

# Mathematics Review

## Algebra

**Using exponents:**  $a^{-x} = \frac{1}{a^x}$        $a^x a^y = a^{(x+y)}$        $\frac{a^x}{a^y} = a^{(x-y)}$        $(a^x)^y = a^{xy}$

$a^0 = 1$        $a^1 = a$        $a^{1/n} = \sqrt[n]{a}$

**Fractions:**  $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$        $\frac{a/b}{c/d} = \frac{ad}{bc}$        $\frac{1}{1/a} = a$

**Logarithms:** Natural (base  $e$ ) logarithms: If  $a = e^x$ , then  $\ln(a) = x$        $\ln(e^x) = x$        $e^{\ln(x)} = x$

Base 10 logarithms: If  $a = 10^x$ , then  $\log_{10}(a) = x$        $\log_{10}(10^x) = x$        $10^{\log_{10}(x)} = x$

The following rules hold for both natural and base 10 logarithms:

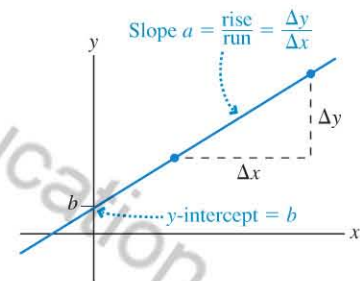
$\ln(ab) = \ln(a) + \ln(b)$        $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$        $\ln(a^n) = n \ln(a)$

The expression  $\ln(a + b)$  cannot be simplified.

**Linear equations:** The graph of the equation  $y = ax + b$  is a straight line.  $a$  is the slope of the graph.  $b$  is the  $y$ -intercept.

**Proportionality:** To say that  $y$  is proportional to  $x$ , written  $y \propto x$ , means that  $y = ax$ , where  $a$  is a constant. Proportionality is a special case of linearity. A graph of a proportional relationship is a straight line that passes through the origin. If  $y \propto x$ , then

$$\frac{y_1}{y_2} = \frac{x_1}{x_2}$$



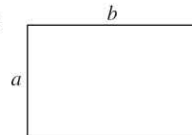
**Quadratic equation:** The quadratic equation  $ax^2 + bx + c = 0$  has the two solutions  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

## Geometry and Trigonometry

**Area and volume:**

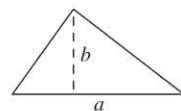
Rectangle

$$A = ab$$



Triangle

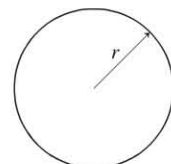
$$A = \frac{1}{2}ab$$



Circle

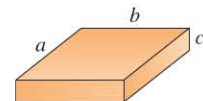
$$C = 2\pi r$$

$$A = \pi r^2$$



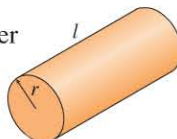
Rectangular box

$$V = abc$$



Right circular cylinder

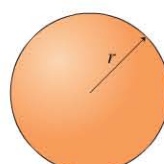
$$V = \pi r^2 l$$



Sphere

$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

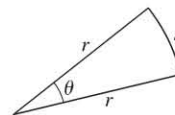


# APPENDIX A

**Arc length and angle:** The angle  $\theta$  in radians is defined as  $\theta = s/r$ .

The arc length that spans angle  $\theta$  is  $s = r\theta$ .

$$2\pi \text{ rad} = 360^\circ$$



**Right triangle:**

Pythagorean theorem  $c = \sqrt{a^2 + b^2}$  or  $a^2 + b^2 = c^2$

$$\sin \theta = \frac{b}{c} = \frac{\text{far side}}{\text{hypotenuse}}$$

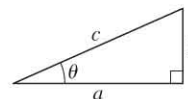
$$\theta = \sin^{-1}\left(\frac{b}{c}\right)$$

$$\cos \theta = \frac{a}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\theta = \cos^{-1}\left(\frac{a}{c}\right)$$

$$\tan \theta = \frac{b}{a} = \frac{\text{far side}}{\text{adjacent side}}$$

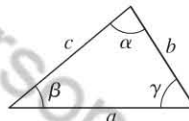
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$



In general, if it is known that sine of an angle  $\theta$  is  $x$ , so  $x = \sin \theta$ , then we can find  $\theta$  by taking the *inverse sine* of  $x$ , denoted  $\sin^{-1} x$ . Thus  $\theta = \sin^{-1} x$ . Similar relations apply for cosines and tangents.

**General triangle:**

$$\alpha + \beta + \gamma = 180^\circ = \pi \text{ rad}$$



**Identities:**

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin(-\alpha) = -\sin \alpha$$

$$\cos(-\alpha) = \cos \alpha$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

## Expansions and Approximations

**Binomial approximation:**  $(1 + x)^n \approx 1 + nx$  if  $x \ll 1$

**Small-angle approximation:** If  $\alpha \ll 1$  rad, then  $\sin \alpha \approx \tan \alpha \approx \alpha$  and  $\cos \alpha \approx 1$ .

The small-angle approximation is excellent for  $\alpha < 5^\circ$  ( $\approx 0.1$  rad) and generally acceptable up to  $\alpha \approx 10^\circ$ .

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## ActivPhysics OnLine™ Activities


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|---|--|--|
| 1.1 Analyzing Motion Using Diagrams           | 7.6 Rotational Inertia                                     | 11.12 Electric Potential, Field, and Force               |
| 1.2 Analyzing Motion Using Graphs             | 7.7 Rotational Kinematics                                  | 11.13 Electrical Potential Energy and Potential          |
| 1.3 Predicting Motion from Graphs             | 7.8 Rotoride: Dynamics Approach                            | 12.1 DC Series Circuits (Qualitative)                    |
| 1.4 Predicting Motion from Equations          | 7.9 Falling Ladder   | 12.2 DC Parallel Circuits                                |
| 1.5 Problem-Solving Strategies for Kinematics | 7.10 Woman and Flywheel Elevator: Dynamics Approach        | 12.3 DC Circuit Puzzles                                  |
| 1.6 Skier Races Downhill                      | 7.11 Race Between a Block and a Disk                       | 12.4 Using Ammeters and Voltmeters                       |
| 1.7 Balloonist Drops Lemonade                 | 7.12 Woman and Flywheel Elevator: Energy Approach          | 12.5 Using Kirchhoff's Laws                              |
| 1.8 Seat Belts Save Lives                     | 7.13 Rotoride: Energy Approach                             | 12.6 Capacitance   |
| 1.9 Screeching to a Halt                      | 7.14 Ball Hits Bat   | 12.7 Series and Parallel Capacitors                      |
| 1.10 Pole-Vaulter Lands                       | 8.1 Characteristics of a Gas                               | 12.8 RC Circuit Time Constants                           |
| 1.11 Car Starts, Then Stops                   | 8.2 Maxwell-Boltzmann Distribution: Conceptual Analysis    | 13.1 Magnetic Field of a Wire                            |
| 1.12 Solving Two-Vehicle Problems             | 8.3 Maxwell-Boltzmann Distribution: Quantitative Analysis  | 13.2 Magnetic Field of a Loop                            |
| 1.13 Car Catches Truck                        | 8.4 State Variables and Ideal Gas Law                      | 13.3 Magnetic Field of a Solenoid                        |
| 1.14 Avoiding a Rear-End Collision            | 8.5 Work Done by a Gas                                     | 13.4 Magnetic Force on a Particle                        |
| 2.1.1 Force Magnitudes                        | 8.6 Heat, Internal Energy, and First Law of Thermodynamics | 13.5 Magnetic Force on a Wire                            |
| 2.1.2 Skydiver                                | 8.7 Heat Capacity  | 13.6 Magnetic Torque on a Loop                           |
| 2.1.3 Tension Change                          | 8.8 Isochoric Process                                      | 13.7 Mass Spectrometer                                   |
| 2.1.4 Sliding on an Incline                   | 8.9 Isobaric Process                                       | 13.8 Velocity Selector                                   |
| 2.1.5 Car Race                                | 8.10 Isothermal Process                                    | 13.9 Electromagnetic Induction                           |
| 2.2 Lifting a Crate                           | 8.11 Adiabatic Process                                     | 13.10 Motional emf                                       |
| 2.3 Lowering a Crate                          | 8.12 Cyclic Process: Strategies                            | 14.1 The RL Circuit                                      |
| 2.4 Rocket Blasts Off                         | 8.13 Cyclic Process: Problems                              | 14.2 The RLC Oscillator                                  |
| 2.5 Truck Pulls Crate                         | 8.14 Carnot Cycle  | 14.3 The Driven Oscillator                               |
| 2.6 Pushing a Crate Up a Wall                 | 9.1 Position Graphs and Equations                          | 15.1 Reflection and Refraction                           |
| 2.7 Skier Goes Down a Slope                   | 9.2 Describing Vibrational Motion                          | 15.2 Total Internal Reflection                           |
| 2.8 Skier and Rope Tow                        | 9.3 Vibrational Energy                                     | 15.3 Refraction Applications                             |
| 2.9 Pole-Vaulter Vaults                       | 9.4 Two Ways to Weigh Young Tarzan                         | 15.4 Plane Mirrors                                       |
| 2.10 Truck Pulls Two Crates                   | 9.5 Ape Drops Tarzan                                       | 15.5 Spherical Mirrors: Ray Diagrams                     |
| 2.11 Modified Atwood Machine                  | 9.6 Releasing a Vibrating Skier I                          | 15.6 Spherical Mirror: The Mirror Equation               |
| 3.1 Solving Projectile Motion Problems        | 9.7 Releasing a Vibrating Skier II                         | 15.7 Spherical Mirror: Linear Magnification              |
| 3.2 Two Balls Falling                         | 9.8 One- and Two-Spring Vibrating Systems                  | 15.8 Spherical Mirror: Problems                          |
| 3.3 Changing the $x$ -Velocity                | 9.9 Vibro-Ride   | 15.9 Thin-Lens Ray Diagrams                              |
| 3.4 Projectile $x$ - and $y$ -Accelerations   | 9.10 Pendulum Frequency                                    | 15.10 Converging Lens Problems                           |
| 3.5 Initial Velocity Components               | 9.12 Risky Pendulum Walk                                   | 15.11 Diverging Lens Problems                            |
| 3.6 Target Practice I                         | 9.13 Physical Pendulum                                     | 15.12 Two-Lens Optical Systems                           |
| 3.7 Target Practice II                        | 10.1 Properties of Mechanical Waves                        | 16.1 Two-Source Interference: Introduction               |
| 4.1 Magnitude of Centripetal Acceleration     | 10.2 Speed of Waves on a String                            | 16.2 Two-Source Interference: Qualitative Questions      |
| 4.2 Circular Motion Problem Solving           | 10.3 Speed of Sound in a Gas                               | 16.3 Two-Source Interference: Problems                   |
| 4.3 Cart Goes Over Circular Path              | 10.4 Standing Waves on Strings                             | 16.4 The Grating: Introduction and Qualitative Questions |
| 4.4 Ball Swings on a String                   | 10.5 Tuning a Stringed Instrument: Standing Waves          | 16.5 The Grating: Problems                               |
| 4.5 Car Circles a Track                       | 10.6 String Mass and Standing Waves                        | 16.6 Single-Slit Diffraction                             |
| 4.6 Satellites Orbit                          | 10.7 Beats and Beat Frequency                              | 16.7 Circular Hole Diffraction                           |
| 5.1 Work Calculations                         | 10.8 Doppler Effect: Conceptual Introduction               | 16.8 Resolving Power                                     |
| 5.2 Upward-Moving Elevator Stops              | 10.9 Doppler Effect: Problems                              | 16.9 Polarization  |
| 5.3 Stopping a Downward-Moving Elevator       | 10.10 Complex Waves: Fourier Analysis                      | 17.1 Relativity of Time                                  |
| 5.4 Inverse Bungee Jumper                     | 11.1 Electric Force: Coulomb's Law                         | 17.2 Relativity of Length                                |
| 5.5 Spring-Launched Bowler                    | 11.2 Electric Force: Superposition Principle               | 17.3 Photoelectric Effect                                |
| 5.6 Skier Speed                               | 11.3 Electric Force Superposition Principle (Quantitative) | 17.4 Compton Scattering                                  |
| 5.7 Modified Atwood Machine                   | 11.4 Electric Field: Point Charge                          | 17.5 Electron Interference                               |
| 6.1 Momentum and Energy Change                | 11.5 Electric Field Due to a Dipole                        | 17.6 Uncertainty Principle                               |
| 6.2 Collisions and Elasticity                 | 11.6 Electric Field: Problems                              | 17.7 Wave Packets  |
| 6.3 Momentum Conservation and Collisions      | 11.7 Electric Flux   | 18.1 The Bohr Model                                      |
| 6.4 Collision Problems                        | 11.8 Gauss's Law   | 18.2 Spectroscopy  |
| 6.5 Car Collision: Two Dimensions             | 11.9 Motion of a Charge in an Electric Field: Introduction | 18.3 The Laser   |
| 6.6 Saving an Astronaut                       | 11.10 Motion in an Electric Field: Problems                | 19.1 Particle Scattering                                 |
| 6.7 Explosion Problems                        | 11.11 Electric Potential: Qualitative Introduction         | 19.2 Nuclear Binding Energy                              |
| 6.8 Skier and Cart                            |  | 19.3 Fusion  |
| 6.9 Pendulum Bashes Box                       |  | 19.4 Radioactivity                                       |
| 6.10 Pendulum Person-Projectile Bowling       |  | 19.5 Particle Physics                                    |
| 7.1 Calculating Torques                       |  | 20.1 Potential Energy Diagrams                           |
| 7.2 A Tilted Beam: Torques and Equilibrium    |  | 20.2 Particle in a Box                                   |
| 7.3 Arm Levers                                |  | 20.3 Potential Wells                                     |
| 7.4 Two Painters on a Beam                    |  | 20.4 Potential Barriers                                  |
| 7.5 Lecturing from a Beam                     |  |  |

## Atomic and Nuclear Data

Atomic Number (Z)	Element	Symbol	Mass Number (A)	Atomic Mass (u)	Percent Abundance	Decay Mode	Half-Life $t_{1/2}$
0	(Neutron)	n	1	1.008 665		$\beta^-$	10.4 min
1	Hydrogen	H	1	1.007 825	99.985	stable	
	Deuterium	D	2	2.014 102	0.015	stable	
	Tritium	T	3	3.016 049		$\beta^-$	12.33 yr
2	Helium	He	3	3.016 029	0.000 1	stable	
			4	4.002 602	99.999 9	stable	
			6	6.018 886		$\beta^-$	0.81 s
3	Lithium	Li	6	6.015 121	7.50	stable	
			7	7.016 003	92.50	stable	
			8	8.022 486		$\beta^-$	0.84 s
4	Beryllium	Be	9	9.012 174	100	stable	
			10	10.013 534		$\beta^-$	$1.5 \times 10^6$ yr
5	Boron	B	10	10.012 936	19.90	stable	
			11	11.009 305	80.10	stable	
			12	12.014 352		$\beta^-$	0.020 2 s
6	Carbon	C	10	10.016 854		$\beta^+$	19.3 s
			11	11.011 433		$\beta^+$	20.4 min
			12	12.000 000	98.90	stable	
			13	13.003 355	1.10	stable	
			14	14.003 242		$\beta^-$	5 730 yr
			15	15.010 599		$\beta^-$	2.45 s
7	Nitrogen	N	12	12.018 613		$\beta^+$	0.011 0 s
			13	13.005 738		$\beta^+$	9.96 min
			14	14.003 074	99.63	stable	
			15	15.000 108	0.37	stable	
			16	16.006 100		$\beta^-$	7.13 s
			17	17.008 450		$\beta^-$	4.17 s
8	Oxygen	O	15	15.003 065		$\beta^+$	122 s
			16	15.994 915	99.76	stable	
			17	16.999 132	0.04	stable	
			18	17.999 160	0.20	stable	
			19	19.003 577		$\beta^-$	26.9 s
9	Fluorine	F	18	18.000 937		$\beta^+$	109.8 min
			19	18.998 404	100	stable	
			20	19.999 982		$\beta^-$	11.0 s
10	Neon	Ne	19	19.001 880		$\beta^+$	17.2 s
			20	19.992 435	90.48	stable	
			21	20.993 841	0.27	stable	
			22	21.991 383	9.25	stable	
17	Chlorine	Cl	35	34.968 853	75.77	stable	
			36	35.968 307		$\beta^-$	$3.0 \times 10^5$ yr
			37	36.965 903	24.23	stable	

# APPENDIX D

Atomic Number (Z)	Element	Symbol	Mass Number (A)	Atomic Mass (u)	Percent Abundance	Decay Mode	Half-Life $t_{1/2}$
18	Argon	Ar	36	35.967 547	0.34	stable	269 yr
			38	37.962 732	0.06	stable	
			39	38.964 314		$\beta^-$	
			40	39.962 384	99.60	stable	
			42	41.963 049		$\beta^-$	
19	Potassium	K	39	38.963 708	93.26	stable	$1.28 \times 10^9$ yr
			40	39.964 000	0.01	$\beta^+$	
			41	40.961 827	6.73	stable	
26	Iron	Fe	54	54.939 613	5.9	stable	$1.5 \times 10^6$ yr
			56	55.934 940	91.72	stable	
			57	56.935 396	2.1	stable	
			58	57.933 278	0.28	stable	
			60	59.934 072		$\beta^-$	
27	Cobalt	Co	59	58.933 198	100	stable	5.27 yr
			60	59.933 820		$\beta^-$	
38	Strontium	Sr	84	83.913 425	0.56%	stable	50.53 days
			86	85.909 262	9.86%	stable	
			87	86.908 879	7.00%	stable	
			88	87.905 614	82.58%	stable	
			89	88.907 450		$\beta^-$	
53	Iodine	I	90	89.907 738		$\beta^-$	27.78 yr
			127	126.904 474	100	stable	$1.6 \times 10^7$ yr
			129	128.904 984		$\beta^-$	
54	Xenon	Xe	131	130.906 124		$\beta^-$	8 days
			128	127.903 531	1.9	stable	5.4 days
			129	128.904 779	26.4	stable	
			130	129.903 509	4.1	stable	
			131	130.905 069	21.2	stable	
			132	131.904 141	26.9	stable	
			133	132.905 906		$\beta^-$	
			134	133.905 394	10.4	stable	
55	Cesium	Cs	136	135.907 215	8.9	stable	30 yr
			133	132.905 436	100	stable	
82	Lead	Pb	137	136.907 078		$\beta^-$	22.3 yr
			204	203.973 020	1.4	stable	
			206	205.974 440	24.1	stable	
			207	206.975 871	22.1	stable	
			208	207.976 627	52.4	stable	
			210	209.984 163		$\alpha, \beta^-$	
83	Bismuth	Bi	211	210.988 734		$\beta^-$	36.1 min
			209	208.980 374	100	stable	2.14 min
			211	210.987 254		$\alpha$	
86	Radon	Rn	215	215.001 836		$\beta^-$	7.4 min
			219	219.009 477		$\alpha$	3.96 s
			220	220.011 369		$\alpha$	55.6 s
			222	222.017 571		$\alpha, \beta^-$	3.823 days



Atomic Number (Z)	Element	Symbol	Mass Number (A)	Atomic Mass (u)	Percent Abundance	Decay Mode	Half-Life $t_{1/2}$
88	Radium	Ra	223	223.018 499		$\alpha$	11.43 days
			224	224.020 187		$\alpha$	3.66 days
			226	226.025 402		$\alpha$	1 600 yr
			228	228.031 064		$\beta^-$	5.75 yr
90	Thorium	Th	227	227.027 701		$\alpha$	18.72 days
			228	228.028 716		$\alpha$	1.913 yr
			229	229.031 757		$\alpha$	7 300 yr
			230	230.033 127		$\alpha$	75.000 yr
			231	231.036 299		$\alpha, \beta^-$	25.52 hr
			232	232.038 051	100	$\alpha$	$1.40 \times 10^{10}$ yr
			234	234.043 593		$\beta^-$	24.1 days
			234	234.043 593		$\alpha$	24.1 days
92	Uranium	U	233	233.039 630		$\alpha$	$1.59 \times 10^5$ yr
			234	234.040 946		$\alpha$	$2.45 \times 10^5$ yr
			235	235.043 924	0.72	$\alpha$	$7.04 \times 10^8$ yr
			236	236.045 562		$\alpha$	$2.34 \times 10^7$ yr
			238	238.050 784	99.28	$\alpha$	$4.47 \times 10^9$ yr
93	Neptunium	Np	237	237.048 168		$\alpha$	$2.14 \times 10^6$ yr
			238	238.050 946		$\beta^-$	2.12 days
			239	239.052 939		$\beta^-$	2.36 days
94	Plutonium	Pu	238	238.049 555		$\alpha$	87.7 yr
			239	239.052 157		$\alpha$	$2.412 \times 10^4$ yr
			240	240.053 808		$\alpha$	6 560 yr
			242	242.058 737		$\alpha$	$3.73 \times 10^6$ yr
			244	244.064 200		$\alpha$	$8.1 \times 10^7$ yr
95	Americium	Am	241	241.056 823		$\alpha$	432.21 yr
			243	243.061 375		$\alpha$	73.070 yr