19 Optical Instruments

This *anableps* is called the "four-eyed fish." How must the top half of its eye differ from the lower half so that it has clear vision both above and below the waterline at the same time?

LOOKING AHEAD

The goal of Chapter 19 is to understand how common optical instruments work.

Cameras

A **camera** uses a lens to project a real image onto a light-sensitive detector.

Although a modern digital camera is an extremely complex device, at its heart it is just a light-tight box with a lens to focus an image.

Looking Back
18.5 Lenses, images, and magnification

Optical Systems That Magnify

Lenses and mirrors can be used to magnify nearby objects (magnifiers and microscopes) or distant ones (telescopes).

Looking Back
18.6 Image formation with curved mirrors

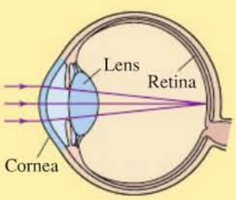
A **magnifier** uses a convex lens to achieve magnifications up to about 20.

Microscopes use two sets of lenses in combination to magnify objects as much as 1000 times.

Large **telescopes** use a curved mirror as their main optical element. The mirror of this telescope is 8 m in diameter!

The Human Eye

The eye functions much like a camera: The cornea and lens focus a real image onto the light-sensitive surface of the retina.



You'll learn how the lenses in eyeglasses and contacts can correct for near- and farsightedness.

Color and Dispersion

White light entering a prism is **dispersed** into a rainbow of colors.

A prism separates white light into its constituent colors.

The dispersion of sunlight by raindrops causes the colors seen in a rainbow.

Resolution of Optical Instruments

There are limits to the ability of a microscope or telescope to make out the fine details of an object being viewed.

Seen at high power, the diffraction patterns of these two nearby stars almost merge, making it hard to tell them apart.

This simple lens suffers from distortions and chromatic (color) aberrations.

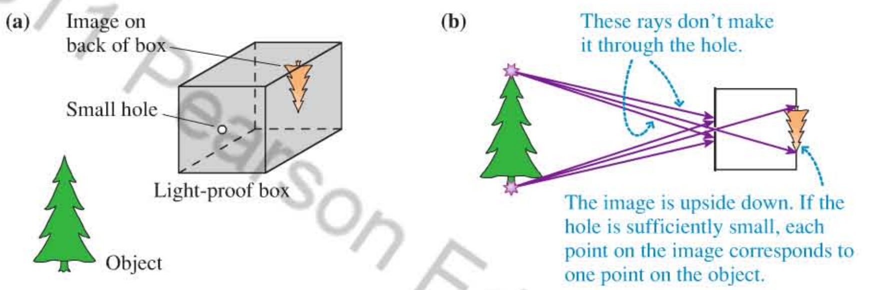
Looking Back
17.6 Circular-aperture diffraction

A pinhole eye BIO The chambered nautilus is the only animal with a true pinhole "camera" as an eye. Light rays passing through the small opening form a crude image on the back surface of the eye, where the rays strike light-sensitive cells. The image may be poor, but it's sufficient to allow the nautilus to catch prey and escape predators.

19.1 The Camera

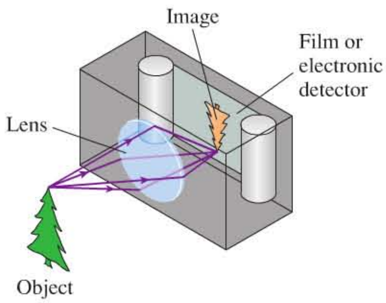
This chapter will investigate a number of optical instruments in which a combination of lenses and mirrors performs a useful function. We'll start with an instrument familiar to everyone: the camera. A **camera** is a device that projects a real image onto a plane surface, where the image can be recorded onto film or, in today's digital cameras, an electronic detector. Although modern cameras are marvels of optical engineering, it is possible to produce decent images using only a light-proof box with a small hole punched in it. Such a **pinhole camera** is shown in FIGURE 19.1a. FIGURE 19.1b uses the ray model of light passing through a small hole to illustrate how the pinhole camera works. Each point on an object emits light rays in all directions, but, ideally, only one of these rays passes through the hole and reaches the film. Each point on the object thus illuminates just one point on the film, forming the image. As the figure illustrates, the geometry of the rays causes the image to be upside down.

FIGURE 19.1 A pinhole camera.



Actually, as you may have realized, each *point* on the object illuminates a small but finite *patch* on the film. This is because the finite size of the hole allows several rays from each point on the object to pass through at slightly different angles. As a result, the image is slightly blurred. Maximum sharpness is achieved by making the hole smaller and smaller, which makes the image dimmer and dimmer. (Diffraction also becomes an issue if the hole gets too small.) A real pinhole camera has to accept a small amount of blurring as the trade-off for having an image bright enough to be practical.

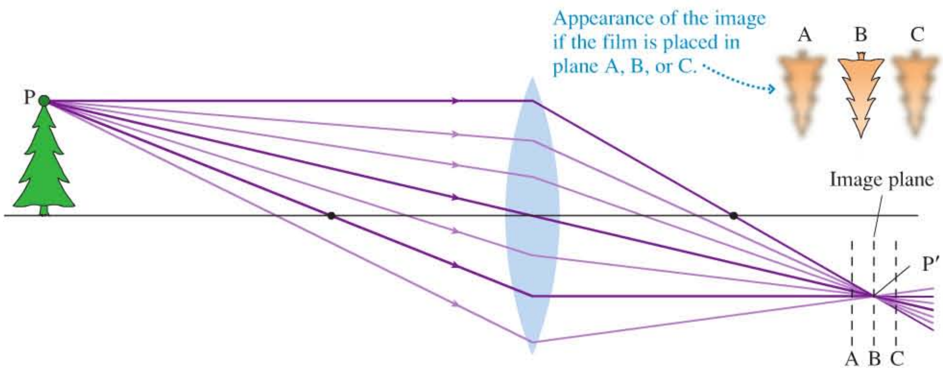
FIGURE 19.2 A camera.



In a standard camera, a dramatic improvement is made possible by using a *lens* in place of a pinhole. FIGURE 19.2 shows how a camera's converging lens projects an inverted real image onto the film, just as a pinhole camera does. Unlike a pinhole, however, a lens can be large, letting in plenty of light while still giving a sharply focused image. A shutter (not shown), an opaque barrier, is briefly moved out of the way in order for light to pass through the lens.

FIGURE 19.3 shows light rays from an object passing through the lens and converging at the image plane B. Here, a single point P on the object focuses to a single point P′

FIGURE 19.3 Focusing a camera.



in the image plane. If the film or electronic detector is located in this plane, a sharp image will form on it. If, however, the film had been located a bit in *front* of the image plane, at position A, rays from point P would not yet have completely converged and would form a small blurry *circle* on the film instead of a sharp point. Thus the image would appear blurred, as shown. Similarly, if the film is placed at C, *behind* the image plane, the rays will be diverging from their perfect focus and again form a blurry image.

Thus to get a sharp image, the film must be accurately located in the image plane. Figure 19.3 shows that one way to do this is to move the film until it coincides with the image plane. More commonly, however, a camera is **focused** by moving the *lens* either toward or away from the film plane until the image is sharp. In either case, the lens-film distance is varied.

EXAMPLE 19.1 Focusing a camera

A digital camera whose lens has a focal length of 8.0 mm is used to take a picture of an object 30 cm away. What must be the distance from the lens to the light-sensitive detector in order for the image to be in focus?

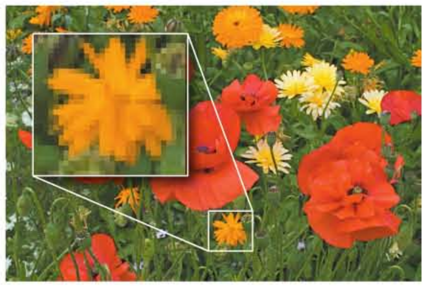
PREPARE As shown in Figure 19.3, the image will be in focus when the detector is in the image plane. Thus we need to find the image distance, knowing the object distance $s = 30$ cm and the lens's focal length $f = 8.0$ mm.

SOLVE We can rearrange the thin-lens equation, Equation 18.11, to solve for the image distance s':

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{0.0080 \text{ m}} - \frac{1}{0.30 \text{ m}} = 122 \text{ m}^{-1}$$

Thus $s' = 1/122 \text{ m}^{-1} = 0.0082 \text{ m} = 8.2$ mm. The lens-detector distance has to be 8.2 mm.

ASSESS When the object is infinitely far away, the image, by definition, is at the focal length: $s' = f = 8.0$ mm. If the object is brought to 30 cm, the lens has to move forward a distance of only 8.2 mm − 8.0 mm = 0.2 mm to bring the object into focus. In general, camera lenses don't need to move far.

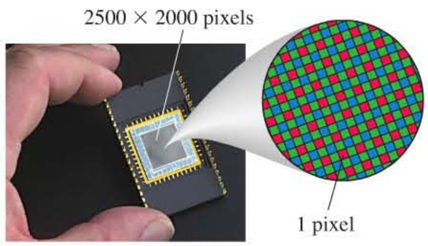


If a digital picture is magnified enough, you can see the individual pixels that make it up.

For traditional cameras, the light-sensitive surface is **film.** Light striking microscopic silver halide crystals creates small particles of silver metal on the crystals. The development process converts the entire crystal to dark metallic silver. Thus the film appears *dark* where light has hit it, forming a *negative* image. To get a positive image—your print—the negative image is projected onto paper with its own photosensitive coating. This results in a second negative process, leading to a positive print. Today's *digital cameras* use an electronic light-sensitive detector called a *charge-coupled device* or **CCD.** A CCD consists of a rectangular array of many millions of small detectors called *pixels.* When light hits one of these pixels, it generates an electric charge proportional to the light intensity. Thus an image is recorded on the CCD in terms of little packets of charge. After the CCD has been exposed, the charges are read out, the signal levels are digitized, and the picture is stored in the digital memory of the camera.

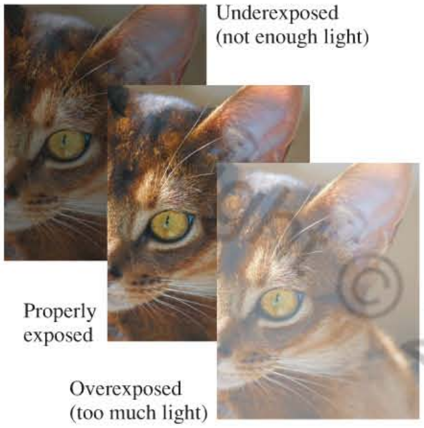
FIGURE 19.4 shows a CCD "chip" and, schematically, the magnified appearance of the pixels on its surface. To record color information, different pixels are covered by red, green, or blue filters; a pixel covered by a green filter, for instance, records only the intensity of the green light hitting it. Later, the camera's microprocessor interpolates nearby colors to give each pixel an overall true color. The structure of the retina of the eye is remarkably similar, as we'll see in Chapter 25.

FIGURE 19.4 A CCD chip used in a digital camera.



Controlling the Exposure

FIGURE 19.5 Photos with increasing amounts of light reaching the film.



The camera also must control the amount of light reaching the detector. As FIGURE 19.5 shows, too little light results in photos that are *underexposed;* too much light gives *overexposed* pictures. Both the shutter and the lens diameter help control the exposure.

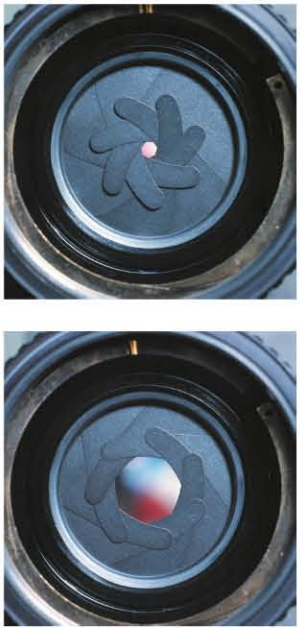
The *shutter* is "opened" for a selected amount of time as the image is recorded. Older cameras used a spring-loaded mechanical shutter that literally opened and closed; digital cameras electronically control the amount of time the detector is active. Either way, the exposure—the amount of light captured by the detector—is directly proportional to the time the shutter is open. Typical exposure times range from 1/1000 s or less for a sunny scene to 1/30 s or more for dimly lit or indoor scenes. The exposure time is generally referred to as the *shutter speed;* a very short exposure, such as 1/1000 s is called a "fast shutter speed," while a much longer exposure is a "slow shutter speed."

A second means of controlling the exposure is to effectively change the diameter d of the lens. A small lens will let in less light than a large one. Of course, the actual diameter of the lens cannot be changed. Instead, an *iris diaphragm,* placed behind the lens, is adjusted to change the amount of light entering the camera, as shown in FIGURE 19.6.

An important measure of the light-gathering ability of a lens is its ***f*-number,** defined as

$$f\text{-number} = \frac{\text{lens focal length}}{\text{lens diameter}} = \frac{f}{d}$$

FIGURE 19.6 A camera's iris can change the effective diameter of the lens.



We use the notation $f/11$ ("f 11"), for instance, to indicate an f-number of 11. The f-number of a lens directly determines the *brightness* of the image on the film. Somewhat contrary to common sense, the *smaller* the f-number, the *brighter* the image. Here's why: For a given focal length, more light enters a lens with a larger diameter than one with a smaller diameter. But since the f-number is inversely proportional to d, the larger-diameter, brighter lens has the lower f-number. Further, a lens of a given diameter will give a brighter image when the focal length f is *small.* To see why, consider a lens imaging a distant object, so that the image is focused at the far focal point. If f is large, the image is large and its light is spread out and dim. If f is small, the image is small and the light is concentrated and bright.

A lens with its diaphragm fully open will let in the most possible light. But under bright light conditions we may need to close, or *stop down,* the diaphragm to control the exposure. How much must the diaphragm be closed in order to let in half as much light? The amount of light entering the lens is proportional to the *area* of the lens, or to d^2. To let in half as much light, we must reduce d^2 by a factor of 2 or, equivalently, reduce the diameter of the lens by a factor of $1/\sqrt{2}$. This means that the f-number, which is inversely proportional to the diameter, will *increase* by a factor of $\sqrt{2} \approx 1.4$. On most camera lenses, the ***f*-stops** increase in the series 1.4, 2, 2.8, 4, 5.6, 8, 11, 16. Each f-number is $\sqrt{2}$ times larger than the preceding one, so an increase of one f-stop cuts the light intensity in half.

CONCEPTUAL EXAMPLE 19.2 Adjusting a camera lens

A camera takes a perfectly exposed picture when the shutter speed is 1/60 s and the lens diaphragm is set to $f/11$. What f-stop should be used to get a correctly exposed picture with a shutter speed of 1/250 s?

REASON We need the same amount of light hitting the film or CCD in both cases. For the second picture the shutter is open only about one-fourth as long, since

$$\frac{1/250\text{ s}}{1/60\text{ s}} = \frac{60}{250} = \frac{1}{4.17} \approx \frac{1}{4}$$

To compensate for the reduced time, we need to increase the light intensity by a factor of 4. Because each stop lets in twice as much light as the preceding stop, the light intensity will be 4 times as great if we open up the diaphragm by two stops. From the sequence given above, opening the iris by two stops from $f/11$ is $f/5.6$.

ASSESS Camera film and CCD detectors yield a good image over a rather wide range of exposures. Thus the fact that shutter speeds and f-numbers change the exposure by factors of 2 still allows us to get the exposure close enough for a good picture.

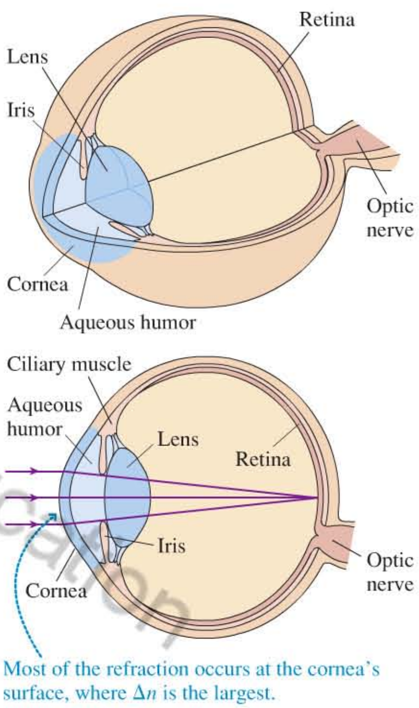
STOP TO THINK 19.1 A camera takes a correctly exposed picture with a certain lens. The lens is then replaced by one with the same diameter but twice the focal length. To get the correct exposure when the focal length is doubled, the shutter speed should

A. Be increased. B. Be decreased. C. Remain the same.

19.2 The Human Eye

The human eye functions much like a camera. Like the camera, it has three main functional groups: an optical system to focus the incoming light, a diaphragm to adjust the amount of light entering the eye, and a light-sensitive surface to detect the resulting image. The parts of the eye making up these three groups are shown in **FIGURE 19.7**. The *cornea,* the *aqueous humor,* and the *lens* are together responsible for refracting incoming light rays and producing an image. The adjustable *iris* determines how much light enters the eye, in much the same way as does the diaphragm of a camera. And the *retina* is the light-sensitive surface on which the image is formed. The retina is the biological equivalent of the CCD in a digital camera.

FIGURE 19.7 The human eye.

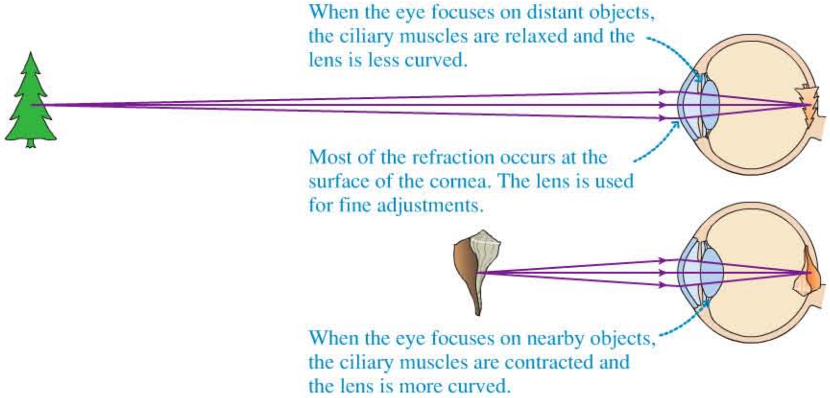


Focusing and Accommodation

Like a camera, the eye works by focusing incoming rays onto a light-sensitive surface, here the retina. To do so, light is refracted by, in turn, the cornea, the aqueous humor, and the lens, as shown in Figure 19.7. The indices of refraction of these parts of the eye vary somewhat, but average around 1.4. Perhaps surprisingly, most of the eye's refraction occurs not in the lens but at the surface of the cornea. This is due both to the strong curvature of the cornea and to the large difference between the indices of refraction on either side of the surface. Light refracts less as it passes through the lens because the lens's index of refraction doesn't differ much from that of the fluid in which it is embedded. If the lens is surgically removed, which it often is for people with cataracts (a clouding of the lens), the cornea alone still provides a marginal level of vision.

The eye must constantly refocus as it views distant objects, then closer ones. It does this so automatically that we're not normally aware of the process. A camera focuses by changing the distance between the lens and the film, but your eye focuses in a different way: by changing the focal length of the lens itself. As shown in **FIGURE 19.8**, it does so by using the ciliary muscles to *change the shape* of the lens. This process of changing the lens shape as the eye focuses at different distances is called **accommodation.**

FIGURE 19.8 Accommodation by the eye.



TRY IT YOURSELF

Inverted vision Just like a camera, the lens of the eye produces an *inverted* image on the retina. The brain is wired to "flip" this inverted image and interpret it as being upright. To show this directly, try this simple experiment. Poke a small hole in a card using a pin and, holding the card a few inches away, look through the hole at a lightbulb. While doing so, move the head of the pin between the hole and your eye; you'll see an *upside-down* pinhead. The hole acts as a point source that casts an *erect* shadow of the pin on your retina. The brain then inverts this erect shadow, making it appear inverted.

Seeing underwater BIO When you swim underwater, the difference in refractive indices between the cornea ($n = 1.38$) and water ($n = 1.33$) is too small to allow significant refraction, so the eye cannot focus. If you wear goggles, the surface of the cornea is in contact with air, not water, and the eye can focus normally. Animals that live underwater generally have more sharply curved corneas to compensate for the small difference in refractive indices. The *anableps* fish shown at the beginning of this chapter is particularly unusual in that it lives at the water's surface. To focus simultaneously on objects on both sides of the waterline, it has evolved a very asymmetrical cornea that is more strongly curved below the waterline.

The optometrist's prescription is -2.25 D for the right eye (top) and -2.50 D for the left (bottom).

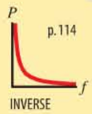
The most distant point on which the completely relaxed eye can focus is called the eye's **far point** (FP). For normal vision, the far point is at infinity. The closest point on which the eye can focus, with the ciliary muscles fully contracted, is called the **near point** (NP). Objects closer than the near point cannot be brought into sharp focus.

Vision Defects and Their Correction

The near point of normal vision is considered to be 25 cm, but the near point of an individual changes with age. The near point of young children can be as little as 10 cm. The "normal" 25 cm near point is characteristic of young adults, but the near point of most individuals begins to move outward by age 40 or 45 and can reach 200 cm by age 60. This loss of accommodation, which arises because the lens loses flexibility, is called **presbyopia.** Even if their vision is otherwise normal, individuals with presbyopia need reading glasses to bring their near point back to 25 or 30 cm, a comfortable distance for reading.

Presbyopia is known as a *refractive error* of the eye. Two other common refractive errors are *hyperopia* and *myopia*. All three can be corrected with lenses—either eye-glasses or contact lenses—that assist the eye's focusing. Corrective lenses are prescribed not by their focal length but by their **refractive power.** The refractive power of a lens is the inverse of its focal length:

$$P = \frac{1}{f} \tag{19.1}$$



Refractive power of a lens with focal length f

A lens with higher refractive power (shorter focal length) causes light rays to refract through a larger angle. The SI unit of lens refractive power is the **diopter,** abbreviated D, defined as $1\,\text{D} = 1\ \text{m}^{-1}$. Thus a lens with $f = 50$ cm $= 0.50$ m has refractive power $P = 2.0$ D.

When writing prescriptions, optometrists don't write the D because the lens maker already knows that prescriptions are in diopters. If you look at your eyeglass prescription next time you visit the optometrist, it will look something like +2.5/+2.7. This says that your right eye needs a corrective lens with $P = +2.5$ D, the + indicating a converging lens with a positive focal length. Your left eye needs a lens with $P = +2.7$ D. Most people's eyes are not the same, so each eye usually gets a slightly different lens. Prescriptions with negative numbers indicate diverging lenses with negative focal lengths.

A person who is *farsighted* can see faraway objects (but even then must use some accommodation rather than a relaxed eye), but his near point is larger than 25 cm, often much larger, so he cannot focus on nearby objects. The cause of farsightedness—called **hyperopia**—is an eyeball that is too short for the refractive power of the cornea and lens. As **FIGURES 19.9a** and **b** on the next page show, no amount of accommodation allows the eye to focus on an object 25 cm away, the normal near point.

With hyperopia, the eye needs assistance to focus the rays from a nearby object onto the closer-than-normal retina. This assistance is obtained by adding refractive power with the positive (i.e., converging) lens shown in **FIGURE 19.9c**. To understand why this works, recall that the goal is to allow the person to focus on an object 25 cm away. If a corrective lens forms an upright, virtual image at the person's actual near point, that virtual image acts as an object for the eye itself and, with maximum accommodation, the eye can focus these rays onto the retina. Presbyopia, the loss of accommodation with age, is corrected in the same way.

A person who is *nearsighted* can clearly see nearby objects when the eye is relaxed (and extremely close objects by using accommodation), but no amount of relaxation allows her to see distant objects. Nearsightedness—called **myopia**—is caused by an eyeball that is too long. As **FIGURE 19.10a** on the next page shows, rays from a distant

FIGURE 19.9 Hyperopia.

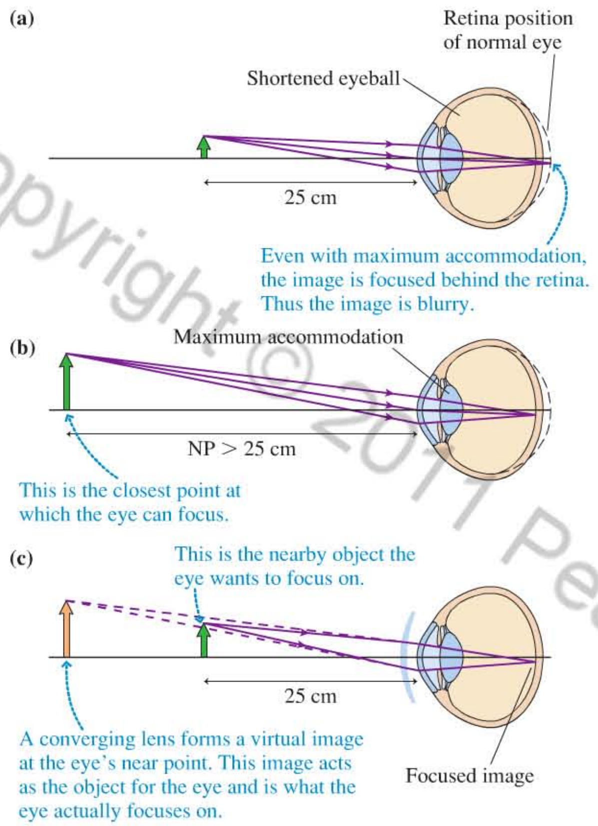
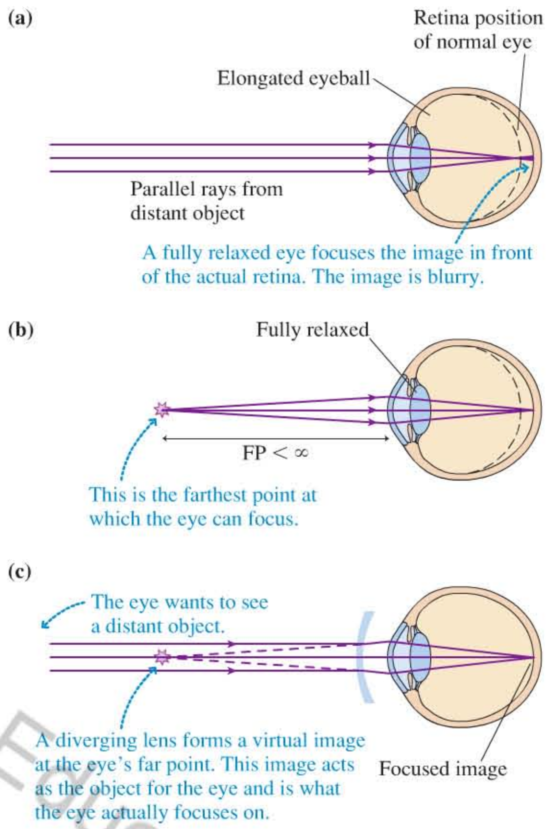


FIGURE 19.10 Myopia.



object come to a focus in front of the retina and have begun to diverge by the time they reach the retina. The eye's far point, shown in **FIGURE 19.10b**, is less than infinity.

To correct myopia, we needed a diverging lens, as shown in **FIGURE 19.10c**, to slightly defocus the rays and move the image point back to the retina. To focus on a very distant object, the person needs a corrective lens that forms an upright, virtual image at her actual far point. That virtual image acts as an object for the eye itself and, when fully relaxed, the eye can focus these rays onto the retina.

EXAMPLE 19.3 Correcting hyperopia

Sanjay has hyperopia, The near point of his left eye is 150 cm. What prescription lens will restore normal vision?

PREPARE Normal vision will allow Sanjay to focus on an object 25 cm away. In measuring distances, we'll ignore the small space between the lens and his eye.

SOLVE Because Sanjay can see objects at 150 cm, using maximum accommodation, we want a lens that creates a virtual image at position $s' = -150$ cm (negative because it's a virtual image) of an object at $s = 25$ cm. From the thin-lens equation,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{0.25\text{ m}} + \frac{1}{-1.50\text{ m}} = 3.3\text{ m}^{-1}$$

$1/f$ is the lens power, and m^{-1} are diopters. Thus the prescription is for a lens with power $P = 3.3$ D.

ASSESS Hyperopia is always corrected with a converging lens.

EXAMPLE 19.4 Correcting myopia

Martina has myopia. The far point of her left eye is 200 cm. What prescription lens will restore normal vision?

PREPARE Normal vision will allow Martina to focus on a very distant object. In measuring distances, we'll ignore the small space between the lens and her eye.

SOLVE Because Martina can see objects at 200 cm with a fully relaxed eye, we want a lens that will create a virtual image at position $s' = -200$ cm (negative because it's a virtual image) of an object at $s = \infty$ cm. From the thin-lens equation,

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{\infty\text{ m}} + \frac{1}{-2.0\text{ m}} = -0.5\text{ m}^{-1}$$

Thus the prescription is for a lens with power $P = -0.5$ D.

ASSESS Myopia is always corrected with a diverging lens.

STOP TO THINK 19.2 With her right eye, Maria can focus on a vase 0.5 m away, but not on a tree 10 m away. Which of the following could be the eyeglass prescription for her right eye?

A. +3.0 D B. +10 D C. −5.0 D D. −1.5 D

19.3 The Magnifier

You've no doubt used a magnifier, or magnifying glass, to get a better look at a small object such as an insect or a coin. As we saw in Chapter 18, a magnifier is a simple converging lens, but why objects appear larger when viewed through such a lens is actually rather subtle.

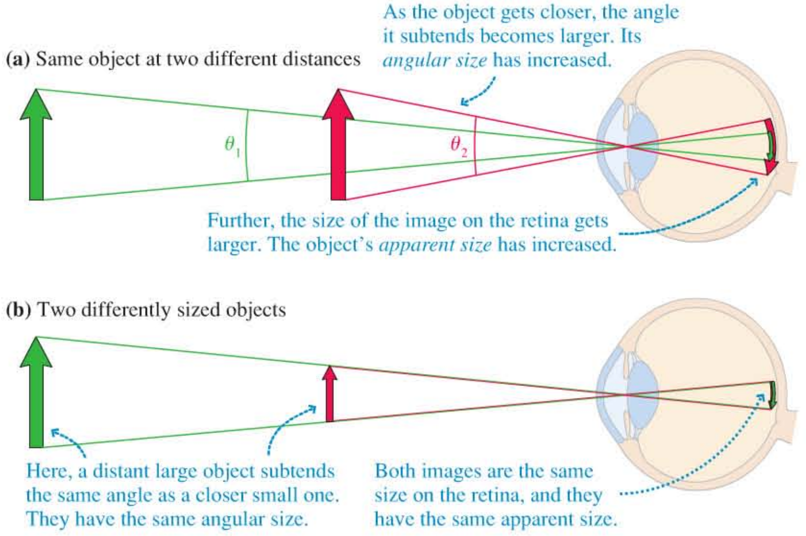
Let's begin by considering the simplest way to magnify an object, one that requires no extra optics at all. You simply get closer to the object you're interested in. The closer you get, the bigger the object appears. Obviously the actual size of the object is unchanged as you approach it, so what exactly is getting "bigger"? A penny, held at arm's length, will more than cover the distant moon. In what sense is the penny "larger" than the moon?

Angular Size and Apparent Size

Consider an object such as the green arrow in FIGURE 19.11a. To find the size of its image on the retina, we can trace the two rays shown that go through the center of the eye lens. (Here, we'll use the thin-lens approximation to reduce the eye's entire optical system to one thin lens.) As we've learned, such rays are undeviated as they pass through the lens, so we can use them to locate the image, shown in green, on the retina. If the arrow is then brought closer to the eye, as shown in red, ray tracing reveals that the size of the arrow's image is *larger.* Our brain interprets a larger image on the retina as representing a larger-appearing *object.* As the object is moved closer, its size doesn't change, but its **apparent size** gets larger.

Angles θ_1 and θ_2 in Figure 19.11a are the angles *subtended* by the green and red arrows. The angle subtended by an object is called its **angular size.** As you

FIGURE 19.11 How the apparent size of an object is determined.

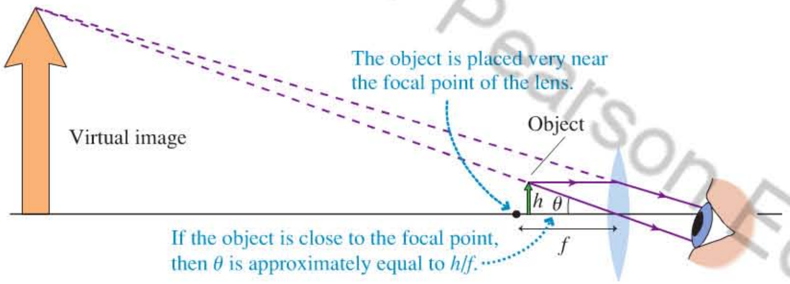


can see from the figure, **objects that subtend a larger angle appear larger to the eye.** It's also possible, as FIGURE 19.11b shows, for objects with different actual sizes to have the same angular size and thus the same apparent size.

Using a Magnifier

There are many ways to use a magnifier. You can hold the lens close to or far from the object, and you can bring your eye right up to the lens or hold it somewhat farther away. However, there is a simple case that reflects the way we usually use a magnifier. The lens is held such that the object is at or just inside the lens's focal point. As shown graphically in FIGURE 19.12, this produces a virtual image that is quite far from the lens. Your eye, looking through the lens, "sees" the virtual image. This is a convenient image location, because your eye's muscles are fully relaxed when looking at a distant image. Thus you can use the magnifier in this way for a long time without eye strain.

FIGURE 19.12 The magnifier.



We can analyze the situation using Figure 19.12. Suppose that the object is almost exactly at the focal point, a distance f from the lens. Then, tracing the ray that goes through the lens's center, we can see that the angular size θ of the image is such that $\tan\theta = h/f$. If this is a fairly small angle, which it usually is, we can use the small-angle approximation $\tan\theta \approx \theta$ to write

$$\theta \approx \frac{h}{f} \tag{19.2}$$

Is this an improvement over using no magnifier? With no magnifier, the object has its largest angular size when we bring it as close as possible to the eye. The closest position at which we can focus is the near point of the eye, as shown in FIGURE 19.13. The object at the near point would have angular size

$$\theta_0 \approx \frac{h}{25\text{ cm}}$$

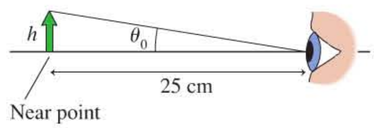
where we have taken the conventional value of 25 cm as the near-point distance. Thus the angular size θ when using the magnifier is larger than that without the magnifier by a factor of

$$M = \frac{\theta}{\theta_0} = \frac{h/f}{h/25\text{ cm}} = \frac{25\text{ cm}}{f} \tag{19.3}$$

where, for this calculation, the focal length f must be in cm. M is called the **angular magnification** of the magnifier. With a lens of short focal length it is possible to get magnifications as high as about 20.

Movie magic? Even if the more distant of two equally sized objects *appears* smaller, we don't usually believe it actually *is* smaller because there are abundant visual clues that tell our brain that it's farther away. If those clues are removed, however, the brain readily accepts the illusion that the farther object is smaller. The technique of *forced perspective* is a special effect used in movies to give this illusion. Here, Camelia, who is actually closer to the camera, looks like a giant compared to Kevin. The lower photo shows how the trick was done.

FIGURE 19.13 Angular size without a magnifier.



CONCEPTUAL EXAMPLE 19.5 **The angular size of a magnified image**

An object is placed right at the focal point of a magnifier. How does the apparent size of the image depend on where the *eye* is placed relative to the lens?

REASON When the object is precisely at the focal point of the lens, Equation 19.2 holds exactly. The angular size is equal to h/f *independent* of the position of the eye. Thus the object's apparent size is independent of the eye's position as well. **FIGURE 19.14** shows a calculator at the focal point of a magnifier. The apparent size of the COS button is the same whether the camera taking the picture is close to or far from the lens.

ASSESS When the object is at the magnifier's focus, we've seen that the image is at infinity. The situation is similar to observing any "infinitely" distant object, such as the moon. If you walk closer to or farther from the moon, its apparent size doesn't change at all. The same holds for a virtual *image* at infinity: Its apparent size is independent of the point from which you observe it.

FIGURE 19.14 Viewing a magnifier with the object at its focus.

Eye close to magnifier   Eye far from magnifier

STOP TO THINK 19.3 A student tries to use a *diverging* lens as a magnifier. She observes a coin placed at the focal point of the lens. She sees

A. An upright image, smaller than the object.
B. An upright image, larger than the object.
C. An inverted image, smaller than the object.
D. An inverted image, larger than the object.
E. A blurry image.

19.4 The Microscope

To get higher magnifications than are possible using a simple magnifier, a *combination* of lenses must be used. This is how microscopes and telescopes are constructed. A simple rule governs how two lenses work in combination: **The image from the first lens acts as the object for the second lens.** The following example illustrates this rule.

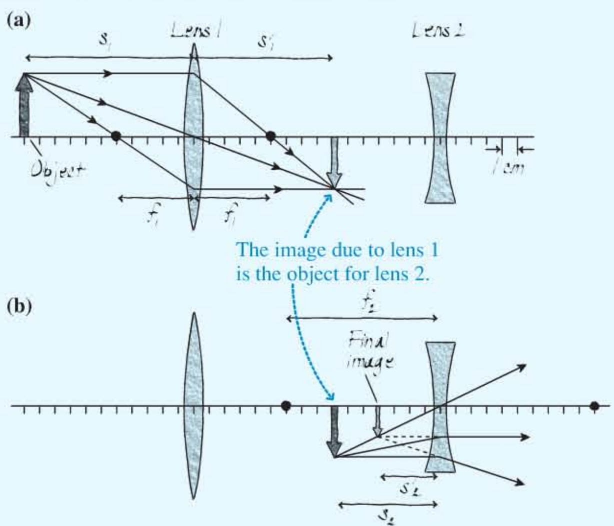
EXAMPLE 19.6 **Finding the image for two lenses in combination**

A 5.0-cm-focal-length converging lens is 16.0 cm in front of a 10.0-cm-focal-length diverging lens. A 4.0-cm-tall object is placed 11.0 cm in front of the converging lens. What are the position and size of the final image?

PREPARE Let's start with ray tracing. A ray-tracing diagram helps us understand the situation and tells us what to expect for an answer. A diagram can often alert you to a calculation error. **FIGURE 19.15a** first uses the three special rays of the converging lens to locate its image. We see that the image of the first lens is a real image falling between the two lenses. According to the rule, we then use this image as the object for the second lens. This is done in **FIGURE 19.15b**, where we see that the final image is inverted, virtual, and roughly 4 cm to the left of the diverging lens.

Mathematically, we can use the thin-lens equation to find the image location and size due to the first lens, then use this as the object for the second lens in a second use of the lens equation.

FIGURE 19.15 Two lenses in combination.



SOLVE We first solve for the image due to lens 1. We have

$$\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s_1} = \frac{1}{5.0\text{ cm}} - \frac{1}{11.0\text{ cm}}$$

from which $s_1' = 9.2$ cm. Because this is a positive image distance, the image is real and located to the right of the first lens. The magnification of the first lens is $m_1 = -s_1'/s_1 = -(9.2\text{ cm})/(11.0\text{ cm}) = -0.84$.

This image is the object for the second lens. Because it is 9.2 cm to the right of the first lens, and the lenses are 16.0 cm apart, it is 16.0 cm − 9.2 cm = 6.8 cm in front of the second lens. Thus $s_2 = 6.8$ cm.

Applying the thin-lens equation again, we have

$$\frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} = \frac{1}{-10.0\text{ cm}} - \frac{1}{6.8\text{ cm}}$$

from which we find $s_2' = -4.0$ cm. Because this image distance is negative, the image is virtual and located to the left of lens 2, as shown in Figure 19.15b. The magnification of the second lens is $m_2 = -s_2'/s_2 = -(-4.0\text{ cm})/(6.8\text{ cm}) = 0.59$.

Thus the final image size is

$$h_2' = m_2h_2 = m_2h_1' = m_2(m_1h_1) = m_1m_2h_1$$
$$= (-0.84)(0.59)(4.0\text{ cm}) = -2.0\text{ cm}$$

Here we used the fact that the object height h_2 of the second lens is equal to the image height h_1' of the first lens.

ASSESS When calculating the final image size, we found that $h_2' = m_1m_2h_1$. This shows the important fact that **the total magnification of a combination of lenses is the *product* of the magnifications for each lens alone.**

A microscope, whose major parts are shown in **FIGURE 19.16**, attains a magnification of up to 1000 by using two lenses in combination. A specimen to be observed is placed on the *stage* of the microscope, directly beneath the **objective lens** (or simply the **objective**), a converging lens with a relatively short focal length. The objective creates a magnified real image that is further enlarged by the **eyepiece,** a lens used as an ordinary magnifier. In most modern microscopes a prism bends the path of the rays from the object so that the eyepiece can be held at a comfortable angle. However, we'll consider a simplified version of a microscope without a prism. The light then travels along a straight tube.

Let's examine the magnification process in more detail. In **FIGURE 19.17** we draw a microscope tilted horizontally. The object distance is just slightly greater than the focal length f_o of the objective lens, so the objective forms a highly magnified real image of the object at a distance $s' = L$. This distance, known as the **tube length,** has been standardized for most biological microscopes at $L = 160$ mm. Most microscopes, such as the one shown in Figure 19.16, are focused by moving the sample stage up and down, using the focusing knob, until the object distance is correct for placing the image at L.

FIGURE 19.16 A microscope.

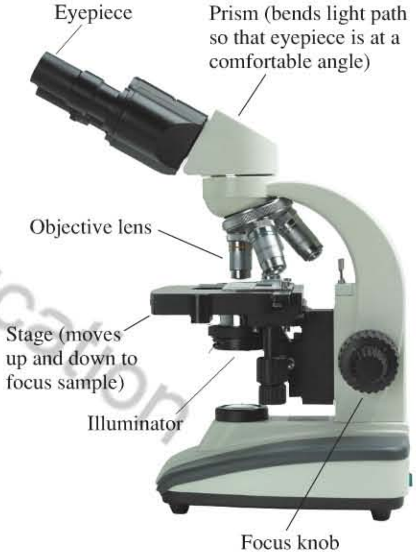
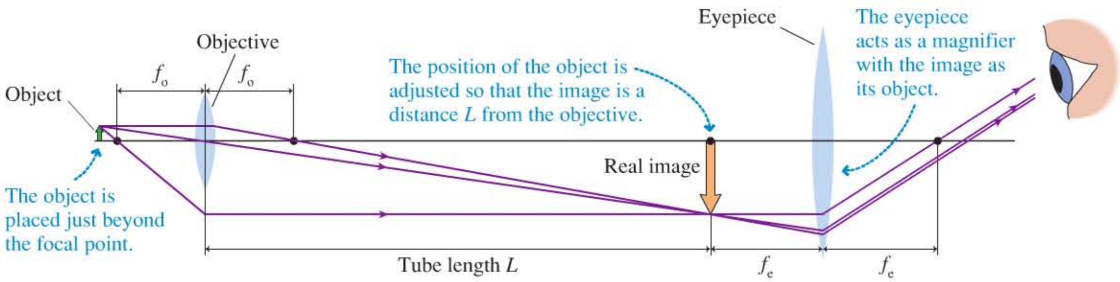


FIGURE 19.17 A horizontal view of the optics in a microscope.



From the magnification equation, Equation 18.8, the magnification of the objective lens is

$$m_o = -\frac{s'}{s} \approx -\frac{L}{f_o} \tag{19.4}$$

Here we used the fact that the image distance s' is equal to the tube length L and the object distance s is very close to the focal length f_o of the objective. The minus sign tells us that the image is inverted with respect to the object.

The image of the objective acts as the object for the eyepiece, which functions as a simple magnifier. The angular magnification of the eyepiece is given by Equation 19.3: $M_e = (25\text{ cm})/f_e$. Together, the objective and eyepiece produce a total angular magnification

$$M = m_o M_e = -\frac{L}{f_o}\frac{25\text{ cm}}{f_e} \tag{19.5}$$

The minus sign shows that the image seen in a microscope is inverted.

EXAMPLE 19.7 Finding the focal length of a microscope objective

A biological microscope objective is labeled "20×." What is its focal length?

PREPARE The "20×" means that the objective has a magnification m_o of -20. We can use Equation 19.4 with L as 160 mm, which we've seen is the standard length for a biological microscope.

SOLVE From Equation 19.4 we have

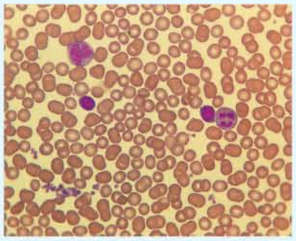
$$f_o = -\frac{L}{m_o} = -\frac{160\text{ mm}}{-20} = 8.0\text{ mm}$$

ASSESS Microscope objectives are specified by their magnification, or "power," not by their focal length. Equation 19.4 relates these two important specifications.

Many microscopes have a set of objectives that can be pivoted into place to change the overall magnification. A complete set of objectives might include 5×, 10×, 20×, 40×, and 100×. Eyepieces are also specified by their magnification and are available with magnifications in the range of 10× to 20×. With these lenses, the lowest magnification available would be $5 \times 10 = 50\times$, while the highest magnification would be $100 \times 20 = 2000\times$.

EXAMPLE 19.8 Viewing blood cells

A pathologist inspects a sample of 7-μm-diameter human blood cells under a microscope. She selects a 40× objective and a 10× eyepiece. What size object, viewed from 25 cm, has the same apparent size as a blood cell seen through the microscope?



PREPARE Angular magnification compares the magnified angular size to the angular size seen at the near-point distance of 25 cm.

SOLVE The microscope's angular magnification is $M = -(40) \times (10) = -400$. The magnified cells will have the same apparent size as an object $400 \times 7\ \mu\text{m} \approx 3$ mm in diameter seen from a distance of 25 cm.

ASSESS 3 mm is about the size of a capital O in this textbook, so a blood cell seen through the microscope will have about the same apparent size as an O seen from a comfortable reading distance.

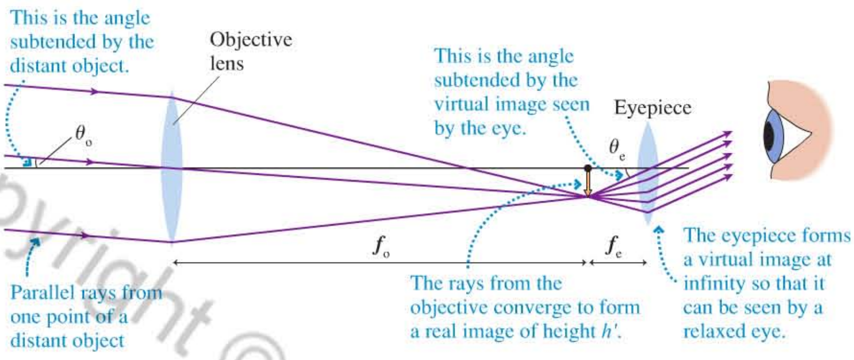
STOP TO THINK 19.4 A biologist rotates the turret of a microscope to replace the 20× objective with a 10× objective. To keep the magnification the same, the focal length of the eyepiece must

A. Be doubled. B. Be halved. C. Remain the same.
D. The magnification cannot stay the same if the objective power is changed.

19.5 The Telescope

The microscope magnifies small objects that can be placed near its objective lens. A *telescope* is used to magnify distant objects. The two-lens arrangement, shown in **FIGURE 19.18** on the next page, is similar to that of the microscope, but the objective lens has a long focal length instead of the very short focal length of a microscope objective. Because the object is very far away ($s \approx \infty$), the converging objective lens forms a real image of the distant object at the lens's focal point. A second lens,

FIGURE 19.18 The telescope.



the eyepiece, is then used as a simple magnifier to enlarge this real image for final viewing by the eye.

We can use Figure 19.18 to find the magnification of a telescope. The original object subtends an angle θ_o. Because the object is distant, its image is formed in the focal plane of the objective lens, a distance f_o from the objective. From the geometry of Figure 19.18, the height of this image (negative because it's inverted) is

$$h' \approx -f_o\theta_o$$

where we have used the small-angle approximation $\tan\theta_o \approx \theta_o$. This image is now the object for the eyepiece lens, which functions as a magnifier. The height of the "object" it views is h', so, from Equation 19.2, the angular size θ_e of the virtual image formed by the eyepiece is

$$\theta_e = \frac{h'}{f_e} = \frac{-f_o\theta_o}{f_e}$$

where f_e is the focal length of the eyepiece. The telescope's angular magnification is the ratio of the angular size seen when looking through the telescope to that seen without the telescope, so we have

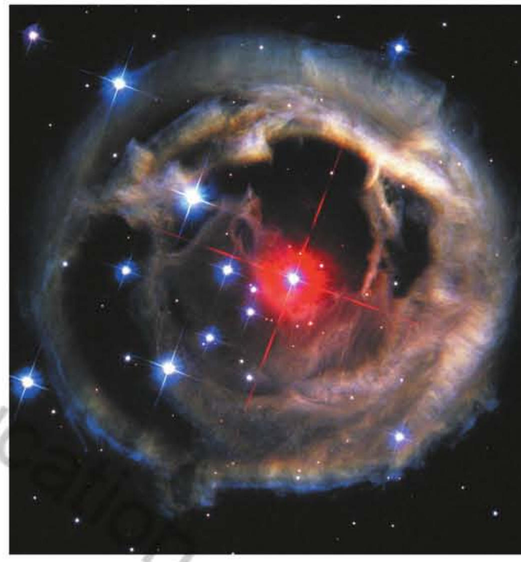
$$M = \frac{\theta_e}{\theta_o} = -\frac{f_o}{f_e} \tag{19.6}$$

The minus sign indicates that you see an upside-down image when looking through a simple telescope. This is not a problem when looking at astronomical objects, but it could be disconcerting to a bird watcher. More sophisticated telescope designs produce an upright image.

To get a high magnification with a telescope, the focal length of the objective should be large and that of the eyepiece small. Contrast this with the magnification of a microscope, Equation 19.5, which is high when the focal lengths of both objective and eyepiece are small.

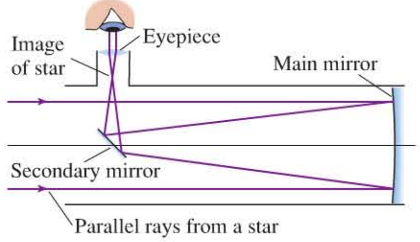
The telescope shown in Figure 19.18 uses a lens as its objective and hence is known as a *refracting telescope*. It is also possible to make a *reflecting telescope* with a concave mirror instead of a lens, as shown in FIGURE 19.19. One problem with this arrangement, for small telescopes, is that the image is formed in front of the mirror where it's hard to magnify with an eyepiece without getting one's head in the way. Newton, who built the first such telescope, used a small angled plane mirror, called a *secondary mirror,* to deflect the image to an eyepiece on the side of the telescope.

For large telescopes, such as those used in astronomy, mirrors have two important advantages over lenses. First, objectives of astronomical telescopes must be quite large in order to gather as much light from faint objects as possible. The Subaru Telescope in Hawaii is the world's largest single-mirror telescope; its mirror has a diameter of 8.3 m (27 ft)! A giant lens of this diameter would sag under its own weight.



A clearer view The performance of telescopes on the earth is limited by the atmosphere. Even at night, the atmosphere glows faintly, interfering with the long exposures needed to photograph faint astronomical objects. Further, atmospheric turbulence—visible to the naked eye as the twinkling of stars—obscures the finest details of the object being observed. Because of this, the Hubble Space Telescope orbits the earth high above the atmosphere. With its 2.4-m-diameter mirror, it has produced some of the most spectacular images of astronomical objects, such as this gas cloud surrounding the star V838 Monocerotis.

FIGURE 19.19 A reflecting telescope.



A mirror, on the other hand, can be supported along its entire back surface. Second, mirrors are free from chromatic aberration, the tendency that lenses have of splitting light into its constituent colors, as we'll see in a later section.

19.6 Color and Dispersion

I procured me a triangular glass prism to try therewith the celebrated phenomena of colors.

Isaac Newton

One of the most obvious visual aspects of light is the phenomenon of color. Yet color, for all its vivid sensation, is not inherent in the light itself. Color is a *perception,* not a physical quantity. Color is associated with the wavelength of light, but the fact that we see light with a wavelength of 650 nm as "red" tells us how our visual system responds to electromagnetic waves of this wavelength. There is no "redness" associated with the light wave itself.

Most of the results of optics do not depend on color. We generally don't need to know the color of light—or, to be more precise, its wavelength—to use the laws of reflection and refraction. Nonetheless, color is an interesting subject, one worthy of a short digression.

Color

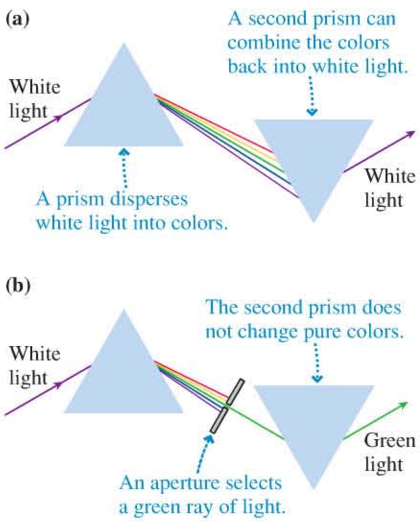


FIGURE 19.20 Newton used prisms to study color.

It has been known since antiquity that irregularly shaped glass and crystals cause sunlight to be broken into various colors. A common idea was that the glass or crystal somehow altered the properties of the light by *adding* color to the light. Newton suggested a different explanation. He first passed a sunbeam through a prism, producing the familiar rainbow of light. We say that the prism *disperses* the light. Newton's novel idea, shown in FIGURE 19.20a, was to use a second prism, inverted with respect to the first, to "reassemble" the colors. He found that the light emerging from the second prism was a beam of pure white light.

But the emerging light beam is white only if *all* the rays are allowed to move between the two prisms. Blocking some of the rays with small obstacles, as in FIGURE 19.20b, causes the emerging light beam to have color. This suggests that color is associated with the light itself, not with anything that the prism is "doing" to the light. Newton tested this idea by inserting a small aperture between the prisms to pass only the rays of a particular color, such as green. If the prism alters the properties of light, then the second prism should change the green light to other colors. Instead, the light emerging from the second prism is unchanged from the green light entering the prism.

These and similar experiments show that:

1. What we perceive as white light is a mixture of all colors. White light can be dispersed into its various colors and, equally important, mixing all the colors produces white light.
2. The index of refraction of a transparent material differs slightly for different colors of light. Glass has a slightly higher index of refraction for violet light than for green light or red light. Consequently, different colors of light refract at slightly different angles. A prism does not alter the light or add anything to the light; it simply causes the different colors that are inherent in white light to follow slightly different trajectories.

TABLE 19.1 A brief summary of the visible spectrum of light

Color	Approximate wavelength
Deepest red	700 nm
Red	650 nm
Yellow	600 nm
Green	550 nm
Blue	450 nm
Deepest violet	400 nm

Dispersion

It was Thomas Young, with his two-slit interference experiment, who showed that different colors are associated with light of different wavelengths. The longest wavelengths are perceived as red light and the shortest wavelengths are perceived as violet light. Table 19.1 is a brief summary of the *visible spectrum* of light. Visible-light wavelengths are used so frequently that it is well worth committing this short table to memory.

The slight variation of index of refraction with wavelength is known as **dispersion.** FIGURE 19.21 shows the *dispersion curves* of two common glasses. Notice that ***n* is *higher* when the wavelength is *shorter;*** thus violet light refracts more than red light.

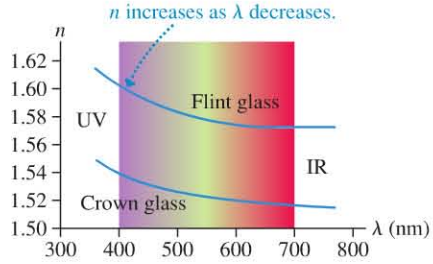


FIGURE 19.21 Dispersion curves show how the index of refraction varies with wavelength.

EXAMPLE 19.9 Dispersing light with a prism

Example 18.4 in Chapter 18 found that a ray incident on a 30° prism is deflected by 22.6° if the prism's index of refraction is 1.59. Suppose this is the index of refraction of deep violet light, and that deep red light has an index of refraction of 1.54.

a. What is the deflection angle for deep red light?
b. If a beam of white light is dispersed by this prism, how wide is the rainbow spectrum on a screen 2.0 m away?

PREPARE Figure 18.18 in Example 18.4 showed the geometry. A ray is incident on the hypotenuse of the prism at $\theta_1 = 30°$.

SOLVE a. If $n_1 = 1.54$ for deep red light, the refraction angle is

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.54 \sin 30°}{1.00}\right) = 50.4°$$

Example 18.4 showed that the deflection angle is $\phi = \theta_2 - \theta_1$, so deep red light is deflected by $\phi_{red} = 20.4°$. This angle is slightly smaller than the previously observed $\phi_{violet} = 22.6°$.

b. The entire spectrum is spread between $\phi_{red} = 20.4°$ and $\phi_{violet} = 22.6°$. The angular spread is

$$\delta = \phi_{violet} - \phi_{red} = 2.2° = 0.038 \text{ rad}$$

At distance r, the spectrum spans an arc length

$$s = r\delta = (2.0 \text{ m})(0.038 \text{ rad}) = 0.076 \text{ m} = 7.6 \text{ cm}$$

ASSESS Notice that we needed three significant figures for ϕ_{red} and ϕ_{violet} in order to determine δ, the *difference* between the two angles, to two significant figures. The angle is so small that there's no appreciable difference between arc length and a straight line. The spectrum will be 7.6 cm wide at a distance of 2.0 m.

Rainbows

One of the most interesting sources of color in nature is the rainbow. The details get somewhat complicated, but FIGURE 19.22a shows that the basic cause of the rainbow is a combination of refraction, reflection, and dispersion.

FIGURE 19.22 Light seen in a rainbow has undergone refraction + reflection + refraction in a raindrop.

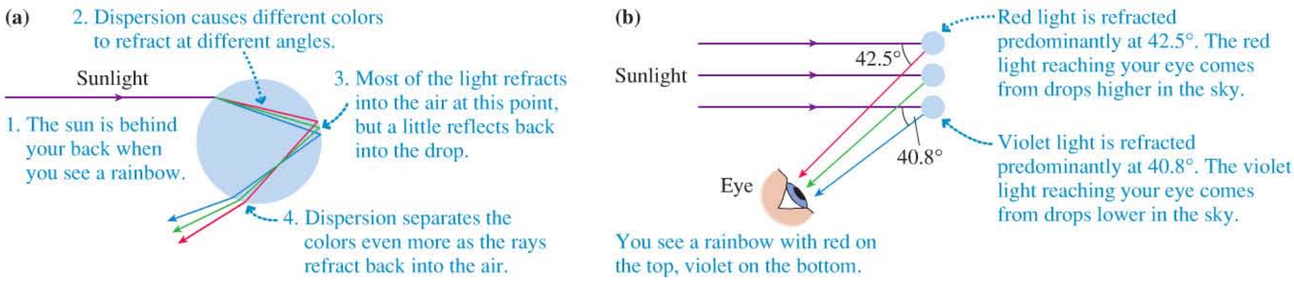
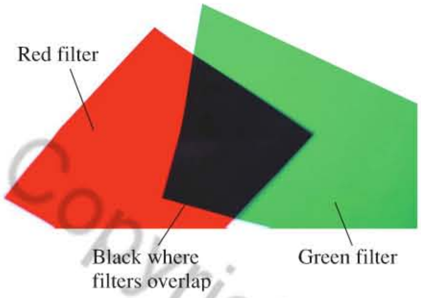


Figure 19.22a might lead you to think that the top edge of a rainbow is violet. In fact, the top edge is red, and violet is on the bottom. The rays leaving the drop in Figure 19.22a are spreading apart, so they can't all reach your eye. As FIGURE 19.22b shows, a ray of red light reaching your eye comes from a drop *higher* in the sky than a ray of violet light. In other words, the colors you see in a rainbow refract toward



No light at all passes through both a green and a red filter.

your eye from different raindrops, not from the same drop. You have to look higher in the sky to see the red light than to see the violet light.

Colored Filters and Colored Objects

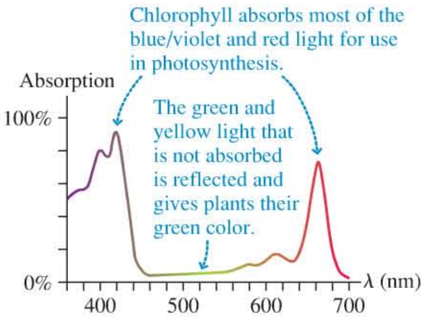
White light passing through a piece of green glass emerges as green light. A possible explanation would be that the green glass *adds* "greenness" to the white light, but Newton found otherwise. Green glass is green because it *removes* any light that is "not green." More precisely, a piece of colored glass *absorbs* all wavelengths except those of one color, and that color is transmitted through the glass without hindrance. We can think of a piece of colored glass or plastic as a *filter* that removes all wavelengths except a chosen few.

CONCEPTUAL EXAMPLE 19.10 Filtering light

White light passes through a green filter and is observed on a screen. Describe how the screen will look if a second green filter is placed between the first filter and the screen. Describe how the screen will look if a red filter is placed between the green filter and the screen.

REASON The first filter removes all light except for wavelengths near 550 nm that we perceive as green light. A second green filter doesn't have anything to do. The nongreen wavelengths have already been removed, and the green light emerging from the first filter will pass through the second filter without difficulty. The screen will continue to be green and its intensity will not change. A red filter, by contrast, absorbs all wavelengths except those near 650 nm. The red filter will absorb the green light, and *no* light will reach the screen. The screen will be dark.

FIGURE 19.23 The absorption curve of chlorophyll.



Opaque objects appear colored by virtue of *pigments* that absorb light of some wavelengths but *reflect* light of other wavelengths. For example, red paint contains pigments that reflect light of wavelengths near 650 nm while absorbing all other wavelengths. Pigments in paints, inks, and natural objects are responsible for most of the color we observe in the world, from the red of lipstick to the blue of a bluebird's feathers.

As an example, **FIGURE 19.23** shows the absorption curve of *chlorophyll.* Chlorophyll is essential for photosynthesis in green plants. The chemical reactions of photosynthesis are able to use red light and blue/violet light; thus chlorophyll has evolved to absorb red light and blue/violet light from sunlight and put it to use. But green and yellow light are not absorbed. Instead, these wavelengths are mostly *reflected* to give the object a greenish-yellow color. When you look at the green leaves on a tree, you're seeing the light that was reflected because it *wasn't* needed for photosynthesis.

STOP TO THINK 19.5 A red apple is viewed through a green filter. The apple appears

A. Red. B. Green. C. Yellow. D. Black.

19.7 Resolution of Optical Instruments

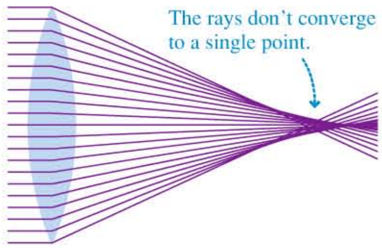
16.8 ActivPhysics

Suppose you wanted to study the *E. coli* bacterium. It's quite small, about 2 μm long and 0.5 μm wide. You might imagine that you could pair a 150× objective (the highest magnification available) with a 25× eyepiece to get a total magnification of 3750! At that magnification, the *E. coli* would appear about 8 mm across—about the size of Lincoln's head on a penny—with much fine detail revealed. But if you tried this, you'd be disappointed. Although you would see the general shape of a bacterium, you wouldn't be able to make out any real details. All real optical instruments are limited in the details they can observe. Some limits are practical: Lenses are never perfect, suffering from **aberrations.** But even a perfect lens would have a fundamental limit to the smallest details that could be seen. As we'll see, this limit is set by the diffraction of light, and so is intimately related to the wave nature of light itself. Together, lens aberrations and diffraction set a limit on an optical system's **resolution**—its ability to make out the fine details of an object.

Aberrations

Consider the simple lens shown in FIGURE 19.24 imaging an object located at infinity, so that the incoming rays are parallel. An ideal lens would focus all the rays to a single point. However, for a real lens with spherical surfaces, the rays that pass near the lens's center come to a focus a bit farther from the lens than those that pass near its edge. There is no single focal point; even at the "best" focus the image is a bit blurred. This inability of a real lens to focus perfectly is called **spherical aberration.**

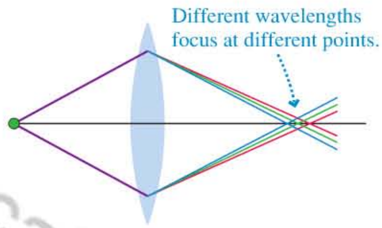
FIGURE 19.24 Spherical aberration.



A careful examination of Figure 19.24 shows that the outer rays are most responsible for the poor focus. Consequently, the effects of spherical aberration can be minimized by using an iris diaphragm to pass only rays near the optical axis. This "stopping down" of a lens improves its imaging characteristics at the expense of its light-gathering capabilities. Part of the function of the iris of the human eye is to improve vision in this way. Our vision is poorer at night with the iris wide open—but our ancestors probably avoided many a predator with this poor but sensitive night vision.

As we learned in the previous section, glass has *dispersion;* that is, the index of refraction of glass varies slightly with wavelength. The higher a lens's index of refraction, the more it bends incoming light rays. Because the index of refraction for violet light is higher than that for red light, a lens's focal length is slightly shorter for violet light than for red light. Consequently, different colors of light come to a focus at slightly different distances from the lens. If red light is sharply focused on a viewing screen, then blue and violet wavelengths are not well focused. This imaging error, illustrated in FIGURE 19.25, is called **chromatic aberration.**

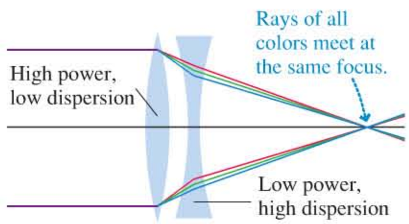
FIGURE 19.25 Chromatic aberration.



Correcting Aberrations

Single lenses always have aberrations of some kind. For high-quality optics, such as those used in microscopes or telescopes, the aberrations are minimized by using a careful *combination* of lenses. An important example is the **achromatic doublet** (achromatic = "without color"), two lenses used in combination to greatly reduce chromatic aberration. FIGURE 19.26 shows how this works. A converging lens is paired with a weaker diverging lens; the combination has an overall positive refractive power and so is converging. However, the glasses are chosen so that the weaker diverging lens has a greater dispersion than the stronger converging lens. In this way the colors of white light, separated at first by the converging lens, are brought back together by the diverging lens. Achromatic doublets also minimize spherical aberration. Real microscope objectives are even more complex, but are based on the same principle as the achromatic doublet.

FIGURE 19.26 An achromatic lens.



Resolution and the Wave Nature of Light

Modern lenses can be well corrected for aberrations, so they might be expected to focus perfectly. According to the ray model of light, a perfect lens should focus parallel rays to a single point in the focal plane. However, we've already hinted that there's a more fundamental limit to the performance of an optical instrument, a limit set by the wave nature of light.

▶ **Eyeglasses in space** After launch, it was discovered that the mirror of the Hubble Space Telescope had been ground to the wrong shape, giving it severe spherical aberration. In a later service mission, corrective optics—in essence, very high-tech glasses—were put in place to correct for this spherical aberration. The photos to the right show an image of a galaxy before and after the corrective optics were added.

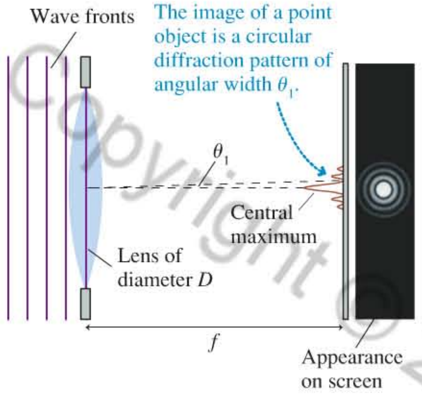


FIGURE 19.27 The image of a distant point source is a circular diffraction pattern.

FIGURE 19.27 shows a plane wave from a distant point source such as a star being focused by a lens of diameter D. Only those waves passing *through* the lens can be focused, so the lens acts like a circular aperture of diameter D in an opaque barrier. In other words, the lens both focuses *and diffracts* light waves.

You learned in Chapter 17 that a circular aperture produces a diffraction pattern with a bright central maximum surrounded by dimmer circular fringes. Consequently, as Figure 19.27 shows, light from a distant point source focuses not to a perfect point but, instead, to a small circular diffraction pattern. Equation 17.22 in Chapter 17 gave the angle θ_1 of the outer edge of the central maximum as

$$\theta_1 = \frac{1.22\lambda}{D} \tag{19.7}$$

Because the wavelength of light λ is so much shorter than the lens diameter D of an ordinary lens, the angular size of the central maximum is very small—but it is not zero.

The fact that light is focused to a small spot, not a perfect point, has important consequences for how well a telescope can resolve two stars separated by only a small angle in the sky. FIGURE 19.28a shows how two nearby stars would appear in a telescope. Instead of perfect points, they appear as two diffraction images. Nonetheless, because they're clearly two separate stars, we say they are *resolved*. FIGURE 19.28b shows two stars that are closer together. Here the two diffraction patterns overlap, and it is becoming difficult to see them as two independent stars: They are barely resolved. The two very nearby stars in Figure 19.28c are so close together that we can't resolve them at all.

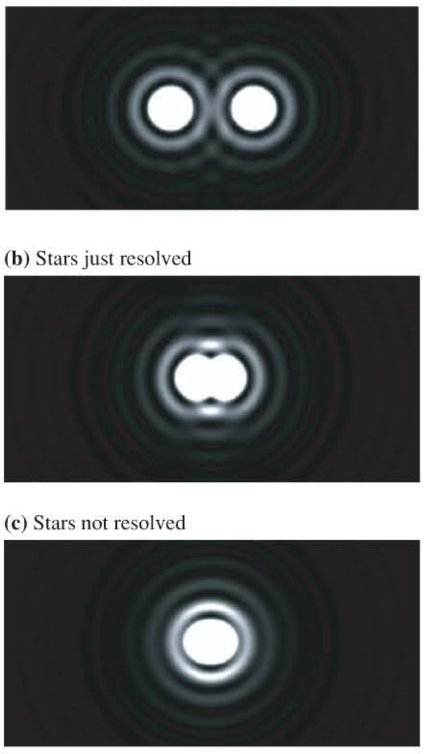


FIGURE 19.28 The resolution of a telescope.

How close can the two diffraction patterns be before you can no longer resolve them? One of the major scientists of the 19th century, Lord Rayleigh, studied this problem and suggested a reasonable rule that today is called **Rayleigh's criterion.** In Figure 19.28b, where the two stars are just resolved, *the central maximum of the diffraction pattern of one star lies on top of the first dark fringe of the diffraction pattern of the other star.* Because the angle between the central maximum and the first dark fringe is θ_1, this means that the centers of the two stars are separated by angle $\theta_1 = 1.22\lambda/D$. Thus Rayleigh's criterion is:

> Two objects are resolvable if they are separated by an angle θ that is greater than $\theta_1 = 1.22\lambda/D$. If their angular separation is less than θ_1, then they are not resolvable. If their separation is equal to θ_1, then they are just barely resolvable.

For telescopes, the angle $\theta_1 = 1.22\lambda/D$ is called the *angular resolution* of the telescope. The angular resolution depends only on the lens diameter and the wavelength; the magnification is not a factor. Two overlapped, unresolved images will remain overlapped and unresolved no matter what the magnification. For visible light, where λ is pretty much fixed, the only parameter over which the astronomer has any control is the diameter of the lens or mirror of the telescope. The urge to build ever-larger telescopes is motivated, in part, by a desire to improve the angular resolution. (Another important motivation is to increase the light-gathering power so as to see objects farther away.)

The Resolution of a Microscope

A microscope differs from a telescope in that it magnifies objects that are very close to the lens, not far away. Nonetheless, the wave nature of light still sets a limit on the ultimate resolution of a microscope. FIGURE 19.29 shows the objective lens of a microscope that is observing two small objects. An analysis based on Rayleigh's criterion finds that the smallest resolvable separation between the two objects is

$$d_{min} = \frac{0.61\lambda}{n \sin\phi_0} \tag{19.8}$$

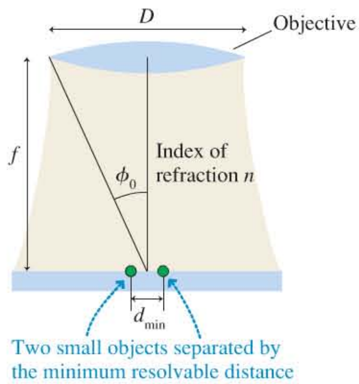
Here, ϕ_0, defined in Figure 19.29, is the angular size of the objective lens and n is the index of refraction of the medium between the objective lens and the specimen being observed. Usually this medium is air, so that $n = 1$, but biologists often use an *oil-immersion microscope* in which this space is filled with oil having $n \approx 1.5$. From Equation 19.8, you can see that this higher value of n reduces d_{min}, allowing objects that are closer together to be resolved.

The quantity $n \sin\phi_0$ is called the **numerical aperture** NA of the objective when immersed in a fluid of index n. For a microscope whose objective has numerical aperture NA, the minimum resolvable distance, also called the **resolving power** RP, is

$$\mathrm{RP} = d_{min} = \frac{0.61\lambda_0}{\mathrm{NA}} \qquad (19.9)$$

Resolving power of a microscope with numerical aperture NA

FIGURE 19.29 The resolution of a microscope.



The lower the resolving power, the *better* the objective is at seeing small details.

In principle, it would appear from Equation 19.9 that the resolving power of a microscope could be made as low as desired simply by increasing the numerical aperture. But there are rather severe practical limits on how high the numerical aperture can be made. The highest possible numerical aperture for a high-magnification 100× objective used in air is about 0.95. For an oil-immersion objective, the numerical aperture might be as high as 1.3. With such an objective, the resolving power would be

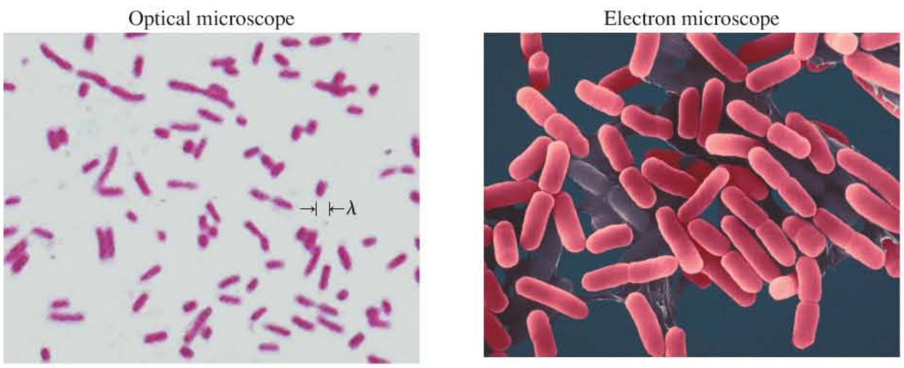
$$\mathrm{RP} \approx 0.5\lambda_0$$

This illustrates the fundamental fact that **the minimum resolving power of a microscope, and thus the size of the smallest detail observable, is about half the wavelength of light.** This is a *fundamental limit* set by the wave nature of light. For $\lambda \approx 400$ nm, at the short-wavelength edge of the visible spectrum, the maximum possible resolving power is RP $\approx$ 200 nm.

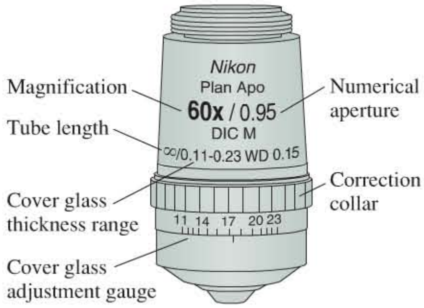
FIGURE 19.30 shows an actual micrograph of the bacillus *E. coli*. The width is about equal to the wavelength of light in the middle of the spectrum, about 500 nm, and the smallest resolved features are about half this. A higher magnification would not reveal any more detail because this micrograph is at the resolution limit set by the diffraction of light.

In contrast, the **electron microscope** micrograph of *E. coli* shows a wealth of detail unobservable in the optical picture. In Chapter 28 we'll find out why the resolving power of an electron microscope is so much lower than that of an optical microscope.

FIGURE 19.30 Optical and electron micrographs of *E. coli*.



▶ The anatomy of a microscope objective
The two most important specifications of a microscope objective, its magnification and its numerical aperture, are prominently displayed on its barrel. Other important information is shown as well. This objective is designed to project its real image at a tube length of infinity (∞), instead of at the standard distance of 160 mm. Extra lenses in the microscope move this image to just in front of the eyepiece. Many biological studies are conducted through a cover glass. This cover glass can introduce spherical aberration, blurring the image. By turning the correction collar, you can adjust this objective to correct for the exact thickness of the cover glass used.



EXAMPLE 19.11 Finding the resolving power of a microscope

A microscope objective lens has a diameter of 6.8 mm and a focal length of 4.0 mm. For a sample viewed in air, what is the resolving power of this objective in red light? In blue light?

PREPARE We can use Equation 19.9 to find the resolving power. We'll need the numerical aperture of the objective, given as NA $= n \sin\phi_0$.

SOLVE From the geometry of Figure 19.29,

$$\tan\phi_0 = \frac{D/2}{f} = \frac{3.4\text{ mm}}{4.0\text{ mm}} = 0.85$$

from which $\phi_0 = \tan^{-1}0.85 = 40.4°$ and $\sin\phi_0 = \sin 40.4° = 0.65$. Hence the numerical aperture is (since $n = 1$ in air)

$$\text{NA} = n\sin\phi_0 = 1 \times 0.65 = 0.65$$

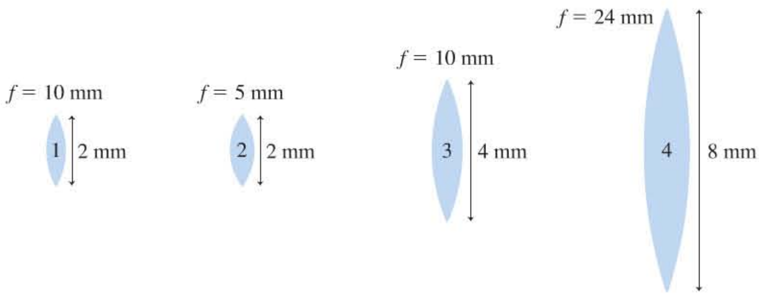
Then, from Equation 19.9, the resolving power is

$$\text{RP} = \frac{0.61\lambda}{0.65} = 0.94\lambda_0$$

Wavelengths of different colors of light were listed in Table 19.1. For red light, with $\lambda_0 = 650$ nm, RP $= 610$ nm, while blue light, with $\lambda_0 = 450$ nm, has RP $= 420$ nm.

ASSESS We see that shorter-wavelength light yields a higher resolution (lower RP). Unfortunately, wavelengths much shorter than 400 nm are invisible, and glass lenses are opaque to light of very short wavelength.

STOP TO THINK 19.6 Four lenses are used as microscope objectives, all for light with the same wavelength λ. Rank in order, from highest to lowest, the resolving powers RP_1 to RP_4 of the lenses.



INTEGRATED EXAMPLE 19.12 The visual acuity of a kestrel

Like most birds of prey, the American kestrel has excellent eyesight. The smallest angular separation between two objects that an eye can resolve is called its *visual acuity;* a smaller visual acuity means better eyesight because objects closer together can be resolved. The eye of a particular kestrel has a pupil diameter of 3.0 mm. The fixed distance from its lens to the retina is 9.0 mm, and the space within the eye is filled with a clear liquid whose index of refraction is 1.31. Assume that the kestrel's optical system can be adequately modeled as a thin lens and a detector.

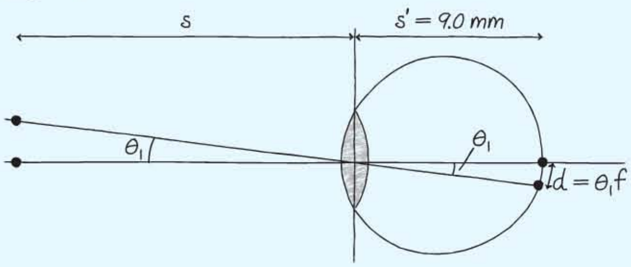
a. As the bird focuses on an insect sitting 0.80 m away, what is the focal length of its lens?

b. Laboratory measurements indicate that the kestrel can just resolve two small objects that have an angular separation of only 0.013°. How does this result compare with the visual acuity predicted by Rayleigh's criterion? Take the wavelength of light in air to be 550 nm.

c. What is the distance on the retina between the images of two small objects that can just be resolved? How does this distance compare to the 2.0 μm distance between two photoreceptors, the light-sensitive cells on the retina? Does this comparison make sense from the standpoint of vision?

PREPARE a. Recall that an eye focuses by changing the focal length of its lens. The image distance s' from the lens to the image plane (at the retina) is unchanged as the bird focuses from a distant object to a nearby one, so, as **FIGURE 19.31** shows, $s' = 9.0$ mm. We can then use the thin-lens equation to find f.

FIGURE 19.31 The kestrel's eye observing two closely spaced objects.



b. By Rayleigh's criterion, visual acuity—the smallest resolvable angle of an ideal lens—is proportional to the wavelength of light used. Inside the eye, however, the wavelength of light is shorter than its wavelength λ_0 in air because of the index of refraction of the liquid within the eye. Thus we must use $\lambda = \lambda_0/n = (550\text{ nm})/(1.31) = 420$ nm in Rayleigh's criterion.

c. Figure 19.31 shows that, within the small-angle approximation, the distance d on the retina between the images of two small objects subtending an angle θ_1 is simply $d = \theta_1 f$, where θ_1 is in radians.

SOLVE a. From the thin-lens equation, Equation 18.11, we have

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{800\text{ mm}} + \frac{1}{9.0\text{ mm}} = 0.112\text{ mm}^{-1}$$

Thus $f = 1/0.112\text{ mm}^{-1} = 8.9$ mm.

b. For a perfect lens, with no aberrations, the smallest resolvable angular separation between two objects is given by Rayleigh's criterion as

$$\theta_1 = \frac{1.22\lambda}{D} = \frac{1.22(420 \times 10^{-9}\text{ m})}{3.0 \times 10^{-3}\text{ m}} = 1.7 \times 10^{-4}\text{ rad}$$

Recalling that there are 360° in 2π rad, this angle is

$$\theta_1 = (1.7 \times 10^{-4}\text{ rad}) \times \frac{360°}{2\pi\text{ rad}} = 0.0098°$$

The observed visual acuity of 0.013° is about 30% greater than this theoretical value. Presumably this is due to aberrations in the optical system of the eye.

c. The angle of 0.013° corresponds to 2.3×10^{-4} rad. Thus the distance between the images of the two small objects is

$$d = \theta_1 f = (2.3 \times 10^{-4}\text{ rad})(9.0\text{ mm})$$
$$= 2.1 \times 10^{-3}\text{ mm} = 2.1\ \mu\text{m}$$

This is just about the same as the photoreceptor distance. This makes sense. If d were significantly greater than the photoreceptor distance, then many receptors would be wasted. If d were smaller, the eye's resolution would be determined by the photoreceptor spacing and would not reach the visual acuity predicted by Rayleigh's criterion.

ASSESS An object far from a converging lens gives an image distance close to the lens's focal length. Thus our focal length of 8.9 mm—close to the image distance of 9.0 mm—is reasonable.

Although the visual acuity of the kestrel is impressive—equivalent to being able to resolve a mouse at a distance of 600 feet—it turns out not to be significantly greater than that for the human eye. Raptors are also aided in hunting their prey by a highly evolved cerebral function that allows them to pick out small movements that humans would miss.

SUMMARY

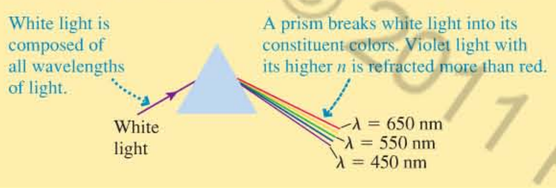
The goal of Chapter 19 has been to understand how common optical instruments work.

IMPORTANT CONCEPTS

Color and dispersion

The eye perceives light of different wavelengths as having different colors.

Dispersion is the dependence of the index of refraction n of a transparent medium on the wavelength of light: Long wavelengths have the lowest n, short wavelengths the highest n.



Resolution of optical instruments

The **resolution** of a telescope or microscope is limited by imperfections, or **aberrations,** in the optical elements, and by the more fundamental limits imposed by diffraction.

For a *microscope,* the minimum resolvable distance between two objects is

$$d_{\min} = \frac{0.61\lambda}{\text{NA}}$$

For a *telescope,* the minimum resolvable angular separation between two objects is

$$\theta_1 = \frac{1.22\lambda}{D}$$

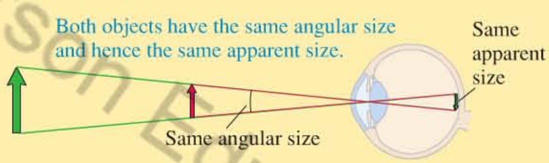
Lenses in combination

When two lenses are used in combination, the image from the first lens serves as the object for the second.

The **refractive power** P of a lens is the inverse of its focal length: $P = 1/f$. Refractive power is measured in diopters:

$$1\ \text{D} = 1\ \text{m}^{-1}$$

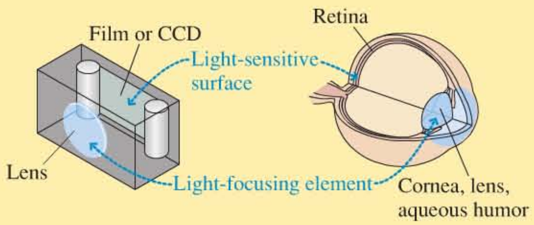
Angular and apparent size



APPLICATIONS

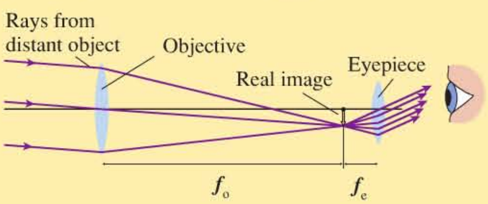
The camera and the eye

Both the camera and the eye work by focusing an image on a light-sensitive surface.



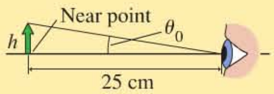
The camera focuses by changing the lens-film distance, while the eye focuses by changing the focal length of its lens.

The telescope magnifies distant objects. The objective lens creates a real image of the distant object. This real image is then magnified by the eyepiece lens, which acts as a simple magnifier. The angular magnification is $M = -f_o/f_e$.

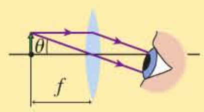


The magnifier

Without a lens, an object cannot be viewed closer than the eye's near point of ≈25 cm. Its angular size θ_0 is h/25 cm.



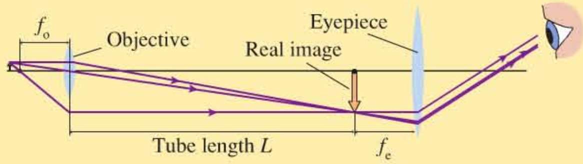
If the object is now placed at the focal point of a converging lens, its angular size is increased to $\theta = h/f$.



The angular magnification is $M = \theta/\theta_0 = 25\ \text{cm}/f$.

The microscope magnifies a small, nearby object. The objective lens creates a real image of the object. This real image is then further magnified by the eyepiece lens, which acts as a simple magnifier. The angular magnification is

$$M = -\frac{L \times 25\ \text{cm}}{f_o f_e}$$



Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. On a sunny summer day, with the sun overhead, you can stand under a tree and look on the ground at the pattern of light that has passed through gaps between the leaves. You may see illuminated circles of varying brightness. Why are there circles, when the gaps between the leaves have irregular shapes?

2. Suppose you have two pinhole cameras. The first has a small round hole in the front of the camera. The second is identical in every regard, except that it has a square hole of the same area as the round hole in the first camera. Would the pictures taken by these two cameras, under the same conditions, be different in any obvious way? Explain.
3. A photographer focuses his camera on his subject. The subject then moves closer to the camera. To refocus, should the lens be moved closer to or farther from the film? Explain.
4. Many cameras have a *zoom lens*. This is a lens whose focal length and distance from the film can be varied. If the camera's exposure is correct when the lens has a focal length of 8.0 mm, will it be overexposed, underexposed, or still correct when the focal length is increased to 16.0 mm (assume the lens diameter remains constant)? Explain.
5. A nature photographer taking a close-up shot of an insect replaces the standard lens on his camera with a lens that has a shorter focal length and is positioned farther from the film. Explain why he does this.
6. The CCD detector in a certain camera has a width of 8 mm. The photographer realizes that with the lens she is currently using, she can't fit the entire landscape she is trying to photograph into her picture. Should she switch to a lens with a longer or shorter focal length? Explain.
7. BIO All humans have what is known as a *blind spot,* where the optic nerve exits the eye and no light-sensitive cells exist. To locate your blind spot, look at the figure of the cross. Close your left eye and place your index finger on the cross. Slowly move your finger to the left while following it with your right eye. At a certain point the cross will disappear. Is your right eye's blind spot on the right or left side of your retina? Explain.

8. Suppose you wanted special glasses designed to wear underwater, without a face mask. Should the glasses use a converging or diverging lens in order for you to be able to focus under water? Explain.
9. You have lenses with the following focal lengths: $f = 25$ mm, 50 mm, 100 mm, and 200 mm. Which lens or pair of lenses would you use, and in what arrangement, to get the highest-power magnifier, microscope, and telescope? Explain.
10. BIO An 8-year-old child and a 75-year-old man both use the same magnifier to observe a bug. For whom does the magnifier more likely have the higher magnification? Explain.
11. A friend lends you the eyepiece of his microscope to use on your own microscope. He claims that since his eyepiece has the same diameter as yours but twice the focal length, the resolving power of your microscope will be doubled. Is his claim valid? Explain.
12. An astronomer is using a telescope to observe two distant stars. The stars are marginally resolved when she looks at them through a filter that passes green light near 550 nm. Which of the following actions would improve the resolution? Assume that the resolution is not limited by the atmosphere.
    a. Changing the filter to a different wavelength? If so, should she use a shorter or a longer wavelength?
    b. Using a telescope with an objective lens of the same diameter but a different focal length? If so, should she select a shorter or a longer focal length?
    c. Using a telescope with an objective lens of the same focal length but a different diameter? If so, should she select a larger or a smaller diameter?
    d. Using an eyepiece with a different magnification? If so, should she select an eyepiece with more or less magnification?
13. A pair of binoculars has a magnification of 7×. What would be their magnification if you were to look through them the wrong way, that is, through one of their objective lenses instead of the eyepieces?
14. BIO Is the wearer of the glasses in Figure Q19.14 nearsighted or farsighted? How can you tell?
15. A red card is illuminated by red light. What color does it appear to be? What if it's illuminated by blue light?

FIGURE Q19.14

Multiple-Choice Questions

16. | A photographer takes a perfectly exposed picture at an f-number of $f/4.0$ and a shutter speed of 1/125 s. Now he wishes to use a shutter speed of 1/250 s. What f-number should he choose to get a correctly exposed picture?
    A. $f/2.0$ B. $f/2.8$ C. $f/5.6$ D. $f/8.0$
17. | A microscope has a tube length of 20 cm. What combination of objective and eyepiece focal lengths will give an overall magnification of 100?
    A. 1.5 cm, 3 cm B. 2 cm, 2 cm
    C. 1 cm, 5 cm D. 3 cm, 8 cm

18. || The distance between the objective and eyepiece of a telescope is 55 cm. The focal length of the eyepiece is 5.0 cm. What is the angular magnification of this telescope?
A. −10 B. −11 C. −50 D. −275

19. | BIO A nearsighted person has a near point of 20 cm and a far point of 40 cm. When he is wearing glasses to correct his distant vision, what is his near point?
A. 10 cm B. 20 cm C. 40 cm D. 1.0 m

20. | BIO A nearsighted person has a near point of 20 cm and a far point of 40 cm. What power lens is necessary to correct this person's vision to allow her to see distant objects?
A. −5.0 D B. −2.5 D C. +2.5 D D. +5.0 D

21. | BIO A 60-year-old man has a near point of 100 cm, making it impossible to read. What power reading glasses would he need to focus on a newspaper held at a comfortable distance of 40 cm?
A. −2.5 D B. −1.5 D C. +1.5 D D. +2.5 D

22. | A person looking through a −10 D lens sees an image that appears 8.0 cm from the lens. How far from the lens is the object?
A. 10 cm B. 20 cm C. 25 cm D. 40 cm

23. | In a darkened room, red light shines on a red cup, a white card, and a blue toy. The cup, card, and toy will appear, respectively,
A. Red, red, blue.
B. Red, white, blue.
C. Red, red, black.
D. Red, black, blue.

24. || An amateur astronomer looks at the moon through a telescope with a 15-cm-diameter objective. What is the minimum separation between two objects on the moon that she can resolve with this telescope? Assume her eye is most sensitive to light with a wavelength of 550 nm.
A. 120 m B. 1.7 km C. 26 km D. 520 km

PROBLEMS

Section 19.1 The Camera

1. | BIO The human eye has a lot in common with a pinhole camera, being essentially a small box with a hole in the front (the pupil) and "film" at the back (the retina). The distance from the pupil to the retina is approximately 24 mm.
   a. Suppose you look at a 180-cm-tall friend who is standing 7.4 m in front of you. Assuming your eye functions like a pinhole camera, what will be the height, in mm, of your friend's image on your retina?
   b. Suppose your friend's image begins to get bigger. How does your brain interpret this information?

2. | A student has built a 20-cm-long pinhole camera for a science fair project. She wants to photograph the Washington Monument, which is 167 m (550 ft) tall, and to have the image on the film be 5.0 cm high. How far should she stand from the Washington Monument?

3. || A pinhole camera is made from an 80-cm-long box with a small hole in one end. If the hole is 5.0 m from a 1.8-m-tall person, how tall will the image of the person on the film be?

4. || A photographer uses his camera, whose lens has a 50 mm focal length, to focus on an object 2.0 m away. He then wants to take a picture of an object that is 40 cm away. How far, and in which direction, must the lens move to focus on this second object?

5. ||| A camera takes a perfectly exposed picture when the lens diaphragm is set to *f*/4 and the shutter speed is 1/250 s. If the diaphragm is changed to *f*/11, what should the new shutter speed be so that the exposure is still correct? (Standard camera shutter speeds include 1/250 s, 1/125 s, 1/60 s, 1/30 s, and 1/15 s.)

6. ||| In Figure P19.6 the camera lens has a 50 mm focal length. How high is the man's well-focused image on the film?

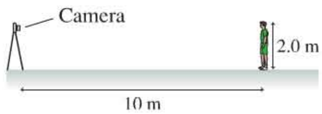


FIGURE P19.6

7. | A telephoto lens with focal length of 135 mm has *f*-numbers ranging from *f*/2.8 to *f*/22. What is the diameter of the lens aperture at these two *f*-numbers?

Section 19.2 The Human Eye

8. | BIO
   a. Estimate the diameter of your eyeball.
   b. Bring this page up to the closest distance at which the text is sharp—not the closest at which you can still read it, but the closest at which the letters remain sharp. If you wear glasses or contact lenses, leave them on. This distance is called the *near point* of your (possibly corrected) eye. Record it.
   c. Estimate the effective focal length of your eye. The effective focal length includes the focusing due to the lens, the curvature of the cornea, and any corrections you wear. Ignore the effects of the fluid in your eye.

9. || BIO A farsighted person has a near point of 50 cm rather than the normal 25 cm. What strength lens, in diopters, should be prescribed to correct this vision problem?

10. | BIO A nearsighted woman has a far point of 300 cm. What kind of lens, converging or diverging, should be prescribed for her to see distant objects more clearly? What power should the lens have?

11. | BIO The relaxed human eye is about 2 cm from front to back. If the iris of the human eye can be opened to 7 mm at its widest, what is the *f*-number of the human eye?

12. | The near point for your myopic uncle is 10 cm. Your own vision is normal; that is, your near point is 25 cm. Suppose you and your uncle hold dimes (which are 1.7 cm in diameter) at your respective near points.
    a. What is the dime's angular size, in radians, according to you?
    b. What is the dime's angular size, in radians, according to your uncle?
    c. Do these calculations suggest any benefit to near-sightedness?

13. For a patient, a doctor prescribes glasses with a converging lens having a power of 4.0 D.
BIO
   a. Is the patient nearsighted or farsighted?
   b. If the patient is nearsighted, what is the location of her eye's far point? If she is farsighted, what is the location of her near point?

14. Rank the following people from the most nearsighted to the most farsighted, indicating any ties:
BIO
   A. Bernie has a prescription of +2.0 D.
   B. Carol needs diverging lenses with a focal length of −0.35 m.
   C. Maria Elena wears converging lenses with a focal length of 0.50 m.
   D. Janet has a prescription of +2.5 D.
   E. Warren's prescription is −3.2 D.

Section 19.3 The Magnifier

15. The diameter of a penny is 19 mm. How far from your eye must it be held so that it has the same apparent size as the moon? (Use the astronomical data inside the back cover.)
16. What is the angular size of the moon? (Use the astronomical data inside the back cover.)
17. A magnifier has a magnification of 5×. How far from the lens should an object be placed so that its (virtual) image is at the near-point distance of 25 cm?
18. A farsighted man has a near point of 40 cm. What power lens should he use as a magnifier to see clearly at a distance of 10 cm without wearing his glasses?

Section 19.4 The Microscope

19. An inexpensive microscope has a tube length of 12.0 cm, and its objective lens is labeled with a magnification of 10×.
   a. Calculate the focal length of the objective lens.
   b. What focal length eyepiece lens should the microscope have to give an overall magnification of 150×?
20. A standard biological microscope is required to have a magnification of 200×.
   a. When paired with a 10× eyepiece, what power objective is needed to get this magnification?
   b. What is the focal length of the objective?
21. A forensic scientist is using a standard biological microscope with a 15× objective and a 5× eyepiece to examine a hair from a crime scene. How far from the objective is the hair?
22. A microscope with an 8.0-mm-focal-length objective has a tube length of 16.0 cm. For the microscope to be in focus, how far should the objective lens be from the specimen?
23. The distance between the objective and eyepiece lenses in a microscope is 20 cm. The objective lens has a focal length of 5.0 mm. What eyepiece focal length will give the microscope an overall angular magnification of 350?

Section 19.5 The Telescope

24. For the combination of two identical lenses shown in Figure P19.24, find the position, size, and orientation of the final image of the 2.0-cm-tall object.

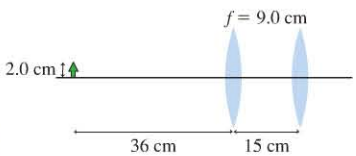


FIGURE P19.24

25. For the combination of two lenses shown in Figure P19.25, find the position, size, and orientation of the final image of the 1.0-cm-tall object.

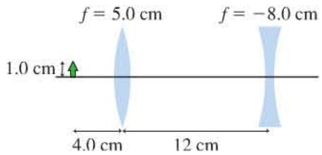


FIGURE P19.25

26. A researcher is trying to shoot a tranquilizer dart at a 2.0-m-tall rhino that is 150 m away. Its angular size as seen through the rifle telescope is 9.1°. What is the magnification of the telescope?
27. The objective lens of the refracting telescope at the Lick Observatory in California has a focal length of 57 ft.
   a. What is the refractive power of this lens?
   b. What focal length (mm) eyepiece would give a magnification of 1000× for this telescope?
28. You use your 8× binoculars to focus on a yellow-rumped warbler (length 14 cm) in a tree 18 m away from you. What angle (in degrees) does the image of the warbler subtend on your retina?
29. Your telescope has a 700-mm-focal-length objective and a 26-mm-focal-length eyepiece. One evening you decide to look at the full moon, which has an angular size of 0.52° when viewed with the naked eye.

   a. What angle (in degrees) does the image of the moon subtend when you look at it through your telescope?
   b. Suppose you decide to take a photograph of the moon using your telescope. You position film so that it captures the image produced by the objective lens. What is the diameter of that image?

Section 19.6 Color and Dispersion

30. A narrow beam of light with wavelengths from 450 nm to 700 nm is incident perpendicular to one face of a 40.00° prism made of crown glass, for which the index of refraction ranges from $n = 1.533$ to $n = 1.517$ for those wavelengths. What is the angular spread of the beam after passing through the prism?
31. A ray of white light strikes the surface of a 4.0-cm-thick slab of flint glass as shown in Figure P19.31. As the ray enters the glass, it is dispersed into its constituent colors. Estimate how far apart the rays of deepest red and deepest violet light are as they exit the bottom surface. Which exiting ray is closer to point P?

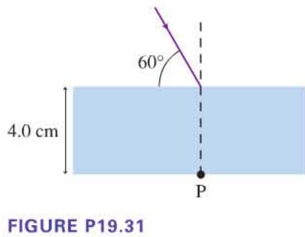


FIGURE P19.31

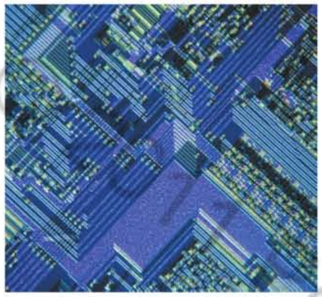
32. A ray of red light, for which $n = 1.54$, and a ray of violet light, for which $n = 1.59$, travel through a piece of glass. They meet right at the boundary between the glass and the air, and emerge into the air as one ray with an angle of refraction of 22.5°. What is the angle between the two rays in the glass?

Section 19.7 Resolution of Optical Instruments

33. Two lightbulbs are 1.0 m apart. From what distance can these light bulbs be marginally resolved by a small telescope with a 4.0-cm-diameter objective lens? Assume that the lens is limited only by diffraction and $\lambda = 600$ nm.

34. A 1.0-cm-diameter microscope objective has a focal length of 2.8 mm. It is used in visible light with a wavelength of 550 nm.
   a. What is the objective's resolving power if used in air?
   b. What is the resolving power of the objective if it is used in an oil-immersion microscope with $n_{oil} = 1.45$?

35. A microscope with an objective of focal length 1.6 mm is used to inspect the tiny features of a computer chip. It is desired to resolve two objects only 400 nm apart. What diameter objective is needed if the microscope is used in air with light of wavelength 550 nm?



General Problems

36. Suppose you point a pinhole camera at a 15-m-tall tree that is 75 m away.
   a. If the film is 22 cm behind the pinhole, what will be the size of the tree's image on the film?
   b. If you would like the image to be larger, should you get closer to the tree or farther from the tree? Explain.
   c. If you had time, you could make the image larger by rebuilding the camera, changing the length or the pinhole size. What one change would give a larger image?

37. "Jason uses a lens with focal length of 10.0 cm as a magnifier by holding it right up to his eye. He is observing an object that is 8.0 cm from the lens. What is the angular magnification of the lens used this way if Jason's near-point distance is 25 cm?

38. A magnifier is labeled "5×." What would its magnification be if used by a person with a near-point distance of 50 cm?

39. A 20× microscope objective is designed for use in a microscope with a 16 cm tube length. The objective is marked NA = 0.40. What is the diameter of the objective lens?

40. Two converging lenses with focal lengths of 40 cm and 20 cm
INT are 10 cm apart. A 2.0-cm-tall object is 15 cm in front of the 40-cm-focal-length lens.
   a. Use ray tracing to find the position and height of the image. To do this accurately use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.
   b. Calculate the image height and image position relative to the second lens. Compare with your ray-tracing answers in part a.

41. A converging lens with a focal length of 40 cm and a diverging
INT lens with a focal length of −40 cm are 160 cm apart. A 2.0-cm-tall object is 60 cm in front of the converging lens.
   a. Use ray tracing to find the position and height of the image. To do this, accurately use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.
   b. Calculate the image height and image position relative to the second lens. Compare with your ray-tracing answers in part a.

42. A lens with a focal length of 25 cm is placed 40 cm in front of a lens with a focal length of 5.0 cm. How far from the second lens is the final image of an object infinitely far from the first lens? Is this image in front of or behind the second lens?

43. A microscope with a 5× objective lens images a 1.0-mm-diameter specimen. What is the diameter of the real image of this specimen formed by the objective lens?

44. Your task in physics lab is to make a microscope from two lenses. One lens has a focal length of 10 cm, the other a focal length of 3.0 cm. You plan to use the more powerful lens as the objective, and you want its image to be 16 cm from the lens, as in a standard biological microscope.
   a. How far should the objective lens be from the object to produce a real image 16 cm from the objective?
   b. What will be the magnification of your microscope?

45. A 20× objective and 10× eyepiece give an angular magnification of 200× when used in a microscope with a 160 mm tube length. What magnification would this objective and eyepiece give if used in a microscope with a 200 mm tube length?

46. The objective lens and the eyepiece lens of a telescope are 1.0 m apart. The telescope has an angular magnification of 50. Find the focal lengths of the eyepiece and the objective.

47. Your telescope has an objective lens with a focal length of 1.0 m. You point the telescope at the moon, only to realize that the eyepiece is missing. Even so, you can still see the real image of the moon formed by the objective lens if you place your eye a little past the image so as to view the rays diverging from the image plane, just as rays would diverge from an object at that location. What is the angular magnification of the moon if you view its real image from 25 cm away, your near-point distance?

48. The 200-inch-diameter objective mirror of the reflecting telescope at the Mt. Palomar Observatory has a focal length of 17 m.
   a. The *f*-number of a mirror is defined exactly the same as the *f*-number of a lens. What is the *f*-number of this mirror?
   b. The *f*-number of the 200-inch telescope is well within the range of *f*-numbers of a cheap camera. So why not just use the camera to take pictures of distant galaxies, instead of constructing this very expensive telescope?

49. Marooned on a desert island and with a lot of time on your hands, you decide to disassemble your glasses to make a crude telescope with which you can scan the horizon for rescuers. Luckily you're farsighted, and as for most people your two eyes have somewhat different lens prescriptions. Your left eye uses a lens of power +4.5 D and your right eye's lens is +3.0 D.
   a. Which lens should you use for the objective and which for the eyepiece? Explain.
   b. What will be the magnification of your telescope?
   c. Approximately how far apart should the two lenses be when you focus on distant objects?

50. A spy satellite uses a telescope with a 2.0-m-diameter mirror. It orbits the earth at a height of 220 km. What minimum spacing must there be between two objects on the earth's surface if they are to be resolved as distinct objects by this telescope? Assume the telescope's resolution is limited only by diffraction and that it is recording light with a wavelength of 500 nm.

51. Two stars have an angular separation of 3.3×10^{-6} rad. What diameter telescope objective is necessary to just resolve these two stars, using light with a wavelength of 650 nm?

52. ||| The planet Neptune is 4.5×10^{12} m from the earth. Its diameter is 4.9×10^{7} m. What diameter telescope objective would be necessary to just barely see Neptune as a disk rather than as a point of light? Assume a wavelength of 550 nm.

53. |||| What is the angular resolution of the Hubble Space Telescope's 2.4-m-diameter mirror when viewing light with a wavelength of 550 nm? The resolution of a reflecting telescope is calculated exactly the same as for a refracting telescope.

54. ||| The Hubble Space Telescope has a mirror diameter of 2.4 m. Suppose the telescope is used to photograph stars near the center of our galaxy, 30,000 light years away, using red light with a wavelength of 650 nm.
   a. What is the distance (in km) between two stars that are marginally resolved? The resolution of a reflecting telescope is calculated exactly the same as for a refracting telescope.
   b. For comparison, what is this distance as a multiple of the distance of Jupiter from the sun?

55. ||| BIO Once dark adapted, the pupil of your eye is approximately 7 mm in diameter. The headlights of an oncoming car are 120 cm apart. If the lens of your eye is limited only by diffraction, at what distance are the two headlights marginally resolved? Assume the light's wavelength in air is 600 nm and the index of refraction inside the eye is 1.33. (Your eye is not really good enough to resolve headlights at this distance, due both to aberrations in the lens and to the size of the receptors in your retina, but it comes reasonably close.)

56. |||| BIO The normal human eye has maximum visual acuity with a pupil size of about 3 mm. For larger pupils, acuity decreases due to increasing aberrations; for smaller pupils, acuity decreases due to increasing effects of diffraction. If your pupil diameter is 2.0 mm, as it would be in fairly bright light, what is the smallest diameter circle that you can barely see as a circle, rather than just a dot, if the circle is at your near point, 25 cm from your eye? Assume the light's wavelength in air is 600 nm and the index of refraction inside the eye is 1.33.

57. ||| BIO Microtubules are filamentous structures in cells that maintain cell shape and facilitate the movement of molecules within the cell. They are long, hollow cylinders with a diameter of about 25 nm. It is possible to incorporate fluorescent molecules into microtubules; when illuminated by an ultraviolet light, the fluorescent molecules emit visible light that can be imaged by the optical system of a microscope. If the emitted light has a wavelength of 500 nm and the NA of the microscope objective is 1.4, can a biologist looking through the microscope tell whether she is looking at a single microtubule or at two microtubules lying side by side?

Passage Problems

Surgical Vision Correction BIO

Light that enters your eyes is focused to form an image on your retina. The optics of your visual system have a total power of about +60 D—about +20 D from the lens in your eye and +40 D from the curved shape of your cornea. Surgical procedures to correct vision generally do not work on the lens; they work to reshape the cornea. In the most common procedure, a laser is used to remove tissue from the center of the cornea, reducing its curvature. This change in shape can correct certain kinds of vision problems.

58. | Flattening the cornea would be a good solution for someone who was
   A. Nearsighted. B. Farsighted.
   C. Either nearsighted or farsighted.

59. | Suppose a woman has a far point of 50 cm. How much should the focusing power of her cornea be changed to correct her vision?
   A. −2.0 D B. −1.0 D C. +1.0 D D. +2.0 D

60. | A *cataract* is a clouding or opacity that develops in the eye's lens, often in older people. In extreme cases, the lens of the eye may need to be removed. This would have the effect of leaving a person
   A. Nearsighted. B. Farsighted.
   C. Neither nearsighted nor farsighted.

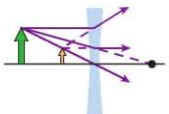
61. | The length of your eye decreases slightly as you age, making the lens a bit closer to the retina. Suppose a man had his vision surgically corrected at age 30. At age 70, once his eyes had decreased slightly in length, he would be
   A. Nearsighted. B. Farsighted.
   C. Neither nearsighted nor farsighted.

STOP TO THINK ANSWERS

Stop to Think 19.1: B. The diameter d of the lens is constant, so increasing the focal length increases the *f*-number f/d of the lens. A lens with a higher *f*-number needs *more* light for a correct exposure, requiring a *slower* shutter speed.

Stop to Think 19.2: D. Because Maria can focus on an object 0.5 m away, but not on one 10 m away, her far point must lie between these two distances. Following Example 19.4, we see that the prescription for her lens must then lie between $1/(-10\text{ m}) = -0.1$ D and $1/(-0.5\text{ m}) = -2$ D. Only the −1.5 D prescription falls in this range.

Stop to Think 19.3: A. Ray tracing shows why.



Stop to Think 19.4: B. The total magnification is the product of the objective magnification m_o and the eyepiece angular magnification M_e. If m_o is halved, from 20× to 10×, M_e must be doubled. Because M_e is inversely proportional to the eyepiece focal length, the focal length of the eyepiece must be halved.

Stop to Think 19.5: D. A green filter lets through only green light, so it blocks the red light from the apple. No light from the apple can pass through the filter, so it appears black.

Stop to Think 19.6: $RP_1 > RP_4 > RP_2 = RP_3$. The resolving power is $RP = 0.61\lambda/\sin\phi_0$ for objectives used in air ($n = 1$), so the resolving power is higher (worse resolution) when the angle ϕ_0 is smaller. From Figure 19.29 you can see that ϕ_0 is smaller when the ratio D/f is smaller. These ratios are $(D/f)_1 = 1/5$, $(D/f)_2 = 2/5$, $(D/f)_3 = 2/5$, and $(D/f)_4 = 1/3$.

Optics

Light is an elusive entity. It is everywhere around us, but exactly what *is* it? One of the more curious aspects of light is that its basic properties depend on the circumstances under which it's studied. Thus it's difficult to develop a single theory of light that applies under all circumstances. Because of this, we have developed two *models* of light in Part V, the wave model and the ray model. We found that each model has its particular realm of applicability.

Many experiments show that light has distinct wave-like properties. Light waves exhibit interference and diffraction, just as water and sound waves do. However, we're usually not aware of the wave aspects of light because the wavelengths of visible light are so short. Wave phenomena become apparent only when light interacts with objects or holes whose size is less than about 0.1 mm.

We can usually ignore the wave nature of light when we consider the propagation of light on larger length scales. In this case, we model light as traveling outward in straight lines, or *rays,* from its source. Light rays change direction at an interface between two media with different indices of refraction (different light speeds). At this interface the rays both reflect, heading back into their original medium, and refract, moving into the new medium but in a new direction. These processes are governed by the laws of reflection and refraction.

Despite light's subtle nature, the practical applications of optics are crucial to many of today's technologies. Cameras, telescopes, and microscopes all employ basic ideas of image formation with lenses and mirrors. We found that the ultimate resolution of an optical instrument is set by the wave nature of light, bringing our study of optics full circle.

KNOWLEDGE STRUCTURE V Optics

BASIC GOALS

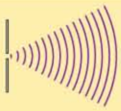
What are the consequences of the wave nature of light?
In the ray model, how do light rays refract and reflect to form images?

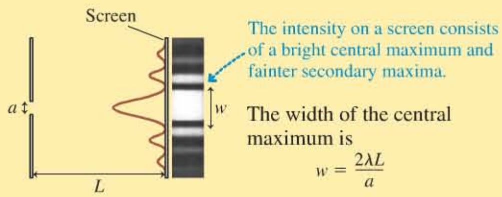
GENERAL PRINCIPLES

Light is understood using two models, the **wave model,** in which light exhibits wave properties such as interference and diffraction, and the **ray model,** in which light travels in straight lines until it reflects or refracts.

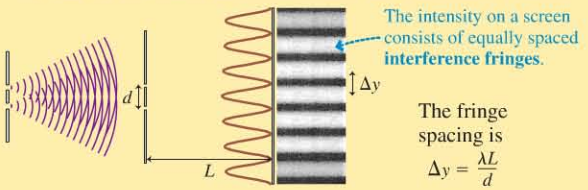
Wave model

- Light spreads out when passing through a narrow opening. This is **diffraction.**





- Light waves from multiple slits in a screen **interfere** where they overlap. The light intensity is large where the interfering waves are in phase, and small where they are out of phase.



- Light waves reflected from the two surfaces of a thin transparent film also interfere.

The resolution of optical instruments

Diffraction limits how close together two point objects can be and still be resolved.

For a *telescope,* the minimum resolvable angular separation between two objects is

$$\theta_1 = \frac{1.22\lambda}{D}$$

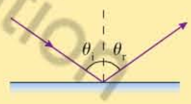
For a *microscope,* the minimum resolvable distance between two objects is

$$d_{min} = \frac{0.61\lambda}{\text{NA}}$$

where the *numerical aperture* NA is a characteristic of the microscope objective.

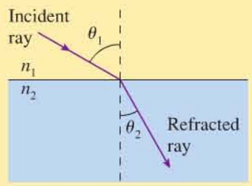
Ray model

- Light travels out from its source in straight lines, called **rays.**
- Rays reflect off a surface between two media, obeying the **law of reflection,** $\theta_i = \theta_r$.



- Light rays change direction as they cross the surface between two media. The angles of incidence and refraction are related by **Snell's law:**

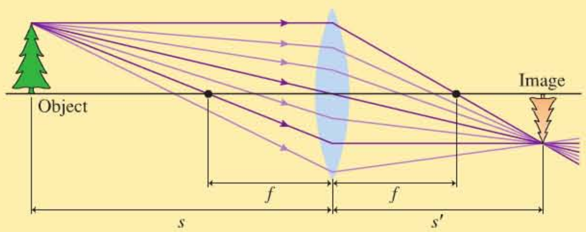
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



where n is the **index of refraction.** The speed of light in a transparent material is $v = c/n$.

Image formation by lenses and mirrors

A lens or mirror has a characteristic **focal length** f. Rays parallel to the optical axis come to focus a distance f from the lens or mirror.



The **object distance** s, the **image distance** s', and the focal length are related by the *thin-lens equation,* which also works for mirrors:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Scanning Confocal Microscopy

Although modern microscopes are marvels of optical engineering, their basic design is not too different from the 1665 compound microscope of Robert Hooke. Recently, advances in optics, lasers, and computer technology have made practical a new kind of optical microscope, the *scanning confocal microscope*. This microscope is capable of taking images of breathtaking clarity.

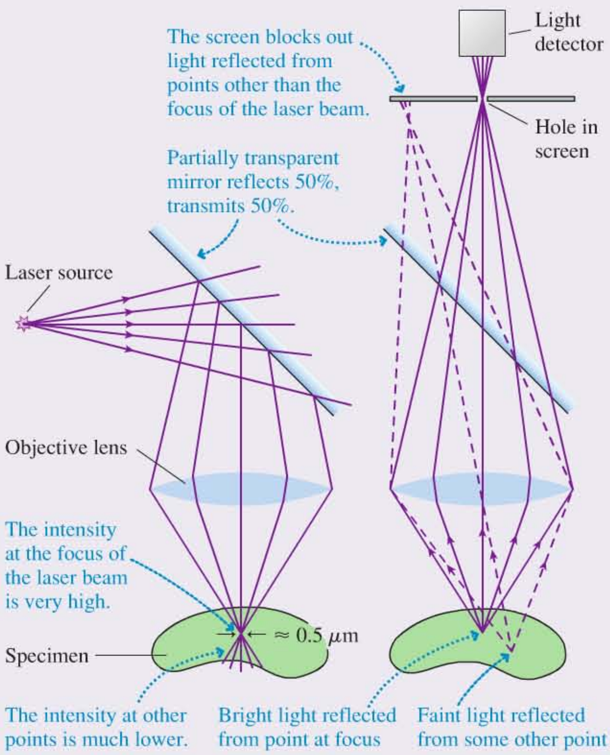
The figure shows the microscope's basic principle of operation. The left part of the figure shows how the translucent specimen is illuminated by light from a laser. The laser beam is converted to a diverging bundle of rays by suitable optics, reflected off a mirror, then directed through a microscope objective lens to a focus within the sample. The microscope objective focuses the laser beam to a very small ($\approx 0.5\ \mu$m) spot. Note that light from the laser passes through other regions of the specimen but, because the rays are not focused in those regions, they are not as intensely illuminated as is the point at the focus. This is the first important aspect of the design: Very intensely illuminate one very small volume of the sample while leaving other regions only weakly illuminated.

As shown in the right half of the figure, light is reflected from all illuminated points in the sample and passes back through the objective lens. The mirror that had reflected the laser light downward is actually a *partially transparent mirror* that reflects 50% of the light and transmits 50%. Thus half of the light reflected upward from the sample passes through the mirror and is focused on a screen containing a small hole. Because of the hole, only light rays that emanate from the brightly illuminated volume in the sample can completely pass through the hole and reach the light detector behind it. Rays from other points in the sample either miss the hole completely or are out of focus when they reach the screen, so that only a small fraction of them pass through the hole. This second key design aspect limits the detected light to only those rays that are emitted from the point in the sample at which the laser light was originally focused.

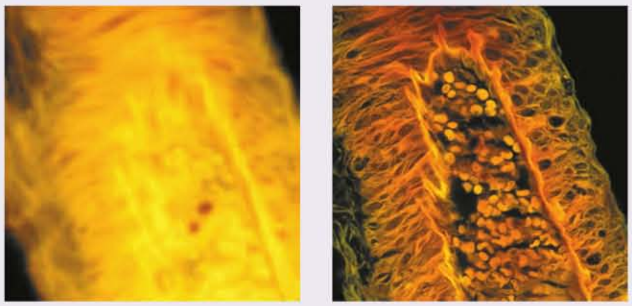
So we see that (a) the point in the sample that is at the focus of the objective is much more intensely illuminated than any other point, so it reflects more rays than any other point, and (b) the hole serves to further limit the detected rays to only those that emanate from the focus. Taken together, these design aspects ensure the detected light comes from a very small, very well-defined volume in the sample.

The microscope as shown would only be useful for examining one small point in the sample. To make an actual *image*, the objective is *scanned* across the sample while the intensity is recorded by a computer. This procedure builds up an image of the sample one *scan line* at a time. The final result is a picture of the sample in the very narrow plane in which the laser beam is focused. Different planes within the sample can be imaged by moving the objective up or down before scanning. It is actually possible to make three-dimensional images of a specimen in this way.

The improvement in contrast and resolution over conventional microscopy can be striking. The images show a section of a human medulla taken using conventional and confocal microscopy. Because light reflected from all parts of the specimen reach the camera in a conventional microscope, that image appears blurred and has low contrast. The confocal microscope image represents a single plane or slice of the sample, and many details become apparent that are invisible in the conventional image.



A confocal microscope.


A thick section of fluorescently stained human medulla imaged using standard optical microscopy (left) and scanning confocal microscopy (right).

The following questions are related to the passage "Scanning Confocal Microscopy" on the previous page.

1. A laser beam consists of parallel rays of light. To convert this light to the diverging rays required for a scanning confocal microscope requires
   A. A converging lens. B. A diverging lens.
   C. Either a converging or a diverging lens.
2. If, because of a poor-quality objective, the light from the laser illuminating the sample in a scanning confocal microscope is focused to a larger spot,
   A. The image would be dimmer because the light illuminating the point imaged would be dimmer.
   B. The image would be blurry because light from more than one point would reach the detector.
   C. The image would be dimmer and blurry—both of the above problems would exist.
3. The resolution of a scanning confocal microscope is limited by diffraction, just as for a regular microscope. In principle, switching to a laser with a shorter wavelength would provide
   A. Greater resolution.
   B. Lesser resolution.
   C. The same resolution.
4. In the optical system shown in the passage, the distance from the source of the diverging light rays to the sample is ________ the distance from the sample to the screen.
   A. greater than
   B. the same as
   C. less than

The following passages and associated questions are based on the material of Part V.

Horse Sense BIO

The ciliary muscles in a horse's eye can make only small changes to the shape of the lens, so a horse can't change the shape of the lens to focus on objects at different distances as humans do. Instead, a horse relies on the fact that its eyes aren't spherical. As Figure V.1 shows, different points at the back of the eye are at somewhat different distances from the front of the eye. We say that the eye has a "ramped retina"; images that form on the top of the retina are farther from the cornea and lens than those that form at lower positions. The horse uses this ramped retina to focus on objects at different distances, tipping its head so that light from an object forms an image at a vertical location on the retina that is at the correct distance for sharp focus.

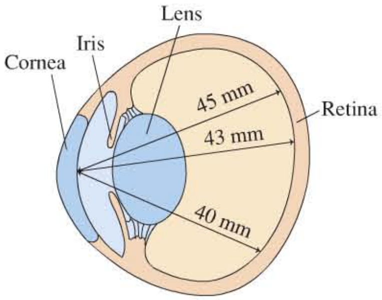


FIGURE V.1

5. In a horse's eye, the image of a close object will be in focus
   A. At the top of the retina.
   B. At the bottom of the retina.
6. In a horse's eye, the image of a distant object will be in focus
   A. At the top of the retina.
   B. At the bottom of the retina.
7. A horse is looking straight ahead at a person who is standing quite close. The image of the person spans much of the vertical extent of the retina. What can we say about the image on the retina?
   A. The person's head is in focus; the feet are out of focus.
   B. The person's feet are in focus; the head is out of focus.
   C. The person's head and feet are both in focus.
   D. The person's head and feet are both out of focus.
8. Certain medical conditions can change the shape of a horse's eyeball; these changes can affect vision. If the lens and cornea are not changed but all of the distances in the Figure V.1 are increased slightly, then the horse will be
   A. Nearsighted.
   B. Farsighted.
   C. Unable to focus clearly at any distance.

The Fire in the Eye BIO

You have certainly seen the reflected light from the eyes of a cat or a dog at night. This "eye shine" is the reflection of light from a layer at the back of the eye called the *tapetum lucidum* (Latin for "bright carpet"). The tapetum is a common structure in the eyes of animals that must see in low light. Light that passes through the retina is reflected by the tapetum back through the cells of the retina, giving them a second chance to detect the light.

Sharks and related fish have a very well-developed tapetum. Figure V.2a shows a camera flash reflected from a shark's eye back toward the camera. This reflected light is much brighter than the diffuse reflection from the body of the shark. How is this bright reflection created?

Figure V.2b shows a typical tapetum structure for a fish. (The tapetum in land animals such as cats, dogs, and deer uses similar principles but has a different structure.) The reflection comes from the interfaces between two layers of nearly transparent cells (whose index of refraction is essentially that of water) and a stack of guanine crystals sandwiched between. Light is reflected from the interface at both sides of the stack of crystals. For certain wavelengths, constructive interference leads to an especially strong reflection.

Bright light from a distant source is focused by the lens of a shark's eye to a point on the retina, as shown in Figure V.2c. The tapetum reflects these rays back through the lens, where refraction bends them into parallel rays traveling back toward the source of the light. Because the reflected light from the tapetum is directional, it is much brighter than the diffuse reflection from the shark's body. But the bright reflection is seen by an observer—or a camera—only at or near the source of the flash that produced the reflection.

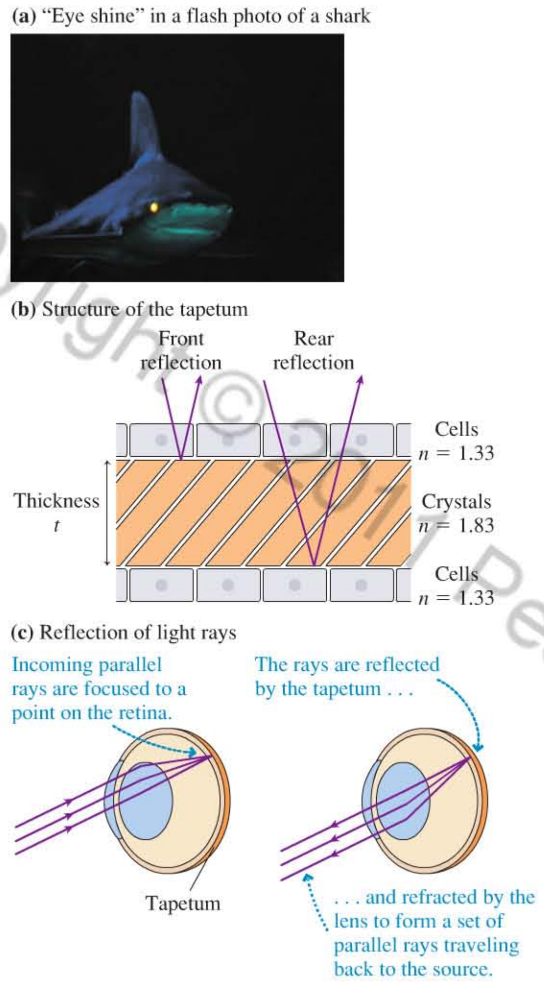


FIGURE V.2

9. Light of wavelength 600 nm in air passes into the layer of guanine crystals. What is the wavelength of the light in this layer?
   A. 1100 nm
   B. 600 nm
   C. 450 nm
   D. 330 nm
10. Figure V.2b shows rays that reflect from the two interfaces in the tapetum. Given the indices of refraction of the cells and the crystals, there will be a phase shift on reflection for
    A. The front reflection and the rear reflection.
    B. The front reflection only.
    C. The rear reflection only.
    D. Neither the front nor the rear reflection.
11. What is the (approximate) smallest thickness of the crystal layer that would lead to constructive interference between the front reflection and the rear reflection for light of wavelength 600 nm?
    A. 80 nm
    B. 160 nm
    C. 240 nm
    D. 320 nm
12. In human vision, the curvature of the cornea provides much of the power of the visual system. This is not the case in fish; in Figure V.2c, the light rays are bent by the lens but are not bent when they enter the cornea. This is because
    A. Fish eyes work in water, and the index of refraction of the fluids in the eye is similar to that of water.
    B. Fish eyes have a much smaller curvature of the cornea.
    C. Most fish have eyes that are more sensitive to light than the eyes of typical land animals.
    D. The reflection of the tapetum interferes with the refraction of the cornea.
13. Flash photographs of cats will generally show the tapetum reflection unless you are careful to avoid it. If you want to take a flash photograph of your cat while minimizing the "eye shine," which of the following strategies will *not* work?
    A. Take the photographs in dim light so that the irises of your cat's eyes are wide open.
    B. Use a flash on a stand at some distance from the camera.
    C. Use a diffuser so that the light from the flash is spread over a wide area.
    D. Use multiple flashes at different positions around the room.
14. Figure V.2c shows the lens of the eye bringing parallel rays together right at the retina. The retina is located
    A. In front of the focal point of the lens.
    B. At the focal point of the lens.
    C. Behind the focal point of the lens.

Additional Integrated Problems

15. BIO The pupil of your eye is smaller in bright light than in dim light. Explain how this makes images seen in bright light appear sharper than images seen in dim light.
16. BIO People with good vision can make out an 8.8-mm-tall letter on an eye chart at a distance of 6.1 m. Approximately how large is the image of the letter on the retina? Assume that the distance from the lens to the retina is 24 mm.
17. A photographer uses a lens with $f = 50$ mm to form an image of a distant object on the CCD detector in a digital camera. The image is 1.2 mm high, and the intensity of light on the detector is $2.5\ \text{W/m}^2$. She then switches to a lens with $f = 300$ mm that is the same diameter as the first lens. What are the height of the image and the intensity now?
18. Sound and other waves undergo diffraction just as light does. Suppose a loudspeaker in a 20°C room is emitting a steady tone of 1200 Hz. A 1.0-m-wide doorway in front of the speaker diffracts the sound wave. A person on the other side walks parallel to the wall in which the door is set, staying 12 m from the wall. When he is directly in front of the doorway, he can hear the sound clearly and loudly. As he continues walking, the sound intensity decreases. How far must he walk from the point where he was directly in front of the door until he reaches the first quiet spot?