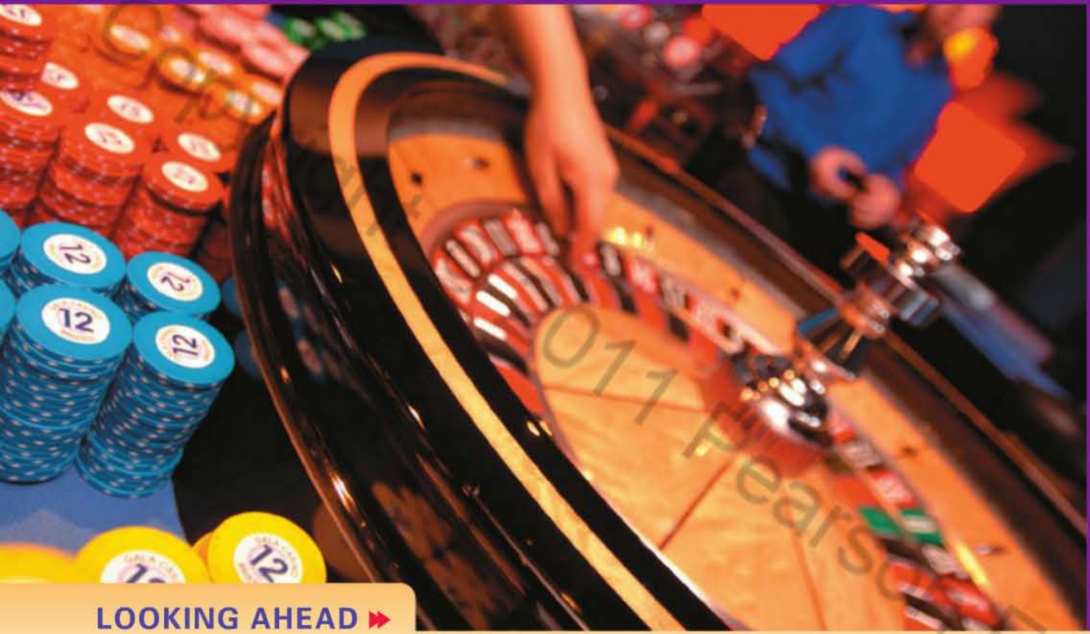
7 Rotational Motion



To get the roulette wheel spinning, the croupier must give the wheel a push in the direction of its motion. Does it matter if she pushes closer to the rim or nearer the center?

LOOKING AHEAD ►►

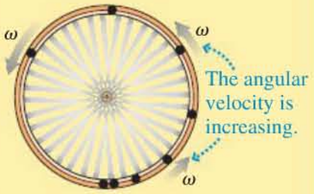
The goal of Chapter 7 is to understand the physics of rotating objects.

The Rotation of a Rigid Body

A **rigid body** is an extended object whose size and shape do not change as it moves.

Boomerangs and bicycle wheels are examples of rigid bodies.

A rigid body whose angular velocity is changing has an **angular acceleration.**



Looking Back ◄◄
6.1–6.2 Uniform circular motion

Newton's Second Law for Rotational Motion

You've learned Newton's second law of motion for *translational* motion: **A net force causes an object to accelerate.** In this chapter, we'll study Newton's second law for *rotational* motion: **A net torque causes an object to have an *angular* acceleration.**

To make the merry-go-round speed up, the girl has to apply a torque to it by pushing at its edge.

Looking Back ◄◄
4.6 Newton's second law

Torque

Torque is the rotational equivalent of force. To get an object rotating, you need to apply a torque to it. The farther from the axis of rotation a force is applied, the greater the torque.

By applying forces at the edge of the large wheel, the sailor can exert a large torque upon it.

Gravitational Torque

For the purpose of calculating torque, the entire weight of an object can be considered as acting at a single point, the **center of gravity.**

The weight of this tree, acting at its center of gravity, tries to rotate the tree about its base.

Moment of Inertia

We have learned that *mass* is the property of an object that resists acceleration. The property of an object that resists angular acceleration is its *moment of inertia*. The moment of inertia of an object depends not only on its mass but also on how that mass is distributed.

By extending its tail, this cat increases its moment of inertia. This increases its resistance to angular acceleration, making it harder for it to fall.

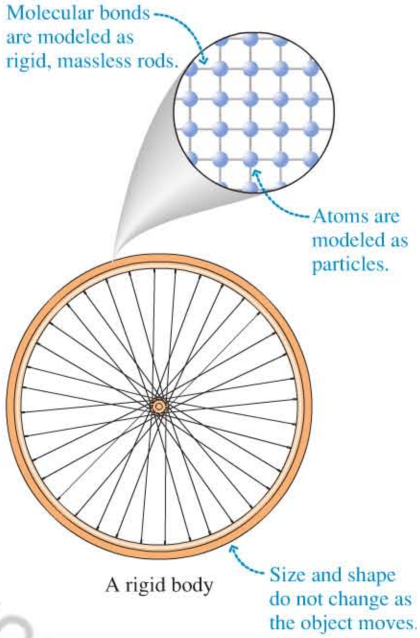
7.1 The Rotation of a Rigid Body

So far, our study of physics has focused almost exclusively on the *particle model* in which an entire object is represented as a single point in space. The particle model is entirely adequate for understanding motion in a wide variety of situations, but there are also cases for which we need to consider the motion of an *extended object*—a system of particles for which the size and shape *do* make a difference and cannot be neglected.

ActivPhysics 7.7

A **rigid body** is an extended object whose size and shape do not change as it moves. For example, a bicycle wheel can be thought of as a rigid body. **FIGURE 7.1** shows a rigid body as a collection of atoms held together by the rigid "massless rods" of molecular bonds.

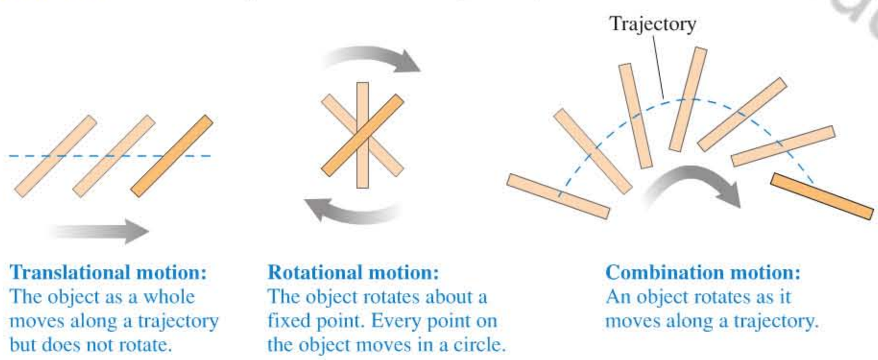
FIGURE 7.1 The rigid-body model of an extended object.



Real molecular bonds are, of course, not perfectly rigid. That's why an object seemingly as rigid as a bicycle wheel can flex and bend. Thus Figure 7.1 is really a simplified *model* of an extended object, the **rigid-body model.** The rigid-body model is a very good approximation for many real objects of practical interest, such as wheels and axles. Even nonrigid objects can often be modeled as rigid bodies during segments of their motion. For example, a diver is well described as a rotating rigid body while she's in the tuck position.

FIGURE 7.2 illustrates the three basic types of motion of a rigid body: **translational motion, rotational motion,** and **combination motion.** We've already studied translational motion of a rigid body using the particle model. If a rigid body doesn't rotate, this model is often adequate for describing its motion. The rotational motion of a rigid body will be the main focus of this chapter. We'll also discuss an important case of combination motion—that of a *rolling* object—later in this chapter.

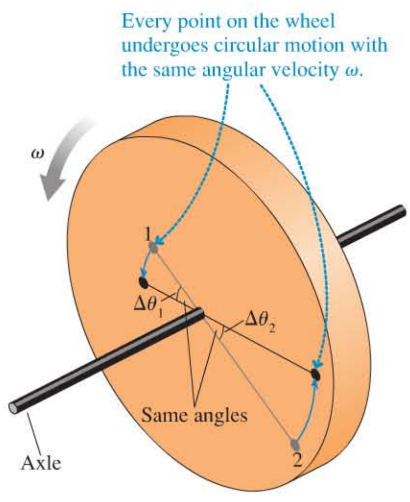
FIGURE 7.2 Three basic types of motion of a rigid body.



Rotational Motion of a Rigid Body

FIGURE 7.3 shows a wheel rotating on an axle. Notice that as the wheel rotates for a time interval Δt, two points 1 and 2 on the wheel, marked with dots, turn through the *same angle,* even through their distances r from the axis of rotation may be different; that is, $\Delta\theta_1 = \Delta\theta_2$ during the time interval Δt. As a consequence, the two points have equal angular velocities: $\omega_1 = \omega_2$. In general, **every point on a rotating rigid body has the same angular velocity.** Because of this, we can refer to the angular velocity ω *of the wheel.*

FIGURE 7.3 All points on a wheel rotate with the same angular velocity.



Recall from Chapter 6 that the speed of a particle moving in a circle is $v = \omega r$, so two points of a rotating object will have different *speeds* if they have different distances from the axis of rotation, but *all* points have the *same* angular velocity ω. Thus angular velocity is one of the most important parameters of a rotating object.

Because every point on a rotating object moves in a circle, we can carry forward all the results for circular motion from Chapter 6. Thus the angular displacement of any point on the wheel shown in Figure 7.3 is found from Equation 6.4 as $\Delta\theta = \omega\,\Delta t$; the speed of any particle in the wheel is $v = \omega r$, where r is the particle's distance from the axis; and the particle's centripetal acceleration is $a = \omega^2 r$.

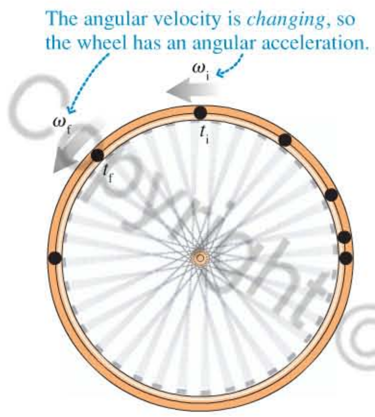


FIGURE 7.4 A rotating wheel with a changing angular velocity.

Angular Acceleration

If you push on the edge of a bicycle wheel, it begins to rotate. If you continue to push, it rotates ever faster. Its angular velocity is *changing*. To understand the dynamics of rotating objects, we'll need to be able to describe this case of changing angular velocity—that is, the case of *nonuniform* circular motion.

FIGURE 7.4 shows a bicycle wheel whose angular velocity is changing. The dot represents a particular point on the wheel at successive times. At time t_i the angular velocity is ω_i; at a later time $t_f = t_i + \Delta t$ the angular velocity has changed to ω_f. The change in angular velocity during this time interval is

$$\Delta\omega = \omega_f - \omega_i$$

Recall that in Chapter 2 we defined the *linear* acceleration as

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t}$$

By analogy, we now define the **angular acceleration** as

$$\alpha = \frac{\text{change in angular velocity}}{\text{time interval}} = \frac{\Delta\omega}{\Delta t} \tag{7.1}$$

Angular acceleration for a particle in nonuniform circular motion

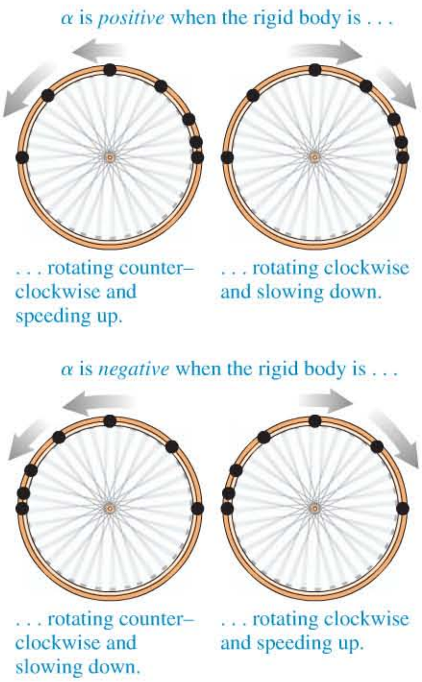


FIGURE 7.5 Determining the sign of the angular acceleration.

We use the symbol α (Greek alpha) for angular acceleration. Because the units of ω are rad/s, the units of angular acceleration are (rad/s)/s, or rad/s^2. From Equation 7.1, the sign of α is the same as the sign of $\Delta\omega$. **FIGURE 7.5** shows how to determine the sign of α. Be careful with the sign of α; just as with linear acceleration, positive and negative values of α can't be interpreted as simply "speeding up" and "slowing down." Like ω, the angular acceleration α is the same for every point on a rotating rigid body.

In Chapter 6 we found analogies between linear and angular positions and velocities. Here we've extended those analogies to include linear and angular accelerations. Table 7.1 summarizes all of these analogies between linear and circular motion.

NOTE ► Don't confuse the angular acceleration with the centripetal acceleration introduced in Chapter 6. The angular acceleration indicates how rapidly the *angular* velocity is changing. The centripetal acceleration is a vector quantity that points toward the center of a particle's circular path; it is nonzero even if the angular velocity is constant. ◄

In addition, the various equations of one-dimensional kinematics have analogs for rotational or circular motion. Table 7.2 lists the equations for one-dimensional motion and the analogous equations for the kinematics of circular motion.

TABLE 7.1 Linear and circular motion variables

Linear motion	Circular motion
Position x	Angular position θ
Velocity $v_x = \Delta x/\Delta t$	Angular velocity $\omega = \Delta\theta/\Delta t$
Acceleration $a_x = \Delta v_x/\Delta t$	Angular acceleration $\alpha = \Delta\omega/\Delta t$

TABLE 7.2 Linear and circular motion equations

Linear motion	Circular motion
Displacement at constant speed: $\Delta x = v\,\Delta t$	Angular displacement at constant angular speed: $\Delta\theta = \omega\,\Delta t$
Change in velocity at constant acceleration: $\Delta v = a\,\Delta t$	Change in angular velocity at constant angular acceleration: $\Delta\omega = \alpha\,\Delta t$
Displacement at constant acceleration: $\Delta x = v_i\,\Delta t + \frac{1}{2}a\,\Delta t^2$	Angular displacement at constant angular acceleration: $\Delta\theta = \omega_i\,\Delta t + \frac{1}{2}\alpha\,\Delta t^2$

EXAMPLE 7.1 Spinning up a computer disk

The disk in a computer disk drive spins up to 5400 rpm in 2.00 s. What is the angular acceleration of the disk? At the end of 2.00 s, how many revolutions has the disk made?

PREPARE The initial angular velocity is $\omega_i = 0$ rad/s. The final angular velocity is $\omega_f = 5400$ rpm. However, this value is not in the correct SI units of rad/s. The conversion is

$$\omega_f = \frac{5400 \text{ rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 565 \text{ rad/s}$$

SOLVE From the definition of angular acceleration, we have

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{565 \text{ rad/s} - 0 \text{ rad/s}}{2.00 \text{ s}} = 283 \text{ rad/s}^2$$

We can compute the angular displacement during this acceleration by using the angular displacement equation from Table 7.2:

$$\begin{aligned}\Delta\theta &= \omega_i \Delta t + \tfrac{1}{2}\alpha\,\Delta t^2 \\ &= (0 \text{ rad/s})(2.00 \text{ s}) + \tfrac{1}{2}(283 \text{ rad/s}^2)(2.00 \text{ s})^2 \\ &= 566 \text{ rad}\end{aligned}$$

Each revolution corresponds to an angular displacement of 2π, so we have

$$\begin{aligned}\text{number of revolutions} &= \frac{566 \text{ rad}}{2\pi \text{ rad/revolution}} \\ &= 90 \text{ revolutions}\end{aligned}$$

The disk completes 90 revolutions during the first 2 seconds.

ASSESS It seems reasonable that a fast-spinning disk would turn 90 times in a few seconds.

Graphs for Rotational Motion with Constant Angular Acceleration

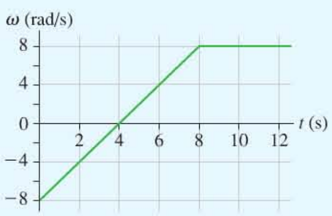
In Chapter 2 we studied position, velocity, and acceleration graphs for motion with constant acceleration. A review of Section 2.5 is highly recommended. Because of the analogies between linear and angular quantities in Table 7.1, the rules for graphing angular variables are identical with those for linear variables. In particular, **the angular velocity is the slope of the angular position-versus-time graph** (as we discussed in Chapter 6), and **the angular acceleration is the slope of the angular velocity-versus-time graph.** When the angular acceleration is constant, the equations for circular motion in Table 7.2 show that the angular velocity graph is linear while the angular position graph is parabolic.

EXAMPLE 7.2 Graphing angular quantities

FIGURE 7.6 shows the angular velocity-versus-time graph for the propeller of a ship.

a. Describe the motion of the propeller.
b. Draw the angular acceleration graph for the propeller.

FIGURE 7.6 The propeller's angular velocity.



PREPARE The angular acceleration graph is the slope of the angular velocity graph.

SOLVE

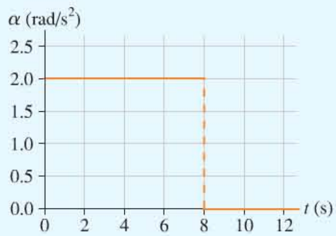
a. Initially the propeller has a negative angular velocity, so it is turning clockwise. It slows down until, at $t = 4$ s, it is instantaneously stopped. It then speeds up in the opposite direction until it is turning counterclockwise at a constant angular velocity.

b. The angular acceleration graph is the slope of the angular velocity graph. From $t = 0$ s, to $t = 8$ s, the slope is

$$\frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(8.0 \text{ rad/s}) - (-8.0 \text{ rad/s})}{8.0 \text{ s}} = 2.0 \text{ rad/s}^2$$

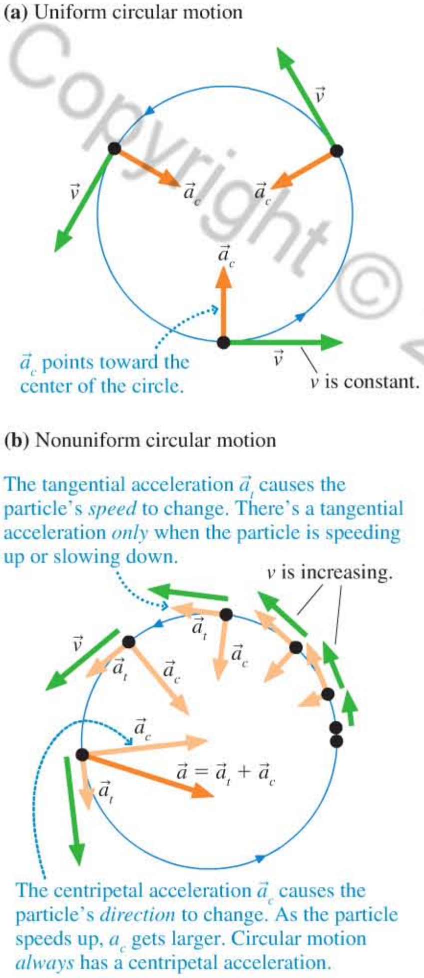
After $t = 8$ s, the slope is zero, so the angular acceleration is zero. This graph is plotted in **FIGURE 7.7**.

FIGURE 7.7 Angular acceleration graph for a propeller.



ASSESS A comparison of these graphs with their linear analogs in Figure 2.24 suggests that we're on the right track.

FIGURE 7.8 Uniform and nonuniform circular motion.



Tangential Acceleration

As you learned in Chapter 6, and as FIGURE 7.8a reminds you, a particle undergoing uniform circular motion has an acceleration directed inward toward the center of the circle. This centripetal acceleration $\vec{a}_c$ is due to the change in the *direction* of the particle's velocity. Recall that the magnitude of the centripetal acceleration is $a_c = v^2/r = \omega^2 r$.

NOTE ► Centripetal acceleration will now be denoted a_c to distinguish it from tangential acceleration a_t, discussed below. ◄

If the particle's circular motion is *nonuniform,* so that the particle's speed is changing, then the particle will have another component to its acceleration. FIGURE 7.8b shows a particle whose speed is increasing as it moves around its circular path. Because the *magnitude* of the velocity is increasing, this second component of the acceleration is directed *tangentially* to the circle, in the same direction as the velocity. This component of acceleration is called the **tangential acceleration.** As shown in Figure 7.8b, **the full acceleration $\vec{a}$ is then the vector sum of these two components:** the centripetal acceleration $\vec{a}_c$ and the tangential acceleration $\vec{a}_t$.

The tangential acceleration measures the rate at which the particle's speed around the circle increases. Thus its magnitude is

$$a_t = \frac{\Delta v}{\Delta t}$$

We can relate the tangential acceleration to the *angular* acceleration by using the relation $v = \omega r$ between the speed of a particle moving in a circle of radius r and its angular velocity ω. We have

$$a_t = \frac{\Delta v}{\Delta t} = \frac{\Delta(\omega r)}{\Delta t} = \frac{\Delta \omega}{\Delta t} r$$

or, because $\alpha = \Delta\omega/\Delta t$ from Equation 7.1,

$$a_t = \alpha r \tag{7.2}$$

Relationship between tangential and angular acceleration

We've seen that all points on a rotating rigid body have the same angular acceleration. From Equation 7.2, however, the centripetal and tangential accelerations of a point on a rotating object depends on the point's distance r from the axis, so that these accelerations are *not* the same for all points.

STOP TO THINK 7.1 A ball on the end of a string swings in a horizontal circle once every second. State whether the magnitude of each of the following quantities is zero, constant (but not zero), or changing.

a. Velocity
b. Angular velocity
c. Centripetal acceleration
d. Angular acceleration
e. Tangential acceleration

7.2 Torque

Newton's genius, summarized in his second law of motion, was to recognize force as the cause of acceleration. But what about *angular* acceleration? What do Newton's laws tell us about rotational motion? To begin our study of rotational motion, we'll need to find a rotational equivalent of force.

Consider the common experience of pushing open a heavy door. **FIGURE 7.9** is a top view of a door that is hinged on the left. Four forces are shown, all of equal strength. Which of these will be most effective at opening the door?

Force $\vec{F}_1$ will open the door, but force $\vec{F}_2$, which pushes straight at the hinge, will not. Force $\vec{F}_3$ will open the door, but not as easily as $\vec{F}_1$. What about $\vec{F}_4$? It is perpendicular to the door and it has the same magnitude as $\vec{F}_1$, but you know from experience that pushing close to the hinge is not as effective as pushing at the outer edge of the door.

The ability of a force to cause a rotation thus depends on three factors:

1. The magnitude F of the force
2. The distance r from the pivot—the axis about which the object can rotate—to the point at which the force is applied
3. The angle at which the force is applied

We can incorporate these three observations into a single quantity called the **torque τ** (Greek tau). Loosely speaking, τ measures the "effectiveness" of a force at causing an object to rotate about a pivot. **Torque is the rotational equivalent of force.** In Figure 7.9, for instance, the torque τ_1 due to $\vec{F}_1$ is greater than τ_4 due to $\vec{F}_4$.

To make these ideas specific, **FIGURE 7.10** shows a force $\vec{F}$ applied at one point of a wrench that's loosening a nut. Figure 7.10 defines the distance r from the pivot to the point at which the force is applied; the **radial line,** the line starting at the pivot and extending through this point; and the angle ϕ (Greek phi) measured from the radial line to the direction of the force.

We saw in Figure 7.9 that force $\vec{F}_1$, which was directed perpendicular to the door, was effective in opening it, but force $\vec{F}_2$, directed toward the hinges, had no effect on its rotation. As shown in **FIGURE 7.11**, this suggests breaking the force $\vec{F}$ applied to the wrench into two component vectors: $\vec{F}_\perp$ directed perpendicular to the radial line, and $\vec{F}_\parallel$ directed parallel to it. Because $\vec{F}_\parallel$ points either directly toward or away from the pivot, it has no effect on the wrench's rotation, and thus contributes nothing to the torque. Only $\vec{F}_\perp$ tends to cause rotation of the wrench, so it is this component of the force that determines the torque.

NOTE ▶ The perpendicular component $\vec{F}_\perp$ is pronounced "F perpendicular" and the parallel component $\vec{F}_\parallel$ is "F parallel." ◀

We've seen that a force applied at a greater distance r from the pivot has a greater effect on rotation, so we expect a larger value of r to give a greater torque. We also saw that only $\vec{F}_\perp$ contributes to the torque. Both these observations are contained in our first expression for torque:

$$\tau = rF_\perp \tag{7.3}$$

Torque due to a force with perpendicular component $F_\perp$ acting at a distance r from the pivot

From this equation, we see that the SI units of torque are newton-meters, abbreviated N · m.

FIGURE 7.9 The four forces are the same strength, but they have different effects on the swinging door.

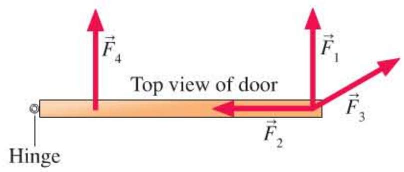


FIGURE 7.10 Force $\vec{F}$ exerts a torque about the pivot point.

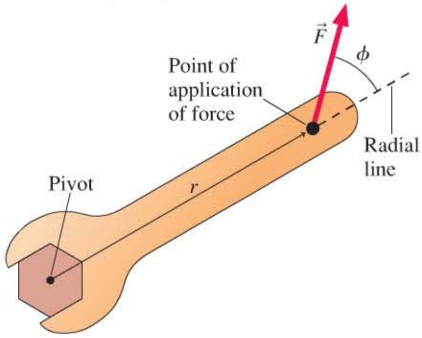
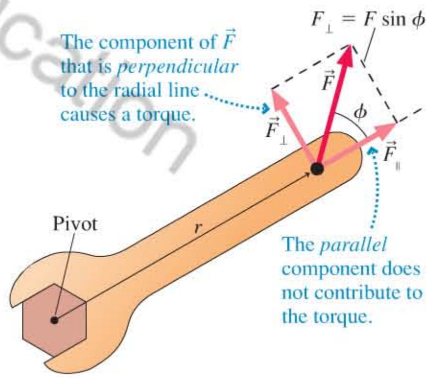


FIGURE 7.11 Torque is due to the component of the force perpendicular to the radial line.



Activ Physics 7.1

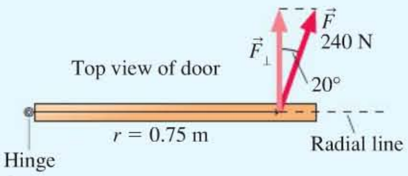
EXAMPLE 7.3 **Torque in opening a door**

In trying to open a stuck door, Ryan pushes it at a point 0.75 m from the hinges with a 240 N force directed 20° away from being perpendicular to the door. What torque does Ryan exert on the door?

PREPARE In **FIGURE 7.12** on the next page the radial line is shown drawn from the pivot—the hinge—through the point at which the force $\vec{F}$ is applied. We see that the component of $\vec{F}$ that is perpendicular to the radial line is $F_\perp = F\cos 20° = 226$ N. The distance from the hinge to the point at which the force is applied is $r = 0.75$ m.

Continued

FIGURE 7.12 Ryan's force exerts a torque on the door.



SOLVE We can find the torque on the door from Equation 7.3:

$$\tau = rF_\perp = (0.75\text{ m})(226\text{ N}) = 170\text{ N}\cdot\text{m}$$

ASSESS Ryan could slightly increase the torque he exerts by pushing with the same force but exactly perpendicular to the door.

FIGURE 7.13 You can also calculate torque in terms of the moment arm between the pivot and the line of action.

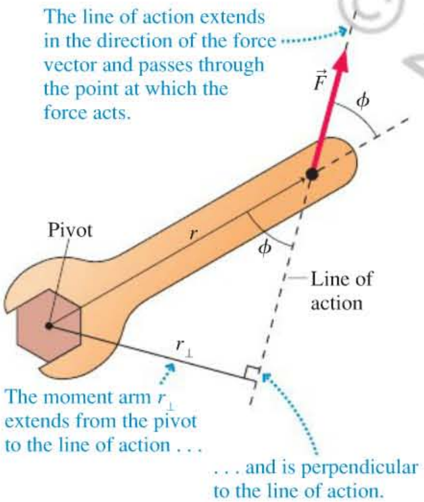


FIGURE 7.13 shows an alternative way to calculate torque. The line that is in the direction of the force, and passes through the point at which the force acts, is called the *line of action*. The perpendicular distance from this line to the pivot is called the **moment arm** (or *lever arm*) $r_\perp$. You can see from the figure that $r_\perp = r\sin\phi$. Further, Figure 7.11 showed that $F_\perp = F\sin\phi$. We can then write Equation 7.3 as $\tau = rF\sin\phi = F(r\sin\phi) = Fr_\perp$. Thus an equivalent expression for the torque is

$$\tau = r_\perp F \qquad (7.4)$$

Torque due to a force F with moment arm $r_\perp$

CONCEPTUAL EXAMPLE 7.4 Starting a bike

It is hard to get going if you try to start your bike with the pedal at the highest point. Why is this?

REASON Aided by the weight of the body, the greatest force can be applied to the pedal straight down. But with the pedal at the top, this force is exerted almost directly toward the pivot, causing only a small torque. We could say either that the perpendicular component of the force is small or that the moment arm is small.

ASSESS If you've ever climbed a steep hill while standing on the pedals, you know that you get the greatest forward motion when one pedal is completely forward with the crank parallel to the ground. This gives the maximum possible torque because the force you apply is entirely perpendicular to the radial line, and the moment arm is as long as it can be.

We've seen that Equation 7.3 can be written as $\tau = rF_\perp = r(F\sin\phi)$, and Equation 7.4 as $\tau = r_\perp F = (r\sin\phi)F$. This shows that both methods of calculating torque lead to the same expression for torque—namely:

$$\tau = rF\sin\phi \qquad (7.5)$$

Torque due to a force F applied at a distance r from the pivot, at an angle ϕ to the radial line

◀ **Torque versus speed** To start and stop quickly, the basketball player needs to apply a large torque to her wheel. To make the torque as large as possible, the handrim—the outside wheel that she actually grabs—is almost as big as the wheel itself. The racer needs to move continuously at high speed, so his wheel spins much faster. To allow his hands to keep up, his handrim is much smaller than his chair's wheel, making its linear velocity correspondingly lower. The smaller radius means, however, that the torque he can apply is lower as well.

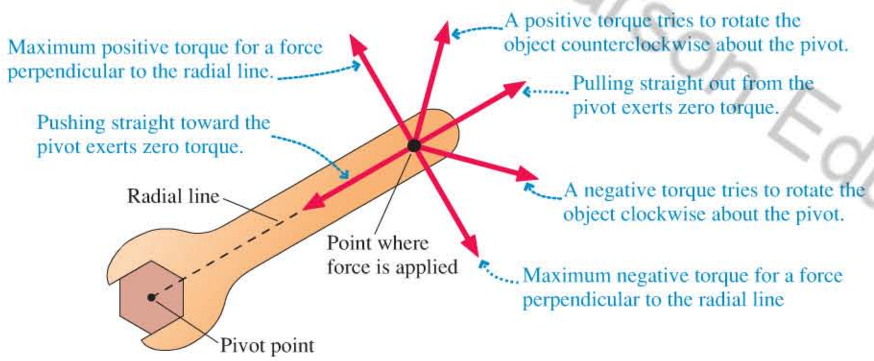
NOTE ► Torque differs from force in a very important way. Torque is calculated or measured *about a particular point.* To say that a torque is 20 N · m is meaningless without specifying the point about which the torque is calculated. Torque can be calculated about any point, but its value depends on the point chosen because this choice determines r and ϕ. In practice, we usually calculate torques about a hinge, pivot, or axle. ◄

Equations 7.3–7.5 are three different ways of thinking about—and calculating—the torque due to a force. Depending on the problem at hand, one might be easier to use than the others. But they all calculate the *same* torque, and all will give the same value for the torque.

These equations give only the magnitude of the torque. But torque, like a force component, has a sign. **A torque that tends to rotate the object in a counterclockwise direction is positive, while a torque that tends to rotate the object in a clockwise direction is negative.** FIGURE 7.14 summarizes the signs. Notice that a force pushing straight toward the pivot or pulling straight out from the pivot exerts *no* torque.

NOTE ► When calculating a torque, you must supply the appropriate sign by observing the direction in which the torque acts. ◄

FIGURE 7.14 Signs and strengths of the torque.

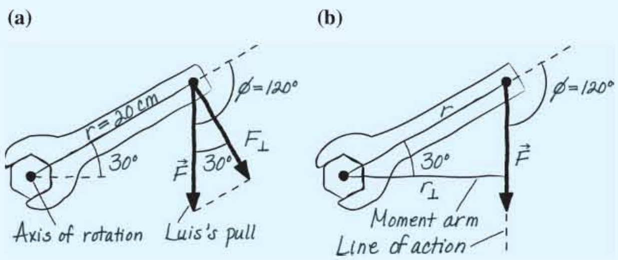


EXAMPLE 7.5 Calculating the torque on a nut

Luis uses a 20-cm-long wrench to turn a nut. The wrench handle is tilted 30° above the horizontal, and Luis pulls straight down on the end with a force of 100 N. How much torque does Luis exert on the nut?

PREPARE FIGURE 7.15 shows the situation. The two illustrations correspond to two methods of calculating torque, corresponding to Equations 7.3 and 7.4.

FIGURE 7.15 A wrench being used to turn a nut.



SOLVE According to Equation 7.3 and 7.5, the torque can be calculated as $\tau = rF_\perp = rF\sin\phi$. From Figure 7.15a we see that the angle between the force and the radial line is $\phi = 30° + 90° = 120°$. The torque is then

$$\tau = -rF\sin\phi = -(0.20\text{ m})(100\text{ N})(\sin 120°) = -17\text{ N}\cdot\text{m}$$

We put in the minus sign because the torque is negative—it tries to rotate the nut in a *clockwise* direction.

Alternatively, we can use Equation 7.4 to find the torque. Figure 7.15b shows the moment arm $r_\perp$, the perpendicular distance from the pivot to the line of action. From the figure we see that

$$r_\perp = r\cos 30° = (0.20\text{ m})(\cos 30°) = 0.17\text{ m}$$

Then the torque is

$$\tau = -r_\perp F = -(0.17\text{ m})(100\text{ N}) = -17\text{ N}\cdot\text{m}$$

Again, we insert the minus sign because the torque acts to give a clockwise rotation.

ASSESS Both methods give the same answer for the torque, as expected. In general, however, you need use only one of Equations 7.3–7.5 to find the torque in any given situation. In using any of these methods to find the torque, remember to include the minus sign if the torque acts to rotate the object in a clockwise direction.

STOP TO THINK 7.2 Rank in order, from largest to smallest, the five torques τ_A to τ_E. The rods all have the same length and are pivoted at the dot.

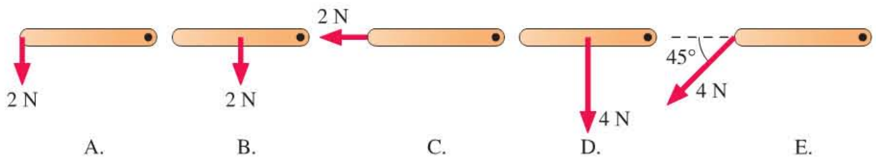
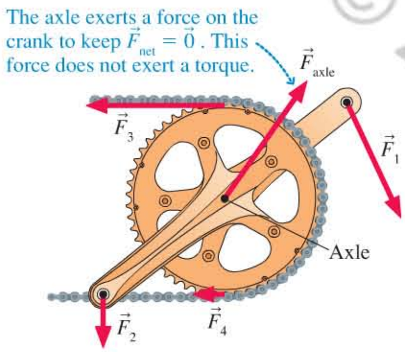


FIGURE 7.16 The forces exert a net torque about the pivot point.



Net Torque

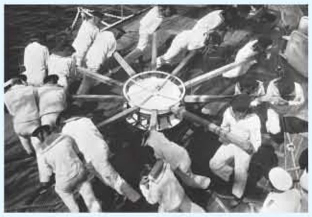
FIGURE 7.16 shows the forces acting on the crankset of a bicycle. Forces $\vec{F}_1$ and $\vec{F}_2$ are due to the rider pushing on the pedals, and $\vec{F}_3$ and $\vec{F}_4$ are tension forces from the chain. The crankset is free to rotate about a fixed axle, but the axle prevents it from having any translational motion with respect to the bike frame. It does so by exerting force $\vec{F}_{axle}$ on the object to balance the other forces and keep $\vec{F}_{net} = \vec{0}$.

Forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and $\vec{F}_4$ exert torques τ_1, τ_2, τ_3, and τ_4 on the crank (measured about the axle), but $\vec{F}_{axle}$ does *not* exert a torque because it is applied at the pivot point—the axle—and so has zero moment arm. Thus the *net* torque about the axle is the sum of the torques due to the *applied* forces:

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4 + \cdots = \sum \tau \tag{7.6}$$

EXAMPLE 7.6 Force in turning a capstan

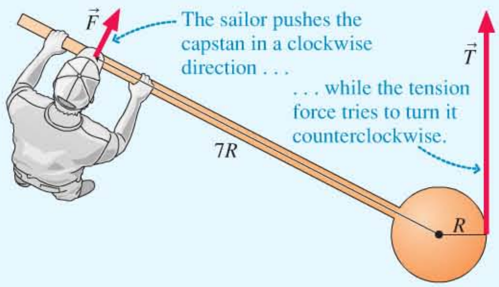
A capstan is a device used on old sailing ships to raise the anchor. A sailor pushes the long lever, turning the capstan and winding up the anchor rope. If the capstan turns at a constant speed, the net torque on it, as we'll learn later in the chapter, is zero.



Suppose the rope tension due to the weight of the anchor is 1500 N. If the distance from the axis to the point on the lever where the sailor pushes is exactly seven times the radius of the capstan around which the rope is wound, with what force must the sailor push if the net torque on the capstan is to be zero?

PREPARE Shown in FIGURE 7.17 is a view looking down from above the capstan. The rope pulls with a tension force $\vec{T}$ at distance R from the axis of rotation. The sailor pushes with a force $\vec{F}$ at distance $7R$ from the axis. Both forces are perpendicular to their radial lines, so ϕ in Equation 7.5 is 90°.

FIGURE 7.17 Top view of a sailor turning a capstan.



SOLVE The torque due to the tension in the rope is

$$\tau_T = RT \sin 90° = RT$$

We don't know the capstan radius, so we'll just leave it as R for now. This torque is positive because it tries to turn the capstan counterclockwise. The torque due to the sailor is

$$\tau_S = -(7R)F \sin 90° = -7RF$$

We put the minus sign in because this torque acts in the clockwise (negative) direction. The net torque is zero, so we have $\tau_T + \tau_S = 0$, or

$$RT - 7RF = 0$$

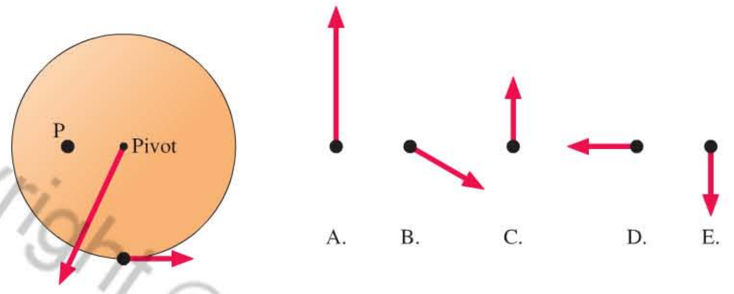
Note that the radius R cancels, leaving

$$F = \frac{T}{7} = \frac{1500 \text{ N}}{7} = 210 \text{ N}$$

ASSESS 210 N is about 50 lb, a reasonable number. The force the sailor must exert is one-seventh the force the rope exerts: The long lever helps him lift the heavy anchor. In the HMS *Warrior,* built in 1860, it took 200 men turning the capstan to lift the huge anchor that weighed close to 55,000 N!

Note that forces $\vec{F}$ and $\vec{T}$ point in different directions. Their torques depend only on their directions with respect to their own radial lines, not on the directions of the forces with respect to each other. The force the sailor needs to apply remains unchanged as he circles the capstan.

STOP TO THINK 7.3 Two forces act on the wheel shown. What third force, acting at point P, will make the net torque on the wheel zero?

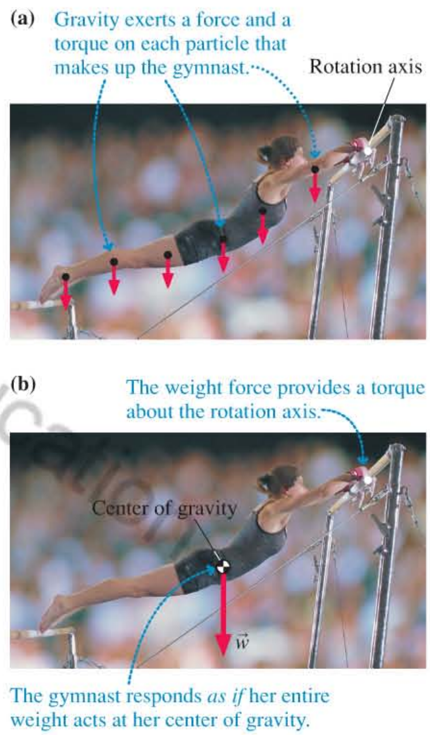


7.3 Gravitational Torque and the Center of Gravity

As the gymnast in **FIGURE 7.18** pivots around the bar, a torque due to the force of gravity causes her to rotate toward a vertical position. A falling tree and a car hood slamming shut are other examples where gravity exerts a torque on an object. Stationary objects can also experience a torque due to gravity. A diving board experiences a gravitational torque about its fixed end. It doesn't rotate because of a counteracting torque provided by forces from the base at its fixed end.

We've learned how to calculate the torque due to a single force acting on an object. But gravity doesn't act at a single point on an object. It pulls downward on *every particle* that makes up the object, as shown for the gymnast in Figure 7.18a, and so each particle experiences a small torque due to the force of gravity that acts upon it. The gravitational torque on the object as a whole is then the *net* torque exerted on all the particles. We won't prove it, but the gravitational torque can be calculated by assuming that the net force of gravity—that is, the object's weight $\vec{w}$—acts at a single special point on the object called its **center of gravity** (symbol ⊕). Then we can calculate the torque due to gravity by the methods learned earlier for a single force ($\vec{w}$) acting at a single point (the center of gravity). Figure 7.18b shows how we can consider the gymnast's weight as acting at her center of gravity.

FIGURE 7.18 The center of gravity is the point where the weight appears to act.

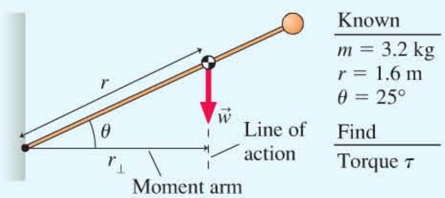


EXAMPLE 7.7 The torque on a flagpole

A 3.2 kg flagpole extends from a wall at an angle of 25° from the horizontal. Its center of gravity is 1.6 m from the point where the pole is attached to the wall. What is the gravitational torque on the flagpole about the point of attachment?

PREPARE **FIGURE 7.19** shows the situation. For the purpose of calculating torque, we can consider the entire weight of the pole as acting at the center of gravity. We can use any of the three methods discussed in Section 7.2 to calculate the torque. Because the moment arm $r_\perp$ is simple to visualize here, we'll use Equation 7.4 for the torque.

FIGURE 7.19 Visual overview of the flagpole.



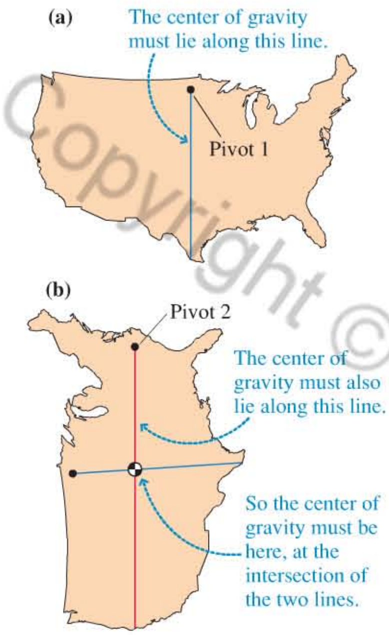
SOLVE From Figure 7.19, we see that the moment arm is $r_\perp = (1.6\text{ m})\cos 25° = 1.5\text{ m}$. Thus the gravitational torque on the flagpole, about the point where it attaches to the wall, is

$$\tau = -r_\perp w = -r_\perp mg = -(1.5\text{ m})(3.2\text{ kg})(9.8\text{ m/s}^2) = -47\text{ N}\cdot\text{m}$$

We inserted the minus sign because the torque tries to rotate the pole in a clockwise direction.

ASSESS If the pole were attached to the wall by a hinge, the gravitational torque would cause the pole to fall. However, the actual rigid connection provides a counteracting (positive) torque to the pole that prevents this.

FIGURE 7.20 Method for finding the center of gravity of an object.



Finding the Center of Gravity

To calculate the gravitational torque, we need to locate the object's center of gravity. There is a simple experimental method for finding the center of gravity of any object, based on the observation that **any object free to rotate about a pivot will come to rest with its center of gravity directly below the pivot.** To see this, consider the cutout map of the continental United States in **FIGURE 7.20a**. If the center of gravity is to the right or left of the blue line, a gravitational torque will cause the map to swing. If the center of gravity lies directly *below* the pivot, however, the weight force lies along the line of action and the torque is zero. The map can then remain at rest with no tendency to swing.

We know that the center of gravity lies somewhere along the blue line, but we don't yet know where. To find out, we need to suspend the map from a second pivot, as shown in **FIGURE 7.20b**. Then the center of gravity will fall somewhere along the red line shown. Because the center of gravity must lie on both the blue and red lines, it must be at their *intersection.* Interestingly, the geographical center of the continental United States is defined in just this way, as the center of gravity of a map of the contiguous United States. This point is one mile northwest of Lebanon, Kansas.

For a simple symmetrical object, such as a rod, sphere, or cube made of a uniform material, **FIGURE 7.21** shows that **the center of gravity of a symmetrical object lies at its geometrical center.** A particularly simple case of this is a point particle, whose center of gravity lies at the position of the particle.

FIGURE 7.21 The center of gravity of a symmetrical object lies at its center.

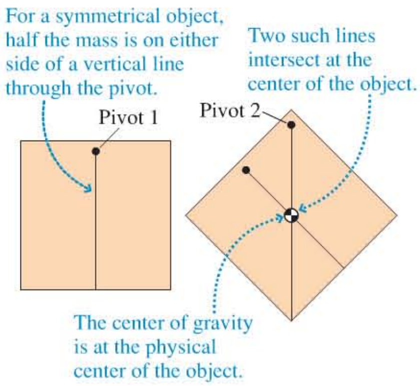
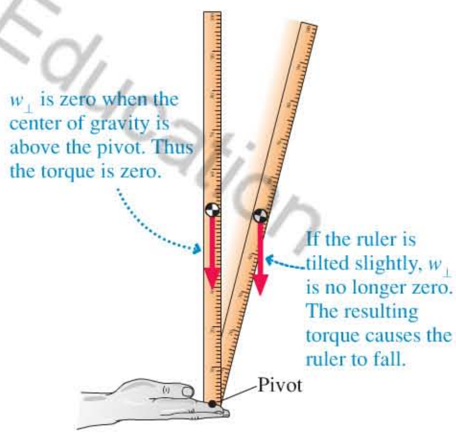


FIGURE 7.22 Balancing a ruler.



As we've seen, an object free to pivot will rotate until its center of gravity is directly below the pivot. If the center of gravity lies directly *above* the pivot, as in **FIGURE 7.22**, there is no torque due to the object's weight and it can remain balanced. However, if the object is even slightly displaced to either side, the gravitational torque will no longer be zero and the object will begin to rotate. This question of *balance*—the behavior of an object whose center of gravity lies above the pivot—will be explored in depth in Chapter 8.

Calculating the Position of the Center of Gravity

It is nice to know you can locate an object's center of gravity by suspending it from a pivot, but it is rarely a practical technique. More often, we would like to calculate the center of gravity of an object made up of a combination of particles and objects whose center-of-gravity positions are known.

Because there's no gravitational torque when the center of gravity lies either directly above or directly below the pivot, it must be the case that **the torque due to gravity when the pivot is *at* the center of gravity is zero.** We can use this fact to find a general expression for the position of the center of gravity.

Consider the dumbbell shown in **FIGURE 7.23**. If we slide the triangular pivot back and forth until the dumbbell balances, the pivot must then be at the center of gravity (at position x_{cg}), and the torque due to gravity must therefore be zero. But we can calculate the gravitational torque directly by calculating and summing the torques about this point due to the two individual weights. Gravity acts on weight 1 with moment arm r_1, so the torque about the pivot at position x_{cg} is

$$\tau_1 = r_1 w_1 = (x_{cg} - x_1) m_1 g$$

Similarly, the torque due to weight 2 is

$$\tau_2 = -r_2 w_2 = -(x_2 - x_{cg}) m_2 g$$

This torque is negative because it tends to rotate the dumbbell in a clockwise direction. We've just argued that the net torque must be zero because the pivot is directly under the center of gravity, so

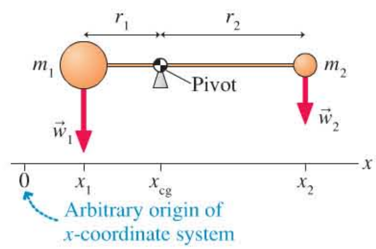
$$\tau_{net} = 0 = \tau_1 + \tau_2 = (x_{cg} - x_1) m_1 g - (x_2 - x_{cg}) m_2 g$$

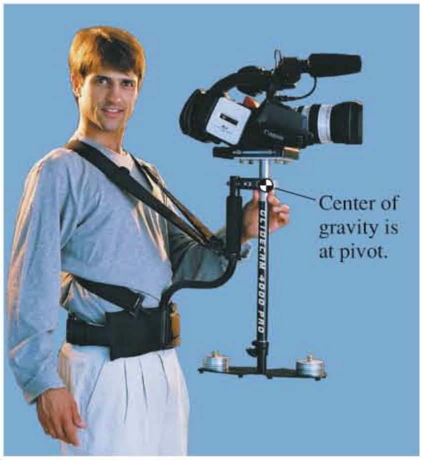
We can solve this equation for the position of the center of gravity x_{cg}:

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} \qquad (7.7)$$

The following Tactics Box shows how Equation 7.7 can be generalized to find the center of gravity of *any* number of particles. If the particles don't all lie along the x-axis, then we'll also need to find the y-coordinate of the center of gravity.

FIGURE 7.23 Finding the center of gravity of a dumbbell.





Holding steady Many movie-making scenes require handheld shots for which the cameraman must walk along, following the action. The stabilizer shown reduces unwanted camera motion as the cameraman walks. The center of gravity of the camera and its hanging weight arm is located exactly at a pivot that can swing freely in any direction. The frictionless pivot exerts no torque on the camera and arm. Neither does the weight, because it acts at the pivot. With no torque acting on it, the camera has no tendency to rotate. The long arm also increases the system's *moment of inertia*, further decreasing unwanted rotations. More about this later!

TACTICS BOX 7.1 **Finding the center of gravity**

1. Choose an origin for your coordinate system. You can choose any convenient point as the origin.
2. Determine the coordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots$ for the particles of masses $m_1, m_2, m_3, \ldots$, respectively.
3. The x-coordinate of the center of gravity is

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \qquad (7.8)$$

4. Similarly, the y-coordinate of the center of gravity is

$$y_{cg} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \qquad (7.9)$$

Exercises 12–15

Because the center of gravity depends on products such as $x_1 m_1$, objects with large masses count more heavily than objects with small masses. Consequently, **the center of gravity tends to lie closer to the heavier objects or particles** that make up the entire object.

EXAMPLE 7.8 **Where should the dumbbell be lifted?**

A 1.0-m-long dumbbell has a 10 kg mass on the left and a 5.0 kg mass on the right. Find the position of the center of gravity, the point where the dumbbell should be lifted in order to remain balanced.

PREPARE We'll treat the two masses as point particles separated by a massless rod. Then we can use the steps from Tactics Box 7.1 to find the center of gravity. Let's choose the origin to be at the position of the 10 kg mass on the left, making $x_1 = 0$ m and $x_2 = 1.0$ m. Because the dumbbell masses lie on the x-axis, the y-coordinate of the center of gravity must also lie on the x-axis. Thus we only need to solve for the x-coordinate of the center of gravity.

Continued

SOLVE The x-coordinate of the center of gravity is found from Equation 7.8:

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{(0\text{ m})(10\text{ kg}) + (1.0\text{ m})(5.0\text{ kg})}{(10\text{ kg}) + (5.0\text{ kg})}$$
$$= 0.33\text{ m}$$

The center of gravity is 0.33 m from the 10 kg mass or, equivalently, 0.17 m left of the center of the bar.

ASSESS The position of the center of gravity is closer to the larger mass. This agrees with our general statement that the center of gravity tends to lie closer to the heavier particles.

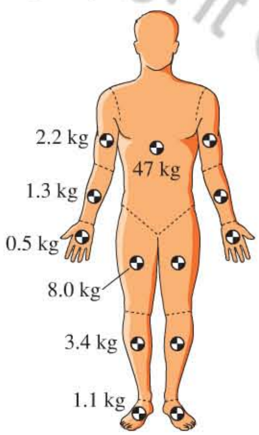


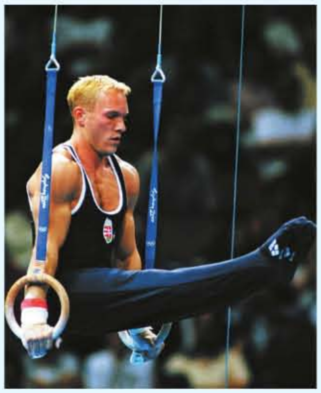
FIGURE 7.24 Body segment masses and centers of gravity.

The center of gravity of an extended object can often be found by considering the object as made up of pieces, each with mass and center of gravity that are known or can be found. Then the coordinates of the entire object's center of gravity are given by Equations 7.8 and 7.9, with (x_1, y_1), (x_2, y_2), (x_3, y_3), . . . the coordinates of the center of gravity of each piece and m_1, m_2, m_3, . . . their masses.

This method is widely used in biomechanics and kinesiology to calculate the center of gravity of the human body. FIGURE 7.24 shows how the body can be considered to be made of segments, each of whose mass and center of gravity have been measured. The numbers shown are appropriate for a man with a total mass of 80 kg. For a given posture the positions of the segments and their centers of gravity can be found, and thus the whole-body center of gravity from Equations 7.8 and 7.9 (and a third equation for the z-coordinate). Example 7.9 explores a simplified version of this method.

EXAMPLE 7.9 Finding the center of gravity of a gymnast

A gymnast performing on the rings holds himself in the pike position. FIGURE 7.25 shows how we can consider his body to be made up of two segments whose masses and center-of-gravity positions are shown. The upper segment includes his head, trunk, and arms, while the lower segment consists of his legs. Locate the overall center of gravity of the gymnast.



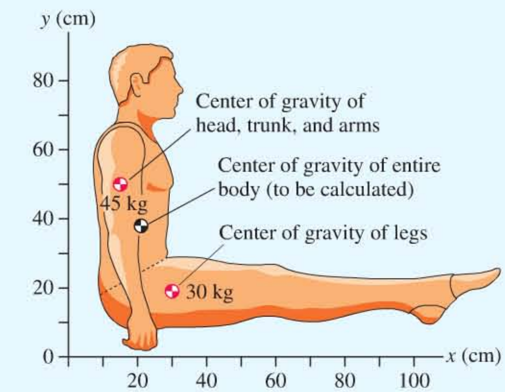


FIGURE 7.25 Centers of gravity of two segments of a gymnast.

PREPARE From Figure 7.25 we can find the x- and y-coordinates of the segment centers of gravity:

$$x_{trunk} = 15\text{ cm} \qquad y_{trunk} = 50\text{ cm}$$
$$x_{legs} = 30\text{ cm} \qquad y_{legs} = 20\text{ cm}$$

SOLVE The x- and y-coordinates of the center of gravity are given by Equations 7.8 and 7.9:

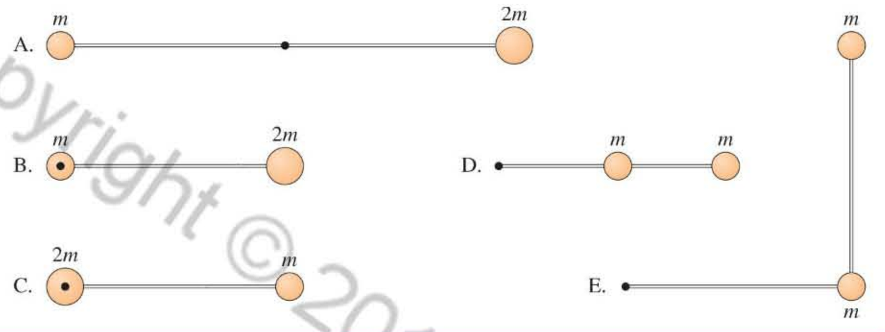
$$x_{cg} = \frac{x_{trunk} m_{trunk} + x_{legs} m_{legs}}{m_{trunk} + m_{legs}}$$
$$= \frac{(15\text{ cm})(45\text{ kg}) + (30\text{ cm})(30\text{ kg})}{45\text{ kg} + 30\text{ kg}} = 21\text{ cm}$$

and

$$y_{cg} = \frac{y_{trunk} m_{trunk} + y_{legs} m_{legs}}{m_{trunk} + m_{legs}}$$
$$= \frac{(50\text{ cm})(45\text{ kg}) + (20\text{ cm})(30\text{ kg})}{45\text{ kg} + 30\text{ kg}} = 38\text{ cm}$$

ASSESS The center-of-gravity position of the entire body, shown in Figure 7.25, is closer to that of the heavier trunk segment than to that of the lighter legs. It also lies along a line connecting the two segment centers of gravity, just as it would for the center of gravity of two point particles. Note also that the gymnast's hands—the pivot point—must lie directly below his center of gravity. Otherwise he would rotate forward or backward.

STOP TO THINK 7.4 The balls are connected by very lightweight rods pivoted at the point indicated by a dot. The rod lengths are all equal except for A, which is twice as long. Rank in order, from least to greatest, the magnitudes of the gravitational torques about the pivots for arrangements A to E.



7.4 Rotational Dynamics and Moment of Inertia

In Section 7.2 we asked: What do Newton's laws tell us about rotational motion? We can now answer that question: **A torque causes an angular acceleration.** This is the rotational equivalent of our earlier discovery, for motion along a line, that a force causes an acceleration.

To see where this connection between torque and angular acceleration comes from, let's start by examining a *single particle* subject to a torque. FIGURE 7.26 shows a particle of mass m attached to a lightweight, rigid rod of length r that constrains the particle to move in a circle. The particle is subject to two forces. Because it's moving in a circle, there must be a force—here, the tension $\vec{T}$ from the rod—directed toward the center of the circle. As we learned in Chapter 6, this is the force responsible for changing the *direction* of the particle's velocity. The acceleration associated with this change in the particle's velocity is the centripetal acceleration $\vec{a}_c$.

But the particle in Figure 7.26 is also subject to the force $\vec{F}$ that changes the *speed* of the particle. This force causes a tangential acceleration $\vec{a}_t$. Applying Newton's second law in the direction tangent to the circle gives

$$a_t = \frac{F}{m} \tag{7.10}$$

Now the tangential and angular accelerations are related by $a_t = \alpha r$, so we can rewrite Equation 7.10 as $\alpha r = F/m$, or

$$\alpha = \frac{F}{mr} \tag{7.11}$$

We can now connect this angular acceleration to the torque because force $\vec{F}$, which is perpendicular to the radial line, exerts torque

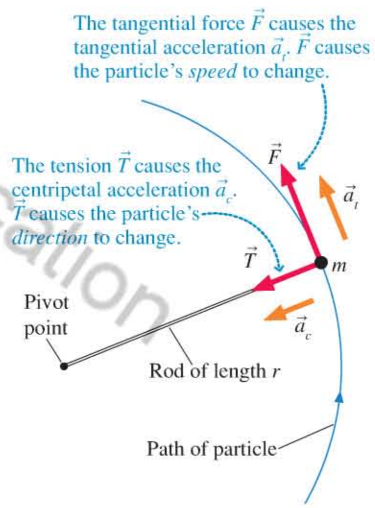
$$\tau = rF$$

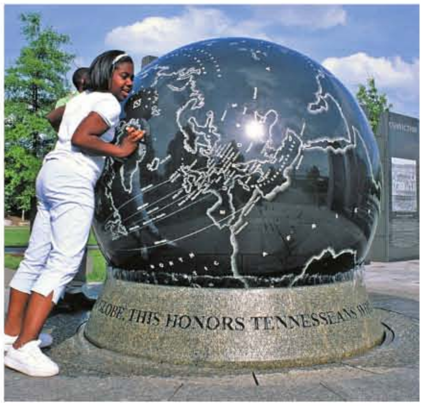
With this relation between F and τ, we can write Equation 7.11 as

$$\alpha = \frac{\tau}{mr^2} \tag{7.12}$$

Equation 7.12 gives a relationship between the torque on a single particle and its angular acceleration. Now all that remains is to expand this idea from a single particle to an extended object.

FIGURE 7.26 A tangential force $\vec{F}$ exerts a torque on the particle and causes an angular acceleration.





This large granite ball, with a mass of 26,400 kg, floats with nearly zero friction on a thin layer of pressurized water. Even though the girl exerts a large torque on the ball, its angular acceleration is small because of its large moment of inertia.

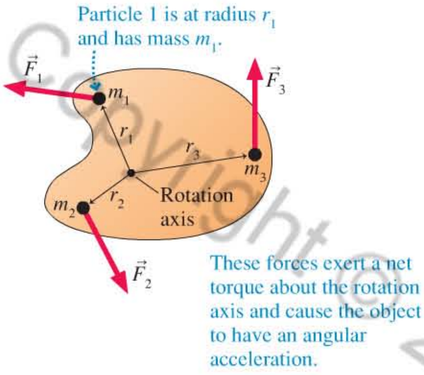


FIGURE 7.27 The forces on a rigid body exert a torque about the rotation axis.

Newton's Second Law for Rotational Motion

FIGURE 7.27 shows a rigid body that undergoes rotation about a fixed and unmoving axis. According to the rigid-body model, we can think of the object as consisting of particles with masses $m_1, m_2, m_3, \ldots$ at fixed distances $r_1, r_2, r_3, \ldots$ from the axis. Suppose forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$ act on these particles. These forces exert torques around the rotation axis, so the object will undergo an angular acceleration α. Because all the particles that make up the object rotate together, each particle has this *same* angular acceleration α. Rearranging Equation 7.12 slightly, we can write the torques on the particles as

$$\tau_1 = m_1 r_1^2 \alpha \qquad \tau_2 = m_2 r_2^2 \alpha \qquad \tau_3 = m_3 r_3^2 \alpha$$

and so on for every particle in the object. If we add up all these torques, the *net* torque on the object is

$$\begin{aligned}\tau_{\text{net}} &= \tau_1 + \tau_2 + \tau_3 + \cdots = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \cdots \\ &= \alpha(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots) = \alpha \sum m_i r_i^2 \end{aligned} \tag{7.13}$$

By factoring α out of the sum, we're making explicit use of the fact that every particle in a rotating rigid body has the *same* angular acceleration α.

7.6 ActivPhysics

The quantity $\sum mr^2$ in Equation 7.13, which is the proportionality constant between angular acceleration and net torque, is called the object's **moment of inertia** I:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum m_i r_i^2 \tag{7.14}$$

Moment of inertia of a collection of particles

The units of moment of inertia are mass times distance squared, or $\text{kg} \cdot \text{m}^2$. An object's moment of inertia, like torque, *depends on the axis of rotation.* Once the axis is specified, allowing the values of $r_1, r_2, r_3, \ldots$ to be determined, the moment of inertia *about that axis* can be calculated from Equation 7.14.

NOTE ► The word "moment" in "moment of inertia" and "moment arm" has nothing to do with time. It stems from the Latin *momentum,* meaning "motion." ◄

Substituting the moment of inertia into Equation 7.13 puts the final piece of the puzzle into place, giving us the fundamental equation for rigid-body dynamics:

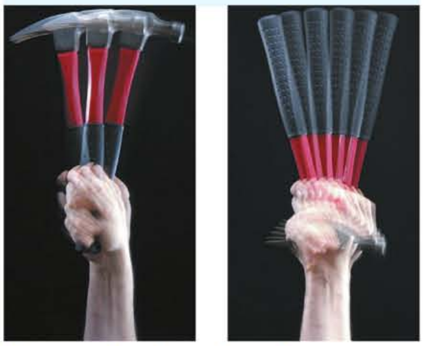
Newton's second law for rotation An object that experiences a net torque τ_{net} about the axis of rotation undergoes an angular acceleration

$$\alpha = \frac{\tau_{\text{net}}}{I} \tag{7.15}$$

where I is the moment of inertia of the object *about the rotation axis.*

In practice we often write $\tau_{\text{net}} = I\alpha$, but Equation 7.15 better conveys the idea that **a net torque is the cause of angular acceleration.** In the absence of a net torque ($\tau_{\text{net}} = 0$), the object has zero angular acceleration α, so it either does not rotate ($\omega = 0$) or rotates with *constant* angular velocity ($\omega = \text{constant}$).

TRY IT YOURSELF



Hammering home inertia Most of the mass of a hammer is in its head, so the hammer's moment of inertia is large when calculated about an axis passing through the end of the handle (far from the head), but small when calculated about an axis passing through the head itself. You can *feel* this difference by attempting to wave a hammer back and forth about the handle end and the head end. It's much harder to do about the handle end because the large moment of inertia keeps the angular acceleration small.

Interpreting the Moment of Inertia

Before rushing to calculate moments of inertia, let's get a better understanding of its meaning. First, notice that **moment of inertia is the rotational equivalent of mass.** It plays the same role in Equation 7.15 as does mass m in the now-familiar $\vec{a} = \vec{F}_{\text{net}}/m$. Recall that objects with larger mass have a larger *inertia,* meaning that they're harder to accelerate. Similarly, an object with a larger moment of inertia is harder to get rotating: It takes a larger torque to spin up an object with a larger moment of inertia than an object with a smaller moment of inertia. The fact that "moment of inertia" retains the word "inertia" reminds us of this.

But why does the moment of inertia depend on the distances r from the rotation axis? Think about trying to start a merry-go-round from rest, as shown in **FIGURE 7.28**. By pushing on the rim of the merry-go-round, you exert a torque on it, and its angular velocity begins to increase. If your friends sit at the rim of the merry-go-round, as in Figure 7.28a, their distances r from the axle are large. The moment of inertia is large, according to Equation 7.14, and it will be difficult to get the merry-go-round rotating. If, however, your friends sit near the axle, as in Figure 7.28b, then r and the moment of inertia are small. You'll find it's much easier to get the merry-go-round going.

Thus an object's moment of inertia depends not only on the object's mass but also on *how the mass is distributed* around the rotation axis. This is well known to bicycle racers. Every time a cyclist accelerates, she has to "spin up" the wheels and tires. The larger the moment of inertia, the more effort it takes and the slower her acceleration. For this reason, racers use the lightest possible tires, and they put those tires on wheels that have been designed to keep the mass as close as possible to the center without sacrificing the necessary strength and rigidity.

Table 7.3 summarizes the analogies between linear and rotational dynamics.

FIGURE 7.28 Moment of inertia depends on both the mass and how the mass is distributed.

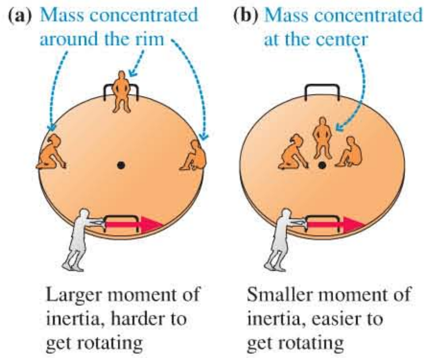


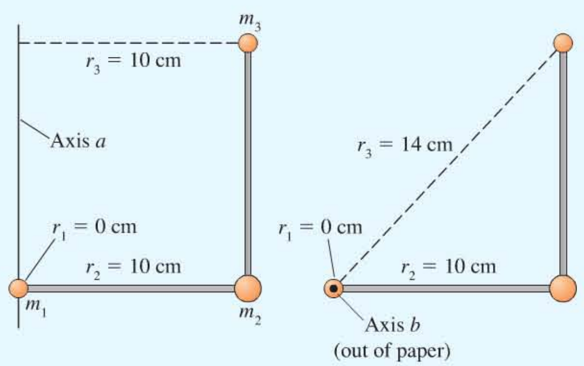
TABLE 7.3 Rotational and linear dynamics

Rotational dynamics		Linear dynamics	
Torque	τ_{net}	Force	$\vec{F}_{net}$
Moment of inertia	I	Mass	m
Angular acceleration	α	Acceleration	$\vec{a}$
Second law	$\alpha = \tau_{net}/I$	Second law	$\vec{a} = \vec{F}_{net}/m$

EXAMPLE 7.10 Calculating the moment of inertia

Your friend is creating an abstract sculpture that consists of three small, heavy spheres attached by very lightweight 10-cm-long rods as shown in **FIGURE 7.29**. The spheres have masses $m_1 = 1.0$ kg, $m_2 = 1.5$ kg, and $m_3 = 1.0$ kg. What is the object's moment of inertia if it is rotated about axis a? About axis b?

FIGURE 7.29 Three point particles separated by lightweight rods.



PREPARE We'll use Equation 7.14 for the moment of inertia:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

In this expression, r_1, r_2, and r_3 are the distances of each particle from the axis of rotation, so they depend on the axis chosen. Particle 1 lies on both axes, so $r_1 = 0$ cm in both cases. Particle 2 lies 10 cm (0.10 m) from both axes. Particle 3 is 10 cm from axis a, but farther from axis b. We can find r_3 for axis b by using the Pythagorean theorem, which gives $r_3 = 14$ cm. These distances are indicated in the figure.

SOLVE For each axis, we can prepare a table of the values of r, m, and mr^2 for each particle, then add the values of mr^2. For axis a we have

Particle	r	m	mr^2
1	0 m	1.0 kg	$0\ \text{kg}\cdot\text{m}^2$
2	0.10 m	1.5 kg	$0.015\ \text{kg}\cdot\text{m}^2$
3	0.10 m	1.0 kg	$0.010\ \text{kg}\cdot\text{m}^2$
			$I_a = 0.025\ \text{kg}\cdot\text{m}^2$

For axis b we have

Particle	r	m	mr^2
1	0 m	1.0 kg	$0\ \text{kg}\cdot\text{m}^2$
2	0.10 m	1.5 kg	$0.015\ \text{kg}\cdot\text{m}^2$
3	0.14 m	1.0 kg	$0.020\ \text{kg}\cdot\text{m}^2$
			$I_b = 0.035\ \text{kg}\cdot\text{m}^2$

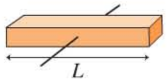
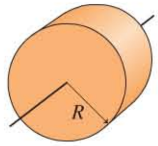
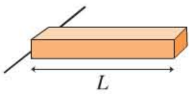
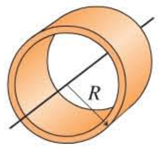
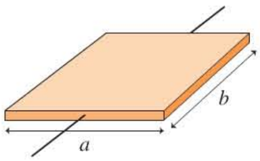
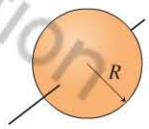
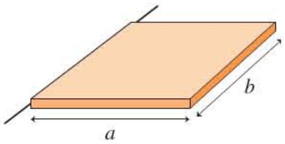
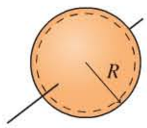
ASSESS We've already noted that the moment of inertia of an object is higher when its mass is distributed farther from the axis of rotation. Here, m_3 is farther from axis b than from axis a, leading to a higher moment of inertia about that axis.

The Moments of Inertia of Common Shapes

Newton's second law for rotational motion is easy to write, but we can't make use of it without knowing an object's moment of inertia. Unlike mass, we can't measure moment of inertia by putting an object on a scale. And although we can guess that the center of gravity of a symmetrical object is at the physical center of the object, we can *not* guess the moment of inertia of even a simple object.

For an object consisting of only a few point particles connected by massless rods, we can use Equation 7.14 to directly calculate I. But such an object is pretty unrealistic. All real objects are made up of solid material that is itself composed of countless atoms. To calculate the moment of inertia of even a simple object requires integral calculus and is beyond the scope of this text. A short list of common moments of inertia is given in Table 7.4. We use a capital M for the total mass of an extended object.

TABLE 7.4 Moments of inertia of objects with uniform density and total mass M

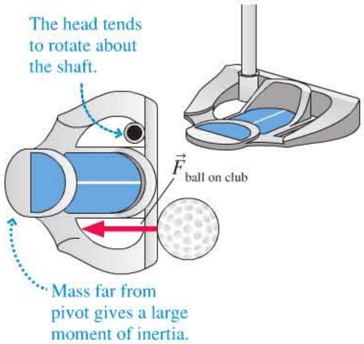
Object and axis	Picture	I	Object and axis	Picture	I
Thin rod (of any cross section), about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod (of any cross section), about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

We can make some general observations about the moments of inertia in Table 7.4. For instance, the cylindrical hoop is composed of particles that are all the same distance R from the axis. Thus each particle of mass m makes the *same* contribution mR^2 to the hoop's moment of inertia. Adding up all these contributions gives

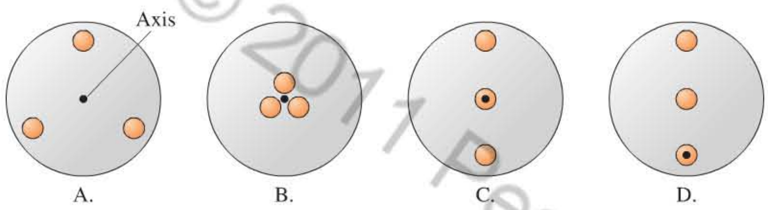
$$I = m_1R^2 + m_2R^2 + m_3R^2 + \cdots = (m_1 + m_2 + m_3 + \cdots)R^2 = MR^2$$

as given in the table. The solid cylinder of the same mass and radius has a *lower* moment of inertia than the hoop because much of the cylinder's mass is nearer its center. In the same way we can see why a slab rotated about its center has a lower moment of inertia than the same slab rotated about its edge: In the latter case, some of the mass is twice as far from the axis as the farthest mass in the former case. Those particles contribute *four times* as much to the moment of inertia, leading to an overall larger moment of inertia for the slab rotated about its edge.

▶ **Novel golf clubs** The latest craze in golf putters is heads with high moments of inertia. When the putter hits the ball, the ball—by Newton's third law—exerts a force on the putter and thus exerts a torque that causes the head of the putter to rotate around the shaft. Any rotation while the putter is still in contact with the ball will affect the ball's direction. If the putter's mass is largely placed rather far from the shaft (the rotation axis), the moment of inertia about the shaft can be greatly increased. The large moment of inertia of the head will keep its angular acceleration small—reducing unwanted rotation and allowing a truer putt.



STOP TO THINK 7.5 Four very lightweight disks of equal radii each have three identical heavy marbles glued to them as shown. Rank in order, from largest to smallest, the moments of inertia of the disks about the indicated axis.



7.5 Using Newton's Second Law for Rotation

In this section we'll look at several examples of rotational dynamics for rigid bodies that rotate about a *fixed axis*. The restriction to a fixed axis avoids complications that arise for an object undergoing a combination of rotational and translational motion. The problem-solving strategy for rotational dynamics is very similar to that for linear dynamics in Chapter 5.

ActivPhysics 7.8, 7.9, 7.10

PROBLEM-SOLVING STRATEGY 7.1 **Rotational dynamics problems** MP™

PREPARE Model the object as a simple shape. Draw a pictorial representation to clarify the situation, define coordinates and symbols, and list known information.

- Identify the axis about which the object rotates.
- Identify the forces and determine their distance from the axis.
- Calculate the torques caused by the forces, and find the signs of the torques.

SOLVE The mathematical representation is based on Newton's second law for rotational motion:

$$\tau_{\text{net}} = I\alpha \qquad \text{or} \qquad \alpha = \frac{\tau_{\text{net}}}{I}$$

- Find the moment of inertia either by direct calculation using Equation 7.14 or from Table 7.4 for common shapes of objects.
- Use rotational kinematics to find angular positions and velocities.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

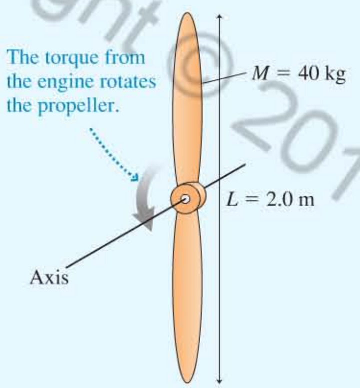
Exercise 25

EXAMPLE 7.11 Starting an airplane engine

The engine in a small airplane is specified to have a torque of 500 N · m. This engine drives a 2.0-m-long, 40 kg single-blade propeller. On start-up, how long does it take the propeller to reach 2000 rpm?

PREPARE The propeller can be modeled as a rod that rotates about its center. The engine exerts a torque on the propeller. FIGURE 7.30 shows the propeller and the rotation axis.

FIGURE 7.30 A rotating airplane propeller.



SOLVE The moment of inertia of a rod rotating about its center is found from Table 7.4:

$$I = \tfrac{1}{12}ML^2 = \tfrac{1}{12}(40 \text{ kg})(2.0 \text{ m})^2 = 13.3 \text{ kg} \cdot \text{m}^2$$

The 500 N · m torque of the engine causes an angular acceleration of

$$\alpha = \frac{\tau}{I} = \frac{500 \text{ N} \cdot \text{m}}{13.3 \text{ kg} \cdot \text{m}^2} = 37.5 \text{ rad/s}^2$$

The time needed to reach $\omega_f = 2000 \text{ rpm} = 33.3 \text{ rev/s} = 209 \text{ rad/s}$ is

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{\omega_f - \omega_i}{\alpha} = \frac{209 \text{ rad/s} - 0 \text{ rad/s}}{37.5 \text{ rad/s}^2} = 5.6 \text{ s}$$

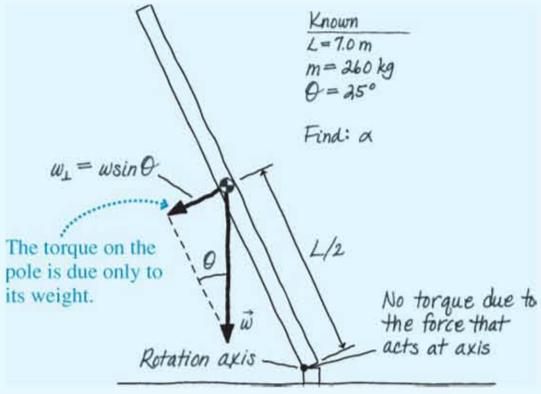
ASSESS We've assumed a constant angular acceleration, which is reasonable for the first few seconds while the propeller is still turning slowly. Eventually, air resistance and friction will cause opposing torques and the angular acceleration will decrease. At full speed, the negative torque due to air resistance and friction cancels the torque of the engine. Then $\tau_{net} = 0$ and the propeller turns at *constant* angular velocity with no angular acceleration.

EXAMPLE 7.12 Angular acceleration of a falling pole

A 7.0-m-tall telephone pole with a mass of 260 kg has just been placed in the ground. Before the wires can be connected, the pole is hit by lightning, nearly severing the pole at its base. The pole begins to fall, rotating about the part still connected to the base. Estimate the pole's angular acceleration when it has fallen by 25° from the vertical.

PREPARE The situation is shown in FIGURE 7.31, where we define our symbols and list the known information. Two forces are acting on the pole: the pole's weight $\vec{w}$, which acts at the center of gravity, and the force of the base on the pole (not shown).

FIGURE 7.31 A falling telephone pole undergoes an angular acceleration due to a gravitational torque.



This second force exerts no torque because it acts at the axis of rotation. The torque on the pole is thus due only to gravity. From the figure we see that this torque tends to rotate the pole in a counterclockwise direction, so the torque is positive.

SOLVE We'll model the pole as a uniform thin rod rotating about one end. Its center of gravity is at its center, a distance $L/2$ from the axis. You can see from the figure that the perpendicular component of $\vec{w}$ is $w_\perp = w\sin\theta$. Thus the torque due to gravity is

$$\tau_{net} = \left(\frac{L}{2}\right)w_\perp = \left(\frac{L}{2}\right)w\sin\theta = \frac{mgL}{2}\sin\theta$$

From Table 7.4, the moment of inertia of a thin rod rotated about its end is $I = \frac{1}{3}mL^2$. Thus, from Newton's second law for rotational motion, the angular acceleration is

$$\alpha = \frac{\tau_{net}}{I} = \frac{\frac{1}{2}mgL\sin\theta}{\frac{1}{3}mL^2} = \frac{3g\sin\theta}{2L}$$

$$= \frac{3(9.8 \text{ m/s}^2)\sin 25°}{2(7.0 \text{ m})} = 0.9 \text{ rad/s}^2$$

ASSESS The answer is given to only one significant figure because the problem asked for an *estimate* of the angular acceleration. This is usually a hint that you should make some simplifying assumptions, as we did here in modeling the pole as a thin rod.

CONCEPTUAL EXAMPLE 7.13 Balancing a meter stick

You've probably tried balancing a rod-shaped object vertically on your fingertip. If the object is very long, like a meter stick or a baseball bat, it's not too hard. But if it's short, like a pencil, it's almost impossible. Why is this?

REASON Suppose you've managed to balance a vertical stick on your fingertip, but then it starts to fall. You'll need to quickly adjust your finger to bring the stick back into balance. As Example 7.12 showed, the angular acceleration α of a thin rod is *inversely proportional* to L. Thus a long object like a meter stick topples much more slowly than a short one like a pencil. Your reaction time is fast enough to correct for a slowly falling meter stick but not for a rapidly falling pencil.

ASSESS If we double the length of a rod, its mass doubles and its center of gravity is twice as high, so the gravitational torque τ on it is four times as much. But because a rod's moment of inertia is $I = \frac{1}{3}ML^2$, the longer rod's moment of inertia will be *eight* times greater, so the angular acceleration will be only half as large.

Constraints Due to Ropes and Pulleys

Many important applications of rotational dynamics involve objects that are attached to ropes that pass over pulleys. **FIGURE 7.32** shows a rope passing over a pulley and connected to an object in linear motion. If the pulley turns *without the rope slipping on it,* then the rope's speed v_{rope} must exactly match the speed of the rim of the pulley, which is $v_{rim} = \omega R$. If the pulley has an angular acceleration, the rope's acceleration a_{rope} must match the *tangential* acceleration of the rim of the pulley, $a_t = \alpha R$.

The object attached to the other end of the rope has the same speed and acceleration as the rope. Consequently, the object must obey the constraints

$$v_{obj} = \omega R$$
$$a_{obj} = \alpha R \tag{7.16}$$

Motion constraints for an object connected to a pulley of radius R by a nonslipping rope

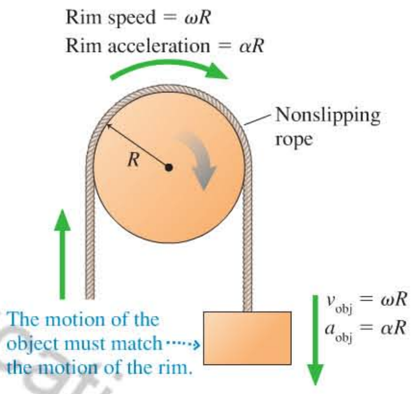


FIGURE 7.32 The rope's motion must match the motion of the rim of the pulley.

These constraints are similar to the acceleration constraints introduced in Chapter 5 for two objects connected by a string or rope.

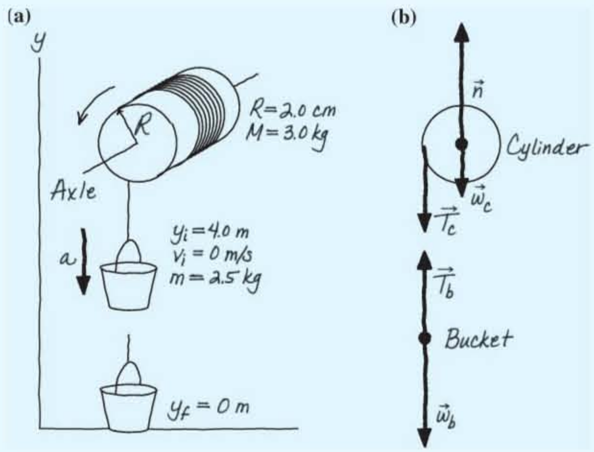
NOTE ► The constraints are given as magnitudes. Specific problems will require you to specify signs that depend on the direction of motion and on the choice of coordinate system. ◄

EXAMPLE 7.14 Time for a bucket to fall

Josh has just raised a 2.5 kg bucket of water using a well's winch when he accidentally lets go of the handle. The winch consists of a rope wrapped around a 3.0 kg, 4.0-cm-diameter cylinder, which rotates on an axle through the center. The bucket is released from rest 4.0 m above the water level of the well. How long does it take to reach the water?

PREPARE Assume the rope is massless and does not slip. **FIGURE 7.33a** gives a visual overview of the falling bucket. **FIGURE 7.33b** shows the free-body diagrams for the cylinder and the bucket. The rope tension exerts an upward force on the bucket and a downward force on the outer edge of the cylinder. The rope is massless, so these two tension forces have equal magnitudes, which we'll call T.

FIGURE 7.33 Visual overview of a falling bucket.



Continued

SOLVE Newton's second law applied to the linear motion of the bucket is

$$ma_y = T - mg$$

where, as usual, the y-axis points upward. What about the cylinder? There is a normal force $\vec{n}$ on the cylinder due to the axle and the weight of the cylinder $\vec{w}_c$. However, neither of these forces exerts a torque because each passes through the rotation axis. The only torque comes from the rope tension. The moment arm for the tension is $r_\perp = R$, and the torque is positive because the rope turns the cylinder counterclockwise. Thus $\tau_{rope} = TR$, and Newton's second law for the rotational motion is

$$\alpha = \frac{\tau_{net}}{I} = \frac{TR}{\frac{1}{2}MR^2} = \frac{2T}{MR}$$

The moment of inertia of a cylinder rotating about a center axis was taken from Table 7.4.

The last piece of information we need is the constraint due to the fact that the rope doesn't slip. Equation 7.16 relates only the magnitudes of the linear and angular accelerations, but in this problem α is positive (counterclockwise acceleration), while a_y is negative (downward acceleration). Hence

$$a_y = -\alpha R$$

Using α from the cylinder's equation in the constraint, we find

$$a_y = -\alpha R = -\frac{2T}{MR}R = -\frac{2T}{M}$$

Thus the tension is $T = -\frac{1}{2}Ma_y$. If we use this value of the tension in the bucket's equation, we can solve for the acceleration:

$$ma_y = -\tfrac{1}{2}Ma_y - mg$$

$$a_y = -\frac{g}{(1 + M/2m)} = -6.1 \text{ m/s}^2$$

The time to fall through $\Delta y = y_f - y_i = -4.0$ m is found from kinematics:

$$\Delta y = \tfrac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-4.0 \text{ m})}{-6.1 \text{ m/s}^2}} = 1.1 \text{ s}$$

ASSESS The expression for the acceleration gives $a_y = -g$ if $M = 0$. This makes sense because the bucket would be in free fall if there were no cylinder. When the cylinder has mass, the downward force of gravity on the bucket has to accelerate the bucket *and* spin the cylinder. Consequently, the acceleration is reduced and the bucket takes longer to fall.

7.6 Rolling Motion

Rolling is a *combination motion* in which an object rotates about an axis that is moving along a straight-line trajectory. For example, FIGURE 7.34 is a time-exposure photo of a rolling wheel with one lightbulb on the axis and a second lightbulb at the edge. The axis light moves straight ahead, but the edge light follows a curve called a *cycloid.* Let's see if we can understand this interesting motion. We'll consider only objects that roll without slipping.

To understand rolling motion, consider FIGURE 7.35, which shows a round object—a wheel or a sphere—that rolls forward, *without slipping,* exactly one revolution. The point initially at the bottom follows the blue curve to the top and then back to the bottom. The overall position of the object is measured by the position x of the object's center. Because the object doesn't slip, in one revolution the center moves forward exactly one circumference, so that $\Delta x = 2\pi R$. The time for the

FIGURE 7.34 The trajectories of the center of a wheel and of a point on the rim are seen in a time-exposure photograph.

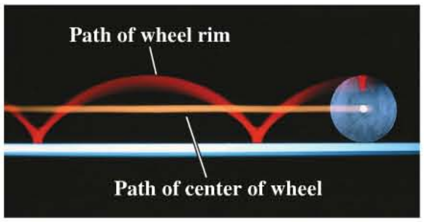
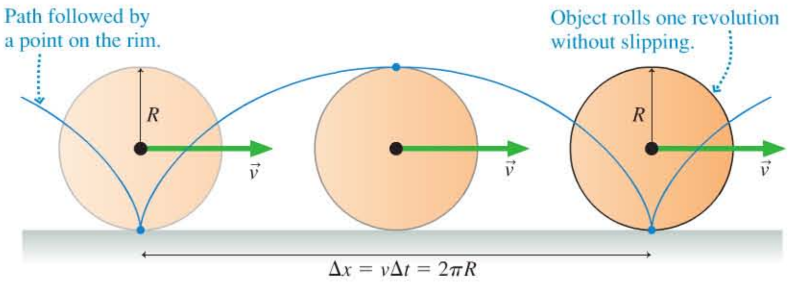


FIGURE 7.35 An object rolling through one revolution.



object to turn one revolution is its period T, so we can compute the speed of the object's center as

$$v = \frac{\Delta x}{T} = \frac{2\pi R}{T} \tag{7.17}$$

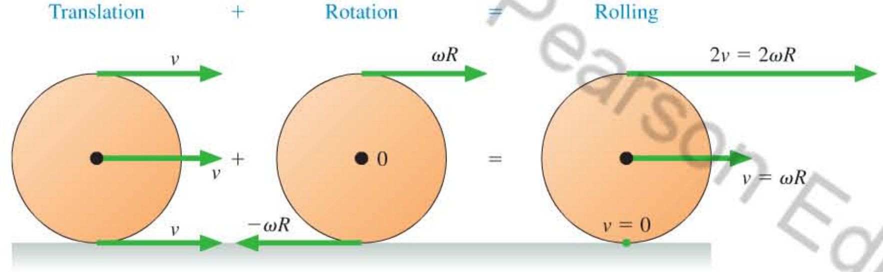
But $2\pi/T$ is the angular velocity ω, as you learned in Chapter 6, which leads to

$$v = \omega R \tag{7.18}$$

Equation 7.18 is the **rolling constraint,** the basic link between translation and rotation for objects that roll without slipping.

We can find the velocity for any point on a rolling object by adding the velocity of that point when the object is in pure translation, without rolling, to the velocity of the point when the object is in pure rotation, without translating. **FIGURE 7.36** shows how the velocity vectors at the top, center, and bottom of a rotating wheel are found in this way.

FIGURE 7.36 Rolling is a combination of translation and rotation.



Thus the point at the top of the wheel has a forward speed of v due to its translational motion plus a forward speed of $\omega R = v$ due to its rotational motion. The speed of a point at the top of a wheel is then $2v = 2\omega R$, or *twice* the speed of its center of mass. On the other hand, the point at the bottom of the wheel, where it touches the ground, still has a forward speed of v due to its translational motion. But its velocity due to rotation points *backward* with a magnitude of $\omega R = v$. Adding these, we find that the velocity of this lowest point is *zero*. In other words, **the point on the bottom of a rolling object is instantaneously at rest.**

Although this seems surprising, it is really what we mean by "rolling without slipping." If the bottom point had a velocity, it would be moving horizontally relative to the surface. In other words, it would be slipping or sliding across the surface. To roll without slipping, the bottom point, the point touching the surface, must be at rest.

Ancient movers The great stone *moai* of Easter Island were moved as far as 16 km from a quarry to their final positions. Archeologists believe that one possible method of moving these 14 ton statues was to place them on rollers. One disadvantage of this method is that the statues, placed on top of the rollers, move twice as fast as the rollers themselves. Thus rollers are continuously left behind and have to be carried back to the front and reinserted. Sadly, the indiscriminate cutting of trees for moving *moai* may have hastened the demise of this island civilization.

EXAMPLE 7.15 **Rotating your tires**

The diameter of your tires is 0.60 m. You take a 60 mile trip at a speed of 45 mph.

a. During this trip, what was your tires' angular speed?
b. How many times did they revolve?

PREPARE The angular speed is related to the speed of a wheel's center by Equation 7.18: $v = \omega R$. Because the center of the wheel turns on an axle fixed to the car, the speed v of the wheel's center is the same as that of the car. We prepare by converting the car's speed to SI units:

$$v = (45 \text{ mph}) \times \left(0.447 \frac{\text{m/s}}{\text{mph}}\right) = 20 \text{ m/s}$$

Once we know the angular speed, we can find the number of times the tires turned from the rotational-kinematic equation $\Delta\theta = \omega\,\Delta t$. We'll need to find the time traveled Δt from $v = \Delta x/\Delta t$.

SOLVE a. From Equation 7.18 we have

$$\omega = \frac{v}{R} = \frac{20 \text{ m/s}}{0.30 \text{ m}} = 67 \text{ rad/s}$$

b. The time of the trip is

$$\Delta t = \frac{\Delta x}{v} = \frac{60 \text{ mi}}{45 \text{ mi/h}} = 1.33 \text{ h} \times \frac{3600 \text{ s}}{1 \text{ h}} = 4800 \text{ s}$$

Continued

Thus the total angle through which the tires turn is

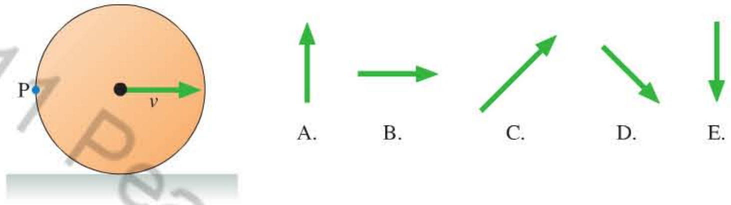
$$\Delta\theta = \omega\,\Delta t = (67\text{ rad/s})(4800\text{ s}) = 3.2 \times 10^5\text{ rad}$$

Because each turn of the wheel is 2π rad, the number of turns is

$$\frac{3.2 \times 10^5\text{ rad}}{2\pi\text{ rad}} = 51{,}000\text{ turns}$$

ASSESS You probably know from seeing tires on passing cars that a tire rotates several times a second at 45 mph. Because there are 3600 s in an hour, and your 60 mile trip at 45 mph is going to take over an hour—say, ≈5000 s—you would expect the tire to make many thousands of revolutions. So 51,000 turns seems to be a reasonable answer. You can see that your tires rotate roughly a thousand times per mile. During the lifetime of a tire, about 50,000 miles, it will rotate about 50 million times!

STOP TO THINK 7.6 A wheel rolls without slipping. Which is the correct velocity vector for point P on the wheel?



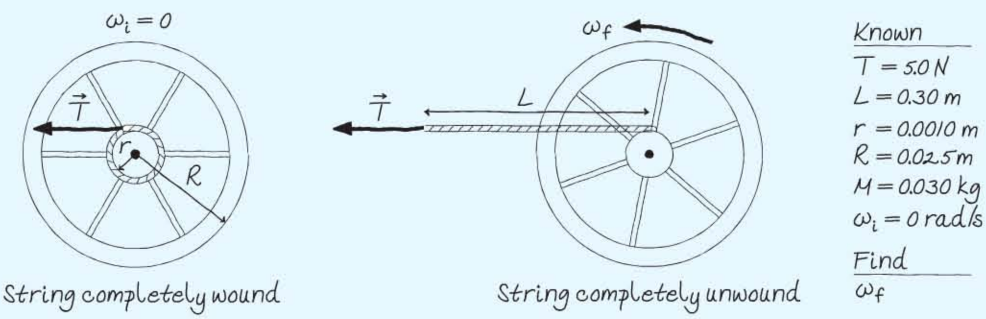
INTEGRATED EXAMPLE 7.16 Spinning a gyroscope

A gyroscope is a top-like toy consisting of a heavy ring attached by light spokes to a central axle. The axle and ring are free to turn on bearings. To get the gyroscope spinning, a 30-cm-long string is wrapped around the 2.0-mm-diameter axle, then pulled with a constant force of 5.0 N. If the ring's diameter is 5.0 cm and its mass is 30 g, at what rate is it spinning, in rpm, once the string is completely unwound?

PREPARE Because the ring is heavy compared to the spokes and the axle, we'll model it as a cylindrical hoop, taking its moment of inertia from Table 7.4 to be $I = MR^2$. **FIGURE 7.37** shows a visual overview of the problem. Two points are worth noting. First, rule 2 of Tactics Box 5.2 tells us that the tension in the string has the same magnitude as the force that pulls on the string, so the tension is $T = 5.0$ N. Second, it is a good idea to convert all the known quantities in the problem statement to SI units, and to collect them all in one place as we have done in the visual overview of Figure 7.37. Here, radius R is half the 5.0 cm ring diameter, and radius r is half the 2.0 mm axle diameter.

We are asked at what rate the ring is spinning when the string is unwound. This is a question about the ring's final *angular velocity*, which we've labeled ω_f. We've assumed that the initial angular velocity is $\omega_i = 0$ rad/s. Because the angular velocity is changing, the ring must have an angular acceleration that, as we know, is caused by a torque. So a good strategy will be to find the torque on the ring, from which we can find its angular acceleration and, using kinematics, the final angular velocity.

FIGURE 7.37 Visual overview of a gyroscope being spun.



SOLVE The torque on the ring is due to the tension in the string. Because the string—and the line of action of the tension—is tangent to the axle, the moment arm of the tension force is the radius r of the axle. Thus $\tau = r_\perp T = rT$. Now we can apply Newton's second law for rotational motion, Equation 7.15, to find the angular acceleration:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{rT}{MR^2} = \frac{(0.0010\text{ m})(5.0\text{ N})}{(0.030\text{ kg})(0.025\text{ m})^2} = 267\text{ rad/s}^2$$

We next use constant-angular-acceleration kinematics to find the final angular velocity. For the equation $\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$ of Table 7.2, we know α and ω_i, and we should be able to find $\Delta\theta$ from the length of string unwound, but we don't know Δt. For the equation $\Delta\omega = \omega_f - \omega_i = \alpha\Delta t$, we know α and ω_i, and ω_f is what we want to find, but again we don't know Δt. To find an equation that doesn't contain Δt, we first write

$$\Delta t = \frac{\omega_f - \omega_i}{\alpha}$$

from the second kinematic equation. Inserting this value for Δt into the first equation gives

$$\Delta\theta = \omega_i \frac{\omega_f - \omega_i}{\alpha} + \frac{1}{2}\alpha\left(\frac{\omega_f - \omega_i}{\alpha}\right)^2$$

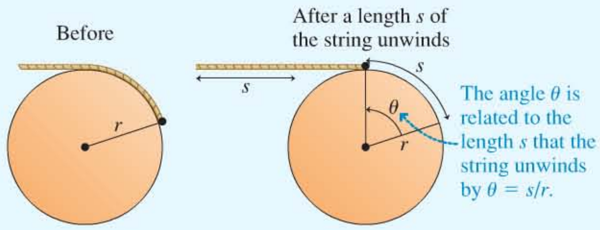
which can be simplified to

$$\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$$

This equation, which is the rotational analog of the linear motion Equation 2.13, will allow us to find ω_f once $\Delta\theta$ is known.

FIGURE 7.38 shows how to find $\Delta\theta$. As a segment of string of length s unwinds, the axle turns through an angle (based on the definition of radian measure) $\theta = s/r$. Thus as the whole string, of length L, unwinds, the axle (and the ring) turns through an angular displacement

FIGURE 7.38 Relating the angle turned to the length of string unwound.



$$\Delta\theta = \frac{L}{r} = \frac{0.30\text{ m}}{0.0010\text{ m}} = 300\text{ rad}$$

Now we can use our kinematic equation to find that

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta = (0\text{ rad/s})^2 + 2(267\text{ rad/s}^2)(300\text{ rad})$$
$$= 160{,}000\text{ (rad/s)}^2$$

from which we find that $\omega_f = 400$ rad/s. Converting rad/s to rpm, we find that the gyroscope ring is spinning at

$$400\text{ rad/s} = \left(\frac{400\text{ rad}}{\text{s}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 3800\text{ rpm}$$

ASSESS This is fast, about the speed of your car engine when its on the highway, but if you've ever played with a gyroscope or a string-wound top, you know you can really get it spinning fast.

SUMMARY

The goal of Chapter 7 has been to understand the physics of rotating objects.

GENERAL PRINCIPLES

Newton's Second Law for Rotational Motion

If a net torque τ_{net} acts on an object, the object will experience an angular acceleration given by $\alpha = \tau_{net}/I$, where I is the object's moment of inertia about the rotation axis.

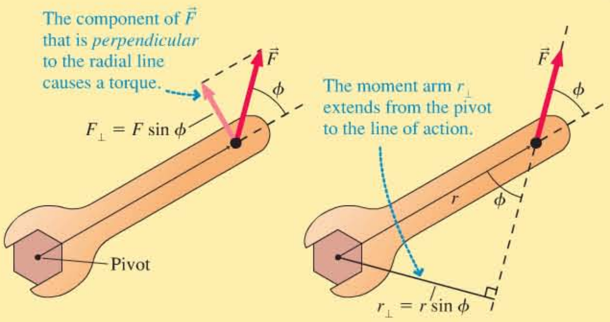
This law is analogous to Newton's second law for linear motion, $\vec{a} = \vec{F}_{net}/m$.

IMPORTANT CONCEPTS

Torque is the rotational analog of force. Just as a force causes an object to undergo a linear acceleration, a torque causes an object to undergo an angular acceleration.

There are two interpretations of torque:

Interpretation 1: $\tau = rF_{\perp}$ Interpretation 2: $\tau = r_{\perp}F$



Both interpretations give the same expression for the magnitude of the torque: $\tau = rF\sin\phi$.

A torque is positive if it tends to rotate the object counterclockwise; negative if it tends to rotate the object clockwise.

The **moment of inertia** is the rotational equivalent of mass. The larger an object's moment of inertia, the more difficult it is to get the object rotating. For an object made up of particles of masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis, the moment of inertia is

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \cdots = \sum mr^2$$

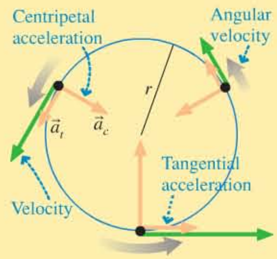
Angular and tangential acceleration

A particle moving in a circle has

- A velocity tangent to the circle.
- A centripetal acceleration $\vec{a}_c$ directed toward the center of the circle.

If the particle's speed is increasing, it will also have

- A tangential acceleration $\vec{a}_t$ directed tangent to the circle.
- An angular acceleration α.

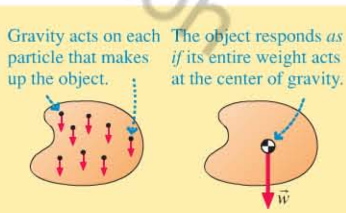


The angular and tangential accelerations are related by $a_t = \alpha r$.

For a **rigid body,** the angular velocity and angular acceleration are the same for every point on the object.

Center of gravity

The **center of gravity** of an object is the point at which gravity can be considered as acting.

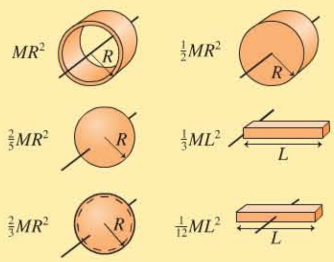


The **position of the center of gravity** depends on the distance $x_1, x_2, \ldots$ of each particle of mass $m_1, m_2, \ldots$ from the origin:

$$x_{cg} = \frac{x_1m_1 + x_2m_2 + x_3m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

APPLICATIONS

Moments of inertia of common shapes



Rotation about a fixed axis

When a net torque is applied to an object that rotates about a fixed axis, the object will undergo an **angular acceleration** given by

$$\alpha = \frac{\tau_{net}}{I}$$

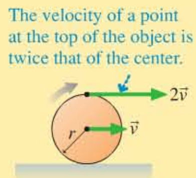
If a rope unwinds from a pulley of radius R, the linear motion of an object tied to the rope is related to the angular motion of the pulley by

$$a_{obj} = \alpha R \qquad v_{obj} = \omega R$$

Rolling motion

For an object that rolls without slipping,

$$v = \omega R$$



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. Figure Q7.1 shows four pulleys, each with a heavy and a light block strung over them. The blocks' velocities are shown. What are the signs (+ or −) of the angular velocity and angular acceleration of the pulley in each case?

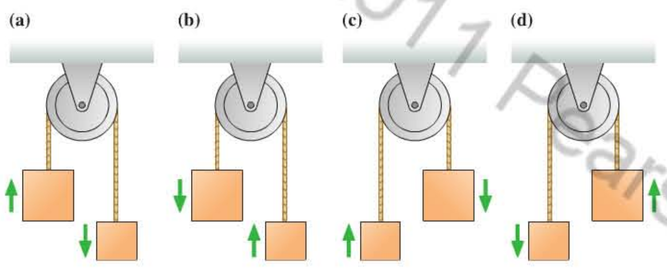


FIGURE Q7.1

2. If you are using a wrench to loosen a very stubborn nut, you can make the job easier by using a "cheater pipe." This is a piece of pipe that slides over the handle of the wrench, as shown in Figure Q7.2, making it effectively much longer. Explain why this would help you loosen the nut.

FIGURE Q7.2

3. Five forces are applied to a door, as seen from above in Figure Q7.3. For each force, is the torque about the hinge positive, negative, or zero?

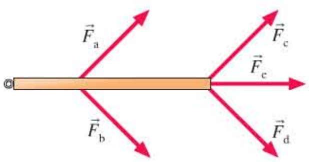


FIGURE Q7.3

4. A screwdriver with a very thick handle requires less force to operate than one with a very skinny handle. Explain why this is so.
5. If you have ever driven a truck, you likely found that it had a steering wheel with a larger diameter than that of a passenger car. Why is this?
6. A common type of door stop is a wedge made of rubber. Is such a stop more effective when jammed under the door near or far from the hinges? Why?
7. Suppose you are hanging from a tree branch. If you move out along the branch, farther away from the trunk, the branch will be more likely to break. Explain why this is so.
8. A student gives a quick push to a ball at the end of a massless, rigid rod, causing the ball to rotate clockwise in a *horizontal* circle as shown in Figure Q7.8. The rod's pivot is frictionless.
   a. As the student is pushing, is the torque about the pivot positive, negative, or zero?
   b. After the push has ended, what does the ball's angular velocity do? Steadily increase? Increase for a while, then hold steady? Hold steady? Decrease for a while, then hold steady? Steadily decrease? Explain.
   c. Right after the push has ended, is the torque positive, negative, or zero?

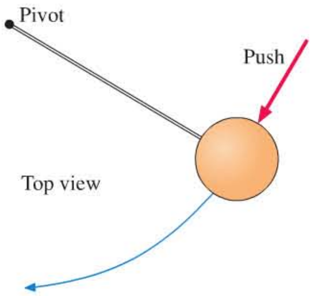


FIGURE Q7.8

9. The two ends of the dumbbell shown in Figure Q7.9 are made of the same material. Is the dumbbell's center of gravity at point 1, 2, or 3? Explain.

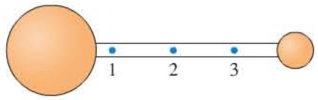


FIGURE Q7.9

10. When you rise from a chair, you have to lean quite far forward (try it!). Why is this?
11. Suppose you have two identical-looking metal spheres of the same size and the same mass. One of them is solid, the other is hollow. How can you tell which is which?
12. The moment of inertia of a uniform rod about an axis through its center is $ML^2/12$. The moment of inertia about an axis at one end is $ML^2/3$. Explain *why* the moment of inertia is larger about the end than about the center.
13. A heavy steel rod, 1.0 m long, and a light pencil, 0.15 m long, are held 15° from the vertical with one end on a table, then released simultaneously. Which will hit the table first? Or will it be a tie? Explain.
14. The wheel in Figure Q7.14 is rolling to the right without slipping. Rank in order, from fastest to slowest, the *speeds* of the points labeled 1 through 5. Explain your reasoning.

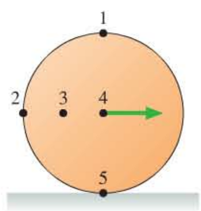


FIGURE Q7.14

15. A car traveling at 60 mph has a pebble stuck in one of its tires. Eventually the pebble works loose, and at the instant of release it is at the top of the tire. Explain why the pebble then slams *hard* into the *front* of the wheel well.

Multiple-Choice Questions

16. | A nut needs to be tightened with a wrench. Which force shown in Figure Q7.16 will apply the greatest torque to the nut?

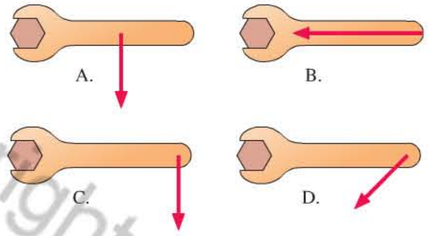


FIGURE Q7.16

17. | Suppose a bolt on your car engine needs to be tightened to a torque of 20 N·m. You are using a 15-cm-long wrench, and you apply a force at the very end in the direction that produces maximum torque. What force should you apply?
A. 1300 N B. 260 N C. 130 N D. 26 N
18. | A machine part is made up of two pieces, with centers of gravity shown in Figure Q7.18. Which point could be the center of gravity of the entire part?

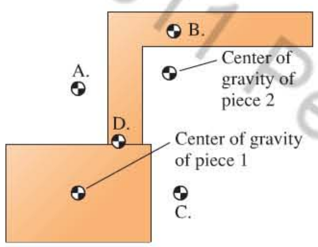


FIGURE Q7.18

19. || A typical compact disk has a mass of 15 g and a diameter of 120 mm. What is its moment of inertia about an axis through its center, perpendicular to the disk?
A. $2.7 \times 10^{-5}\ \text{kg} \cdot \text{m}^2$ B. $5.4 \times 10^{-5}\ \text{kg} \cdot \text{m}^2$
C. $1.1 \times 10^{-4}\ \text{kg} \cdot \text{m}^2$ D. $2.2 \times 10^{-4}\ \text{kg} \cdot \text{m}^2$
20. ||| Suppose you make a new kind of compact disk that is the same thickness as a current disk but twice the diameter. By what factor will the moment of inertia increase?
A. 2 B. 4 C. 8 D. 16
21. | Doors 1 and 2 have the same mass, height, and thickness. Door 2 is twice as wide as door 1. Bob pushes straight against the outer edge of door 2 with force F, and Barb pushes straight against the outer edge of door 1 with force $2F$. How do the angular accelerations α_1 and α_2 of the two doors compare?
A. $\alpha_1 > \alpha_2$ B. $\alpha_1 = \alpha_2$ C. $\alpha_1 < \alpha_2$
22. | A baseball bat has a heavy barrel and a thin handle. If you want to hold a baseball bat on your palm so that it balances vertically, you should
A. Put the end of the handle in your palm, with the barrel up.
B. Put the end of the barrel in your palm, with the handle up.
C. The bat will be equally easy to balance in either configuration.
23. | A car traveling at a steady 30 m/s has 74-cm-diameter tires. What is the approximate acceleration of a piece of the tread on any of the tires?
A. $24\ \text{m/s}^2$ B. $48\ \text{m/s}^2$ C. $2400\ \text{m/s}^2$ D. $4800\ \text{m/s}^2$

PROBLEMS

Section 7.1 The Rotation of a Rigid Body

1. ||| To throw a discus, the thrower holds it with a fully outstretched arm. Starting from rest, he begins to turn with a constant angular acceleration, releasing the discus after making one complete revolution. The diameter of the circle in which the discus moves is about 1.8 m. If the thrower takes 1.0 s to complete one revolution, starting from rest, what will be the speed of the discus at release?
2. ||| A computer hard disk starts from rest, then speeds up with an angular acceleration of $190\ \text{rad/s}^2$ until it reaches its final angular speed of 7200 rpm. How many revolutions has the disk made 10.0 s after it starts up?
3. ||| The crankshaft in a race car goes from rest to 3000 rpm in 2.0 s.
a. What is the crankshaft's angular acceleration?
b. How many revolutions does it make while reaching 3000 rpm?

Section 7.2 Torque

4. | Reconsider the situation in Example 7.5. If Luis pulls straight down on the end of a wrench that is in the same orientation but is 35 cm long, rather than 20 cm, what force must he apply to exert the same torque?
5. ||| Balls are attached to light rods and can move in horizontal circles as shown in Figure P7.5. Rank in order, from smallest to largest, the torques τ_1 to τ_4 about the centers of the circles. Explain.

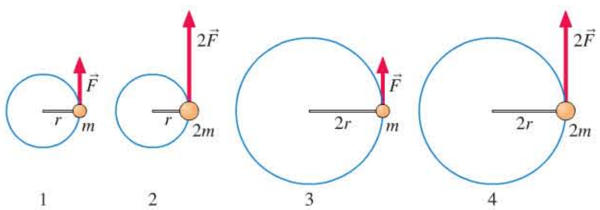


FIGURE P7.5

6. |||| Six forces, each of magnitude either F or $2F$, are applied to a door as seen from above in Figure P7.6. Rank in order, from smallest to largest, the six torques τ_1 to τ_6 about the hinge.

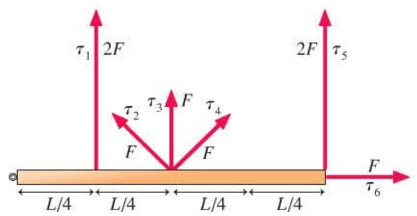


FIGURE P7.6

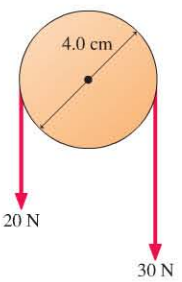


FIGURE P7.7

7. | What is the net torque about the axle on the pulley in Figure P7.7?
8. ||| The tune-up specifications of a car call for the spark plugs to be tightened to a torque of 38 N·m. You plan to tighten the plugs by pulling on the end of a 25-cm-long wrench. Because of the cramped space under the hood, you'll need to pull at an angle of 120° with respect to the wrench shaft. With what force must you pull?
9. ||| A professor's office door is 0.91 m wide, 2.0 m high, and 4.0 cm thick; has a mass of 25 kg; and pivots on frictionless hinges. A "door closer" is attached to the door and the top of the door frame. When the door is open and at rest, the door closer exerts a torque of 5.2 N·m. What is the least force that you need to apply to the door to hold it open?

10. ‖ In Figure P7.10, force $\vec{F}_2$ acts half as far from the pivot as $\vec{F}_1$. What magnitude of $\vec{F}_2$ causes the net torque on the rod to be zero?

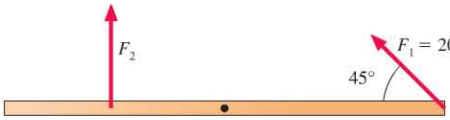


FIGURE P7.10

11. ‖‖ Tom and Jerry both push on the 3.00-m-diameter merry-go-round shown in Figure P7.11.
   a. If Tom pushes with a force of 40.0 N and Jerry pushes with a force of 35.2 N, what is the net torque on the merry-go-round?
   b. What is the net torque if Jerry reverses the direction he pushes by 180° without changing the magnitude of his force?

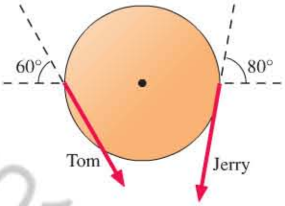


FIGURE P7.11

12. ‖ What is the net torque of the bar shown in Figure P7.12, about the axis indicated by the dot?

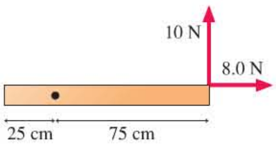


FIGURE P7.12

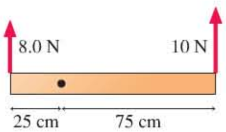


FIGURE P7.13

13. ‖ What is the net torque of the bar shown in Figure P7.13, about the axis indicated by the dot?
14. ‖ What is the net torque of the bar shown in Figure P7.14, about the axis indicated by the dot?

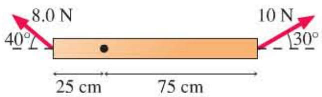


FIGURE P7.14

15. ‖ A 1.7-m-long barbell has a 20 kg weight on its left end and a 35 kg weight on its right end.
   a. If you ignore the weight of the bar itself, how far from the left end of the barbell is the center of gravity?
   b. Where is the center of gravity if the 8.0 kg mass of the barbell itself is taken into account?

Section 7.3 Gravitational Torque and the Center of Gravity

16. ‖ Three identical coins lie on three corners of a square 10.0 cm on a side, as shown in Figure P7.16. Determine the x and y coordinates of the center of gravity of the three coins.

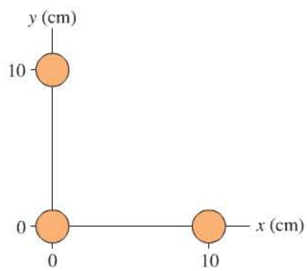


FIGURE P7.16

17. | Hold your arm outstretched so that it is horizontal. Estimate the mass of your arm and the position of its center of gravity. What is the gravitational torque on your arm in this position, computed around the shoulder joint?
BIO

18. ‖‖ A solid cylinder sits on top of a solid cube as shown in Figure P7.18. How far above the table's surface is the center of gravity of the combined object?

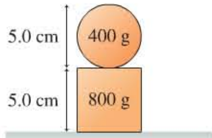


FIGURE P7.18

FIGURE P7.19

19. ‖ The 2.0 kg, uniform, horizontal rod in Figure P7.19 is seen from the side. What is the gravitational torque about the point shown?
20. ‖‖‖ A 4.00-m-long, 500 kg steel beam extends horizontally from the point where it has been bolted to the framework of a new building under construction. A 70.0 kg construction worker stands at the far end of the beam. What is the magnitude of the torque about the point where the beam is bolted into place?
21. ‖‖ An athlete at the gym holds a 3.0 kg steel ball in his hand. His arm is 70 cm long and has a mass of 4.0 kg. What is the magnitude of the torque about his shoulder if he holds his arm
BIO
   a. Straight out to his side, parallel to the floor?
   b. Straight, but 45° below horizontal?
22. ‖‖ The 2.0-m-long, 15 kg beam in Figure P7.22 is hinged at its left end. It is "falling" (rotating clockwise, under the influence of gravity), and the figure shows its position at three different times. What is the gravitational torque on the beam about an axis through the hinged end when the beam is at the
   a. Upper position?
   b. Middle position?
   c. Lower position?

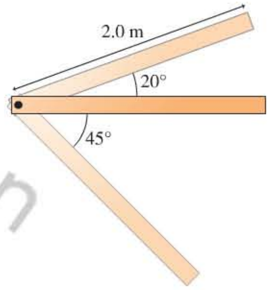


FIGURE P7.22

23. ‖‖ Two thin beams are joined end-to-end as shown in Figure P7.23 to make a single object. The left beam is 10.0 kg and 1.00 m long and the right one is 40.0 kg and 2.00 m long.
   a. How far from the left end of the left beam is the center of gravity of the object?
   b. What is the gravitational torque on the object about an axis through its left end? The object is seen from the side.

FIGURE P7.23

24. ‖‖ Figure P7.24 shows two thin beams joined at right angles. The vertical beam is 15.0 kg and 1.00 m long and the horizontal beam is 25.0 kg and 2.00 m long.
   a. Find the center of gravity of the two joined beams. Express your answer in the form (x, y), taking the origin at the corner where the beams join.
   b. Calculate the gravitational torque on the joined beams about an axis through the corner. The beams are seen from the side.

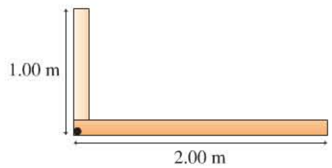


FIGURE P7.24

Section 7.4 Rotational Dynamics and Moment of Inertia

25. ||| A regulation table tennis ball has a mass of 2.7 g and is 40 mm in diameter. What is its moment of inertia about an axis that passes through its center?

26. || Three pairs of balls are connected by very light rods as shown in Figure P7.26. Rank in order, from smallest to largest, the moments of inertia I_1, I_2, and I_3 about axes through the centers of the rods.

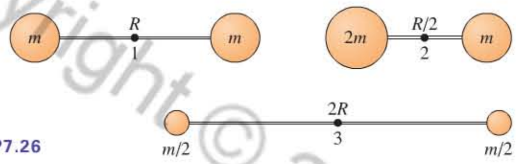


FIGURE P7.26

27. || A playground toy has four seats, each 5.0 kg, attached to very light, 1.5-m-long rods, as seen from above in Figure P7.27. If two children, with masses of 15 kg and 20 kg, sit in seats opposite one another, what is the moment of inertia about the rotation axis?

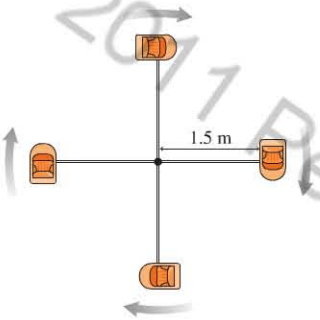


FIGURE P7.27

28. |||| A solid cylinder with a radius of 4.0 cm has the same mass as a solid sphere of radius R. If the cylinder and sphere have the same moment of inertia about their centers, what is the sphere's radius?

29. || A bicycle rim has a diameter of 0.65 m and a moment of inertia, measured about its center, of 0.19 kg $\cdot$ m^2. What is the mass of the rim?

Section 7.5 Using Newton's Second Law for Rotation

30. || The left part of Figure P7.30 shows a bird's-eye view of two identical balls connected by a light rod that rotates about a vertical axis through its center. The right part shows a ball of twice the mass connected to a light rod of half the length, that rotates about its left end. If equal forces are applied as shown in the figure, which of the two rods will have the greater angular acceleration?

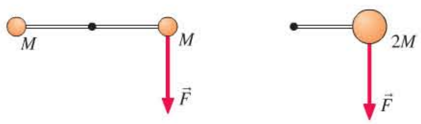


FIGURE P7.30

31. |||| a. What is the moment of inertia of the door in Problem 9?
    b. If you let go of the open door, what is its angular acceleration immediately afterward?

32. | A small grinding wheel has a moment of inertia of 4.0×10^{-5} kg $\cdot$ m^2. What net torque must be applied to the wheel for its angular acceleration to be 150 rad/s^2?

33. || While sitting in a swivel chair, you push against the floor with your heel to make the chair spin. The 7.0 N frictional force is applied at a point 40 cm from the chair's rotation axis, in the direction that causes the greatest angular acceleration. If that angular acceleration is 1.8 rad/s^2, what is the total moment of inertia about the axis of you and the chair?

34. | An object's moment of inertia is 2.0 kg $\cdot$ m^2. Its angular velocity is increasing at the rate of 4.0 rad/s per second. What is the net torque on the object?

35. |||| A 200 g, 20-cm-diameter plastic disk is spun on an axle through its center by an electric motor. What torque must the motor supply to take the disk from 0 to 1800 rpm in 4.0 s?

36. |||| The 2.5 kg object shown in Figure P7.36 has a moment of inertia about the rotation axis of 0.085 kg $\cdot$ m^2. The rotation axis is horizontal. When released, what will be the object's initial angular acceleration?

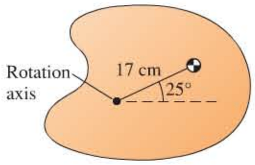


FIGURE P7.36

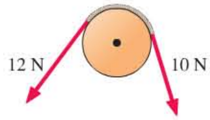


FIGURE P7.37

37. ||| A frictionless pulley, which can be modeled as a 0.80 kg solid cylinder with a 0.30 m radius, has a rope going over it, as shown in Figure P7.37. The tension in the rope is 10 N on one side and 12 N on the other. What is the angular acceleration of the pulley?

38. || If you lift the front wheel of a poorly maintained bicycle off the ground and then start it spinning at 0.72 rev/s, friction in the bearings causes the wheel to stop in just 12 s. If the moment of inertia of the wheel about its axle is 0.30 kg $\cdot$ m^2, what is the magnitude of the frictional torque?

39. ||||| A toy top with a spool of diameter 5.0 cm has a moment of inertia of 3.0×10^{-5} kg $\cdot$ m^2 about its rotation axis. To get the top spinning, its string is pulled with a tension of 0.30 N. How long does it take for the top to complete the first five revolutions? The string is long enough that it is wrapped around the top more than five turns.

40. ||| A 34-cm-diameter potter's wheel with a mass of 20 kg is spinning at 180 rpm. Using her hands, a potter forms a pot, centered on the wheel, with a 14 cm diameter. Her hands apply a net friction force of 1.3 N to the edge of the pot. If the power goes out, so that the wheel's motor no longer provides any torque, how long will it take for the wheel to come to a stop in her hands?

41. |||| A 1.5 kg block and a 2.5 kg block are attached to opposite ends of a light rope. The rope hangs over a solid, frictionless pulley that is 30 cm in diameter and has a mass of 0.75 kg. When the blocks are released, what is the acceleration of the lighter block?

Section 7.6 Rolling Motion

42. ||| A bicycle with 0.80-m-diameter tires is coasting on a level road at 5.6 m/s. A small blue dot has been painted on the tread of the rear tire.
    a. What is the angular speed of the tires?
    b. What is the speed of the blue dot when it is 0.80 m above the road?
    c. What is the speed of the blue dot when it is 0.40 m above the road?

43. |||| A 1.2 g pebble is stuck in a tread of a 0.76-m-diameter automobile tire, held in place by static friction that can be at most 3.6 N. The car starts from rest and gradually accelerates on a straight road. How fast is the car moving when the pebble flies out of the tire tread?

General Problems

44. INT Figure P7.44 shows the angular position-versus-time graph for a particle moving in a circle.
 a. Write a description of the particle's motion.
 b. Draw the angular velocity-versus-time graph.

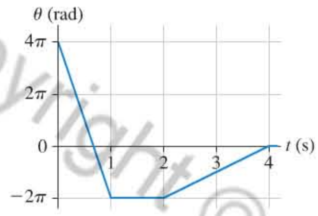


FIGURE P7.44

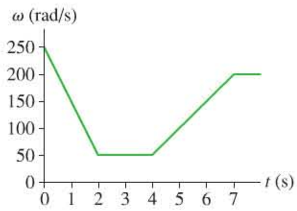


FIGURE P7.45

45. INT The graph in Figure P7.45 shows the angular velocity of the crankshaft in a car. Draw a graph of the angular acceleration versus time. Include appropriate numerical scales on both axes.

46. A computer disk is 8.0 cm in diameter. A reference dot on the edge of the disk is initially located at $\theta = 45°$. The disk accelerates steadily for 0.50 s, reaching 2000 rpm, then coasts at steady angular velocity for another 0.50 s.
 a. What is the tangential acceleration of the reference dot at $t = 0.25$ s?
 b. What is the centripetal acceleration of the reference dot at $t = 0.25$ s?
 c. What is the angular position of the reference dot at $t = 1.0$ s?
 d. What is the speed of the reference dot at $t = 1.0$ s?

47. INT A car with 58-cm-diameter tires accelerates uniformly from rest to 20 m/s in 10 s. How many times does each tire rotate?

48. The cable lifting an elevator is wrapped around a 1.0-m-diameter cylinder that is turned by the elevator's motor. The elevator is moving upward at a speed of 1.6 m/s. It then slows to a stop, while the cylinder turns one complete revolution. How long does it take for the elevator to stop?

49. The 20-cm-diameter disk in Figure P7.49 can rotate on an axle through its center. What is the net torque about the axle?

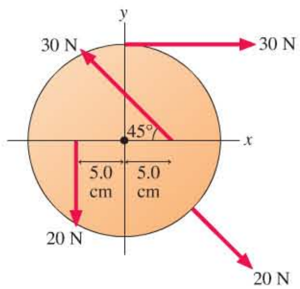


FIGURE P7.49

50. INT A combination lock has a 1.0-cm-diameter knob that is part of the dial you turn to unlock the lock. To turn that knob, you grip it between your thumb and forefinger with a force of 0.60 N as you twist your wrist. Suppose the coefficient of static friction between the knob and your fingers is only 0.12 because some oil accidentally got onto the knob. What is the most torque you can exert on the knob without having it slip between your fingers?

51. BIO A 70 kg man's arm, including the hand, can be modeled as a 75-cm-long uniform cylinder with a mass of 3.5 kg. In raising both his arms, from hanging down to straight up, by how much does he raise his center of gravity?

52. A penny has a mass of 2.5 g and is 1.5 mm thick; a nickel has a mass of 5.7 g and is 1.9 mm thick. If you make a stack of coins on a table, starting with five nickels and finishing with four pennies, how far above the tabletop is the center of gravity of the stack?

53. The machinist's square shown in Figure P7.53 consists of a thin, rectangular blade connected to a rectangular handle.
 a. Determine the x and y coordinates of the center of gravity. Let the lower left corner be $x = 0$, $y = 0$.
 b. Sketch how the tool would hang if it were allowed to freely pivot about the point $x = 0$, $y = 0$.
 c. When hanging from that point, what angle would the long side of the blade make with the vertical?

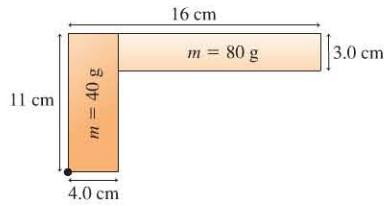


FIGURE P7.53

54. The four masses shown in Figure P7.54 are connected by massless, rigid rods.
 a. Find the coordinates of the center of gravity.
 b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.
 c. Find the moment of inertia about a diagonal axis that passes through masses B and D.

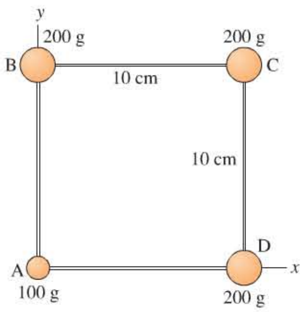


FIGURE P7.54

55. Three 0.10 kg balls are connected by light rods to form an equilateral triangle with a side length of 0.30 m. What is the moment of inertia of this triangle about an axis perpendicular to its plane and passing through one of the balls?

56. The three masses shown in Figure P7.56 are connected by massless, rigid rods.
 a. Find the coordinates of the center of gravity.
 b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.
 c. Find the moment of inertia about an axis that passes through masses B and C.

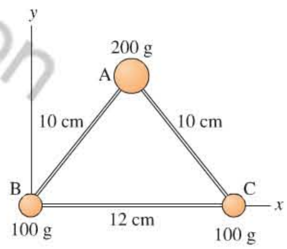


FIGURE P7.56

57. BIO A reasonable estimate of the moment of inertia of an ice skater spinning with her arms at her sides can be made by modeling most of her body as a uniform cylinder. Suppose the skater has a mass of 64 kg. One-eighth of that mass is in her arms, which are 60 cm long and 20 cm from the vertical axis about which she rotates. The rest of her mass is approximately in the form of a 20-cm-radius cylinder.
 a. Estimate the skater's moment of inertia to two significant figures.
 b. If she were to hold her arms outward, rather than at her sides, would her moment of inertia increase, decrease, or remain unchanged? Explain.

58. Starting from rest, a 12-cm-diameter compact disk takes 3.0 s to reach its operating angular velocity of 2000 rpm. Assume that the angular acceleration is constant. The disk's moment of inertia is $2.5 \times 10^{-5}\ \mathrm{kg \cdot m^2}$.
 a. How much torque is applied to the disk?
 b. How many revolutions does it make before reaching full speed?

59. The ropes in Figure P7.59 are each wrapped around a cylinder, and the two cylinders are fastened together. The smaller cylinder has a diameter of 10 cm and a mass of 5.0 kg; the larger cylinder has a diameter of 20 cm and a mass of 20 kg. What is the angular acceleration of the cylinders? Assume that the cylinders turn on a frictionless axle.

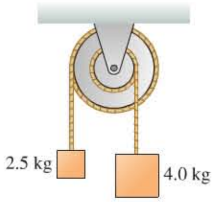


FIGURE P7.59

60. Flywheels are large, massive wheels used to store energy. They can be spun up slowly, then the wheel's energy can be released quickly to accomplish a task that demands high power. An industrial flywheel has a 1.5 m diameter and a mass of 250 kg. A motor spins up the flywheel with a constant torque of 50 N · m. How long does it take the flywheel to reach top angular speed of 1200 rpm?

61. A 1.0 kg ball and a 2.0 kg ball are connected by a 1.0-m-long rigid, massless rod. The rod and balls are rotating clockwise about their center of gravity at 20 rpm. What torque will bring the balls to a halt in 5.0 s?

62. INT A 1.5 kg block is connected by a rope across a 50-cm-diameter, 2.0 kg, frictionless pulley, as shown in Figure P7.62. A constant 10 N tension is applied to the other end of the rope. Starting from rest, how long does it take the block to move 30 cm?

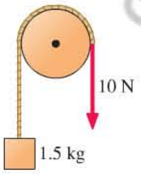


FIGURE P7.62

63. INT The two blocks in Figure P7.63 are connected by a massless rope that passes over a pulley. The pulley is 12 cm in diameter and has a mass of 2.0 kg. As the pulley turns, friction at the axle exerts a torque of magnitude 0.50 N · m. If the blocks are released from rest, how long does it take the 4.0 kg block to reach the floor?

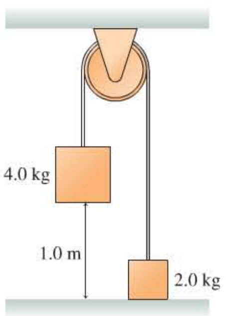


FIGURE P7.63

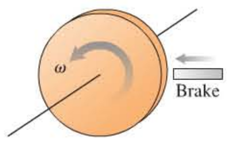


FIGURE P7.64

64. The 2.0 kg, 30-cm-diameter disk in Figure P7.64 is spinning at 300 rpm. How much friction force must the brake apply to the rim to bring the disk to a halt in 3.0 s?

65. INT A tradesman sharpens a knife by pushing it against the rim of a grindstone. The 30-cm-diameter stone is spinning at 200 rpm and has a mass of 28 kg. The coefficient of kinetic friction between the knife and the stone is 0.20. If the stone loses 10% of its speed in 10 s of grinding, what is the force with which the man presses the knife against the stone?

66. BIO The bunchberry flower has the fastest-moving parts ever seen in a plant. Initially, the stamens are held by the petals in a bent position, storing elastic energy like a coiled spring. As the petals release, the tips of the stamens act like medieval catapults, flipping through a 60° angle in just 0.30 ms to launch pollen from the anther sacs at their ends. The human eye just sees a burst of pollen; careful photography (see Figure P7.66a) reveals the details. As shown in Figure P7.66b, we can model a stamen tip as a 1.0-mm-long, 10 μg rigid rod with a 10 μg anther sac at one end and a pivot point at the opposite end. Although oversimplifying, we will assume that the angular acceleration is constant throughout the motion.

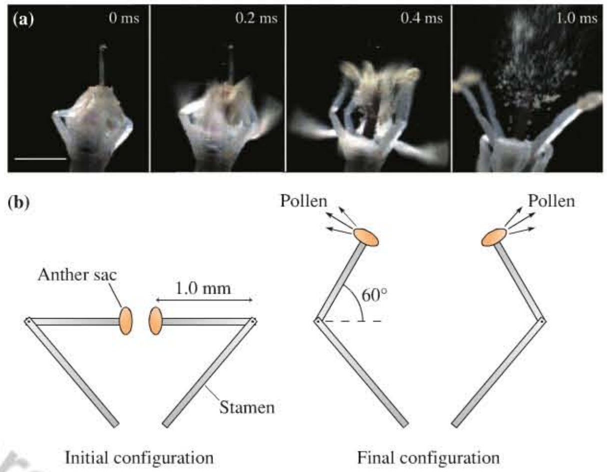


FIGURE P7.66

a. What is the tangential acceleration of the anther sac during the motion?
b. What is the speed of the anther sac as it releases its pollen?
c. How large is the "straightening torque"? Neglect gravitational forces in your calculation.
d. Compute the gravitational torque on the stamen tip (including the anther sac) in its initial orientation. Was it reasonable to neglect the gravitational torque in part c?

Passage Problems

The Illusion of Flight

The grand jeté is a classic ballet maneuver in which a dancer executes a horizontal leap while moving her arms and legs up and then down. At the center of the leap, the arms and legs are gracefully extended, as we see in Figure P7.67a. The goal of the leap is to create the illusion of flight. As discussed in Section 7.3, the center of mass—and hence the center of gravity—of an extended object follows a parabolic trajectory when undergoing projectile motion. But when you watch a dancer leap through the air, you don't watch her center of gravity, you watch her head. If the translational motion of her head is horizontal—not parabolic—this creates the illusion that she is flying through the air, held up by unseen forces.

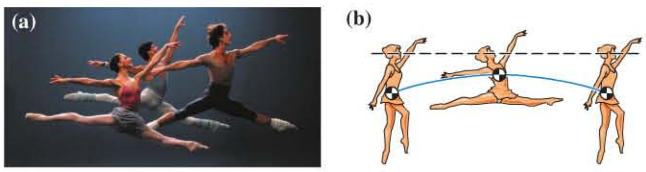


FIGURE P7.67

Figure P7.67b illustrates how the dancer creates this illusion. While in the air, she changes the position of her center of gravity relative to her body by moving her arms and legs up, then down. Her center of gravity moves in a parabolic path, but her head moves in a straight line. It's not flight, but it will appear that way, at least for a moment.

67. | To perform this maneuver, the dancer relies on the fact that the position of her center of gravity
   A. Is near the center of the torso.
   B. Is determined by the positions of her arms and legs.
   C. Moves in a horizontal path.
   D. Is outside of her body.

68. | Suppose you wish to make a vertical leap with the goal of getting your head as high as possible above the ground. At the top of your leap, your arms should be
   A. Held at your sides.
   B. Raised above your head.
   C. Outstretched, away from your body.

69. | When the dancer is in the air, is there a gravitational torque on her? Take the dancer's rotation axis to be through her center of gravity.
   A. Yes, there is a gravitational torque.
   B. No, there is not a gravitational torque.
   C. It depends on the positions of her arms and legs.

70. | In addition to changing her center of gravity, a dancer may change her moment of inertia. Consider her moment of inertia about a vertical axis through the center of her body. When she raises her arms and legs, this
   A. Increases her moment of inertia.
   B. Decreases her moment of inertia.
   C. Does not change her moment of inertia.

STOP TO THINK ANSWERS

Stop to Think 7.1: a. constant (but not zero), b. constant (but not zero), c. constant (but not zero), d. zero, e. zero. The angular velocity ω is constant. Thus the magnitude of the velocity $v = \omega r$ and the centripetal acceleration $a_c = \omega^2 r$ are constant. This also means that the ball's angular acceleration α and tangential acceleration $a_t = \alpha r$ are both zero.

Stop to Think 7.2: $\tau_E > \tau_A = \tau_D > \tau_B > \tau_C$. The perpendicular component in E is larger than 2 N.

Stop to Think 7.3: A. The force acting at the axis exerts no torque. Thus the third force needs to exert an equal but opposite torque to that exerted by the force acting at the rim. Force A, which has twice the magnitude but acts at half the distance from the axis, does so.

Stop to Think 7.4: $\tau_E = \tau_B > \tau_D > \tau_A = \tau_C$. The torques are $\tau_B = \tau_E = 2mgL$, $\tau_D = \frac{3}{2}mgL$, and $\tau_A = \tau_C = mgL$, where L is the length of the rod in B.

Stop to Think 7.5: $I_D > I_A > I_C > I_B$. The moments of inertia are $I_B \approx 0$, $I_C = 2mr^2$, $I_A = 3mr^2$, and $I_D = mr^2 + m(2r)^2 = 5mr^2$.

Stop to Think 7.6: C. The velocity of P is the vector sum of $\vec{v}$ directed to the right and an upward velocity of the same magnitude due to the rotation of the wheel.