, so we'll make only a very slight error in treating it *as if* it were in free fall. If you drop a feather, air resistance is *not* negligible, and we can't make this approximation. Motion with air resistance is a problem we will study in Chapter 5. Until then, we will restrict our attention to situations in which air resistance can be ignored, and we will make the reasonable assumption that falling objects are in free fall.

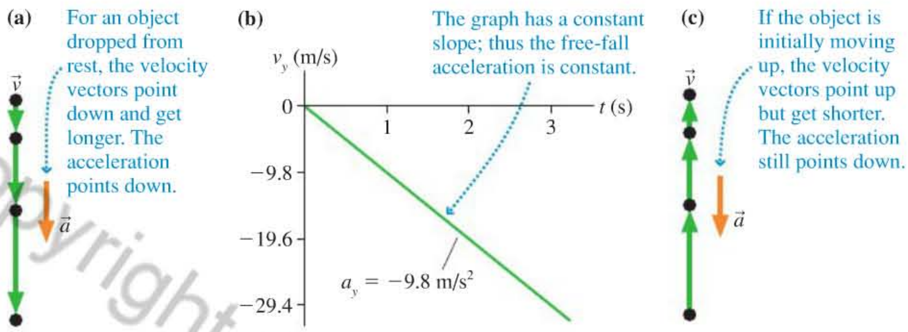
"Looks like Mr. Galileo was correct . . ." was the comment made by Apollo 15 astronaut David Scott, who dropped a hammer and a feather on the moon. The objects were dropped from the same height at the same time and hit the ground simultaneously—something that would not happen in the atmosphere of the earth!

As part of his early studies of motion, Galileo did the first careful experiments on free fall and made the surprising observation that two objects of different weight dropped from the same height will, if air resistance can be neglected, hit the ground at the same time and with the same speed. In fact—as Galileo surmised, and as a famous demonstration on the moon showed—in a vacuum, where there is no air resistance, this holds true for *any* two objects.

Galileo's discovery about free fall means that **any two objects in free fall, regardless of their mass, have the same acceleration.** This is an especially important conclusion. **FIGURE 2.36a** shows the motion diagram of an object that was released from rest and falls freely. The motion diagram and graph would be identical for a falling baseball or a falling boulder! **FIGURE 2.36b** shows the object's velocity graph. As we can see, the velocity changes at a steady rate. The slope of the velocity-versus-time graph is the free-fall acceleration $a_{\text{free fall}}$.

Instead of dropping the object, suppose we throw it upward. What happens then? You know that the object will move up and that its speed will decrease as it rises.

FIGURE 2.36 Motion of an object in free fall.



This is illustrated in the motion diagram of **FIGURE 2.36c**, which shows a surprising result: Even though the object is moving up, its acceleration still points down. In fact, **the free-fall acceleration always points down,** no matter what direction an object is moving.

NOTE ► Despite the name, free fall is not restricted to objects that are literally falling. Any object moving under the influence of gravity only, and no other forces, is in free fall. This includes objects falling straight down, objects that have been tossed or shot straight up, objects in projectile motion (such as a passed football), and, as we will see, satellites in orbit. In this chapter we consider only objects that move up and down along a vertical line; projectile motion will be studied in Chapter 3. ◄

Some of the children are moving up and some are moving down, but all are in free fall—and so are accelerating downward at $9.8\ \mathrm{m/s^2}$.

The free-fall acceleration is always in the same direction, and on earth, it always has approximately the same magnitude. Careful measurements show that the value of the free-fall acceleration varies slightly at different places on the earth, but for the calculations in this book we will use the the following average value:

$$\vec{a}_{\text{free fall}} = (9.80\ \mathrm{m/s^2},\ \text{vertically downward}) \tag{2.15}$$

Standard value for the acceleration of an object in free fall

The magnitude of the **free-fall acceleration** has the special symbol g:

$$g = 9.80\ \mathrm{m/s^2}$$

We will generally work with two significant figures and so will use $g = 9.8\ \mathrm{m/s^2}$.

Several points about free fall are worthy of note:

- g, by definition, is *always* positive. **There will never be a problem that uses a negative value for g.**
- The velocity graph in Figure 2.36b has a negative slope. Even though a falling object speeds up, it has *negative* acceleration. Alternatively, notice that the acceleration vector $\vec{a}_{\text{free fall}}$ points down. Thus g is *not* the object's acceleration, simply the magnitude of the acceleration. The one-dimensional acceleration is

$$a_y = a_{\text{free fall}} = -g$$

  It is a_y that is negative, not g.
- Because free fall is motion with constant acceleration, we can use the kinematic equations of Table 2.4 with the acceleration being due to gravity, $a_y = -g$.
- g is not called "gravity." Gravity is a force, not an acceleration. g is the *free-fall acceleration.*
- $g = 9.80\ \mathrm{m/s^2}$ only on earth. Other planets have different values of g. You will learn in Chapter 6 how to determine g for other planets.

TRY IT YOURSELF

A reaction time challenge Hold a $1 (or larger!) bill by an upper corner. Have a friend prepare to grasp a lower corner, putting her fingers *near but not touching* the bill. Tell her to try to catch the bill when you drop it by simply closing her fingers without moving her hand downward—and that if she can catch it, she can keep it. Don't worry; the bill's free fall will keep your money safe. In the few tenths of a second that it takes your friend to react, free fall will take the bill beyond her grasp.

- We will sometimes compute acceleration in units of g. An acceleration of 9.8 m/s^2 is an acceleration of $1g$; an acceleration of 19.6 m/s^2 is $2g$. Generally, we can compute

$$\text{acceleration (in units of } g) = \frac{\text{acceleration (in units of m/s}^2)}{9.8 \text{ m/s}^2} \qquad (2.16)$$

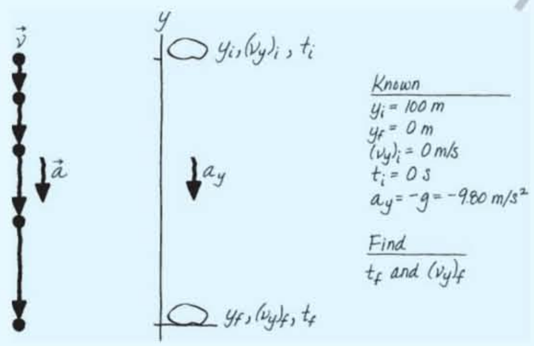
This allows us to express accelerations in units that have a definite physical reference.

EXAMPLE 2.13 Analyzing a rock's fall

A heavy rock is dropped from rest at the top of a cliff and falls 100 m before hitting the ground. How long does the rock take to fall to the ground, and what is its velocity when it hits?

PREPARE FIGURE 2.37 shows a visual overview with all necessary data. We have placed the origin at the ground, which makes $y_i = 100$ m.

FIGURE 2.37 Visual overview of a falling rock.



SOLVE Free fall is motion with the specific constant acceleration $a_y = -g$. The first question involves a relation between time and distance, a relation expressed by the second equation in Table 2.4. Using $(v_y)_i = 0$ m/s and $t_i = 0$ s, we find

$$y_f = y_i + (v_y)_i\,\Delta t + \tfrac{1}{2}a_y\,\Delta t^2 = y_i - \tfrac{1}{2}g\,\Delta t^2 = y_i - \tfrac{1}{2}g t_f^2$$

We can now solve for t_f:

$$t_f = \sqrt{\frac{2(y_i - y_f)}{g}} = \sqrt{\frac{2(100 \text{ m} - 0 \text{ m})}{9.80 \text{ m/s}^2}} = 4.52 \text{ s}$$

Now that we know the fall time, we can use the first kinematic equation to find $(v_y)_f$:

$$(v_y)_f = (v_y)_i - g\,\Delta t = -g t_f = -(9.80 \text{ m/s}^2)(4.52 \text{ s})$$
$$= -44.3 \text{ m/s}$$

ASSESS Are the answers reasonable? Well, 100 m is about 300 feet, which is about the height of a 30-floor building. How long does it take something to fall 30 floors? Four or five seconds seems pretty reasonable. How fast would it be going at the bottom? Using an approximate version of our conversion factor 1 m/s $\approx$ 2 mph, we find that 44.3 m/s $\approx$ 90 mph. That also seems like a pretty reasonable speed for something that has fallen 30 floors. Suppose we had made a mistake. If we misplaced a decimal point we could have calculated a speed of 443 m/s, or about 900 mph! This is clearly *not* reasonable. If we had misplaced the decimal point in the other direction, we would have calculated a speed of 4.3 m/s $\approx$ 9 mph. This is another unreasonable result, because this is slower than a typical bicycling speed.

CONCEPTUAL EXAMPLE 2.14 Analyzing the motion of a ball tossed upward

Draw a motion diagram and a velocity-versus-time graph for a ball tossed straight up in the air from the point that it leaves the hand until just before it is caught.

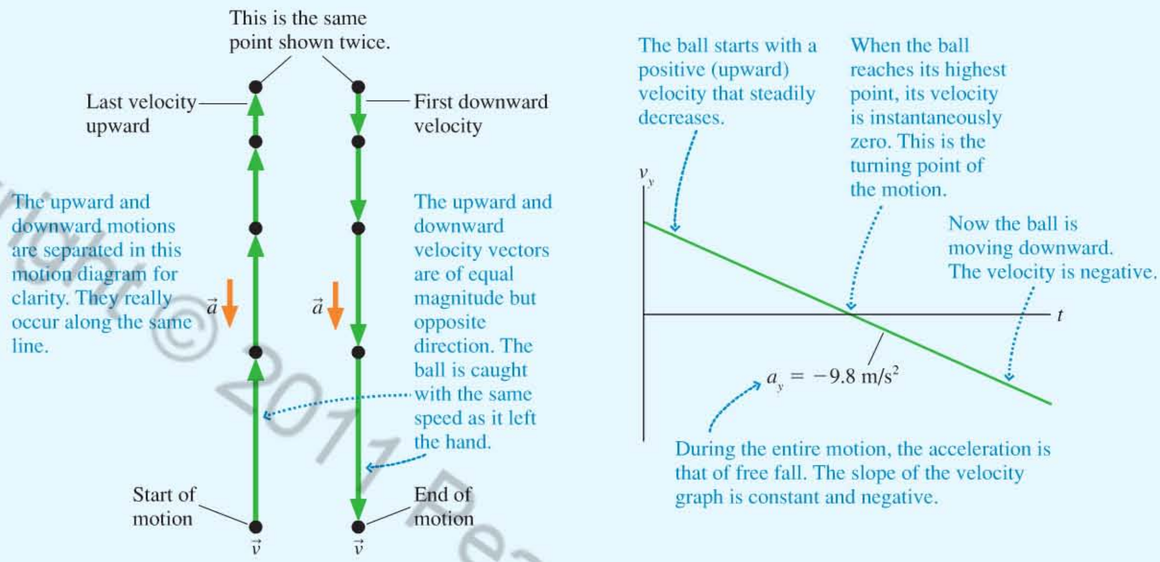
REASON You know what the motion of the ball looks like: The ball goes up, and then it comes back down again. This complicates the drawing of a motion diagram a bit, as the ball retraces its route as it falls. A literal motion diagram would show the upward motion and downward motion on top of each other, leading to confusion. We can avoid this difficulty by horizontally separating the upward motion and downward motion diagrams. This will not affect our conclusions because it does not change any of the vectors. The motion diagram and velocity-versus-time graph appear as in FIGURE 2.38 on the next page.

ASSESS The highest point in the ball's motion, where it reverses direction, is called a *turning point.* What are the velocity and the acceleration at this point? We can see from the motion diagram that the velocity vectors are pointing upward but getting shorter as the ball approaches the top. As it starts to fall, the velocity vectors are pointing downward and getting longer. There must be a moment—just an instant as $\vec{v}$ switches from pointing up to pointing down—when the velocity is zero. Indeed, the ball's velocity *is* zero for an instant at the precise top of the motion! We can also see on the velocity graph that there is one instant of time when $v_y = 0$. This is the turning point.

But what about the acceleration at the top? Many people expect the acceleration to be zero at the highest point. But recall that the velocity at the top point is changing—from up to down. If the velocity is changing, there *must* be an acceleration. The slope of the velocity graph at the instant when $v_y = 0$—that is, at the highest point—is no different than at any other point in the motion. The ball is still in free fall with acceleration $a_y = -g$!

Another way to think about this is to note that zero acceleration would mean no change of velocity. When the ball reached zero velocity at the top, it would hang there and not fall if the acceleration were also zero!

FIGURE 2.38 Motion diagram and velocity graph of a ball tossed straight up in the air.



EXAMPLE 2.15 Finding the height of a leap

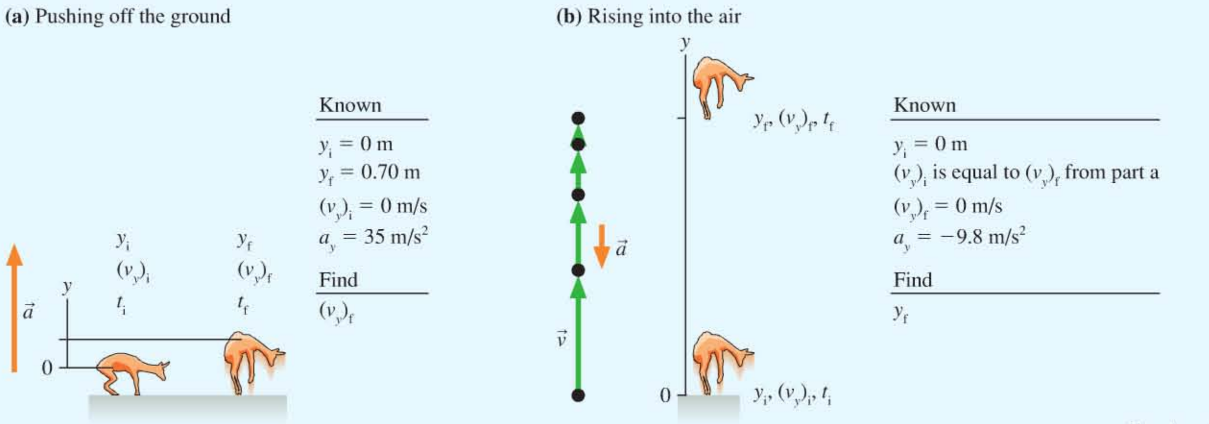
A springbok is an antelope found in southern Africa that gets its name from its remarkable jumping ability. When a springbok is startled, it will leap straight up into the air—a maneuver called a "pronk." A springbok goes into a crouch to perform a pronk. It then extends its legs forcefully, accelerating at 35 m/s^2 for 0.70 m as its legs straighten. Legs fully extended, it leaves the ground and rises into the air.

a. At what speed does the springbok leave the ground?
b. How high does it go?

PREPARE We begin with the visual overview shown in **FIGURE 2.39**, where we've identified two different phases of the motion: the springbok pushing off the ground and the springbok rising into the air. We'll treat these as two separate problems that we solve in turn. We will "re-use" the variables y_i, y_f, $(v_y)_i$, and $(v_y)_f$ for the two phases of the motion.

For the first part of our solution, in Figure 2.39a we choose the origin of the y-axis at the position of the springbok deep in the crouch. The final position is the top extent of the push, at the instant the springbok leaves the ground. We want to find the velocity at this position because that's how fast the springbok is moving as it leaves the ground. Figure 2.39b essentially starts over—we have defined a new vertical axis with its origin at the ground, so the highest point of the springbok's motion is a

FIGURE 2.39 A visual overview of the springbok's leap.



Continued

distance above the ground. The table of values shows the key piece of information for this second part of the problem: The initial velocity for part b is the final velocity from part a.

After the springbok leaves the ground, this is a free-fall problem because the springbok is moving under the influence of gravity only. We want to know the height of the leap, so we are looking for the height at the top point of the motion. This is a turning point of the motion, with the instantaneous velocity equal to zero. Thus y_f, the height of the leap, is the springbok's position at the instant $(v_y)_f = 0$.

SOLVE a. For the first phase, pushing off the ground, we have information about displacement, initial velocity, and acceleration, but we don't know anything about the time interval. The third equation in Table 2.4 is perfect for this type of situation. We can rearrange it to solve for the velocity with which the springbok lifts off the ground:

$$(v_y)_f^2 = v_i^2 + 2a_y\Delta y = (0\text{ m/s})^2 + 2(35\text{ m/s}^2)(0.70\text{ m}) = 49\text{ m}^2/\text{s}^2$$

$$(v_y)_f = \sqrt{49\text{ m}^2/\text{s}^2} = 7.0\text{ m/s}$$

The springbok leaves the ground with a speed of 7.0 m/s.

b. Now we are ready for the second phase of the motion, the vertical motion after leaving the ground. The third equation in Table 2.4 is again appropriate because again we don't know the time. Because $y_i = 0$, the springbok's displacement is $\Delta y = y_f - y_i = y_f$, the height of the vertical leap. From part a, the initial velocity is $(v_y)_i = 7.0$ m/s, and the final velocity is $(v_y)_f = 0$. This is free-fall motion, with $a_y = -g$; thus

$$(v_y)_f^2 = 0 = (v_y)_i^2 - 2g\Delta y = (v_y)_i^2 - 2gy_f$$

which gives

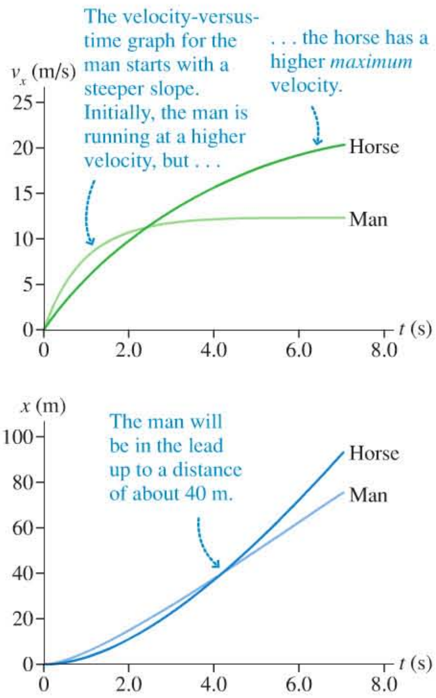
$$(v_y)_i^2 = 2gy_f$$

Solving for y_f, we get a jump height of

$$y_f = \frac{(7.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 2.5\text{ m}$$

ASSESS 2.5 m is a remarkable leap—a bit over 8 ft—but these animals are known for their jumping ability, so this seems reasonable.

FIGURE 2.40 Velocity-versus-time and position-versus-time graphs for a sprint between a man and a horse.



The caption accompanying the photo at the start of the chapter asked a question about animals and their athletic abilities: Who is the winner in a race between a horse and a man? The surprising answer is "It depends." Specifically, the winner depends on the length of the race.

Some animals are capable of high speed; others are capable of great acceleration. Horses can run much faster than humans, but, when starting from rest, humans are capable of much greater initial acceleration. **FIGURE 2.40** shows velocity and position graphs for an elite male sprinter and a thoroughbred racehorse. The horse's maximum velocity is about twice that of the man, but the man's initial acceleration—the slope of the velocity graph at early times—is about twice that of the horse. As the second graph shows, a man could win a *very* short race. For a longer race, the horse's higher maximum velocity will put it in the lead; the men's world-record time for the mile is a bit under 4 min, but a horse can easily run this distance in less than 2 min.

For a race of many miles, another factor comes into play: energy. A very long race is less about velocity and acceleration than about endurance—the ability to continue expending energy for a long time. In such endurance trials, humans often win. We will explore such energy issues in Chapter 11.

STOP TO THINK 2.6 A volcano ejects a chunk of rock straight up at a velocity of $v_y = 30$ m/s. Ignoring air resistance, what will be the velocity v_y of the rock when it falls back into the volcano's crater?

A. > 30 m/s B. 30 m/s C. 0 m/s D. -30 m/s E. < -30 m/s

INTEGRATED EXAMPLE 2.16 Speed versus endurance

Cheetahs have the highest top speed of any land animal, but they usually fail in their attempts to catch their prey because their endurance is limited. They can maintain their maximum speed of 30 m/s for only about 15 s before they need to stop.

Thomson's gazelles, their preferred prey, have a lower top speed than cheetahs, but they can maintain this speed for a few minutes. When a cheetah goes after a gazelle, success or failure is a simple matter of kinematics: Is the cheetah's high speed enough to allow it to reach its prey before the cheetah runs out of steam? The following problem uses realistic data for such a chase.

A cheetah has spotted a gazelle. The cheetah leaps into action, reaching its top speed of 30 m/s in a few seconds. At this instant, the gazelle, 160 m from the running cheetah, notices the danger and heads directly away. The gazelle accelerates at 4.5 m/s² for 6.0 s, then continues running at a constant speed. After reaching its maximum speed, the cheetah can continue running for only 15 s. Does the cheetah catch the gazelle, or does the gazelle escape?

PREPARE The example asks, "Does the cheetah catch the gazelle?" Our most challenging task is to translate these words into a graphical and mathematical problem that we can solve using the techniques of the chapter.

There are two related problems: the motion of the cheetah and the motion of the gazelle, for which we'll use the subscripts "C" and "G". Let's take our starting time, $t_1 = 0$ s, as the instant that the gazelle notices the cheetah and begins to run. We'll take the position of the cheetah at this instant as the origin of our coordinate system, so $x_{1C} = 0$ m and $x_{1G} = 160$ m—the gazelle is 160 m away when it notices the cheetah. We've used this information to draw the visual overview in **FIGURE 2.41**, which includes motion diagrams and velocity graphs for the cheetah and the gazelle. The visual overview sums up everything we know about the problem.

With a clear picture of the situation, we can now rephrase the problem this way: Compute the position of the cheetah and the position of the gazelle at $t_3 = 15$ s, the time when the cheetah needs to break off the chase. If $x_{3G} \geq x_{3C}$, then the gazelle stays out in front and escapes. If $x_{3G} < x_{3C}$, the cheetah wins the race—and gets its dinner.

SOLVE The cheetah is in uniform motion for the entire duration of the problem, so we can use Equation 2.4 to solve for its position at $t_3 = 15$ s:

$$x_{3C} = x_{1C} + (v_x)_{1C}\Delta t = 0 \text{ m} + (30 \text{ m/s})(15 \text{ s}) = 450 \text{ m}$$

The gazelle's motion has two phases: one of constant acceleration and then one of constant velocity. We can solve for the position and the velocity at t_2, the end of the first phase, using the first two equations in Table 2.4. Let's find the velocity first:

$$(v_x)_{2G} = (v_x)_{1G} + (a_x)_G\Delta t = 0 \text{ m/s} + (4.5 \text{ m/s}^2)(6.0 \text{ s}) = 27 \text{ m/s}$$

The gazelle's position at t_2 is:

Δt is the time for this phase of the motion, $t_2 - t_1 = 6.0$ s.

$$x_{2G} = x_{1G} + (v_x)_{1G}\,\Delta t + \tfrac{1}{2}(a_x)_G(\Delta t)^2$$
$$= 160 \text{ m} + 0 + \tfrac{1}{2}(4.5 \text{ m/s}^2)(6.0 \text{ s})^2 = 240 \text{ m}$$

The gazelle has a head start; it begins at $x_{1G} = 160$ m.

From t_2 to t_3 the gazelle moves at a constant speed, so we can use the uniform motion equation, Equation 2.4, to find its final position:

The gazelle begins this phase of the motion at $x_{2G} = 240$ m.

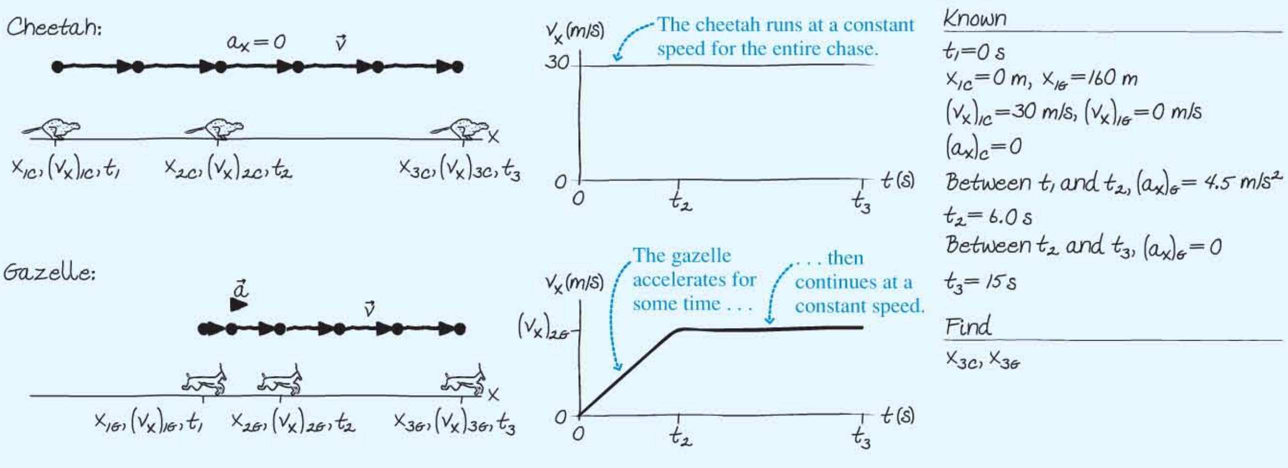
Δt for this phase of the motion is $t_3 - t_2 = 9.0$ s.

$$x_{3G} = x_{2G} + (v_x)_{2G}\,\Delta t = 240 \text{ m} + (27 \text{ m/s})(9.0 \text{ s}) = 480 \text{ m}$$

x_{3C} is 450 m; x_{3G} is 480 m. The gazelle is 30 m ahead of the cheetah when the cheetah has to break off the chase, so the gazelle escapes.

ASSESS Does our solution make sense? Let's look at the final result. The numbers in the problem statement are realistic, so we expect our results to mirror real life. The speed for the gazelle is close to that of the cheetah, which seems reasonable for two animals known for their speed. And the result is the most common occurrence—the chase is close, but the gazelle gets away.

FIGURE 2.41 Visual overview for the cheetah and for the gazelle.



SUMMARY

The goal of Chapter 2 has been to describe and analyze linear motion.

GENERAL STRATEGIES

Problem-Solving Strategy

Our general problem-solving strategy has three parts:

PREPARE Set up the problem:

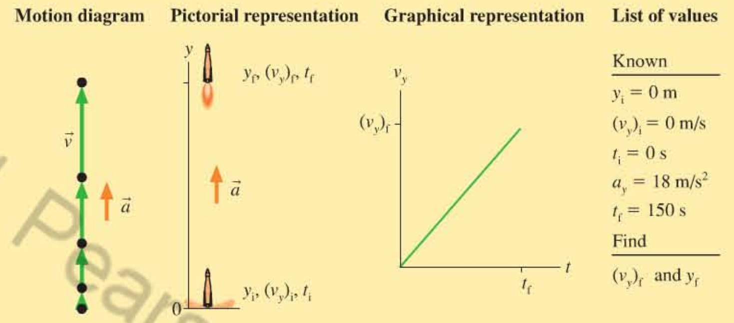
- Draw a picture.
- Collect necessary information.
- Do preliminary calculations.

SOLVE Do the necessary mathematics or reasoning.

ASSESS Check your answer to see if it is complete in all details and makes physical sense.

Visual Overview

A visual overview consists of several pieces that completely specify a problem. This may include any or all of the elements below:



IMPORTANT CONCEPTS

Velocity is the rate of change of position:

$$v_x = \frac{\Delta x}{\Delta t}$$

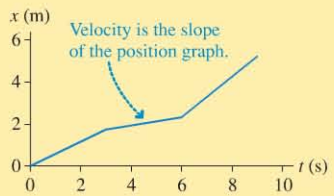
Acceleration is the rate of change of velocity:

$$a_x = \frac{\Delta v_x}{\Delta t}$$

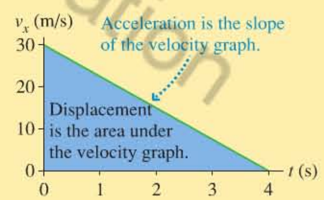
The units of acceleration are m/s^2.

An object is speeding up if v_x and a_x have the same sign, slowing down if they have opposite signs.

A **position-versus-time graph** plots position on the vertical axis against time on the horizontal axis.



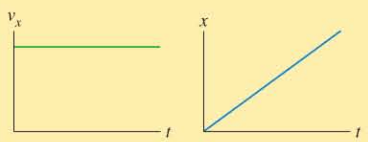
A **velocity-versus-time graph** plots velocity on the vertical axis against time on the horizontal axis.



APPLICATIONS

Uniform motion

An object in uniform motion has a constant velocity. Its velocity graph is a horizontal line; its position graph is linear.



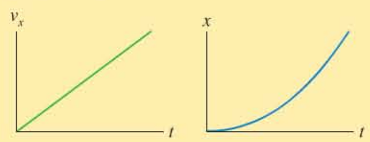
Kinematic equation for uniform motion:

$$x_f = x_i + v_x\,\Delta t$$

Uniform motion is a special case of constant-acceleration motion, with $a_x = 0$.

Motion with constant acceleration

An object with constant acceleration has a constantly changing velocity. Its velocity graph is linear; its position graph is a parabola.



Kinematic equations for motion with constant acceleration:

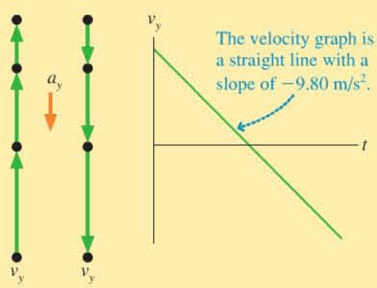
$$(v_x)_f = (v_x)_i + a_x\,\Delta t$$

$$x_f = x_i + (v_x)_i\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x\,\Delta x$$

Free fall

Free fall is a special case of constant-acceleration motion; the acceleration has magnitude $g = 9.80$ m/s^2 and is always directed vertically downward whether an object is moving up or down.



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. A person gets in an elevator on the ground floor and rides it to the top floor of a building. Sketch a velocity-versus-time graph for this motion.
2. a. Give an example of a vertical motion with a positive velocity and a negative acceleration.
   b. Give an example of a vertical motion with a negative velocity and a negative acceleration.
3. Sketch a velocity-versus-time graph for a rock that is thrown straight upward, from the instant it leaves the hand until the instant it hits the ground.
4. You are driving down the road at a constant speed. Another car going a bit faster catches up with you and passes you. Draw a position graph for both vehicles on the same set of axes, and note the point on the graph where the other vehicle passes you.
5. A car is traveling north. Can its acceleration vector ever point south? Explain.
6. BIO Certain animals are capable of running at great speeds; other animals are capable of tremendous accelerations. Speculate on which would be more beneficial to a predator—large maximum speed or large acceleration.
7. A ball is thrown straight up into the air. At each of the following instants, is the ball's acceleration a_y equal to g, $-g$, 0, $< g$, or $> g$?
   a. Just after leaving your hand?
   b. At the very top (maximum height)?
   c. Just before hitting the ground?
8. A rock is *thrown* (not dropped) straight down from a bridge into the river below.
   a. Immediately after being released, is the magnitude of the rock's acceleration greater than g, less than g, or equal to g? Explain.
   b. Immediately before hitting the water, is the magnitude of the the rock's acceleration greater than g, less than g, or equal to g? Explain.
9. Figure Q2.9 shows an object's position-versus-time graph. The letters A to E correspond to various segments of the motion in which the graph has constant slope.

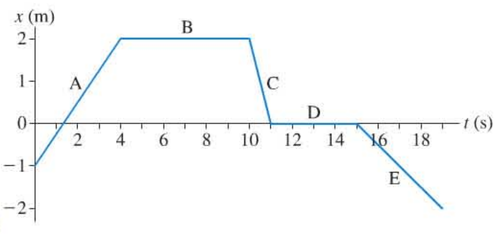


FIGURE Q2.9

   a. Write a realistic motion short story for an object that would have this position graph.
   b. In which segment(s) is the object at rest?
   c. In which segment(s) is the object moving to the right?
   d. Is the speed of the object during segment C greater than, equal to, or less than its speed during segment E? Explain.
10. Figure Q2.10 shows the position graph for an object moving along the horizontal axis.
   a. Write a realistic motion short story for an object that would have this position graph.
   b. Draw the corresponding velocity graph.

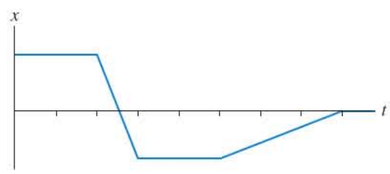


FIGURE Q2.10

11. Figure Q2.11 shows the position-versus-time graphs for two objects, A and B, that are moving along the same axis.
   a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.
   b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

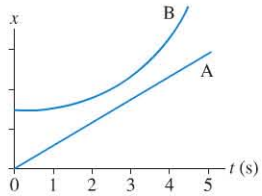


FIGURE Q2.11

12. Figure Q2.12 shows a position-versus-time graph. At which lettered point or points is the object
   a. Moving the fastest?
   b. Moving to the left?
   c. Speeding up?
   d. Slowing down?
   e. Turning around?

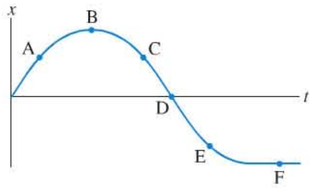


FIGURE Q2.12

13. Figure Q2.13 is the velocity-versus-time graph for an object moving along the x-axis.
   a. During which segment(s) is the velocity constant?
   b. During which segment(s) is the object speeding up?
   c. During which segment(s) is the object slowing down?
   d. During which segment(s) is the object standing still?
   e. During which segment(s) is the object moving to the right?

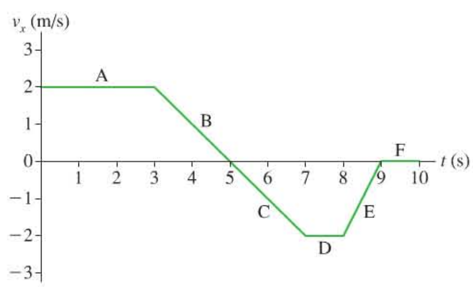


FIGURE Q2.13

14. A car traveling at velocity v takes distance d to stop after the brakes are applied. What is the stopping distance if the car is initially traveling at velocity $2v$? Assume that the acceleration due to the braking is the same in both cases.

Multiple-Choice Questions

15. | Figure Q2.15 shows the position graph of a car traveling on a straight road. At which labeled instant is the speed of the car greatest?

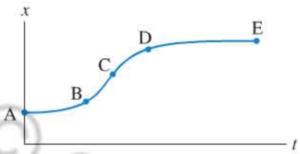


FIGURE Q2.15

16. | Figure Q2.16 shows the position graph of a car traveling on a straight road. The velocity at instant 1 is _________ and the velocity at instant 2 is _________.
   A. positive, negative
   B. positive, positive
   C. negative, negative
   D. negative, zero
   E. positive, zero

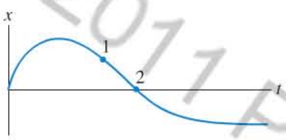


FIGURE Q2.16

17. | Figure Q2.17 shows an object's position-versus-time graph. What is the velocity of the object at $t = 6$ s?
   A. 0.67 m/s
   B. 0.83 m/s
   C. 3.3 m/s
   D. 4.2 m/s
   E. 25 m/s

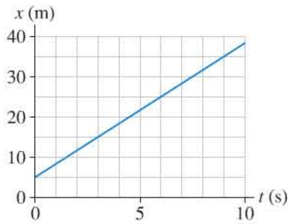


FIGURE Q2.17

18. | The following options describe the motion of four cars A–D. Which car has the largest acceleration?
   A. Goes from 0 m/s to 10 m/s in 5.0 s
   B. Goes from 0 m/s to 5.0 m/s in 2.0 s
   C. Goes from 0 m/s to 20 m/s in 7.0 s
   D. Goes from 0 m/s to 3.0 m/s in 1.0 s

19. | A car is traveling at $v_x = 20$ m/s. The driver applies the brakes, and the car slows with $a_x = -4.0$ m/s^2. What is the stopping distance?
   A. 5.0 m
   B. 25 m
   C. 40 m
   D. 50 m

20. || Velocity-versus-time graphs for three drag racers are shown in Figure Q2.20. At $t = 5.0$ s, which car has traveled the furthest?
   A. Andy
   B. Betty
   C. Carl
   D. All have traveled the same distance

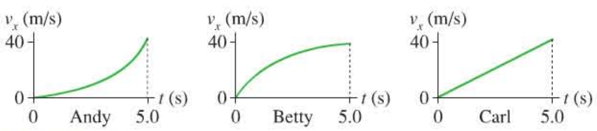


FIGURE Q2.20

21. | Which of the three drag racers in Question 20 had the greatest acceleration at $t = 0$ s?
   A. Andy
   B. Betty
   C. Carl
   D. All had the same acceleration

22. || Ball 1 is thrown straight up in the air and, at the same instant, ball 2 is released from rest and allowed to fall. Which velocity graph in Figure Q2.22 best represents the motion of the two balls?

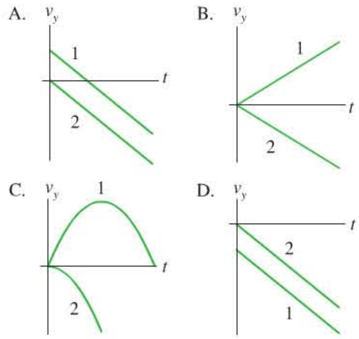


FIGURE Q2.22

23. || Figure Q2.23 shows a motion diagram with the clock reading (in seconds) shown at each position. From $t = 9$ s to $t = 15$ s the object is at the same position. After that, it returns along the same track. The positions of the dots for $t \geq 16$ s are offset for clarity. Which graph best represents the object's *velocity?*

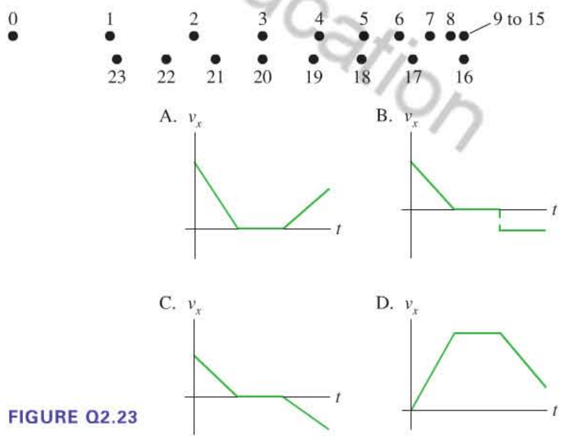


FIGURE Q2.23

24. || A car can go from 0 to 60 mph in 7.0 s. Assuming that it could maintain the same acceleration at higher speeds, how long would it take the car to go from 0 to 120 mph?
   A. 10 s
   B. 14 s
   C. 21 s
   D. 28 s

25. || A car can go from 0 to 60 mph in 12 s. A second car is capable of twice the acceleration of the first car. Assuming that it could maintain the same acceleration at higher speeds, how much time will this second car take to go from 0 to 120 mph?
   A. 12 s
   B. 9.0 s
   C. 6.0 s
   D. 3.0 s

PROBLEMS

Section 2.1 Describing Motion

1. ||| Figure P2.1 shows a motion diagram of a car traveling down a street. The camera took one frame every second. A distance scale is provided.
   a. Measure the x-value of the car at each dot. Place your data in a table, similar to Table 2.1, showing each position and the instant of time at which it occurred.
   b. Make a graph of x versus t, using the data in your table. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.

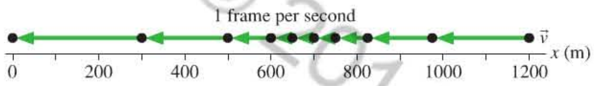


FIGURE P2.1

2. | For each motion diagram in Figure P2.2, determine the sign (positive or negative) of the position and the velocity.

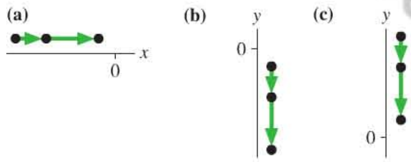


FIGURE P2.2

3. | Write a short description of the motion of a real object for which Figure P2.3 would be a realistic position-versus-time graph.

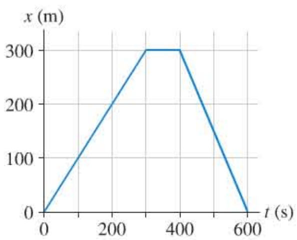


FIGURE P2.3

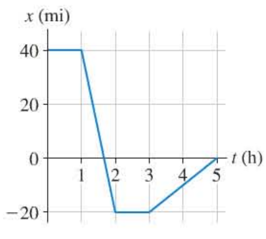


FIGURE P2.4

4. | Write a short description of the motion of a real object for which Figure P2.4 would be a realistic position-versus-time graph.

5. || The position graph of Figure P2.5 shows a dog slowly sneaking up on a squirrel, then putting on a burst of speed.
   a. For how many seconds does the dog move at the slower speed?
   b. Draw the dog's velocity-versus-time graph. Include a numerical scale on both axes.

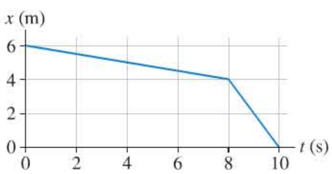


FIGURE P2.5

6. || The position graph of Figure P2.6 represents the motion of a ball being rolled back and forth by two children.
   a. At what positions are the two children sitting?
   b. Draw the ball's velocity-versus-time graph. Include a numerical scale on both axes.

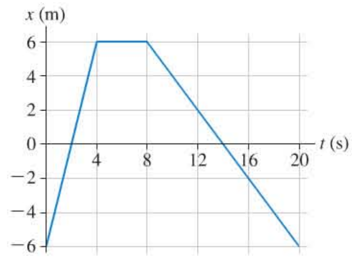


FIGURE P2.6

7. || A rural mail carrier is driving slowly, putting mail in mailboxes near the road. He overshoots one mailbox, stops, shifts into reverse, and then backs up until he is at the right spot. The velocity graph of Figure P2.7 represents his motion.
   a. Draw the mail carrier's position-versus-time graph. Assume that $x = 0$ m at $t = 0$ s.
   b. What is the position of the mailbox?

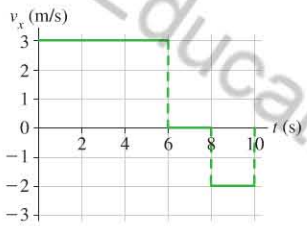


FIGURE P2.7

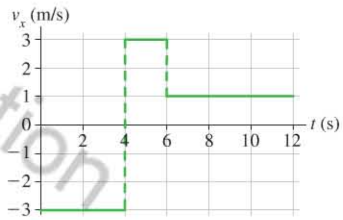


FIGURE P2.8

8. || For the velocity-versus-time graph of Figure P2.8:
   a. Draw the corresponding position-versus-time graph. Assume that $x = 0$ m at $t = 0$ s.
   b. What is the object's position at $t = 12$ s?
   c. Describe a moving object that could have these graphs.

9. || A bicyclist has the position-versus-time graph shown in Figure P2.9. What is the bicyclist's velocity at $t = 10$ s, at $t = 25$ s, and at $t = 35$ s?

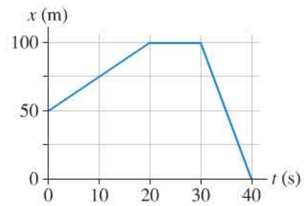


FIGURE P2.9

Section 2.2 Uniform Motion

10. | In college softball, the distance from the pitcher's mound to the batter is 43 feet. If the ball leaves the bat at 100 mph, how much time elapses between the hit and the ball reaching the pitcher?

11. | Alan leaves Los Angeles at 8:00 A.M. to drive to San Francisco, 400 mi away. He travels at a steady 50 mph. Beth leaves Los Angeles at 9:00 A.M. and drives a steady 60 mph.
   a. Who gets to San Francisco first?
   b. How long does the first to arrive have to wait for the second?
12. | Richard is driving home to visit his parents. 125 mi of the trip are on the interstate highway where the speed limit is 65 mph. Normally Richard drives at the speed limit, but today he is running late and decides to take his chances by driving at 70 mph. How many minutes does he save?
13. ||| In a 5.00 km race, one runner runs at a steady 12.0 km/h and another runs at 14.5 km/h. How long does the faster runner have to wait at the finish line to see the slower runner cross?
14. |||| In an 8.00 km race, one runner runs at a steady 11.0 km/h and another runs at 14.0 km/h. How far from the finish line is the slower runner when the faster runner finishes the race?
15. | A car moves with constant velocity along a straight road. Its position is $x_1 = 0$ m at $t_1 = 0$ s and is $x_2 = 30$ m at $t_2 = 3.0$ s. Answer the following by considering ratios, without computing the car's velocity.
   a. What is the car's position at $t = 1.5$ s?
   b. What will be its position at $t = 9.0$ s?
16. | While running a marathon, a long-distance runner uses a stopwatch to time herself over a distance of 100 m. She finds that she runs this distance in 18 s. Answer the following by considering ratios, without computing her velocity.
   a. If she maintains her speed, how much time will it take her to run the next 400 m?
   b. How long will it take her to run a mile at this speed?

Section 2.3 Instantaneous Velocity

17. | Figure P2.17 shows the position graph of a particle.
   a. Draw the particle's velocity graph for the interval $0\text{ s} \le t \le 4$ s.
   b. Does this particle have a turning point or points? If so, at what time or times?

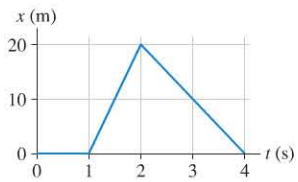


FIGURE P2.17

18. | A somewhat idealized graph of the speed of the blood in the
BIO ascending aorta during one beat of the heart appears as in Figure P2.18.
   a. Approximately how far, in cm, does the blood move during one beat?
   b. Assume similar data for the motion of the blood in your aorta. Estimate how many beats of the heart it will it take the blood to get from your heart to your brain.

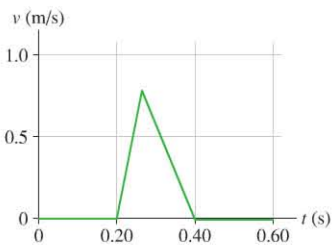


FIGURE P2.18

19. ||| A car starts from $x_i = 10$ m at $t_i = 0$ s and moves with the velocity graph shown in Figure P2.19.
   a. What is the object's position at $t = 2$ s, 3 s, and 4 s?
   b. Does this car ever change direction? If so, at what time?

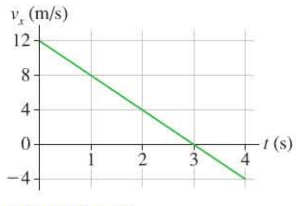


FIGURE P2.19

20. | Figure P2.20 shows a graph of actual position-versus-time data for a particular type of drag racer known as a "funny car."
   a. Estimate the car's velocity at 2.0 s.
   b. Estimate the car's velocity at 4.0 s.

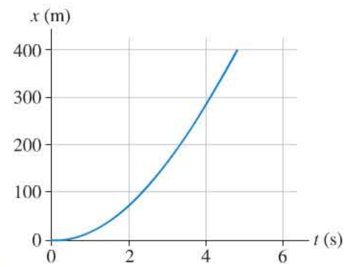


FIGURE P2.20

Section 2.4 Acceleration

21. | Figure P2.21 shows the velocity graph of a bicycle. Draw the bicycle's acceleration graph for the interval $0\text{ s} \le t \le 4$ s. Give both axes an appropriate numerical scale.

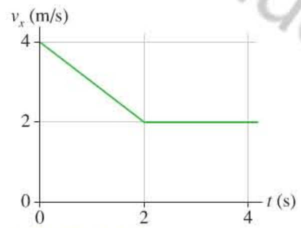


FIGURE P2.21

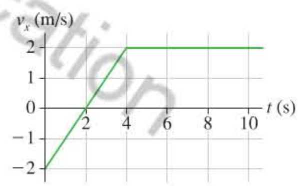


FIGURE P2.22

22. ||| Figure P2.22 shows the velocity graph of a train that starts from the origin at $t = 0$ s.
   a. Draw position and acceleration graphs for the train.
   b. Find the acceleration of the train at $t = 3.0$ s.
23. | For each motion diagram shown earlier in Figure P2.2, determine the sign (positive or negative) of the acceleration.
24. | Figure P2.18 showed data for the speed of blood in the aorta.
BIO Determine the magnitude of the acceleration for both phases, speeding up and slowing down.
25. | Figure P2.25 is a somewhat simplified velocity graph for Olympic sprinter Carl Lewis starting a 100 m dash. Estimate his acceleration during each of the intervals A, B, and C.

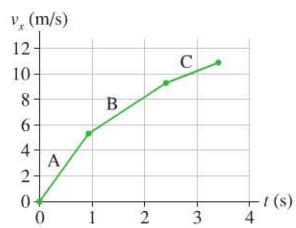


FIGURE P2.25

Section 2.5 Motion with Constant Acceleration

26. | A Thomson's gazelle can reach a speed of 13 m/s in 3.0 s. A
BIO lion can reach a speed of 9.5 m/s in 1.0 s. A trout can reach a speed of 2.8 m/s in 0.12 s. Which animal has the largest acceleration?

27. ||| BIO When striking, the pike, a predatory fish, can accelerate from rest to a speed of 4.0 m/s in 0.11 s.

a. What is the acceleration of the pike during this strike?
b. How far does the pike move during this strike?

28. || a. What constant acceleration, in SI units, must a car have to go from zero to 60 mph in 10 s?
b. What fraction of g is this?
c. How far has the car traveled when it reaches 60 mph? Give your answer both in SI units and in feet.

29. || Light-rail passenger trains that provide transportation within and between cities are capable of modest accelerations. The magnitude of the maximum acceleration is typically 1.3 m/s^2, but the driver will usually maintain a constant acceleration that is less than the maximum. A train travels through a congested part of town at 5.0 m/s. Once free of this area, it speeds up to 12 m/s in 8.0 s. At the edge of town, the driver again accelerates, with the same acceleration, for another 16 s to reach a higher cruising speed. What is the final speed?

30. ||| A speed skater moving across frictionless ice at 8.0 m/s hits a 5.0-m-wide patch of rough ice. She slows steadily, then continues on at 6.0 m/s. What is her acceleration on the rough ice?

31. || A small propeller airplane can comfortably achieve a high enough speed to take off on a runway that is 1/4 mile long. A large, fully loaded passenger jet has about the same acceleration from rest, but it needs to achieve twice the speed to take off. What is the minimum runway length that will serve? **Hint:** You can solve this problem using ratios without having any additional information.

32. || Figure P2.32 shows a velocity-versus-time graph for a particle moving along the x-axis. At $t = 0$ s, assume that $x = 0$ m.
a. What are the particle's position, velocity, and acceleration at $t = 1.0$ s?
b. What are the particle's position, velocity, and acceleration at $t = 3.0$ s?

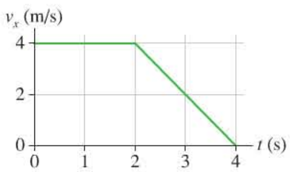


FIGURE P2.32

Section 2.6 Solving One-Dimensional Motion Problems

33. || A driver has a reaction time of 0.50 s, and the maximum deceleration of her car is 6.0 m/s^2. She is driving at 20 m/s when suddenly she sees an obstacle in the road 50 m in front of her. Can she stop the car in time to avoid a collision?

34. || BIO Chameleons catch insects with their tongues, which they can rapidly extend to great lengths. In a typical strike, the chameleon's tongue accelerates at a remarkable 250 m/s^2 for 20 ms, then travels at constant speed for another 30 ms. During this total time of 50 ms, 1/20 of a second, how far does the tongue reach?

35. ||| You're driving down the highway late one night at 20 m/s when a deer steps onto the road 35 m in front of you. Your reaction time before stepping on the brakes is 0.50 s, and the maximum deceleration of your car is 10 m/s^2.
a. How much distance is between you and the deer when you come to a stop?
b. What is the maximum speed you could have and still not hit the deer?

36. ||| A light-rail train going from one station to the next on a straight section of track accelerates from rest at 1.1 m/s^2 for 20 s. It then proceeds at constant speed for 1100 m before slowing down at 2.2 m/s^2 until it stops at the station.
a. What is the distance between the stations?
b. How much time does it take the train to go between the stations?

37. ||| A simple model for a person running the 100 m dash is to assume the sprinter runs with constant acceleration until reaching top speed, then maintains that speed through the finish line. If a sprinter reaches his top speed of 11.2 m/s in 2.14 s, what will be his total time?

Section 2.7 Free Fall

38. ||| Ball bearings can be made by letting spherical drops of molten metal fall inside a tall tower—called a *shot tower*—and solidify as they fall.
a. If a bearing needs 4.0 s to solidify enough for impact, how high must the tower be?
b. What is the bearing's impact velocity?

39. ||| BIO In the chapter, we saw that a person's reaction time is generally not quick enough to allow the person to catch a dollar bill dropped between the fingers. If a typical reaction time in this case is 0.25 s, how long would a bill need to be for a person to have a good chance of catching it?

40. || A ball is thrown vertically upward with a speed of 19.6 m/s.
a. What are the ball's velocity and height after 1.00, 2.00, 3.00, and 4.00 s?
b. Draw the ball's velocity-versus-time graph. Give both axes an appropriate numerical scale.

41. | A student at the top of a building of height h throws ball A straight upward with speed v_0 and throws ball B straight downward with the same initial speed.
a. Compare the balls' accelerations, both direction and magnitude, immediately after they leave her hand. Is one acceleration larger than the other? Or are the magnitudes equal?
b. Compare the final speeds of the balls as they reach the ground. Is one larger than the other? Or are they equal?

42. || Excellent human jumpers can leap straight up to a height of 110 cm off the ground. To reach this height, with what speed would a person need to leave the ground?

43. || A football is kicked straight up into the air; it hits the ground 5.2 s later.
a. What was the greatest height reached by the ball? Assume it is kicked from ground level.
b. With what speed did it leave the kicker's foot?

44. ||||| In an action movie, the villain is rescued from the ocean by grabbing onto the ladder hanging from a helicopter. He is so intent on gripping the ladder that he lets go of his briefcase of counterfeit money when he is 130 m above the water. If the briefcase hits the water 6.0 s later, what was the speed at which the helicopter was ascending?

45. |||| A rock climber stands on top of a 50-m-high cliff overhanging a pool of water. He throws two stones vertically downward 1.0 s apart and observes that they cause a single splash. The initial speed of the first stone was 2.0 m/s.
a. How long after the release of the first stone does the second stone hit the water?
b. What was the initial speed of the second stone?
c. What is the speed of each stone as they hit the water?

General Problems

46. ||| BIO Actual velocity data for a lion pursuing prey are shown in Figure P2.46. Estimate:
   a. The initial acceleration of the lion.
   b. The acceleration of the lion at 2 s and at 4 s.
   c. The distance traveled by the lion between 0 s and 8 s.

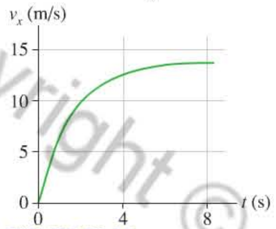


FIGURE P2.46

Problems 47 and 48 concern *nerve impulses*, electrical signals propagated along nerve fibers consisting of many *axons* (fiberlike extensions of nerve cells) connected end-to-end. Axons come in two varieties: insulated axons with a sheath made of myelin, and uninsulated axons with no such sheath. Myelinated (sheathed) axons conduct nerve impulses much faster than unmyelinated (unsheathed) axons. The impulse speed depends on the diameter of the axons and the sheath, but a typical myelinated axon transmits nerve impulses at a speed of about 25 m/s, much faster than the typical 2.0 m/s for an unmyelinated axon. Figure P2.47 shows small portions of three nerve fibers consisting of axons of equal size. Two-thirds of the axons in fiber B are myelinated.

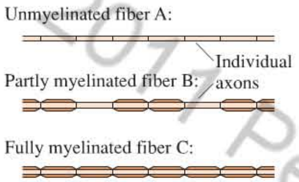


FIGURE P2.47

47. | BIO Suppose nerve impulses simultaneously enter the left side of the nerve fibers sketched in Figure P2.47, then propagate to the right. Draw qualitatively accurate position and velocity graphs for the nerve impulses in all three cases. A nerve fiber is made up of many axons, but show the propagation of the impulses only over the six axons shown here.
48. | BIO Suppose that the nerve fibers in Figure P2.47 connect a finger to your brain, a distance of 1.2 m.
   a. What are the travel times of a nerve impulse from finger to brain along fibers A and C?
   b. For fiber B, 2/3 of the length is composed of myelinated axons, 1/3 unmyelinated axons. Compute the travel time for a nerve impulse on this fiber.
   c. When you touch a hot stove with your finger, the sensation of pain must reach your brain as a nerve signal along a nerve fiber before your muscles can react. Which of the three fibers gives you the best protection against a burn? Are any of these fibers unsuitable for transmitting urgent sensory information?
49. || A truck driver has a shipment of apples to deliver to a destination 440 miles away. The trip usually takes him 8 hours. Today he finds himself daydreaming and realizes 120 miles into his trip that he is running 15 minutes later than his usual pace at this point. At what speed must he drive for the remainder of the trip to complete the trip in the usual amount of time?
50. || BIO When you sneeze, the air in your lungs accelerates from rest to approximately 150 km/h in about 0.50 seconds.
   a. What is the acceleration of the air in m/s^2?
   b. What is this acceleration, in units of g?
51. || Figure P2.51 shows the motion diagram, made at two frames of film per second, of a ball rolling along a track. The track has a 3.0-m-long sticky section.

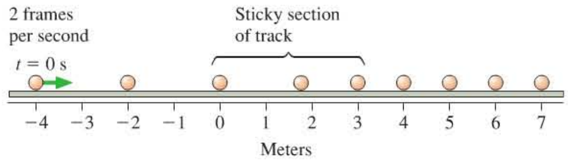


FIGURE P2.51

   a. Use the scale to determine the positions of the center of the ball. Place your data in a table, similar to Table 2.1, showing each position and the instant of time at which it occurred.
   b. Make a graph of x versus t for the ball. Because you have data only at certain instants of time, your graph should consist of dots that are not connected together.
   c. What is the *change* in the ball's position from $t = 0$ s to $t = 1.0$ s?
   d. What is the *change* in the ball's position from $t = 2.0$ s to $t = 4.0$ s?
   e. What is the ball's velocity before reaching the sticky section?
   f. What is the ball's velocity after passing the sticky section?
   g. Determine the ball's acceleration on the sticky section of the track.
52. || Julie drives 100 mi to Grandmother's house. On the way to Grandmother's, Julie drives half the *distance* at 40 mph and half the distance at 60 mph. On her return trip, she drives half the *time* at 40 mph and half the time at 60 mph.
   a. How long does it take Julie to complete the trip to Grandmother's house?
   b. How long does the return trip take?
53. || The takeoff speed for an Airbus A320 jetliner is 80 m/s. Velocity data measured during takeoff are as shown in the table.

t(s)	v_x(m/s)
0	0
10	23
20	46
30	69

   a. What is the takeoff speed in miles per hour?
   b. What is the jetliner's acceleration during takeoff?
   c. At what time do the wheels leave the ground?
   d. For safety reasons, in case of an aborted takeoff, the runway must be three times the takeoff distance. Can an A320 take off safely on a 2.5-mi-long runway?
54. |||| Does a real automobile have constant acceleration? Measured data for a Porsche 944 Turbo at maximum acceleration are as shown in the table.

t(s)	v_x(mph)
0	0
2	28
4	46
6	60
8	70
10	78

   a. Convert the velocities to m/s, then make a graph of velocity versus time. Based on your graph, is the acceleration constant? Explain.
   b. Draw a smooth curve through the points on your graph, then use your graph to *estimate* the car's acceleration at 2.0 s and 8.0 s. Give your answer in SI units. **Hint:** Remember that acceleration is the slope of the velocity graph.

55. | People hoping to travel to other worlds are faced with huge challenges. One of the biggest is the time required for a journey. The nearest star is 4.1×10^{16} m away. Suppose you had a spacecraft that could accelerate at $1.0g$ for half a year, then continue at a constant speed. (This is far beyond what can be achieved with any known technology.) How long would it take you to reach the nearest star to earth?

56. ||| You are driving to the grocery store at 20 m/s. You are 110 m from an intersection when the traffic light turns red. Assume that your reaction time is 0.70 s and that your car brakes with constant acceleration.
   a. How far are you from the intersection when you begin to apply the brakes?
   b. What acceleration will bring you to rest right at the intersection?
   c. How long does it take you to stop?

57. | When you blink your eye, the upper lid goes from rest with your
BIO eye open to completely covering your eye in a time of 0.024 s.
   a. Estimate the distance that the top lid of your eye moves during a blink.
   b. What is the acceleration of your eyelid? Assume it to be constant.
   c. What is your upper eyelid's final speed as it hits the bottom eyelid?

58. ||| A bush baby, an African
BIO primate, is capable of leaping vertically to the remarkable height of 2.3 m. To jump this high, the bush baby accelerates over a distance of 0.16 m while rapidly extending its legs. The acceleration during the jump is approximately constant. What is the acceleration in m/s^2 and in g's?

59. |||| When jumping, a flea reaches a takeoff speed of 1.0 m/s over
BIO a distance of 0.50 mm.
   a. What is the flea's acceleration during the jump phase?
   b. How long does the acceleration phase last?
   c. If the flea jumps straight up, how high will it go? (Ignore air resistance for this problem; in reality, air resistance plays a large role, and the flea will not reach this height.)

60. ||| Certain insects can achieve
BIO seemingly impossible accelerations while jumping. The click beetle accelerates at an astonishing $400g$ over a distance of 0.60 cm as it rapidly bends its thorax, making the "click" that gives it its name.

   a. Assuming the beetle jumps straight up, at what speed does it leave the ground?
   b. How much time is required for the beetle to reach this speed?
   c. Ignoring air resistance, how high would it go?

61. |||| Divers compete by diving into a 3.0-m-deep pool from a platform 10 m above the water. What is the magnitude of the minimum acceleration in the water needed to keep a diver from hitting the bottom of the pool? Assume the acceleration is constant.

62. |||| A student standing on the ground throws a ball straight up. The ball leaves the student's hand with a speed of 15 m/s when the hand is 2.0 m above the ground. How long is the ball in the air before it hits the ground? (The student moves her hand out of the way.)

63. |||| A rock is tossed straight up with a speed of 20 m/s. When it returns, it falls into a hole 10 m deep.
   a. What is the rock's velocity as it hits the bottom of the hole?
   b. How long is the rock in the air, from the instant it is released until it hits the bottom of the hole?

64. ||||| A 200 kg weather rocket is loaded with 100 kg of fuel and fired straight up. It accelerates upward at 30.0 m/s^2 for 30.0 s, then runs out of fuel. Ignore any air resistance effects.
   a. What is the rocket's maximum altitude?
   b. How long is the rocket in the air?
   c. Draw a velocity-versus-time graph for the rocket from liftoff until it hits the ground.

65. ||||| A juggler throws a ball straight up into the air with a speed of 10 m/s. With what speed would she need to throw a second ball half a second later, starting from the same position as the first, in order to hit the first ball at the top of its trajectory?

66. |||| A hotel elevator ascends 200 m with a maximum speed of 5.0 m/s. Its acceleration and deceleration both have a magnitude of 1.0 m/s^2.
   a. How far does the elevator move while accelerating to full speed from rest?
   b. How long does it take to make the complete trip from bottom to top?

67. |||| A car starts from rest at a stop sign. It accelerates at 2.0 m/s^2 for 6.0 seconds, coasts for 2.0 s, and then slows down at a rate of 1.5 m/s^2 for the next stop sign. How far apart are the stop signs?

68. |||| A toy train is pushed forward and released at $x_i = 2.0$ m with a speed of 2.0 m/s. It rolls at a steady speed for 2.0 s, then one wheel begins to stick. The train comes to a stop 6.0 m from the point at which it was released. What is the train's acceleration after its wheel begins to stick?

69. ||| Heather and Jerry are standing on a bridge 50 m above a river. Heather throws a rock straight down with a speed of 20 m/s. Jerry, at exactly the same instant of time, throws a rock straight up with the same speed. Ignore air resistance.
   a. How much time elapses between the first splash and the second splash?
   b. Which rock has the faster speed as it hits the water?

70. |||| A motorist is driving at 20 m/s when she sees that a traffic light 200 m ahead has just turned red. She knows that this light stays red for 15 s, and she wants to reach the light just as it turns green again. It takes her 1.0 s to step on the brakes and begin slowing at a constant deceleration. What is her speed as she reaches the light at the instant it turns green?

71. ||||| A "rocket car" is launched along a long straight track at $t = 0$ s. It moves with constant acceleration $a_1 = 2.0$ m/s^2. At $t = 2.0$ s, a second car is launched along a parallel track, from the same starting point, with constant acceleration $a_2 = 8.0$ m/s^2.
   a. At what time does the second car catch up with the first one?
   b. How far have the cars traveled when the second passes the first?

72. ||| A Porsche challenges a Honda to a 400 m race. Because the Porsche's acceleration of 3.5 m/s^2 is larger than the Honda's 3.0 m/s^2, the Honda gets a 50-m head start. Assume, somewhat unrealistically, that both cars can maintain these accelerations the entire distance. Who wins, and by how much time?

73. ||||| The minimum stopping distance for a car traveling at a speed of 30 m/s is 60 m, including the distance traveled during the driver's reaction time of 0.50 s.
    a. What is the minimum stopping distance for the same car traveling at a speed of 40 m/s?
    b. Draw a position-versus-time graph for the motion of the car in part a. Assume the car is at $x_i = 0$ m when the driver first sees the emergency situation ahead that calls for a rapid halt.
74. ||||| A rocket is launched straight up with constant acceleration. Four seconds after liftoff, a bolt falls off the side of the rocket. The bolt hits the ground 6.0 s later. What was the rocket's acceleration?

Passage Problems

Free Fall on Different Worlds

Objects in free fall on the earth have acceleration $a_y = -9.8$ m/s^2. On the moon, free-fall acceleration is approximately 1/6 of the acceleration on earth. This changes the scale of problems involving free fall. For instance, suppose you jump straight upward, leaving the ground with velocity v_i and then steadily slowing until reaching zero velocity at your highest point. Because your initial velocity is determined mostly by the strength of your leg muscles, we can assume your initial velocity would be the same on the moon. But considering the final equation in Table 2.4 we can see that, with a smaller free-fall acceleration, your maximum height would be greater. The following questions ask you to think about how certain athletic feats might be performed in this reduced-gravity environment.

75. | If an astronaut can jump straight up to a height of 0.50 m on earth, how high could he jump on the moon?
    A. 1.2 m B. 3.0 m C. 3.6 m D. 18 m
76. | On the earth, an astronaut can safely jump to the ground from a height of 1.0 m; her velocity when reaching the ground is slow enough to not cause injury. From what height could the astronaut safely jump to the ground on the moon?
    A. 2.4 m B. 6.0 m C. 7.2 m D. 36 m
77. | On the earth, an astronaut throws a ball straight upward; it stays in the air for a total time of 3.0 s before reaching the ground again. If a ball were to be thrown upward with the same initial speed on the moon, how much time would pass before it hit the ground?
    A. 7.3 s B. 18 s C. 44 s D. 108 s

STOP TO THINK ANSWERS

Stop to Think 2.1: D. The motion consists of two constant-velocity phases; the second one has a greater velocity. The correct graph has two straight-line segments, with the second one having a steeper slope.

Stop to Think 2.2: B. The displacement is the area under a velocity-versus-time curve. In all four cases, the graph is a straight line, so the area under the curve is a rectangle. The area is the product of the length times the height, so the largest displacement belongs to the graph with the largest product of the length (the time interval, in s) times the height (the velocity, in m/s).

Stop to Think 2.3: C. Consider the slope of the position-versus-time graph; it starts out positive and constant, then decreases to zero. Thus the velocity graph must start with a constant positive value, then decrease to zero.

Stop to Think 2.4: C. Acceleration is the slope of the velocity-versus-time graph. The largest magnitude of the slope is at point C.

Stop to Think 2.5: E. An acceleration of 1.2 m/s^2 corresponds to an increase of 1.2 m/s every second. At this rate, the cruising speed of 6.0 m/s will be reached after 5.0 s.

Stop to Think 2.6: D. The final velocity will have the same *magnitude* as the initial velocity, but the velocity is negative because the rock will be moving downward.