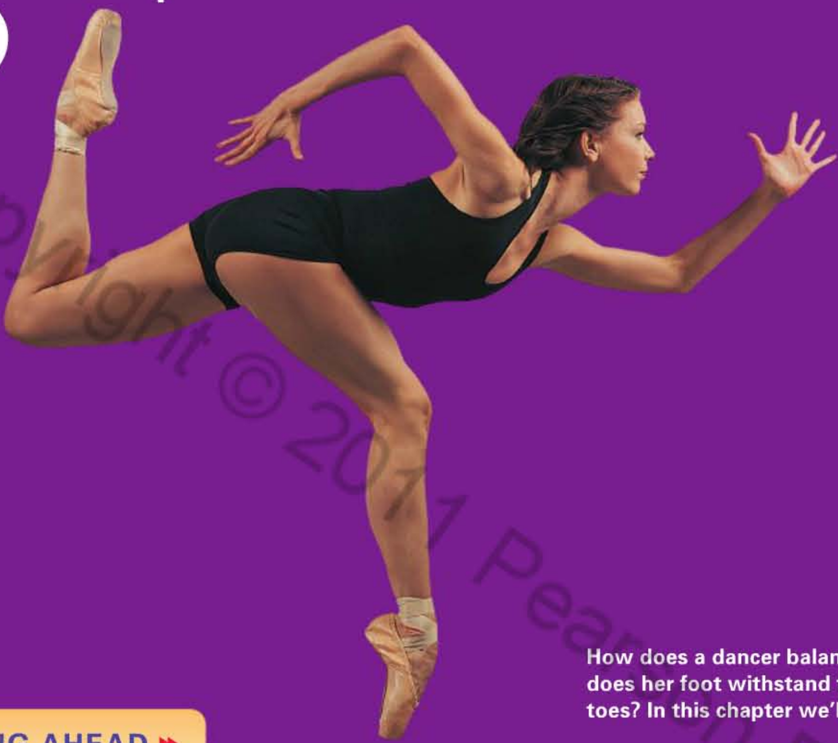
8 Equilibrium and Elasticity



How does a dancer balance so gracefully *en pointe*? And how does her foot withstand the great stresses concentrated on her toes? In this chapter we'll find answers to both these questions.

LOOKING AHEAD ➤➤

The goals of Chapter 8 are to learn about the static equilibrium of extended objects and to understand the basic properties of springs and elastic materials.

Torque and Static Equilibrium

In Chapter 5 we found that a particle can be in static equilibrium only if the net force acting on it is zero. For an extended object such as the dancer shown above, we'll learn that the net torque on the object must also be zero for it to be in equilibrium.

For this cyclist and his bike to stand motionless, the net force *and* net torque on them must be zero.

Looking Back ◀◀
5.1 Equilibrium

Springs

We'll study springs and similar elastic objects, and we'll find the important result that the spring force is proportional to the distance the spring is stretched or compressed from its natural length.

You can get a better workout by stretching the band farther, because the farther it stretches, the harder it is to pull.

Looking Back ◀◀
4.3 Spring forces

Stability and Balance

Why are some objects more *stable* than others; that is, why are some easy to tip over while others are more solidly planted? We'll learn that an object is more stable when its center of gravity is low and its contact points with the ground are widely separated.

For maximal stability, this football player keeps his center of gravity low and his stance wide.

The girls on this tree trunk have a hard time balancing because of their tall stance and narrow footprint.

Looking Back ◀◀
7.2–7.3 Torque, center of gravity, gravitational torque

Elastic Materials

All materials, even seemingly rigid ones like glass or steel, stretch slightly when you pull on them—they act like very stiff springs. We'll learn how to calculate the amount a solid object stretches or compresses as forces are applied to it.

Each of the steel cables suspending this bridge is 24" in diameter, yet the designers must carefully compensate for the slight stretch in each cable due to the enormous load of the bridge.

Biological Materials

The elastic properties of biological materials such as bone, tendon, and even spider silk play an important role in the world of living things. Bone, of course, is quite rigid, but did you know that spider silk is as strong as steel?

8.1 Torque and Static Equilibrium

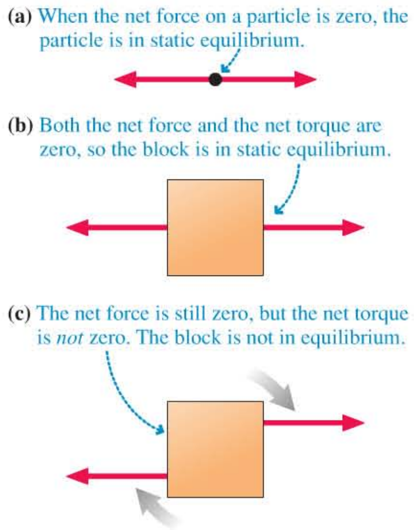
We have now spent several chapters studying motion and its causes. In many disciplines, it is just as important to understand the conditions under which objects do *not* move. In structural engineering, buildings and dams must be designed such that they remain motionless, even when huge forces act on them. In sports science, a correct stationary position is often the starting point for a successful athletic event. And joints in the body must sustain large forces when the body is supporting heavy loads, as in holding or carrying heavy objects.

Recall from Section 5.1 that an object at rest is in *static equilibrium.* As long as an object can be modeled as a *particle,* the condition necessary for static equilibrium is that the net force $\vec{F}_{net}$ on the particle is zero. Such a situation is shown in **FIGURE 8.1a**, where the two forces applied to the particle balance and the particle can remain at rest.

But in Chapter 7 we moved beyond the particle model to study extended objects that can rotate. Consider, for example, the block in **FIGURE 8.1b**. In this case the two forces act along the same line, the net force is zero, and the block is in equilibrium. But what about the block in **FIGURE 8.1c**? The net force is still zero, but this time the block begins to rotate because the two forces exert a net *torque*. For an extended object, $\vec{F}_{net} = \vec{0}$ is not by itself enough to ensure static equilibrium. There is a second condition for static equilibrium of an extended object: The net torque τ_{net} on the object must also be zero.

If we write the net force in component form, the conditions for static equilibrium of an extended object are

FIGURE 8.1 A block with no net force acting on it may still be out of equilibrium.



$$\left.\begin{aligned}\sum F_x &= 0\\ \sum F_y &= 0\end{aligned}\right\}\ \text{No net force}\qquad\qquad (8.1)$$
$$\sum \tau = 0 \ \}\ \text{No net torque}$$

Conditions for static equilibrium of an extended object

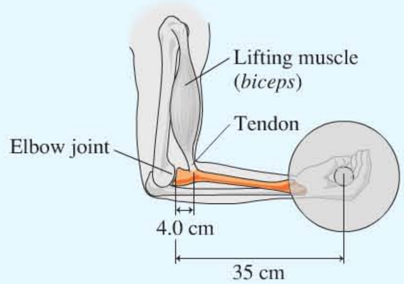
If motion is possible in the z-direction, we'd also require that $\sum F_z = 0$. In this chapter, however, we'll consider only motion restricted to the xy-plane.

EXAMPLE 8.1 Finding the force from the biceps tendon

Weightlifting can exert extremely large forces on the body's joints and tendons. In the *strict curl* event, a standing athlete lifts a barbell by moving only his forearms, which pivot at the elbow. The record weight lifted in the strict curl is over 200 pounds (about 900 N). **FIGURE 8.2** shows the arm bones and the main lifting muscle when the forearm is horizontal. The distance from the tendon to the elbow joint is 4.0 cm, and from the barbell to the elbow 35 cm.

a. What is the tension in the tendon connecting the biceps muscle to the bone while a 900 N barbell is held stationary in this position?
b. What is the force exerted by the elbow on the forearm bones?

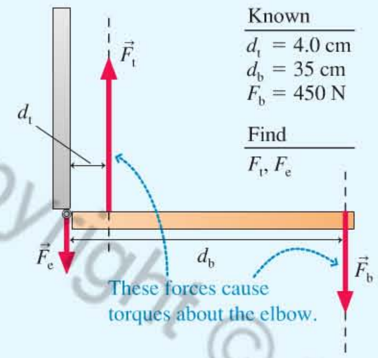
FIGURE 8.2 An arm holding a barbell.



PREPARE **FIGURE 8.3** shows a simplified model of the arm and the forces acting on the forearm. $\vec{F}_t$ is the tension force due to the muscle, $\vec{F}_b$ is the downward force of the barbell, and $\vec{F}_e$ is the force of the elbow joint on the forearm. As a simplification, we've neglected the weight of the arm itself because it is so much less than the weight of the barbell. Because $\vec{F}_t$ and $\vec{F}_b$ have no x-component, neither can $\vec{F}_e$. If it did, the net force in the x-direction would not be zero, and the forearm could not be in equilibrium. Because each arm supports half the weight of the barbell, the magnitude of the barbell force is $F_b = 450$ N.

Continued

FIGURE 8.3 Visual overview of holding a barbell.



SOLVE a. For the forearm to be in static equilibrium, the net force and net torque on it must both be zero. Setting the net force to zero gives

$$\sum F_y = F_t - F_e - F_b = 0$$

We don't know either of the forces F_t and F_e, nor does the force equation give us enough information to find them. But the fact that in static equilibrium the torque also must be zero gives us the extra information that we need.

Recall that the torque must be calculated about a particular point. Here, a natural choice is the elbow joint, about which the forearm can pivot. Given this pivot, we can calculate the torque due to each of the three forces in terms of their magnitudes F and moment arms $r_\perp$ as $\tau = r_\perp F$. The moment arm is the perpendicular distance between the pivot and the "line of action" along which the force is applied. Figure 8.3 shows that the moment arms for $\vec{F}_t$ and $\vec{F}_b$ are the distances d_t and d_b, respectively, measured along the beam representing the forearm. The moment arm for $\vec{F}_e$ is zero, because this force acts directly at the pivot. Thus we have

$$\tau_{net} = F_e \times 0 + F_t d_t - F_b d_b = 0$$

The tension in the tendon tries to rotate the arm counterclockwise, so it produces a positive torque; the torque due to the barbell, which tries to rotate the arm in a clockwise direction, is negative. We can solve the torque equation for F_t:

$$F_t = F_b \frac{d_b}{d_t} = (450 \text{ N}) \frac{35 \text{ cm}}{4.0 \text{ cm}} = 3900 \text{ N}$$

b. We now need to make use of the force equation:

$$F_e = F_t - F_b = 3900 \text{ N} - 450 \text{ N} = 3450 \text{ N}$$

ASSESS This large value for F_t makes sense: The short distance d_t from the tendon to the elbow joint means that the force supplied by the biceps has to be very large to counter the torque generated by a force applied at the opposite end of the forearm.

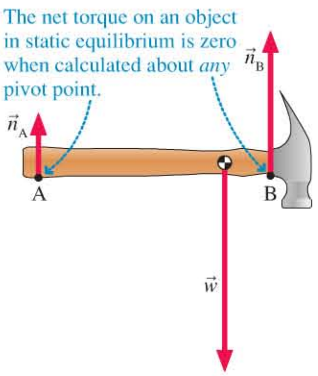
STOP TO THINK 8.1 Which of these objects is in static equilibrium?

Choosing the Pivot Point

In Example 8.1, we calculated the net torque using the elbow joint as the axis of rotation or pivot point. But we learned in Chapter 7 that the torque depends on which point is chosen as the pivot point. Was there something special about our choice of the elbow joint?

Consider the hammer shown in FIGURE 8.4, supported on a pegboard by two pegs A and B. Because the hammer is in static equilibrium, the net torque around the pivot at peg A must be zero: The clockwise torque due to the weight $\vec{w}$ is exactly balanced by the counterclockwise torque due to the force $\vec{n}_B$ of peg B. (Recall that the torque due to $\vec{n}_A$ is zero, because here $\vec{n}_A$ acts at the pivot A.) But if instead we take B as the pivot, the net torque is still zero. The counterclockwise torque due to $\vec{w}$ (with a large force but small moment arm) balances the clockwise torque due to $\vec{n}_A$ (with a small force but large moment arm). Indeed, **for an object in static equilibrium, the net torque about *every* point must be zero.** This means you can pick *any* point you wish as a pivot point for calculating the torque.

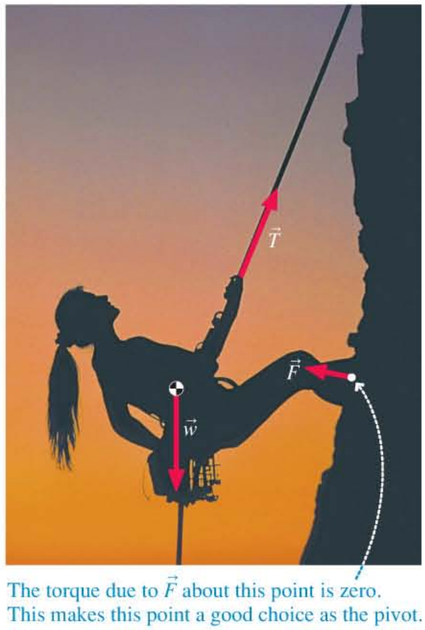
FIGURE 8.4 A hammer resting on two pegs.



Although any choice of a pivot point will work, some choices are better because they simplify the calculations. Often, there is a "natural" axis of rotation in the problem, an axis about which rotation *would* occur if the object were not in static equilibrium. Example 8.1 is of this type, with the elbow joint as a natural axis of rotation.

If no point naturally suggests itself as an axis, look for a point on the object at which several forces act, or at which a force acts whose magnitude you don't know. Such a point is a good choice because any force acting at that point does not contribute to the torque. For instance, the woman in **FIGURE 8.5** is in equilibrium as she rests on the rock wall. A good choice of pivot point would be where her foot contacts the wall because this choice eliminates the torque due to the force $\vec{F}$ of the wall on her foot. But don't agonize over the choice of a pivot point! You can still solve the problem no matter which point you choose.

FIGURE 8.5 Choosing the pivot for a woman rappelling down a rock wall.



PROBLEM-SOLVING STRATEGY 8.1 **Static equilibrium problems** MP™

PREPARE Model the object as a simple shape. Draw a visual overview that shows all forces and distances. List known information.

- Pick an axis or pivot about which the torques will be calculated.
- Determine the torque about this pivot point due to each force acting on the object. The torques due to any forces acting *at* the pivot are zero.
- Determine the sign of each torque about this pivot point.

SOLVE The mathematical steps are based on the fact that an object in static equilibrium has no net force and no net torque:

$$\vec{F}_{\text{net}} = \vec{0} \qquad \text{and} \qquad \tau_{\text{net}} = 0$$

- Write equations for $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau = 0$.
- Solve the resulting equations.

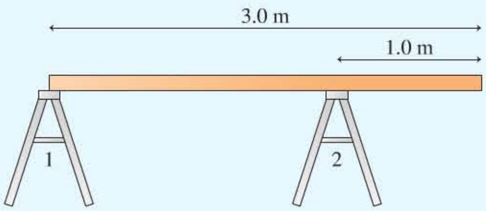
ASSESS Check that your result is reasonable and answers the question.

ActivPhysics 7.2, 7.3, 7.4, 7.5

EXAMPLE 8.2 **Forces on a board on sawhorses**

A board weighing 100 N sits across two sawhorses, as shown in **FIGURE 8.6**. What are the magnitudes of the normal forces of the sawhorses acting on the board?

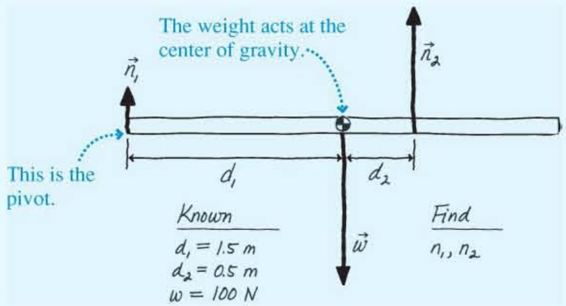
FIGURE 8.6 A board sitting on two sawhorses.



PREPARE The board and the forces acting on it are shown in **FIGURE 8.7**. $\vec{n}_1$ and $\vec{n}_2$ are the normal forces on the board due to the sawhorses, and $\vec{w}$ is the weight of the board acting at the center of gravity. The distance d_1 to the center of the board is half the board's length, or 1.5 m. Then d_2 is $d_1 - 1.0$ m, or 0.5 m.

As discussed above, a good choice for the pivot is a point at which an unknown force acts, because that force contributes nothing to the torque. Either the point where $\vec{n}_1$ acts or the point where $\vec{n}_2$ acts will work; let's choose the left end of the board, where $\vec{n}_1$ acts, for this example. With this choice of pivot point, the moment arm for $\vec{w}$ is $d_1 = 1.5$ m. Because $\vec{w}$ tends to rotate the board clockwise, its torque is negative. The moment arm for $\vec{n}_2$ is the distance $d_1 + d_2 = 2.0$ m, and its torque is positive.

FIGURE 8.7 Visual overview of a board on two sawhorses.



SOLVE The board is in static equilibrium, so the net force $\vec{F}_{\text{net}}$ and the net torque τ_{net} must both be zero. The forces have only y-components, so the force equation is

$$\sum F_y = n_1 - w + n_2 = 0$$

The torque equation, computed around the left end of the board, is

$$\tau_{\text{net}} = -d_1 w + (d_1 + d_2) n_2 = 0$$

Continued

We now have two simultaneous equations with the two unknowns n_1 and n_2. To solve these, let's solve for n_2 in the torque equation and then substitute that result into the force equation. From the torque equation,

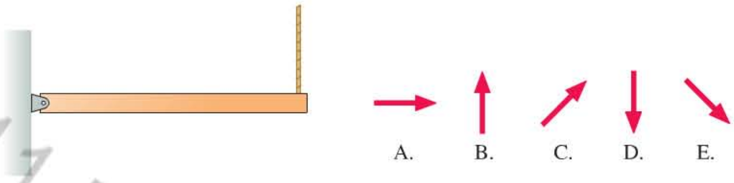
$$n_2 = \frac{d_1 w}{d_1 + d_2} = \frac{(1.5\text{ m})(100\text{ N})}{2.0\text{ m}} = 75\text{ N}$$

The force equation is then $n_1 - 100\text{ N} + 75\text{ N} = 0$, which we can solve for n_1:

$$n_1 = w - n_2 = 100\text{ N} - 75\text{ N} = 25\text{ N}$$

ASSESS It seems reasonable that $n_2 > n_1$ because more of the board sits over the right sawhorse.

STOP TO THINK 8.2 A beam with a pivot on its left end is suspended from a rope. In which direction is the force of the pivot on the beam?



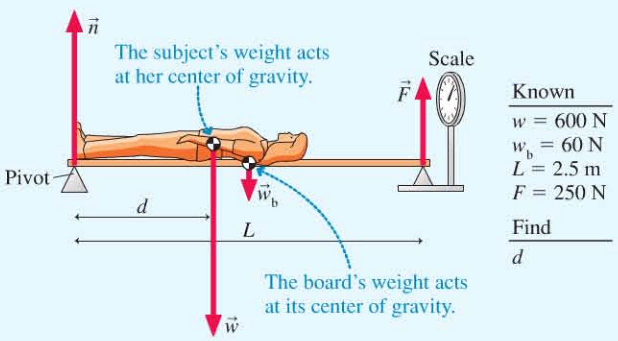
An interesting application of static equilibrium is to find the center of gravity of the human body. Because the human body is highly flexible, the position of the center of gravity is quite variable and depends on just how the body is posed. The horizontal position of the body's center of gravity can be located accurately from simple measurements with a *reaction board* and a scale. The following example shows how this is done.

EXAMPLE 8.3 Finding the center of gravity of the human body

A woman weighing 600 N lies on a 2.5-m-long, 60 N reaction board with her feet over the pivot. The scale on the right reads 250 N. What is the distance d from the woman's feet to her center of gravity?

PREPARE The forces and distances in the problem are shown in FIGURE 8.8. We'll consider the board and woman as a single object. We've assumed that the board is uniform, so its center of gravity is at its midpoint. To eliminate the unknown magnitude of $\vec{n}$ from the torque equation, we'll choose the pivot to be the left end of the board. The torque due to $\vec{F}$ is positive, and those due to $\vec{w}$ and $\vec{w}_b$ are negative.

FIGURE 8.8 Visual overview of the reaction board and woman.



SOLVE Because the board and woman are in static equilibrium, the net force and net torque on them must be zero. The force equation reads

$$\sum F_y = n - w_b - w + F = 0$$

and the torque equation gives

$$\sum \tau = -\frac{L}{2} w_b - dw + LF = 0$$

In this case, the force equation isn't needed because we can solve the torque equation for d:

$$d = \frac{LF - \frac{1}{2}Lw_b}{w} = \frac{(2.5\text{ m})(250\text{ N}) - \frac{1}{2}(2.5\text{ m})(60\text{ N})}{600\text{ N}}$$
$$= 0.92\text{ m}$$

ASSESS If the woman is 5′ 6″ (1.68 m) tall, her center of gravity is (0.92 m)/(1.68 m) = 55% of her height, or a little more than halfway up her body. This seems reasonable.

EXAMPLE 8.4 Will the ladder slip?

A 3.0-m-long ladder leans against a frictionless wall at an angle of 60° with respect to the floor. What is the minimum value of μ_s, the coefficient of static friction with the ground, that will prevent the ladder from slipping?

PREPARE The ladder is a rigid rod of length L. To not slip, both the net force and net torque on the ladder must be zero. FIGURE 8.9 shows the ladder and the forces acting on it. The bottom corner of the ladder is a good choice of a pivot point because two of the

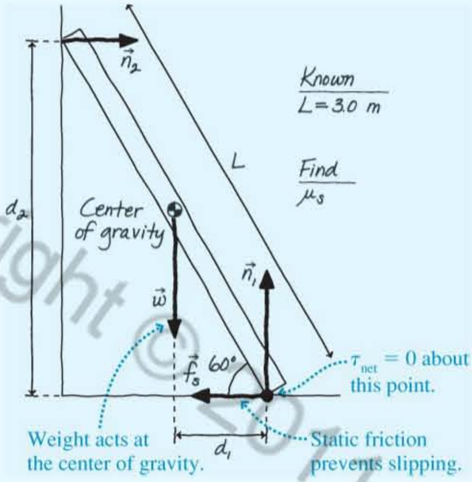


FIGURE 8.9 Visual overview of a ladder in static equilibrium.

forces pass through this point and thus produce no torque about it. With this choice, the weight of the ladder, acting at the center of gravity, exerts torque $d_1 w$ and the force of the wall exerts torque $-d_2 n_2$. The signs are based on the observation that $\vec{w}$ would cause the ladder to rotate counterclockwise, while $\vec{n}_2$ would cause it to rotate clockwise.

SOLVE The x- and y-components of $\vec{F}_{net} = \vec{0}$ are

$$\sum F_x = n_2 - f_s = 0$$
$$\sum F_y = n_1 - w = n_1 - Mg = 0$$

The torque about the bottom corner is

$$\tau_{net} = d_1 w - d_2 n_2 = \frac{1}{2}(L\cos 60°)Mg - (L\sin 60°)n_2 = 0$$

Altogether, we have three equations with the three unknowns n_1, n_2, and f_s. If we solve the third equation for n_2,

$$n_2 = \frac{\frac{1}{2}(L\cos 60°)Mg}{L\sin 60°} = \frac{Mg}{2\tan 60°}$$

we can then substitute this into the first equation to find

$$f_s = \frac{Mg}{2\tan 60°}$$

Our model of static friction is $f_s \le f_{s\max} = \mu_s n_1$. We can find n_1 from the second equation: $n_1 = Mg$. From this, the model of friction tells us that

$$f_s \le \mu_s Mg$$

Comparing these two expressions for f_s, we see that μ_s must obey

$$\mu_s \ge \frac{1}{2\tan 60°} = 0.29$$

Thus the minimum value of the coefficient of static friction is 0.29.

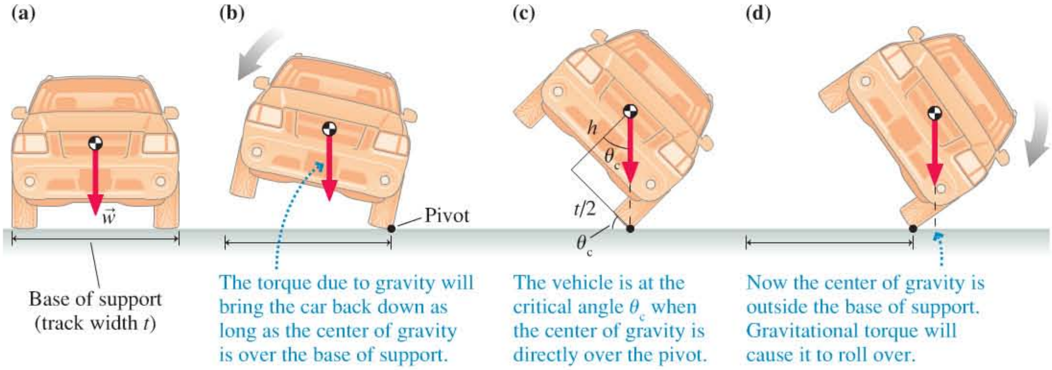
ASSESS You know from experience that you can lean a ladder or other object against a wall if the ground is "rough," but it slips if the surface is too smooth. 0.29 is a "medium" value for the coefficient of static friction, which is reasonable.

8.2 Stability and Balance

If you tilt a box up on one edge by a small amount and let go, it falls back down. If you tilt it too much, it falls over. And if you tilt it "just right," you can get the box to balance on its edge. What determines these three possible outcomes?

FIGURE 8.10 illustrates the idea with a car, but the results are general and apply in many situations. An extended object, whether it's a car, a box, or a person, has a *base of support* on which it rests when in static equilibrium. If you tilt the object, one edge of the base of support becomes a pivot point. As long as the object's center of gravity remains over the base of support, torque due to gravity will rotate the object back toward its stable equilibrium position; we say that the object is **stable.** This is the

FIGURE 8.10 A car—or any object—will fall over when tilted too far.



situation in Figure 8.10b. But if the center of gravity gets outside the base of support, as in Figure 8.10d, the gravitational torque causes a rotation in the opposite direction. Now the car rolls over; it is **unstable.**

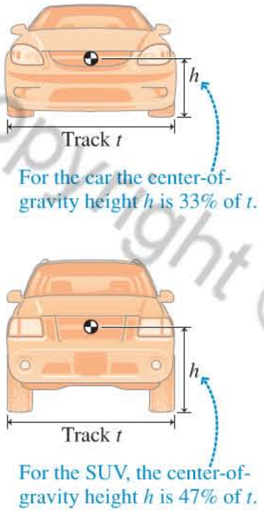
A *critical angle* θ_c is reached when the center of gravity is directly over the pivot point. This is the point of balance, with no net torque. For vehicles, the distance between the tires—the base of support—is called the *track width t*. If the height of the center of gravity is h, you can see from Figure 8.10c that the critical angle is

$$\theta_c = \tan^{-1}\left(\frac{t/2}{h}\right) = \tan^{-1}\left(\frac{t}{2h}\right) \tag{8.2}$$

If an accident (or taking a corner too fast) causes a vehicle to pivot up onto two wheels, it will roll back to an upright position as long as $\theta < \theta_c$ but it will roll over if $\theta > \theta_c$. Notice that it's the height-to-width ratio that's important, not the absolute height of the center of gravity.

FIGURE 8.11 compares a passenger car and a sport utility vehicle (SUV). For the passenger car, with $h \approx 0.33t$, the critical angle is $\theta_c \approx 57°$. But for the SUV, with its higher center of gravity ($h \approx 0.47t$), the critical angle is only $\theta_c \approx 47°$. Loading an SUV with cargo further raises the center of gravity, especially if the roof rack is used, thus reducing θ_c even more. Various automobile safety groups have determined that a vehicle with $\theta_c > 50°$ is unlikely to roll over in an accident. A rollover becomes increasingly likely when θ_c is less than 50°. The same argument that leads to Equation 8.2 for tilted vehicles can be made for any object, leading to the general rule that **a wider base of support and/or a lower center of gravity improve stability.**

FIGURE 8.11 Compared to a passenger car, an SUV has a high center of gravity relative to its width.



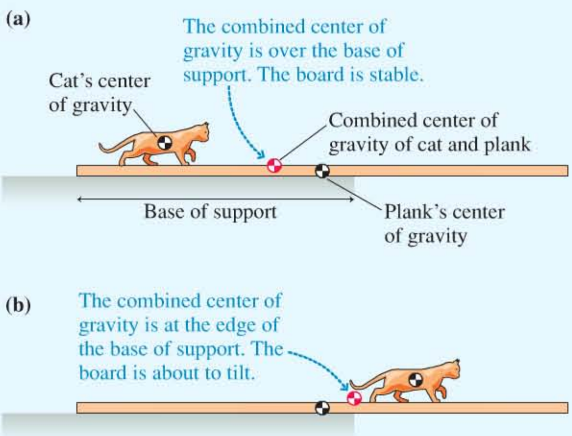
CONCEPTUAL EXAMPLE 8.5 How far to walk the plank?

A cat walks along a plank that extends out from a table. If the cat walks too far out on the plank, the plank will begin to tilt. What determines when this happens?

REASON An object is stable if its center of gravity lies over its base of support, and unstable otherwise. Let's take the cat and the plank to be one combined object whose center of gravity lies along a line between the cat's center of gravity and that of the plank.

In **FIGURE 8.12a**, when the cat is near the left end of the plank, the combined center of gravity is over the base of support and the plank is stable. As the cat moves to the right, he reaches a point where the combined center of gravity is directly over the edge of the table, as shown in **FIGURE 8.12b**. If the cat takes one more step, the cat and plank will become unstable and the plank will begin to tilt.

FIGURE 8.12 Changing stability as a cat walks on a plank.



ASSESS Because the plank's center of gravity must be to the left of the edge for it to be stable by itself, the cat can actually walk a short distance out onto the unsupported part of the plank before it starts to tilt. The heavier the plank is, the farther the cat can walk.

TRY IT YOURSELF

Balancing a soda can Try to balance a soda can—full or empty—on the narrow bevel at the bottom. It can't be done because, either full or empty, the center of gravity is near the center of the can. If the can is tilted enough to sit on the bevel, the center of gravity lies far outside this small base of support. But if you put about 2 ounces (60 ml) of water in an empty can, the center of gravity will be right over the bevel and the can will balance.

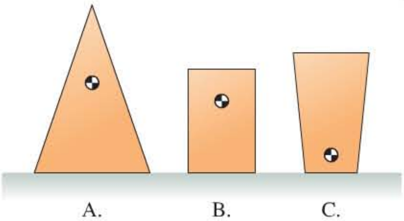
Stability and Balance of the Human Body

The human body is remarkable for its ability to constantly adjust its stance to remain stable on just two points of support. In walking, running, or even in the simple act of rising from a chair, the position of the body's center of gravity is constantly changing. To maintain stability, we unconsciously adjust the positions of our arms and legs to keep our center of gravity over our base of support.

A simple example of how the body naturally realigns its center of gravity is found in the act of standing up on tiptoes. **FIGURE 8.13a** shows the body in its normal standing position. Notice that the center of gravity is well centered over the base of support (the feet), ensuring stability. If the subject were now to stand on tiptoes *without* otherwise adjusting the body position, her center of gravity would fall behind the base of support, which is now the balls of the feet, and she would fall backward. To prevent this, as shown in **FIGURE 8.13b**, the body naturally leans forward, regaining stability by moving the center of gravity over the balls of the feet.

FIGURE 8.13 Standing on tiptoes.

STOP TO THINK 8.3 Rank in order, from least stable to most stable, the three objects shown in the figure. The positions of their centers of gravity are marked. (For the centers of gravity to be positioned like this, the objects must have a nonuniform composition.)



TRY IT YOURSELF

Impossible balance Stand facing a wall with your toes touching the base of the wall. Now rise onto your tiptoes. You will not be able to do so without falling backward. As we see from Figure 8.13b, your body has to lean forward to stand on tiptoes. With the wall in your way, you cannot lean enough to maintain your balance, and you will begin to topple backward.

8.3 Springs and Hooke's Law

We have assumed that objects in equilibrium maintain their shape as forces and torques are applied to them. In reality this is an oversimplification. Every solid object stretches, compresses, or deforms when a force acts upon it. This change is easy to see when you press on a green twig on a tree, but even the largest branch on the tree will bend slightly under your weight.

If you stretch a rubber band, there is a force that tries to pull the rubber band back to its equilibrium, or unstretched, length. A force that restores a system to an equilibrium position is called a **restoring force.** Systems that exhibit such restoring forces are called **elastic.** The most basic examples of **elasticity** are things like springs and rubber bands. If you stretch a spring, a tension-like force pulls back. Similarly, a compressed spring tries to re-expand to its equilibrium length. Elasticity and restoring forces are properties of much stiffer systems as well. The steel beams of a bridge bend slightly as you drive your car over it, but they are restored to equilibrium after your car passes by. Your leg bones flex a bit during each step you take. Nearly everything that stretches, compresses, bends, or twists exhibits a restoring force and can be called elastic.

The behavior of a simple spring illustrates the basic ideas of elasticity. When no forces act on a spring to compress or extend it, it will relax to its **equilibrium length.** If we now stretch the spring by a displacement Δx, how hard does it pull back? **FIGURE 8.14** shows what happens: The farther we stretch the spring, the harder the restoring force of the spring pulls back.

FIGURE 8.14 The spring force depends on how far the spring is stretched.

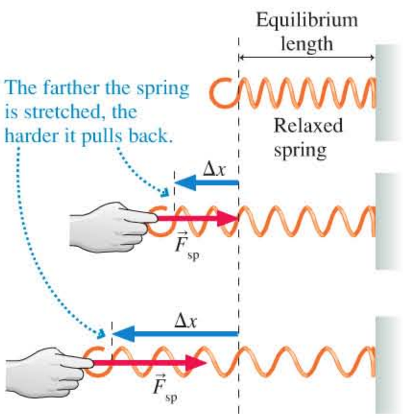


FIGURE 8.15 Measured data for the restoring force of a real spring.

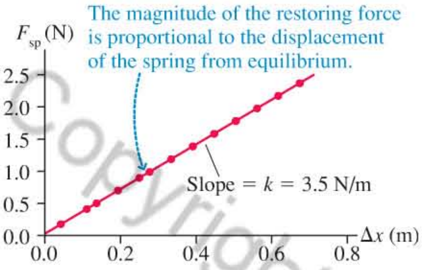
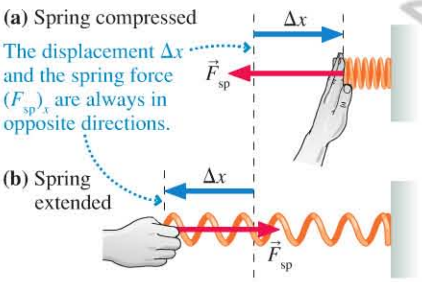


FIGURE 8.16 The spring force is always directed opposite the displacement.



Elasticity in action A golf ball compresses quite a bit when struck. The restoring force that pushes the ball back into its original shape helps launch the ball off the face of the club, making for a longer drive.

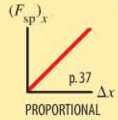
In FIGURE 8.15, data for the magnitude of the restoring force of a real spring show that **the force of the spring is *proportional* to the displacement of the end of the spring.** That is, compressing or stretching the spring twice as far results in a restoring force that is twice as large. This is a *linear relationship,* and the slope k of the line is the proportionality constant:

$$F_{sp} = k\,\Delta x \tag{8.3}$$

A second important fact about spring forces is illustrated in FIGURE 8.16. If the spring is compressed, as in Figure 8.16a, Δx is positive and, because $\vec{F}_{sp}$ points to the left, its component $(F_{sp})_x$ is negative. If, however, the spring is stretched, as in Figure 8.16b, Δx is negative and, because $\vec{F}_{sp}$ points to the right, its component $(F_{sp})_x$ is positive. In general, **the spring force always points in the opposite direction to the displacement from equilibrium.** We can express this fact, along with what we've learned about the magnitude of the spring force, by rewriting Equation 8.3 in terms of the *component* of the spring force:

$$(F_{sp})_x = -k\,\Delta x \tag{8.4}$$

Hooke's law for the force due to a spring



The minus sign in Equation 8.4 reflects the fact that $(F_{sp})_x$ and Δx are always of opposite sign. (For motion in the vertical (y) direction, Hooke's law is $(F_{sp})_y = -k\,\Delta y$.)

The proportionality constant k is called the **spring constant.** The units of the spring constant are N/m. The spring constant k is a property that characterizes a spring, just as mass m characterizes a particle. If k is large, it takes a large pull to cause a significant stretch, and we call the spring a "stiff" spring. If k is small, we can stretch the spring with very little force, and we call it a "soft" spring. Every spring has its own, unique value of k. The spring constant for the spring in Figure 8.15 can be determined from the slope of the straight line to be $k = 3.5$ N/m.

Equation 8.4 for the restoring force of a spring was first suggested by Robert Hooke, a contemporary (and sometimes bitter rival) of Newton. Hooke's law is not a true "law of nature," in the sense that Newton's laws are, but is actually just a *model* of a restoring force. It works extremely well for some springs, as in Figure 8.15, but less well for others. Hooke's law will fail for any spring if it is compressed or stretched too far.

NOTE ► Just as we used massless strings, we will adopt the idealization of a *massless spring*. Though not a perfect description, it is a good approximation if the mass attached to a spring is much greater than the mass of the spring itself. ◄

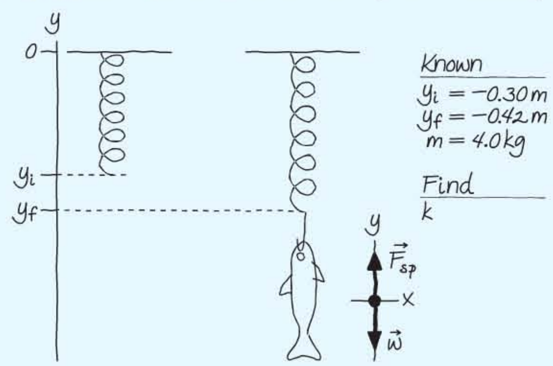
EXAMPLE 8.6 Weighing a fish

A scale used to weigh fish consists of a spring connected to the ceiling. The spring's equilibrium length is 30 cm. When a 4.0 kg fish is suspended from the end of the spring, it stretches to a length of 42 cm.

a. What is the spring constant k for this spring?
b. If an 8.0 kg fish is suspended from the spring, what will be the length of the spring?

PREPARE The visual overview in FIGURE 8.17 shows the details for the first part of the problem. The fish hangs in static equilibrium, so the net force in the y-direction and the net torque must be zero.

FIGURE 8.17 Visual overview of a mass suspended from a spring.



SOLVE a. Because the fish is in static equilibrium, we have

$$\sum F_y = (F_{sp})_y + w_y = -k\,\Delta y - mg = 0$$

so that $k = -mg/\Delta y$. (The net torque is zero because the fish's center of gravity comes to rest directly under the pivot point of the hook.) From Figure 8.17, the displacement of the spring from equilibrium is $\Delta y = y_f - y_i = (-0.42\text{ m}) - (-0.30\text{ m}) = -0.12\text{ m}$. This displacement is *negative* because the fish moves in the $-y$-direction. We can now solve for the spring constant:

$$k = -\frac{mg}{\Delta y} = -\frac{(4.0\text{ kg})(9.8\text{ m/s}^2)}{-0.12\text{ m}} = 330\text{ N/m}$$

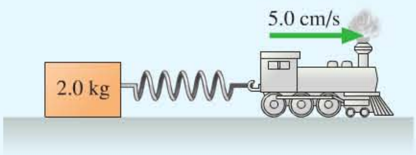
b. The restoring force is proportional to the displacement of the spring from its equilibrium length. If we double the mass (and thus the weight) of the fish, the displacement of the end of the spring will double as well, to $\Delta y = -0.24\text{ m}$. Thus the spring will be 0.24 m longer, so its new length is $0.30\text{ m} + 0.24\text{ m} = 0.54\text{ m}$.

ASSESS A spring constant of 330 N/m means that when the spring is stretched by 1.0 m it will exert a force of 330 N (about 75 lb). This seems reasonable for a spring used to weigh objects of 10 or 20 lb.

EXAMPLE 8.7 When does the block slip?

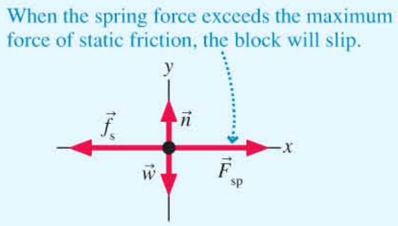
FIGURE 8.18 shows a spring attached to a 2.0 kg block. The other end of the spring is pulled by a motorized toy train that moves forward at 5.0 cm/s. The spring constant is 50 N/m, and the coefficient of static friction between the block and the surface is 0.60. The spring is at its equilibrium length at $t = 0$ s when the train starts to move. When does the block slip?

FIGURE 8.18 A toy train stretches the spring until the block slips.



PREPARE We model the block as a particle and the spring as a massless spring. **FIGURE 8.19** is a free-body diagram for the block. We convert the speed of the train into m/s: $v = 0.050$ m/s.

FIGURE 8.19 Free-body diagram for the block.



SOLVE Recall that the tension in a massless string pulls equally at *both* ends of the string. The same is true for the spring force: It pulls (or pushes) equally at *both* ends. Imagine holding a rubber band with your left hand and stretching it with your right hand. Your left hand feels the pulling force, even though it was the right end of the rubber band that moved.

This is the key to solving the problem. As the right end of the spring moves, stretching the spring, the spring pulls backward on the train *and* forward on the block with equal strength. The train is moving to the right, and so the spring force pulls to the left on the train—as we would expect. But the block is at the other end of the spring; the spring force pulls to the right on the block, as shown in Figure 8.19. As the spring stretches, the static friction force on the block increases in magnitude to keep the block at rest. The block is in static equilibrium, so

$$\sum F_x = (F_{sp})_x + (f_s)_x = F_{sp} - f_s = 0$$

where F_{sp} is the magnitude of the spring force. This magnitude is $F_{sp} = k\,\Delta x$, where $\Delta x = vt$ is the distance the train has moved. Thus

$$f_s = F_{sp} = k\,\Delta x$$

The block slips when the static friction force reaches its maximum value $f_{s\max} = \mu_s n = \mu_s mg$. This occurs when the train has moved a distance

$$\Delta x = \frac{f_{s\max}}{k} = \frac{\mu_s mg}{k} = \frac{(0.60)(2.0\text{ kg})(9.8\text{ m/s}^2)}{50\text{ N/m}} = 0.235\text{ m}$$

The time at which the block slips is

$$t = \frac{\Delta x}{v} = \frac{0.235\text{ m}}{0.050\text{ m/s}} = 4.7\text{ s}$$

ASSESS The result of about 5 s seems reasonable for a slowly moving toy train to stretch the spring enough for the block to slip.

STOP TO THINK 8.4 A 1.0 kg weight is suspended from a spring, stretching it by 5.0 cm. How much does the spring stretch if the 1.0 kg weight is replaced by a 3.0 kg weight?

A. 5.0 cm B. 10.0 cm C. 15.0 cm D. 20.0 cm

8.4 Stretching and Compressing Materials

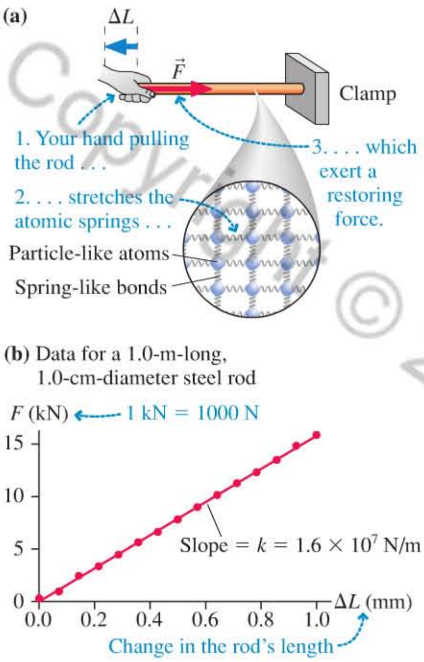


FIGURE 8.20 Stretching a steel rod.

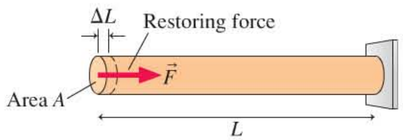


FIGURE 8.21 A rod stretched by length ΔL.

TABLE 8.1 Young's modulus for rigid materials

Material	Young's modulus (10^{10} N/m^2)
Cast iron	20
Steel	20
Silicon	13
Copper	11
Aluminum	7
Glass	7
Concrete	3
Wood (Douglas Fir)	1

In Chapter 4 we noted that we could model most solid materials as being made of particle-like atoms connected by spring-like bonds. We can model a steel rod this way, as illustrated in **FIGURE 8.20a**. The spring-like bonds between the atoms in steel are quite stiff, but they can be stretched or compressed, meaning that even a steel rod is elastic. If you pull on the end of a steel rod, as in Figure 8.20a, you will slightly stretch the bonds between the particles that make it up, and the rod itself will stretch. The stretched bonds pull back on your hand with a restoring force that causes the rod to return to its original length when released. In this sense, the entire rod acts like a very stiff spring. As is the case for a spring, a restoring force is also produced by compressing the rod.

In **FIGURE 8.20b**, real data for a 1.0-m-long, 1.0-cm-diameter steel rod show that, just as for a spring, the restoring force is proportional to the change in length. However, the *scale* of the stretch of the rod and the restoring force is much different from that for a spring. It would take a force of 16,000 N to stretch the rod by only 1 mm, corresponding to a spring constant of 1.6×10^7 N/m! Steel is elastic, but under normal forces, it experiences only very small changes in dimension. Materials of this sort are called **rigid.**

The behavior of other materials, such as the rubber in a rubber band, can be quite different. A rubber band can be stretched quite far—several times its equilibrium length—with a very small force, and then snaps back to its original shape when released. Materials that show large deformations with small forces are called **pliant.**

A rod's spring constant depends on several factors, as shown in **FIGURE 8.21**. First, we expect that a thick rod, with a large cross-section area A, will be more difficult to stretch than a thinner rod. Second, a rod with a long length L will be easier to stretch by a given amount than a short rod (think of trying to stretch a rope by 1 cm—this would be easy to do for a 10-m-long rope, but it would be pretty hard for a piece of rope only 10 cm long). Finally, the stiffness of the rod will depend on the material that it's made of. Experiments bear out these observations, and it is found that the spring constant of the rod can be written as

$$k = \frac{YA}{L} \tag{8.5}$$

where the constant Y is called **Young's modulus.** Young's modulus is a property of the *material* from which the rod is made—it does not depend on the object's shape or size. All rods made from steel have the same Young's modulus, regardless of their length or area, while aluminum rods have a different Young's modulus.

From Equation 8.3, the magnitude of the restoring force for a spring is related to the change in its length as $F_{sp} = k\,\Delta x$. Writing the change in the length of a rod as ΔL, as shown in Figure 8.21, we can use Equation 8.5 to write the restoring force F of a rod as

$$F = \frac{YA}{L}\,\Delta L \tag{8.6}$$

Equation 8.6 applies both to elongation (stretching) and to compression.

It's useful to rearrange Equation 8.6 in terms of two new ratios, the *stress* and the *strain*:

The ratio of force to cross-section area is called **stress**.

$$\frac{F}{A} = Y\left(\frac{\Delta L}{L}\right) \tag{8.7}$$

The ratio of the change in length to the original length is called **strain**.

The unit of stress is N/m^2. If the stress is due to stretching, we call it a **tensile stress.** The strain is the fractional change in the rod's length. If the rod's length changes by 1%, the strain is 0.01. Because strain is dimensionless, Young's modulus Y has the same units as stress. Table 8.1 gives values of Young's modulus for several rigid materials. Large values of Y characterize materials that are stiff. "Softer" materials have smaller values of Y. Because the values of Young's modulus for materials such as steel or aluminum are very large, it takes a significant stress to produce even a small strain.

EXAMPLE 8.8 Finding the stretch of a wire

A *Foucault pendulum* in a physics department (used to prove that the earth rotates) consists of a 120 kg steel ball that swings at the end of a 6.0-m-long steel cable. The cable has a diameter of 2.5 mm. When the ball was first hung from the cable, by how much did the cable stretch?

PREPARE The amount by which the cable stretches depends on the elasticity of the steel cable. Young's modulus for steel is given in Table 8.1 as $Y = 20 \times 10^{10}\ \text{N/m}^2$.

SOLVE Equation 8.7 relates the stretch of the cable ΔL to the restoring force F and to the properties of the cable. Rearranging terms, we find that the cable stretches by

$$\Delta L = \frac{LF}{AY}$$

The cross-section area of the cable is

$$A = \pi r^2 = \pi(0.00125\ \text{m})^2 = 4.91 \times 10^{-6}\ \text{m}^2$$

The restoring force of the cable is equal to the ball's weight:

$$F = w = mg = (120\ \text{kg})(9.8\ \text{m/s}^2) = 1180\ \text{N}$$

The change in length is thus

$$\Delta L = \frac{(6.0\ \text{m})(1180\ \text{N})}{(4.91 \times 10^{-6}\ \text{m}^2)(20 \times 10^{10}\ \text{N/m}^2)} = 0.0072\ \text{m} = 7.2\ \text{mm}$$

ASSESS If you've ever strung a guitar with steel strings, you know that the strings stretch several millimeters with the force you can apply by turning the tuning pegs. So a stretch of 7 mm under a 120 kg load seems reasonable.

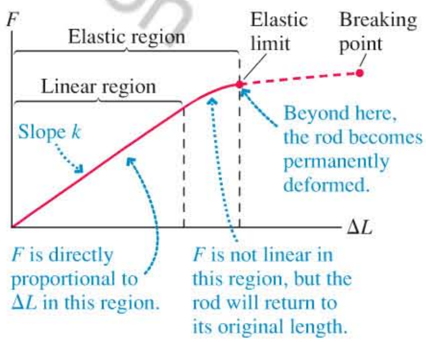
Beyond the Elastic Limit

In the previous section, we found that if we stretch a rod by a small amount ΔL, it will pull back with a restoring force F, according to Equation 8.6. But if we continue to stretch the rod, this simple linear relationship between ΔL and F will eventually break down. **FIGURE 8.22** is a graph of the rod's restoring force from the start of the stretch until the rod finally breaks.

As you can see, the graph has a *linear region,* the region where F and ΔL are proportional to each other, obeying Hooke's law: $F = k\,\Delta L$. **As long as the stretch stays within the linear region, a solid rod acts like a spring and obeys Hooke's law.**

How far can you stretch the rod before damaging it? As long as the stretch is less than the **elastic limit,** the rod will return to its initial length L when the force is removed. The elastic limit is the end of the **elastic region.** Stretching the rod beyond the elastic limit will permanently deform it, and the rod won't return to its original length. Finally, at a certain point the rod will reach a breaking point, where it will snap in two. The maximum stress that a material can be subjected to before failing is called the **tensile strength.** Table 8.2 lists values of tensile strength for rigid materials. When we speak of the *strength* of a material, we are referring to its tensile strength.

FIGURE 8.22 Stretch data for a steel rod.



EXAMPLE 8.9 Breaking a pendulum cable

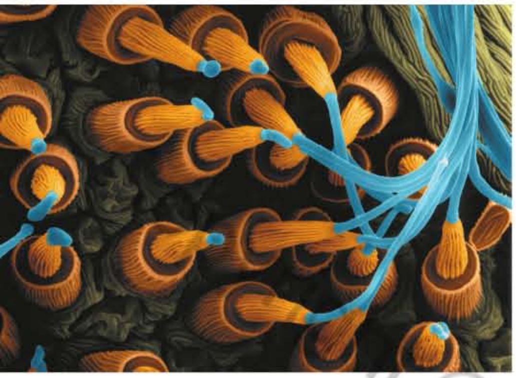
After a late night of studying physics, several 80 kg students decide it would be fun to swing on the Foucault pendulum of Example 8.8. What's the maximum number of students that the pendulum cable could support?

PREPARE The tensile strength, given for steel in Table 8.2 as $1000 \times 10^6\ \text{N/m}^2$, or $1.0 \times 10^9\ \text{N/m}^2$, is the largest stress the cable can sustain. Because the stress in the cable is F/A, we can find the maximum force F_{max} that can be applied to the cable before it fails.

Continued

TABLE 8.2 Tensile strengths of rigid materials

Material	Tensile strength (N/m^2)
Polypropylene	20×10^6
Glass	60×10^6
Cast iron	150×10^6
Aluminum	400×10^6
Steel	1000×10^6



Spider silk BIO The glands on the abdomen of a spider produce different kinds of silk. The silk that is used in webs can be quite stretchy; that used to subdue prey is generally not. An individual strand of silk may be a mix of fibers of different types, allowing spiders great flexibility in their material.

SOLVE We have

$$F_{max} = A(1.0 \times 10^9 \text{ N/m}^2)$$

From Example 8.8, the diameter of the cable is 2.5 mm, so its radius is 0.00125 m. Thus

$$F_{max} = \left(\pi(0.00125 \text{ m})^2\right)(1.0 \times 10^9 \text{ N/m}^2) = 4.9 \times 10^3 \text{ N}$$

This force is the weight of the heaviest mass the cable can support: $w = m_{max}g$. The maximum mass that can be supported is

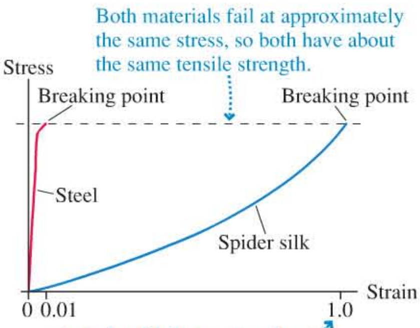
$$m_{max} = \frac{F_{max}}{g} = 500 \text{ kg}$$

The ball has a mass of 120 kg, leaving 380 kg for the students. Four students have a mass of 320 kg, which is less than this value. But five students, totaling 400 kg, would cause the cable to break.

ASSESS Steel has a very large tensile strength. This very narrow wire can still support 4900 N ≈ 1100 lb.

Biological Materials

FIGURE 8.23 Stress-versus-strain graphs for steel and spider silk.

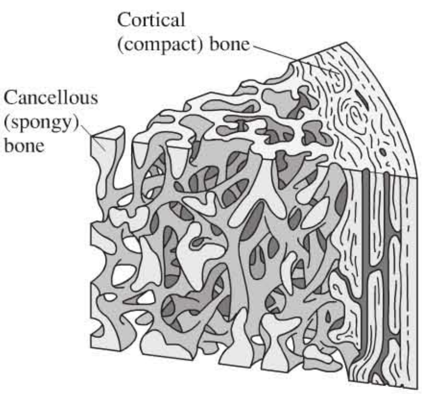


Suppose we take equal lengths of spider silk and steel wire, stretch each, and measure the restoring force of each until it breaks. The graph of stress versus strain might appear as in **FIGURE 8.23**.

The spider silk is certainly less stiff: For a given stress, the silk will stretch about 100 times farther than steel. Interestingly, though, spider silk and steel eventually fail at approximately the same stress. In this sense, spider silk is "as strong as steel." Many pliant biological materials share this combination of low stiffness and large tensile strength. These materials can undergo significant deformations without failing. Tendons, the walls of arteries, and the web of a spider are all quite strong but nonetheless capable of significant stretch.

Bone is an interesting example of a rigid biological material. Most bones in your body are made of two different kinds of bony material: dense and rigid cortical (or compact) bone on the outside, and porous, flexible cancellous (or spongy) bone on the inside. **FIGURE 8.24** shows a cross section of a typical bone. Cortical and cancellous bones have very different values of Young's modulus. Young's modulus for cortical bone approaches that of concrete, so it is very rigid with little ability to stretch or compress. In contrast, cancellous bone has a much lower Young's modulus. Consequently, the elastic properties of bones can be well modeled as those of a hollow cylinder.

FIGURE 8.24 Cross section of a long bone.



The structure of bones in birds actually approximates a hollow cylinder quite well. **FIGURE 8.25** shows that a typical bone is a thin-walled tube of cortical bone with a tenuous structure of cancellous bone inside. Most of a cylinder's rigidity comes from the material near its surface. A hollow cylinder retains most of the rigidity of a solid one, but it is much lighter. Bird bones carry this idea to its extreme.

FIGURE 8.25 Section of a bone from a bird.

Table 8.3 gives values of Young's modulus for biological materials. Note the large difference between pliant and rigid materials. Table 8.4 shows the tensile strengths for biological materials. Interestingly, spider silk, a pliant material, has a greater tensile strength than bone!

The values in Table 8.4 are for static forces—forces applied for a long time in a testing machine. Bone can withstand significantly greater stresses if the forces are applied for only a very short period of time.

EXAMPLE 8.10 Finding the compression of a bone

The femur, the long bone in the thigh, can be modeled as a tube of cortical bone for most of its length. A 70 kg person has a femur with a cross-section area (of the cortical bone) of 4.8×10^{-4} m^2, a typical value.

a. If this person supports his entire weight on one leg, what fraction of the tensile strength of the bone does this stress represent?
b. By what fraction of its length does the femur shorten?

PREPARE The stress on the femur is F/A. Here F, the force compressing the femur, is the person's weight, so $F = mg$. The fractional change $\Delta L/L$ in the femur is the strain, which we can find using Equation 8.7, taking the value of Young's modulus for cortical bone from Table 8.3.

SOLVE

a. The person's weight is $mg = (70 \text{ kg})(9.8 \text{ m/s}^2) = 690$ N. The resulting stress on the femur is

$$\frac{F}{A} = \frac{690 \text{ N}}{4.8 \times 10^{-4} \text{ m}^2} = 1.4 \times 10^6 \text{ N/m}^2$$

A stress of 1.4×10^6 N/m^2 is 1.4% of the tensile strength of cortical bone given in Table 8.4.

b. We can compute the strain as

$$\frac{\Delta L}{L} = \left(\frac{1}{Y}\right)\frac{F}{A} = \left(\frac{1}{1.6 \times 10^{10} \text{ N/m}^2}\right)(1.4 \times 10^6 \text{ N/m}^2) = 8.8 \times 10^{-5} \approx 0.0001$$

The femur compression is $\Delta L \approx 0.0001L$, or $\approx 0.01\%$ of its length.

ASSESS It makes sense that, under ordinary standing conditions, the stress on the femur is only a percent or so of the maximum value it can sustain.

TABLE 8.3 Young's modulus for biological materials

Material	Young's modulus (10^{10} N/m^2)
Tooth enamel	6
Cortical bone	1.6
Cancellous bone	0.02–0.3
Spider silk	0.2
Tendon	0.15
Cartilage	0.0001
Blood vessel (aorta)	0.00005

TABLE 8.4 Tensile strength of biological materials

Material	Tensile strength (N/m^2)
Cancellous bone	5×10^6
Cortical bone	100×10^6
Tendon	100×10^6
Spider silk	1000×10^6

The dancer in the chapter-opening photo stands *en pointe,* balanced delicately on the tip of her shoe with her entire weight supported on a very small area. The stress on the bones in her toes is very large, but it is still much less than the tensile strength of bone.

STOP TO THINK 8.5 A 10 kg mass is hung from a 1-m-long cable, causing the cable to stretch by 2 mm. Suppose a 10 kg mass is hung from a 2 m length of the same cable. By how much does the cable stretch?

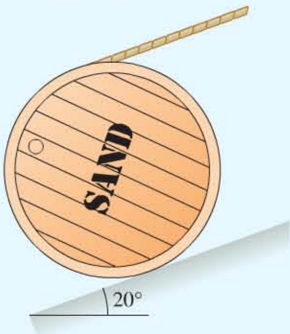
A. 0.5 mm B. 1 mm C. 2 mm D. 3 mm E. 4 mm

INTEGRATED EXAMPLE 8.11 Holding a barrel on a hill

FIGURE 8.26 shows a 60-cm-diameter barrel of sand, with a mass of 600 kg, being held in place on a hill by a polypropylene rope wrapped around the barrel. The coefficient of static friction between the barrel and the hill is 0.25.

a. What is the tension in the rope?
b. What is the steepest hill that the barrel could rest on without slipping?

FIGURE 8.26 A barrel being held by a rope.



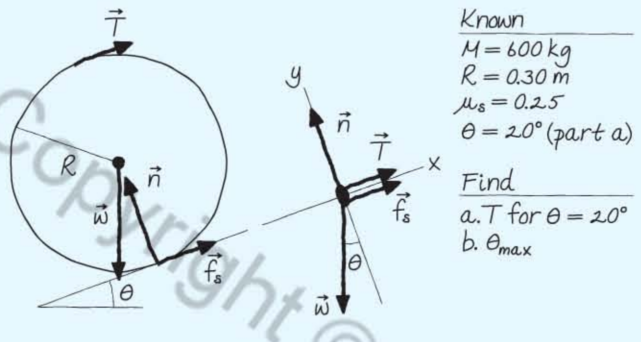
c. What is the smallest-diameter rope that can be used without the rope breaking?

PREPARE We'll follow Problem-Solving Strategy 8.1: For an object in static equilibrium, the net torque is zero, $\Sigma\tau = 0$, and the net force is zero, $\Sigma F_x = 0$ and $\Sigma F_y = 0$. To find the net torque on the barrel, we'll redraw it in **FIGURE 8.27** on the next page with all the forces shown at the points at which they act. To find the components of the net force, we'll draw the free-body diagram of Figure 8.27. As usual, we tilt our x-axis so that it's parallel to the surface of the hill. Recall from Figure 5.14 that the angle between the weight vector and the $-y$-axis is the same as the angle of the slope.

The direction of the static friction force is chosen to keep the bottom of the barrel from slipping down the hill. Imagine what

Continued

FIGURE 8.27 Visual overview of the barrel.



would happen if friction suddenly vanished while the rope was holding the top of the barrel in place.

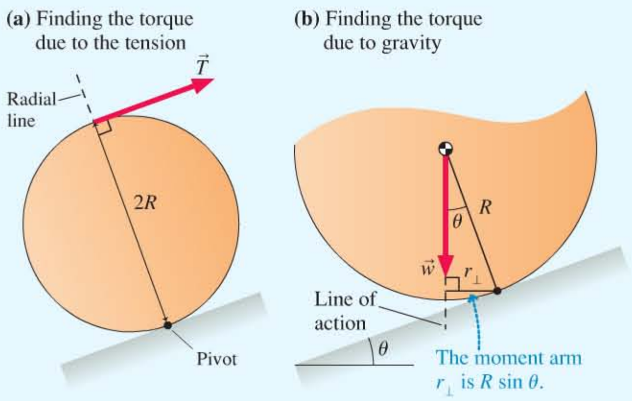
We need to pick a pivot about which the torque will be calculated. If we choose the point of contact of the barrel with the hill as our pivot, then the unknown forces $\vec{n}$ and $\vec{f}_s$, which act at that point, make no contribution to the torque. Only the weight, whose magnitude we know, and the tension, which is what we want to find, contribute to the torque.

For part c, the rope will fail if the stress exceeds the tensile strength of polypropylene, given in Table 8.2 as 2.0×10^7 N/m^2.

SOLVE

a. FIGURE 8.28 shows how to calculate the torque. Forces $\vec{n}$ and $\vec{f}_s$ are not shown because, as just discussed, they act at the pivot and do not contribute to the torque. In Figure 8.28a, the tension $\vec{T}$ acts perpendicular to the radial line, at a distance $2R$ from the pivot, so the torque due to the tension is $\tau_T = -2RT$. It's negative because the tension tries to rotate the barrel clockwise.

FIGURE 8.28 Calculating the torque.



We'll use Equation 7.4, $\tau = r_\perp F$, to find the torque due to the weight, which acts at the center of gravity. From Figure 8.28b we see that the moment arm $r_\perp$, the perpendicular distance from the line of action to the pivot, is $r_\perp = R\sin\theta$. The magnitude of the weight force is Mg, so the torque due to the weight is $\tau_w = MgR\sin\theta$.

We can now write the condition that the net torque is zero as

$$\tau_{net} = \tau_w + \tau_T = MgR\sin\theta - 2RT = 0$$

Solving this equation for the tension gives

$$T = \tfrac{1}{2}Mg\sin\theta = \frac{(600\text{ kg})(9.8\text{ m/s}^2)}{2}\sin 20° = 1000\text{ N}$$

Note that R cancels, so the tension does not depend on the radius of the barrel.

b. Part a was solved using only the net torque equation. For this part, we'll need the two force equations as well. From the free-body diagram of Figure 8.27, we can write

$$\sum F_x = T + f_s - Mg\sin\theta = 0$$
$$\sum F_y = n - Mg\cos\theta = 0$$

We can solve the first of these equations for the friction force:

$$f_s = Mg\sin\theta - T = Mg\sin\theta - \tfrac{1}{2}Mg\sin\theta = \tfrac{1}{2}Mg\sin\theta$$

Here we used the result for the tension T found in part a.

The static friction must have this value to keep the barrel from slipping. But static friction can't exceed the maximum possible value, $f_{s\,max} = \mu_s n$. From the y force equation, the normal force is $n = Mg\cos\theta$, so $f_{s\,max} = \mu_s Mg\cos\theta$. The barrel will slip when the friction force equals its maximum possible value, or when

$$f_s = \tfrac{1}{2}Mg\sin\theta = f_{s\,max} = \mu_s Mg\cos\theta$$

The factor Mg cancels from both sides of this equation, giving $\frac{1}{2}\sin\theta = \mu_s\cos\theta$, which, after dividing both sides by $\cos\theta$, can be written as $\frac{1}{2}\tan\theta = \mu_s$. Thus the angle at which the barrel will slip is given by $\tan\theta = 2\mu_s$, or

$$\theta = \tan^{-1}(2\mu_s) = \tan^{-1}(2 \cdot 0.25) = 27°$$

c. The maximum possible stress in the rope, when it is at its breaking point, is

$$\frac{F}{A} = \frac{T}{\pi r^2} = \frac{1000\text{ N}}{\pi r^2} = 2.0 \times 10^7\text{ N/m}^2$$

The radius of the rope at this level of stress is

$$r = \sqrt{\frac{1000\text{ N}}{\pi(2.0 \times 10^7\text{ N/m}^2)}} = 4.0 \times 10^{-3}\text{ m}$$

The minimum rope diameter is twice this radius, or 8.0 mm. If the rope were any smaller, the stress would exceed the tensile strength of polypropylene and the rope would break.

ASSESS Back in Example 5.14, we found that an object will *slide* (without rolling) down a slope when the angle exceeds $\tan^{-1}\mu_s$, a *smaller* angle than for our rolling object. This makes sense because for the barrel there is an extra uphill force—the tension—that is absent for a sliding object. This uphill force allows the slope to be steeper before a round object begins to slip.

SUMMARY

The goals of Chapter 8 have been to learn about the static equilibrium of extended objects and to understand the basic properties of springs and elastic materials.

GENERAL PRINCIPLES

Static Equilibrium

An object in **static equilibrium** must have no net force on it and no net torque. Mathematically, we express this as

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau = 0$$

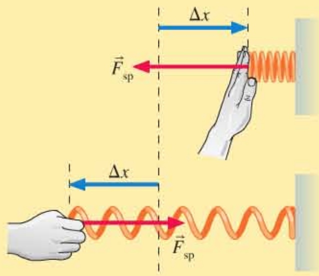
Since the net torque is zero about *any* point, the pivot point for calculating the torque can be chosen at any convenient location.

Springs and Hooke's Law

When a spring is stretched or compressed, it exerts a force proportional to the change Δx in its length but in the opposite direction. This is known as **Hooke's law:**

$$(F_{sp})_x = -k\,\Delta x$$

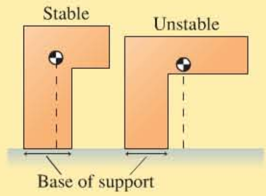
The constant of proportionality k is called the **spring constant.** It is larger for a "stiff" spring.



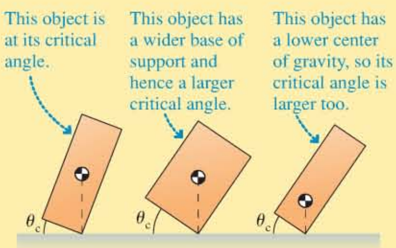
IMPORTANT CONCEPTS

Stability

An object is **stable** if its center of gravity is over its base of support; otherwise, it is **unstable.**

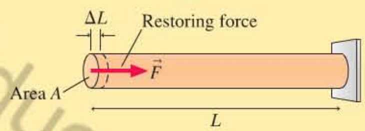


If an object is tipped, it will reach the limit of its stability when its center of gravity is over the edge of the base. This defines the **critical angle** θ_c.



Greater stability is possible with a lower center of gravity or a broader base of support.

Elastic materials and Young's modulus



A solid rod illustrates how materials respond when stretched or compressed.

Stress is the restoring force of the rod divided by its cross-section area.

$$\left(\frac{F}{A}\right) = Y\left(\frac{\Delta L}{L}\right)$$

Strain is the fractional change in the rod's length.

Young's modulus

This equation can also be written as

This is the "spring constant" k for the rod.

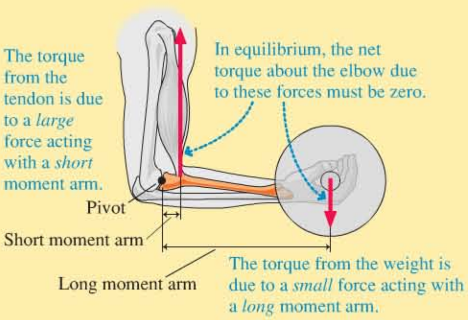
$$F = \left(\frac{YA}{L}\right)\Delta L$$

showing that a rod obeys Hooke's law and acts like a very stiff spring.

APPLICATIONS

Forces in the body

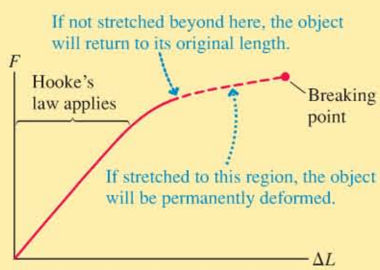
Muscles and tendons apply the forces and torques needed to maintain static equilibrium. These forces may be quite large.



The elastic limit and beyond

If a rod or other object is not stretched too far, when released it will return to its original shape.

If stretched too far, an object will permanently deform, and finally break. The stress at which an object breaks is its **tensile stress.**



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. An object is acted upon by two (and only two) forces that are of equal magnitude and oppositely directed. Is the object necessarily in static equilibrium?
2. Sketch a force acting at point P in Figure Q8.2 that would make the rod be in static equilibrium. Is there only one such force?
3. Could a ladder on a level floor lean against a wall in static equilibrium if there were no friction forces? Explain.
4. Suppose you are hanging from a tree branch. If you move out the branch, farther away from the trunk, the branch will be more likely to break. Explain why this is so.

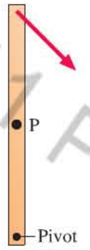


FIGURE Q8.2

5. As divers stand on tiptoes on the edge of a diving platform, in preparation for a high dive, as shown in Figure Q8.5, they usually extend their arms in front of them. Why do they do this?
6. Where are the centers of gravity of the two people doing the classic yoga poses shown in Figure Q8.6?

FIGURE Q8.5

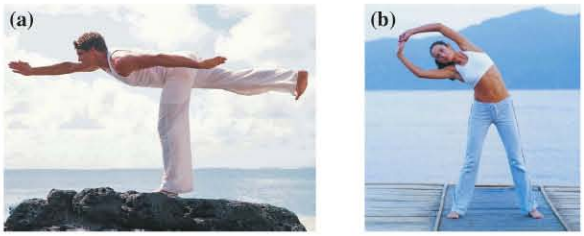


FIGURE Q8.6

7. You must lean quite far forward as you rise from a chair (try it!). Explain why.
8. A spring exerts a 10 N force after being stretched by 1 cm from its equilibrium length. By how much will the spring force *increase* if the spring is stretched from 4 cm away from equilibrium to 5 cm from equilibrium?
9. The left end of a spring is attached to a wall. When Bob pulls on the right end with a 200 N force, he stretches the spring by 20 cm. The same spring is then used for a tug-of-war between Bob and Carlos. Each pulls on his end of the spring with a 200 N force.
   a. How far does Bob's end of the spring move? Explain.
   b. How far does Carlos's end of the spring move? Explain.
10. A spring is attached to the floor and pulled straight up by a string. The string's tension is measured. The graph in Figure Q8.10 shows the tension in the spring as a function of the spring's length L.
    a. Does this spring obey Hooke's law? Explain.
    b. If it does, what is the spring constant?

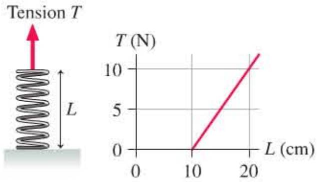


FIGURE Q8.10

11. Take a spring and cut it in half to make two springs. Is the spring constant of these smaller springs larger, smaller, or the same as the spring constant of the original spring? Explain.
12. A wire is stretched right to its breaking point by a 5000 N force. A longer wire made of the same material has the same diameter. Is the force that will stretch it right to its breaking point larger than, smaller than, or equal to 5000 N? Explain.
13. Steel nails are rigid and unbending. Steel wool is soft and squishy. How would you account for this difference?

Multiple-Choice Questions

14. || Two children carry a lightweight 1.8-m-long horizontal pole with a water bucket hanging from it. The older child supports twice as much weight as the younger child. How far is the bucket from the older child?
    A. 0.3 m B. 0.6 m
    C. 0.9 m D. 1.2 m
15. || The uniform rod in Figure Q8.15 has a weight of 14.0 N. What is the magnitude of the normal force exerted on the rod by the surface?
    A. 7 N B. 14 N
    C. 20 N D. 28 N

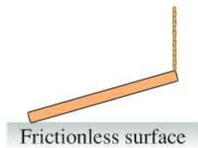


FIGURE Q8.15

16. | A student lies on a very light, rigid board with a scale under each end. Her feet are directly over one scale, and her body is positioned as shown in Figure Q8.16. The two scales read the values shown in the figure. What is the student's weight?
    A. 65 lb B. 75 lb
    C. 100 lb D. 165 lb

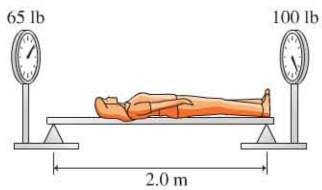


FIGURE Q8.16

17. | For the student in Figure Q8.16, approximately how far from her feet is her center of gravity?
    A. 0.6 m B. 0.8 m
    C. 1.0 m D. 1.2 m

Questions 18 through 20 use the information in the following paragraph and figure.

Suppose you stand on one foot while holding your other leg up behind you. Your muscles will have to apply a force to hold your leg in this raised position. We can model this situation as in Figure Q8.18. The leg pivots at the knee joint, and the force to hold the leg up is provided by a tendon attached to the lower leg as shown. Assume that the lower leg and the foot together have a combined mass of 4.0 kg, and that their combined center of gravity is at the center of the lower leg.

FIGURE Q8.18

18. BIO How much force must the tendon exert to keep the leg in this position?
    A. 40 N B. 200 N C. 400 N D. 1000 N
19. BIO As you hold your leg in this position, the upper leg exerts a force on the lower leg at the knee joint. What is the direction of this force?
    A. Up B. Down C. Right D. Left
20. BIO What is the magnitude of the force of the upper leg on the lower leg at the knee joint?
    A. 40 N B. 160 N C. 200 N D. 240 N
21. You have a heavy piece of equipment hanging from a 1.0-mm-diameter wire. Your supervisor asks that the length of the wire be doubled without changing how far the wire stretches. What diameter must the new wire have?
    A. 1.0 mm B. 1.4 mm C. 2.0 mm D. 4.0 mm
22. A 30.0-cm-long board is placed on a table such that its right end hangs over the edge by 8.0 cm. A second identical board is stacked on top of the first, as shown in Figure Q8.22. What is the largest that the distance x can be before both boards topple over?
    A. 4.0 cm B. 8.0 cm
    C. 14 cm D. 15 cm

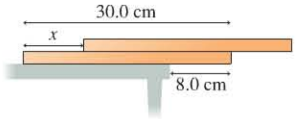


FIGURE Q8.22

23. Two 20 kg blocks are connected by a 2.0-m-long, 5.0-mm-diameter rope. Young's modulus for this rope is $1.5 \times 10^9\ \text{N/m}^2$. The rope is then hung over a pulley, so that the blocks, hanging from each side of the pulley, are in static equilibrium. By how much does the rope stretch?
    A. 3.0 mm B. 6.3 mm
    C. 9.3 mm D. 13 mm

PROBLEMS

Section 8.1 Torque and Static Equilibrium

1. A 64 kg woman stands on a very light, rigid board that rests on a bathroom scale at each end, as shown in Figure P8.1. What is the reading on each of the scales?

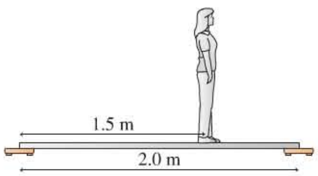


FIGURE P8.1

2. Suppose the woman in Figure P8.1 is 54 kg, and the board she is standing on has a 10 kg mass. What is the reading on each of the scales?
3. How close to the right edge of the 56 kg picnic table shown in Figure P8.3 can a 70 kg man stand without the table tipping over?

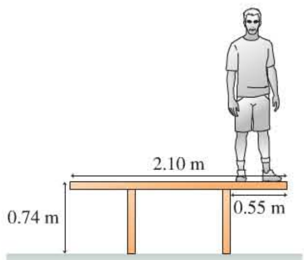


FIGURE P8.3

4. In Figure P8.4, a 70 kg man walks out on a 10 kg beam that rests on, but is not attached to, two supports. When the beam just starts to tip, what is the force exerted on the beam by the right support?

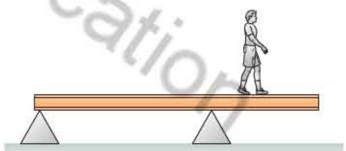


FIGURE P8.4

5. You're carrying a 3.6-m-long, 25 kg pole to a construction site when you decide to stop for a rest. You place one end of the pole on a fence post and hold the other end of the pole 35 cm from its tip. How much force must you exert to keep the pole motionless in a horizontal position?
6. How much torque must the pin exert to keep the rod in Figure P8.6 from rotating? Calculate this torque about an axis that passes through the point where the pin enters the rod and is perpendicular to the plane of the figure.

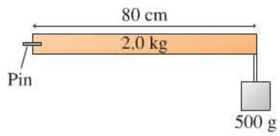


FIGURE P8.6

7. Is the object in Figure P8.7 in equilibrium? Explain.

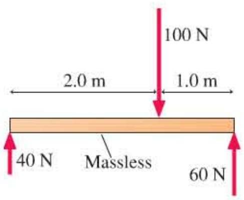


FIGURE P8.7

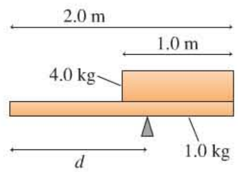


FIGURE P8.8

8. The two objects in Figure P8.8 are balanced on the pivot. What is distance d?

9. ||| A 60 kg diver stands at the end of a 30 kg springboard, as shown in Figure P8.9. The board is attached to a hinge at the left end but simply rests on the right support. What is the magnitude of the vertical force exerted by the hinge on the board?

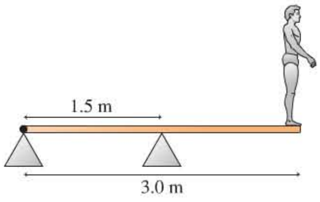


FIGURE P8.9

10. ||| A uniform beam of length 1.0 m and mass 10 kg is attached to a wall by a cable, as shown in Figure P8.10. The beam is free to pivot at the point where it attaches to the wall. What is the tension in the cable?

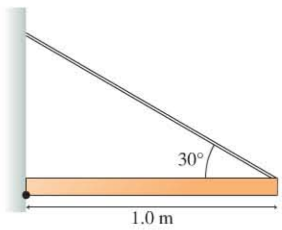


FIGURE P8.10

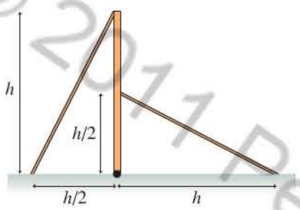


FIGURE P8.11

11. || Figure P8.11 shows a vertical pole of height h that can rotate about a hinge at the bottom. The pole is held in position by two wires under tension. What is the ratio of the tension in the left wire to the tension in the right wire?

Section 8.2 Stability and Balance

12. || You want to slowly push a stiff board across a 20 cm gap between two tabletops that are at the same height. If you apply only a horizontal force, how long must the board be so that it doesn't tilt down into the gap before reaching the other side?

13. | A magazine rack has a center of gravity 16 cm above the floor, as shown in Figure P8.13. Through what maximum angle, in degrees, can the rack be tilted without falling over?

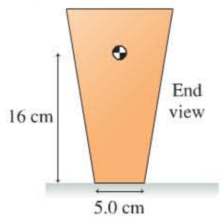


FIGURE P8.13

14. || A car manufacturer claims that you can drive its new vehicle across a hill with a 47° slope before the vehicle starts to tip. If the vehicle is 2.0 m wide, how high is its center of gravity?

15. || A thin 2.00 kg box rests on a 6.00 kg board that hangs over the end of a table, as shown in Figure P8.15. How far can the center of the box be from the end of the table before the board begins to tilt?

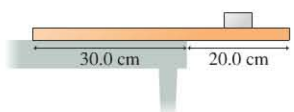


FIGURE P8.15

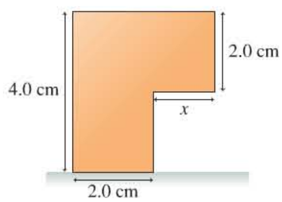


FIGURE P8.16

16. ||| The object shown in Figure P8.16 is made of a uniform material. What is the greatest that x can be without the object tipping over?

Section 8.3 Springs and Hooke's Law

17. | One end of a spring is attached to a wall. A 25 N pull on the other end causes the spring to stretch by 3.0 cm. What is the spring constant?

18. | BIO Experiments using "optical tweezers" measure the elasticity of individual DNA molecules. For small enough changes in length, the elasticity has the same form as that of a spring. A DNA molecule is anchored at one end, then a force of 1.5 nN (1.5×10^{-9} N) pulls on the other end, causing the molecule to stretch by 5.0 nm (5.0×10^{-9} m). What is the spring constant of that DNA molecule?

19. || A spring has an unstretched length of 10 cm. It exerts a restoring force F when stretched to a length of 11 cm.
   a. For what total stretched length of the spring is its restoring force $3F$?
   b. At what compressed length is the restoring force $2F$?

20. || A 10-cm-long spring is attached to the ceiling. When a 2.0 kg mass is hung from it, the spring stretches to a length of 15 cm.
   a. What is the spring constant?
   b. How long is the spring when a 3.0 kg mass is suspended from it?

21. || A spring stretches 5.0 cm when a 0.20 kg block is hung from it. If a 0.70 kg block replaces the 0.20 kg block, how far does the spring stretch?

22. || A 1.2 kg block is hung from a vertical spring, causing the spring to stretch by 2.4 cm. How much farther will the spring stretch if a 0.60 kg block is added to the 1.2 kg block?

23. || A runner wearing spiked shoes pulls a 20 kg sled across frictionless ice using a horizontal spring with spring constant 1.5×10^2 N/m. The spring is stretched 20 cm from its equilibrium length. What is the acceleration of the sled?

24. | You need to make a spring scale to measure the mass of objects hung from it. You want each 1.0 cm length along the scale to correspond to a mass difference of 0.10 kg. What should be the value of the spring constant?

Section 8.4 Stretching and Compressing Materials

25. || A force stretches a wire by 1.0 mm.
   a. A second wire of the same material has the same cross section and twice the length. How far will it be stretched by the same force?
   b. A third wire of the same material has the same length and twice the diameter as the first. How far will it be stretched by the same force?

26. |||| What hanging mass will stretch a 2.0-m-long, 0.50-mm-diameter steel wire by 1.0 mm?

27. ||| How much force does it take to stretch a 10-m-long, 1.0-cm-diameter steel cable by 5.0 mm?

28. ||| An 80-cm-long, 1.0-mm-diameter steel guitar string must be tightened to a tension of 2.0 kN by turning the tuning screws. By how much is the string stretched?

29. ||| A 2000 N force stretches a wire by 1.0 mm.
   a. A second wire of the same material is twice as long and has twice the diameter. How much force is needed to stretch it by 1.0 mm? Explain.
   b. A third wire of the same material is twice as long as the first and has the same diameter. How far is it stretched by a 4000 N force?

30. ||| A 1.2-m-long steel rod with a diameter of 0.50 cm hangs vertically from the ceiling. An auto engine weighing 4.7 kN is hung from the rod. By how much does the rod stretch?

31. ||| A mine shaft has an elevator hung from a single steel-wire cable of diameter 2.5 cm. When the cable is fully extended, the end of the cable is 500 m below the support. How much does the fully extended cable stretch when 3000 kg of ore is loaded into the elevator?

32. || The normal force of the ground on the foot can reach three BIO times a runner's body weight when the foot strikes the pavement. By what amount does the 52-cm-long femur of an 80 kg runner compress at this moment? The cross-section area of the bone of the femur can be taken as 5.2×10^{-4} m^2.

33. ||| A three-legged wooden bar stool made out of solid Douglas fir has legs that are 2.0 cm in diameter. When a 75 kg man sits on the stool, by what percent does the length of the legs decrease? Assume, for simplicity, that the stool's legs are vertical and that each bears the same load.

34. ||| A 3.0-m-tall, 50-cm-diameter concrete column supports a 200,000 kg load. By how much is the column compressed?

General Problems

35. |||| A 3.0-m-long rigid beam with a mass of 100 kg is supported at each end, as shown in Figure P8.35. An 80 kg student stands 2.0 m from support 1. How much upward force does each support exert on the beam?

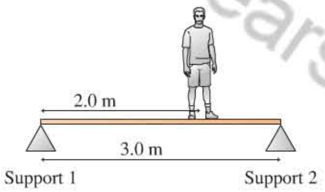


FIGURE P8.35

36. ||| An 80 kg construction worker sits down 2.0 m from the end of a 1450 kg steel beam to eat his lunch, as shown in Figure P8.36. The cable supporting the beam is rated at 15,000 N. Should the worker be worried?

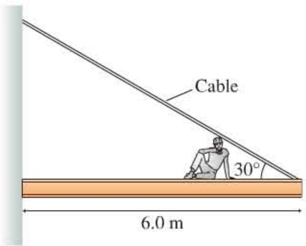


FIGURE P8.36

37. |||| Using the information in Figure 8.2, calculate the tension in BIO the biceps tendon if the hand is holding a 10 kg ball while the forearm is held 45° below horizontal.

38. ||| A woman weighing 580 N does a BIO pushup from her knees, as shown in Figure P8.38. What are the normal forces of the floor on (a) each of her hands and (b) each of her knees?

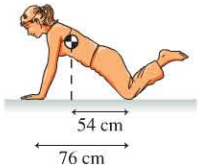


FIGURE P8.38

39. |||| When you bend over, a series of large BIO muscles, the erector spinae, pull on your spine to hold you up. Figure P8.39 shows a simplified model of the spine as a rod of length L that pivots at its lower end. In this model, the center of gravity of the 320 N weight of the upper torso is at the center of the spine. The 160 N weight of the head and arms acts at the top of the spine. The erector spinae muscles are modeled as a single muscle that acts at an 12° angle to the spine. Suppose the person in Figure P8.39 bends over to an angle of 30° from the horizontal.

a. What is the tension in the erector muscle?

Hint: Align your x-axis with the axis of the spine.

b. A force from the pelvic girdle acts on the base of the spine. What is the component of this force in the direction of the spine? (This large force is the cause of many back injuries).

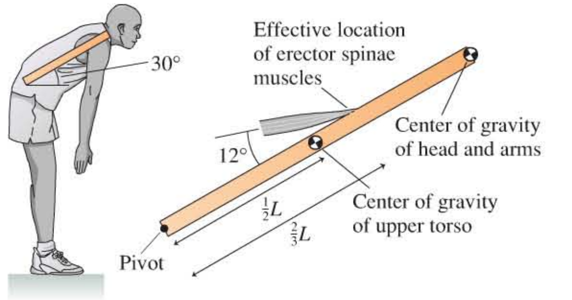


FIGURE P8.39

40. | The woman lying on the reaction board in Example 8.3 spreads her arms out in the plane of the board. The reading of the scale is observed to increase by 10 N. By how much does the distance from her feet to her center of gravity change?

41. || A man is attempting to raise a 7.5-m-long, 28 kg flagpole that has a hinge at the base by pulling on a rope attached to the top of the pole, as shown in Figure P8.41. With what force does the man have to pull on the rope to hold the pole motionless in this position?

FIGURE P8.41

42. |||| A library ladder of length L rolls on wheels as shown in Figure P8.42. The two legs of the ladder freely pivot at the hinge at the top. The legs are kept from splaying apart by a lightweight chain that is attached halfway up the ladder. If the ladder weighs 200 N, and the angle between each leg and the vertical is 25°, what is the tension in the chain?

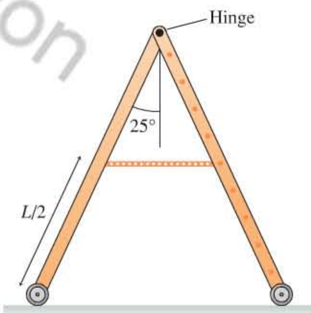


FIGURE P8.42

43. |||| A 40 kg, 5.0-m-long beam is supported by, but not attached to, the two posts in Figure P8.43. A 20 kg boy starts walking along the beam. How close can he get to the right end of the beam without it tipping?

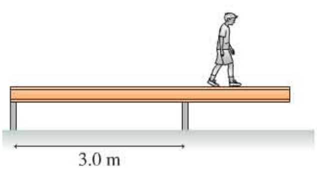


FIGURE P8.43

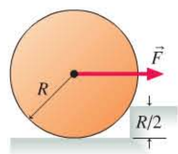


FIGURE P8.44

44. ||| The wheel of mass m in Figure P8.44 is pulled on by a horizontal force applied at its center. The wheel is touching a curb whose height is half the wheel's radius. What is the minimum force required to just raise the wheel off the ground?

45. ‖ A 5.0 kg mass hanging from a spring scale is slowly lowered onto a vertical spring, as shown in Figure P8.45. The scale reads in newtons.
   a. What does the spring scale read just before the mass touches the lower spring?
   b. The scale reads 20 N when the lower spring has been compressed by 2.0 cm. What is the value of the spring constant for the lower spring?
   c. At what compression distance will the scale read zero?

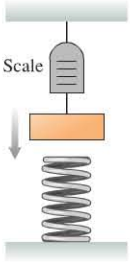


FIGURE P8.45

46. ||| Two identical, side-by-side springs with spring constant 240 N/m support a 2.00 kg hanging box. By how much is each spring stretched?

47. | Two springs have the same equilibrium length but different spring constants. They are arranged as shown in Figure P8.47, then a block is pushed against them, compressing both by 1.00 cm. With what net force do they push back on the block?

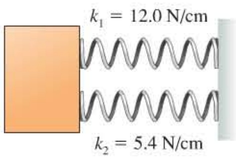


FIGURE P8.47

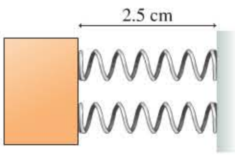


FIGURE P8.48

48. ‖ Two springs have the same spring constant $k = 130$ N/m but different equilibrium lengths, 3.0 cm and 5.0 cm. They are arranged as shown in Figure P8.48, then a block is pushed against them, compressing both to a length of 2.5 cm. With what net force do they push back on the block?

49. ‖ Figure P8.49 shows two springs attached to a block that can slide on a frictionless surface. In the block's equilibrium position, the left spring is compressed by 2.0 cm.
   a. By how much is the right spring compressed?
   b. What is the net force on the block if it is moved 15 cm to the right of its equilibrium position?

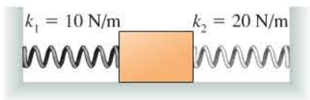


FIGURE P8.49

50. ||| Figure P8.50 shows two springs attached to each other, and also attached to a box that can slide on a frictionless surface. In the block's equilibrium position, neither spring is stretched. What is the net force on the block if it is moved 15 cm to the right of its equilibrium position?
   Hint: There is zero net force on the point where the two springs meet. This implies a relationship between the amounts the two springs stretch.

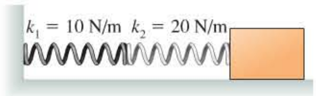


FIGURE P8.50

51. ‖ A 60 kg student is standing atop a spring in an elevator that is accelerating upward at 3.0 m/s^2. The spring constant is 2.5×10^3 N/m. By how much is the spring compressed?
INT

52. ||| A 25 kg child bounces on a pogo stick. The pogo stick has a spring with spring constant 2.0×10^4 N/m. When the child makes a nice big bounce, she finds that at the bottom of the bounce she is accelerating *upward* at 9.8 m/s^2. How much is the spring compressed?
INT

53. |||| Two 3.0 kg blocks on a level, frictionless surface are connected by a spring with spring constant 1000 N/m, as shown in Figure P8.53. The left block is pushed by a horizontal force $\vec{F}$. At $t = 0$ s, both blocks have velocity 3.2 m/s to the right. For the next second, the spring's compression is a constant 1.5 cm.
INT
   a. What is the velocity of the right block at $t = 1.0$ s?
   b. What is the magnitude of $\vec{F}$ during that 1.0 s interval?

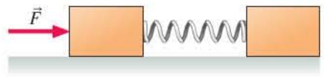


FIGURE P8.53

54. ‖ What is the effective spring constant (that is, the ratio of force to change in length) of a copper cable that is 5.0 mm in diameter and 5.0 m long?

55. ||| Figure P8.55 shows a 100 kg plank supported at its right end by a 7.0-mm-diameter rope with a tensile strength of $6.0 \times 10^7 \text{ N/m}^2$. How far along the plank, measured from the pivot, can the center of gravity of an 800 kg piece of heavy machinery be placed before the rope snaps?

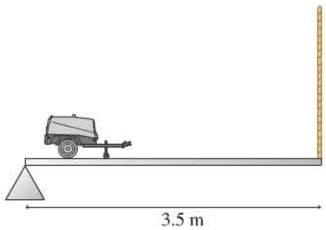


FIGURE P8.55

56. ||| When you walk, your Achilles tendon, which connects your heel to your calf muscles, repeatedly stretches and contracts, much like a spring. This helps make walking more efficient. Suppose your Achilles tendon is 15 cm long and has a cross-section area of 110 mm^2, typical values. If you model the Achilles tendon as a spring, what is its spring constant?
BIO

57. ||| There is a disk of cartilage between each pair of vertebrae in your spine. Suppose a disk is 0.50 cm thick and 4.0 cm in diameter. If this disk supports half the weight of a 65 kg person, by what fraction of its thickness does the disk compress?
BIO

58. | In Example 8.1, the tension in the biceps tendon for a person doing a strict curl of a 900 N barbell was found to be 3900 N. What fraction does this represent of the maximum possible tension the biceps tendon can support? You can assume a typical cross-section area of 130 mm^2.
BIO

59. ||| Larger animals have sturdier bones than smaller animals. A mouse's skeleton is only a few percent of its body weight, compared to 16% for an elephant. To see why this must be so, recall, from Example 8.10, that the stress on the femur for a man standing on one leg is 1.4% of the bone's tensile strength. Suppose we scale this man up by a factor of 10 in all dimensions, keeping the same body proportions. Use the data for Example 8.10 to compute the following.
BIO
   a. Both the inside and outside diameter of the femur, the region of cortical bone, will increase by a factor of 10. What will be the new cross-section area?
   b. The man's body will increase by a factor of 10 in each dimension. What will be his new mass?
   c. If the scaled-up man now stands on one leg, what fraction of the tensile strength is the stress on the femur?

60. ‖ Orb spiders make silk with a typical diameter of 0.15 mm.
BIO
   a. A typical large orb spider has a mass of 0.50 g. If this spider suspends itself from a single 12-cm-long strand of silk, by how much will the silk stretch?
   b. What is the maximum weight that a single thread of this silk could support?

Passage Problems

Standing on Tiptoes BIO

When you stand on your tiptoes, your feet pivot about your ankle. As shown in Figure P8.61, the forces on your foot are an upward force on your toes from the floor, a downward force on your ankle from the lower leg bone, and an upward force on the heel of your foot from your Achilles tendon. Suppose a 60 kg woman stands on tiptoes with the sole of her foot making a 25° angle with the floor. Assume that each foot supports half her weight.

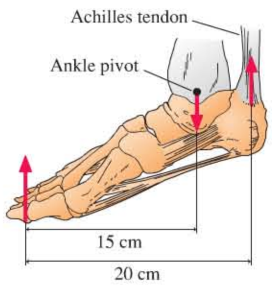


FIGURE P8.61

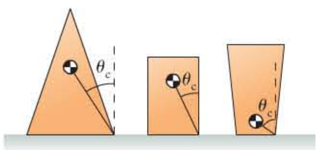
61. ||| What is the upward force of the floor on the toes of one foot?
   A. 140 N B. 290 N
   C. 420 N D. 590 N
62. || What upward force does the Achilles tendon exert on the heel of her foot?
   A. 290 N B. 420 N
   C. 590 N D. 880 N
63. ||| The tension in the Achilles tendon will cause it to stretch. If the Achilles tendon is 15 cm long and has a cross-section area of 110 mm^2, by how much will it stretch under this force?
   A. 0.2 mm B. 0.8 mm
   C. 2.3 mm D. 5.2 mm

STOP TO THINK ANSWERS

Stop to Think 8.1: D. Only object D has both zero net force and zero net torque.

Stop to Think 8.2: B. The tension in the rope and the weight have no horizontal component. To make the net force zero, the force due to the pivot must also have no horizontal component, so we know it points either up or down. Now consider the torque about the point where the rope is attached. The tension provides no torque. The weight exerts a counterclockwise torque. To make the net torque zero, the pivot force must exert a *clockwise* torque, which it can do only if it points *up*.

Stop to Think 8.3: B, A, C. The critical angle θ_c, shown in the figure, measures how far the object can be tipped before falling. B has the smallest critical angle, followed by A, then C.



Stop to Think 8.4: C. The restoring force of the spring is proportional to the stretch. Increasing the restoring force by a factor of 3 requires increasing the stretch by a factor of 3.

Stop to Think 8.5: E. The cables have the same diameter, and the force is the same, so the stress is the same in both cases. This means that the strain, $\Delta L/L$, is the same. The 2 m cable will experience twice the change in length of the 1 m cable.

PART I SUMMARY

Force and Motion

The goal of Part I has been to discover the connection between force and motion. We started with kinematics, the mathematical description of motion; then we proceeded to dynamics, the explanation of motion in terms of forces. We then used these descriptions to analyze and explain motions ranging from the motion of the moon about the earth to the forces in your elbow when you lift a weight. Newton's three laws of motion formed the basis of all of our explanations.

The table below is a *knowledge structure* for force and motion. The knowledge structure does not represent everything you have learned over the past eight chapters. It's a summary of the "big picture," outlining the basic goals, the general principles, and the primary applications of the part of the book we have just finished. When you are immersed in a chapter, it may be hard to see the connections among all of the different topics. Before we move on to new topics, we will finish each part of the book with a knowledge structure to make these connections clear.

Work through the knowledge structure from top to bottom. First are the goals and general principles. There aren't that many general principles, but we can use them along with the general problem-solving strategy to solve a wide range of problems. Once you recognize a problem as a dynamics problem, you immediately know to start with Newton's laws. You can then determine the category of motion and apply Newton's second law in the appropriate form. The kinematic equations for that category of motion then allow you to reach the solution you seek. These equations and other detailed information from the chapters are summarized in the bottom section.

KNOWLEDGE STRUCTURE I Force and Motion

BASIC GOALS

How can we describe motion? How does an object respond to a force?
How do systems interact? What is the nature of the force of gravity?
How can we analyze the motion and deformation of extended objects?

GENERAL PRINCIPLES

Newton's first law	An object with no forces acting on it will remain at rest or move in a straight line at a constant speed.
Newton's second law	$\vec{F}_{\text{net}} = m\vec{a}$
Newton's third law	$\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}$
Newton's law of gravity	$F_{1\text{ on }2} = F_{2\text{ on }1} = \dfrac{Gm_1m_2}{r^2}$

BASIC PROBLEM-SOLVING STRATEGY

Use Newton's second law for each particle or system. Use Newton's third law to equate the magnitudes of the two members of an action/reaction pair.

Types of forces:
$\vec{w} = (mg, \text{downward})$
$\vec{f}_k = (\mu_k n, \text{opposite motion})$
$(F_{sp})_x = -k\,\Delta x$

Linear and projectile motion:

$$\left.\begin{array}{l}\sum F_x = ma_x \\ \sum F_y = 0\end{array}\right\} \text{ or } \left\{\begin{array}{l}\sum F_x = 0 \\ \sum F_y = ma_y\end{array}\right.$$

Circular motion:

The force is directed to the center:

$$\vec{F}_{\text{net}} = \left(\frac{mv^2}{r}, \text{toward center of circle}\right)$$

Rigid-body motion:

When a torque is exerted on an object with moment of inertia I,

$$\tau_{\text{net}} = I\alpha$$

Equilibrium:

For an object at rest,

$$\sum F_x = 0 \quad \sum \tau = 0$$
$$\sum F_y = 0$$

Linear and projectile kinematics

Uniform motion: $x_f = x_i + v_x\,\Delta t$
($a_x = 0$, $v_x = \text{constant}$)

Constant acceleration: $(v_x)_f = (v_x)_i + a_x\,\Delta t$
($a_x = \text{constant}$)

$$x_f = x_i + (v_x)_i\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$
$$(v_x)_f^2 = (v_x)_i^2 + 2a_x\,\Delta x$$

Projectile motion: Projectile motion is uniform horizontal motion and constant-acceleration vertical motion with $a_y = -g$.

Velocity is the slope of the position-versus-time graph.
Acceleration is the slope of the velocity-versus-time graph.

Circular kinematics

Uniform circular motion:

$$f = \frac{1}{T} \qquad \omega = 2\pi f$$
$$v = \frac{2\pi r}{T} = \omega r \qquad a = \frac{v^2}{r} = \omega^2 r$$

Rigid bodies

Torque $\tau = rF_\perp = r_\perp F$

Center of gravity $x_{cg} = \dfrac{x_1m_1 + x_2m_2 + \cdots}{m_1 + m_2 + \cdots}$

Moment of inertia $I = \sum mr^2$

Dark Matter and the Structure of the Universe

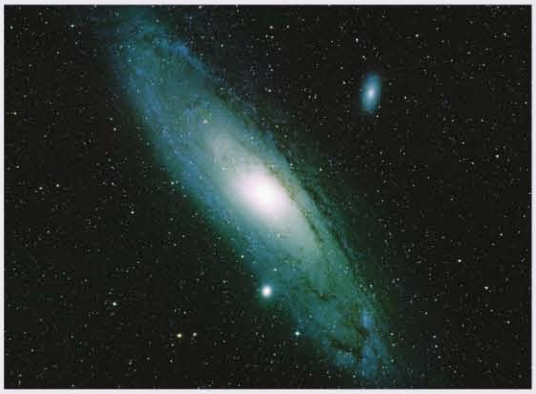
The idea that the earth exerts a gravitational force on us is something we now accept without questioning. But when Isaac Newton developed this idea to show that the gravitational force also holds the moon in its orbit, it was a remarkable, ground-breaking insight. It changed the way that we look at the universe we live in.

Newton's laws of motion and gravity are tools that allow us to continue Newton's quest to better understand our place in the cosmos. But it sometimes seems that the more we learn, the more we realize how little we actually know and understand.

Here's an example. Advances in astronomy over the past 100 years have given us great insight into the structure of the universe. But everything our telescopes can see appears to be only a small fraction of what is out there. As much as 90% of the mass in the universe is *dark matter*—matter that gives off no light or other radiation that we can detect. Everything that we have ever seen through a telescope is merely the tip of the cosmic iceberg.

What is this dark matter? Black holes? Neutrinos? Some form of exotic particle? No one knows. It could be any of these, or all of them—or something entirely different that no one has yet dreamed of. You might wonder how we know that such matter exists if no one has seen it. Even though we can't directly observe dark matter, we see its effects. And you now know enough physics to understand why.

Whatever dark matter is, it has mass, and so it has gravity. This picture of the Andromeda galaxy shows a typical spiral galaxy structure: a dense collection of stars in the center surrounded by a disk of stars and other matter. This is the shape of our own Milky Way galaxy.



The spiral Andromeda galaxy.

This structure is reminiscent of the structure of the solar system: a dense mass (the sun) in the center surrounded by a disk of other matter (the planets, asteroids, and comets). The sun's gravity keeps the planets in their orbits, but the planets would fall into the sun unless they were in constant motion around it. The same is true of a spiral galaxy; everything in the galaxy orbits its center. Our solar system orbits the center of our galaxy with a period of about 200 million years.

The orbital speed of an object depends on the mass that pulls on it. If you analyze our sun's motion about the center of the Milky Way, or the motion of stars in the Andromeda galaxy about its center, you find that the orbits are much faster than they should be, based on how many stars we see. There must be some other mass present.

There's another problem with the orbital motion of stars around the center of their galaxies. We know that the orbital speeds of planets decrease with distance from the sun; Neptune orbits at a much slower speed than the earth. We might expect something similar for galaxies: Stars farther from the center should orbit at reduced speeds. But they don't. As we measure outward from the center of the galaxy, the orbital speed stays about the same—even as we get to the edge of the visible disk. There must be some other mass—the invisible dark matter—exerting a gravitational force on the stars. This dark matter, which far outweighs the matter we can see, seems to form a halo around the centers of galaxies, providing the gravitational force necessary to produce the observed rotation. Other observations of the motions of galaxies with respect to each other verify this basic idea.

On a cosmic scale, the picture is even stranger. The universe is currently expanding. The mutual gravitational attraction of all matter—regular and dark—in the universe should slow this expansion. But recent observations of the speeds of distant galaxies imply that the expansion of the universe is accelerating, so there must be yet another component to the universe, something that "pushes out". The best explanation at present is that the acceleration is caused by *dark energy*. The nature of dark matter isn't known, but the nature of dark energy is even more mysterious. If current theories hold, it's the most abundant stuff in the universe. And we don't know what it is.

This sort of mystery is what drives scientific investigation. It's what drove Newton to wonder about the connection between the fall of an apple and the motion of the moon, and what drove investigators to develop all of the techniques and theories you will learn about in the coming chapters.

VIEW ALL SOLUTIONS

The following questions are related to the passage "Dark Matter and the Structure of the Universe" on the previous page.

1. As noted in the passage, our solar system orbits the center of the Milky Way galaxy in about 200 million years. If there were no dark matter in our galaxy, this period would be
   A. Longer.
   B. The same.
   C. Shorter.
2. Saturn is approximately 10 times as far away from the sun as the earth. This means that its orbital acceleration is ______ that of the earth.
   A. 1/10
   B. 1/100
   C. 1/1000
   D. 1/10,000
3. Saturn is approximately 10 times as far away from the sun as the earth. If dark matter changed the orbital properties of the planets so that Saturn had the same orbital speed as the earth, Saturn's orbital acceleration would be ______ that of the earth.
   A. 1/10
   B. 1/100
   C. 1/1000
   D. 1/10,000
4. Which of the following might you expect to be an additional consequence of the fact that galaxies contain more mass than expected?
   A. The gravitational force between galaxies is greater than expected.
   B. Galaxies appear less bright than expected.
   C. Galaxies are farther away than expected.
   D. There are more galaxies than expected.

The following passages and associated questions are based on the material of Part I.

Animal Athletes BIO

Different animals have very different capacities for running. A horse can maintain a top speed of 20 m/s for a long distance but has a maximum acceleration of only 6.0 m/s^2, half what a good human sprinter can achieve with a block to push against. Greyhounds, dogs especially bred for feats of running, have a top speed of 17 m/s, but their acceleration is much greater than that of the horse. Greyhounds are particularly adept at turning corners at a run.

FIGURE I.1

5. If a horse starts from rest and accelerates at the maximum value until reaching its top speed, how much time elapses, to the nearest second?
   A. 1 s B. 2 s
   C. 3 s D. 4 s
6. If a horse starts from rest and accelerates at the maximum value until reaching its top speed, how far does it run, to the nearest 10 m?
   A. 40 m B. 30 m
   C. 20 m D. 10 m
7. A greyhound on a racetrack turns a corner at a constant speed of 15 m/s with an acceleration of 7.1 m/s^2. What is the radius of the turn?
   A. 40 m B. 30 m
   C. 20 m D. 10 m
8. A human sprinter of mass 70 kg starts a run at the maximum possible acceleration, pushing backward against a block set in the track. What is the force of his foot on the block?
   A. 1500 N B. 840 N
   C. 690 N D. 420 N
9. In the photograph of the greyhounds in Figure I.1, what is the direction of the net force on each dog?
   A. Up
   B. Down
   C. Left, toward the outside of the turn
   D. Right, toward the inside of the turn

Sticky Liquids BIO

The drag force on an object moving in a liquid is quite different from that in air. Drag forces in air are largely the result of the object having to push the air out of its way as it moves. For an object moving slowly through a liquid, however, the drag force is mostly due to the *viscosity* of the liquid, a measure of how much resistance to flow the fluid has. Honey, which drizzles slowly out of its container, has a much higher viscosity than water, which flows fairly freely.

The *viscous drag* force in a liquid depends on the shape of the object, but there is a simple result called *Stokes's law* for the drag on a sphere. The drag force on a sphere of radius r moving at speed v through a fluid with viscosity η is

$$\vec{D} = (6\pi\eta r v,\ \text{direction opposite motion})$$

At small scales, viscous drag becomes very important. To a paramecium (Figure I.2), a single-celled animal that can propel itself through water with fine hairs on its body, swimming through water feels like swimming through honey would to you. We can model a paramecium as a sphere of diameter 50 μm, with a mass of 6.5×10^{-11} kg. Water has a viscosity of 0.0010 N · s/m^2.

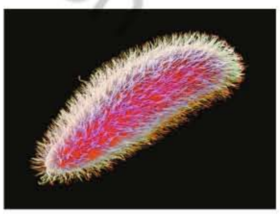


FIGURE I.2

10. A paramecium swimming at a constant speed of 0.25 mm/s ceases propelling itself and slows to a stop. At the instant it stops swimming, what is the magnitude of its acceleration?
    A. $0.2g$ B. $0.5g$
    C. $2g$ D. $5g$
11. If the acceleration of the paramecium in Problem 10 were to stay constant as it comes to rest, approximately how far would it travel before stopping?
    A. 0.02 μm B. 0.2 μm
    C. 2 μm D. 20 μm
12. If the paramecium doubles its swimming speed, how does this change the drag force?
    A. The drag force decreases by a factor of 2.
    B. The drag force is unaffected.
    C. The drag force increases by a factor of 2.
    D. The drag force increases by a factor of 4.

13. You can test the viscosity of a liquid by dropping a steel sphere into it and measuring the speed at which it sinks. For viscous fluids, the sphere will rapidly reach a terminal speed. At this terminal speed, the net force on the sphere is
    A. Directed downward.
    B. Zero.
    C. Directed upward.

Pulling Out of a Dive BIO

Falcons are excellent fliers that can reach very high speeds by diving nearly straight down. To pull out of such a dive, a falcon extends its wings and flies through a circular arc that redirects its motion. The forces on the falcon that control its motion are its weight and an upward lift force—like an airplane—due to the air flowing over its wings. At the bottom of the arc, as in Figure I.3, a falcon can easily achieve an acceleration of 15 m/s^2.

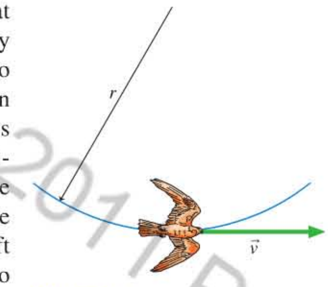


FIGURE I.3

14. At the bottom of the arc, as in Figure I.3, what is the direction of the net force on the falcon?
    A. To the left, opposite the motion
    B. To the right, in the direction of the motion
    C. Up
    D. Down
    E. The net force is zero.
15. Suppose the falcon weighs 8.0 N and is turning with an acceleration of 15 m/s^2 at the lowest point of the arc. What is the magnitude of the upward lift force at this instant?
    A. 8.0 N
    B. 12 N
    C. 16 N
    D. 20 N
16. A falcon starts from rest, does a free-fall dive from a height of 30 m, and then pulls out by flying in a circular arc of radius 50 m. Which segment of the motion has a higher acceleration?
    A. The free-fall dive
    B. The circular arc
    C. The two accelerations are equal.

Bending Beams

If you bend a rod down, it compresses the lower side of the rod and stretches the top, resulting in a restoring force. Figure I.4 shows a beam of length L, width w, and thickness t fixed at one end and free to move at the other. Deflecting the end of the beam causes a restoring force F at the end of the beam. The magnitude of the restoring force F depends on the dimensions of the beam, the Young's modulus Y for the material, and the deflection d. For small values of the deflection, the restoring force is

$$F = \left[\frac{Ywt^3}{4L^3}\right]d$$

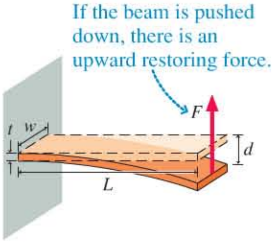


FIGURE I.4

This is similar to the formula for the restoring force of a spring, with the quantity in brackets playing the role of the spring constant k.

When a 70 kg man stands on the end of a springboard (a type of diving board), the board deflects by 4.0 cm.

17. If a 35 kg child stands at the end of the board, the deflection is
    A. 1.0 cm.
    B. 2.0 cm.
    C. 3.0 cm.
    D. 4.0 cm.
18. A 70 kg man jumps up and lands on the end of the board, deflecting it by 12 cm. At this instant, what is the approximate magnitude of the upward force the board exerts on his feet?
    A. 700 N
    B. 1400 N
    C. 2100 N
    D. 2800 N
19. If the board is replaced by one that is half the length but otherwise identical, how much will it deflect when a 70 kg man stands on the end?
    A. 0.50 cm
    B. 1.0 cm
    C. 2.0 cm
    D. 4.0 cm

Additional Integrated Problems

20. You go to the playground and slide down the slide, a 3.0-m-long ramp at an angle of 40° with respect to horizontal. The pants that you've worn aren't very slippery; the coefficient of kinetic friction between your pants and the slide is $\mu_k = 0.45$. A friend gives you a very slight push to get you started. How long does it take you to reach the bottom of the slide?
21. If you stand on a scale at the equator, the scale will read slightly less than your true weight due to your circular motion with the rotation of the earth.
    a. Draw a free-body diagram to show why this is so.
    b. By how much is the scale reading reduced for a person with a true weight of 800 N?
22. Dolphins and other sea creatures can leap to great heights by swimming straight up and exiting the water at a high speed. A 210 kg dolphin leaps straight up to a height of 7.0 m. When the dolphin reenters the water, drag from the water brings it to a stop in 1.5 m. Assuming that the force of the water on the dolphin stays constant as it slows down,
    a. How much time does it take for the dolphin to come to rest?
    b. What is the force of the water on the dolphin as it is coming to rest?