PART II

Conservation Laws

Copyright © 2011 Pearson Education

The kestrel is pulling in its wings to begin a steep dive, in which it can achieve a speed of 60 mph. How does the bird achieve such a speed, and why does this speed help the kestrel catch its prey? Such questions are best answered by considering the conservation of energy and momentum.

Copyright © 2011 Pearson Education

Why Some Things Stay the Same

Part I of this textbook was about *change*. Simple observations show us that most things in the world around us are changing. Even so, there are some things that *don't* change even as everything else is changing around them. Our emphasis in Part II will be on things that stay the same.

Consider, for example, a strong, sealed box in which you have replaced all the air with a mixture of hydrogen and oxygen. The mass of the box plus the gases inside is 600.0 g. Now, suppose you use a spark to ignite the hydrogen and oxygen. As you know, this is an explosive reaction, with the hydrogen and oxygen combining to create water—and quite a bang. But the strong box contains the explosion and all of its products.

What is the mass of the box after the reaction? The gas inside the box is different now, but a careful measurement would reveal that the mass hasn't changed—it's still 600.0 g! We say that the mass is *conserved*. Of course, this is true only if the box has stayed sealed. For conservation of mass to apply, the system must be *closed.*

Conservation Laws

A closed system of interacting particles has another remarkable property. Each system is characterized by a certain number, and no matter how complex the interactions, the value of this number never changes. This number is called the *energy* of the system, and the fact that it never changes is called the *law of conservation of energy*. It is, perhaps, the single most important physical law ever discovered.

The law of conservation of energy is much more general than Newton's laws. Energy can be converted to many different forms, and, in all cases, the total energy stays the same:

- Gasoline, diesel, and jet engines convert the energy of a fuel into the mechanical energy of moving pistons, wheels, and gears.
- A solar cell converts the electromagnetic energy of light into electrical energy.
- An organism converts the chemical energy of food into a variety of other forms of energy, including kinetic energy, sound energy, and thermal energy.

Energy will be *the* most important concept throughout the remainder of this textbook, and much of Part II will focus on understanding what energy is and how it is used.

But energy is not the only conserved quantity. We will begin Part II with the study of two other quantities that are conserved in a closed system: *momentum* and *angular momentum.* Their conservation will help us understand a wide range of physical processes, from the forces when two rams butt heads to the graceful spins of ice skaters.

Conservation laws will give us a new and different *perspective* on motion. Some situations are most easily analyzed from the perspective of Newton's laws, but others make much more sense when analyzed from a conservation-law perspective. An important goal of Part II is to learn which perspective is best for a given problem.

9 Momentum

Male rams butt heads at high speeds in a ritual to assert their dominance. How can the force of this collision be minimized so as to avoid damage to their brains?

Copyright © 2011 Pearson Education

LOOKING AHEAD ▸▸

The goals of Chapter 9 are to introduce the ideas of impulse, momentum, and angular momentum and to learn a new problem-solving strategy based on conservation laws.

Impulse

We'll begin by studying what happens to an object subject to a strong but short-duration **impulsive force.**

We say that the club has delivered an **impulse** to the ball.

A golf ball being struck by a club is subject to a brief but very large force.

Looking Back ◂◂
5.2 Newton's second law

Momentum

The **momentum** of an object is the product of its mass and its velocity. A heavy, fast-moving object such as a car has much greater momentum than a light, slow-moving one such as a falling raindrop.

The momentum of a moving car is a billion times greater than that of a falling raindrop.

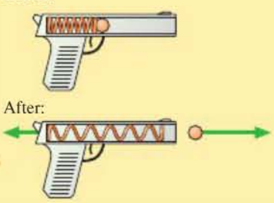
Momentum and Impulse

We'll discover how an impulse delivered to an object *changes* the object's momentum.

The player's head delivers an impulse to the ball, changing its momentum and thus its direction.

Conservation of Momentum

When two or more objects interact only with each other, the total momentum of these objects is conserved—it is the same after the interaction as it was before. This **law of conservation of momentum** will lead us to a powerful new *before-and-after* problem-solving strategy.



Conservation of momentum is an important tool for analyzing *explosions*, like the ball shot from this toy gun, or *collisions*, like two pool balls hitting and flying apart.

Looking Back ◂◂
4.8 Newton's third law
5.7 Interacting objects

Angular Momentum

Rotating objects have *angular* momentum. Angular momentum is changed by an applied torque. When no torques act on a rotating object, its angular momentum is conserved.

Conservation of angular momentum causes this skater's spin to increase as she pulls her arms in toward her body.

Looking Back ◂◂
7.2, 7.4 Torque and moment of inertia

9.1 Impulse

Suppose that two or more objects have an intense and perhaps complex interaction, such as a collision or an explosion. Our goal is to find a relationship between the velocities of the objects before the interaction and their velocities after the interaction. We'll start by looking at collisions.

A **collision** is a short-duration interaction between two objects. The collision between a tennis ball and a racket, or a baseball and a bat, may seem instantaneous to your eye, but that is a limitation of your perception. A careful look at the tennis ball/racket collision in **FIGURE 9.1** reveals that the right side of the ball is flattened and pressed up against the strings of the racket. It takes time to compress the ball, and more time for the ball to re-expand as it leaves the racket.

The duration of a collision depends on the materials from which the objects are made, but 1 to 10 ms (0.001 to 0.010 s) is typical. This is the time during which the two objects are in contact with each other. The harder the objects, the shorter the contact time. A collision between two steel balls lasts less than 1 ms, while that between a tennis ball and racket might last 10 ms.

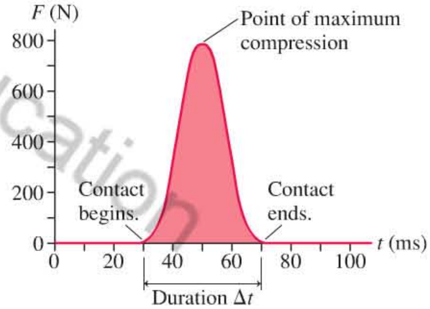
FIGURE 9.1 A tennis ball collides with a racket. Notice that the right side of the ball is flattened.

FIGURE 9.2 A sequence of high-speed photos of a soccer ball being kicked.

Let's begin our discussion by considering a collision that most of us have experienced: kicking a soccer ball. A sequence of high-speed photos of a soccer kick is shown in **FIGURE 9.2**. As the foot and the ball just come into contact, as shown in the left frame, the ball is just beginning to compress. By the middle frame of Figure 9.2, the ball has sped up and become greatly compressed. Finally, as shown in the right frame, the ball, now moving very fast, is again only slightly compressed.

The amount by which the ball is compressed is a measure of the magnitude of the force the foot exerts on the ball; more compression indicates a greater force. If we were to graph this force versus time, it would look something like **FIGURE 9.3**. The force is zero until the foot first contacts the ball, rises quickly to a maximum value, and then falls back to zero as the ball leaves the foot. Thus there is a well-defined duration Δt of the force. A large force like this exerted during a short interval of time is called an **impulsive force.** The forces of a hammer on a nail and of a bat on a baseball are other examples of impulsive forces.

FIGURE 9.3 The force on a soccer ball changes rapidly.



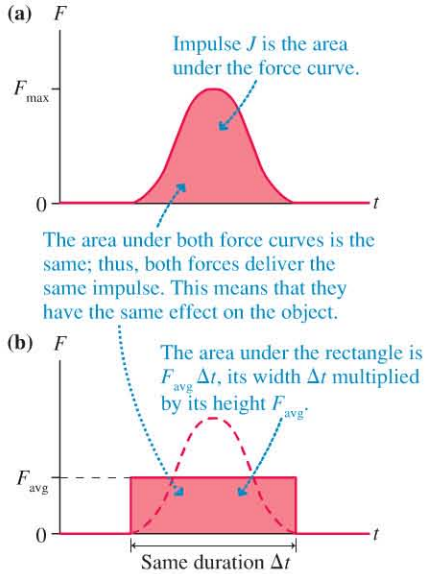
A harder kick (i.e., a taller force curve) or a kick of longer duration (a wider force curve) causes the ball to leave the kicker's foot with a higher speed; that is, the *effect* of the kick is larger. Now a taller or wider force-versus-time curve has a larger *area* between the curve and the axis (i.e., the area "under" the force curve is larger), so we can say that **the effect of an impulsive force is proportional to the area under the force-versus-time curve.** This area, shown in **FIGURE 9.4a**, is called the **impulse** J of the force.

Impulsive forces can be complex, and the shape of the force-versus-time graph often changes in a complicated way. Consequently, it is often useful to think of the collision in terms of an *average* force F_{avg}. As **FIGURE 9.4b** shows, F_{avg} is defined to be the constant force that has the same duration Δt and the same area under the force curve as the real force. You can see from the figure that the area under the force curve can be written simply as $F_{avg}\,\Delta t$. Thus

$$\text{impulse } J = \text{area under the force curve} = F_{avg}\,\Delta t \qquad (9.1)$$

Impulse due to a force acting for a duration Δt

FIGURE 9.4 Looking at the impulse graphically.



Copyright © 2017 Pearson Education

From Equation 9.1 we see that impulse has units of N · s, but you should be able to show that N · s are equivalent to kg · m/s. We'll see shortly why the latter are the preferred units for impulse.

So far, we've been assuming the force is directed along a coordinate axis, such as the x-axis. In this case impulse is a *signed* quantity—it can be positive or negative. A positive impulse results from an average force directed in the positive x-direction (that is, F_{avg} is positive), while a negative impulse is due to a force directed in the negative x-direction (F_{avg} is negative). More generally, the impulse is a *vector* quantity pointing in the direction of the average force vector:

$$\vec{J} = \vec{F}_{avg}\,\Delta t \tag{9.2}$$

Copyright © 2011 Pearson Education

EXAMPLE 9.1 Finding the impulse on a bouncing ball

A rubber ball experiences the force shown in **FIGURE 9.5** as it bounces off the floor.

a. What is the impulse on the ball?
b. What is the average force on the ball?

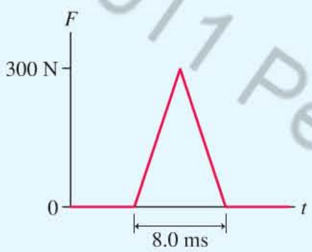


FIGURE 9.5 The force of the floor on a bouncing ball.

PREPARE The impulse is the area under the force curve. Here the shape of the graph is triangular, so we'll need to use the fact that the area of a triangle is $\frac{1}{2}$ × height × base.

SOLVE a. The impulse is

$$J = \tfrac{1}{2}(300\text{ N})(0.0080\text{ s}) = 1.2\text{ N}\cdot\text{s} = 1.2\text{ kg}\cdot\text{m/s}$$

b. From Equation 9.1, $J = F_{avg}\,\Delta t$, we can find the average force that would give this same impulse:

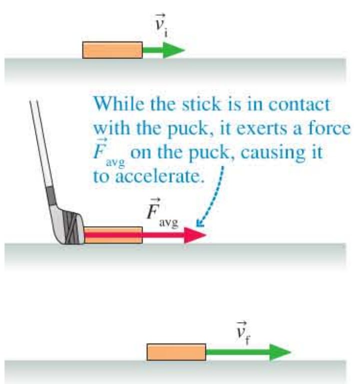
$$F_{avg} = \frac{J}{\Delta t} = \frac{1.2\text{ N}\cdot\text{s}}{0.0080\text{ s}} = 150\text{ N}$$

ASSESS In this particular example, the average value of the force is half the maximum value. This is not surprising for a triangular force because the area of a triangle is *half* the base times the height.

9.2 Momentum and the Impulse-Momentum Theorem

We've noted that the effect of an impulsive force depends on the impulse delivered to the object. The effect also depends on the object's mass. Our experience tells us that giving a kick to a heavy object will change its velocity much less than giving the same kick to a light object. We want now to find a quantitative relationship for impulse, mass, and velocity change.

FIGURE 9.6 The stick exerts an impulse on the puck, changing its speed.



Consider the puck of mass m in **FIGURE 9.6**, sliding with an initial velocity $\vec{v}_i$. It is struck by a hockey stick that delivers an impulse $\vec{J} = \vec{F}_{avg}\,\Delta t$ to the puck. After the impulse, the puck leaves the stick with a final velocity $\vec{v}_f$. How is this final velocity related to the initial velocity?

From Newton's second law, the average acceleration of the puck during the time the stick is in contact with it is

$$\vec{a}_{avg} = \frac{\vec{F}_{avg}}{m} \tag{9.3}$$

The average acceleration is related to the change in the velocity by

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \tag{9.4}$$

Combining Equations 9.3 and 9.4, we have

$$\frac{\vec{F}_{avg}}{m} = \vec{a}_{avg} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

or, rearranging,

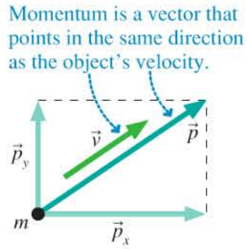
$$\vec{F}_{avg}\,\Delta t = m\vec{v}_f - m\vec{v}_i \tag{9.5}$$

We recognize the left side of this equation as the impulse $\vec{J}$. The right side is the *change* in the quantity $m\vec{v}$. This quantity, the product of the object's mass and velocity, is called the **momentum** of the object. The symbol for momentum is $\vec{p}$:

$$\vec{p} = m\vec{v} \tag{9.6}$$

Momentum of an object of mass m and velocity $\vec{v}$

FIGURE 9.7 A particle's momentum vector $\vec{p}$ can be decomposed into x- and y-components.



From Equation 9.6, the units of momentum are those of mass times velocity, or kg · m/s. We noted above that kg · m/s are the preferred units of impulse. Now we see that the reason for that preference is to match the units of momentum.

FIGURE 9.7 shows that the momentum $\vec{p}$ is a *vector* quantity that points in the same direction as the velocity vector $\vec{v}$. Like any vector, $\vec{p}$ can be decomposed into x- and y-components. Equation 9.6, which is a vector equation, is a shorthand way to write the two equations

$$\begin{aligned} p_x &= mv_x \\ p_y &= mv_y \end{aligned} \tag{9.7}$$

NOTE ▶ One of the most common errors in momentum problems is failure to use the correct signs. The momentum component p_x has the same sign as v_x. Just like velocity, momentum is positive for a particle moving to the right (on the x-axis) or up (on the y-axis), but *negative* for a particle moving to the left or down. ◀

The *magnitude* of an object's momentum is simply the product of the object's mass and speed, or $p = mv$. A heavy, fast-moving object will have a great deal of momentum, while a light, slow-moving object will have very little. Two objects with very different masses can have similar momenta if their speeds are very different as well. Table 9.1 gives some typical values of the momenta (the plural of *momentum*) of various moving objects. You can see that the momenta of a bullet and a fastball are similar. The momentum of a moving car is almost a billion times greater than that of a falling raindrop.

TABLE 9.1 Some typical momenta (approximate)

Object	Mass (kg)	Speed (m/s)	Momentum (kg · m/s)
Falling raindrop	2×10^{-5}	5	10^{-4}
Bullet	0.004	500	2
Pitched baseball	0.15	40	6
Running person	70	3	200
Car on highway	1000	30	3×10^4

Copyright © 2011 Pearson Education

The Impulse-Momentum Theorem

We can now write Equation 9.5 in terms of impulse and momentum:

$$\vec{J} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} \tag{9.8}$$

Impulse-momentum theorem

where $\vec{p}_i = m\vec{v}_i$ is the object's initial momentum, $\vec{p}_f = m\vec{v}_f$ is its final momentum after the impulse, and $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$ is the *change* in its momentum. This expression is known as the **impulse-momentum theorem.** It states that **an impulse delivered to an object causes the object's momentum to change.** That is, the *effect* of an impulsive force is to change the object's momentum from $\vec{p}_i$ to

$$\vec{p}_f = \vec{p}_i + \vec{J} \tag{9.9}$$

Equation 9.8 can also be written in terms of its x- and y-components as

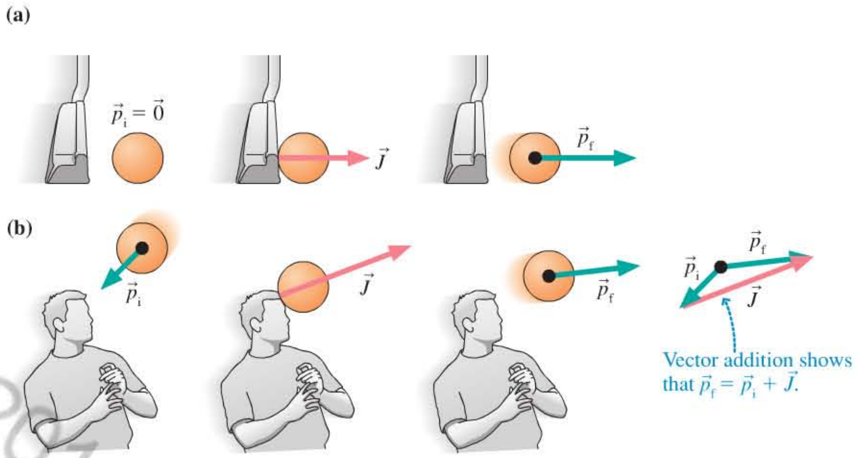
$$\begin{aligned} J_x &= \Delta p_x = (p_x)_f - (p_x)_i = m(v_x)_f - m(v_x)_i \\ J_y &= \Delta p_y = (p_y)_f - (p_y)_i = m(v_y)_f - m(v_y)_i \end{aligned} \tag{9.10}$$

The impulse-momentum theorem is illustrated by two examples in FIGURE 9.8 on the next page. In the first, the putter strikes the ball, exerting a force on it and delivering an impulse $\vec{J} = \vec{F}_{avg}\,\Delta t$. Notice that the direction of the impulse is the same as

Legging it BIO A frog making a jump wants to gain as much momentum as possible before leaving the ground. This means that he wants the greatest impulse $J = F_{avg}\,\Delta t$ delivered to him by the ground. There is a maximum force that muscles can exert, limiting F_{avg}. But the time interval Δt over which the force is exerted can be greatly increased by having long legs. Many animals that are good jumpers have particularly long legs.

Copyright © 2011 Pearson Education

FIGURE 9.8 Impulse causes a *change* in momentum.



that of the force. Because $\vec{p}_i = \vec{0}$ in this situation, we can use the impulse-momentum theorem to find that the ball leaves the putter with momentum $\vec{p}_f = \vec{p}_i + \vec{J} = \vec{J}$.

NOTE ► You can think of the putter as changing the ball's momentum by transferring momentum to it as an impulse. Thus we say the putter *delivers* an impulse to the ball, and the ball *receives* an impulse from the putter. ◄

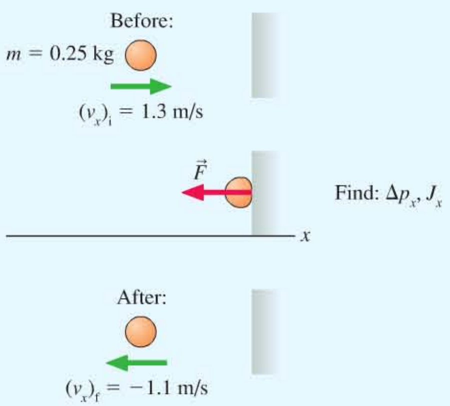
The soccer player in Figure 9.8b presents a more complicated case. Here, the initial momentum of the ball is directed downward to the left. The impulse delivered to it by the player's head, upward to the right, is strong enough to reverse the ball's motion and send it off in a new direction. The graphical addition of vectors in Figure 9.8b again shows that $\vec{p}_f = \vec{p}_i + \vec{J}$.

EXAMPLE 9.2 Calculating the change in momentum

A ball of mass $m = 0.25$ kg rolling to the right at 1.3 m/s strikes a wall and rebounds to the left at 1.1 m/s. What is the change in the ball's momentum? What is the impulse delivered to it by the wall?

PREPARE A visual overview of the ball bouncing is shown in **FIGURE 9.9**. This is a new kind of visual overview, one in which we show the situation "before" and "after" the interaction. We'll have more to say about before-and-after pictures in the next section. The ball is moving along the x-axis, so we'll write the momentum in component form, as in Equation 9.7. The change in momentum is then the difference between the final and initial values of the momentum. By the impulse-momentum theorem, the impulse is equal to this change in momentum.

FIGURE 9.9 Visual overview for a ball bouncing off a wall.



SOLVE The x-component of the initial momentum is

$$(p_x)_i = m(v_x)_i = (0.25 \text{ kg})(1.3 \text{ m/s}) = 0.33 \text{ kg} \cdot \text{m/s}$$

The y-component of the momentum is zero both before and after the bounce. After the ball rebounds, the x-component of its momentum is

$$(p_x)_f = m(v_x)_f = (0.25 \text{ kg})(-1.1 \text{ m/s}) = -0.28 \text{ kg} \cdot \text{m/s}$$

It is particularly important to notice that the x-component of the momentum, like that of the velocity, is negative. This indicates that the ball is moving to the *left*. The change in momentum is

$$\Delta p_x = (p_x)_f - (p_x)_i = (-0.28 \text{ kg} \cdot \text{m/s}) - (0.33 \text{ kg} \cdot \text{m/s})$$
$$= -0.61 \text{ kg} \cdot \text{m/s}$$

The change in the momentum is negative. By the impulse-momentum theorem, the impulse delivered to the ball by the wall is equal to this change, so

$$J_x = \Delta p_x = -0.61 \text{ kg} \cdot \text{m/s}$$

ASSESS The impulse is negative, indicating that the force causing the impulse is pointing to the left, which makes sense.

▶ Water balloon catch If you've ever tried to catch a water balloon, you may have learned the hard way not to catch it with your arms rigidly extended. The brief collision time implies a large, balloon-bursting force. A better way to catch a water balloon is to pull your arms in toward your body as you catch it, lengthening the collision time and hence reducing the force on the balloon.

TRY IT YOURSELF

Copyright © 2011 Pearson Education

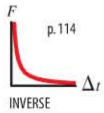
An interesting application of the impulse-momentum theorem is to the question of how to slow down a fast-moving object in the gentlest possible way. For instance, a car is headed for a collision with a bridge abutment. How can this crash be made survivable? How do the rams in the chapter-opening photo avoid injury when they collide?

In these examples, the object has momentum $\vec{p}_i$ just before impact and zero momentum after (i.e., $\vec{p}_f = \vec{0}$). The impulse-momentum theorem tells us that

$$\vec{J} = \vec{F}_{avg}\,\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = -\vec{p}_i$$

or

$$\vec{F}_{avg} = -\frac{\vec{p}_i}{\Delta t} \qquad (9.11)$$



That is, the average force needed to stop an object is *inversely proportional* to the duration Δt of the collision. **If the duration of the collision can be increased, the force of the impact will be decreased.** This is the principle used in most impact-lessening techniques.

For example, obstacles such as bridge abutments are made safer by placing a line of water-filled barrels in front of them. Water is heavy but deformable. In case of a collision, the time it takes for the car to plow through these barrels is much longer than the time it would take it to stop if it hit the abutment head-on. The force on the car (and on the driver from his or her seat belt) is greatly reduced by the longer-duration collision with the barrels.

A hedgehog is its own crash cushion!

The spines of a hedgehog obviously help protect it from predators. But they serve another function as well. If a hedgehog falls from a tree—a not uncommon occurrence—it simply rolls itself into a ball before it lands. Indeed, hedgehogs have been observed to purposely descend to the ground by simply dropping from the tree. Its thick spines then cushion the blow by increasing the time it takes for the animal to come to rest. Along with its small size, this adaptation allows the hedgehog to easily survive long falls unhurt.

The butting rams shown in the photo at the beginning of this chapter also have adaptations that allow them to collide at high speeds without injury to their brains. The cranium has a double wall to prevent skull injuries, and there is a thick spongy mass that increases the time it takes for the brain to come to rest upon impact, again reducing the magnitude of the force on the brain.

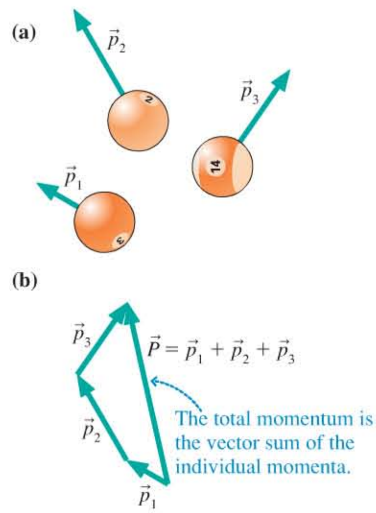
Total Momentum

If we have more than one object moving—a *system* of particles—then the system as a whole has an overall momentum. The **total momentum** $\vec{P}$ (note the capital P) of a system of particles is the vector sum of the momenta of the individual particles:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots$$

FIGURE 9.10 shows how the momentum vectors of three moving pool balls are graphically added to find the total momentum. The concept of total momentum will be of key importance when we discuss the conservation law for momentum in Section 9.4.

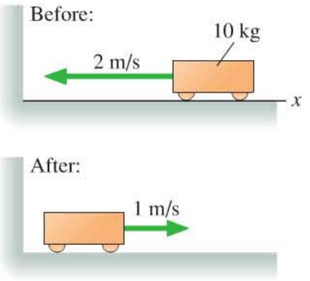
FIGURE 9.10 The total momentum of three pool balls.



Copyright © 2011 Pearson Education

STOP TO THINK 9.1 The cart's change of momentum is

A. $-30\ \text{kg} \cdot \text{m/s}$.
B. $-20\ \text{kg} \cdot \text{m/s}$.
C. $-10\ \text{kg} \cdot \text{m/s}$.
D. $10\ \text{kg} \cdot \text{m/s}$.
E. $20\ \text{kg} \cdot \text{m/s}$.
F. $30\ \text{kg} \cdot \text{m/s}$.



9.3 Solving Impulse and Momentum Problems

Visual overviews have become an important problem-solving tool. The visual overviews and free-body diagrams that you learned to draw in Chapters 1–8 were oriented toward the use of Newton's laws and a subsequent kinematical analysis. Now we are interested in making a connection between "before" and "after."

TACTICS BOX 9.1 Drawing a before-and-after visual overview

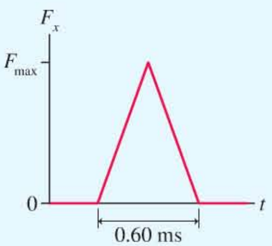
1. **Sketch the situation.** Use two drawings, labeled "Before" and "After," to show the objects *immediately before* they interact and again *immediately after* they interact.
2. **Establish a coordinate system.** Select your axes to match the motion.
3. **Define symbols.** Define symbols for the masses and for the velocities before and after the interaction. Position and time are not needed.
4. **List known information.** List the values of quantities known from the problem statement or that can be found quickly with simple geometry or unit conversions. Before-and-after pictures are usually simpler than the pictures you used for dynamics problems, so listing known information on the sketch is often adequate.
5. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined as symbols in step 3.

Exercises 9–11

EXAMPLE 9.3 Force in hitting a baseball

A 150 g baseball is thrown with a speed of 20 m/s. It is hit straight back toward the pitcher at a speed of 40 m/s. The impulsive force of the bat on the ball has the shape shown in FIGURE 9.11. What is the *maximum* force F_{max} that the bat exerts on the ball? What is the *average* force that the bat exerts on the ball?

FIGURE 9.11 The interaction force between the baseball and the bat.



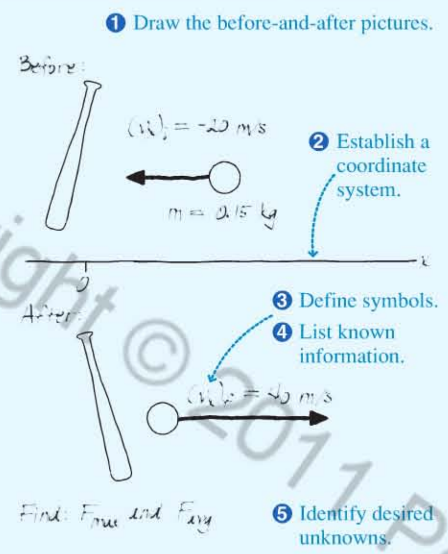
PREPARE We can model the interaction as a collision. FIGURE 9.12 is a before-and-after visual overview in which the steps from Tactics Box 9.1 are explicitly noted. Because F_x is positive (a force to the right), we know the ball was initially moving toward the left and is hit back toward the right. Thus we converted the statements about *speeds* into information about *velocities*, with $(v_x)_i$ negative.

SOLVE In the last several chapters we've started the mathematical solution with Newton's second law. Now we want to use the impulse-momentum theorem:

$$\Delta p_x = J_x = \text{area under the force curve}$$

Copyright © 2011 Pearson Education

FIGURE 9.12 A before-and-after visual overview.



We know the velocities before and after the collision, so we can find the change in the ball's momentum:

$$\Delta p_x = m(v_x)_f - m(v_x)_i = (0.15\text{ kg})(40\text{ m/s} - (-20\text{ m/s}))$$
$$= 9.0\text{ kg}\cdot\text{m/s}$$

The force curve is a triangle with height F_{max} and width 0.60 ms. As in Example 9.1, the area under the curve is

$$J_x = \text{area} = \tfrac{1}{2} \times F_{max} \times (6.0 \times 10^{-4}\text{ s})$$
$$= (F_{max})(3.0 \times 10^{-4})$$

According to the impulse-momentum theorem, $\Delta p_x = J_x$, so we have

$$9.0\text{ kg}\cdot\text{m/s} = (F_{max})(3.0 \times 10^{-4}\text{ s})$$

Thus the *maximum* force is

$$F_{max} = \frac{9.0\text{ kg}\cdot\text{m/s}}{3.0 \times 10^{-4}\text{ s}} = 30{,}000\text{ N}$$

Using Equation 9.1, we find that the *average* force, which depends on the collision duration $\Delta t = 6.0 \times 10^{-4}$ s, has the smaller value:

$$F_{avg} = \frac{J_x}{\Delta t} = \frac{\Delta p_x}{\Delta t} = \frac{9.0\text{ kg}\cdot\text{m/s}}{6.0 \times 10^{-4}\text{ s}} = 15{,}000\text{ N}$$

ASSESS F_{max} is a large force, but quite typical of the impulsive forces during collisions.

The Impulse Approximation

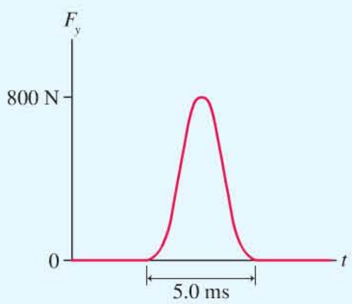
When two objects interact during a collision or other brief interaction, such as that between the bat and ball of Example 9.3, the forces *between* them are generally quite large. Other forces may also act on the interacting objects, but usually these forces are *much* smaller than the interaction forces. In Example 9.3, for example, the 1.5 N weight of the ball is vastly less than the 30,000 N force of the bat on the ball. We can reasonably neglect these small forces *during* the brief time of the impulsive force. Doing so is called the **impulse approximation.**

When we use the impulse approximation, $(p_x)_i$ and $(p_x)_f$—and $(v_x)_i$ and $(v_x)_f$—are then the momenta (and velocities) *immediately* before and *immediately* after the collision. For example, the velocities in Example 9.3 are those of the ball just before and after it collides with the bat. We could then do a follow-up problem, including weight and drag, to find the ball's speed a second later as the second baseman catches it.

EXAMPLE 9.4 Height of a bouncing ball

A 100 g rubber ball is thrown straight down onto a hard floor so that it strikes the floor with a speed of 11 m/s. **FIGURE 9.13** shows the force that the floor exerts on the ball. Estimate the height of the ball's bounce.

FIGURE 9.13 The force of the floor on a bouncing rubber ball.



PREPARE The ball experiences an impulsive force while in contact with the floor. Using the impulse approximation, we'll neglect the ball's weight during these 5.0 ms. The ball's rise after the bounce is free-fall motion—that is, motion subject only to the force of gravity. We'll use free-fall kinematics to describe the motion after the bounce.

FIGURE 9.14 on the next page is a visual overview. Here we have a two-part problem, an impulsive collision followed by upward free fall. The overview thus shows the ball just before the collision, where we label its velocity as v_{1y}; just after the collision, where its velocity is v_{2y}; and at the highest point of its rising free fall, where its velocity is $v_{3y} = 0$.

Continued

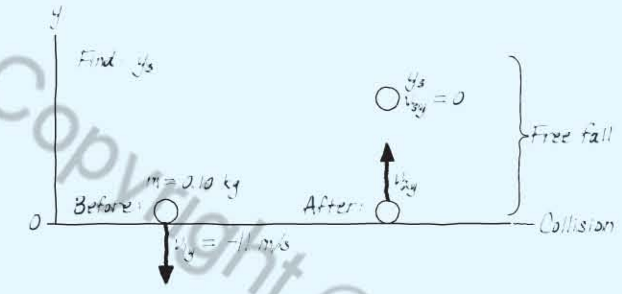


FIGURE 9.14 Before-and-after visual overview for a bouncing ball.

SOLVE The impulse-momentum theorem tells us that $J_y = \Delta p_y = p_{2y} - p_{1y}$, so that $p_{2y} = p_{1y} + J_y$. The initial momentum, just before the collision, is $p_{1y} = mv_{1y} = (0.10\text{ kg})(-11\text{ m/s}) = -1.1\text{ kg}\cdot\text{m/s}$.

Next, we need to find the impulse J_y, which is the area under the curve in Figure 9.13. Because the force is given as a smooth curve, we'll have to *estimate* this area. Recall that the area can be written as $F_{avg}\,\Delta t$. From the curve, we might estimate F_{avg} to be about 400 N, or half the maximum value of the force. With this estimate we have

$$J_y = \text{area under the force curve} \approx (400\text{ N}) \times (0.0050\text{ s})$$
$$= 2.0\text{ N}\cdot\text{s} = 2.0\text{ kg}\cdot\text{m/s}$$

Thus

$$p_{2y} = p_{1y} + J_y = (-1.1\text{ kg}\cdot\text{m/s}) + 2.0\text{ kg}\cdot\text{m/s}$$
$$= 0.9\text{ kg}\cdot\text{m/s}$$

and the post-collision velocity is

$$v_{2y} = \frac{p_{2y}}{m} = \frac{0.9\text{ kg}\cdot\text{m/s}}{0.10\text{ kg}} = 9\text{ m/s}$$

The rebound speed is less than the impact speed, as expected. Finally, we can use free-fall kinematics to find

$$v_{3y}^2 = 0 = v_{2y}^2 - 2g\,\Delta y = v_{2y}^2 - 2gy_3$$
$$y_3 = \frac{v_{2y}^2}{2g} = \frac{(9\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 4\text{ m}$$

We estimate that the ball bounces to a height of 4 m.

ASSESS This is a reasonable height for a rubber ball thrown down quite hard.

Copyright © 2011 Pearson Education

STOP TO THINK 9.2 A 10 g rubber ball and a 10 g clay ball are each thrown at a wall with equal speeds. The rubber ball bounces; the clay ball sticks. Which ball receives the greater impulse from the wall?

A. The clay ball receives a greater impulse because it sticks.
B. The rubber ball receives a greater impulse because it bounces.
C. They receive equal impulses because they have equal momenta.
D. Neither receives an impulse because the wall doesn't move.

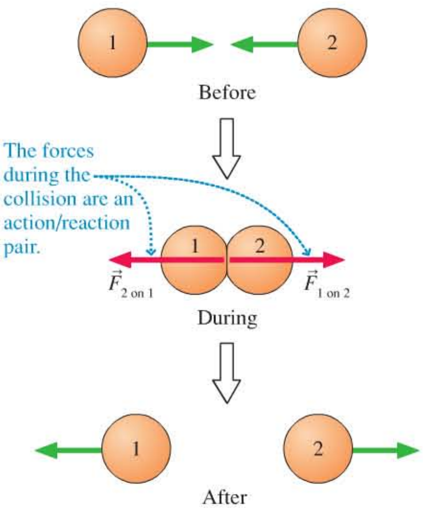


FIGURE 9.15 A collision between two balls.

9.4 Conservation of Momentum

The impulse-momentum theorem was derived from Newton's second law and is really just an alternative way of looking at that law. It is used in the context of single-particle dynamics, much as we used Newton's law in Chapters 4–7.

However, consider two objects, such as the rams shown in the opening photo of this chapter, that interact during the brief moment of a collision. During a collision, two objects exert forces on each other that vary in a complex way. We usually don't even know the magnitudes of these forces. Using Newton's second law alone to predict the outcome of such a collision would thus be a daunting challenge. However, by using Newton's *third* law in the language of impulse and momentum, we'll find that it's possible to describe the *outcome* of a collision—the final speeds and directions of the colliding objects—in a simple way. Newton's third law will lead us to one of the most important conservation laws in physics.

FIGURE 9.15 shows two balls initially headed toward each other. The balls collide, then bounce apart. The forces during the collision, when the balls are interacting, are the action/reaction pair $\vec{F}_{1\text{ on }2}$ and $\vec{F}_{2\text{ on }1}$. For now, we'll continue to assume that the motion is one dimensional along the x-axis.

During the collision, the impulse J_{2x} delivered to ball 2 by ball 1 is the average value of $\vec{F}_{1\text{ on }2}$ multiplied by the collision time Δt. Likewise, the impulse J_{1x} delivered to ball 1 by ball 2 is the average value of $\vec{F}_{2\text{ on }1}$ multiplied by Δt. Because $\vec{F}_{1\text{ on }2}$ and $\vec{F}_{2\text{ on }1}$ form an action/reaction pair, they have equal magnitudes but opposite directions. As a result, the two impulses J_{1x} and J_{2x} are also equal in magnitude but opposite in sign, so that $J_{1x} = -J_{2x}$.

According to the impulse-momentum theorem, the change in the momentum of ball 1 is $\Delta p_{1x} = J_{1x}$ and the change in the momentum of ball 2 is $\Delta p_{2x} = J_{2x}$. Because $J_{1x} = -J_{2x}$, the change in the momentum of ball 1 is equal in magnitude but opposite in sign to the change in momentum of ball 2. If ball 1's momentum increases by a certain amount during the collision, ball 2's momentum will *decrease* by exactly the same amount. This implies that **the total momentum $P_x = p_{1x} + p_{2x}$ of the two balls is *unchanged* by the collision;** that is,

$$(P_x)_\mathrm{f} = (P_x)_\mathrm{i} \tag{9.12}$$

Because it doesn't change during the collision, we say that the x-component of total momentum is *conserved*. Equation 9.12 is our first example of a *conservation law*.

Copyright © 2011 Pearson Education

Law of Conservation of Momentum

The same arguments just presented for the two colliding balls can be extended to systems containing any number of objects. FIGURE 9.16 shows the idea. Each pair of particles in the system (the boundary of which is denoted by the red line) interacts via forces that are an action/reaction pair. Exactly as for the two-particle collision, the change in momentum of particle 2 due to the force from particle 3 is equal in magnitude, but opposite in direction, to the change in particle 3's momentum due to particle 2. The *net* change in the momentum of these two particles due to their interaction forces is thus zero. The same argument holds for every pair, with the result that, no matter how complicated the forces between the particles, **there is no change in the *total* momentum $\vec{P}$ of the system.** The total momentum of the system remains constant: It is *conserved*.

FIGURE 9.16 A system of three particles.

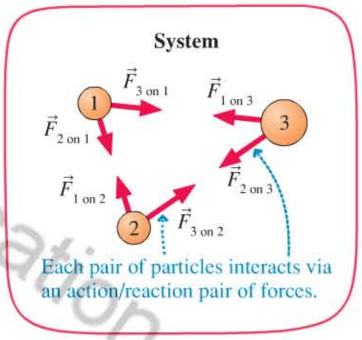


Figure 9.16 showed particles interacting only with other particles inside the system. Forces that act only between particles within the system are called **internal forces.** As we've just seen, **the total momentum of a system subject only to internal forces is conserved.**

Most systems are also subject to forces from agents outside the system. These forces are called **external forces.** For example, the system consisting of a student on a skateboard is subject to three external forces—the normal force of the ground on the skateboard, the force of gravity on the student, and the force of gravity on the board. How do external forces affect the momentum of a system of particles?

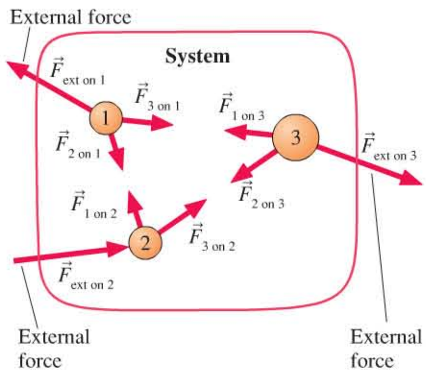
In FIGURE 9.17 we show the same three-particle system of Figure 9.16, but now with *external* forces acting on the three particles. These external forces *can* change the momentum of the system. During a time interval Δt, for instance, the external force $\vec{F}_{\text{ext on }1}$ acting on particle 1 changes its momentum, according to the impulse-momentum theorem, by $\Delta\vec{p}_1 = (\vec{F}_{\text{ext on }1})\Delta t$. The momenta of the other two particles change similarly. Thus the change in the total momentum is

$$\begin{aligned}\Delta\vec{P} &= \Delta\vec{p}_1 + \Delta\vec{p}_2 + \Delta\vec{p}_3 \\ &= (\vec{F}_{\text{ext on }1}\Delta t) + (\vec{F}_{\text{ext on }2}\Delta t) + (\vec{F}_{\text{ext on }3}\Delta t) \\ &= (\vec{F}_{\text{ext on }1} + \vec{F}_{\text{ext on }2} + \vec{F}_{\text{ext on }3})\Delta t \\ &= \vec{F}_{\text{net}}\Delta t\end{aligned} \tag{9.13}$$

where $\vec{F}_{\text{net}}$ is the net force due to *external forces*.

Equation 9.13 has a very important implication in the case where the net force on a system is zero: **If $\vec{F}_{\text{net}} = \vec{0}$ the *total* momentum $\vec{P}$ of the system does not change.** The total momentum remains constant, *regardless* of whatever interactions are going on *inside* the system.

FIGURE 9.17 A system of particles subject to external forces.



Copyright © 2011 Pearson Education

Earlier, we found that a system's total momentum is conserved when the system has no external forces acting on it. Now we've found that the system's total momentum is also conserved when the net external force acting on it is zero. With no external forces acting that can change its momentum, we call a system with $\vec{F}_{\text{net}} = \vec{0}$ an **isolated system.**

The importance of these results is sufficient to elevate them to a law of nature, alongside Newton's laws.

Law of conservation of momentum The total momentum $\vec{P}$ of an isolated system is a constant. Interactions within the system do not change the system's total momentum.

NOTE ► It is worth emphasizing the critical role of Newton's third law in the derivation of Equation 9.13. The law of conservation of momentum is a direct consequence of the fact that interactions within an isolated system are action/reaction pairs. ◄

Mathematically, the law of conservation of momentum for an isolated system is

$$\vec{P}_{\text{f}} = \vec{P}_{\text{i}} \tag{9.14}$$

Law of conservation of momentum for an isolated system

The total momentum after an interaction is equal to the total momentum before the interaction. Because Equation 9.14 is a vector equation, the equality is true for each of the components of the momentum vector; that is,

$$\text{x-component: } \overbrace{(p_{1x})_{\text{f}} + (p_{2x})_{\text{f}} + (p_{3x})_{\text{f}} + \cdots}^{\text{Final momentum}} = \overbrace{(p_{1x})_{\text{i}} + (p_{2x})_{\text{i}} + (p_{3x})_{\text{i}} + \cdots}^{\text{Initial momentum}}$$

Particle 1 Particle 2 Particle 3

$$\text{y-component: } (p_{1y})_{\text{f}} + (p_{2y})_{\text{f}} + (p_{3y})_{\text{f}} + \cdots = (p_{1y})_{\text{i}} + (p_{2y})_{\text{i}} + (p_{3y})_{\text{i}} + \cdots \tag{9.15}$$

EXAMPLE 9.5 **Speed of ice skaters pushing off**

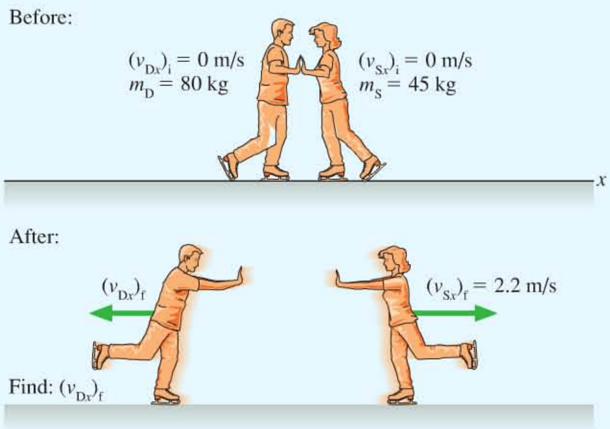
Two ice skaters, Sandra and David, stand facing each other on frictionless ice. Sandra has a mass of 45 kg, David a mass of 80 kg. They then push off from each other. After the push, Sandra moves off at a speed of 2.2 m/s. What is David's speed?

PREPARE The two skaters interact with each other, but they form an isolated system because, for each skater, the upward normal force of the ice balances their downward weight force to make $\vec{F}_{\text{net}} = \vec{0}$. Thus the total momentum of the system of the two skaters is conserved.

FIGURE 9.18 shows a before-and-after visual overview for the two skaters. The total momentum before they push off is $\vec{P}_{\text{i}} = \vec{0}$ because both skaters are at rest. Consequently, the total momentum will still be $\vec{0}$ *after* they push off.

SOLVE Since the motion is only in the x-direction, we'll only need to consider x-components of momentum. We write Sandra's initial momentum as $(p_{Sx})_{\text{i}} = m_{\text{S}}(v_{Sx})_{\text{i}}$, where m_{S} is her mass and

FIGURE 9.18 Before-and-after visual overview for two skaters pushing off from each other.



$(v_{Sx})_i$ her initial velocity. Similarly, we write David's initial momentum as $(p_{Dx})_i = m_D(v_{Dx})_i$. Both these momenta are zero because both skaters are initially at rest.

We can now apply the mathematical statement of momentum conservation, Equation 9.15. Writing the final momentum of Sandra as $m_S(v_{Sx})_f$ and that of David as $m_D(v_{Dx})_f$, we have

$$\underbrace{m_S(v_{Sx})_f + m_D(v_{Dx})_f}_{\text{The skaters' final momentum . . .}} = \underbrace{m_S(v_{Sx})_i + m_D(v_{Dx})_i}_{\text{. . . equals their initial momentum . . .}} = \underbrace{0}_{\text{. . . which was zero.}}$$

Solving for $(v_{Dx})_f$, we find

$$(v_{Dx})_f = -\frac{m_S}{m_D}(v_{Sx})_f = -\frac{45 \text{ kg}}{80 \text{ kg}} \times 2.2 \text{ m/s} = -1.2 \text{ m/s}$$

David moves backward with a *speed* of 1.2 m/s.

Notice that we didn't need to know any details about the force between David and Sandra in order to find David's final speed. Conservation of momentum *mandates* this result.

ASSESS The *total* momentum of the system is zero both before and after they push off, but the individual momenta are not zero. Because $(p_{Sx})_f$ is positive (Sandra moves to the right), $(p_{Dx})_f$ must have the same magnitude but the opposite sign (David moves to the left).

Copyright © 2017 Pearson Education

A Strategy for Conservation of Momentum Problems

Our derivation of the law of conservation of momentum, and the conditions under which it holds, suggests a problem-solving strategy.

Activ Online Physics 6.3, 6.4, 6.6, 6.7, 6.10

PROBLEM-SOLVING STRATEGY 9.1 **Conservation of momentum problems** MP™

PREPARE Clearly define *the system*.

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is then conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or, as you'll learn in Chapter 10, conservation of energy.

Following Tactics Box 9.1, draw a before-and-after visual overview. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum: $\vec{P}_f = \vec{P}_i$. In component form, this is

$$(p_{1x})_f + (p_{2x})_f + (p_{3x})_f + \cdots = (p_{1x})_i + (p_{2x})_i + (p_{3x})_i + \cdots$$
$$(p_{1y})_f + (p_{2y})_f + (p_{3y})_f + \cdots = (p_{1y})_i + (p_{2y})_i + (p_{3y})_i + \cdots$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 17

EXAMPLE 9.6 **Getaway speed of a cart**

Bob is running from the police and thinks he can make a faster getaway by jumping on a stationary cart in front of him. He runs toward the cart, jumps on, and rolls along the horizontal street. Bob has a mass of 75 kg and the cart's mass is 25 kg. If Bob's speed is 4.0 m/s when he jumps onto the cart, what is the cart's speed after Bob jumps on?

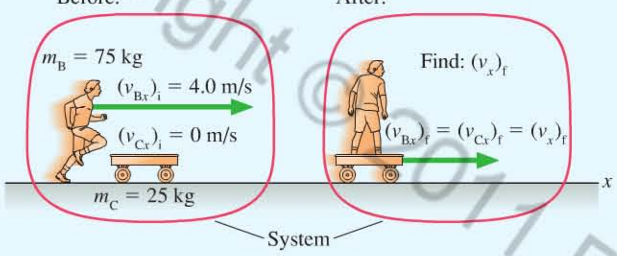
PREPARE When Bob lands on and sticks to the cart, a "collision" occurs between Bob and the cart. If we take Bob and the cart together to be the system, the forces involved in this collision—friction forces between Bob's feet and the cart—are internal forces. Because the normal force balances the weight of both Bob and the cart, the net external force on the system is zero, so the

Continued

total momentum of Bob + cart is conserved: It is the same before and after the collision.

The visual overview in FIGURE 9.19 shows the important point that Bob and the cart move together after he lands on the cart, so $(v_x)_f$ is their common final velocity.

FIGURE 9.19 Before-and-after visual overview of Bob and the cart.



SOLVE To solve for the final velocity of Bob and the cart, we'll use conservation of momentum: $(P_x)_f = (P_x)_i$. Written in terms of the individual momenta, we have

$$(P_x)_i = m_B(v_{Bx})_i + m_C\underbrace{(v_{Cx})_i}_{0\text{ m/s}} = m_B(v_{Bx})_i$$

$$(P_x)_f = m_B(v_x)_f + m_C(v_x)_f = (m_B + m_C)(v_x)_f$$

In the second equation, we've used the fact that both Bob and the cart travel at the common velocity of $(v_x)_f$. Equating the final and initial total momenta gives

$$(m_B + m_C)(v_x)_f = m_B(v_{Bx})_i$$

Solving this for $(v_x)_f$, we find

$$(v_x)_f = \frac{m_B}{m_B + m_C}(v_{Bx})_i = \frac{75\text{ kg}}{100\text{ kg}} \times 4.0\text{ m/s} = 3.0\text{ m/s}$$

The cart's speed is 3.0 m/s immediately after Bob jumps on.

ASSESS It makes sense that Bob has *lost* speed because he had to share his initial momentum with the cart. Not a good way to make a getaway!

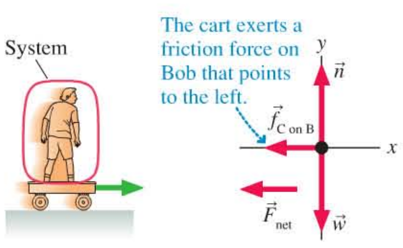
Copyright © 2011 Pearson Education

Notice how easy this was! No forces, no kinematic equations, no simultaneous equations. Why didn't we think of this before? Although conservation laws are indeed powerful, they can answer only certain questions. Had we wanted to know how far Bob slid across the cart before sticking to it, how long the slide took, or what the cart's acceleration was during the collision, we would not have been able to answer such questions on the basis of the conservation law. There is a price to pay for finding a simple connection between before and after, and that price is the loss of information about the details of the interaction. If we are satisfied with knowing only about before and after, then conservation laws are a simple and straightforward way to proceed. But many problems *do* require us to understand the interaction, and for these there is no avoiding Newton's laws and all they entail.

It Depends on the System

The first step in the problem-solving strategy asks you to clearly define *the system*. This is worth emphasizing, because many problem-solving errors arise from trying to apply momentum conservation to an inappropriate system. **The goal is to choose a system whose momentum will be conserved.** Even then, it is the *total* momentum of the system that is conserved, not the momenta of the individual particles within the system.

FIGURE 9.20 An analysis of the system consisting of Bob alone.



In Example 9.6, we chose the system to be Bob and the cart. Why this choice? Let's see what would happen if we had chosen the system to be Bob alone, as shown in FIGURE 9.20. As the free-body diagram shows, as Bob lands on the cart, there are three forces acting on him: the normal force $\vec{n}$ of the cart on Bob, his weight $\vec{w}$, and a friction force $\vec{f}_{\text{C on B}}$ of the cart on Bob. This last force is subtle. We know that Bob's feet must exert a rightward-directed friction force $\vec{f}_{\text{B on C}}$ *on the cart* as he lands; it is this friction force that causes the cart to speed up. By Newton's third law, then, the cart exerts a leftward directed force $\vec{f}_{\text{C on B}}$ on Bob.

The free-body diagram of Figure 9.20 then shows that there is a net force on Bob directed to the left. Thus the system consisting of Bob alone is *not* isolated, and Bob's momentum will not be conserved. Indeed, we know that Bob slows down after landing on the cart, so that his momentum clearly *decreases*.

If we had chosen the cart to be the system, the unbalanced rightward force $\vec{f}_{\text{B on C}}$ of Bob on the cart would also lead to a nonzero net force. Thus the cart's momentum would not be conserved; in fact, we know it *increases* because the cart speeds up.

Only by choosing the system to be Bob and the cart *together* is the net force on the system zero and the total momentum conserved. The momentum lost by Bob is gained by the cart, so the total momentum of the two is unchanged.

Explosions

An **explosion,** where the particles of the system move apart after a brief, intense interaction, is the opposite of a collision. The explosive forces, which could be from an expanding spring or from expanding hot gases, are *internal* forces. If the system is isolated, its total momentum during the explosion will be conserved.

Copyright © 2011 Pearson Education

EXAMPLE 9.7 Recoil speed of a rifle

A 30 g ball is fired from a 1.2 kg spring-loaded toy rifle with a speed of 15 m/s. What is the recoil speed of the rifle?

PREPARE As the ball moves down the barrel, there are complicated forces exerted on the ball and on the rifle. However, if we take the system to be the ball + rifle, these are *internal* forces that do not change the total momentum.

The *external* forces of the rifle's and ball's weights are balanced by the external force exerted by the person holding the rifle, so $\vec{F}_{\text{net}} = \vec{0}$. This is an isolated system and the law of conservation of momentum applies.

FIGURE 9.21 Before-and-after visual overview for a toy rifle.

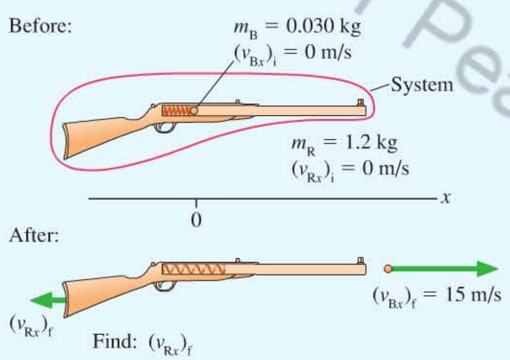


FIGURE 9.21 shows a visual overview before and after the ball is fired. We'll assume the ball is fired in the $+x$-direction.

SOLVE The x-component of the total momentum is $P_x = p_{Bx} + p_{Rx}$. Everything is at rest before the trigger is pulled, so the initial momentum is zero. After the trigger is pulled, the internal force of the spring pushes the ball down the barrel *and* pushes the rifle backward. Conservation of momentum gives

$$(P_x)_f = m_B(v_{Bx})_f + m_R(v_{Rx})_f = (P_x)_i = 0$$

Solving for the rifle's velocity, we find

$$(v_{Rx})_f = -\frac{m_B}{m_R}(v_{Bx})_f = -\frac{0.030 \text{ kg}}{1.2 \text{ kg}} \times 15 \text{ m/s} = -0.38 \text{ m/s}$$

The minus sign indicates that the rifle's recoil is to the left. The recoil *speed* is 0.38 m/s.

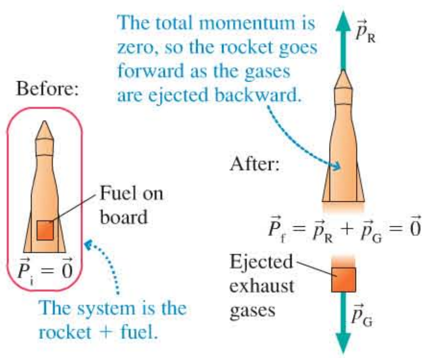
ASSESS Real rifles fire their bullets at much higher velocities, and their recoil is correspondingly higher. Shooters need to brace themselves against the "kick" of the rifle back against their shoulder.

We would not know where to begin to solve a problem such as this using Newton's laws. But Example 9.7 is a simple problem when approached from the before-and-after perspective of a conservation law. The selection of ball + rifle as "the system" was the critical step. For momentum conservation to be a useful principle, we had to select a system in which the complicated forces due to the spring and to friction were all internal forces. The rifle by itself is *not* an isolated system, so its momentum is *not* conserved.

Much the same reasoning explains how a rocket or jet aircraft accelerates. FIGURE 9.22 shows a rocket with a parcel of fuel on board. Burning converts the fuel to hot gases that are expelled from the rocket motor. If we choose rocket + gases to be the system, then the burning and expulsion are internal forces. In deep space there are no other forces, so the total momentum of the rocket + gases system must be conserved. The rocket gains forward velocity and momentum as the exhaust gases are shot out the back, but the *total* momentum of the system remains zero.

Many people find it hard to understand how a rocket can accelerate in the vacuum of space because there is nothing to "push against." Thinking in terms of momentum, you can see that the rocket does not push against anything *external,* but only against the gases that it pushes out the back. In return, in accordance with Newton's third law, the gases push forward on the rocket.

FIGURE 9.22 Rocket propulsion is an example of conservation of momentum.



▶ **Squid propulsion** BIO Squids use a form of "rocket propulsion" to make quick movements to escape enemies or catch prey. The squid draws in water through a pair of valves in its outer sheath, or mantle, and then quickly expels the water through a funnel, propelling the squid backward. The funnel's direction is adjustable, allowing the squid to move in any backward direction.

STOP TO THINK 9.3 An explosion in a rigid pipe shoots three balls out of its ends. A 6 g ball comes out the right end. A 4 g ball comes out the left end with twice the speed of the 6 g ball. From which end, left or right, does the third ball emerge?

Copyright © 2011 Pearson Education

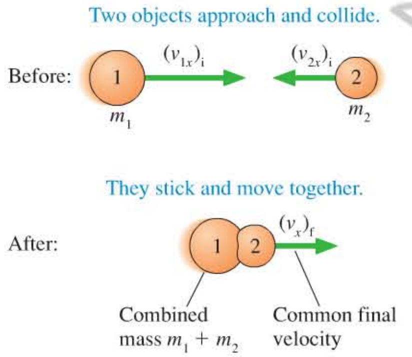
9.5 Inelastic Collisions

Collisions can have different possible outcomes. A rubber ball dropped on the floor bounces—it's *elastic*—but a ball of clay sticks to the floor without bouncing; we call such a collision *inelastic*. A golf club hitting a golf ball causes the ball to rebound away from the club (elastic), but a bullet striking a block of wood becomes embedded in the block (inelastic).

A collision in which the two objects stick together and move with a common final velocity is called a **perfectly inelastic collision**. The clay hitting the floor and the bullet embedding itself in the wood are examples of perfectly inelastic collisions. Other examples include railroad cars coupling together upon impact and darts hitting a dart board. FIGURE 9.23 emphasizes the fact that the two objects have a common final velocity after they collide. (We have drawn the combined object moving to the right, but it could have ended up moving to the left, depending on the objects' masses and initial velocities.)

In an *elastic collision,* by contrast, the two objects bounce apart. We've looked at some examples of elastic collisions, but a full analysis requires some ideas about energy. We will return to elastic collisions in Chapter 10.

FIGURE 9.23 A perfectly inelastic collision.



EXAMPLE 9.8 **Speeds in a perfectly inelastic glider collision**

In a laboratory experiment, a 200 g air-track glider and a 400 g air-track glider are pushed toward each other from opposite ends of the track. The gliders have Velcro tabs on their fronts so that they will stick together when they collide. The 200 g glider is pushed with an initial speed of 3.0 m/s. The collision causes it to reverse direction at 0.50 m/s. What was the initial speed of the 400 g glider?

PREPARE We model the gliders as particles and define the two gliders as the system. This is an isolated system, so its total momentum is conserved in the collision. The gliders stick together, so this is a perfectly inelastic collision.

FIGURE 9.24 Before-and-after visual overview for two gliders colliding on an air track.

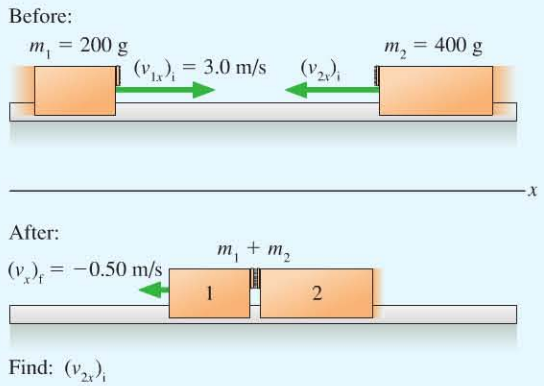


FIGURE 9.24 shows a visual overview. We've chosen to let the 200 g glider (glider 1) start out moving to the right, so $(v_{1x})_i$ is a positive 3.0 m/s. The gliders move to the left after the collision, so their common final velocity is $(v_x)_f = -0.50$ m/s. You can see that velocity $(v_{2x})_i$ must be negative in order to "turn around" both gliders.

SOLVE The law of conservation of momentum, $(P_x)_f = (P_x)_i$, is

$$(m_1 + m_2)(v_x)_f = m_1(v_{1x})_i + m_2(v_{2x})_i$$

where we made use of the fact that the combined mass $m_1 + m_2$ moves together after the collision. We can easily solve for the initial velocity of the 400 g glider:

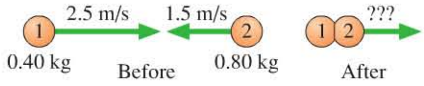
$$\begin{aligned}(v_{2x})_i &= \frac{(m_1 + m_2)(v_x)_f - m_1(v_{1x})_i}{m_2}\\ &= \frac{(0.60\text{ kg})(-0.50\text{ m/s}) - (0.20\text{ kg})(3.0\text{ m/s})}{0.40\text{ kg}}\\ &= -2.3\text{ m/s}\end{aligned}$$

The negative sign, which we anticipated, indicates that the 400 g glider started out moving to the left. The initial *speed* of the glider, which we were asked to find, is 2.3 m/s.

ASSESS The key step in solving inelastic collision problems is that both objects move after the collision with the same velocity. You should thus choose a single symbol (here, $(v_x)_f$) for this common velocity.

STOP TO THINK 9.4 The two particles shown collide and stick together. After the collision, the combined particles

A. Move to the right as shown.
B. Move to the left.
C. Are at rest.



Copyright © 2011 Pearson Education

9.6 Momentum and Collisions in Two Dimensions

Our examples thus far have been confined to motion along a one-dimensional axis. Many practical examples of momentum conservation involve motion in a plane. The total momentum $\vec{P}$ is the *vector* sum of the momenta $\vec{p} = m\vec{v}$ of the individual particles. Consequently, as Equation 9.15 showed, momentum is conserved only if each component of $\vec{P}$ is conserved:

$$\begin{aligned} (p_{1x})_f + (p_{2x})_f + (p_{3x})_f + \cdots &= (p_{1x})_i + (p_{2x})_i + (p_{3x})_i + \cdots \\ (p_{1y})_f + (p_{2y})_f + (p_{3y})_f + \cdots &= (p_{1y})_i + (p_{2y})_i + (p_{3y})_i + \cdots \end{aligned} \qquad (9.16)$$

In this section we'll apply momentum conservation to motion in two dimensions.

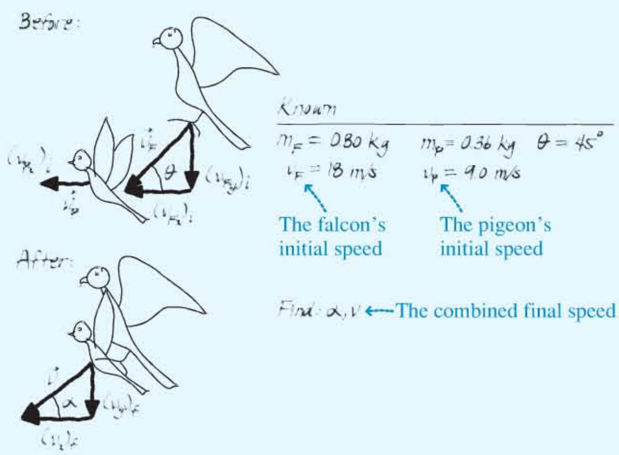
Collisions and explosions often involve motion in two dimensions.

EXAMPLE 9.9 Analyzing a peregrine falcon strike

Peregrine falcons often grab their prey from above while both falcon and prey are in flight. A falcon, flying at 18 m/s, swoops down at a 45° angle from behind a pigeon flying horizontally at 9.0 m/s. The falcon has a mass of 0.80 kg and the pigeon a mass of 0.36 kg. What are the speed and direction of the falcon (now holding the pigeon) immediately after impact?

PREPARE This is a perfectly inelastic collision because after the collision the falcon and pigeon move at a common velocity. The total momentum of the falcon + pigeon system is conserved. For a two-dimensional collision, this means that the x-component of the total momentum before the collision must equal the x-component of the total momentum after the collision, and similarly for the y-components. **FIGURE 9.25** is a before-and-after visual overview.

FIGURE 9.25 Before-and-after visual overview for a falcon catching a pigeon.



SOLVE We'll start by finding the x- and y-components of the momentum before the collision. For the x-component we have

The x-component of the initial momentum . . .

(Both velocity components are negative, since they point to the left.)

$$(P_x)_i = \underbrace{m_F(v_{Fx})_i}_{\text{. . . equals the } x\text{-component of the initial momentum of the falcon . . .}} + \underbrace{m_P(v_{Px})_i}_{\text{. . . plus the } x\text{-component of the initial momentum of the pigeon.}} = m_F(-v_F\cos\theta) + m_P(-v_P)$$

$$= (0.80\text{ kg})(-18\text{ m/s})(\cos 45°) + (0.36\text{ kg})(-9.0\text{ m/s})$$

$$= -13.4\text{ kg}\cdot\text{m/s}$$

Similarly, for the y-component of the initial momentum we have

$$(P_y)_i = m_F(v_{Fy})_i + m_P(v_{Py})_i = m_F(-v_F\sin\theta) + 0$$

$$= (0.80\text{ kg})(-18.0\text{ m/s})(\sin 45°) = -10.2\text{ kg}\cdot\text{m/s}$$

After the collision, the two birds move with a common velocity $\vec{v}$ that is directed at an angle α from the horizontal. The x-component of the final momentum is then

$$(P_x)_f = (m_F + m_P)(v_x)_f$$

Continued

Momentum conservation requires $(P_x)_f = (P_x)_i$, so

$$(v_x)_f = \frac{(P_x)_i}{m_F + m_P} = \frac{-13.4\ \text{kg}\cdot\text{m/s}}{(0.80\ \text{kg}) + (0.36\ \text{kg})} = -11.6\ \text{m/s}$$

Similarly, $(P_y)_f = (P_y)_i$ gives

$$(v_y)_f = \frac{(P_y)_i}{m_F + m_P} = \frac{-10.2\ \text{kg}\cdot\text{m/s}}{(0.80\ \text{kg}) + (0.36\ \text{kg})} = -8.79\ \text{m/s}$$

From the figure we see that $\tan\alpha = (v_y)_f/(v_x)_f$, so that

$$\alpha = \tan^{-1}\left(\frac{(v_y)_f}{(v_x)_f}\right) = \tan^{-1}\left(\frac{-8.79\ \text{m/s}}{-11.6\ \text{m/s}}\right) = 37°$$

The magnitude of the final velocity (i.e., the speed) can be found from the Pythagorean theorem as

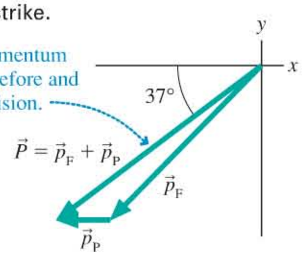
$$v = \sqrt{(v_x)_f^2 + (v_y)_f^2}$$
$$= \sqrt{(-11.6\ \text{m/s})^2 + (-8.79\ \text{m/s})^2} = 15\ \text{m/s}$$

Thus immediately after impact the falcon, with its meal, is moving 37° below horizontal at a speed of 15 m/s.

ASSESS It makes sense that the falcon slows down after catching the slower-moving pigeon. Also, the final angle is closer to the horizontal than the falcon's initial angle. This seems reasonable because the pigeon was initially flying horizontally, making the total momentum vector more horizontal than the direction of the falcon's initial momentum.

Copyright © 2011 Pearson Education

FIGURE 9.26 The momentum vectors of the falcon strike.



It's instructive to examine this collision with a picture of the momentum vectors. The vectors $\vec{p}_F$ and $\vec{p}_P$ before the collision, and their sum $\vec{P} = \vec{p}_F + \vec{p}_P$, are shown in FIGURE 9.26. You can see that the total momentum vector makes a 53° angle with the negative y-axis. The individual momenta change in the collision, *but the total momentum does not*.

9.7 Angular Momentum

For a single particle, we can think of the law of conservation of momentum as an alternative way of stating Newton's first law. Rather than saying that a particle will continue to move in a straight line at constant velocity unless acted on by a net force, we can say that the momentum of an isolated particle is conserved. Both express the idea that a particle moving in a straight line tends to "keep going" unless something acts on it to change its motion.

Another important motion you've studied is motion in a circle. The momentum $\vec{p}$ is *not* conserved for a particle undergoing circular motion. Momentum is a vector, and the momentum of a particle in circular motion changes as the direction of motion changes.

Nonetheless, a spinning bicycle wheel would keep turning if it were not for friction, and a ball moving in a circle at the end of a string tends to "keep going" in a circular path. The quantity that expresses this idea for circular motion is called *angular momentum*.

FIGURE 9.27 By applying a torque to the merry-go-round, the girl is increasing its angular momentum.

Let's start by looking at an example from everyday life: pushing a merry-go-round, as in FIGURE 9.27. If you push tangentially to the rim, you are applying a *torque* to the merry-go-round. As we learned in Chapter 7, the merry-go-round's angular speed will continue to increase for as long as you apply this torque. If you push *harder* (greater torque) or for a *longer time,* the greater the increase in its angular velocity will be. How can we quantify these observations?

Let's apply a constant torque τ_{net} to the merry-go-round for a time Δt. By how much will the merry-go-round's angular speed increase? In Section 7.4 we found that the angular acceleration α is given by the rotational equivalent of Newton's second law, or

$$\alpha = \frac{\tau_{net}}{I} \tag{9.17}$$

where I is the merry-go-round's moment of inertia.

Now the angular acceleration is the rate of change of the angular velocity, so

$$\alpha = \frac{\Delta\omega}{\Delta t} \tag{9.18}$$

Setting Equations 9.17 and 9.18 equal to each other gives

$$\frac{\Delta\omega}{\Delta t} = \frac{\tau_{\text{net}}}{I}$$

or, rearranging,

$$\tau_{\text{net}}\,\Delta t = I\,\Delta\omega \tag{9.19}$$

If you recall the impulse-momentum theorem for *linear* motion, which is

$$\vec{F}_{\text{net}}\,\Delta t = m\,\Delta\vec{v} = \Delta\vec{p} \tag{9.20}$$

you can see that Equation 9.19 is an analogous statement about rotational motion. Because the quantity $I\omega$ is evidently the rotational equivalent of $m\vec{v}$, the linear momentum $\vec{p}$, it seems reasonable to define the **angular momentum** L to be

$$L = I\omega \tag{9.21}$$

Angular momentum of an object with moment of inertia I rotating at angular velocity ω

The SI units of angular momentum are those of moment of inertia times angular velocity, or $\text{kg} \cdot \text{m}^2/\text{s}$.

Just as an object in linear motion can have a large momentum by having either a large mass or a high speed, a rotating object can have a large angular momentum by having a large moment of inertia or a large angular velocity. The merry-go-round in Figure 9.27 has a larger angular momentum if it's spinning fast than if it's spinning slowly. Also, the merry-go-round (large I) has a much larger angular momentum than a toy top (small I) spinning with the same angular velocity.

Table 9.2 summarizes the analogies between linear and rotational quantities that you learned in Chapter 7 and adds the analogy between linear momentum and angular momentum.

TABLE 9.2 Rotational and linear dynamics

Rotational dynamics	Linear dynamics
Torque τ_{net}	Force $\vec{F}_{\text{net}}$
Moment of inertia I	Mass m
Angular velocity ω	Velocity $\vec{v}$
Angular momentum $L = I\omega$	Linear momentum $\vec{p} = m\vec{v}$

Conservation of Angular Momentum

Having now defined angular momentum, we can write Equation 9.19 as

$$\tau_{\text{net}}\,\Delta t = \Delta L \tag{9.22}$$

in exact analogy with its linear dynamics equivalent, Equation 9.20. This equation states that the change in the angular momentum of an object is proportional to the net torque applied to the object. If the net external torque on an object is *zero,* the rotational analog of an isolated system, then the change in the angular momentum is zero as well. That is, a rotating object will continue to rotate with *constant* angular velocity—to "keep going"—unless acted upon by an external torque. We can state this conclusion as the *law of conservation of angular momentum*:

ActivPhysics ONLINE 7.14

Law of conservation of angular momentum The angular momentum of a rotating object subject to no net external torque ($\tau_{\text{net}} = 0$) is a constant. The final angular momentum L_f is equal to the initial angular momentum L_i.

This law is analogous to that for the conservation of linear momentum; there, linear momentum is conserved if the net *force* is zero. Because the angular momentum is $L = I\omega$, the mathematical statement of the law of conservation of angular momentum is

$$I_f\omega_f = I_i\,\omega_i \tag{9.23}$$

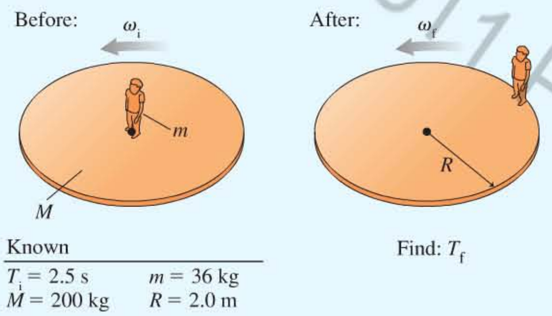
Copyright © 2011 Pearson Education

EXAMPLE 9.10 Period of a merry-go-round

Joey, whose mass is 36 kg, stands at the center of a 200 kg merry-go-round that is rotating once every 2.5 s. While it is rotating, Joey walks out to the edge of the merry-go-round, 2.0 m from its center. What is the rotational period of the merry-go-round when Joey gets to the edge?

PREPARE Take the system to be Joey + merry-go-round and assume frictionless bearings. There is no external torque on this system, so the angular momentum of the system will be conserved. As shown in the visual overview of FIGURE 9.28, we model the merry-go-round as a uniform disk of radius $R = 2.0$ m. From Table 7.2, the moment of inertia of a disk is $I_{\text{disk}} = \frac{1}{2}MR^2$. If we model Joey as a particle of mass m, his moment of inertia is zero when he is at the center, but it increases to mR^2 when he reaches the edge.

FIGURE 9.28 Visual overview of the merry-go-round.



SOLVE The mathematical statement of the law of conservation of momentum is $L_i = L_f$ or, from Equation 9.23, $I_f\omega_f = I_i\omega_i$, which we can rewrite as

$$\omega_f = \frac{I_i}{I_f}\omega_i$$

As Joey moves out to the edge, the moment of inertia of the system increases and, as a result, the angular velocity decreases. Initially, the moment of inertia of the system is just that of the merry-go-round because Joey's contribution is zero. Thus

$$I_i = I_{\text{disk}} = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(200\text{ kg})(2.0\text{ m})^2 = 400\text{ kg}\cdot\text{m}^2$$

When Joey reaches the edge, the total moment of inertia becomes

$$I_f = I_{\text{disk}} + mR^2 = 400\text{ kg}\cdot\text{m}^2 + (36\text{ kg})(2.0\text{ m})^2$$
$$= 540\text{ kg}\cdot\text{m}^2$$

The initial angular velocity is related to the initial period of rotation T_i by

$$\omega_i = \frac{2\pi}{T_i} = \frac{2\pi}{2.5\text{ s}} = 2.5\text{ rad/s}$$

Thus the final angular velocity is

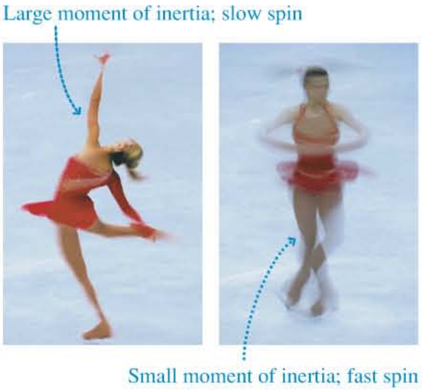
$$\omega_f = \frac{I_i}{I_f}\omega_i = \frac{400\text{ kg}\cdot\text{m}^2}{540\text{ kg}\cdot\text{m}^2}(2.5\text{ rad/s}) = 1.9\text{ rad/s}$$

When Joey reaches the edge, the period of the merry-go-round has increased to

$$T_f = \frac{2\pi}{\omega_f} = \frac{2\pi}{1.9\text{ rad/s}} = 3.3\text{ s}$$

ASSESS The merry-go-round rotates *more slowly* after Joey moves out to the edge. This makes sense because if the system's moment of inertia increases, as it does when Joey moves out, the angular velocity must decrease to keep the angular momentum constant.

FIGURE 9.29 A spinning figure skater.



Conservation of momentum enters into many aspects of sports. Because no external torques act, the angular momentum of a platform diver is conserved while she's in the air. Just as for Joey and the merry-go-round of Example 9.10, she spins slowly when her moment of inertia is large; by decreasing her moment of inertia, she increases her rate of spin. Divers can thus markedly increase their spin rate by changing their body from an extended posture to a tuck position. Figure skaters also increase their spin rate by decreasing their moment of inertia, as shown in FIGURE 9.29. The following example gives a simplified treatment of this process.

EXAMPLE 9.11 Analyzing a spinning ice skater

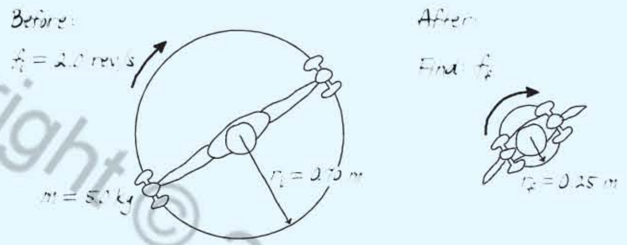
An ice skater spins around on the tips of his blades while holding a 5.0 kg weight in each hand. He begins with his arms straight out from his body and his hands 140 cm apart. While spinning at 2.0 rev/s, he pulls the weights in and holds them 50 cm apart against his shoulders. If we neglect the mass of the skater, how fast is he spinning after pulling the weights in?

PREPARE Although the mass of the skater is larger than the mass of the weights, neglecting the skater's mass is not a bad approximation. Moment of inertia depends on the *square* of the distance of the mass from the axis of rotation. The skater's mass is concentrated in his torso, which has an effective radius (i.e., where most of the mass is concentrated) of only 9 or 10 cm. The weights move in much larger circles and have a disproportionate influence on his motion. The skater's arms exert radial forces on the

Copyright © 2011 Pearson Education

weights just to keep them moving in circles, and even larger radial forces as he pulls them in. But there is no external torque on the weights, so their total angular momentum is conserved. **FIGURE 9.30** shows a before-and-after visual overview, as seen from above.

FIGURE 9.30 Top view visual overview of the spinning ice skater.



Copyright © 2011 Pearson Education

SOLVE The two weights have the same mass, move in circles with the same radius, and have the same angular velocity. Thus the total angular momentum is twice that of one weight. The mathematical statement of angular momentum conservation, $I_f\omega_f = I_i\omega_i$, is

There are two weights.

$$\underbrace{(2\,mr_f^2)}_{I_f}\omega_f = \underbrace{(2\,mr_i^2)}_{I_i}\omega_i$$

Because the angular velocity is related to the rotation frequency f by $\omega = 2\pi f$, this equation simplifies to

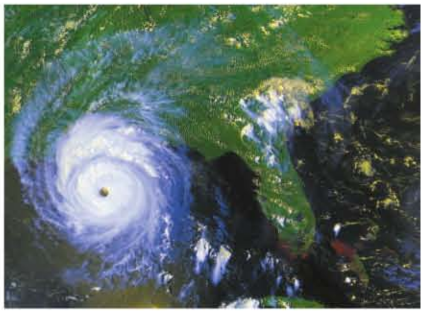
$$f_f = \left(\frac{r_i}{r_f}\right)^2 f_i$$

When he pulls the weights in, his rotation frequency increases to

$$f_f = \left(\frac{0.70\text{ m}}{0.25\text{ m}}\right)^2 \times 2.0\text{ rev/s} = 16\text{ rev/s}$$

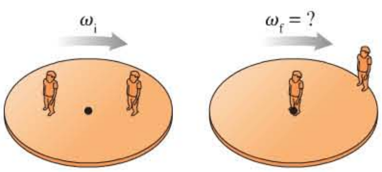
ASSESS Pulling in the weights increases the skater's spin from 2 rev/s to 16 rev/s. This is somewhat high, because we neglected the mass of the skater, but it illustrates how skaters do "spin up" by pulling their mass in toward the rotation axis.

Solving either of these two examples using Newton's laws would be quite difficult. We would have to deal with internal forces, such as Joey's feet against the merry-go-round, and other complications. For problems like these, where we're interested only in the before-and-after aspects of the motion, using a conservation law makes the solution much simpler.



▶ **The eye of a hurricane** As air masses from the slowing rotating outer zones are drawn toward the low-pressure center, their moment of inertia decreases. Because the angular momentum of these air masses is conserved, their speed must *increase* as they approach the center, leading to the high wind speeds near the center of the storm.

STOP TO THINK 9.5 The left figure shows two boys of equal mass standing halfway to the edge on a turntable that is freely rotating at angular speed ω_i. They then walk to the positions shown in the right figure. The final angular speed ω_f is



A. Greater than ω_i.   B. Less than ω_i.   C. Equal to ω_i.

INTEGRATED EXAMPLE 9.12 Aerial firefighting

A forest fire is easiest to attack when it's just getting started. In remote locations, this often means using airplanes to rapidly deliver large quantities of water and fire suppressant to the blaze.

The "Superscooper" is an amphibious aircraft that can pick up a 6000 kg load of water by skimming over the surface of a river or lake and scooping water directly into its storage tanks. As it approaches the water's surface at a speed of 35 m/s, an empty Superscooper has a mass of 13,000 kg.

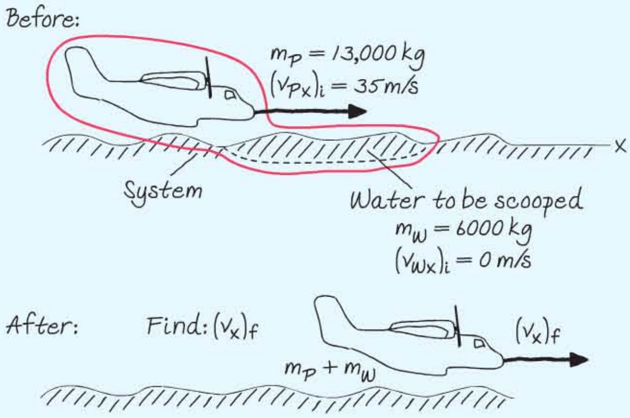
a. It takes the plane 12 s to pick up a full load of water. If we ignore the force on the plane due to the thrust of its propellers, what is its speed immediately after picking up the water?
b. What is the impulse delivered to the plane by the water?
c. What is the average force of the water on the plane?
d. The plane then flies over the fire zone at 40 m/s. It releases water by opening doors in the belly of the plane, allowing the water to fall straight down with respect to the plane. What is the plane's speed after dropping the water if it takes 5.0 s to do so?

PREPARE We can solve part a, and later part d, using conservation of momentum, following Problem-Solving Strategy 9.1. We'll need to choose the system with care, so that $\vec{F}_{\text{net}} = \vec{0}$. The plane alone is not an appropriate system for using conservation of momentum: As the plane scoops up the water, the water exerts a large external drag force on the plane, so $\vec{F}_{\text{net}}$ is definitely not zero. Instead, we should choose the plane *and* the water it is going to scoop up as the system. Then there are no external forces in the x-direction, and the net force in the y-direction is zero, since neither plane nor water accelerates appreciably in this direction during the scooping process. The complicated forces between plane and water are now *internal* forces that do not change the total momentum of the plane + water system.

With the system chosen, we follow the steps of Tactics Box 9.1 to prepare the before-and-after visual overview shown in FIGURE 9.31.

Parts b and c are impulse-and-momentum problems, so to solve them we'll use the impulse-momentum theorem, Equation 9.8. The impulse-momentum theorem considers the dynamics of a *single* object—here, the plane—subject to external forces—in this case, from the water.

FIGURE 9.31 Visual overview of the plane and water.



SOLVE a. Conservation of momentum in the x-direction is

$$(P_x)_f = (P_x)_i$$

or

$$(m_P + m_W)(v_x)_f = m_P(v_{Px})_i + m_W(v_{Wx})_i = m_P(v_{Px})_i + 0$$

Here we've used the facts that the initial velocity of the water is zero and that the final situation, as in an inelastic collision, has the combined mass of the plane and water moving with the same velocity $(v_x)_f$. Solving for $(v_x)_f$, we find

$$(v_x)_f = \frac{m_P(v_{Px})_i}{m_P + m_W} = \frac{(13{,}000\text{ kg})(35\text{ m/s})}{(13{,}000\text{ kg}) + (6000\text{ kg})} = 24\text{ m/s}$$

b. The impulse-momentum theorem is $J_x = \Delta p_x$, where $\Delta p_x = m_P \Delta v_x$ is the change in the plane's momentum. Thus

$$J_x = m_P \Delta v_x = m_P[(v_x)_f - (v_{Px})_i]$$
$$= (13{,}000\text{ kg})(24\text{ m/s} - 35\text{ m/s}) = -1.4 \times 10^5\text{ kg}\cdot\text{m/s}$$

c. From Equation 9.1, the definition of impulse, we have

$$(F_{\text{avg}})_x = \frac{J_x}{\Delta t} = \frac{-1.4 \times 10^5\text{ kg}\cdot\text{m/s}}{12\text{ s}} = -12{,}000\text{ N}$$

d. Because the water drops straight down *relative to the plane*, it has the same x-component of velocity immediately after being dropped as before being dropped. That is, simply opening the doors doesn't cause the water to speed up or slow down horizontally, so the water's horizontal momentum doesn't change upon being dropped. Because the total momentum of the plane + water system is conserved, the momentum of the plane doesn't change either. The plane's speed after the drop is still 40 m/s.

ASSESS The mass of the water is nearly half that of the plane, so the significant decrease in the plane's velocity as it scoops up the water is reasonable. The force of the water on the plane is large, but is still only about 10% of the plane's weight, $mg = 130{,}000$ N, so the answer seems to be reasonable.

Copyright © 2011 Pearson Education

SUMMARY

The goals of Chapter 9 have been to introduce the ideas of impulse, momentum, and angular momentum and to learn a new problem-solving strategy based on conservation laws.

Copyright © 2011 Pearson Education

GENERAL PRINCIPLES

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Conservation of Angular Momentum

The angular momentum L of a rotating object subject to zero external torque does not change. Thus

$$L_f = L_i$$

This can be written in terms of the moment of inertia and angular velocity as

$$I_f\omega_f = I_i\omega_i$$

Solving Momentum Conservation Problems

PREPARE Choose an isolated system or a system that is isolated during at least part of the problem. Draw a visual overview of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components:

$$(p_{1x})_f + (p_{2x})_f + \cdots = (p_{1x})_i + (p_{2x})_i + \cdots$$

$$(p_{1y})_f + (p_{2y})_f + \cdots = (p_{1y})_i + (p_{2y})_i + \cdots$$

In terms of masses and velocities, this is

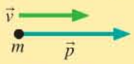
$$m_1(v_{1x})_f + m_2(v_{2x})_f + \cdots = m_1(v_{1x})_i + m_2(v_{2x})_i + \cdots$$

$$m_1(v_{1y})_f + m_2(v_{2y})_f + \cdots = m_1(v_{1y})_i + m_2(v_{2y})_i + \cdots$$

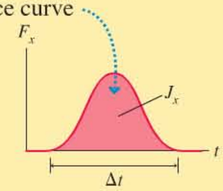
ASSESS Is the result reasonable?

IMPORTANT CONCEPTS

Momentum $\vec{p} = m\vec{v}$



Impulse J_x = area under force curve



Impulse and momentum are related by the **impulse-momentum theorem**

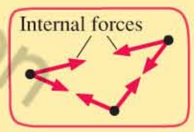
$$\Delta p_x = J_x$$

This is an alternative statement of Newton's second law.

Angular momentum $L = I\omega$ is the rotational analog of linear momentum $\vec{p} = m\vec{v}$.

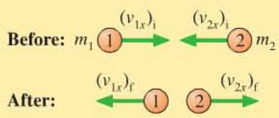
System A group of interacting particles.

Isolated system A system on which the net external force is zero.



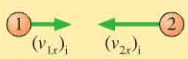
Before-and-after visual overview

- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.



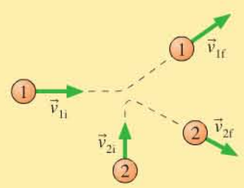
APPLICATIONS

Collisions Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.



Explosions Two or more particles move away from each other.

Two dimensions Both the x- and y-components of the total momentum P must be conserved, giving two simultaneous equations.



MP™ For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to |||| (challenging).

Problems labeled can be done on a Workbook Momentum Worksheet; INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

VIEW ALL SOLUTIONS

QUESTIONS

Conceptual Questions

1. Rank in order, from largest to smallest, the momenta p_{1x} through p_{5x} of the objects presented in Figure Q9.1. Explain.

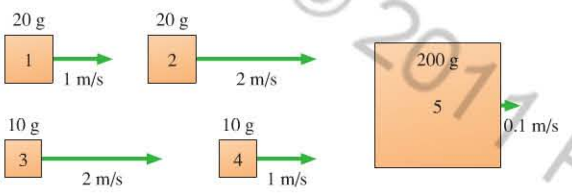


FIGURE Q9.1

2. Starting from rest, object 1 is subject to a 12 N force for 2.0 s. Object 2, with twice the mass, is subject to a 15 N force for 3.0 s. Which object has the greater final speed? Explain.
3. A 0.2 kg plastic cart and a 20 kg lead cart can roll without friction on a horizontal surface. Equal forces are used to push both carts forward for a time of 1 s, starting from rest. After the force is removed at $t = 1$ s, is the momentum of the plastic cart greater than, less than, or equal to the momentum of the lead cart? Explain.
4. Two pucks, of mass m and $4m$, lie on a frictionless table. Equal forces are used to push both pucks forward a distance of 1 m.
   a. Which puck takes longer to travel the distance? Explain.
   b. Which puck has the greater momentum upon completing the distance? Explain.
5. A stationary firecracker explodes into three pieces. One piece travels off to the east; a second travels to the north. Which of the vectors of Figure Q9.5 could be the velocity of the third piece? Explain.

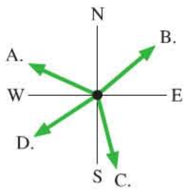


FIGURE Q9.5

6. Two students stand at rest, facing each other on frictionless skates. They then start tossing a heavy ball back and forth between them. Describe their subsequent motion.
7. Two particles collide, one of which was initially moving and the other initially at rest.
   a. Is it possible for *both* particles to be at rest after the collision? Give an example in which this happens, or explain why it can't happen.
   b. Is it possible for *one* particle to be at rest after the collision? Give an example in which this happens, or explain why it can't happen.
8. Automobiles are designed with "crumple zones" intended to collapse in a collision. Why would a manufacturer design part of a car so that it collapses in a collision?
9. You probably know that it feels better to catch a baseball if you are wearing a padded glove. Explain why this is so, using the ideas of momentum and impulse.
10. In the early days of rocketry, some people claimed that rockets couldn't fly in outer space as there was no air for the rockets to push against. Suppose you were an early investigator in the field of rocketry and met someone who made this argument. How would you convince the person that rockets could travel in space?

11. Two ice skaters, Megan and Jason, push off from each other on frictionless ice. Jason's mass is twice that of Megan.
    a. Which skater, if either, experiences the greater impulse during the push? Explain.
    b. Which skater, if either, has the greater speed after the push-off? Explain.
12. Suppose a rubber ball and a steel ball collide. Which, if either, receives the larger impulse? Explain.
13. While standing still on a basketball court, you throw the ball to a teammate. Why do you not move backward as a result? Is the law of conservation of momentum violated?
14. To win a prize at the county fair, you're trying to knock down a heavy bowling pin by hitting it with a thrown object. Should you choose to throw a rubber ball or a beanbag of equal size and weight? Explain.
15. Rank in order, from largest to smallest, the angular momenta L_1 through L_5 of the balls shown in Figure Q9.15. Explain.

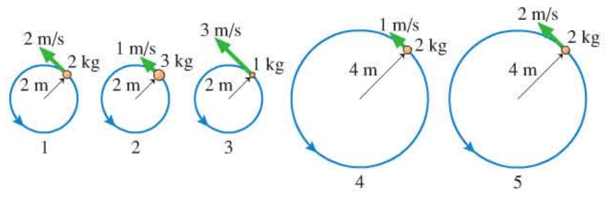


FIGURE Q9.15

16. Figure Q9.16 shows two masses held together by a thread on a rod that is rotating about its center with angular velocity ω. If the thread breaks, the masses will slide out to the ends of the rod. If that happens, will the rod's angular velocity increase, decrease, or remain unchanged? Explain.

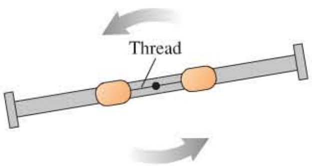


FIGURE Q9.16

17. If the earth warms significantly, the polar ice caps will melt. Water will move from the poles, near the earth's rotation axis, and will spread out around the globe. In principle, this will change the length of the day. Why? Will the length of the day increase or decrease?

Copyright © 2014 Pearson Education

18. The disks shown in Figure Q9.18 have equal mass. Is the angular momentum of disk 2, on the right, larger than, smaller than, or equal to the angular momentum of disk 1? Explain.

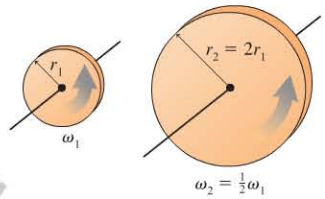


FIGURE Q9.18

Multiple-Choice Questions

19. | Curling is a sport played with 20 kg stones that slide across an ice surface. Suppose a curling stone sliding at 1 m/s strikes another stone and comes to rest in 2 ms. Approximately how much force is there on the stone during the impact?
A. 200 N B. 1000 N C. 2000 N D. 10,000 N

20. | Two balls are hung from cords. The first ball, of mass 1.0 kg, is pulled to the side and released, reaching a speed of 2.0 m/s at the bottom of its arc. Then, as shown in Figure Q9.20, it hits and sticks to another ball. The speed of the pair just after the collision is 1.2 m/s. What is the mass of the second ball?
A. 0.67 kg
B. 2.0 kg
C. 1.7 kg
D. 1.0 kg

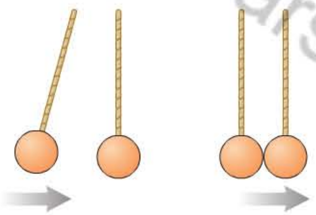


FIGURE Q9.20

21. | Figure Q9.21 shows two blocks sliding on a frictionless surface. Eventually the smaller block overtakes the larger one, collides with it, and sticks. What is the speed of the two blocks after the collision?
A. $v_i/2$ B. $4v_i/5$ C. v_i D. $5v_i/4$ E. $2v_i$

FIGURE Q9.21

22. | Two friends are sitting in a stationary canoe. At $t = 3.0$ s the person at the front tosses a sack to the person in the rear, who catches the sack 0.2 s later. Which plot in Figure Q9.22 shows the velocity of the boat as a function of time? Positive velocity is forward, negative velocity is backward. Neglect any drag force on the canoe from the water.

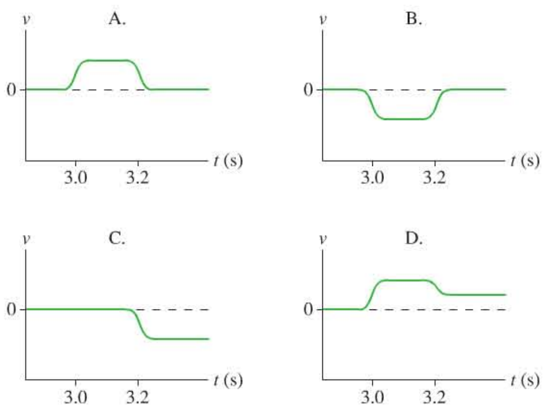


FIGURE Q9.22

23. || Two blocks, with masses $m_1 = 2.5$ kg and $m_2 = 14$ kg, approach each other along a horizontal, frictionless track. The initial velocities of the blocks are $v_1 = 12.0$ m/s to the right and $v_2 = 3.4$ m/s to the left. The two blocks then collide and stick together. Which of the graphs could represent the force of block 1 on block 2 during the collision?

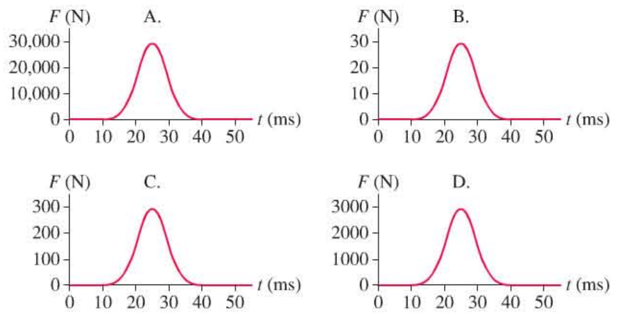


FIGURE Q9.23

24. | A small puck is sliding to the right with momentum $\vec{p}_i$ on a horizontal, frictionless surface, as shown in Figure Q9.24. A force is applied to the puck for a short time and its momentum afterward is $\vec{p}_f$. Which lettered arrow shows the direction of the impulse that was delivered to the puck?

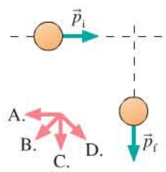


FIGURE Q9.24

25. | A red ball, initially at rest, is simultaneously hit by a blue ball traveling from west to east at 3 m/s and a green ball traveling east to west at 3 m/s. All three balls have equal mass. Afterward, the red ball is traveling south and the green ball is moving to the east. In which direction is the blue ball traveling?
A. West
B. North
C. Between north and west
D. Between north and east
E. Between south and west

26. | A 24 g, 3-cm-diameter thin, hollow sphere rotates at 30 rpm about a vertical, frictionless axis through its center. A 4 g bug stands at the top of the sphere. He then walks along the surface of the sphere until he reaches its "equator." When he reaches the equator, the sphere is rotating at
A. 15 rpm
B. 24 rpm
C. 30 rpm
D. 37 rpm
E. 45 rpm

27. | A 5.0 kg solid cylinder of radius 12 cm rotates with $\omega_i = 3.7$ rad/s about an axis through its center. A torque of 0.040 N · m is applied to the cylinder for 5.0 s. By how much does the cylinder's angular momentum change?
A. 0.12 kg · m²/s
B. 0.20 kg · m²/s
C. 0.38 kg · m²/s
D. 0.52 kg · m²/s
E. 0.88 kg · m²/s

Copyright © 2017 Pearson Education

VIEW ALL SOLUTIONS

PROBLEMS

Section 9.1 Impulse

Section 9.2 Momentum and the Impulse-Momentum Theorem

1. At what speed do a bicycle and its rider, with a combined mass of 100 kg, have the same momentum as a 1500 kg car traveling at 5.0 m/s?
2. A 57 g tennis ball is served at 45 m/s. If the ball started from rest, what impulse was applied to the ball by the racket?
3. A student throws a 120 g snowball at 7.5 m/s at the side of the schoolhouse, where it hits and sticks. What is the magnitude of the average force on the wall if the duration of the collision is 0.15 s?
4. In Figure P9.4, what value of F_{max} gives an impulse of 6.0 N · s?

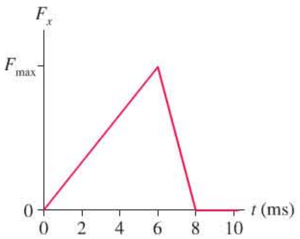


FIGURE P9.4

5. A sled and rider, gliding over horizontal, frictionless ice at 4.0 m/s, have a combined mass of 80 kg. The sled then slides over a rough spot in the ice, slowing down to 3.0 m/s. What impulse was delivered to the sled by the friction force from the rough spot?

Section 9.3 Solving Impulse and Momentum Problems

6. Use the impulse-momentum theorem to find how long a stone falling straight down takes to increase its speed from 5.5 m/s to 10.4 m/s.
7. a. A 2.0 kg object is moving to the right with a speed of 1.0 m/s when it experiences the force shown in Figure P9.7a. What are the object's speed and direction after the force ends?
   b. Answer this question for the force shown in Figure P9.7b.

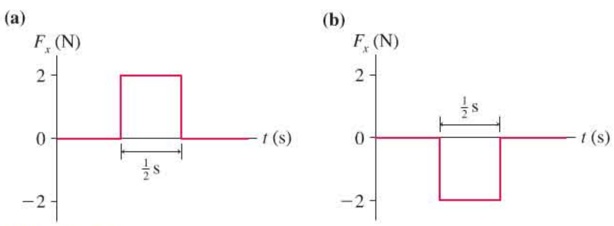


FIGURE P9.7

8. A 60 g tennis ball with an initial speed of 32 m/s hits a wall and rebounds with the same speed. Figure P9.8 shows the force of the wall on the ball during the collision. What is the value of F_{max}, the maximum value of the contact force during the collision?

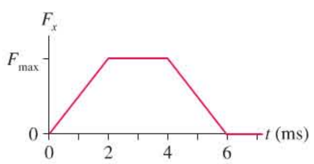


FIGURE P9.8

9. A child is sliding on a sled at 1.5 m/s to the right. You stop the sled by pushing on it for 0.50 s in a direction opposite to its motion. If the mass of the child and sled is 35 kg, what average force do you need to apply to stop the sled? Use the concepts of impulse and momentum.
10. An ice hockey puck slides along the ice at 12 m/s. A hockey stick delivers an impulse of 4.0 kg · m/s, causing the puck to move off in the opposite direction with the same speed. What is the mass of the puck?
11. As part of a safety investigation, two 1400 kg cars traveling at 20 m/s are crashed into different barriers. Find the average forces exerted on (a) the car that hits a line of water barrels and takes 1.5 s to stop, and (b) the car that hits a concrete barrier and takes 0.1 s to stop.
12. In a Little League baseball game, the 145 g ball enters the strike zone with a speed of 15.0 m/s. The batter hits the ball, and it leaves his bat with a speed of 20.0 m/s in exactly the opposite direction.
    a. What is the magnitude of the impulse delivered by the bat to the ball?
    b. If the bat is in contact with the ball for 1.5 ms, what is the magnitude of the average force exerted by the bat on the ball?

Section 9.4 Conservation of Momentum

13. A small, 100 g cart is moving at 1.20 m/s on an air track when it collides with a larger, 1.00 kg cart at rest. After the collision, the small cart recoils at 0.850 m/s. What is the speed of the large cart after the collision?
14. A man standing on very slick ice fires a rifle horizontally. The mass of the man together with the rifle is 70 kg, and the mass of the bullet is 10 g. If the bullet leaves the muzzle at a speed of 500 m/s, what is the final speed of the man?
15. A 2.7 kg block of wood sits on a table. A 3.0 g bullet, fired horizontally at a speed of 500 m/s, goes completely through the block, emerging at a speed of 220 m/s. What is the speed of the block immediately after the bullet exits?
16. A strong man is compressing a lightweight spring between two weights. One weight has a mass of 2.3 kg, the other a mass of 5.3 kg. He is holding the weights stationary, but then he loses his grip and the weights fly off in opposite directions. The lighter of the two is shot out at a speed of 6.0 m/s. What is the speed of the heavier weight?
17. A 10,000 kg railroad car is rolling at 2.00 m/s when a 4000 kg load of gravel is suddenly dropped in. What is the car's speed just after the gravel is loaded?
18. A 5000 kg open train car is rolling on frictionless rails at 22.0 m/s when it starts pouring rain. A few minutes later, the car's speed is 20.0 m/s. What mass of water has collected in the car?
19. A 50.0 kg archer, standing on frictionless ice, shoots a 40 g arrow at a speed of 60 m/s. What is the recoil speed of the archer?
20. A 9.5 kg dog takes a nap in a canoe and wakes up to find the canoe has drifted out onto the lake but now is stationary. He walks along the length of the canoe at 0.50 m/s, relative to the water, and the canoe simultaneously moves in the opposite direction at 0.15 m/s. What is the mass of the canoe?

Copyright © 2011 Pearson Education

Section 9.5 Inelastic Collisions

21. A 300 g bird flying along at 6.0 m/s sees a 10 g insect heading straight toward it with a speed of 30 m/s. The bird opens its mouth wide and enjoys a nice lunch. What is the bird's speed immediately after swallowing?
22. A 71 kg baseball player jumps straight up to catch a line drive. If the 140 g ball is moving horizontally at 28 m/s, and the catch is made when the ballplayer is at the highest point of his leap, what is his speed immediately after stopping the ball?
23. A kid at the junior high cafeteria wants to propel an empty milk carton along a lunch table by hitting it with a 3.0 g spit ball. If he wants the speed of the 20 g carton just after the spit ball hits it to be 0.30 m/s, at what speed should his spit ball hit the carton?
24. The parking brake on a 2000 kg Cadillac has failed, and it is rolling slowly, at 1 mph, toward a group of small children. Seeing the situation, you realize you have just enough time to drive your 1000 kg Volkswagen head-on into the Cadillac and save the children. With what speed should you impact the Cadillac to bring it to a halt?
25. A 2.0 kg block slides along a frictionless surface at 1.0 m/s. A second block, sliding at a faster 4.0 m/s, collides with the first from behind and sticks to it. The final velocity of the combined blocks is 2.0 m/s. What was the mass of the second block?

Section 9.6 Momentum and Collisions in Two Dimensions

26. A 20 g ball of clay traveling east at 3.0 m/s collides with a 30 g ball of clay traveling north at 2.0 m/s. What are the speed and the direction of the resulting 50 g ball of clay?
27. Two particles collide and bounce apart. Figure P9.27 shows the initial momenta of both and the final momentum of particle 2. What is the final momentum of particle 1? Show your answer by copying the figure and drawing the final momentum vector on the figure.

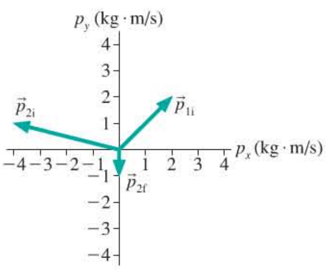


FIGURE P9.27

28. A 20 g ball of clay traveling east at 2.0 m/s collides with a 30 g ball of clay traveling 30° south of west at 1.0 m/s. What are the speed and direction of the resulting 50 g blob of clay?
29. A firecracker in a coconut blows the coconut into three pieces. Two pieces of equal mass fly off south and west, perpendicular to each other, at 20 m/s. The third piece has twice the mass as the other two. What are the speed and direction of the third piece?

Section 9.7 Angular Momentum

30. INT What is the angular momentum of the moon around the earth? The moon's mass is 7.4×10^{22} kg and it orbits 3.8×10^{8} m from the earth.
31. A little girl is going on the merry-go-round for the first time, and wants her 47 kg mother to stand next to her on the ride, 2.6 m from the merry-go-round's center. If her mother's speed is 4.2 m/s when the ride is in motion, what is her angular momentum around the center of the merry-go-round?
32. What is the angular momentum about the axle of the 500 g rotating bar in Figure P9.32?

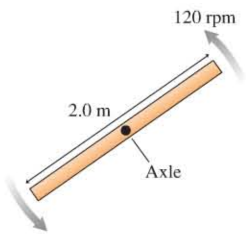


FIGURE P9.32

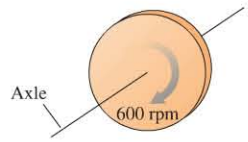


FIGURE P9.33

33. What is the angular momentum about the axle of the 2.0 kg, 4.0-cm-diameter rotating disk in Figure P9.33?
34. BIO Divers change their body position in midair while rotating about their center of mass. In one dive, the diver leaves the board with her body nearly straight, then tucks into a somersault position. If the moment of inertia of the diver in a straight position is 14 kg · m^2 and in a tucked position is 4.0 kg · m^2, by what factor is her angular velocity when tucked greater than when straight?
35. BIO Ice skaters often end their performances with spin turns, where they spin very fast about their center of mass with their arms folded in and legs together. Upon ending, their arms extend outward, proclaiming their finish. Not quite as noticeably, one leg goes out as well. Suppose that the moment of inertia of a skater with arms out and one leg extended is 3.2 kg · m^2 and for arms and legs in is 0.80 kg · m^2. If she starts out spinning at 5.0 rev/s, what is her angular speed (in rev/s) when her arms and one leg open outward?

General Problems

36. What is the impulse on a 3.0 kg particle that experiences the force described by the graph in Figure P9.36?

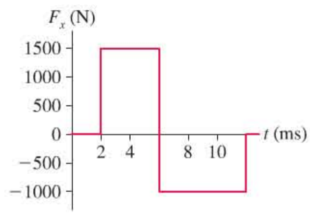


FIGURE P9.36

37. A 600 g air-track glider collides with a spring at one end of the track. Figure P9.37 shows the glider's velocity and the force exerted on the glider by the spring. How long is the glider in contact with the spring?

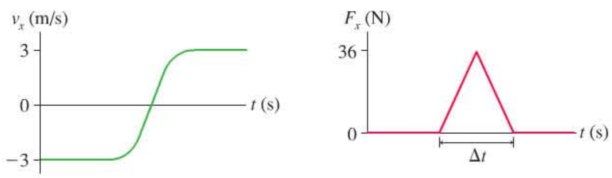


FIGURE P9.37

Copyright © 2014 Pearson Education

38. Far in space, where gravity is negligible, a 425 kg rocket traveling at 75.0 m/s in the positive x-direction fires its engines. Figure P9.38 shows the thrust force as a function of time. The mass lost by the rocket during these 30.0 s is negligible.
   a. What impulse does the engine impart to the rocket?
   b. At what time does the rocket reach its maximum speed? What is the maximum speed?

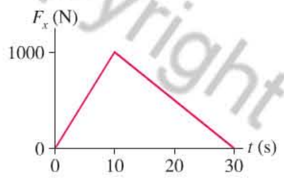


FIGURE P9.38

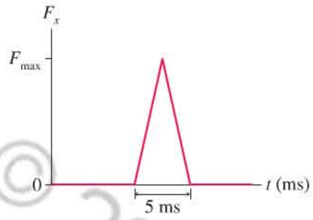


FIGURE P9.39

39. A 200 g ball is dropped from a height of 2.0 m, bounces on a hard floor, and rebounds to a height of 1.5 m. Figure P9.39 shows the impulse received from the floor. What maximum force does the floor exert on the ball?

40. A 200 g ball is dropped from a height of 2.0 m and bounces on a hard floor. The force on the ball from the floor is shown in Figure P9.40. How high does the ball rebound?

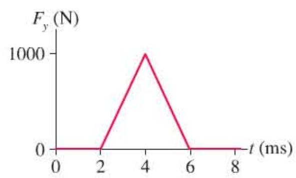


FIGURE P9.40

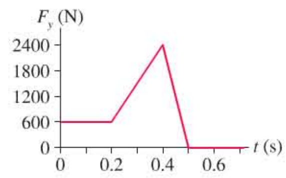


FIGURE P9.41

41. Figure P9.41 is a graph of the force exerted by the floor on a woman making a vertical jump. At what speed does she leave the ground?
Hint: The force of the floor is not the only force acting on the woman.

42. A sled slides along a horizontal surface for which the coefficient of kinetic friction is 0.25. Its velocity at point A is 8.0 m/s and at point B is 5.0 m/s. Use the impulse-momentum theorem to find how long the sled takes to travel from A to B.

43. A 140 g baseball is moving horizontally to the right at 35 m/s when it is hit by the bat. The ball flies off to the left at 55 m/s, at an angle of 25° above the horizontal. What are the magnitude and direction of the impulse that the bat delivers to the ball?

44. Squids rely on jet propulsion, a versatile technique to move around in water. A 1.5 kg squid at rest suddenly expels 0.10 kg of water backward to quickly get itself moving forward at 3.0 m/s. If other forces (such as the drag force on the squid) are ignored, what is the speed with which the squid expels the water?

45. The flowers of the bunchberry plant open with astonishing force and speed, causing the pollen grains to be ejected out of the flower in a mere 0.30 ms at an acceleration of 2.5×10^4 m/s^2. If the acceleration is constant, what impulse is delivered to a pollen grain with a mass of 1.0×10^{-7} g?

46. a. With what speed are pollen grains ejected from a bunchberry flower? See Problem 45 for information.
   b. Suppose that 1000 ejected pollen grains slam into the abdomen of a 5.0 g bee that is hovering just above the flower. If the collision is perfectly inelastic, what is the bee's speed immediately afterward? Is the bee likely to notice?

47. A tennis player swings her 1000 g racket with a speed of 10 m/s. She hits a 60 g tennis ball that was approaching her at a speed of 20 m/s. The ball rebounds at 40 m/s.
   a. How fast is her racket moving immediately after the impact? You can ignore the interaction of the racket with her hand for the brief duration of the collision.
   b. If the tennis ball and racket are in contact for 10 ms, what is the average force that the racket exerts on the ball?

48. A 20 g ball of clay is thrown horizontally at 30 m/s toward a 1.0 kg block sitting at rest on a frictionless surface. The clay hits and sticks to the block.
   a. What is the speed of the block and clay right after the collision?
   b. Use the block's initial and final speeds to calculate the impulse the clay exerts on the block.
   c. Use the clay's initial and final speeds to calculate the impulse the block exerts on the clay.
   d. Does $\vec{J}_{\text{block on clay}} = -\vec{J}_{\text{clay on block}}$?

49. Dan is gliding on his skateboard at 4.0 m/s. He suddenly jumps backward off the skateboard, kicking the skateboard forward at 8.0 m/s. How fast is Dan going as his feet hit the ground? Dan's mass is 50 kg and the skateboard's mass is 5.0 kg.

50. James and Sarah stand on a stationary cart with frictionless wheels. The total mass of the cart and riders is 130 kg. At the same instant, James throws a 1.0 kg ball to Sarah at 4.5 m/s, while Sarah throws a 0.50 kg ball to James at 1.0 m/s. James's throw is to the right and Sarah's is to the left.
   a. While the two balls are in the air, what are the speed and direction of the cart and its riders?
   b. After the balls are caught, what are the speed and direction of the cart and riders?

51. Ethan, whose mass is 80 kg, stands at one end of a very long, stationary wheeled cart that has a mass of 500 kg. He then starts sprinting toward the other end of the cart. He soon reaches his top speed of 8.0 m/s, measured relative to the cart. What is the speed of the cart when Ethan has reached his top speed?

52. The cars of a long coal train are filled by pulling them under a hopper, from which coal falls into the cars at a rate of 10,000 kg/s. Ignoring friction due to the rails, what is the average force that the engine must exert on the coal train to keep it moving under the hopper at a speed of 0.50 m/s?

53. Three identical train cars, coupled together, are rolling east at 2.0 m/s. A fourth car traveling east at 4.0 m/s catches up with the three and couples to make a four-car train. A moment later, the train cars hit a fifth car that was at rest on the tracks, and it couples to make a five-car train. What is the speed of the five-car train?

54. A 110 kg linebacker running at 2.0 m/s and an 82 kg quarterback running at 3.0 m/s have a head-on collision in midair. The linebacker grabs and holds onto the quarterback. Who ends up moving forward after they hit?

55. Most geologists believe that the dinosaurs became extinct 65 million years ago when a large comet or asteroid struck the earth, throwing up so much dust that the sun was blocked out for a period of many months. Suppose an asteroid with a diameter of 2.0 km and a mass of 1.0×10^{13} kg hits the earth with an impact speed of 4.0×10^4 m/s.
   a. What is the earth's recoil speed after such a collision? (Use a reference frame in which the earth was initially at rest.)
   b. What percentage is this of the earth's speed around the sun? (Use the astronomical data inside the back cover.)

Copyright © 2011 Pearson Education

56. At the center of a 50-m-diameter circular ice rink, a 75 kg skater traveling north at 2.5 m/s collides with and holds onto a 60 kg skater who had been heading west at 3.5 m/s.
   a. How long will it take them to glide to the edge of the rink?
   b. Where will they reach it? Give your answer as an angle north of west.

57. Two ice skaters, with masses of 50 kg and 75 kg, are at the center of a 60-m-diameter circular rink. The skaters push off against each other and glide to opposite edges of the rink. If the heavier skater reaches the edge in 20 s, how long does the lighter skater take to reach the edge?

58. One billiard ball is shot east at 2.00 m/s. A second, identical billiard ball is shot west at 1.00 m/s. The balls have a glancing collision, not a head-on collision, deflecting the second ball by 90° and sending it north at 1.41 m/s. What are the speed and direction of the first ball after the collision?

59. A 10 g bullet is fired into a 10 kg wood block that is at rest on a wood table. The block, with the bullet embedded, slides 5.0 cm across the table. What was the speed of the bullet?

60. You are part of a search-and-rescue mission that has been called out to look for a lost explorer. You've found the missing explorer, but you're separated from him by a 200-m-high cliff and a 30-m-wide raging river, as shown in Figure P9.60. To save his life, you need to get a 5.0 kg package of emergency supplies across the river. Unfortunately, you can't throw the package hard enough to make it across. Fortunately, you happen to have a 1.0 kg rocket intended for launching flares. Improvising quickly, you attach a sharpened stick to the front of the rocket, so that it will impale itself into the package of supplies, then fire the rocket at ground level toward the supplies. What minimum speed must the rocket have just before impact in order to save the explorer's life?

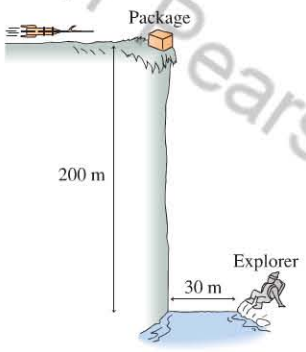


FIGURE P9.60

61. A 1500 kg weather rocket accelerates upward at 10.0 m/s^2. It explodes 2.00 s after liftoff and breaks into two fragments, one twice as massive as the other. Photos reveal that the lighter fragment traveled straight up and reached a maximum height of 530 m. What were the speed and direction of the heavier fragment just after the explosion?

62. Two 500 g blocks of wood are 2.0 m apart on a frictionless table. A 10 g bullet is fired at 400 m/s toward the blocks. It passes all the way through the first block, then embeds itself in the second block. The speed of the first block immediately afterward is 6.0 m/s. What is the speed of the second block after the bullet stops?

63. A 500 kg cannon fires a 10 kg cannonball with a speed of 200 m/s relative to the muzzle. The cannon is on wheels that roll without friction. When the cannon fires, what is the speed of the cannonball relative to the earth?

64. Laura, whose mass is 35 kg, jumps horizontally off a 55 kg canoe at 1.5 m/s relative to the canoe. What is the canoe's speed just after she jumps?

65. A spaceship of mass 2.0×10^6 kg is cruising at a speed of 5.0×10^6 m/s when the antimatter reactor fails, blowing the ship into three pieces. One section, having a mass of 5.0×10^5 kg, is blown straight backward with a speed of 2.0×10^6 m/s. A second piece, with mass 8.0×10^5 kg, continues forward at 1.0×10^6 m/s. What are the direction and speed of the third piece?

66. A proton is shot at 5.0×10^7 m/s toward a gold target. The nucleus of a gold atom, with a mass 197 times that of the proton, repels the proton and deflects it straight back with 90% of its initial speed. What is the recoil speed of the gold nucleus?

67. Figure P9.67 shows a collision between three balls of clay. The three hit simultaneously and stick together. What are the speed and direction of the resulting blob of clay?

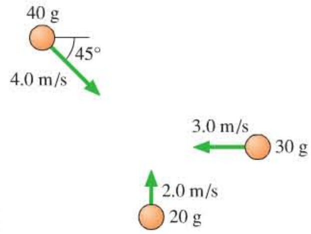


FIGURE P9.67

68. The carbon isotope ^{14}C is used for carbon dating of archeological artifacts. ^{14}C (mass 2.34×10^{-26} kg) decays by the process known as *beta decay* in which the nucleus emits an electron (the beta particle) and a subatomic particle called a neutrino. In one such decay, the electron and the neutrino are emitted at right angles to each other. The electron (mass 9.11×10^{-31} kg) has a speed of 5.00×10^7 m/s and the neutrino has a momentum of 8.00×10^{-24} kg · m/s. What is the recoil speed of the nucleus?

69. A 1.0-m-long massless rod is pivoted at one end and swings around in a circle on a frictionless table. A block with a hole through the center can slide in and out along the rod. Initially, a small piece of wax holds the block 30 cm from the pivot. The block is spun at 50 rpm, then the temperature of the rod is slowly increased. When the wax melts, the block slides out to the end of the rod. What is the final angular speed? Give your answer in rpm.

70. A 200 g puck revolves in a circle on a frictionless table at the end of a 50.0-cm-long string. The puck's angular momentum about the center of the circle is 3.00 kg · m^2/s. What is the tension in the string?

71. Figure P9.71 shows a 100 g puck revolving in a 20-cm-radius circle on a frictionless table. The string passes through a hole in the center of the table and is tied to two 200 g weights.
   a. What speed does the puck need to support the two weights?
   b. The lower weight is a light bag filled with sand. Suppose a pin pokes a hole in the bag and the sand slowly leaks out while the puck is revolving. What will be the puck's speed and the radius of its trajectory after all of the sand is gone?

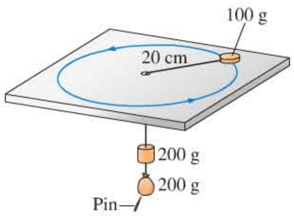


FIGURE P9.71

72. A 2.0 kg, 20-cm-diameter turntable rotates at 100 rpm on frictionless bearings. Two 500 g blocks fall from above, hit the turntable simultaneously at opposite ends of a diagonal, and stick. What is the turntable's angular speed, in rpm, just after this event?

73. Joey, from Example 9.10, stands at rest at the outer edge of the frictionless merry-go-round of Figure 9.28. The merry-go-round is also at rest. Joey then begins to run around the perimeter of the merry-go-round, finally reaching a constant speed, measured relative to the ground, of 5.0 m/s. What is the final angular speed of the merry-go-round?

Copyright © 2014 Pearson Education

74. ||| A 3.0-m-diameter merry-go-round with a mass of 250 kg is spinning at 20 rpm. John runs around the merry-go-round at 5.0 m/s, in the same direction that it is turning, and jumps onto the outer edge. John's mass is 30 kg. What is the merry-go-round's angular speed, in rpm, after John jumps on?

75. ||| Disk A, with a mass of 2.0 kg and a radius of 40 cm, rotates clockwise about a frictionless vertical axle at 30 rev/s. Disk B, also 2.0 kg but with a radius of 20 cm, rotates counterclockwise about that same axle, but at a greater height than disk A, at 30 rev/s. Disk B slides down the axle until it lands on top of disk A, after which they rotate together. After the collision, what is their common angular speed (in rev/s) and in which direction do they rotate?

Passage Problems

Hitting a Golf Ball

Consider a golf club hitting a golf ball. To a good approximation, we can model this as a collision between the rapidly moving head of the golf club and the stationary golf ball, ignoring the shaft of the club and the golfer.

A golf ball has a mass of 46 g. Suppose a 200 g club head is moving at a speed of 40 m/s just before striking the golf ball. After the collision, the golf ball's speed is 60 m/s.

76. | What is the momentum of the club + ball system right before the collision?
   A. 1.8 kg · m/s
   B. 8.0 kg · m/s
   C. 3220 kg · m/s
   D. 8000 kg · m/s

77. | Immediately after the collision, the momentum of the club + ball system will be
   A. Less than before the collision.
   B. The same as before the collision.
   C. More than before the collision.

78. | A manufacturer makes a golf ball that compresses more than a traditional golf ball when struck by a club. How will this affect the average force during the collision?
   A. The force will decrease.
   B. The force will not be affected.
   C. The force will increase.

79. | By approximately how much does the club head slow down as a result of hitting the ball?
   A. 4 m/s B. 6 m/s C. 14 m/s D. 26 m/s

Copyright © 2011 Pearson Education

STOP TO THINK ANSWERS

Stop to Think 9.1: F. The cart is initially moving in the negative x-direction, so $(p_x)_i = -20\ \text{kg}\cdot\text{m/s}$. After it bounces, $(p_x)_f = 10\ \text{kg}\cdot\text{m/s}$. Thus $\Delta p = (10\ \text{kg}\cdot\text{m/s}) - (-20\ \text{kg}\cdot\text{m/s}) = 30\ \text{kg}\cdot\text{m/s}$.

Stop to Think 9.2: B. The clay ball goes from $(v_x)_i = v$ to $(v_x)_f = 0$, so $J_{\text{clay}} = \Delta p_x = -mv$. The rubber ball rebounds, going from $(v_x)_i = v$ to $(v_x)_f = -v$ (same speed, opposite direction). Thus $J_{\text{rubber}} = \Delta p_x = -2mv$. The rubber ball has a greater momentum change, and this requires a greater impulse.

Stop to Think 9.3: Right end. The balls started at rest, so the total momentum of the system is zero. It's an isolated system, so the total momentum after the explosion is still zero. The 6 g ball has momentum $6v$. The 4 g ball, with velocity $-2v$, has momentum $-8v$. The combined momentum of these two balls is $-2v$. In order for P to be zero, the third ball must have a *positive* momentum $(+2v)$ and thus a positive velocity.

Stop to Think 9.4: B. The momentum of particle 1 is $(0.40\ \text{kg})(2.5\ \text{m/s}) = 1.0\ \text{kg}\cdot\text{m/s}$, while that of particle 2 is $(0.80\ \text{kg})(-1.5\ \text{m/s}) = -1.2\ \text{kg}\cdot\text{m/s}$. The total momentum is then $1.0\ \text{kg}\cdot\text{m/s} - 1.2\ \text{kg}\cdot\text{m/s} = -0.2\ \text{kg}\cdot\text{m/s}$. Because it's negative, the total momentum, and hence the final velocity of the particles, is directed to the left.

Stop to Think 9.5: B. Angular momentum $L = I\omega$ is conserved. Both boys have mass m and initially stand distance $R/2$ from the axis. Thus the initial moment of inertia is $I_i = I_{\text{disk}} + 2 \times m(R/2)^2 = I_{\text{disk}} + \frac{1}{2}mR^2$. The final moment of inertia is $I_f = I_{\text{disk}} + 0 + mR^2$, because the boy standing at the axis contributes nothing to the moment of inertia. Because $I_f > I_i$ we must have $\omega_f < \omega_i$.