10 Energy and Work

As this bungee jumper falls, he gains kinetic energy, the energy of motion. Where does this energy come from? And where does it go as he slows at the bottom of his fall?

LOOKING AHEAD ➤

The goals of Chapter 10 are to introduce the concept of energy and to learn a new problem-solving strategy based on conservation of energy.

Forms of Energy

A principal goal of this chapter is to learn about several important forms of energy.

Kinetic energy is the energy of motion. This heavy, fast-moving elephant has lots of kinetic energy.

These passengers gain **potential energy,** the energy of position, as they ride up the escalator.

The **thermal energy** of this red-hot horseshoe is associated with the microscopic motion of its molecules.

Looking Back ◀◀
2.5 Motion with constant acceleration
7.1, 7.4 Rotation and moment of inertia
8.3 Hooke's law

The Law of Conservation of Energy

One of the most fundamental laws of physics, the **law of conservation of energy** states that the total energy of an isolated system is a constant.

How fast are these water sliders moving at the bottom? How fast does the rock fly out of the slingshot? We'll use conservation of energy and the before-and-after analysis introduced in Chapter 9 to solve these kinds of problems.

Looking Back ◀◀
9.2–9.3 Before-and after visual overviews

Transferring Energy

Energy can be *transferred* into a system by pushing on it, a process called **work.**

The bobsledders do work on the sled, *transferring* energy to it and causing it to speed up.

Transforming Energy

Energy of one kind can change into energy of a different kind. These **energy transformations** are what make the world an interesting place.

As this race car skids to a stop, its kinetic energy is being *transformed* into thermal energy, making the tires hot enough to smoke.

Power

We're very often interested in **power,** the *rate* at which energy is transformed from one kind into another.

As they climb, this truck and these jets both transform the chemical energy of their fuel into potential energy. But the jet engines transform energy at a rate 70 times that of the truck's engine—

10.1 The Basic Energy Model

Energy. It's a word you hear all the time. We use chemical energy to heat our homes and bodies, electric energy to run our lights and computers, and solar energy to grow our crops and forests. We're told to use energy wisely and not to waste it. Athletes and weary students consume "energy bars" and "energy drinks."

But just what is energy? The concept of energy has grown and changed over time, and it is not easy to define in a general way just what energy is. Rather than starting with a formal definition, we'll let the concept of energy expand slowly over the course of several chapters. In this chapter we introduce several fundamental forms of energy, including kinetic energy, potential energy, and thermal energy. Our goal is to understand the characteristics of energy, how energy is used, and, especially important, how energy is transformed from one form into another. Much of modern technology is concerned with transforming energy, such as changing the chemical energy of oil molecules into electric energy or into the kinetic energy of your car.

We'll also learn how energy can be transferred to or from a system by the application of mechanical forces. By pushing on a sled, you increase its speed, and hence its energy of motion. By lifting a heavy object, you increase its gravitational potential energy.

These observations will lead us to discover a very powerful conservation law for energy. Energy is neither created nor destroyed: If one form of energy in a system decreases, it must appear in an equal amount in another form. Many scientists consider the law of conservation of energy to be the most important of all the laws of nature. This law will have implications throughout the rest of this book.

Systems and Energy

In Chapter 9 we introduced the idea of a *system* of interacting objects. A system can be as simple as a falling acorn or as complex as a city. But whether simple or complex, every system in nature has associated with it a quantity we call its **total energy** E. The total energy is the sum of the different kinds of energies present in the system. In the table below, we give a brief overview of some of the more important forms of energy; in the rest of the chapter, we'll look at several of these forms of energy in much greater detail.

A system may have many of these kinds of energy at one time. For instance, a moving car has kinetic energy of motion, chemical energy stored in its gasoline, thermal energy in its hot engine, and many other forms of energy. The total energy of the system, E, is the *sum* of all the different energies present in the system:

$$E = K + U_g + U_s + E_{th} + E_{chem} + \cdots \qquad (10.1)$$

The energies shown in this sum are the forms of energy in which we'll be most interested in this and the next chapter. The ellipses ($\cdots$) stand for other forms of energy, such as nuclear or electric, that also might be present. We'll treat these and others in later chapters.

Some important forms of energy

Kinetic energy K

Kinetic energy is the energy of *motion.* All moving objects have kinetic energy. The heavier an object and the faster it moves, the more kinetic energy it has. The wrecking ball in this picture is effective in part because of its large kinetic energy.

Gravitational potential energy U_g

Gravitational potential energy is *stored* energy associated with an object's *height above the ground.* As this coaster ascends, energy is stored as gravitational potential energy. As it descends, this stored energy is converted into kinetic energy.

Elastic or spring potential energy U_s

Elastic potential energy is energy stored when a spring or other elastic object, such as this archer's bow, is *stretched.* This energy can later be transformed into the kinetic energy of the arrow.

Continued

Thermal energy E_{th}

Hot objects have more *thermal energy* than cold ones because the molecules in a hot object jiggle around more than those in a cold object. Thermal energy is the sum of the microscopic kinetic and potential energies of all the molecules in an object. In boiling water, some molecules have enough energy to escape the water as steam.

Chemical energy E_{chem}

Electric forces cause atoms to bind together to make molecules. Energy can be stored in these bonds, energy that can later be released as the bonds are rearranged during chemical reactions. When we burn fuel to run our car or eat food to power our bodies, we are using *chemical energy.*

Nuclear energy $E_{nuclear}$

An enormous amount of energy is stored in the *nucleus,* the tiny core of an atom. Certain nuclei can be made to break apart, releasing some of this *nuclear energy,* which is transformed into the kinetic energy of the fragments and then into thermal energy. The ghostly blue glow of a nuclear reactor results from high-energy fragments as they travel through water.

Energy Transformations

We've seen that all systems contain energy in many different forms. But if the amounts of each form of energy never changed, the world would be a very dull place. What makes the world interesting is that **energy of one kind can be *transformed* into energy of another kind.** The gravitational potential energy of the roller coaster at the top of the track is rapidly transformed into kinetic energy as the coaster descends; the chemical energy of gasoline is transformed into the kinetic energy of your moving car. The following table illustrates a few common energy transformations. In this table, we use an arrow $\rightarrow$ as a shorthand way of representing an energy transformation.

Some energy transformations

A weightlifter lifts a barbell over her head
The barbell has much more gravitational potential energy when high above her head than when on the floor. To lift the barbell, she is transforming chemical energy in her body into gravitational potential energy of the barbell.

$$E_{chem} \rightarrow U_g$$

A base runner slides into the base
When running, he has lots of kinetic energy. After sliding, he has none. His kinetic energy is transformed mainly into thermal energy: The ground and his legs are slightly warmer.

$$K \rightarrow E_{th}$$

A burning campfire
The wood contains considerable chemical energy. When the carbon in the wood combines chemically with oxygen in the air, this chemical energy is transformed largely into thermal energy of the hot gases and embers.

$$E_{chem} \rightarrow E_{th}$$

A springboard diver
Here's a two-step energy transformation. At the instant shown, the board is flexed to its maximum extent, so that elastic potential energy stored in the board. Soon this energy will begin to be transformed into kinetic energy; as the diver rises into the air and slows, this kinetic energy will be transformed into gravitational potential energy.

$$U_s \rightarrow K \rightarrow U_g$$

FIGURE 10.1 Energy transformations occur within the system.

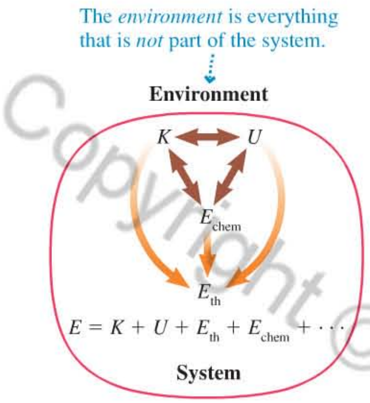


FIGURE 10.1 reinforces the idea that **energy transformations are changes of energy *within* the system from one form to another.** (The U in this figure is a generic potential energy; it could be gravitational potential energy U_g, spring potential energy U_s, or some other form of potential energy.) Note that it is easy to convert kinetic, potential, and chemical energies into thermal energy, but converting thermal energy back into these other forms is not so easy. How it can be done, and what possible limitations there might be in doing so, will form a large part of the next chapter.

Energy Transfers and Work

We've just seen that energy *transformations* occur between forms of energy *within* a system. But every physical system also interacts with the world around it—that is, with its *environment.* In the course of these interactions, the system can exchange energy with the environment. **An exchange of energy between system and environment is called an energy *transfer*.** There are two primary energy-transfer processes: **work,** the *mechanical* transfer of energy to or from a system by pushing or pulling on it, and **heat,** the *nonmechanical* transfer of energy from the environment to the system (or vice versa) because of a temperature difference between the two.

FIGURE 10.2 The basic energy model shows that work and heat are energy transfers into and out of the system, while energy transformations occur within the system.

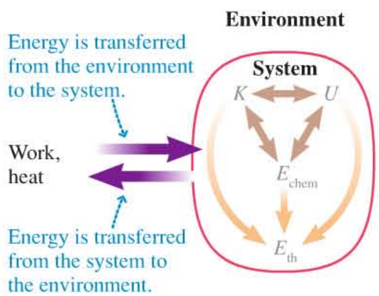


FIGURE 10.2, which we call the **basic energy model,** shows how our energy model is modified to include energy transfers into and out of the system as well as energy transformations within the system. In this chapter we'll consider only energy transfers by means of work; the concept of heat will be developed much further in Chapters 11 and 12.

"Work" is a common word in the English language, with many meanings. When you first think of work, you probably think of physical effort or the job you do to make a living. After all, we talk about "working out," or we say, "I just got home from work." But that is not what work means in physics.

In physics, "work" is the process of *transferring* energy from the environment to a system, or from a system to the environment, by the application of mechanical forces—pushes and pulls—to the system. Once the energy has been transferred to the system, it can appear in many forms. Exactly what form it takes depends on the details of the system and how the forces are applied. The table below gives three examples of energy transfers due to work. We use W as the symbol for work.

Energy transfers: work

Putting a shot

The system: The shot

The environment: The athlete

As the athlete pushes on the shot to get it moving, he is doing work on the system; that is, he is transferring energy from himself to the ball. The energy transferred to the system appears as kinetic energy.

The transfer: $W \rightarrow K$

Striking a match

The system: The match and matchbox

The environment: The hand

As the hand quickly pulls the match across the box, the hand does work on the system, increasing its thermal energy. The match head becomes hot enough to ignite.

The transfer: $W \rightarrow E_{th}$

Firing a slingshot

The system: The slingshot

The environment: The boy

As the boy pulls back on the elastic bands, he does work on the system, increasing its elastic potential energy.

The transfer: $W \rightarrow U_s$

Notice that in each example on the previous page, the environment applies a force while the system undergoes a *displacement.* Energy is transferred as work only when the system *moves* while the force acts. A force applied to a stationary object, such as when you push against a wall, transfers no energy to the object and thus does no work.

NOTE ► In the table on the previous page, energy is being transferred *from* the athlete *to* the shot by the force of his hand. We say he "does work" on the shot, or "work is done" by the force of his hand. ◄

The Law of Conservation of Energy

Work done on a system represents energy that is transferred into or out of the system. This transferred energy *changes* the system's energy by exactly the amount of work W that was done. Writing the change in the system's energy as ΔE, we can represent this idea mathematically as

$$\Delta E = W \tag{10.2}$$

Now the total energy E of a system is, according to Equation 10.1, the sum of the different energies present in the system. Thus the change in E is the sum of the *changes* of the different energies present. Then Equation 10.2 gives what is called the *work-energy equation:*

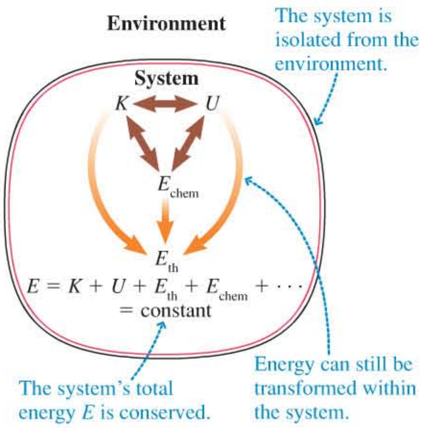
The work-energy equation The total energy of a system changes by the amount of work done on it:

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = W \tag{10.3}$$

NOTE ► Equation 10.3, the work-energy equation, is the mathematical representation of the basic energy model of Figure 10.2. Together, they are the heart of what the subject of energy is all about. ◄

Suppose now we have an **isolated system,** one that is separated from its surrounding environment in such a way that no energy is transferred into or out of the system. This means that *no work is done on the system*. The energy within the system may be transformed from one form into another, but it is a deep and remarkable fact of nature that, during these transformations, the total energy of an isolated system—the *sum* of all the individual kinds of energy—remains *constant,* as shown in **FIGURE 10.3**. We say that **the total energy of an isolated system is *conserved*.**

FIGURE 10.3 An isolated system.



For an isolated system, we must set $W = 0$ in Equation 10.3, leading to the following *law of conservation of energy:*

Law of conservation of energy The total energy of an isolated system remains constant:

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = 0 \tag{10.4}$$

The law of conservation of energy is similar to the law of conservation of momentum. Momentum changes when an impulse acts on a system; the total momentum of an isolated system doesn't change. Similarly, energy changes when external forces do work on a system; the total energy of an isolated system doesn't change.

In solving momentum problems, we adopted a new before-and-after perspective: The momentum *after* an interaction was the same as the momentum *before* the interaction. We will introduce a similar before-and-after perspective for energy that will lead to an extremely powerful problem-solving strategy.

Before using energy ideas to solve problems, however, we first need to develop quantitative expressions for work, kinetic energy, potential energy, and thermal energy. This will be our task in the next several sections.

STOP TO THINK 10.1 A child slides down a playground slide at constant speed. The energy transformation is

A. $U_g \rightarrow K$ B. $K \rightarrow U_g$ C. $W \rightarrow K$ D. $U_g \rightarrow E_{th}$ E. $K \rightarrow E_{th}$

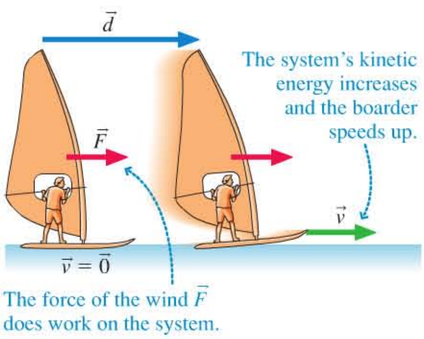
10.2 Work

Our first task is to learn how work is calculated. We've just seen that work is the transfer of energy to or from a system by the application of forces exerted on the system by the environment. Thus work is done on a system by forces from *outside* the system; we call such forces *external forces*. Only external forces can change the energy of a system. *Internal forces*—forces between objects *within* the system—cause energy transformations within the system but don't change the system's total energy.

We also learned that in order for energy to be transferred as work, the system must undergo a displacement—it must *move*—during the time that the force is applied. Let's further investigate the relationship among work, force, and displacement.

Consider a system consisting of a windsurfer at rest, as shown on the left in FIGURE 10.4. Let's assume that there is no friction between his board and the water. Initially the system has no kinetic energy. But if a force from outside the system, such as the force due to the wind, begins to act on the system, the surfer will begin to speed up, and his kinetic energy will increase. In terms of energy transfers, we would say that the energy of the system has increased because of the work done on the system by the force of the wind.

FIGURE 10.4 The force of the wind does work on the system, increasing its kinetic energy K.



What determines how much work is done by the force of the wind? First, we note that the greater the distance over which the wind pushes the surfer, the faster the surfer goes, and the more his kinetic energy increases. This implies a greater transfer of energy. So, **the larger the displacement, the greater the work done.** Second, if the wind pushes with a stronger force, the surfer speeds up more rapidly, and the change in his kinetic energy is greater than with a weaker force. **The stronger the force, the greater the work done.**

This experiment suggests that the amount of energy transferred to a system by a force $\vec{F}$—that is, the amount of work done by $\vec{F}$—depends on both the magnitude F of the force *and* the displacement d of the system. Many experiments of this kind have established that the amount of work done by $\vec{F}$ is *proportional* to both F and d. For the simplest case described above, where the force $\vec{F}$ is constant and points in the direction of the object's displacement, the expression for the work done is found to be

$$W = Fd \tag{10.5}$$

Work done by a constant force $\vec{F}$ in the direction of a displacement $\vec{d}$

The unit of work, that of force multiplied by distance, is $N \cdot m$. This unit is so important that it has been given its own name, the **joule** (rhymes with *tool*). We define:

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ N} \cdot \text{m}$$

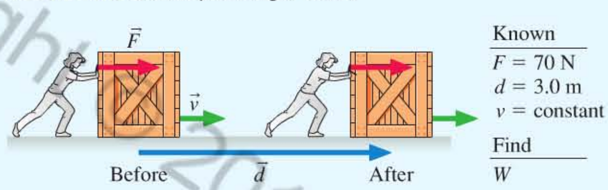
Because work is simply energy being transferred, **the joule is the unit of *all* forms of energy.** Note that work is a *scalar* quantity.

EXAMPLE 10.1 **Work done in pushing a crate**

Sarah pushes a heavy crate 3.0 m along the floor at a constant speed. She pushes with a constant horizontal force of magnitude 70 N. How much work does Sarah do on the crate?

PREPARE We begin with the visual overview in FIGURE 10.5. Sarah pushes with a constant force in the direction of the crate's motion, so we can use Equation 10.5 to find the work done.

FIGURE 10.5 Sarah pushing a crate.



SOLVE The work done by Sarah is

$$W = Fd = (70 \text{ N})(3.0 \text{ m}) = 210 \text{ J}$$

ASSESS Work represents a transfer of energy into a system, so here the energy of the system—the box and the floor—increases. Unlike the windsurfer, the box doesn't speed up, so its kinetic energy doesn't increase. Instead, the work increases the thermal energy in the crate and the part of the floor along which it slides, increasing the temperature of both. Using the notation of Equation 10.3, we can write this energy transfer as $\Delta E_{th} = W$.

Force at an Angle to the Displacement

A force does the greatest possible amount of work on an object when the force points in the same direction as the object's displacement. Less work is done when the force acts at an angle to the displacement. To see this, consider the kite buggy of FIGURE 10.6a, pulled along a horizontal path by the angled force of the kite string $\vec{F}$. As shown in FIGURE 10.6b, we can divide $\vec{F}$ into a component $F_\perp$ perpendicular to the motion, and a component $F_\parallel$ parallel to the motion. Only the parallel component acts to accelerate the rider and increase his kinetic energy, so only the parallel component does work on the rider. From Figure 10.6b, we see that if the angle between $\vec{F}$ and the displacement is θ, then the parallel component is $F_\parallel = F\cos\theta$. So, when the force acts at an angle θ to the direction of the displacement, we have

$$W = F_\parallel d = Fd\cos\theta \qquad (10.6)$$

Work done by a constant force $\vec{F}$ at an angle θ to the displacement $\vec{d}$

Notice that this more general definition of work agrees with Equation 10.5 if $\theta = 0°$.

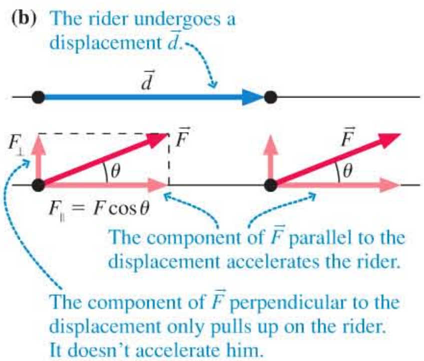
Tactics Box 10.1 shows how to calculate the work done by a force at any angle to the direction of motion. The system illustrated is a block sliding on a frictionless, horizontal surface, so that only the kinetic energy is changing. However, the same relationships hold for any object undergoing a displacement.

The quantities F and d are always positive, so **the sign of W is determined entirely by the angle θ between the force and the displacement.** Note that Equation 10.6, $W = Fd\cos\theta$, is valid for any angle θ. In three special cases, $\theta = 0°$, $\theta = 90°$, and $\theta = 180°$, however, there are simple versions of Equation 10.6 that you can use. These are noted in Tactics Box 10.1.

FIGURE 10.6 Finding the work done when the force is at an angle to the displacement.

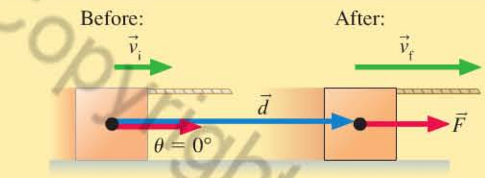
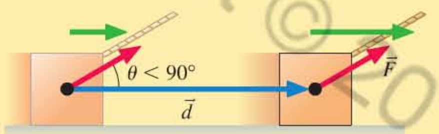
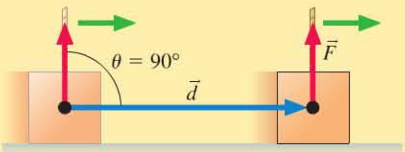
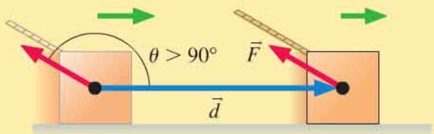
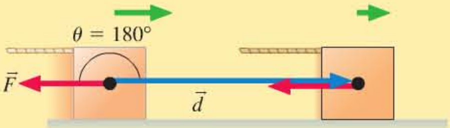
(a)

(b)



ActivPhysics ONLINE 5.1

TACTICS BOX 10.1 Calculating the work done by a constant force

Direction of force relative to displacement	Angles and work done	Sign of W	Energy transfer
	$\theta = 0°$ $\cos\theta = 1$ $W = Fd$	+	The force is in the direction of motion. The block has its greatest positive acceleration. K increases the most: **Maximum energy transfer to system.**
	$\theta < 90°$ $W = Fd\cos\theta$	+	The component of force parallel to the displacement is less than F. The block has a smaller positive acceleration. K increases less: **Decreased energy transfer to system.**
	$\theta = 90°$ $\cos\theta = 0$ $W = 0$	0	There is no component of force in the direction of motion. The block moves at constant speed. No change in K: **No energy transferred.**
	$\theta > 90°$ $W = Fd\cos\theta$	−	The component of force parallel to the displacement is opposite the motion. The block slows down, and K decreases: **Decreased energy transfer *out* of system.**
	$\theta = 180°$ $\cos\theta = -1$ $W = -Fd$	−	The force is directly opposite the motion. The block has its greatest deceleration. K decreases the most: **Maximum energy transfer *out* of system.**

Exercises 5–6

EXAMPLE 10.2 Work done in pulling a suitcase

A strap inclined upward at a 45° angle pulls a suitcase through the airport. The tension in the strap is 20 N. How much work does the tension do if the suitcase is pulled 100 m at a constant speed?

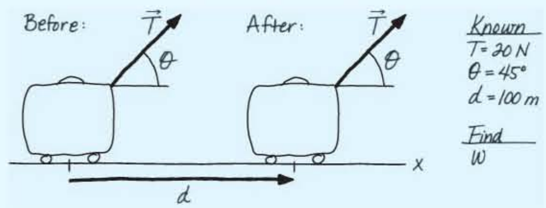
PREPARE FIGURE 10.7 shows a visual overview. Since the suitcase moves at a constant speed, there must be a rolling friction force acting to the left.

SOLVE We can use Equation 10.6, with force $F = T$, to find that the tension does work:

$$W = Td\cos\theta = (20\text{ N})(100\text{ m})\cos 45° = 1400\text{ J}$$

ASSESS Because a person is pulling on the other end of the strap, causing the tension, we would say informally that the person does 1400 J of work on the suitcase. This work represents energy transferred into the suitcase + floor system. Since the suitcase moves at a constant speed, the system's kinetic energy doesn't change. Thus, just as for Sarah pushing the crate in Example 10.1, the work done goes entirely into increasing the thermal energy E_{th} of the suitcase and the floor.

FIGURE 10.7 A suitcase pulled by a strap.

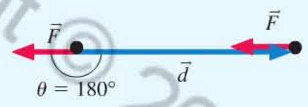


CONCEPTUAL EXAMPLE 10.3 **Work done by a parachute**

A drag racer is slowed by a parachute. What is the sign of the work done?

REASON The drag force on the drag racer is shown in **FIGURE 10.8**, along with the dragster's displacement as it slows. The force points in the direction opposite the displacement, so that the angle θ in

FIGURE 10.8 The force acting on a drag racer.



Equation 10.6 is 180°. Then $\cos\theta = \cos(180°) = -1$. Because F and d in Equation 10.6 are magnitudes, and hence positive, the work $W = Fd\cos\theta = -Fd$ done by the drag force is *negative.*

ASSESS Applying Equation 10.3 to this situation, we have

$$\Delta K = W$$

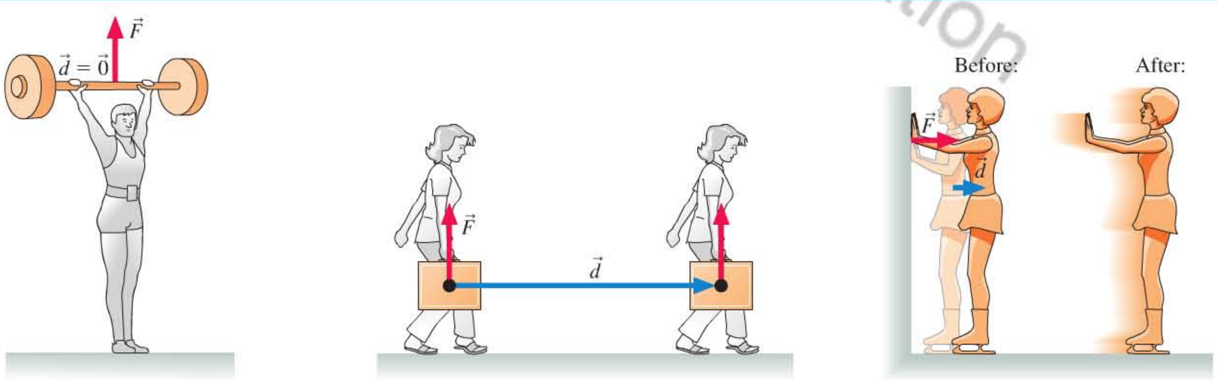
because the only system energy that changes is the racer's kinetic energy K. Because the kinetic energy is decreasing, its change ΔK is negative. This agrees with the sign of W. This example illustrates the general principle that **negative work represents a transfer of energy out of the system.**

If several forces act on an object that undergoes a displacement, each does work on the object. The **total** (or **net**) **work** W_{total} is the sum of the work done by each force. The total work represents the total energy transfer *to* the system from the environment (if $W_{\text{total}} > 0$) or *from* the system to the environment (if $W_{\text{total}} < 0$).

Forces That Do No Work

The fact that a force acts on an object doesn't mean that the force will do work on the object. The table below shows three common cases where a force does no work.

Forces that do no work



If the object undergoes no displacement while the force acts, no work is done.

This can sometimes seem counterintuitive. The weightlifter struggles mightily to hold the barbell over his head. But during the time the barbell remains stationary, he does no work on it because its displacement is zero. Why then is it so hard for him to hold it there? We'll see in Chapter 11 that it takes a rapid conversion of his internal chemical energy to keep his arms extended under this great load.

A force perpendicular to the displacement does no work.

The woman exerts only a vertical force on the briefcase she's carrying. This force has no component in the direction of the displacement, so the briefcase moves at a constant velocity and its kinetic energy remains constant. Since the energy of the briefcase doesn't change, it must be that no energy is being transferred to it as work.

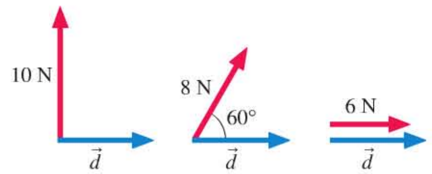
(This is the case where $\theta = 90°$ in Tactics Box 10.1.)

If the part of the object on which the force acts undergoes no displacement, no work is done.

Even though the wall pushes on the skater with a normal force $\vec{n}$ and she undergoes a displacement $\vec{d}$, the wall does no work on her, because the point of her body on which $\vec{n}$ acts—her hands—undergoes no displacement. This makes sense: How could energy be transferred as work from an inert, stationary object? So where does her kinetic energy come from? This will be the subject of much of Chapter 11. Can you guess?

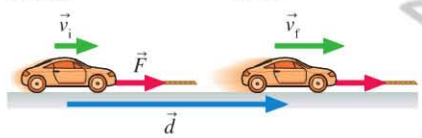
STOP TO THINK 10.2 Which force does the most work?

A. The 10 N force.
B. The 8 N force.
C. The 6 N force.
D. They all do the same amount of work.



10.3 Kinetic Energy

FIGURE 10.9 The work done by the tow rope increases the car's kinetic energy.



We've already qualitatively discussed kinetic energy, an object's energy of motion. Let's now use what we've learned about work, and some simple kinematics, to find a quantitative expression for kinetic energy. Consider a car being pulled by a tow rope, as in FIGURE 10.9. The rope pulls with a constant force $\vec{F}$ while the car undergoes a displacement $\vec{d}$, so the force does work $W = Fd$ on the car. If we ignore friction and drag, the work done by $\vec{F}$ is transferred entirely into the car's energy of motion—its kinetic energy. In this case, the change in the car's kinetic energy is given by the work-energy equation, Equation 10.3, as

$$W = \Delta K = K_f - K_i \tag{10.7}$$

Using kinematics, we can find another expression for the work done, in terms of the car's initial and final speeds. Recall from Chapter 2 the kinematic equation

$$v_f^2 = v_i^2 + 2a\Delta x$$

Applied to the motion of our car, $\Delta x = d$ is the car's displacement and, from Newton's second law, the acceleration is $a = F/m$. Thus we can write

$$v_f^2 = v_i^2 + \frac{2Fd}{m} = v_i^2 + \frac{2W}{m}$$

where we have replaced Fd with the work W. If we now solve for the work, we find

$$W = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

If we compare this result with Equation 10.7, we see that

$$K_f = \frac{1}{2}mv_f^2 \qquad \text{and} \qquad K_i = \frac{1}{2}mv_i^2$$

6.1 

In general, then, an object of mass m moving with speed v has kinetic energy

$$K = \frac{1}{2}mv^2 \tag{10.8}$$

Kinetic energy of an object of mass m moving with speed v

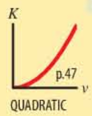


TABLE 10.1 Some approximate kinetic energies

Object	Kinetic energy
Ant walking	1×10^{-8} J
Penny dropped 1 m	2.5×10^{-3} J
Person walking	70 J
Fastball, 100 mph	150 J
Bullet	5000 J
Car, 60 mph	5×10^5 J
Supertanker, 20 mph	2×10^{10} J

From Equation 10.8, the units of kinetic energy are those of mass times speed squared, or $\text{kg} \cdot (\text{m/s})^2$. But

$$1\ \text{kg} \cdot (\text{m/s})^2 = \underbrace{1\ \text{kg} \cdot (\text{m/s}^2)}_{1\ \text{N}} \cdot \text{m} = 1\ \text{N} \cdot \text{m} = 1\ \text{J}$$

We see that the units of kinetic energy are the same as those of work, as they must be. Table 10.1 gives some approximate kinetic energies. Everyday kinetic energies range from a tiny fraction of a fraction of a joule to nearly a million joules for a speeding car.

CONCEPTUAL EXAMPLE 10.4 **Kinetic energy changes for a car**

Compare the increase in a 1000 kg car's kinetic energy as it speeds up by 5.0 m/s, starting from 5.0 m/s, to its increase in kinetic energy as it speeds up by 5.0 m/s, starting from 10 m/s.

REASON The change in the car's kinetic energy in going from 5.0 m/s to 10 m/s is

$$\Delta K_{5\rightarrow 10} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This gives

$$\Delta K_{5\rightarrow 10} = \frac{1}{2}(1000\text{ kg})(10\text{ m/s})^2 - \frac{1}{2}(1000\text{ kg})(5.0\text{ m/s})^2$$
$$= 3.8 \times 10^4\text{ J}$$

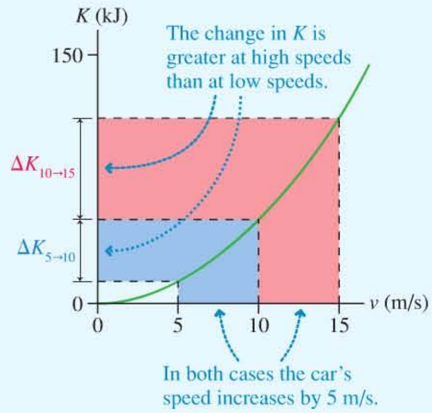
Similarly, increasing from 10 m/s to 15 m/s requires

$$\Delta K_{10\rightarrow 15} = \frac{1}{2}(1000\text{ kg})(15\text{ m/s})^2 - \frac{1}{2}(1000\text{ kg})(10\text{ m/s})^2$$
$$= 6.3 \times 10^4\text{ J}$$

Even though the increase in the car's *speed* is the same in both cases, the increase in kinetic energy is substantially greater in the second case.

ASSESS Kinetic energy depends on the *square* of the speed v. In **FIGURE 10.10**, which plots kinetic energy versus speed, we see that the energy of the car increases rapidly with speed. We can also see graphically why the change in K for a 5 m/s change in v is greater at high speeds than at low speeds. In part this is why it's harder to accelerate your car at high speeds than at low speeds.

FIGURE 10.10 The kinetic energy increases as the *square* of the speed.

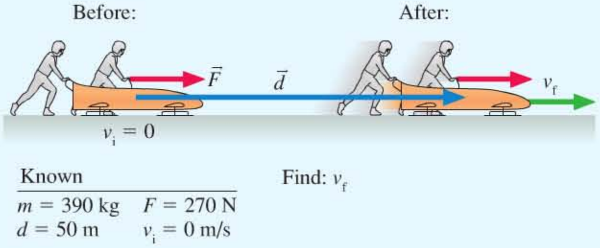


EXAMPLE 10.5 **Speed of a bobsled after pushing**

A two-man bobsled has a mass of 390 kg. Starting from rest, the two racers push the sled for the first 50 m with a net force of 270 N. Neglecting friction, what is the sled's speed at the end of the 50 m?

PREPARE We can find the sled's final speed if we can find its final kinetic energy. We can do so by equating the work done by the racers as they push on the sled to the change in its kinetic energy. **FIGURE 10.11** lists the known quantities and the quantity (v_f) that we want to find.

FIGURE 10.11 The work done by the pushers increases the sled's kinetic energy.



SOLVE From Equation 10.3, the work-energy equation, the change in the sled's kinetic energy is $\Delta K = K_f - K_i = W$. The sled's final kinetic energy is thus

$$K_f = K_i + W$$

Using our expressions for kinetic energy and work, we get

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + Fd$$

Because $v_i = 0$, the work-energy equation reduces to

$$\frac{1}{2}mv_f^2 = Fd$$

We can solve for the final speed to get

$$v_f = \sqrt{\frac{2Fd}{m}} = \sqrt{\frac{2(270\text{ N})(50\text{ m})}{390\text{ kg}}} = 8.3\text{ m/s}$$

ASSESS 8.3 m/s, about 18 mph, seems a reasonable speed for two fast pushers to attain.

STOP TO THINK 10.3 Rank in order, from greatest to least, the kinetic energies of the sliding pucks.

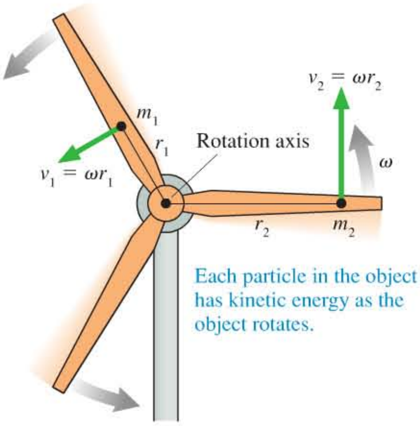
FIGURE 10.12 The large rotating blades of a windmill have a great deal of kinetic energy.

Rotational Kinetic Energy

We've just found an expression for the kinetic energy of an object moving along a line or some other path. This energy is called **translational kinetic energy.** Consider now an object rotating about a fixed axis, such as the windmill blades in **FIGURE 10.12**. Although the blades have no overall translational motion, each particle in the blade is moving and hence has kinetic energy. Adding up the kinetic energy for each particle that makes up the blades, we find that the blades have **rotational kinetic energy,** the kinetic energy due to rotation.

FIGURE 10.13 shows two of the particles making up a windmill blade that rotates with angular velocity ω. Recall from Section 6.2 that a particle moving with angular velocity ω in a circle of radius r has a speed $v = \omega r$. Thus particle 1, which rotates in a circle of radius r_1, moves with speed $v_1 = r_1\omega$ and so has kinetic energy $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1r_1^2\omega^2$. Similarly, particle 2, which rotates in a circle with a larger radius r_2, has kinetic energy $\frac{1}{2}m_2r_2^2\omega^2$. The object's rotational kinetic energy is the sum of the kinetic energies of *all* the particles:

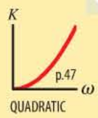
FIGURE 10.13 Rotational kinetic energy is due to the circular motion of the particles.



$$K_{rot} = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \cdots = \frac{1}{2}\left(\sum mr^2\right)\omega^2$$

You will recognize the term in parentheses as our old friend, the moment of inertia I. Thus the rotational kinetic energy is

$$K_{rot} = \frac{1}{2}I\omega^2 \tag{10.9}$$



Rotational kinetic energy of an object with moment of inertia I and angular velocity ω

NOTE ► Rotational kinetic energy is *not* a new form of energy. It is the ordinary kinetic energy of motion, only now expressed in a form that is especially convenient for rotational motion. Comparison with the familiar $\frac{1}{2}mv^2$ shows again that the moment of inertia I is the rotational equivalent of mass. ◄

A rolling object, such as a wheel, is undergoing both rotational *and* translational motions. Consequently, its total kinetic energy is the sum of its rotational and translational kinetic energies:

$$K = K_{trans} + K_{rot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{10.10}$$

This illustrates an important fact: **The kinetic energy of a rolling object is always greater than that of a nonrotating object moving at the same speed.**

◄ Rotational recharge The International Space Station (ISS) gets its electric power from solar panels. But during each 92-minute orbit, the ISS is in the earth's shadow for 30 minutes. The batteries that currently provide power during these blackouts need periodic replacement, which is very expensive in space. A promising new technology would replace the batteries with a *flywheel*—a cylinder rotating at a very high angular speed. Energy from the solar panels is used to speed up the flywheel, storing energy as rotational kinetic energy, which can then be converted back into electric energy when the ISS is in shadow.

EXAMPLE 10.6 Kinetic energy of a bicycle

Bike 1 has a 10.0 kg frame and 1.00 kg wheels; bike 2 has a 9.00 kg frame and 1.50 kg wheels. Both bikes thus have the same 12.0 kg total mass. What is the kinetic energy of each bike when they are ridden at 12.0 m/s? Model each wheel as a hoop of radius 35.0 cm.

PREPARE Each bike's frame has only translational kinetic energy $K_{frame} = \frac{1}{2}mv^2$, where m is the mass of the frame. The kinetic energy of each rolling wheel is given by Equation 10.10. From Table 7.4, we find that I for a hoop is MR^2, where M is the mass of one wheel.

SOLVE From Equation 10.10 the kinetic energy of each rolling wheel is

$$K_{\text{wheel}} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\underbrace{(MR^2)}_{I}\underbrace{\left(\frac{v}{R}\right)^2}_{\omega^2} = Mv^2$$

Then the total kinetic energy of a bike is

$$K = K_{\text{frame}} + 2K_{\text{wheel}} = \frac{1}{2}mv^2 + 2Mv^2$$

The factor of 2 in the second term occurs because each bike has two wheels. Thus the kinetic energies of the two bikes are

$$K_1 = \frac{1}{2}(10.0\text{ kg})(12.0\text{ m/s})^2 + 2(1.00\text{ kg})(12.0\text{ m/s})^2 = 1010\text{ J}$$

$$K_2 = \frac{1}{2}(9.00\text{ kg})(12.0\text{ m/s})^2 + 2(1.50\text{ kg})(12.0\text{ m/s})^2 = 1080\text{ J}$$

The kinetic energy of bike 2 is about 7% higher than that of bike 1. Note that the radius of the wheels was not needed in this calculation.

ASSESS As the cyclists on these bikes accelerate from rest to 12 m/s, they must convert some of their internal chemical energy into the kinetic energy of the bikes. Racing cyclists want to use as little of their own energy as possible. Although both bikes have the same total mass, the one with the lighter wheels will take less energy to get it moving. Shaving a little extra weight off your wheels is more useful than taking that same weight off your frame.

It's important that racing bike wheels are as light as possible.

10.4 Potential Energy

When two or more objects in a system interact, it is sometimes possible to *store* energy in the system in a way that the energy can be easily recovered. For instance, the earth and a ball interact by the gravitational force between them. If the ball is lifted up into the air, energy is stored in the ball + earth system, energy that can later be recovered as kinetic energy when the ball is released and falls. Similarly, a spring is a system made up of countless atoms that interact via their atomic "springs." If we push a box against a spring, energy is stored that can be recovered when the spring later pushes the box across the table. This sort of stored energy is called **potential energy,** since it has the *potential* to be converted into other forms of energy, such as kinetic or thermal energy.

The forces due to gravity and springs are special in that they allow for the storage of energy. Other interaction forces do not. When a crate is pushed across the floor, the crate and the floor interact via the force of friction, and the work done on the system is converted into thermal energy. But this energy is *not* stored up for later recovery—it slowly diffuses into the environment and cannot be recovered.

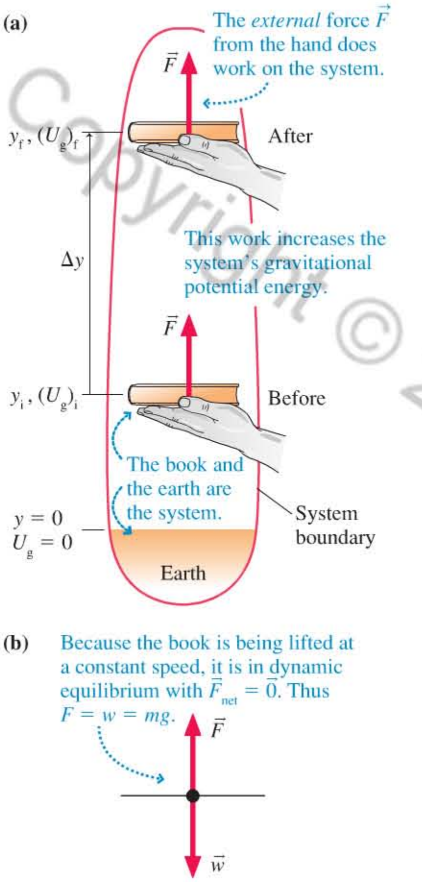
Interaction forces that can store useful energy are called **conservative forces.** The name comes from the important fact that, as we'll see, the mechanical energy of a system is *conserved* when only conservative forces act. Gravity and elastic forces are conservative forces, and later we'll find that the electric force is a conservative force as well. Friction, on the other hand, is a **nonconservative force.** When two objects interact via a friction force, energy is not stored. It is usually transformed into thermal energy.

Let's look more closely at the potential energies associated with the two conservative forces—gravity and springs—that we'll study in this chapter.

Gravitational Potential Energy

To find an expression for gravitational potential energy, let's consider the system of the book and the earth shown in **FIGURE 10.14a** on the next page. The book is lifted at a constant speed from its initial position at y_i to a final height y_f. The lifting force of the hand is external to the system and so does work W on the system, increasing its energy. The book is lifted at a constant speed, so its kinetic energy doesn't change. Because there's no friction, the book's thermal energy doesn't change either. Thus

FIGURE 10.14 Lifting a book increases the system's gravitational potential energy.



the work done goes entirely into increasing the gravitational potential energy of the system. According to Equation 10.3, the work-energy equation, $\Delta U_g = W$. Because $\Delta U_g = (U_g)_f - (U_g)_i$, Equation 10.3 can be written

$$(U_g)_f = (U_g)_i + W \tag{10.11}$$

The work done is $W = Fd$, where $d = \Delta y = y_f - y_i$ is the vertical distance that the book is lifted. From the free-body diagram of **FIGURE 10.14b**, we see that $F = mg$. Thus $W = mg\Delta y$, and so

$$(U_g)_f = (U_g)_i + mg\Delta y \tag{10.12}$$

Because our final height was greater than our initial height, Δy is positive and $(U_g)_f > (U_g)_i$. **The higher the object is lifted, the greater the gravitational potential energy in the object + earth system.**

Equation 10.12 gives the final gravitational potential energy $(U_g)_f$ in terms of its initial value $(U_g)_i$. But what is the value of $(U_g)_i$? We can gain some insight by writing Equation 10.12 in terms of energy *changes:*

$$(U_g)_f - (U_g)_i = \Delta U_g = mg\Delta y$$

For example, if we lift a 1.5 kg book up by $\Delta y = 2.0$ m, we increase the system's gravitational potential energy by $\Delta U_g = (1.5 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m}) = 29.4$ J. This increase is *independent* of the book's starting height: The gravitational potential energy increases by 29.4 J whether we lift the book 2.0 m starting at sea level or starting at the top of the Washington Monument. This illustrates an important general fact about *every* form of potential energy: **Only *changes* in potential energy are significant.**

Because of this fact, we are free to choose a *reference level* where we define U_g to be zero. Our expression for U_g is particularly simple if we choose this reference level to be at $y = 0$. We then have

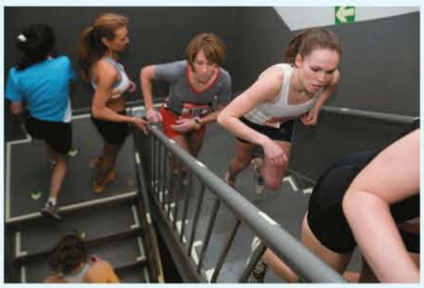
$$U_g = mgy \tag{10.13}$$

Gravitational potential energy of an object of mass m at height y (assuming $U_g = 0$ when the object is at $y = 0$)

NOTE ► We've emphasized that gravitational potential energy is an energy of the earth + object *system*. In solving problems using the law of conservation of energy, you'll need to include the earth as part of your system. For simplicity, we'll usually speak of "the gravitational potential energy of the ball," but what we really mean is the potential energy of the earth + ball system. ◄

EXAMPLE 10.7 Racing up a skyscraper

In the Empire State Building Run-Up, competitors race up the 1576 steps of the Empire State Building, climbing a total vertical distance of 320 m. How much gravitational potential energy does a 70 kg racer gain during this race?



Racers head up the staircase in the Empire State Building Run-Up.

PREPARE We choose $y = 0$ m and $U_g = 0$ J at the ground floor of the building.

SOLVE At the top, the racer's gravitational potential energy is

$$U_g = mgy = (70 \text{ kg})(9.8 \text{ m/s}^2)(320 \text{ m}) = 2.2 \times 10^5 \text{ J}$$

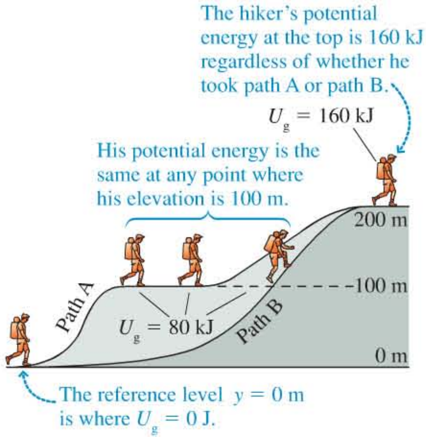
Because the racer's gravitational potential energy was 0 J at the ground floor, the change in his potential energy is 2.2×10^5 J.

ASSESS This is a large amount of energy. According to Table 10.1, it's comparable to the energy of a speeding car. But if you think how hard it would be to climb the Empire State Building, it seems like a plausible result.

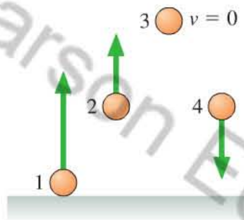
An important conclusion from Equation 10.13 is that gravitational potential energy depends only on the height of the object above the reference level $y = 0$, not on the object's horizontal position. To understand why, consider carrying a briefcase while walking on level ground at a constant speed. As shown in the table on page 297, the vertical force of your hand on the briefcase is *perpendicular* to the displacement. *No work* is done on the briefcase, so its gravitational potential energy remains constant as long as its height above the ground doesn't change.

This idea can be applied to more complicated cases, such as the 82 kg hiker in **FIGURE 10.15**. His gravitational potential energy depends *only* on his height y above the reference level. Along path A, it's the same value $U_g = mgy = 80$ kJ at any point where he is at height $y = 100$ m above the reference level. If he had instead taken path B, his gravitational potential energy at $y = 100$ m would be the same 80 kJ. It doesn't matter *how* he gets to the 100 m elevation; his potential energy at that height is always the same. **Gravitational potential energy depends only on the *height* of an object and not on the path the object took to get to that position.** This fact will allow us to use the law of conservation of energy to easily solve a variety of problems that would be very difficult to solve using Newton's laws alone.

FIGURE 10.15 The hiker's gravitational potential energy depends only on his height above the $y = 0$ m reference level.



STOP TO THINK 10.4 Rank in order, from largest to smallest, the gravitational potential energies of identical balls 1 through 4.



Elastic Potential Energy

Energy can also be stored in a compressed or extended spring as **elastic** (or **spring**) **potential energy** U_s. We can find out how much energy is stored in a spring by using an external force to slowly compress the spring. This external force does work on the spring, transferring energy to the spring. Since only the elastic potential energy of the spring is changing, Equation 10.3 reads

$$\Delta U_s = W \tag{10.14}$$

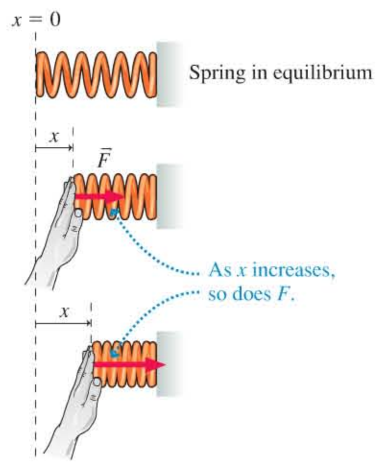
That is, we can find out how much elastic potential energy is stored in the spring by calculating the amount of work needed to compress the spring.

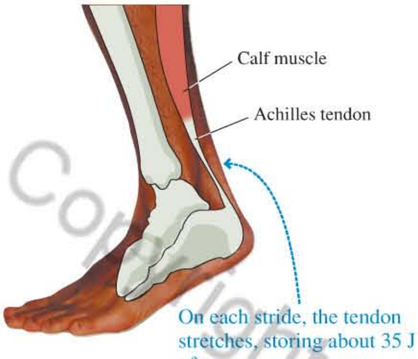
FIGURE 10.16 shows a spring being compressed by a hand. In Section 8.3 we found that the force the spring exerts on the hand is $F_s = -k\Delta x$ (Hooke's law), where Δx is the displacement of the end of the spring from its equilibrium position and k is the spring constant. In Figure 10.16 we have set the origin of our coordinate system at the equilibrium position. The displacement from equilibrium Δx is therefore equal to x, and the spring force is then $-kx$. By Newton's third law, the force that the hand exerts on the spring is thus $F = +kx$.

As the hand pushes the end of the spring from its equilibrium position to a final position x, the applied force increases from 0 to kx. This is not a constant force, so we can't use Equation 10.5, $W = Fd$, to find the work done. However, it seems reasonable to calculate the work by using the *average* force in Equation 10.5. Because the force varies from $F_i = 0$ to $F_f = kx$, the average force used to compress the spring is $F_{avg} = \frac{1}{2}kx$. Thus the work done by the hand is

$$W = F_{avg}d = F_{avg}x = \left(\frac{1}{2}kx\right)x = \frac{1}{2}kx^2$$

FIGURE 10.16 The force required to compress a spring is not constant.





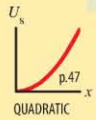
Spring in your step BIO As you run, you lose some of your mechanical energy each time your foot strikes the ground; this energy is transformed into unrecoverable thermal energy. Luckily, about 35% of the decrease of your mechanical energy when your foot lands is stored as elastic potential energy in the stretchable Achilles tendon of the lower leg. On each plant of the foot, the tendon is stretched, storing some energy. The tendon springs back as you push off the ground again, helping to propel you forward. This recovered energy reduces the amount of internal chemical energy you use, increasing your efficiency.

This work is stored as potential energy in the spring, so we can use Equation 10.14 to find that as the spring is compressed, the elastic potential energy increases by

$$\Delta U_s = \frac{1}{2}kx^2$$

Just as in the case of gravitational potential energy, we have found an expression for the *change* in U_s, not U_s itself. Again, we are free to set $U_s = 0$ at any convenient spring extension. An obvious choice is to set $U_s = 0$ at the point where the spring is in equilibrium, neither compressed nor stretched—that is, at $x = 0$. With this choice we have

$$U_s = \frac{1}{2}kx^2 \qquad (10.15)$$



Elastic potential energy of a spring displaced a distance x from equilibrium (assuming $U_s = 0$ when the end of the spring is at $x = 0$)

NOTE ► Because U_s depends on the *square* of the displacement x, U_s is the same whether x is positive (the spring is compressed as in Figure 10.16) or negative (the spring is stretched). ◄

EXAMPLE 10.8 Pulling back on a bow

An archer pulls back the string on her bow to a distance of 70 cm from its equilibrium position. To hold the string at this position takes a force of 140 N. How much elastic potential energy is stored in the bow?

PREPARE A bow is an elastic material, so we will model it as obeying Hooke's law, $F_s = -kx$, where x is the distance the string is pulled back. We can use the force required to hold the string, and the distance it is pulled back, to find the bow's spring constant k. Then we can use Equation 10.15 to find the elastic potential energy.

SOLVE From Hooke's law, the spring constant is

$$k = \frac{F}{x} = \frac{140\ \text{N}}{0.70\ \text{m}} = 200\ \text{N/m}$$

Then the elastic potential energy of the flexed bow is

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}(200\ \text{N/m})(0.70\ \text{m})^2 = 49\ \text{J}$$

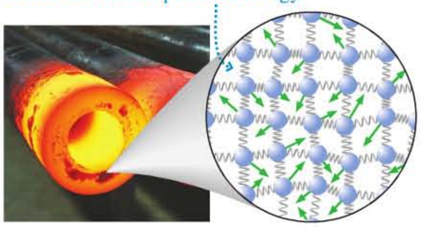
ASSESS When the arrow is released, this elastic potential energy will be transformed into the kinetic energy of the arrow. Because arrows are quite light, 49 J of kinetic energy will correspond to a very high speed.

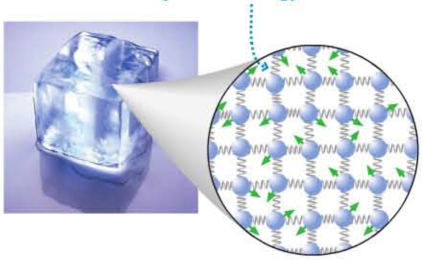
STOP TO THINK 10.5 When a spring is stretched by 5 cm, its elastic potential energy is 1 J. What will its elastic potential energy be if it is *compressed* by 10 cm?

A. −4 J B. −2 J C. 2 J D. 4 J

10.5 Thermal Energy

FIGURE 10.17 A molecular view of thermal energy.





We noted earlier that thermal energy is related to the microscopic motion of the molecules of an object. As **FIGURE 10.17** shows, the molecules in a hot object jiggle around their average positions more than the molecules in a cold object. This has two consequences. First, each atom is on average moving faster in the hot object. This means that each atom has a higher *kinetic energy*. Second, each atom in the hot object tends to stray farther from its equilibrium position, leading to a greater stretching or compressing of the spring-like molecular bonds. This means that each atom has on average a higher *potential energy*. The potential energy stored in any one bond and the kinetic energy of any one atom are both exceedingly small, but there are incredibly many bonds and atoms. The sum of all these microscopic potential and kinetic energies is what we call **thermal energy.** Increasing an object's thermal energy corresponds to increasing its temperature.

Creating Thermal Energy

FIGURE 10.18 shows a thermogram of a heavy box and the floor across which it has just been dragged. In this image, warmer areas appear light blue or green. You can see that the bottom of the box and the region of the floor that the box moved over are noticeably warmer than their surroundings. In the process of dragging the box, thermal energy has appeared in the box and the floor.

We can find a quantitative expression for the change in thermal energy by considering such a box pulled by a rope at a constant speed. As the box is pulled across the floor, the rope exerts a constant forward force $\vec{F}$ on the box, while the friction force $\vec{f}_k$ exerts a constant force on the box that is directed backward. Because the box moves at a constant speed, the magnitudes of these two forces are equal: $F = f_k$.

As the box moves through a displacement $d = \Delta x$, the rope does work $W = F\Delta x$ on the box. This work represents energy transferred into the system, so the system's energy must *increase*. In what form is this increased energy? The box's speed remains constant, so there is no change in its kinetic energy ($\Delta K = 0$). And its height doesn't change, so its gravitational potential energy is unchanged as well ($\Delta U_g = 0$). Instead, the increased energy must be in the form of *thermal* energy E_{th}. As Figure 10.18 shows, this energy appears as an increased temperature of both the box *and* the floor across which it was dragged.

We can write the work-energy equation, Equation 10.3, for the case where only thermal energy changes:

$$\Delta E_{th} = W$$

or, because the work is $W = F\Delta x = f_k \Delta x$,

$$\Delta E_{th} = f_k \Delta x \tag{10.16}$$

This increase in thermal energy is a general feature of any system where friction between sliding objects is present. An atomic-level explanation is shown in FIGURE 10.19. Although we arrived at Equation 10.16 by considering energy transferred into the system via work done by an external force, the equation is equally valid for the transformation of mechanical energy into thermal energy when, for instance, an object slides to a halt on a rough surface. Equation 10.16 also applies to rolling friction; we need only replace f_k by f_r.

FIGURE 10.18 A thermograph of a box that's been dragged across the floor.

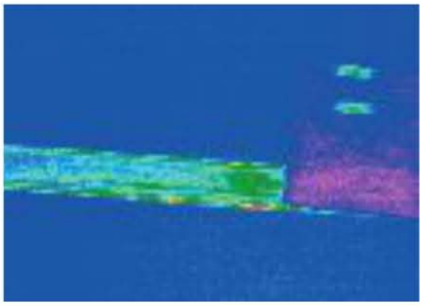
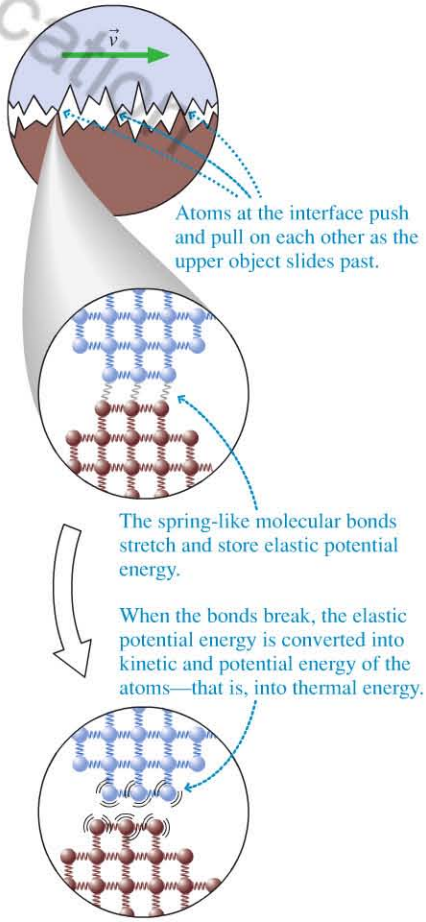


FIGURE 10.19 How friction causes an increase in thermal energy.



TRY IT YOURSELF

◀**Agitating atoms** Vigorously rub a somewhat soft object such as a blackboard eraser on your desktop for about 10 seconds. If you then pass your fingers over the spot where you rubbed, you'll feel a distinct warm area. Congratulations: You've just set some 100,000,000,000,000,000,000,000 atoms into motion!

EXAMPLE 10.9 **Creating thermal energy by rubbing**

A 0.30 kg block of wood is rubbed back and forth against a wood table 30 times in each direction. The block is moved 8.0 cm during each stroke and pressed against the table with a force of 22 N. How much thermal energy is created in this process?

PREPARE The hand holding the block does work to push the block back and forth. Work transfers energy into the block + table system, where it appears as thermal energy according to Equation 10.16. The force of friction can be found from the model of kinetic friction introduced in Chapter 5, $f_k = \mu_k n$; from Table 5.1 the coefficient of kinetic friction for wood sliding on wood is $\mu_k = 0.20$. To find the normal force n acting on the block, we draw the free-body diagram of **FIGURE 10.20**, which shows only the *vertical* forces acting on the block.

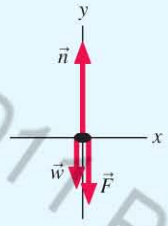


FIGURE 10.20 Free-body diagram (vertical forces only) for a block being rubbed against a table.

SOLVE From Equation 10.16 we have $\Delta E_{th} = f_k \Delta x$, where $f_k = \mu_k n$. The block is not accelerating in the y-direction, so from the free-body diagram Newton's second law gives

$$\sum F_y = n - w - F = ma_y = 0$$

or

$$n = w + F = mg + F = (0.30 \text{ kg})(9.8 \text{ m/s}^2) + 22 \text{ N} = 25 \text{ N}$$

The friction force is then $f_k = \mu_k n = (0.20)(25 \text{ N}) = 5.0 \text{ N}$. The total displacement of the block is $2 \times 30 \times 8.0 \text{ cm} = 4.8 \text{ m}$. Thus the thermal energy created is

$$\Delta E_{th} = f_k \Delta x = (5.0 \text{ N})(4.8 \text{ m}) = 24 \text{ J}$$

ASSESS This modest amount of thermal energy seems reasonable for a person to create by rubbing.

10.6 Using the Law of Conservation of Energy

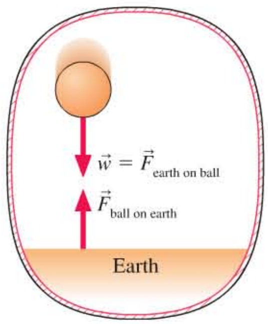
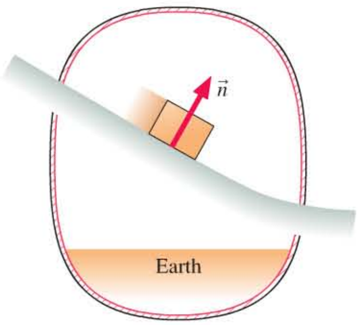
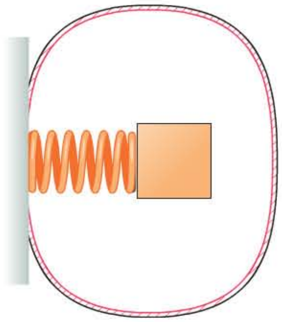
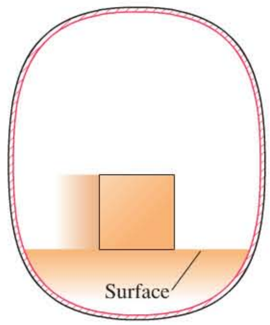
The law of conservation of energy, Equation 10.4, states that **the total energy of an *isolated system* is conserved** so that its change is zero:

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = 0 \qquad (10.17)$$

This law applies to every form of energy, from kinetic to chemical to nuclear. For the rest of this chapter, however, we'll narrow our focus and concern ourselves with only the forms of energy typically transformed during the motion of ordinary objects. These forms are kinetic energy K, gravitational and elastic potential energies U_g and U_s, and thermal energy E_{th}.

We defined an isolated system as one on which external forces do no work, so that no energy is transferred into or out of the system. The following table shows how to choose an isolated system for four common situations.

TABLE 10.2 Choosing an isolated system

An object in free fall	An object sliding down a frictionless ramp	An object compressing a spring	An object sliding along a surface with friction
			
We choose the ball *and* the earth as the system, so that the forces between them are *internal* forces. There are no external forces to do work, so the system is isolated.	The external force the ramp exerts on the object is perpendicular to the motion, and so does no work. The object and the earth together form an isolated system.	We choose the object and the spring to be the system. The forces between them are internal forces, so no work is done.	The block and the surface interact via kinetic friction forces, but these forces are internal to the system. There are no external forces to do work, so the system is isolated.

Just as for momentum conservation, we wish to develop a before-and-after perspective for energy conservation. We can do so by noting that $\Delta K = K_f - K_i$, $\Delta U_g = (U_g)_f - (U_g)_i$, and so on. Then Equation 10.17 can be written as

$$K_f + (U_g)_f + (U_s)_f + \Delta E_{th} = K_i + (U_g)_i + (U_s)_i \quad (10.18)$$

Equation 10.18 is the before-and-after version of the law of conservation of energy: It equates the final value of an isolated system's energy to its initial energy. This equation will be the basis for a powerful problem-solving strategy.

NOTE ► We don't write ΔE_{th} as $(E_{th})_f - (E_{th})_i$ because the initial and final values of the thermal energy are typically unknown; only their *difference* ΔE_{th} can be measured. ◄

Conservation of Mechanical Energy

If we further restrict ourselves to cases where friction can be neglected, so that $\Delta E_{th} = 0$, the law of conservation of energy, Equation 10.18, becomes

$$K_f + (U_g)_f + (U_s)_f = K_i + (U_g)_i + (U_s)_i \quad (10.19)$$

The sum of the kinetic and potential energies, $K + U_g + U_s$, is called the **mechanical energy** of the system, so Equation 10.19 says that **the mechanical energy is conserved for an isolated system without friction.**

These observations about the conservation of energy suggest the following problem-solving strategy.

Activ Physics ONLINE 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 7.11, 7.12, 7.13

Spring into action BIO A locust can jump as far as 1 meter, an impressive distance for such a small animal. To make such a jump, its legs must extend much more rapidly than muscles can ordinarily contract. Thus, instead of using its muscles to make the jump directly, the locust uses them to more slowly stretch an internal "spring" near its knee joint. This stores elastic potential energy in the spring. When the muscles relax, the spring is suddenly released, and its energy is rapidly converted into kinetic energy of the insect.

PROBLEM-SOLVING STRATEGY 10.1 Conservation of energy problems MP™

PREPARE Referring to Table 10.2, choose your system so that it is isolated. Draw a before-and-after visual overview, as was outlined in Tactics Box 9.1. Note the known quantities, and identify what you're trying to find.

SOLVE There are two important situations:

- If the system is isolated *and* there's no friction, the mechanical energy is conserved:

$$K_f + (U_g)_f + (U_s)_f = K_i + (U_g)_i + (U_s)_i$$

- If the system is isolated but there is friction within the system, the total energy is conserved:

$$K_f + (U_g)_f + (U_s)_f + \Delta E_{th} = K_i + (U_g)_i + (U_s)_i$$

Depending on the problem, you'll need to calculate the initial and/or final values of these energies; you can then solve for the unknown energies, and from these any unknown speeds (from K), heights (from U_g and U_s), or displacements or friction forces (from $\Delta E_{th} = f_k \Delta x$).

ASSESS Check the signs of your energies. Kinetic energy is always positive, as is the change in thermal energy. Check that your result has the correct units, is reasonable, and answers the question.

Exercise 23

EXAMPLE 10.10 Hitting the bell

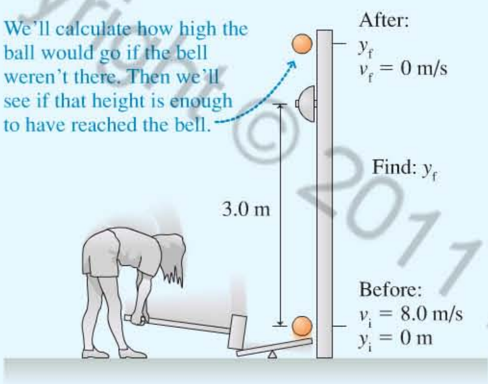
At the county fair, Katie tries her hand at the ring-the-bell attraction, as shown in **FIGURE 10.21** on the next page. She swings the mallet hard enough to give the ball an initial upward speed of 8.0 m/s. Will the ball ring the bell, 3.0 m from the bottom?

PREPARE We'll follow the steps of Problem-Solving Strategy 10.1. From Table 10.2, we see that once the ball is in the air, the system consisting of the ball and the earth is isolated. If we assume that the track along which the ball moves is frictionless, then the

Continued

system's mechanical energy is conserved. Figure 10.21 shows a before-and-after visual overview in which we've chosen $y = 0$ m to be at the ball's starting point. We can then use conservation of mechanical energy, Equation 10.19.

FIGURE 10.21 Before-and-after visual overview of the ring-the-bell attraction.



SOLVE Equation 10.19 tells us that $K_f + (U_g)_f = K_i + (U_g)_i$. We can use our expressions for kinetic and potential energy to write this as

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Let's ignore the bell for the moment and figure out how far the ball would rise if there were nothing in its way. We know that the ball starts at $y_i = 0$ m and that its speed v_f at the highest point is 0 m/s. Thus the energy equation simplifies to

$$mgy_f = \frac{1}{2}mv_i^2$$

This is easily solved for the height y_f:

$$y_f = \frac{v_i^2}{2g} = \frac{(8.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 3.3 \text{ m}$$

This is higher than the point where the bell sits, so the ball would actually hit it on the way up.

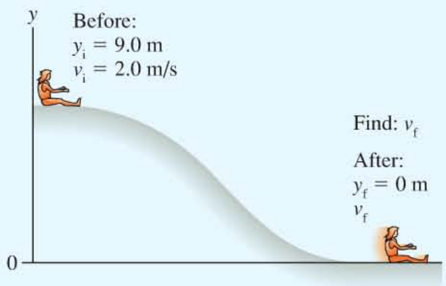
ASSESS It seems reasonable that Katie could swing the mallet hard enough to make the ball rise by about 3 m.

EXAMPLE 10.11 Speed at the bottom of a water slide

Still at the county fair, Katie tries the water slide, whose shape is shown in **FIGURE 10.22**. The starting point is 9.0 m above the ground. She pushes off with an initial speed of 2.0 m/s. If the slide is frictionless, how fast will Katie be traveling at the bottom?

PREPARE Table 10.2 showed that the system consisting of Katie and the earth is isolated because the normal force of the slide is perpendicular to Katie's motion and does no work. If we assume the slide is frictionless, we can use the conservation of mechanical energy equation. Figure 10.22 is a visual overview of the problem.

FIGURE 10.22 Before-and-after visual overview of Katie on the water slide.



SOLVE Conservation of mechanical energy gives

$$K_f + (U_g)_f = K_i + (U_g)_i$$

or

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Taking $y_f = 0$ m, we have

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i$$

which we can solve to get

$$v_f = \sqrt{v_i^2 + 2gy_i}$$
$$= \sqrt{(2.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(9.0 \text{ m})} = 13 \text{ m/s}$$

ASSESS This speed is about 30 mph. This is probably faster than you really would go on a water slide but, because we have ignored friction, our answer is reasonable. It is important to realize that the *shape* of the slide does not matter because gravitational potential energy depends only on the *height* above a reference level. **If you slide down any (frictionless) slide of the same height, your speed at the bottom is the same.**

EXAMPLE 10.12 Speed of a spring-launched ball

A spring-loaded toy gun is used to launch a 10 g plastic ball. The spring, which has a spring constant of 10 N/m, is compressed by 10 cm as the ball is pushed into the barrel. When the trigger is pulled, the spring is released and shoots the ball back out. What is the ball's speed as it leaves the barrel? Assume that friction is negligible.

PREPARE Assume the spring obeys Hooke's law, $F_s = -kx$, and is massless so that it has no kinetic energy of its own. Using Table 10.2, we choose the isolated system to be the spring and the ball. There's no friction; hence the system's mechanical energy $K + U_s$ is conserved.

FIGURE 10.23 Before-and-after visual overview of a ball being shot out of a spring-loaded toy gun.

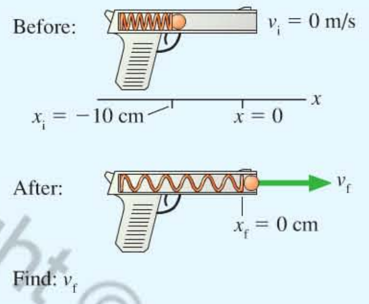


FIGURE 10.23 shows a before-and-after visual overview. The compressed spring will push on the ball until the spring has returned to its equilibrium length. We have chosen the origin of the coordinate system at the equilibrium position of the free end of the spring, making $x_i = -10$ cm and $x_f = 0$ cm.

SOLVE The energy conservation equation is $K_f + (U_s)_f = K_i + (U_s)_i$. We can use the elastic potential energy of the spring, Equation 10.15, to write this as

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}kx_f^2 = \tfrac{1}{2}mv_i^2 + \tfrac{1}{2}kx_i^2$$

We know that $x_f = 0$ m and $v_i = 0$ m/s, so this simplifies to

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}kx_i^2$$

It is now straightforward to solve for the ball's speed:

$$v_f = \sqrt{\frac{kx_i^2}{m}} = \sqrt{\frac{(10\text{ N/m})(-0.10\text{ m})^2}{0.010\text{ kg}}} = 3.2\text{ m/s}$$

ASSESS This is *not* a problem that we could have easily solved with Newton's laws. The acceleration is not constant, and we have not learned how to handle the kinematics of nonconstant acceleration. But with conservation of energy—it's easy!

Friction and Thermal Energy

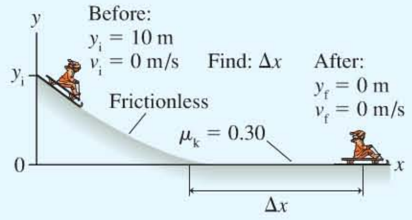
Thermal energy is always created when kinetic friction is present, so we must use the more general conservation of energy equation, Equation 10.18, which includes thermal-energy changes ΔE_{th}. Furthermore, we know from Section 10.5 that the change in the thermal energy when an object slides a distance Δx while subject to a friction force f_k is $\Delta E_{th} = f_k \Delta x$.

EXAMPLE 10.13 **Where will the sled stop?**

A sledder, starting from rest, slides down a 10-m-high hill. At the bottom of the hill is a long horizontal patch of rough snow. The hill is nearly frictionless, but the coefficient of friction between the sled and the rough snow at the bottom is $\mu_k = 0.30$. How far will the sled slide along the rough patch?

PREPARE In order to be isolated, the system must include the sled, the earth, *and* the rough snow. As Table 10.2 shows, this makes the friction force an internal force so that no work is done on the system. We can use conservation of energy, but we will need to include thermal energy. A visual overview of the problem is shown in **FIGURE 10.24**.

FIGURE 10.24 Visual overview of a sledder sliding downhill.



SOLVE At the top of the hill the sled has only gravitational potential energy $(U_g)_i = mgy_i$. It has no kinetic or potential energy after stopping at the bottom of the hill, so $K_f = (U_g)_f = 0$. However, friction in the rough patch causes an increase in thermal energy. Thus our conservation of energy equation $K_f + (U_g)_f + \Delta E_{th} = K_i + (U_g)_i$ is

$$\Delta E_{th} = (U_g)_i = mgy_i$$

The change in thermal energy is $\Delta E_{th} = f_k \Delta x = \mu_k n \Delta x$. The normal force $\vec{n}$ balances the sled's weight $\vec{w}$ as it crosses the rough patch, so $n = w = mg$. Thus

$$\Delta E_{th} = \mu_k n \Delta x = \mu_k (mg) \Delta x = mgy_i$$

from which we find

$$\Delta x = \frac{y_i}{\mu_k} = \frac{10\text{ m}}{0.30} = 33\text{ m}$$

ASSESS It seems reasonable that the sledder would slide a distance that is greater than the height of the hill he started down.

10.7 Energy in Collisions

In Chapter 9 we studied collisions between two objects. We found that if no external forces are acting on the objects, the total *momentum* of the objects will be conserved. Now we wish to study what happens to *energy* in collisions. The energetics of

ActivPhysics ONLINE 6.2, 6.5, 6.8, 6.9

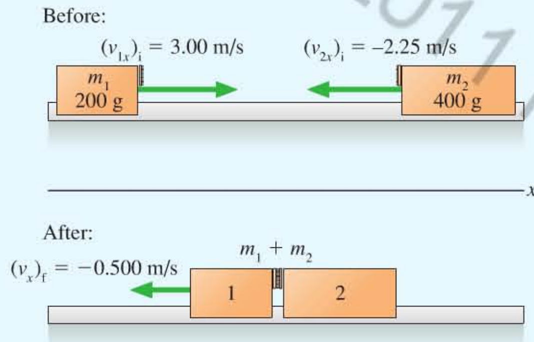
collisions are important in many applications in biokinetics, such as designing safer automobiles and bicycle helmets.

Let's first re-examine a perfectly inelastic collision. We studied just such a collision in Example 9.8. Recall that in such a collision the two objects stick together and then move with a common final velocity. What happens to the energy?

EXAMPLE 10.14 Energy transformations in a perfectly inelastic collision

FIGURE 10.25 shows two air-track gliders that are pushed toward each other, collide, and stick together. In Example 9.8, we used conservation of momentum to find the final velocity shown in Figure 10.25 from the given initial velocities. How much thermal energy is created in this collision?

FIGURE 10.25 Before-and-after visual overview of a completely inelastic collision.



PREPARE We'll choose our system to be the two gliders. Because the track is horizontal, there is no change in potential energy. Thus the law of conservation of energy, Equation 10.18, is $K_f + \Delta E_{th} = K_i$. The total energy before the collision must equal the total energy afterward, but the *mechanical* energies need not be equal.

SOLVE The initial kinetic energy is

$$K_i = \frac{1}{2}m_1(v_{1x})_i^2 + \frac{1}{2}m_2(v_{2x})_i^2$$

$$= \frac{1}{2}(0.200\text{ kg})(3.00\text{ m/s})^2 + \frac{1}{2}(0.400\text{ kg})(-2.25\text{ m/s})^2$$

$$= 1.91\text{ J}$$

Because the gliders stick together and move as a single object with mass $m_1 + m_2$, the final kinetic energy is

$$K_f = \frac{1}{2}(m_1 + m_2)(v_x)_f^2$$

$$= \frac{1}{2}(0.600\text{ kg})(-0.500\text{ m/s})^2 = 0.0750\text{ J}$$

From the conservation of energy equation above, we find that the thermal energy increases by

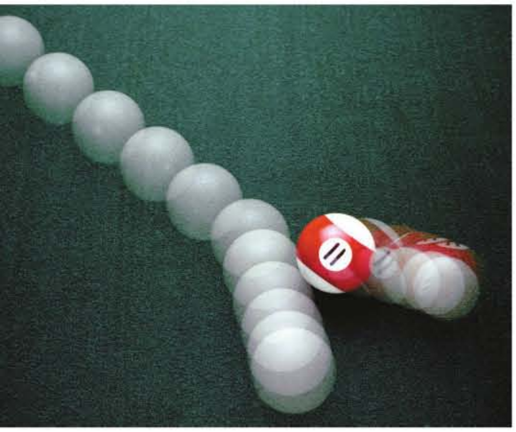
$$\Delta E_{th} = K_i - K_f = 1.91\text{ J} - 0.075\text{ J} = 1.84\text{ J}$$

This amount of the initial kinetic energy is transformed into thermal energy during the impact of the collision.

ASSESS About 96% of the initial kinetic energy is transformed into thermal energy. This is typical of many real-world collisions.

Elastic Collisions

Figure 9.1 showed a collision of a tennis ball with a racket. The ball is compressed and the racket strings stretch as the two collide, then the ball expands and the strings relax as the two are pushed apart. In the language of energy, the kinetic energy of the objects is transformed into the elastic potential energy of the ball and strings, then back into kinetic energy as the two objects spring apart. If *all* of the kinetic energy is stored as elastic potential energy, and *all* of the elastic potential energy is transformed back into the post-collision kinetic energy of the objects, then mechanical energy is conserved. A collision for which mechanical energy is conserved is called a **perfectly elastic collision.**



In a collision between a cue ball and a stationary ball, the mechanical energy of the balls is almost perfectly conserved.

Needless to say, most real collisions fall somewhere between perfectly elastic and perfectly inelastic. A rubber ball bouncing on the floor might "lose" 20% of its kinetic energy on each bounce and return to only 80% of the height of the preceding bounce. But collisions between two very hard objects, such as two pool balls or two steel balls, come close to being perfectly elastic. And collisions between microscopic particles, such as atoms or electrons, can be perfectly elastic.

FIGURE 10.26 on the next page shows a head-on, perfectly elastic collision of a ball of mass m_1, having initial velocity $(v_{1x})_i$, with a ball of mass m_2 that is initially at rest. The balls' velocities after the collision are $(v_{1x})_f$ and $(v_{2x})_f$. These are velocities, not speeds, and have signs. Ball 1, in particular, might bounce backward and have a negative value for $(v_{1x})_f$.

The collision must obey two conservation laws: conservation of momentum (obeyed in any collision) and conservation of mechanical energy (because the collision is perfectly elastic). Although the energy is transformed into potential energy during the collision, the mechanical energy before and after the collision is purely kinetic energy. Thus,

$$\text{momentum conservation:} \quad m_1(v_{1x})_i = m_1(v_{1x})_f + m_2(v_{2x})_f$$

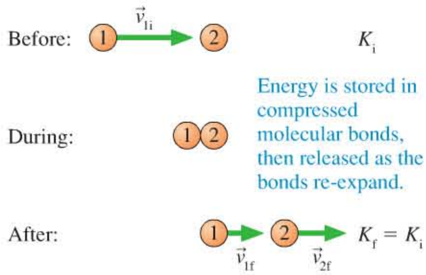
$$\text{energy conservation:} \quad \frac{1}{2}m_1(v_{1x})_i^2 = \frac{1}{2}m_1(v_{1x})_f^2 + \frac{1}{2}m_2(v_{2x})_f^2$$

Momentum conservation alone is not sufficient to analyze the collision because there are two unknowns: the two final velocities. That is why we did not consider perfectly elastic collisions in Chapter 9. Energy conservation gives us another condition. The complete solution of these two equations involves straightforward but rather lengthy algebra. We'll just give the solution here:

$$(v_{1x})_f = \frac{m_1 - m_2}{m_1 + m_2}(v_{1x})_i \qquad (v_{2x})_f = \frac{2m_1}{m_1 + m_2}(v_{1x})_i \tag{10.20}$$

Perfectly elastic collision with object 2 initially at rest

FIGURE 10.26 A perfectly elastic collision.



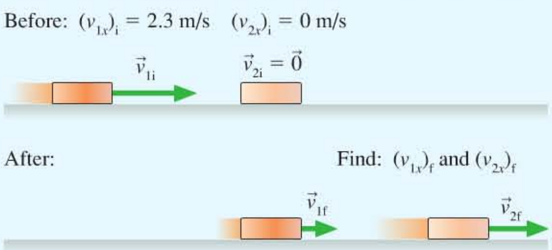
Equations 10.20 allow us to compute the final velocity of each object. Let's look at a common and important example: a perfectly elastic collision between two objects of equal mass.

EXAMPLE 10.15 Velocities in an air hockey collision

On an air hockey table, a moving puck, traveling to the right at 2.3 m/s, makes a head-on collision with an identical puck at rest. What is the final velocity of each puck?

PREPARE The before-and-after visual overview is shown in FIGURE 10.27. We've shown the final velocities in the picture, but we don't really know yet which way the pucks will move. Because one puck was initially at rest, we can use Equation 10.20 to find the final velocities of the pucks. The pucks are identical, so we have $m_1 = m_2 = m$.

FIGURE 10.27 A moving puck collides with a stationary puck.



SOLVE We use Equation 10.20 with $m_1 = m_2 = m$ to get

$$(v_{1x})_f = \frac{m - m}{m + m}(v_{1x})_i = 0 \text{ m/s}$$

$$(v_{2x})_f = \frac{2m}{m + m}(v_{1x})_i = (v_{1x})_i = 2.3 \text{ m/s}$$

The incoming puck stops dead, and the initially stationary puck goes off with the same velocity that the incoming one had.

ASSESS You can see that momentum and energy are conserved: The incoming puck's momentum and energy are completely transferred to the outgoing puck. If you've ever played pool, you've probably seen this sort of collision when you hit a ball head-on with the cue ball. The cue ball stops and the other ball picks up the cue ball's velocity.

Other cases where the colliding objects have unequal masses will be treated in the end-of-chapter problems.

Forces in Collisions

The collision between two pool balls occurs very quickly, and the forces are typically very large and difficult to calculate. Fortunately, by using the concepts of momentum and energy conservation, we can often calculate the final velocities of the balls without having to know the forces between them. There are collisions, however, where knowing the forces involved is of critical importance. The following example shows how a helmet helps protect the head from the large forces involved in a bicycle accident.

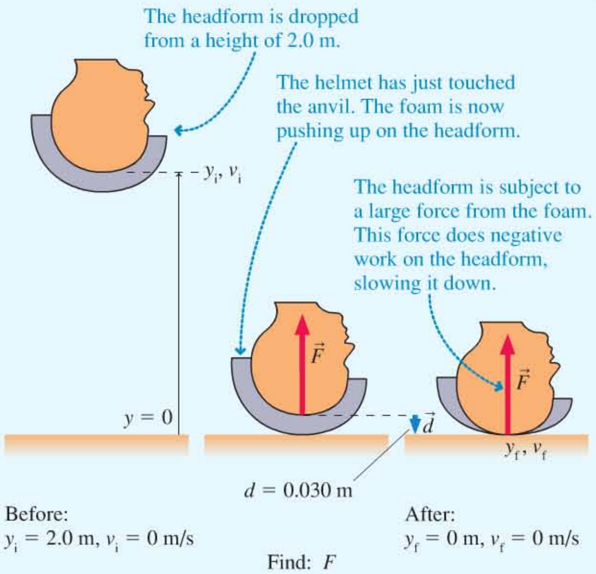
EXAMPLE 10.16 Protecting your head

A bike helmet—basically a shell of hard, crushable foam—is tested by being strapped onto a 5.0 kg headform and dropped from a height of 2.0 m onto a hard anvil. What force is encountered by the headform if the impact crushes the foam by 3.0 cm?

The foam inside a bike helmet is designed to crush upon impact.

PREPARE A before-and-after visual overview of the test is shown in **FIGURE 10.28**. We've chosen the endpoint of the problem to be when the headform comes to rest with the foam crushed. We can use the work-energy equation, Equation 10.3, to calculate the force on the headform. We'll choose the headform and the earth to be the system; the foam in the helmet is part of the environment. We make this choice so that the force on the headform due to the foam is an *external* force that does work W on the headform.

FIGURE 10.28 Before-and-after visual overview of the bike helmet test.



SOLVE The work-energy equation $\Delta K + \Delta U_g + \Delta E_{th} = W$ tells us that the work done by external forces—in this case, the force of the foam on the headform—changes the energy of the system. The headform starts at rest, speeds up as it falls, then returns to rest during the impact. Overall, then, $\Delta K = 0$. Furthermore, $\Delta E_{th} = 0$ because there's no friction to increase the thermal energy. Only the gravitational potential energy changes, giving

$$\Delta U_g = (U_g)_f - (U_g)_i = W$$

The upward force of the foam on the headform is opposite the downward displacement of the headform. Referring to Tactics Box 10.1, we see that the work done is negative: $W = -Fd$, where we've assumed that the force is relatively constant. Using this result in the work-energy equation and solving for F, we find

$$F = -\frac{(U_g)_f - (U_g)_i}{d} = \frac{(U_g)_i - (U_g)_f}{d}$$

Taking our reference height to be $y = 0$ m at the anvil, we have $(U_g)_f = 0$. We're left with $(U_g)_i = mgy_i$, so

$$F = \frac{mgy_i}{d} = \frac{(5.0\text{ kg})(9.8\text{ m/s})(2.0\text{ m})}{0.030\text{ m}} = 3300\text{ N}$$

This is the force that acts on the head to bring it to a halt in 3.0 cm. More important from the perspective of possible brain injury is the head's *acceleration:*

$$a = \frac{F}{m} = \frac{3300\text{ N}}{5.0\text{ kg}} = 660\text{ m/s}^2 = 67g$$

ASSESS The accepted threshold for serious brain injury is around $300g$, so this helmet would protect the rider in all but the most serious accidents. Without the helmet, the rider's head would come to a stop in a much shorter distance and thus be subjected to a much larger acceleration.

10.8 Power

We've now studied how energy can be transformed from one kind into another and how it can be transferred between the environment and the system as work. In many situations we would like to know *how quickly* the energy is transformed or transferred. Is a transfer of energy very rapid, or does it take place over a long time? In passing a truck, your car needs to transform a certain amount of the chemical energy in its fuel into kinetic energy. It makes a *big* difference whether your engine can do this in 20 s or 60 s!

The question How quickly? implies that we are talking about a *rate*. For example, the velocity of an object—how fast it is going—is the *rate of change* of position. So, when we raise the issue of how fast the energy is transformed, we are talking about the *rate of transformation* of energy. Suppose in a time interval Δt an amount of energy ΔE is transformed from one form to another. The rate at which this energy is transformed is called the **power** P and is defined as

$$P = \frac{\Delta E}{\Delta t} \qquad (10.21)$$

Power when an amount of energy ΔE is transformed in a time interval Δt

The unit of power is the **watt,** which is defined as 1 watt = 1 W = 1 J/s.

Power also measures the rate at which energy is transferred into or out of a system as work W. If work W is done in time interval Δt, the rate of energy *transfer* is

$$P = \frac{W}{\Delta t} \qquad (10.22)$$

Power when an amount of work W is done in a time interval Δt

A force that is doing work (i.e., transferring energy) at a rate of 3 J/s has an "output power" of 3 W. A system that is gaining energy at the rate of 3 J/s is said to "consume" 3 W of power. Common prefixes used for power are mW (milliwatts), kW (kilowatts), and MW (megawatts).

We can express Equation 10.22 in a different form. If in the time interval Δt an object undergoes a displacement Δx, the work done by a force acting on the object is $W = F\Delta x$. Then Equation 10.22 can be written as

$$P = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\frac{\Delta x}{\Delta t} = Fv$$

The rate at which energy is transferred to an object as work—the power—is the product of the force that does the work and the velocity of the object:

$$P = Fv \qquad (10.23)$$

Rate of energy transfer due to a force F acting on an object moving at velocity v

Both these cars take about the same energy to reach 60 mph, but the race car gets there in a much shorter time, so its *power* is much greater.

The English unit of power is the *horsepower.* The conversion factor to watts is

1 horsepower = 1 hp = 746 W

Many common appliances, such as motors, are rated in hp.

EXAMPLE 10.17 Power to pass a truck

Your 1500 kg car is behind a truck traveling at 60 mph (27 m/s). To pass it, you speed up to 75 mph (34 m/s) in 6.0 s. What engine power is required to do this?

PREPARE Your engine is transforming the chemical energy of its fuel into the kinetic energy of the car. We can calculate the rate of transformation by finding the change ΔK in the kinetic energy and using the known time interval.

SOLVE We have

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(1500\text{ kg})(27\text{ m/s})^2 = 5.47 \times 10^5\text{ J}$$
$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(1500\text{ kg})(34\text{ m/s})^2 = 8.67 \times 10^5\text{ J}$$

so that

$$\Delta K = K_f - K_i$$
$$= (8.67 \times 10^5\text{ J}) - (5.47 \times 10^5\text{ J}) = 3.20 \times 10^5\text{ J}$$

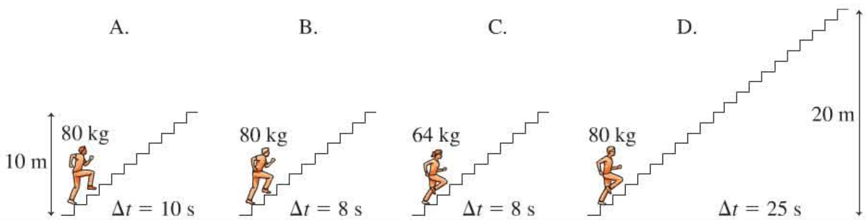
To transform this amount of energy in 6 s, the power required is

$$P = \frac{\Delta K}{\Delta t} = \frac{3.20 \times 10^5\text{ J}}{6.0\text{ s}} = 53{,}000\text{ W} = 53\text{ kW}$$

This is about 71 hp. This power is in addition to the power needed to overcome drag and friction and cruise at 60 mph, so the total power required from the engine will be even greater than this.

ASSESS You use a large amount of energy to perform a simple driving maneuver such as this. 3.20×10^5 J is enough energy to lift an 80 kg person 410 m in the air—the height of a tall skyscraper. And 53 kW would lift him there in only 6 s!

STOP TO THINK 10.6 Four students run up the stairs in the times shown. Rank in order, from largest to smallest, their power outputs P_A through P_D.



INTEGRATED EXAMPLE 10.18 Stopping a runaway truck

A truck's brakes can overheat and fail while descending mountain highways, leading to an extremely dangerous runaway truck. Some highways have *runaway-truck ramps* to safely bring out-of-control trucks to a stop. These uphill ramps are covered with a deep bed of gravel. The uphill slope and the large coefficient of rolling friction as the tires sink into the gravel bring the truck to a safe halt.

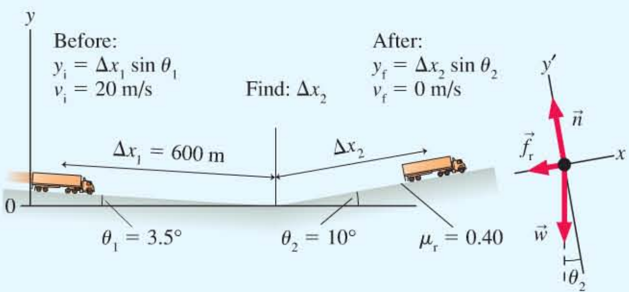
A runaway-truck ramp along Interstate 70 in Colorado.

A 22,000 kg truck heading down a 3.5° slope at 20 m/s (≈45 mph) suddenly has its brakes fail. Fortunately, there's a runaway-truck ramp 600 m ahead. The ramp slopes upward at an angle of 10°, and the coefficient of rolling friction between the truck's tires and the loose gravel is $\mu_r = 0.40$. Ignore air resistance and rolling friction as the truck rolls down the highway.

a. Use conservation of energy to find how far along the ramp the truck travels before stopping.
b. By how much does the thermal energy of the truck and ramp increase as the truck stops?

PREPARE Parts a and b can be solved using energy conservation by following Problem-Solving Strategy 10.1. **FIGURE 10.29** shows a before-and-after visual overview. Because we're going to need to determine friction forces to calculate the increase in thermal energy, we've also drawn a free-body diagram for the truck as it moves up the ramp. One slight complication is that the y-axis of free-body diagrams is drawn perpendicular to the slope, whereas the calculation of gravitational potential energy needs a vertical y-axis to measure height. We've dealt with this by labeling the free-body diagram axis the y'-axis.

FIGURE 10.29 Visual overview of the runaway truck.



SOLVE a. The law of conservation of energy for the motion of the truck, from the moment its brakes fail to when it finally stops, is

$$K_f + (U_g)_f + \Delta E_{th} = K_i + (U_g)_i$$

Because friction is present only along the ramp, thermal energy will be created only as the truck moves up the ramp. This thermal energy is then given by $\Delta E_{th} = f_r \Delta x_2$, because Δx_2 is the length of the ramp. The conservation of energy equation then is

$$\frac{1}{2}mv_f^2 + mgy_f + f_r\Delta x_2 = \frac{1}{2}mv_i^2 + mgy_i$$

From Figure 10.29 we have $y_i = \Delta x_1 \sin\theta_1$, $y_f = \Delta x_2 \sin\theta_2$, and $v_f = 0$, so the equation becomes

$$mg\Delta x_2 \sin\theta_2 + f_r\Delta x_2 = \frac{1}{2}mv_i^2 + mg\Delta x_1 \sin\theta_1$$

To find $f_r = \mu_r n$ we need to find the normal force n. The free-body diagram shows that

$$\sum F_{y'} = n - mg\cos\theta_2 = a_{y'} = 0$$

from which $f_r = \mu_r n = \mu_r mg\cos\theta_2$. With this result for f_r, our conservation of energy equation is

$$mg\Delta x_2 \sin\theta_2 + \mu_r mg\cos\theta_2 \Delta x_2 = \frac{1}{2}mv_i^2 + mg\Delta x_1 \sin\theta_1$$

which, after we divide both sides by mg, simplifies to

$$\Delta x_2 \sin\theta_2 + \mu_r \cos\theta_2 \Delta x_2 = \frac{v_i^2}{2g} + \Delta x_1 \sin\theta_1$$

Solving this for Δx_2 gives

$$\Delta x_2 = \frac{\dfrac{v_i^2}{2g} + \Delta x_1 \sin\theta_1}{\sin\theta_2 + \mu_r\cos\theta_2}$$

$$= \frac{\dfrac{(20\text{ m/s})^2}{2(9.8\text{ m/s}^2)} + (600\text{ m})(\sin 3.5°)}{\sin 10° + 0.40(\cos 10°)} = 100\text{ m}$$

b. We know that $\Delta E_{th} = f_r\Delta x_2 = (\mu_r mg\cos\theta_2)\Delta x_2$, so that

$$\Delta E_{th} = (0.40)(22{,}000\text{ kg})(9.8\text{ m/s}^2)(\cos 10°)(100\text{ m})$$
$$= 8.5 \times 10^6\text{ J}$$

ASSESS It seems reasonable that a truck that speeds up as it rolls 600 m downhill takes only 100 m to stop on a steeper, high-friction ramp. We also expect the thermal energy to be roughly comparable to the kinetic energy of the truck, since it's largely the kinetic energy that is transformed into thermal energy. At the top of the hill the truck's kinetic energy is $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(22{,}000\text{ kg})(20\text{ m/s})^2 = 4.4 \times 10^6$ J, which is of the same order of magnitude as ΔE_{th}. Our answer is reasonable.

SUMMARY

The goals of Chapter 10 are to introduce the concept of energy and to learn a new problem-solving strategy based on conservation of energy.

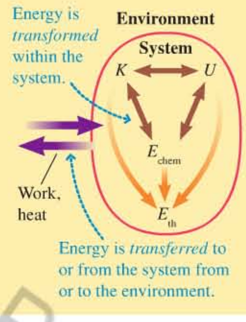
GENERAL PRINCIPLES

Basic Energy Model

Within a system, energy can be **transformed** between various forms.

Energy can be **transferred** into or out of a system in two basic ways:

- **Work:** The transfer of energy by mechanical forces.
- **Heat:** The nonmechanical transfer of energy from a hotter to a colder object.



Conservation of Energy

When work W is done on a system, the system's total energy changes by the amount of work done. In mathematical form, this is the **work-energy equation:**

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = W$$

A system is **isolated** when no energy is transferred into or out of the system. This means the work is zero, giving the **law of conservation of energy:**

$$\Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = 0$$

Solving Energy Conservation Problems

PREPARE Choose your system so that it's isolated. Draw a before-and-after visual overview.

SOLVE

- If the system is isolated and there's no friction, then mechanical energy is conserved:

$$K_f + (U_g)_f + (U_s)_f = K_i + (U_g)_i + (U_s)_i$$

- If the system is isolated but there's friction present, then the total energy is conserved:

$$K_f + (U_g)_f + (U_s)_f + \Delta E_{th} = K_i + (U_g)_i + (U_s)_i$$

ASSESS Kinetic energy is always positive, as is the change in thermal energy.

IMPORTANT CONCEPTS

Kinetic energy is an energy of motion:

$$K = \underbrace{\tfrac{1}{2}mv^2}_{\text{Translational}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{Rotational}}$$

Potential energy is energy stored in a system of interacting objects.

- **Gravitational potential energy:** $U_g = mgy$
- **Elastic potential energy:** $U_s = \frac{1}{2}kx^2$

Mechanical energy is the sum of a system's kinetic and potential energies:

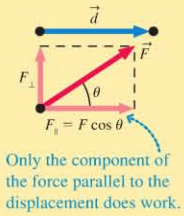
$$\text{Mechanical energy} = K + U = K + U_g + U_s$$

Thermal energy is the sum of the microscopic kinetic and potential energies of all the molecules in an object. The hotter an object, the more thermal energy it has. When kinetic (sliding) friction is present, the increase in the thermal energy is $\Delta E_{th} = f_k \Delta x$.

Work is the process by which energy is transferred to or from a system by the application of mechanical forces.

If a particle moves through a displacement $\vec{d}$ while acted upon by a constant force $\vec{F}$, the force does work

$$W = F_{\parallel} d = Fd\cos\theta$$



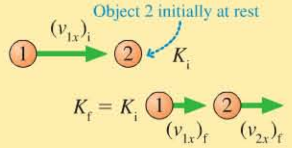
APPLICATIONS

Perfectly elastic collisions

Both mechanical energy and momentum are conserved.

$$(v_{1x})_f = \frac{m_1 - m_2}{m_1 + m_2}(v_{1x})_i$$

$$(v_{2x})_f = \frac{2m_1}{m_1 + m_2}(v_{1x})_i$$



Power is the rate at which energy is transformed . . .

$$P = \frac{\Delta E \leftarrow \text{Amount of energy transformed}}{\Delta t \leftarrow \text{Time required to transform it}}$$

. . . or at which work is done.

$$P = \frac{W \leftarrow \text{Amount of work done}}{\Delta t \leftarrow \text{Time required to do work}}$$

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

Problems labeled can be done on a Workbook Energy Worksheet; INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

QUESTIONS

Conceptual Questions

1. The brake shoes of your car are made of a material that can tolerate very high temperatures without being damaged. Why is this so?
2. When you pound a nail with a hammer, the nail gets quite warm. Describe the energy transformations that lead to the addition of thermal energy in the nail.

For Questions 3 through 10, give a specific example of a system with the energy transformation shown. In these questions, W is the work done on the system, and K, U, and E_{th} are the kinetic, potential, and thermal energies of the system, respectively. Any energy not mentioned in the transformation is assumed to remain constant; if work is not mentioned, it is assumed to be zero.

3. $W \rightarrow K$
4. $W \rightarrow U$
5. $K \rightarrow U$
6. $K \rightarrow W$
7. $U \rightarrow K$
8. $W \rightarrow \Delta E_{th}$
9. $U \rightarrow \Delta E_{th}$
10. $K \rightarrow \Delta E_{th}$
11. A ball of putty is dropped from a height of 2 m onto a hard floor, where it sticks. What object or objects need to be included within the system if the system is to be isolated during this process?
12. A 0.5 kg mass on a 1-m-long string swings in a circle on a horizontal, frictionless table at a steady speed of 2 m/s. How much work does the tension in the string do on the mass during one revolution? Explain.
13. Particle A has less mass than particle B. Both are pushed forward across a frictionless surface by equal forces for 1 s. Both start from rest.
    a. Compare the amount of work done on each particle. That is, is the work done on A greater than, less than, or equal to the work done on B? Explain.
    b. Compare the impulses delivered to particles A and B. Explain.
    c. Compare the final speeds of particles A and B. Explain.
14. The meaning of the word "work" is quite different in physics from its everyday usage. Give an example of an action a person could do that "feels like work" but that does not involve any work as we've defined it in this chapter.
15. To change a tire, you need to use a jack to raise one corner of your car. While doing so, you happen to notice that pushing the jack handle down 20 cm raises the car only 0.2 cm. Use energy concepts to explain why the handle must be moved so far to raise the car by such a small amount.
16. You drop two balls from a tower, one of mass m and the other of mass $2m$. Just before they hit the ground, which ball, if either, has the larger kinetic energy? Explain.
17. A roller coaster car rolls down a frictionless track, reaching speed v at the bottom.
    a. If you want the car to go twice as fast at the bottom, by what factor must you increase the height of the track?
    b. Does your answer to part a depend on whether the track is straight or not? Explain.
18. A spring gun shoots out a plastic ball at speed v. The spring is then compressed twice the distance it was on the first shot.
    a. By what factor is the spring's potential energy increased?
    b. By what factor is the ball's speed increased? Explain.
19. Sandy and Chris stand on the edge of a cliff and throw identical mass rocks at the same speed. Sandy throws her rock horizontally while Chris throws his upward at an angle of 45° to the horizontal. Are the rocks moving at the same speed when they hit the ground, or is one moving faster than the other? If one is moving faster, which one? Explain.
20. A solid cylinder and a cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axles. (The cylindrical shell has lightweight spokes connecting the shell to the axle.) A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground as shown in Figure Q10.20. Both blocks are released simultaneously. The ropes do not slip. Which block hits the ground first? Or is it a tie? Explain.

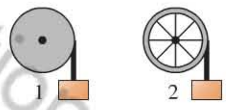


FIGURE Q10.20

21. BIO You are much more likely to be injured if you fall and your head strikes the ground than if your head strikes a gymnastics pad. Use energy and work concepts to explain why this is so.

Multiple-Choice Questions

22. || If you walk up a flight of stairs at constant speed, gaining vertical height h, the work done on you (the system, of mass m) is
    A. $+mgh$, by the normal force of the stairs.
    B. $-mgh$, by the normal force of the stairs.
    C. $+mgh$, by the gravitational force of the earth.
    D. $-mgh$, by the gravitational force of the earth.
23. | You and a friend each carry a 15 kg suitcase up two flights of stairs, walking at a constant speed. Take each suitcase to be the system. Suppose you carry your suitcase up the stairs in 30 s while your friend takes 60 s. Which of the following is true?
    A. You did more work, but both of you expended the same power.
    B. You did more work and expended more power.
    C. Both of you did equal work, but you expended more power.
    D. Both of you did equal work, but you expended less power.

24. A woman uses a pulley and a rope to raise a 20 kg weight to a height of 2 m. If it takes 4 s to do this, about how much power is she supplying?
    A. 100 W B. 200 W C. 300 W D. 400 W
25. A hockey puck sliding along frictionless ice with speed v to the right collides with a horizontal spring and compresses it by 2.0 cm before coming to a momentary stop. What will be the spring's maximum compression if the same puck hits it at a speed of $2v$?
    A. 2.0 cm B. 2.8 cm C. 4.0 cm
    D. 5.6 cm E. 8.0 cm
26. A block slides down a smooth ramp, starting from rest at a height h. When it reaches the bottom it's moving at speed v. It then continues to slide up a second smooth ramp. At what height is its speed equal to $v/2$?
    A. $h/4$ B. $h/2$ C. $3h/4$ D. $2h$
27. A wrecking ball is suspended from a 5.0-m-long cable that makes a 30° angle with the vertical. The ball is released and swings down. What is the ball's speed at the lowest point?
    A. 7.7 m/s B. 4.4 m/s C. 3.6 m/s D. 3.1 m/s

PROBLEMS

Section 10.2 Work

1. During an etiquette class, you walk slowly and steadily at 0.20 m/s for 2.5 m with a 0.75 kg book balanced on top of your head. How much work does your head do on the book?
2. A 2.0 kg book is lying on a 0.75-m-high table. You pick it up and place it on a bookshelf 2.3 m above the floor.
    a. How much work does gravity do on the book?
    b. How much work does your hand do on the book?
3. The two ropes seen in Figure P10.3 are used to lower a 255 kg piano exactly 5 m from a second-story window to the ground. How much work is done by each of the three forces?

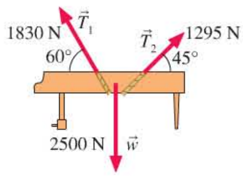


FIGURE P10.3

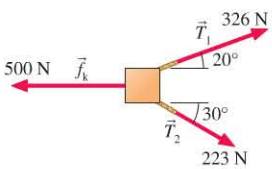


FIGURE P10.4

4. The two ropes shown in the bird's-eye view of Figure P10.4 are used to drag a crate exactly 3 m across the floor. How much work is done by each of the ropes on the crate?
5. a. At the airport, you ride a "moving sidewalk" that carries you horizontally for 25 m at 0.70 m/s. Assuming that you were moving at 0.70 m/s before stepping onto the moving sidewalk and continue at 0.70 m/s afterward, how much work does the moving sidewalk do on you? Your mass is 60 kg.
    b. An escalator carries you from one level to the next in the airport terminal. The upper level is 4.5 m above the lower level, and the length of the escalator is 7.0 m. How much work does the up escalator do on you when you ride it from the lower level to the upper level?
    c. How much work does the down escalator do on you when you ride it from the upper level to the lower level?
6. A boy flies a kite with the string at a 30° angle to the horizontal. The tension in the string is 4.5 N. How much work does the string do on the boy if the boy
    a. Stands still?
    b. Walks a horizontal distance of 11 m away from the kite?
    c. Walks a horizontal distance of 11 m toward the kite?

Section 10.3 Kinetic Energy

7. Which has the larger kinetic energy, a 10 g bullet fired at 500 m/s or a 10 kg bowling ball sliding at 10 m/s?
8. At what speed does a 1000 kg compact car have the same kinetic energy as a 20,000 kg truck going 25 km/hr?
9. A car is traveling at 10 m/s.
    a. How fast would the car need to go to double its kinetic energy?
    b. By what factor does the car's kinetic energy increase if its speed is doubled to 20 m/s?
10. Sam's job at the amusement park is to slow down and bring to a stop the boats in the log ride. If a boat and its riders have a mass of 1200 kg and the boat drifts in at 1.2 m/s, how much work does Sam do to stop it?
11. A 20 g plastic ball is moving to the left at 30 m/s. How much work must be done on the ball to cause it to move to the right at 30 m/s?
12. The turntable in a microwave oven has a moment of inertia of $0.040\ \text{kg} \cdot \text{m}^2$ and is rotating once every 4.0 s. What is its kinetic energy?
13. An energy storage system based on a flywheel (a rotating disk) can store a maximum of 4.0 MJ when the flywheel is rotating at 20,000 revolutions per minute. What is the moment of inertia of the flywheel?

Section 10.4 Potential Energy

14. The lowest point in Death Valley is 85.0 m below sea level. The summit of nearby Mt. Whitney has an elevation of 4420 m. What is the change in gravitational potential energy of an energetic 65.0 kg hiker who makes it from the floor of Death Valley to the top of Mt. Whitney?
15. a. What is the kinetic energy of a 1500 kg car traveling at a speed of 30 m/s ($\approx$65 mph)?
    b. From what height should the car be dropped to have this same amount of kinetic energy just before impact?
    c. Does your answer to part b depend on the car's mass?
16. The world's fastest humans can reach speeds of about 11 m/s. In order to increase his gravitational potential energy by an amount equal to his kinetic energy at full speed, how high would such a sprinter need to climb?
17. A 72 kg bike racer climbs a 1200-m-long section of road that has a slope of 4.3°. By how much does his gravitational potential energy change during this climb?
18. A 1000 kg wrecking ball hangs from a 15-m-long cable. The ball is pulled back until the cable makes an angle of 25° with the vertical. By how much has the gravitational potential energy of the ball changed?
19. How far must you stretch a spring with $k = 1000$ N/m to store 200 J of energy?

20. ‖ How much energy can be stored in a spring with a spring constant of 500 N/m if its maximum possible stretch is 20 cm?

21. ‖‖‖ BIO The elastic energy stored in your tendons can contribute up to 35% of your energy needs when running. Sports scientists have studied the change in length of the knee extensor tendon in sprinters and nonathletes. They find (on average) that the sprinters' tendons stretch 41 mm, while nonathletes' stretch only 33 mm. The spring constant for the tendon is the same for both groups, 33 N/mm. What is the difference in maximum stored energy between the sprinters and the nonathletes?

Section 10.5 Thermal Energy

22. | Marissa drags a 23 kg duffel bag 14 m across the gym floor. If the coefficient of kinetic friction between the floor and bag is 0.15, how much thermal energy does Marissa create?

23. ‖ Mark pushes his broken car 150 m down the block to his friend's house. He has to exert a 110 N horizontal force to push the car at a constant speed. How much thermal energy is created in the tires and road during this short trip?

24. ‖| A 900 N crate slides 12 m down a ramp that makes an angle of 35° with the horizontal. If the crate slides at a constant speed, how much thermal energy is created?

25. ‖| A 25 kg child slides down a playground slide at a *constant speed*. The slide has a height of 3.0 m and is 7.0 m long. Using energy considerations, find the magnitude of the kinetic friction force acting on the child.

Section 10.6 Using the Law of Conservation of Energy

26. ‖ A boy reaches out of a window and tosses a ball straight up with a speed of 10 m/s. The ball is 20 m above the ground as he releases it. Use conservation of energy to find
    a. The ball's maximum height above the ground.
    b. The ball's speed as it passes the window on its way down.
    c. The speed of impact on the ground.

27. ‖ a. With what minimum speed must you toss a 100 g ball straight up to just barely hit the 10-m-high ceiling of the gymnasium if you release the ball 1.5 m above the floor? Solve this problem using energy.
    b. With what speed does the ball hit the floor?

28. ‖| What minimum speed does a 100 g puck need to make it to the top of a frictionless ramp that is 3.0 m long and inclined at 20°?

29. ‖ A car is parked at the top of a 50-m-high hill. It slips out of gear and rolls down the hill. How fast will it be going at the bottom? (Ignore friction.)

30. ‖| A 1500 kg car is approaching the hill shown in Figure P10.30 at 10 m/s when it suddenly runs out of gas.
    a. Can the car make it to the top of the hill by coasting?
    b. If your answer to part a is yes, what is the car's speed after coasting down the other side?

FIGURE P10.30

31. ‖ A 10 kg runaway grocery cart runs into a spring with spring constant 250 N/m and compresses it by 60 cm. What was the speed of the cart just before it hit the spring?

32. ‖ As a 15,000 kg jet lands on an aircraft carrier, its tail hook snags a cable to slow it down. The cable is attached to a spring with spring constant 60,000 N/m. If the spring stretches 30 m to stop the plane, what was the plane's landing speed?

33. ‖ Your friend's Frisbee has become stuck 16 m above the ground in a tree. You want to dislodge the Frisbee by throwing a rock at it. The Frisbee is stuck pretty tight, so you figure the rock needs to be traveling at least 5.0 m/s when it hits the Frisbee. If you release the rock 2.0 m above the ground, with what minimum speed must you throw it?

34. ‖ A fireman of mass 80 kg slides down a pole. When he reaches the bottom, 4.2 m below his starting point, his speed is 2.2 m/s. By how much has thermal energy increased during his slide?

35. ‖ A 20 kg child slides down a 3.0-m-high playground slide. She starts from rest, and her speed at the bottom is 2.0 m/s.
    a. What energy transfers and transformations occur during the slide?
    b. What is the total change in the thermal energy of the slide and the seat of her pants?

36. ‖ A hockey puck is given an initial speed of 5.0 m/s. If the coefficient of kinetic friction between the puck and the ice is 0.05, how far does the puck slide before coming to rest? Solve this problem using conservation of energy.

Section 10.7 Energy in Collisions

37. ‖ A 50 g marble moving at 2.0 m/s strikes a 20 g marble at rest. What is the speed of each marble immediately after the collision? Assume the collision is perfectly elastic and the marbles collide head-on.

38. ‖ Ball 1, with a mass of 100 g and traveling at 10 m/s, collides head-on with ball 2, which has a mass of 300 g and is initially at rest. What are the final velocities of each ball if the collision is (a) perfectly elastic? (b) perfectly inelastic?

39. | An air-track glider undergoes a perfectly inelastic collision with an identical glider that is initially at rest. What fraction of the first glider's initial kinetic energy is transformed into thermal energy in this collision?

40. | Two balls undergo a perfectly elastic head-on collision, with one ball initially at rest. If the incoming ball has a speed of 200 m/s, what are the final speed and direction of each ball if
    a. The incoming ball is *much* more massive than the stationary ball?
    b. The stationary ball is *much* more massive than the incoming ball?

Section 10.8 Power

41. ‖ a. How much work must you do to push a 10 kg block of steel across a steel table at a steady speed of 1.0 m/s for 3.0 s? The coefficient of kinetic friction for steel on steel is 0.60.
    b. What is your power output while doing so?

42. | a. How much work does an elevator motor do to lift a 1000 kg elevator a height of 100 m?
    b. How much power must the motor supply to do this in 50 s at constant speed?

43. ‖| A 1000 kg sports car accelerates from 0 to 30 m/s in 10 s. What is the average power of the engine?

44. ‖| In just 0.30 s, you compress a spring (spring constant 5000 N/m), which is initially at its equilibrium length, by 4.0 cm. What is your average power output?

45. ‖| In the winter sport of curling, players give a 20 kg stone a push across a sheet of ice. A curler accelerates a stone to a speed of 3.0 m/s over a time of 2.0 s.
    a. How much force does the curler exert on the stone?
    b. What average power does the curler use to bring the stone up to speed?

46. ‖ A 710 kg car drives at a constant speed of 23 m/s. It is subject to a drag force of 500 N. What power is required from the car's engine to drive the car
   a. On level ground?
   b. Up a hill with a slope of 2.0°?
47. ||| An elevator weighing 2500 N ascends at a constant speed of 8.0 m/s. How much power must the motor supply to do this?

General Problems

48. ‖ A 2.3 kg box, starting from rest, is pushed up a ramp by a 10 N force parallel to the ramp. The ramp is 2.0 m long and tilted at 17°. The speed of the box at the top of the ramp is 0.80 m/s. Consider the system to be the box + ramp + earth.
   a. How much work W does the force do on the system?
   b. What is the change ΔK in the kinetic energy of the system?
   c. What is the change ΔU_g in the gravitational potential energy of the system?
   d. What is the change ΔE_{th} in the thermal energy of the system?
49. | A 55 kg skateboarder wants to just make it to the upper edge of a "half-pipe" with a radius of 3.0 m, as shown in Figure P10.49. What speed v_i does he need at the bottom if he is to coast all the way up?

FIGURE P10.49

   a. First do the calculation treating the skateboarder and board as a point particle, with the entire mass nearly in contact with the half-pipe.
   b. More realistically, the mass of the skateboarder in a deep crouch might be thought of as concentrated 0.75 m from the half-pipe. Assuming he remains in that position all the way up, what v_i is needed to reach the upper edge?
50. ||| BIO Fleas have remarkable jumping ability. A 0.50 mg flea, jumping straight up, would reach a height of 40 cm if there were no air resistance. In reality, air resistance limits the height to 20 cm.
   a. What is the flea's kinetic energy as it leaves the ground?
   b. At its highest point, what fraction of the initial kinetic energy has been converted to potential energy?
51. ||| INT A marble slides without friction in a *vertical* plane around the inside of a smooth, 20-cm-diameter horizontal pipe. The marble's speed at the bottom is 3.0 m/s; this is fast enough so that the marble makes a complete loop, never losing contact with the pipe. What is its speed at the top?
52. ‖ A 20 kg child is on a swing that hangs from 3.0-m-long chains, as shown in Figure P10.52. What is her speed v_i at the bottom of the arc if she swings out to a 45° angle before reversing direction?

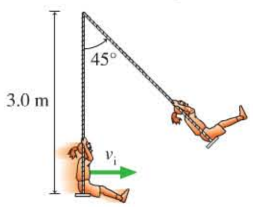


FIGURE P10.52

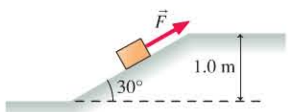


FIGURE P10.53

53. ‖ Suppose you lift a 20 kg box by a height of 1.0 m.
   a. How much work do you do in lifting the box?
   Instead of lifting the box straight up, suppose you push it up a 1.0-m-high ramp that makes a 30° degree angle with the horizontal, as shown in Figure P10.53. Being clever, you choose a ramp with no friction.
   b. How much force F is required to push the box straight up the slope at a constant speed?
   c. How long is the ramp?
   d. Use your force and distance results to calculate the work you do in pushing the box up the ramp. How does this compare to your answer to part a?
54. ‖ A cannon tilted up at a 30° angle fires a cannon ball at 80 m/s from atop a 10-m-high fortress wall. What is the ball's impact speed on the ground below? Ignore air resistance.
55. | The sledder shown in Figure P10.55 starts from the top of a frictionless hill and slides down into the valley. What initial speed v_i does the sledder need to just make it over the next hill?

FIGURE P10.55

56. |||| INT In a physics lab experiment, a spring clamped to the table shoots a 20 g ball horizontally. When the spring is compressed 20 cm, the ball travels horizontally 5.0 m and lands on the floor 1.5 m below the point at which it left the spring. What is the spring constant?
57. |||| A 50 g ice cube can slide without friction up and down a 30° slope. The ice cube is pressed against a spring at the bottom of the slope, compressing the spring 10 cm. The spring constant is 25 N/m. When the ice cube is released, what distance will it travel up the slope before reversing direction?
58. |||| BIO The maximum energy a bone can absorb without breaking is surprisingly small. For a healthy human of mass 60 kg, experimental data show that the leg bones can absorb about 200 J.
   a. From what maximum height could a person jump and land rigidly upright on both feet without breaking his legs? Assume that all the energy is absorbed in the leg bones in a rigid landing.
   b. People jump from much greater heights than this; explain how this is possible.
   Hint: Think about how people land when they jump from greater heights.
59. ‖ INT In an amusement park water slide, people slide down an essentially frictionless tube. They drop 3.0 m and exit the slide, moving horizontally, 1.2 m above a swimming pool. What horizontal distance do they travel from the exit point before hitting the water? Does the mass of the person make any difference?
60. ‖ The 5.0-m-long rope in Figure P10.60 hangs vertically from a tree right at the edge of a ravine. A woman wants to use the rope to swing to the other side of the ravine. She runs as fast as she can, grabs the rope, and swings out over the ravine.

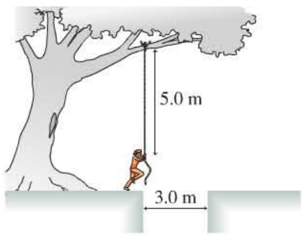


FIGURE P10.60

   a. As she swings, what energy conversion is taking place?
   b. When she's directly over the far edge of the ravine, how much higher is she than when she started?
   c. Given your answers to parts a and b, how fast must she be running when she grabs the rope in order to swing all the way across the ravine?

61. ||| INT You have been asked to design a "ballistic spring system" to measure the speed of bullets. A bullet of mass m is fired into a block of mass M. The block, with the embedded bullet, then slides across a frictionless table and collides with a horizontal spring whose spring constant is k. The opposite end of the spring is anchored to a wall. The spring's maximum compression d is measured.
   a. Find an expression for the bullet's initial speed v_B in terms of m, M, k, and d.

   Hint: This is a two-part problem. The bullet's collision with the block is an inelastic collision. What quantity is conserved in an inelastic collision? Subsequently the block hits a spring on a frictionless surface. What quantity is conserved in this collision?
   b. What was the speed of a 5.0 g bullet if the block's mass is 2.0 kg and if the spring, with $k = 50$ N/m, was compressed by 10 cm?
   c. What fraction of the bullet's initial kinetic energy is "lost"? Where did it go?

62. ||| INT A new event, shown in Figure P10.62, has been proposed for the Winter Olympics. An athlete will sprint 100 m, starting from rest, then leap onto a 20 kg bobsled. The person and bobsled will then slide down a 50-m-long ice-covered ramp, sloped at 20°, and into a spring with a carefully calibrated spring constant of 2000 N/m. The athlete who compresses the spring the farthest wins the gold medal. Lisa, whose mass is 40 kg, has been training for this event. She can reach a maximum speed of 12 m/s in the 100 m dash.

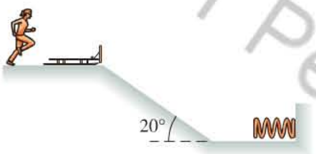


FIGURE P10.62

   a. How far will Lisa compress the spring?
   b. The Olympic committee has very exact specifications about the shape and angle of the ramp. Is this necessary? If the committee asks your opinion, what factors about the ramp will you tell them are important?

63. || Boxes A and B in Figure P10.63 have masses of 12.0 kg and 4.0 kg, respectively. The two boxes are released from rest. Use conservation of energy to find the boxes' speed when box B has fallen a distance of 0.50 m. Assume a frictionless upper surface.

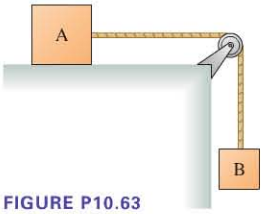


FIGURE P10.63

64. |||| What would be the speed of the boxes in Problem 63 if the coefficient of kinetic friction between box A and the surface it slides on were 0.20? Use conservation of energy.

65. ||||| A 20 g ball is fired horizontally with initial speed v_i toward a 100 g ball that is hanging motionless from a 1.0-m-long string. The balls undergo a head-on, perfectly elastic collision, after which the 100 g ball swings out to a maximum angle $\theta_{max} = 50°$. What was v_i?

66. || INT Two coupled boxcars are rolling along at 2.5 m/s when they collide with and couple to a third, stationary boxcar.
   a. What is the final speed of the three coupled boxcars?
   b. What fraction of the cars' initial kinetic energy is transformed into thermal energy?

67. || A fish scale, consisting of a spring with spring constant $k =$ 200 N/m, is hung vertically from the ceiling. A 5.0 kg fish is attached to the end of the unstretched spring and then released. The fish moves downward until the spring is fully stretched, then starts to move back up as the spring begins to contract. What is the maximum distance through which the fish falls?

68. | BIO A 70 kg human sprinter can accelerate from rest to 10 m/s in 3.0 s. During the same interval, a 30 kg greyhound can accelerate from rest to 20 m/s. Compute (a) the change in kinetic energy and (b) the average power output for each.

69. ||| INT A 50 g ball of clay traveling at speed v_i hits and sticks to a 1.0 kg block sitting at rest on a frictionless surface.
   a. What is the speed of the block after the collision?
   b. Show that the mechanical energy is *not* conserved in this collision. What percentage of the ball's initial kinetic energy is "lost"? Where did this kinetic energy go?

70. || INT A package of mass m is released from rest at a warehouse loading dock and slides down a 3.0-m-high frictionless chute to a waiting truck. Unfortunately, the truck driver went on a break without having removed the previous package, of mass $2m$, from the bottom of the chute as shown in Figure P10.70.
   a. Suppose the packages stick together. What is their common speed after the collision?
   b. Suppose the collision between the packages is perfectly elastic. To what height does the package of mass m rebound?

FIGURE P10.70

71. |||| INT A 50 kg sprinter, starting from rest, runs 50 m in 7.0 s at constant acceleration.
   a. What is the magnitude of the horizontal force acting on the sprinter?
   b. What is the sprinter's average power output during the first 2.0 s of his run?
   c. What is the sprinter's average power output during the final 2.0 s?

72. || INT Bob can throw a 500 g rock with a speed of 30 m/s. He moves his hand forward 1.0 m while doing so.
   a. How much force, assumed to be constant, does Bob apply to the rock?
   b. How much work does Bob do on the rock?

73. ||| A 2.0 hp electric motor on a water well pumps water from 10 m below the surface. The density of water is 1.0 kg per L. How many liters of water can the motor pump in 1 h?

74. || BIO The human heart has to pump the average adult's 6.0 L of blood through the body every minute. The heart must do work to overcome frictional forces that resist the blood flow. The average blood pressure is 1.3×10^4 N/m^2.
   a. Compute the work done moving the 6.0 L of blood completely through the body, assuming the blood pressure always takes its average value.
   b. What power output must the heart have to do this task once a minute?

   Hint: When the heart contracts, it applies force to the blood. Pressure is just force/area, so we can write work = (pressure)(area)(distance). But (area)(distance) is just the blood volume passing through the heart.

Passage Problems

Tennis Ball Testing

A tennis ball bouncing on a hard surface compresses and then rebounds. The details of the rebound are specified in tennis regulations. Tennis balls, to be acceptable for tournament play, must have a mass of 57.5 g. When dropped from a height of 2.5 m onto a concrete surface, a ball must rebound to a height of 1.4 m. During impact, the ball compresses by approximately 6 mm.

75. | How fast is the ball moving when it hits the concrete surface? (Ignore air resistance.)
A. 5 m/s B. 7 m/s C. 25 m/s D. 50 m/s
76. | If the ball accelerates uniformly when it hits the floor, what is its approximate acceleration as it comes to rest before rebounding?
A. 1000 m/s^2 B. 2000 m/s^2 C. 3000 m/s^2 D. 4000 m/s^2
77. | The ball's kinetic energy just after the bounce is less than just before the bounce. In what form does this lost energy end up?
A. Elastic potential energy
B. Gravitational potential energy
C. Thermal energy
D. Rotational kinetic energy
78. | By approximately what percent does the kinetic energy decrease?
A. 35% B. 45% C. 55% D. 65%
79. | When a tennis ball bounces from a racket, the ball loses approximately 30% of its kinetic energy to thermal energy. A ball that hits a racket at a speed of 10 m/s will rebound with approximately what speed?
A. 8.5 m/s B. 7.0 m/s C. 4.5 m/s D. 3.0 m/s

Work and Power in Cycling

When you ride a bicycle at constant speed, almost all of the energy you expend goes into the work you do against the drag force of the air. In this problem, assume that *all* of the energy expended goes into working against drag. As we saw in Section 5.7, the drag force on an object is approximately proportional to the square of its speed with respect to the air. For this problem, assume that $F \propto v^2$ exactly and that the air is motionless with respect to the ground unless noted otherwise. Suppose a cyclist and her bicycle have a combined mass of 60 kg and she is cycling along at a speed of 5 m/s.

80. | If the drag force on the cyclist is 10 N, how much energy does she use in cycling 1 km?
A. 6 kJ B. 10 kJ C. 50 kJ D. 100 kJ
81. | Under these conditions, how much power does she expend as she cycles?
A. 10 W B. 50 W C. 100 W D. 200 W
82. | If she doubles her speed to 10 m/s, how much energy does she use in cycling 1 km?
A. 20 kJ B. 40 kJ C. 200 kJ D. 400 kJ
83. | How much power does she expend when cycling at that speed?
A. 100 W B. 200 W C. 400 W D. 1000 W
84. | Upon reducing her speed back down to 5 m/s, she hits a headwind of 5 m/s. How much power is she expending now?
A. 100 W B. 200 W C. 500 W D. 1000 W

STOP TO THINK ANSWERS

Stop to Think 10.1: D. Since the child slides at a constant speed, his kinetic energy doesn't change. But his gravitational potential energy decreases as he descends. It is transformed into thermal energy in the slide and his bottom.

Stop to Think 10.2: C. $W = Fd\cos\theta$. The 10 N force at 90° does no work at all. $\cos 60° = \frac{1}{2}$, so the 8 N force does less work than the 6 N force.

Stop to Think 10.3: B > D > A = C. $K = (1/2)mv^2$. Using the given masses and velocities, we find $K_A = 2.0$ J, $K_B = 4.5$ J, $K_C = 2.0$ J, $K_D = 4.0$ J.

Stop to Think 10.4: $(U_g)_3 > (U_g)_2 = (U_g)_4 > (U_g)_1$. Gravitational potential energy depends only on height, not speed.

Stop to Think 10.5: D. The potential energy of a spring depends on the *square* of the displacement x, so the energy is positive whether the spring is compressed or extended. Furthermore, if the spring is compressed by twice the amount it had been stretched, the energy will increase by a factor of $2^2 = 4$. So the energy will be $4 \times 1\ \text{J} = 4\ \text{J}$.

Stop to Think 10.6: $P_B > P_A = P_C > P_D$. The power here is the rate at which each runner's internal chemical energy is converted into gravitational potential energy. The change in gravitational potential energy is $mg\Delta y$, so the power is $mg\Delta y/\Delta t$. For runner A, the ratio $m\Delta y/\Delta t$ equals (80 kg)(10 m)/(10 s) = 80 kg · m/s. For C, it's the same. For B, it's 100 kg · m/s, while for D the ratio is 64 kg · m/s.