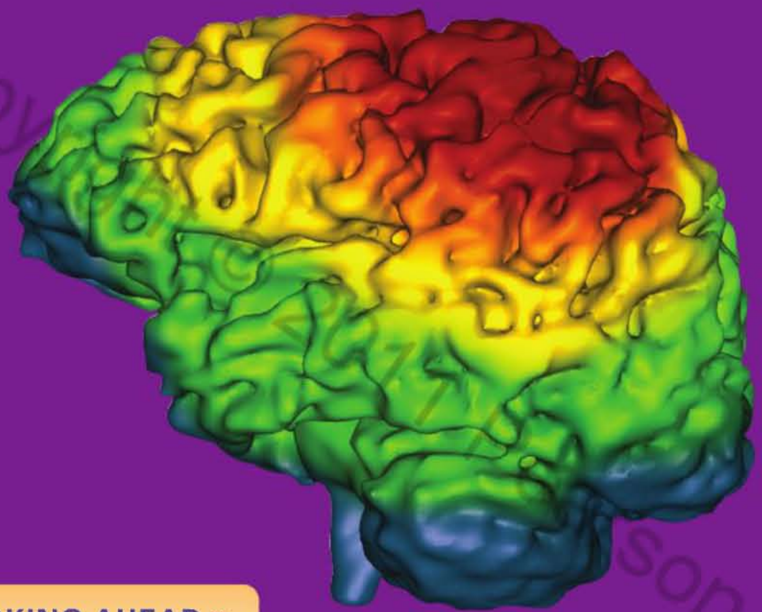
21 Electric Potential



The colors on this patient's brain are a map of the electric potential on the brain's surface after the patient was given a particular sensory stimulus. How does visualizing the electric potential help us understand the electric properties of the brain?

LOOKING AHEAD

The goal of Chapter 21 is to calculate and use the electric potential and electric potential energy.

Electric Potential Energy

A charged particle acquires **electric potential energy** when it is brought near other charges.

Lightning is a dramatic example of conversion of electric potential energy to light and thermal energy.

Looking Back
10.2–10.4 Work, kinetic energy, and potential energy

Electric Potential

The **electric potential** at a given location determines the electric potential energy that a charge would have if placed there. Electric potential differences are caused by the *separation of charge.*

Batteries create a charge separation by chemical means, leading to a potential difference between their terminals.

Using the Electric Potential

Depending on the sign of its charge, a charged particle speeds up or slows down as it moves through a potential difference.

X rays are produced when electrons, accelerated through a large potential difference, collide with a metal target.

Looking Back
20.3 Conservation of energy

Capacitors

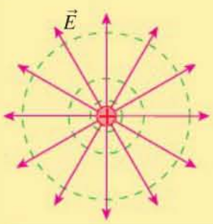
Capacitors store charge and electric potential energy. They're used in devices ranging from computers to defibrillators.

Each cylindrical element in this circuit is a capacitor, capable of rapidly storing and releasing charge and energy.

Looking Back
20.5 Parallel-plate capacitors

Connecting Potential and Field

The electric field and electric potential are intimately connected. You'll learn how to move from the field representation to the potential representation and back again.

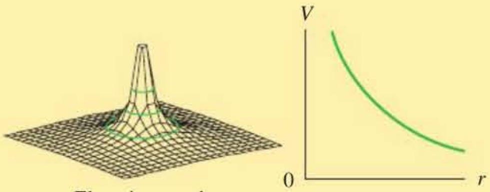


The electric field and the electric potential can be related to each other graphically.

Looking Back
20.4 The electric field

Calculating the Electric Potential

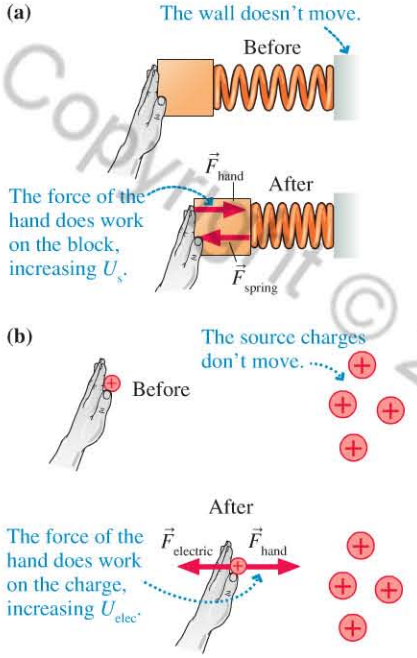
You'll learn to calculate the electric potential for several important charge distributions.



Elevation graph Potential graph

Because the electric potential is an abstract idea, we'll develop several different ways of *visualizing* the electric potential, as shown here for the important case of the potential due to a point charge.

FIGURE 21.1 The elastic potential energy of a spring and the electric potential energy of a system of charges.



21.1 Electric Potential Energy and Electric Potential

Conservation of energy was a powerful tool for understanding the motion of mechanical systems. In Chapter 10 you learned that the total energy of an isolated system remains constant. However, a system's energy can be changed by doing *work* on it. You will recall that work is the *transfer* of energy to or from a system by external forces that act on it as it undergoes a displacement. Depending on how the work is done, the energy transferred to the system can appear as kinetic energy K, potential energy U, or, as we'll investigate in this chapter, forms of energy associated with *electric* forces acting on charges.

To remind ourselves of how conservation of energy works for a mechanical system, **FIGURE 21.1a** shows a hand pushing a block against a spring in such a way that the block moves at a constant speed. (The spring also exerts a force on the block, directed opposite the hand force, so that the net force is zero and the block's speed is constant.) The force of the hand is a force *external* to the block + spring system, so this force does work on the system, increasing its energy. In this case, the energy transferred to the system by the hand appears as the elastic potential energy U_s stored in the spring. If the hand is removed, this stored energy will shoot the block away as the elastic potential energy is transformed into kinetic energy.

Let's apply these same ideas to the system of charged particles in **FIGURE 21.1b**. Several charges have been identified as source charges, and these—like the wall in Figure 21.1a—don't move. Suppose the hand pushes charge q at a constant speed toward the source charges. (The charge q is also subject to the electric force due to the source charges.) The force of the hand does work as it pushes the charge through a displacement, increasing the system's energy. Just as in the block + spring system, this energy appears as increased potential energy—in this case, as **electric potential energy** U_{elec}. If the hand is removed, this stored energy will shoot the charge q back out, in exact analogy with how the block was shot out by the spring.

In both cases, the energy transfer is $\Delta U = W$; that is, the work done increases the system's potential energy. This means that **we can determine electric potential energy by computing how much work must be done to assemble a set of charged particles.**

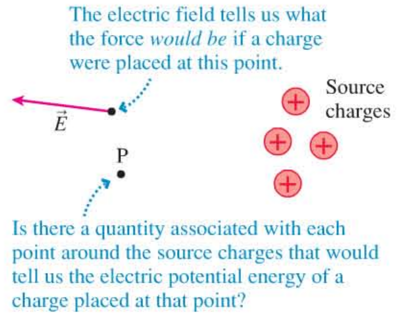
NOTE ► The electric potential energy in Figure 21.1b is an energy of the *system*—of charge q *and* the fixed source charges. However, because we're usually focused only on the moving charge, we often speak of the electric potential energy *of the charge*. ◄

Electric Potential

We introduced the concept of the *electric field* in Chapter 20. In the field model, the electric field is the agent by which charges exert a long-range force on another charge q. As shown in **FIGURE 21.2**, the source charges first alter the space around them by creating an electric field $\vec{E}$ at every point in space. It is then this electric field—not the source charges themselves—that exerts a force $\vec{F}_{elec} = q\vec{E}$ on charge q. An important idea was that the electric field of the source charges is present throughout space whether or not charge q is present to experience it. In other words, the electric field tells us what the force on the charge *would be* if the charge were placed there.

Can we apply similar reasoning to electric potential energy? Consider again the source charges in Figure 21.2. If we place a charge q at point P near the source charges, charge q will have electric potential energy. If we place a different charge q' at P, charge q' will have a different electric potential energy. Is there a quantity associated with P that would tell us what the electric potential energy of q, q', or any other charge *would have* at point P, without actually having to place the charge there?

FIGURE 21.2 What is the electric potential energy near some source charges?



To find out if this is possible, we'll have to understand in a bit more detail how to find the electric potential energy of a charge q. Suppose, to be specific, we take $q = 10$ nC in **FIGURE 21.3a** and, for convenience, we let $(U_{\text{elec}})_{\text{A}} = 0$ J when the charge is at point A.

As we've already seen, to find charge q's electric potential energy at any other point, such as point B or C, we need to find the amount of work it takes to move the charge from A to B, or A to C. In **FIGURE 21.3b** it takes the hand 4 μJ of work to move the charge from A to B; thus its electric potential energy at B is $(U_{\text{elec}})_{\text{B}} = 4\ \mu\text{J}$. (Recall that the SI unit of energy is the joule, J.) Similarly, q's electric potential energy at point C is $(U_{\text{elec}})_{\text{C}} = 6\ \mu\text{J}$ because it took 6 μJ of work to move it to point C.

What if we were to repeat this experiment with a different charge—say, $q = 20$ nC? According to Coulomb's law, the electric force on this charge due to the source charges will be twice that on the 10 nC charge. Consequently, the hand will have to push with twice as much force and thus do twice as much work in moving this charged particle from A to B. As a result, the 20 nC particle has $(U_{\text{elec}})_{\text{B}} = 8\ \mu\text{J}$ at B. A 5 nC particle would have $(U_{\text{elec}})_{\text{B}} = 2\ \mu\text{J}$ at B because the hand would have to work only half as hard to move it to B as it did to move the 10 nC particle to B. In general, **a charged particle's potential energy is proportional to its charge.**

When two quantities are proportional to each other, their *ratio* is constant. We can see this directly for the electric potential energy of a charged particle at point B by calculating the ratio

$$\frac{U_{(\text{elec})\text{B}}}{q} = \frac{2\ \mu\text{J}}{\underset{\substack{U/q \text{ for}\\ q = 5 \text{ nC}}}{5\ \text{nC}}} = \frac{4\ \mu\text{J}}{\underset{\substack{U/q \text{ for}\\ q = 10 \text{ nC}}}{10\ \text{nC}}} = \frac{8\ \mu\text{J}}{\underset{\substack{U/q \text{ for}\\ q = 20 \text{ nC}}}{20\ \text{nC}}} = \underset{\substack{\text{All three ratios}\\ \text{are the } same.}}{400\ \frac{\text{J}}{\text{C}}}$$

Thus we can write a simple expression for the electric potential energy that *any* charge q *would have* if placed at point B:

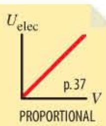
$$(U_{\text{elec}})_{\text{B}} = \underset{\substack{\text{This number is asso-}\\ \text{ciated with point B.}}}{\left(400\ \frac{\text{J}}{\text{C}}\right)}\underset{\substack{\text{This part depends on the}\\ \text{charge we place at B.}}}{q}$$

The number 400 J/C, which is associated with point B, tells us the *potential* for creating potential energy (there's a mouthful!) if a charge q is placed at point B. Thus this value is called the **electric potential,** and it is given the symbol V. At B, then, the electric potential is 400 J/C, as shown in **FIGURE 21.3c**. By similar reasoning, the electric potential at point C is 600 J/C.

This idea can be generalized. Any source charges create an electric potential at every point in the space around them. At a point where the potential is V, the electric potential energy of a charged particle q is

$$U_{\text{elec}} = qV \tag{21.1}$$

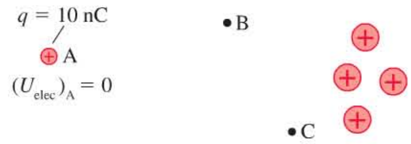
Relationship between electric potential and electric potential energy



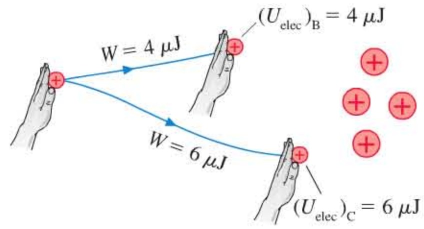
Notice the similarity to $\vec{F}_{\text{elec}} = q\vec{E}$. The electric potential, like the electric field, is created by the source charges and is present at all points in space. The electric potential is there whether or not charge q is present to experience it. While the electric field tells us how the source charges would exert a *force* on q, the electric potential tells us how the source charges would provide q with *potential energy*. Although we used the work done on a positive charge to justify Equation 21.1, it is also valid if q is negative.

FIGURE 21.3 Finding the electric potential energy and the electric potential.

(a) The electric potential energy of a 10 nC charge at A is zero. What is its potential energy at point B or C?



(b) The charge's electric potential energy at any point is equal to the amount of work done in moving it there from point A.



(c) The electric potential is created by the source charges. It exists at *every* point in space, not only at A, B, and C.

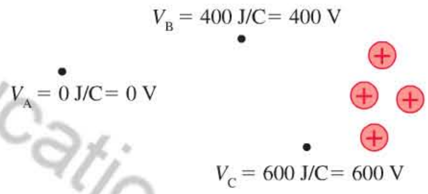


TABLE 21.1 Typical electric potentials

Source of potential	Approximate potential
Cells in human body	100 mV
Battery	1–10 V
Household electricity	100 V
Static electricity	10 kV
Transmission lines	500 kV

NOTE ► For potential energy, we found that we could choose a particular configuration of the system to have $U = 0$, the zero of potential energy. The same idea holds for electric potential. We can choose any point in space, wherever is convenient, to be $V = 0$. It will turn out that only *changes* in the electric potential are important, so the choice of a point to be $V = 0$ has no physical consequences. ◄

The unit of electric potential is the joule per coulomb, called the **volt** V:

$$1 \text{ volt} = 1 \text{ V} = 1 \text{ J/C}$$

This unit is named for Alessandro Volta, who invented the battery in 1800. Microvolts (μV), millivolts (mV), and kilovolts (kV) are commonly used units. Table 21.1 lists some typical electric potentials. We can now recognize that the electric potential in Figure 21.2—a potential due to the source charges—is 0 V at A and 400 V at B. This is shown in Figure 21.3c.

NOTE ► The symbol V is widely used to represent the *volume* of an object, and now we're introducing the same symbol to mean *potential*. To make matters more confusing, V is the abbreviation for *volts*. In printed text, V for potential is italicized while V for volts is not, but you can't make such a distinction in handwritten work. This is not a pleasant state of affairs, but these are the commonly accepted symbols. You must be especially alert to the *context* in which a symbol is used. ◄

EXAMPLE 21.1 Finding the change in a charge's electric potential energy

A 15 nC charged particle moves from point A, where the electric potential is 300 V, to point B, where the electric potential is –200 V. By how much does the electric potential change? By how much does the particle's electric potential energy change? How would your answers differ if the particle's charge were –15 nC?

PREPARE The change in the electric potential ΔV is the potential at the final point B minus the potential at the initial point A. From Equation 21.1, we can find the change in the electric potential energy by noting that $\Delta U_{\text{elec}} = (U_{\text{elec}})_B - (U_{\text{elec}})_A = q(V_B - V_A) = q\Delta V$.

SOLVE We have

$$\Delta V = V_B - V_A = (-200 \text{ V}) - (300 \text{ V}) = -500 \text{ V}$$

This change is *independent* of the charge q because the electric potential is created by source charges.

The change in the particle's electric potential energy is

$$\Delta U_{\text{elec}} = q\Delta V = (15 \times 10^{-9} \text{ C})(-500 \text{ V}) = -7.5\ \mu\text{J}$$

A -15 nC charge would have ΔU_{elec} $+7.5\ \mu$J because q changes sign while ΔV remains unchanged.

ASSESS Because the electric potential at B is lower than that at A, the positive (+15 nC) charge will lose electric potential energy, while the negative (-15 nC) charge will gain energy.

STOP TO THINK 21.1 A positively charged particle moves from point 1 to point 2. As it does, its electric potential energy

A. Increases.
B. Decreases.
C. Stays the same.

21.2 Sources of Electric Potential

How is an electric potential created in the first place? Consider the uncharged capacitor shown in **FIGURE 21.4a**. There's no force on charge q, so no work is required to move it from A to B. Consequently, charge q's electric potential energy remains *unchanged* as it is moved from A to B, so that $(U_{\text{elec}})_B = (U_{\text{elec}})_A$. Then, because $U_{\text{elec}} = qV$, it must be the case that $V_B = V_A$. We say that the **potential difference** $\Delta V = V_B - V_A$ is *zero*.

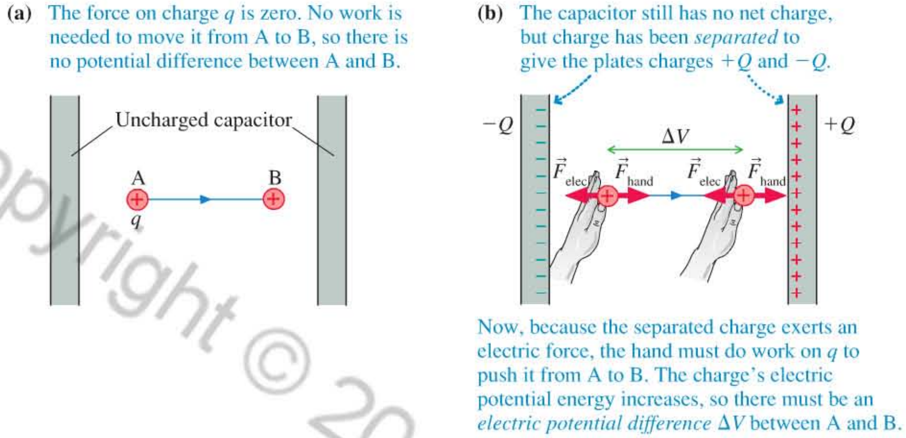


FIGURE 21.4 Potential differences are created by charge separation.

Now consider what happens if electrons are transferred from the right side of the capacitor to the left, giving the left electrode charge $-Q$ and the right electrode charge Q. The capacitor still has no net charge, but the charge has been *separated*. These separated charges exert a force $\vec{F}_{elec}$ on q, as **FIGURE 21.4b** shows, so that the hand must now do work on q to move it from A to B, increasing its electric potential energy so that $(U_{elec})_B > (U_{elec})_A$. This means that the potential difference ΔV between A and B is no longer zero. What we've shown here is quite general: **A potential difference is created by *separating* positive charge from negative charge.**

Perhaps the most straightforward way to create a separation of charge is by the frictional transfer of charge discussed in Chapter 20. As you shuffle your feet across a carpet, the friction between your feet and the carpet transfers charge to your body, causing a potential difference between your body and, say, a nearby doorknob. The potential difference between you and a doorknob can be many tens of thousands of volts—enough to create a spark as the excess charge on your body moves from higher to lower potential.

Lightning is the result of large potential differences built up by charge separation within clouds.

Lightning is the result of a charge separation that occurs in clouds. As small ice particles in the clouds collide, they become charged by frictional rubbing. The details are still not well understood, but heavier particles, which fall to the bottom of the cloud, gain a negative charge, while lighter particles, which are lifted to the cloud's top, become positive. Thus the top of the cloud becomes positively charged and the bottom negatively charged. This natural charge separation creates a huge potential difference—as much as 100 millions volts—between the top and bottom of the cloud. The negative charge in the bottom of the cloud causes positive charge to accumulate in the ground below. A lightning strike occurs when the potential difference between the cloud and the ground becomes too large for the air to sustain.

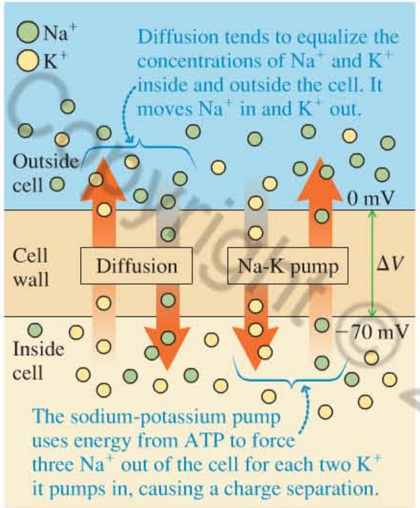
A car battery maintains a fixed potential difference of 12 V between its + and − terminals.

Charge separation, and hence potential differences, can also be created by chemical processes. A common and important means of creating a fixed potential difference is the **battery.** We'll study batteries in Chapter 22, but all batteries use chemical reactions to create an internal charge separation. This separation proceeds until a characteristic potential difference—about 1.5 V for a standard alkaline battery—appears between the two terminals of the battery. Different kinds of batteries maintain different potential differences between their terminals.

NOTE ► The potential difference between two points is often called the **voltage.** Thus we say that a battery's voltage is 1.5 V or 12 V, and we speak of the potential difference between a battery's terminals as the voltage "across" the battery. ◄

Chemical means of producing potential differences are also crucial in biological systems. For example, there's a potential difference of about 70 mV between the inside and outside of a cell, with the inside of the cell more negative than the outside.

FIGURE 21.5 The membrane potential of a cell is due to a charge separation.



A voltmeter always uses two probes to measure a potential difference. Here, we see that the potential difference of a fresh 9 V battery is closer to 9.7 V.

As illustrated in FIGURE 21.5, this *membrane potential* is caused by an imbalance of potassium (K^+) and sodium (Na^+) ions. The molar concentration of K^+ is higher inside the cell than outside, while the molar concentration of Na^+ is higher outside than inside. To keep the charge separated in the face of diffusion, which tends to equalize the ion concentrations, a *sodium-potassium exchange pump* continuously pumps sodium out of the cell and potassium into the cell. During one pumping cycle, three Na^+ are pushed out of the cell but only two K^+ are pushed in, giving a net transfer of one positive charge out of the cell. This continuous pumping leads to the charge separation that causes the membrane potential. We'll have a careful look at the electrical properties of nerve cells in Chapter 23.

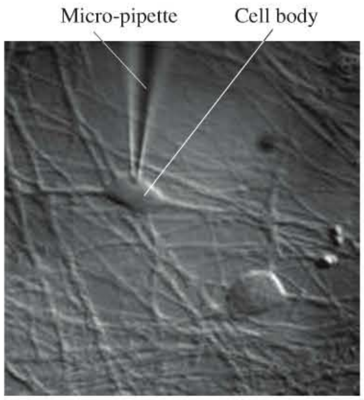
Measuring Electric Potential

Measurements of the electric potential play an important role in a broad range of applications. An electrocardiogram measures the potential difference between several locations on the body to diagnose possible heart problems; temperature is often measured using a *thermocouple,* a device that develops a potential difference proportional to temperature; your digital camera can sense when its battery is low by measuring the potential difference between its terminals.

Note that in all these applications, it's the potential *difference* between two points that's measured. The actual value of the potential at a given point depends on where we choose V to be zero, but the difference in potential between two points is independent of this choice. Because of this, a **voltmeter,** the basic instrument for measuring potential differences, always has *two* inputs. Probes are connected from these inputs to the two points between which the potential difference is to be measured. We'll learn more in Chapter 23 about how voltmeters work.

As small as cells are, the membrane potential difference between the inside and outside of a cell can be measured by a (very small) probe connected to a voltmeter. FIGURE 21.6 is a micrograph of a nerve cell whose membrane potential is being measured. A very small glass pipette, filled with conductive fluid, is actually inserted through the cell's membrane. This pipette is one of the probes. The second probe need not be so small; it is simply immersed in the conducting fluid that surrounds the cell and can be quite far from the cell.

FIGURE 21.6 Measuring the membrane potential.



21.3 Electric Potential and Conservation of Energy

The potential energy of a charged particle is determined by the electric potential: $U_{elec} = qV$. Although potential and potential energy are related and have similar names, they are not the same thing. Table 21.2 will help you distinguish between electric potential and electric potential energy.

Because energy is conserved, a particle of charge q speeds up or slows down as it moves through a region of changing potential. The conservation of energy equation is

$$K_f + (U_{elec})_f = K_i + (U_{elec})_i$$

which we can write in terms of the electric potential V as

$$K_f + qV_f = K_i + qV_i \tag{21.2}$$

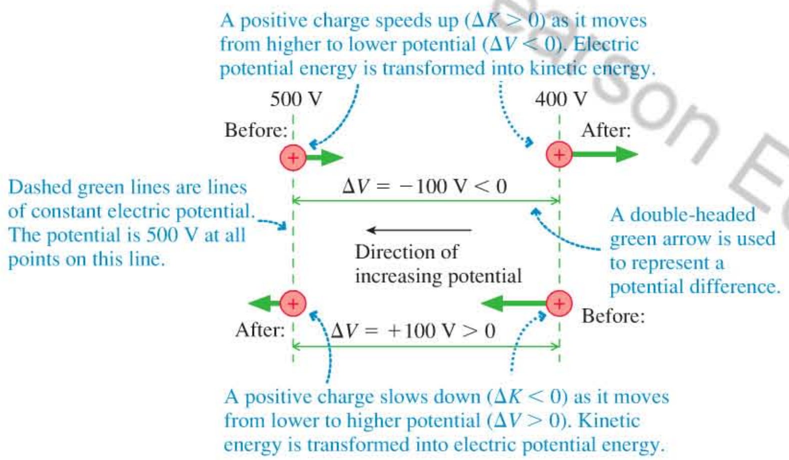
where, as usual, the subscripts i and f stand for the initial and final situations.

FIGURE 21.7 shows two positive charges moving through a region of changing electric potential. This potential has been created by source charges that aren't shown; our concern is only with the *effect* of this potential on the moving charge. Notice that we've used the before-and-after visual overview introduced earlier when we studied conservation of momentum and energy.

TABLE 21.2 Distinguishing electric potential and potential energy

The *electric potential* is a property of the source charges. The electric potential is present whether or not a charged particle is there to experience it. Potential is measured in J/C, or V.
The *electric potential energy* is the interaction energy of a charged particle with the source charges. Potential energy is measured in J.

FIGURE 21.7 A charged particle speeds up or slows down as it moves through a potential difference.



We can understand the motion of the charges if we rewrite Equation 21.2 as $K_f - K_i = -q(V_f - V_i)$, or

$$\Delta K = -q\Delta V \tag{21.3}$$

For the upper charge in Figure 21.7, the change in potential—that is, the potential difference—as it moves from left to right is

$$\Delta V = V_f - V_i = 400\text{ V} - 500\text{ V} = -100\text{ V}$$

which is negative. Equation 21.3 then shows that ΔK is *positive,* indicating that the particle *speeds up* as it moves from higher to lower potential. Conversely, for the lower charge in Figure 21.7 the potential difference is $+100$ V; Equation 21.3 then shows that ΔK is negative, so the particle *slows down* in moving from lower to higher potential.

NOTE ▶ The situation is reversed for a negative charge. If $q < 0$, Equation 21.3 requires K to increase as V increases. A negative charge speeds up if it moves into a region of higher potential. ◀

Conservation of energy is the basis of a powerful problem-solving strategy.

PROBLEM-SOLVING STRATEGY 21.1 **Conservation of energy in charge interactions**

PREPARE Draw a before-and-after visual overview. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + qV_f = K_i + qV_i$$

- Find the electric potential at both the initial and final positions. You may need to calculate it from a known expression for the potential, such as that of a point charge.
- K_i and K_f are the total kinetic energies of all moving particles.
- Some problems may need additional conservation laws, such as conservation of charge or conservation of momentum.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 18

EXAMPLE 21.2 **A speeding proton**

A proton moves through an electric potential created by a number of source charges. Its speed is 2.5×10^5 m/s at a point where the potential is 1500 V. What will be the proton's speed a short time later when it reaches a point where the potential is –500 V?

PREPARE The mass of a proton is $m = 1.7 \times 10^{-27}$ kg. The positively charged proton moves from a region of higher potential to one of lower potential, so the change in potential is negative and the proton loses electric potential energy. Conservation of energy then requires its kinetic energy to increase. We can use Equation 21.2 to find the proton's final speed.

SOLVE Conservation of energy gives

$$K_f + qV_f = K_i + qV_i$$

or

$$\tfrac{1}{2}mv_f^2 + qV_f = \tfrac{1}{2}mv_i^2 + qV_i$$

Solving for v_f^2 gives

$$v_f^2 = v_i^2 + \frac{2}{m}(qV_i - qV_f) = v_i^2 + \frac{2q}{m}(V_i - V_f)$$

or

$$v_f^2 = (2.5 \times 10^5\ \text{m/s})^2 + \frac{2(1.6 \times 10^{-19}\ \text{C})}{1.7 \times 10^{-27}\ \text{kg}}[1500\ \text{V} - (-500\ \text{V})]$$

$$= 4.4 \times 10^{11}\ (\text{m/s})^2$$

Solving for the final speed gives

$$v_f = 6.6 \times 10^5\ \text{m/s}$$

ASSESS This problem is very similar to the situation in the upper part of Figure 21.7. A positively charged particle speeds up as it moves from higher to lower potential, analogous to a particle speeding up as it slides down a hill from higher gravitational potential energy to lower gravitational potential energy.

FIGURE 21.8 A transformation of electric potential energy into thermal energy.

So far we've considered only the transformation of electric potential energy into kinetic energy as a charged particle moves from higher to lower electric potential. That is, we've studied the energy transformation $\Delta K = -q\Delta V$. But it's worth noting that electric potential energy can also be transformed into other kinds of energy; this is the basis of many applications of electricity. In **FIGURE 21.8**, for example, charges move in the wires from the high-potential terminal of the battery, through the lightbulb, and back to the low-potential terminal. In the bulb, their electric potential energy is transformed into thermal energy E_{th}, making the bulb hot enough to glow brightly. This energy transformation is $\Delta E_{th} = -q\Delta V$. Or, as charges move from the high to the low potential terminals of an elevator motor, their electric potential energy is transformed into gravitational potential energy as the elevator and its passengers are lifted, so in this case $\Delta U_g = -q\Delta V$.

We'll have much more to say about these and other transformations of electric energy in chapters to come!

The Electron Volt

The joule is a unit of appropriate size in mechanics and thermodynamics, where we deal with macroscopic objects, but it will be very useful to have an energy unit appropriate to atomic and nuclear events.

Suppose an electron accelerates through a potential difference $\Delta V = 1$ V. The electron might be accelerating from 0 V to 1 V, or from 1000 V to 1001 V. Regardless of the actual voltages, an electron, being negative, *speeds up* when it moves toward a higher potential, so that, according to Equation 21.3, the 1 V potential difference causes the electron, with $q = -e$, to gain kinetic energy

$$\Delta K = -q\Delta V = e\Delta V = (1.60 \times 10^{-19}\ \text{C})(1\ \text{V}) = 1.60 \times 10^{-19}\ \text{J}$$

Let us define a new unit of energy, called the **electron volt,** as

$$1\ \text{electron volt} = 1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$$

With this definition, the kinetic energy gained by the electron in our example is

$$\Delta K = 1\ \text{eV}$$

In other words, **1 electron volt is the kinetic energy gained by an electron (or proton) if it accelerates through a potential difference of 1 volt.**

NOTE ► The abbreviation eV uses a lowercase e but an uppercase V. Units of keV (10^3 eV), MeV (10^6 eV), and GeV (10^9 eV) are common. ◄

The electron volt can be a troublesome unit. One difficulty is its unusual name, which looks less like a unit than, say, "meter" or "second." A more significant difficulty is that the name suggests a relationship to volts. But *volts* are units of electric potential, whereas this new unit is a unit of energy! It is crucial to distinguish between the *potential V*, measured in volts, and an *energy* that can be measured either in joules or in electron volts. You can now use electron volts anywhere that you would previously have used joules. Doing so is no different from converting back and forth between pressure units of pascals and atmospheres.

NOTE ► The joule remains the SI unit of energy. It will be useful to express energies in eV, but you *must* convert this energy to joules before doing most calculations. ◄

Cancer-fighting electrons BIO Tightly focused beams of x rays can be used in radiation therapy for cancer patients. The x rays are generated by directing a high-energy beam of electrons at a metal target. The electrons gain their energy by being accelerated in a *linear accelerator* through a potential difference of 20 MV, so their final kinetic energy is 20 MeV.

EXAMPLE 21.3 The speed of a proton

Atomic particles are often characterized by their kinetic energy in MeV. What is the speed of an 8.7 MeV proton?

SOLVE The kinetic energy of this particle is 8.7×10^6 eV. First, we convert the energy to joules:

$$K = 8.7 \times 10^6\ \text{eV} \times \frac{1.60 \times 10^{-19}\ \text{J}}{1.0\ \text{eV}} = 1.4 \times 10^{-12}\ \text{J}$$

Now we can find the speed from

$$K = \frac{1}{2}mv^2$$

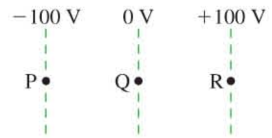
which gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.4 \times 10^{-12}\ \text{J})}{1.7 \times 10^{-27}\ \text{kg}}} = 4.1 \times 10^7\ \text{m/s}$$

Because the proton's charge and the electron's charge have the same magnitude, a general rule is that a proton or electron that accelerates (decelerates) through a potential difference of V volts gains (loses) V eV of kinetic energy. In Example 21.2, with a 2000 V potential difference, the proton gained 2000 eV of kinetic energy. In Example 21.3, the proton had to accelerate through a 8.7×10^6 V = 8.7 MV potential difference to acquire 8.7 MeV of kinetic energy.

STOP TO THINK 21.2 A proton is released from rest at point Q, where the potential is 0 V. Afterward, the proton

A. Remains at rest at Q.
B. Moves toward P with a steady speed.
C. Moves toward P with an increasing speed.
D. Moves toward R with a steady speed.
E. Moves toward R with an increasing speed.



21.4 Calculating the Electric Potential

11.11 ActivPhysics

Let's put these ideas to work by calculating the electric potential for some important cases. We'll do so using Equation 21.1, the relationship between the potential energy of a charge q at a point in space and the electric potential at that point. Rewriting Equation 21.1 slightly, we have

$$V = \frac{U_{\text{elec}}}{q} \tag{21.4}$$

Our prescription for finding the potential at a certain point in space, then, is to first calculate the electric potential *energy* of a charge q placed at that point. Then we can use Equation 21.4 to find the electric potential.

The Electric Potential Inside a Parallel-Plate Capacitor

In Chapter 20 we learned that a *uniform* electric field can be created by placing equal but opposite charges on two parallel conducting plates—the **parallel-plate capacitor.** Thus finding the electric potential inside a parallel-plate capacitor is equivalent to finding the potential for the very important case of a uniform electric field.

FIGURE 21.9 Finding the potential of a parallel-plate capacitor.

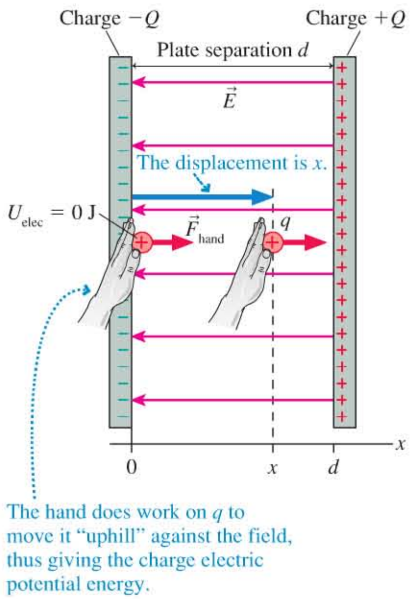


FIGURE 21.9 shows a cross-section view of a charged parallel-plate capacitor with separation d between the plates. The charges $\pm Q$ on the plates are the source charges that create both the electric field and an electric potential in the space between the plates. As we found in Chapter 20, the electric field is $\vec{E} = (Q/\epsilon_0 A$, from positive to negative). We'll choose a coordinate system with $x = 0$ at the negative plate and $x = d$ at the positive plate.

We're free to choose the point of zero potential energy anywhere that's convenient, so let $U_{\text{elec}} = 0$ when a mobile charge q is at the negative plate. The charge's potential energy at any other position x is then the amount of work an external force must do to move the charge at steady speed from the negative plate to that position. We'll represent the external force by a hand, although that's not really how charges get moved around.

The electric field in Figure 21.9 points to the left, so the force $\vec{F}_{\text{elec}} = q\vec{E}$ of the field on the charge is also to the left. To move the charge to the right at constant speed ($\vec{F}_{\text{net}} = \vec{0}$), the external force $\vec{F}_{\text{hand}}$ must push to the right with a force of the same magnitude: $F_{\text{hand}} = qE$. This force does work on the system as the charge is moved, changing the potential energy of the system.

Because the external force is constant and is parallel to the displacement, the work to move the charge to position x is

$$W = \text{force} \times \text{displacement} = F_{\text{hand}}x = qEx$$

Consequently, the electric potential energy when charge q is at position x is

$$U_{\text{elec}} = W = qEx$$

As the final step, we can use Equation 21.4 to find that the electric potential of the parallel-plate capacitor at position x, measured from the positive plate, is

$$V = \frac{U_{\text{elec}}}{q} = Ex = \frac{Q}{\epsilon_0 A}x \tag{21.5}$$

where, in the last step, we wrote the electric field strength in terms of the amount of charge on the capacitor plates. Keep in mind that this is the potential due to the source charges on the plates. We used the movable charge q to find the potential, but q does not cause or contribute to the potential.

A first point to notice is that the electric potential increases linearly from the negative plate at $x = 0$, where $V = V_- = 0$, to the positive plate at $x = d$, where $V = V_+ = Ed$. Let's define the potential difference ΔV_C between the two capacitor plates to be

$$\Delta V_C = V_+ - V_- = Ed \tag{21.6}$$

People who work with circuits would call ΔV_C "the voltage across the capacitor" or simply "the voltage."

In many cases, the capacitor voltage is fixed at some value ΔV_C by connecting its plates to a battery with a known voltage. In this case, the electric field strength inside the capacitor is determined from Equation 21.6 as

$$E = \frac{\Delta V_C}{d} \tag{21.7}$$

This means that we can establish an electric field of known strength by applying a voltage across a capacitor whose plate spacing is known.

Equation 21.7 implies that the units of electric field are volts per meter, or V/m. We have been using electric field units of newtons per coulomb. In fact, as you can show as a homework problem, these units are equivalent to each other; that is,

$$1 \text{ N/C} = 1 \text{ V/m}$$

NOTE ► Volts per meter are the electric field units used by scientists and engineers in practice. We will now adopt them as our standard electric field units. ◄

Returning to the electric potential, we can substitute Equation 21.7 for E into Equation 21.5 for V. In terms of the capacitor voltage ΔV_C, the electric potential at position x inside the capacitor is

$$V = \frac{x}{d}\Delta V_C \tag{21.8}$$

You can see that the potential increases linearly from $V = 0$ at $x = 0$ (the negative plate) to $V = \Delta V_C$ at $x = d$ (the positive plate).

Let's explore the electric potential inside the capacitor by looking at several different, but related, ways that the potential can be represented. In this example, a battery has established a 1.5 V potential difference across a parallel-plate capacitor with a 3 mm plate spacing.

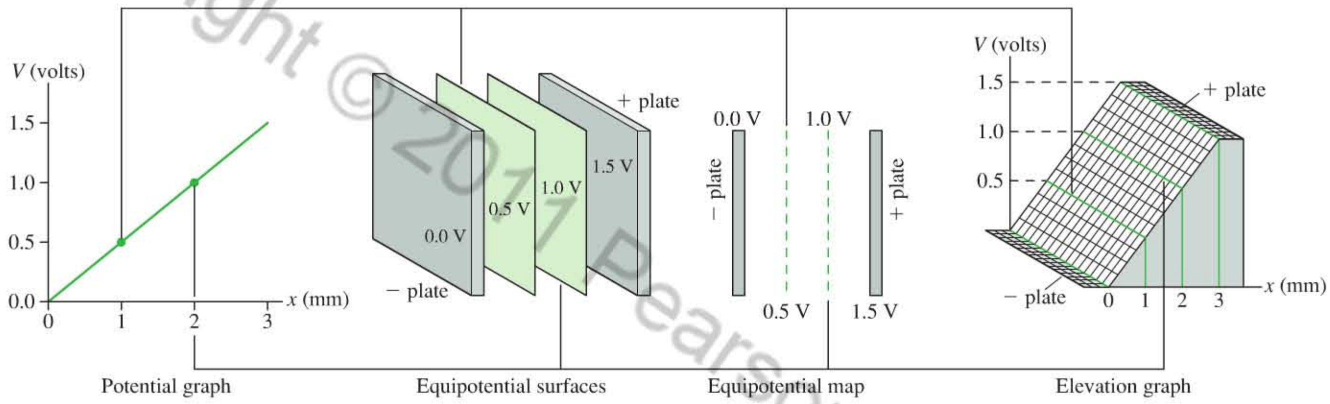
Graphical representations of the electric potential inside a capacitor

A graph of potential versus x. You can see the potential increasing from 0 V at the negative plate to 1.5 V at the positive plate.

A three-dimensional view showing **equipotential surfaces.** These are mathematical surfaces, not physical surfaces, that have the same value of V at every point. The equipotential surfaces of a capacitor are planes parallel to the capacitor plates. The capacitor plates are also equipotential surfaces.

A two-dimensional **equipotential map.** The green dashed lines represent slices through the equipotential surfaces, so V has the same value everywhere along such a line. We call these lines of constant potential **equipotential lines** or simply **equipotentials.**

A three-dimensional elevation graph. The potential is graphed vertically versus the x- and y-coordinates on the other axes. Viewing the front face of the elevation graph gives you the potential graph.



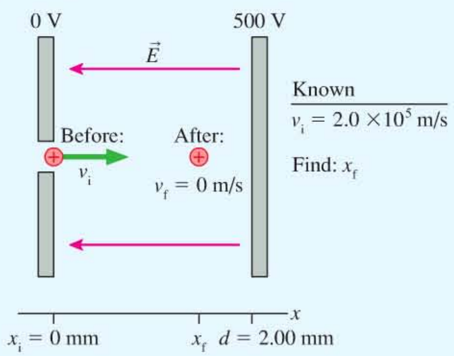
NOTE ► Equipotential lines are just the intersections of equipotential surfaces with the two-dimensional plane of the paper, and so are really just another way of representing equipotential surfaces. Because of this, we'll often use the terms "equipotential lines," "equipotential surfaces," and "equipotentials" interchangeably. ◄

EXAMPLE 21.4 A proton in a capacitor

A parallel-plate capacitor is constructed of two disks spaced 2.00 mm apart. It is charged to a potential difference of 500 V. A proton is shot through a small hole in the negative plate with a speed of 2.0×10^5 m/s. Does it reach the other side? If not, what is the farthest distance from the negative plate that the proton reaches?

PREPARE Energy is conserved. The proton's potential energy inside the capacitor can be found from the capacitor's electric potential. **FIGURE 21.10** is a before-and-after visual overview of the proton in the capacitor.

FIGURE 21.10 A before-and-after visual overview of a proton moving in a capacitor.



SOLVE The proton has charge $q = e$, and its potential energy at a point where the capacitor's potential is V is $U_{elec} = eV$. It will gain potential energy $\Delta U_{elec} = e\,\Delta V_C$ if it moves all the way across the capacitor. The increase in potential energy comes at the expense of kinetic energy, so the proton has sufficient kinetic energy to make it all the way across only if

$$K_i \geq e\,\Delta V_C$$

We can calculate that $K_i = \frac{1}{2}mv_i^2 = 3.34 \times 10^{-17}$ J and that $e\,\Delta V_C = 8.00 \times 10^{-17}$ J. The proton does *not* have sufficient kinetic energy to be able to gain 8.00×10^{-17} J of potential energy, so it will not make it across. Instead, the proton will reach a turning point and reverse direction.

The proton starts at the negative plate, where $x_i = 0$ mm. Let the turning point be at x_f. The potential inside the capacitor is given by $V = \Delta V_C x/d$ with $d = 0.0020$ m and $\Delta V_C = 500$ V. Conservation of energy requires $K_f + eV_f = K_i + eV_i$. This is

$$0 + e\,\Delta V_C \frac{x_f}{d} = \frac{1}{2}mv_i^2 + 0$$

where we used $V_i = 0$ V at the negative plate ($x_i = 0$) and $K_f = 0$ at the turning point. The solution for the turning point is

$$x_f = \frac{mdv_i^2}{2e\,\Delta V_C} = 0.84 \text{ mm}$$

The proton travels 0.84 mm, less than halfway across, before being turned back.

ASSESS We were able to use the electric potential inside the capacitor to determine the proton's potential energy.

The Potential of a Point Charge

The simplest possible source charge is a single fixed point charge q. To find the electric potential, we'll again start by first finding the electric potential energy when a second charge, which we'll call q', is distance r from charge q. As usual, we'll do this by calculating the work needed to bring q' from a point where $U_{elec} = 0$ to distance r from q. We're free to choose $U_{elec} = 0$ at any point that's convenient. Because the influence of a point charge extends infinitely far, our result for the electric potential will have an especially simple form if we choose $U_{elec} = 0$ (and hence $V = 0$) at a point that is infinitely distant from q.

We can't use the simple expression $W = Fd$ to find the work done in moving q'; this expression is valid only for a *constant* force F and, as we know from Coulomb's law, the force on q' gets larger and larger as it approaches q. To do this calculation properly requires the methods of calculus. However, we can understand *qualitatively* how the potential energy depends on the distance r between the two charges.

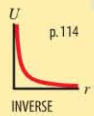
FIGURE 21.11 shows q' at two different distances r from the fixed charge q. When q' is relatively far from q, the electric force on q' is small. Not much external force is needed to push q' closer to q by a small displacement d, so the work done on q' is small and the *change* ΔU_{elec} in the electric potential energy is small as well. This implies, as Figure 21.11 shows, that the graph of U_{elec} versus r is fairly flat when q' is far from q.

On the other hand, the force on q' is quite large when it gets near q, so the work required to move it through the same small displacement is much greater than before. The change in U_{elec} is large, so the graph of U_{elec} versus r is steeper when q' is near q.

The general shape of the graph of U_{elec} must be as shown in Figure 21.11. When q' is far from q, the potential energy is small (U_{elec} must go to zero as r goes to infinity). As q' approaches q, the potential energy gets larger and larger. An exact calculation finds the potential energy of two point charges to be

$$U_{elec} = K\frac{qq'}{r} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r} \tag{21.9}$$

Electric potential energy of two charges q and q' separated by distance r



NOTE ▶ This expression is very similar to Coulomb's law. The difference is that the electric potential energy depends on the *inverse* of r—that is, on $1/r$—instead of the inverse-square dependence of Coulomb's law. Make sure you remember which is which! ◀

Figure 21.11 was a graph of the potential energy for two like charges, where the product qq' is positive. But Equation 21.9 is equally valid for *opposite* charges. In this case, the potential energy of the charges is *negative*. As **FIGURE 21.12** shows, the potential energy of the two charges *decreases* as they get closer together. A particle speeds up as its potential energy decreases ($U \rightarrow K$), so charge q' accelerates toward the fixed charge q. The graph lets us think about the attractive force between opposite charges from the perspective of energy.

FIGURE 21.11 The electric potential energy of two point charges.

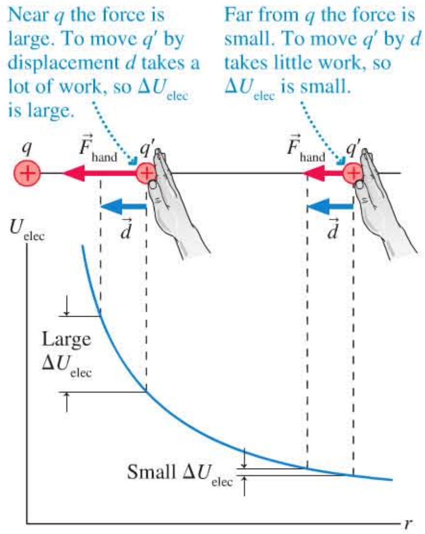
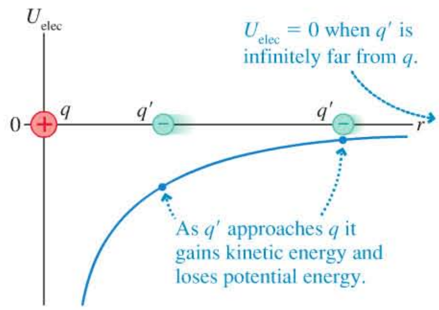


FIGURE 21.12 Potential-energy diagram for two opposite charges.

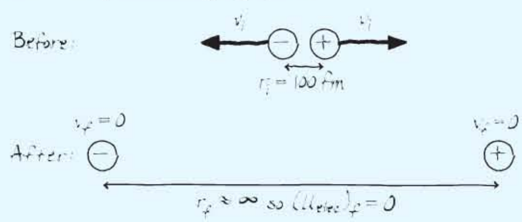


EXAMPLE 21.5 Finding the escape velocity

An interaction between two elementary particles causes an electron and a positron (a positively charged electron) to be shot out back-to-back with equal speeds. What minimum speed must each particle have when they are 100 fm apart in order to end up far from each other? (Recall that 1 fm = 10^{-15} m.)

PREPARE Energy is conserved. The particles end up "far from each other," which we interpret as sufficiently far to make $(U_{elec})_f \approx 0$ J. **FIGURE 21.13** shows the before-and-after visual

FIGURE 21.13 The before-and-after visual overview of an electron and a positron flying apart.



Continued

overview. The minimum speed to escape is the speed that allows the particles to reach $r_f = \infty$ with $v_f = 0$.

SOLVE Here it is essential to interpret U_{elec} as the potential energy of the electron + positron system. Similarly, K is the *total* kinetic energy of the system. The electron and the positron, with equal masses and equal speeds, have equal kinetic energies. Conservation of energy $K_f + U_f = K_i + U_i$ is

$$0 + 0 = \left(\frac{1}{2}mv_i^2 + \frac{1}{2}mv_i^2\right) + K\frac{q_e q_p}{r_i} = mv_i^2 - \frac{Ke^2}{r_i}$$

Using $r_i = 100 \text{ fm} = 1.0 \times 10^{-13}$ m, we can calculate the minimum initial speed to be

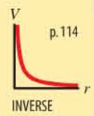
$$v_i = \sqrt{\frac{Ke^2}{mr_i}} = 5.0 \times 10^7 \text{ m/s}$$

ASSESS v_i is a little more than 10% the speed of light, just about the limit of what a "classical" calculation can predict. We would need to use the theory of relativity if v_i were much larger.

Equation 21.9 gives the potential energy of a charge q' when it is a distance r from a point charge q. We know that the electric *potential* is related to the potential energy by $V = U_{elec}/q'$. Thus the electric potential of charge q is

$$V = K\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad (21.10)$$

Electric potential at distance r from a point charge q



Notice that only the *source* charge q appears in this expression. This is in line with our picture that a source charge *creates* the electric potential around it.

EXAMPLE 21.6 Calculating the potential of a point charge

What is the electric potential 1.0 cm from a 1.0 nC charge? What is the potential difference between a point 1.0 cm away and a second point 3.0 cm away?

PREPARE We can use Equation 21.10 to find the potential at the two distances from the charge.

SOLVE The potential at $r = 1.0$ cm is

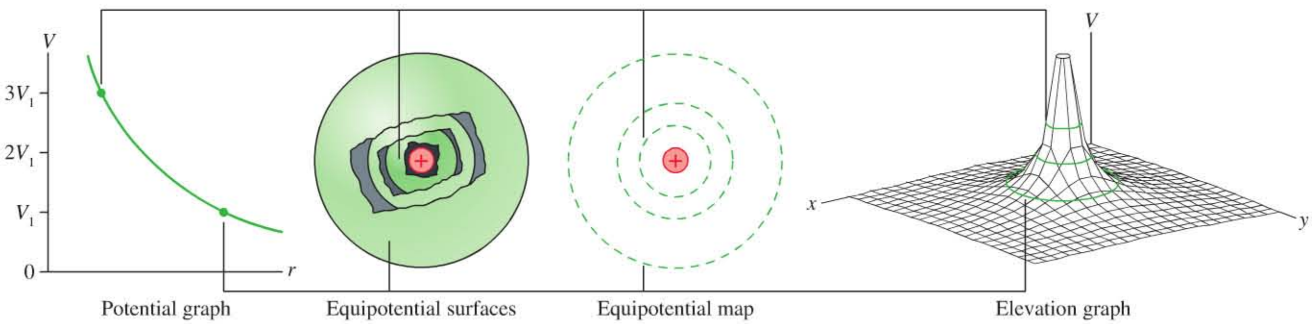
$$V_{1\,\text{cm}} = K\frac{q}{r} = (9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)\left(\frac{1.0 \times 10^{-9} \text{ C}}{0.010 \text{ m}}\right)$$
$$= 900 \text{ V}$$

We can similarly calculate $V_{3\,\text{cm}} = 300$ V. Thus the potential difference between these two points is $\Delta V = V_{1\,\text{cm}} - V_{3\,\text{cm}} = 600$ V.

ASSESS 1 nC is typical of the electrostatic charge produced by rubbing, and you can see that such a charge creates a fairly large potential nearby. Why aren't we shocked and injured when working with the "high voltages" of such charges? As we'll learn in Chapter 26, the sensation of being shocked is a result of current, not potential. Some high-potential sources simply do not have the ability to generate much current.

FIGURE 21.14 shows four graphical representations of the electric potential of a point charge. These match the four representations of the electric potential inside a capacitor,

FIGURE 21.14 Four graphical representations of the electric potential of a point charge.

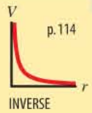


and a comparison of the two is worthwhile. This figure assumes that q is positive; you may want to think about how the representations would change if q were negative.

The Electric Potential of a Charged Sphere

Equation 21.10 gives the electric potential of a point charge. It can be shown that the electric potential outside a charged sphere is the *same* as that of a point charge; that is,

$$V = K\frac{Q}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \qquad (21.11)$$



Electric potential at a distance $r > R$ from the center of a sphere of radius R and with charge Q

We can cast this result in a more useful form. It is common to charge a metal object, such as a sphere, "to" a certain potential; for instance, this can be done by connecting the sphere to, say, a 30 volt battery. This potential, which we will call V_0, is the potential right on the surface of the sphere. We can see from Equation 21.11 that

$$V_0 = V(\text{at } r = R) = \frac{Q}{4\pi\epsilon_0 R} \qquad (21.12)$$

Consequently, a sphere of radius R that is charged to potential V_0 has total charge

$$Q = 4\pi\epsilon_0 R V_0 \qquad (21.13)$$

If we substitute this expression for Q into Equation 21.11, we can write the potential outside a sphere that is charged to potential V_0 as

$$V = \frac{R}{r}V_0 \qquad (21.14)$$

Equation 21.14 tells us that the potential of a sphere is V_0 on the surface and decreases inversely with the distance. Thus the potential at $r = 3R$ is $\frac{1}{3}V_0$.

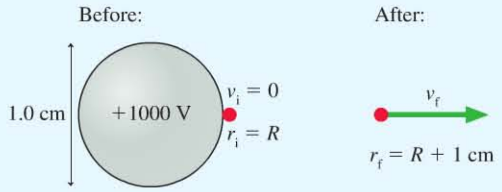
EXAMPLE 21.7 **Releasing a proton from a charged sphere**

A proton is released from rest at the surface of a 1.0-cm-diameter sphere that has been charged to +1000 V.

a. What is the charge of the sphere?
b. What is the proton's speed when it is 1.0 cm from the sphere?
c. When the proton is 1.0 cm from the sphere, what is its kinetic energy in eV?

PREPARE Energy is conserved. The potential outside the charged sphere is the same as the potential of a point charge at the center. **FIGURE 21.15** is a before-and-after visual overview.

FIGURE 21.15 A before-and-after visual overview of a sphere and a proton.



SOLVE

a. We can use the sphere's potential in Equation 21.13 to find that the charge of the sphere is

$$Q = 4\pi\epsilon_0 R V_0 = 0.56 \times 10^{-9}\text{ C} = 0.56\text{ nC}$$

b. A sphere charged to $V_0 = +1000$ V is positively charged. The proton will be repelled by this charge and move away from the sphere. The conservation of energy equation $K_f + eV_f = K_i + eV_i$, with Equation 21.14 for the potential of a sphere, is

$$\frac{1}{2}mv_f^2 + \frac{eR}{r_f}V_0 = \frac{1}{2}mv_i^2 + \frac{eR}{r_i}V_0$$

The proton starts from the surface of the sphere, $r_i = R$, with $v_i = 0$. When the proton is 1.0 cm from the *surface* of the sphere, it has $r_f = 1.0\text{ cm} + R = 1.5$ cm. Using these values, we can solve for v_f:

$$v_f = \sqrt{\frac{2eV_0}{m}\left(1 - \frac{R}{r_f}\right)} = 3.6 \times 10^5\text{ m/s}$$

c. The kinetic energy is

$$K_f = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}(1.7 \times 10^{-27}\text{ kg})(3.6 \times 10^5\text{ m/s})^2$$
$$= 1.1 \times 10^{-16}\text{ J}$$

Converting to electron volts, we find

$$1.1 \times 10^{-16}\text{ J} \times \frac{1\text{ eV}}{1.6 \times 10^{-19}\text{ J}} = 690\text{ eV}$$

ASSESS A proton at the surface of the sphere, where $V = 1000$ V would, by definition, have 1000 eV of electric potential energy. It is reasonable that after it's moved some distance away, it has transformed 690 eV of this to kinetic energy.

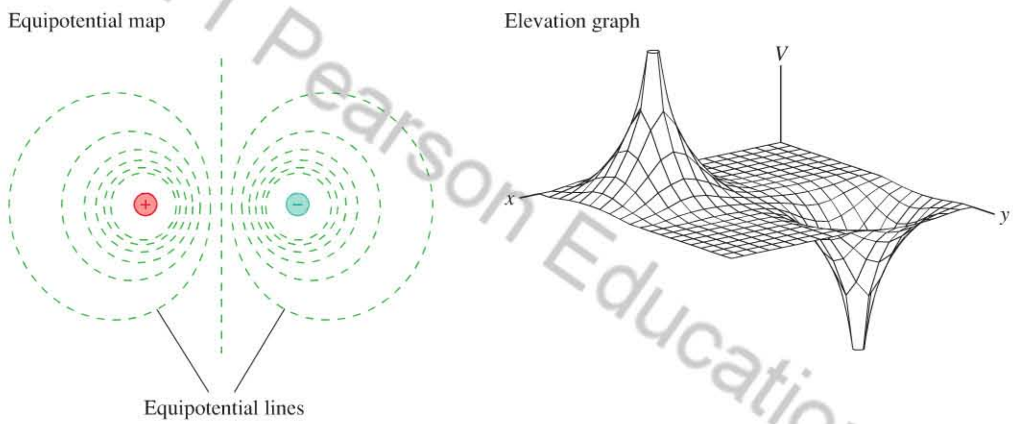
The Electric Potential of Many Charges

Suppose there are many source charges $q_1, q_2, \ldots$. The electric potential V at a point in space is the *sum* of the potentials due to each charge:

$$V = \sum_i K\frac{q_i}{r_i} = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i} \tag{21.15}$$

where r_i is the distance from charge q_i to the point in space where the potential is being calculated. Unlike the electric field of multiple charges, which required vector addition, the electric potential is a simple scalar sum. This makes finding the potential of many charges considerably easier than finding the corresponding electric field. As an example, the equipotential map and elevation graph in FIGURE 21.16 show that the potential of an electric dipole is the sum of the potentials of the positive and negative charges. We'll see later that the electric potential of the heart has the form of an electric dipole.

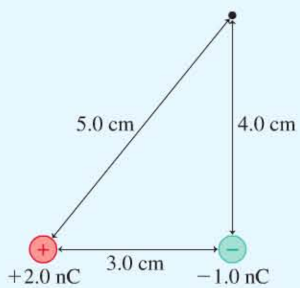
FIGURE 21.16 The electric potential of an electric dipole.



EXAMPLE 21.8 Finding the potential of two charges

What is the electric potential at the point indicated in FIGURE 21.17?

FIGURE 21.17 Finding the potential of two charges.



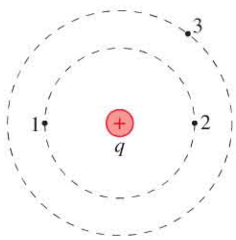
PREPARE The potential is the sum of the potentials due to each charge.

SOLVE The potential at the indicated point is

$$\begin{aligned} V &= \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} \\ &= (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\left(\frac{2.0 \times 10^{-9}\ \mathrm{C}}{0.050\ \mathrm{m}} + \frac{-1.0 \times 10^{-9}\ \mathrm{C}}{0.040\ \mathrm{m}}\right) \\ &= 140\ \mathrm{V} \end{aligned}$$

ASSESS As noted, the potential is a *scalar,* so we found the net potential by adding two scalars. We don't need any angles or components to calculate the potential.

STOP TO THINK 21.3 Rank in order, from largest to smallest, the potential differences ΔV_{12}, ΔV_{13}, and ΔV_{23} between points 1 and 2, points 1 and 3, and points 2 and 3.



21.5 Connecting Potential and Field

In Chapter 20, we learned how source charges create an electric field around them; in this chapter, we found that source charges also create an electric potential everywhere in the space around them. But these two concepts of field and potential are clearly linked. For instance, we can calculate potential differences by considering the work done on a charge as it is pushed against the electric force due to the field. In this section, we'll build on this idea and will find that **the electric potential and electric field are not two distinct entities but, instead, two different perspectives or two different mathematical representations of how source charges alter the space around them.**

To make the connection between potential and field, FIGURE 21.18 shows an equipotential map of the electric potential due to some source charges (which aren't shown here). Suppose a charge q moves a short distance along one of the equipotential surfaces. Because it moves along an equipotential, its potential and hence its potential energy are the *same* at the beginning and end of its displacement. This means that no work is done in moving the charge. As Figure 21.18 shows, the only way that no work can be done is if the electric field is *perpendicular* to the equipotential. (You should recall, from Chapter 10, that no work is done by a force perpendicular to a particle's displacement.) This, then, is our first discovery about the connection between field and potential: **The electric field at a point is perpendicular to the equipotential surface at that point.**

In Figure 21.18, there are actually two directions that are perpendicular to the equipotential surface: the one shown by $\vec{E}$ in the figure, and the other pointing opposite $\vec{E}$. Which direction is correct?

FIGURE 21.19 shows a positive charge released from rest starting at the 10 V equipotential. You learned in Section 21.3 that a positive charge speeds up as it moves from higher to lower potential, so this charge will speed up as it moves toward the lower 0 V equipotential. Because it is the electric field $\vec{E}$ in Figure 21.19 that pushes on the charge, causing it to speed up, $\vec{E}$ must point as shown, from higher potential (10 V) to lower potential (0 V). This is our second discovery about the connection between field and potential: **The electric field points in the direction of *decreasing* potential.**

Finally, we can find an expression for the *magnitude* of $\vec{E}$ by considering the work required to move the charge, at constant speed, through a displacement that is directed *opposite* $\vec{E}$, as shown in FIGURE 21.20. The displacement is small enough that the electric field in this region can be considered as nearly constant. Conservation of energy requires that

$$W = \Delta U_{\text{elec}} = q\Delta V \qquad (21.16)$$

Because the charge moves at a constant speed, the magnitude of the force of the hand $\vec{F}_{\text{hand}}$ is exactly equal to that of the electric force $q\vec{E}$. Thus, the work done on the charge in moving it through displacement d is

$$W = F_{\text{hand}}d = qEd$$

Comparing this result with Equation 21.16 shows that the strength of the electric field is

$$E = \frac{\Delta V}{d} \qquad (21.17)$$

Electric field strength in terms of the potential difference ΔV between two equipotential surfaces a distance d apart

FIGURE 21.18 The electric field is always perpendicular to an equipotential surface.

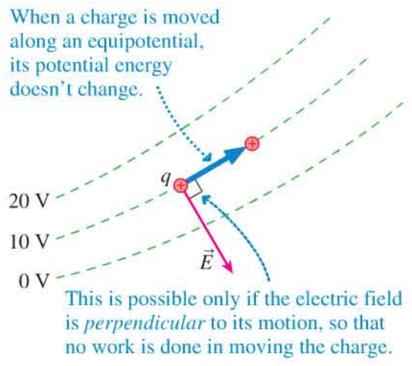


FIGURE 21.19 The electric field points "downhill."

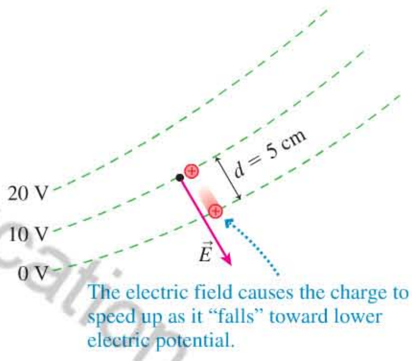
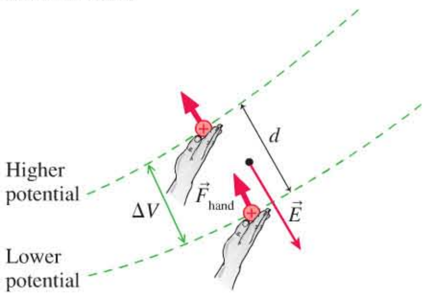


FIGURE 21.20 Finding the strength of the electric field.



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◀ **Electroreception in sharks** BIO Sharks have a "sixth sense" that aids them in detecting prey: They are able to detect the weak electric potentials created by other fish. A shark has an array of sensor cells, the *ampullae of Lorenzini,* distributed around its snout. These appear as small black spots on the blue shark shown. These cells are highly sensitive to potential differences and can measure voltages as small as a few billionths of a volt! As we saw in Chapter 20, there are electric fields and potentials associated with every beat of the heart; muscle contractions also create potential differences. The shark can detect these small potential differences, even using them to sense animals buried in the sand.

For example, in Figure 21.19 the magnitude of the potential difference is $\Delta V = (10\text{ V}) - (0\text{ V}) = 10\text{ V}$, while $d = 5\text{ cm} = 0.05\text{ m}$. Thus the electric field strength between these two equipotential surfaces is

$$E = \frac{\Delta V}{d} = \frac{10\text{ V}}{0.05\text{ m}} = 200\text{ V/m}$$

NOTE ▶ This expression for the electric field strength is similar to the result $E = \Delta V_C/d$ for the electric field strength inside a parallel-plate capacitor. A capacitor has a uniform field, the same at all points, so we can calculate the field using the full potential difference ΔV_C and the full spacing d. In contrast, Equation 21.17 applies only *locally,* at a point where the spacing between two adjacent equipotential lines is d. As the spacing varies, so too does the field strength. ◀

FIGURE 21.21 summarizes what we've learned about the connection between potential and field.

FIGURE 21.21 The geometry of potential and field.

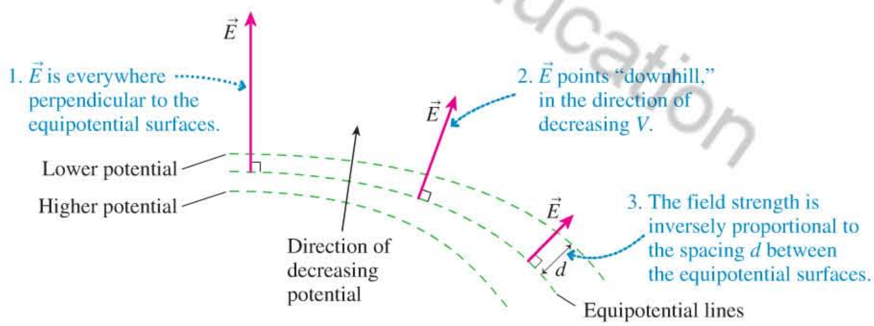
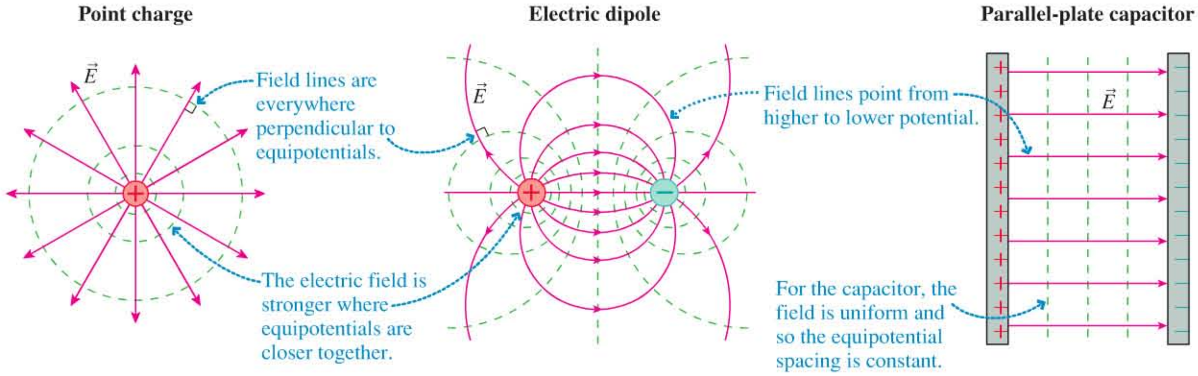


FIGURE 21.22 shows three important arrangements of charges that we've studied in this chapter and Chapter 20. Both electric field lines and equipotentials are shown, and you can see how the connections between field and potential summarized in Figure 21.21 apply in each case.

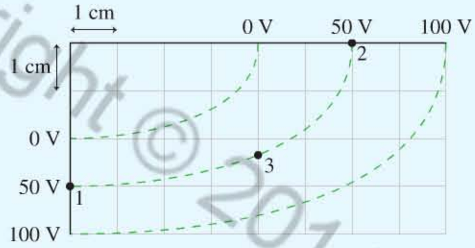
FIGURE 21.22 Electric field lines and equipotentials for three important cases.



EXAMPLE 21.9 Finding the electric field from equipotential lines

In **FIGURE 21.23** a 1 cm × 1 cm grid is superimposed on an equipotential map of the potential. Estimate the strength and direction of the electric field at points 1, 2, and 3. Show your results graphically by drawing the electric field vectors on the equipotential map.

FIGURE 21.23 Equipotential lines.

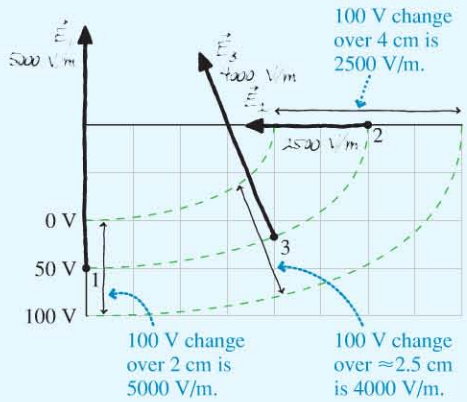


PREPARE The electric field is perpendicular to the equipotential lines, points "downhill," and depends on the spacing between the equipotential lines. The potential is highest on the bottom and the right. An elevation graph of the potential would look like the lower-right quarter of a bowl or a football stadium.

SOLVE Some distant but unseen source charges have created an electric field and potential. We do not need to see the source charges to relate the field to the potential. Because $E = \Delta V/d$, the electric field is stronger where the equipotential lines are closer together and weaker where they are farther apart.

FIGURE 21.24 shows how measurements of d from the grid are combined with values of ΔV to determine $\vec{E}$. Point 3 requires an estimate of the spacing between the 0 V and the 100 V lines. Notice that we're using the 0 V and 100 V equipotential lines to determine $\vec{E}$ at a point on the 50 V equipotential.

FIGURE 21.24 The electric field at points 1, 2, and 3.



ASSESS The *directions* of $\vec{E}$ are found by drawing downhill vectors perpendicular to the equipotentials. The distances between the equipotential lines are needed to determine the field strengths.

A Conductor in Electrostatic Equilibrium

In Chapter 20, you learned four important properties about conductors in electrostatic equilibrium:

1. Any excess charge is on the surface.
2. The electric field inside is zero.
3. The exterior electric field is perpendicular to the surface.
4. The field strength is largest at sharp corners.

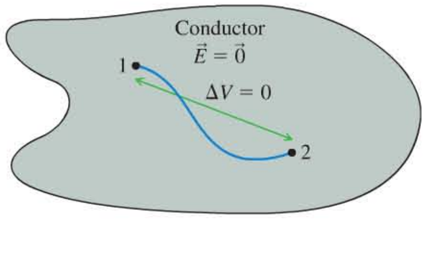
Now we can add a fifth important property:

5. The entire conductor is at the same potential, and thus the surface is an equipotential surface.

At pointed metal tips, the electric field can be strong enough to ionize air molecules. The field accelerates these ions, which collide, releasing energy in the bluish glow seen here.

To see why this is so, **FIGURE 21.25** shows two points inside a conductor connected by a line that remains entirely inside the conductor. We can find the potential difference $\Delta V = V_2 - V_1$ between these points by using an external force to push a charge along the line from 1 to 2 and calculating the work done. But because $\vec{E} = \vec{0}$, there is no force on the charge. The work is zero, and so $\Delta V = 0$. In other words, **any two points inside a conductor in electrostatic equilibrium are at the same potential.**

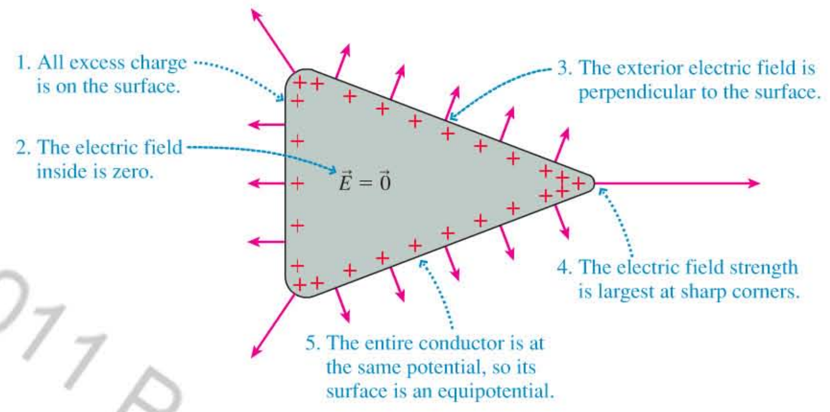
When a conductor is in electrostatic equilibrium, the *entire conductor* is at the same potential. If we charge a metal electrode, the entire electrode is at a single potential. The facts that the surface is an equipotential surface and that the exterior electric field is perpendicular to the surface can now be seen as a special case of our conclusion from the preceding section that electric fields are always perpendicular to equipotentials.



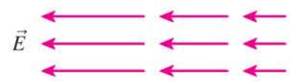
▶ **FIGURE 21.25** All points inside a conductor in electrostatic equilibrium are at the same potential.

FIGURE 21.26 summarizes what we know about conductors in electrostatic equilibrium. These are important and practical conclusions because conductors are the primary components of electrical devices.

FIGURE 21.26 Electrical properties of a conductor in electrostatic equilibrium.



STOP TO THINK 21.4 Which set of equipotential surfaces matches this electric field?



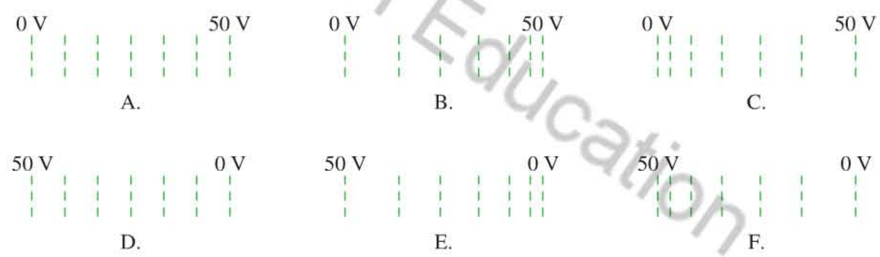
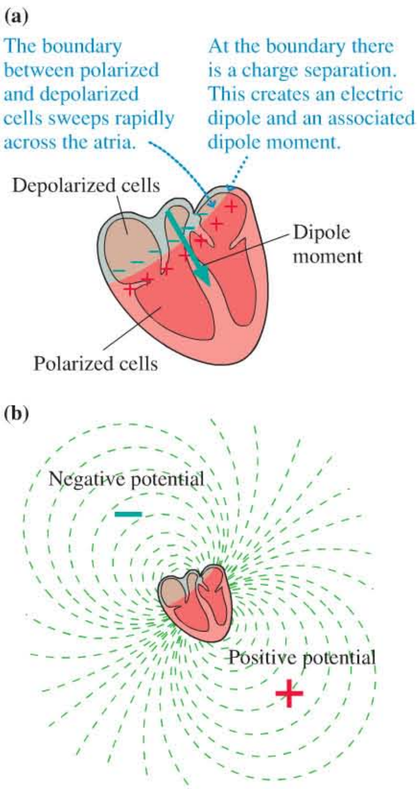


FIGURE 21.27 A contracting heart is an electric dipole.



21.6 The Electrocardiogram

As we saw in Chapter 20, the electrical activity of cardiac muscle cells makes the beating heart an electric dipole. A resting nerve or muscle cell is *polarized,* meaning that the outside is positive and the inside negative. Figure 21.5 showed this situation. Initially, all muscle cells in the heart are polarized. When triggered by an electrical impulse from the heart's sino-auricular node in the right atrium, heart cells begin to *depolarize,* moving ions through the cell wall until the outside becomes negative. This causes the muscle to contract. The depolarization of one cell triggers depolarization in an adjacent cell, causing a "wave" of depolarization to spread across the tissues of the heart.

At any instant during this process, a boundary divides the negative charges of depolarized cells from the positive charges of cells that have not depolarized. As FIGURE 21.27a shows, this separation of charges creates an electric dipole and produces a dipole electric field and potential. FIGURE 21.27b shows the equipotential surfaces of the heart's dipole at one instant of time. These equipotential surfaces match those shown earlier in Figure 21.16.

NOTE ► The convention is to draw diagrams of the heart as if you were facing the person whose heart is being drawn. The left side of the heart is thus on the right side of the diagram. ◄

A measurement of the electric potential of the heart is an invaluable diagnostic tool. Recall, however, that only potential *differences* are meaningful, so we need to measure the potential difference between two points on the torso. In practice, as **FIGURE 21.28** shows, the potential difference is measured between several pairs of *electrodes* (often called *leads*). A chart of the potential differences is known as an **electrocardiogram,** abbreviated either ECG or, from its European origin, EKG. A common method of performing an EKG uses 12 leads and records 12 pairs of potential differences.

FIGURE 21.29 shows a simplified model of electrocardiogram measurement using only two electrodes, one on each arm. As the wave of depolarization moves across the heart muscle during each heart beat, the dipole moment vector of the heart changes its magnitude and direction. As Figure 21.29 shows, both of these affect the potential difference between the electrodes, so each point on the EKG graph corresponds to a particular magnitude and orientation of the dipole moment.

FIGURE 21.28 Measuring an EKG.

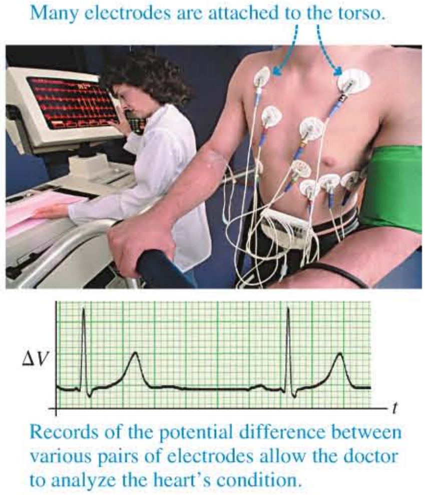
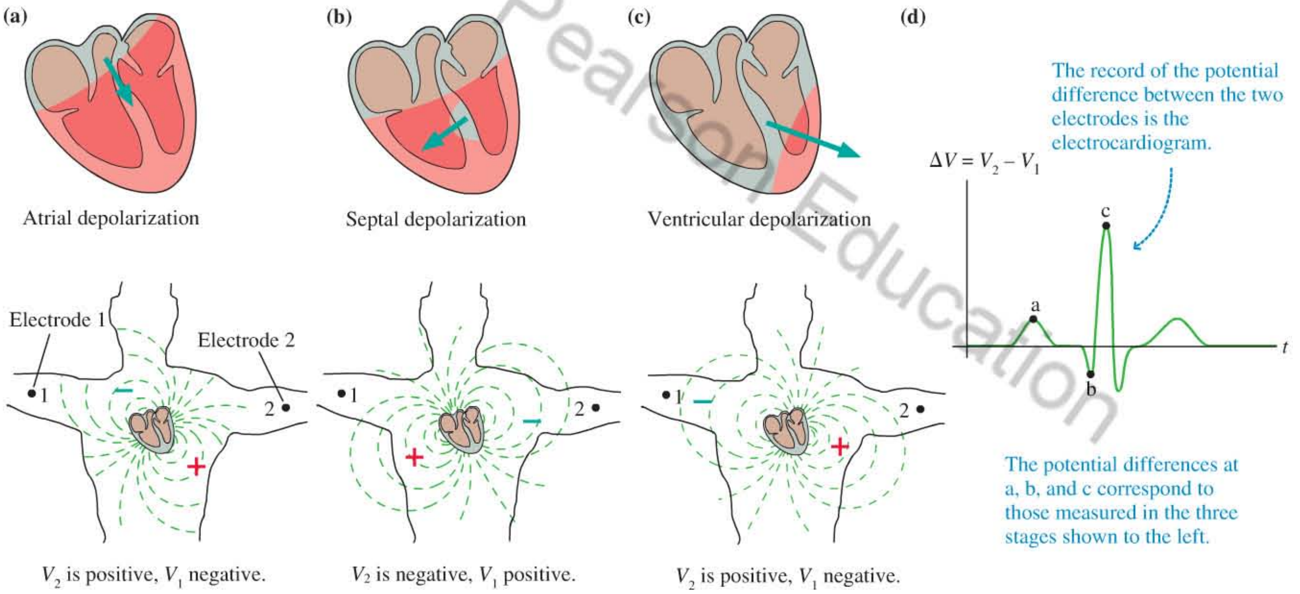


FIGURE 21.29 The potential difference between the electrodes changes as the heart beats.



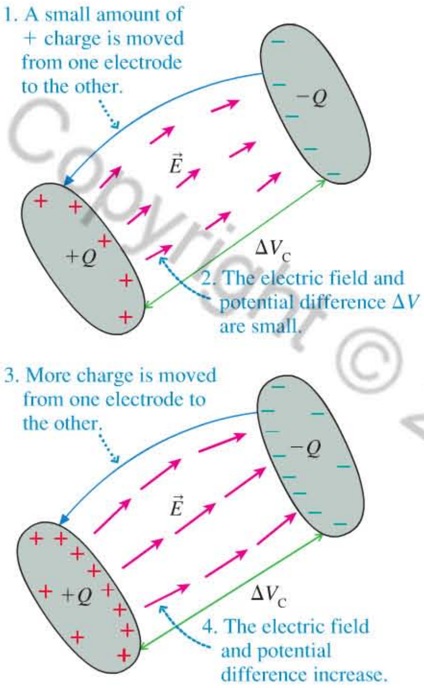
21.7 Capacitance and Capacitors

In Section 21.2 we found that potential differences are caused by the separation of charge. One common method of creating a charge separation, shown in **FIGURE 21.30** on the next page, is to move charge Q from one initially uncharged conductor to a second initially uncharged conductor. This results in charge $+Q$ on one conductor and $-Q$ on the other. Two conductors with equal but opposite charge form a **capacitor.** The two conductors that make up a capacitor are its *electrodes* or *plates.* We've already looked at some of the properties of parallel-plate capacitors; now we want to study capacitors that might have any shape. Capacitors can be used to store charge, making them invaluable in all kinds of electronic circuits.

As Figure 21.30 on the next page shows, the electric field strength E and the potential difference ΔV_C increase as the charge on each electrode increases. If we double the amount of charge on each electrode, the work required to move a charge from one electrode to the other doubles. This implies a doubling of the potential difference between the electrodes. Thus **the potential difference between the electrodes is directly proportional to their charge.**

Capacitors are important elements in electric circuits. They come in a wide variety of sizes and shapes.

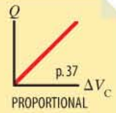
FIGURE 21.30 Charging a capacitor.



Stated another way, **the charge of a capacitor is directly proportional to the potential difference between its electrodes.** As we'll see, this is actually the most common way that a capacitor is used: A source of potential difference, such as a battery, is connected between the electrodes, causing a charge proportional to the potential difference to be moved from one electrode to the other. We can write the relationship between charge and potential difference as

$$Q = C\,\Delta V_C \qquad (21.18)$$

Charge on a capacitor with potential difference ΔV_C



The constant of proportionality C between Q and ΔV_C is called the **capacitance** of the capacitor. Capacitance depends on the shape, size, and spacing of the two electrodes. A capacitor with a large capacitance holds more charge for a given potential difference than one with a small capacitance.

NOTE ▶ We will consider only situations where the charge on the electrodes is equal in magnitude but opposite in sign. When we say "A capacitor has charge Q," we mean that one electrode has charge $+Q$ and the other charge $-Q$. The potential difference between the electrodes is called the potential difference *of* the capacitor. ◀

The SI unit of capacitance is the **farad,** named in honor of Michael Faraday, the originator of the idea of electric and magnetic fields. One farad is defined as

$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb/volt} = 1 \text{ C/V}$$

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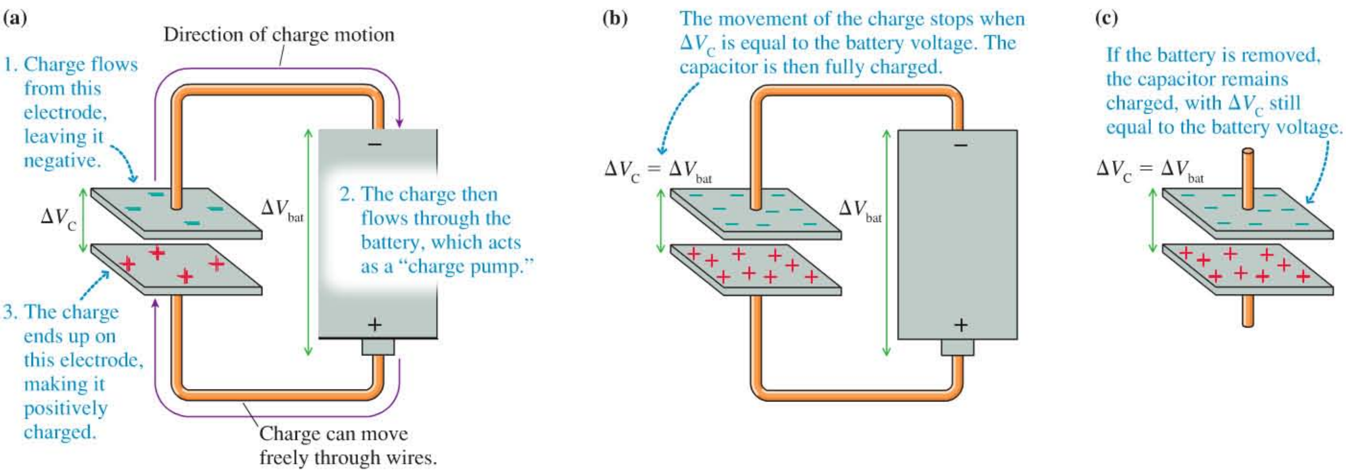
One farad turns out to be a very large capacitance. Practical capacitors are usually measured in units of microfarads (μF) or picofarads (1 pF $= 10^{-12}$ F).

Charging a Capacitor

To "charge" a capacitor, we need to move charge from one electrode to the other. The simplest way to do this is to use a source of potential difference such as a battery, as shown in FIGURE 21.31. We learned earlier that a battery uses its internal chemistry to maintain a fixed potential difference between its terminals. If we connect a capacitor to a battery, charge flows from the negative electrode of the capacitor, through the battery, and onto the positive electrode. This flow of charge continues until the potential difference between the capacitor's electrodes is the same as the fixed potential difference of the battery. If the battery is then removed, the capacitor remains charged with a potential equal to that of the battery that charged it because there's no conducting path for charge on the positive electrode to move back to the negative electrode. Thus **a capacitor can be used to store charge.**

FIGURE 21.31 Charging a capacitor using a battery.



EXAMPLE 21.10 **Charging a capacitor**

A 1.3 μF capacitor is connected to a 1.5 V battery. What is the charge on the capacitor?

PREPARE Charge flows through the battery from one capacitor electrode to the other until the potential difference ΔV_C between the electrodes equals that of the battery, or 1.5 V.

SOLVE The charge on the capacitor is given by Equation 21.18:

$$Q = C\,\Delta V_C = (1.3 \times 10^{-6}\ \text{F})(1.5\ \text{V}) = 2.0 \times 10^{-6}\ \text{C}$$

ASSESS This is the charge on the positive electrode; the other electrode has a charge of -2.0×10^{-6} C.

The Parallel-Plate Capacitor

As we've seen, the *parallel-plate capacitor* is important because it creates a uniform electric field between its flat electrodes. In Chapter 20, we found that the electric field of a parallel-plate capacitor is

$$\vec{E} = \left(\frac{Q}{\epsilon_0 A}, \text{from positive to negative}\right)$$

where A is the surface area of the electrodes and Q is the charge on the capacitor. We can use this result to find the capacitance of a parallel-plate capacitor.

Earlier in this chapter, we found that the electric field strength of a parallel-plate capacitor is related to the potential difference ΔV_C and the plate spacing d by

$$E = \frac{\Delta V_C}{d}$$

Combining these two results, we see that

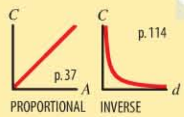
$$\frac{Q}{\epsilon_0 A} = \frac{\Delta V_C}{d}$$

or, equivalently,

$$Q = \frac{\epsilon_0 A}{d}\Delta V_C \tag{21.19}$$

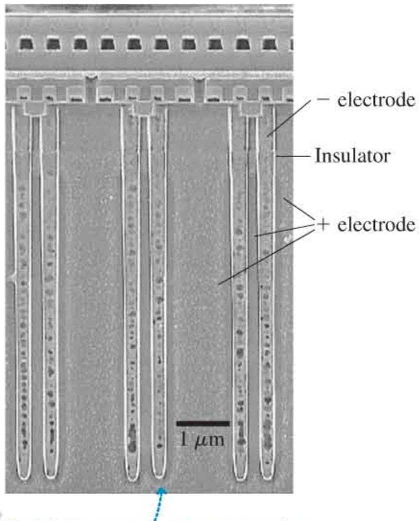
If we compare Equation 21.19 to Equation 21.18, the definition of capacitance, we see that the capacitance of the parallel-plate capacitor is

$$C = \frac{\epsilon_0 A}{d} \tag{21.20}$$



Capacitance of a parallel-plate capacitor with plate area A and separation d

NOTE ► From Equation 21.20 you can see that the units of ϵ_0 can be written as F/m. These units are useful when working with capacitors. ◄



Each long structure is one capacitor.

A capacity for memory Your computer's random-access memory, or RAM, uses tiny capacitors to store the digital ones and zeroes that make up your data. A charged capacitor represents a one and an uncharged capacitor a zero. For a billion or more capacitors to fit on a single chip they must be very small. The micrograph is a cross section through the silicon wafer that makes up the memory chip. Each capacitor consists of a very long electrode separated by a thin insulating layer from the common electrode shared by all capacitors. Each capacitor's capacitance is only about 30×10^{-15} F!

EXAMPLE 21.11 **Charging a parallel-plate capacitor**

The spacing between the plates of a 1.0 μF parallel-plate capacitor is 0.070 mm.

a. What is the surface area of the plates?
b. How much charge is on the plates if this capacitor is attached to a 1.5 V battery?

SOLVE

a. From the definition of capacitance,

$$A = \frac{dC}{\epsilon_0} = \frac{(0.070 \times 10^{-3}\ \text{m})(1.0 \times 10^{-6}\ \text{F})}{8.85 \times 10^{-12}\ \text{F/m}} = 7.9\ \text{m}^2$$

b. The charge is $Q = C\,\Delta V_C = (1.0 \times 10^{-6}\ \text{F})(1.5\ \text{V}) = 1.5 \times 10^{-6}\ \text{C} = 1.5\ \mu\text{C}$.

ASSESS The surface area needed to construct a 1.0 μF capacitor (a fairly typical value) is enormous and hardly practical. We'll see in the next section that real capacitors can be reduced to a more manageable size by placing an insulator between the capacitor plates.

STOP TO THINK 21.5 If the potential difference across a capacitor is doubled, its capacitance

A. Doubles. B. Halves. C. Remains the same.

FIGURE 21.32 An insulator in an electric field becomes polarized.

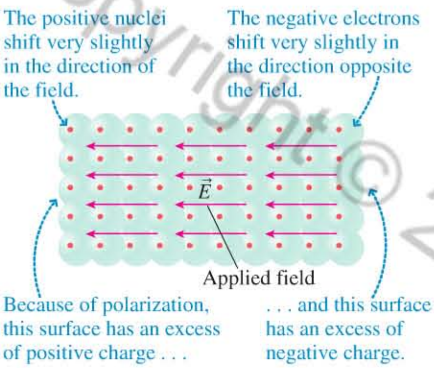


FIGURE 21.33 The electric field inside a dielectric.

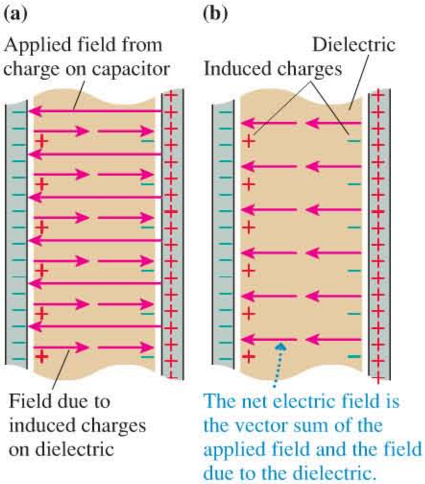


TABLE 21.3 Dielectric constants of some materials at 20°C

Material	Dielectric constant κ
Vacuum	1 (exactly)
Air	1.00054*
Teflon	2.0
Paper	3.0
Pyrex glass	4.8
Cell membrane	9.0
Ethanol	24
Water	80
Strontium titanate	300

*Use 1.00 in all calculations.

21.8 Dielectrics and Capacitors

An insulator consists of vast numbers of atoms. When an insulator is placed in an electric field, each of its atoms polarizes. Recall, from Chapter 20, that *polarization* occurs when an atom's negative electron cloud and positive nucleus shift very slightly in opposite directions in response to an applied electric field. The net effect of all these tiny atomic polarizations is shown in FIGURE 21.32: An *induced* positive charge builds up on one surface of the insulator, and an induced negative charge on the other surface. The insulator has no net charge, but it has become polarized.

An insulator placed between the plates of a capacitor is called a **dielectric.** FIGURE 21.33a shows a dielectric in the uniform field of a parallel-plate capacitor. The capacitor's electric field polarizes the dielectric, and positive and negative charges build up on opposite surfaces. Notice that this distribution of charge—two equal but opposite layers—is identical to that of a parallel-plate capacitor, so this induced charge will create an electric field identical to that of a parallel-plate capacitor.

However, as Figure 21.33a shows, the field due to the induced charge on the surface of the dielectric is *opposite* the applied electric field due to the charge on the capacitor plates that established the polarization in the first place. The *net* electric field inside the dielectric is the vector sum of these two contributions: the applied field of the capacitor plates plus the field due to the polarization of the dielectric. As FIGURE 21.33b shows, these two fields add to give a net field in the same direction as the applied field, but smaller. Thus **the electric field inside a dielectric is smaller than the applied field.**

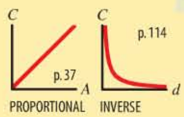
Because some atoms are more easily polarized than others, the factor by which the electric field is reduced depends on the dielectric material. This factor is called the **dielectric constant** of the material, and it is given the symbol κ (Greek letter kappa). If E is the strength of the electric field inside the capacitor without the dielectric present, the field strength E' with the dielectric present is

$$E' = \frac{E}{\kappa} \tag{21.21}$$

Table 21.3 lists the dielectric constants for a number of common substances. Note the high dielectric constant of water, which is of great importance in regulating the chemistry of biological processes. As we saw in Section 20.2, a molecule of water is a *permanent* electric dipole because the oxygen atom has a slight negative charge while the two hydrogen atoms have a slight positive charge. Because the charge in a water molecule is *already* separated, the molecules' dipoles easily turn to line up with an applied electric field, leading to water's very high dielectric constant.

If we insert a dielectric between the plates of a capacitor, the electric field between the plates will be reduced by a factor of κ. The electric potential difference between the plates, given by $\Delta V_C = Ed$, is reduced by the same factor, and thus the capacitance $C = Q/\Delta V_C$ is *increased* by a factor of κ. Therefore the capacitance of a dielectric-containing parallel-plate capacitor is

$$C = \frac{\kappa \epsilon_0 A}{d} \tag{21.22}$$



Capacitance of a parallel-plate capacitor with a dielectric of dielectric constant κ

EXAMPLE 21.12 Finding the dielectric constant

A parallel-plate capacitor is charged using a 100 V battery; then the battery is removed. If a dielectric slab is slid between the plates, filling the space inside, the capacitor voltage drops to 30 V. What is the dielectric constant of the dielectric?

PREPARE The capacitor voltage remains $(\Delta V_C)_1 = 100$ V when it is disconnected from the battery. Placing the dielectric between the plates reduces the voltage to $(\Delta V_C)_2 = 30$ V.

SOLVE The electric field strength between the capacitor plates is $E = \Delta V_C/d$. The plate separation d doesn't change, so

$$\frac{E_1}{E_2} = \frac{(\Delta V_C)_1/d}{(\Delta V_C)_2/d} = \frac{(\Delta V_C)_1}{(\Delta V_C)_2} = \frac{100\text{ V}}{30\text{ V}} = 3.3$$

But $E_2 = E_1/\kappa$, from Equation 21.21, so

$$\kappa = \frac{E_1}{E_2} = 3.3$$

ASSESS The amount of charge on the capacitor didn't change. The capacitor voltage decreased because the dielectric reduced the electric field strength inside the capacitor.

In Example 21.11, we found that impractically large electrodes would be needed to create a parallel-plate capacitor with $C = 1\ \mu$F. Practical capacitors take advantage of dielectrics with $\kappa > 100$. As shown in **FIGURE 21.34**, a typical capacitor is a "sandwich" of two modest-sized pieces of aluminum foil separated by a very thin dielectric that increases the capacitance by a large factor. This sandwich is then folded, rolled up, and sealed in a small plastic cylinder. The two wires extending from a capacitor connect to the two electrodes and allow the capacitor to be charged. Even though the electrodes are no longer planes, the capacitance is reasonably well predicted from the parallel-plate-capacitor equation if A is the area of the foils before being folded and rolled.

FIGURE 21.34 A capacitor disassembled to show its internal rolled-up structure.

STOP TO THINK 21.6 A 100 V battery is connected across the plates of a parallel-plate capacitor. If a Teflon slab is slid between the plates, without disconnecting the battery, the electric field between the plates

A. Increases. B. Decreases. C. Remains the same.

21.9 Energy and Capacitors

We've seen that a practical way of charging a capacitor is to attach its plates to a battery. Charge then flows from one plate, through the battery, and onto the other plate, leaving one plate with charge $-Q$ and the other with charge $+Q$. The battery must do *work* to transfer the charge; this work increases the electric potential energy of the charge on the capacitor. **A charged capacitor stores energy as electric potential energy.**

To find out how much energy is stored in a charged capacitor, recall that a charge q moved through a potential difference ΔV gains potential energy $U = q\Delta V$. When a capacitor is charged, total charge Q is moved from the negative plate to the positive plate. At first, when the capacitor is uncharged, the potential difference is $\Delta V = 0$. The last little bit of charge to be moved, as the capacitor reaches full charge, has to move through a potential difference $\Delta V \approx \Delta V_C$. *On average,* the potential difference across a capacitor while it's being charged is $\Delta V_{\text{average}} = \frac{1}{2}\Delta V_C$. Thus it seems plausible

Taking a picture in a flash When you take a flash picture, the flash is fired using electric potential energy stored in a capacitor. Batteries are unable to deliver the required energy rapidly enough, but capacitors can discharge all their energy in only microseconds. A battery is used to slowly charge up the capacitor, which then rapidly discharges through the flashlamp. This slow recharging process is why you must wait some time between taking flash pictures.

(and can be proved in a more advanced treatment) that the potential energy U_C stored in a charged capacitor is

$$U_C = Q\Delta V_{\text{average}} = \frac{1}{2}Q\Delta V_C$$

We can use $Q = C\Delta V_C$ to write this in two different ways:

$$U_C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C(\Delta V_C)^2 \qquad (21.23)$$

Electric potential energy of a capacitor with charge Q and potential difference ΔV_C

The potential energy stored in a capacitor depends on the *square* of the potential difference across it. This result is reminiscent of the potential energy $U_s = \frac{1}{2}k(\Delta x)^2$ stored in a spring, and a charged capacitor really is analogous to a stretched spring. A stretched spring holds energy until we release it; then that potential energy is transformed into kinetic energy. Likewise, a charged capacitor holds energy until we discharge it.

EXAMPLE 21.13 Energy in a camera flash

How much energy is stored in a 220 μF camera-flash capacitor that has been charged to 330 V? What is the average power delivered to the flash lamp if this capacitor is discharged in 1.0 ms?

SOLVE The energy stored in the capacitor is

$$U_C = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}(220 \times 10^{-6}\ \text{F})(330\ \text{V})^2 = 12\ \text{J}$$

If this energy is released in 1.0 ms, the average power is

$$P = \frac{\Delta E}{\Delta t} = \frac{12\ \text{J}}{1.0 \times 10^{-3}\ \text{s}} = 12{,}000\ \text{W}$$

ASSESS The stored energy is equivalent to raising a 1 kg mass by 1.2 m. This is a rather large amount of energy; imagine the damage a 1 kg object could do after falling 1.2 m. When this energy is released very quickly, as is possible in an electronic circuit, the power is very high.

A defibrillator, which can restore a heartbeat, discharges a capacitor through the patient's chest.

The usefulness of a capacitor stems from the fact that it can be charged very slowly, over many seconds, and then can release the energy very quickly. A mechanical analogy would be using a crank to slowly stretch the spring of a catapult, then quickly releasing the energy to launch a massive rock.

An important medical application of the ability of capacitors to rapidly deliver energy is the *defibrillator.* A heart attack or a serious injury can cause the heart to enter a state known as *fibrillation* in which the heart muscles twitch randomly and cannot pump blood. A strong electric shock through the chest can sometimes stop the fibrillation and allow a normal heart rhythm to be restored. A defibrillator has a large capacitor that can store up to 360 J of energy. This energy is released in about 2 ms through two "paddles" pressed against the patient's chest. It takes several seconds to charge the capacitor, which is why, on television medical shows, you hear an emergency room doctor or nurse shout, "Charging!"

The Energy in the Electric Field

We can "see" the potential energy of a stretched spring in the tension of the coils. If a charged capacitor is analogous to a stretched spring, where is the stored energy? It's in the electric field!

FIGURE 21.35 shows a parallel-plate capacitor in which the plates have area A and are separated by distance d. The potential difference across the capacitor is related to the electric field inside the capacitor by $\Delta V_C = Ed$. The capacitance, which we found in Equation 21.22, is $C = \kappa\epsilon_0 A/d$. Substituting these into Equation 21.23, we find that the energy stored in the capacitor is

$$U_C = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}\frac{\kappa\epsilon_0 A}{d}(Ed)^2 = \frac{\kappa\epsilon_0}{2}(Ad)E^2 \qquad (21.24)$$

The quantity Ad is the volume *inside* the capacitor, the region in which the capacitor's electric field exists. (Recall that an ideal capacitor has $\vec{E} = \vec{0}$ everywhere except between the plates.) Although we talk about "the energy stored in the capacitor," Equation 21.24 suggests that, strictly speaking, **the energy is stored in the capacitor's electric field.**

Because Ad is the volume in which the energy is stored, we can define an **energy density** u_E of the electric field:

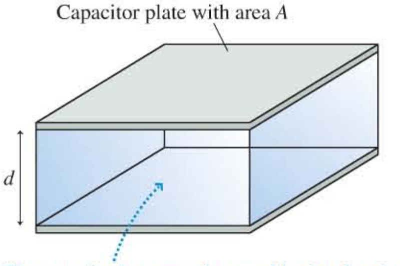
$$u_E = \frac{\text{energy stored}}{\text{volume in which it is stored}} = \frac{U_C}{Ad} = \frac{1}{2}\kappa\epsilon_0 E^2 \qquad (21.25)$$

The energy density has units J/m^3. We've derived Equation 21.25 for a parallel-plate capacitor, but it turns out to be the correct expression for any electric field.

From this perspective, charging a capacitor stores energy in the capacitor's electric field as the field grows in strength. Later, when the capacitor is discharged, the energy is released as the field collapses.

We first introduced the electric field as a way to visualize how a long-range force operates. But if the field can store energy, the field must be real, not merely a pictorial device. We'll explore this idea further in Chapter 25, where we'll find that the energy transported by a light wave—the very real energy of warm sunshine—is the energy of electric and magnetic fields.

FIGURE 21.35 A capacitor's energy is stored in the electric field.



EXAMPLE 21.14 **Finding the energy density for a defibrillator**

A defibrillator unit contains a 150 μF capacitor that is charged to 2000 V. The capacitor plates are separated by a 0.010-mm-thick dielectric with $\kappa = 300$.

a. What is the total area of the capacitor plates?
b. What is the energy density stored in the electric field when the capacitor is charged?

PREPARE Assume the capacitor can be modeled as a parallel-plate capacitor with a dielectric.

SOLVE a. Equation 21.22 for the capacitance of a dielectric-filled parallel-plate capacitor gives the surface area of the electrodes:

$$A = \frac{dC}{\kappa\epsilon_0} = \frac{(1.0 \times 10^{-5}\text{ m})(150 \times 10^{-6}\text{ F})}{(300)(8.85 \times 10^{-12}\text{ F/m})} = 0.56\text{ m}^2$$

b. The electric field strength is

$$E = \frac{\Delta V_C}{d} = \frac{2000\text{ V}}{1.0 \times 10^{-5}\text{ m}} = 2.0 \times 10^8\text{ V/m}$$

Consequently, the energy density in the electric field is

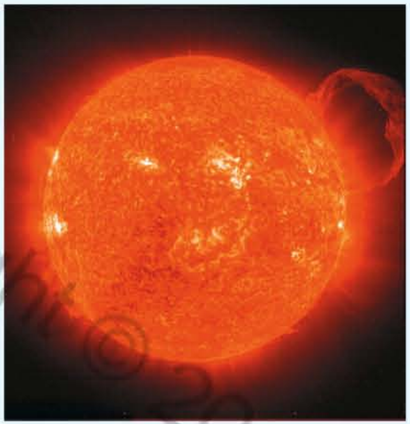
$$\begin{aligned} u_E &= \frac{1}{2}\kappa\epsilon_0 E^2 \\ &= \frac{1}{2}(300)(8.85 \times 10^{-12}\text{ F/m})(2.0 \times 10^8\text{ V/m})^2 \\ &= 5.3 \times 10^7\text{ J/m}^3 \end{aligned}$$

ASSESS For comparison, the energy density of gasoline is about 3×10^9 J/m^3, about 60 times higher than this capacitor. Capacitors store less energy than some other devices, but they can deliver this energy *very* rapidly.

STOP TO THINK 21.7 The plates of a parallel-plate capacitor are connected to a battery. If the distance between the plates is halved, the energy of the capacitor

A. Increases by a factor of 4.
B. Doubles.
C. Remains the same.
D. Is halved.
E. Decreases by a factor of 4.

INTEGRATED EXAMPLE 21.15 Proton fusion in the sun



The sun's energy comes from nuclear reactions that fuse lighter nuclei into heavier ones, releasing energy in the process. The solar fusion process begins when two protons (the nuclei of hydrogen atoms) merge to produce a *deuterium* nucleus. Deuterium is the "heavy" isotope of hydrogen, with a nucleus consisting of a proton *and* a neutron. To become deuterium, one of the protons that fused has to turn into a neutron. The nuclear-physics process by which this occurs—and which releases the energy—will be studied in Chapter 30. Our interest for now lies not with the nuclear physics but with the conditions that allow fusion to occur.

Before two protons can fuse, they must come into contact. However, the energy required to bring two protons into contact is considerable because the electric potential energy of the two protons increases rapidly as they approach each other. Fusion occurs in the core of the sun because the ultra-high temperature there gives the protons the kinetic energy they need to come together.

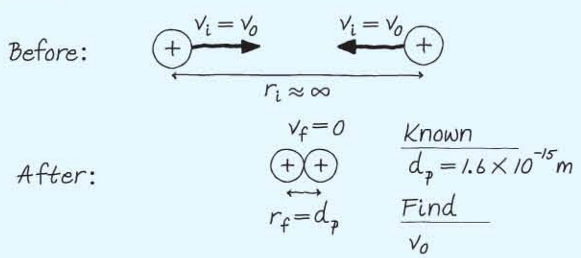
a. A proton can be modeled as a charged sphere of diameter $d_p = 1.6 \times 10^{-15}$ m with total charge e. When two protons are in contact, what is the electric potential of one proton at the center of the other?

b. Two protons are approaching each other head-on, each with the same speed v_0. What value of v_0 is required for the protons to just come into contact with each other?

c. What does the temperature of the sun's core need to be so that the rms speed v_{rms} of protons is equal to v_0?

PREPARE Energy is conserved, so Problem-Solving Strategy 21.1 is the basis of our solution. **FIGURE 21.36** shows a before-and-after visual overview. Both protons are initially moving with speeds $v_i = v_0$, so both contribute to the initial kinetic energy. We will assume that they start out so far apart that $U_i \approx 0$. To "just touch" means that they've instantaneously come to rest ($K_f = 0$) at the point where the distance between their centers is equal to the diameter of a proton. We can use the potential of a charged sphere and the energy-conservation equation to find the speed v_0 required to achieve contact. Then we can use the results of Chapter 12 to find the temperature at which v_0 is the rms speed of the protons.

FIGURE 21.36 Visual overview of two protons coming into contact.



SOLVE a. The electric potential at distance r from a charged sphere was found to be $V = KQ/r$. When the protons are in contact, the distance between their centers is $r_f = d_p = 1.6 \times 10^{-15}$ m. Thus the potential of one proton, with $Q = e$, at the center of the other is

$$V = \frac{Ke}{r_f} = \frac{Ke}{d_p} = \frac{(9.0 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})}{1.6 \times 10^{-15}\ \text{m}}$$
$$= 9.0 \times 10^5\ \text{V}$$

b. The conservation of energy equation $K_f + qV_f = K_i + qV_i$ is

$$(0 + 0) + eV_f = \left(\frac{1}{2}mv_0^2 + \frac{1}{2}mv_0^2\right) + 0$$

where, as noted above, both protons contribute to the initial kinetic energy, both end up at rest as the protons touch, and they started far enough apart that the initial potential energy (and potential) is effectively zero. When the protons meet, their potential energy is the charge of one proton (e) multiplied by the electric potential of the other—namely, the potential found in part a: $V_f = 9.0 \times 10^5$ V. Solving the energy equation for v_0, we get

$$v_0 = \sqrt{\frac{eV_f}{m}} = \sqrt{\frac{(1.6 \times 10^{-19}\ \text{C})(9.0 \times 10^5\ \text{V})}{1.7 \times 10^{-27}\ \text{kg}}}$$
$$= 9.2 \times 10^6\ \text{m/s}$$

c. In Chapter 12, we found that the temperature of a gas is related to the average kinetic energy of the particles and thus to the rms speed of the particles by the equation

$$T = \frac{mv_{rms}^2}{3k_B}$$

It may seem strange to think of protons as a gas, but in the center of the sun, where all the atoms are ionized into nuclei and electrons, the protons are zooming around and do, indeed, act like a gas. For v_{rms} of the protons to be equal to v_0 that we calculated in part b, the temperature would have to be

$$T = \frac{mv_0^2}{3k_B} = \frac{(1.7 \times 10^{-27}\ \text{kg})(9.2 \times 10^6\ \text{m/s})^2}{3(1.4 \times 10^{-23}\ \text{J/K})} = 3.4 \times 10^9\ \text{K}$$

ASSESS An extraordinarily high temperature—over 3 billion kelvin—is required to give an average solar proton a speed of 9.2×10^6 m/s. In fact, the core temperature of the sun is "only" about 14 million kelvin, a factor of ≈200 less than we calculated. Protons can fuse at this lower temperature both because there are always a few protons moving much faster than average and because protons can reach each other even if their speeds are too low by the quantum-mechanical process of *tunneling*, which you'll learn about in Chapter 28. Still, because of the core's relatively "low" temperature, most protons bounce around in the sun for several billion years before fusing!

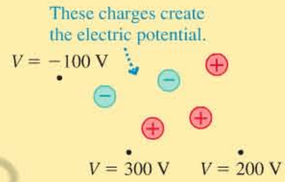
SUMMARY

The goal of Chapter 21 has been to calculate and use the electric potential and electric potential energy.

GENERAL PRINCIPLES

Electric Potential and Potential Energy

The electric potential V is created by charges and exists at every point surrounding those charges.



When a charge q is brought near these charges, it acquires an electric potential energy

$$U_{\text{elec}} = qV$$

at a point where the other charges have created an electric potential V.

Energy is conserved for a charged particle in an electric potential:

$$K_f + qV_f = K_i + qV_i$$

or

$$\Delta K = -q\Delta V$$

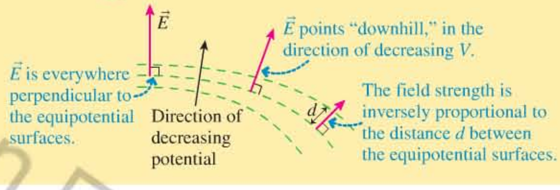
Sources of Potential

Potential differences ΔV are created by a *separation of charge*. Two important sources of potential difference are

- A *battery,* which uses chemical means to separate charge and produce a potential difference.
- The opposite charges on the plates of a *capacitor,* which create a potential difference between the plates.

The electric potential of a point charge q is $V = K\dfrac{q}{r}$

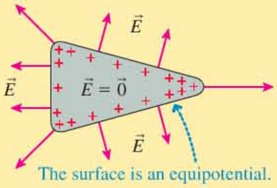
Connecting potential and field



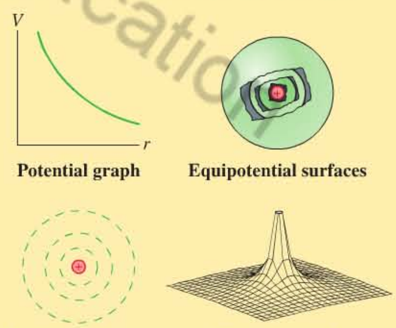
IMPORTANT CONCEPTS

For a **conductor in electrostatic equilibrium**

- Any excess charge is on the surface.
- The electric field inside is zero.
- The exterior electric field is perpendicular to the surface.
- The field strength is largest at sharp corners.
- The entire conductor is at the same potential and so the surface is an equipotential.



Graphical representations of the potential



Potential graph

Equipotential surfaces

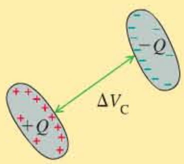
Equipotential map

Elevation graph

APPLICATIONS

Capacitors and dielectrics

The potential difference ΔV_C between two conductors charged to $\pm Q$ is proportional to the charge:



$$\Delta V_C = Q/C$$

where C is the **capacitance** of the two conductors.

A **parallel-plate capacitor** with plates of area A and separation d has a capacitance

$$C = \kappa\epsilon_0 A/d$$

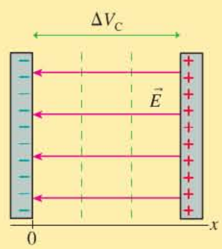
When a **dielectric** is inserted between the plates of a capacitor, its capacitance is increased by a factor κ, the **dielectric constant** of the material.

The **energy stored in a capacitor** is $U_C = \frac{1}{2}C(\Delta V_C)^2$.

This energy is stored in the electric field, which has energy density $u_E = \frac{1}{2}\kappa\epsilon_0 E^2$

Parallel-plate capacitor

For a capacitor charged to ΔV_C the potential at distance x from the negative plate is



$$V = \frac{x}{d}\Delta V_C$$

The electric field inside is

$$E = \Delta V_C/d$$

Units

- Electric potential: 1 V = 1 J/C
- Electric field: 1 V/m = 1 N/C
- Energy: 1 electron volt = 1 eV = 1.60×10^{-19} J is the kinetic energy gained by an electron upon accelerating through a potential difference of 1 V.

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. By moving a 10 nC charge from point A to point B, you determine that the electric potential at B is 150 V. What would be the potential at B if a 20 nC charge were moved from A to B?
2. Charge q is fired through a small hole in the positive plate of a capacitor, as shown in Figure Q21.2.
   a. If q is a positive charge, does it speed up or slow down inside the capacitor? Answer this question twice: (i) Using the concept of force. (ii) Using the concept of energy.
   b. Repeat part a if q is a negative charge.

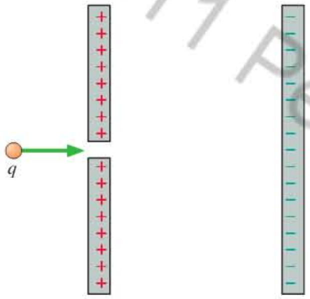


FIGURE Q21.2

3. *Why* is the potential energy of two opposite charges a negative number? (Note: Saying that the formula gives a negative number is not an explanation.)
4. An electron ($q = -e$) completes half of a circular orbit of radius r around a nucleus with $Q = +3e$, as shown in Figure Q21.4.
   a. How much work is done on the electron as it moves from i to f? Give either a numerical value or an expression from which you could calculate the value if you knew the radius. Justify your answer.
   b. By how much does the electric potential energy change as the electron moves from i to f?
   c. Is the electron's speed at f greater than, less than, or equal to its speed at i?
   d. Are your answers to parts a and c consistent with each other?

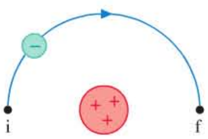


FIGURE Q21.4

5. An electron moves along the trajectory from i to f in Figure Q21.5.
   a. Does the electric potential energy increase, decrease, or stay the same? Explain.
   b. Is the electron's speed at f greater than, less than, or equal to its speed at i? Explain.

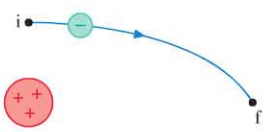


FIGURE Q21.5

6. The graph in Figure Q21.6 shows the electric potential along the x-axis. Draw a graph of the potential energy of a 0.10 C charged particle in this region of space, providing a numerical scale on the energy axis.

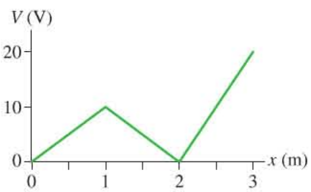


FIGURE Q21.6

7. As shown in Figure Q21.7, two protons are launched with the same speed from point 1 inside a parallel-plate capacitor. One proton moves along the path from 1 to 2, the other from 1 to 3. Points 2 and 3 are the same distance from the positive plate.
   a. Is $\Delta U_{1\to 2}$, the change in potential energy along the path $1 \to 2$, larger than, smaller than, or equal to $\Delta U_{1\to 3}$? Explain.
   b. Is the proton's speed v_2 at point 2 larger than, smaller than, or equal to the proton's speed v_3 at point 3? Explain.

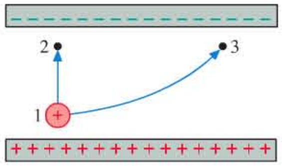


FIGURE Q21.7

8. Figure Q21.8 shows two points inside a capacitor. Let $V = 0$ V at the negative plate.
   a. What is the ratio V_2/V_1 of the electric potential at these two points? Explain.
   b. What is the ratio E_2/E_1 of the electric field strength at these two points? Explain.

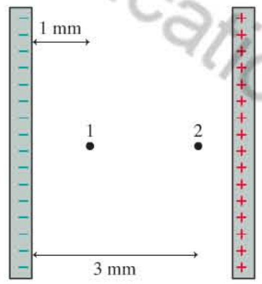


FIGURE Q21.8

9. A capacitor with plates separated by distance d is charged to a potential difference ΔV_C. All wires and batteries are disconnected, then the two plates are pulled apart (with insulated handles) to a new separation of distance $2d$.
   a. Does the capacitor charge Q change as the separation increases? If so, by what factor? If not, why not?
   b. Does the electric field strength E change as the separation increases? If so, by what factor? If not, why not?
   c. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?
10. Rank in order, from most positive to most negative, the electric potentials V_1 to V_5 at points 1 to 5 in Figure Q21.10. Explain.

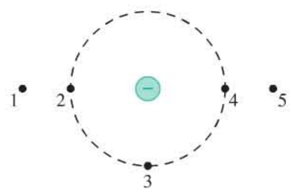


FIGURE Q21.10

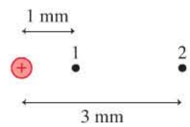


FIGURE Q21.11

11. Figure Q21.11 shows two points near a positive point charge.
 a. What is the ratio V_1/V_2 of the electric potentials at these two points? Explain.
 b. What is the ratio E_1/E_2 of the electric field strengths at these two points? Explain.
12. Each part of Figure Q21.12 shows three points in the vicinity of two point charges. The charges have equal magnitudes. Rank in order, from largest to smallest, the potentials V_1, V_2, and V_3.

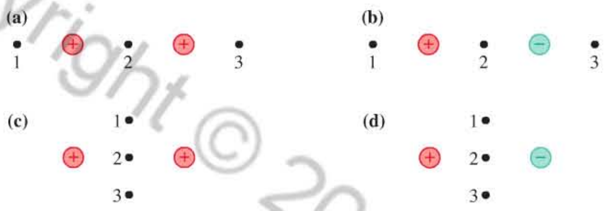


FIGURE Q21.12

13. a. Suppose that $\vec{E}=\vec{0}$ throughout some region of space. Can you conclude that $V = 0$ V in this region? Explain.
 b. Suppose that $V = 0$ V throughout some region of space. Can you conclude that $\vec{E}=\vec{0}$ in this region? Explain.
14. Rank in order, from largest to smallest, the electric field strengths E_1, E_2, E_3, and E_4 at the four labeled points in Figure Q21.14. Explain.

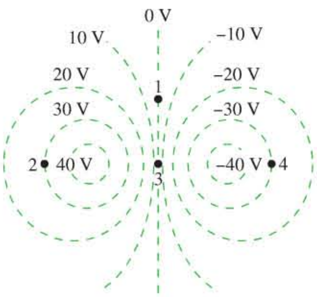


FIGURE Q21.14

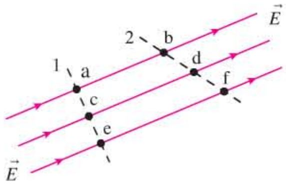


FIGURE Q21.15

15. Figure Q21.15 shows an electric field diagram. Dotted lines 1 and 2 are two surfaces in space, not physical objects.
 a. Is the electric potential at point a higher than, lower than, or equal to the electric potential at point b? Explain.
 b. Rank in order, from largest to smallest, the potential differences ΔV_{ab}, ΔV_{cd}, and ΔV_{ef}. Explain.
 c. Is surface 1 an equipotential surface? What about surface 2? Explain why or why not.
16. Figure Q21.16 shows a negatively charged electroscope. The gold leaf stands away from the rigid metal post. Is the electric potential of the leaf higher than, less than, or equal to the potential of the post? Explain.

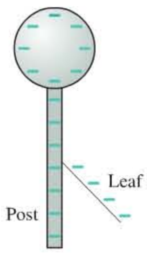


FIGURE Q21.16

17. Rank in order, from largest to smallest, the energies $(U_C)_1$ to $(U_C)_4$ stored in each of the capacitors in Figure Q21.17. Explain.

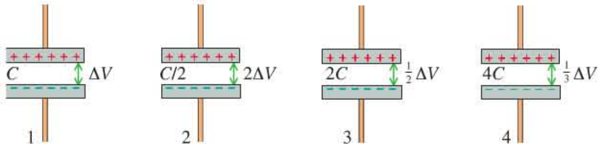


FIGURE Q21.17

18. A parallel-plate capacitor with plate separation d is connected to a battery that has potential difference ΔV_{bat}. Without breaking any of the connections, insulating handles are used to increase the plate separation to $2d$.
 a. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?
 b. Does the capacitance change? If so, by what factor? If not, why not?
 c. Does the capacitor charge Q change? If so, by what factor? If not, why not?
19. The gap between the capacitor plates shown in Figure Q21.19 is *partially* filled with a dielectric. The capacitor was charged by a 9 V battery, then disconnected from the battery. Rank in order, from smallest to largest, the electric field strengths E_1, E_2, and E_3 at the points labeled in the figure, as well as the field strength E_4 between the plates if the dielectric is removed. Explain.

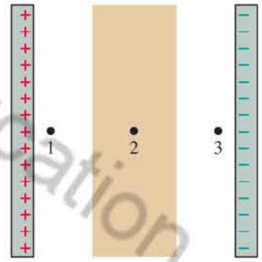


FIGURE Q21.19

Multiple-Choice Questions

20. | A 1.0 nC positive point charge is located at point A in Figure Q21.20. The electric potential at point B is
 A. 9.0 V
 B. $9.0\sin 30°$ V
 C. $9.0\cos 30°$ V
 D. $9.0\tan 30°$ V

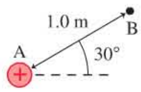


FIGURE Q21.20

21. | For the capacitor shown in Figure Q21.21, the potential difference ΔV_{ab} between points a and b is
 A. 6 V
 B. $6\sin 30°$ V
 C. $6\cos 30°$ V
 D. $6/\sin 30°$ V
 E. $6/\cos 30°$ V

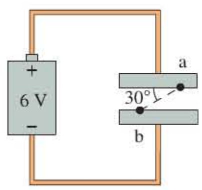


FIGURE Q21.21

22. | The electric potential is 300 V at $x = 0$ cm, is −100 V at $x = 5$ cm, and varies linearly with x. If a positive charge is released from rest at $x = 2.5$ cm, and is subject only to electric forces, the charge will
 A. Move to the right.
 B. Move to the left.
 C. Stay at $x = 2.5$ cm.
 D. Not enough information to tell.

Questions 23 through 27 refer to Figure Q21.23, which shows equipotential lines in a region of space. The equipotential lines are spaced by the same difference in potential, and several of the potentials are given.

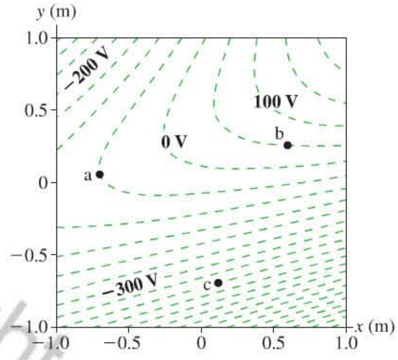


FIGURE Q21.23

23. | What is the potential at point c?
    A. −400 V
    B. −350 V
    C. −100 V
    D. 350 V
    E. 400 V
24. | At which point, a, b, or c, is the magnitude of the electric field the greatest?
25. | What is the approximate magnitude of the electric field at point c?
    A. 100 V/m B. 300 V/m C. 800 V/m
    D. 1500 V/m E. 3000 V/m
26. | The direction of the electric field at point b is closest to which direction?
    A. Right B. Up C. Left D. Down
27. || A +10 nC charge is moved from point c to point a. How much work is required in order to do this?
    A. 3.5×10^{-6} J B. 4.0×10^{-6} J C. 3.5×10^{-3} J
    D. 4.0×10^{-3} J E. 3.5 J
28. | A bug zapper consists of two metal plates connected to a high-voltage power supply. The voltage between the plates is set to give an electric field slightly less than 1×10^6 V/m. When a bug flies between the two plates, it increases the field enough to initiate a spark that incinerates the bug. If a bug zapper has a 4000 V power supply, what is the approximate separation between the plates?
    A. 0.05 cm B. 0.5 cm
    C. 5 cm D. 50 cm
29. | An atom of helium and one of argon are singly ionized—one electron is removed from each. The two ions are then accelerated from rest by the electric field between two plates with a potential difference of 150 V. After accelerating from one plate to the other,
    A. The helium ion has more kinetic energy.
    B. The argon ion has more kinetic energy.
    C. Both ions have the same kinetic energy.
    D. There is not enough information to say which ion has more kinetic energy.
30. || BIO The dipole moment of the heart is shown at a particular instant in Figure Q21.30. Which of the following potential differences will have the largest positive value?
    A. $V_1 - V_2$
    B. $V_1 - V_3$
    C. $V_2 - V_1$
    D. $V_3 - V_1$

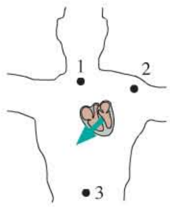


FIGURE Q21.30

PROBLEMS

Section 21.1 Electric Potential Energy and Electric Potential

Section 21.2 Sources of Electric Potential

1. ||| Moving a charge from point A, where the potential is 300 V, to point B, where the potential is 150 V, takes 4.5×10^{-4} J of work. What is the value of the charge?
2. |||| The graph in Figure P21.2 shows the electric potential energy as a function of separation for two point charges. If one charge is +0.44 nC, what is the other charge?

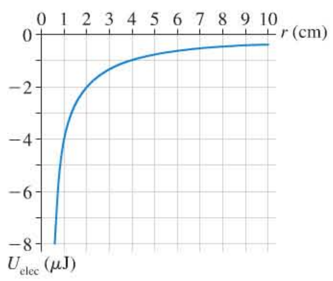


FIGURE P21.2

3. ||| It takes 3.0 μJ of work to move a 15 nC charge from point A to B. It takes −5.0 μJ of work to move the charge from C to B. What is the potential difference $V_C - V_A$?
4. | A 20 nC charge is moved from a point where $V = 150$ V to a point where $V = -50$ V. How much work is done by the force that moves the charge?
5. | At one point in space, the electric potential energy of a 15 nC charge is 45 μJ.
    a. What is the electric potential at this point?
    b. If a 25 nC charge were placed at this point, what would its electric potential energy be?

Section 21.3 Electric Potential and Conservation of Energy

6. | An electron has been accelerated from rest through a potential difference of 1000 V.
    a. What is its kinetic energy, in electron volts?
    b. What is its kinetic energy, in joules?
    c. What is its speed?
7. | A proton has been accelerated from rest through a potential difference of −1000 V.
    a. What is its kinetic energy, in electron volts?
    b. What is its kinetic energy, in joules?
    c. What is its speed?
8. ||||| What potential difference is needed to accelerate a He^+ ion (charge $+e$, mass 4 u) from rest to a speed of 1.0×10^6 m/s?
9. || An electron with an initial speed of 500,000 m/s is brought to rest by an electric field.
    a. Did the electron move into a region of higher potential or lower potential?
    b. What was the potential difference that stopped the electron?
    c. What was the initial kinetic energy of the electron, in electron volts?

10. ||| A proton with an initial speed of 800,000 m/s is brought to rest by an electric field.
    a. Did the proton move into a region of higher potential or lower potential?
    b. What was the potential difference that stopped the proton?
    c. What was the initial kinetic energy of the proton, in electron volts?

Section 21.4 Calculating the Electric Potential

11. || The electric potential at a point that is halfway between two identical charged particles is 300 V. What is the potential at a point that is 25% of the way from one particle to the other?
12. || A $2.0 \text{ cm} \times 2.0 \text{ cm}$ parallel-plate capacitor has a 2.0 mm spacing. The electric field strength inside the capacitor is 1.0×10^5 V/m.
    a. What is the potential difference across the capacitor?
    b. How much charge is on each plate?
13. ||| Two $2.00 \text{ cm} \times 2.00 \text{ cm}$ plates that form a parallel-plate capacitor are charged to ± 0.708 nC. What are the electric field strength inside and the potential difference across the capacitor if the spacing between the plates is (a) 1.00 mm and (b) 2.00 mm?
14. | a. In Figure P21.14, which capacitor plate, left or right, is the positive plate?
    b. What is the electric field strength inside the capacitor?
    c. What is the potential energy of a proton at the midpoint of the capacitor?

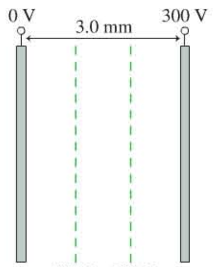


FIGURE P21.14

15. | A +25 nC charge is at the origin.
    a. What are the radii of the 1000 V, 2000 V, 3000 V, and 4000 V equipotential surfaces?
    b. Draw an equipotential map in the xy-plane showing the charge and these four surfaces.
16. || a. What is the electric potential at points A, B, and C in Figure P21.16?
    b. What is the potential energy of an electron at each of these points?
    c. What are the potential differences ΔV_{AB} and ΔV_{BC}?

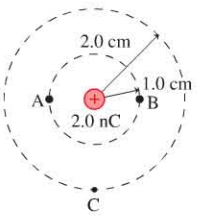


FIGURE P21.16

17. ||| A 1.0-mm-diameter ball bearing has 2.0×10^9 excess electrons. What is the ball bearing's potential?
18. || What is the electric potential at the point indicated with the dot in Figure P21.18?

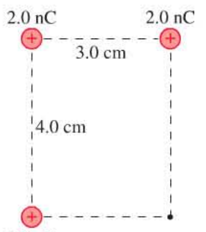


FIGURE P21.18

19. || a. What is the potential difference between the terminals of an ordinary AA or AAA battery? (If you're not sure, find one and look at the label.)
    b. An AA battery is connected to a parallel-plate capacitor having 4.0-cm-diameter plates spaced 2 mm apart. How much charge does the battery move from one plate to the other?

Section 21.5 Connecting Potential and Field

20. || a. In Figure P21.20, which point, A or B, has a higher electric potential?
    b. What is the potential difference between A and B?

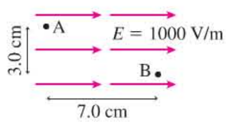


FIGURE P21.20

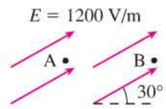


FIGURE P21.21

21. ||| In Figure P21.21, the electric potential at point A is −300 V. What is the potential at point B, which is 5.0 cm to the right of A?
22. || What is the potential difference between $x_i = 10$ cm and $x_f = 30$ cm in the uniform electric field $E_x = 1000$ V/m?
23. | What are the magnitude and direction of the electric field at the dot in Figure P21.23?

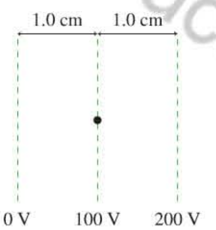


FIGURE P21.23

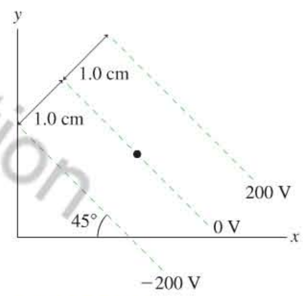


FIGURE P21.24

24. | What are the magnitude and direction of the electric field at the dot in Figure P21.24?

Section 21.6 The Electrocardiogram

25. | BIO One standard location for a pair of electrodes during an EKG is shown in Figure P21.25. The potential difference $\Delta V_{31} = V_3 - V_1$ is recorded. For each of the three instants a, b, and c during the heart's cycle shown in Figure 21.29, will ΔV_{31} be positive or negative? Explain.

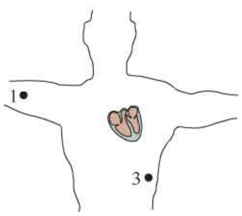


FIGURE P21.25

26. | BIO Three electrodes, 1–3, are attached to a patient as shown in Figure P21.26. During ventricular depolarization (see Figure 21.29), across which pair of electrodes is the magnitude of the potential difference likely to be the smallest? Explain.

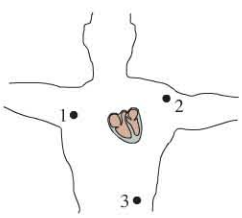


FIGURE P21.26

Section 21.7 Capacitance and Capacitors

27. ||| Two 2.0 cm × 2.0 cm square aluminum electrodes, spaced 0.50 mm apart, are connected to a 100 V battery.
    a. What is the capacitance?
    b. What is the charge on the positive electrode?
28. ||| An uncharged capacitor is connected to the terminals of a 3.0 V battery, and 6.0 μC flows to the positive plate. The 3.0 V battery is then disconnected and replaced with a 5.0 V battery, with the positive and negative terminals connected in the same manner as before. How much additional charge flows to the positive plate?
29. ||| You need to construct a 100 pF capacitor for a science project. You plan to cut two $L \times L$ metal squares and place spacers between them. The thinnest spacers you have are 0.20 mm thick. What is the proper value of L?
30. | A switch that connects a battery to a 10 μF capacitor is closed. Several seconds later you find that the capacitor plates are charged to ± 30 μC. What is the battery voltage?
31. | What is the voltage of a battery that will charge a 2.0 μF capacitor to ± 48 μC?
32. | Two electrodes connected to a 9.0 V battery are charged to ± 45 nC. What is the capacitance of the electrodes?
33. | Initially, the switch in Figure P21.33 is open and the capacitor is uncharged. How much charge flows through the switch after the switch is closed?

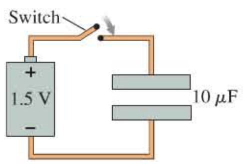


FIGURE P21.33

Section 21.8 Dielectrics and Capacitors

34. || A 1.2 nF parallel-plate capacitor has an air gap between its plates. Its capacitance increases by 3.0 nF when the gap is filled by a dielectric. What is the dielectric constant of that dielectric?
35. ||| A science-fair radio uses a homemade capacitor made of two 35 cm × 35 cm sheets of aluminum foil separated by a 0.25-mm-thick sheet of paper. What is its capacitance?
36. ||| A 25 pF parallel-plate capacitor with an air gap between the plates is connected to a 100 V battery. A Teflon slab is then inserted between the plates and completely fills the gap. What is the change in the charge on the positive plate when the Teflon is inserted?
37. || Two 2.0-cm-diameter electrodes with a 0.10-mm-thick sheet of Teflon between them are attached to a 9.0 V battery. Without disconnecting the battery, the Teflon is removed. What are the charge, potential difference, and electric field (a) before and (b) after the Teflon is removed?
38. |||| A capacitor with its plates separated by paper stores 4.4 nC of charge when it is connected to a particular battery. An otherwise identical capacitor, but with its plates separated by Pyrex glass, is connected to the same battery. How much charge does that capacitor store?

Section 21.9 Energy and Capacitors

39. ||| To what potential should you charge a 1.0 μF capacitor to store 1.0 J of energy?
40. || A pair of 10 μF capacitors in a high-power laser are charged to 1.7 kV.
    a. What charge is stored in each capacitor?
    b. How much energy is stored in each capacitor?
41. | Capacitor 2 has half the capacitance and twice the potential difference as capacitor 1. What is the ratio $(U_C)_1/(U_C)_2$?
42. |||| Two uncharged metal spheres, spaced 15.0 cm apart, have a capacitance of 24.0 pF. How much work would it take to move 12.0 nC of charge from one sphere to the other?
43. |||| 50 pJ of energy is stored in a 2.0 cm × 2.0 cm × 2.0 cm region of uniform electric field. What is the electric field strength?

General Problems

44. || A 2.0-cm-diameter parallel-plate capacitor with a spacing of 0.50 mm is charged to 200 V. What are (a) the total energy stored in the electric field and (b) the energy density?
45. ||| What is the change in electric potential energy of a 3.0 nC point charge when it is moved from point A to point B in Figure P21.45?

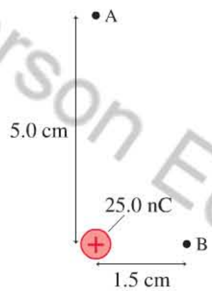


FIGURE P21.45

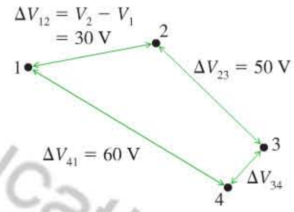


FIGURE P21.46

46. || What is the potential difference ΔV_{34} in Figure P21.46?
47. || INT A −50 nC charged particle is in a uniform electric field $\vec{E} = (10 \text{ V/m, east})$. An external force moves the particle 1.0 m north, then 5.0 m east, then 2.0 m south, and finally 3.0 m west. The particle begins and ends its motion with zero velocity.
    a. How much work is done on it by the external force?
    b. What is the potential difference between the particle's final and initial positions?
48. || INT At a distance r from a point charge, the electric potential is 3000 V and the magnitude of the electric field is 2.0×10^5 V/m.
    a. What is the distance r?
    b. What are the electric potential and the magnitude of the electric field at distance $r/2$ from the charge?
49. || What is the electric potential energy of the electron in Figure P21.49? The protons are fixed and can't move.

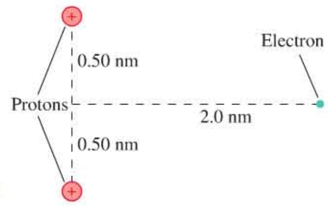


FIGURE P21.49

50. |||| Two point charges 2.0 cm apart have an electric potential energy −180 μJ. The total charge is 30 nC. What are the two charges?

51. [INT] Two positive point charges are 5.0 cm apart. If the electric potential energy is 72 μJ, what is the magnitude of the force between the two charges?

52. A $+3.0$ nC charge is at $x = 0$ cm and a -1.0 nC charge is at $x = 4$ cm. At what point or points on the x-axis is the electric potential zero?

53. A -3.0 nC charge is on the x-axis at $x = -9$ cm and a $+4.0$ nC charge is on the x-axis at $x = 16$ cm. At what point or points on the y-axis is the electric potential zero?

54. [INT] A -2.0 nC charge and a $+2.0$ nC charge are located on the x-axis at $x = -1.0$ cm and $x = +1.0$ cm, respectively.
   a. At what position or positions on the x-axis is the electric field zero?
   b. At what position or positions on the x-axis is the electric potential zero?
   c. Draw graphs of the electric field strength and the electric potential along the x-axis.

55. [INT] A -10.0 nC point charge and a $+20.0$ nC point charge are 15.0 cm apart on the x-axis.
   a. What is the electric potential at the point on the x-axis where the electric field is zero?
   b. What are the magnitude and direction of the electric field at the point on the x-axis, between the charges, where the electric potential is zero?

56. A 2.0-mm-diameter glass bead is positively charged. The potential difference between a point 2.0 mm from the bead and a point 4.0 mm from the bead is 500 V. What is the charge on the bead?

57. In a semiclassical model of the hydrogen atom, the electron orbits the proton at a distance of 0.053 nm.
   a. What is the electric potential of the proton at the position of the electron?
   b. What is the electron's potential energy?

58. What is the electric potential at the point indicated with the dot in Figure P21.58?

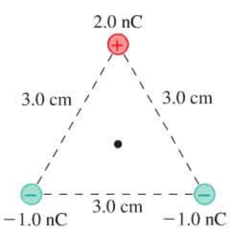


FIGURE P21.58

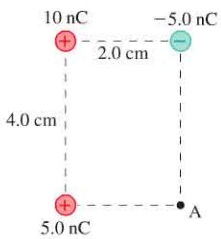


FIGURE P21.59

59. a. What is the electric potential at point A in Figure P21.59?
   b. What is the potential energy of a proton at point A?

60. A proton's speed as it passes point A is 50,000 m/s. It follows the trajectory shown in Figure P21.60. What is the proton's speed at point B?

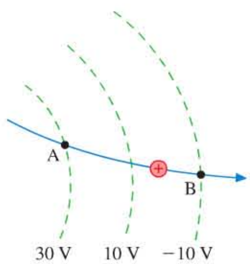


FIGURE P21.60

61. Electric outlets have a voltage of approximately 120 V between the two parallel slots. Estimate the electric field strength between these two slots.

62. [BIO] Estimate the magnitude of the electric field in a cell membrane with a thickness of 8 nm.

63. [BIO] A Na^+ ion moves from inside a cell, where the electric potential is -70 mV, to outside the cell, where the potential is 0 V. What is the change in the ion's electric potential energy as it moves from inside to outside the cell? Does its energy increase or decrease?

64. [BIO] Suppose that a molecular ion with charge $-10e$ is embedded within the 5.0-nm-thick cell membrane of a cell with membrane potential -70 mV. What is the electric force on the molecule?

65. The electric field strength is 50,000 V/m inside a parallel-plate capacitor with a 2.0 mm spacing. A proton is released from rest at the positive plate. What is the proton's speed when it reaches the negative plate?

66. [INT] An alpha particle (the nucleus of a helium atom, with charge $+2e$ and a mass four times that of a proton) and an antiproton (which has the same mass as a proton but charge $-e$) are released from rest a great distance apart. They are oppositely charged, so each accelerates toward the other. What are the speeds of the two particles when they are 2.5 nm apart? **Hint:** You'll need to use *two* conservation laws. And what does "a great distance" suggest about the initial value of r?

67. A proton is released from rest at the positive plate of a parallel-plate capacitor. It crosses the capacitor and reaches the negative plate with a speed of 50,000 m/s. What will be the proton's final speed if the experiment is repeated with double the amount of charge on each capacitor plate?

68. The electric field strength is 20,000 V/m inside a parallel-plate capacitor with a 1.0 mm spacing. An electron is released from rest at the negative plate. What is the electron's speed when it reaches the positive plate?

69. [INT] In the early 1900s, Robert Millikan used small charged droplets of oil, suspended in an electric field, to make the first quantitative measurements of the electron's charge. A 0.70-μm-diameter droplet of oil, having a charge of $+e$, is suspended in midair between two horizontal plates of a parallel-plate capacitor. The upward electric force on the droplet is exactly balanced by the downward force of gravity. The oil has a density of 860 kg/m^3, and the capacitor plates are 5.0 mm apart. What must the potential difference between the plates be to hold the droplet in equilibrium?

70. Two 2.0-cm-diameter disks spaced 2.0 mm apart form a parallel-plate capacitor. The electric field between the disks is 5.0×10^5 V/m.
   a. What is the voltage across the capacitor?
   b. How much charge is on each disk?
   c. An electron is launched from the negative plate. It strikes the positive plate at a speed of 2.0×10^7 m/s. What was the electron's speed as it left the negative plate?

71. [BIO] In *proton-beam therapy,* a high-energy beam of protons is fired at a tumor. The protons come to rest in the tumor, depositing their kinetic energy and breaking apart the tumor's DNA, thus killing its cells. For one patient, it is desired that 0.10 J of proton energy be deposited in a tumor. To create the proton beam, the protons are accelerated from rest through a 10 MV potential difference. What is the total charge of the protons that must be fired at the tumor to deposit the required energy?

72. |||| A 2.5-mm-diameter sphere is charged to -4.5 nC. An electron fired directly at the sphere from far away comes to within 0.30 mm of the surface of the target before being reflected.
  a. What was the electron's initial speed?
  b. At what distance from the surface of the sphere is the electron's speed half of its initial value?
  c. What is the acceleration of the electron at its turning point?

73. || A proton is fired from far away toward the nucleus of an iron atom. Iron is element number 26, and the diameter of the nucleus is 9.0 fm. (1 fm $= 10^{-15}$ m.) What initial speed does the proton need to just reach the surface of the nucleus? Assume the nucleus remains at rest.

74. || Two 10.0-cm-diameter electrodes 0.50 cm apart form a parallel-plate capacitor. The electrodes are attached by metal wires to the terminals of a 15 V battery. After a long time, the capacitor is disconnected from the battery but is not discharged. What are the charge on each electrode, the electric field strength inside the capacitor, and the potential difference between the electrodes
  a. Right after the battery is disconnected?
  b. After insulating handles are used to pull the electrodes away from each other until they are 1.0 cm apart?

75. || Two 10.0-cm-diameter electrodes 0.50 cm apart form a parallel-plate capacitor. The electrodes are attached by metal wires to the terminals of a 15 V battery. What are the charge on each electrode, the electric field strength inside the capacitor, and the potential difference between the electrodes
  a. While the capacitor is attached to the battery?
  b. After insulating handles are used to pull the electrodes away from each other until they are 1.0 cm apart? The electrodes remain connected to the battery during this process.

76. ||| Determine the magnitude and direction of the electric field at points 1 and 2 in Figure P21.76.

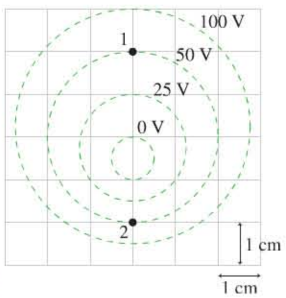


FIGURE P21.76

77. | Figure P21.77 shows a series of equipotential curves.
  a. Is the electric field strength at point A larger than, smaller than, or equal to the field strength at point B? Explain.
  b. Is the electric field strength at point C larger than, smaller than, or equal to the field strength at point D? Explain.
  c. Determine the electric field $\vec{E}$ at point D. Express your answer as a magnitude and direction.

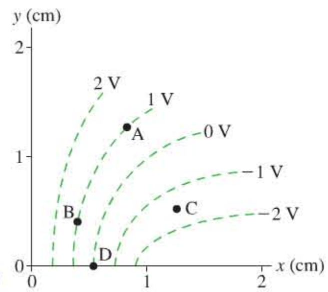


FIGURE P21.77

78. || Figure P21.78 shows the electric potential on a grid whose squares are 5.0 cm on a side.
  a. Reproduce this figure on your paper, then draw the 50 V, 75 V, and 100 V equipotential surfaces.
  b. Estimate the electric field (strength and direction) at points A, B, C, and D.
  c. Draw the electric field vectors at points A, B, C, and D on your diagram.

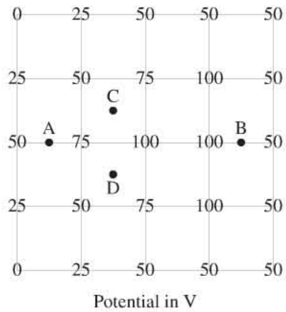


FIGURE P21.78

79. || The plates of a 3.0 nF parallel-plate capacitor are each 0.27 m^2 in area.
  a. How far apart are the plates if there's air between them?
  b. If the plates are separated by a Teflon sheet, how thick is the sheet?

80. |||| The dielectric in a capacitor serves two purposes. It increases the capacitance, compared to an otherwise identical capacitor with an air gap, and it increases the maximum potential difference the capacitor can support. If the electric field in a material is sufficiently strong, the material will suddenly become able to conduct, creating a spark. The critical field strength, at which breakdown occurs, is 3.0 MV/m for air, but 60 MV/m for Teflon.
  a. A parallel-plate capacitor consists of two square plates, 15 cm on a side, spaced 0.50 mm apart with only air between them. What is the maximum energy that can be stored by the capacitor?
  b. What is the maximum energy that can be stored if the plates are separated by a 0.50-mm-thick Teflon sheet?

81. ||||| INT The flash unit in a camera uses a special circuit to "step up" the 3.0 V from the batteries to 300 V, which charges a capacitor. The capacitor is then discharged through a flashlamp. The discharge takes 10 μs, and the average power dissipated in the flashlamp is 10^5 W. What is the capacitance of the capacitor?

In Problems 82 through 85 you are given the equation(s) used to solve a problem. For each of these,
  a. Write a realistic problem for which this is the correct equation(s).
  b. Finish the solution of the problem.

82. | $$\frac{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})q_1q_2}{0.030\ \mathrm{m}} = 90\times 10^{-6}\ \mathrm{J}$$
$$q_1 + q_2 = 40\ \mathrm{nC}$$

83. || $$\tfrac{1}{2}(1.67\times 10^{-27}\ \mathrm{kg})(2.5\times 10^6\ \mathrm{m/s})^2 + 0 = \tfrac{1}{2}(1.67\times 10^{-27}\ \mathrm{kg})v_i^2 + \frac{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})(2.0\times 10^{-9}\ \mathrm{C})(1.60\times 10^{-19}\ \mathrm{C})}{0.0010\ \mathrm{m}}$$

84. || $$\frac{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})(3.0\times 10^{-9}\ \mathrm{C})}{0.030\ \mathrm{m}} + \frac{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})(3.0\times 10^{-9}\ \mathrm{C})}{(0.030\ \mathrm{m}) + d} = 1200\ \mathrm{V}$$

85. || $$400\ \mathrm{nC} = (100\ \mathrm{V})\,C$$
$$C = \frac{(8.85\times 10^{-12}\ \mathrm{F/m})(0.10\ \mathrm{m}\times 0.10\ \mathrm{m})}{d}$$

Passage Problems

A Lightning Strike

Storm clouds build up large negative charges, as described in the chapter. The charges dwell in *charge centers,* regions of concentrated charge. Suppose a cloud has −25 C in a 1.0-km-diameter spherical charge center located 10 km above the ground, as sketched in Figure P21.88. The negative charge center attracts a similar amount of positive charge that is spread on the ground below the cloud.

The charge center and the ground function as a charged capacitor, with a potential difference of approximately 4×10^8 V. The large electric field between these two "electrodes" may ionize the air, leading to a conducting path between the cloud and the ground. Charges will flow along this conducting path, causing a discharge of the capacitor—a lightning strike.

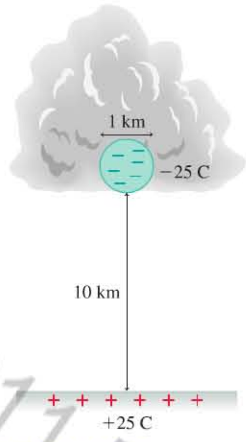


FIGURE P21.88

86. | What is the approximate magnitude of the electric field between the charge center and the ground?
A. 4×10^4 V/m B. 4×10^5 V/m
C. 4×10^6 V/m D. 4×10^7 V/m

87. | Which of the curves sketched in Figure P21.87 best approximates the shape of an equipotential drawn halfway between the charge center and the ground?

FIGURE P21.87

88. | What is the approximate capacitance of the charge center + ground system?
A. 6×10^{-8} F B. 2×10^7 F
C. 4×10^6 F D. 8×10^6 F

89. | If 12.5 C of charge is transferred from the cloud to the ground in a lightning strike, what fraction of the stored energy is dissipated?
A. 12% B. 25% C. 50% D. 75%

90. | If the cloud transfers all of its charge to the ground via several rapid lightning flashes lasting a total of 1 s, what is the average power?
A. 1 GW B. 2 GW C. 5 GW D. 10 GW

STOP TO THINK ANSWERS

Stop to Think 21.1: B If the charge were moved from 1 to 2 at a constant speed by a hand, the force exerted by the hand would need to be to the left, to oppose the rightward-directed electric force on the charge due to the source charges. Because the force due to the hand would be opposite the displacement, the hand would do *negative* work on the charge, decreasing its electric potential energy.

Stop to Think 21.2: C. The proton gains speed by losing potential energy. It loses potential energy by moving in the direction of decreasing electric potential.

Stop to Think 21.3: $\Delta V_{13} = \Delta V_{23} > \Delta V_{12}$. The potential depends only on the *distance* from the charge, not the direction. $\Delta V_{12} = 0$ because these points are at the same distance.

Stop to Think 21.4: C. $\vec{E}$ points "downhill," so V must decrease from right to left. E is larger on the left than on the right, so the equipotential lines must be closer together on the left.

Stop to Think 21.5: C. Capacitance is a property of the shape and position of the electrodes. It does not depend on the potential difference or charge.

Stop to Think 21.6: C. The electric field is $\Delta V_C/d$. With ΔV_C fixed by the battery, introducing the dielectric does not change E. More charge flows from the battery to compensate for the dielectric.

Stop to Think 21.7: B. The energy is $\frac{1}{2}C(\Delta V_C)^2$. ΔV_C is constant, but C doubles when the distance is halved.