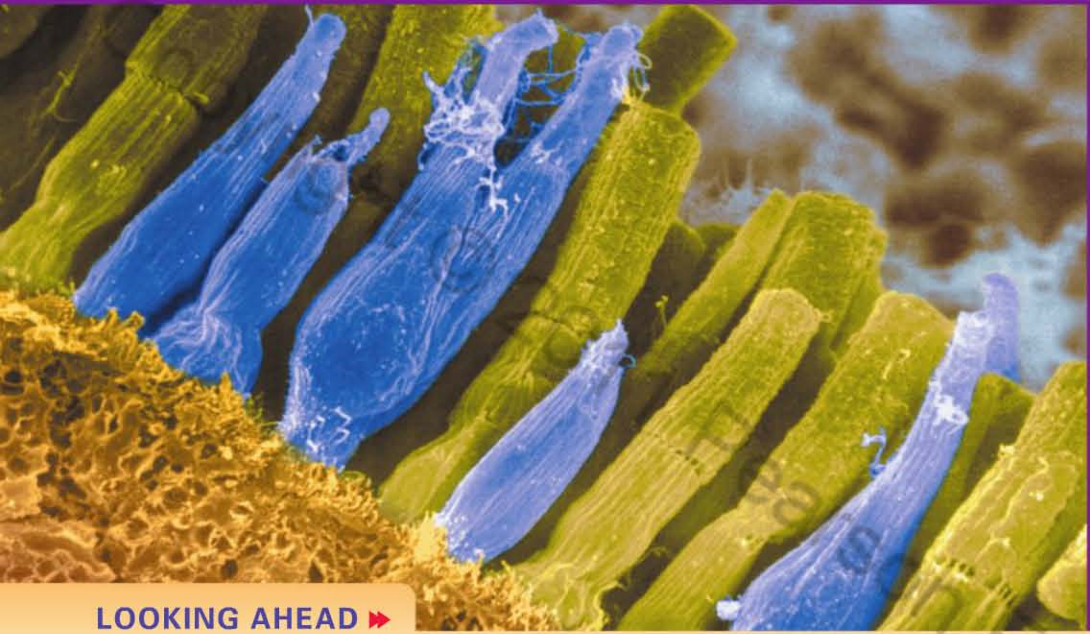
28 Quantum Physics



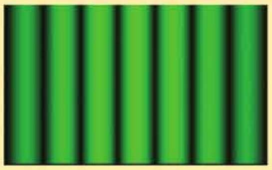
This false-color image showing individual rod cells (green) and cone cells (blue) on the human retina was made with an electron microscope. Such exquisite detail would not be possible in an image created with a light microscope. Why is greater resolution possible in an image made with a beam of electrons?

LOOKING AHEAD ▸▸

The goal of Chapter 28 is to understand the quantization of energy for light and matter.

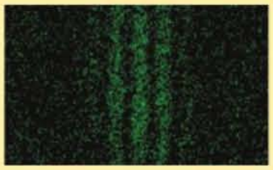
Waves and Particles

Electromagnetic waves like light are waves of electric and magnetic fields. Light has the properties of a wave.



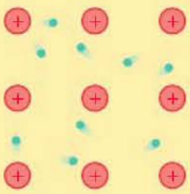
As we saw in Chapter 17, understanding diffraction and interference of light requires us to think of light as a wave.

But the picture is more complicated. Light and other electromagnetic waves also have a particle nature.



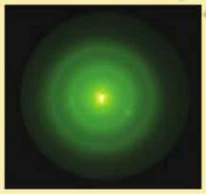
An interference pattern made with very low-intensity light clearly shows that the light hits the screen in "chunks." Sometimes, light looks like a particle.

As we've seen in earlier chapters, an electron has the properties of a particle.



Our model of conduction in metals was based on the motion of particle-like electrons moving among fixed ions.

But the picture is more complicated. Electrons and other particles have a wave nature as well.

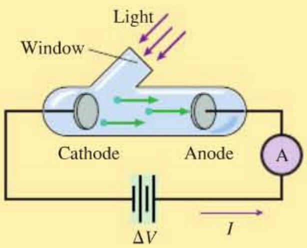


Shooting a beam of electrons through a crystal produces a diffraction pattern, something we'd expect for a wave.

Looking Back ◂◂
17.2–17.5 Diffraction and interference of light

Photons and the Photoelectric Effect

The particle nature of light requires us to think in terms of photons. As we've seen, the energy of a photon is proportional to the frequency of the light.

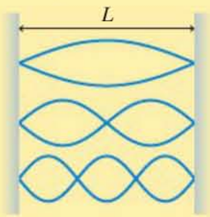


Light shining on a metal surface will eject electrons if the photons have sufficient energy.

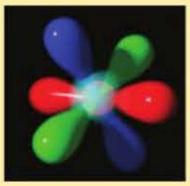
Looking Back ◂◂
25.7 The photon model of EM waves
25.8 X rays

Matter Waves and Quantization

The wave nature of electrons has far-reaching consequences.



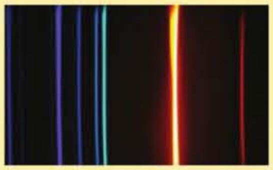
Just as a standing wave on a string stretched between fixed ends has only certain allowed modes ...



... an electron's wave nature means that electrons in atoms exist in only certain standing-wave modes.

Energy Levels

Atomic particles can have only certain allowed energies; their energy is **quantized.**



Electrons emit photons as they make quantum jumps between allowed energy levels. Only certain photon energies are possible, so we see a spectrum of discrete wavelengths.

Except for relativity, everything we have studied until this point in the book was known by 1900. Newtonian mechanics, thermodynamics, and the theory of electromagnetism form what we call *classical physics*. It is an impressive body of knowledge with immense explanatory power and a vast number of applications.

But a spate of discoveries right around 1900 showed that classical physics, though remarkable, was incomplete. Investigations into the nature of light and matter led to many astonishing discoveries that classical physics simply could not explain. Sometimes, as you will see, light refuses to act like a wave and seems more like a collection of particles. Other experiments found that electrons sometimes behave like waves. These discoveries eventually led to a radical new theory of light and matter called *quantum physics*.

This chapter will introduce you to this strange but wonderful quantum world. We will take a more historical approach than in previous chapters. As we introduce new ideas, we will describe in some detail the key experiments and the evolution of theories to explain them.

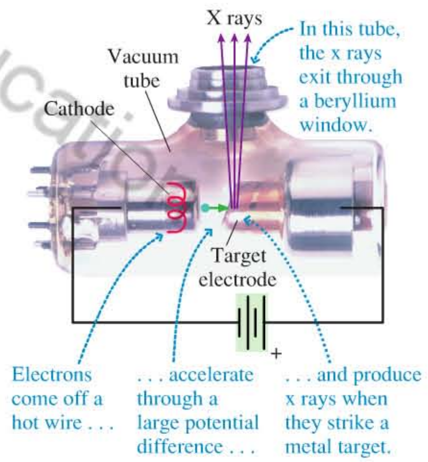
28.1 X Rays and X-Ray Diffraction

The rules of quantum physics apply at the scale of atoms and electrons. Experiments to elucidate the nature of the atom and the physics of atomic particles produced results that defied explanation with classical theories. Investigators saw things no one had ever seen before, phenomena that needed new principles and theories to explain them.

In 1895, the German physicist Wilhelm Röntgen was studying how electrons (called cathode rays at the time) travel through a vacuum. He sealed an electron-producing cathode and a metal target electrode into a vacuum tube. A high voltage pulled electrons from the cathode and accelerated them to very high speed before they struck the target electrode. One day, by chance, Röntgen left a sealed envelope containing film near the vacuum tube. He was later surprised to discover that the film had been exposed even though it had never been removed from the envelope. Some sort of penetrating radiation from the tube had exposed the film.

Röntgen had no idea what was coming from the tube, so he called them x rays, using the algebraic symbol x meaning "unknown." X rays were unlike anything, particle or wave, ever discovered before. Röntgen was not successful at reflecting the rays or at focusing them with a lens. He showed that they travel in straight lines, like particles, but they also pass right through most solid materials with very little absorption, something no known particle could do. The experiments of Röntgen and others led scientists to conclude that these mysterious rays were electromagnetic waves with very short wavelengths, as we learned in Chapter 25. These short-wavelength waves were produced in Röntgen's apparatus by the collision of fast electrons with a metal target. X rays are still produced this way, as shown in the illustration of the operation of a modern x-ray tube in **FIGURE 28.1**.

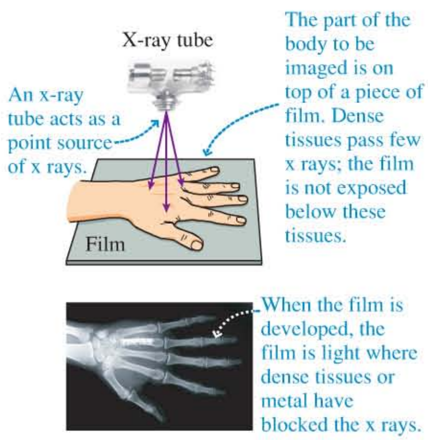
FIGURE 28.1 The operation of a modern x-ray tube.



X-Ray Images

X rays are penetrating, and Röntgen immediately realized that x rays could be used to create an image of the interior of the body. One of Röntgen's first images showed the bones in his wife's hand, dramatically demonstrating the medical potential of these newly discovered rays. Substances with high atomic numbers, such as lead or the minerals in bone, are effective at stopping them; materials with low atomic numbers, such as the beryllium window of the x-ray tube in Figure 28.1 or the water and organic compounds of soft tissues in the body, diminish them only slightly. As illustrated in **FIGURE 28.2**, an x-ray image is essentially a shadow of the bones and dense components of the body; where these tissues stop the x rays, the film is not exposed. The basic procedure for producing an x-ray image on film is little changed from Röntgen's day.

FIGURE 28.2 Creating an x-ray image.

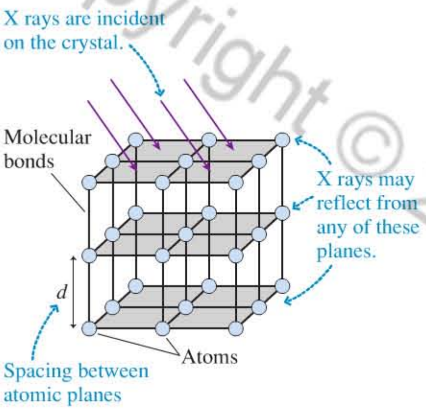


This use of x rays was of tremendous practical importance, but more important to the development of our story is the use of x rays to probe the structure of matter at an atomic scale.

X-Ray Diffraction

At about the same time scientists were first concluding that x rays were very-short-wavelength electromagnetic waves, researchers were also deducing that the size of an atom is ≈ 0.1 nm, and it was suggested that solids might consist of atoms arranged in a regular crystalline *lattice.* In 1912, the German scientist Max von Laue noted that x rays passing through a crystal ought to undergo diffraction from the "three-dimensional grating" of the crystal in much the same way that visible light diffracts from a diffraction grating. Such x-ray diffraction by crystals was soon confirmed experimentally, and measurements confirmed that x rays are indeed electromagnetic waves with wavelengths in the range 0.01 nm to 10 nm—a much shorter wavelength than visible light.

FIGURE 28.3 X rays incident on a simple cubic lattice crystal.



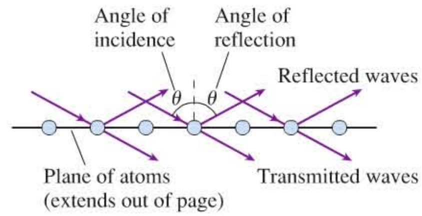
To understand x-ray diffraction, we begin by looking at the arrangement of atoms in a solid. FIGURE 28.3 shows x rays striking a crystal with a *simple cubic lattice.* This is a very straightforward arrangement, with the atoms in planes with spacing d between them.

FIGURE 28.4a shows a side view of the x rays striking the crystal, with the x rays incident at angle θ. Most of the x rays are transmitted through the plane, but a small fraction of the wave is reflected, much like the weak reflection of light from a sheet of glass. The reflected wave obeys the law of reflection—the angle of reflection equals the angle of incidence—and the figure has been drawn accordingly.

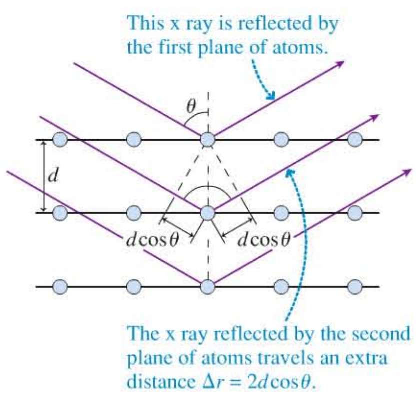
As we saw in Figure 28.3, a solid has not one single plane of atoms but many parallel planes. As x rays pass through a solid, a small fraction of the wave reflects from each of the parallel planes of atoms shown in FIGURE 28.4b. The *net* reflection from the solid is the *superposition* of the waves reflected by each atomic plane. For most angles of incidence, the reflected waves are out of phase and their superposition is very nearly zero. However, as in the thin-film interference we studied in Chapter 17, there are a few specific angles of incidence for which the reflected waves are in phase. For these angles of incidence, the reflected waves interfere constructively to produce a strong reflection. This strong x-ray reflection at a few specific angles of incidence is called **x-ray diffraction.**

FIGURE 28.4 X-ray reflections from parallel atomic planes.

(a) X rays are transmitted and reflected at one plane of atoms.



(b) The reflections from parallel planes interfere.



You can see from Figure 28.4b that the wave reflecting from any particular plane travels an extra distance $\Delta r = 2d\cos\theta$ before combining with the reflection from the plane immediately above it, where d is the spacing between the atomic planes. If Δr is a whole number of wavelengths, then these two waves will be in phase when they recombine. But if the reflections from two neighboring planes are in phase, then *all* the reflections from *all* the planes are in phase and will interfere constructively to produce a strong reflection. Consequently, x rays will reflect from the crystal when the angle of incidence θ_m satisfies the **Bragg condition:**

$$\Delta r = 2d\cos\theta_m = m\lambda \qquad m = 1, 2, 3, \ldots \tag{28.1}$$

The Bragg condition for constructive interference of x rays reflected from a solid

NOTE ► This formula is similar to that for constructive interference for light passed through a grating that we saw in Chapter 17. In both cases, we get constructive interference at only a few well-defined angles. ◄

EXAMPLE 28.1 Analyzing x-ray diffraction

X rays with a wavelength of 0.105 nm are diffracted by a crystal with a simple cubic lattice. Diffraction maxima are observed at angles 31.6° and 55.4° and at no angles between these two. What is the spacing between the atomic planes causing this diffraction?

PREPARE The angles must satisfy the Bragg condition. We don't know the values of m, but we know that they are two consecutive integers. In Equation 28.1 θ_m *decreases* as m increases, so 31.6° corresponds to the larger value of m. We will assume that 55.4° corresponds to m and 31.6° to $m + 1$.

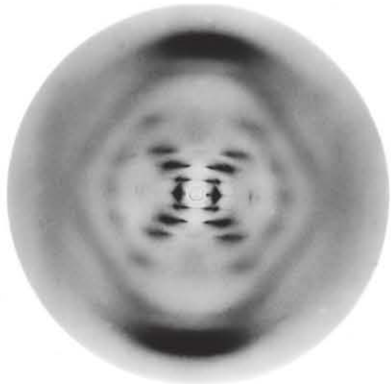
SOLVE The values of d and λ are the same for both diffractions, so we can use the Bragg condition to find

$$\frac{m+1}{m} = \frac{\cos 31.6°}{\cos 55.4°} = 1.50 = \frac{3}{2}$$

Thus 55.4° is the second-order diffraction and 31.6° is the third-order diffraction. With this information we can use the Bragg condition again to find

$$d = \frac{2\lambda}{2\cos\theta_2} = \frac{0.105\ \text{nm}}{\cos 55.4°} = 0.185\ \text{nm}$$

ASSESS We learned above that the size of atoms is ≈ 0.1 nm, so this is a reasonable value for the atomic spacing in a crystal.



X marks the spot BIO Rosalind Franklin obtained this x-ray diffraction pattern for DNA in 1953. The cross of dark bands in the center of the diffraction pattern reveals something about the arrangement of atoms in the DNA molecule—that the molecule has the structure of a helix. This x-ray diffraction image was a key piece of information in the effort to unravel the structure of the DNA molecule.

Example 28.1 shows that an x-ray diffraction pattern reveals details of the crystal that produced it. The structure of the crystal was quite simple, so the example was straightforward. More complex crystals produce correspondingly complex patterns that can help reveal the structure of the crystals that produced them. As investigators developed theories of atoms and atomic structure, x rays were an invaluable tool—as they still are. X-ray diffraction is still widely used to decipher the three-dimensional structure of biological molecules such as proteins.

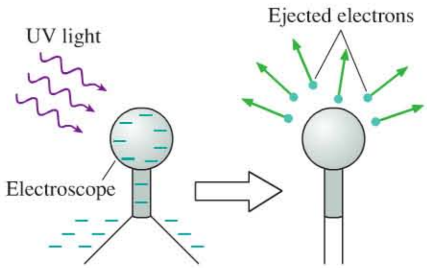
STOP TO THINK 28.1 The first-order diffraction of x rays from two crystals with simple cubic structure is measured. The first-order diffraction from crystal A occurs at an angle of 20°. The first-order diffraction of the same x rays from crystal B occurs at 30°. Which crystal has the larger atomic spacing?

28.2 The Photoelectric Effect

In Chapter 25, we introduced the idea that light can be thought of as *photons,* packets of energy of a particular size. This is an idea that you have likely heard before, but when it was first introduced, it was truly revolutionary. For such an odd idea to find broad acceptance, compelling experimental evidence was needed. This evidence was provided by studies of the *photoelectric effect,* which we will explore in detail in this section to recognize the rationale for and the impact of this startling new concept.

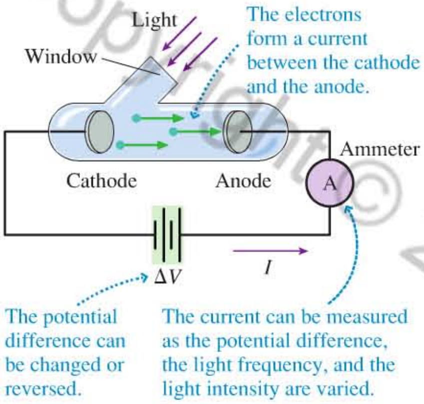
The first hints about the photon nature of light came in the late 1800s with the discovery that a negatively charged electroscope could be discharged by shining ultraviolet light on it. The English physicist J. J. Thomson found that the ultraviolet light was causing the electroscope to emit electrons, as illustrated in **FIGURE 28.5**. The emission of electrons from a substance due to light striking its surface came to be called the **photoelectric effect.** This seemingly minor discovery became a pivotal event that opened the door to the new ideas we discuss in this chapter.

FIGURE 28.5 Ultraviolet light discharges a negatively charged electroscope.



17.3 ActivPhysics Online

FIGURE 28.6 An experimental device to study the photoelectric effect.



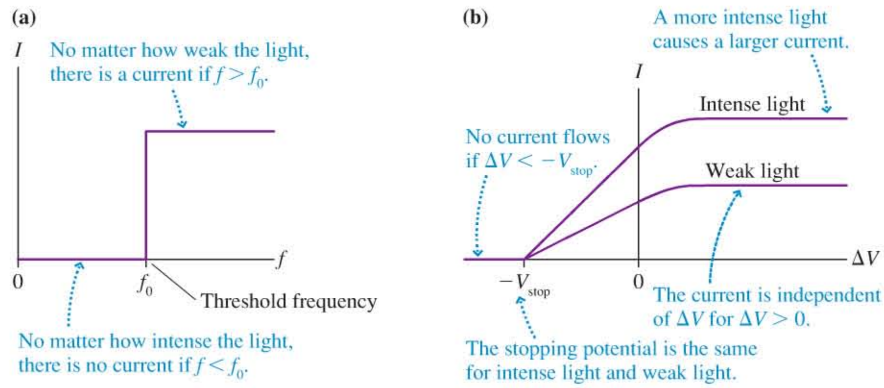
Characteristics of the Photoelectric Effect

FIGURE 28.6 shows an evacuated glass tube with two facing electrodes and a window. When ultraviolet light shines on the cathode, a steady counterclockwise current (clockwise flow of electrons) passes through the ammeter. There are no junctions in this circuit, so the current must be the same all the way around the loop. The current in the space between the cathode and the anode consists of electrons moving freely through space (i.e., not inside a wire) at the *same rate* as the current in the wire. There is no current if the electrodes are in the dark, so electrons don't spontaneously leap off the cathode. Instead, the light causes electrons to be ejected from the cathode at a steady rate.

The battery in Figure 28.6 establishes an adjustable potential difference ΔV between the two electrodes. With it, we can study how the current I varies as the potential difference and the light's wavelength and intensity are changed. Doing so reveals the following characteristics of the photoelectric effect:

1. The current I is directly proportional to the light intensity. If the light intensity is doubled, the current also doubles.
2. The current appears without delay when the light is applied.
3. Electrons are emitted *only* if the light frequency f exceeds a **threshold frequency** f_0. This is shown in the graph of FIGURE 28.7a.
4. The value of the threshold frequency f_0 depends on the type of metal from which the cathode is made.
5. If the potential difference ΔV is positive (anode positive with respect to the cathode), the current changes very little as ΔV is increased. If ΔV is made negative (anode negative with respect to the cathode), by reversing the battery, the current decreases until at some voltage $\Delta V = -V_{stop}$ the current reaches zero. The value of V_{stop} is called the **stopping potential.** This behavior is shown in FIGURE 28.7b.
6. The value of V_{stop} is the same for both weak light and intense light. A more intense light causes a larger current, but in both cases the current ceases when $\Delta V = -V_{stop}$.

FIGURE 28.7 The photoelectric current dependence on the light frequency f and the battery potential difference ΔV.



NOTE ► We're defining V_{stop} to be a *positive* number. The potential difference that stops the electrons is $\Delta V = -V_{stop}$, with an explicit minus sign. ◄

Understanding the Photoelectric Effect

You learned in Chapter 22 that electrons are the charge carriers in a metal and move around freely inside like a sea of negatively charged particles. The electrons are bound inside the metal and do not spontaneously spill out of an electrode at room temperature.

A useful analogy, shown in **FIGURE 28.8**, is the water in a swimming pool. Water molecules do not spontaneously leap out of the pool if the water is calm. To remove a water molecule, you must do *work* on it to lift it upward, against the force of gravity, to the edge of the pool. A minimum energy is needed to extract a water molecule—namely, the energy needed to lift a molecule that is right at the surface. Removing a water molecule that is deeper requires more than the minimum energy.

Similarly, a *minimum* energy is needed to free an electron from a metal. To extract an electron, you need to exert a force on it (i.e., do *work* on it) until its speed is fast enough to escape. The minimum energy E_0 needed to free an electron is called the **work function** of the metal. Some electrons, like deeper water molecules, may require more energy than E_0 to escape, but all will require *at least* E_0. Table 28.1 lists the work functions in eV of some elements. (Recall that the conversion to joules is $1\text{ eV} = 1.60 \times 10^{-19}\text{ J}$.)

Now, let's return to the photoelectric effect experiment of Figure 28.6. When ultraviolet light shines on the cathode, electrons leave with some kinetic energy. An electron with energy E_{elec} inside the metal loses energy ΔE as it escapes, so it emerges as an electron with kinetic energy $K = E_{\text{elec}} - \Delta E$. The work function energy E_0 is the *minimum* energy needed to remove an electron, so the *maximum* possible kinetic energy of an ejected electron is

$$K_{\text{max}} = E_{\text{elec}} - E_0$$

The electrons, after leaving the cathode, move out in all directions, as shown in **FIGURE 28.9**. If the potential difference between the cathode and the anode is $\Delta V = 0$, there will be no electric field between the plates. Some electrons will reach the anode, creating a measurable current, but many do not. The panels in the figure also show:

- If the anode is positive, it attracts *all* of the electrons to the anode. A further increase in ΔV does not cause any more electrons to reach the anode and thus does not cause a further increase in the current I. This is why the curves in Figure 28.7b become horizontal for positive ΔV.
- If the anode is negative, it repels the electrons. However, an electron leaving the cathode with sufficient kinetic energy can still reach the anode, just as a ball hits the ceiling if you toss it upward with sufficient kinetic energy. A slightly negative anode voltage turns back only the slowest electrons. The current steadily decreases as the anode voltage becomes increasingly negative until, as the left side of Figure 28.7b shows, at the stopping potential, *all* electrons are turned back and the current ceases.

We can use conservation of energy to relate the maximum kinetic energy to the stopping potential. When ΔV is negative, as in the bottom panel of Figure 28.9, electrons are "going uphill," converting kinetic energy to potential energy as they slow down. That is, $\Delta U = -e\,\Delta V = -\Delta K$, where we've used $q = -e$ for electrons and ΔK is negative because the electrons are losing kinetic energy. When $\Delta V = -V_{\text{stop}}$, where the current ceases, the very fastest electrons, with K_{max}, are being turned back *just* as they reach the anode. They're converting 100% of their kinetic energy into potential energy, so $\Delta K = -K_{\text{max}}$. Thus $e\,\Delta V_{\text{stop}} = K_{\text{max}}$, or

$$V_{\text{stop}} = \frac{K_{\text{max}}}{e} \qquad (28.2)$$

In other words, **measuring the stopping potential tells us the maximum kinetic energy of the electrons.**

Einstein's Explanation

When light shines on the cathode in a photoelectric effect experiment, why do electrons leave the metal at all? Early investigators suggested explanations based on classical physics. A heated electrode spontaneously emits electrons, so it was natural to suggest that the light falling on the cathode simply heated it, causing it to emit

FIGURE 28.8 A swimming pool analogy of electrons in a metal.

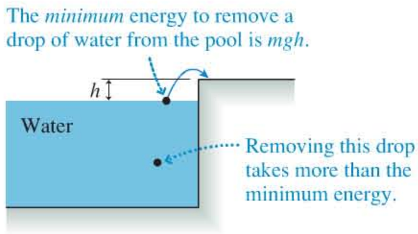
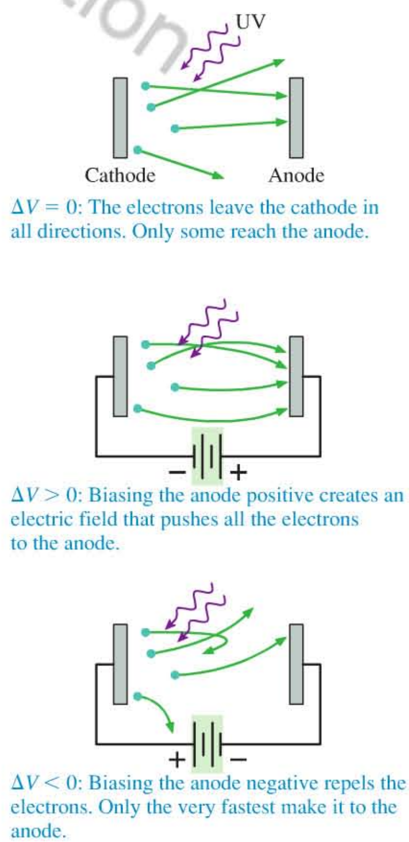


TABLE 28.1 The work functions for some metals

Element	E_0 (eV)
Potassium	2.30
Sodium	2.75
Aluminum	4.28
Tungsten	4.55
Copper	4.65
Iron	4.70
Gold	5.10

FIGURE 28.9 The effect of different voltages between the anode and cathode.



Einstein's "Miracle Year" Albert Einstein was a little-known young man of 26 in 1905. This photograph from the time bears little resemblance to the familiar picture of a white-haired older Einstein. In 1905, within the span of a single year, Einstein published three papers on three different topics, each of which would revolutionize physics. One was his initial paper on the theory of relativity, a subject you learned about in Chapter 27. Though relativity is the subject with which Einstein is most associated in the public mind, this paper received less attention at the time than the other two. A second paper used statistical mechanics to explain a phenomenon called *Brownian motion,* the random motion of small particles suspended in water. It is Einstein's third paper of 1905, on the nature of light, in which we are most interested in this chapter.

electrons. This would explain the photoelectric effect in terms of physics that was well accepted and understood.

But this simple explanation can't be correct. One way to see this is to consider the threshold frequency. If a weak intensity at a frequency just slightly above the threshold can generate a current, then certainly a strong intensity at a frequency just slightly below the threshold should be able to do so—it will heat the metal even more. There is no reason that a slight change in frequency should matter. Yet the experimental evidence shows a sharp frequency threshold, as we've seen.

A new physical theory was needed to fully explain the photoelectric-effect data. The currently accepted solution came in a 1905 paper by Albert Einstein in which he offered an exceedingly simple but amazingly bold idea that explained all of the noted features of the data.

Einstein's paper extended the work of the German physicist Max Planck, who had found that he could explain the form of the spectrum of a glowing, incandescent object that we saw in Chapter 25 only if he assumed that the oscillating atoms inside the heated solid vibrated in a particular way. The energy of an atom vibrating with frequency f had to be one of the specific energies $E = 0, hf, 2hf, 3hf, \ldots,$ where h is a constant. That is, the vibration energies are **quantized.** The constant h, now called **Planck's constant,** is

$$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$$

The first value, with SI units, is the proper one for most calculations, but you will find the second to be useful when energies are expressed in eV.

Einstein was the first to take Planck's idea seriously. Einstein went even further and suggested that **electromagnetic radiation itself is quantized!** That is, light is not really a continuous wave but, instead, arrives in small packets or bundles of energy. Einstein called each packet of energy a **light quantum,** and he postulated that the energy of one light quantum is directly proportional to the frequency of the light. That is, each quantum of light, which is now known as a **photon,** has energy

$$E = hf \qquad (28.3)$$

The energy of a photon, a quantum of light, of frequency f

where h is Planck's constant. Higher-frequency light is composed of higher-energy photons—it is composed of bundles of greater energy. This seemingly simple assumption allowed Einstein to explain all of the properties of the photoelectric effect. As we've seen, it can also explain many other observations about electromagnetic waves of different frequencies.

Not all ultraviolet is created equal BIO
The sharp threshold for ultraviolet damage to tissue means that ultraviolet light sources with small differences in wavelength can have very different biological effects. Tanning beds emit nearly all of their energy at wavelengths greater than 315 nm. This light stimulates cells to produce melanin—resulting in a tan—but produces little short-term cell damage. Germicidal lamps use ultraviolet peaked at 254 nm, which will damage and even kill cells. Exposure to such a source will result in very painful sunburn.

EXAMPLE 28.2 Finding the energy of ultraviolet photons

Ultraviolet light at 290 nm does 250 times as much cellular damage as an equal intensity of ultraviolet at 310 nm; there is a clear threshold for damage at about 300 nm. What is the energy, in eV, of photons with a wavelength of 300 nm?

PREPARE The energy of a photon is related to its frequency by $E = hf$.

SOLVE The frequency at wavelength 300 nm is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \mathrm{m/s}}{300 \times 10^{-9}\ \mathrm{m}} = 1.00 \times 10^{15}\ \mathrm{Hz}$$

We can now use Equation 28.3 to calculate the energy, using the value of h in $\mathrm{eV \cdot s}$:

$$E = hf = (4.14 \times 10^{-15}\ \mathrm{eV \cdot s})(1.00 \times 10^{15}\ \mathrm{Hz}) = 4.14\ \mathrm{eV}$$

ASSESS This number seems reasonable. We saw in Chapter 25 that splitting a bond in a water molecule requires an energy of 4.7 eV. We'd expect photons with energies in this range to be able to damage the complex organic molecules in a cell. As the problem notes, there is a sharp threshold for this damage. For energies larger than about 4.1 eV, photons can disrupt the genetic material of cells. Lower energies have little effect.

Einstein's Postulates and the Photoelectric Effect

The idea that light is quantized is now widely understood and accepted. But at the time of Einstein's paper, it was a truly revolutionary idea. Though we have used the photon model before, it is worthwhile to look at the theoretical underpinnings in more detail. In his 1905 paper, Einstein framed three postulates about light quanta and their interaction with matter:

1. Light of frequency f consists of discrete quanta, each of energy $E = hf$. Each photon travels at the speed of light c.
2. Light quanta are emitted or absorbed on an all-or-nothing basis. A substance can emit 1 or 2 or 3 quanta, but not 1.5. Similarly, an electron in a metal cannot absorb half a quantum but, instead, only an integer number.
3. A light quantum, when absorbed by a metal, delivers its entire energy to *one* electron.

NOTE ► These three postulates—that light comes in chunks, that the chunks cannot be divided, and that the energy of one chunk is delivered to one electron—are crucial for understanding the new ideas that will lead to quantum physics. ◄

Let's look at how Einstein's postulates apply to the photoelectric effect. We now think of the light shining on the metal as a torrent of photons, each of energy hf. Each photon is absorbed by *one* electron, giving that electron an energy $E_{elec} = hf$. This leads us to several interesting conclusions:

1. An electron that has just absorbed a quantum of light energy has $E_{elec} = hf$. **FIGURE 28.10** shows that this electron can escape from the metal if its energy exceeds the work function E_0, or if

$$E_{elec} = hf \geq E_0 \tag{28.4}$$

In other words, there is a *threshold frequency*

$$f_0 = \frac{E_0}{h} \tag{28.5}$$

for the ejection of electrons. If f is less than f_0, even by just a small amount, none of the electrons will have sufficient energy to escape no matter how intense the light. But even very weak light with $f \geq f_0$ will give a few electrons sufficient energy to escape **because each photon delivers all of its energy to one electron.** This threshold behavior is exactly what the data show.

2. A more intense light delivers a larger number of photons to the surface. These eject a larger number of electrons and cause a larger current, exactly as observed.
3. There is a distribution of kinetic energies, because different electrons require different amounts of energy to escape, but the *maximum* kinetic energy is

$$K_{max} = E_{elec} - E_0 = hf - E_0 \tag{28.6}$$

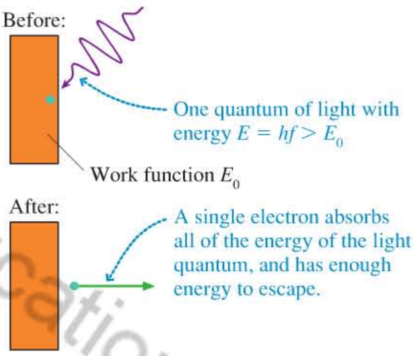
As we noted in Equation 28.2, the stopping potential V_{stop} is a measure of K_{max}. Einstein's theory predicts that the stopping potential is related to the light frequency by

$$V_{stop} = \frac{K_{max}}{e} = \frac{hf - E_0}{e} \tag{28.7}$$

According to Equation 28.7, the stopping potential does *not* depend on the intensity of the light. Both weak light and intense light will have the same stopping potential. This agrees with the data.

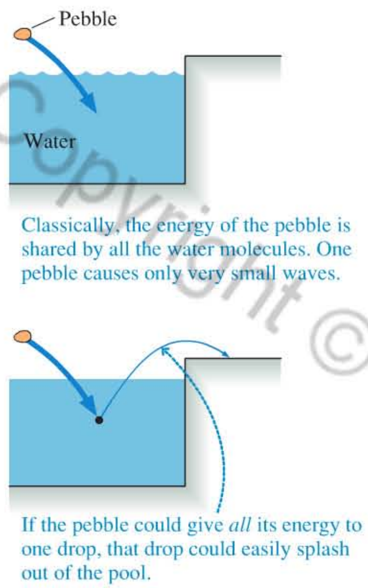
4. If each photon transfers its energy hf to just one electron, that electron immediately has enough energy to escape. The current should begin instantly, with no delay, exactly as experiments had found.

FIGURE 28.10 The ejection of an electron.



Seeing the world in a different light BIO
Many processes that are triggered by light have threshold frequencies. Plants use photosynthesis to convert the energy of light to chemical energy. Photons of visible light have sufficient energy to trigger the necessary molecular transitions, but photons of infrared do not. The leaves of trees absorb most of the visible light that falls on them, so trees will appear quite dark in a normal black-and-white landscape photo. But this photo wasn't made by visible light—the film was exposed by infrared. Infrared photons do not have enough energy to cause photosynthesis, so they are reflected, not absorbed, making the trees appear a ghostly white.

FIGURE 28.11 A pebble transfers energy to the water.



Ultimately, Einstein's postulates are able to explain all of the observed features of the data for the photoelectric effect, though they require us to think of light in a very different way.

Let's use the swimming pool analogy again to help us visualize the photon model. FIGURE 28.11 shows a pebble being thrown into the pool. The pebble increases the energy of the water, but the increase is shared among all the molecules in the pool. The increase in the water's energy is barely enough to make ripples, not nearly enough to splash water out of the pool. But suppose *all* the pebble's energy could go to *one drop* of water that didn't have to share it. That one drop of water would easily have enough energy to leap out of the pool. Einstein's hypothesis that a light quantum transfers all its energy to one electron is equivalent to the pebble transferring all its energy to one drop of water.

Einstein was awarded the Nobel Prize in 1921 not for his theory of relativity, as many would suppose, but for his explanation of the photoelectric effect. Einstein showed convincingly that energy is quantized and that light, even though it exhibits wave-like interference, comes in the particle-like packets of energy we now call photons. This was the first big step in the development of the theory of quantum physics.

EXAMPLE 28.3 **Finding the photoelectric threshold frequency**

What are the threshold frequencies and wavelengths for electron emission from sodium and from aluminum?

PREPARE Table 28.1 gives the work function for sodium as $E_0 = 2.75$ eV and that for aluminum as $E_0 = 4.28$ eV.

SOLVE We can use Equation 28.5, with h in units of eV · s, to calculate

$$f_0 = \frac{E_0}{h} = \begin{cases} 6.64 \times 10^{14}\ \text{Hz} & \text{sodium} \\ 10.34 \times 10^{14}\ \text{Hz} & \text{aluminum} \end{cases}$$

These frequencies are converted to wavelengths with $\lambda = c/f$, giving

$$\lambda = \begin{cases} 452\ \text{nm} & \text{sodium} \\ 290\ \text{nm} & \text{aluminum} \end{cases}$$

ASSESS The photoelectric effect can be observed with sodium for $\lambda < 452$ nm. This includes blue and violet visible light but not red, orange, yellow, or green. Aluminum, with a larger work function, needs ultraviolet wavelengths, $\lambda < 290$ nm.

EXAMPLE 28.4 **Determining the maximum electron speed**

What are the maximum electron speed and the stopping potential if sodium is illuminated with light of 300 nm?

PREPARE The kinetic energy of the emitted electrons—and the potential difference necessary to stop them—depends on the energy of the incoming photons, $E = hf$, and the work function of the metal from which they are emitted, $E_0 = 2.75$ eV.

SOLVE The light frequency is $f = c/\lambda = 1.00 \times 10^{15}$ Hz, so each light quantum has energy $hf = 4.14$ eV. The maximum kinetic energy of an electron is

$$\begin{aligned} K_{max} &= hf - E_0 = 4.14\ \text{eV} - 2.75\ \text{eV} = 1.39\ \text{eV} \\ &= 2.22 \times 10^{-19}\ \text{J} \end{aligned}$$

Because $K = \frac{1}{2}mv^2$, where m is the electron's mass, not the mass of the sodium atom, the maximum speed of an electron leaving the cathode is

$$v_{max} = \sqrt{\frac{2K_{max}}{m}} = 6.99 \times 10^5\ \text{m/s}$$

Note that K_{max} must be in J, the SI unit of energy, in order to calculate a speed in m/s.

Now that we know the maximum kinetic energy of the electrons, we can use Equation 28.7 to calculate the stopping potential:

$$V_{stop} = \frac{K_{max}}{e} = 1.39\ \text{V}$$

An anode voltage of -1.39 V will be just sufficient to stop the fastest electrons and thus reduce the current to zero.

ASSESS The stopping potential has the *same numerical value* as K_{max} expressed in eV, which makes sense. An electron with a kinetic energy of 1.39 eV can go "uphill" against a potential difference of 1.39 V, but no more.

STOP TO THINK 28.2 The work functions of metals A, B, and C are 3.0 eV, 4.0 eV, and 5.0 eV, respectively. Ultraviolet light shines on all three metals, causing electrons to be emitted. Rank in order, from largest to smallest, the stopping voltages for A, B, and C.

28.3 Photons

We've now seen compelling evidence for the photon nature of light, but this leaves an important question: Just what *are* photons? To begin our explanation, let's return to the experiment that showed most dramatically the wave nature of light—Young's double-slit interference experiment. We will make a change, though: We will dramatically lower the light intensity by inserting filters between the light source and the slits. The fringes will be too dim to see with the naked eye, so we will use a detector that can build up an image over time. (This is the same sort of detector we imagined using for the extremely low-light photograph in Chapter 25.)

FIGURE 28.12 shows the outcome of such an experiment at four different times. At early times, very little light has reached the detector, and it does not show bands at all. Instead, it shows dots; the detector is registering the arrival of particle-like objects.

As the detector builds up the image for a longer time, we see that the positions of the dots are not entirely random. They are grouped into bands at *exactly* the positions where we expect to see bright constructive-interference fringes. As the detector continues to gather light, the light and dark fringes become quite distinct. After a long time, the individual dots overlap and the image looks exactly like those we saw in Chapter 17.

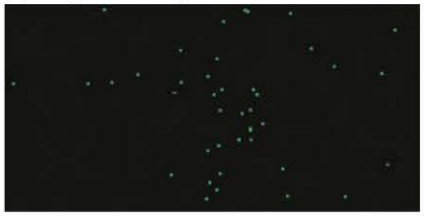
The dots of light on the screen, which we'll attribute to the arrival of individual photons, are particle-like, but the overall picture clearly does not mesh with the classical idea of a particle. A classical particle, when faced with Young's double-slit apparatus, would go through one slit or the other. If light consisted of classical particles, we would see two bright areas on the screen, corresponding to light that has gone through one or the other slit. Instead, we see particle-like dots forming wave-like interference fringes.

This experiment was performed with a light level so low that only one photon at a time passed through the apparatus. If particle-like photons arrive at the detector in a banded pattern as a consequence of wave-like interference, as Figure 28.12 shows, but if only one photon at a time is passing through the experiment, what is it interfering with? The only possible answer is that the photon is somehow interfering *with itself.* Nothing else is present. But if each photon interferes with itself, rather than with other photons, then each photon, despite the fact that it is a particle-like object, must somehow go through *both* slits! This is something only a wave could do.

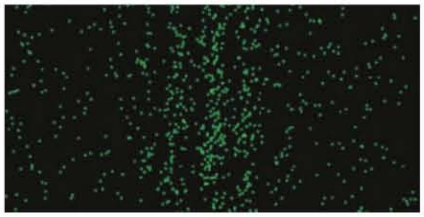
This all seems pretty crazy. But crazy or not, this is the way light behaves in real experiments. **Sometimes it exhibits particle-like behavior and sometimes it exhibits wave-like behavior.** The thing we call *light* is stranger and more complex than it first appeared, and there is no way to reconcile these seemingly contradictory behaviors. We have to accept nature as it is, rather than hoping that nature will conform to our expectations. Furthermore, as we will see, this half-wave/half-particle behavior is not restricted to light.

FIGURE 28.12 A double-slit experiment performed with light of very low intensity.

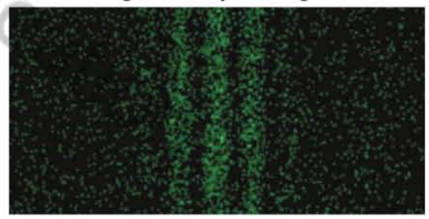
(a) Image after a very short time



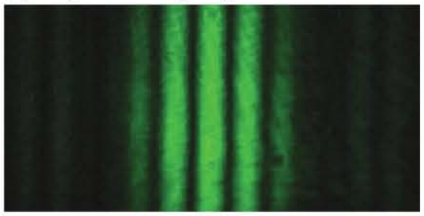
(b) Image after a slightly longer time



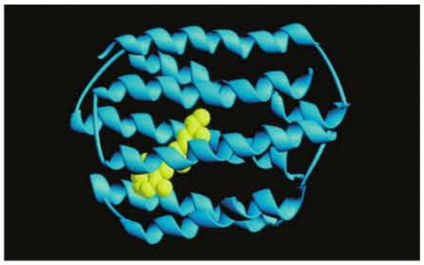
(c) Continuing to build up the image



(d) Image after a very long time

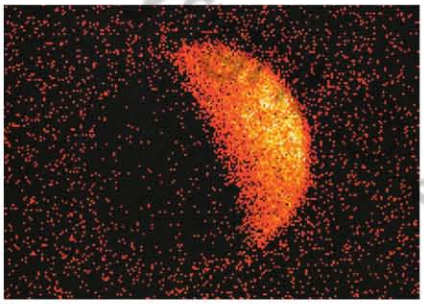


▶ **Seeing photons** The basis of vision is the detection of single photons by specially adapted molecules in the rod and cone cells of the eye. This image shows a molecule of *rhodopsin* (blue) with a molecule called *retinal* (yellow) nested inside. A single photon of the right energy triggers a transition of the retinal molecule, changing its shape so that it no longer fits inside the rhodopsin "cage". The rhodopsin then changes shape to eject the retinal, and this motion leads to an electrical signal in a nerve fiber. Slightly different versions of these molecules are "tuned" to different photon energies and thus different colors of light.



The Photon Rate

The photon nature of light isn't apparent in most cases. Most light sources with which you are familiar emit such vast numbers of photons that you are aware of only their wave-like superposition, just as you notice only the roar of a heavy rain on your roof and not the individual raindrops. Only at extremely low intensities does the light begin to appear as a stream of individual photons, like the random patter of raindrops when it is barely sprinkling.



High-energy moonlight The sun emits vast quantities of visible-light photons. It also emits high-energy photons well beyond the range of the visible spectrum, but in much smaller numbers. This image of the moon was made with an orbiting telescope that detects x rays. Each dot in the image shows where one x-ray photon hit a detector in an orbiting telescope. The sunlit half of the moon "glows" with the reflection of x rays from the sun. The random dots seen everywhere are individual x-ray photons from our Milky Way galaxy, the *x-ray radiation background* of the cosmos.

EXAMPLE 28.5 How many photons per second does a laser emit?

The 1.0 mW light beam from a laser pointer ($\lambda = 670$ nm) shines on a screen. How many photons strike the screen each second?

PREPARE The power of the beam is 1.0 mW, or 1.0×10^{-3} J/s. Each second, 1.0×10^{-3} J of energy reaches the screen. It arrives as individual photons of energy given by Equation 28.3.

SOLVE The frequency of the photons is $f = c/\lambda = 4.5 \times 10^{14}$ Hz, so the energy of an individual photon is $E = hf = (6.6 \times 10^{-34}\ \text{J}\cdot\text{s})(4.5 \times 10^{14}\ \text{Hz}) = 3.0 \times 10^{-19}$ J. The number of photons reaching the screen each second is the total energy reaching the screen each second divided by the energy of an individual photon:

$$\frac{1.0 \times 10^{-3}\ \text{J/s}}{3.0 \times 10^{-19}\ \text{J/photon}} = 3.3 \times 10^{15}\ \text{photons per second}$$

ASSESS Each photon carries a small amount of energy, so there must be a huge number of photons per second to produce even this modest power.

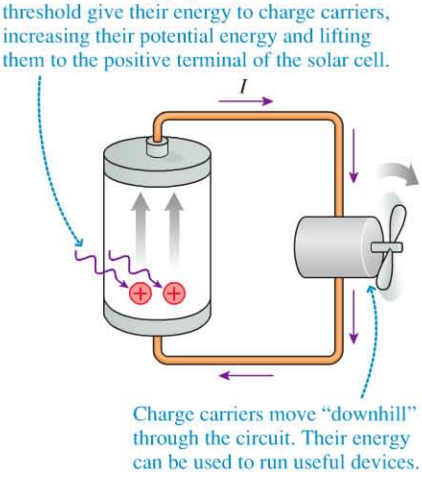
CONCEPTUAL EXAMPLE 28.6 Comparing photon rates

A red laser pointer and a green laser pointer have the same power. Which one emits a larger number of photons per second?

REASON Red light has a longer wavelength and thus a lower frequency than green light, so the energy of a photon of red light is less than the energy of a photon of green light. The two pointers emit the same amount of light energy per second. Because the red laser emits light in smaller "chunks" of energy, it must emit more chunks per second to have the same power. The red laser emits more photons each second.

ASSESS This result can seem counterintuitive if you haven't thought hard about the implications of the photon model. Light of different wavelengths is made of photons of different energies, so these two lasers with different wavelengths—though they have the same power—must emit photons at different rates.

FIGURE 28.13 The operation of a solar cell.



Detecting Photons

Early light detectors, which used the photoelectric effect directly, consisted of a polished metal plate in a vacuum tube. When light fell on the plate, an electron current was generated that could trigger an action, such as sounding an alarm, or could provide a measurement of the light intensity.

Modern devices work on similar principles. In a *solar cell*, incoming photons give their energy to charge carriers, lifting them into higher-energy states. Recall the charge escalator model of a battery in Chapter 22. The solar cell works much like a battery, but the energy to lift charges to a higher potential comes from photons, not chemical reactions, as shown in FIGURE 28.13. The photon energy must exceed some minimum value to cause this transition, so solar cells have a threshold frequency, just like a device that uses the photoelectric effect directly. For a silicon-based solar cell, the most common type, the energy threshold is about 1.1 eV, corresponding to a wavelength of about 1200 nm, just beyond the range of the visible light spectrum, in the infrared.

EXAMPLE 28.7 Finding the current from a solar cell

1.0 W of monochromatic light of wavelength 550 nm illuminates a silicon solar cell, driving a current in a circuit. What is the maximum possible current this light could produce?

PREPARE The wavelength is shorter than the 1200 nm threshold wavelength noted for a silicon solar cell, so the photons will have sufficient energy to cause charge carriers to flow. Each photon of the incident light will give its energy to a single charge carrier. The maximum number of charge carriers that can possibly flow in each second is thus equal to the number of photons that arrive each second.

SOLVE The power of the light is $P = 1.0\text{ W} = 1.0\text{ J/s}$. The frequency of the light is $f = c/\lambda = 5.5 \times 10^{14}\text{ Hz}$, so the energy of individual photons is $E = hf = 3.6 \times 10^{-19}\text{ J}$. The number of photons arriving per second is $(1.0\text{ J/s})/(3.6 \times 10^{-19}\text{ J/photon}) = 2.8 \times 10^{18}$. Each photon can set at most one charge carrier into motion, so the maximum current is 2.8×10^{18} electrons/s. The current in amps—coulombs per second—is the electron flow rate multiplied by the charge per electron:

$$I_{max} = (2.8 \times 10^{18}\text{ electrons/s})(1.6 \times 10^{-19}\text{ C})$$
$$= 0.45\text{ C/s} = 0.45\text{ A}$$

ASSESS The key concept underlying the solution is that one photon gives its energy to a single charge carrier. We've calculated the current if all photons give their energy to charge carriers. The current in a real solar cell will be less than this because some photons will be reflected or otherwise "lost" and will not transfer their energy to charge carriers.

The *charge-coupled device* (CCD) or *complementary metal oxide semiconductor* (CMOS) detector in a digital camera consists of millions of *pixels,* each a microscopic silicon-based photodetector. Each photon hitting a pixel (if its frequency exceeds the threshold frequency) liberates one electron. These electrons are stored inside the pixel, and the total accumulated charge is directly proportional to the light intensity—the number of photons—hitting the pixel. After the exposure, the charge in each pixel is read and the value stored in memory; then the pixel is reset to be ready for the next picture.

STOP TO THINK 28.3 The intensity of a beam of light is increased but the light's frequency is unchanged. Which one (or perhaps more than one) of the following is true?

A. The photons travel faster.
B. Each photon has more energy.
C. There are more photons per second.

TRY IT YOURSELF

Photographing photons
Photodetectors based on silicon can be triggered by photons with energy as low as 1.1. eV, corresponding to a wavelength in the infrared. The light-sensing chip in your digital camera can detect the infrared signal given off by a remote control. Press a button on your remote control, aim it at your digital camera, and snap a picture. The picture will clearly show the infrared emitted by the remote, though this signal is invisible to your eye. (Some cameras have infrared filters that may block most or nearly all of the signal.)

28.4 Matter Waves

Prince Louis-Victor de Broglie was a French graduate student in 1924. It had been 19 years since Einstein had shaken the world of physics by introducing photons and thus blurring the distinction between a particle and a wave. As de Broglie thought about these issues, it seemed that nature should have some kind of symmetry. If light waves could have a particle-like nature, why shouldn't material particles have some kind of wave-like nature? In other words, could **matter waves** exist?

With no experimental evidence to go on, de Broglie reasoned by analogy with Einstein's equation $E = hf$ for the photon and with some of the ideas of his theory of relativity. De Broglie determined that *if* a material particle of momentum $p = mv$ has a wave-like nature, its wavelength must be given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{28.8}$$

De Broglie wavelength for a moving particle

where h is Planck's constant. This wavelength is called the **de Broglie wavelength.**

EXAMPLE 28.8 Calculating the de Broglie wavelength of an electron

What is the de Broglie wavelength of an electron with a kinetic energy of 1.0 eV?

SOLVE An electron with kinetic energy $K = \frac{1}{2}mv^2 = 1.0\text{ eV} = 1.6 \times 10^{-19}\text{ J}$ has speed

$$v = \sqrt{\frac{2K}{m}} = 5.9 \times 10^5 \text{ m/s}$$

Although fast by macroscopic standards, the electron gains this speed by accelerating through a potential difference of a mere 1 V. The de Broglie wavelength is

$$\lambda = \frac{h}{mv} = 1.2 \times 10^{-9}\text{ m} = 1.2\text{ nm}$$

ASSESS The electron's wavelength is small, but it is larger than the wavelengths of x rays and larger than the approximately 0.1 nm spacing of atoms in a crystal. We can observe x-ray diffraction, so if an electron has a wave nature, it should be easily observable.

FIGURE 28.14 A double-slit interference pattern created with electrons.

What would it mean for matter—an electron or a proton or a baseball—to have a wavelength? Would it obey the principle of superposition? Would it exhibit diffraction and interference? Surprisingly, **matter exhibits all of the properties that we associate with waves.** For example, FIGURE 28.14 shows the intensity pattern recorded after 50 keV electrons passed through two narrow slits separated by 1.0 μm. The pattern is clearly a double-slit interference pattern, and the spacing of the fringes is exactly as the theory of Chapter 17 would predict for a wavelength given by de Broglie's formula. **The electrons are behaving like waves!**

But if matter waves are real, why don't we see baseballs and other macroscopic objects exhibiting wave-like behavior? The key is the wavelength. We found in Chapter 17 that diffraction, interference, and other wave-like phenomena are observed when the wavelength is comparable to or larger than the size of an opening a wave must pass through. As Example 28.8 just showed, a typical electron wavelength is somewhat larger than the spacing between atoms in a crystal, so we expect to see wave-like behavior as electrons pass through matter or through microscopic slits. But the de Broglie wavelength is inversely proportional to an object's mass, so the wavelengths of macroscopic objects are millions or billions of times smaller than the wavelengths of electrons—vastly smaller than the size of any openings these objects might pass through. The wave nature of macroscopic objects is unimportant and undetectable because their wavelengths are so incredibly small, as the following example shows.

EXAMPLE 28.9 Calculating the de Broglie wavelength of a smoke particle

One of the smallest macroscopic particles we could imagine using for an experiment would be a very small smoke or soot particle. These are $\approx 1\ \mu$m in diameter, too small to see with the naked eye and just barely at the limits of resolution of a microscope. A particle this size has mass $m \approx 10^{-18}$ kg. Estimate the de Broglie wavelength for a 1-μm-diameter particle moving at the very slow speed of 1 mm/s.

SOLVE The particle's momentum is $p = mv \approx 10^{-21}\text{ kg}\cdot\text{m/s}$. The de Broglie wavelength of a particle with this momentum is

$$\lambda = \frac{h}{p} \approx 7 \times 10^{-13}\text{ m}$$

ASSESS The wavelength is much, much smaller than the particle itself—much smaller than an individual atom! We don't expect to see this particle exhibiting wave-like behavior.

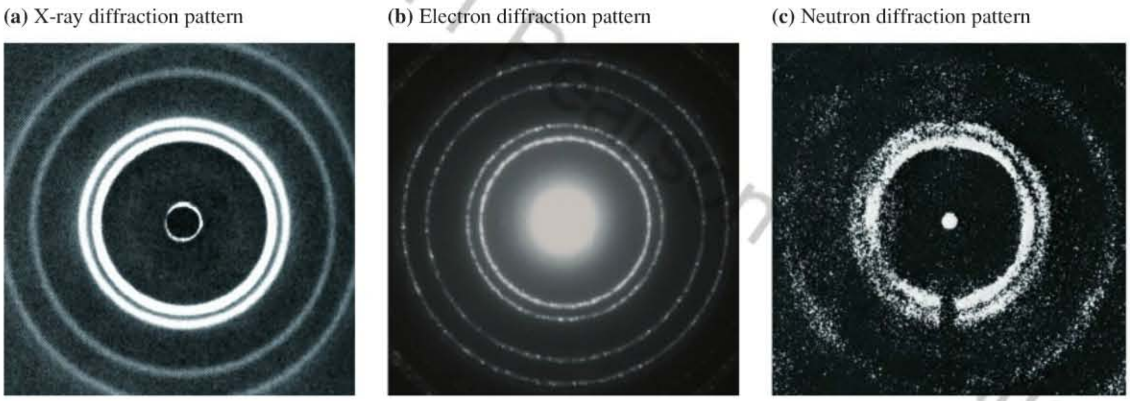
The preceding example shows that a very small particle moving at a very slow speed has a wavelength that is too small to be of consequence. For larger objects moving at higher speeds, the wavelength is even smaller. A pitched baseball will have a wavelength of about 10^{-34} m, so a batter cannot use the wave nature of the ball as an excuse for not getting a hit. With such unimaginably small wavelengths, it is little wonder that we do not see macroscopic objects exhibiting wave-like behavior.

The Interference and Diffraction of Matter

Though de Broglie made his hypothesis in the absence of experimental data, experimental evidence was soon forthcoming. **FIGURES 28.15a** and **b** show diffraction patterns produced by x rays and electrons passing through an aluminum-foil target. The primary observation to make from Figure 28.15 is that **electrons diffract and interfere exactly like x rays.**

ActivPhysics 17.5

FIGURE 28.15 The diffraction patterns produced by x rays, electrons, and neutrons passing through an aluminum-foil target.



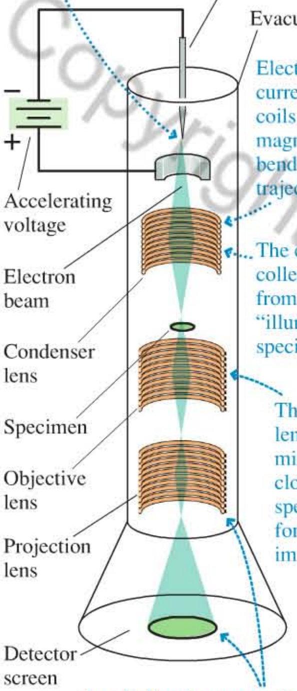
Later experiments demonstrated that de Broglie's hypothesis applies to other material particles as well. Neutrons have a much larger mass than electrons, which tends to decrease their de Broglie wavelength, but it is possible to generate very slow neutrons. The much smaller speed compensates for the heavier mass, so neutron wavelengths can be made comparable to electron wavelengths. **FIGURE 28.15c** shows a neutron diffraction pattern. It is similar to the x-ray and electron diffraction patterns, although of lower quality because neutrons are harder to detect. A neutron, too, is a matter wave. In recent years it has become possible to observe the interference and diffraction of atoms and even large molecules!

The Electron Microscope

Ray optics is based on the idea that light travels in straight lines—light rays—except when it crosses the boundary between two transparent media. Refraction at the boundary bends the rays, and we can use this idea to design lenses that bring parallel rays to a focus at a single point. If light really followed the ray model, carefully designed lenses would allow us to build a microscope with unlimited resolution and magnification. However, real microscopes are limited by the fact that light has wave-like properties. We learned in Chapter 19 that diffraction, a wave behavior, limits the resolving power of a microscope to, at best, about half the wavelength. For visible light, the smallest feature that can be resolved, even with perfect lenses, is about 200 or 250 nm. But the picture of the retina at the start of the chapter can show details much finer than this because it wasn't made with light—it was made with a beam of electrons.

The electron microscope, invented in the 1930s, works much like a light microscope. In the absence of electric or magnetic fields, electrons travel through a vacuum

FIGURE 28.16 The electron microscope.



in straight lines much like light rays. Electron trajectories can be bent with electric or magnetic fields. A coil of wire carrying a current can produce a magnetic field that bends parallel electron trajectories so that they all cross at a single point; we call this an *electron lens*. An electron lens focuses electrons in the same way a glass lens bends and focuses light rays.

FIGURE 28.16 shows how a *transmission electron microscope (TEM)* works. This is purely classical physics; the electrons experience electric and magnetic forces, and they follow trajectories given by Newton's second law. Our ability to control electron trajectories allows electron microscopes to have magnifications far exceeding those of light microscopes. But just as in a light microscope, the resolution is ultimately limited by wave effects. Electrons are not classical point particles; they have wave-like properties and a de Broglie wavelength $\lambda = h/p$.

CONCEPTUAL EXAMPLE 28.10 Which wavelength is shorter?

An electron is accelerated through a potential difference ΔV. A second electron is accelerated through a potential difference that is twice as large. Which electron has a shorter de Broglie wavelength?

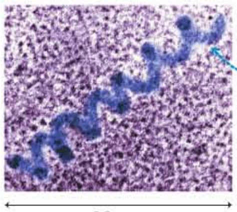
REASON The wavelength is inversely proportional to the speed. The electron that is accelerated through the larger potential difference will be moving faster and so will have a shorter de Broglie wavelength.

ASSESS Creating an electron micrograph requires high-speed electrons. Higher accelerating voltages mean higher speeds and shorter wavelengths, which would—in principle—allow for better resolution.

The reasoning used in Chapter 19 to determine maximum resolution applies equally well to electrons. Thus the resolving power is, at best, about half the electrons' de Broglie wavelength. For a 100 kV accelerating voltage, which is fairly typical, the de Broglie wavelength is $\lambda \approx 0.004$ nm (the electrons are moving fast enough that the momentum has to be calculated using relativity) and thus the theoretical resolving power of an electron microscope is about 0.002 nm.

In practice, the resolving powers of the best electron microscopes are limited by imperfections in the electron lenses to about 0.2 nm, not quite sufficient to resolve individual atoms with diameters of about 0.1 nm. This resolving power is about 1000 times smaller than can be achieved with light microscopes, as noted in FIGURE 28.17. Good light microscopes function at their theoretical limit, but there's still room to improve electron microscopes if a clever scientist or engineer can make a better electron lens.

FIGURE 28.17 TEM image of a pigment molecule from a crustacean shell.



The great resolving power of the microscope allows the imaging of incredibly fine detail, in this case the actual structure of a molecule.

The total image size is 1/10 the size of the smallest feature that can be resolved by a light microscope. A light microscope could not detect this individual molecule, let alone show its structure.

STOP TO THINK 28.4 A beam of electrons, a beam of protons, and a beam of oxygen atoms each pass at the same speed through a 1-μm-wide slit. Which will produce the widest central maximum on a detector behind the slit?

A. The beam of electrons.
B. The beam of protons.
C. The beam of oxygen atoms.
D. All three patterns will be the same.
E. None of the beams will produce a diffraction pattern.

28.5 Energy Is Quantized

De Broglie hypothesized that material particles have wave-like properties, and you've now seen experimental evidence that this must be true. Not only is this bizarre, the implications are profound.

You learned in Chapter 16 that the waves on a string fixed at both ends form standing waves. Wave reflections from both ends create waves traveling in both directions, and the superposition of two oppositely directed waves produces a standing wave. Could we do something like this with particles? Is there such a thing as a "standing matter wave"? In fact, you are probably already familiar with standing matter waves—the atomic electron orbitals that you learned about in chemistry.

We'll have more to say about these orbitals in Chapter 29. For now, we'll start our discussion of standing matter waves with a simpler physical system called a "particle in a box." For simplicity, we'll consider one-dimensional motion, a particle that moves back and forth along the x-axis. The "box" is defined by two fixed ends, and the particle bounces back and forth between these boundaries as in **FIGURE 28.18**. We'll assume that collisions with the ends of the box are perfectly elastic, with no loss of kinetic energy.

Figure 28.18a shows a classical particle, such as a ball or a dust particle, in the box. This particle simply bounces back and forth at constant speed. But if particles have wave-like properties, perhaps we should consider a *wave* reflecting back and forth from the ends of the box. The reflections will create the standing wave shown in Figure 28.18b. This standing wave is analogous to the standing wave on a string that is tied at both ends.

What can we say about the properties of this standing matter wave? We can use what we know about matter waves and standing waves to make some deductions.

For waves on a string, we saw that there were only certain possible modes. The same will be true for the particle in a box; only certain states are possible. In Chapter 16, we found that the wavelength of a standing wave is related to the length L of the string by

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots \tag{28.9}$$

The wavelength of the particle in a box will follow the same formula, but the wave describing the particle must also satisfy the de Broglie condition $\lambda = h/p$. Equating these two expressions for the wavelength gives

$$\frac{h}{p} = \frac{2L}{n} \tag{28.10}$$

Solving Equation 28.10 for the particle's momentum p, we find

$$p_n = n\left(\frac{h}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots \tag{28.11}$$

This is a remarkable result; it is telling us that the momentum of the particle can have only certain values, the ones given by the equation. Other values simply aren't possible. The energy of the particle is related to its momentum by

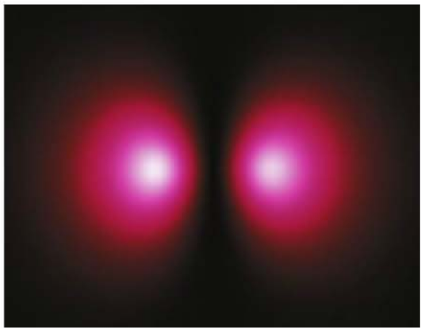
$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{28.12}$$

If we use Equation 28.11 for the momentum, we find that the particle's energy is also restricted to a specific set of values:

$$E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, 4, \ldots \tag{28.13}$$

Allowed energies of a particle in a box

This conclusion is one of the most profound discoveries of physics. Because of the wave nature of matter, **a confined particle can have only certain energies.** This result—that a confined particle can have only discrete values of energy—is called the **quantization** of energy. More informally, we say that energy is *quantized*. The

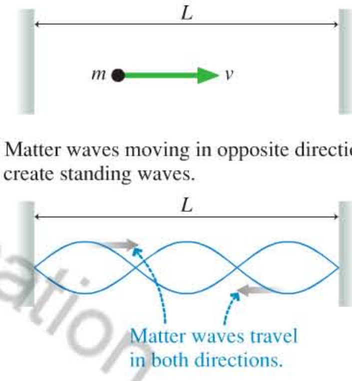


This computer simulation shows the p orbital of an atom. This orbital is an electron standing wave with a clear node at the center.

FIGURE 28.18 A particle of mass m confined in a box of length L.

(a) A classical particle of mass m bounces back and forth between two boundaries.

(b) Matter waves moving in opposite directions create standing waves.



number n is called the **quantum number,** and each value of n characterizes one **energy level** of the particle in the box.

The lowest possible energy the particle in the box can have is

$$E_1 = \frac{h^2}{8mL^2} \tag{28.14}$$

We saw that, for a standing wave, the only possible frequencies were multiples of a lowest, fundamental frequency, $f_n = nf_1$. Similarly, for the particle in a box, the only possible energies are multiples of the lowest possible energy given by Equation 28.14; the only possible energies are

$$E_n = n^2 E_1 \tag{28.15}$$

This quantization is in stark contrast to the behavior of classical objects. It would be as if a baseball pitcher could throw a baseball only at 10 m/s, or 20 m/s, or 30 m/s, and so on, but at no speed in between. Baseball speeds aren't quantized, but the energy levels of a confined electron are—a result that has far-reaching implications.

EXAMPLE 28.11 Finding the allowed energies of a confined electron

An electron is confined to a region of space of length 0.19 nm—comparable in size to an atom. What are the first three allowed energies of the electron?

PREPARE We'll model this system as a particle in a box, with a box of length 0.19 m. The possible energies are given by Equation 28.13.

SOLVE The mass of an electron is $m = 9.11 \times 10^{-31}$ kg. Thus the first allowed energy is

$$E_1 = \frac{h^2}{8mL^2} = 1.6 \times 10^{-18}\ \text{J} = 10\ \text{eV}$$

This is the lowest allowed energy. The next two allowed energies are

$$E_2 = 2^2 E_1 = 40\ \text{eV}$$
$$E_3 = 3^2 E_1 = 90\ \text{eV}$$

ASSESS These energies are significant; E_1 is larger than the work function of any metal in Table 28.1. Confining an electron to a region the size of an atom limits its energy to states separated by significant differences in energy. Clearly, our treatment of electrons in atoms must be a quantum treatment.

The energies allowed by Equation 28.13 are inversely proportional to both m and L^2. Both m and L have to be exceedingly small before energy quantization has any significance. Classical physics still works for baseballs! It is only at the atomic scale that quantization effects become important, as the following example shows.

EXAMPLE 28.12 Determining the minimum energy of a smoke particle

What is the first allowed energy of the very small 1-μm-diameter particle of Example 28.9 if it is confined to a very small box 10 μm in length?

PREPARE As in Example 28.11, we'll model the system as a particle in a box, with the energy levels given by Equation 28.13.

SOLVE The particle's mass is given in Example 28.9 as $m \approx 10^{-18}$ kg; the length of the box is given in the problem statement as $L = 1.0 \times 10^{-6}$ m. The first allowed energy, $n = 1$, is

$$E_1 = \frac{h^2}{8mL^2} \approx 5 \times 10^{-40}\ \text{J}$$

ASSESS This is an unimaginably small amount of energy. By comparison, the kinetic energy of a 1-μm-diameter particle moving at a barely perceptible speed of 1 mm/s is $K = 5 \times 10^{-25}$ J, a factor of 10^{15} larger. There is no way we could ever observe or measure discrete energies this small, so it is not surprising that we are unaware of energy quantization for macroscopic objects.

An atom is certainly more complicated than a simple one-dimensional box, but an electron is "confined" within an atom. Thus the electron orbits must, in some sense, be standing waves, and **the energy of the electrons in an atom must be quantized.** This has important implications for the physics of atomic systems, as we'll see in the next section.

28.6 Energy Levels and Quantum Jumps

Einstein and de Broglie introduced revolutionary new ideas—a blurring of the distinction between waves and particles, and the quantization of energy—but the first to develop a full-blown theory of quantum physics, in 1925, was the Austrian physicist Erwin Schrödinger. Schrödinger's theory is now called *quantum mechanics*. It describes how to calculate the quantized energy levels of systems from the particle in a box to electrons in atoms. Quantum mechanics also describes another important piece of the puzzle: How does a quantized system gain or lose energy?

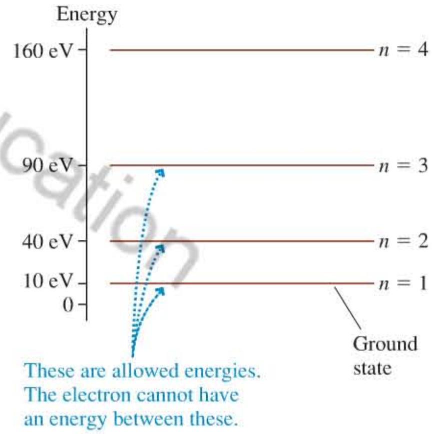
Erwin Schrödinger, one of the early architects of quantum mechanics.

Energy-Level Diagrams

We used the idea of a standing de Broglie wave to find the allowed energies of a particle in a one-dimensional box. The full theory of quantum mechanics is needed to predict the allowed energy of more realistic physical systems, such as atoms or semiconductors, but the final results share a key property: The energy is quantized. Only certain energies are allowed while all other energies are forbidden.

An **energy-level diagram** is a useful visual representation of the quantized energies. As an example, **FIGURE 28.19** is the energy-level diagram for an electron in a 0.19-nm-long box. We computed these energies in Example 28.11. An energy-level diagram is less a graph than it is a picture. The vertical axis represents energy, but the horizontal axis is not a scale. Think of this as a ladder in which the energies are the rungs of the ladder. The lowest rung, with energy E_1, is called the **ground state.** Higher rungs, called **excited states,** are labeled by their quantum numbers, $n = 2, 3, 4, \ldots$. Whether it is a particle in a box, an atom, or the nucleus of an atom, quantum physics requires the system to be on one of the rungs of the ladder.

FIGURE 28.19 The energy-level diagram of an electron in a 0.19-nm-long box.

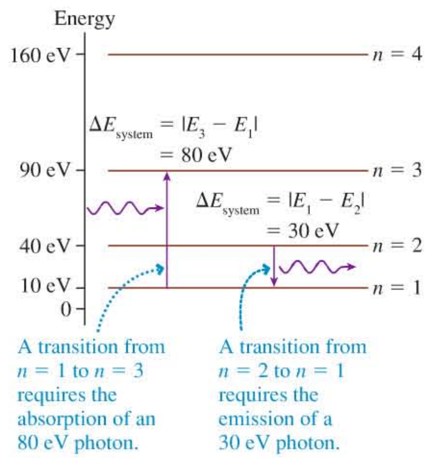


If a quantum system changes from one state to another, there is a change in energy. One thing that has not changed in quantum physics is the conservation of energy—energy is still conserved in the quantum world. If a system drops from a higher energy level to a lower, the excess energy ΔE_{system} must go somewhere. In the systems we will consider, this energy generally ends up in the form of an emitted photon. A quantum system in energy level E_i that "jumps down" to energy level E_f loses an energy $\Delta E_{\text{system}} = |E_f - E_i|$. This jump must correspond to the emission of a photon of frequency

$$f_{\text{photon}} = \frac{\Delta E_{\text{system}}}{h} \tag{28.16}$$

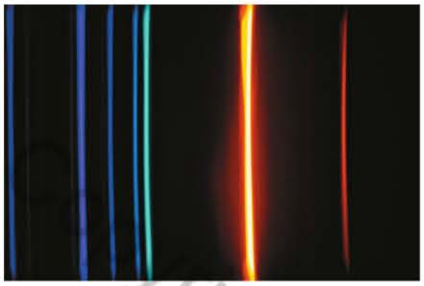
Conversely, if the system absorbs a photon, it can "jump up" to a higher energy level. In this case, the frequency of the absorbed photon must follow Equation 28.16 as well. Such jumps are called **transitions** or **quantum jumps.** **FIGURE 28.20** shows two transitions for the particle in a box system of Figure 28.19.

FIGURE 28.20 A particle can jump between energy levels by emitting or absorbing a photon.



Notice that Equation 28.16 links Schrödinger's quantum theory to Einstein's earlier idea about the quantization of light energy. According to Einstein, a photon of frequency f has energy $E_{\text{photon}} = hf$. If a particle jumps from an initial state with energy E_i to a final state with *lower* energy E_f, energy will be conserved if the system emits a photon with $E_{\text{photon}} = \Delta E_{\text{system}}$. The photon must have exactly the frequency given by Equation 28.16 if it is to carry away exactly the right amount of energy. As we'll see in the next chapter, these photons form the *emission spectrum* of the quantum system.

Similarly, a particle can conserve energy while jumping to a higher-energy state, for which additional energy is needed, by absorbing a photon of frequency $f_{\text{photon}} = \Delta E_{\text{system}}/h$. The photon will not be absorbed unless it has exactly this frequency. The frequencies absorbed in these upward transitions form the system's *absorption spectrum.*



The spectrum of helium gas shows a discrete set of wavelengths, corresponding to the energies of possible transitions.

Let's summarize what quantum physics has to say about the properties of atomic-level systems:

1. **The energies are quantized.** Only certain energies are allowed, all others are forbidden. This is a consequence of the wave-like properties of matter.
2. **The ground state is stable.** Quantum systems seek the lowest possible energy state. A particle in an excited state, if left alone, will jump to lower and lower energy states until it reaches the ground state. Once in its ground state, there are no states of any lower energy to which a particle can jump.
3. **Quantum systems emit and absorb a *discrete spectrum* of light.** Only those photons whose frequencies match the energy *intervals* between the allowed energy levels can be emitted or absorbed. Photons of other frequencies cannot be emitted or absorbed without violating energy conservation.

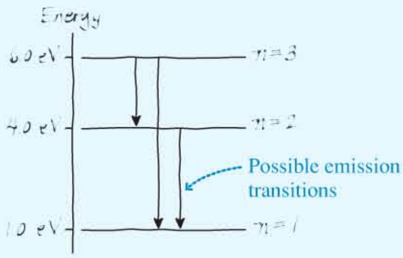
We'll use these ideas in the next two chapters to understand the properties of atoms and nuclei.

EXAMPLE 28.13 Determining an emission spectrum from quantum states

An electron in a quantum system has allowed energies $E_1 = 1.0$ eV, $E_2 = 4.0$ eV, and $E_3 = 6.0$ eV. What wavelengths are observed in the emission spectrum of this system?

PREPARE FIGURE 28.21 shows the energy-level diagram for this system. Photons are emitted when the system undergoes a quantum jump from a higher energy level to a lower energy level. There are three possible transitions.

FIGURE 28.21 The system's energy-level diagram and quantum jumps.



SOLVE This system will emit photons on the $3 \rightarrow 1$, $2 \rightarrow 1$, and $3 \rightarrow 2$ transitions, with $\Delta E_{3\rightarrow 1} = 5.0$ eV, $\Delta E_{2\rightarrow 1} = 3.0$ eV, and $\Delta E_{3\rightarrow 2} = 2.0$ eV. From $f_{\text{photon}} = \Delta E_{\text{system}}/h$ and $\lambda = c/f$, we find that the wavelengths in the emission spectrum are

$3 \rightarrow 1$ $\quad f = 5.0 \text{ eV}/h = 1.2 \times 10^{15}$ Hz

$\lambda = 250$ nm (ultraviolet)

$2 \rightarrow 1$ $\quad f = 3.0 \text{ eV}/h = 7.2 \times 10^{14}$ Hz

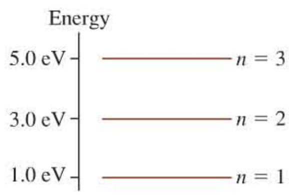
$\lambda = 420$ nm (blue)

$3 \rightarrow 2$ $\quad f = 2.0 \text{ eV}/h = 4.8 \times 10^{14}$ Hz

$\lambda = 620$ nm (orange)

ASSESS Transitions with a small energy difference, like $3 \rightarrow 2$, correspond to lower photon energies and thus longer wavelengths than transitions with a large energy difference like $3 \rightarrow 1$, as we would expect.

STOP TO THINK 28.5 A photon with a wavelength of 420 nm has energy $E_{\text{photon}} = 3.0$ eV. Do you expect to see a spectral line with $\lambda = 420$ nm in the emission spectrum of the system represented by this energy-level diagram?



28.7 The Uncertainty Principle

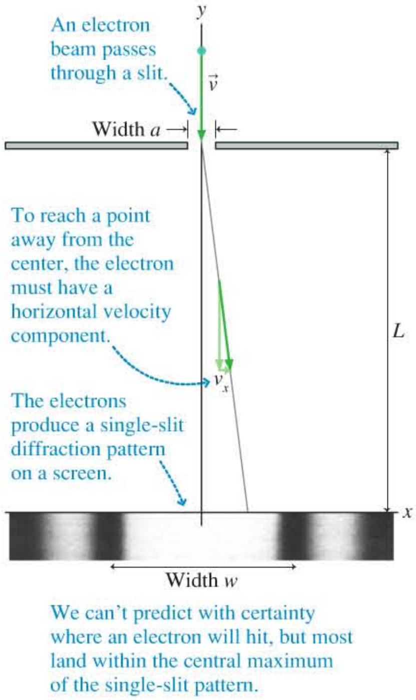
One of the strangest aspects of the quantum view of the world is an inherent limitation on our knowledge: **For a particle such as an electron, if you know where it is, you can't know exactly where it is going.** This very counterintuitive notion is a result of the wave nature of matter and is worth a bit of explanation.

FIGURE 28.22 on the next page shows an experiment in which electrons moving along the y-axis pass through a slit of width a. We know the result: Because of the wave nature of electrons, the slit causes them to spread out and produce a diffraction pattern.

But we can think of the experiment in a different way—as making a measurement of the position of the electrons. As an electron goes through the slit, we know something about its horizontal position. Our knowledge isn't perfect; we just know it is somewhere within the slit. We can establish an *uncertainty*, a limit on our knowledge. The uncertainty in the horizontal position is $\Delta x = a$, the width of the slit.

But, after passing through the slit, the electrons spread out, as they must to produce a diffraction pattern. The electrons, which had been moving along the y-axis before reaching the slit, now have a component of velocity along the x-axis. Because the electrons strike the screen over a range of positions, the value of v_x must vary from electron to electron. By sending the electrons through a slit—by trying to pin down their horizontal position—we've created an uncertainty in their horizontal velocity. Gaining knowledge of the *position* of the electrons has introduced uncertainty into our knowledge of the *velocity* of the electrons.

FIGURE 28.22 An experiment to illustrate the uncertainty principle.



CONCEPTUAL EXAMPLE 28.14 Changing the uncertainty

Suppose we narrow the slit in the above experiment, allowing us to determine the electron's horizontal position more precisely. How does this affect the diffraction pattern? How does this change in the diffraction pattern affect the uncertainty in the velocity?

REASON We learned in Chapter 17 that the width of the central maximum of the single-slit diffraction pattern is $w = 2\lambda L/a$. Making the slit narrower—decreasing the value of a—increases the value of w, making the central fringe wider. If the fringe is wider, the spread of horizontal velocities must be greater, so there is a greater uncertainty in the horizontal velocity.

ASSESS Improving our knowledge of the position decreases our knowledge of the velocity. This is the hallmark of the *uncertainty principle*.

We've made this argument by considering a particular experiment, but it is an example of a general principle. In 1927, the German physicist Werner Heisenberg proved that, for any particle, the product of the uncertainty Δx in its position and the uncertainty Δp_x in its x-momentum has a lower limit fixed by the expression

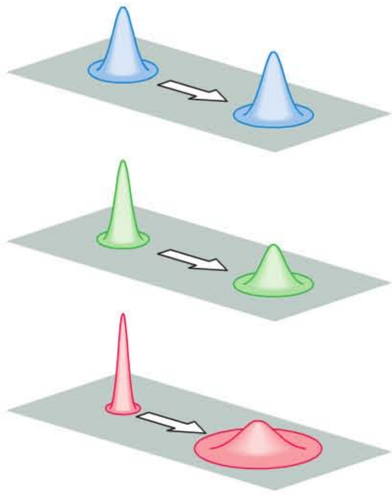
ActivPhysics 17.6

$$\Delta x\, \Delta p_x \ge \frac{h}{2\pi} \qquad (28.17)$$

Heisenberg uncertainty principle for position and momentum

A decreased uncertainty in position—knowing more precisely where a particle is—comes at the expense of an increased uncertainty in velocity and thus in momentum. But the relationship also goes the other way: Knowing a particle's velocity or momentum more precisely requires an increase in the uncertainty about its position.

NOTE ► In statements of the uncertainty principle, the right side is sometimes $h/2\pi$, as we have it, but other formulations have $h/4\pi$ or $h/2$ because of different conventions for the definition of Δx and Δp_x. Don't worry about these small differences. The important idea is that the product of Δx and Δp_x for a particle cannot be significantly less than Planck's constant h. ◄



► If I know where you are, I don't know where you're going In quantum physics, we represent particles by a *wave function* that describes their wave nature. This series of diagrams shows simulations of the evolution of three traveling wave functions. The top diagram shows the broad wave function of a particle whose position is not precisely defined. The uncertainty in momentum (and thus velocity) is small, so the wave function doesn't spread out much as it travels. The lower graphs show particles with more sharply peaked wave functions, implying less uncertainty in their initial positions. A reduced uncertainty in position means a larger uncertainty in velocity, so the wave functions spread out more quickly.

Uncertainties are associated with all experimental measurements, but better procedures and techniques can reduce those uncertainties. Classical physics places no limits on how small the uncertainties can be. A classical particle at any instant of time has an exact position x and an exact momentum p_x, and with sufficient care we can measure both x and p_x with such precision that we can make the product $\Delta x\, \Delta p_x$ as small as we like. There are no inherent limits to our knowledge.

In the quantum world, it's not so simple. No matter how clever you are, and no matter how good your experiment, you *cannot* measure both x and p_x simultaneously with arbitrarily good precision. Any measurements you make are limited by the condition that $\Delta x\, \Delta p_x \geq h/2\pi$. **The position and the momentum of a particle are *inherently* uncertain.**

Why? Because of the wave-like nature of matter! The "particle" is spread out in space, so there simply is not a precise value of its position x. Our belief that position and momentum have precise values is tied to our classical concept of a particle. As we revise our ideas of what atomic particles are like, we must also revise our ideas about position and momentum.

Let's revisit particles in a one-dimensional "box," now looking at uncertainties.

EXAMPLE 28.15 Determining uncertainties

a. What range of velocities might an electron have if confined to a 0.10-nm-wide region, about the size of an atom?

b. A 1.0-μm-diameter dust particle ($m \approx 10^{-15}$ kg) is confined within a 10-μm-long box. Can we know with certainty if the particle is at rest? If not, within what range is its velocity likely to be found?

PREPARE Localizing a particle means specifying its position with some accuracy—so there must be an uncertainty in the velocity. We can estimate the uncertainty by using Heisenberg's uncertainty principle.

SOLVE a. We aren't given the exact position of the particle, only that it is within a 0.10-nm-wide region. This means that we have specified the electron's position within a range $\Delta x = 1.0 \times 10^{-10}$ m. The uncertainty principle thus specifies that the least possible uncertainty in the momentum is

$$\Delta p_x = \frac{h}{2\pi\, \Delta x}$$

The uncertainty in the velocity is thus approximately

$$\Delta v_x = \frac{\Delta p_x}{m} \approx \frac{h}{2\pi m\, \Delta x} \approx 1 \times 10^6 \text{ m/s}$$

Because the *average* velocity is zero, (the particle is equally likely to be moving right or left) the best we can do is to say that the electron's velocity is somewhere in the interval $-5 \times 10^5 \text{ m/s} \leq v_x \leq 5 \times 10^5$ m/s. **It is simply not possible to specify the electron's velocity more precisely than this.**

b. If we know *for sure* that the dust particle is at rest, then $p_x = 0$ with no uncertainty. That is, $\Delta p_x = 0$. But then, according to the uncertainty principle, the uncertainty in our knowledge of the particle's position would have to be $\Delta x = \infty$. In other words, we would have no knowledge at all about the particle's position—it could be anywhere! But that is not the case. We know the particle is *somewhere* in the box, so the uncertainty in our knowledge of its position is at most $\Delta x = L = 10\ \mu$m. With a finite Δx, the uncertainty Δp_x *cannot* be zero. **We cannot know with certainty if the particle is at rest inside the box.** No matter how hard we try to bring the particle to rest, the uncertainty in our knowledge of the particle's momentum will be approximately $\Delta p_x \approx h/(2\pi\, \Delta x) = h/(2\pi L)$. Consequently, the range of possible velocities is

$$\Delta v_x = \frac{\Delta p_x}{m} \approx \frac{h}{2\pi m L} \approx 1.0 \times 10^{-14} \text{ m/s}$$

This range of possible velocities will be centered on $v_x = 0$ m/s if we have done our best to have the particle be at rest. Therefore all we can know with certainty is that the particle's velocity is somewhere within the interval $-5 \times 10^{-15} \text{ m/s} \leq v_x \leq 5 \times 10^{-15}$ m/s.

ASSESS Our uncertainty about the electron's velocity is enormous—nearly 1% of the speed of light. For an electron confined to a region of this size, the best we can do is to state that its speed is less than one million miles per hour! The uncertainty principle clearly sets real, practical limits on our ability to describe electrons. The situation for the dust particle is different. We can't say for certain that the particle is absolutely at rest. But knowing that its speed is less than 5×10^{-15} m/s means that the particle is at rest for all practical purposes. At this speed, the dust particle would require nearly 6 hours to travel the width of one atom! Again we see that the quantum view has profound implications at the atomic scale but need not affect the way we think of macroscopic objects.

STOP TO THINK 28.6 The speeds of an electron and a proton have been measured to the same uncertainty. Which one has a larger uncertainty in position?

A. The proton, because it's more massive.
B. The electron, because it's less massive.
C. The uncertainty in position is the same, because the uncertainty in velocity is the same.

28.8 Applications and Implications of Quantum Theory

Quantum theory seems bizarre to those of us living at a scale where the rather different rules of classical physics apply. In this section we consider some of the implications of quantum theory and some applications that confirm these unusual notions.

Tunneling and the Scanning Tunneling Microscope

The fact that particles have a wave nature allows for imaging at remarkably small scales—the scale of single atoms! The *scanning tunneling microscope* doesn't work like other microscopes you have seen, but instead builds an image of a solid surface by scanning a probe near the surface.

FIGURE 28.23 shows the tip of a very, very thin metal needle, called the *probe tip,* positioned above the surface of a solid sample. The space between the tip and the surface is about 0.5 nm, only a few atomic diameters. Electrons in the sample are attracted to the positive probe tip, but no current should flow, according to classical physics, because the electrons cannot cross the gap between the sample and the probe; it is an incomplete circuit. As we found with the photoelectric effect, the electrons are bound inside the sample and not free to leave.

However, electrons are not classical particles. The electron has a wave nature, and waves don't have sharp edges. One startling prediction of Schrödinger's quantum mechanics is that the electrons' wave functions extend very slightly beyond the edge of the sample. When the probe tip comes close enough to the surface, close enough to poke into an electron's wave function, an electron that had been in the sample might suddenly find itself in the probe tip. In other words, a quantum electron *can* cross the gap between the sample and the probe tip, thus causing a current to flow in the circuit. This process is called **tunneling** because it is rather like tunneling through an uncrossable mountain barrier to get to the other side. Tunneling is completely forbidden by the laws of classical physics, so the fact that it occurs is a testament to the reality of quantum ideas.

The probability that an electron will tunnel across the gap is very sensitive to the size of the gap, which makes the **scanning tunneling microscope,** or STM, possible. When the probe tip passes over an atom or over an atomic-level bump on the surface, the gap narrows and the current increases as more electrons are able to tunnel across. Similarly, the tunneling probability decreases when the probe tip passes across an atomic-size valley, and the current falls. The current-versus-position data are used to construct an image of the surface.

The STM was the first technology that allowed imaging of individual atoms, and it was one of a handful of inventions in the 1980s that jump-started the current interest in nanotechnology. STM images offer a remarkable view of the world at an atomic scale. The STM image of **FIGURE 28.24a** clearly shows the hexagonal arrangement of the individual atoms on the surface of pyrolytic graphite. The image of a DNA molecule in **FIGURE 28.24b** shows the actual twists of the double-helix structure. Current research efforts aim to develop methods for sequencing DNA with scanning tunneling microscopes and other nanoprobes—to directly "read" a single strand of DNA!

Wave–Particle Duality

One common theme that has run through this chapter is the idea that, in quantum theory, things we think of as being waves have a particle nature, while things we think of as being particles have a wave nature. What is the true nature of light, or an electron? Are they particles or waves?

The various objects of classical physics are *either* particles *or* waves. There's no middle ground. Planets and baseballs are particles or collections of particles, while sound and light are clearly waves. Particles follow trajectories given by Newton's

FIGURE 28.23 The scanning tunneling microscope.

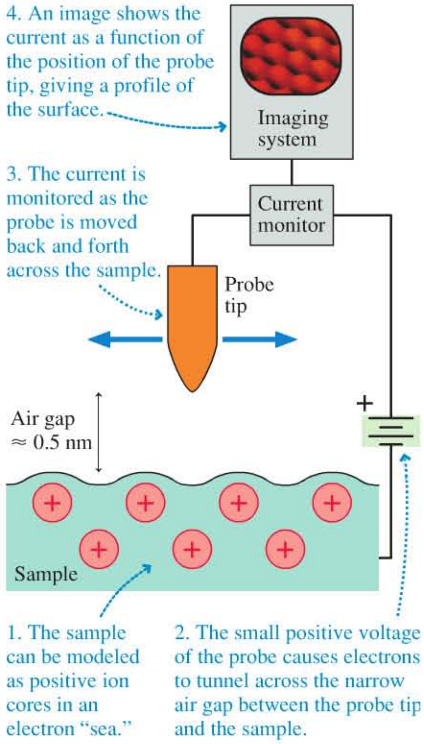
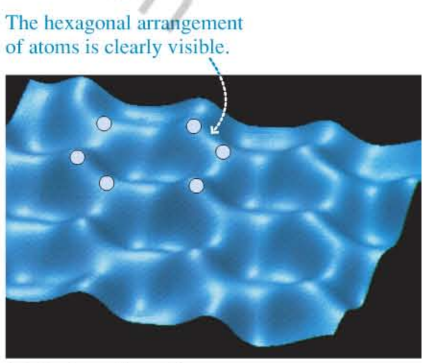
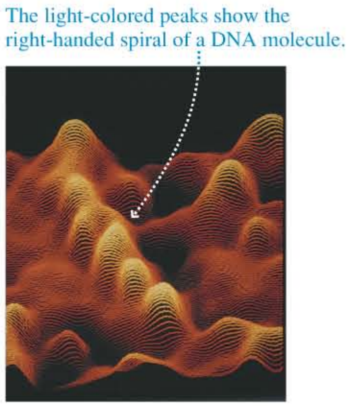


FIGURE 28.24 STM images.



(a) Surface of graphite



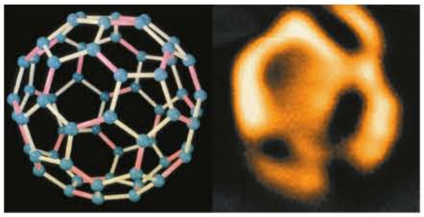
(b) DNA molecule

laws; waves obey the principle of superposition and exhibit interference. This wave–particle dichotomy seemed obvious until physicists encountered irrefutable evidence that light sometimes acts like a particle and, even stranger, that matter sometimes acts like a wave.

You might at first think that light and matter are *both* a wave *and* a particle, but that idea doesn't quite work. The basic definitions of particleness and waviness are mutually exclusive. Two sound waves can pass through each other and can overlap to produce a larger-amplitude sound wave; two baseballs can't. It is more profitable to conclude that light and matter are *neither* a wave *nor* a particle. At the microscopic scale of atoms and their constituents—a physical scale not directly accessible to our five senses—the classical concepts of particles and waves turn out to be simply too limited to explain the subtleties of nature.

Although matter and light have both wave-like aspects and particle-like aspects, they show us only one face at a time. If we arrange an experiment to measure a wave-like property, such as interference, we find photons and electrons acting like waves, not particles. An experiment to look for particles will find photons and electrons acting like particles, not waves. These two aspects of light and matter are *complementary* to each other, like a two-piece jigsaw puzzle. Neither the wave nor the particle model alone provides an adequate picture of light or matter, but taken together they provide us with a basis for understanding these elusive but most fundamental constituents of nature. This two-sided point of view is called *wave–particle duality.*

For over two hundred years, scientists and nonscientists alike felt that the clockwork universe of Newtonian physics was a fundamental description of reality. But wave–particle duality, along with Einstein's relativity, undermines the basic assumptions of the Newtonian worldview. The certainty and predictability of classical physics have given way to a new understanding of the universe in which chance and uncertainty play key roles—the universe of quantum physics.



◀ The dual nature of a buckyball Treating atomic-level structures involves frequent shifts between particle and wave views. 60 carbon atoms can create the molecule diagrammed at left, known as C_{60}, or *buckminsterfullerene.* The scanning electron microscope image of a C_{60} molecule shown on the right is a particle-like view of the molecule with individual carbon atoms clearly visible. The C_{60} molecule, though we can make a picture of it—showing the atoms that make it up—also has a wave nature. A beam of C_{60} sent through a grating will produce a diffraction pattern!

INTEGRATED EXAMPLE 28.16 Magnetic resonance imaging

In Chapter 24, we learned that the magnetism of permanent magnets arises because the inherent magnetic moment of electrons causes them to act like little compass needles. Protons also have an inherent magnetic moment, and this is the basis for magnetic resonance imaging (MRI) in medicine.

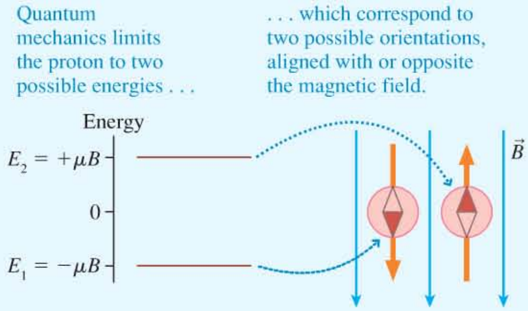
Although a compass needle would prefer to align with a magnetic field, the needle can point in *any* direction. This isn't the case for the magnetic moment of a proton. Quantum physics tells us that the proton's energy must be quantized. There are only two possible energy levels—and thus two possible orientations—for protons in a magnetic field:

$E_1 = -\mu B$ magnetic moment aligned with the field

$E_2 = +\mu B$ magnetic moment aligned opposite the field

where $\mu = 1.41 \times 10^{-26}$ J/T is the known value of the proton's magnetic moment. FIGURE 28.25 shows the two possible energy states. The magnetic moment, like a compass needle, "wants" to align with the field, so that is the lower-energy state.

FIGURE 28.25 Energy levels for a proton in a magnetic field.



Human tissue is mostly water. Each water molecule has two hydrogen atoms whose nuclei are single protons. In a magnetic field, the protons go into one or the other quantum state. A photon of just the right energy can "flip" the orientation of a proton's magnetic moment by causing a quantum jump from one state to the other. The energy difference between the states is small, so the relatively low-frequency photons are in the radio portion of the electromagnetic spectrum. These photons are provided by a *probe coil* that emits radio waves. When the probe is tuned to just the right frequency, the waves are *in resonance* with the energy levels of the protons, thus giving us the name magnetic *resonance* imaging.

The rate of absorption of these low-energy photons is proportional to the density of hydrogen atoms. Hydrogen density varies with tissue type, so an MRI image—showing different tissues—is formed by measuring the variation across the body of the rate at which photons cause quantum jumps between the two proton energy levels. A figure showing the absorption rate versus position in the body is an image of a "slice" through a patient's body, as in **FIGURE 28.26**.

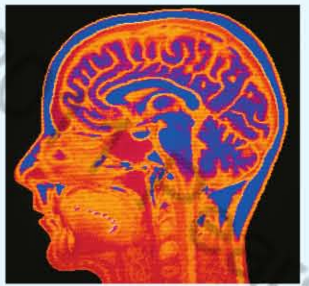


FIGURE 28.26 An MRI image shows the cross section of a patient's head.

a. An MRI patient is placed inside a solenoid that creates a strong magnetic field. If the field strength is 2.00 T, to what frequency must the probe coil be set? What is the wavelength of the photons produced?

b. In a uniform magnetic field, all protons in the body would absorb photons of the same frequency. To form an image of the body, the magnetic field is designed to vary from point to point in a known way. Because the field is different at each point in the body, each point has a unique frequency of photons that will be absorbed. The actual procedure is complex, but consider a simple model in which the field strength varies only along the axis of the patient's body, which we will call the x-axis. In particular, suppose that the magnetic field strength in tesla is given by $B = 2.00 + 1.60x$, where x, measured from a known reference point, is in meters. The probe coil is first tuned to the resonance frequency at the reference point. As the frequency is increased, a strong signal is observed at a frequency 4.7 MHz above the starting frequency. What is the location in the body, relative to the reference point, of the tissue creating this strong signal?

PREPARE If a photon has energy equal to the energy difference between the high- and low-energy states, it will be able to cause a quantum jump to the higher state—it will be absorbed. The photon energy E_{photon} must be equal to the energy difference between the two states: $\Delta E_{\text{system}} = 2\mu B$.

SOLVE a. At 2.00 T, the energy difference between the two proton states is

$$E_{\text{system}} = E_2 - E_1 = 2\mu B = 2(1.41 \times 10^{-26}\ \text{J/T})(2.00\ \text{T})$$
$$= 5.64 \times 10^{-26}\ \text{J}$$

This is a very low energy—only 3.5×10^{-7} eV. A photon will be absorbed if $E_{\text{photon}} = hf = E_{\text{system}}$. Thus the photon frequency must be

$$f = \frac{\Delta E_{\text{system}}}{h} = \frac{5.64 \times 10^{-26}\ \text{J}}{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 85.1 \times 10^{6}\ \text{Hz} = 85.1\ \text{MHz}$$

This corresponds to a wavelength of $\lambda = c/f = 3.52$ m.

b. The magnetic field at the reference point ($x = 0$ m) is 2.00 T, so the probe frequency at this point is the 85.1 MHz we found in part a. The strong signal is 4.7 MHz above this, or 89.8 Hz. We can solve $E_{\text{photon}} = hf = E_{\text{system}} = 2\mu B$ to find the magnetic field at the point creating this strong signal:

$$B = \frac{hf}{2\mu} = 2.11\ \text{T}$$

We can then use the field-versus-distance formula given in the problem to find the position of this signal:

$$B = 2.11\ \text{T} = 2.00 + 1.60x$$
$$x = 0.069\ \text{m}$$

Thus there is a high density of protons 6.9 cm from the reference point.

ASSESS The frequency of the probe coil is in the radio portion of the electromagnetic spectrum, as we expected. The strong signal of part b is at a higher frequency, so this corresponds to a higher field and a positive value of x, as we found. The frequency is only slightly different from the original frequency, so we expect the point to be close to the reference position, as we found.

This is a simplified model of MRI, but the key features are present: A magnetic field that varies with position creates different energy levels for protons at different positions in the body, then tuned radio-wave photons measure the proton density at these different positions by causing and detecting quantum jumps between the two proton energy levels.

SUMMARY

The goal of Chapter 28 has been to understand the quantization of energy for light and matter.

GENERAL PRINCIPLES

Light has particle-like properties

- The energy of a light wave comes in discrete packets (light quanta) we call **photons.**
- For light of frequency f, the energy of each photon is $E = hf$, where h is **Planck's constant**.
- When light strikes a metal surface, all of the energy of a single photon is given to a single electron.

Matter has wave-like properties

- The **de Broglie wavelength** of a particle of mass m is $\lambda = h/mv$.
- The wave-like nature of matter is seen in the interference patterns of electrons, protons, and other particles.

Quantization of energy

When a particle is confined, it sets up a de Broglie standing wave.

The fact that standing waves can have only certain allowed wavelengths leads to the conclusion that a confined particle can have only certain allowed energies.

Wave–particle duality

- Experiments designed to measure wave properties will show the wave nature of light and matter.
- Experiments designed to measure particle properties will show the particle nature of light and matter.

Heisenberg uncertainty principle

A particle with wave-like characteristics does not have a precise value of position x or a precise value of momentum p_x. Both are uncertain. The position uncertainty Δx and momentum uncertainty Δp_x are related by

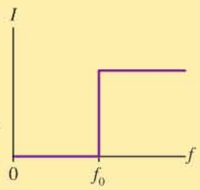
$$\Delta x \, \Delta p_x \geq \frac{h}{2\pi}$$

The more you pin down the value of one, the less precisely the other can be known.

IMPORTANT CONCEPTS

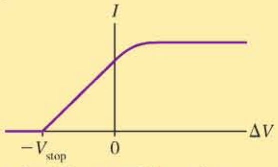
Photoelectric effect

Light with frequency f can eject electrons from a metal only if $f \geq f_0 = E_0/h$, where E_0 is the metal's **work function.** Electrons will be ejected even if the intensity of the light is very small.



The **stopping potential** that stops even the fastest electrons is

$$V_{\text{stop}} = \frac{K_{\text{max}}}{e} = \frac{hf - E_0}{e}$$



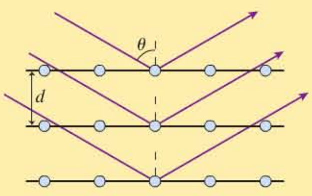
The details of the photoelectric effect could not be explained with classical physics. New models were needed.

X-ray diffraction

X rays with wavelength λ undergo strong reflections from atomic planes spaced by d when the angle of incidence satisfies the **Bragg condition:**

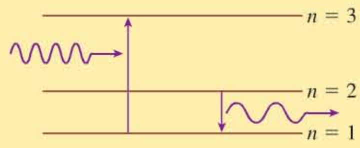
$$2d\cos\theta = m\lambda$$

$$m = 1, 2, 3, \ldots$$



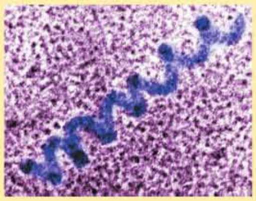
Energy levels and quantum jumps

The localization of electrons leads to quantized energy levels. An electron can exist only in certain energy states. An electron can jump to a higher level if a photon is absorbed, or to a lower level if a photon is emitted. The energy difference between the levels equals the photon energy.

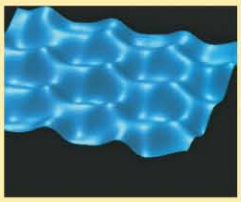


APPLICATIONS

The wave nature of light limits the resolution of a light microscope. A more detailed image may be made with an **electron microscope** because of the very small de Broglie wavelength of fast electrons.



The wave nature of electrons allows them to **tunnel** across an insulating layer of air to the tip of a **scanning tunneling microscope,** revealing details of the atoms on a surface.



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. The first-order x-ray diffraction of monochromatic x rays from a crystal occurs at angle θ_1. The crystal is then compressed, causing a slight reduction in its volume. Does θ_1 increase, decrease, or stay the same? Explain.
2. Explain the reasoning by which we claim that the stopping potential V_{stop} measures the maximum kinetic energy of the electrons in a photoelectric-effect experiment.
3. How does Einstein's explanation account for each of these characteristics of the photoelectric effect?
   A. The photoelectric current is zero for frequencies below some threshold.
   B. The photoelectric current increases with increasing light intensity.
   C. The photoelectric current is independent of ΔV for $\Delta V > 0$.
   D. The photoelectric current decreases slowly as ΔV becomes more negative.
   E. The stopping potential is independent of the light intensity.

   Which of these *cannot* be explained by classical physics? Explain.
4. How would the graph of Figure 28.7a look if the emission of electrons from the cathode was due to the heating of the metal by light falling on it? Draw the graph and explain your reasoning. Assume that the light intensity remains constant as its frequency and wavelength are varied.
5. Figure Q28.5 shows the typical photoelectric behavior of a metal as the anode-cathode potential difference ΔV is varied.
   a. Why do the curves become horizontal for $\Delta V > 0$ V? Shouldn't the current increase as the potential difference increases? Explain.
   b. Why doesn't the current immediately drop to zero for $\Delta V < 0$ V? Shouldn't $\Delta V < 0$ V prevent the electrons from reaching the anode? Explain.
   c. The current is zero for $\Delta V < -2.0$ V. Where do the electrons go? Are no electrons emitted if $\Delta V < -2.0$ V? Or if they are, why is there no current? Explain.

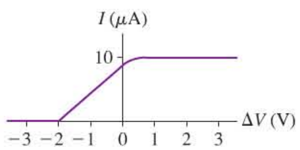


FIGURE Q28.5

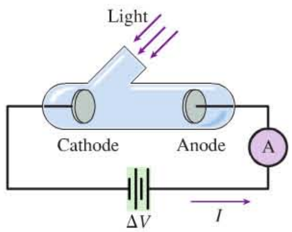


FIGURE Q28.6

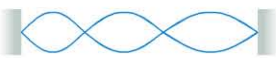
6. In the photoelectric effect experiment, as illustrated by Figure Q28.6, a current is measured while light is shining on the cathode. But this does not appear to be a complete circuit, so how can there be a current? Explain.
7. Metal surfaces on spacecraft in bright sunlight develop a net electric charge. Do they develop a negative or a positive charge? Explain.
8. Metal 1 has a larger work function than metal 2. Both are illuminated with the same short-wavelength ultraviolet light. Do electrons from metal 1 have a higher speed, a lower speed, or the same speed as electrons from metal 2? Explain.
9. A gold cathode is illuminated with light of wavelength 250 nm. It is found that the current is zero when $\Delta V = 1.0$ V. Would the current change if
   a. The light intensity is doubled?
   b. The anode-cathode potential difference is increased to $\Delta V = 5.5$ V?
10. Three laser beams have wavelengths $\lambda_1 = 400$ nm, $\lambda_2 = 600$ nm, and $\lambda_3 = 800$ nm. The power of each laser beam is 1 W.
    a. Rank in order, from largest to smallest, the photon energies E_1, E_2, and E_3 in these three laser beams. Explain.
    b. Rank in order, from largest to smallest, the number of photons per second N_1, N_2, and N_3 delivered by the three laser beams. Explain.
11. When we say that a photon is a "quantum of light," what does that mean? What is quantized?
12. An investigator is measuring the current in a photoelectric effect experiment. The cathode is illuminated by light of a single wavelength. What happens to the current if the intensity of the light is doubled while the wavelength is held constant?
13. An investigator is measuring the current in a photoelectric effect experiment. The cathode is illuminated by light of a single wavelength. What happens to the current if the wavelength of the light is reduced by a factor of two while keeping the intensity constant?
14. To have the best resolution, should an electron microscope use very fast electrons or very slow electrons? Explain.
15. An electron and a proton are accelerated from rest through potential differences of the same magnitude. Afterward, which particle has the larger de Broglie wavelength? Explain.
16. A neutron is shot straight up with an initial speed of 100 m/s. As it rises, does its de Broglie wavelength increase, decrease, or not change? Explain.
17. Double-slit interference of electrons occurs because:
    A. The electrons passing through the two slits repel each other.
    B. Electrons collide with each other behind the slits.
    C. Electrons collide with the edges of the slits.
    D. Each electron goes through both slits.
    E. The energy of the electrons is quantized.
    F. Only certain wavelengths of the electrons fit through the slits.

    Which of these (perhaps none, perhaps more than one) are correct? Explain.
18. Can an electron with a de Broglie wavelength of 2 μm pass through a slit that is 1 μm wide? Explain.

19. a. For the allowed energies of a particle in a box to be large, should the box be very big or very small? Explain.
 b. Which is likely to have larger values for the allowed energies: an atom in a molecule, an electron in an atom, or a proton in a nucleus? Explain.
20. Figure Q28.20 shows the standing de Broglie wave of a particle in a box.
 a. What is the quantum number?
 b. Can you determine from this picture whether the "classical" particle is moving to the right or to the left? If so, which is it? If not, why not?

FIGURE Q28.20


21. A particle in a box of length L_a has $E_1 = 2$ eV. The same particle in a box of length L_b has $E_2 = 50$ eV. What is the ratio L_a/L_b?
22. Imagine that the horizontal box of Figure 28.18 is instead oriented vertically. Also imagine the box to be on a neutron star where the gravitational field is so strong that the particle in the box slows significantly, nearly stopping, before it hits the top of the box. Make a *qualitative* sketch of the $n = 3$ de Broglie standing wave of a particle in this box.
 Hint: The nodes are *not* uniformly spaced.
23. Figure Q28.23 shows a standing de Broglie wave.
 a. Does this standing wave represent a particle that travels back and forth between the boundaries with a constant speed or a changing speed? Explain.
 b. If the speed is changing, at which end is the particle moving faster and at which end is it moving slower?

FIGURE Q28.23


24. BIO The molecules in the rods and cones in the eye are tuned to absorb photons of particular energies. The retinal molecule, like many molecules, is a long chain. Electrons can freely move along one stretch of the chain but are reflected at the ends, thus behaving like a particle in a one-dimensional box. The absorption of a photon lifts an electron from the ground state into the first excited state. Do the molecules in a red cone (which are tuned to absorb red light) or the molecules in a blue cone (tuned to absorb blue light) have a longer "box"?
25. Science fiction movies often use devices that transport people and objects rapidly from one position to another. To "beam" people in this fashion means taking them apart atom by atom, carefully measuring each position, and then sending the atoms in a beam to the desired final location where they reassemble. How do the principles of quantum mechanics pose problems for this futuristic means of transportation?

Multiple-Choice Questions

26. A light sensor is based on a photodiode that requires a minimum photon energy of 1.7 eV to create mobile electrons. What is the longest wavelength of electromagnetic radiation that the sensor can detect?
 A. 500 nm
 B. 730 nm
 C. 1200 nm
 D. 2000 nm
27. In a photoelectric effect experiment, the frequency of the light is increased while the intensity is held constant. As a result,
 A. There are more electrons. B. The electrons are faster.
 C. Both A and B. D. Neither A nor B.
28. In a photoelectric effect experiment, the intensity of the light is increased while the frequency is held constant. As a result,
 A. There are more electrons.
 B. The electrons are faster.
 C. Both A and B.
 D. Neither A nor B.
29. In the photoelectric effect, electrons are never emitted from a metal if the frequency of the incoming light is below a certain threshold value. This is because
 A. Photons of lower-frequency light don't have enough energy to eject an electron.
 B. The electric field of low-frequency light does not vibrate the electrons rapidly enough to eject them.
 C. The number of photons in low-frequency light is too small to eject electrons.
 D. Low-frequency light does not penetrate far enough into the metal to eject electrons.
30. Visible light has a wavelength of about 500 nm. A typical radio wave has a wavelength of about 1.0 m. How many photons of the radio wave are needed to equal the energy of one photon of visible light?
 A. 2,000 B. 20,000
 C. 200,000 D. 2,000,000
31. Two radio stations have the same power output from their antennas. One broadcasts AM at a frequency of 1000 kHz and one broadcasts FM at a frequency of 100 MHz. Which statement is true?
 A. The FM station emits more photons per second.
 B. The AM station emits more photons per second.
 C. The two stations emit the same number of photons per second.
32. An electron is accelerated through a 5000 V potential difference, strikes a metal target, and causes an x ray to be emitted. What is the (approximate) minimum wavelength of the emitted x ray?
 A. 0.25 nm B. 1.0 nm
 C. 2.5 nm D. 4.0 nm
33. How many photons does a 5.0 mW helium-neon laser ($\lambda = 633$ nm) emit in 1 second?
 A. 1.2×10^{19} B. 4.0×10^{18}
 C. 8.0×10^{16} D. 1.6×10^{16}
34. You shoot a beam of electrons through a double slit to make an interference pattern. After noting the properties of the pattern, you then double the speed of the electrons. What effect would this have?
 A. The fringes would get closer together.
 B. The fringes would get farther apart.
 C. The positions of the fringes would not change.
35. Photon P in Figure Q28.35 moves an electron from energy level $n = 1$ to energy level $n = 3$. The electron jumps down to $n = 2$, emitting photon Q, and then jumps down to $n = 1$, emitting photon R. The spacing be-tween energy levels is drawn to scale. What is the correct relationship among the wavelengths of the photons?

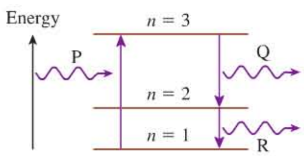


FIGURE Q28.35

 A. $\lambda_P < \lambda_Q < \lambda_R$ B. $\lambda_R < \lambda_P < \lambda_Q$
 C. $\lambda_Q < \lambda_P < \lambda_R$ D. $\lambda_R < \lambda_Q < \lambda_P$

PROBLEMS

Section 28.1 X Rays and X-Ray Diffraction

1. | X rays with a wavelength of 0.12 nm undergo first-order diffraction from a crystal at a 68° angle of incidence. What is the angle of second-order diffraction?
2. | X rays with a wavelength of 0.20 nm undergo first-order diffraction from a crystal at a 54° angle of incidence. At what angle does first-order diffraction occur for x rays with a wavelength of 0.15 nm?
3. | X rays diffract from a crystal in which the spacing between atomic planes is 0.175 nm. The second-order diffraction occurs at 45.0°. What is the angle of the first-order diffraction?
4. || The spacing between atomic planes in a crystal is 0.110 nm. If 12.0 keV x rays are diffracted by this crystal, what are the angles of (a) first-order and (b) second-order diffraction?
5. || X rays with a wavelength of 0.085 nm diffract from a crystal in which the spacing between atomic planes is 0.18 nm. How many diffraction orders are observed?

Section 28.2 The Photoelectric Effect

6. | Which metals in Table 28.1 exhibit the photoelectric effect for (a) light with $\lambda = 400$ nm and (b) light with $\lambda = 250$ nm?
7. | Electrons are emitted when a metal is illuminated by light with a wavelength less than 388 nm but for no greater wavelength. What is the metal's work function?
8. || Electrons in a photoelectric-effect experiment emerge from a copper surface with a maximum kinetic energy of 1.10 eV. What is the wavelength of the light?
9. || You need to design a photodetector that can respond to the entire range of visible light. What is the maximum possible work function of the cathode?
10. | A photoelectric-effect experiment finds a stopping potential of 1.93 V when light of 200 nm wavelength is used to illuminate the cathode.
    a. From what metal is the cathode made?
    b. What is the stopping potential if the intensity of the light is doubled?
11. || Zinc has a work function of 4.3 eV.
    a. What is the longest wavelength of light that will release an electron from a zinc surface?
    b. A 4.7 eV photon strikes the surface and an electron is emitted. What is the maximum possible speed of the electron?
12. || Image intensifiers used in night-vision devices create a bright image from dim light by letting the light first fall on a *photocathode*. Electrons emitted by the photoelectric effect are accelerated and then strike a phosphorescent screen, causing it to glow more brightly than the original scene. Recent devices are sensitive to wavelengths as long as 900 nm, in the infrared:

    a. If the threshold wavelength is 900 nm, what is the work function of the photocathode?
    b. If light of wavelength 700 nm strikes such a photocathode, what will be the maximum kinetic energy, in eV, of the emitted electrons?
13. || Light with a wavelength of 350 nm shines on a metal surface, which emits electrons. The stopping potential is measured to be 1.25 V.
    a. What is the maximum speed of emitted electrons?
    b. Calculate the work function and identify the metal.

Section 28.3 Photons

14. | BIO When an ultraviolet photon is absorbed by a molecule of DNA, the photon's energy can be converted into vibrational energy of the molecular bonds. Excessive vibration damages the molecule by causing the bonds to break. Ultraviolet light of wavelength less than 290 nm causes significant damage to DNA; ultraviolet light of longer wavelength causes minimal damage. What is the threshold photon energy, in eV, for DNA damage?
15. | The spacing between atoms in graphite is approximately 0.25 nm. What is the energy of an x-ray photon with this wavelength?
16. || BIO A firefly glows by the direct conversion of chemical energy to light. The light emitted by a firefly has peak intensity at a wavelength of 550 nm.

    a. What is the minimum chemical energy, in eV, required to generate each photon?
    b. One molecule of ATP provides 0.30 eV of energy when it is metabolized in a cell. What is the minimum number of ATP molecules that must be consumed in the reactions that lead to the emission of one photon of 550 nm light?
17. | BIO Your eyes have three different types of cones with maximum absorption at 437 nm, 533 nm, and 564 nm. What photon energies correspond to these wavelengths?
18. | What is the wavelength, in nm, of a photon with energy (a) 0.30 eV, (b) 3.0 eV, and (c) 30 eV? For each, is this wavelength visible light, ultraviolet, or infrared?
19. | What is the ratio of the energy of a photon of light at the far red end of the visible spectrum (700 nm) to that of a photon at the far blue end of the visible spectrum (400 nm)?
20. || BIO INT The wavelengths of light emitted by a firefly span the visible spectrum but have maximum intensity near 550 nm. A typical flash lasts for 100 ms and has a power of 1.2 mW. If we assume that all of the light is emitted at the peak-intensity wavelength of 550 nm, how many photons are emitted in one flash?
21. || Station KAIM in Hawaii broadcasts on the AM dial at 870 kHz, with a maximum power of 50,000 W. At maximun power, how many photons does the transmitting antenna emit each second?
22. ||| BIO At 510 nm, the wavelength of maximum sensitivity of the human eye, the dark-adapted eye can sense a 100-ms-long flash of light of total energy 4.0×10^{-17} J. (Weaker flashes of light may be detected, but not reliably.) If 60% of the incident light is lost to reflection and absorption by tissues of the eye, how many photons reach the retina from this flash?

23. | 550 nm is the average wavelength of visible light.
   a. What is the energy of a photon with a wavelength of 550 nm?
   b. A typical incandescent lightbulb emits about 1 J of visible light energy every second. Estimate the number of visible photons emitted per second.

24. || *Dinoflagellates* are single-cell creatures that float in the world's oceans; many types are bioluminescent. When disturbed by motion in the water, a typical bioluminescent dinoflagellate emits 100,000,000 photons in a 0.10-s-long flash of light of wavelength 460 nm. What is the power of the flash in watts?
BIO

25. || A circuit employs a silicon solar cell to detect flashes of light lasting 0.25 s. The smallest current the circuit can detect reliably is 0.42 μA. Assuming that all photons reaching the solar cell give their energy to a charge carrier, what is the minimum power of a flash of light of wavelength 550 nm that can be detected?

Section 28.4 Matter Waves

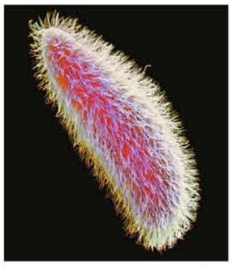
26. | Estimate your de Broglie wavelength while walking at a speed of 1 m/s.

27. | a. What is the de Broglie wavelength of a 200 g baseball with a speed of 30 m/s?
   b. What is the speed of a 200 g baseball with a de Broglie wavelength of 0.20 nm?

28. | a. What is the speed of an electron with a de Broglie wavelength of 0.20 nm?
   b. What is the speed of a proton with a de Broglie wavelength of 0.20 nm?

29. || What is the kinetic energy, in eV, of an electron with a de Broglie wavelength of 1.0 nm?

30. || A paramecium is covered with motile hairs called cilia that propel it at a speed of 1 mm/s. If the paramecium has a volume of 2×10^{-13} m^3 and a density equal that of water, what is its de Broglie wavelength when in motion? What fraction of the paramecium's 150 μm length does this wavelength represent?
BIO INT



31. || The diameter of an atomic nucleus is about 10 fm (1 fm $= 10^{-15}$ m). What is the kinetic energy, in MeV, of a proton with a de Broglie wavelength of 10 fm?

32. || Rubidium atoms are cooled to 0.10 μK in an atom trap. What is their de Broglie wavelength? How many times larger is this than the 0.25 nm diameter of the atoms?
INT

33. || Through what potential difference must an electron be accelerated from rest to have a de Broglie wavelength of 500 nm?

Section 28.5 Energy Is Quantized

34. || What is the length of a box in which the minimum energy of an electron is 1.5×10^{-18} J?

35. ||| What is the length of a one-dimensional box in which an electron in the $n = 1$ state has the same energy as a photon with a wavelength of 600 nm?

36. ||| An electron confined in a one-dimensional box is observed, at different times, to have energies of 12 eV, 27 eV, and 48 eV. What is the length of the box?

37. | The nucleus of a typical atom is 5.0 fm (1 fm $= 10^{-15}$ m) in diameter. A very simple model of the nucleus is a one-dimensional box in which protons are confined. Estimate the energy of a proton in the nucleus by finding the first three allowed energies of a proton in a 5.0-fm-long box.

Section 28.6 Energy Levels and Quantum Jumps

38. || The allowed energies of a quantum system are 1.0 eV, 2.0 eV, 4.0 eV, and 7.0 eV. What wavelengths appear in the system's emission spectrum?

39. || Figure P28.39 is an energy-level diagram for a quantum system. What wavelengths appear in the system's emission spectrum?

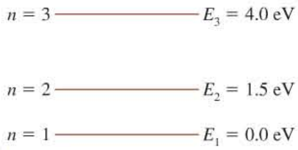


FIGURE P28.39

40. ||| The allowed energies of a quantum system are 0.0 eV, 4.0 eV, and 6.0 eV.
   a. Draw the system's energy-level diagram. Label each level with the energy and the quantum number.
   b. What wavelengths appear in the system's emission spectrum?

41. ||| The allowed energies of a quantum system are 0.0 eV, 1.5 eV, 3.0 eV, and 6.0 eV. How many different wavelengths appear in the emission spectrum?

Section 28.7 The Uncertainty Principle

42. || The speed of an electron is known to be between 3.0×10^6 m/s and 3.2×10^6 m/s. Estimate the uncertainty in its position.

43. || What is the smallest box in which you can confine an electron if you want to know for certain that the electron's speed is no more than 10 m/s?

44. ||| A spherical virus has a diameter of 50 nm. It is contained inside a long, narrow cell of length 1×10^{-4} m. What uncertainty does this imply for the velocity of the virus along the length of the cell? Assume the virus has a density equal to that of water.
BIO INT

45. ||| A thin solid barrier in the *xy*-plane has a 10-μm-diameter circular hole. An electron traveling in the *z*-direction with $v_x = 0$ m/s passes through the hole. Afterward, is v_x still zero? If not, within what range is v_x likely to be?

46. ||| A proton is confined within an atomic nucleus of diameter 4 fm (1 fm $= 10^{-15}$ m). Estimate the smallest range of speeds you might find for a proton in the nucleus.

General Problems

47. || X rays with a wavelength of 0.0700 nm diffract from a crystal. Two adjacent angles of x-ray diffraction are 45.6° and 21.0°. What is the distance in nm between the atomic planes responsible for the diffraction?

48. || Potassium and gold cathodes are used in a photoelectric-effect experiment. For each cathode, find:
   a. The threshold frequency
   b. The threshold wavelength
   c. The maximum electron ejection speed if the light has a wavelength of 220 nm
   d. The stopping potential if the wavelength is 220 nm

49. |||| In a photoelectric-effect experiment, the maximum kinetic energy of electrons is 2.8 eV. When the wavelength of the light is increased by 50%, the maximum energy decreases to 1.1 eV. What are (a) the work function of the cathode and (b) the initial wavelength?

50. |||| In a photoelectric-effect experiment, the stopping potential at a wavelength of 400 nm is 25.7% of the stopping potential at a wavelength of 300 nm. Of what metal is the cathode made?

51. || Light of constant intensity but varying wavelength was used to illuminate the cathode in a photoelectric-effect experiment. The graph of Figure P28.51 shows how the stopping potential depended on the frequency of the light. What is the work function, in eV, of the cathode?

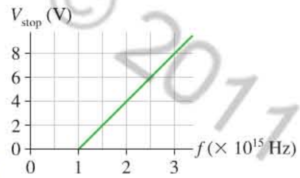


FIGURE P28.51

52. || BIO What is the de Broglie wavelength of a red blood cell with a mass of 1.00×10^{-11} g that is moving with a speed of 0.400 cm/s? Do we need to be concerned with the wave nature of the blood cells when we describe the flow of blood in the body?

53. || BIO Suppose you need to image the structure of a virus with a diameter of 50 nm. For a sharp image, the wavelength of the probing wave must be 5.0 nm or less. We have seen that, for imaging such small objects, this short wavelength is obtained by using an electron beam in an electron microscope. Why don't we simply use short-wavelength electromagnetic waves? There's a problem with this approach: As the wavelength gets shorter, the energy of a photon of light gets greater and could damage or destroy the object being studied. Let's compare the energy of a photon and an electron that can provide the same resolution.
   a. For light of wavelength 5.0 nm, what is the energy (in eV) of a single photon? In what part of the electromagnetic spectrum is this?
   b. For an electron with a de Broglie wavelength of 5.0 nm, what is the kinetic energy (in eV).

54. | Gamma rays are photons with very high energy.
   a. What is the wavelength of a gamma-ray photon with energy 625 keV?
   b. How many visible-light photons with a wavelength of 500 nm would you need to match the energy of this one gamma-ray photon?

55. || A red laser with a wavelength of 650 nm and a blue laser with a wavelength of 450 nm emit laser beams with the same light power. What is the ratio of the red laser's photon emission rate (photons per second) to the blue laser's photon emission rate?

56. || BIO INT A typical incandescent lightbulb emits approximately 3×10^{18} visible-light photons per second. Your eye, when it is fully dark adapted, can barely see the light from an incandescent lightbulb 10 km away. How many photons per second are incident at the image point on your retina? The diameter of a dark-adapted pupil is 6 mm.

57. || BIO The intensity of sunlight hitting the surface of the earth on a cloudy day is about 0.50 kW/m^2. Assuming your pupil can close down to a diameter of 2.0 mm and that the average wavelength of visible light is 550 nm, how many photons per second of visible light enter your eye if you look up at the sky on a cloudy day?

58. ||| INT A red LED (light emitting diode) is connected to a battery; it carries a current. As electrons move through the diode, they jump between states, emitting photons in the process. Assume that each electron that travels through the diode causes the emission of a single 630 nm photon. What current is necessary to produce 5.0 mW of emitted light?

59. || A ruby laser emits an intense pulse of light that lasts a mere 10 ns. The light has a wavelength of 690 nm, and each pulse has an energy of 500 mJ.
   a. How many photons are emitted in each pulse?
   b. What is the *rate* of photon emission, in photons per second, during the 10 ns that the laser is "on"?

60. ||| BIO INT The human body emits thermal electromagnetic radiation, as we've seen. Assuming that all radiation is emitted at the wavelength of peak intensity, for a skin temperature of 33°C and a surface area of 1.8 m^2, how many photons per second does the body emit?

61. ||| INT The wavelength of the radiation in a microwave oven is 12 cm. How many photons are absorbed by 200 g of water as it's heated from 20°C to 90°C?

62. ||| BIO Exposure to a sufficient quantity of ultraviolet will redden the skin, producing *erythema*—a sunburn. The amount of exposure necessary to produce this reddening depends on the wavelength. For a 1.0 cm^2 patch of skin, 3.7 mJ of ultraviolet light at a wavelength of 254 nm will produce reddening; at 300 nm wavelength, 13 mJ are required.
   a. What is the photon energy corresponding to each of these wavelengths?
   b. How many total photons does each of these exposures correspond to?
   c. Explain why there is a difference in the number of photons needed to provoke a response in the two cases.

63. ||| INT A silicon solar cell looks like a battery with a 0.50 V terminal voltage. Suppose that 1.0 W of light of wavelength 600 nm falls on a solar cell and that 50% of the photons give their energy to charge carriers, creating a current. What is the solar cell's efficiency—that is, what percentage of the energy incident on the cell is converted to electric energy?

64. ||| INT Electrons with a speed of 2.0×10^6 m/s pass through a double-slit apparatus. Interference fringes are detected with a fringe spacing of 1.5 mm.
   a. What will the fringe spacing be if the electrons are replaced by neutrons with the same speed?
   b. What speed must neutrons have to produce interference fringes with a fringe spacing of 1.5 mm?

65. | INT Electrons pass through a 1.0-μm-wide slit with a speed of 1.5×10^6 m/s. How wide is the electron diffraction pattern on a detector 1.0 m behind the slit?

66. | INT The electron interference pattern of Figure 28.14 was made by shooting electrons with 50 keV of kinetic energy through two slits spaced 1.0 μm apart. The fringes were recorded on a detector 1.0 m behind the slits.
   a. What was the speed of the electrons? (The speed is large enough to justify using relativity, but for simplicity do this as a nonrelativistic calculation.)
   b. Figure 28.14 is greatly magnified. What was the actual spacing on the detector between adjacent bright fringes?

67. |||| INT It is stated in the text that special relativity must be used to calculate the de Broglie wavelength of electrons in an electron microscope. Let us discover how much of an effect relativity has. Consider an electron accelerated through a potential difference of 1.00×10^5 V.
   a. Using the Newtonian (nonrelativistic) expressions for kinetic energy and momentum, what is the electron's de Broglie wavelength?
   b. The de Broglie wavelength is $\lambda = h/p$, but the momentum of a relativistic particle is not mv. Using the relativistic expressions for kinetic energy and momentum, what is the electron's de Broglie wavelength?

68. || An electron confined to a one-dimensional box of length 0.70 nm jumps from the $n = 2$ level to the ground state. What is the wavelength (in nm) of the emitted photon?

69. | a. What is the minimum energy of a 2.7 g Ping-Pong ball in a 10-cm-long box?
   b. What speed corresponds to this kinetic energy?

70. ||| The color of dyes results from the preferential absorption of certain wavelengths of light. Certain dye molecules consist of symmetric pairs of rings joined at the center by a chain of carbon atoms, as shown in Figure P28.70. Electrons of the bonds along the chain of carbon atoms are shared among the atoms in the chain, but are repelled by the nitrogen-containing rings at the end of the chain. These electrons are thus free to move along the chain but not beyond its ends. They look very much like a particle in a one-dimensional box. For the molecule shown, the effective length of the "box" is 0.85 nm. Assuming that the electrons start in the lowest energy state, what are the three longest wavelengths this molecule will absorb?

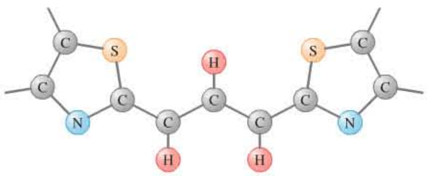


FIGURE P28.70

71. || What is the length of a box in which the difference between an electron's first and second allowed energies is 1.0×10^{-19} J?

72. || Two adjacent allowed energies of an electron in a one-dimensional box are 2.0 eV and 4.5 eV. What is the length of the box?

73. ||| An electron confined to a box has an energy of 1.28 eV. Another electron confined to an identical box has an energy of 2.88 eV. What is the smallest possible length for those boxes?

74. || BIO INT Consider a small virus having a diameter of 10 nm. The atoms of the intracellular fluid are confined within this "box." Suppose we model the virus as a one-dimensional box of length 10 nm. What is the ground-state energy (in eV) of a sodium ion confined in such a box?

75. ||| It can be shown that the allowed energies of a particle of mass m in a two-dimensional square box of side L are

$$E_{nm} = \frac{h^2}{8mL^2}(n^2 + l^2)$$

The energy depends on two quantum numbers, n and l, both of which must have an integer value 1, 2, 3, ...
   a. What is the minimum energy for a particle in a two-dimensional square box of side L?
   b. What are the five lowest allowed energies? Give your values as multiples of $E_{\min}$.

76. ||| An electron confined in a one-dimensional box emits a 200 nm photon in a quantum jump from $n = 2$ to $n = 1$. What is the length of the box?

77. ||| A proton confined in a one-dimensional box emits a 2.0 MeV gamma-ray photon in a quantum jump from $n = 2$ to $n = 1$. What is the length of the box?

78. ||| As an electron in a one-dimensional box of length 0.600 nm jumps between two energy levels, a photon of energy 8.36 eV is emitted. What are the quantum numbers of the two levels?

79. || Magnetic resonance is used in imaging; it is also a useful tool for analyzing chemical samples. Magnets for magnetic resonance experiments are often characterized by the proton resonance frequency they create. What is the field strength of an 800 MHz magnet?

80. ||| INT The electron has a magnetic moment, so you can do magnetic resonance measurements on substances with unpaired electron spins. The electron has a magnetic moment $\mu = 9.3 \times 10^{-24}$ J/T. A sample is placed in a solenoid of length 15 cm with 1200 turns of wire carrying a current of 3.5 A. A probe coil provides radio waves to "flip" the spins. What is the necessary frequency for the probe coil?

Passage Problems

Compton Scattering

Further support for the photon model of electromagnetic waves comes from *Compton scattering,* in which x rays scatter from electrons, changing direction and frequency in the process. Classical electromagnetic wave theory cannot explain the change in frequency of the x rays on scattering, but the photon model can.

Suppose an x-ray photon is moving to the right. It has a collision with a slow-moving electron, as in Figure P28.81. The photon transfers energy and momentum to the electron, which recoils at a high speed. The x-ray photon loses energy, and the photon energy formula $E = hf$ tells us that its frequency must decrease. The collision looks very much like the collision between two particles.

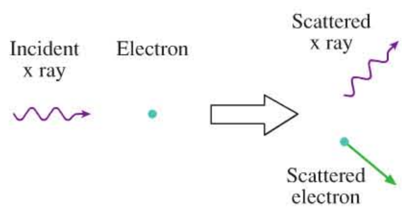


FIGURE P28.81

81. | When the x-ray photon scatters from the electron,
   A. Its speed increases.
   B. Its speed decreases.
   C. Its speed stays the same.

82. | When the x-ray photon scatters from the electron,
   A. Its wavelength increases.
   B. Its wavelength decreases.
   C. Its wavelength stays the same.

83. | When the electron is struck by the x-ray photon,
   A. Its de Broglie wavelength increases.
   B. Its de Broglie wavelength decreases.
   C. Its de Broglie wavelength stays the same.

84. | X-ray diffraction can also change the direction of a beam of x rays. Which statement offers the best comparison between Compton scattering and x-ray diffraction?
   A. X-ray diffraction changes the wavelength of x rays; Compton scattering does not.
   B. Compton scattering changes the speed of x rays; x-ray diffraction does not.
   C. X-ray diffraction relies on the particle nature of the x rays; Compton scattering relies on the wave nature.
   D. X-ray diffraction relies on the wave nature of the x rays; Compton scattering relies on the particle nature.

STOP TO THINK ANSWERS

Stop to Think 28.1: A. The Bragg condition $2d\sin\theta_1 = \lambda$ tells us that larger values of d go with smaller values of θ_1.

Stop to Think 28.2: $V_A > V_B > V_C$. For a given wavelength of light, electrons are ejected faster from metals with smaller work functions because it takes less energy to remove an electron. Faster electrons need a larger negative voltage to stop them.

Stop to Think 28.3: C. Photons always travel at c, and a photon's energy depends only on the light's frequency, not its intensity. Greater intensity means more energy each second, which means more photons.

Stop to Think 28.4: A. The widest diffraction pattern occurs for the largest wavelength. The de Broglie wavelength is inversely proportional to the particle's mass, and so will be largest for the least massive particle.

Stop to Think 28.5: No. The energy of an emitted photon is the energy *difference* between two allowed energies. The three possible quantum jumps have energy differences of 2.0 eV, 2.0 eV, and 4.0 eV.

Stop to Think 28.6: B. Because $\Delta p_x = m\Delta v_x$, the uncertainty in position is $\Delta x = \dfrac{h}{\Delta p_x} = \dfrac{h}{m\Delta v_x}$. A more massive particle has a smaller position uncertainty.