16 Superposition and Standing Waves

The didgeridoo is a simple musical instrument played by aboriginal tribes in Australia. It consists of a hollow tube that is open at both ends. How can the player get a wide range of notes out of such a simple device?

LOOKING AHEAD ▸▸

The goal of Chapter 16 is to use the idea of superposition to understand the phenomena of interference and standing waves.

Superposition

Traveling waves can pass through each other. As they do, their displacements add together. This is the **principle of superposition.**

The surface of the water supports multiple waves. It looks like the waves simply stack on top of each other, which, in fact, they do.

Looking Back ◂◂
15.2–15.3 The fundamental properties of traveling waves

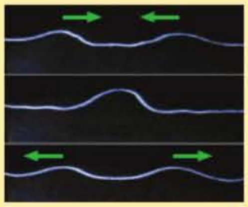
Standing Waves

Traveling waves bounce back and forth between the ends of a string that is clamped at both ends. The superposition of these reflected waves makes the string vibrate up and down. We call this a **standing wave.**

Standing waves occur only as well-defined patterns called **modes,** each with its own distinctive frequency. These are the **resonant modes** of the medium. Some points on the wave, called **nodes**, do not oscillate at all.

Looking Back ◂◂
14.7 The principle of resonance

Constructive and Destructive Interference



Two waves on a string each displace the string upward. Where the two waves overlap, the displacement is twice that of the individual waves. This is **constructive interference.**

Noise-canceling headphones create a sound wave that is inverted from the ambient sound. When the waves are added, they cancel, producing a much smaller wave. This is **destructive interference.**

Music and Speech

Standing waves on the strings of the guitar allow it to produce different musical notes. The frequency is determined by the length, mass, and tension of the string.

A tube can support a standing wave as well—a standing sound wave. We'll see how to calculate the possible standing waves and how these determine the notes a wind instrument can produce.

Your vocal system also depends on standing waves. A vibration of your vocal cords is amplified by the resonances of the tube of your vocal tract. We'll see how these elements work together to make speech.

Looking Back ◂◂
15.4 The nature of sound waves

16.1 The Principle of Superposition

FIGURE 16.1a shows two baseball players, Alan and Bill, at batting practice. Unfortunately, someone has turned the pitching machines so that pitching machine A throws baseballs toward Bill while machine B throws toward Alan. If two baseballs are launched at the same time and with the same speed, they collide at the crossing point and bounce away. Two baseballs cannot occupy the same point of space at the same time.

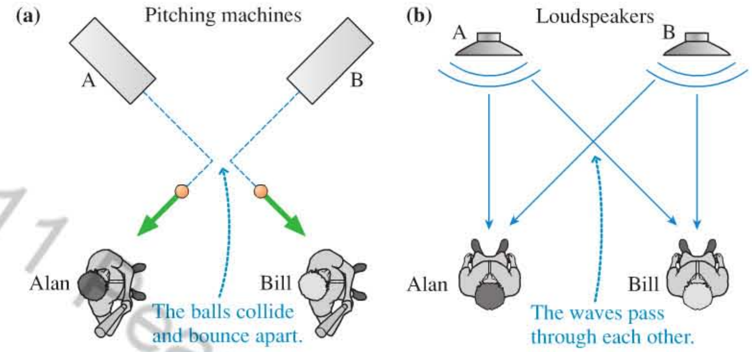


FIGURE 16.1 Two baseballs cannot pass through each other. Two waves can.

FIGURE 16.2 Two wave pulses on a stretched string pass through each other.

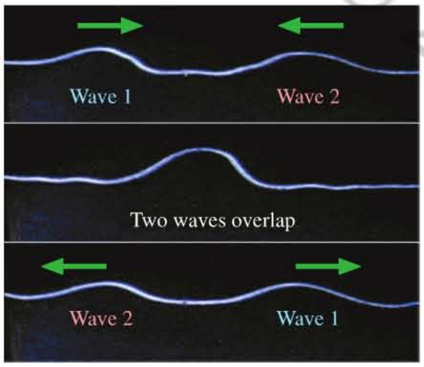
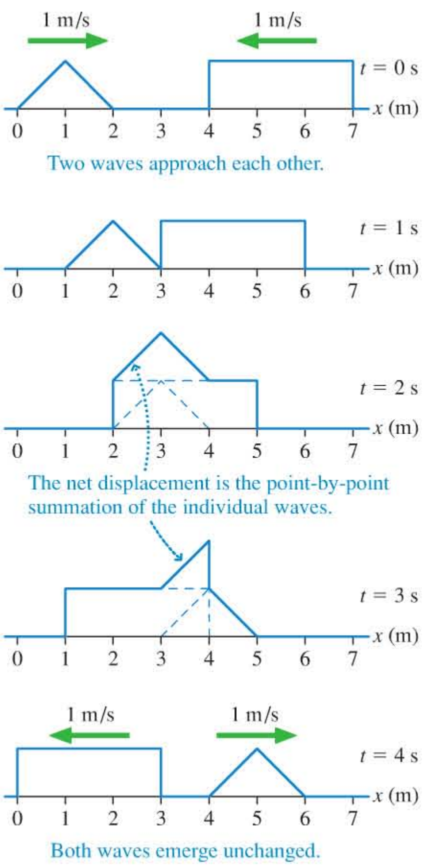


FIGURE 16.3 The superposition of two waves on a string as they pass through each other.



But unlike baseballs, sound waves *can* pass directly through each other. In **FIGURE 16.1b**, Alan and Bill are listening to the stereo system in the locker room after practice. Both hear the music quite well, without distortion or missing sound, so the sound wave that travels from speaker A toward Bill must pass through the wave traveling from speaker B toward Alan, with no effect on either. This is a basic property of waves.

What happens to the medium at a point where two waves are present simultaneously? What is the displacement of the medium at this point? **FIGURE 16.2** shows a sequence of photos of two wave pulses traveling along a stretched string. In the first photo, the waves are approaching each other. In the second, the waves overlap, and the displacement of the string is larger than it was for either of the individual waves. A careful measurement would reveal that the displacement is the sum of the displacements of the two individual waves. In the third frame, the waves have passed through each other and continue on as if nothing had happened.

This result is not limited to stretched strings; the outcome is the same whenever two waves of any type pass through each other. This is known as the *principle of superposition:*

> **Principle of superposition** When two or more waves are *simultaneously* present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.

To use the principle of superposition you must know the displacement that each wave would cause if it traveled through the medium alone. Then you go through the medium *point by point* and add the displacements due to each wave *at that point* to find the net displacement at that point. The outcome will be different at each point in the medium because the displacements are different at each point.

Let's illustrate this principle with an idealized example. **FIGURE 16.3** shows five snapshot graphs taken 1 s apart of two waves traveling at the same speed (1 m/s) in opposite directions along a string. The displacement of each wave is shown as a dotted line. The solid line is the sum *at each point* of the two displacements at that point. This is the displacement that you would actually observe as the two waves pass through each other.

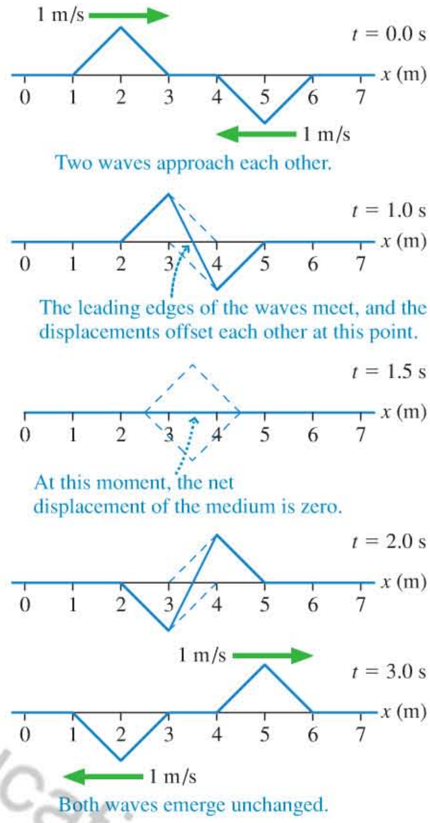
Constructive and Destructive Interference

The superposition of two waves is often called **interference.** The displacements of the waves in Figure 16.3 are both positive, so the total displacement of the medium where they overlap is larger than it would be due to either of the waves separately. We call this **constructive interference.**

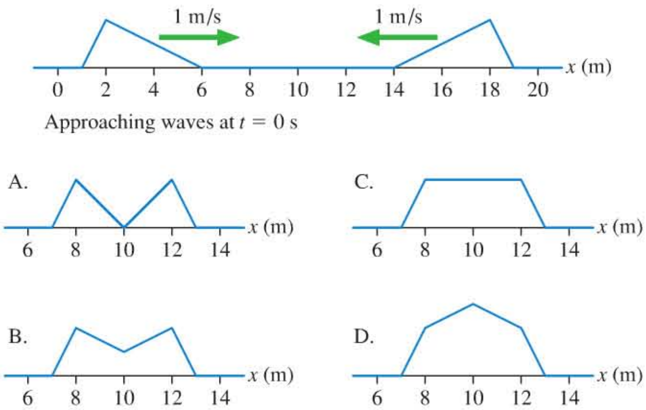
FIGURE 16.4 shows another series of snapshot graphs of two counterpropagating waves, but this time one has a negative displacement. The principle of superposition still applies, but now the displacements are opposite each other. The displacement of the medium where the waves overlap is *less* than it would be due to either of the waves separately. We call this **destructive interference.**

In the series of graphs in Figure 16.4 the displacement of the medium at $x = 3.5$ m is always zero. The positive displacement of the wave traveling to the right and the negative displacement of the wave traveling to the left always exactly cancel at this spot. The complete cancellation at one point of two waves traveling in opposite directions is something we will see again. When the displacements of the waves cancel, where does the energy of the wave go? We know that the waves continue on unchanged after their interaction, so no energy is dissipated. Consider the graph at $t = 1.5$ s. There is no net displacement at any point of the medium at this instant, but the *string is moving rapidly.* The energy of the waves hasn't vanished—it is in the form of the kinetic energy of the medium.

FIGURE 16.4 Two waves with opposite displacements produce destructive interference.



STOP TO THINK 16.1 Two pulses on a string approach each other at speeds of 1 m/s. What is the shape of the string at $t = 6$ s?



▶ **Breaking stones with sound** BIO As we saw in Chapter 15, waves carry energy. The energy of high-intensity ultrasonic waves can be used to break up kidney stones so that they can be cleared from the body, a technique known as *shock wave lithotripsy.* This machine uses two generators, each of which produces ultrasonic waves. The two waves, which enter the body at different points, are directed so that they overlap and produce constructive interference at the position of a stone. This allows the individual waves to have lower intensity, minimizing tissue damage as they pass through the body, while still providing a region of high intensity right where it is needed.

16.2 Standing Waves

When you pluck a guitar string or a rubber band stretched between your fingers, you create waves. But how is this possible? There isn't really anywhere for the waves to go, because the string or the rubber band is held between two fixed ends. **FIGURE 16.5** shows a strobe photograph of waves on a stretched elastic cord. This is a wave, though it may not look like one, because it doesn't "travel" either right or left. Waves that are "trapped" between two boundaries, like those in the photo or on a guitar

FIGURE 16.5 The motion of a standing wave on a string.

string, are what we call *standing waves.* **Individual points on the string oscillate up and down, but the wave itself does not travel.** It is called a **standing wave** because the crests and troughs "stand in place" as it oscillates. As we'll see, a standing wave isn't a totally new kind of wave; it is simply the superposition of two traveling waves moving in opposite directions.

Superposition Creates a Standing Wave

Suppose we have a string on which two sinusoidal waves of equal wavelength and amplitude travel in opposite directions, as in FIGURE 16.6a. When the waves meet, the displacement of the string will be a superposition of these two waves. FIGURE 16.6b shows nine snapshot graphs, at intervals of $\frac{1}{8}T$, of the two waves as they move through each other. The red and orange dots identify particular crests of each of the waves to help you see that the red wave is traveling to the right and the orange wave to the left. At *each point,* the net displacement of the medium is found by adding the red displacement and the orange displacement. The resulting blue wave is the superposition of the two traveling waves.

FIGURE 16.6 Two sinusoidal waves traveling in opposite directions.

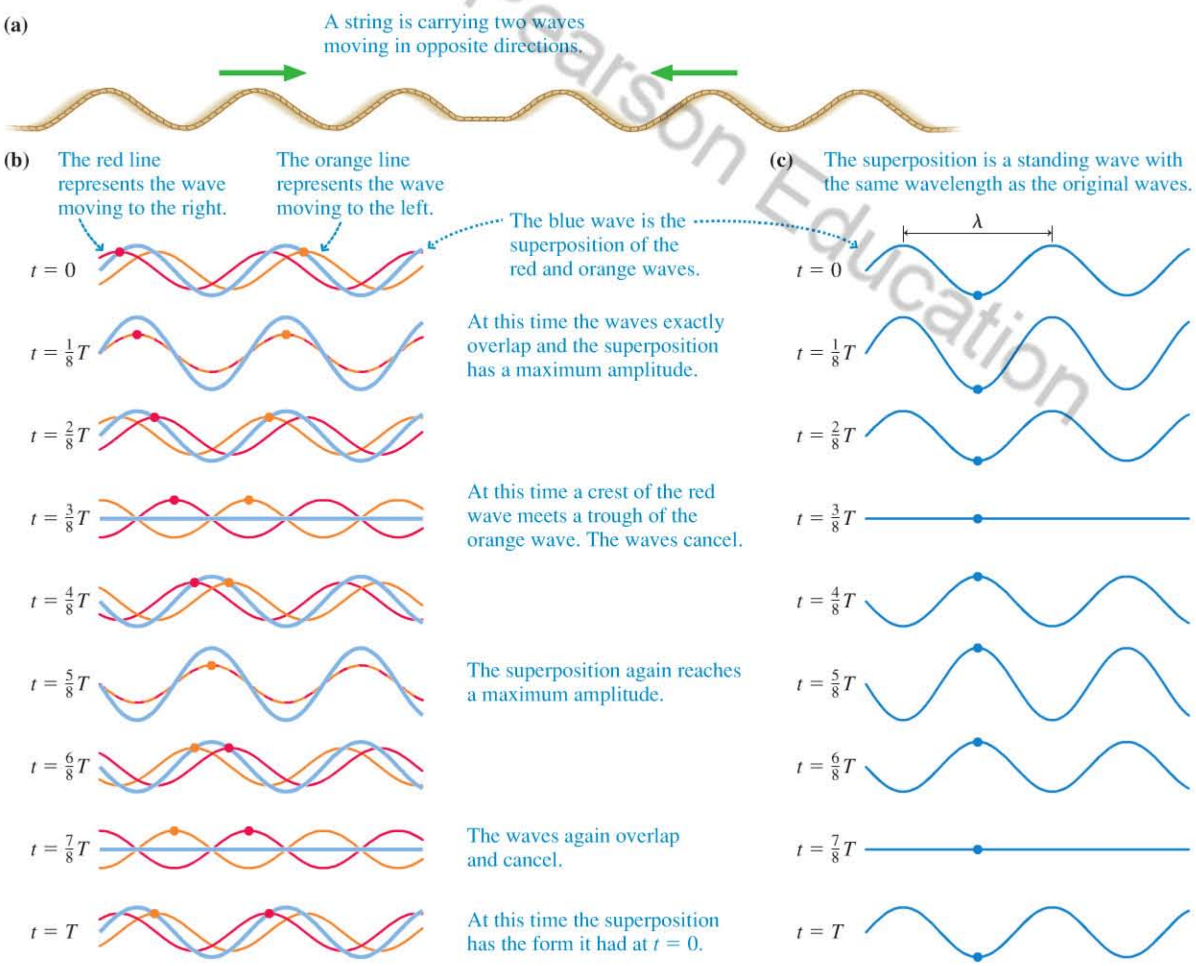


FIGURE 16.6c shows just the superposition of the two waves. This is the wave that you would actually observe in the medium. The blue dot shows that the wave in Figure 16.6c is moving neither right nor left. The superposition of the two counter-propagating traveling waves is a standing wave. Notice that the wavelength of the standing wave, the distance between two crests or two troughs, is the same as the wavelengths of the two traveling waves that combine to produce it.

Nodes and Antinodes

In **FIGURE 16.7** we have superimposed the nine snapshot graphs of Figure 16.6c into a single graphical representation of this standing wave. The graphs at different times overlap, much as the photos of the string at different times in the strobe photograph of Figure 16.5. The motion of individual points on the standing wave is now clearly seen. A striking feature of a standing-wave pattern is points that *never move!* These points, which are spaced $\lambda/2$ apart, are called **nodes.** Halfway between the nodes are points where the particles in the medium oscillate with maximum displacement. These points of maximum amplitude are called **antinodes,** and you can see that they are also spaced $\lambda/2$ apart. This means that **the wavelength of a standing wave is *twice* the distance between successive nodes or successive antinodes.**

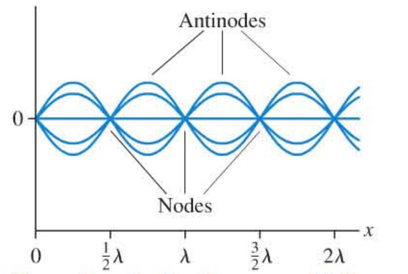
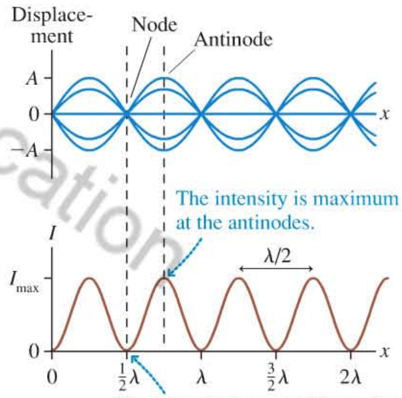


FIGURE 16.7 Superimposing multiple snapshot graphs of a standing wave clearly shows the nodes and antinodes.

It seems surprising and counterintuitive that some particles in the medium have no motion at all. This happens for the same reason we saw in Figure 16.4: The two waves exactly offset each other at that point. Look carefully at the two traveling waves in Figure 16.6b. You will see that the nodes occur at points where at *every instant* of time the displacements of the two traveling waves have equal magnitudes but *opposite signs.* Thus the superposition of the displacements at these points is always zero—they are points of destructive interference. The antinodes have large displacements. They correspond to points where the two displacements have equal magnitudes and the *same sign* at all times. Constructive interference at these points gives a displacement twice that of each individual wave.

The intensity of a wave is largest at points where it oscillates with maximum amplitude. **FIGURE 16.8** shows that the points of maximum intensity along the standing wave occur at the antinodes; the intensity is zero at the nodes. For a standing sound wave, the loudness varies from zero (no sound) at the nodes to a maximum at the antinodes and then back to zero. The key idea is that **the intensity is maximum at points of constructive interference and zero at points of destructive interference.**

FIGURE 16.8 Intensity of a standing wave.



EXAMPLE 16.1 Setting up a standing wave

Two children hold an elastic cord at each end. Each child shakes her end of the cord 2.0 times per second, sending waves at 3.0 m/s toward the middle, where the two waves combine to create a standing wave. What is the distance between adjacent nodes?

SOLVE The distance between adjacent nodes is $\lambda/2$. The wavelength, frequency, and speed are related as $v = f\lambda$, as we saw in Chapter 15, so the wavelength is

$$\lambda = \frac{v}{f} = \frac{3.0\text{ m/s}}{2.0\text{ Hz}} = 1.5\text{ m}$$

The distance between adjacent nodes is $\lambda/2$ and thus is 0.75 m.

16.3 Standing Waves on a String

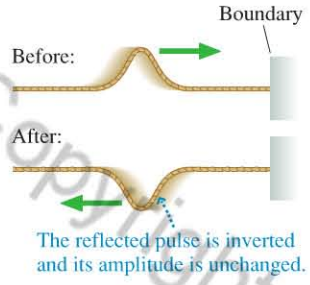
The oscillation of a guitar string is a standing wave. A standing wave is naturally produced on a string when both ends are fixed (i.e., tied down), as in the case of a guitar string or the string in the photo of Figure 16.5. We also know that a standing wave is produced when there are two counterpropagating traveling waves. But you don't shake both ends of a guitar string to produce the standing wave! How do we actually get two traveling waves on a string with both ends fixed? Before we can answer this question, we need a brief explanation of what happens when a traveling wave encounters a boundary or a discontinuity.

ActivPhysics ONLINE 10.4, 10.5, 10.6

Reflections

We know that light reflects from mirrors; it can also reflect from the surface of a pond or from a pane of glass. As we saw in Chapter 15, sound waves reflect as well;

FIGURE 16.9 A wave reflects when it encounters a boundary.

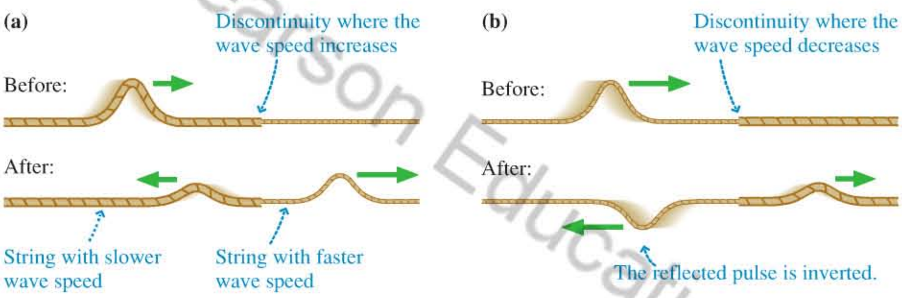


that's how an echo is produced. To understand reflections, we'll look at waves on a string, but the results will be completely general and can be applied to other waves as well.

Suppose we have a string that is attached to a wall or other fixed support, as in FIGURE 16.9. The wall is what we will call a *boundary*—it's the end of the medium. When the pulse reaches this boundary, it reflects, moving away from the wall. *All* the wave's energy is reflected; hence **the amplitude of a wave reflected from a boundary is unchanged.** Figure 16.9 shows that the amplitude doesn't change when the pulse reflects, but the pulse is inverted.

Waves also reflect from what we will call a *discontinuity,* a point where there is a change in the properties of the medium. FIGURE 16.10a shows a discontinuity where a string with a large linear density connects with a string with a small linear density. The tension is the same in both strings, so the wave speed is slower on the left, faster on the right. Whenever a wave encounters a discontinuity, some of the wave's energy is *transmitted* forward and some is reflected. Because energy must be conserved, both the transmitted and the reflected pulses have a smaller amplitude than the initial pulse in this case.

FIGURE 16.10 The reflection of a wave at a discontinuity.



In FIGURE 16.10b, an incident wave encounters a discontinuity at which the wave speed decreases. Once again, some of the wave's energy is transmitted and some is reflected.

In Figure 16.10a, the reflected pulse is right-side up. The string on the right is light and provides little resistance, so the junction moves up and down as the pulse passes. This motion of the string is like the original "snap" of the string that started the pulse, so the reflected pulse has the same orientation as the original pulse. In Figure 16.10b, the string on the right is more massive, so it looks more like the fixed boundary in Figure 16.9 and the reflected pulse is again inverted.

TRY IT YOURSELF

Through the glass darkly A piece of window glass is a discontinuity to a light wave, so it both transmits and reflects light. To verify this, look at the windows in a brightly lit room at night. The small percentage of the interior light that reflects from windows is more intense than the light coming in from outside, so reflection dominates and the windows show a mirror-like reflection of the room. Now turn out the lights. With no more reflected interior light you will be able to see the transmitted light from outside.

Creating a Standing Wave

Now that we understand reflections, let's create a standing wave. FIGURE 16.11 shows a string of length L that is tied at $x = 0$ and $x = L$. This string has *two* boundaries where reflections can occur. If you wiggle the string in the middle, sinusoidal waves travel outward in both directions and soon reach the boundaries, where they reflect. The reflections at the ends of the string cause two waves of *equal amplitude and wavelength* to travel in opposite directions along the string. As we've just seen, these are the conditions that cause a standing wave!

What kind of standing waves might develop on the string? There are two conditions that must be met:

- Because the string is fixed at the ends, the displacements at $x = 0$ and $x = L$ must be zero at all times. Stated another way, we require nodes at both ends of the string.
- We know that standing waves have a spacing of $\lambda/2$ between nodes. This means that the nodes must be equally spaced.

FIGURE 16.11 Reflections at the two boundaries cause a standing wave on the string.

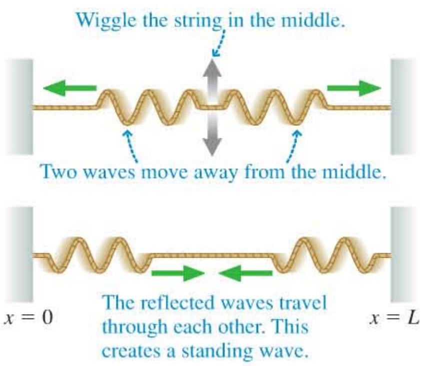


FIGURE 16.12 shows the first three possible waves that meet these conditions. These are called the standing-wave **modes** of the string. To help quantify the possible waves, we can assign a **mode number** m to each. The first wave in Figure 16.12, with a node at each end, has mode number $m = 1$. The next wave is $m = 2$, and so on.

NOTE ► Figure 16.12 shows only the first three modes, for $m = 1$, $m = 2$, and $m = 3$. But there are many more modes, for all possible values of m. ◄

The distance between adjacent nodes is $\lambda/2$, so the different modes have different wavelengths. For the first mode in Figure 16.12, the distance between nodes is the length of the string, so we can write

$$\lambda_1 = 2L$$

The subscript identifies the mode number; in this case $m = 1$. For $m = 2$, the distance between nodes is $L/2$; this means that $\lambda_2 = L$. Generally, for any mode m the wavelength is given by the following equation:

$$\lambda_m = \frac{2L}{m} \qquad m = 1, 2, 3, 4, \ldots \tag{16.1}$$

Wavelengths of standing-wave modes of a string of length L

These are the only possible wavelengths for standing waves on the string. **A standing wave can exist on the string *only* if its wavelength is one of the values given by Equation 16.1.**

NOTE ► Other wavelengths, which would be perfectly acceptable wavelengths for a traveling wave, cannot exist as a *standing* wave of length L because they do not meet the constraint of having a node at each end of the string. ◄

If standing waves are possible only for certain wavelengths, then only specific oscillation frequencies are allowed. Because $\lambda f = v$ for a sinusoidal wave, the oscillation frequency corresponding to wavelength λ_m is

$$f_m = \frac{v}{\lambda_m} = \frac{v}{2L/m} = m\left(\frac{v}{2L}\right) \qquad m = 1, 2, 3, 4, \ldots \tag{16.2}$$

Frequencies of standing-wave modes of a string of length L

FIGURE 16.13 shows the first three modes with their wavelengths and frequencies. You can see that **the mode number m is equal to the number of antinodes of the standing wave.** You can therefore tell a string's mode of oscillation by counting the number of antinodes (*not* the number of nodes).

In Chapter 14, we looked at the concept of *resonance*. A mass on a spring has a certain frequency at which it "wants" to oscillate. If the system is driven at its resonance frequency, it will develop a large amplitude of oscillation. A stretched string will support standing waves, meaning it has a series of frequencies at which it "wants" to oscillate: the frequencies of the different standing-wave modes. We can call these **resonant modes,** or more simply, **resonances.** A small oscillation of a stretched string at a frequency near one of its resonant modes will cause it to develop a standing wave with a large amplitude. **FIGURE 16.14** shows photographs of the first four standing-wave modes on a string, corresponding to four different driving frequencies.

NOTE ► When we draw standing-wave modes, as in Figure 16.13, we usually show only the *envelope* of the wave, the greatest extent of the motion of the string. The string's motion is actually continuous and goes through all intermediate positions as well, as we see from the time-exposure photographs of standing waves in Figure 16.14. ◄

FIGURE 16.12 The first three possible standing waves on a string of length L.

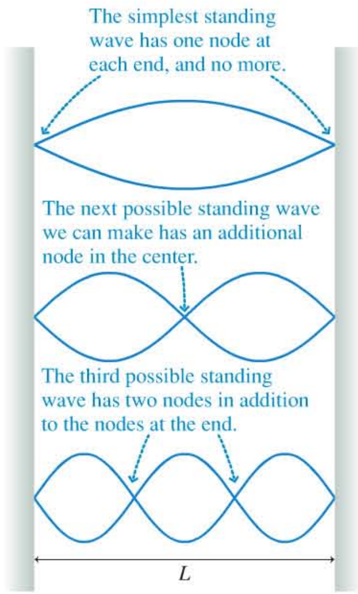


FIGURE 16.13 Possible standing waves of a string fixed at both ends.

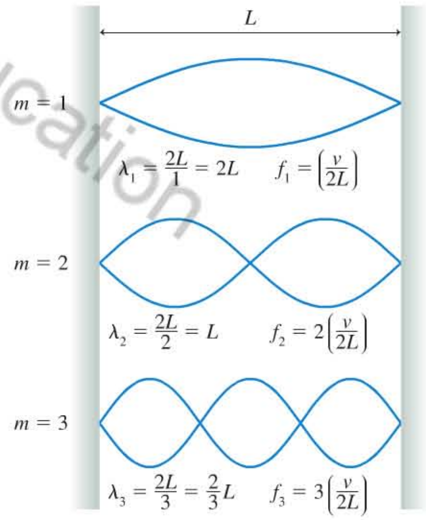
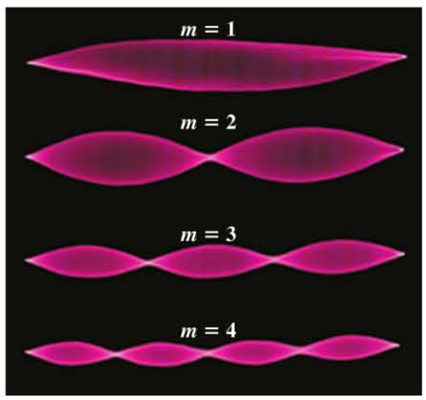


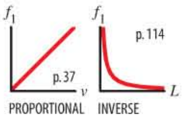
FIGURE 16.14 Resonant modes of a stretched string.



The Fundamental and the Higher Harmonics

The sequence of possible frequencies for a standing wave on a string has an interesting pattern that is worth exploring. The first mode has frequency

$$f_1 = \frac{v}{2L} \tag{16.3}$$



We call this the **fundamental frequency** of the string. All of the other modes have frequencies that are multiples of this fundamental frequency. We can rewrite Equation 16.2 in terms of the fundamental frequency as

$$f_m = mf_1 \qquad m = 1, 2, 3, 4, \ldots \tag{16.4}$$

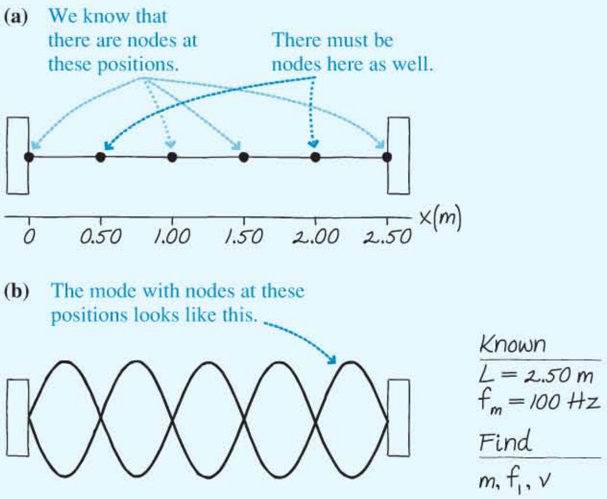
The allowed standing-wave frequencies are all integer multiples of the fundamental frequency. This sequence of possible frequencies is called a set of **harmonics.** The fundamental frequency f_1 is also known as the *first harmonic,* the $m = 2$ wave at frequency f_2 is called the *second harmonic,* the $m = 3$ wave is called the *third harmonic,* and so on. The frequencies above the fundamental frequency, the harmonics with $m = 2, 3, 4, \ldots$, are referred to as the **higher harmonics.**

EXAMPLE 16.2 Identifying harmonics on a string

A 2.50-m-long string vibrates as a 100 Hz standing wave with nodes at 1.00 m and 1.50 m from one end of the string and at no points in between these two. Which harmonic is this? What is the string's fundamental frequency? And what is the speed of the traveling waves on the string?

PREPARE We begin with the visual overview in **FIGURE 16.15**, in which we sketch this particular standing wave and note the known and unknown quantities. We set up an x-axis with one end of the string at $x = 0$ m and the other end at $x = 2.50$ m. The ends of the string are nodes, and there are nodes at 1.00 m and 1.50 m as well, with no nodes in between. We know that standing-wave nodes are equally spaced, so there must be other nodes on the string, as shown in Figure 16.15a. Figure 16.15b is a sketch of the standing-wave mode with this node structure.

FIGURE 16.15 A visual overview of the string.



SOLVE We count the number of antinodes of the standing wave to deduce the mode number; this is mode $m = 5$. This is the fifth harmonic. The frequencies of the harmonics are given by $f_m = mf_1$, so the fundamental frequency is

$$f_1 = \frac{f_5}{5} = \frac{100\text{ Hz}}{5} = 20\text{ Hz}$$

The wavelength of the fundamental mode is $\lambda_1 = 2L = 2(2.50\text{ m}) = 5.00$ m, so can find the wave speed using the fundamental relationship for sinusoidal waves:

$$v = f_1\lambda_1 = (20\text{ Hz})(5.00\text{ m}) = 100\text{ m/s}$$

ASSESS We can calculate the speed of the wave using any possible mode, which gives us a way to check our work. The distance between successive nodes is $\lambda/2$. Figure 16.15 shows that the nodes are spaced by 0.50 m, so the wavelength of the $m = 5$ mode is 1.00 m. The frequency of this mode is 100 Hz, so we calculate

$$v = f_5\lambda_5 = (100\text{ Hz})(1.00\text{ m}) = 100\text{ m/s}$$

This is the same speed that we calculated earlier, which gives us confidence in our results.

Stringed Musical Instruments

Think about stringed musical instruments, such as the guitar, the piano, and the violin. These instruments all have strings that are fixed at both ends and tightened to create tension. A disturbance is generated on the string by plucking, striking, or bowing. Regardless of how it is generated, the disturbance creates standing waves on the string. Understanding the sound of a stringed musical instrument means understanding standing waves.

In Chapter 15, we saw that the speed of a wave on a stretched string depended on T_s, the tension in the string, and μ, the linear density, as $v = \sqrt{T_s/\mu}$. Combining this with Equation 16.3, we find that the fundamental frequency of a stretched string is

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T_s}{\mu}} \tag{16.5}$$

When you pluck or bow a string, you initially excite a wide range of frequencies. However, resonance sees to it that the only frequencies to persist are those of the possible standing waves. The string will support a wave of the fundamental frequency f_1 plus waves of all the higher harmonics f_2, f_3, f_4, and so on. Your brain interprets the sound as a musical note of frequency f_1; the higher harmonics determine the *tone quality*, a concept we will explore later in the chapter.

NOTE ► In Equation 16.5, v is the wave speed *on the string,* not the speed of sound in air. ◄

For instruments like the guitar or the violin, the strings are all the same length and under approximately the same tension. Were that not the case, the neck of the instrument would tend to twist toward the side of higher tension. The strings have different frequencies because they differ in linear density. The lower-pitched strings are "fat" while the higher-pitched strings are "skinny." This difference changes the frequency by changing the wave speed. Small adjustments are then made in the tension to bring each string to the exact desired frequency.

Standing waves on a bridge This photo shows the Tacoma Narrows suspension bridge on the day in 1940 when it experienced a catastrophic oscillation that led to its collapse. Aerodynamic forces caused the amplitude of a particular resonant mode of the bridge to increase dramatically until the bridge failed. In this photo, the red line shows the original line of the deck of the bridge. You can clearly see the large amplitude of the oscillation and the node at the center of the span.

CONCEPTUAL EXAMPLE 16.3 Tuning and playing a guitar

A guitar has strings of a fixed length. Plucking a string makes a particular musical note. A player can make other notes by pressing the string against frets, metal bars on the neck of the guitar, as shown in **FIGURE 16.16**. The fret becomes the new end of the string, making the effective length shorter.

FIGURE 16.16 Guitar frets.

a. A guitar player plucks a string to play a note. He then presses down on a fret to make the string shorter. Does the new note have a higher or lower frequency?

b. The frequency of one string is too low. (Musically, we say the note is "flat.") How must the tension be adjusted to bring the string to the right frequency?

REASON

a. The fundamental frequency, the note we hear, is $f_1 = (1/2L)\sqrt{T_s/\mu}$. Because f_1 is inversely proportional to L, decreasing the string length increases the frequency.

b. Because f is proportional to the square root of T_s, the player must increase the tension to increase the fundamental frequency.

ASSESS If you watch someone play a guitar, you can see that he or she plays higher notes by moving the fingers to shorten the strings.

EXAMPLE 16.4 Setting the tension in a guitar string

The fifth string on a guitar plays the musical note A, at a frequency of 110 Hz. On a typical guitar, this string is stretched between two fixed points 0.640 m apart, and this length of string has a mass of 2.86 g. What is the tension in the string?

PREPARE Strings sound at their fundamental frequency, so 110 Hz is f_1.

SOLVE The linear density of the string is

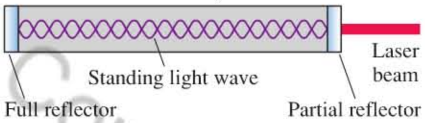
$$\mu = \frac{m}{L} = \frac{2.86 \times 10^{-3}\ \text{kg}}{0.640\ \text{m}} = 4.47 \times 10^{-3}\ \text{kg/m}$$

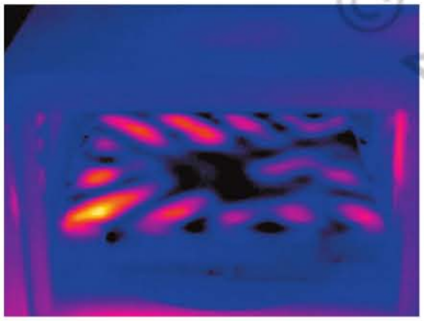
We can rearrange Equation 16.5 for the fundamental frequency to solve for the tension in terms of the other variables:

$$T_s = (2Lf_1)^2\mu = [2(0.640\ \text{m})(110\ \text{Hz})]^2\,(4.47 \times 10^{-3}\ \text{kg/m})$$
$$= 88.6\ \text{N}$$

ASSESS If you have ever strummed a guitar, you know that the tension is quite large, so this result seems reasonable. If each of the guitar's six strings has approximately the same tension, the total force on the neck of the guitar is a bit more than 500 N.

FIGURE 16.17 A laser contains a standing light wave between two parallel mirrors.





Microwave modes A microwave oven uses a type of electromagnetic wave—microwaves, with a wavelength of about 12 cm—to heat food. The inside walls of a microwave oven are reflective to microwaves, so we have the correct conditions to set up a standing wave. This isn't a good thing! A standing wave has high intensity at the antinodes and low intensity at the nodes, so your oven has hot spots and cold spots, as we see in this thermal image showing the interior of an oven with a thin layer of water that has been "cooked" for a short time. A turntable in a microwave oven keeps the food moving so that no part of your dinner remains at a node or an antinode.

Standing Electromagnetic Waves

The standing-wave descriptions we've found for a vibrating string are valid for any transverse wave, including an electromagnetic wave. For example, standing light waves can be established between two parallel mirrors that reflect the light back and forth. The mirrors are boundaries, analogous to the boundaries at the ends of a string. In fact, this is exactly how a laser works. The two facing mirrors in **FIGURE 16.17** form what is called a *laser cavity.*

Because the mirrors act exactly like the points to which a string is tied, the light wave must have a node at the surface of each mirror. (To allow some of the light to escape the laser cavity and form the *laser beam,* one of the mirrors lets some of the light through. This doesn't affect the node.)

EXAMPLE 16.5 Finding the mode number for a laser

A helium-neon laser emits light of wavelength $\lambda = 633$ nm. A typical cavity for such a laser is 15.0 cm long. What is the mode number of the standing wave in this cavity?

PREPARE Because a light wave is a transverse wave, Equation 16.1 for λ_m applies to a laser as well as a vibrating string.

SOLVE The standing light wave in a laser cavity has a mode number m that is roughly

$$m = \frac{2L}{\lambda} = \frac{2 \times 0.150\ \text{m}}{633 \times 10^{-9}\ \text{m}} = 474{,}000$$

ASSESS The wavelength of light is very short, so we'd expect the nodes to be closely spaced. A high mode number seems reasonable.

STOP TO THINK 16.2 A standing wave on a string is shown. Which of the modes shown below (on the same string) has twice the frequency of the original wave?

16.4 Standing Sound Waves

Wind instruments like flutes, trumpets, and didgeridoos work very differently from stringed instruments. The player blows air into one end of a tube, producing standing waves of sound that make the notes we hear. In this section, we will look at the properties of such standing sound waves.

Recall that a sound wave is a longitudinal pressure wave. The air molecules oscillate back and forth parallel to the direction in which the wave is traveling, creating *compressions* (regions of higher pressure) and *rarefactions* (regions of lower pressure). Consider a sound wave confined to a long, narrow column of air, such as the air in a tube. A wave traveling down the tube eventually reaches the end, where it encounters the atmospheric pressure of the surrounding environment. This is a discontinuity, much like the small rope meeting the big rope in Figure 16.10b. Part of the wave's energy is transmitted out into the environment, allowing you to hear the sound, and part is reflected back into the tube. Reflections at both ends of the tube create waves traveling both directions inside the tube, and their superposition, like that of the reflecting waves on a string, is a standing wave.

We start by looking at a sound wave in a tube open at both ends. What kind of standing waves can exist in such a tube? Because the ends of the tube are open to the atmosphere, the pressure at the ends is fixed at atmospheric pressure and cannot vary. This is analogous to a stretched string that is fixed at the end. As a result, **the open end of a column of air must be a node of the pressure wave.**

FIGURE 16.18a shows a column of air open at both ends. We call this an *open-open tube*. Whereas the antinodes of a standing wave on a string are points where the string oscillates with maximum displacement, the antinodes of a standing sound wave are where the pressure has the largest variation, creating, alternately, the maximum compression and the maximum rarefaction. In Figure 16.18a, the air molecules squeeze together on the left side of the tube. Then, in **FIGURE 16.18b**, half a cycle later, the air molecules squeeze together on the right side. The varying density creates a variation in pressure across the tube, as the graphs show. **FIGURE 16.18c** combines the information of Figures 16.18a and 16.18b into a graph of the pressure of the standing sound wave in the tube.

As the standing wave oscillates, the air molecules "slosh" back and forth along the tube with the wave frequency, alternately pushing together (maximum pressure at the antinode) and pulling apart (minimum pressure at the antinode). This makes sense, because sound is a longitudinal wave in which the air molecules oscillate parallel to the tube.

NOTE ► The variation in pressure from atmospheric pressure in a real standing sound wave is much smaller than Figure 16.18 implies. When we display graphs of the pressure in sound waves, we won't generally graph the pressure p. Instead, we will graph Δp, the variation from atmospheric pressure. ◄

Many musical instruments, such as a flute, can be modeled as open-open tubes. The flutist blows across one end to create a standing wave inside the tube, and a note of this frequency is emitted from both ends of the flute. The possible standing waves in tubes, like standing waves on strings, are resonances of the system. A gentle puff of air across the mouthpiece of a flute can cause large standing waves at these resonant frequencies.

Other instruments work differently from the flute. A trumpet or a clarinet is a column of air open at the bell end but *closed* by the player's lips at the mouthpiece. To be complete in our treatment of sound waves in tubes, we need to consider tubes that are closed at one or both ends. At a closed end, the air molecules can alternately rush toward the wall, creating a compression, and then rush away from the wall, leaving a rarefaction. Thus **a closed end of an air column is an antinode of pressure.**

FIGURE 16.19 shows graphs of the first three standing-wave modes of a tube open at both ends (an *open-open tube*), a tube closed at both ends (a *closed-closed tube*), and a tube open at one end but closed at the other (an *open-closed tube*), all with the same length L. These are graphs of the pressure wave, with a node at open ends and an antinode at closed ends. The standing wave in the closed-closed tube looks like the wave in the open-open tube except that the positions of the nodes and antinodes are interchanged. In both cases there are m half-wavelength segments between the

FIGURE 16.18 The $m = 2$ standing sound wave inside an open-open column of air.

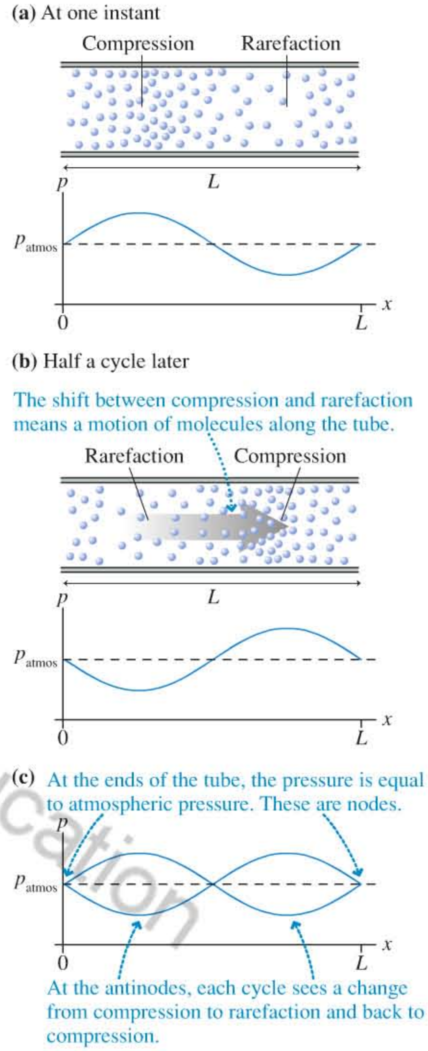
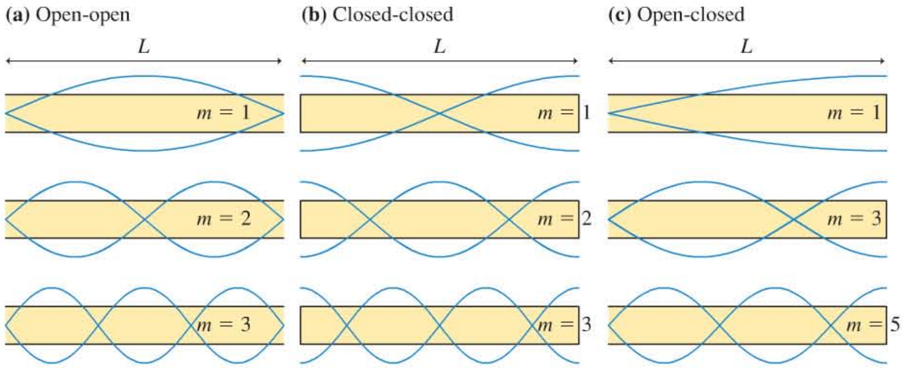


FIGURE 16.19 The first three standing sound wave modes in columns of air with different ends. These graphs show the pressure variation in the tube.



ends; thus the wavelengths and frequencies of an open-open tube and a closed-closed tube are the same as those of a string tied at both ends:

$$\lambda_m = \frac{2L}{m} \qquad f_m = m\left(\frac{v}{2L}\right) = mf_1 \qquad m = 1, 2, 3, 4, \ldots \tag{16.6}$$

Wavelengths and frequencies of standing sound wave modes in an open-open or closed-closed tube

The open-closed tube is different, as we can see from Figure 16.19. The $m = 1$ mode has a node at one and an antinode at the other, and so has only one-quarter of a wavelength in a tube of length L. The $m = 1$ wavelength is $\lambda_1 = 4L$, twice the $m = 1$ wavelength of an open-open or a closed-closed tube. Consequently, **the fundamental frequency of an open-closed tube is half that of an open-open or a closed-closed tube of the same length.**

The wavelength of the next mode of the open-closed tube is $4L/3$. Because this is 1/3 of λ_1, we assign $m = 3$ to this mode. The wavelength of the subsequent mode is $4L/5$, so this is $m = 5$. In other words, an open-closed tube allows only odd-numbered modes. Consequently, the possible wavelengths and frequencies are

$$\lambda_m = \frac{4L}{m} \qquad f_m = m\left(\frac{v}{4L}\right) = mf_1 \qquad m = 1, 3, 5, 7, \ldots \tag{16.7}$$

Wavelengths and frequencies of standing sound wave modes in an open-closed tube

Fiery interference In this apparatus, a speaker at one end of the metal tube emits a sinusoidal wave. The wave reflects from the other end, which is closed, to make a counter-propagating wave and set up a standing sound wave in the tube. The tube is filled with propane gas that exits through small holes on top. The burning propane allows us to easily discern the nodes and the antinodes of the standing sound wave. An exciting demonstration—but one you shouldn't try yourself!

NOTE ► Because sound is a pressure wave, the graphs of Figure 16.19 are *not* "pictures" of the wave as they are for a string wave. The graphs show the pressure variation versus position x. The tube itself is shown merely to indicate the location of the open and closed ends, but the diameter of the tube is *not* related to the amplitude of the wave. ◄

We are now in a position to suggest the following problem-solving strategy, not just for sound waves, but for any standing wave.

PROBLEM-SOLVING STRATEGY 16.1 Standing waves

MP™

PREPARE

- For sound waves, determine what sort of pipe or tube you have: open-open, closed-closed, or open-closed.
- For string or light waves, the ends will be fixed points.
- Determine known values: length of the tube or string, frequency, wavelength, positions of nodes or antinodes.
- It may be useful to sketch a visual overview, including a picture of the relevant mode.

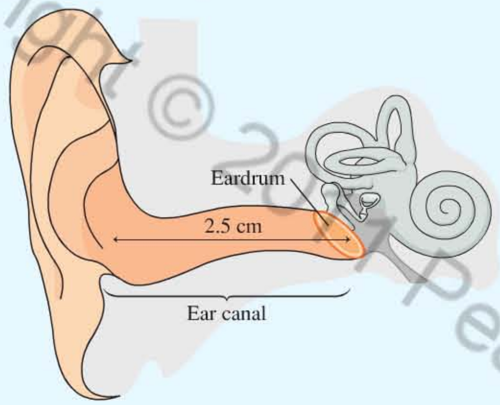
SOLVE For a string, the allowed frequencies and wavelengths are given by Equations 16.1 and 16.2. For sound waves in an open-open or closed-closed tube, the allowed frequencies and wavelengths are given by Equation 16.6; for an open-closed tube, by Equation 16.7.

ASSESS Does your final answer seem reasonable? Is there another way to check on your results? For example, the frequency times the wavelength for any mode should equal the wave speed—you can check to see that it does.

EXAMPLE 16.6 Resonances of the ear canal

The eardrum, which transmits vibrations to the sensory organs of your ear, lies at the end of the ear canal. As FIGURE 16.20 shows, the ear canal in adults is about 2.5 cm in length. What frequency standing waves can occur within the ear canal that are within the range of human hearing? The speed of sound in the warm air of the ear canal is 350 m/s.

FIGURE 16.20 The anatomy of the ear.



PREPARE We proceed according to the steps in Problem-Solving Strategy 16.1. We can treat the ear canal as an open-closed tube: open to the atmosphere at the external end, closed by the eardrum at the other end. The possible standing-wave modes appear as in Figure 16.19c. The length of the tube is 2.5 cm.

SOLVE Equation 16.7 gives the allowed frequencies in an open-closed tube. We are looking for the frequencies in the range 20 Hz–20,000 Hz. The fundamental frequency is

$$f_1 = \frac{v}{4L} = \frac{350\text{ m/s}}{4(0.025\text{ m})} = 3500\text{ Hz}$$

The higher harmonics are odd multiples of this frequency:

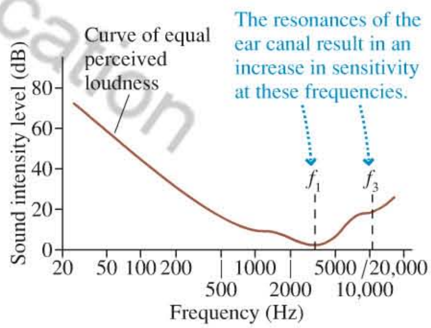
$$f_3 = 3(3500\text{ Hz}) = 10{,}500\text{ Hz}$$
$$f_5 = 5(3500\text{ Hz}) = 17{,}500\text{ Hz}$$

These three modes lie within the range of human hearing; higher modes are greater than 20,000 Hz.

ASSESS The ear canal is short, so we expect the resonant frequencies to be high; our answers seem reasonable.

How important are the resonances of the ear canal calculated in Example 16.6? In Chapter 15, you learned about the decibel scale for measuring loudness of sound. In fact, your ears have varying sensitivity to sounds of different frequencies. FIGURE 16.21 shows a curve of *equal perceived loudness,* the sound intensity level (in dB) required to give the *impression* of equal loudness for sinusoidal waves of the noted frequency. Lower values mean that your ear is more sensitive at that frequency. In general, the curve decreases to about 1000 Hz, the frequency at which your hearing is most acute, then slowly rises at higher frequencies. However, this general trend is punctuated by two dips in the curve, showing two frequencies at which a quieter sound produces the same perceived loudness. As you can see, these two dips correspond to the resonances f_1 and f_3 of the ear canal. Incoming sounds at these frequencies produce a larger oscillation, resulting in an increased sensitivity to these frequencies.

FIGURE 16.21 A curve of equal perceived loudness.



Wind Instruments

With a wind instrument, blowing into the mouthpiece creates a standing sound wave inside a tube of air. The player changes the notes by using her fingers to cover holes or open valves, changing the effective length of the tube. The first open hole becomes a node because the tube is open to the atmosphere at that point. The fact that the holes are on the side, rather than literally at the end, makes very little difference. The length of the tube determines the standing-wave resonances, and thus the musical note that the instrument produces.

Many wind instruments have a "buzzer" at one end of the tube, such as a vibrating reed on a saxophone or clarinet, or the musician's vibrating lips on a trumpet or trombone. Buzzers like these generate a continuous range of frequencies rather than single notes, which is why they sound like a "squawk" if you play on just the mouthpiece without the rest of the instrument. When the buzzer is connected to the body of the instrument, most of those frequencies cause little response. But the frequencies from the buzzer that match the resonant frequencies of the instrument cause the buildup of large amplitudes at these frequencies—standing-wave resonances. The combination of these frequencies makes the musical note that we hear.

Truly classical music The oldest known musical instruments are bone flutes from burial sites in central China. The flutes in the photo are up to 9000 years old and are made from naturally hollow bones from crowned cranes. The positions of the holes determine the frequencies that the flutes can produce. Soon after the first flutes were created, the design was standardized so that different flutes would play the same notes—including the notes in the modern Chinese musical scale.

CONCEPTUAL EXAMPLE 16.7 **Comparing the flute and the clarinet**

A flute and a clarinet have about the same length, but the lowest note that can be played on the clarinet is much lower than the lowest note that can be played on the flute. Why is this?

REASON A flute is an open-open tube; the frequency of the fundamental mode is $f_1 = v/2L$. A clarinet is open at one end, but the player's lips and the reed close it at the other end. The clarinet is thus an open-closed tube with a fundamental frequency $f_1 = v/4L$. This is about half the fundamental frequency of the flute, so the lowest note on the clarinet has a much lower pitch. In musical terms, it's about an octave lower than the flute.

ASSESS A quick glance at Figure 16.19 shows that the wavelength of the lowest mode of the open-closed tube is longer than that of the open-open tube, so we expect a lower frequency for the clarinet.

A clarinet has a lower pitch than a flute because its lowest mode has a longer wavelength and thus a lower frequency. But the higher harmonics are different as well. An open-open tube like a flute has all of the harmonics; an open-closed tube like a clarinet has only those with odd mode numbers. This gives the two instruments a very different tone quality—it's easy to distinguish the sound of a flute from that of a clarinet. We'll explore this connection between the harmonics an instrument produces and its tone quality in the next section.

EXAMPLE 16.8 **The importance of warming up**

Wind instruments have an adjustable joint to change the tube length. Players know that they may need to adjust this joint to stay in tune—that is, to stay at the correct frequency. To see why, suppose a "cold" flute plays the note A at 440 Hz when the air temperature is 20°C.

a. How long is the tube? At 20°C, the speed of sound in air is 343 m/s.
b. As the player blows air through the flute, the air inside the instrument warms up. Once the air temperature inside the flute has risen to 32°C, increasing the speed of sound to 350 m/s, what is the frequency?
c. At the higher temperature, how must the length of the tube be changed to bring the frequency back to 440 Hz?

SOLVE A flute is an open-open tube with fundamental frequency $f_1 = v/2L$.

a. At 20°C, the length corresponding to 440 Hz is

$$L = \frac{v}{2f} = \frac{343\text{ m/s}}{2(440\text{ Hz})} = 0.390\text{ m}$$

b. As the speed of sound increases, the frequency changes to

$$f_1(\text{at }32°\text{C}) = \frac{350\text{ m/s}}{2(0.390\text{ m})} = 449\text{ Hz}$$

c. To bring the flute back into tune, the length must be increased to give a frequency of 440 Hz with a speed of 350 m/s. The new length is

$$L = \frac{v}{2f_1} = \frac{350\text{ m/s}}{2(440\text{ Hz})} = 0.398\text{ m}$$

Thus the flute must be increased in length by 8 mm.

ASSESS A small change in the absolute temperature produces a correspondingly small change in the speed of sound. We expect that this will require a small change in length, so our answer makes sense.

STOP TO THINK 16.3 A tube that is open at both ends supports a standing wave with harmonics at 300 Hz and 400 Hz, with no harmonics between. What is the fundamental frequency of this tube?

A. 50 Hz B. 100 Hz C. 150 Hz D. 200 Hz E. 300 Hz

16.5 Speech and Hearing

When you hear a particular note played on a guitar, it sounds very different from the same note played on a trumpet. And you have perhaps been to a lecture in which the speaker talked at essentially the same pitch the entire time—but you could still understand what was being said. Clearly, there is more to your brain's perception of sound than pitch alone. How do you tell the difference between a guitar and a trumpet? How do you distinguish between an "oo" vowel sound and an "ee" vowel sound at the same pitch?

The Frequency Spectrum

To this point, we have pictured sound waves as sinusoidal waves, with a well-defined frequency. In fact, most of the sounds that you hear are not pure sinusoidal waves. Most sounds are a mix, or superposition, of different frequencies. For example, we have seen how certain standing-wave modes are possible on a stretched string. When you pluck a string on a guitar, you generally don't excite just one standing-wave mode—you simultaneously excite many different modes.

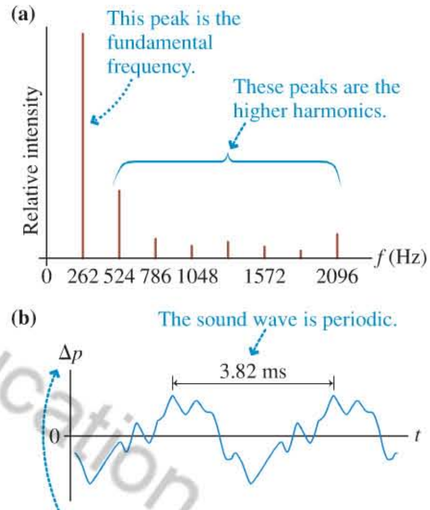
If you play the note "middle C" on a guitar, the fundamental frequency is 262 Hz. There will be a standing wave at this frequency, but there will also be standing waves at the frequencies 524 Hz, 786 Hz, 1048 Hz, . . . , all the higher harmonics predicted by Equation 16.4.

FIGURE 16.22a is a bar chart showing all the frequencies present in the sound of the vibrating guitar string. The height of each bar shows the relative intensity of that harmonic. The fundamental frequency has the highest intensity, but many other harmonics have significant intensities as well. A bar chart showing the relative intensities of the different frequencies is called the **frequency spectrum** of the sound.

When your brain interprets the mix of frequencies from the guitar in Figure 16.22a, it identifies the fundamental frequency as the *pitch*. 262 Hz corresponds to middle C, so you will identify the pitch as middle C, even though the sound consists of many different frequencies. Your brain uses the higher harmonics to determine the **tone quality,** which is also called the *timbre*. The tone quality—and therefore the higher harmonics—is what makes a middle C played on a guitar sound quite different from a middle C played on a trumpet. The frequency spectrum of a trumpet would show a very different pattern of the relative intensities of the higher harmonics, and this different mix of higher harmonics gives the trumpet a different sound.

The actual sound wave produced by a guitar playing middle C is shown in **FIGURE 16.22b**. The sound wave is periodic, with a period of 3.82 ms that corresponds to the 262 Hz fundamental frequency. But the wave doesn't have a simple sinusoidal shape; it is more complex. **The higher harmonics don't change the period of the sound wave; they change only its shape.** The sound wave of a trumpet playing middle C would also have a 3.82 ms period, but its shape would be entirely different.

FIGURE 16.22 The frequency spectrum and a graph of the sound wave of a guitar playing a note with fundamental frequency 262 Hz.



CONCEPTUAL EXAMPLE 16.9 Playing the didgeridoo

The didgeridoo, a musical instrument developed by aboriginal Australians, is deceptively simple. It consists of a tube (a eucalyptus stem or branch hollowed out by termites) of $1\frac{1}{2}$m or more in length. The player presses his lips against the end and blows air through his lips as with a trumpet. He may also make sounds with his vocal cords. Skilled players can make a wide variety of sounds. How is this possible with such a simple instrument?

REASON Because the lips seal one end, a didgeridoo has the resonances of an open-closed tube, given by Equation 16.7. The instrument can therefore produce many different frequencies. Changing the vibration of the lips and the sounds from the vocal cords can change the mix of standing-wave modes that are produced, leading to very different sounds.

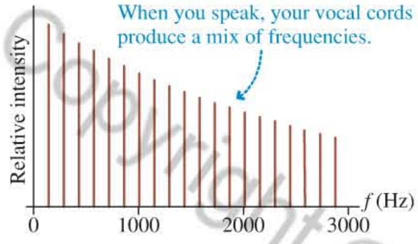
One instrument, many sounds A synthesizer can be adjusted to sound like a flute, a clarinet, a trumpet, a piano—or any other musical instrument. The keys on a synthesizer determine what fundamental frequency to produce. The other controls adjust the mix of higher harmonics to match the frequency spectrum of various musical instruments, effectively mimicking their sounds.

Vowels and Formants

Try this: Keep your voice at the same pitch and say the "ee" sound, as in "beet," then the "oo" sound, as in "boot." Pay attention to how you reshape your mouth as you

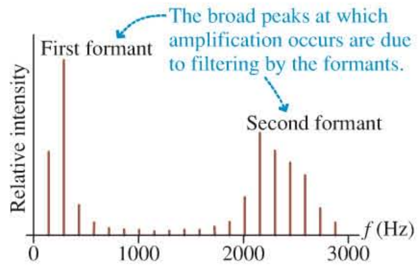
FIGURE 16.23 The frequency spectrum from the vocal cords, and after passing through the vocal tract.

(a) Frequencies from the vocal cords



(b) Actual spoken frequencies (vowel sound "ee")

When you form your vocal tract to make a certain vowel sound, it increases the amplitudes of certain frequencies and suppresses others.



move back and forth between the two sounds. The two vowel sounds are at the same pitch, and thus have the same period, but they sound quite different. The difference in sound arises from the difference in the higher harmonics, just as for musical instruments. As you speak, you adjust the properties of your vocal tract to produce different mixes of harmonics that make the "ee," "oo," "ah," and other vowel sounds.

Speech begins with the vibration of your vocal cords, stretched bands of tissue in your throat. The vibration is similar to that of a wave on a stretched string. In ordinary speech, the average fundamental frequency for adult males and females is about 150 Hz and 250 Hz, respectively, but you can change the vibration frequency by changing the tension of your vocal cords. That's how you make your voice higher or lower as you sing.

Your vocal cords produce a mix of different frequencies as they vibrate—the fundamental frequency and a rich mixture of higher harmonics. If you put a microphone in your throat and measured the sound waves right at your vocal cords the frequency spectrum would appear as in **FIGURE 16.23a**.

There is more to the story though. Before reaching the opening of your mouth, sound from your vocal cords must pass through your vocal tract—a series of hollow cavities including your throat, mouth, and nose. The vocal tract acts like a series of tubes, and, as in any tube, certain frequencies will set up standing-wave resonances. The rather broad standing-wave resonances of the vocal tract are called **formants.** Harmonics of your vocal cords at or near the formant frequencies will be amplified; harmonics far from a formant will be suppressed. **FIGURE 16.23b** shows the formants of an adult male making an "ee" sound. The filtering of the vocal cord harmonics by the formants is clear.

You can change the shape and frequencies of the formants, and thus the sound you make, by changing the shape and length of your vocal tract. You do this by changing your mouth opening and the shape and position of your tongue. The first two formants for an "ee" sound are at roughly 270 Hz and 2300 Hz, but for an "oo" sound they are 300 Hz and 870 Hz. The much lower second formant of the "oo" emphasizes midrange frequencies, making a "calming" sound, while the more strident sound of "ee" comes from enhancing the higher frequencies.

Saying "ah" BIO Why, during a throat exam, does a doctor ask you to say "ah"? This particular vowel sound is formed by opening the mouth and the back of the throat wide—giving a clear view of the tissues of the throat.

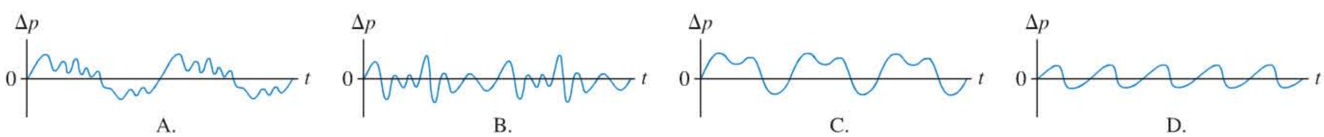
CONCEPTUAL EXAMPLE 16.10 High-frequency hearing loss

As you age, your hearing sensitivity will decrease. This decrease is not uniform; for most people, the loss of sensitivity is greater for higher frequencies. The loss of sensitivity at high frequencies may make it difficult to understand what others are saying. Why is this?

REASON It is the high-frequency components of speech that allow us to distinguish different vowel sounds. A decrease in sensitivity to these higher frequencies makes it more difficult to make such distinctions.

ASSESS This result makes sense. In Figure 16.23b, the lowest frequency is less than 200 Hz, but the second formant is over 2000 Hz. There's a big difference between what we hear as the pitch of someone's voice and the frequencies we use to interpret speech.

STOP TO THINK 16.4 These sound waves represent notes played on different musical instruments. Which has the highest pitch?



16.6 The Interference of Waves from Two Sources

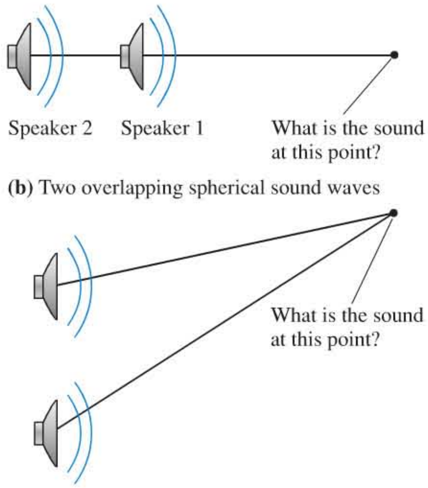
Perhaps you have seen headphones that offer "active noise reduction." When you turn on the headphones, they produce sound that somehow *cancels* noise from the external environment. How does adding sound to a system make it quieter?

We began the chapter by noting that waves, unlike particles, can pass through each other. Where they do, the principle of superposition tells us that the displacement of the medium is the sum of the displacements due to each wave acting alone. Consider the two loudspeakers in **FIGURE 16.24**, both emitting sound waves with the same frequency. In Figure 16.24a, sound from loudspeaker 2 passes loudspeaker 1, then two overlapped sound waves travel to the right along the x-axis. What sound is heard at the point indicated with the dot? And what about at the dot in Figure 16.24b, where the speakers are side by side? These are two cases we will consider in this section. Although we'll use sound waves for our discussion, the results are general and apply to all waves. In Chapter 17, we will use these ideas to study the interference of light waves.

FIGURE 16.24 Interference of waves from two sources.

(a) Two sound waves overlapping along a line

(b) Two overlapping spherical sound waves

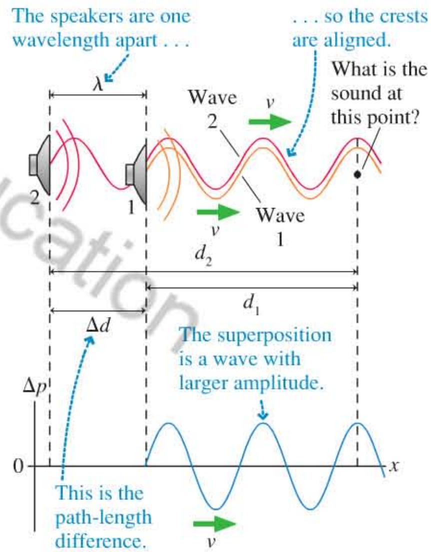


Interference Along a Line

FIGURE 16.25 shows traveling waves from two loudspeakers spaced exactly one wavelength apart. The graphs are slightly displaced from each other so that you can see what each wave is doing, but the *physical situation* is one in which the waves are traveling *on top of* each other. We assume that the two speakers emit sound waves of identical frequency f, wavelength λ, and amplitude A.

At every point along the line, the net sound pressure wave will be the sum of the pressures from the individual waves. That's the principle of superposition. Because the two speakers are separated by one wavelength, the two waves are aligned crest-to-crest and trough-to-trough. Waves aligned this way are said to be **in phase;** waves that are in phase march along "in step" with each other. The superposition is a traveling wave with wavelength λ and twice the amplitude of the individual waves. This is constructive interference.

FIGURE 16.25 Constructive interference of two waves traveling along the x-axis.



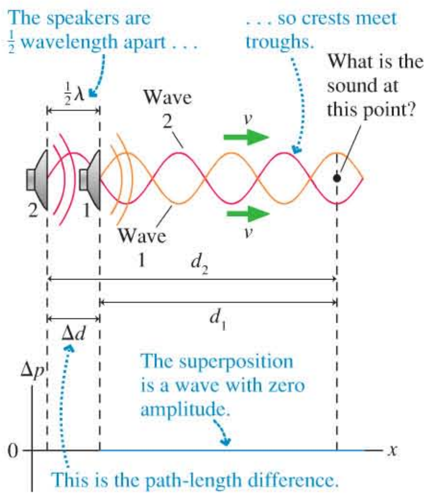
NOTE ► Textbook pictures can be misleading because they're frozen in time. The net sound wave in Figure 16.25 is a *traveling* wave, moving to the right with the speed of sound v. It differs from the two individual waves only by having twice the amplitude. This is not a standing wave with nodes and antinodes that remain in one place. ◄

If d_1 and d_2 are the distances from loudspeakers 1 and 2 to a point at which we want to know the combined sound wave, their difference $\Delta d = d_2 - d_1$ is called the **path-length difference.** It is the *extra* distance traveled by wave 2 on the way to the point where the two waves are combined. In Figure 16.25, we see that constructive interference results from a path-length difference $\Delta d = \lambda$. But increasing Δd by an additional λ would produce exactly the same result, so we will also have constructive interference for $\Delta d = 2\lambda$, $\Delta d = 3\lambda$, and so on. In other words, **two waves will be in phase and will produce constructive interference any time their path-length difference is a whole number of wavelengths.**

In **FIGURE 16.26**, the two speakers are separated by half a wavelength. Now the crests of one wave align with the troughs of the other, and the waves march along "out of step" with each other. We say that the two waves are **out of phase.** When two waves are out of phase, they are equal and opposite at every point. Consequently, the sum of the two waves is zero *at every point.* This is destructive interference.

The destructive interference of Figure 16.26 results from a path-length difference $\Delta d = \frac{1}{2}\lambda$. Again, increasing Δd by an additional λ would produce a picture that looks exactly the same, so we will also have destructive interference for $\Delta d = 1\frac{1}{2}\lambda$, $\Delta d = 2\frac{1}{2}\lambda$, and so on. That is, **two waves will be out of phase and will produce destructive interference any time their path-length difference is a whole number of wavelengths plus half a wavelength.**

FIGURE 16.26 Destructive interference of two waves traveling along the x-axis.



Summing up, for two identical sources of waves, constructive interference occurs when the path-length difference is

$$\Delta d = m\lambda \qquad m = 0, 1, 2, 3, \ldots \tag{16.8}$$

and destructive interference occurs when the path-length difference is

$$\Delta d = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, 1, 2, 3, \ldots \tag{16.9}$$

NOTE ► The path-length difference needed for constructive or destructive interference depends on the wavelength and hence the frequency. If one particular frequency interferes destructively, another may not. ◄

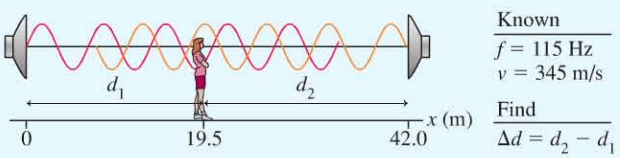
The path-length difference is not necessarily the distance between the speakers, as we see in the next example. It is simply the difference in the distances traveled by the two waves.

EXAMPLE 16.11 **Is the sound loud or quiet? Part I**

Two loudspeakers 42.0 m apart and facing each other emit identical 115 Hz sinusoidal sound waves. Susan is walking along a line between the speakers. As she walks, she finds herself moving through loud and quiet spots. If Susan stands 19.5 m from one speaker, is she standing at a quiet spot or a loud spot? Assume that the speed of sound is 345 m/s.

PREPARE As Susan walks along the line between the speakers, she moves between points of constructive interference (loud spots) and destructive interference (quiet spots). Is her current position one of constructive or destructive interference? This will depend on the path-length difference. We start with a visual overview of the situation in **FIGURE 16.27**.

FIGURE 16.27 Visual overview of loudspeakers.



SOLVE At Susan's position, the distances the two waves travel to reach her are

$$d_1 = 19.5 \text{ m} \qquad d_2 = 42.0 \text{ m} - 19.5 \text{ m} = 22.5 \text{ m}$$

At the point where the two waves reach Susan and interfere, their path-length difference is

$$\Delta d = d_2 - d_1 = 3.0 \text{ m}$$

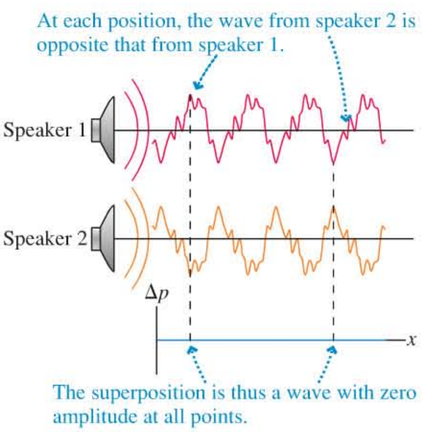
To know if we have constructive or destructive interference, we need to compare this with the wavelength:

$$\lambda = \frac{v}{f} = \frac{345 \text{ m/s}}{115 \text{ Hz}} = 3.0 \text{ m}$$

Because the path-length difference is exactly one wavelength, Susan is standing at a point of constructive interference; that is, she is standing at a loud spot.

ASSESS There's a nice way that we can check to see that our answer makes sense. As you'll recall from earlier in the chapter, two counterpropagating waves create a standing wave. In this case, the conditions for constructive and destructive interference are the same as the conditions for antinodes and nodes, respectively, that we found earlier. Susan is at an antinode of the standing wave. The ideas of interference give us a different perspective on standing waves.

FIGURE 16.28 Opposite waves cancel.



The analysis above assumed that the two loudspeakers were emitting identical waves. Another interesting and important case of interference, illustrated in **FIGURE 16.28**, occurs when one loudspeaker emits a sound wave that is *the exact inverse* of the wave from the other speaker. If the speakers are side by side, so that $\Delta d = 0$, the superposition of these two waves will result in destructive interference; they will completely cancel. This destructive interference does not require the waves to have any particular frequency or any particular shape.

Headphones with *active noise reduction* use this technique. A microphone on the outside of the headphones measures ambient sound. A circuit inside the headphones produces an inverted version of the microphone signal and sends it to the headphone speakers. The ambient sound and the inverted version of the sound from the speakers arrive at the ears together and interfere destructively, reducing the sound intensity. In this case, *adding* sound results in a *lower* overall intensity inside the headphones!

Interference of Spherical Waves

Interference along a line illustrates the idea of interference, but it's not very realistic. In practice, sound waves from a loudspeaker or light waves from a lightbulb spread out as spherical waves. **FIGURE 16.29** shows a wave-front diagram for a spherical wave. Recall that the wave fronts represent the *crests* of the wave and are spaced by the wavelength λ. Halfway between two wave fronts is a trough of the wave. What happens when two spherical waves overlap? For example, imagine two loudspeakers emitting identical waves radiating sound in all directions. **FIGURE 16.30** shows the wave fronts of the two waves. This is a static picture, of course, so you have to imagine the wave fronts spreading out as new circular rings are born at the speakers. The waves overlap as they travel, and, as was the case in one dimension, this causes interference.

Consider a particular point like that marked by the red dot in Figure 16.30. The two waves each have a crest at this point, so there is constructive interference here. But at other points, such as that marked by the black dot, a crest overlaps a trough, so this is a point of destructive interference.

Notice—simply by counting the wave fronts—that the red dot is three wavelengths from speaker 2 ($r_2 = 3\lambda$) but only two wavelengths from speaker 1 ($r_1 = 2\lambda$). The path-length difference of the two waves arriving at the red dot is $\Delta r = r_2 - r_1 = \lambda$. That is, the wave from speaker 2 has to travel one full wavelength more than the wave from speaker 1, so the waves are in phase (crest aligned with crest) and interfere constructively. You should convince yourself that Δr is a *whole number of wavelengths* at every point where two wave fronts intersect.

Similarly, the path-length difference at the black dot, where the interference is destructive, is $\Delta r = \frac{1}{2}\lambda$. As with interference along a line, destructive interference results when the path-length difference is a whole number of wavelengths plus half a wavelength.

Thus the general rule for determining whether there is constructive or destructive interference at any point is the same for spherical waves as for waves traveling along a line. For identical sources, constructive interference occurs when the path-length difference is

$$\Delta r = m\lambda \qquad m = 0, 1, 2, 3, \ldots \tag{16.10}$$

Destructive interference occurs when the path-length difference is

$$\Delta r = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, 1, 2, 3, \ldots \tag{16.11}$$

The conditions for constructive and destructive interference are the same for spherical waves as for waves along a line. And the treatment we have seen for sound waves can be applied to any wave, as we have noted. For any two wave sources, the following Tactics Box sums up how to determine if the interference at a point is constructive or destructive.

TACTICS BOX 16.1 Identifying constructive and destructive interference MP™

1. Identify the path length from each source to the point of interest. Compute the path-length difference $\Delta r = |r_2 - r_1|$.
2. Find the wavelength, if it is not specified.
3. If the path-length difference is a whole number of wavelengths ($\lambda, 2\lambda, 3\lambda, \ldots$), crests are aligned with crests and there is constructive interference.
4. If the path-length difference is a whole number of wavelengths plus a half wavelength $\left(1\frac{1}{2}\lambda, 2\frac{1}{2}\lambda, 3\frac{1}{2}\lambda, \ldots\right)$, crests are aligned with troughs and there is destructive interference.

Exercises 9,10

NOTE ► Keep in mind that interference is determined by Δr, the path-length *difference*, not by r_1 or r_2. ◄

FIGURE 16.29 A spherical wave.

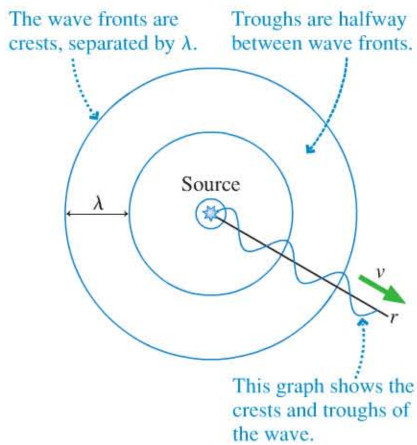
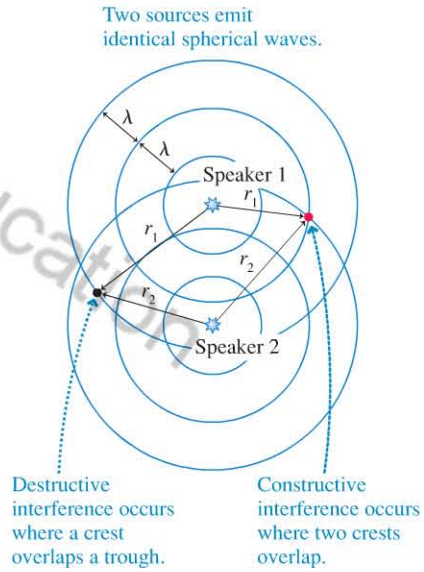


FIGURE 16.30 The overlapping wave patterns of two sources.

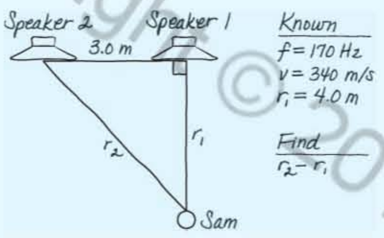


EXAMPLE 16.12 **Is the sound loud or quiet? Part II**

Two speakers are 3.0 m apart and play identical tones of frequency 170 Hz. Sam stands directly in front of one speaker at a distance of 4.0 m. Is this a loud spot or a quiet spot? Assume that the speed of sound in air is 340 m/s.

PREPARE FIGURE 16.31 shows a visual overview of the situation, showing the positions of and path lengths from each speaker.

FIGURE 16.31 Visual overview of two speakers.



SOLVE Following the steps in Tactics Box 16.1, we first compute the path-length difference. r_1, r_2, and the distance between the speakers form a right triangle, so we can use the Pythagorean theorem to find

$$r_2 = \sqrt{(4.0\text{ m})^2 + (3.0\text{ m})^2} = 5.0\text{ m}$$

Thus the path-length difference is

$$\Delta r = r_2 - r_1 = 1.0\text{ m}$$

Next, we compute the wavelength:

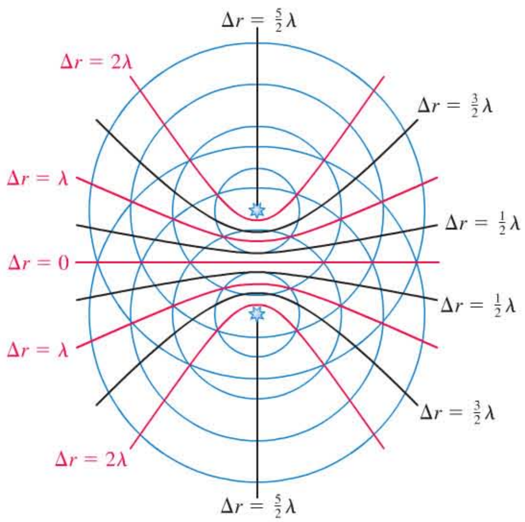
$$\lambda = \frac{v}{f} = \frac{340\text{ m/s}}{170\text{ Hz}} = 2.0\text{ m}$$

The path-length difference is $\frac{1}{2}\lambda$, so this is a point of destructive interference. Sam is at a quiet spot.

Taming and tuning exhaust noise It's possible to use destructive interference to cut automobile exhaust noise using a device called a *resonator*. The wide section of pipe at the end of the exhaust system is a tube with one closed end. A sound wave enters the tube, reflects from the end, and reenters the exhaust pipe. If the length of the tube is just right, the reflected wave will be out of phase with the sound wave in the pipe, producing destructive interference. The resonator is tuned to eliminate the loudest frequencies from the engine, but other frequencies will produce constructive interference, enhancing them and giving the exhaust a certain "note."

So far, we have looked at interference only at particular points. What can we say about the overall pattern of points at which we have constructive or destructive interference? For instance, the red dot in Figure 16.30 is only one point where $\Delta r = \lambda$; you should be able to locate several more. Taken together, all the points with $\Delta r = \lambda$ form a curved line along which constructive interference is occurring. Another curved line of constructive interference connects all the points at which $\Delta r = 2\lambda$, another connects the $\Delta r = 3\lambda$ points, and so on. These lines, shown as red lines in FIGURE 16.32, are called **antinodal lines**. They are analogous to the antinodes of a standing wave—hence the name. An antinode is a *point* of constructive interference; for spherical waves, oscillation at maximum amplitude occurs along a continuous *line*. To understand this idea better, imagine the static picture of Figure 16.32 evolving with time. As the wave fronts expand, the *intersection point* of two rings moves outward along one of the red lines.

FIGURE 16.32 The points of constructive and destructive interference fall along antinodal and nodal lines.



Antinodal lines, constructive interference. Intensity is at its maximum value.

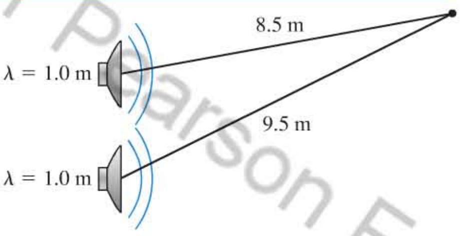
Nodal lines, destructive interference. Intensity is zero.

Similarly, we can connect together points where Δr is a multiple of λ plus $\frac{1}{2}\lambda$. The black dot in Figure 16.30 is just one of many points with $\Delta r = \frac{1}{2}\lambda$. Together, these points of destructive interference form a **nodal line** along which the displacement is always zero. The nodal lines in Figure 16.32 are shown in black.

You are regularly exposed to sound from two separated sources: stereo speakers. When you walk across a room in which a stereo is playing, why don't you hear a pattern of loud and soft sounds as you cross antinodal and nodal lines? First, we don't listen to single frequencies. Music is a complex sound wave with many frequencies, but only one frequency at a time satisfies the condition for constructive or destructive interference. Most of the sound frequencies are not affected. Second, reflections of sound waves from walls and furniture make the situation much more complex than the idealized two-source picture in Figure 16.32. Sound wave interference can be heard, but it takes careful selection of a pure tone and a room with no hard, reflecting surfaces. Interference that's rather tricky to demonstrate with sound waves is easy to produce with light waves, as we'll see in the next chapter.

The two water waves overlap, leading to patterns of constructive and destructive interference.

STOP TO THINK 16.5 These speakers emit identical sound waves with a wavelength of 1.0 m. At the point indicated, is the interference constructive, destructive, or something in between?



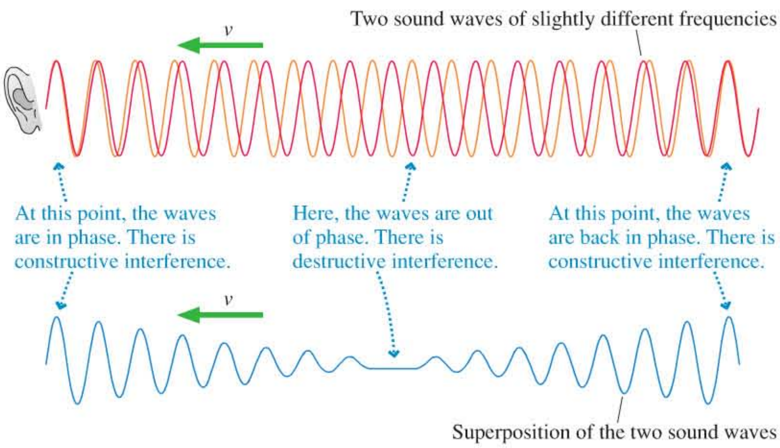
16.7 Beats

Activ Physics ONLINE 10.7

Thus far we have looked at the superposition of waves from sources having the same frequency. We can also use the principle of superposition to investigate a phenomenon that is easily demonstrated with two sources of slightly *different* frequencies.

Suppose two sinusoidal waves are traveling toward your ear, as shown in **FIGURE 16.33**. The two waves have the same amplitude but slightly different frequencies: The red wave has a slightly higher frequency (and thus a slightly shorter wavelength) than the orange wave. This slight difference causes the waves to combine in a manner that alternates between constructive and destructive interference. Their superposition, drawn in blue below the two waves, is a wave whose amplitude shows

FIGURE 16.33 The superposition of two sound waves with slightly different frequencies.



a periodic variation. As the waves reach your ear, you will hear a single tone whose intensity is *modulated.* That is, the sound goes up and down in volume, loud, soft, loud, soft, . . . , making a distinctive sound pattern called **beats.**

Suppose the two waves have frequencies f_1 and f_2 that differ only slightly, so that $f_1 \approx f_2$. A complete mathematical analysis would show that the air oscillates against your eardrum at frequency

$$f_{osc} = \frac{1}{2}(f_1 + f_2)$$

This is the *average* of f_1 and f_2, and it differs little from either since the two frequencies are nearly equal. Further, the intensity of the sound is modulated at a frequency called the *beat frequency:*

$$f_{beat} = |f_1 - f_2| \tag{16.12}$$

The beat frequency is the *difference* between the two individual frequencies.

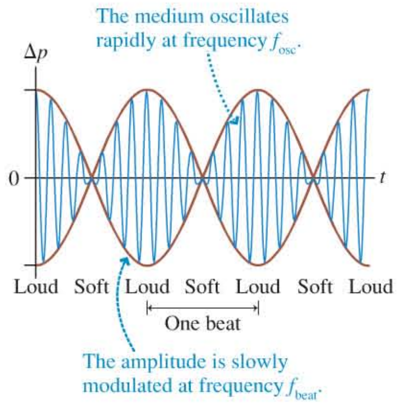


FIGURE 16.34 The modulated sound of beats.

FIGURE 16.34 is a history graph of the wave at the position of your ear. You can see both the sound wave oscillation at frequency f_{osc} and the much slower intensity oscillation at frequency f_{beat}. Frequency f_{osc} determines the pitch you hear, while f_{beat} determines the frequency of the loud-soft-loud modulations of the sound intensity.

Musicians can use beats to tune their instruments. If one flute is properly tuned at 440 Hz but another plays at 438 Hz, the flutists will hear two loud-soft-loud beats per second. The second flutist is "flat" and needs to shorten her flute slightly to bring the frequency up to 440 Hz.

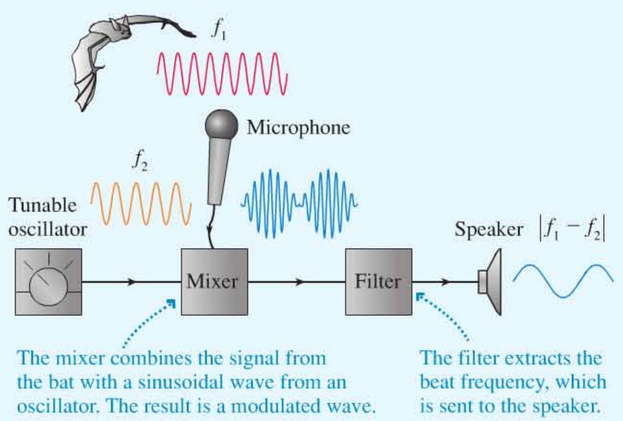
Many measurement devices use beats to determine an unknown frequency by comparing it to a known frequency. For example, Chapter 15 described a Doppler blood flow meter that used the Doppler shift of ultrasound reflected from moving blood to determine its speed. The meter determines this very small frequency shift by combining the emitted wave and the reflected wave and measuring the resulting beat frequency. The beat frequency is equal to the shift in frequency on reflection, exactly what is needed to determine the blood speed. Another example of using beats to make a measurement is given in the following example.

EXAMPLE 16.13 Detecting bats using beats

The little brown bat is a common bat species in North America. It emits echolocation pulses at a frequency of 40 kHz, well above the range of human hearing. To allow observers to "hear" these bats, the bat detector shown in FIGURE 16.35 combines the bat's sound wave at frequency f_1 with a wave of frequency f_2 from a tunable oscillator. The resulting beat frequency is isolated with a filter, then amplified and sent to a loudspeaker. To what frequency should the tunable oscillator be set to produce an audible beat frequency of 3 kHz?

SOLVE The beat frequency is $f_{beat} = |f_1 - f_2|$, so the oscillator frequency and the bat frequency need to *differ* by 3 kHz. An oscillator frequency of either 37 kHz or 43 kHz will work nicely.

FIGURE 16.35 The operation of a bat detector.



STOP TO THINK 16.6 You hear three beats per second when two sound tones are generated. The frequency of one tone is known to be 610 Hz. The frequency of the other is

A. 604 Hz
B. 607 Hz
C. 613 Hz
D. 616 Hz
E. Either A or D
F. Either B or C

INTEGRATED EXAMPLE 16.14 The size of a dog determines the sound of its growl

The sounds of the human vocal system result from the interplay of two different oscillations: the oscillation of the vocal cords and the standing-wave resonances of the vocal tract. A dog's vocalizations are based on similar principles, but the canine vocal tract is quite a bit simpler than that of a human. When a dog growls or howls, the shape of the vocal tract is essentially a tube closed at the larynx and open at the lips.

All dogs growl at a low pitch because the fundamental frequency of the vocal cords is quite low. But the growls of small dogs and big dogs differ because they have very different formants. The frequency of the formants is determined by the length of the vocal tract, and the vocal-tract length is a pretty good measure of the size of a dog. A larger dog has a longer vocal tract and a correspondingly lower-frequency formant, which sends an important auditory message to other dogs.

The masses of two different dogs and the frequencies of their formants are given in Table 16.1.

a. What is the approximate vocal-tract length of these two dogs? Assume that the speed of sound is 350 m/s at a dog's body temperature.
b. Growls aren't especially loud; at a distance of 1.0 m, a dog's growl is about 60 dB—the same as normal conversation. What is the acoustic power emitted by a 60 dB growl?
c. The lower-pitched growl of the Doberman will certainly sound more menacing, but which dog's 60 dB growl sounds louder to a human? **Hint:** Assume that most of the acoustic energy is at frequencies near the first formant; then look at the curve of equal perceived loudness in Figure 16.21.

TABLE 16.1 Mass and acoustic data for two growling dogs

Breed	Mass (kg)	First formant (Hz)	Second formant (Hz)
West Highland Terrier (Westie)	8.0	650	1950
Doberman	38	350	1050

PREPARE FIGURE 16.36 shows an idealized model of a dog's vocal tract as an open-closed tube. The formants will correspond to the standing-wave resonances of this system. The first two modes are shown in the figure. The first formant corresponds to $m = 1$; the second formant corresponds to $m = 3$ because an open-closed tube has only odd harmonics. The frequencies of the formants will allow us to determine the length of the tube that produces them.

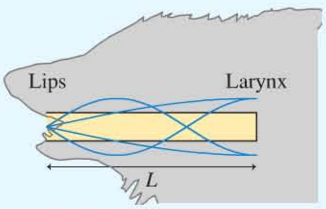


FIGURE 16.36 A model of the canine vocal tract.

For the question about loudness, we can determine the sound intensity from the sound intensity level. Knowing the distance, we can use this value to determine the emitted acoustic power.

SOLVE a. We can use the frequency of the first formant to find the length of the vocal tract. Recall that the standing-wave frequencies of an open-closed tube are given by

$$f_m = m\frac{v}{4L} \qquad m = 1, 3, 5, \ldots$$

Rearranging this to solve for L, with $m = 1$, we find

$$L(\text{Westie}) = m\frac{v}{4f_m} = (1)\frac{350 \text{ m/s}}{4(650 \text{ Hz})} = 0.13 \text{ m}$$

$$L(\text{Doberman}) = m\frac{v}{4f_m} = (1)\frac{350 \text{ m/s}}{4(350 \text{ Hz})} = 0.25 \text{ m}$$

The length is greater for the Doberman, as we would predict, given the relative masses of the dogs.

b. Equation 15.15 lets us compute the sound intensity for a given sound intensity level:

$$I = (1.0 \times 10^{-12} \text{ W/m}^2)10^{(\beta/10 \text{ dB})}$$
$$= (1.0 \times 10^{-12} \text{ W/m}^2)10^{(60 \text{ dB}/10 \text{ dB})} = 1.0 \times 10^{-6} \text{ W/m}^2$$

This is the intensity at a distance of 1.0 m. The sound spreads out in all directions, so we can use Equation 15.12, $I = P_{\text{source}}/4\pi r^2$, to compute the power of the source:

$$P_{\text{source}} = I \cdot 4\pi r^2 = (1.0 \times 10^{-6} \text{ W/m}^2) \cdot 4\pi(1.0 \text{ m})^2 = 13\ \mu\text{W}$$

A growl may sound menacing, but that's not because of its power!

c. Figure 16.21 is a curve of equal perceived loudness. The curve steadily decreases until reaching $\approx$3500 Hz, so the acoustic power needed to produce the same sensation of loudness decreases with frequency up to this point. That is, below 3500 Hz your ear is more sensitive to higher frequencies than to lower frequencies. Much of the acoustic power of a dog growl is concentrated at the frequencies of the first two formants. The Westie's growl has its energy at higher frequencies than that of the Doberman, so the Westie's growl will sound louder—though the lower formants of the Doberman's growl will make it sound more formidable.

ASSESS The lengths of the vocal tract that we calculated—13 cm (5 in) for a small terrier and 25 cm (10 in) for a Doberman—seem reasonable given the size of the dogs. We can also check our work by looking at the second formant, corresponding to $m = 3$; using this harmonic in the second expression in Equation 16.7, we find

$$L(\text{Westie}) = m\frac{v}{4f_m} = (3)\frac{350 \text{ m/s}}{4(1950 \text{ Hz})} = 0.13 \text{ m}$$

$$L(\text{Doberman}) = m\frac{v}{4f_m} = (3)\frac{350 \text{ m/s}}{4(1050 \text{ Hz})} = 0.25 \text{ m}$$

This exact match to our earlier calculations gives us confidence in our model and in our results.

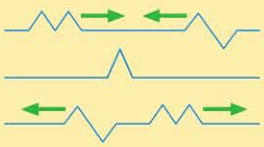
SUMMARY

The goal of Chapter 16 has been to use the idea of superposition to understand the phenomena of interference and standing waves.

GENERAL PRINCIPLES

Principle of Superposition

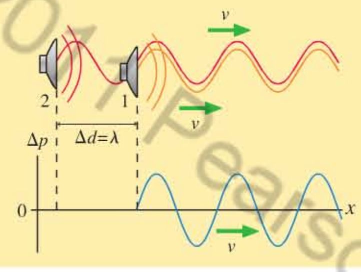
The displacement of a medium when more than one wave is present is the sum of the displacements due to each individual wave.



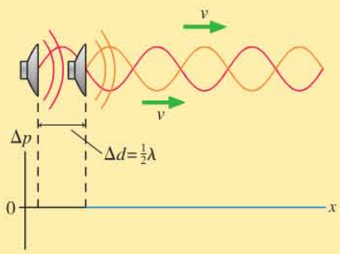
Interference

In general, the superposition of two or more waves into a single wave is called interference.

Constructive interference occurs when crests are aligned with crests and troughs with troughs. We say the waves are in phase. It occurs when the path-length difference Δd is a whole number of wavelengths.



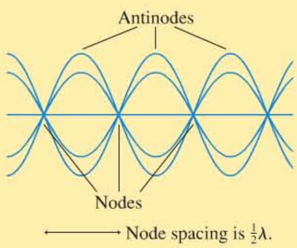
Destructive interference occurs when crests are aligned with troughs. We say the waves are out of phase. It occurs when the path-length difference Δd is a whole number of wavelengths plus half a wavelength.



IMPORTANT CONCEPTS

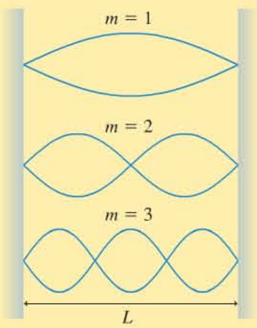
Standing Waves

Two identical traveling waves moving in opposite directions create a standing wave.



The boundary conditions determine which standing-wave frequencies and wavelengths are allowed. The allowed standing waves are **modes** of the system.

A **standing wave on a string** has a node at each end. Possible modes:



$$\lambda_m = \frac{2L}{m} \qquad f_m = m\left(\frac{v}{2L}\right) = mf_1$$

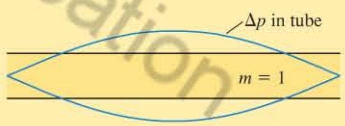
$$m = 1, 2, 3, \ldots$$

A **standing sound wave in a tube** can have different boundary conditions: open-open, closed-closed, or open-closed.

Open-open

$$f_m = m\left(\frac{v}{2L}\right)$$

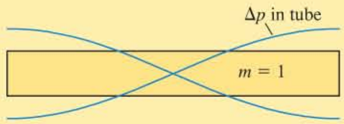
$$m = 1, 2, 3, \ldots$$



Closed-closed

$$f_m = m\left(\frac{v}{2L}\right)$$

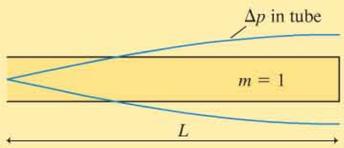
$$m = 1, 2, 3, \ldots$$



Open-closed

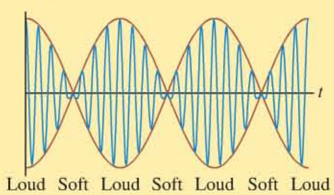
$$f_m = m\left(\frac{v}{4L}\right)$$

$$m = 1, 3, 5, \ldots$$



APPLICATIONS

Beats (loud-soft-loud-soft modulations of intensity) are produced when two waves of slightly different frequencies are superimposed.

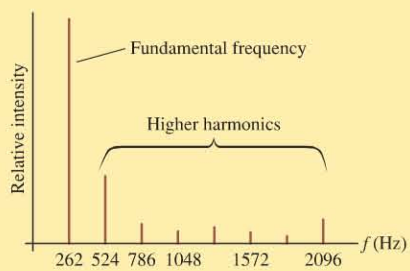


$$f_{beat} = |f_1 - f_2|$$

Standing waves are multiples of a **fundamental frequency,** the frequency of the lowest mode. The higher modes are the higher **harmonics.**

For sound, the fundamental frequency determines the perceived **pitch;** the higher harmonics determine the **tone quality.**

Our vocal cords create a range of harmonics. The mix of higher harmonics is changed by our vocal tract to create different vowel sounds.



QUESTIONS

Conceptual Questions

1. Light can pass easily through water and through air, but light will reflect from the surface of a lake. What does this tell you about the speed of light in air and in water?

2. Ocean waves are partially reflected from the entrance to a harbor, where the depth of the water is suddenly less. What does this tell you about the speed of waves in water of different depths?
3. A string has an abrupt change in linear density at its midpoint so that the speed of a pulse on the left side is 2/3 of that on the right side.
   a. On which side is the linear density greater? Explain.
   b. From which side would you start a pulse so that its reflection from the midpoint would not be inverted? Explain.
4. A guitarist finds that the frequency of one of her strings is too low by 1.4%. Should she increase or decrease the tension of the string? Explain.
5. BIO Certain illnesses inflame your vocal cords, causing them to swell. How does this affect the pitch of your voice? Explain.
6. Figure Q16.6 shows a standing wave on a string that is oscillating at frequency f_0. How many antinodes will there be if the frequency is doubled to $2f_0$? Explain.

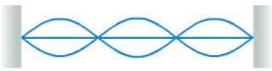


FIGURE Q16.6

7. Figure Q16.7 shows a standing sound wave in a tube of air that is open at both ends.
   a. Which mode (value of m) standing wave is this?
   b. Is the air vibrating horizontally or vertically?

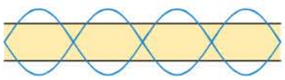


FIGURE Q16.7

8. A typical flute is about 66 cm long. A piccolo is a very similar instrument, though it is smaller, with a length of about 32 cm. How does the pitch of a piccolo compare to that of a flute?
9. Some pipes on a pipe organ are open at both ends, others are closed at one end. For pipes that play low-frequency notes, there is an advantage to using pipes that are closed at one end. What is the advantage?
10. A flute player tunes her instrument when the air (and the flute) is cold. As she plays, the flute and the air inside it warm up. Both the changing speed of sound in the air inside and the thermal expansion of the flute affect the frequency of the sound wave produced by the flute. After some time, she finds that her playing is "sharp"—the frequencies are too high. Which change produced this effect: the warming of the air or the warming of the body of the flute?
11. BIO A friend's voice sounds different over the telephone than it does in person. This is because telephones do not transmit frequencies over about 3000 Hz. 3000 Hz is well above the normal frequency of speech, so why does eliminating these high frequencies change the sound of a person's voice?
12. Suppose you were to play a trumpet after breathing helium, in which the speed of sound is much greater than in air. Would the pitch of the instrument be higher or lower than normal, or would it be unaffected by being played with helium inside the tube rather than air?
13. If you pour liquid in a tall, narrow glass, you may hear sound with a steadily rising pitch. What is the source of the sound, and why does the pitch rise as the glass fills?
14. BIO When you speak after breathing helium, in which the speed of sound is much greater than in air, your voice sounds quite different. The frequencies emitted by your vocal cords do not change since they are determined by the mass and tension of your vocal cords. So what *does* change when your vocal tract is filled with helium rather than air?
15. BIO Sopranos can sing notes at very high frequencies—over 1000 Hz. When they sing such high notes, it can be difficult to understand the words they are singing. Use the concepts of harmonics and formants to explain this.
16. When you hit a baseball with a bat, the bat flexes and then vibrates. We can model this vibration as a transverse standing wave. The modes of this standing wave are similar to the modes of a stretched string, but with one important difference: The ends of the bat are antinodes instead of nodes, because the ends of the bat are free to move. The modes thus look like the modes of a stretched string with antinodes replacing nodes and nodes replacing antinodes. If the ball hits the bat near an antinode of a standing-wave mode, the bat will start oscillating in this mode. The batter holds the bat at one end, which is also an antinode, so a large vibration of the bat causes an unpleasant vibration in the batter's hands. This can be avoided if the ball hits the bat at what players call the "sweet spot," which is a node of the standing-wave pattern. The first standing-wave mode of a vibrating bat is the $m = 2$ mode. Sketch the appearance of this vibrational mode of the bat, then estimate the approximate distance of the sweet spot (as a fraction of the bat's length) from the end of the bat.
17. BIO If a cold gives you a stuffed-up nose, it changes the way your voice sounds, even if your vocal cords are not affected. Explain why this is so.
18. BIO A small boy and a grown woman both speak at approximately the same pitch. Nonetheless, it's easy to tell which is which from listening to the sounds of their voices. How are you able to make this determination?

19. Figure Q16.19 shows wave fronts of a circular wave. Are the displacements at the following pairs of positions *in phase* or *out of phase?* Explain.
a. A and B b. C and D c. E and F

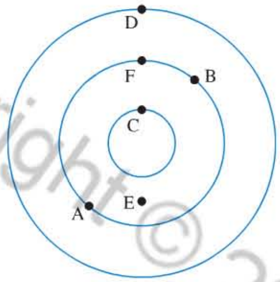


FIGURE Q16.19

Multiple-Choice Questions

Questions 20 through 22 refer to the snapshot graph Figure Q16.20.

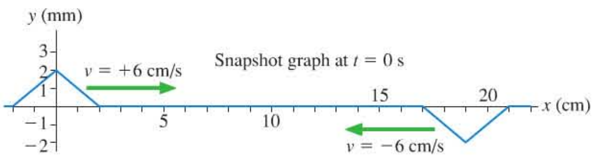


FIGURE Q16.20

20. | At $t = 1$ s, what is the displacement y of the string at $x = 7$ cm?
A. −1.0 mm B. 0 mm C. 0.5 mm
D. 1.0 mm E. 2.0 mm

21. | At $x = 3$ cm, what is the earliest time that y will equal 2 mm?
A. 0.5 s B. 0.7 s C. 1.0 s
D. 1.5 s E. 2.5 s

22. | At $t = 1.5$ s, what is the value of y at $x = 10$ cm?
A. −2.0 mm B. −1.0 mm C. −0.5 mm
D. 0 mm E. 1.0 mm

23. || Two sinusoidal waves with the same amplitude A and frequency f travel in opposite directions along a long string. You stand at one point and watch the string. The maximum displacement of the string at that point is
A. A B. $2A$ C. 0
D. There is not enough information to decide.

24. | A student in her physics lab measures the standing-wave modes of a tube. The lowest frequency that makes a resonance is 20 Hz. As the frequency is increased, the next resonance is at 60 Hz. What will be the next resonance after this?
A. 80 Hz B. 100 Hz C. 120 Hz D. 180 Hz

25. | An organ pipe is tuned to exactly 384 Hz when the temperature in the room is 20°C. Later, when the air has warmed up to 25°C, the frequency is
A. Greater than 384 Hz.
B. 384 Hz.
C. Less than 384 Hz.

26. || Two guitar strings made of the same type of wire have the same length. String 1 has a higher pitch than string 2. Which of the following is true?
A. The wave speed of string 1 is greater than that of string 2.
B. The tension in string 2 is greater than that in string 1.
C. The wavelength of the lowest standing-wave mode on string 2 is longer than that on string 1.
D. The wavelength of the lowest standing-wave mode on string 1 is longer than that on string 2.

27. | The frequency of the lowest standing-wave mode on a 1.0-m-long string is 20 Hz. What is the wave speed on the string?
A. 10 m/s B. 20 m/s C. 30 m/s D. 40 m/s

28. | Suppose you pluck a string on a guitar and it produces the note A at a frequency of 440 Hz. Now you press your finger down on the string against one of the frets, making this point the new end of the string. The newly shortened string has 4/5 the length of the full string. When you pluck the string, its frequency will be
A. 350 Hz B. 440 Hz C. 490 Hz D. 550 Hz

VIEW ALL SOLUTIONS

PROBLEMS

Section 16.1 The Principle of Superposition

1. | Figure P16.1 is a snapshot graph at $t = 0$ s of two waves on a taut string approaching each other at 1 m/s. Draw six snapshot graphs, stacked vertically, showing the string at 1 s intervals from $t = 1$ s to $t = 6$ s.

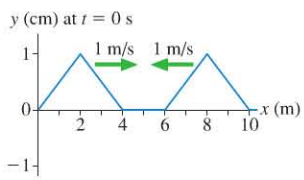


FIGURE P16.1

2. || Figure P16.2 is a snapshot graph at $t = 0$ s of two waves approaching each other at 1 m/s. Draw four snapshot graphs, stacked vertically, showing the string at $t = 2$, 4, 6, and 8 s.

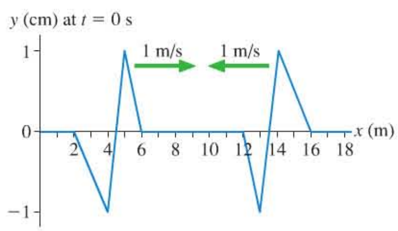


FIGURE P16.2

3. || Figure P16.3a is a snapshot graph at $t = 0$ s of two waves on a string approaching each other at At what time was the snapshot graph in Figure P16.3b taken?

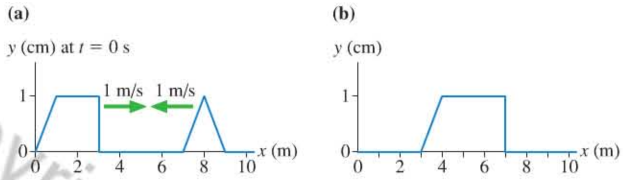


FIGURE P16.3

Section 16.2 Standing Waves

Section 16.3 Standing Waves on a String

4. ||| Figure P16.4 is a snapshot graph at $t = 0$ s of a pulse on a string moving to the right at 1 m/s. The string is fixed at $x = 5$ m. Draw a history graph spanning the time interval $t = 0$ s to $t = 10$ s for the location $x = 3$ m on the string.

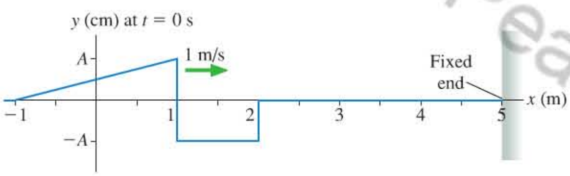


FIGURE P16.4

5. ||| At $t = 0$ s, a small "upward" (positive y) pulse centered at $x = 6.0$ m is moving to the right on a string with fixed ends at $x = 0.0$ m and $x = 10.0$ m. The wave speed on the string is 4.0 m/s. At what time will the string next have the same appearance that it did at $t = 0$ s?
6. ||| You are holding one end of an elastic cord that is fastened to a wall 3.0 m away. You begin shaking the end of the cord at 3.5 Hz, creating a continuous sinusoidal wave of wavelength 1.0 m. How much time will pass until a standing wave fills the entire length of the string?
7. ||| A 2.0-m-long string is fixed at both ends and tightened until the wave speed is 40 m/s. What is the frequency of the standing wave shown in Figure P16.7?

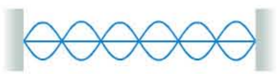


FIGURE P16.7

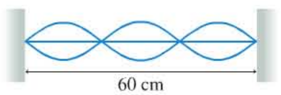


FIGURE P16.8

8. || Figure P16.8 shows a standing wave oscillating at 100 Hz on a string. What is the wave speed?
9. || A bass guitar string is 89 cm long with a fundamental frequency of 30 Hz. What is the wave speed on this string?
10. ||| The fundamental frequency of a guitar string is 384 Hz. What is the fundamental frequency if the tension in the string is halved?
11. || a. What are the three longest wavelengths for standing waves on a 240-cm-long string that is fixed at both ends?
    b. If the frequency of the second-longest wavelength is 50.0 Hz, what is the frequency of the third-longest wavelength?
12. || A 121-cm-long, 4.00 g string oscillates in its $m = 3$ mode with a frequency of 180 Hz and a maximum amplitude of 5.00 mm. What are (a) the wavelength and (b) the tension in the string?
13. || A guitar string with a linear density of 2.0 g/m is stretched between supports that are 60 cm apart. The string is observed to form a standing wave with three antinodes when driven at a frequency of 420 Hz. What are (a) the frequency of the fifth harmonic of this string and (b) the tension in the string?
14. || A violin string has a standard length of 32.8 cm. It sounds the musical note A (440 Hz) when played without fingering. How far from the end of the string should you place your finger to play the note C (523 Hz)?
15. || The lowest note on a grand piano has a frequency of 27.5 Hz. The entire string is 2.00 m long and has a mass of 400 g. The vibrating section of the string is 1.90 m long. What tension is needed to tune this string properly?

Section 16.4 Standing Sound Waves

16. | The lowest frequency in the audible range is 20 Hz. (a) What are the lengths of (a) the shortest open-open tube and (b) the shortest open-closed tube needed to produce this frequency?
17. | The contrabassoon is the wind instrument capable of sounding the lowest pitch in an orchestra. It is folded over several times to fit its impressive 18 ft length into a reasonable size instrument.
    a. If we model the instrument as an open-closed tube, what is its fundamental frequency? The sound speed inside is 350 m/s because the air is warmed by the player's breath.
    b. The actual fundamental frequency of the contrabassoon is 27.5 Hz, which should be different from your answer in part a. This means the model of the instrument as an open-closed tube is a bit too simple. But if you insist on using that model, what is the "effective length" of the instrument?
18. | Figure P16.18 shows a standing sound wave in an 80-cm-long tube. The tube is filled with an unknown gas. What is the speed of sound in this gas?

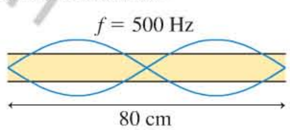


FIGURE P16.18

19. || What are the three longest wavelengths for standing sound waves in a 121-cm-long tube that is (a) open at both ends and (b) open at one end, closed at the other?
20. | The lowest pedal note on a large pipe organ has a fundamental frequency of 16 Hz. This extreme bass note is more felt as a rumble than heard with the ears. What is the length of the open-closed pipe that makes that note?
21. || The fundamental frequency of an open-open tube is 1500 Hz
INT when the tube is filled with 0°C helium. What is its frequency when filled with 0°C air?
22. | *Parasaurolophus* was a
BIO dinosaur whose distinguishing feature was a hollow crest on the head. The 1.5-m-long hollow tube in the crest had connections to the nose and throat, leading some investigators to hypothesize that the tube was a resonant chamber for vocalization. If you model the tube as an open-closed system, what are the first three resonant frequencies?

23. A drainage pipe running under a freeway is 30.0 m long. Both ends of the pipe are open, and wind blowing across one end causes the air inside to vibrate.
    a. If the speed of sound on a particular day is 340 m/s, what will be the fundamental frequency of air vibration in this pipe?
    b. What is the frequency of the lowest harmonic that would be audible to the human ear?
    c. What will happen to the frequency in the later afternoon as the air begins to cool?
24. BIO Although the vocal tract is quite complicated, we can make a simple model of it as an open-closed tube extending from the opening of the mouth to the diaphragm, the large muscle separating the abdomen and the chest cavity. What is the length of this tube if its fundamental frequency equals a typical speech frequency of 200 Hz? Assume a sound speed of 350 m/s. Does this result for the tube length seem reasonable, based on observations on your own body?
25. BIO A child has an ear canal that is 1.3 cm long. At what sound frequencies in the audible range will the child have increased hearing sensitivity?

Section 16.5 Speech and Hearing

26. BIO The first formant of your vocal system can be modeled as the resonance of an open-closed tube, the closed end being your vocal cords and the open end your lips. Estimate the frequency of the first formant from the graph of Figure 16.23, and then estimate the length of the tube of which this is a resonance. Does your result seem reasonable?
27. BIO INT Deep-sea divers often breathe a mixture of helium and oxygen to avoid the complications of breathing high-pressure nitrogen. At great depths the mix is almost entirely helium, which has the side effect of making the divers' voices sound very odd. Breathing helium doesn't affect the frequency at which the vocal cords vibrate, but it does affect the frequencies of the formants. The text gives the frequencies of the first two formants for an "ee" vowel sound as 270 and 2300 Hz. What will these frequencies be for a helium-oxygen mixture in which the speed of sound at body temperature is 750 m/s?

Section 16.6 The Interference of Waves from Two Sources

28. Two loudspeakers in a 20°C room emit 686 Hz sound waves along the x-axis. What is the smallest distance between the speakers for which the interference of the sound waves is destructive?
29. Two loudspeakers emit sound waves along the x-axis. The sound has maximum intensity when the speakers are 20 cm apart. The sound intensity decreases as the distance between the speakers is increased, reaching zero at a separation of 30 cm.
    a. What is the wavelength of the sound?
    b. If the distance between the speakers continues to increase, at what separation will the sound intensity again be a maximum?
30. Two identical loudspeakers separated by distance d emit 170 Hz sound waves along the x-axis. As you walk along the axis, away from the speakers, you don't hear anything even though both speakers are on. What are three possible values for d? Assume a sound speed of 340 m/s.
31. Figure P16.31 shows the circular wave fronts emitted by two sources. Make a table with rows labeled P, Q, and R and columns labeled r_1, r_2, Δr, and C/D. Fill in the table for points P, Q, and R, giving the distances as multiples of λ and indicating, with a C or a D, whether the interference at that point is constructive or destructive.

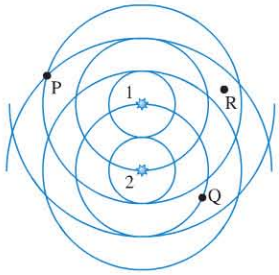


FIGURE P16.31

32. Two identical loudspeakers 2.0 m apart are emitting 1800 Hz sound waves into a room where the speed of sound is 340 m/s. Is the point 4.0 m directly in front of one of the speakers, perpendicular to the plane of the speakers, a point of maximum constructive interference, perfect destructive interference, or something in between?

Section 16.7 Beats

33. Two strings are adjusted to vibrate at exactly 200 Hz. Then the tension in one string is increased slightly. Afterward, three beats per second are heard when the strings vibrate at the same time. What is the new frequency of the string that was tightened?
34. A flute player hears four beats per second when she compares her note to a 523 Hz tuning fork (the note C). She can match the frequency of the tuning fork by pulling out the "tuning joint" to lengthen her flute slightly. What was her initial frequency?

General Problems

35. The fundamental frequency of a standing wave on a 1.0-m-long string is 440 Hz. What would be the wave speed of a pulse moving along this string?
36. BIO INT In addition to producing images, ultrasound can be used to heat tissues of the body for therapeutic purposes. When a sound wave hits the boundary between soft tissue and air, or between soft tissue and bone, most of the energy is reflected; only 0.11% is transmitted. This means that standing waves can be set up in the body, creating excess thermal energy in the tissues at an antinode. Suppose 0.75 MHz ultrasound is directed through a layer of tissue with a bone 0.50 cm below the surface. Will standing waves be created? Explain.
37. INT An 80-cm-long steel string with a linear density of 1.0 g/m is under 200 N tension. It is plucked and vibrates at its fundamental frequency. What is the wavelength of the sound wave that reaches your ear in a 20°C room?
38. BIO INT Tendons are, essentially, elastic cords stretched between two fixed ends; as such, they can support standing waves. These resonances can be undesirable. The Achilles tendon connects the heel with a muscle in the calf. A woman has a 20-cm-long tendon with a cross-section area of 110 mm^2. The density of tendon tissue is 1100 kg/m^3. For a reasonable tension of 500 N, what will be the resonant frequencies of her Achilles tendon?
39. A string, stretched between two fixed posts, forms standing-wave resonances at 325 Hz and 390 Hz. What is the largest possible value of its fundamental frequency?

40. BIO INT Spiders may "tune" strands of their webs to give enhanced response at frequencies corresponding to the frequencies at which desirable prey might struggle. Orb web silk has a typical diameter of 0.0020 mm, and spider silk has a density of 1300 kg/m^3. To give a resonance at 100 Hz, to what tension must a spider adjust a 12-cm-long strand of silk?

41. INT A violinist places her finger so that the vibrating section of a 1.0 g/m string has a length of 30 cm, then she draws her bow across it. A listener nearby in a 20°C room hears a note with a wavelength of 40 cm. What is the tension in the string?

42. INT A particularly beautiful note reaching your ear from a rare Stradivarius violin has a wavelength of 39.1 cm. The room is slightly warm, so the speed of sound is 344 m/s. If the string's linear density is 0.600 g/m and the tension is 150 N, how long is the vibrating section of the violin string?

43. INT A heavy piece of hanging sculpture is suspended by a 90-cm-long, 5.0 g steel wire. When the wind blows hard, the wire hums at its fundamental frequency of 80 Hz. What is the mass of the sculpture?

44. An experimenter finds that standing waves on a 0.80-m-long string, fixed at both ends, occur at 24 Hz and 32 Hz, but at no frequencies in between.
   a. What is the fundamental frequency?
   b. What is the wave speed on the string?
   c. Draw the standing-wave pattern for the string at 32 Hz.

45. INT Astronauts visiting Planet X have a 2.5-m-long string whose mass is 5.0 g. They tie the string to a support, stretch it horizontally over a pulley 2.0 m away, and hang a 1.0 kg mass on the free end. Then the astronauts begin to excite standing waves on the string. Their data show that standing waves exist at frequencies of 64 Hz and 80 Hz, but at no frequencies in between. What is the value of g, the free-fall acceleration, on Planet X?

46. INT A 75 g bungee cord has an equilibrium length of 1.2 m. The cord is stretched to a length of 1.8 m, then vibrated at 20 Hz. This produces a standing wave with two antinodes. What is the spring constant of the bungee cord?

47. A 2.5-cm-diameter steel cable (with density 7900 kg/m^3) that is part of the suspension system for a footbridge stretches 14 m between the tower and the ground. After walking over the bridge, a hiker finds that the cable is vibrating in its fundamental mode with a period of 0.40 s. What is the tension in the cable?

48. Lake Erie is prone to remarkable *seiches*—standing waves that slosh water back and forth in the lake basin from the west end at Toledo to the east end at Buffalo. Figure P16.48 shows smoothed data for the displacement from normal water levels along the lake at the high point of one particular seiche. 3 hours later the water was at normal levels throughout the basin; 6 hours later the water was high in Toledo and low in Buffalo.

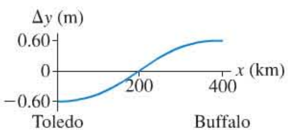


FIGURE P16.48

   a. What is the wavelength of this standing wave?
   b. What is the frequency?
   c. What is the wave speed?

49. INT A steel wire is used to stretch a spring, as shown in Figure P16.49. An oscillating magnetic field drives the steel wire back and forth. A standing wave with three antinodes is created when the spring is stretched 8.0 cm. What stretch of the spring produces a standing wave with two antinodes?

FIGURE P16.49

50. Just as you are about to step into a nice hot bath, a small earthquake rattles your bathroom. Immediately afterward, you notice that the water in the tub is oscillating. The water in the center seems to be motionless while the water at the two ends alternately rises and falls, like a seesaw. You happen to know that your bathtub is 1.4 m long, and you count 10 complete oscillations of the water in 20 s.
   a. What is the wavelength of this standing wave?
   b. What is the speed of the waves that are reflecting back and forth inside the tub to create the standing wave?

51. A microwave generator can produce microwaves at any frequency between 10 GHz and 20 GHz. As Figure P16.51 shows, the microwaves are aimed, through a small hole, into a "microwave cavity" that consists of a 10-cm-long cylinder with reflective ends.

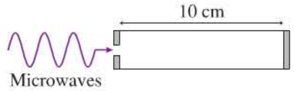


FIGURE P16.51

   a. Which frequencies between 10 GHz and 20 GHz will create standing waves in the microwave cavity?
   b. For which of these frequencies is the cavity midpoint an antinode?

52. An open-open organ pipe is 78.0 cm long. An open-closed pipe has a fundamental frequency equal to the third harmonic of the open-open pipe. How long is the open-closed pipe?

53. INT A carbon-dioxide laser emits infrared light with a wavelength of 10.6 μm.
   a. What is the length of a tube that will oscillate in the $m = 100{,}000$ mode?
   b. What is the frequency?
   c. Imagine a pulse of light bouncing back and forth between the ends of the tube. How many round trips will the pulse make in each second?

54. In 1866, the German scientist Adolph Kundt developed a technique for accurately measuring the speed of sound in various gases. A long glass tube, known today as a Kundt's tube, has a vibrating piston at one end and is closed at the other. Very finely ground particles of cork are sprinkled in the bottom of the tube before the piston is inserted. As the vibrating piston is slowly moved forward, there are a few positions that cause the cork particles to collect in small, regularly spaced piles along the bottom. Figure P16.54 shows an experiment in which the tube is filled with pure oxygen and the piston is driven at 400 Hz.

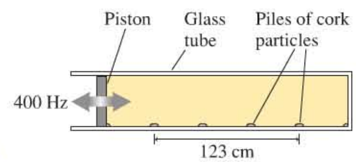


FIGURE P16.54

   a. Do the cork particles collect at standing-wave nodes or antinodes?
      Hint: Consider the appearance of the ends of the tube.
   b. What is the speed of sound in oxygen?

55. A 40-cm-long tube has a 40-cm-long insert that can be pulled in and out, as shown in Figure P16.55. A vibrating tuning fork is held next to the tube. As the insert is slowly pulled out, the sound from the tuning fork creates standing waves in the tube when the total length L is 42.5 cm, 56.7 cm, and 70.9 cm. What is the frequency of the tuning fork? The air temperature is 20°C.

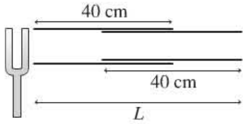


FIGURE P16.55

56. A 1.0-m-tall vertical tube is filled with 20°C water. A tuning fork vibrating at 580 Hz is held just over the top of the tube as the water is slowly drained from the bottom. At what water heights, measured from the bottom of the tube, will there be a standing sound wave in the air at the top of the tube?

57. A 50-cm-long wire with a mass of 1.0 g and a tension of 440 N passes across the open end of an open-closed tube of air. The wire, which is fixed at both ends, is bowed at the center so as to vibrate at its fundamental frequency and generate a sound wave. Then the tube length is adjusted until the fundamental frequency of the tube is heard. What is the length of the tube? Assume the speed of sound is 340 m/s.

58. A 25-cm-long wire with a linear density of 20 g/m passes across the open end of an 85-cm-long open-closed tube of air. If the wire, which is fixed at both ends, vibrates at its fundamental frequency, the sound wave it generates excites the second vibrational mode of the tube of air. What is the tension in the wire? Assume the speed of sound is 340 m/s.

59. Two loudspeakers located along the x-axis as shown in Figure P16.59 produce sounds of equal frequency. Speaker 1 is at the origin, while the location of speaker 2 can be varied by a remote control wielded by the listener. He notices maxima in the sound intensity when speaker 2 is located at $x = 0.75$ m and 1.00 m, but at no points in between. What is the frequency of the sound? Assume the speed of sound is 340 m/s.

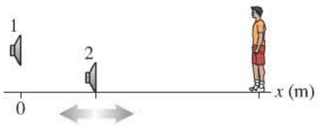


FIGURE P16.59

60. You are standing 2.50 m directly in front of one of the two loudspeakers shown in Figure P16.60. They are 3.00 m apart and both are playing a 686 Hz tone in phase. As you begin to walk directly away from the speaker, at what distances from the speaker do you hear a *minimum* sound intensity? The room temperature is 20°C.

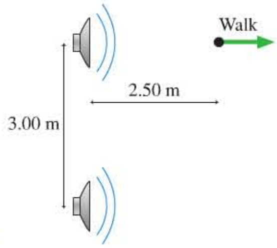


FIGURE P16.60

61. FM station KCOM ("All commercials, all the time") transmits simultaneously, at a frequency of 99.9 MHz, from two broadcast towers placed precisely 31.5 m apart along a north-south line.
   a. What is the wavelength of KCOM's transmissions?
   b. Suppose you stand 90.0 m due east of the point halfway between the two towers with your portable FM radio. Will you receive a strong or weak signal at this position? Why?
   c. You then stand 90.0 m due north of the northern tower with your radio. Will you receive a strong or weak signal at this position? Why?

62. Two loudspeakers, 4.0 m apart and facing each other, play identical sounds of the same frequency. You stand halfway between them, where there is a maximum of sound intensity. Moving from this point toward one of the speakers, you encounter a minimum of sound intensity when you have moved 0.25 m.
   a. What is the frequency of the sound?
   b. If the frequency is then increased while you remain 0.25 m from the center, what is the first frequency for which that location will be a maximum of sound intensity?

63. Two radio antennas are separated by 2.0 m. Both broadcast identical 750 MHz waves. If you walk around the antennas in a circle of radius 10 m, how many maxima will you detect?

64. BIO Certain birds produce vocalizations consisting of two distinct frequencies that are not harmonically related—that is, the two frequencies are not harmonics of a common fundamental frequency. These two frequencies must be produced by two different vibrating structures in the bird's vocal tract.
   a. Wood ducks have been observed to make a call with approximately equal intensities at 850 Hz and 1200 Hz. The membranes that produce the vocalizations do not seem to vibrate at frequencies less than 500 Hz. Given this limitation, could these two frequencies be higher harmonics of a lower-frequency fundamental?
   b. If we model the duck's vocal tract as an open-closed tube, what length has a fundamental frequency equal to the lower of the two frequencies in part a?

65. Piano tuners tune pianos by listening to the beats between the harmonics of two different strings. When properly tuned, the note A should have the frequency 440 Hz and the note E should be at 659 Hz. The tuner can determine this by listening to the beats between the third harmonic of the A and the second harmonic of the E.
   a. A tuner first tunes the A string very precisely by matching it to a 440 Hz tuning fork. She then strikes the A and E strings simultaneously and listens for beats between the harmonics. What beat frequency indicates that the E string is properly tuned?
   b. The tuner starts with the tension in the E string a little low, then tightens it. What is the frequency of the E string when she hears four beats per second?

66. A flutist assembles her flute in a room where the speed of sound is 342 m/s. When she plays the note A, it is in perfect tune with a 440 Hz tuning fork. After a few minutes, the air inside her flute has warmed to where the speed of sound is 346 m/s.
   a. How many beats per second will she hear if she now plays the note A as the tuning fork is sounded?
   b. How far does she need to extend the "tuning joint" of her flute to be in tune with the tuning fork?

67. A student waiting at a stoplight notices that her turn signal, which has a period of 0.85 s, makes one blink exactly in sync with the turn signal of the car in front of her. The blinker of the car ahead then starts to get ahead, but 17 s later the two are exactly in sync again. What is the period of the blinker of the other car?

68. ||| Musicians can use beats to tune their instruments. One flute is properly tuned and plays the musical note A at exactly 440 Hz. A second player sounds the same note and hears that her instrument is slightly "flat" (i.e., at too low a frequency). Playing at the same time as the first flute, she hears two loud-soft-loud beats per second. Must she shorten or lengthen her flute, and by how much, to bring it into tune? Assume a speed of sound of 350 m/s.

69. ||| INT Police radars determine speed by measuring the Doppler shift of radio waves reflected by a moving vehicle. They do so by determining the beat frequency between the reflected wave and the 10.5 GHz emitted wave. Some units can be calibrated by using a tuning fork; holding a vibrating fork in front of the unit causes the display to register a speed corresponding to the vibration frequency. A tuning fork is labeled "55 mph." What is the frequency of the tuning fork?

70. || BIO INT A Doppler blood flow meter emits ultrasound at a frequency of 5.0 MHz. What is the beat frequency between the emitted waves and the waves reflected from blood cells moving away from the emitter at 0.15 m/s?

71. | BIO INT An ultrasound unit is being used to measure a patient's heartbeat by combining the emitted 2.0 MHz signal with the sound waves reflected from the moving tissue of one point on the heart. The beat frequency between the two signals has a maximum value of 520 Hz. What is the maximum speed of the heart tissue?

Passage Problems

Harmonics and Harmony

You know that certain musical notes sound good together—harmonious—whereas others do not. This harmony is related to the various harmonics of the notes.

The musical notes C (262 Hz) and G (392 Hz) make a pleasant sound when played together; we call this consonance. As Figure P16.72 shows, the harmonics of the two notes are either far from each other or very close to each other (within a few Hz). This is the key to consonance: harmonics that are spaced either far apart or very close. The close harmonics have a beat frequency of a few Hz that is perceived as pleasant. If the harmonics of two notes are close but not too close, the rather high beat frequency between the two is quite unpleasant. This is what we hear as dissonance. Exactly how much a difference is maximally dissonant is a matter of opinion, but harmonic separations of 30 or 40 Hz seem to be quite unpleasant for most people.

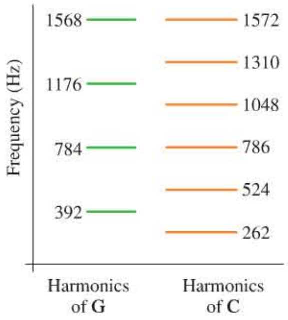


FIGURE P16.72

72. | What is the beat frequency between the second harmonic of G and the third harmonic of C?
A. 1 Hz B. 2 Hz C. 4 Hz D. 6 Hz

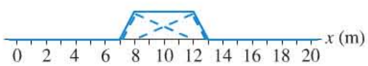
73. | Would a G-flat (frequency 370 Hz) and a C played together be consonant or dissonant?
A. Consonant
B. Dissonant

74. | An organ pipe open at both ends is tuned so that its fundamental frequency is a G. How long is the pipe?
A. 43 cm B. 87 cm C. 130 cm D. 173 cm

75. | If the C were played on an organ pipe that was open at one end and closed at the other, which of the harmonic frequencies in Figure 16.72 would be present?
A. All of the harmonics in the figure would be present.
B. 262, 786, and 1310 Hz
C. 524, 1048, and 1572 Hz
D. 262, 524, and 1048 Hz

STOP TO THINK ANSWERS

Stop to Think 16.1: C. The figure shows the two waves at $t = 6$ s and their superposition. The superposition is the *point-by-point* addition of the displacements of the two individual waves.



Stop to Think 16.2: C. Standing-wave frequencies are $f_m = mf_1$. The original wave has frequency $f_2 = 2f_1$ because it has two antinodes. The wave with frequency $2f_2 = 4f_1$ is the $m = 4$ mode with four antinodes.

Stop to Think 16.3: B. 300 Hz and 400 Hz are not f_1 and f_2 because $400\text{ Hz} \neq 2 \times 300\text{ Hz}$. Instead, both are multiples of the fundamental frequency. Because the difference between them is 100 Hz, we see that $f_3 = 3 \times 100$ Hz and $f_4 = 4 \times 100$ Hz. Thus $f_1 = 100$ Hz.

Stop to Think 16.4: D. Highest pitch, or highest frequency, corresponds to the shortest period. For a complex wave, the period is the time required for the entire wave pattern to repeat.

Stop to Think 16.5: Constructive interference. The path-length difference is $\Delta r = 1.0\text{ m} = \lambda$. Interference is constructive when the path-length difference is a whole number of wavelengths.

Stop to Think 16.6: F. The beat frequency is the difference between the two frequencies.

PART IV SUMMARY

Oscillations and Waves

As we have studied oscillations and waves, one point we have emphasized is the *unity* of the basic physics. The mathematics of oscillation describes the motion of a mass on a spring or the motion of a gibbon swinging from a branch. The same theory of waves works for string waves and sound waves and light waves. A few basic ideas enable us to understand a wide range of physical phenomena.

The physics of oscillations and waves is not quite as easily summarized as the physics of particles. Newton's laws and the conservation laws are two very general sets of principles about particles, principles that allowed us to develop the powerful problem-solving strategies of Parts I and II. The knowledge structure of oscillations and waves, shown below, rests more heavily on *phenomena* than on general principles. This knowledge structure doesn't contain a problem-solving strategy or a wide range of general principles, but is instead a logical grouping of the major topics you studied. This is a different way of structuring knowledge, but it still provides you with a mental framework for analyzing and thinking about wave problems.

The physics of oscillations and waves will be with us for the rest of the book. Part V is an exploration of optics, beginning with a detailed study of light as a wave. In Part VI, we will study the nature of electromagnetic waves in more detail. Finally, in Part VII, we will see that matter has a wave nature. There, the wave ideas we have developed in Part IV will lead us into the exciting world of quantum physics, finishing with some quite remarkable insights about the nature of light and matter.

KNOWLEDGE STRUCTURE IV Oscillations and Waves

BASIC GOALS
How can we describe oscillatory motion?
How does a wave travel through a medium?
What are the distinguishing features of waves?
What happens when two waves meet?

GENERAL PRINCIPLES
Simple harmonic motion occurs when a linear restoring force acts to return a system to equilibrium.
The period of simple harmonic motion depends on physical properties of the system but not the amplitude.
All sinusoidal waves, whether water waves, sound waves, or light waves, have the same functional form.
When two waves meet, they pass through each other, combining where they overlap by superposition.

Simple harmonic motion

$$x(t) = A\cos(2\pi ft)$$

$$v_x(t) = -v_{\max}\sin(2\pi ft)$$
$$v_{\max} = 2\pi fA$$

$$a_x(t) = -a_{\max}\cos(2\pi ft)$$
$$a_{\max} = (2\pi f)^2 A$$

The frequency of a mass on a spring depends on the spring constant and the mass:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The frequency of a pendulum depends on the length and the free-fall acceleration:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Traveling waves

All traveling waves are described by the same equation:

$$y = A\cos\left(2\pi\left(\frac{x}{\lambda} \pm \frac{t}{T}\right)\right)$$

+: wave travels to left
−: wave travels to right

The wave speed depends on the properties of the medium. $v_{\text{string}} = \sqrt{\dfrac{T_s}{\mu}}$

The speed, wavelength, period, and frequency of a wave are related. $T = \dfrac{1}{f}$ $\quad v = f\lambda$

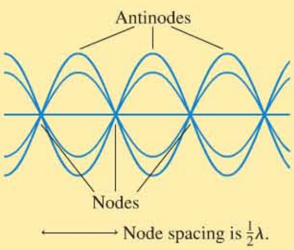
Superposition and interference

The superposition of two waves is the sum of the displacements of the two individual waves.

Constructive interference occurs when crests are aligned with crests and troughs with troughs. Constructive interference occurs when the path-length difference is a whole number of wavelengths.

Destructive interference occurs when crests are aligned with troughs. Destructive interference occurs when the path-length difference is a whole number of wavelengths plus half a wavelength.

Standing waves



Standing waves are due to the superposition of two traveling waves moving in opposite directions.

The boundary conditions determine which standing-wave frequencies and wavelengths are allowed. The allowed standing waves are modes of the system.

For a string of length L, the modes have wavelength and frequency

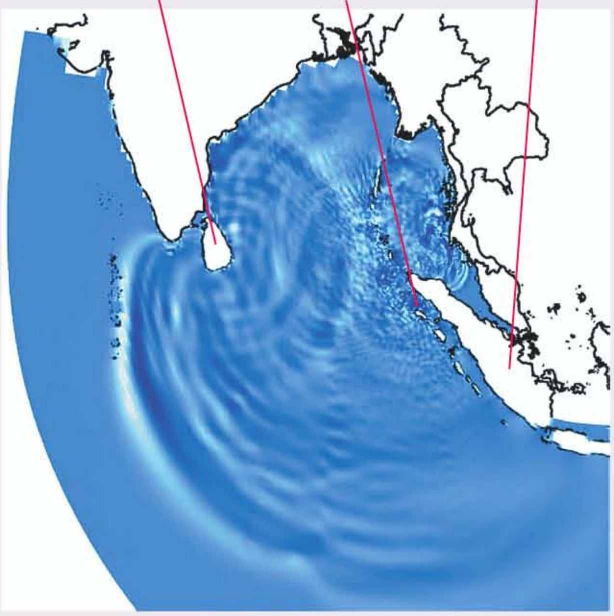
$$\lambda_m = \frac{2L}{m} \qquad f_m = m\left(\frac{v_{\text{string}}}{2L}\right) = mf_1 \qquad m = 1, 2, 3, 4, \ldots$$

Waves in the Earth and the Ocean

In December 2004, a large earthquake off the coast of Indonesia produced a devastating water wave, called a *tsunami,* that caused tremendous destruction thousands of miles away from the earthquake's epicenter. The tsunami was a dramatic illustration of the energy carried by waves.

It was also a call to action. Many of the communities hardest hit by the tsunami were struck hours after the waves were generated, long after seismic waves from the earthquake that passed through the earth had been detected at distant recording stations, long after the possibility of a tsunami was first discussed. With better detection and more accurate models of how a tsunami is formed and how a tsunami propagates, the affected communities could have received advance warning. The study of physics may seem an abstract undertaking with few practical applications, but on this day a better scientific understanding of these waves could have averted tragedy.

Let's use our knowledge of waves to explore the properties of a tsunami. In Chapter 15, we saw that a vigorous shake of one end of a rope causes a pulse to travel along it, carrying energy as it goes. The earthquake that produced the Indian Ocean tsunami of 2004 caused a sudden upward displacement of the seafloor that produced a corresponding rise in the surface of the ocean. This was the *disturbance* that produced the tsunami, very much like a quick shake on the end of a rope. The resulting wave propagated through the ocean, as we see in the figure.



One frame from a computer simulation of the Indian Ocean tsunami three hours after the earthquake that produced it. The disturbance propagating outward from the earthquake is clearly seen, as are wave reflections from the island of Sri Lanka.

This simulation of the tsunami looks much like the ripples that spread when you drop a pebble into a pond. But there is a big difference—the scale. The fact that you can see the individual waves on this diagram that spans 5000 km is quite revealing. To show up so clearly, the individual wave pulses must be very wide—up to hundreds of kilometers from front to back.

A tsunami is actually a "shallow water wave," even in the deep ocean, because the depth of the ocean is much less than the width of the wave. Consequently, a tsunami travels differently than normal ocean waves. In Chapter 15 we learned that wave speeds are fixed by the properties of the medium. That is true for normal ocean waves, but the great width of the wave causes a tsunami to "feel the bottom." Its wave speed is determined by the depth of the ocean: The greater the depth, the greater the speed. In the deep ocean, a tsunami travels at hundreds of kilometers per hour, much faster than a typical ocean wave. Near shore, as the ocean depth decreases, so does the speed of the wave.

The height of the tsunami in the open ocean was about half a meter. Why should such a small wave—one that ships didn't even notice as it passed—be so fearsome? Again, it's the *width* of the wave that matters. Because a tsunami is the wave motion of a considerable mass of water, great energy is involved. As the front of a tsunami wave nears shore, its speed decreases, and the back of the wave moves faster than the front. Consequently, the width decreases. The water begins to pile up, and the wave dramatically increases in height.

The Indian Ocean tsunami had a height of up to 15 m when it reached shore, with a width of up to several kilometers. This tremendous mass of water was still moving at high speed, giving it a great deal of energy. A tsunami reaching the shore isn't like a typical wave that breaks and crashes. It is a kilometers-wide wall of water that moves onto the shore and just keeps on coming. In many places, the water reached 2 km inland.

The impact of the Indian Ocean tsunami was devastating, but it was the first tsunami for which scientists were able to use satellites and ocean sensors to make planet-wide measurements. An analysis of the data has helped us better understand the physics of these ocean waves. We won't be able to stop future tsunamis, but with a better knowledge of how they are formed and how they travel, we will be better able to warn people to get out of their way.

The following questions are related to the passage "Waves in the Earth and the Ocean" on the previous page.

1. Rank from fastest to slowest the following waves according to their speed of propagation:
   A. An earthquake wave B. A tsunami
   C. A sound wave in air D. A light wave
2. The increase in height as a tsunami approaches shore is due to
   A. The increase in frequency as the wave approaches shore.
   B. The increase in speed as the wave approaches shore.
   C. The decrease in speed as the wave approaches shore.
   D. The constructive interference with the wave reflected from shore.
3. In the middle of the Indian Ocean, the tsunami referred to in the passage was a train of pulses approximating a sinusoidal wave with speed 200 m/s and wavelength 150 km. What was the approximate period of these pulses?
   A. 1 min B. 3 min
   C. 5 min D. 15 min
4. If a train of pulses moves into shallower water as it approaches a shore,
   A. The wavelength increases.
   B. The wavelength stays the same.
   C. The wavelength decreases.
5. The tsunami described in the passage produced a very erratic pattern of damage, with some areas seeing very large waves and nearby areas seeing only small waves. Which of the following is a possible explanation?
   A. Certain areas saw the wave from the primary source, others only the reflected waves.
   B. The superposition of waves from the primary source and reflected waves produced regions of constructive and destructive interference.
   C. A tsunami is a standing wave, and certain locations were at nodal positions, others at antinodal positions.

The following passages and associated questions are based on the material of Part IV.

Deep-Water Waves

Water waves are called *deep-water waves* when the depth of the water is much greater than the wavelength of the wave. The speed of deep-water waves depends on the wavelength as follows:

$$v = \sqrt{\frac{g\lambda}{2\pi}}$$

Suppose you are on a ship at rest in the ocean, observing the crests of a passing sinusoidal wave. You estimate that the crests are 75 m apart.

6. Approximately how much time elapses between one crest reaching your ship and the next?
   A. 3 s
   B. 5 s
   C. 7 s
   D. 12 s
7. The captain starts the engines and sails directly opposite the motion of the waves at 4.5 m/s. Now how much time elapses between one crest reaching your ship and the next?
   A. 3 s
   B. 5 s
   C. 7 s
   D. 12 s
8. In the deep ocean, a longer-wavelength wave travels faster than a shorter-wavelength wave. Thus, a higher-frequency wave travels ______ a lower-frequency wave.
   A. Faster than
   B. At the same speed as
   C. Slower than

Attenuation of Ultrasound BIO

Ultrasound is absorbed in the body; this complicates the use of ultrasound to image tissues. The intensity of a beam of ultrasound decreases by a factor of 2 after traveling a distance of 40 wavelengths. Each additional travel of 40 wavelengths results in a decrease by another factor of 2.

9. A beam of 1.0 MHz ultrasound begins with an intensity of 1000 W/m^2. After traveling 12 cm through tissue with no significant reflection, the intensity is about
   A. 750 W/m^2 B. 500 W/m^2
   C. 250 W/m^2 D. 125 W/m^2
10. A physician is making an image with ultrasound of initial intensity 1000 W/m^2. When the frequency is set to 1.0 MHz, the intensity drops to 500 W/m^2 at a certain depth in the patient's body. What will be the intensity at this depth if the physician changes the frequency to 2.0 MHz?
    A. 750 W/m^2 B. 500 W/m^2
    C. 250 W/m^2 D. 125 W/m^2
11. A physician is using ultrasound to make an image of a patient's heart. Increasing the frequency will provide
    A. Better penetration and better resolution.
    B. Less penetration but better resolution.
    C. More penetration but worse resolution.
    D. Less penetration and worse resolution.
12. A physician is using Doppler ultrasound to measure the motion of a patient's heart. The device measures the beat frequency between the emitted and the reflected waves. Increasing the frequency of the ultrasound will
    A. Increase the beat frequency.
    B. Not affect the beat frequency.
    C. Decrease the beat frequency.

Measuring the Speed of Sound

A student investigator is measuring the speed of sound by looking at the time for a brief, sinusoidal pulse from a loudspeaker to travel down a tube, reflect from the closed end, and reach a microphone. The apparatus is shown in Figure IV.1a; typical data recorded by the microphone are graphed in Figure IV.1b. The first pulse is the sound directly from the loudspeaker; the second pulse is the reflection from the closed end. A portion of the returning wave reflects from the open end of the tube and makes another round trip before being detected by the microphone; this is the third pulse seen in the data.

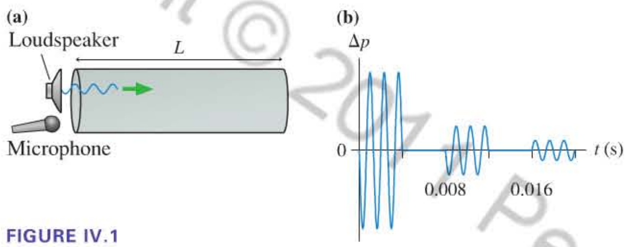


FIGURE IV.1

13. What was the approximate frequency of the sound wave used in this experiment?
    A. 250 Hz
    B. 500 Hz
    C. 750 Hz
    D. 1000 Hz
14. What can you say about the reflection of sound waves at the ends of a tube?
    A. Sound waves are inverted when reflected both from open and closed tube ends.
    B. Sound waves are inverted when reflected from a closed end, not inverted when reflected from an open end.
    C. Sound waves are inverted when reflected from an open end, not inverted when reflected from a closed end.
    D. Sound waves are not inverted when reflected either from open or closed tube ends.
15. What was the approximate length of the tube?
    A. 0.35 m
    B. 0.70 m
    C. 1.4 m
    D. 2.8 m
16. An alternative technique to determine sound speed is to measure the frequency of a standing wave in the tube. What is the wavelength of the lowest resonance of this tube?
    A. $L/2$
    B. L
    C. $2L$
    D. $4L$

In the Swing

A rope swing is hung from a tree right at the edge of a small creek. The rope is 5.0 m long; the creek is 3.0 m wide.

17. You sit on the swing, and your friend gives you a gentle push so that you swing out over the creek. How long will it be until you swing back to where you started?
    A. 4.5 s B. 3.4 s
    C. 2.2 s D. 1.1 s
18. Now you switch places with your friend, who has twice your mass. You give your friend a gentle push so that he swings out over the creek. How long will it be until he swings back to where he started?
    A. 4.5 s B. 3.4 s
    C. 2.2 s D. 1.1 s
19. Your friend now pushes you over and over, so that you swing higher and higher. At some point you are swinging all the way across the creek—at the top point of your arc you are right above the opposite side. How fast are you moving when you get back to the lowest point of your arc?
    A. 6.3 m/s B. 5.4 m/s
    C. 4.4 m/s D. 3.1 m/s

Additional Integrated Problems

20. BIO The jumping gait of the kangaroo is efficient because energy is stored in the stretch of stout tendons in the legs; the kangaroo literally bounces with each stride. We can model the bouncing of a kangaroo as the bouncing of a mass on a spring. A 70 kg kangaroo hits the ground, the tendons stretch to a maximum length, and the rebound causes the kangaroo to leave the ground approximately 0.10 s after its feet first touch.
    a. Modeling this as the motion of a mass on a spring, what is the period of the motion?
    b. Given the kangaroo mass and the period you've calculated, what is the spring constant?
    c. If the kangaroo speeds up, it must bounce higher and farther with each stride, and so must store more energy in each bounce. How does this affect the time and the amplitude of each bounce?
21. BIO A brand of earplugs reduces the sound intensity level by 27 dB. By what factor do these earplugs reduce the acoustic intensity?
22. BIO Sperm whales, just like bats, use echolocation to find prey. A sperm whale's vocal system creates a single sharp click, but the emitted sound consists of several equally spaced clicks of decreasing intensity. Researchers use the time interval between the clicks to estimate the size of the whale that created them. Explain how this might be done.

    Hint: The head of a sperm whale is complex, with air pockets at either end.