13 Fluids

The 20,000 pound boat floats while the 130 pound diver sinks. What determines whether an object floats or sinks?

LOOKING AHEAD ►

The goal of Chapter 13 is to understand the static and dynamic properties of fluids.

Pressure in Fluids

The pressure in a **fluid**—a gas or a liquid—exerts a force on the fluid's container as well as on all parts of the fluid itself.

The pressure in a liquid increases with depth. The high pressure at the base of this water tower allows water to be distributed throughout the city.

Looking Back ◄
5.1 Equilibrium
12.3 Pressure in gases

Measuring Pressure

Pressure is measured by the force it exerts on a known area. An important kind of pressure gauge is the **manometer.**

Blood pressure can be measured by the height of the mercury in this manometer.

Buoyancy

Why do some objects float while others sink? You'll learn that the **bouyant force**, the upward force of a fluid on an immersed object, is described by a simple but powerful concept called **Archimedes' principle.**

Teams from across the country compete in the annual National Concrete Canoe Competition. You'll learn how such a heavy object can stay afloat.

Fluid Dynamics

The volume of a moving fluid is conserved. This requires flowing fluids to speed up as they squeeze through a narrow section of a tube.

You'll learn why holding your thumb over the end of a hose makes the water come out faster.

Looking Back ◄
4.6, 5.2 Newton's second law
10.2 Work
10.6 Conservation of energy

In studying moving fluids, we'll use many of the same ideas we used for studying the motion of particles: Newton's laws and conservation of energy.

Moving fluids can exert large forces on objects. **Bernoulli's equation**, a statement of conservation of energy applied to fluids, gives us a way to calculate pressures and forces due to moving fluids.

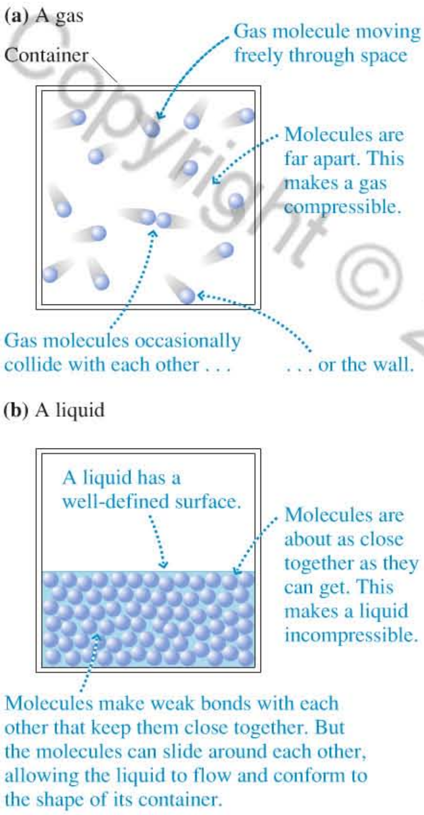
You'll learn how forces due to the motion of fluids can be enough to lift a massive airplane or lift the roof off a house during a hurricane.

Viscosity

You know from experience that some liquids, such as water, flow freely while others, like honey, are hard to pour. Honey has a much higher **viscosity** than water. You'll learn how viscosity affects the flow of fluids.

13.1 Fluids and Density

FIGURE 13.1 Simple atomic-level models of gases and liquids.



A **fluid** is simply a substance that flows. Because they flow, fluids take the shape of their container rather than retaining a shape of their own. You may think that gases and liquids are quite different, but both are fluids, and their similarities are often more important than their differences.

As you learned in Chapter 12, a gas, as shown in FIGURE 13.1a, is a system in which each molecule moves freely through space until, on occasion, it collides with another molecule or with the wall of the container. The gas you are most familiar with is air, a mixture of mostly nitrogen and oxygen molecules. Gases are *compressible;* that is, the volume of a gas is easily increased or decreased, a consequence of the "empty space" between the molecules in a gas.

Liquids are more complicated than either gases or solids. Liquids, like solids, are essentially *incompressible.* This property tells us that the molecules in a liquid, as in a solid, are about as close together as they can get without coming into contact with each other. At the same time, a liquid flows and deforms to fit the shape of its container. The fluid nature of a liquid tells us that the molecules are free to move around.

Together, these observations suggest the model of a liquid shown in FIGURE 13.1b. Here you see a system in which the molecules are loosely held together by weak molecular bonds. The bonds are strong enough that the molecules never get far apart but not strong enough to prevent the molecules from sliding around each other.

Density

An important parameter that characterizes a macroscopic system is its *density.* Suppose you have several blocks of copper, each of different size. Each block has a different mass m and a different volume V. Nonetheless, all the blocks are copper, so there should be some quantity that has the *same* value for all the blocks, telling us, "This is copper, not some other material." The most important such parameter is the *ratio* of mass to volume, which we call the **mass density** ρ (lowercase Greek rho):

$$\rho = \frac{m}{V} \tag{13.1}$$

Mass density of an object of mass m and volume V

Conversely, an object of mass density ρ and volume V has mass

$$m = \rho V \tag{13.2}$$

The SI units of mass density are kg/m^3. Nonetheless, units of g/cm^3 are widely used. You need to convert these to SI units before doing most calculations. You must convert both the grams to kilograms and the cubic centimeters to cubic meters. The net result is the conversion factor

$$1\ g/cm^3 = 1000\ kg/m^3$$

The mass density is independent of the object's size. That is, mass and volume are parameters that characterize a *specific piece* of some substance—say, copper—whereas mass density characterizes the substance itself. All pieces of copper have the same mass density, which differs from the mass density of almost any other substance. Thus mass density allows us to talk about the properties of copper in general without having to refer to any specific piece of copper.

The mass density is usually called simply "the density" if there is no danger of confusion. However, we will meet other types of density as we go along, and sometimes it is important to be explicit about which density you are using. Table 13.1 provides a short list of the mass densities of various fluids. Notice the enormous difference between the densities of gases and liquids. Gases have lower densities because the molecules in gases are farther apart than in liquids. Also, the density of a

TABLE 13.1 Densities of fluids at 1 atm pressure

Substance	ρ (kg/m^3)
Helium gas (20°C)	0.166
Air (20°C)	1.20
Air (0°C)	1.28
Gasoline	680
Ethyl alcohol	790
Oil (typical)	900
Water	1000
Seawater	1030
Blood (whole)	1060
Glycerin	1260
Mercury	13,600

liquid varies only slightly with temperature because its molecules are always nearly in contact. The density of a gas, such as air, has a larger variation with temperature because it's easy to change the already large distance between the molecules.

What does it *mean* to say that the density of gasoline is 680 kg/m^3? Recall in Chapter 1 we discussed the meaning of the word "per." We found that it meant "for each," so that 2 miles per hour means you travel 2 miles *for each* hour that passes. In the same way, saying that the density of gasoline is 680 kg per cubic meter means that there are 680 kg of gasoline *for each* 1 cubic meter of the liquid. If we have 2 m^3 of gasoline, each will have a mass of 680 kg, so the total mass will be 2×680 kg $= 1360$ kg. The product ρV is the number of cubic meters times the mass of each cubic meter—that is, the total mass of the object.

EXAMPLE 13.1 Weighing the air in a living room

What is the mass of air in a living room with dimensions 4.0 m × 6.0 m × 2.5 m?

PREPARE Table 13.1 gives air density at a temperature of 20°C, which is about room temperature.

SOLVE The room's volume is

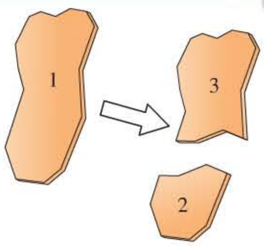
$$V = (4.0\text{ m}) \times (6.0\text{ m}) \times (2.5\text{ m}) = 60\text{ m}^3$$

The mass of the air is

$$m = \rho V = (1.20\text{ kg/m}^3)(60\text{ m}^3) = 72\text{ kg}$$

ASSESS This is perhaps more mass—about that of an adult person—than you might have expected from a substance that hardly seems to be there. For comparison, a swimming pool this size would contain 60,000 kg of water.

STOP TO THINK 13.1 A piece of glass is broken into two pieces of different size. Rank in order, from largest to smallest, the mass densities of pieces 1, 2, and 3.



13.2 Pressure

In Chapter 12, you learned how a gas exerts a force on the walls of its containers. Liquids also exert forces on the walls of their containers, as shown in **FIGURE 13.2**, where a force $\vec{F}$ due to the liquid pushes against a small area A of the wall. Just as for a gas, we define the pressure at this point in the fluid to be the ratio of the force to the area on which the force is exerted:

$$p = \frac{F}{A} \tag{13.3}$$

This is the same as Equation 12.9 of Chapter 12. Recall also from Chapter 12 that the SI unit of pressure, the pascal, is defined as

$$1\text{ pascal} = 1\text{ Pa} = 1\,\frac{\text{N}}{\text{m}^2}$$

It's important to realize that the force due to a fluid's pressure pushes not only on the walls of its container, but on *all* parts of the fluid itself. If you punch holes in a container of water, the water spurts out from the holes, as in **FIGURE 13.3**. It is the force due to the pressure of the water behind each hole that pushes the water forward through the holes.

FIGURE 13.2 The fluid presses against area A with force $\vec{F}$.

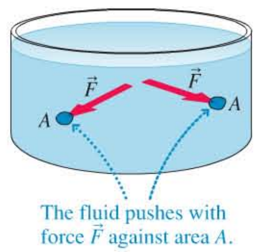
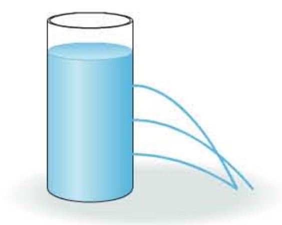
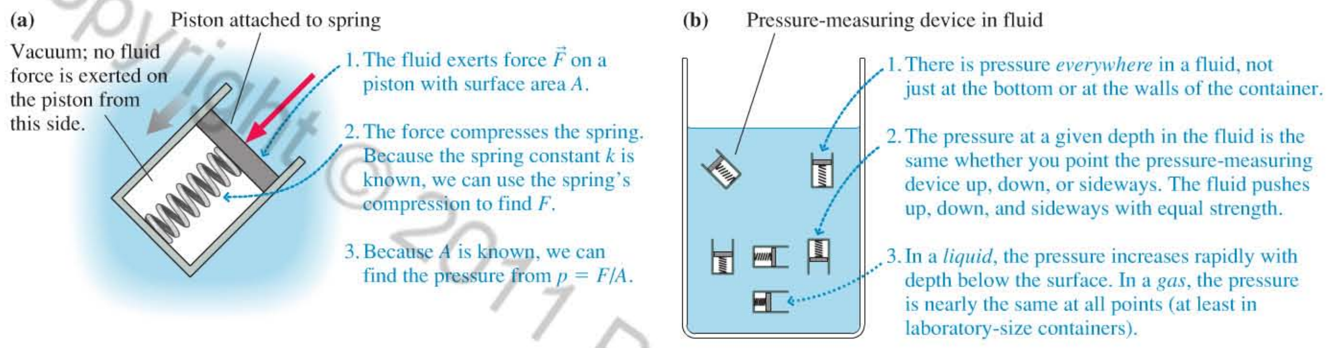


FIGURE 13.3 Pressure pushes the water *sideways*, out of the holes.



To measure the pressure at any point within a fluid we can use the simple pressure-measuring device shown in **FIGURE 13.4a**. Because the spring constant k and the area A are known, we can determine the pressure by measuring the compression of the spring. Once we've built such a device, we can place it in various liquids and gases to learn about pressure. **FIGURE 13.4b** shows what we can learn from simple experiments.

FIGURE 13.4 Learning about pressure.



The first statement in Figure 13.4b emphasizes again that pressure exists at *all* points within a fluid, not just at the walls of the container. You may recall that tension exists at *all* points in a string, not only at its ends where it is tied to an object. We understood tension as the different parts of the string *pulling* against each other. Pressure is an analogous idea, except that the different parts of a fluid are *pushing* against each other.

Pressure in Liquids

If you introduce a liquid into a container, the force of gravity pulls the liquid down, causing it to fill the bottom of the container. It is this force of gravity—that is, the weight of the liquid—that is responsible for the pressure in a liquid. Pressure increases with depth in a liquid because the liquid below is being squeezed by all the liquid above, including any other liquid floating on the first liquid, as well as the pressure of the air above the liquid.

We'd like to determine the pressure at a depth d below the surface of the liquid. We will assume that the liquid is at rest; flowing liquids will be considered later in this chapter. The darker shaded cylinder of liquid in **FIGURE 13.5** extends from the surface to depth d. This cylinder, like the rest of the liquid, is in static equilibrium with $\vec{F}_{net} = \vec{0}$. Several forces act on this cylinder: its weight mg, a downward force p_0A due to the pressure p_0 at the surface of the liquid, an upward force pA due to the liquid beneath the cylinder pushing up on the bottom of the cylinder, and inward-directed forces due to the liquid pushing in on the sides of the cylinder. The forces due to the liquid pushing on the cylinder are a consequence of our earlier observation that different parts of a fluid push against each other. Pressure p, the pressure at the bottom of the cylinder, is what we're trying to find.

The horizontal forces cancel each other. The upward force balances the two downward forces, so

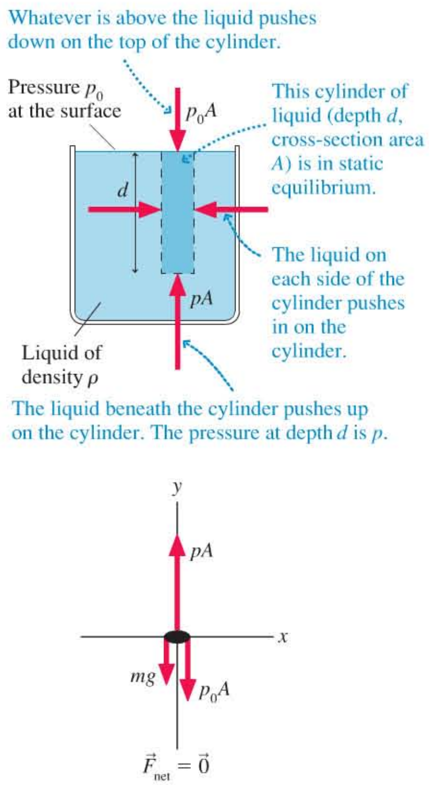
$$pA = p_0A + mg \tag{13.4}$$

The liquid is a cylinder of cross-section area A and height d. Its volume is $V = Ad$ and its mass is $m = \rho V = \rho Ad$. Substituting this expression for the mass of the liquid into Equation 13.4, we find that the area A cancels from all terms. The pressure at depth d in a liquid is then

$$p = p_0 + \rho gd \tag{13.5}$$

Pressure of a liquid with density ρ at depth d

FIGURE 13.5 Measuring the pressure at depth d in a liquid.



Free-body diagram of the column of liquid. The horizontal forces cancel and are not shown.

Because of our assumption that the fluid is at rest, the pressure given by Equation 13.5 is called the **hydrostatic pressure.** The fact that g appears in Equation 13.5 reminds us that the origin of this pressure is the gravitational force on the fluid.

As expected, $p = p_0$ at the surface, where $d = 0$. Pressure p_0 is usually due to the air or other gas above the liquid. For a liquid that is open to the air at sea level, $p_0 = 1$ atm $= 101.3$ kPa, as we learned in Section 12.2. In other situations, p_0 might be the pressure due to a piston or a closed surface pushing down on the top of the liquid.

NOTE ► Equation 13.5 assumes that the fluid is *incompressible;* that is, its density ρ doesn't increase with depth. This is an excellent assumption for liquids, but not a good one for a gas. Equation 13.5 should not be used for calculating the pressure of a gas. Gas pressure is found with the ideal-gas law. ◄

EXAMPLE 13.2 The pressure on a submarine

A submarine cruises at a depth of 300 m. What is the pressure at this depth? Give the answer in both pascals and atmospheres.

SOLVE The density of seawater, from Table 13.1, is $\rho = 1030$ kg/m^3. At the surface, $p_0 = 1$ atm $= 101.3$ kPa. The pressure at depth $d = 300$ m is found from Equation 13.5 to be

$$p = p_0 + \rho g d$$
$$= (1.013 \times 10^5 \text{ Pa}) + (1030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(300 \text{ m})$$
$$= 3.13 \times 10^6 \text{ Pa}$$

Converting the answer to atmospheres gives

$$p = (3.13 \times 10^6 \text{ Pa}) \times \frac{1 \text{ atm}}{1.013 \times 10^5 \text{ Pa}} = 30.9 \text{ atm}$$

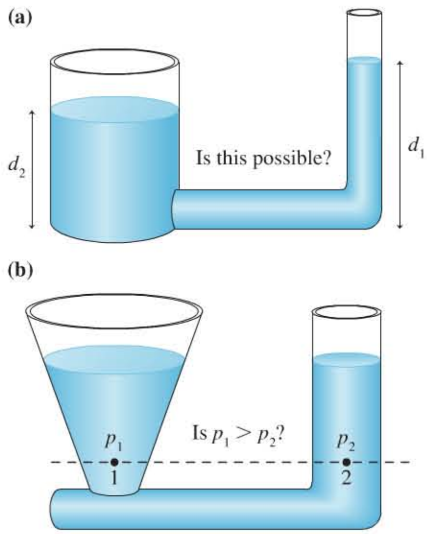
ASSESS The pressure deep in the ocean is very great. The research submarine *Alvin,* shown in the left photo, can safely dive as deep as 4500 m, where the pressure is over 450 atm! Its viewports are over 3.5 inches thick to withstand this pressure. As shown in the right photo, each viewport is tapered, with its larger face toward the sea. The water pressure then pushes the viewports firmly into their conical seats, helping to seal them tightly.

According to Equation 13.5, the hydrostatic pressure in a liquid depends on only the depth and the pressure at the surface. This observation has some important implications. **FIGURE 13.6a** shows two connected tubes. It's certainly true that the larger volume of liquid in the wide tube weighs more than the liquid in the narrow tube. You might think that this extra weight would push the liquid in the narrow tube higher than in the wide tube. But it doesn't. If d_1 were larger than d_2, then, according to the hydrostatic pressure equation, the pressure at the bottom of the narrow tube would be higher than the pressure at the bottom of the wide tube. This *pressure difference* would cause the liquid to *flow* from right to left until the heights were equal.

Thus a first conclusion: **A connected liquid in hydrostatic equilibrium rises to the same height in all open regions of the container.** This is the familiar observation that "water seeks its own level."

FIGURE 13.6b shows two connected tubes of different shape. The conical tube holds more liquid above the dotted line, so you might think that $p_1 > p_2$. But it isn't. Both points are at the same depth; thus $p_1 = p_2$. You can arrive at the same conclusion by thinking about the pressure at the bottoms of the tubes. If p_1 were greater than p_2, the pressure at the bottom of the left tube would be greater than the pressure at the bottom of the right tube. This would cause the liquid to flow until the pressures were equal. Thus a second conclusion: **In hydrostatic equilibrium, the pressure is the same at all points on a horizontal line through a connected liquid of a single kind.** (When the liquid is not the same kind at different points on the line, the pressure need not be the same.)

FIGURE 13.6 Some properties of a liquid in hydrostatic equilibrium are not what you might expect.

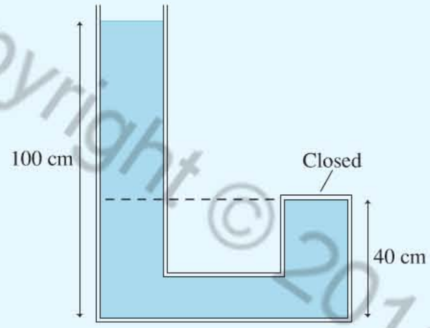


NOTE ► Both of these conclusions are restricted to liquids in hydrostatic equilibrium. The situation is entirely different for flowing fluids, as we'll see later on. ◄

EXAMPLE 13.3 **Pressure in a closed tube**

Water fills the tube shown in FIGURE 13.7. What is the pressure at the top of the closed tube?

FIGURE 13.7 A bent tube closed at one end.



PREPARE This is a liquid in hydrostatic equilibrium. The closed tube is not an open region of the container, so the water cannot rise to an equal height. Nevertheless, the pressure is still the same at all points on a horizontal line. In particular, the pressure at the top of the closed tube equals the pressure in the open tube at the height of the dotted line. Assume $p_0 = 1$ atm.

SOLVE A point 40 cm above the bottom of the open tube is at a depth of 60 cm. The pressure at this depth is

$$
\begin{aligned}
p &= p_0 + \rho g d \\
&= (1.01 \times 10^5\ \text{Pa}) + (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.60\ \text{m}) \\
&= 1.07 \times 10^5\ \text{Pa} = 1.06\ \text{atm}
\end{aligned}
$$

ASSESS The water in the open tube *pushes* the water in the closed tube up against the top of the tube. Consequently, in accordance with Newton's third law, the top of the tube *presses down on the liquid* with a force of magnitude $F = pA$. This explains why the pressure at the top of the closed tube is greater than atmospheric pressure.

We can draw one more conclusion from the hydrostatic pressure equation $p = p_0 + \rho g d$. If we change the pressure at the surface to $p_1 = p_0 + \Delta p$, so that Δp is the *change* in pressure, then the pressure at a point at a depth d becomes

$$p' = p_1 + \rho g d = (p_0 + \Delta p) + \rho g d = (p_0 + \rho g d) + \Delta p = p + \Delta p$$

That is, the pressure at depth d changes by the same amount as it did at the surface. This idea was first recognized by Blaise Pascal (the same Pascal for whom the pressure unit is named) and is called *Pascal's principle:*

Pascal's principle If the pressure at one point in an incompressible fluid is changed, the pressure at every other point in the fluid changes by the same amount.

For example, if we compress the air above the open tube in Example 13.3 to a pressure of 1.50 atm, an increase of 0.50 atm, the pressure at the top of the closed tube will increase to 1.56 atm.

STOP TO THINK 13.2 Water is slowly poured into the container until the water level has risen into tubes 1, 2, and 3. The water doesn't overflow from any of the tubes. How do the water depths in the three columns compare to each other?

A. $d_1 > d_2 > d_3$
B. $d_1 < d_2 < d_3$
C. $d_1 = d_2 = d_3$
D. $d_1 = d_2 > d_3$
E. $d_1 = d_2 < d_3$

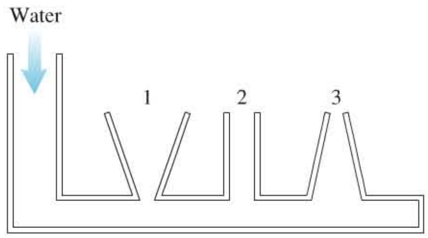
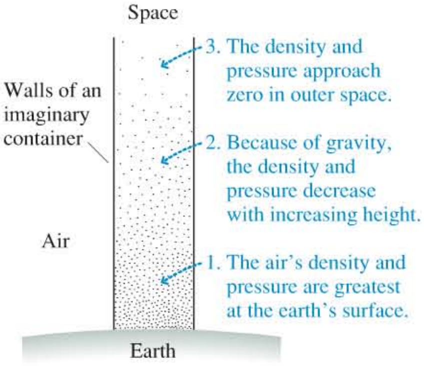


FIGURE 13.8 Atmospheric pressure and density.



Atmospheric Pressure

We live at the bottom of a "sea" of air that extends up many kilometers. As FIGURE 13.8 shows, there is no well-defined top to the atmosphere; it just gets less and less dense with increasing height until reaching zero in the vacuum of space. Nonetheless, 99% of the air in the atmosphere is below about 30 km.

If we recall that a gas like air is quite compressible, we can see why the atmosphere becomes less dense with increasing altitude. In a liquid, pressure increases with depth because of the weight of the liquid above. The same holds true for the air in the atmosphere, but because the air is compressible, the weight of the air above compresses the air below, increasing its density. At high altitudes there is very little air above to push down, so the density is less.

We learned in Chapter 12 that the global average sea-level pressure, the *standard atmosphere,* is 1 atm = 101,300 Pa. The standard atmosphere, usually referred to simply as "atmospheres," is a commonly used unit of pressure. But it is not an SI unit, so you must convert atmospheres to pascals before doing most calculations with pressure.

NOTE ► Unless you happen to live right at sea level, the atmospheric pressure around you is not exactly 1 atm. A pressure gauge must be used to determine the actual atmospheric pressure. For simplicity, this textbook will always assume that the pressure of the air is $p_{\text{atmos}} = 1$ atm unless stated otherwise. ◄

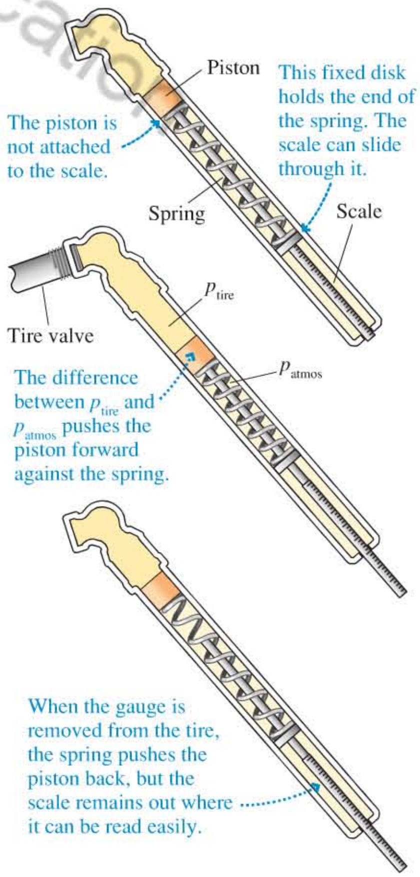
Atmospheric pressure varies not only with altitude, but also with changes in the weather. Large-scale regions of low-pressure air are created at the equator, where hot air rises and flows to the north and south temperate zones. There the air falls, creating high-pressure zones. Local winds and weather are largely determined by presence and movement of air masses of differing pressure. You may have seen weather maps like the one shown in **FIGURE 13.9** on the evening news. The letters H and L denote regions of high and low atmospheric pressure.

FIGURE 13.9 High- and low-pressure zones on a weather map.

13.3 Measuring and Using Pressure

The pressure in a fluid is measured with a *pressure gauge,* which is often a device very similar to that shown in Figure 13.4. The fluid pushes against a spring, and the displacement of the spring is indicated on a scale. The familiar tire-pressure gauge shown in **FIGURE 13.10** works in the same way. The pressure p_{tire} exerts a force $p_{\text{tire}}A$ on the front area A of the piston, while atmospheric pressure exerts a force $p_{\text{atmos}}A$ on the back of the piston. Thus the *net* pressure force on the piston is $(p_{\text{tire}} - p_{\text{atmos}})A$. This force compresses the spring until equilibrium is reached. Thus the movement of the scale depends not on the absolute pressure in the tire, but on the *difference* between the tire pressure and atmospheric pressure. This type of gauge measures the *gauge pressure* $p_g = p_{\text{tire}} - p_{\text{atmos}}$, an idea introduced in Chapter 12.

FIGURE 13.10 A tire gauge measures the difference between the tire's pressure and atmospheric pressure.



Solving Hydrostatic Problems

We now have enough information to formulate a set of rules for working with hydrostatic problems.

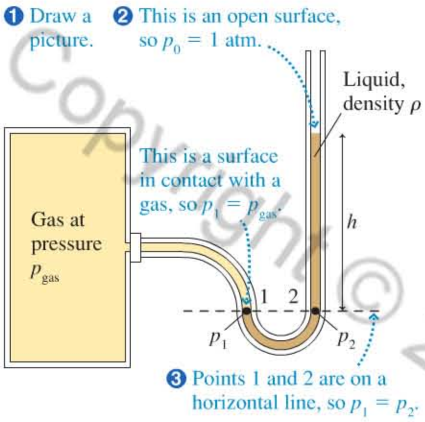
TACTICS BOX 13.1 Hydrostatics MP™

1. **Draw a picture.** Show open surfaces, pistons, boundaries, and other features that affect pressure. Include height and area measurements and fluid densities. Identify the points at which you need to find the pressure.
2. **Determine the pressure p_0 at surfaces.**
   - **Surface open to the air:** $p_0 = p_{\text{atmos}}$, usually 1 atm.
   - **Surface in contact with a gas:** $p_0 = p_{\text{gas}}$.
   - **Closed surface:** $p_0 = F/A$, where F is the force that the surface, such as a piston, exerts on the fluid.
3. **Use horizontal lines.** The pressure in a connected fluid (of one kind) is the same at any point along a horizontal line.
4. **Allow for gauge pressure.** Pressure gauges read $p_g = p - 1$ atm.
5. **Use the hydrostatic pressure equation:** $p = p_0 + \rho g d$.

Exercises 5–7

Manometers and Barometers

FIGURE 13.11 A manometer is used to measure gas pressure.



Gas pressure is sometimes measured with a device called a *manometer.* A manometer, shown in FIGURE 13.11, is a U-shaped tube connected to the gas at one end and open to the air at the other end. The tube is filled with a liquid—often mercury—of density ρ. The liquid is in static equilibrium. A scale allows the user to measure the height h of the right side of the liquid above the left side.

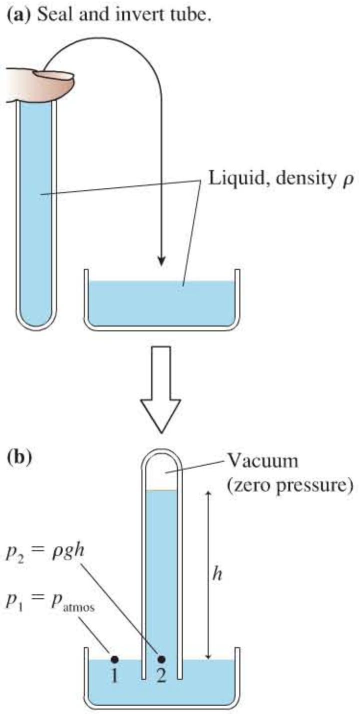
Steps 1–3 from Tactics Box 13.1 lead to the conclusion that the pressures p_1 and p_2 must be equal. Pressure p_1, at the surface on the left, is simply the gas pressure: $p_1 = p_{gas}$. Pressure p_2 is the hydrostatic pressure at depth $d = h$ in the liquid on the right: $p_2 = 1\text{ atm} + \rho gh$. Equating these two pressures gives

$$p_{gas} = 1\text{ atm} + \rho gh \tag{13.6}$$

NOTE ► The height h has a *positive* value when the liquid is *higher* on the right than on the left ($p_{gas} > 1$ atm) and a *negative* value when the liquid is *lower* on the right than on the left ($p_{gas} < 1$ atm). ◄

Another important pressure-measuring instrument is the *barometer,* which is used to measure atmospheric pressure p_{atmos}. FIGURE 13.12a shows a glass tube, sealed at the bottom, that has been completely filled with a liquid. If we temporarily seal the top end, we can invert the tube and place it in a beaker of the same liquid. When the temporary seal is removed, some, but not all, of the liquid runs out, leaving a liquid column in the tube that is a height h above the surface of the liquid in the beaker. This device, shown in FIGURE 13.12b, is a barometer. What does it measure? And why doesn't *all* the liquid in the tube run out?

FIGURE 13.12 A barometer.



We can analyze the barometer much as we did the manometer. Point 1 in Figure 13.12b is open to the atmosphere, so $p_1 = p_{atmos}$. The pressure at point 2 is the pressure due to the weight of the liquid in the tube plus the pressure of the gas above the liquid. But in this case there is no gas above the liquid! Because the tube had been completely full of liquid when it was inverted, the space left behind when the liquid ran out is essentially a vacuum, with $p_0 = 0$. Thus pressure p_2 is simply ρgh.

Because points 1 and 2 are on a horizontal line, and the liquid is in hydrostatic equilibrium, the pressures at these two points must be equal. Equating these two pressures gives

$$p_{atmos} = \rho gh \tag{13.7}$$

Thus we can measure the atmosphere's pressure by measuring the height of the liquid column in a barometer.

Equation 13.7 shows that the liquid height is $h = p_{atmos}/\rho g$. If a barometer were made using water, with $\rho = 1000\text{ kg/m}^3$, the liquid column would be more than 10 m high, which is impractical. Instead, mercury, with its high density of $13{,}600\text{ kg/m}^3$, is usually used. The average air pressure at sea level causes a column of mercury in a mercury barometer to stand 760 mm above the surface. We can then use Equation 13.7 to find that the average atmospheric pressure is

$$\begin{aligned} p_{atmos} &= \rho_{Hg} gh = (13{,}600\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.760\text{ m}) \\ &= 1.013 \times 10^5\text{ Pa} = 101.3\text{ kPa} \end{aligned}$$

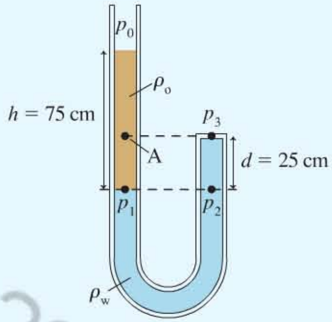
This is the value given earlier as "1 standard atmosphere."

Because of the importance of mercury-filled barometers in measuring pressure, the height of a column of mercury in millimeters is a common unit of pressure. From our discussion of barometers, 760 millimeters of mercury (abbreviated "mm Hg") corresponds to a pressure of 1 atm.

EXAMPLE 13.4 Pressure in a tube with two liquids

A U-shaped tube is closed at one end; the other end is open to the atmosphere. Water fills the side of the tube that includes the closed end, while oil, floating on the water, fills the side of the tube open to the atmosphere. The two liquids do not mix. The height of the oil above the point where the two liquids touch is 75 cm, while the height of the closed end of the tube above this point is 25 cm. What is the gauge pressure at the closed end?

FIGURE 13.13 A tube containing two different liquids.



PREPARE Following the steps in Tactics Box 13.1, we start by drawing the picture shown in **FIGURE 13.13**. We know that the pressure at the open surface of the oil is $p_0 = 1$ atm. Pressures p_1 and p_2 are the same because they are on a horizontal line that connects two points in the *same* fluid. (The pressure at point A is *not* equal to p_3, even though point A and the closed end are on the same horizontal line, because the two points are in *different* fluids.)

We can apply the hydrostatic pressure equation twice: once to find the pressure p_1 by its known depth below the open end at pressure p_0, and again to find the pressure p_3 at the closed end once we know p_2 a distance d below it. We'll need the densities of water and oil, which are found in Table 13.1 to be $\rho_w = 1000$ kg/m^3 and $\rho_o = 900$ kg/m^3.

SOLVE The pressure at point 1, 75 cm below the open end, is

$$\begin{aligned} p_1 &= p_0 + \rho_o g h \\ &= 1 \text{ atm} + (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.75 \text{ m}) \\ &= 1 \text{ atm} + 6600 \text{ Pa} \end{aligned}$$

(We will keep $p_0 = 1$ atm separate in this result because we'll eventually need to subtract exactly 1 atm to calculate the gauge pressure.) We can also use the hydrostatic pressure equation to find

$$\begin{aligned} p_2 &= p_3 + \rho_w g d \\ &= p_3 + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.25 \text{ m}) \\ &= p_3 + 2500 \text{ Pa} \end{aligned}$$

But we know that $p_2 = p_1$, so

$$\begin{aligned} p_3 &= p_2 - 2500 \text{ Pa} = p_1 - 2500 \text{ Pa} \\ &= 1 \text{ atm} + 6600 \text{ Pa} - 2500 \text{ Pa} \\ &= 1 \text{ atm} + 4100 \text{ Pa} \end{aligned}$$

The gauge pressure at point 3, the closed end of the tube, is $p_3 - 1$ atm or 4100 Pa.

ASSESS The oil's open surface is 50 cm higher than the water's closed surface. Their densities are not too different, so we expect a pressure difference of roughly $\rho g(0.50 \text{ m}) = 5000$ Pa. This is not too far from our answer, giving us confidence that it's correct.

Pressure Units

In practice, pressure is measured in a number of different units. This plethora of units and abbreviations has arisen historically as scientists and engineers working on different subjects (liquids, high-pressure gases, low-pressure gases, weather, etc.) developed what seemed to them the most convenient units. These units continue in use through tradition, so it is necessary to become familiar with converting back and forth among them. Table 13.2 gives the basic conversions.

TABLE 13.2 Pressure units

Unit	Abbreviation	Conversion to 1 atm	Uses
pascal	Pa	101.3 kPa	SI unit: 1 Pa = 1 N/m^2 used in most calculations
atmosphere	atm	1 atm	general
millimeters of mercury	mm Hg	760 mm Hg	gases and barometric pressure
inches of mercury	in	29.92 in	barometric pressure in U.S. weather forecasting
pounds per square inch	psi	14.7 psi	U.S. engineering and industry

FIGURE 13.14 Blood pressure during one cycle of a heartbeat.

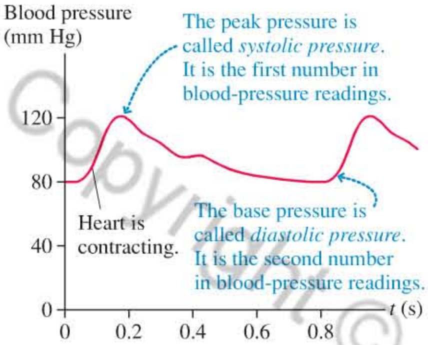


FIGURE 13.15 Measuring blood pressure with a manometer.

Blood Pressure

The last time you had a medical checkup, the doctor may have told you something like, "Your blood pressure is 120 over 80." What does that mean?

About every 0.8 s, assuming a pulse rate of 75 beats per minute, your heart "beats." The heart muscles contract and push blood out into your aorta. This contraction, like squeezing a balloon, raises the pressure in your heart. The pressure increase, in accordance with Pascal's principle, is transmitted through all your arteries.

FIGURE 13.14 is a pressure graph showing how blood pressure changes during one cycle of the heartbeat. The medical condition of *high blood pressure* usually means that your maximum (*systolic*) blood pressure is higher than necessary for blood circulation. The high pressure causes undue stress and strain on your entire circulatory system, often leading to serious medical problems. Low blood pressure can cause you to faint if you stand up quickly because the pressure isn't adequate to pump the blood up to your brain.

As shown in FIGURE 13.15, blood pressure is measured with a cuff that goes around your arm. The doctor or nurse pressurizes the cuff, places a stethoscope over the artery in your arm, then slowly releases the pressure while watching a pressure gauge. Initially, the cuff squeezes the artery shut and cuts off the blood flow. When the cuff pressure drops below the systolic pressure, the pressure pulse during each beat of your heart forces the artery open briefly and a squirt of blood goes through. The doctor or nurse records the pressure when he or she hears the blood start to flow. This is your systolic pressure.

This pulsing of the blood through your artery lasts until the cuff pressure reaches the diastolic pressure. Then the artery remains open continuously and the blood flows smoothly. This transition is easily heard in the stethoscope, and the doctor or nurse records your base or *diastolic* pressure.

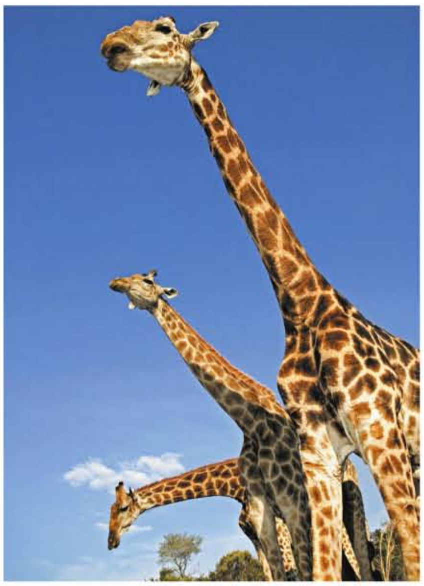
Blood pressure is measured in millimeters of mercury. And it is a gauge pressure, the pressure in excess of 1 atm. A fairly typical blood pressure of a healthy young adult is 120/80, meaning that the systolic pressure is $p_g = 120$ mm Hg (absolute pressure $p = 880$ mm Hg) and the diastolic pressure is 80 mm Hg.

CONCEPTUAL EXAMPLE 13.5 Blood pressure and the height of the arm

In Figure 13.15, the patient's arm is held at about the same height as her heart. Why?

REASON The hydrostatic pressure of a fluid varies with height. Although flowing blood is not in hydrostatic equilibrium, it is still true that blood pressure increases with the distance below the heart and decreases above it. Because the upper arm when held beside the body is at the same height as the heart, the pressure here is the same as the pressure at the heart. If the patient held her arm straight up, the pressure cuff would be a distance $d \approx 25$ cm above her heart and the pressure would be *less* than the pressure at the heart by $\Delta p = \rho_{\text{blood}}\, gd \approx 20$ mm Hg.

ASSESS 20 mm Hg is a substantial fraction of the average blood pressure. Measuring pressure above or below heart level could lead to a misdiagnosis of the patient's condition.



◀ **Pressure at the top** BIO A giraffe's head is some 2.5 m above its heart, compared to a distance of only about 30 cm for humans. To pump blood this extra height requires a blood pressure at the giraffe's heart that is some 170 mm Hg higher than a human's, making its blood pressure more than twice as high as a human's.

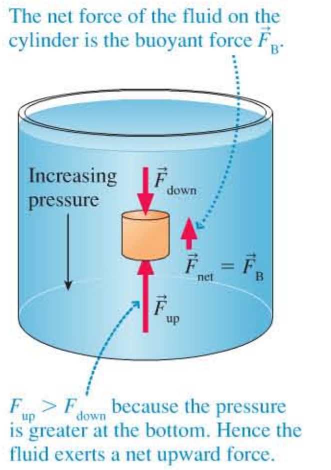
13.4 Buoyancy

A rock, as you know, sinks like a rock. Wood floats on the surface of a lake. A penny with a mass of a few grams sinks, but a massive steel aircraft carrier floats. How can we understand these diverse phenomena?

An air mattress floats effortlessly on the surface of a swimming pool. But if you've ever tried to push an air mattress underwater, you know it is nearly impossible. As you push down, the water pushes up. This upward force of a fluid is called the **buoyant force.**

The basic reason for the buoyant force is easy to understand. FIGURE 13.16 shows a cylinder submerged in a liquid. The pressure in the liquid increases with depth, so the pressure at the bottom of the cylinder is greater than at the top. Both cylinder ends have equal area, so force $\vec{F}_{\text{up}}$ is greater than force $\vec{F}_{\text{down}}$. (Remember that pressure forces push in *all* directions.) Consequently, the pressure in the liquid exerts a *net upward force* on the cylinder of magnitude $F_{\text{net}} = F_{\text{up}} - F_{\text{down}}$. This is the buoyant force.

FIGURE 13.16 The buoyant force arises because the fluid pressure at the bottom of the cylinder is greater than at the top.



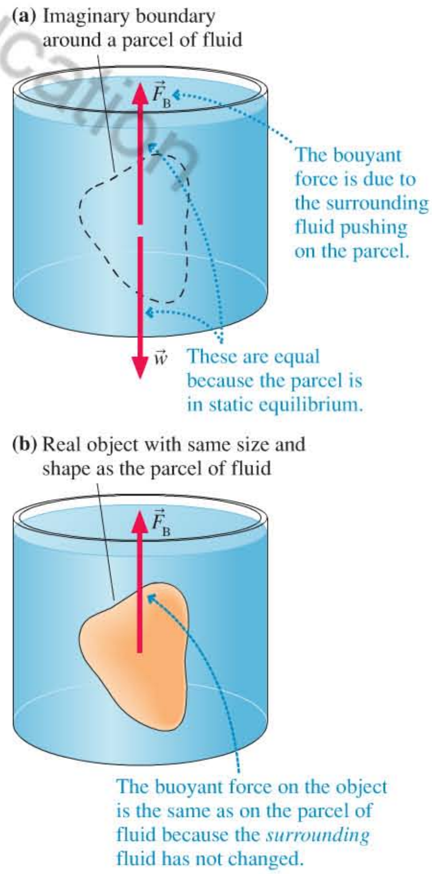
The submerged cylinder illustrates the idea in a simple way, but the result is not limited to cylinders or to liquids. Suppose we isolate a parcel of fluid of arbitrary shape and volume by drawing an imaginary boundary around it, as shown in FIGURE 13.17a. This parcel is in static equilibrium. Consequently, the parcel's weight force pulling it down must be balanced by an upward force. The upward force, which is exerted on this parcel of fluid by the surrounding fluid, is the buoyant force $\vec{F}_B$. The buoyant force matches the weight of the fluid: $F_B = w$.

Now imagine that we could somehow remove this parcel of fluid and instantaneously replace it with an object having exactly the same shape and size, as shown in FIGURE 13.17b. Because the buoyant force is exerted by the *surrounding* fluid, and the surrounding fluid hasn't changed, the buoyant force on this new object is *exactly the same* as the buoyant force on the parcel of fluid that we removed.

When an object (or a portion of an object) is immersed in a fluid, it *displaces* fluid that would otherwise fill that region of space. This fluid is called the **displaced fluid.** The displaced fluid's volume is exactly the volume of the portion of the object that is immersed in the fluid. Figure 13.17 leads us to conclude that the magnitude of the upward buoyant force matches the weight of this displaced fluid.

This idea was first recognized by the ancient Greek mathematician and scientist Archimedes, perhaps the greatest scientist of antiquity, and today we know it as *Archimedes' principle:*

FIGURE 13.17 The buoyant force on an object is the same as the buoyant force on the fluid it displaces.



> **Archimedes' principle** A fluid exerts an upward buoyant force $\vec{F}_B$ on an object immersed in or floating on the fluid. The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Suppose the fluid has density ρ_f and the object displaces a volume V_f of fluid. The mass of the displaced fluid is then $m_f = \rho_f V_f$ and so its weight is $w_f = \rho_f V_f g$. Thus Archimedes' principle in equation form is

$$F_B = \rho_f V_f g \tag{13.8}$$

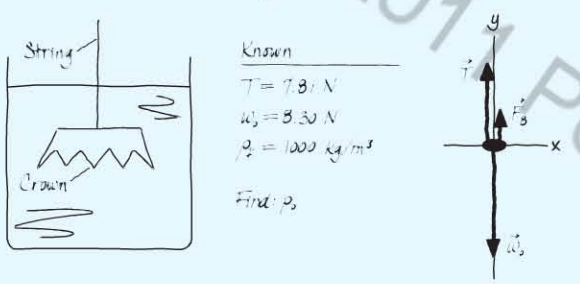
NOTE ► It is important to distinguish the density and volume of the displaced fluid from the density and volume of the object. To do so, we'll use subscript f for the fluid and o for the object. ◄

EXAMPLE 13.6 **Is the crown gold?**

Legend has it that Archimedes was asked by King Hiero of Syracuse to determine whether a crown was of pure gold or had been adulterated with a lesser metal by an unscrupulous goldsmith. It was this problem that led him to the principle that bears his name. In a modern version of his method, a crown weighing 8.30 N is suspended underwater from a string. The tension in the string is measured to be 7.81 N. Is the crown pure gold?

PREPARE To discover whether the crown is pure gold, we need to determine its density ρ_o and compare it to the known density of gold. **FIGURE 13.18** shows the forces acting on the crown. In addition to the familiar tension and weight forces, the water exerts an upward buoyant force on the crown. The size of the buoyant force is given by Archimedes' principle.

FIGURE 13.18 The forces acting on the submerged crown.



SOLVE Because the crown is in static equilibrium, its acceleration and the net force on it are zero. Newton's second law then reads

$$\sum F_y = F_B + T - w_o = 0$$

from which the buoyant force is

$$F_B = w_o - T = 8.30\text{ N} - 7.81\text{ N} = 0.49\text{ N}$$

According to Archimedes' principle, $F_B = \rho_f V_f g$, where V_f is the volume of the fluid displaced. Here, where the crown is completely submerged, the volume of the fluid displaced is equal to the volume V_o of the crown. Now the crown's weight is $w_o = m_o g = \rho_o V_o g$, so its volume is

$$V_o = \frac{w_o}{\rho_o g}$$

Inserting this volume into Archimedes' principle gives

$$F_B = \rho_f V_o g = \rho_f\left(\frac{w_o}{\rho_o g}\right)g = \frac{\rho_f}{\rho_o}w_o$$

or, solving for ρ_0,

$$\rho_o = \frac{\rho_f w_o}{F_B} = \frac{(1000\text{ kg/m}^3)(8.30\text{ N})}{0.49\text{ N}} = 17{,}000\text{ kg/m}^3$$

The crown's density is considerably lower than that of pure gold, which is 19,300 kg/m³. The crown is not pure gold.

ASSESS For an object made of a dense material such as gold, the buoyant force is small compared to its weight.

Float or Sink?

If you *hold* an object underwater and then release it, it rises to the surface, sinks, or remains "hanging" in the water. How can we predict which it will do? Whether it heads for the surface or the bottom depends on whether the upward buoyant force F_B on the object is larger or smaller than the downward weight force w_o.

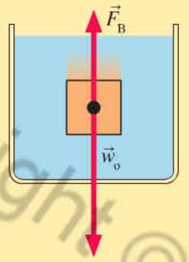
The magnitude of the buoyant force is $\rho_f V_f g$. The weight of a uniform object, such as a block of steel, is simply $\rho_o V_o g$. But a compound object, such as a scuba diver, may have pieces of varying density. If we define the **average density** to be $\rho_{avg} = m_o/V_o$, the weight of a compound object can be written as $w_o = \rho_{avg} V_o g$.

Comparing $\rho_f V_f g$ to $\rho_{avg} V_o g$, and noting that $V_f = V_o$ for an object that is fully submerged, we see that an object floats or sinks depending on whether the fluid density ρ_f is larger or smaller than the object's average density ρ_{avg}. If the densities are equal, the object is in static equilibrium and hangs motionless. This is called **neutral buoyancy.** These conditions are summarized in Tactics Box 13.2.

◄ **Submersible scales** BIO In Example 13.6, we saw how the density of an object could be determined by weighing it both underwater and in air. This idea is the basis of an accurate method for determining a person's percentage of body fat. Fat has a lower density than lean muscle or bone, so a lower overall body density implies a greater proportion of body fat. To determine a person's density, she is first weighed in air and then lowered completely into water and weighed again. Standard tables accurately relate body density to fat percentage.

TACTICS BOX 13.2 **Finding whether an object floats or sinks**

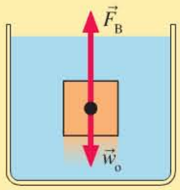
❶ Object sinks



An object sinks if it weighs more than the fluid it displaces—that is, if its average density is greater than the density of the fluid:

$$\rho_{avg} > \rho_f$$

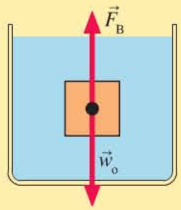
❷ Object floats



An object rises to the surface if it weighs less than the fluid it displaces—that is, if its average density is less than the density of the fluid:

$$\rho_{avg} < \rho_f$$

❸ Object has neutral buoyancy



An object hangs motionless if it weighs exactly the same as the fluid it displaces—that is, if its average density equals the density of the fluid:

$$\rho_{avg} = \rho_f$$

Exercises 10–12

For example, steel is denser than water, so a chunk of steel sinks. Oil is less dense than water, so oil floats on water. Fish use *swim bladders* filled with air and scuba divers use weighted belts to adjust their average density to match the water. Both are examples of neutral buoyancy.

If you release a block of wood underwater, the net upward force causes the block to shoot to the surface. Then what? To understand floating, let's begin with a *uniform* object such as the block shown in **FIGURE 13.19**. This object contains nothing tricky, like indentations or voids. Because it's floating, it must be the case that $\rho_o < \rho_f$.

Now that the object is floating, it's in static equilibrium. Thus, the upward buoyant force, given by Archimedes' principle, exactly balances the downward weight of the object; that is,

$$F_B = \rho_f V_f g = w_o = \rho_o V_o g \tag{13.9}$$

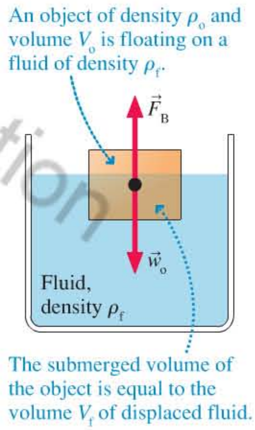
For a floating object, the volume of the displaced fluid is *not* the same as the volume of the object. In fact, we can see from Equation 13.9 that the volume of fluid displaced by a floating object of uniform density is

$$V_f = \frac{\rho_o}{\rho_f} V_o \tag{13.10}$$

which is *less* than V_o because $\rho_o < \rho_f$.

NOTE ► Equation 13.10 applies only to *uniform* objects. It does not apply to boats, hollow spheres, or other objects of nonuniform composition. ◄

FIGURE 13.19 A floating object is in static equilibrium.



► **Hidden depths** You've probably heard it said that "90% of an iceberg is underwater." Equation 13.10 is the basis for that statement. Most icebergs break off glaciers and are fresh-water ice with a density of 917 kg/m^3. The density of seawater is 1030 kg/m^3. Thus

$$V_f = \frac{917 \text{ kg/m}^3}{1030 \text{ kg/m}^3} V_o = 0.89 V_o$$

V_f, the volume of the displaced water, is also the volume of the iceberg that is underwater. You can see that, indeed, 89% of the volume of an iceberg is underwater.

CONCEPTUAL EXAMPLE 13.7 **Which has the greater buoyant force?**

A block of iron sinks to the bottom of a vessel of water while a block of wood of the *same size* floats. On which is the buoyant force greater?

REASON The buoyant force is equal to the volume of water displaced. The iron block is completely submerged, so it displaces a volume of water equal to its own volume. The wood block floats, so it displaces only the fraction of its volume that is under water, which is *less* than its own volume. The buoyant force on the iron block is therefore greater than on the wood block.

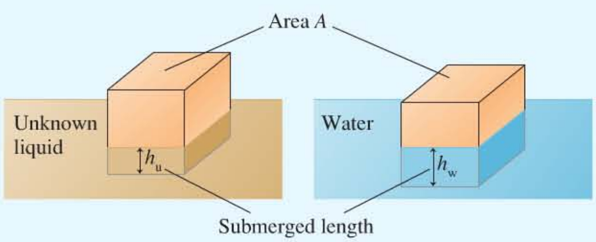
ASSESS This result may seem counterintuitive, but remember that the iron block sinks because of its high density, while the wood block floats because of its low density. A smaller buoyant force is sufficient to keep it floating.

EXAMPLE 13.8 **Measuring the density of an unknown liquid**

You need to determine the density of an unknown liquid. You notice that a block floats in this liquid with 4.6 cm of the side of the block submerged. When the block is placed in water, it also floats but with 5.8 cm submerged. What is the density of the unknown liquid?

PREPARE Assume that the block is an object of uniform composition. **FIGURE 13.20** shows the block as well as the cross-section area A and submerged lengths h_u in the unknown liquid and h_w in water.

FIGURE 13.20 A block floating in two liquids.



SOLVE The block is floating, so Equation 13.10 applies. The block displaces volume $V_u = Ah_u$ of the unknown liquid. Thus

$$V_u = Ah_u = \frac{\rho_o}{\rho_u} V_o$$

Similarly, the block displaces volume $V_w = Ah_w$ of the water, leading to

$$V_w = Ah_w = \frac{\rho_o}{\rho_w} V_o$$

Because there are two fluids, we've used subscripts w for water and u for the unknown in place of the fluid subscript f. The product $\rho_o V_o$ appears in both equations. In the first $\rho_o V_o = \rho_u A h_u$, and in the second $\rho_o V_o = \rho_w A h_w$. Equating the right-hand sides gives

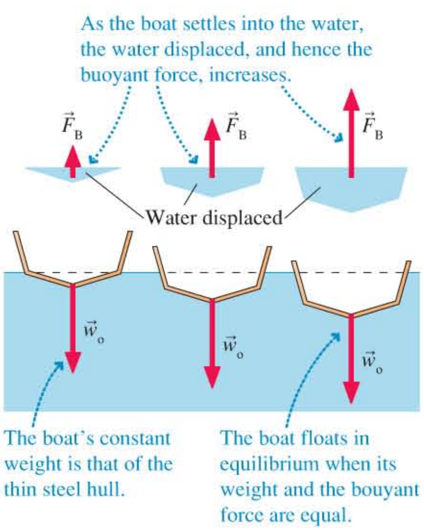
$$\rho_u A h_u = \rho_w A h_w$$

The area A cancels, and the density of the unknown liquid is

$$\rho_u = \frac{h_w}{h_u}\rho_w = \frac{5.8 \text{ cm}}{4.6 \text{ cm}} 1000 \text{ kg/m}^3 = 1300 \text{ kg/m}^3$$

ASSESS Comparison with Table 13.1 shows that the unknown liquid is likely to be glycerin.

FIGURE 13.21 How a boat floats.



Boats and Balloons

A chunk of steel sinks, so how does a steel-hulled boat float? As we've seen, an object floats if the upward buoyant force—the weight of the displaced water—balances the weight of the object. A boat is really a large hollow shell whose weight is determined by the volume of steel in the hull. As **FIGURE 13.21** shows, the volume of water displaced by a shell is *much* larger than the volume of the hull itself. As a boat settles into the water, it sinks until the weight of the displaced water exactly matches the boat's weight. It is then in static equilibrium, so it floats at that level.

The concept of buoyancy and flotation applies to all fluids, not just liquids. An object immersed in a gas such as air feels a buoyant force as well. Because the density of air is so low, this buoyant force is generally negligible. Nonetheless, even though the buoyant force due to air is small, an object will float in air if it weighs less than the air that it displaces. This is why a floating balloon cannot be filled with regular air. If it were, then the weight of the air inside it would equal the weight of the air it displaced, so that it would have no net upward force on it. Adding in the weight of the balloon itself would then lead to a net downward force. For a balloon to float, it must be filled with a gas that has a *lower* density than that of air. The following example illustrates how this works.

EXAMPLE 13.9 How big does a balloon need to be?

What diameter must a helium-filled balloon have to float with neutral buoyancy? The mass of the empty balloon is 2.0 g.

PREPARE We'll model the balloon as a sphere. The balloon will float when its weight—the weight of the empty balloon plus the helium—equals the weight of the air it displaces. The densities of air and helium are given in Table 13.1.

SOLVE The volume of the balloon is V_{balloon}. Its weight is

$$w_{\text{balloon}} = m_{\text{balloon}}g + \rho_{\text{He}}V_{\text{balloon}}g$$

where m_{balloon} is the mass of the empty balloon. The weight of the displaced air is

$$w_{\text{air}} = \rho_{\text{air}}V_{\text{air}}g = \rho_{\text{air}}V_{\text{ballon}}g$$

where we noted that the volume of the displaced air is the volume of the balloon. The balloon will float when these two forces are equal, or when

$$\rho_{\text{air}}V_{\text{balloon}}g = m_{\text{balloon}}g + \rho_{\text{He}}V_{\text{balloon}}g$$

The g cancels, and we can solve for the volume of the balloon:

$$V_{\text{balloon}} = \frac{m_{\text{balloon}}}{\rho_{\text{air}} - \rho_{\text{He}}} = \frac{2.0 \times 10^{-3}\ \text{kg}}{1.28\ \text{kg/m}^3 - 0.17\ \text{kg/m}^3} = 1.8 \times 10^{-3}\ \text{m}^3$$

A sphere has volume $V = (4\pi/3)r^3$. Thus the balloon's radius is

$$r = \left(\frac{3V_{\text{balloon}}}{4\pi}\right)^{\frac{1}{3}} = \left(\frac{3 \times (1.8 \times 10^{-3}\ \text{m}^3)}{4\pi}\right)^{\frac{1}{3}} = 0.075\ \text{m}$$

The diameter of the balloon is twice this, or 15 cm.

ASSESS This example shows why a helium balloon will no longer float once its volume falls below a certain value.

Hot air rising A hot-air balloon is filled with a low-density gas: hot air! You learned in Chapter 12 that gases expand upon heating, thus lowering their density. The air at the top of a hot-air balloon is surprisingly hot—about 100°C, the temperature of boiling water. Using the ideal-gas law, you should be able to show that the density of 100°C air is only 79% that of room-temperature air at 20°C. A burst of flame lowers the density of the air and thus the balloon's weight. The balloon rises when its weight becomes less than the weight of the cooler air it has displaced.

STOP TO THINK 13.3 An ice cube is floating in a glass of water that is filled entirely to the brim. When the ice cube melts, the water level will

A. Fall. B. Stay the same. C. Rise, causing the water to spill.

13.5 Fluids in Motion

The wind blowing through your hair, a white-water river, and oil gushing from an oil well are examples of fluids in motion. We've focused thus far on fluid statics, but it's time to turn our attention to fluid *dynamics.*

Fluid flow is a complex subject. Many aspects of fluid flow, especially turbulence and the formation of eddies, are still not well understood and are areas of current research. We will avoid these difficulties by using a simplified *model* of an *ideal* fluid. Our model can be expressed in three assumptions about the fluid:

1. The fluid is *incompressible.* This is a very good assumption for liquids, but it also holds reasonably well for a moving gas, such as air. For instance, even when a 100 mph wind slams into a wall, its density changes by only about 1%.
2. The flow is *steady.* That is, the fluid velocity at each point in the fluid is constant; it does not fluctuate or change with time. Flow under these conditions is called **laminar flow,** and it is distinguished from *turbulent flow.*
3. The fluid is *nonviscous.* Water flows much more easily than cold pancake syrup because the syrup is a very viscous liquid. Viscosity is resistance to flow, and assuming a fluid is nonviscous is analogous to assuming the motion of a particle is frictionless. Gases have very low viscosity, and even many liquids are well approximated as being nonviscous.

Later, in Section 13.7, we'll relax assumption 3 and consider the effects of viscosity.

The rising smoke in FIGURE 13.22 begins as laminar flow, recognizable by the smooth contours, but at some point undergoes a transition to turbulent flow. A laminar-to-turbulent transition is not uncommon in fluid flow. Our model of fluids can be applied to the laminar flow, but not to the turbulent flow.

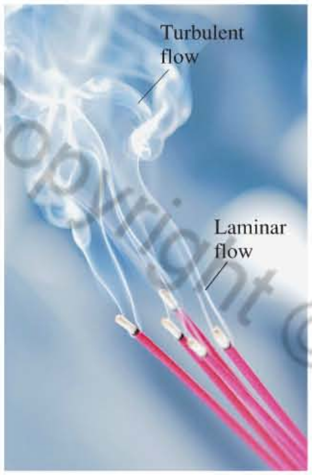


FIGURE 13.22 Rising smoke changes from laminar flow to turbulent flow.

The Equation of Continuity

Consider a fluid flowing through a tube—oil through a pipe or blood through an artery. If the tube's diameter changes, as happens in FIGURE 13.23, what happens to the speed of the fluid?

When you squeeze a toothpaste tube, the volume of toothpaste that emerges matches the amount by which you reduce the volume of the tube. An *incompressible* fluid flowing through a rigid tube or pipe acts the same way. Fluid is neither created nor destroyed within the tube, and there's no place to store any extra fluid introduced into the tube. If volume V enters the tube during some interval of time Δt, then an equal volume of fluid must leave the tube.

To see the implications of this idea, suppose all the molecules of the fluid in Figure 13.23 are moving forward with speed v_1 at a point where the cross-section area is A_1. Farther along the tube, where the cross-section area is A_2, their speed is v_2. During an interval of time Δt, the molecules in the wider section move forward a distance $\Delta x_1 = v_1\, \Delta t$ and those in the narrower section move $\Delta x_2 = v_2\, \Delta t$. Because the fluid is incompressible, the volumes ΔV_1 and ΔV_2 must be equal; that is,

$$\Delta V_1 = A_1\, \Delta x_1 = A_1 v_1\, \Delta t = \Delta V_2 = A_2\, \Delta x_2 = A_2 v_2\, \Delta t \qquad (13.11)$$

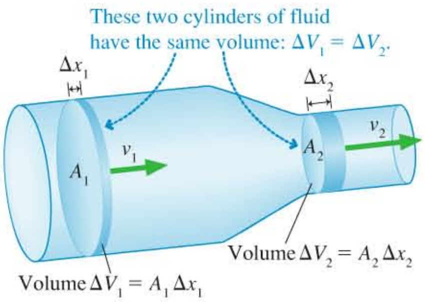


FIGURE 13.23 Flow speed changes through a tapered tube.

Dividing both sides of the equation by Δt gives the **equation of continuity**:

$$v_1 A_1 = v_2 A_2 \qquad (13.12)$$

The equation of continuity relating the speed v of an incompressible fluid to the cross-section area A of the tube in which it flows

Equations 13.11 and 13.12 say that **the volume of an incompressible fluid entering one part of a tube or pipe must be matched by an equal volume leaving downstream.**

An important consequence of the equation of continuity is that **flow is faster in narrower parts of a tube, slower in wider parts.** You're familiar with this conclusion from many everyday observations. The garden hose shown in FIGURE 13.24a squirts farther after you put a nozzle on it. This is because the narrower opening of the nozzle gives the water a higher exit speed. Water flowing from the faucet shown in FIGURE 13.24b picks up speed as it falls. As a result, the flow tube "necks down" to a smaller diameter.

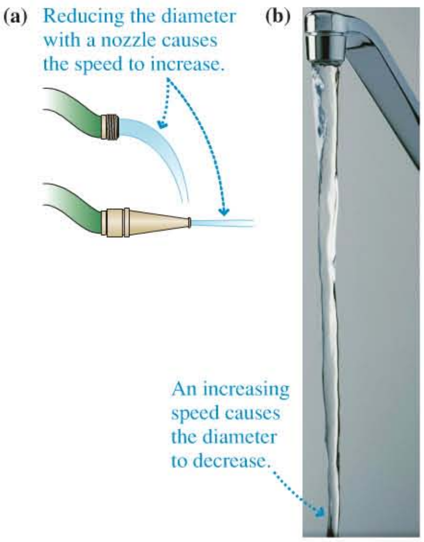


FIGURE 13.24 The speed of the water is inversely proportional to the diameter of the stream.

The *rate* at which fluid flows through the tube—volume per second—is $\Delta V/\Delta t$. This is called the **volume flow rate** Q. We can see from Equation 13.11 that

$$Q = \frac{\Delta V}{\Delta t} = vA \qquad (13.13)$$

The SI units of Q are m^3/s, although in practice Q may be measured in cm^3/s, liters per minute, or, in the United States, gallons per minute and cubic feet per minute. Another way to express the meaning of the equation of continuity is to say that **the volume flow rate is constant at all points in a tube.**

EXAMPLE 13.10 Speed of water through a hose

A garden hose has an inside diameter of 16 mm. The hose can fill a 10 L bucket in 20 s.

a. What is the speed of the water out of the end of the hose?
b. What diameter nozzle would cause the water to exit with a speed 4 times greater than the speed inside the hose?

PREPARE Water is essentially incompressible, so the equation of continuity applies.

SOLVE

a. The volume flow rate is $Q = \Delta V/\Delta t = (10\text{ L})/(20\text{ s}) = 0.50\text{ L/s}$. To convert this to SI units, recall that $1\text{ L} = 1000\text{ mL} = 10^3\text{ cm}^3 = 10^{-3}\text{ m}^3$. Thus $Q = 5.0 \times 10^{-4}\text{ m}^3\text{/s}$. We can find the speed of the water from Equation 13.13:

$$v = \frac{Q}{A} = \frac{Q}{\pi r^2} = \frac{5.0 \times 10^{-4}\text{ m}^3\text{/s}}{\pi(0.0080\text{ m})^2} = 2.5\text{ m/s}$$

b. The quantity $Q = vA$ remains constant as the water flows through the hose and then the nozzle. To increase v by a factor of 4, A must be reduced by a factor of 4. The cross-section area depends on the square of the diameter, so the area is reduced by a factor of 4 if the diameter is reduced by a factor of 2. Thus the necessary nozzle diameter is 8 mm.

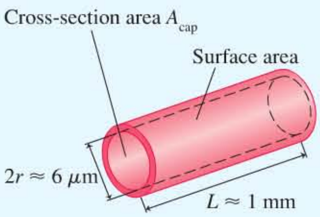
EXAMPLE 13.11 Blood flow in capillaries

The volume flow rate of blood leaving the heart to circulate throughout the body is about 5 L/min for a person at rest. All this blood eventually must pass through the smallest blood vessels, the capillaries. Microscope measurements show that a typical capillary is 6 μm in diameter and 1 mm long, and the blood flows through it at an average speed of 1 mm/s.

a. Estimate the total number of capillaries in the body.
b. Estimate the total surface area of all the capillaries.

The various lengths and areas are shown in **FIGURE 13.25**.

FIGURE 13.25 A capillary (not to scale).



PREPARE We can use the equation of continuity: The 5 L of blood passing through the heart each minute must be the total flow through all the capillaries combined.

SOLVE

a. We'll start by converting Q to SI units:

$$Q = 5\frac{\text{L}}{\text{min}} \times \frac{1\text{ m}^3}{1000\text{ L}} \times \frac{1\text{ min}}{60\text{ s}} = 8 \times 10^{-5}\text{ m}^3\text{/s}$$

to one-significant-figure accuracy. Then the total cross-section area of all the capillaries together is

$$A_{\text{total}} = \frac{Q}{v} = \frac{8 \times 10^{-5}\text{ m}^3\text{/s}}{0.001\text{ m/s}} = 0.08\text{ m}^2$$

The cross-section area of each capillary is $A_{\text{cap}} = \pi r^2$, so the total number of capillaries is approximately

$$N = \frac{A_{\text{total}}}{A_{\text{cap}}} = \frac{0.08\text{ m}^2}{\pi(3 \times 10^{-6}\text{ m})^2} = 3 \times 10^9$$

b. The surface area of one capillary is

$$A = \text{circumference of capillary} \times \text{length}$$
$$= 2\pi rL = 2\pi(3 \times 10^{-6}\text{ m})(0.001\text{ m}) = 2 \times 10^{-8}\text{ m}^2$$

so the total surface area of all the capillaries is about

$$A_{\text{surface}} = NA = (3 \times 10^9)(2 \times 10^{-8}\text{ m}) = 60\text{ m}^2$$

ASSESS The number of capillaries is huge, and the total surface area is about the area of a two-car garage. As we saw in Chapter 12, oxygen and nutrients move from the blood into cells by the slow process of diffusion. Only by having this large surface area available for diffusion can the required rate of gas and nutrient exchange be attained.

Representing Fluid Flow: Streamlines and Fluid Elements

FIGURE 13.26 Smoke reveals the laminar air flow around a car in a wind tunnel.

Representing the flow of fluid is more complicated than representing the motion of a point particle because fluid flow is the collective motion of a vast number of particles. **FIGURE 13.26** gives us an idea of one possible fluid-flow representation. Here smoke is being used to help engineers visualize the air flow around a car in a wind tunnel. The smoothness of the flow tells us this is laminar flow. But notice also how the individual smoke trails retain their identity. They don't cross or get mixed together. Each smoke trail represents a *streamline* in the fluid.

Imagine that we could inject a tiny colored drop of water into a stream of water undergoing laminar flow. Because the flow is steady and the water is incompressible, this colored drop would maintain its identity as it flowed along. The path or trajectory followed by this "particle of fluid" is called a **streamline.** Smoke particles mixed with the air allow you to see the streamlines in the wind-tunnel photograph of Figure 13.26. **FIGURE 13.27** illustrates three important properties of streamlines.

FIGURE 13.27 Particles in a fluid move along streamlines.

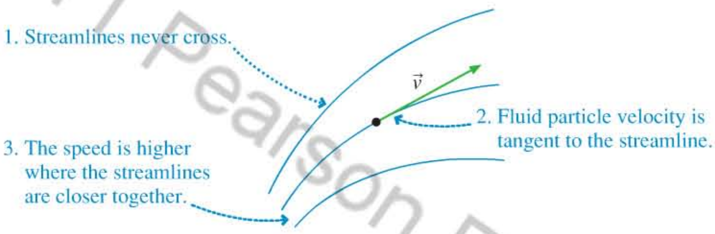
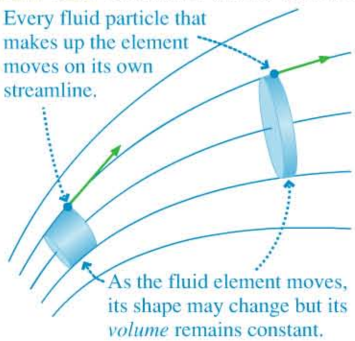
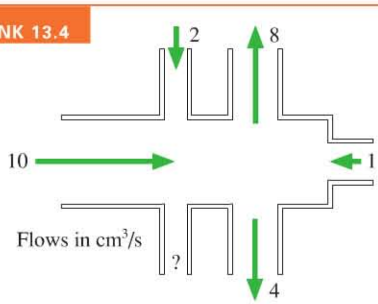


FIGURE 13.28 Motion of a fluid element.



As we study the motion of a fluid, it is often also useful to consider a small *volume* of fluid, a volume containing many particles of fluid. Such a volume is called a **fluid element.** **FIGURE 13.28** shows two important properties of a fluid element. Unlike a particle, a fluid element has an actual shape and volume. Although the shape of a fluid element may change as it moves, the equation of continuity requires that its volume remain constant. The progress of a fluid element as it moves along streamlines and changes shape is another very useful representation of fluid motion.

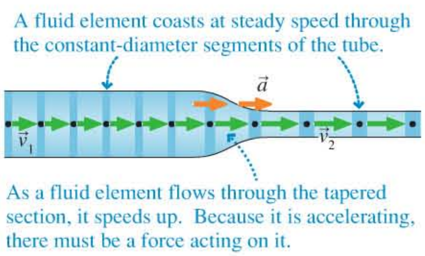
STOP TO THINK 13.4



The figure shows volume flow rates (in cm^3/s) for all but one tube. What is the volume flow rate through the unmarked tube? Is the flow direction in or out?

13.6 Fluid Dynamics

FIGURE 13.29 A motion diagram of a fluid element moving through a narrowing tube.



The equation of continuity describes a moving fluid but doesn't tell us anything about *why* the fluid is in motion. To understand the dynamics, consider the ideal fluid moving from left to right through the tube shown in **FIGURE 13.29**. The fluid moves at a steady speed v_1 in the wider part of the tube. In accordance with the equation of continuity, its speed is a higher, but steady, v_2 in the narrower part of the tube. If we follow a fluid element through the tube, we see that it undergoes an *acceleration* from v_1 to v_2 in the tapered section of the tube.

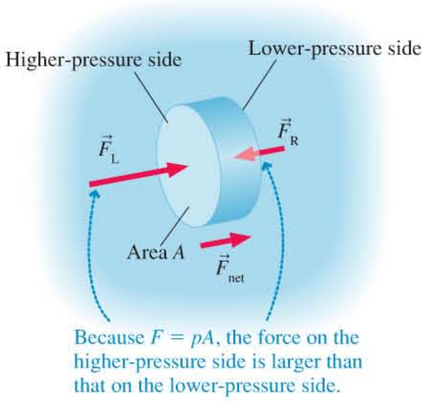
In the absence of friction, Newton's first law tells us that a particle will coast forever at a steady speed. An ideal fluid is nonviscous and thus analogous to a particle moving without friction. This means a fluid element moving through the constant-diameter sections of the tube in Figure 13.29 requires no force to “coast” at steady speed. On the other hand, a fluid element moving through the tapered section of the tube accelerates, speeding up from v_1 to v_2. According to Newton’s second law, there must be a net force acting on this fluid element to accelerate it.

What is the origin of this force? There are no external forces, and the horizontal motion rules out gravity. Instead, the fluid element is pushed from both ends by the *surrounding fluid*—that is, by *pressure forces*. The fluid element with cross-section area A in **FIGURE 13.30** has a higher pressure on its left side than on its right. Thus the force $F_L = p_L A$ of the fluid pushing on its left side—a force pushing to the right—is greater than the force $F_R = p_R A$ of the fluid on its right side. The net force, which points from the higher-pressure side of the element to the lower-pressure side, is

$$F_{net} = F_L - F_R = (p_L - p_R)A = A\,\Delta p$$

In other words, there’s a net force on the fluid element, causing it to change speed, if and only if there’s a pressure *difference* Δp between the two faces.

FIGURE 13.30 The net force on a fluid element due to pressure points from high to low pressure.



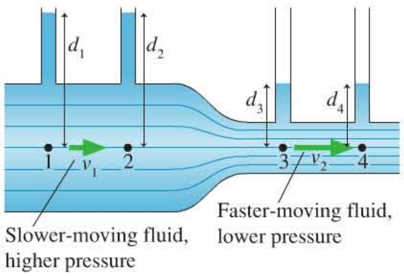
Thus, in order to accelerate the fluid elements in Figure 13.29 through the neck, the pressure p_1 in the wider section of the tube must be higher than the pressure p_2 in the narrower section. When the pressure is changing from one point in a fluid to another, we say that there is a **pressure gradient** in that region. We can also say that pressure forces are caused by pressure gradients, so **an ideal fluid accelerates wherever there is a pressure gradient.**

As a result, **the pressure is higher at a point along a streamline where the fluid is moving slower, lower where the fluid is moving faster.** This property of fluids was discovered in the 18th century by the Swiss scientist Daniel Bernoulli and is called the **Bernoulli effect.**

NOTE ► It is important to realize that it is the change in pressure from high to low that *causes* the fluid to speed up. A high fluid speed doesn’t *cause* a low pressure any more than a fast-moving particle causes the force that accelerated it. ◄

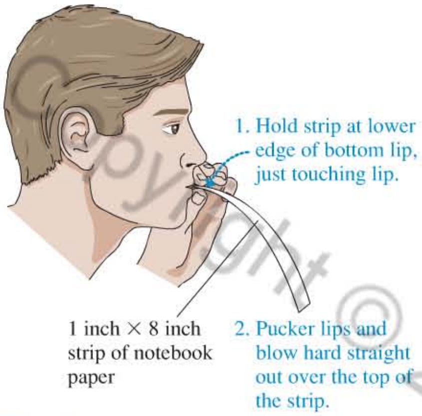
This relationship between pressure and fluid speed can be used to measure the speed of a fluid with a device called a *Venturi tube.* **FIGURE 13.31** shows a simple Venturi tube suitable for a flowing liquid. The high pressure at point 1, where the fluid is moving slowly, causes the fluid to rise in the vertical pipe to a total height d_1. Because there’s no vertical motion of the fluid, we can use the hydrostatic pressure equation to find that the pressure at point 1 is $p_1 = p_0 + \rho g d_1$. At point 3, on the same streamline, the fluid is moving faster and the pressure is lower; thus the fluid in the vertical pipe rises to a lower height d_3. The pressure *difference* is $\Delta p = \rho g(d_1 - d_3)$, so the pressure difference across the neck of the pipe can be found by measuring the difference in heights of the fluid. We’ll see later in this section how to relate this pressure difference to the increase of speed.

FIGURE 13.31 A Venturi tube measures flow speeds of a fluid.



Note that the fluid height d_1 is the same as d_2, and d_3 is the same as d_4. For an ideal fluid, with no viscosity, no pressure difference is needed to keep the fluid moving at a constant speed, as it does in both the large- and small-diameter sections of the tube. In the next section we’ll see how this result changes for a viscous fluid.

TRY IT YOURSELF



Blowing up Try the experiment in the figure. You might expect the strip to be pushed *down* by the force of your breath, but you'll find that the strip actually *rises*. Your breath moving over the curved strip is similar to wind blowing over a hill, and Bernoulli's effect likewise predicts a zone of lower pressure above the strip that causes it to rise.

Applications of the Bernoulli Effect

Many important applications of the Bernoulli effect can be understood by considering the flow of air over a hill, as shown in **FIGURE 13.32**. Far to the left, away from the hill, the wind blows at a constant speed, so its streamlines are equally spaced. But as the air moves over the hill, the hill forces the streamlines to bunch together so that the air speeds up. According to the Bernoulli effect, there exists a zone of *low pressure* at the crest of the hill, where the air is moving the fastest.

Using these ideas, we can understand *lift*, the upward force on the wing of a moving airplane that makes flight possible. **FIGURE 13.33** shows an airplane wing, seen in cross section, for an airplane flying to the left. The figure is drawn in the reference frame of the airplane, so that the wing appears stationary and the air flows past it to the right. The shape of the wing is such that, just as for the hill in Figure 13.32, the streamlines of the air must squeeze together as they pass over the wing. This increased speed is, by the Bernoulli effect, accompanied by a region of low pressure above the wing. The air speed below the wing is actually slowed, so that there's a region of high pressure below. This high pressure pushes up on the wing, while the low-pressure air above it presses down, but less strongly. The result is a net upward force—the lift.

FIGURE 13.32 Wind speeds up as it crests a hill.

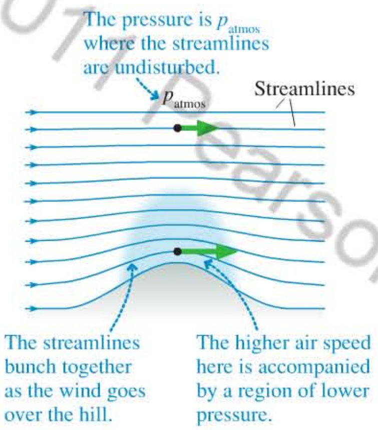
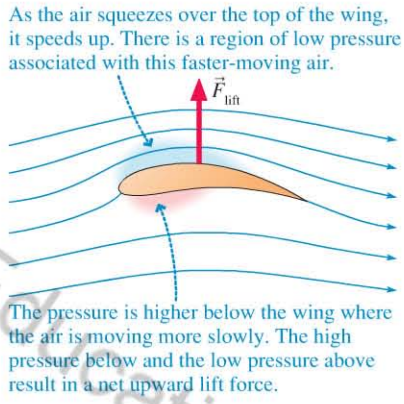
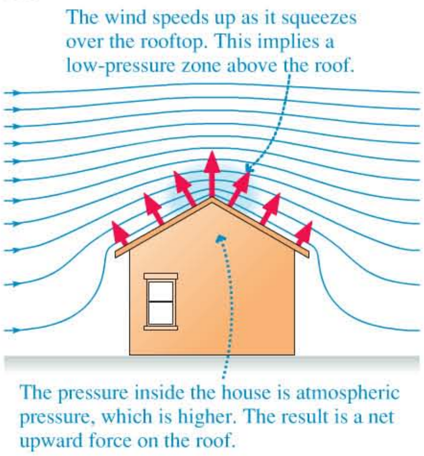


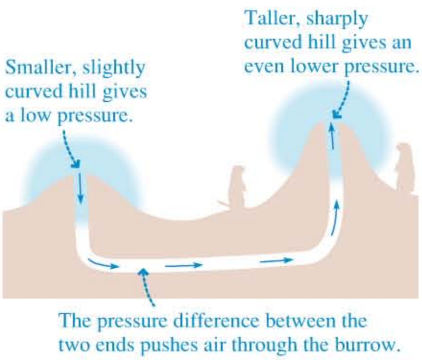
FIGURE 13.33 Air flow over a wing generates lift by creating unequal pressures above and below.



The Bernoulli effect also explains how hurricanes destroy the roofs of houses. The roofs are not "blown" off; they are *lifted* off by pressure differences caused by the Bernoulli effect. **FIGURE 13.34** shows that the situation is similar to that for the hill and the airplane wing. The pressure difference causing the lift force is not very large—but the force is proportional to the *area* of the roof, which, being very large, can produce a lift force large enough to separate the roof from its supporting walls.

FIGURE 13.34 How high winds lift roofs off.





Nature's air conditioning Prairie dogs ventilate their underground burrows with the same aerodynamic forces and pressures that give airplanes lift. The two entrances to their burrows are surrounded by mounds, one higher than the other. When the wind blows across these mounds, the pressure is reduced at the top, just as for an airplane's wing. The taller mound, with its greater curvature, has the lower pressure of the two entrances. Air then is pushed through the burrow toward this lower-pressure side.

Bernoulli's Equation

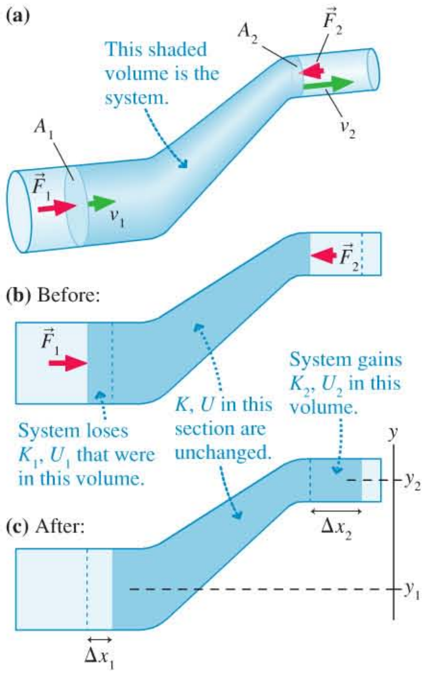
We've seen that a pressure gradient causes a fluid to accelerate horizontally. Not surprisingly, gravity can also cause a fluid to speed up or slow down if the fluid changes elevation. These are the key ideas of fluid dynamics. Now we would like to make these ideas quantitative by finding a numerical relationship for pressure, height, and the speed of a fluid. We can do so by applying the statement of conservation of mechanical energy you learned in Chapter 10,

$$\Delta K + \Delta U = W$$

where U is the gravitational potential energy and W is the work done by other forces—in this case, pressure forces. Recall that we're still considering ideal fluids, with no friction or viscosity, so there's no dissipation of energy to thermal energy.

FIGURE 13.35a shows fluid flowing through a tube. The tube narrows from cross-section area A_1 to area A_2 as it bends uphill. Let's concentrate on the large volume of the fluid that is shaded in Figure 13.35a. **This moving segment of fluid will be our system for the purpose of applying conservation of energy.**

FIGURE 13.35 An ideal fluid flowing through a tube.



To use conservation of energy, we need to draw a before-and-after overview. The "before" situation is shown in FIGURE 13.35b. A short time Δt later, the system has moved along the tube a bit, as shown in the "after" drawing of FIGURE 13.35c. Because the tube is not of uniform diameter, the two ends of the fluid system do not move the same distance during Δt: The lower end moves distance Δx_1, while the upper end moves Δx_2. Thus the system moves *out of* a cylindrical volume $\Delta V_1 = A_1\,\Delta x_1$ at the lower end and *into* a volume $\Delta V_2 = A_2\,\Delta x_2$ at the upper end. The equation of continuity tells us that these two volumes must be the same, so $A_1\,\Delta x_1 = A_2\,\Delta x_2 = \Delta V$.

From the "before" situation to the "after" situation, the system *loses* the kinetic and potential energy it originally had in the volume ΔV_1 but *gains* the kinetic and potential energy in the volume ΔV_2 that it later occupies. (The energy it has in the region between these two small volumes is unchanged.) Let's find the kinetic energy in each of these small volumes. The mass of fluid in each cylinder is $m = \rho\,\Delta V$, where ρ is the density of the fluid. The kinetic energies of the two small volumes 1 and 2 are then

$$K_1 = \frac{1}{2}\underbrace{\rho\,\Delta V}_{m} v_1^2 \quad \text{and} \quad K_2 = \frac{1}{2}\underbrace{\rho\,\Delta V}_{m} v_2^2$$

Thus the net *change* in kinetic energy is

$$\Delta K = K_2 - K_1 = \frac{1}{2}\rho\,\Delta V v_2^2 - \frac{1}{2}\rho\,\Delta V v_1^2$$

Similarly, the net change in the gravitational potential energy of our fluid system is

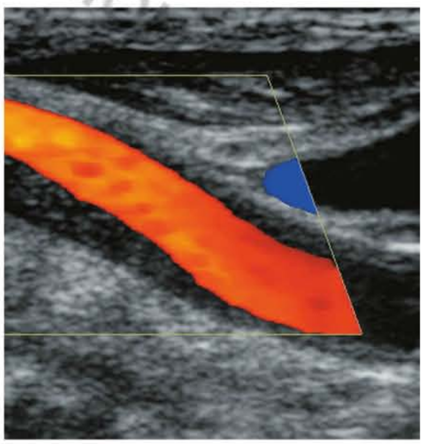
$$\Delta U = U_2 - U_1 = \rho\,\Delta V g y_2 - \rho\,\Delta V g y_1$$

The final piece of our conservation of energy treatment is the work done on the system by the rest of the fluid. As the fluid system moves, positive work is done on it by the force $\vec{F}_1$ due to the pressure p_1 of the fluid to the left of the system, while negative work is done on the system by the force $\vec{F}_2$ due to the pressure p_2 of the fluid to the right of the system. The positive work is

$$W_1 = F_1\,\Delta x_1 = (p_1 A_1)\,\Delta x_1 = p_1(A_1\,\Delta x_1) = p_1\,\Delta V$$

Similarly the negative work is

$$W_2 = -F_2\,\Delta x_2 = -(p_2 A_2)\,\Delta x_2 = -p_2(A_2\,\Delta x_2) = -p_2\,\Delta V$$



Living under pressure BIO When plaque builds up in major arteries, such as the carotid artery that supplies the head, dangerous drops in blood pressure can result. *Doppler ultrasound,* which you'll study in Chapter 15, uses sound waves to produce images of the interior of the body, and can detect the velocity of flowing blood. The image above shows the blood flow through a carotid artery with significant plaque buildup; yellow indicates a higher blood velocity than red. Once the velocities are known at two points along the flow, Bernoulli's equation can be used to deduce the corresponding pressure drop.

Thus the *net* work done on the system is

$$W = W_1 + W_2 = p_1\,\Delta V - p_2\,\Delta V = (p_1 - p_2)\,\Delta V$$

We can now use these expressions for ΔK, ΔU, and W to write the energy equation as

$$\underbrace{\frac{1}{2}\rho\,\Delta V v_2^2 - \frac{1}{2}\rho\,\Delta V v_1^2}_{\Delta K} + \underbrace{\rho\,\Delta V g y_2 - \rho\,\Delta V g y_1}_{\Delta U} = \underbrace{(p_1 - p_2)\,\Delta V}_{W}$$

The ΔV's cancel, and we can rearrange the remaining terms to get **Bernoulli's equation:**

$$p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 \qquad (13.14)$$

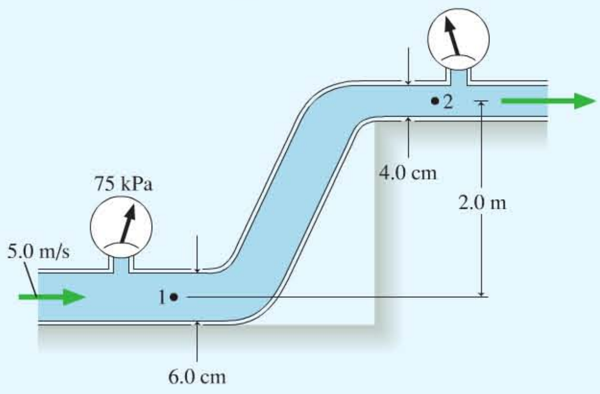
Bernoulli's equation relating pressure p, speed v, and height y at any two points along a streamline in an ideal fluid

Equation 13.14, a quantitative statement of the ideas we developed earlier in this section, is really nothing more than a statement about work and energy. Using Bernoulli's equation is very much like using the law of conservation of energy. Rather than identifying a "before" and "after," you want to identify two points on a streamline. As the following example shows, Bernoulli's equation is often used in conjunction with the equation of continuity.

EXAMPLE 13.12 Pressure in an irrigation system

Water flows through the pipes shown in **FIGURE 13.36**. The water's speed through the lower pipe is 5.0 m/s, and a pressure gauge reads 75 kPa. What is the reading of the pressure gauge on the upper pipe?

FIGURE 13.36 The water pipes of an irrigation system.



PREPARE Treat the water as an ideal fluid obeying Bernoulli's equation. Consider a streamline connecting point 1 in the lower pipe with point 2 in the upper pipe.

SOLVE Bernoulli's equation, Equation 13.14, relates the pressures, fluid speeds, and heights at points 1 and 2. It is easily solved for the pressure p_2 at point 2:

$$p_2 = p_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 + \rho g y_1 - \rho g y_2$$

$$= p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2)$$

All quantities on the right are known except v_2, and that is where the equation of continuity will be useful. The cross-section areas and water speeds at points 1 and 2 are related by

$$v_1 A_1 = v_2 A_2$$

from which we find

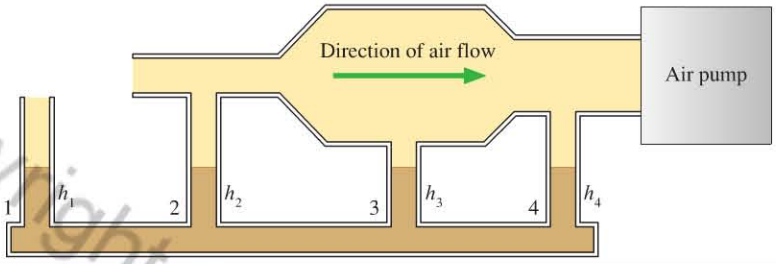
$$v_2 = \frac{A_1}{A_2}v_1 = \frac{r_1^2}{r_2^2}v_1 = \frac{(0.030\text{ m})^2}{(0.020\text{ m})^2}(5.0\text{ m/s}) = 11.25\text{ m/s}$$

The pressure at point 1 is $p_1 = 75\text{ kPa} + 1\text{ atm} = 176{,}300\text{ Pa}$. We can now use the above expression for p_2 to calculate $p_2 = 105{,}900$ Pa. This is the absolute pressure; the pressure gauge on the upper pipe will read

$$p_2 = 105{,}900\text{ Pa} - 1\text{ atm} = 4.6\text{ kPa}$$

ASSESS Reducing the pipe size decreases the pressure because it makes $v_2 > v_1$. Gaining elevation also reduces the pressure.

STOP TO THINK 13.5 Rank in order, from highest to lowest, the liquid heights h_1 to h_4 in tubes 1 to 4. The air flow is from left to right.



13.7 Viscosity and Poiseuille's Equation

Thus far, the only thing we've needed to know about a fluid is its density. It is the density that determines how the pressure in a static fluid increases with depth, and density appears in Bernoulli's equation for the motion of a fluid. But you know from everyday experience that another property of fluids is often crucial in determining how fluids flow. The densities of honey and water are not too different, but there's a huge difference in the way honey and water pour. The honey is much "thicker." This property of a fluid, which measures its resistance to flow, is called **viscosity**. A very viscous fluid flows slowly when poured and is difficult to force through a pipe or tube. The viscous nature of fluids is of key importance in understanding a wide range of real-world applications, from blood flow to the flight of birds to throwing a curveball.

An ideal fluid, with no viscosity, will "coast" at constant speed through a constant-diameter tube with no change in pressure. That's why the fluid heights are the same in the first and second pressure-measuring columns on the Venturi tube of Figure 13.31. But as **FIGURE 13.37** shows, any real fluid requires a *pressure difference* between the ends of a tube to keep the fluid moving at constant speed. The size of the pressure difference depends on the viscosity of the fluid. Think about how much harder you have to suck on a straw to drink a thick milkshake than to drink water or to pull air through the straw.

FIGURE 13.37 A viscous fluid needs a pressure difference to keep it moving. Compare this to Figure 13.31, which showed an ideal fluid.

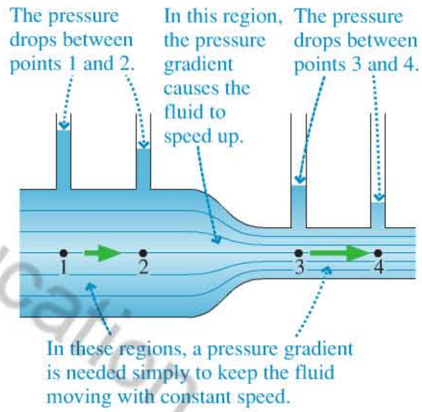


FIGURE 13.38 The pressure difference needed to keep the fluid flowing is proportional to the fluid's viscosity.

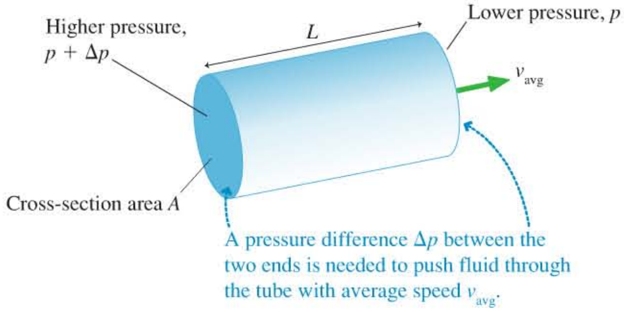


FIGURE 13.38 shows a viscous fluid flowing with a constant average speed v_{avg} through a tube of length L and cross-section area A. Experiments show that the pressure difference needed to keep the fluid moving is proportional to v_{avg} and to L and inversely proportional to A. We can write

$$\Delta p = 8\pi\eta\frac{Lv_{avg}}{A} \tag{13.15}$$

where $8\pi\eta$ is the constant of proportionality and η (lowercase Greek eta) is called the **coefficient of viscosity** (or just the *viscosity*). (The 8π enters the equation from the technical definition of viscosity, which need not concern us.)

Equation 13.15 makes sense. A more viscous fluid needs a larger pressure difference to push it through the tube; an ideal fluid, with $\eta = 0$, will keep flowing without any

TABLE 13.3 Viscosities of fluids

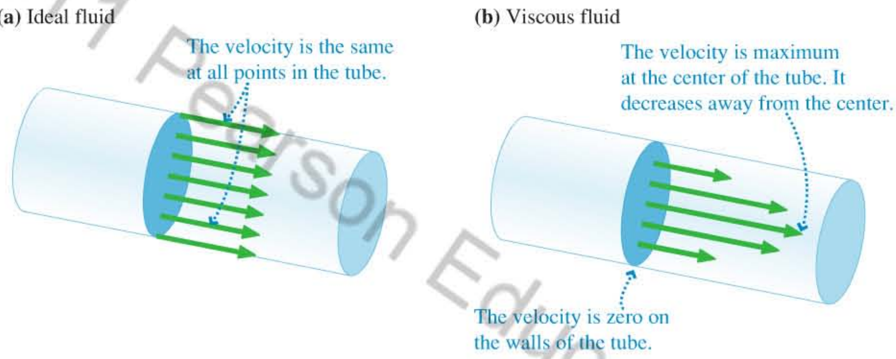
Fluid	η (Pa · s)
Air (20°C)	1.8×10^{-5}
Water (20°C)	1.0×10^{-3}
Water (40°C)	0.7×10^{-3}
Water (60°C)	0.5×10^{-3}
Whole blood (37°C)	2.5×10^{-3}
Motor oil (−30°C)	3×10^{5}
Motor oil (40°C)	0.07
Motor oil (100°C)	0.01
Honey (15°C)	600
Honey (40°C)	20

pressure difference. We can also see from Equation 13.15 that the units of viscosity are N · s/m^2 or, equivalently, Pa · s. Table 13.3 gives values of η for some common fluids. Note that the viscosity of many liquids decreases *very* rapidly with temperature. Cold oil hardly flows at all, but hot oil pours almost like water.

Poiseuille's Equation

Viscosity has a profound effect on how a fluid moves through a tube. **FIGURE 13.39a** shows that in an ideal fluid, all fluid particles move with the same speed v, the speed that appears in the equation of continuity. For a viscous fluid, **FIGURE 13.39b** shows that the fluid moves fastest in the center of the tube. The speed decreases as you move away from the center of the tube until reaching zero on the walls of the tube. That is, the layer of fluid in contact with the tube doesn't move at all. Whether it be water through pipes or blood through arteries, the fact that the fluid at the outer edges "lingers" and barely moves allows deposits to build on the inside walls of a tube.

FIGURE 13.39 Viscosity alters the velocities of the fluid particles.



Although we can't characterize the flow of a viscous liquid by a single flow speed v, we can still define the *average* flow speed. Suppose a fluid with viscosity η flows through a circular pipe with radius R and cross-section area $A = \pi R^2$. From Equation 13.15, a pressure difference Δp between the ends of the pipe causes the fluid to flow with average speed

$$v_{avg} = \frac{R^2}{8\eta L}\Delta p \tag{13.16}$$

The average flow speed is directly proportional to the pressure difference; for the fluid to flow twice as fast, you would need to double the pressure difference between the ends of the pipe.

Equation 13.13 defined the volume flow rate $Q = \Delta V/\Delta t$ and found that $Q = vA$ for an ideal fluid. For viscous flow, where v isn't constant throughout the fluid, we simply need to replace v with the average speed v_{avg} found in Equation 13.16. Using $A = \pi R^2$ for a circular tube, we see that a pressure difference Δp causes a volume flow rate

$$Q = v_{avg}A = \frac{\pi R^4 \Delta p}{8\eta L} \tag{13.17}$$

Poiseuille's equation for viscous flow through a tube of radius R and length L

This result is called **Poiseuille's equation** after the French scientist Jean Poiseuille (1797–1869) who first performed this calculation.

One surprising result of Poiseuille's equation is the very strong dependence of the flow on the tube's radius; the volume flow rate is proportional to the *fourth* power of R. If you double the radius of a tube, the flow rate will increase by a factor of $2^4 = 16$. This strong radius dependence comes from two factors. First, the flow rate depends on the area of the tube, which is proportional to R^2; larger tubes carry more fluid. Second, the average speed is faster in a larger tube because the center of the tube, where the flow is fastest, is farther from the "drag" exerted by the wall. As we've seen, the average speed is also proportional to R^2. These two terms combine to give the R^4 dependence.

CONCEPTUAL EXAMPLE 13.13 **Blood pressure and cardiovascular disease**

Cardiovascular disease is a narrowing of the arteries due to the buildup of plaque deposits on the interior walls. Magnetic resonance imaging, which you'll learn about in Chapter 24, can create exquisite three-dimensional images of the internal structure of the body. Shown are the carotid arteries that supply blood to the head, with a dangerous narrowing—a *stenosis*—indicated by the arrow.

If a section of an artery has narrowed by 8%, not nearly as much as the stenosis shown, by what percentage must the blood-pressure difference between the ends of the narrowed section increase to keep blood flowing at the same rate?

REASON According to Poiseuille's equation, the pressure difference Δp must increase to compensate for a decrease in the artery's radius R if the blood flow rate Q is to remain unchanged. If we write Poiseuille's equation as

$$R^4 \Delta p = \frac{8\eta L Q}{\pi}$$

we see that the product $R^4\Delta p$ must remain unchanged if the artery is to deliver the same flow rate. Let the initial artery radius and pressure difference be R_i and Δp_i. Disease decreases the radius by 8%, meaning that $R_f = 0.92R_i$. The requirement

$$R_i^4\,\Delta p_i = R_f^4\,\Delta p_f$$

can be solved for the new pressure difference:

$$\Delta p_f = \frac{R_i^4}{R_f^4}\,\Delta p_i = \frac{R_i^4}{(0.92R_i)^4}\,\Delta p_i = 1.4\,\Delta p_i$$

The pressure difference must increase by 40% to maintain the flow.

ASSESS Because the flow rate depends on R^4, even a small change in radius requires a large change in Δp to compensate. Either the person's blood pressure must increase, which is dangerous, or he or she will suffer a significant reduction in blood flow. For the stenosis shown in the image, the reduction in radius is much greater than 8%, and the pressure difference will be large and very dangerous.

EXAMPLE 13.14 **Pressure drop along a capillary**

In Example 13.11 we examined blood flow through a capillary. Using the numbers from that example, calculate the pressure "drop" from one end of a capillary to the other.

PREPARE Example 13.11 gives enough information to determine the flow rate through a capillary. We can then use Poiseuille's equation to calculate the pressure difference between the ends.

SOLVE The measured volume flow rate leaving the heart was given as $5\text{ L/min} = 8 \times 10^{-5}\text{ m}^3\text{/s}$. This flow is divided among all the capillaries, which we found to number $N = 3 \times 10^9$. Thus the flow rate through each capillary is

$$Q_{cap} = \frac{Q_{heart}}{N} = \frac{8 \times 10^{-5}\text{ m}^3\text{/s}}{3 \times 10^9} = 2.7 \times 10^{-14}\text{ m}^3\text{/s}$$

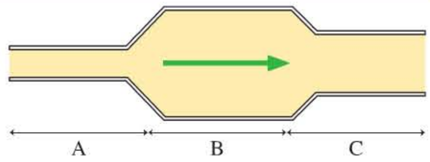
Solving Poiseuille's equation for Δp, we get

$$\Delta p = \frac{8\eta L Q_{cap}}{\pi R^4} = \frac{8(2.5 \times 10^{-3}\text{ Pa}\cdot\text{s})(0.001\text{ m})(2.7 \times 10^{-14}\text{ m}^3\text{/s})}{\pi(3 \times 10^{-6}\text{ m})^4} = 2100\text{ Pa}$$

Converting to mm of mercury, the units of blood pressure, the pressure drop across the capillary is $\Delta p = 16$ mm Hg.

ASSESS The average blood pressure provided by the heart (the average of the systolic and diastolic pressures) is about 100 mm Hg. A physiology textbook will tell you that the pressure has decreased to 35 mm by the time blood enters the capillaries, and it exits from capillaries into the veins at 17 mm. Thus the pressure drop across the capillaries is 18 mm Hg. Our calculation, based on the laws of fluid flow and some simple estimates of capillary size, is in almost perfect agreement with measured values.

STOP TO THINK 13.6 A viscous fluid flows through the pipe shown. The three marked segments are of equal length. Across which segment is the pressure difference the greatest?



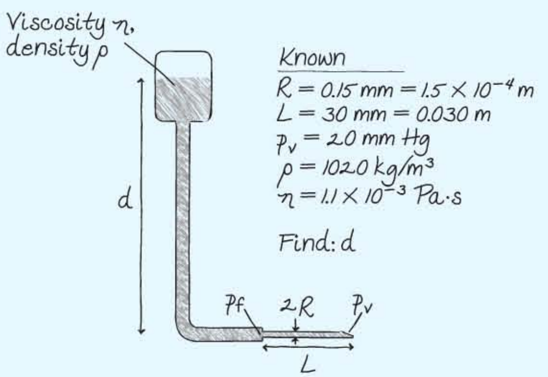
INTEGRATED EXAMPLE 13.15 An intravenous transfusion

At the hospital, a patient often receives fluids via an intravenous (IV) infusion. A bag of the fluid is held at a fixed height above the patient's body. The fluid then travels down a large-diameter, flexible tube to a catheter—a short tube with a small diameter—inserted into the patient's vein.

1.0 L of saline solution, with a density of 1020 kg/m^3 and a viscosity of 1.1×10^{-3} Pa·s, is to be infused into a patient in 8.0 h. The catheter is 30 mm long and has an inner diameter of 0.30 mm. The pressure in the patient's vein is 20 mm Hg. How high above the patient should the bag be positioned to get the desired flow rate?

PREPARE **FIGURE 13.40** shows a sketch of the situation, defines variables, and lists the known information. We're concerned with the flow of a viscous fluid. According to Poiseuille's equation, the flow rate depends inversely on the fourth power of a tube's radius. We are told that the tube from the elevated bag to the catheter has a large diameter, while the diameter of the catheter is small. Thus, we expect the flow rate to be determined entirely by the flow through the narrow catheter; the wide tube has a negligible effect on the rate.

FIGURE 13.40 Visual overview of an IV transfusion.



To use Poiseuille's equation for the catheter, we need the pressure difference Δp between the ends of the catheter. We know the pressure on the end of the catheter in the patient's vein is $p_v = 20$ mm Hg or, converting to SI units using Table 13.2,

$$p_v = 20 \text{ mm Hg} \times \frac{101 \times 10^3 \text{ Pa}}{760 \text{ mm Hg}} = 2700 \text{ Pa}$$

This is a gauge pressure, the pressure in excess of 1 atm. The pressure on the fluid side of the catheter is due to the hydrostatic pressure of the saline solution filling the bag and the flexible tube leading to the catheter. This pressure is given by the hydrostatic pressure equation $p = p_0 + \rho g d$, where d is the "depth" of the catheter below the bag. Thus we'll use Δp to find d.

SOLVE The desired volume flow rate is

$$Q = \frac{\Delta V}{\Delta t} = \frac{1.0 \text{ L}}{8.0 \text{ h}} = 0.125 \text{ L/h}$$

Converting to SI units using 1.0 L = 1.0×10^{-3} m^3, we have

$$Q = 0.125 \frac{\text{L}}{\text{h}} \times \frac{1.0 \times 10^{-3} \text{ m}^3}{\text{L}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 3.5 \times 10^{-8} \text{ m}^3/\text{s}$$

Poiseuille's equation for viscous fluid flow is

$$Q = \frac{\pi R^4 \Delta p}{8 \eta L}$$

Thus the pressure difference needed between the ends of the tube to produce the desired flow rate Q is

$$\Delta p = \frac{8\eta L Q}{\pi R^4} = \frac{8(1.1 \times 10^{-3} \text{ Pa}\cdot\text{s})(0.030 \text{ m})(3.5 \times 10^{-8} \text{ m}^3/\text{s})}{\pi(1.5 \times 10^{-4} \text{ m})^4} = 5800 \text{ Pa}$$

Now Δp is the difference between the fluid pressure p_f at one end of the catheter and the vein pressure p_v at the other end: $\Delta p = p_f - p_v$. We know p_v, so

$$p_f = p_v + \Delta p = 2700 \text{ Pa} + 5800 \text{ Pa} = 8500 \text{ Pa}$$

This pressure, like the vein pressure, is a gauge pressure. The true hydrostatic pressure at the catheter is $p = 1$ atm + 8500 Pa. But the hydrostatic pressure in the fluid is

$$p = p_0 + \rho g d = 1 \text{ atm} + \rho g d$$

We see that $\rho g d = 8500$ Pa. Solving for d, we find the required elevation of the bag above the patient's arm:

$$d = \frac{p_f}{\rho g} = \frac{8500 \text{ Pa}}{(1020 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 0.85 \text{ m}$$

ASSESS This height of about a meter seems reasonable for the height of an IV bag. In practice, the bag can be raised or lowered to adjust the fluid flow rate.

SUMMARY

The goal of Chapter 13 has been to understand the static and dynamic properties of fluids.

GENERAL PRINCIPLES

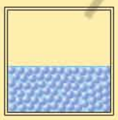
Fluid Statics

Gases

- Freely moving particles
- Compressible
- Pressure mainly due to particle collisions with walls

Liquids

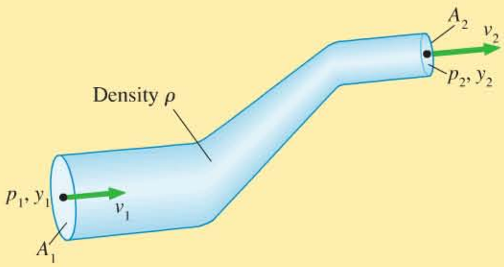
- Loosely bound particles
- Incompressible
- Pressure due to the weight of the liquid
- Hydrostatic pressure at depth d is $p = p_0 + \rho g d$
- The pressure is the same at all points on a horizontal line through a liquid (of one kind) in hydrostatic equilibrium



Fluid Dynamics

Ideal-fluid model

- Incompressible
- Smooth, laminar flow
- Nonviscous



Equation of continuity

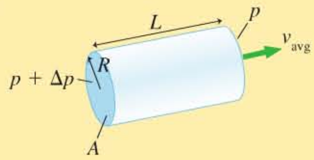
Volume flow rate $Q = \dfrac{\Delta V}{\Delta t} = v_1 A_1 = v_2 A_2$

Bernoulli's equation is a statement of energy conservation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Poiseuille's equation governs viscous flow through a tube:

$$Q = v_{avg} A = \frac{\pi R^4 \Delta p}{8\eta L}$$



IMPORTANT CONCEPTS

Density $\rho = m/V$, where m is mass and V is volume.

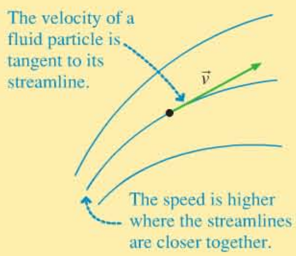
Pressure $p = F/A$, where F is force magnitude and A is the area on which the force acts.

- Pressure exists at all points in a fluid.
- Pressure pushes equally in all directions.
- Gauge pressure $p_g = p - 1$ atm.

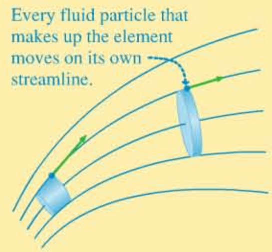
Viscosity η is the property of a fluid that makes it resist flowing.

Representing fluid flow

Streamlines are the paths of individual fluid particles.



Fluid elements contain a fixed volume of fluid. Their shape may change as they move.

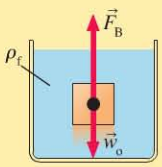


APPLICATIONS

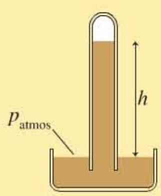
Buoyancy is the upward force of a fluid on an object immersed in the fluid.

Archimedes' principle: The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

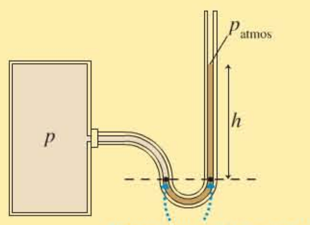
Sink:	$\rho_{avg} > \rho_f$	$F_B < w_o$
Float:	$\rho_{avg} < \rho_f$	$F_B > w_o$
Neutrally buoyant:	$\rho_{avg} = \rho_f$	$F_B = w_o$



Barometers measure atmospheric pressure. Atmospheric pressure is related to the height of the liquid column by $p_{atmos} = \rho g h$.



Manometers measure pressure. The pressure at the closed end of the tube is $p = 1\text{ atm} + \rho g h$.



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

QUESTIONS

Conceptual Questions

1. Which has the greater density, 1 g of mercury or 1000 g of water?
2. A 1×10^{-3} m^3 chunk of material has a mass of 3 kg.
   a. What is the material's density?
   b. Would a 2×10^{-3} m^3 chunk of the same material have the same mass? Explain.
   c. Would a 2×10^{-3} m^3 chunk of the same material have the same density? Explain.
3. You are given an irregularly shaped chunk of material and asked to find its density. List the *specific* steps that you would follow to do so.
4. Object 1 has an irregular shape. Its density is 4000 kg/m^3.
   a. Object 2 has the same shape and dimensions as object 1, but it is twice as massive. What is the density of object 2?
   b. Object 3 has the same mass and the same *shape* as object 1, but its size in all three dimensions is twice that of object 1. What is the density of object 3?
5. BIO When you get a blood transfusion the bag of blood is held above your body, but when you donate blood the collection bag is held below. Why is this?
6. BIO To explore the bottom of a 10-m-deep lake, your friend Tom proposes to get a long garden hose, put one end on land and the other in his mouth for breathing underwater, and descend into the depths. Susan, who overhears the conversation, reacts with horror and warns Tom that he will not be able to inhale when he is at the lake bottom. Why is Susan so worried?
7. Rank in order, from largest to smallest, the pressures at A, B, and C in Figure Q13.7. Explain.

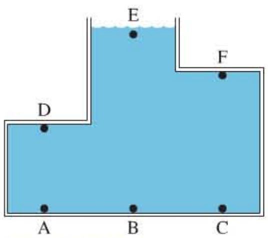


FIGURE Q13.7

8. Refer to Figure Q13.7. Rank in order, from largest to smallest, the pressures at D, E, and F. Explain.
9. Cylinders A and B contain liquids. The pressure p_A at the bottom of A is higher than the pressure p_B at the bottom of B. Is the ratio p_A/p_B of the absolute pressures larger, smaller, or equal to the ratio of the gauge pressures? Explain.
10. In Figure Q13.10, A and B are rectangular tanks full of water. They have equal heights and equal side lengths (the dimension into the page), but different widths.

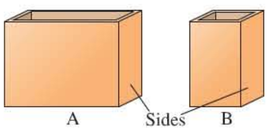


FIGURE Q13.10

   a. Compare the forces the water exerts on the bottoms of the tanks. Is F_A larger, smaller, or equal to F_B? Explain.
   b. Compare the forces the water exerts on the sides of the tanks. Is F_A larger, smaller, or equal to F_B? Explain.
11. Helium-filled weather balloons are spherical when they reach very high altitudes. However, they are only partially inflated with helium before they are released. Explain why this is done.

12. Water expands when heated. Suppose a beaker of water is heated from 10°C to 90°C. Does the pressure at the bottom of the beaker increase, decrease, or stay the same? Explain.
13. In Figure Q13.13, is p_A larger, smaller, or equal to p_B? Explain.

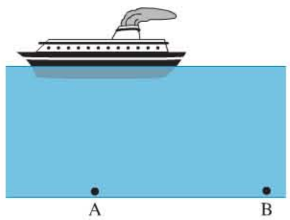


FIGURE Q13.13

14. A beaker of water rests on a scale. A metal ball is then lowered into the beaker using a string tied to the ball. The ball doesn't touch the sides or bottom of the beaker, and no water spills from the beaker. Does the scale reading decrease, increase, or stay the same? Explain.
15. Rank in order, from largest to smallest, the densities of objects A, B, and C in Figure Q13.15. Explain.

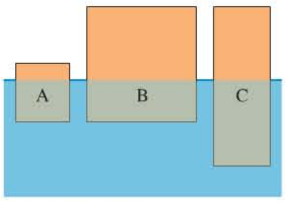


FIGURE Q13.15

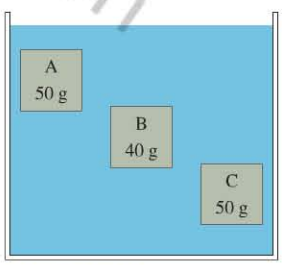


FIGURE Q13.16

16. Objects A, B, and C in Figure Q13.16 have the same volume. Rank in order, from largest to smallest, the sizes of the buoyant forces F_A, F_B, and F_C on A, B, and C. Explain.
17. Refer to Figure Q13.16. Now A, B, and C have the same density, but still have the masses given in the figure. Rank in order, from largest to smallest, the sizes of the buoyant forces on A, B, and C. Explain.
18. When you stand on a bathroom scale, it reads 700 N. Suppose a giant vacuum cleaner sucks half the air out of the room, reducing the pressure to 0.5 atm. Would the scale reading increase, decrease, or stay the same? Explain.
19. Suppose you stand on a bathroom scale that is on the bottom of a swimming pool. The water comes up to your waist. Does the scale read more, less, or the same as your true weight? Explain.
20. When you place an egg in water, it sinks. If you add salt to the water, after some time the egg floats. Explain.

21. Submerged submarines contain tanks filled with water. To rise to the surface, compressed air is used to force the water out of the tanks. Explain why this works.

22. Fish can adjust their buoyancy with an organ called the *swim*
BIO *bladder*. The swim bladder is a flexible gas-filled sac; the fish can increase or decrease the amount of gas in the swim bladder so that it stays neutrally buoyant—neither sinking nor floating. Suppose the fish is neutrally buoyant at some depth and then goes deeper. What needs to happen to the volume of air in the swim bladder? Will the fish need to add or remove gas from the swim bladder to maintain its neutral buoyancy?

23. Figure Q13.23 shows two identical beakers filled to the same height with water. Beaker B has a plastic sphere floating in it. Which beaker, with all its contents, weighs more? Or are they equal? Explain.

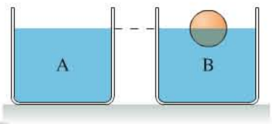


FIGURE Q13.23

24. A tub of water, filled to the brim, sits on a scale. Then a floating block of wood is placed in the tub, pushing some water over the rim. The water that overflows immediately runs off the scale. What happens to the reading of the scale?

25. Ships A and B have the same height and the same mass. Their cross-section profiles are shown in Figure Q13.25. Does one ship ride higher in the water (more height above the water line) than the other? If so, which one? Explain.

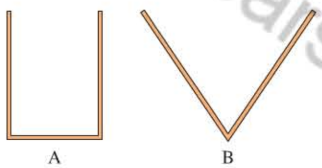


FIGURE Q13.25

26. Gas flows through a pipe, as shown in Figure Q13.26. The pipe's constant outer diameter is shown; you can't see into the pipe to know how the inner diameter changes. Rank in order, from largest to smallest, the gas speeds v_1 to v_3 at points 1, 2, and 3. Explain.

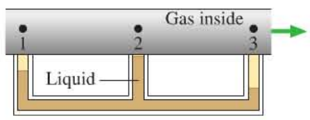


FIGURE Q13.26

27. Liquid flows through a pipe as shown in Figure Q13.27. The pipe's constant outer diameter is shown; you can't see into the pipe to know how the inner diameter changes. Rank in order, from largest to smallest, the flow speeds v_1 to v_3 at points 1, 2, and 3. Explain.

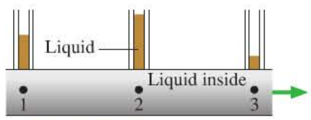


FIGURE Q13.27

28. A liquid with negligible viscosity flows through the pipe shown in Figure Q13.28. This is an overhead view.
    a. Rank in order, from largest to smallest, the flow speeds v_1 to v_4 at points 1 to 4. Explain.
    b. Rank in order, from largest to smallest, the pressures p_1 to p_4 at points 1 to 4. Explain.

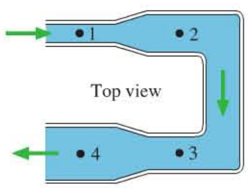


FIGURE Q13.28

FIGURE Q13.29

29. Wind blows over the house shown in Figure Q13.29. A window on the ground floor is open. Is there an air flow through the house? If so, does the air flow in the window and out the chimney, or in the chimney and out the window? Explain.

30. Two pipes have the same inner cross-section area. One has a circular cross section and the other has a rectangular cross section with its height one-tenth its width. Through which pipe, if either, would it be easier to pump a viscous liquid? Explain.

Multiple-Choice Questions

31. Figure Q13.31 shows a 100 g block of copper ($\rho = 8900\ \mathrm{kg/m^3}$) and a 100 g block of aluminum ($\rho = 2700\ \mathrm{kg/m^3}$) connected by a massless string that runs over two massless, frictionless pulleys. The two blocks exactly balance, since they have the same mass. Now suppose that the whole system is submerged in water. What will happen?

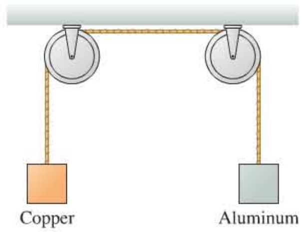


FIGURE Q13.31

    A. The copper block will fall, the aluminum block will rise.
    B. The aluminum block will fall, the copper block will rise.
    C. Nothing will change.
    D. Both blocks will rise.

32. Masses A and B rest on very light pistons that enclose a fluid, as shown in Figure Q13.32. There is no friction between the pistons and the cylinders they fit inside. Which of the following is true?

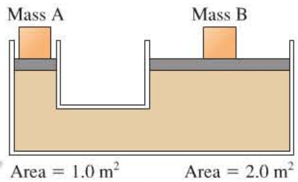


FIGURE Q13.32

    A. Mass A is greater. B. Mass B is greater.
    C. Mass A and mass B are the same.

33. If you dive underwater, you notice an uncomfortable pres-
BIO sure on your eardrums due to the increased pressure. The human eardrum has an area of about 70 $\mathrm{mm^2}$ ($7 \times 10^{-5}\ \mathrm{m^2}$), and it can sustain a force of about 7 N without rupturing. If your body had no means of balancing the extra pressure (which, in reality, it does), what would be the maximum depth you could dive without rupturing your eardrum?
    A. 0.3 m B. 1 m C. 3 m D. 10 m

34. An 8.0 lb bowling ball has a diameter of 8.5 inches. When lowered into water, this ball will
    A. Float. B. Sink. C. Have neutral buoyancy.

35. A basketball has a mass of 0.50 kg and a volume of $8.0 \times 10^{-3}\ \mathrm{m^3}$. What is the magnitude of the net force on a basketball when it is fully submerged in water?
    A. 4.9 N B. 74 N C. 78 N D. 83 N

36. An object floats in water, with 75% of its volume submerged. What is its approximate density?
    A. 250 $\mathrm{kg/m^3}$ B. 750 $\mathrm{kg/m^3}$
    C. 1000 $\mathrm{kg/m^3}$ D. 1250 $\mathrm{kg/m^3}$

37. A syringe is being used to squirt water as shown in Figure Q13.37. The water is ejected from the nozzle at 10 m/s. At what speed is the plunger of the syringe being depressed?

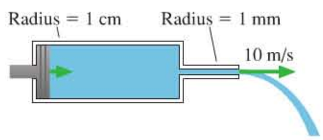


FIGURE Q13.37

    A. 0.01 m/s B. 0.1 m/s C. 1 m/s D. 10 m/s

38. || Water flows through a 4.0-cm-diameter horizontal pipe at a speed of 1.3 m/s. The pipe then narrows down to a diameter of 2.0 cm. Ignoring viscosity, what is the pressure difference between the wide and narrow sections of the pipe?
A. 850 Pa B. 3400 Pa C. 9300 Pa
D. 12,700 Pa E. 13,500 Pa

39. || A 15-m-long garden hose has an inner diameter of 2.5 cm. One end is connected to a spigot; 20°C water flows from the other end at a rate of 1.2 L/s. What is the gauge pressure at the spigot end of the hose?
A. 1900 Pa B. 2700 Pa C. 4200 Pa
D. 5800 Pa E. 7300 Pa

PROBLEMS

Section 13.1 Fluids and Density

1. || A 100 mL beaker holds 120 g of liquid. What is the liquid's density in SI units?
2. | Containers A and B have equal volumes. Container A holds helium gas at 1.0 atm pressure and 20°C. Container B is completely filled with a liquid whose mass is 7600 times the mass of helium gas in container A. Identify the liquid in B.
3. || Air enclosed in a sphere has density $\rho = 1.4\ \text{kg/m}^3$. What will the density be if the radius of the sphere is halved, compressing the air within?
4. || Air enclosed in a cylinder has density $\rho = 1.4\ \text{kg/m}^3$.
   a. What will be the density of the air if the length of the cylinder is doubled while the radius is unchanged?
   b. What will be the density of the air if the radius of the cylinder is halved while the length is unchanged?
5. || a. 50 g of gasoline are mixed with 50 g of water. What is the average density of the mixture?
   b. 50 cm^3 of gasoline are mixed with 50 cm^3 of water. What is the average density of the mixture?
6. ||| Ethyl alcohol has been added to 200 mL of water in a container that has a mass of 150 g when empty. The resulting container and liquid mixture has a mass of 512 g. What volume of alcohol was added to the water?
7. || The average density of the body of a fish is 1080 kg/m^3. To keep BIO from sinking, the fish increases its volume by inflating an internal air bladder, known as a swim bladder, with air. By what percent must the fish increase its volume to be neutrally buoyant in fresh water? Use the Table 13.1 value for the density of air at 20°C.

Section 13.2 Pressure

8. || The deepest point in the ocean is 11 km below sea level, deeper than Mt. Everest is tall. What is the pressure in atmospheres at this depth?
9. || a. What volume of water has the same mass as 8.0 m^3 of ethyl alcohol?
   b. If this volume of water completely fills a cubic tank, what is the pressure at the bottom?
10. ||| A 1.0-m-diameter vat of liquid is 2.0 m deep. The pressure at the bottom of the vat is 1.3 atm. What is the mass of the liquid in the vat?
11. ||| A 35-cm-tall, 5.0-cm-diameter cylindrical beaker is filled to its brim with water. What is the downward force of the water on the bottom of the beaker?
12. || The gauge pressure at the bottom of a cylinder of liquid is $p_g = 0.40$ atm. The liquid is poured into another cylinder with twice the radius of the first cylinder. What is the gauge pressure at the bottom of the second cylinder?
13. |||| A research submarine has a 20-cm-diameter window 8.0 cm thick. The manufacturer says the window can withstand forces up to 1.0×10^6 N. What is the submarine's maximum safe depth in seawater? The pressure inside the submarine is maintained at 1.0 atm.
14. |||| The highest that George can suck water up a very long straw BIO is 2.0 m. (This is a typical value.) What is the lowest pressure that he can maintain in his mouth?
15. ||| The two 60-cm-diameter cylinders in Figure P13.15, closed at one end, open at the other, are joined to form a single cylinder, then the air inside is removed.
   a. How much force does the atmosphere exert on the flat end of each cylinder?
   b. Suppose the upper cylinder is bolted to a sturdy ceiling. How many 100 kg football players would need to hang from the lower cylinder to pull the two cylinders apart?

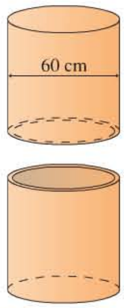


FIGURE P13.15

Section 13.3 Measuring and Using Pressure

16. |||| What is the gas pressure inside the box shown in Figure P13.16?

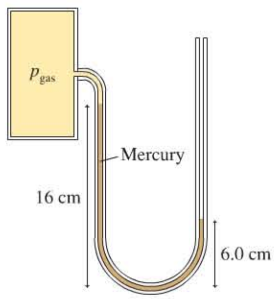


FIGURE P13.16

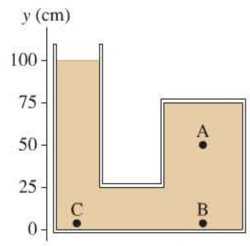


FIGURE P13.17

17. || The container shown in Figure P13.17 is filled with oil. It is open to the atmosphere on the left.
   a. What is the pressure at point A?
   b. What is the pressure difference between points A and B? Between points A and C?
18. ||| Glycerin is poured into an open U-shaped tube until the height in both sides is 20 cm. Ethyl alcohol is then poured into one arm until the height of the alcohol column is 20 cm. The two liquids do not mix. What is the difference in height between the top surface of the glycerin and the top surface of the alcohol?
19. ||| A U-shaped tube, open to the air on both ends, contains mercury. Water is poured into the left arm until the water column is 10.0 cm deep. How far upward from its initial position does the mercury in the right arm rise?

20. What is the height of a water barometer at atmospheric pressure?

21. BIO Postural hypotension is the occurrence of low (systolic) blood pressure when standing up too quickly from a reclined position, causing fainting or lightheadedness. For most people, a systolic pressure less than 90 mm Hg is considered low. If the blood pressure in your brain is 120 mm when you are lying down, what would it be when you stand up? Assume that your brain is 40 cm from your heart and that $\rho = 1060\ \text{kg/m}^3$ for your blood. Note: Normally, your blood vessels constrict and expand to keep your brain blood pressure stable when you change your posture.

Section 13.4 Buoyancy

22. A 6.00-cm-diameter sphere with a mass of 89.3 g is neutrally buoyant in a liquid. Identify the liquid.

23. A cargo barge is loaded in a saltwater harbor for a trip up a freshwater river. If the rectangular barge is 3.0 m by 20.0 m and sits 0.80 m deep in the harbor, how deep will it sit in the river?

24. A 10 cm × 10 cm × 10 cm wood block with a density of 700 kg/m^3 floats in water.
   a. What is the distance from the top of the block to the water if the water is fresh?
   b. If it's seawater?

25. What is the tension in the string in Figure P13.25?

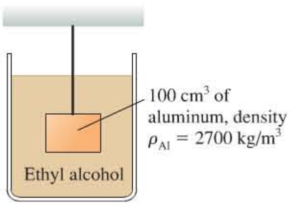


FIGURE P13.25

26. A 10 cm × 10 cm × 10 cm block of steel ($\rho_{steel} = 7900\ \text{kg/m}^3$) is suspended from a spring scale. The scale is in newtons.
   a. What is the scale reading if the block is in air?
   b. What is the scale reading after the block has been lowered into a beaker of oil and is completely submerged?

27. Styrofoam has a density of 300 kg/m^3. What is the maximum mass that can hang without sinking from a 50-cm-diameter Styrofoam sphere in water? Assume the volume of the mass is negligible compared to that of the sphere.

28. Calculate the buoyant force due to the surrounding air on a man weighing 800 N. Assume his average density is the same as that of water.

Section 13.5 Fluids in Motion

29. River Pascal with a volume flow rate of 5.0×10^5 L/s joins with River Archimedes, which carries 10.0×10^5 L/s, to form the Bernoulli River. The Bernoulli River is 150 m wide and 10 m deep. What is the speed of the water in the Bernoulli River?

30. Water flowing through a 2.0-cm-diameter pipe can fill a 300 L bathtub in 5.0 min. What is the speed of the water in the pipe?

31. A pump is used to empty a 6000 L wading pool. The water exits the 2.5-cm-diameter hose at a speed of 2.1 m/s. How long will it take to empty the pool?

32. A 1.0-cm-diameter pipe widens to 2.0 cm, then narrows to 0.50 cm. Liquid flows through the first segment at a speed of 4.0 m/s.
   a. What are the speeds in the second and third segments?
   b. What is the volume flow rate through the pipe?

Section 13.6 Fluid Dynamics

33. What does the top pressure gauge in Figure P13.33 read?

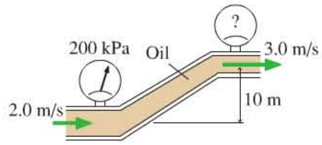


FIGURE P13.33

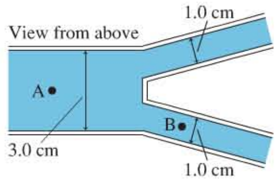


FIGURE P13.34

34. The 3.0-cm-diameter water line in Figure P13.34 splits into two 1.0-cm-diameter pipes. All pipes are circular and at the same elevation. At point A, the water speed is 2.0 m/s and the gauge pressure is 50 kPa. What is the gauge pressure at point B?

35. A rectangular trough, 2.0 m long, 0.60 m wide, and 0.45 m deep, is completely full of water. One end of the trough has a small drain plug right at the bottom edge. When you pull the plug, at what speed does water emerge from the hole?

Section 13.7 Viscosity and Poiseuille's Equation

36. What pressure difference is required between the ends of a 2.0-m-long, 1.0-mm-diameter horizontal tube for 40°C water to flow through it at an average speed of 4.0 m/s?

37. Water flows at 0.25 L/s through a 10-m-long garden hose 2.5 cm in diameter that is lying flat on the ground. The temperature of the water is 20°C. What is the gauge pressure of the water where it enters the hose?

38. Figure P13.38 shows a water-filled syringe with a 4.0-cm-long needle. What is the gauge pressure of the water at the point P, where the needle meets the wider chamber of the syringe?

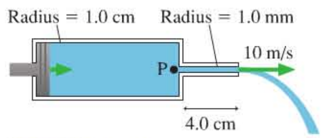


FIGURE P13.38

General Problems

39. The density of gold is 19,300 kg/m^3. 197 g of gold is shaped into a cube. What is the length of each edge?

40. INT The density of copper is 8920 kg/m^3. How many moles are in a 2.0 cm × 2.0 cm × 2.0 cm cube of copper?

41. INT The density of aluminum is 2700 kg/m^3. How many atoms are in a 2.0 cm × 2.0 cm × 2.0 cm cube of aluminum?

42. A 50-cm-thick layer of oil floats on a 120-cm-thick layer of water. What is the pressure at the bottom of the water layer?

43. An oil layer floats on 85 cm of water in a tank. The absolute pressure at the bottom of the tank is 112.0 kPa. How thick is the oil?

44. The little Dutch boy saved Holland by sticking his finger in the leaking dike. If the water level was 2.5 m above his finger, *estimate* the force of the water on his finger.

45. a. In Figure P13.45, how much force does the fluid exert on the end of the cylinder at A?
b. How much force does the fluid exert on the end of the cylinder at B?

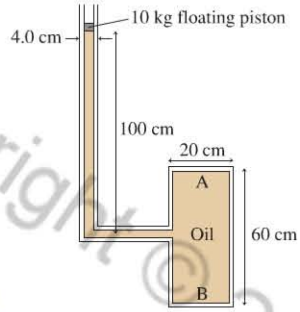


FIGURE P13.45

46. A friend asks you how much pressure is in your car tires. You know that the tire manufacturer recommends 30 psi, but it's been a while since you've checked. You can't find a tire gauge in the car, but you do find the owner's manual and a ruler. Fortunately, you've just finished taking physics, so you tell your friend, "I don't know, but I can figure it out." From the owner's manual you find that the car's mass is 1500 kg. It seems reasonable to assume that each tire supports one-fourth of the weight. With the ruler you find that the tires are 15 cm wide and the flattened segment of the tire in contact with the road is 13 cm long. What answer will you give your friend?

47. INT A diver 50 m deep in 10°C fresh water exhales a 1.0-cm-diameter bubble. What is the bubble's diameter just as it reaches the surface of the lake, where the water temperature is 20°C? **Hint:** Assume that the air bubble is always in thermal equilibrium with the surrounding water.

48. A 6.0-cm-tall cylinder floats in water with its axis perpendicular to the surface. The length of the cylinder above water is 2.0 cm. What is the cylinder's mass density?

49. A sphere completely submerged in water is tethered to the bottom with a string. The tension in the string is one-third the weight of the sphere. What is the density of the sphere?

50. You need to determine the density of a ceramic statue. If you suspend it from a spring scale, the scale reads 28.4 N. If you then lower the statue into a tub of water so that it is completely submerged, the scale reads 17.0 N. What is the density?

51. A 5.0 kg rock whose density is 4800 kg/m^3 is suspended by a string such that half of the rock's volume is under water. What is the tension in the string?

52. A flat slab of styrofoam, with a density of 32 kg/m^3, floats on a lake. What minimum volume must the slab have so that a 40 kg boy can sit on the slab without it sinking?

53. BIO A 2.0 mL syringe has an inner diameter of 6.0 mm, a needle inner diameter of 0.25 mm, and a plunger pad diameter (where you place your finger) of 1.2 cm. A nurse uses the syringe to inject medicine into a patient whose blood pressure is 140/100. Assume the liquid is an ideal fluid.
a. What is the minimum force the nurse needs to apply to the syringe?
b. The nurse empties the syringe in 2.0 s. What is the flow speed of the medicine through the needle?

54. INT A child's water pistol shoots water through a 1.0-mm-diameter hole. If the pistol is fired horizontally 70 cm above the ground, a squirt hits the ground 1.2 m away. What is the volume flow rate during the squirt? Ignore air resistance.

55. BIO The leaves of a tree lose water to the atmosphere via the process of *transpiration*. A particular tree loses water at the rate of 3×10^{-8} m^3/s; this water is replenished by the upward flow of sap through vessels in the trunk. This tree's trunk contains about 2000 vessels, each 100 μm in diameter. What is the speed of the sap flowing in the vessels?

56. A hurricane wind blows across a 6.00 m $\times$ 15.0 m flat roof at a speed of 130 km/h.
a. Is the air pressure above the roof higher or lower than the pressure inside the house? Explain.
b. What is the pressure difference?
c. How much force is exerted on the roof? If the roof cannot withstand this much force, will it "blow in" or "blow out"?

57. Water flows from the pipe shown in Figure P13.57 with a speed of 4.0 m/s.
a. What is the water pressure as it exits into the air?
b. What is the height h of the standing column of water?

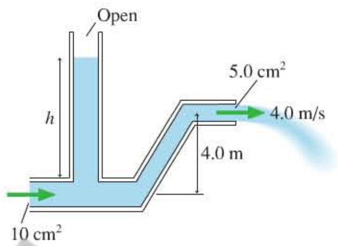


FIGURE P13.57

58. Air flows through the tube shown in Figure P13.58. Assume that air is an ideal fluid.
a. What are the air speeds v_1 and v_2 at points 1 and 2?
b. What is the volume flow rate?

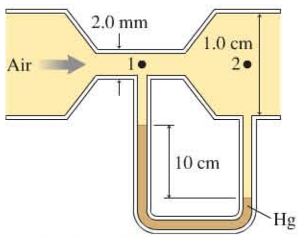


FIGURE P13.58

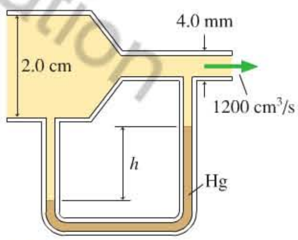


FIGURE P13.59

59. INT Air flows through the tube shown in Figure P13.59 at a rate of 1200 cm^3/s. Assume that air is an ideal fluid. What is the height h of mercury in the right side of the U-tube?

60. Water flows at 5.0 L/s through a horizontal pipe that narrows smoothly from 10 cm diameter to 5.0 cm diameter. A pressure gauge in the narrow section reads 50 kPa. What is the reading of a pressure gauge in the wide section?

61. INT The mercury manometer shown in Figure P13.61 is attached to a gas cell. The mercury height h is 120 mm when the cell is placed in an ice-water mixture. The mercury height drops to 30 mm when the device is carried into an industrial freezer. What is the freezer temperature?
Hint: The right tube of the manometer is much narrower than the left tube. What reasonable assumption can you make about the gas volume?

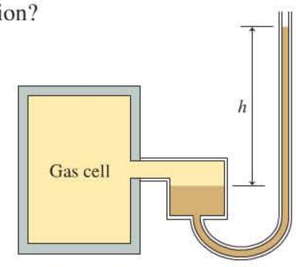


FIGURE P13.61

62. Figure P13.62 shows a section of a long tube that narrows near its open end to a diameter of 1.0 mm. Water at 20°C flows out of the open end at 0.020 L/s. What is the gauge pressure at point P, where the diameter is 4.0 mm?

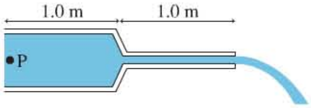


FIGURE P13.62

63. Smoking tobacco is bad for your circulatory health. In an attempt to maintain the blood's capacity to deliver oxygen, the body increases its red blood cell production, and this increases the viscosity of the blood. In addition, nicotine from tobacco causes arteries to constrict. BIO

For a nonsmoker, with blood viscosity of 2.5×10^{-3} Pa·s, normal blood flow requires a pressure difference of 8.0 mm Hg between the two ends of an artery. If this person were to smoke regularly, his blood viscosity would increase to 2.7×10^{-3} Pa·s, and the arterial diameter would constrict to 90% of its normal value. What pressure difference would be needed to maintain the same blood flow?

64. A stiff, 10-cm-long tube with an inner diameter of 3.0 mm is attached to a small hole in the side of a tall beaker. The tube sticks out horizontally. The beaker is filled with 20°C water to a level 45 cm above the hole, and it is continually topped off to maintain that level. What is the volume flow rate through the tube?

Passage Problems

Blood Pressure and Blood Flow BIO

The blood pressure at your heart is approximately 100 mm Hg. As blood is pumped from the left ventricle of your heart, it flows through the aorta, a single large blood vessel with a diameter of about 2.5 cm. The speed of blood flow in the aorta is about 60 cm/s. Any change in pressure as blood flows in the aorta is due to the change in height: the vessel is large enough that viscous drag is not a major factor. As the blood moves through the circulatory system, it flows into successively smaller and smaller blood vessels until it reaches the capillaries. Blood flows in the capillaries at the much lower speed of approximately 0.7 mm/s. The diameter of capillaries and other small blood vessels is so small that viscous drag is a major factor.

65. There is a limit to how long your neck can be. If your neck were too long, no blood would reach your brain! What is the maximum height a person's brain could be above his heart, given the noted pressure and assuming that there are no valves or supplementary pumping mechanisms in the neck? The density of blood is 1060 kg/m^3.
A. 0.97 m B. 1.3 m C. 9.7 m D. 13 m

66. Because the flow speed in your capillaries is much less than in the aorta, the total cross-section area of the capillaries considered together must be much larger than that of the aorta. Given the flow speeds noted, the total area of the capillaries considered together is equivalent to the cross-section area of a single vessel of approximately what diameter?
A. 25 cm B. 50 cm C. 75 cm D. 100 cm

67. Suppose that in response to some stimulus a small blood vessel narrows to 90% its original diameter. If there is no change in the pressure across the vessel, what is the ratio of the new volume flow rate to the original flow rate?
A. 0.66 B. 0.73 C. 0.81 D. 0.90

68. Sustained exercise can increase the blood flow rate of the heart by a factor of 5 with only a modest increase in blood pressure. This is a large change in flow. Although several factors come into play, which of the following physiological changes would most plausibly account for such a large increase in flow with a small change in pressure?
A. A decrease in the viscosity of the blood
B. Dilation of the smaller blood vessels to larger diameters
C. Dilation of the aorta to larger diameter
D. An increase in the oxygen carried by the blood

STOP TO THINK ANSWERS

Stop to Think 13.1: $\rho_1 = \rho_2 = \rho_3$. Density depends only on what the object is made of, not how big the pieces are.

Stop to Think 13.2: C. These are all open tubes, so the liquid rises to the same height in all three despite their different shapes.

Stop to Think 13.3: B. The weight of the displaced water equals the weight of the ice cube. When the ice cube melts and turns into water, that amount of water will exactly fill the volume that the ice cube is now displacing.

Stop to Think 13.4: 1 cm^3/s out. The fluid is incompressible, so the sum of what flows in must match the sum of what flows out. 13 cm^3/s is known to be flowing in while 12 cm^3/s flows out. An additional 1 cm^3/s must flow out to achieve balance.

Stop to Think 13.5: $h_2 > h_4 > h_3 > h_1$. The liquid level is higher where the pressure is lower. The pressure is lower where the flow speed is higher. The flow speed is highest in the narrowest tube, zero in the open air.

Stop to Think 13.6: A. All three segments have the same volume flow rate Q. According to Poiseuille's equation, the segment with the smallest radius R has the greatest pressure difference Δp.

Properties of Matter

The goal of Part III has been to understand macroscopic systems and their properties. We've introduced no new fundamental laws in these chapters. Instead, we've broadened and extended the scope of Newton's laws and the law of conservation of energy. In many ways, Part III has focused on *applications* of the general principles introduced in Parts I and II.

That matter exists in three phases—solids, liquids, gases—is perhaps the most basic fact about macroscopic systems. We looked at an atomic-level picture of the three phases of matter, and we also found a connection between *heat* and *phase changes.* Both liquids and gases are *fluids,* and we spent quite a bit of time investigating the properties of fluids, ranging from pressure and the ideal-gas law to buoyancy, fluid flow, and Bernoulli's equation.

Along the way, we introduced *mechanical properties* of matter, such as density and viscosity, and *thermal properties,* such as thermal-expansion coefficients and specific heats. You should now be able to use these properties to figure out what happens when you heat a system, whether an object will float, and how fast a fluid flows. These are all very practical, useful things to know, with many applications to the world around us.

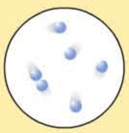
Last, but not least, we discovered that many *macroscopic* properties are determined by the *microscopic* motions of atoms and molecules. For example, we found that the ideal-gas law is based on the idea that gas pressure is due to the collisions of atoms with the walls of the container. Similarly, regularities in the specific heats of gases were explained on the basis of how atoms and molecules share the thermal energy of a system. Atoms are important, even if we can't see them or follow their individual motions.

KNOWLEDGE STRUCTURE III Properties of Matter

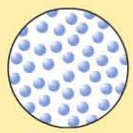
BASIC GOAL How can we describe the macroscopic flows of energy and matter in heat transfer and fluid flow?

GENERAL PRINCIPLES Phases of matter

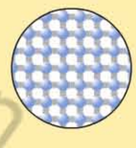
Gas: Particles are freely moving. Thermal energy is kinetic energy of motion of particles. Pressure is due to collisions of particles with walls of container.



Liquid: Particles are loosely bound, but flow is possible. Heat is transferred by convection and conduction. Pressure is due to gravity.



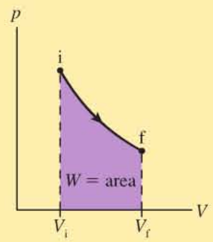
Solid: Particles are joined by spring-like bonds. Increasing temperature causes expansion. Heat is transferred by conduction only.



Heat must be added to change a solid to a liquid and a liquid to a gas. Heat must be removed to reverse these changes.

Ideal-gas processes

We can graph processes on a pV diagram. Work by the gas is the area under the graph.



For a gas process in a sealed container, the ideal-gas law relates values of pressure, volume, and temperature:

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$$

- Constant-volume process $V = \text{constant}$
- Isothermal process $T = \text{constant}$
- Isobaric process $p = \text{constant}$
- Adiabatic process $Q = 0$

Heat and heat transfer

The **specific heat** c of a material is the heat required to raise the temperature of 1 kg by 1 K:

$$Q = Mc\,\Delta T$$

When two or more systems interact thermally, they come to a common final temperature determined by

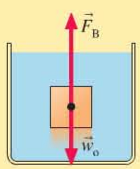
$$Q_{net} = Q_1 + Q_2 + Q_3 + \cdots = 0$$

Conduction transfers heat by direct physical contact.
Convection transfers heat by the motion of a fluid.
Radiation transfers heat by electromagnetic waves.

Fluid statics

Archimedes' principle: The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

An object sinks if it is more dense than the fluid in which it is submerged; if it is less dense, it floats.



Pressure increases with depth in a liquid. The pressure at depth d is

$$p = p_0 + \rho g d$$

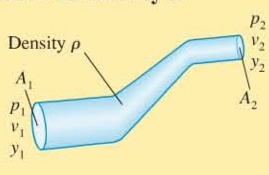
Fluid flow

Ideal fluid flow is laminar, incompressible, and nonviscous. The **equation of continuity** is

$$v_1 A_1 = v_2 A_2$$

Bernoulli's equation is

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

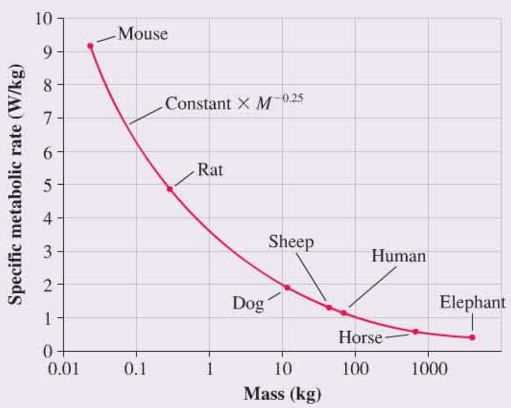


Size and Life

Physicists look for simple models and general principles that underlie and explain diverse physical phenomena. In the first 13 chapters of this book, you've seen that just a handful of general principles and laws can be used to solve a wide range of problems. Can this approach have any relevance to a subject like biology? It may seem surprising, but there *are* general "laws of biology" that apply, with quantitative accuracy, to organisms as diverse as elephants and mice.

Let's look at an example. An elephant uses more metabolic power than a mouse. This is not surprising, as an elephant is quite a bit bigger. But recasting the data shows an interesting trend. When we looked at the energy required to raise the temperature of different substances, we considered *specific heat*. The "specific" meant that we considered the heat required for 1 kilogram. For animals, rather than metabolic rate, we can look at the *specific metabolic rate,* the metabolic power used *per kilogram* of tissue. If we factor out the mass difference between a mouse and an elephant, are their specific metabolic powers the same?

In fact, the specific metabolic rate varies quite a bit among mammals, as the graph of specific metabolic rate versus mass shows. But there is an interesting trend: All of the data points lie on a single smooth curve. In other words, there really is a biological *law* we can use to predict a mammal's metabolic rate knowing only its mass M. In particular, the specific metabolic rate is proportional to $M^{-0.25}$. Because a 4000 kg elephant is 160,000 times more massive than a 25 g mouse, the mouse's specific metabolic power is $(160{,}000)^{0.25} = 20$ times that of the elephant. A law that shows how a property scales with the size of a system is called a *scaling law*.



Specific metabolic rate as a function of body mass follows a simple scaling law.

A similar scaling law holds for birds, reptiles, and even bacteria. Why should a single simple relationship hold true for organisms that range in size from a single cell to a 100 ton blue whale? Interestingly, no one knows for sure. It is a matter of current research to find out just what this and other scaling laws tell us about the nature of life.

Perhaps the metabolic-power scaling law is a result of heat transfer. In Chapter 12, we noted that all metabolic energy used by an animal ends up as heat, which must be transferred to the environment. A 4000 kg elephant has 160,000 times the mass of a 25 g mouse, but it has only about 3,000 times the surface area. The heat transferred to the environment depends on the surface area; the more surface area, the greater the rate of heat transfer. An elephant with a mouse-sized metabolism simply wouldn't be able to dissipate heat fast enough—it would quickly overheat and die.

If heat dissipation were the only factor limiting metabolism, we can show that the specific metabolic rate should scale as $M^{-0.33}$, quite different from the $M^{-0.25}$ scaling observed. Clearly, another factor is at work. Exactly what underlies the $M^{-0.25}$ scaling is still a matter of debate, but some recent analysis suggests the scaling is due to limitations not of heat transfer but of fluid flow. Cells in mice, elephants, and all mammals receive nutrients and oxygen for metabolism from the bloodstream. Because the minimum size of a capillary is about the same for all mammals, the structure of the circulatory system must vary from animal to animal. The human aorta has a diameter of about 1 inch; in a mouse, the diameter is approximately 1/20th of this. Thus a mouse has fewer levels of branching to smaller and smaller blood vessels as we move from the aorta to the capillaries. The smaller blood vessels in mice mean that viscosity is more of a factor throughout the circulatory system. The circulatory system of a mouse is quite different from that of an elephant.

A model of specific metabolic rate based on blood-flow limitations predicts a $M^{-0.25}$ law, exactly as observed. The model also makes other testable predictions. For example, the model predicts that the smallest possible mammal should have a body mass of about 1 gram—exactly the size of the smallest shrew. Even smaller animals have different types of circulatory systems; in the smallest animals, nutrient transport is by diffusion alone. But the model can be extended to predict that the specific metabolic rate for these animals will follow a scaling law similar to that for mammals, exactly as observed. It is too soon to know if this model will ultimately prove to be correct, but it's indisputable that there are large-scale regularities in biology that follow mathematical relationships based on the laws of physics.

 VIEW ALL SOLUTIONS

The following questions are related to the passage "Size and Life" on the previous page. BIO

1. A typical timber wolf has a mass of 40 kg, a typical jackrabbit a mass of 2.5 kg. Given the scaling law presented in the passage, we'd expect the specific metabolic rate of the jackrabbit to be higher by a factor of
   A. 2 B. 4 C. 8 D. 16
2. A typical timber wolf has a mass of 40 kg, a typical jackrabbit a mass of 2.5 kg. Given the scaling law presented in the passage, we'd expect the wolf to use _______ times more energy than a jackrabbit in the course of a day.
   A. 2 B. 4 C. 8 D. 16
3. Given the data of the graph, approximately how much energy, in Calories, would a 200 g rat use during the course of a day?
   A. 10 B. 20 C. 100 D. 200
4. All other things being equal, species that inhabit cold climates tend to be larger than related species that inhabit hot climates. For instance, the Alaskan hare is the largest North American hare, with a typical mass of 5.0 kg, double that of a jackrabbit. A likely explanation is that
   A. Larger animals have more blood flow, allowing for better thermoregulation.
   B. Larger animals need less food to survive than smaller animals.
   C. Larger animals have larger blood volumes than smaller animals.
   D. Larger animals lose heat less quickly than smaller animals.
5. The passage proposes that there are quantitative "laws" of biology that have their basis in physical principles, using the scaling of specific metabolic rate with body mass as an example. Which of the following regularities among animals might also be an example of such a "law"?
   A. As a group, birds have better color vision than mammals.
   B. Reptiles have a much lower specific metabolic rate than mammals.
   C. Predators tend to have very good binocular vision; prey animals tend to be able to see over a very wide angle.
   D. Jump height varies very little among animals. Nearly all animals, ranging in size from a flea to a horse, have a maximum vertical leap that is quite similar.

VIEW ALL SOLUTIONS

The following passages and associated questions are based on the material of Part III.

Keeping Your Cool BIO

A 68 kg cyclist is pedaling down the road at 15 km/h, using a total metabolic power of 480 W. A certain fraction of this energy is used to move the bicycle forward, but the balance ends up as thermal energy in his body, which he must get rid of to keep cool. On a very warm day, conduction, convection, and radiation transfer little energy, and so he does this by perspiring, with the evaporation of water taking away the excess thermal energy.

6. If the cyclist reaches his 15 km/h cruising speed by rolling down a hill, what is the approximate height of the hill?
   A. 22 m B. 11 m C. 2 m D. 1 m
7. As he cycles at a constant speed on level ground, at what rate is chemical energy being converted to thermal energy in his body, assuming a typical efficiency of 25% for the conversion of chemical energy to the mechanical energy of motion?
   A. 480 W B. 360 W C. 240 W D. 120 W
8. To keep from overheating, the cyclist must get rid of the excess thermal energy generated in his body. If he cycles at this rate for 2 hours, how many liters of water must he perspire, to the nearest 0.1 liter?
   A. 0.4 L B. 0.9 L C. 1.1 L D. 1.4 L
9. Being able to exhaust this thermal energy is very important. If he isn't able to get rid of any of the excess heat, by how much will the temperature of his body increase in 10 minutes of riding, to the nearest 0.1°C?
   A. 0.3°C B. 0.6°C C. 0.9°C D. 1.2°C

Weather Balloons

The data used to generate weather forecasts are gathered by hundreds of weather balloons launched from sites throughout the world. A typical balloon is made of latex and filled with hydrogen. A packet of sensing instruments (called a *radiosonde*) transmits information back to earth as the balloon rises into the atmosphere.

At the beginning of its flight, the average density of the weather balloon package (total mass of the balloon plus cargo divided by their volume) is less than the density of the surrounding air, so the balloon rises. As it does, the density of the surrounding air decreases, as shown in Figure III.1. The balloon will rise to the point at which the buoyant force of the air exactly balances its weight. This would not be very high if the balloon couldn't expand. However, the latex envelope of the balloon is very thin and very stretchy, so the balloon can, and does, expand, allowing the volume to increase by a factor of 100 or more. The expanding balloon displaces an ever-larger volume of the lower-density air, keeping the buoyant force greater than the weight force until the balloon rises to an altitude of 40 km or more.

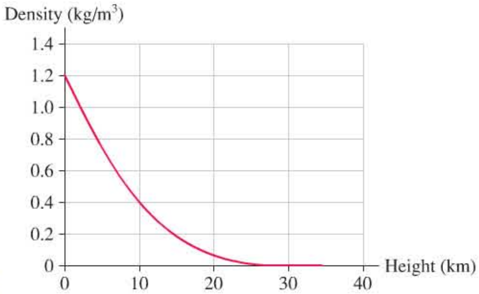


FIGURE III.1

10. A balloon launched from sea level has a volume of approximately 4 m^3. What is the approximate buoyant force on the balloon?
A. 50 N B. 40 N
C. 20 N D. 10 N

11. A balloon launched from sea level with a volume of 4 m^3 will have a volume of about 12 m^3 on reaching an altitude of 10 km. What is the approximate buoyant force now?
A. 50 N B. 40 N
C. 20 N D. 10 N

12. The balloon expands as it rises, keeping the pressures inside and outside the balloon approximately equal. If the balloon rises slowly, heat transfers will keep the temperature inside the same as the outside air temperature. A balloon with a volume of 4.0 m^3 is launched at sea level, where the atmospheric pressure is 100 kPa and the temperature is 15°C. It then rises slowly to a height of 5500 m, where the pressure is 50 kPa and the temperature is –20°C. What is the volume of the balloon at this altitude?
A. 5.0 m^3 B. 6.0 m^3
C. 7.0 m^3 D. 8.0 m^3

13. If the balloon rises quickly, so that no heat transfer is possible, the temperature inside the balloon will drop as the gas expands. If a 4.0 m^3 balloon is launched at a pressure of 100 kPa and rapidly rises to a point where the pressure is 50 kPa, the volume of the balloon will be
A. Greater than 8.0 m^3
B. 8.0 m^3
C. Less than 8.0 m^3

14. At the end of the flight, the radiosonde is dropped and falls to earth by parachute. Suppose the parachute achieves its terminal speed at a height of 30 km. As it descends into the atmosphere, how does the terminal speed change?
A. It increases.
B. It stays the same.
C. It decreases.

Passenger Balloons

Long-distance balloon flights are usually made using a hot-air-balloon/helium-balloon hybrid. The balloon has a sealed, flexible chamber of helium gas that expands or contracts to keep the helium pressure approximately equal to the air pressure outside. The helium chamber sits on top of an open (that is, air can enter or leave), constant-volume chamber of propane-heated air. Assume that the hot air and the helium are kept at a constant temperature by burning propane.

15. A balloon is launched at sea level, where the air pressure is 100 kPa. The helium has a volume of 1000 m^3 at this altitude. What is the volume of the helium when the balloon has risen to a height where the atmospheric pressure is 33 kPa?
A. 330 m^3 B. 500 m^3 C. 1000 m^3 D. 3000 m^3

16. A balloon is launched at sea level, where the air pressure is 100 kPa. The density in the hot-air chamber is 1.0 kg/m^3. What is the density of the air when the balloon has risen to a height where the atmospheric pressure is 33 kPa?
A. 3.0 kg/m^3
B. 1.0 kg/m^3
C. 0.66 kg/m^3
D. 0.33 kg/m^3

17. A balloon is at a height of 5.0 km and is descending at a constant rate. The buoyancy force is directed _______; the drag force is directed _______.
A. Up, up
B. Up, down
C. Down, up
D. Down, down

Additional Integrated Problems

18. BIO When you exhale, all of the air in your lungs must exit through the trachea. If you exhale through your nose, this air subsequently leaves through your nostrils. The area of your nostrils is less than that of your trachea. How does the speed of the air in the trachea compare to that in the nostrils?

19. BIO Sneezing requires an increase in pressure of the air in the lungs; a typical sneeze might result in an extra pressure of 7.0 kPa. Estimate how much force this exerts on the diaphragm, the large muscle at the bottom of the ribcage.

20. A 20 kg block of aluminum sits on the bottom of a tank of water. How much force does the block exert on the bottom of the tank?

21. BIO We've seen that fish can control their buoyancy through the use of a swim bladder, a gas-filled organ inside the body. You can assume that the gas pressure inside the swim bladder is roughly equal to the external water pressure. A fish swimming at a particular depth adjusts the volume of its swim bladder to give it neutral buoyancy. If the fish swims upward or downward, the changing water pressure causes the bladder to expand or contract. Consequently, the fish must adjust the quantity of gas to restore the original volume and thus reestablish neutral buoyancy. Consider a large, 7.0 kg striped bass with a volume of 7.0 L. When neutrally buoyant, 7.0% of the fish's volume is taken up by the swim bladder. Assume a body temperature of 15°C.
a. How many moles of air are in the swim bladder when the fish is at a depth of 80 ft?
b. What will the volume of the swim bladder be if the fish ascends to a 50 ft depth without changing the quantity of gas?
c. To return the swim bladder to its original size, how many moles of gas must be added?