23 Circuits

The electric eel isn't really an eel; it's a fish. But it is electric, producing pulses of up to 600 V that it uses to stun prey. How does the fish produce such a large potential difference?

LOOKING AHEAD ▸▸

The goal of Chapter 23 is to understand the fundamental physical principles that govern electric circuits.

Analyzing Circuits

We've seen different elements that you can use to create a circuit—batteries, resistors, capacitors. In this chapter, we'll explore how to analyze circuits built with these parts.

Most practical circuits consist of many different elements connected together. We'll see how to take complex circuits and break them down into manageable pieces, allowing us to analyze their operation.

Circuit analysis will draw on material of the two preceding chapters:

Looking Back ◂◂

21.5 Relating field and potential
21.7–21.9 Capacitors, dielectrics, and energy in capacitors
22.2 Current
22.4–22.6 Resistors and circuits, energy and power

Series and Parallel Circuits

As we look for ways to simplify circuits, we'll find patterns in how circuit elements are connected. The most basic connections come in two types: **series circuits** and **parallel circuits.**

In a series circuit, the elements are connected one after the other. The simple wiring of a series connection is a good choice for these inexpensive minilights. But there's a cost: If you remove one light, the string goes dark.

In a parallel circuit, each element "sees" the full voltage of the battery. If one is lost, the others stay on. Car headlights are wired as a parallel circuit, for safety.

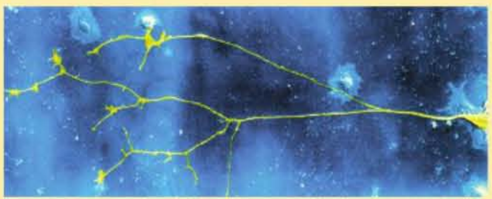
Capacitor Circuits

Capacitors store energy, as we've seen. In this chapter we'll explore different uses of capacitors in circuits.

Your camera flash is powered by a charged capacitor. After the flash fires, it takes a certain amount of time for the battery to recharge the capacitor for another flash. Capacitor circuits often have such a characteristic time.

Electricity in the Body

We will use our knowledge of electric fields, potentials, resistance, and capacitance—and an understanding of basic circuits—to explain how electrical signals propagate in your nervous system.



The long, yellow fibers are called axons. They transmit electrical signals from cell to cell. We'll make a simple model to understand how this transmission works.

23.1 Circuit Elements and Diagrams

In Chapter 22 we analyzed a very simple circuit, a resistor connected to a battery. In this chapter, we will explore more complex circuits involving more and different elements. As was the case with other topics in the book, we will learn a good deal by making appropriate drawings. To do so, we need a system for representing circuits symbolically in a manner that highlights their essential features.

FIGURE 23.1 An electric circuit.

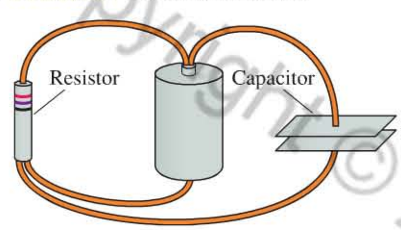


FIGURE 23.1 shows an electric circuit in which a resistor and a capacitor are connected by wires to a battery. To understand the operation of this circuit, we do not need to know whether the wires are bent or straight, or whether the battery is to the right or to the left of the resistor. The literal picture of Figure 23.1 provides many irrelevant details. It is customary when describing or analyzing circuits to use a more abstract picture called a **circuit diagram**. A circuit diagram is a *logical* picture of what is connected to what. The actual circuit, once it is built, may *look* quite different from the circuit diagram, but it will have the same logic and connections.

In a circuit diagram we replace pictures of the circuit elements with symbols. **FIGURE 23.2** shows the basic symbols that we will need.

FIGURE 23.2 A library of basic symbols used for electric circuit drawings.

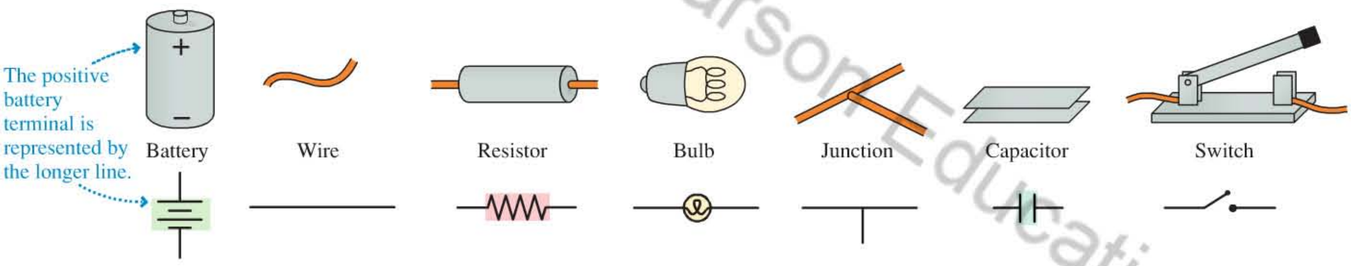


FIGURE 23.3 A circuit diagram for the circuit of Figure 23.1.

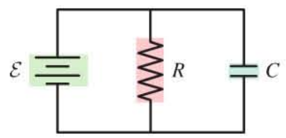
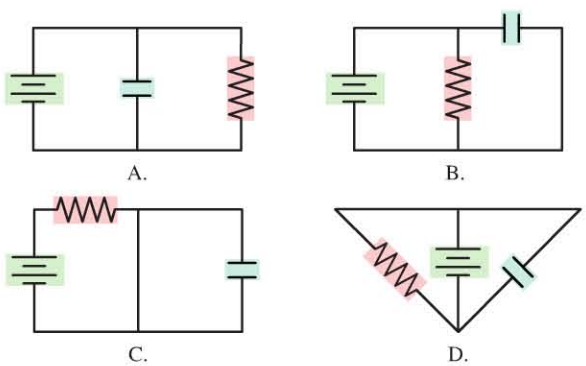


FIGURE 23.3 is a circuit diagram of the circuit shown in Figure 23.1. Notice how circuit elements are labeled. The battery's emf $\mathcal{E}$ is shown beside the battery, and the resistance R of the resistor and capacitance C of the capacitor are written beside them. We would use numerical values for $\mathcal{E}$, R, and C if we knew them. The wires, which in practice may bend and curve, are shown as straight-line connections between the circuit elements. The positive potential of the battery is at the top of the diagram; in general, we try to put higher potentials toward the top. You should get into the habit of drawing your own circuit diagrams in a similar fashion.

STOP TO THINK 23.1 Which of these diagrams represent the same circuit?



23.2 Kirchhoff's Laws

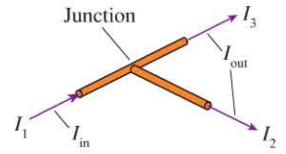
Once we have a diagram for a circuit, we can analyze it. Our tools and techniques for analyzing circuits will be based on the physical principles of potential differences and currents.

You learned in Chapter 22 that, as a result of charge and current conservation, the total current into a junction must equal the total current leaving the junction, as in FIGURE 23.4. This result was called *Kirchhoff's junction law,* which we wrote as

$$\sum I_{\text{in}} = \sum I_{\text{out}} \tag{23.1}$$

Kirchhoff's junction law

FIGURE 23.4 Kirchhoff's junction law.



Junction law: $I_1 = I_2 + I_3$

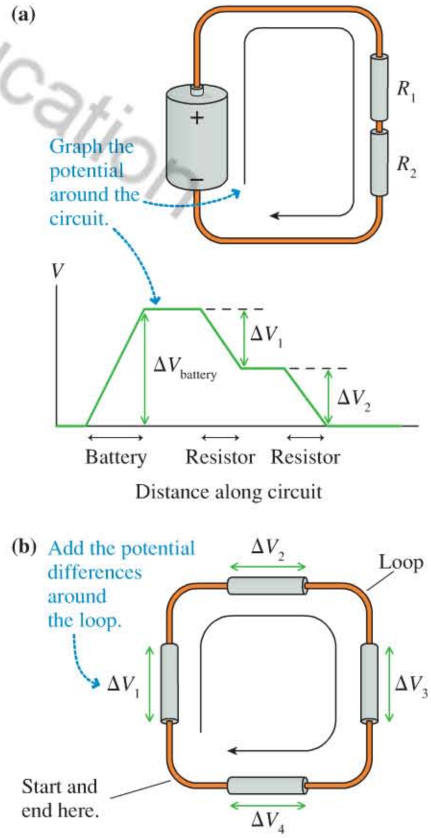
Kirchhoff's junction law isn't a new law of nature. It's an application of a law we already know: the conservation of charge. We can also apply the law of conservation of energy to circuits. When we learned about gravitational potential energy in Chapter 10, we saw that the gravitational potential energy of an object depends on its position, not on the path it took to get to that position. The same is true of electric potential energy, as you learned in Chapter 21 and as we discussed in Chapter 22. If a charged particle moves around a closed loop and returns to its starting point, there is no net change in its electric potential energy: $\Delta U_{\text{elec}} = 0$. Because $V = U_{\text{elec}}/q$, **the net change in the electric potential around any loop or closed path must be zero** as well.

FIGURE 23.5a shows a circuit consisting of a battery and two resistors. If we start at the lower left corner, at the negative terminal of the battery, and plot the potential around the loop, we get the graph shown in the figure. The potential increases as we move "uphill" through the battery, then decreases in two "downhill" steps, one for each resistor. Ultimately, the potential ends up where it started, as it must. This is a general principle that we can apply to any circuit, as shown in FIGURE 23.5b. If we add all of the potential differences around the loop formed by the circuit, the sum must be zero. This result is known as **Kirchhoff's loop law:**

$$\Delta V_{\text{loop}} = \sum_i \Delta V_i = 0 \tag{23.2}$$

Kirchhoff's loop law

FIGURE 23.5 Kirchhoff's loop law.



Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

In Equation 23.2, ΔV_i is the potential difference of the ith component in the loop.

Kirchhoff's loop law can be true only if at least one of the potential differences ΔV_i is negative. To apply the loop law, we need to explicitly identify which potential differences are positive and which are negative.

TACTICS BOX 23.1 **Using Kirchhoff's loop law** MP™

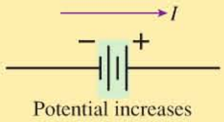
1. **Draw a circuit diagram.** Label all known and unknown quantities.
2. **Assign a direction to the current.** Draw and label a current arrow I to show your choice. Choose the direction of the current based on how the batteries or sources of emf "want" the current to go. If you choose the current direction opposite the actual direction, the final value for the current that you calculate will have the correct magnitude but will be negative, letting you know that the direction is opposite the direction you chose.

Continued

❸ **"Travel" around the loop.** Start at any point in the circuit: then go all the way around the loop in the direction you assigned to the current in step 2. As you go through each circuit element, ΔV is interpreted to mean $\Delta V = V_{\text{downstream}} - V_{\text{upstream}}$.

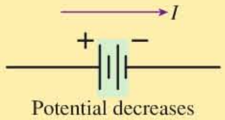
- For a battery with current in the negative-to-positive direction:

$$\Delta V_{\text{bat}} = +\mathcal{E}$$

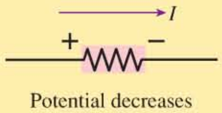


- For a battery in the positive-to-negative direction (i.e., the current is going into the positive terminal of the battery):

$$\Delta V_{\text{bat}} = -\mathcal{E}$$



- For a resistor: $\Delta V_R = -IR$



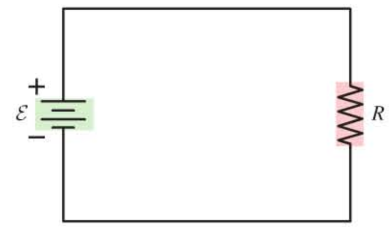
❹ **Apply the loop law:** $\sum \Delta V_i = 0$

Exercises 7, 8

ΔV_{bat} can be positive or negative for a battery, but ΔV_R for a resistor is always negative because the potential in a resistor *decreases* along the direction of the current—charge flows "downhill," as we saw in Chapter 22. Because the potential across a resistor always decreases, we often speak of the *voltage drop* across the resistor.

NOTE ► The equation for ΔV_R in Tactics Box 23.1 seems to be the opposite of Ohm's law, but Ohm's law was concerned only with the *magnitude* of the potential difference. Kirchhoff's law requires us to recognize that the electric potential inside a resistor *decreases* in the direction of the current. ◄

FIGURE 23.6 The basic circuit of a resistor connected to a battery.

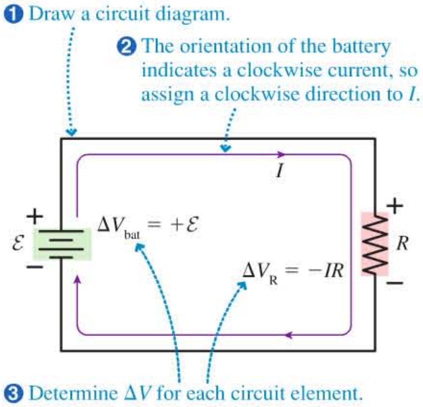


The most basic electric circuit is a single resistor connected to the two terminals of a battery, as in **FIGURE 23.6**. We considered this circuit in Chapter 22, but let's now apply Kirchhoff's laws to its analysis.

This circuit of Figure 23.6 has no junctions, so the current is the same in all parts of the circuit. Kirchhoff's junction law is not needed. Kirchhoff's loop law is the tool we need to analyze this circuit, and **FIGURE 23.7** shows the first three steps of Tactics Box 23.1. Notice that we're assuming the ideal-wire model in which there are no potential differences along the connecting wire. The fourth step is to apply Kirchhoff's loop law, $\sum \Delta V_i = 0$:

$$\Delta V_{\text{loop}} = \sum_i \Delta V_i = \Delta V_{\text{bat}} + \Delta V_R = 0 \tag{23.3}$$

FIGURE 23.7 Analysis of the basic circuit using Kirchhoff's loop law.



Let's look at each of the two terms in Equation 23.3:

1. The potential *increases* as we travel through the battery on our clockwise journey around the loop, as we see in the conventions in Tactics Box 23.1. We enter the negative terminal and, farther downstream, exit the positive terminal after having gained potential $\mathcal{E}$. Thus

$$\Delta V_{\text{bat}} = +\mathcal{E}$$

2. The *magnitude* of the potential difference across the resistor is $\Delta V = IR$, but Ohm's law does not tell us whether this should be positive or negative—and the difference is crucial. The potential of a resistor *decreases* in the direction of the current, which we've indicated with the + and − signs in Figure 23.7. Thus

$$\Delta V_R = -IR$$

With this information about ΔV_{bat} and ΔV_{R}, the loop equation becomes

$$\mathcal{E} - IR = 0 \tag{23.4}$$

We can solve the loop equation to find that the current in the circuit is

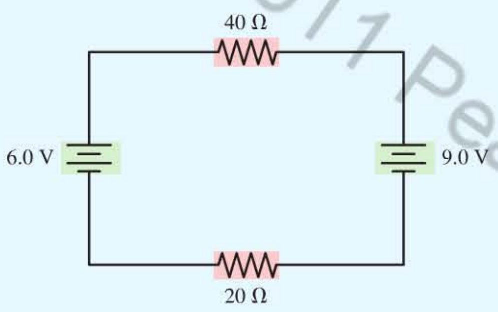
$$I = \frac{\mathcal{E}}{R} \tag{23.5}$$

This is exactly the result we saw in Chapter 22. Notice again that the current in the circuit depends on the size of the resistance. The emf of a battery is a fixed quantity; the current that the battery delivers depends jointly on the emf and the resistance.

EXAMPLE 23.1 **Analyzing a circuit with two batteries**

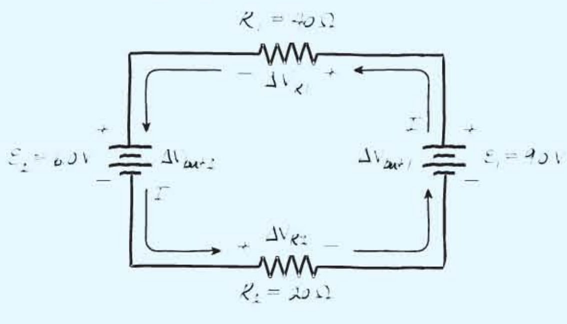
What is the current in the circuit of **FIGURE 23.8**? What is the potential difference across each resistor?

FIGURE 23.8 The circuit with two batteries.



PREPARE We will solve this circuit using Kirchhoff's loop law, as outlined in Tactics Box 23.1. But how do we deal with *two* batteries? What happens when charge flows "backward" through a battery, from positive to negative? Consider the charge escalator analogy. Left to itself, a charge escalator lifts charge from lower to higher potential. But it *is* possible to run down an up escalator, as many of you have probably done. If two escalators are placed "head to head," whichever is "stronger" will, indeed, force the charge to run down the up escalator of the other battery. The current in a battery *can* be from positive to negative if driven in that direction by a larger emf from a second battery. Indeed, this is how batteries are "recharged." In this circuit, the current goes in the direction that the larger emf—the 9.0 V battery—"wants" it to go. We have redrawn the circuit in **FIGURE 23.9**, showing the direction of the current and the direction of the potential difference for each circuit element.

FIGURE 23.9 Analyzing the circuit.



SOLVE Kirchhoff's loop law requires us to add the potential differences as we travel around the circuit in the direction of the current. Let's do this starting at the negative terminal of the 9.0 V battery:

$$\sum_i \Delta V_i = +9.0\text{ V} - I(40\ \Omega) - 6.0\text{ V} - I(20\ \Omega) = 0$$

The 6.0 V battery has $\Delta V_{\text{bat}} = -\mathcal{E}$, in accord with Tactics Box 23.1, because the potential decreases as we travel through this battery in the positive-to-negative direction. We can solve this equation for the current:

$$I = \frac{3.0\text{ V}}{60\ \Omega} = 0.050\text{ A} = 50\text{ mA}$$

Now that the current is known, we can use Ohm's law, $\Delta V = IR$, to find the potential difference across each resistor. For the 40 Ω resistor,

$$\Delta V_1 = (0.050\text{ A})(40\ \Omega) = 2.0\text{ V}$$

and for the 20 Ω resistor,

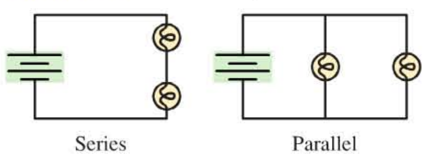
$$\Delta V_2 = (0.050\text{ A})(20\ \Omega) = 1.0\text{ V}$$

ASSESS The Assess step will be very important in circuit problems. There are generally other ways that you can analyze a circuit to check your work. In this case, you can do a final application of the loop law. If we start at the lower right-hand corner of the circuit and travel clockwise around the loop, the potential increases by 9.0 V in the first battery, then decreases by 2.0 V in the first resistor, decreases by 6.0 V in the second battery, and decreases by 1.0 V in the second resistor. The total decrease is 9.0 V, so the charge returns to its starting potential, a good check on our calculations.

23.3 Series and Parallel Circuits

Example 23.1 involved a circuit with multiple elements—two batteries and two resistors. As we introduce more circuit elements, we have possibilities for different types of connections. Suppose you use a single battery to light two lightbulbs. There are two possible ways that you can connect the circuit, as shown in **FIGURE 23.10**. These *series* and *parallel* circuits have very different properties. We will consider these two cases in turn.

FIGURE 23.10 Series and parallel circuits.



We say two bulbs are connected in **series** if they are connected directly to each other with no junction in between. All series circuits share certain characteristics.

CONCEPTUAL EXAMPLE 23.2 **Brightness of bulbs in series**

FIGURE 23.11 shows two identical lightbulbs connected in series. Which bulb is brighter: A or B? Or are they equally bright?

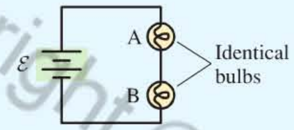


FIGURE 23.11 Two bulbs in series.

REASON Current is conserved, and there are no junctions in the circuit. Thus the current is the same at all points, as we see in FIGURE 23.12.

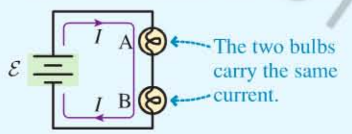


FIGURE 23.12 The current in the series circuit.

We learned in Chapter 22 that the power dissipated by a resistor is $P = I^2R$. If the two bulbs are identical (i.e., the same resistance) and have the same current through them, the power dissipated by each bulb is the same. This means that the brightness of the bulbs must be the same. The voltage across each of the bulbs will be the same as well because $\Delta V = IR$.

ASSESS It's perhaps tempting to think that bulb A will be brighter than bulb B, thinking that something is "used up" before the current gets to bulb B. It is true that *energy* is being transformed in each bulb, but current must be conserved and so both bulbs dissipate energy at the same rate. We can extend this logic to a special case: If one bulb burns out, and no longer lights, the second bulb will go dark as well. If one bulb can no longer carry a current, neither can the other.

Series Resistors

FIGURE 23.13 Replacing two series resistors with an equivalent resistor.

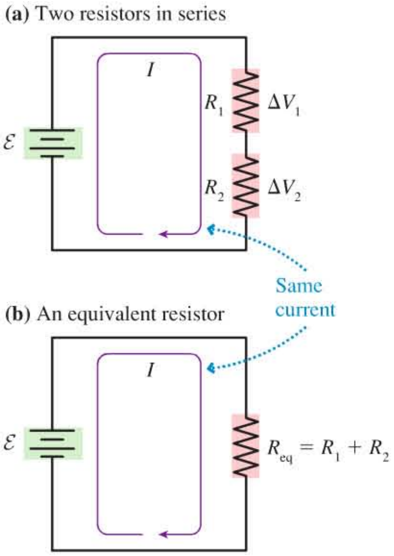


FIGURE 23.13a shows two resistors in series connected to a battery. Because there are no junctions, the current I must be the same in both resistors.

We can use Kirchhoff's loop law to look at the potential differences. Starting at the battery's negative terminal and following the current clockwise around the circuit, we find

$$\sum_i \Delta V_i = \mathcal{E} + \Delta V_1 + \Delta V_2 = 0 \qquad (23.6)$$

The voltage drops across the two resistors, in the direction of the current, are $\Delta V_1 = -IR_1$ and $\Delta V_2 = -IR_2$, so we can use Equation 23.6 to find the current in the circuit:

$$\mathcal{E} = -\Delta V_1 - \Delta V_2 = IR_1 + IR_2$$

$$I = \frac{\mathcal{E}}{R_1 + R_2} \qquad (23.7)$$

Suppose, as in FIGURE 23.13b, we replace the two resistors with a single resistor having the value $R_{eq} = R_1 + R_2$. The total potential difference across this resistor is still $\mathcal{E}$ because the potential difference is established by the battery. Further, the current in this single-resistor circuit is

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R_1 + R_2}$$

which is the same as it had been in the two-resistor circuit. In other words, this single resistor is *equivalent* to the two series resistors in the sense that the circuit's current and potential difference are the same in both cases. Nothing anywhere else in the circuit would differ if we took out resistors R_1 and R_2 and replaced them with resistor R_{eq}.

We can extend this analysis to a case with more resistors. If we have N resistors in series, their **equivalent resistance** is the sum of the N individual resistances:

$$R_{eq} = R_1 + R_2 + \cdots + R_N \qquad (23.8)$$

Equivalent resistance of N series resistors

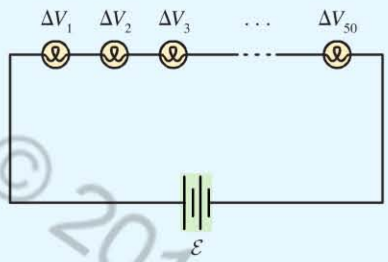
The behavior of the circuit will be unchanged if the N series resistors are replaced by the single resistor R_{eq}.

EXAMPLE 23.3 Potential difference of Christmas-tree minilights

A string of Christmas-tree minilights consists of 50 bulbs wired in series. What is the potential difference across each bulb when the string is plugged into a 120 V outlet?

PREPARE **FIGURE 23.14** shows the minilight circuit, which has 50 bulbs in series. The current in each of the bulbs is the same because they are in series.

FIGURE 23.14 50 bulbs connected in series.



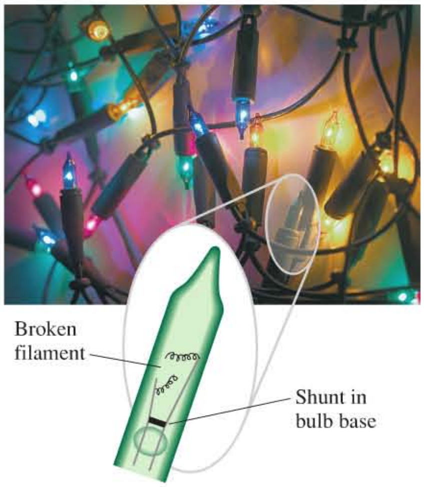
SOLVE Applying Kirchhoff's loop law around the circuit, we find

$$\mathcal{E} = \Delta V_1 + \Delta V_2 + \cdots + \Delta V_{50}$$

The bulbs are all identical and, because the current in the bulbs is the same, all of the bulbs have the same potential difference. The potential difference across a single bulb is thus

$$\Delta V_1 = \frac{\mathcal{E}}{50} = \frac{120\text{ V}}{50} = 2.4\text{ V}$$

ASSESS This result seems reasonable. The potential difference is "shared" by the bulbs in the circuit. Since the potential difference is shared among 50 bulbs, the potential difference across each bulb will be quite small.



A seasonal series circuit puzzle
Christmas-tree minilights are connected in series. This is easy to verify: When you remove one bulb from a string of lights, the circuit is not complete, and the entire string of lights goes out. But when one bulb *burns* out, meaning its filament has broken, the string of lights stays lit. How is this possible? The secret is a *shunt* in the base of the bulb. Initially, the shunt is a good insulator. But if the filament breaks, the shunt is activated, and its resistance drops. The shunt can now carry the current, so the other bulbs will stay lit.

Minilights are wired in series because the bulbs can be inexpensive low-voltage bulbs. But there is a drawback that is true of all series circuits: If one bulb is removed, there is no longer a complete circuit, and there will be no current. Indeed, if you remove a bulb from a string of minilights, the entire string will go dark.

EXAMPLE 23.4 Analyzing a series resistor circuit

What is the current in the circuit of **FIGURE 23.15**?

PREPARE The three resistors are in series, so we can replace them with a single equivalent resistor as shown in **FIGURE 23.16**.

SOLVE The equivalent resistance is calculated using Equation 23.8:

$$R_{eq} = 25\ \Omega + 31\ \Omega + 19\ \Omega = 75\ \Omega$$

The current in the equivalent circuit of Figure 23.16 is

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{9.0\text{ V}}{75\ \Omega} = 0.12\text{ A}$$

This is also the current in the original circuit.

ASSESS The current in the circuit is the same whether there are three resistors or a single equivalent resistor. The equivalent resistance is the sum of the individual resistance values, and so it is always greater than any of the individual values. This is a good check on your work.

FIGURE 23.15 A series resistor circuit.

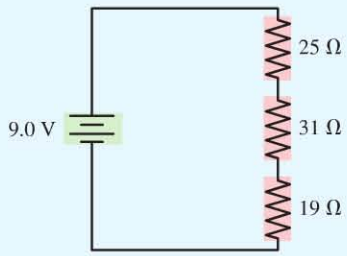


FIGURE 23.16 Analyzing a circuit with series resistors.

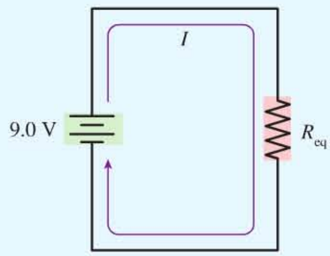
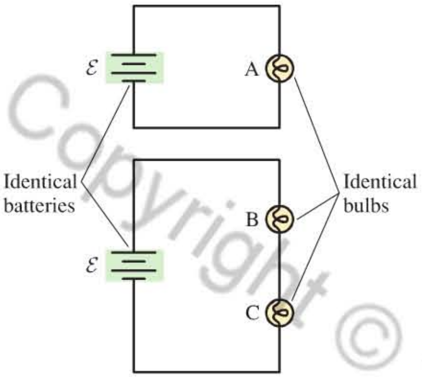


FIGURE 23.17 How does the brightness of bulb B compare to that of bulb A?



Let's use our knowledge of series circuits to look at another lightbulb puzzle. FIGURE 23.17 shows two different circuits, one with one battery and one lightbulb and a second with one battery and two lightbulbs. All of the batteries and bulbs are identical. You now know that B and C, which are connected in series, are equally bright, but how does the brightness of B compare to that of A?

Suppose the resistance of each identical lightbulb is R. In the first circuit, the battery drives current $I_A = \mathcal{E}/R$ through bulb A. In the second circuit, bulbs B and C are in series, with an equivalent resistance $R_{eq} = R_A + R_B = 2R$, but the battery has the same emf $\mathcal{E}$. Thus the current through bulbs B and C is $I_{B+C} = \mathcal{E}/R_{eq} = \mathcal{E}/2R = \frac{1}{2}I_A$. Bulb B has only half the current of bulb A, so B is dimmer.

Many people predict that A and B should be equally bright. It's the same battery, so shouldn't it provide the same current to both circuits? No—recall that **a battery is a source of potential difference, *not* a source of current.** In other words, the battery's emf is the same no matter how the battery is used. When you buy a 1.5 V battery you're buying a device that provides a specified amount of potential difference, not a specified amount of current. The battery does provide the current to the circuit, but the *amount* of current depends on the resistance. Your 1.5 V battery causes 1 A to pass through a 1.5 Ω resistor but only 0.1 A to pass through a 15 Ω resistor.

This is a critical idea for understanding circuits. A battery provides a fixed emf (potential difference). It does *not* provide a fixed and unvarying current. **The amount of current depends jointly on the battery's emf *and* the resistance of the circuit attached to the battery.**

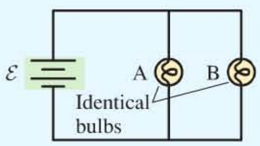
Parallel Resistors

In the next example, we consider the second way of connecting two bulbs in a circuit. The two bulbs in Figure 23.18 are connected at *both* ends. We say that they are connected in **parallel.**

CONCEPTUAL EXAMPLE 23.5 Brightness of bulbs in parallel

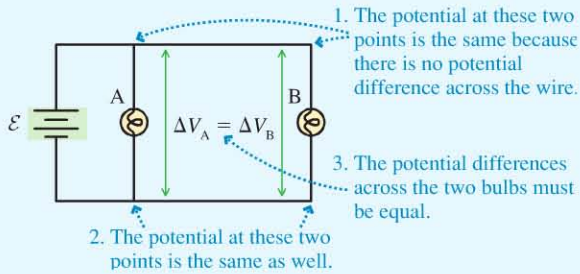
Which lightbulb in the circuit of FIGURE 23.18 is brighter: A or B? Or are they equally bright?

FIGURE 23.18 Two bulbs in parallel.



REASON Both ends of the two lightbulbs are connected together by wires. Because there's no potential difference along ideal wires, the potential at the top of bulb A must be the same as the potential at the top of bulb B. Similarly, the potentials at the bottoms of the bulbs must be the same. This means that the potential *difference* ΔV across the two bulbs must be the same, as we see in FIGURE 23.19. Because the bulbs are identical (i.e., equal resistances), the currents $I = \Delta V/R$ through the two bulbs are equal and thus the bulbs are equally bright.

FIGURE 23.19 The potential differences of the bulbs.



ASSESS One might think that A would be brighter than B because current takes the "shortest route." But current is determined by potential difference, and two bulbs connected in parallel have the same potential difference.

Let's look at parallel circuits in more detail. The circuit of FIGURE 23.20a on the next page has a battery and two resistors connected in parallel. If we assume ideal wires, the potential differences across the two resistors are equal. In fact, the potential difference across each resistor is equal to the emf of the battery because both resistors are connected directly to the battery with ideal wires; that is, $\Delta V_1 = \Delta V_2 = \mathcal{E}$.

Now we apply Kirchhoff's junction law. The current I_{bat} from the battery splits into currents I_1 and I_2 at the top junction noted in **FIGURE 23.20b**. According to the junction law,

$$I_{bat} = I_1 + I_2 \tag{23.9}$$

We can apply Ohm's law to each resistor to find that the battery current is

$$I_{bat} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{23.10}$$

Can we replace a group of parallel resistors with a single equivalent resistor as we did for series resistors? To be equivalent, the potential difference across the equivalent resistor must be $\Delta V = \mathcal{E}$, the same as for the two resistors it replaces. Further, so that the battery can't know there's been any change, the current through the equivalent resistor must be $I = I_{bat}$. A resistor with this current and potential difference must have resistance

$$R_{eq} = \frac{\Delta V}{I} = \frac{\mathcal{E}}{I_{bat}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \tag{23.11}$$

where we used Equation 23.10 for I_{bat}. This is the *equivalent resistance,* so a single resistor R_{eq} acts exactly the same as the two resistors R_1 and R_2 as shown in **FIGURE 23.20c**.

We can extend this analysis to the case of N resistors in parallel. For this circuit, the equivalent resistance is the inverse of the sum of the inverses of the N individual resistances:

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1} \tag{23.12}$$

Equivalent resistance of N parallel resistors

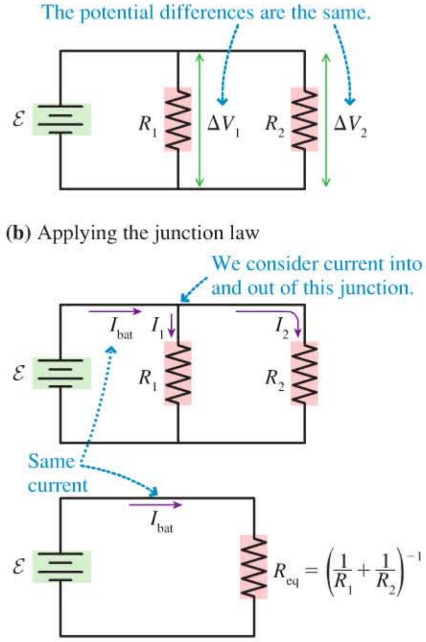
The behavior of the circuit will be unchanged if the N parallel resistors are replaced by the single resistor R_{eq}.

NOTE ► When you use Equation 23.12, don't forget to take the inverse of the sum that you compute. ◄

In Figure 23.20 each of the resistors "sees" the full potential difference of the battery. If one resistor were removed, the conditions of the second resistor would not change. This is an important property of parallel circuits.

FIGURE 23.20 Replacing two parallel resistors with an equivalent resistor.

(a) Two resistors in parallel



(c) An equivalent resistor

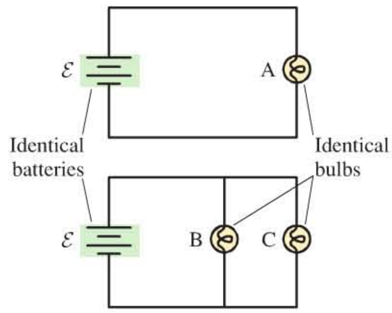
ActivPhysics 12.2, 12.5

► Parallel circuits for safety You have certainly seen cars with only one headlight lit. This tells us that automobile headlights are connected in parallel: The currents in the two bulbs are independent, so the loss of one bulb doesn't affect the other. The parallel wiring is very important so that the failure of one headlight will not leave the car without illumination.

Now, let's look at another lightbulb puzzle. **FIGURE 23.21** shows two different circuits: one with one battery and one lightbulb and a second with one battery and two lightbulbs. As before, the batteries and the bulbs are identical. You know that B and C, which are connected in parallel, are equally bright, but how does the brightness of B compare to that of A?

Each of the bulbs A, B, and C is connected to the same potential difference, that of the battery, so they each have the *same* brightness. Though all of the bulbs have the same brightness, there is a difference between the circuits. In the second circuit, the battery must power two lightbulbs, and so it must provide twice as much current. Recall that the battery is a source of fixed potential difference; the current depends on the circuit that is connected to the battery. Adding a second lightbulb doesn't change the potential difference, but it does increase the current from the battery.

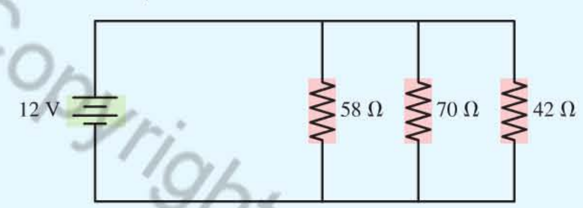
FIGURE 23.21 How does the brightness of bulb B compare to that of A?



EXAMPLE 23.6 **Current in a parallel resistor circuit**

The three resistors of **FIGURE 23.22** are connected to a 12 V battery. What current is provided by the battery?

FIGURE 23.22 A parallel resistor circuit.



PREPARE The three resistors are in parallel, so we can reduce them to a single equivalent resistor, as in **FIGURE 23.23**.

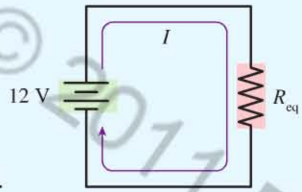


FIGURE 23.23 Analyzing a circuit with parallel resistors.

SOLVE We can use Equation 23.12 to calculate the equivalent resistance:

$$R_{eq} = \left(\frac{1}{58\ \Omega} + \frac{1}{70\ \Omega} + \frac{1}{42\ \Omega}\right)^{-1} = 18\ \Omega$$

Once we know the equivalent resistance, we can use Ohm's law to calculate the current leaving the battery:

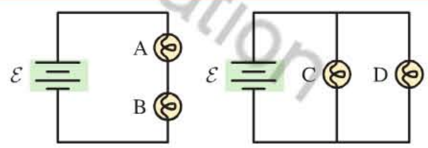
$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{12\ \text{V}}{18\ \Omega} = 0.67\ \text{A}$$

Because the battery can't tell the difference between the original three resistors and this single equivalent resistor, the battery in Figure 23.22 provides a current of 0.67 A to the circuit.

ASSESS As we'll see, the equivalent resistance of a group of parallel resistors is less than the resistance of any of the resistors in the group. 18 Ω is less than any of the individual values, a good check on our work.

The value of the total resistance in this example may seem surprising. The equivalent of a parallel combination of 58 Ω, 70 Ω, and 42 Ω is 18 Ω. Shouldn't more resistors imply more resistance? The answer is yes for resistors in series, but not for resistors in parallel. Even though a resistor is an obstacle to the flow of charge, parallel resistors provide more pathways for charge to get through. Consequently, **the equivalent of several resistors in parallel is always *less* than any single resistor in the group.** As an analogy, think about driving in heavy traffic. If there is an alternate route or an extra lane for cars to travel, more cars will be able to "flow."

STOP TO THINK 23.2 Rank in order, from brightest to dimmest, the identical bulbs A to D.



12.1, 12.4 ActivPhysics

23.4 Measuring Voltage and Current

When you use a meter to measure the voltage or the current in a circuit, how do you connect the meter? The connection depends on the quantity you wish to measure.

A device that measures the current in a circuit element is called an **ammeter.** Because charge flows *through* circuit elements, an ammeter must be placed *in series* with the circuit element whose current is to be measured.

FIGURE 23.24 An ammeter measures the current in a circuit.

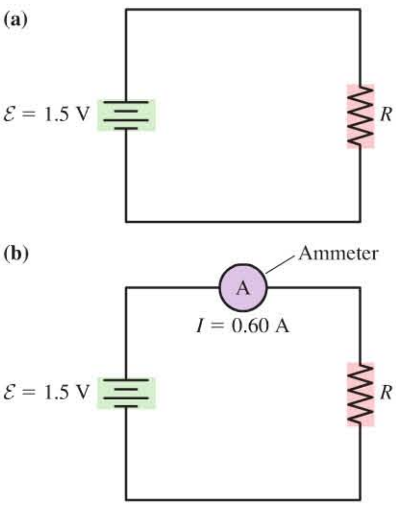


FIGURE 23.24a shows a simple one-resistor circuit with a fixed emf $\mathcal{E} = 1.5$ V and an unknown resistance R. To determine the resistance, we must know the current in the circuit, which we measure using an ammeter. We insert the ammeter in the circuit as shown in **FIGURE 23.24b**. We have to *break the connection* between the battery and the resistor in order to insert the ammeter. The resistor and the ammeter now have the same current because they are in series, so the reading of the ammeter is the current through the resistor.

Because the ammeter is in series with resistor R, the total resistance seen by the battery is $R_{eq} = R + R_{meter}$. In order to *measure* the current without *changing* the current, the ammeter's resistance must be much less than R. Thus **the resistance of an ideal ammeter is zero.** Real ammeters come quite close to this ideal.

The ammeter in Figure 23.24b reads 0.60 A, meaning that the current in the ammeter—and in the resistor—is $I = 0.60$ A. If the ammeter is ideal, which we will assume, then there is no potential difference across the ammeter ($\Delta V = IR = 0$ if $R = 0\ \Omega$) and thus the potential difference across the resistor is $\Delta V = \mathcal{E}$. The resistance can then be calculated as

$$R = \frac{\mathcal{E}}{I} = \frac{1.5\ \text{V}}{0.60\ \text{A}} = 2.5\ \Omega$$

As we saw in Chapter 21, we can use a **voltmeter** to measure potential differences in a circuit. Because a potential difference is measured *across* a circuit element, from one side to the other, a voltmeter is placed in *parallel* with the circuit element whose potential difference is to be measured. We want to *measure* the voltage without *changing* the voltage—without affecting the circuit. Because the voltmeter is in parallel with the resistor, the voltmeter's resistance must be very large so that it draws very little current. **An ideal voltmeter has infinite resistance.** Real voltmeters come quite close to this ideal.

FIGURE 23.25a shows a simple circuit in which a 24 Ω resistor is connected in series with an unknown resistance, with the pair of resistors connected to a 9.0 V battery. To determine the unknown resistance, we first characterize the circuit by measuring the potential difference across the known resistor with a voltmeter as shown in **FIGURE 23.25b**. The voltmeter is connected in parallel with the resistor; using a voltmeter does *not* require that we break the connections. The resistor and the voltmeter have the same potential difference because they are in parallel, so the reading of the voltmeter is the voltage across the resistor.

The voltmeter in Figure 23.25b tells us that the potential difference across the 24 Ω resistor is 6.0 V, so the current through the resistor is

$$I = \frac{\Delta V}{R} = \frac{6.0\ \text{V}}{24\ \Omega} = 0.25\ \text{A} \tag{23.13}$$

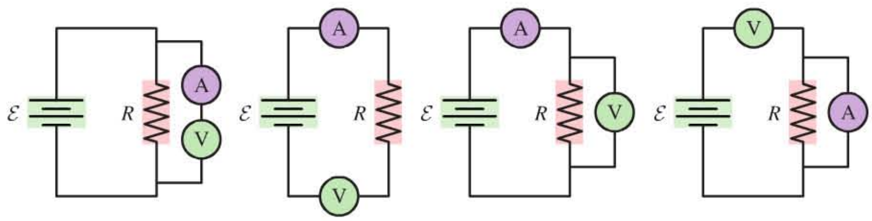
The two resistors are in series, so this is also the current in unknown resistor R. We can use Kirchhoff's loop law and the voltmeter reading to find the potential difference across the unknown resistor:

$$\sum_i \Delta V_i = 9.0\ \text{V} + \Delta V_\text{R} - 6.0\ \text{V} = 0 \tag{23.14}$$

from which we find $\Delta V_\text{R} = -3.0$ V. We can now use $\Delta V_\text{R} = -IR$ to calculate

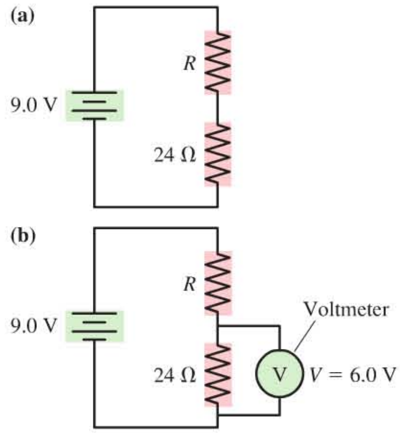
$$R = \frac{-\Delta V_\text{R}}{I} = -\frac{(-3.0\ \text{V})}{0.25\ \text{A}} = 12\ \Omega \tag{23.15}$$

STOP TO THINK 23.3 Which is the right way to connect the meters to measure the potential difference across and the current through the resistor?



A. B. C. D.

FIGURE 23.25 A voltmeter measures the potential difference across a circuit element.



▲ In this text, we speak of ammeters and voltmeters, but in practice we generally make measurements with a *multimeter.* A dial on the front sets the meter to measure voltage, current, or other electrical quantities. When set to measure current, it works as an ammeter and must be connected in series. When set to measure voltage, it works as a voltmeter and must be connected in parallel. It can also work as an *ohmmeter,* directly measuring the resistance of a resistor not in a circuit.

▶ **A circuit for all seasons** This device displays wind speed and temperature, but these are computed from basic measurements of voltage and current. The wind turns a propeller attached to a generator; a rapid spin means a high voltage. A circuit in the device contains a *thermistor,* whose resistance varies with temperature; low temperatures mean high resistance and thus a small current.

23.5 More Complex Circuits

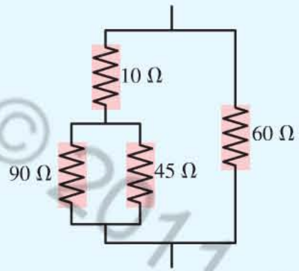
In this section, we will consider circuits that involve both series and parallel resistors. Combinations of resistors can often be reduced to a single equivalent resistance through a step-by-step application of the series and parallel rules.

EXAMPLE 23.7 Combining resistors

What is the equivalent resistance of the group of resistors shown in FIGURE 23.26?

PREPARE We can analyze this circuit by reducing combinations of series and parallel resistors. We will do this in a series of steps, redrawing the circuit after each step.

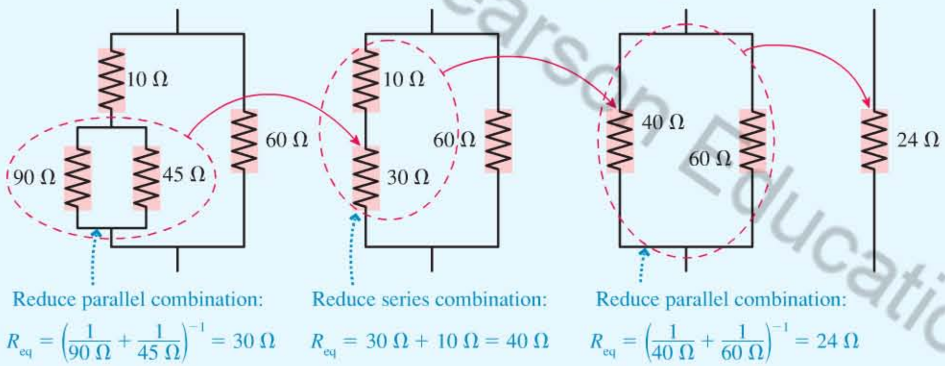
FIGURE 23.26 A resistor circuit.



SOLVE The process of simplifying the circuit is shown in FIGURE 23.27. Note that the 10 Ω and 60 Ω resistors are *not* in parallel. They are connected at their top ends but not at their bottom ends. Resistors must be connected at *both* ends to be in parallel. Similarly, the 10 Ω and 45 Ω resistors are *not* in series because of the junction between them.

ASSESS The last step in the process is to reduce a combination of parallel resistors. The resistance of parallel resistors is always less than the smallest of the individual resistance values, so our final result must be less than 40 Ω. This is a good check on the result.

FIGURE 23.27 A combination of resistors is reduced to a single equivalent resistor.



Two special cases (worth remembering for reducing circuits) are the equivalent resistances of two identical resistors $R_1 = R_2 = R$ in series and in parallel:

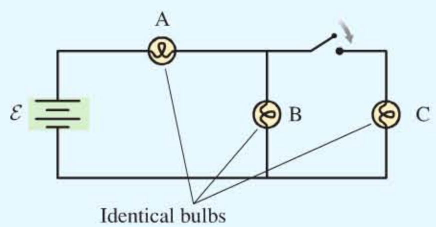
Two identical resistors in series: $R_{eq} = 2R$

Two identical resistors in parallel: $R_{eq} = \dfrac{R}{2}$

EXAMPLE 23.8 How does the brightness change?

Initially the switch in FIGURE 23.28 is open. Bulbs A and B are equally bright, and bulb C is not glowing. What happens to the brightness of A and B when the switch is closed? And how does the brightness of C then compare to that of A and B? Assume that all bulbs are identical.

FIGURE 23.28 A lightbulb circuit.



SOLVE Suppose the resistance of each bulb is R. Initially, before the switch is closed, bulbs A and B are in series; bulb C is not part of the circuit. A and B are identical resistors in series, so their equivalent resistance is $2R$ and the current from the battery is

$$I_{before} = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{2R} = \frac{1}{2}\frac{\mathcal{E}}{R}$$

This is the initial current in bulbs A and B, so they are equally bright.

Closing the switch places bulbs B and C in parallel with each other. The equivalent resistance of the two identical resistors in parallel is $R_{B+C} = R/2$. This equivalent resistance of B and C is in series with bulb A; hence the total resistance of the circuit is $R_{eq} = R + \frac{1}{2}R = \frac{3}{2}R$, and the current leaving the battery is

$$I_{after} = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{3R/2} = \frac{2}{3}\frac{\mathcal{E}}{R} > I_{before}$$

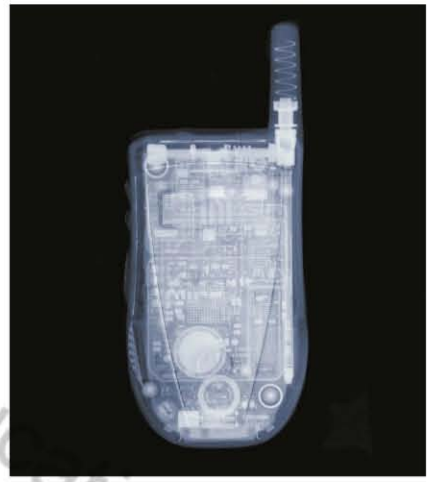
Closing the switch *decreases* the total circuit resistance and thus *increases* the current leaving the battery.

All the current from the battery passes through bulb A, so A *increases* in brightness when the switch is closed. The current I_{after} then splits at the junction. Bulbs B and C have equal resistance, so the current divides equally. The current in B is $\frac{1}{3}(\mathcal{E}/R)$, which is *less* than I_{before}. Thus B *decreases* in brightness when the switch is closed. With the switch closed, bulbs B and C are in parallel, so bulb C has the same brightness as bulb B.

ASSESS Our final results make sense. Initially, bulbs A and B are in series, and all of the current that goes through bulb A goes through bulb B. But when we add bulb C, the current has another option—it can go through bulb C. This will increase the total current, and all that current must go through bulb A, so we expect a brighter bulb A. But now the current through bulb A can go through bulb B or C. The current splits, so we'd expect that bulb B will be dimmer than before.

Analyzing Complex Circuits

We can use the information in this chapter to analyze more complex but more realistic circuits. This will give us a chance to bring together the many ideas of this chapter and to see how they are used in practice. The techniques that we use for this analysis are general, so we present them as a Problem-Solving Strategy. You can use these steps to analyze any resistor circuit, as we show in the next example.



This x-ray image of a cell phone shows the complex circuitry inside. Though there are thousands of components, the analysis of such a circuit starts with the same basic rules we are studying in this chapter.

PROBLEM-SOLVING STRATEGY 23.1 Resistor circuits

PREPARE Draw a circuit diagram. Label all known and unknown quantities.

SOLVE Base your mathematical analysis on Kirchhoff's laws and on the rules for series and parallel resistors:

- Step by step, reduce the circuit to the smallest possible number of equivalent resistors.
- Determine the current through and potential difference across the equivalent resistors.
- Rebuild the circuit, using the facts that the current is the same through all resistors in series and the potential difference is the same across all parallel resistors.

ASSESS Use two important checks as you rebuild the circuit.

- Verify that the sum of the potential differences across series resistors matches ΔV for the equivalent resistor.
- Verify that the sum of the currents through parallel resistors matches I for the equivalent resistor.

Exercise 22

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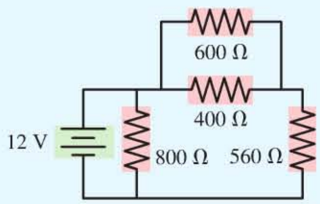
EXAMPLE 23.9 Analyzing a complex circuit

Find the current through and the potential difference across each of the four resistors in the circuit shown in **FIGURE 23.29**.

FIGURE 23.29 A multiple-resistor circuit.



PREPARE **FIGURE 23.30** shows the circuit diagram. We'll keep redrawing the diagram as we analyze the circuit.

SOLVE First, we break down the circuit, step-by-step, into one with a single resistor. Figure 23.30a does this in three steps, using the rules for series and parallel resistors. The final battery-and-resistor circuit is one we know well how to analyze. The potential difference across the 400 Ω equivalent resistor is $\Delta V_{400} = \Delta V_{bat} = \mathcal{E} = 12$ V. The current is

$$I = \frac{\mathcal{E}}{R} = \frac{12 \text{ V}}{400 \text{ Ω}} = 0.030 \text{ A} = 30 \text{ mA}$$

Continued

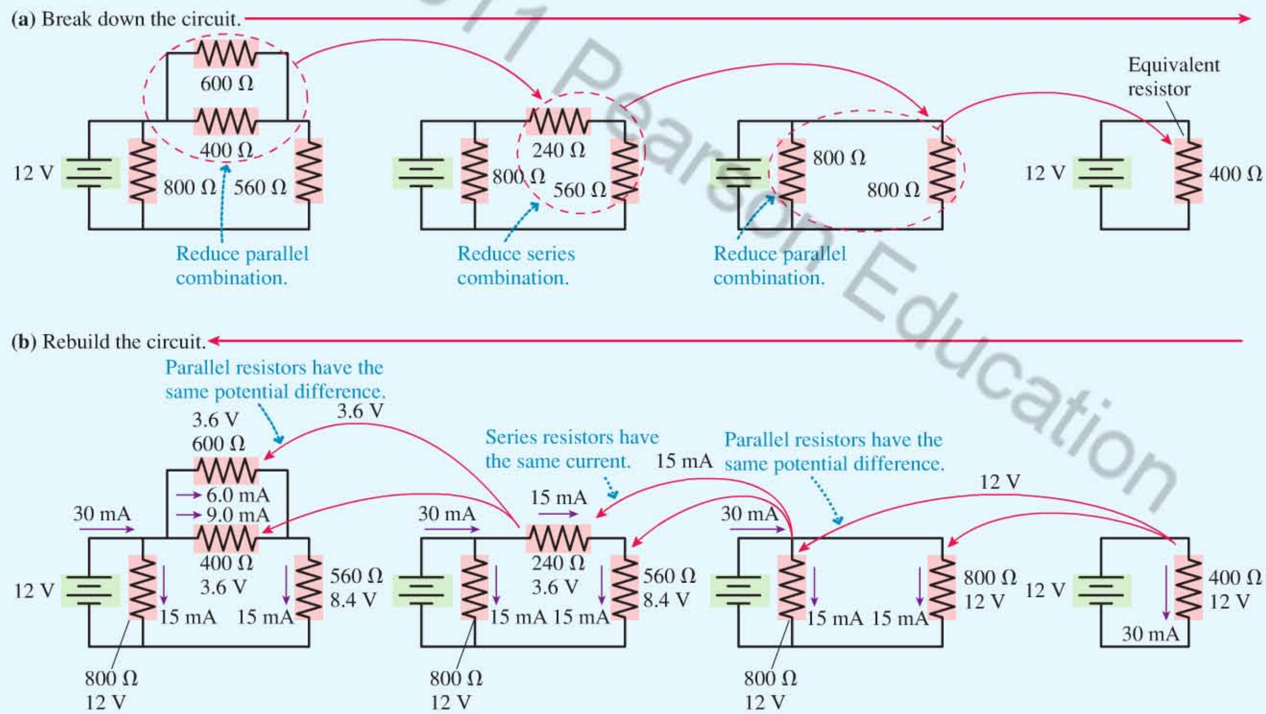
Second, we rebuild the circuit, step-by-step, finding the currents and potential differences at each step. Figure 23.30b repeats the steps of Figure 23.30a exactly, but in reverse order. The 400 Ω resistor came from two 800 Ω resistors in parallel. Because $\Delta V_{400} = 12$ V, it must be true that each $\Delta V_{800} = 12$ V. The current through each 800 Ω is then $I = \Delta V/R = 15$ mA. A check on our work is to note that 15 mA + 15 mA = 30 mA.

The right 800 Ω resistor was formed by combining 240 Ω and 560 Ω in series. Because $I_{800} = 15$ mA, it must be true that $I_{240} = I_{560} = 15$ mA. The potential difference across each is $\Delta V = IR$, so $\Delta V_{240} = 3.6$ V and $\Delta V_{560} = 8.4$ V. Here the check on our work is to note that 3.6 V + 8.4 V = 12 V = ΔV_{800}, so the potential differences add as they should.

Finally, the 240 Ω resistor came from 600 Ω and 400 Ω in parallel, so they each have the same 3.6 V potential difference as their 240 Ω equivalent. The currents are $I_{600} = 6.0$ mA and $I_{400} = 9.0$ mA. Note that 6.0 mA + 9.0 mA = 15 mA, which is a third check on our work. We now know all currents and potential differences.

ASSESS We *checked our work* at each step of the rebuilding process by verifying that currents summed properly at junctions and that potential differences summed properly along a series of resistances. This "check as you go" procedure is extremely important. It provides you, the problem solver, with a built-in error finder that will immediately inform you if a mistake has been made.

FIGURE 23.30 The step-by-step circuit analysis.



STOP TO THINK 23.4 Rank in order, from brightest to dimmest, the identical bulbs A to D.

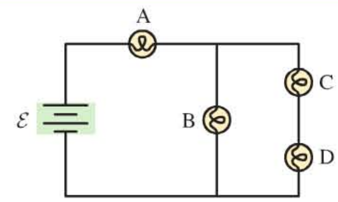
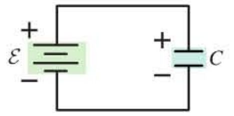


FIGURE 23.31 Simple capacitor circuit.



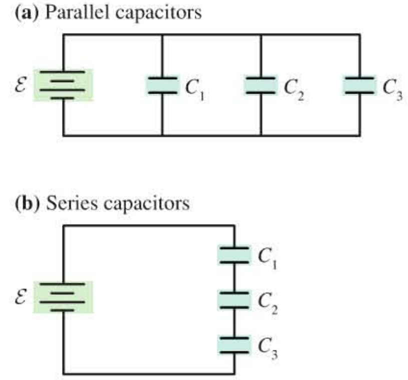
23.6 Capacitors in Parallel and Series

Two conductors separated by an insulating layer make a circuit element called a *capacitor,* a device we have considered in some detail in the past few chapters. FIGURE 23.31 shows a basic circuit consisting of a battery and a capacitor. When we connect the capacitor to the battery, charge will flow to the capacitor, increasing its potential difference until $\Delta V_C = \mathcal{E}$. Once the capacitor is fully charged, there will be no further current. We saw in Chapter 21 that the magnitude of the charge on each plate of the capacitor at this point will be $Q = C\,\Delta V_C = C\mathcal{E}$.

In resistor circuits, we often combine multiple resistors; we can do the same with capacitors. **FIGURE 23.32** illustrates two basic combinations: parallel capacitors and series capacitors.

NOTE ► The terms "parallel capacitors" and "parallel-plate capacitor" do not describe the same thing. The former term describes how two or more capacitors are connected to each other: the latter describes how a particular capacitor is constructed. ◄

FIGURE 23.32 Parallel and series capacitors.



Parallel or series capacitors can be represented by a single **equivalent capacitance,** though the rules for the combinations are different from those for resistors. Let's start our analysis with the two parallel capacitors C_1 and C_2 of **FIGURE 23.33a**.

Suppose we start with a circuit consisting of only the battery and capacitor C_1. Adding capacitor C_2 in parallel won't change the potential difference across or the charge on capacitor C_1, but there will be a change in the circuit. C_2 sees the same potential difference as C_1, so the battery must charge up the second capacitor to ΔV_C. As a result, the combination of two capacitors stores more charge than the single capacitor, at the same voltage. This means an increased overall capacitance.

The total charge is the sum of the charges on the two capacitors:

$$Q = Q_1 + Q_2 = C_1\Delta V_C + C_2\Delta V_C = (C_1 + C_2)\Delta V_C$$

We can replace the two capacitors by a single equivalent capacitance C_{eq}, as shown in **FIGURE 23.33b**. The equivalent capacitance is the sum of the individual capacitance values:

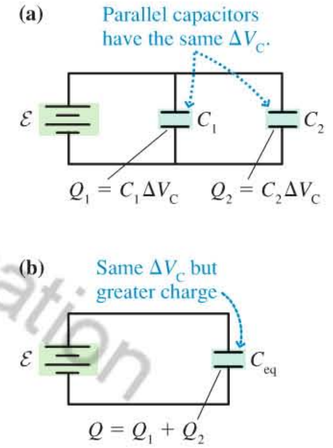
$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{(C_1 + C_2)\Delta V_C}{\Delta V_C} = C_1 + C_2 \qquad (23.16)$$

We could easily extend this analysis to more than two capacitors. If N capacitors are in parallel, their equivalent capacitance is the sum of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N \qquad (23.17)$$

Equivalent capacitance of N parallel capacitors

FIGURE 23.33 Replacing two parallel capacitors with an equivalent capacitor.



Neither the battery nor any other part of a circuit can tell if the parallel capacitors are replaced by a single capacitor having capacitance C_{eq}.

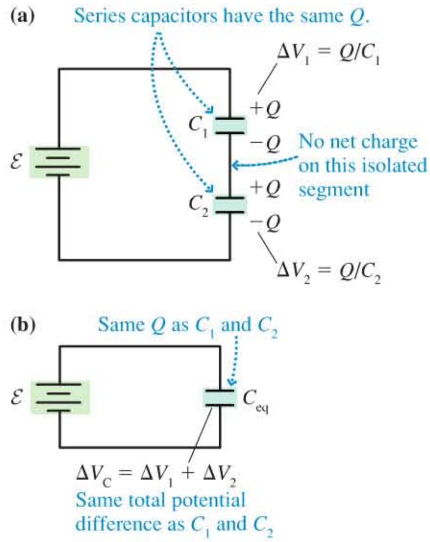
NOTE ► Adding another capacitor in parallel adds more capacitance. The formula for *parallel* capacitors is thus similar to the formula for *series* resistors. ◄

Now let's look at two capacitors connected in series. As we'll see, in this case the capacitors have the same charge, but each sees less than the full voltage of the battery.

Take a look at the circuit consisting of two series capacitors in **FIGURE 23.34a**. The center section, consisting of the bottom plate of C_1, the top plate of C_2, and the connecting wire, is electrically isolated. The battery cannot remove charge from or add charge to this section. If it starts out with no net charge, it must end up with no net charge. As a consequence, the two capacitors in series have equal charges $\pm Q$. The battery transfers Q from the bottom of C_2 to the top of C_1. This transfer polarizes the center section, but it still has $Q_{net} = 0$.

The potential differences across the two capacitors are $\Delta V_1 = Q/C_1$ and $\Delta V_2 = Q/C_2$. The total potential difference across both capacitors is $\Delta V_C = \Delta V_1 + \Delta V_2$. Suppose, as in **FIGURE 23.34b**, we replaced the two capacitors with a single capacitor having charge Q and potential difference $\Delta V_C = \Delta V_1 + \Delta V_2$. This capacitor is equivalent to the original two because the battery has to establish the same potential difference and move the same amount of charge in either case.

FIGURE 23.34 Replacing two series capacitors with an equivalent capacitor.



The inverse of the capacitance of this equivalent capacitor is

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \qquad (23.18)$$

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This analysis hinges on the fact that **series capacitors each have the same charge Q.**

We could easily extend this analysis to more than two capacitors. If N capacitors are in series, their equivalent capacitance is the inverse of the sum of the inverses of the individual capacitances:

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}\right)^{-1} \qquad (23.19)$$

Equivalent capacitance of N series capacitors

For series capacitors, the equivalent capacitance is less than that of the individual capacitors.

NOTE ► The total charge on the capacitors is the charge on each individual capacitor, but each capacitor sees only a fraction of the voltage. Adding capacitors in *series* reduces the total capacitance, just like adding resistors in *parallel*. ◄

EXAMPLE 23.10 Analyzing a capacitor circuit

a. Find the equivalent capacitance of the combination of capacitors in the circuit of FIGURE 23.35.

b. What charge flows through the battery as the capacitors are being charged?

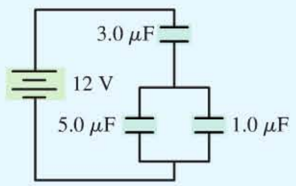


FIGURE 23.35 A capacitor circuit.

PREPARE We can use the relationships for parallel and series capacitors to reduce the capacitors to a single equivalent capacitance, much as we did for resistor circuits. We can then compute the charge through the battery using this value of capacitance.

SOLVE

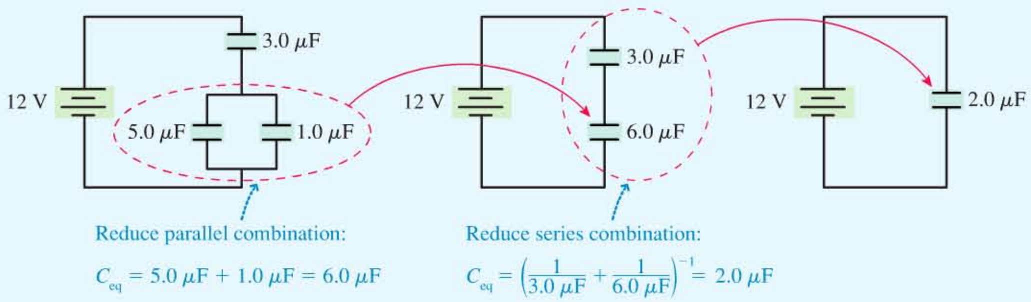
a. FIGURE 23.36 shows how we find the equivalent capacitance by reducing parallel and series combinations.

b. The battery sees a capacitance of 2.0 μF. To establish a potential difference of 12 V, the charge that must flow is

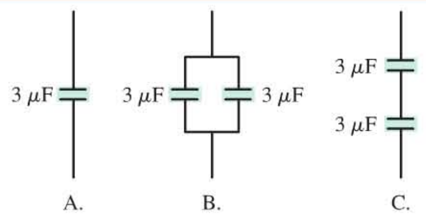
$$Q = C_{eq}\,\Delta V_C = (2.0 \times 10^{-6}\ \text{F})(12\ \text{V}) = 2.4 \times 10^{-5}\ \text{C}$$

ASSESS We solve capacitor circuit problems in a manner very similar to what we followed for resistor circuits.

FIGURE 23.36 Analyzing a capacitor circuit.



STOP TO THINK 23.5 Rank in order, from largest to smallest, the equivalent capacitance $(C_{eq})_A$ to $(C_{eq})_C$ of circuits A to C.



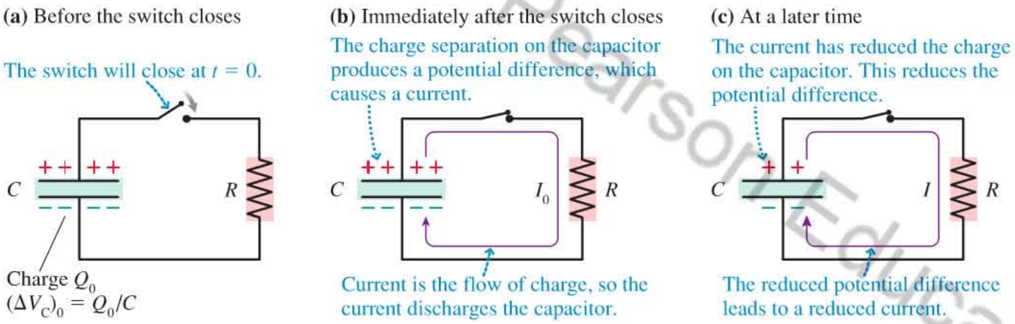
23.7 *RC* Circuits

The rear flasher on a bike blinks on and off. The timing is controlled by an *RC* circuit.

The resistor circuits we have seen have a steady current. If we add a capacitor to a resistor circuit, we can make a circuit in which the current varies with time. Circuits containing resistors and capacitors are known as ***RC* circuits.** They are very common in electronic equipment. A simple example of an *RC* circuit is the flashing bike light in the photograph. As we will see, the values of the resistance and capacitance in an *RC* circuit determine the *time* it takes the capacitor to charge or discharge. In the case of the bike light, this time determines the time between flashes. A large capacitance causes a slow cycle of on-off-on; a smaller capacitance means a more rapid flicker.

FIGURE 23.37a shows an *RC* circuit consisting of a charged capacitor, an open switch, and a resistor. The capacitor has initial charge Q_0 and potential difference $(\Delta V_C)_0 = Q_0/C$. There is no current, so the potential difference across the resistor is zero. Then, at $t = 0$, the switch closes and the capacitor begins to discharge through the resistor, as in **FIGURE 23.37b**.

FIGURE 23.37 Discharging an *RC* circuit.



How long does the capacitor take to discharge? How does the current through the resistor vary as a function of time? Let's look at the behavior of the circuit after the switch is closed.

Figure 23.37b shows the circuit *immediately* after the switch closes. The capacitor voltage is still $(\Delta V_C)_0$ because the capacitor hasn't yet had time to lose any charge, but now there's a current I_0 in the circuit that's starting to discharge the capacitor. Applying Kirchhoff's loop law, going around the loop clockwise, we find

$$\sum_i \Delta V_i = \Delta V_C + \Delta V_R = (\Delta V_C)_0 - I_0 R = 0$$

Thus the *initial* current—the initial rate at which the capacitor begins to discharge—is

$$I_0 = \frac{(\Delta V_C)_0}{R} \tag{23.20}$$

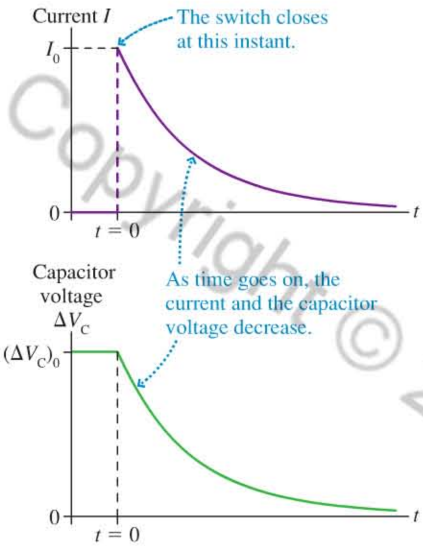
As time goes by, the current continues and the charge on the capacitor decreases. **FIGURE 23.37c** shows the circuit some time after the switch is closed; as we can see, both the charge on the capacitor (and thus the potential difference) and the current in the circuit have decreased. When the capacitor voltage has decreased to ΔV_C, the current has decreased to

$$I = \frac{\Delta V_C}{R} \tag{23.21}$$

The current discharges the capacitor, which causes ΔV_C to decrease. The capacitor voltage ΔV_C drives the current, so the current I decreases as well. The current I and the voltage ΔV_C both decrease until the capacitor is fully discharged and the current is zero.

If we use a voltmeter and an ammeter to measure the capacitor voltage and the current in the circuit of Figure 23.37 as a function of time, we find the variation

FIGURE 23.38 Current and capacitor voltage in an *RC* discharge circuit.



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shown in the graphs of **FIGURE 23.38**. At $t = 0$, when the switch closes, the potential difference across the capacitor is $(\Delta V_C)_0$ and the current suddenly jumps to I_0. The current and the capacitor voltage then "decay" to zero, but *not* linearly.

The graphs in Figure 23.38 have the same shape as that for the decay of the amplitude of a damped simple harmonic oscillator we saw in Chapter 14. Both the voltage and the current are *exponential decays* given by the equations

$$I = I_0 e^{-t/RC}$$
$$\Delta V_C = (\Delta V_C)_0 e^{-t/RC} \qquad (23.22)$$

Current and voltage during a capacitor discharge

In Chapter 14, we saw that we could characterize exponential decay by a **time constant** τ. The time constant is really a *characteristic time* for the circuit. A long time constant implies a slow decay; a short time constant, a rapid decay. The time constant for the decay of current and voltage in an *RC* circuit is

$$\tau = RC \qquad (23.23)$$

If you work with the units, you can show that the product of ohms and farads is seconds, so the quantity *RC* really is a time. In terms of this time constant, the current and voltage equations are

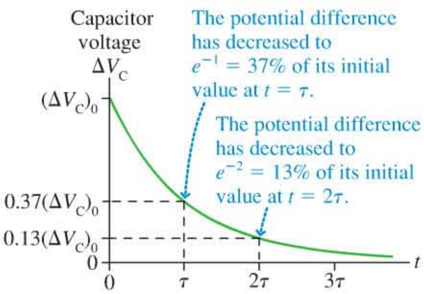
$$I = I_0 e^{-t/\tau}$$
$$\Delta V_C = (\Delta V_C)_0 e^{-t/\tau} \qquad (23.24)$$

The current and voltage in the circuit do not drop to zero after one time constant; that's not what the time constant means. Instead, each increase in time by one time constant causes the voltage and current to decrease by a factor of $e^{-1} = 0.37$, as we see in **FIGURE 23.39**.

We can understand why the time constant has the form $\tau = RC$. A large value of resistance opposes the flow of charge, so increasing R increases the decay time. A larger capacitance stores more charge, so increasing C also increases the decay time.

After one time constant, the current and voltage in a capacitor circuit have decreased to 37% of their initial values. When is the capacitor fully discharged? There is no exact time that we can specify because ΔV_C approaches zero gradually. But after 5τ the voltage and current have decayed to less than 1% of their initial values. For most purposes, we can say that the capacitor is discharged at this time.

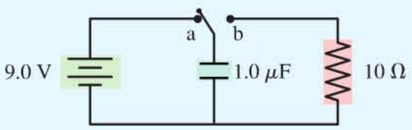
FIGURE 23.39 The meaning of the time constant in an *RC* circuit.



EXAMPLE 23.11 Finding the current in an *RC* circuit

The switch in the circuit of **FIGURE 23.40** has been in position a for a long time, so the capacitor is fully charged. The switch is changed to position b at $t = 0$. What is the current in the circuit immediately after the switch is closed? What is the current in the circuit 25 μs later?

FIGURE 23.40 The *RC* circuit.



SOLVE The capacitor is connected across the battery terminals, so initially it is charged to $(\Delta V_C)_0 = 9.0$ V. When the switch is closed, the initial current is given by Equation 23.20:

$$I_0 = \frac{(\Delta V_C)_0}{R} = \frac{9.0\ \text{V}}{10\ \Omega} = 0.90\ \text{A}$$

As charge flows, the capacitor discharges. The time constant for the decay is given by Equation 23.23:

$$\tau = (10\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 1.0 \times 10^{-5}\ \text{s} = 10\ \mu\text{s}$$

The current in the circuit as a function of time is given by Equation 23.22. 25 μs after the switch is closed, the current is

$$I = I_0 e^{-t/\tau} = (0.90\ \text{A})e^{-(25\ \mu\text{s})/(10\ \mu\text{s})} = 0.074\ \text{A}$$

ASSESS This result makes sense. 25 μs after the switch has closed is 2.5 time constants, so we expect the current to decrease to a small fraction of the initial current. Notice that we left times in units of μs; this is one of the rare cases where we needn't convert to SI units. Because the exponent is $-t/\tau$, which involves a ratio of two times, we need only be certain that both t and τ are in the same units.

Charging a Capacitor

FIGURE 23.41a shows a circuit that charges a capacitor. After the switch is closed, the potential difference of the battery causes a current in the circuit, and the capacitor begins to charge. As the capacitor charges, it develops a potential difference that opposes the current, so the current decreases. As the current decreases, so does the rate of charging of the capacitor. The capacitor charges until $\Delta V_C = \mathcal{E}$, when the charging current ceases.

If we measure the current in the circuit and the potential difference across the capacitor as a function of time, we find that they vary according to the graphs in FIGURE 23.41b. The characteristic time for this charging circuit is the same as for the discharge, the time constant $\tau = RC$.

When the switch is first closed, the potential difference across the uncharged capacitor is zero, so the initial current is

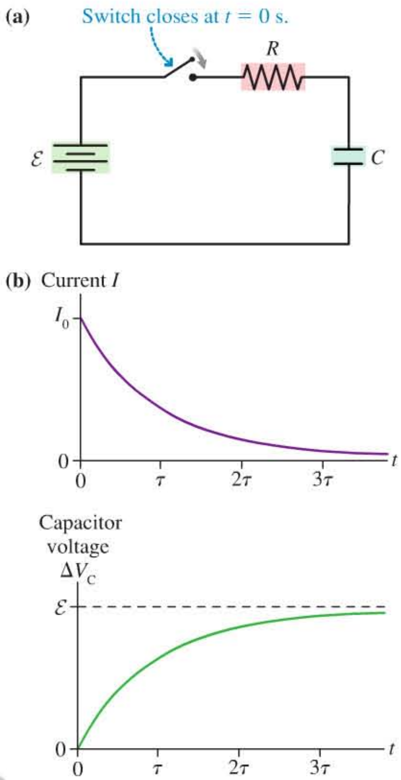
$$I_0 = \frac{\mathcal{E}}{R}$$

The equations that describe the capacitor voltage and the current as a function of time are

$$\Delta V_C = \mathcal{E}(1 - e^{-t/\tau})$$
$$I = I_0 e^{-t/\tau} \tag{23.25}$$

Current and voltage while charging a capacitor

FIGURE 23.41 A circuit for charging a capacitor.

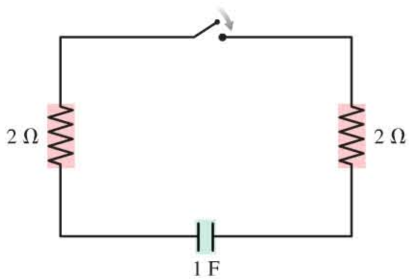


The time constant τ in an RC circuit can be used to control the behavior of a circuit. For example, a bike flasher uses an RC circuit that alternately charges and discharges, over and over, as a switch opens and closes. A separate circuit turns the light on when the capacitor voltage exceeds some threshold voltage and turns the light off when the capacitor voltage goes below this threshold. The time constant of the RC circuit determines how long the capacitor voltage stays above the threshold and thus sets the length of the flashes. More complex RC circuits provide timing in computers and other digital electronics. As we will see in the next section, we can also use RC circuits to model the transmission of nerve impulses, and the time constant will be a key factor in determining the speed at which signals can be propagated in the nervous system.

A rainy-day *RC* circuit When you adjust the dial to control the delay of the intermittent windshield wipers in your car, you are adjusting a variable resistor in an RC circuit that triggers the wipers. Increasing the resistance increases the time constant and thus produces a longer delay between swipes of the blades. A light mist calls for a long time constant and thus a large resistance.

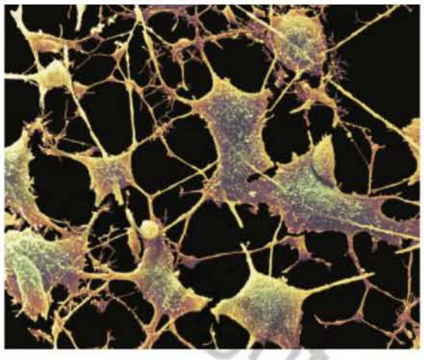
STOP TO THINK 23.6 The time constant for the discharge of this capacitor is

A. 5 s.
B. 4 s.
C. 2 s.
D. 1 s.

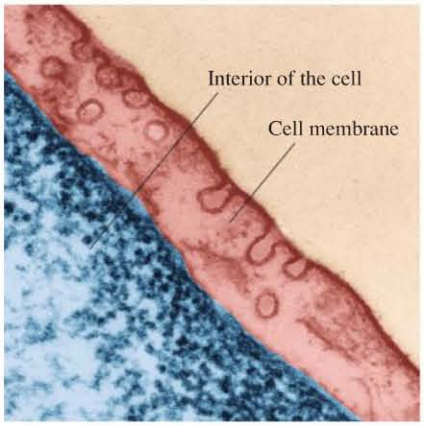


23.8 Electricity in the Nervous System

In the late 1700s, the Italian scientist Galvani discovered that animal tissue has an electrical nature. He found that a frog's leg would twitch when stimulated with electricity, even when no longer attached to the frog. Further investigations by Galvani and others revealed that electrical signals can animate muscle cells, and that a small



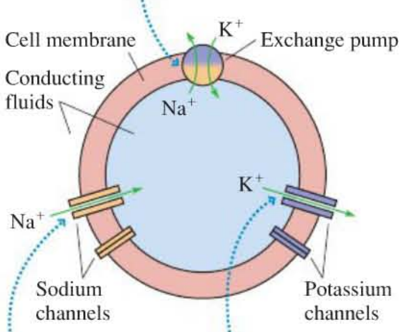
The long fibers connecting these nerve cells are *axons,* the transmission lines for electrical signals between cells.



A close-up view of the cell membrane, the insulating layer that divides the interior of a cell from the conducting fluid outside.

FIGURE 23.42 A simple model of a nerve cell.

The pump moves sodium out of the cell and potassium in, so the sodium concentration is higher outside the cell, the potassium concentration is higher inside.



When a sodium channel is open, the higher sodium concentration outside the cell causes ions to flow into the cell.

When a potassium channel is open, the higher potassium concentration inside the cell causes ions to flow out of the cell.

potential applied to the *axon* of a nerve cell can produce a signal that propagates down its length.

Our goal in this section will be to understand the nature of electrical signals in the nervous system. When your brain orders your hand to move, how does the signal get from your brain to your hand? Answering this question will use our knowledge of fields, potential, resistance, capacitance, and circuits, all of the knowledge and techniques that we have learned so far in Part VI.

The Electrical Nature of Nerve Cells

We start our analysis with a very simple *model* of a nerve cell that allows us to describe its electrical properties. The model begins with a *cell membrane,* an insulating layer of lipids approximately 7 nm thick that separates regions of conducting fluid inside and outside the cell.

As we saw in Chapter 21, the cell membrane is not a passive structure. It has channels and pumps that transport ions between the inside and the outside of the cell. In our simple model we will consider the transport of only two positive ions, sodium (Na^+) and potassium (K^+), though other ions are also important to cell function. Ions, rather than electrons, are the charge carriers of the cell. These ions can slowly diffuse across the cell membrane. In addition, sodium and potassium ions are transported via the following structures:

- *Sodium-potassium exchange pumps.* These pump Na^+ ions out of the cell and K^+ ions in. In the cell's resting state, the concentration of sodium ions outside the cell is about ten times the concentration on the inside. Potassium ions are more concentrated on the inside.
- *Sodium and potassium channels.* These channels in the cell membrane are usually closed. When they are open, ions move in the direction of lower concentration. Thus Na^+ ions flow into the cell and K^+ ions flow out.

Our simple model, illustrated in **FIGURE 23.42**, neglects many of the features of real cells, but it allows us to accurately describe the reaction of nerve cells to a stimulus and the conduction of electrical signals.

The ion exchange pumps act much like the charge escalator of a battery, using chemical energy to separate charge by transporting ions. The transport and subsequent diffusion of charged ions lead to a separation in charge across the cell membrane. Consequently, **a living cell generates an emf.** This emf takes energy to create and maintain. The ion pumps that produce the emf of neural cells account for 25–40% of the energy usage of the brain.

The charge separation produces an electric field inside the cell membrane and results in a potential difference between the inside and the outside of the cell, as shown in **FIGURE 23.43** on the next page. The potential inside a nerve cell is typically 70 mV less than that outside the cell. This is called the cell's *resting potential.* Because this potential difference is produced by a charge separation across the membrane, we say that the membrane is *polarized.* Because the potential difference is entirely across the membrane, we may call this potential difference the *membrane potential.*

EXAMPLE 23.12 Electric field in a cell membrane

The thickness of a typical nerve cell membrane is 7.0 nm. What is the electric field inside the membrane of a resting nerve cell?

PREPARE The potential difference across the membrane of a resting nerve cell is -70 mV. The inner and outer surfaces of the membrane are equipotentials. We learned in Chapter 21 that the electric field is perpendicular to the equipotentials and is related to the potential difference by $E = \Delta V/d$.

SOLVE The magnitude of the potential difference between the inside and the outside of the cell is 70 mV. The field strength is thus

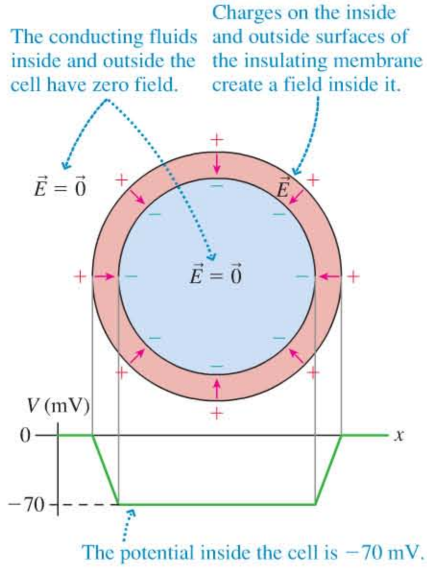
$$E = \frac{\Delta V}{d} = \frac{70 \times 10^{-3}\ \text{V}}{7.0 \times 10^{-9}\ \text{m}} = 1.0 \times 10^{7}\ \text{V/m}$$

The field points from positive to negative, so the electric field is

$$\vec{E} = (1.0 \times 10^{7}\ \text{V/m, inward})$$

ASSESS This is a very large electric field; in air it would be large enough to cause a spark! But we expect the fields to be large to explain the cell's strong electrical character.

FIGURE 23.43 The resting potential of a nerve cell.

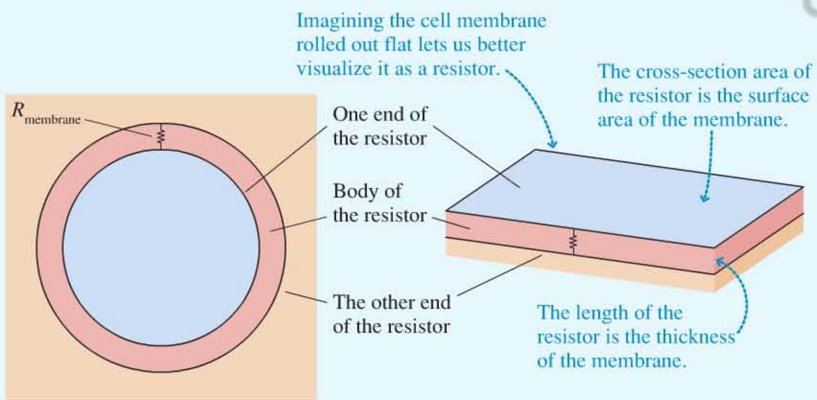


EXAMPLE 23.13 Finding the resistance of a cell membrane

Charges can move across the cell membrane, so it is not a perfect insulator; the cell membrane will have a certain resistance. The resistivity of the cell membrane was given in Chapter 22 as $36 \times 10^{6}\ \Omega \cdot \text{m}$. What is the resistance of the 7.0-nm-thick membrane of a spherical cell with diameter 0.050 mm?

PREPARE The membrane potential will cause charges to move *through* the membrane. As we learned in Chapter 22, an object's resistance depends on its resistivity, length, and cross-section area. What this means for a cell membrane is noted in **FIGURE 23.44**.

FIGURE 23.44 The cell membrane can be modeled as a resistor.



SOLVE The area of the membrane is the surface area of a sphere, $4\pi r^2$. We can calculate the resistance using the equation for the resistance of a conductor of length L and cross-section area A from Chapter 22:

$$R_{\text{membrane}} = \frac{\rho L}{A} = \frac{(3.6 \times 10^{7}\ \Omega \cdot \text{m})(7.0 \times 10^{-9}\ \text{m})}{4\pi(2.5 \times 10^{-5}\ \text{m})^2}$$

$$= 3.2 \times 10^{7}\ \Omega = 32\ \text{M}\Omega$$

ASSESS The resistance is quite high; the membrane is a good insulator, as we noted.

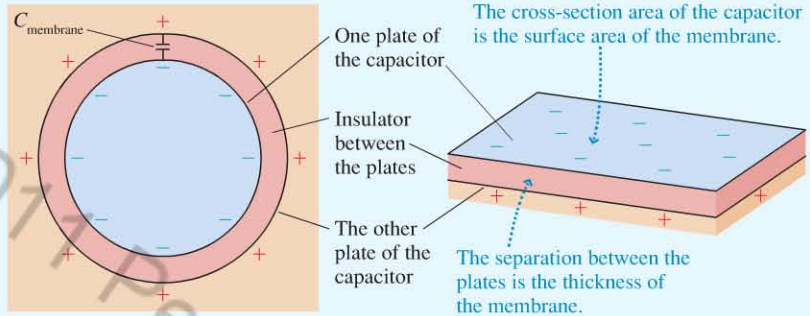
We can associate a resistance with the cell membrane, but we can associate other electrical quantities as well. The fluids inside and outside of the membrane are good conductors; they are separated by the membrane, which is not. Charges therefore accumulate on the inside and outside surfaces of the membrane. A cell thus looks like two charged conductors separated by an insulator—a capacitor.

EXAMPLE 23.14 Finding the capacitance of a cell membrane

What is the capacitance of the membrane of the spherical cell specified in Example 23.13? The dielectric constant of a cell membrane is approximately 9.0.

PREPARE If we imagine opening up a cell membrane and flattening it out, we would get something that looks like a parallel-plate capacitor with the plates separated by a dielectric as illustrated in FIGURE 23.45. The relevant dimensions are the same as those in Example 23.13.

FIGURE 23.45 The cell membrane can also be modeled as a capacitor.

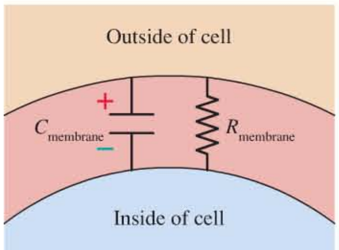


SOLVE The capacitance of the membrane is that of a parallel-plate capacitor filled with a dielectric, which was given in Equation 21.22. Inserting the dimensions from Example 23.13, we find

$$C_{membrane} = \frac{\kappa\epsilon_0 A}{d} = \frac{9.0(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})4\pi(2.5\times10^{-5}\ \mathrm{m})^2}{7.0\times10^{-9}\ \mathrm{m}}$$
$$= 8.9\times10^{-11}\ \mathrm{F}$$

ASSESS Though the cell is small, the cell membrane has a reasonably large capacitance of ≈90 pF. This makes sense because the membrane is quite thin.

FIGURE 23.46 The cell membrane can be modeled as an *RC* circuit.



Because the cell membrane has both resistance and capacitance, it can be modeled as an *RC* circuit, as shown in FIGURE 23.46. The membrane, like any *RC* circuit, has a time constant. The previous examples calculated the resistance and capacitance of the 7.0-nm-thick membrane of a 0.050-mm-diameter cell. We can use these numbers to compute the membrane's time constant:

$$\tau = RC = (3.2\times10^{7}\ \Omega)(8.9\times10^{-11}\ \mathrm{F}) = 2.8\times10^{-3}\ \mathrm{s} \approx 3\ \mathrm{ms}$$

Indeed, if we raise the membrane potential of a real nerve cell by 10 mV (large enough to easily measure but not enough to trigger a response in the cell), the potential will decay back to its resting value with a time constant of a few ms.

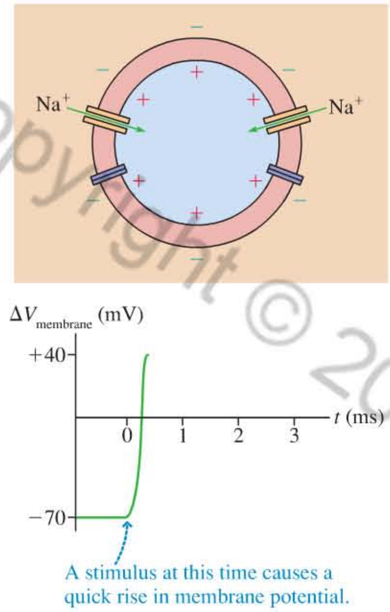
But the real action happens when some stimulus *is* large enough to trigger a response in the cell. In this case, ion channels open and the potential changes in much less time than the cell's time constant, as we will see next.

The Action Potential

Suppose a nerve cell is sitting quietly at its resting potential. The membrane potential is approximately −70 mV. However, this potential can change drastically in response to a *stimulus. Neurons*—nerve cells—can be stimulated by neurotransmitter chemicals released at synapse junctions. A neuron can also be electrically stimulated by a changing potential, which is why Galvani saw the frog's leg jump. Whatever the stimulus, the result is a rapid change called an *action potential;* this is the "firing" of a nerve cell. There are three phases in the action potential, as outlined below.

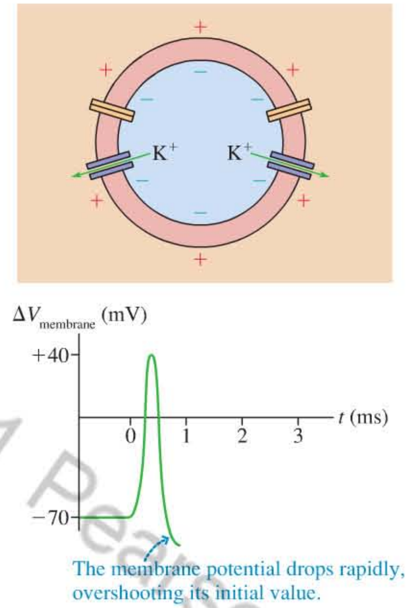
The action potential

Depolarization



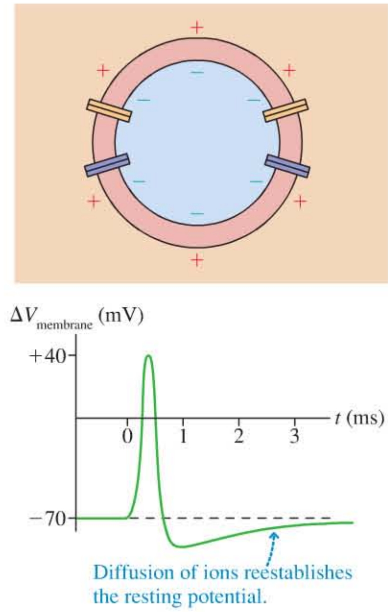
A stimulus causes the cell to "fire"; the first step is the opening of the sodium channels. The concentration of sodium ions is much higher outside the cell, so sodium ions flow rapidly into the cell. In less than 1 ms, this influx of positive ions raises the membrane potential from −70 mV to +40 mV, at which point the sodium channels close. This phase of the action potential is called *depolarization*.

Repolarization



The changing membrane potential now causes the potassium channels to open. The higher potassium concentration inside the cell drives these ions out of the cell, making the membrane potential negative. The negative potential closes the potassium channels, but a delayed response leads to a slight *overshoot* of the resting potential to about −80 mV. This phase of the action potential is called *repolarization*.

Reestablishing resting potential



The reestablishment of the resting potential after the sodium and potassium channels close is a relatively slow process controlled by the motion of ions across the membrane.

After the action potential is complete, there is a brief resting period, after which the cell is ready to be triggered again. The action potential is driven by ionic conduction through sodium and potassium channels, so the potential changes are quite rapid. The time for the potential to rise and then to fall is much less than the 3 ms time constant of the membrane.

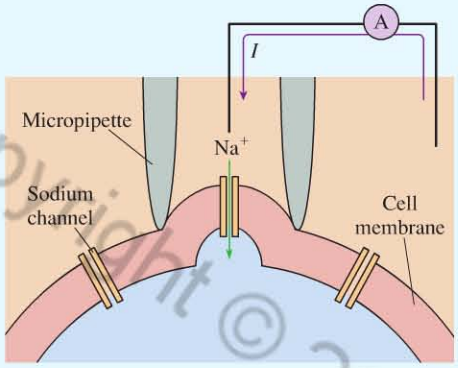
The above discussion concerned nerve cells, but muscle cells undergo a similar cycle of depolarization and repolarization. The resulting potential changes are responsible for the signal that is measured by an electrocardiogram, which we learned about in Chapter 21. The potential differences in the human body are small because the changes in potential are small. But some fish have electric organs in which the action potentials of thousands of specially adapted cells are added in series, leading to very large potential differences—hundreds of volts in the case of the electric eel.

EXAMPLE 23.15 **Counting ions through a channel**

Investigators can measure the ion flow through a single ion channel with the *patch clamp* technique, as illustrated in **FIGURE 23.47**. A micropipette, a glass tube $\approx 1\ \mu$m in diameter, makes a seal on a patch of cell membrane that includes one sodium channel. This tube is filled with a conducting saltwater solution, and a very sensitive ammeter measures the current as sodium ions flow into the cell. A sodium channel passes an average current of 4.0 pA during the 0.40 ms that the channel is open during an action potential. How many sodium ions pass through the channel?

Continued

FIGURE 23.47 Measuring the current in a single sodium channel.



PREPARE Current is the rate of flow of charge. Each ion has charge $q = e$.

SOLVE In Chapter 22, we saw that the charge delivered by a steady current in time Δt is $Q = I\Delta t$. The amount of charge flowing through the channel in $\Delta t = 4.0 \times 10^{-4}$ s is

$$Q = I\Delta t = (4.0 \times 10^{-12}\ \text{A})(4.0 \times 10^{-4}\ \text{s}) = 1.6 \times 10^{-15}\ \text{C}$$

This charge is due to N ions, each with $q = e$, so the number of ions is

$$N = \frac{Q}{e} = \frac{1.6 \times 10^{-15}\ \text{C}}{1.6 \times 10^{-19}\ \text{C}} = 10{,}000$$

ASSESS The number of ions flowing through one channel is not large, but a cell has a great many channels. The patch clamp technique and other similar procedures have allowed investigators to elucidate the details of the response of the cell membrane to a stimulus.

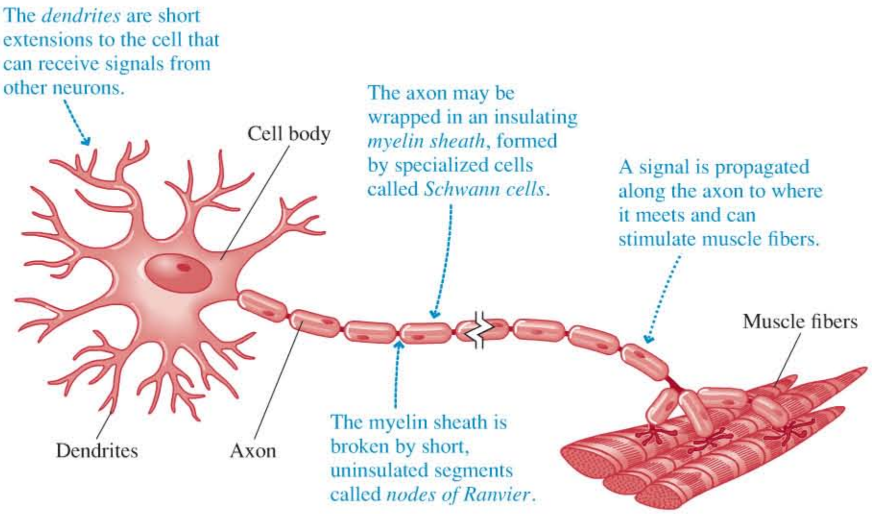
◀ **Touchless typing** Different thought processes lead to different patterns of action potentials among the many neurons of the brain. The electrical activity of the cells and the motion of ions through the conducting fluid surrounding them lead to measurable differences in potential between points on the scalp. You can't use these potential differences to "read someone's mind," but it is possible to program a computer to recognize patterns and perform actions when they are detected. This man is using his thoughts—and the resulting pattern of electric potentials—to select and enter letters.

The Propagation of Nerve Impulses

Let's return to the question posed at the start of the section: How is a signal transmitted from the brain to a muscle in the hand? The primary cells of the nervous system responsible for signal transmission are known as *neurons.* The transmission of a signal to a muscle is the function of a *motor neuron,* whose structure is sketched in FIGURE 23.48. The transmission of signals takes place along the *axon* of the neuron, a long fiber—up to 1 m in length—that connects the cell body to a muscle fiber. This particular neuron has a myelin sheath around the axon, though not all neurons do.

How is a signal transmitted along an axon? The axon is long enough that different points on its membrane may have different potentials. When one point on

FIGURE 23.48 A motor neuron.



the axon's membrane is stimulated, the membrane will depolarize at this point. The resulting action potential may trigger depolarization in adjacent parts of the membrane. Stimulating the axon's membrane at one point can trigger a *wave* of action potential—a nerve impulse—that travels along the axon. When this signal reaches a muscle cell, the muscle cell depolarizes and produces a mechanical response.

Let's look at this process in more detail. We will start with a simple model of an axon with no myelin sheath in FIGURE 23.49a. The sodium channels are normally closed, but if the potential at some point is raised by ≈ 15 mV, from the resting potential of -70 mV to ≈ -55 mV, the sodium channels suddenly open, sodium ions rush into the cell, and an action potential is triggered. This is the key idea: **A small increase in the potential difference across the membrane causes the sodium channels to open, triggering a large action-potential response.**

This process begins at the cell body, in response to signals the neuron receives at its dendrites. If the cell body potential goes up by ≈ 15 mV, an action potential is initiated in the cell body. As the cell body potential quickly rises to a peak of $+40$ mV, it causes the potential on the nearest section of the axon—where the axon attaches to the cell body—to rise by 15 mV. This triggers an action potential in this first section of the axon. The action potential in the first section of the axon triggers an action potential in the next section of the axon, which triggers an action potential in the next section, and so on down the axon until reaching the end.

As FIGURE 23.49b shows, this causes a wave of action potential to propagate down the axon. The signal moves relatively slowly. At each point on the membrane, channels must open and ions must diffuse through, which takes time. On a typical axon with no myelin sheath, the action potential propagates at a speed of about 1 m/s. If all nerve signals traveled at this speed, a signal telling your hand to move would take about 1 s to travel from your brain to your hand. Clearly, at least some neurons in the nervous system must transmit signals at a higher speed than this!

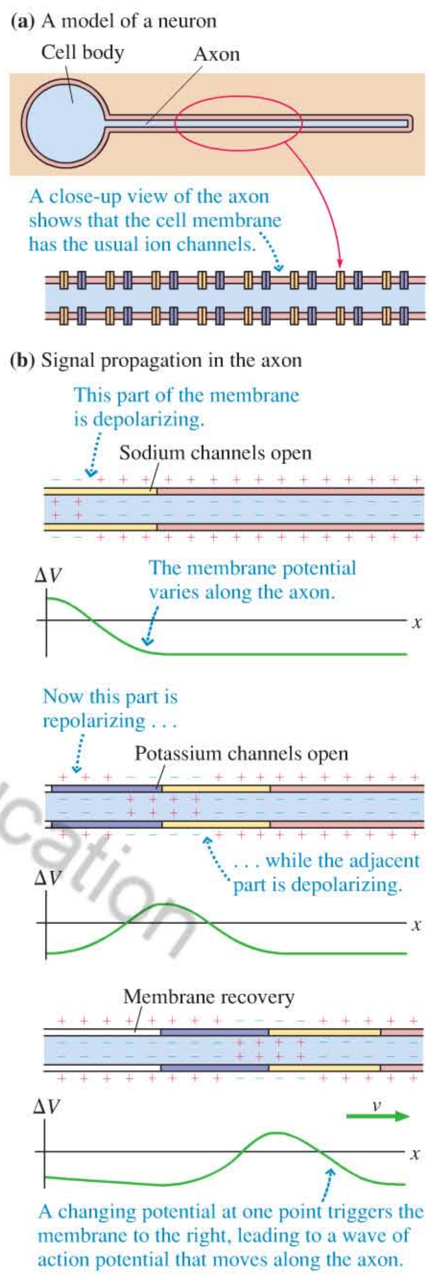
One way to make the signals travel more quickly is to increase an axon's diameter. The giant axon in the squid triggers a rapid escape response when the squid is threatened. This axon may have a diameter of 1 mm, a thousand times that of a typical axon, providing for the necessary rapid signal transmission. But your nervous system consists of 300 billion neurons, and they can't all be 1 mm in diameter—there simply isn't enough space in your body. In your nervous system, higher neuron signal speed is achieved in a totally different manner.

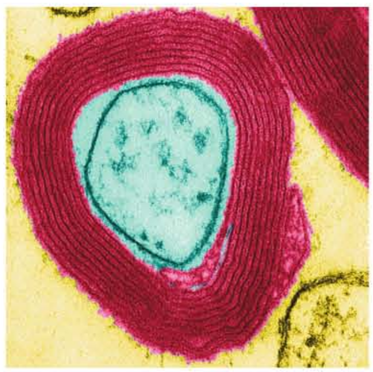
Increasing Speed by Insulation

The axons of motor neurons and most other neurons in your body can transmit signals at very high speeds because they are insulated with a myelin sheath. Look back at the diagram of a motor neuron in Figure 23.48. Schwann cells wrap the axon with myelin, insulating it electrically and chemically, with breaks at the nodes of Ranvier. The ion channels are concentrated in these nodes because this is the only place where the extracellular fluid is in contact with the cell membrane. In an insulated axon, a signal propagates by jumping from one node to the next. This process is called *saltatory conduction,* from the Latin *saltare,* "to leap."

FIGURE 23.50 on the next page shows a model for saltatory conduction. The membrane is triggered to depolarize at the left node. An action potential is produced here, but it can't travel down the axon as before because the axon is insulated. Instead, the

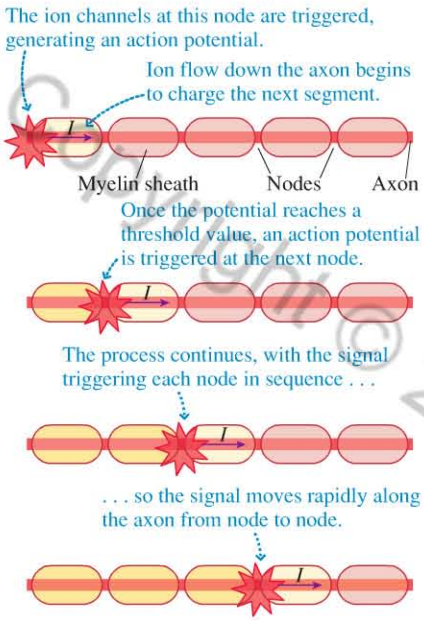
FIGURE 23.49 Propagation of a nerve impulse.

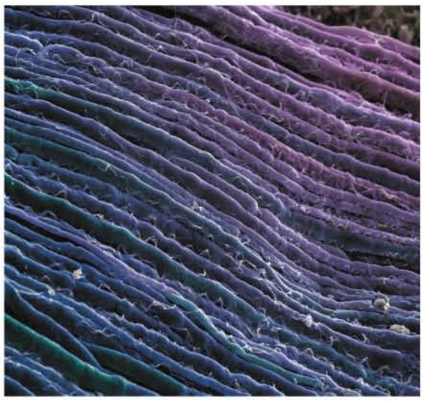




▶ This cross-section view of a myelinated axon shows that the axon (colored aqua) is wrapped several times by a Schwann cell (colored red). There is insulating myelin between each layer.

FIGURE 23.50 Nerve propagation along a myelinated axon.





Myelinated axons in the spinal cord carry electrical signals between the brain and body. You can see the nodes of Ranvier on some of the axons.

potential difference between this node and the next causes ions to flow in the body of the axon. As the charges flow down the axon, the potential at the next node rises. When the potential has risen by ≈ 15 mV, the next node is triggered and an action potential is produced at this node. This process continues, with the depolarization "jumping" from node to node down the axon. This dramatically increases the speed of propagation, as we can show.

How rapidly does a pulse move down a myelinated axon? **FIGURE 23.51** provides a model of the process based on *RC* circuits. The critical time for propagation is the time constant $\tau = RC$ for charging the capacitance of the segments of the axon.

The resistance of an axon between one node and the next is ≈ 25 MΩ. The myelin insulation increases the separation between the inner conducting fluid and the outer conducting fluid. Because the capacitance of a capacitor depends inversely on the electrode spacing d, the myelin reduces the capacitance of the membrane from the ≈ 90 pF we calculated earlier to ≈ 1.6 pF per segment. With these values, the time constant for charging the capacitor in one segment is

$$\tau = R_{\text{axon}} C_{\text{membrane}} = (25 \times 10^{6}\ \Omega)(1.6 \times 10^{-12}\ \text{F}) = 40\ \mu\text{s}$$

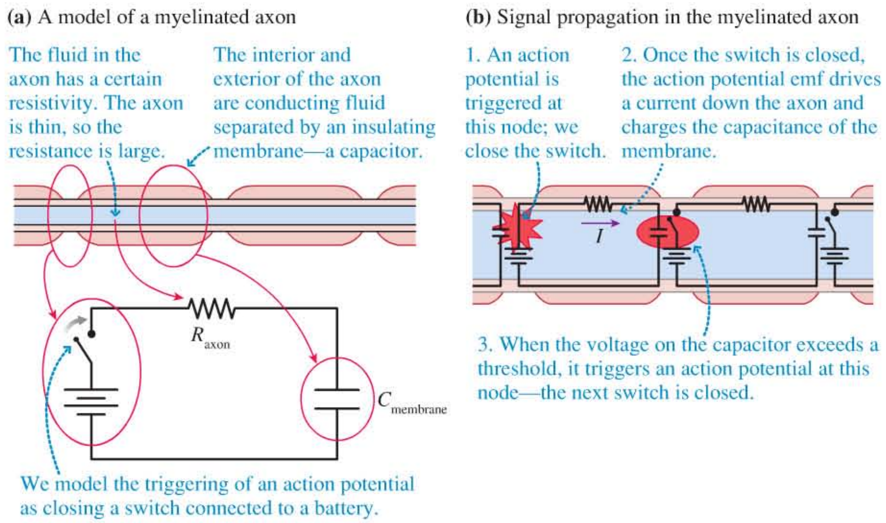
We've modeled the axon as a series of such segments, and the time constant is a good estimate of how much time it takes for a signal to jump from one node to the next. Because the nodes of Ranvier are spaced about 1 mm apart, the speed at which the nerve impulse travels down the axon is approximately

$$v = \frac{L_{\text{node}}}{\tau} = \frac{1.0 \times 10^{-3}\ \text{m}}{40 \times 10^{-6}\ \text{s}} = 25\ \text{m/s}$$

Although our model of nerve-impulse propagation is very simple, this predicted speed is just about right for saltatory conduction of signals in myelinated axons. This speed is 25 times faster than that in unmyelinated axons; at this speed, your brain can send a signal to your hand in $\approx \frac{1}{25}$ s.

Your electrical nature might not be as apparent as that of the electric eel, but the operation of your nervous system is inherently electrical. When you decide to move your hand, the signal from your brain travels to your hand in a process that is governed by the electrical nature of the cells in your body.

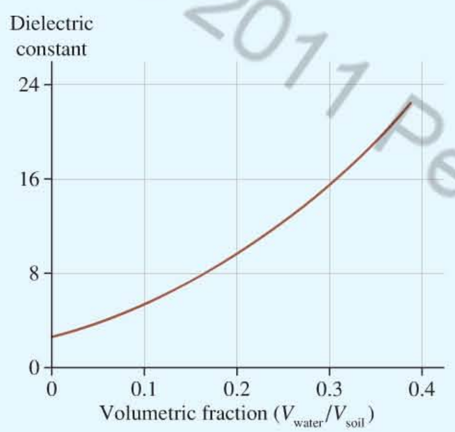
FIGURE 23.51 A circuit model of nerve-impulse propagation along myelinated axons.



INTEGRATED EXAMPLE 23.16 Soil moisture measurement

The moisture content of soil is given in terms of its *volumetric fraction,* the ratio of the volume of water in the soil to the volume of the soil itself. Making this measurement directly is quite time-consuming, so soil scientists are eager to find other means to reliably measure soil moisture. Water has a very large dielectric constant, so the dielectric constant of soil increases as its moisture content increases. **FIGURE 23.52** shows data for the dielectric constant of soil versus the volumetric fraction; the increase in dielectric constant with soil moisture is quite clear. This strong dependence of dielectric constant on soil moisture allows a very sensitive—and simple—electrical test of soil moisture.

FIGURE 23.52 Variation of the dielectric constant with soil moisture.



A soil moisture meter has a probe with two separated electrodes. When the probe is inserted into the soil, the electrodes form a capacitor whose capacitance depends on the dielectric constant of the soil between them. A circuit charges the capacitor probe to 3.0 V, then discharges it through a resistor. The decay time depends on the capacitance—and thus the soil moisture—so a measurement of the time for the capacitor to discharge allows a determination of the amount of moisture in the soil.

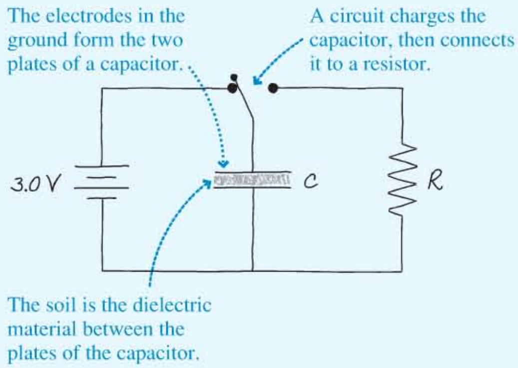
In air, the probe's capacitance takes 15 μs to discharge from 3.0 V to 1.0 V. In one particular test, when the probe was inserted into the ground, this discharge required 150 μs. What was the approximate volumetric fraction of water for the soil in this test?

PREPARE **FIGURE 23.53** is a sketch of the measurement circuit of the soil moisture meter. The capacitor is first charged, then connected across a resistor to form an *RC* circuit. The decay of the capacitor voltage is governed by the time constant for the circuit. The time constant depends on the resistance and on the electrode capacitance, which depends on the dielectric constant of the soil between the electrodes.

The capacitance of the probe in air is that of a parallel-plate capacitor: $C_{air} = \epsilon_0 A/d$. The capacitance of the probe in soil differs only by the additional factor of the dielectric constant of the medium between the plates—in this case, the soil:

$$C_{soil} = \kappa_{soil}\left(\frac{\epsilon_0 A}{d}\right) = \kappa_{soil} C_{air}$$

FIGURE 23.53 The measurement circuit of the soil moisture meter.



The ratio of the capacitance values gives the dielectric constant:

$$\kappa_{soil} = \frac{C_{soil}}{C_{air}}$$

Once we know the dielectric constant, we can determine the volumetric fraction of water from the graph.

SOLVE We are given the times for the decay in air and in soil, but not the capacitance or the resistance of the probe. That's not a problem, though; we don't actually need the capacitance, only the *ratio* of the capacitances in air and in soil. Equation 23.22 gives the voltage decay of an *RC* circuit: $\Delta V_C = (\Delta V_C)_0 e^{-t/RC}$. In air, the decay is

$$1.0\text{ V} = (3.0\text{ V})e^{-(15\times10^{-6}\text{ s})/RC_{air}}$$

In soil, the decay is

$$1.0\text{ V} = (3.0\text{ V})e^{-(150\times10^{-6}\text{ s})/RC_{soil}}$$

Because the starting and ending points for the decay are the same, the exponents of the two expressions must be equal:

$$\frac{15\times10^{-6}\text{ s}}{RC_{air}} = \frac{150\times10^{-6}\text{ s}}{RC_{soil}}$$

We can solve this for the ratio of the capacitances in soil and air:

$$\frac{C_{soil}}{C_{air}} = \frac{150\times10^{-6}\text{ s}}{15\times10^{-6}\text{ s}} = 10$$

We saw above that this ratio is the dielectric constant, so $\kappa_{soil} = 10$. We then use the graph of Figure 23.52 to determine that this dielectric constant corresponds to a volumetric water fraction of approximately 0.20.

ASSESS The decay times in air and in soil differ by a factor of 10, so the capacitance in the soil is much larger than that in air. This implies a large dielectric constant, meaning that there is a lot of water in the soil. A volumetric fraction of 0.20 means that 20% of the soil's volume is water (that is, 1.0 cm^3 of soil contains 0.20 cm^3 of water)—which is quite a bit, so our result seems reasonable.

SUMMARY

The goal of Chapter 23 has been to understand the fundamental physical principles that govern electric circuits.

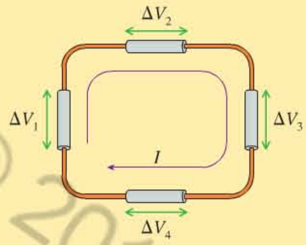
GENERAL PRINCIPLES

Kirchhoff's loop law

For a closed loop:

- Assign a direction to the current.
- Add potential differences around the loop:

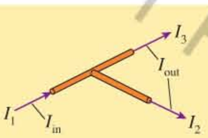
$$\sum_i \Delta V_i = 0$$



Kirchhoff's junction law

For a junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$



Analyzing Circuits

PREPARE Draw a circuit diagram.

SOLVE *Break the circuit down:*

- Reduce the circuit to the smallest possible number of equivalent resistors.
- Find the current and potential difference.

Rebuild the circuit:

- Find current and potential difference for each resistor.

ASSESS Verify that

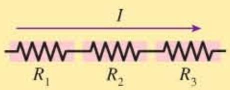
- The sum of the potential differences across series resistors matches that for the equivalent resistor.
- The sum of the currents through parallel resistors matches that for the equivalent resistor.

IMPORTANT CONCEPTS

Series elements

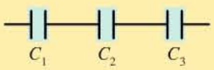
A series connection has no junction.
The current in each element is the same.

Resistors in series can be reduced to an equivalent resistance:



$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots$$

Capacitors in series can be reduced to an equivalent capacitance:

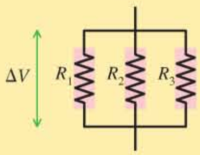


$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

Parallel elements

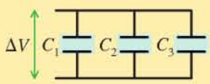
Elements connected in parallel are connected by wires at both ends.
The potential difference across each element is the same.

Resistors in parallel can be reduced to an equivalent resistance:



$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)^{-1}$$

Capacitors in parallel can be reduced to an equivalent capacitance:



$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$$

APPLICATIONS

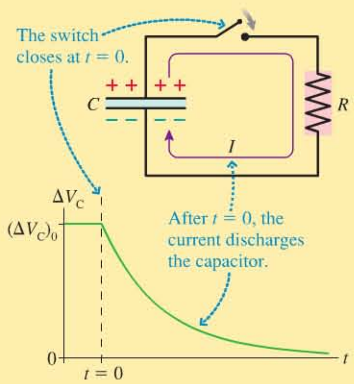
RC circuits

The discharge of a capacitor through a resistor is an exponential decay:

$$\Delta V_C = (\Delta V_C)_0 e^{-t/\tau}$$

The **time constant** for the decay is

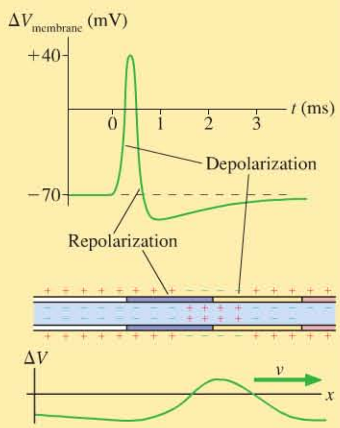
$$\tau = RC$$



Electricity in the nervous system

Cells in the nervous system maintain a negative potential inside the cell membrane. When triggered, the membrane depolarizes and generates an *action potential.*

An action potential travels as a wave along the axon of a neuron. More rapid saltatory conduction can be achieved by insulating the axon with myelin, causing the action potential to jump from node to node.



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

VIEW ALL SOLUTIONS

QUESTIONS

Conceptual Questions

1. The tip of a flashlight bulb is touching the top of a 3 V battery as shown in Figure Q23.1. Does the bulb light? Why or why not?

FIGURE Q23.1

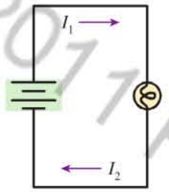


FIGURE Q23.2

2. A flashlight bulb is connected to a battery and is glowing; the circuit is shown in Figure Q23.2. Is current I_2 greater than, less than, or equal to current I_1? Explain.
3. Current I_{in} flows into three resistors connected together one after the other as shown in Figure Q23.3. The accompanying graph shows the value of the potential as a function of position.
   a. Is I_{out} greater than, less than, or equal to I_{in}? Explain.
   b. Rank in order, from largest to smallest, the three resistances R_1, R_2, and R_3. Explain.

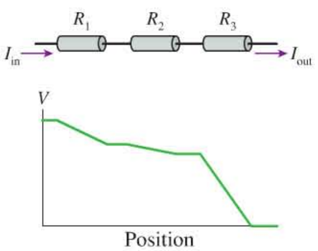


FIGURE Q23.3

4. The circuit in Figure Q23.4 has two resistors, with $R_1 > R_2$. Which resistor dissipates the larger amount of power? Explain.

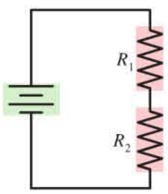


FIGURE Q23.4

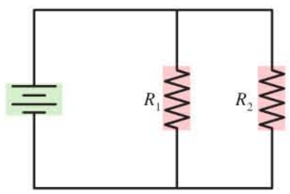


FIGURE Q23.5

5. The circuit in Figure Q23.5 has a battery and two resistors, with $R_1 > R_2$. Which resistor dissipates the larger amount of power? Explain.
6. In the circuit shown in Figure Q23.6, bulbs A and B are glowing. Then the switch is closed. What happens to each bulb? Does it get brighter, stay the same, get dimmer, or go out? Explain.

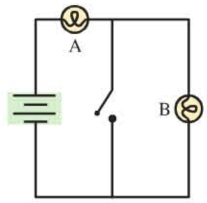


FIGURE Q23.6

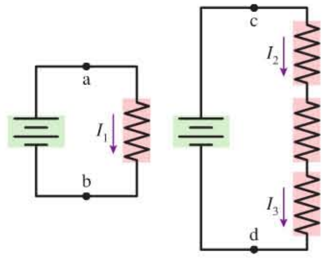


FIGURE Q23.7

7. Figure Q23.7 shows two circuits. The two batteries are identical and the four resistors all have exactly the same resistance.
   a. Is ΔV_{ab} larger than, smaller than, or equal to ΔV_{cd}? Explain.
   b. Rank in order, from largest to smallest, the currents I_1, I_2, and I_3. Explain.
8. Figure Q23.8 shows two circuits. The two batteries are identical and the four resistors all have exactly the same resistance.
   a. Compare ΔV_{ab}, ΔV_{cd}, and ΔV_{ef}. Are they all the same? If not, rank them in order from largest to smallest. Explain.
   b. Rank in order, from largest to smallest, the five currents I_1 to I_5. Explain.

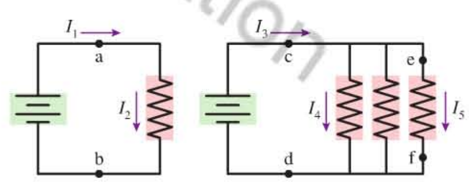


FIGURE Q23.8

9. a. In Figure Q23.9, what fraction of current I goes through the 3 Ω resistor?
   b. If the 9 Ω resistor is replaced with a larger resistor, will the fraction of current going through the 3 Ω resistor increase, decrease, or stay the same?

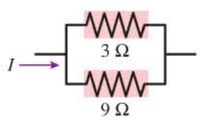


FIGURE Q23.9

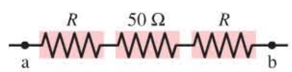


FIGURE Q23.10

10. Two of the three resistors in Figure Q23.10 are unknown but equal. Is the total resistance between points a and b less than, greater than, or equal to 50 Ω? Explain.
11. Two of the three resistors in Figure Q23.11 are unknown but equal. Is the total resistance between points a and b less than, greater than, or equal to 200 Ω? Explain.

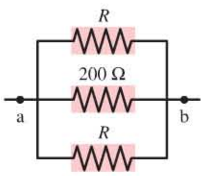


FIGURE Q23.11

12. Rank in order, from largest to smallest, the currents I_1, I_2, and I_3 in the circuit diagram in Figure Q23.12.

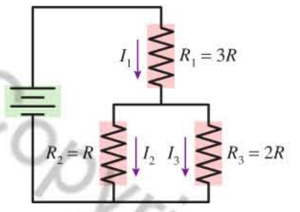


FIGURE Q23.12

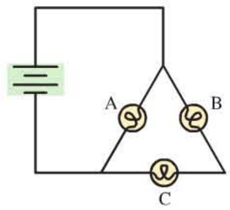


FIGURE Q23.13

13. The three bulbs in Figure Q23.13 are identical. Rank the bulbs from brightest to dimmest. Explain.
14. The four bulbs in Figure Q23.14 are identical. Rank the bulbs from brightest to dimmest. Explain.

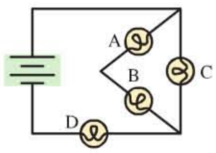


FIGURE Q23.14

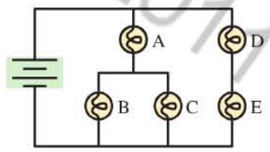


FIGURE Q23.15

15. Figure Q23.15 shows five identical bulbs connected to a battery. All the bulbs are glowing. Rank the bulbs from brightest to dimmest. Explain.
16. a. The three bulbs in Figure Q23.16 are identical. Rank the bulbs from brightest to dimmest. Explain.
    b. Suppose a wire is connected between points 1 and 2. What happens to each bulb? Does it get brighter, stay the same, get dimmer, or go out? Explain.

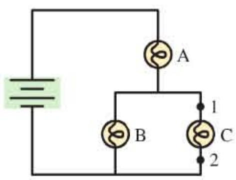


FIGURE Q23.16

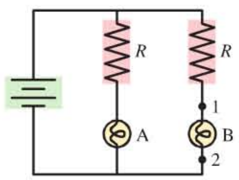


FIGURE Q23.17

17. Initially, bulbs A and B in Figure Q23.17 are both glowing. Bulb B is then removed from its socket. Does removing bulb B cause the potential difference ΔV_{12} between points 1 and 2 to increase, decrease, stay the same, or become zero? Explain.
18. a. Consider the points a and b in Figure Q23.18. Is the potential difference ΔV_{ab} between points a and b zero? If so, why? If not, which point is more positive?
    b. If a wire is connected between points a and b, does it carry a current? If so, in which direction—to the right or to the left? Explain.

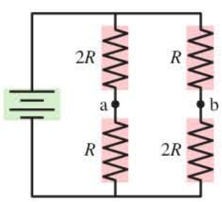


FIGURE Q23.18

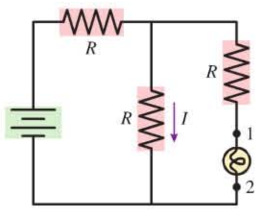


FIGURE Q23.19

19. Initially the lightbulb in Figure Q23.19 is glowing. It is then removed from its socket.
    a. What happens to the current I when the bulb is removed? Does it increase, stay the same, or decrease? Explain.
    b. What happens to the potential difference ΔV_{12} between points 1 and 2? Does it increase, stay the same, decrease, or become zero? Explain.
20. A voltmeter is (incorrectly) inserted into a circuit as shown in Figure Q23.20.
    a. What is the current in the circuit?
    b. What does the voltmeter read?
    c. How would you change the circuit to correctly connect the voltmeter to measure the potential difference across the resistor?

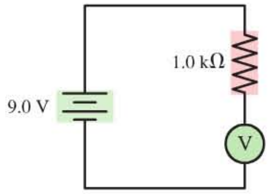


FIGURE Q23.20

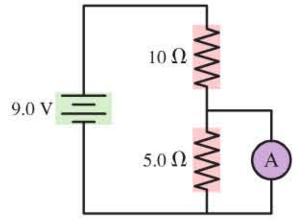


FIGURE Q23.21

21. An ammeter is (incorrectly) inserted into a circuit as shown in Figure Q23.21.
    a. What is the current through the 5.0 Ω resistor?
    b. How would you change the circuit to correctly connect the ammeter to measure the current through the 5.0 Ω resistor?
22. Rank in order, from largest to smallest, the equivalent capacitances $(C_{eq})_1$ to $(C_{eq})_4$ of the four groups of capacitors shown in Figure Q23.22.

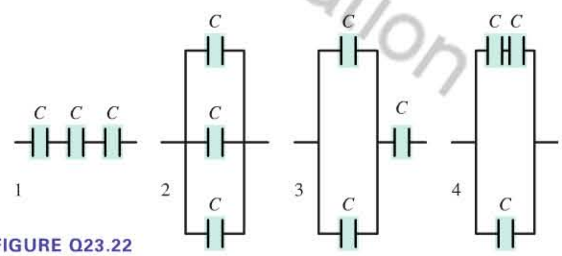


FIGURE Q23.22

23. Figure Q23.23 shows a circuit consisting of a battery, a switch, two identical lightbulbs, and a capacitor that is initially uncharged.
    a. *Immediately* after the switch is closed, are either or both bulbs glowing? Explain.
    b. If both bulbs are glowing, which is brighter? Or are they equally bright? Explain.
    c. For any bulb (A or B or both) that lights up immediately after the switch is closed, does its brightness increase with time, decrease with time, or remain unchanged? Explain.

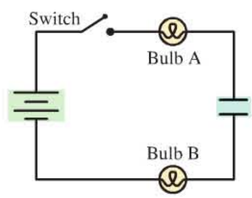


FIGURE Q23.23

24. Figure Q23.24 shows the voltage as a function of time across a capacitor as it is discharged (separately) through three different resistors. Rank in order, from largest to smallest, the values of the resistances R_1 to R_3.

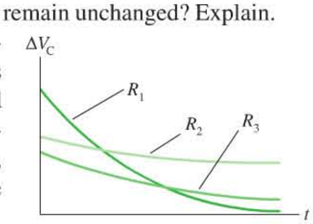


FIGURE Q23.24

25. A charged capacitor could be connected to two identical resistors in either of the two ways shown in Figure Q23.25. Which configuration will discharge of the capacitor in the shortest time once the switch is closed? Explain.

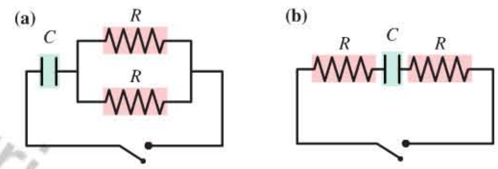


FIGURE Q23.25

26. A flashing light is controlled by the charging and discharging of an *RC* circuit. If the light is flashing too rapidly, describe two changes that you could make to the circuit to reduce the flash rate.

27. BIO A device to make an electrical measurement of skin moisture has electrodes that form two plates of a capacitor; the skin is the dielectric between the plates. Adding moisture to the skin means adding water, which has a large dielectric constant. If a circuit repeatedly charges and discharges the capacitor to determine the capacitance, how will an increase in skin moisture affect the charging and discharging time? Explain.

28. BIO Consider the model of nerve conduction in myelinated axons presented in the chapter. Suppose the distance between the nodes of Ranvier was halved for a particular axon.
   a. How would this affect the resistance and the capacitance of one segment of the axon?
   b. How would this affect the time constant for the charging of one segment?
   c. How would this affect the signal propagation speed for the axon?

29. BIO Adding a myelin sheath to an axon results in faster signal propagation. It also means that less energy is required for a signal to propagate down the axon. Explain why this is so.

Multiple-Choice Questions

30. | What is the current in the circuit of Figure Q23.30?
   A. 1.0 A B. 1.7 A
   C. 2.5 A D. 4.2 A

31. | Which resistor in Figure Q23.30 dissipates the most power?
   A. The 4.0 Ω resistor.
   B. The 6.0 Ω resistor.
   C. Both dissipate the same power.

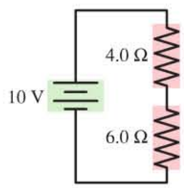


FIGURE Q23.30

32. || Normally, household lightbulbs are connected in parallel to a power supply. Suppose a 40 W and a 60 W lightbulb are, instead, connected in series, as shown in Figure Q23.32. Which bulb is brighter?
   A. The 60 W bulb.
   B. The 40 W bulb.
   C. The bulbs are equally bright.

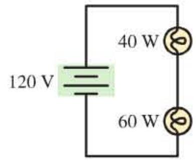


FIGURE Q23.32

33. ||| A metal wire of resistance *R* is cut into two pieces of equal length. The two pieces are connected together side by side. What is the resistance of the two connected wires?
   A. $R/4$ B. $R/2$ C. R
   D. $2R$ E. $4R$

34. | What is the value of resistor *R* in Figure Q23.34?
   A. 4.0 Ω
   B. 12 Ω
   C. 36 Ω
   D. 72 Ω
   E. 96 Ω

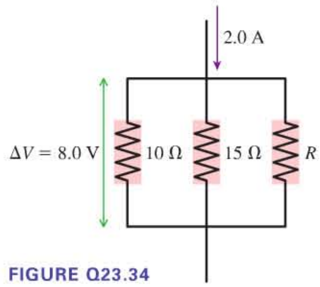


FIGURE Q23.34

35. | Two capacitors are connected in series. They are then reconnected to be in parallel. The capacitance of the parallel combination
   A. Is less than that of the series combination.
   B. Is more than that of the series combination.
   C. Is the same as that of the series combination.
   D. Could be more or less than that of the series combination depending on the values of the capacitances.

36. | BIO If a cell's membrane thickness doubles but the cell stays the same size, how do the resistance and the capacitance of the cell membrane change?
   A. The resistance and the capacitance would increase.
   B. The resistance would increase, the capacitance would decrease.
   C. The resistance would decrease, the capacitance would increase.
   D. The resistance and the capacitance would decrease.

37. ||| BIO If a cell's diameter is reduced by 50% without changing the membrane thickness, how do the resistance and capacitance of the cell membrane change?
   A. The resistance and the capacitance would increase.
   B. The resistance would increase, the capacitance would decrease.
   C. The resistance would decrease, the capacitance would increase.
   D. The resistance and the capacitance would decrease.

VIEW ALL SOLUTIONS

PROBLEMS

Section 23.1 Circuit Elements and Diagrams

1. || Draw a circuit diagram for the circuit of Figure P23.1.

2. || Draw a circuit diagram for the circuit of Figure P23.2.

3. || Draw a circuit diagram for the circuit of Figure P23.3.

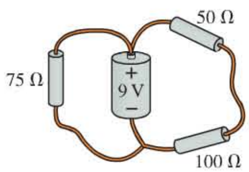


FIGURE P23.1

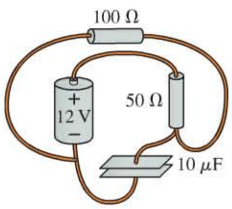


FIGURE P23.2

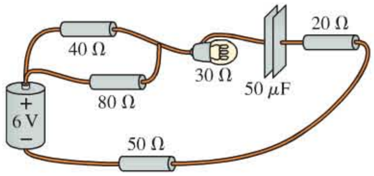


FIGURE P23.3

Section 23.2 Kirchhoff's Laws

4. || In Figure P23.4, what is the current in the wire above the junction? Does charge flow toward or away from the junction?

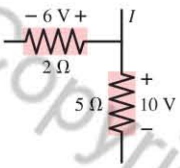


FIGURE P23.4

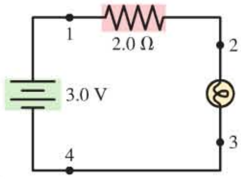


FIGURE P23.5

5. || The lightbulb in the circuit diagram of Figure P23.5 has a resistance of 1.0 Ω. Consider the potential difference between pairs of points in the figure.
   a. What are the values of ΔV_{12}, ΔV_{23}, and ΔV_{34}?
   b. What are the values if the bulb is removed?
6. | a. What are the magnitude and direction of the current in the 30 Ω resistor in Figure P23.6?
   b. Draw a graph of the potential as a function of the distance traveled through the circuit, traveling clockwise from $V = 0$ V at the lower left corner. See Figure P23.9 for an example of such a graph.

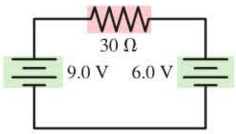


FIGURE P23.6

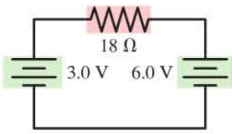


FIGURE P23.7

7. || a. What are the magnitude and direction of the current in the 18 Ω resistor in Figure P23.7?
   b. Draw a graph of the potential as a function of the distance traveled through the circuit, traveling clockwise from $V = 0$ V at the lower left corner. See Figure P23.9 for an example of such a graph.
8. | a. What is the potential difference across each resistor in Figure P23.8?
   b. Draw a graph of the potential as a function of the distance traveled through the circuit, traveling clockwise from $V = 0$ V at the lower left corner. See Figure P23.9 for an example of such a graph.

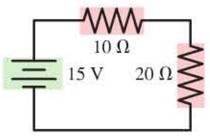


FIGURE P23.8

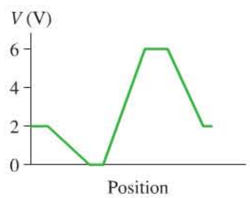


FIGURE P23.9

9. | The current in a circuit with only one battery is 2.0 A. Figure P23.9 shows how the potential changes when going around the circuit in the clockwise direction, starting from the lower left corner. Draw the circuit diagram.

Section 23.3 Series and Parallel Circuits

10. | What is the equivalent resistance of each group of resistors shown in Figure P23.10?

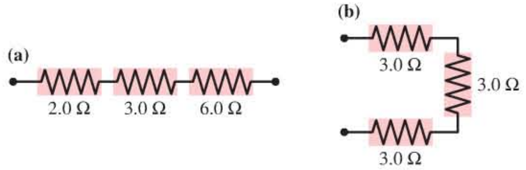


FIGURE P23.10

11. | What is the equivalent resistance of each group of resistors shown in Figure P23.11?

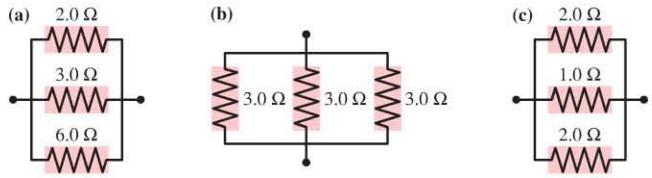


FIGURE P23.11

12. |||| An 80-cm-long wire is made by welding a 1.0-mm-diameter, INT 20-cm-long copper wire to a 1.0-mm-diameter, 60-cm-long iron wire. What is the resistance of the composite wire?
13. | You have a collection of 1.0 kΩ resistors. How can you connect four of them to produce an equivalent resistance of 0.25 kΩ?
14. | You have a collection of six 1.0 kΩ resistors. What is the smallest resistance you can make by combining them?
15. | You have three 6.0 Ω resistors and one 3.0 Ω resistor. How can you connect them to produce an equivalent resistance of 5.0 Ω?
16. || You have six 1.0 kΩ resistors. How can you connect them to produce a total equivalent resistance of 1.5 kΩ?

Section 23.4 Measuring Voltage and Current

Section 23.5 More Complex Circuits

17. || What is the equivalent resistance between points a and b in Figure P23.17?

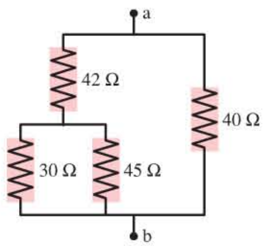


FIGURE P23.17

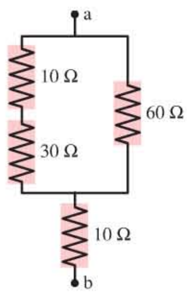


FIGURE P23.18

18. | What is the equivalent resistance between points a and b in Figure P23.18?
19. || The currents in two resistors in a circuit are shown in Figure P23.19. What is the value of resistor R?

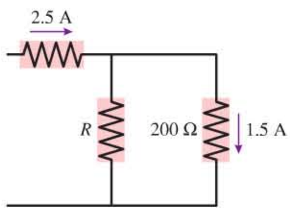


FIGURE P23.19

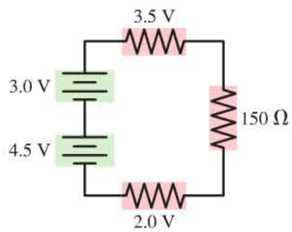


FIGURE P23.20

20. ‖ Two batteries supply current to the circuit in Figure P23.20. The figure shows the potential difference across two of the resistors and the value of the third resistor. What current is supplied by the batteries?

21. | Part of a circuit is shown in Figure P23.21.
   a. What is the current through the 3.0 Ω resistor?
   b. What is the value of the current I?

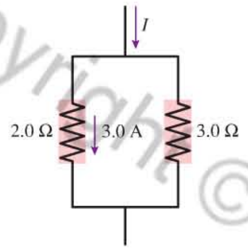


FIGURE P23.21

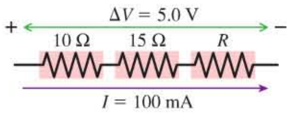


FIGURE P23.22

22. | What is the value of resistor R in Figure P23.22?

23. ‖ What are the resistance R and the emf of the battery in Figure P23.23?

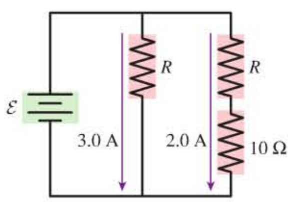


FIGURE P23.23

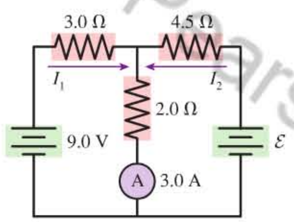


FIGURE P23.24

24. ‖ The ammeter in Figure P23.24 reads 3.0 A. Find I_1, I_2, and $\mathcal{E}$.

25. ‖| Find the current through and the potential difference across each resistor in Figure P23.25.

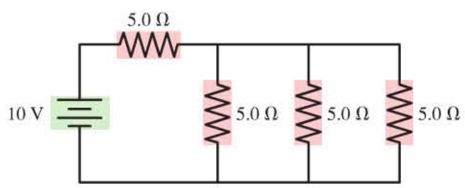


FIGURE P23.25

26. ‖| Find the current through and the potential difference across each resistor in Figure P23.26.

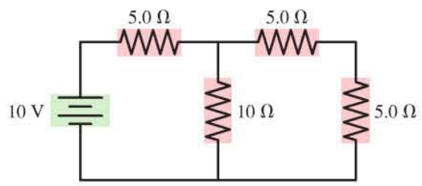


FIGURE P23.26

27. ‖ For the circuit shown in Figure P23.27, find the current through and the potential difference across each resistor. Place your results in a table for ease of reading.

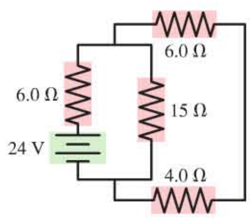


FIGURE P23.27

28. ‖‖ In the circuit of Figure P23.28, what are the values of ΔV_{14}, ΔV_{24}, and ΔV_{34}?

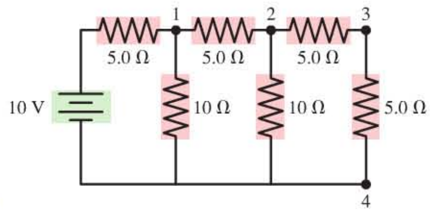


FIGURE P23.28

29. ‖ For the circuit shown in Figure P23.29, find the current through and the potential difference across each resistor. Place your results in a table for ease of reading.

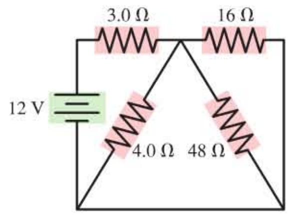


FIGURE P23.29

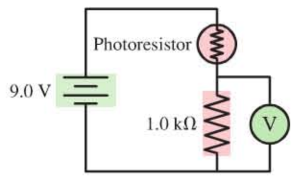


FIGURE P23.30

30. ‖ A photoresistor, whose resistance decreases with light intensity, is connected in the circuit of Figure P23.30. On a sunny day, the photoresistor has a resistance of 0.56 Ω. On a cloudy day, the resistance rises to 4.0 kΩ. At night, the resistance is 20 kΩ.
   a. What does the voltmeter read for each of these conditions?
   b. Does the voltmeter reading increase or decrease as the light intensity increases?

31. ‖ A photoresistor, whose resistance decreases with light intensity, is connected in the circuit of Figure P23.31.
   a. Draw a circuit diagram to illustrate how you would use a voltmeter and an ammeter to determine the resistance of the photoresistor in this circuit.
   b. What do the two meters read when the resistance of the photoresistor is 2.5 kΩ?

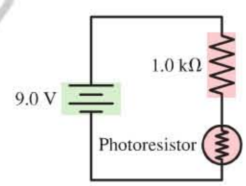


FIGURE P23.31

Section 23.6 Capacitors in Parallel and Series

32. | A 6.0 μF capacitor, a 10 μF capacitor, and a 16 μF capacitor are connected in parallel. What is their equivalent capacitance?

33. | A 6.0 μF capacitor, a 10 μF capacitor, and a 16 μF capacitor are connected in series. What is their equivalent capacitance?

34. | You need a capacitance of 50 μF, but you don't happen to have a 50 μF capacitor. You do have a 30 μF capacitor. What additional capacitor do you need to produce a total capacitance of 50 μF? Should you join the two capacitors in parallel or in series?

35. | You need a capacitance of 50 μF, but you don't happen to have a 50 μF capacitor. You do have a 75 μF capacitor. What additional capacitor do you need to produce a total capacitance of 50 μF? Should you join the two capacitors in parallel or in series?

36. || What is the equivalent capacitance of the three capacitors in Figure P23.36?

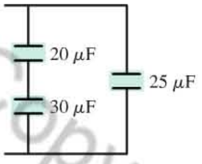


FIGURE P23.36

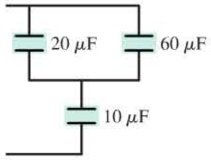


FIGURE P23.37

37. | What is the equivalent capacitance of the three capacitors in Figure P23.37?

38. ||| For the circuit of Figure P23.38,
  a. What is the equivalent capacitance?
  b. How much charge flows through the battery as the capacitors are being charged?

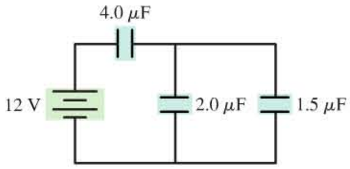


FIGURE P23.38

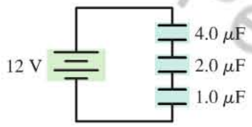


FIGURE P23.39

39. ||| For the circuit of Figure P23.39,
  a. What is the equivalent capacitance?
  b. What is the charge of each of the capacitors?

Section 23.7 *RC* Circuits

40. || What is the time constant for the discharge of the capacitor in Figure P23.40?

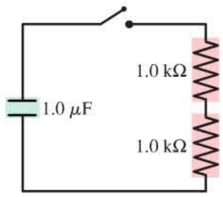


FIGURE P23.40

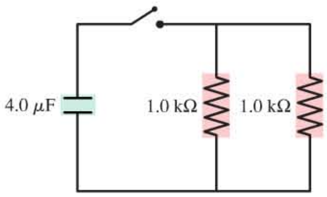


FIGURE P23.41

41. || What is the time constant for the discharge of the capacitor in Figure P23.41?

42. |||| Capacitors won't hold a charge indefinitely; as time goes on, charge gradually migrates from the positive to the negative plate. We can model this as a discharge of the capacitor through an internal "leakage resistance." A 0.47 F capacitor charged to 2.5 V will initially discharge with a leakage current of 0.25 mA.
  a. What is the leakage resistance?
  b. How long will it take for the capacitor voltage to drop to 1.0 V?

43. ||| A 10 μF capacitor initially charged to 20 μC is discharged through a 1.0 kΩ resistor. How long does it take to reduce the capacitor's charge to 10 μC?

44. || The switch in Figure P23.44 has been in position a for a long time. It is changed to position b at $t = 0$ s. What are the charge Q on the capacitor and the current I through the resistor (a) immediately after the switch is closed? (b) At $t = 50\ \mu$s? (c) At $t = 200\ \mu$s?

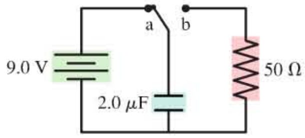


FIGURE P23.44

Section 23.8 Electricity in the Nervous System

45. | BIO A 9.0-nm-thick cell membrane undergoes an action potential that follows the curve in the table on page 761. What is the strength of the electric field inside the membrane just before the action potential and at the peak of the depolarization?

46. ||| BIO A cell membrane has a resistance and a capacitance and thus a characteristic time constant. What is the time constant of a 9.0-nm-thick membrane surrounding a 0.040-mm-diameter spherical cell?

47. | BIO Changing the thickness of the myelin sheath surrounding an axon changes its capacitance and thus the conduction speed. A myelinated nerve fiber has a conduction speed of 55 m/s. If the spacing between nodes is 1.0 mm and the resistance of segments between nodes is 25 MΩ, what is the capacitance of each segment?

48. ||| BIO A particular myelinated axon has nodes spaced 0.80 mm apart. The resistance between nodes is 20 MΩ; the capacitance of each insulated segment is 1.2 pF. What is the conduction speed of a nerve impulse along this axon?

49. | BIO To measure signal propagation in a nerve in the arm, the nerve is triggered near the armpit. The peak of the action potential is measured at the elbow and then, 4.0 ms later, 24 cm away from the elbow at the wrist.
  a. What is the speed of propagation along this nerve?
  b. A determination of the speed made by measuring the time between the application of a stimulus at the armpit and the peak of an action potential at the elbow or the wrist would be inaccurate. Explain the problem with this approach, and why the noted technique is preferable.

50. || BIO A myelinated axon conducts nerve impulses at a speed of 40 m/s. What is the signal speed if the thickness of the myelin sheath is halved but no other changes are made to the axon?

General Problems

51. || INT How much power is dissipated by each resistor in Figure P23.51?

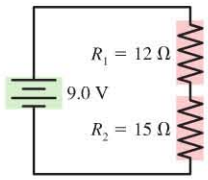


FIGURE P23.51

52. |||| INT Two 75 W (120 V) lightbulbs are wired in series, then the combination is connected to a 120 V supply. How much power is dissipated by each bulb?

53. |||| INT The corroded contacts in a lightbulb socket have 5.0 Ω total resistance. How much actual power is dissipated by a 100 W (120V) lightbulb screwed into this socket?

54. |||| INT A real battery is not just an emf. We can model a real 1.5 V battery as a 1.5 V emf in series with a resistor known as the "internal resistance," as shown in Figure P23.54. A typical bat-

tery has 1.0 Ω internal resistance due to imperfections that limit current through the battery. When there's no current through the battery, and thus no voltage drop across the internal resistance, the potential difference between its terminals is 1.5 V, the value of the emf. Suppose the terminals of this battery are connected to a 2.0 Ω resistor.

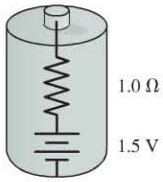


FIGURE P23.54

a. What is the potential difference between the terminals of the battery?
b. What fraction of the battery's power is dissipated by the internal resistance?

55. |||| For the real battery shown in Figure P23.54, calculate the power dissipated by a resistor R connected to the battery when (a) $R = 0.25\ \Omega$, (b) $R = 0.50\ \Omega$, (c) $R = 1.0\ \Omega$, (d) $R = 2.0\ \Omega$, and (e) $R = 4.0\ \Omega$. (Your results should suggest that maximum power dissipation is achieved when the external resistance R equals the internal resistance. This is true in general.)

56. |||| INT Batteries are recharged by connecting them to a power supply (i.e., another battery) of greater emf in such a way that the current flows *into* the positive terminal of the battery being recharged, as was shown in Example 23.1. This reverse current through the battery replenishes its chemicals. The current is kept fairly low so as not to overheat the battery being recharged by dissipating energy in its internal resistance.
a. Suppose the real battery of Figure P23.54 is rechargeable. What emf power supply should be used for a 0.75 A recharging current?
b. If this power supply charges the battery for 10 minutes, how much energy goes into the battery? How much is dissipated as thermal energy in the internal resistance?

57. ||| When two resistors are connected in parallel across a battery of unknown voltage, one resistor carries a current of 3.2 A while the second carries a current of 1.8 A. What current will be supplied by the same battery if these two resistors are connected to it in series?

58. || INT The 10 Ω resistor in Figure P23.58 is dissipating 40 W of power. How much power are the other two resistors dissipating?

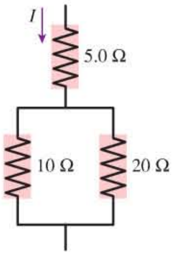


FIGURE P23.58

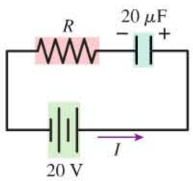


FIGURE P23.59

59. |||| INT At this instant, the current in the circuit of Figure P23.59 is 20 mA in the direction shown and the capacitor charge is 200 μC. What is the resistance R?

60. | What is the equivalent resistance between points a and b in Figure P23.60?

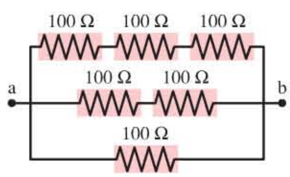


FIGURE P23.60

61. || You have three 12 Ω resistors. Draw diagrams showing how you could arrange all three so that their equivalent resistance is (a) 4.0 Ω, (b) 8.0 Ω, (c) 18 Ω, and (d) 36 Ω.

62. |||| INT A 9.0 V battery is connected to a wire made of three segments of different metals connected one after another: 10 cm of copper wire, then 12 cm of iron wire, then 18 cm of tungsten wire. All of the wires are 0.26 mm in diameter. Find the potential difference across each piece of wire.

63. ||| You have a device that needs a voltage reference of 3.0 V, but you have only a 9.0 V battery. Fortunately, you also have several 10 kΩ resistors. Show how you can use the resistors and the battery to make a circuit that provides a potential difference of 3.0 V.

64. | INT There is a current of 0.25 A in the circuit of Figure P23.64.
a. What is the direction of the current? Explain.
b. What is the value of the resistance R?
c. What is the power dissipated by R?
d. Make a graph of potential versus position, starting from $V = 0$ V in the lower left corner and proceeding clockwise. See Figure P23.9 for an example.

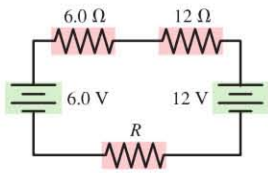


FIGURE P23.64

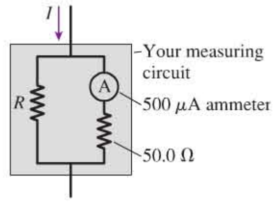


FIGURE P23.65

65. || A circuit you're building needs an ammeter that goes from 0 mA to a full-scale reading of 50.0 mA. Unfortunately, the only ammeter in the storeroom goes from 0 μA to a full-scale reading of only 500 μA. Fortunately, you can make this ammeter work by putting it in a measuring circuit, as shown in Figure P23.65 This lets a certain fraction of the current pass through the meter; knowing this value, you can deduce the total current. Assume that the ammeter is ideal.
a. What value of R must you use so that the meter will go to full scale when the current I is 50.0 mA?

Hint: When $I = 50.0$ mA, the ammeter should be reading its maximum value.

b. What is the equivalent resistance of your measuring circuit?

66. || A circuit you're building needs a voltmeter that goes from 0 V to a full-scale reading of 5.0 V. Unfortunately, the only meter in the storeroom is an *ammeter* that goes from 0 μA to a full-scale reading of 500 μA. It is possible to use this meter to measure voltages by putting in a measuring circuit as shown in Figure P23.66. What value of R must you use so that the meter will go to full scale when the potential difference ΔV is 5.0 V? Assume that the ammeter is ideal.

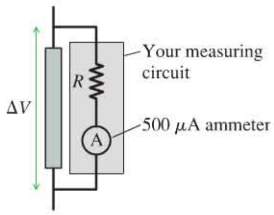


FIGURE P23.66

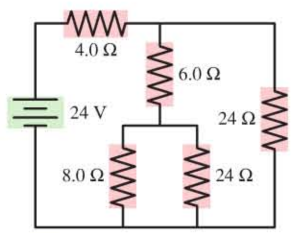


FIGURE P23.67

67. || For the circuit shown in Figure P23.67, find the current through and the potential difference across each resistor. Place your results in a table for ease of reading.

68. || You have three capacitors. Draw diagrams showing how you could arrange all three so that their equivalent capacitance is (a) 4.0 μF, (b) 8.0 μF, (c) 18 μF, and (d) 36 μF.

69. Initially, the switch in Figure P23.69 is in position a and capacitors C_2 and C_3 are uncharged. Then the switch is flipped to position b. Afterward, what are the charge on and the potential difference across each capacitor?

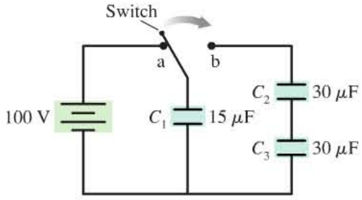


FIGURE P23.69

70. INT The capacitor in an RC circuit with a time constant of 15 ms is charged to 10 V. The capacitor begins to discharge at $t = 0$ s.
   a. At what time will the charge on the capacitor be reduced to half its initial value?
   b. At what time will the energy stored in the capacitor be reduced to half its initial value?

71. What value resistor will discharge a 1.0 μF capacitor to 10% of its initial charge in 2.0 ms?

72. The charging circuit for the flash system of a camera uses a 100 μF capacitor that is charged from a 250 V power supply. What is the most resistance that can be in series with the capacitor if the capacitor is to charge to at least 87% of its final voltage in no more than 8.0 s?

73. A capacitor is discharged through a 100 Ω resistor. The discharge current decreases to 25% of its initial value in 2.5 ms. What is the value of the capacitor?

74. A 50 μF capacitor that had been charged to 30 V is discharged through a resistor. Figure P23.74 shows the capacitor voltage as a function of time. What is the value of the resistance?

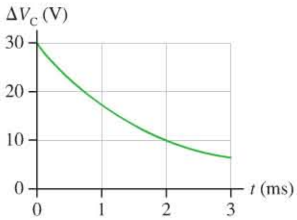


FIGURE P23.74

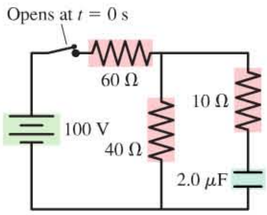


FIGURE P23.75

75. The switch in Figure P23.75 has been closed for a very long time.
   a. What is the charge on the capacitor?
   b. The switch is opened at $t = 0$ s. At what time has the charge on the capacitor decreased to 10% of its initial value?

76. Intermittent windshield wipers use a variable resistor in an RC circuit to set the delay between successive passes of the wipers. A typical circuit is shown in Figure P23.76. When the switch closes, the capacitor (initially uncharged) begins to charge and the potential at point b begins to increase. A sensor measures the potential difference between points a and b, triggering a pass of the wipers when $V_b = V_a$. (Another part of the circuit, not shown, discharges the capacitor at this time so that the cycle can start again.)

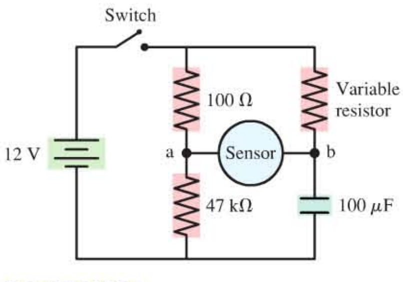


FIGURE P23.76

   a. What value of the variable resistor will give 12 seconds from the start of a cycle to a pass of the wipers?
   b. To decrease the time, should the variable resistance be increased or decreased?

77. BIO In Example 23.14 we estimated the capacitance of the cell membrane to be 89 pF, and in Example 23.15 we found that approximately 10,000 Na^+ ions flow through an ion channel when it opens. Based on this information and what you learned in this chapter about the action potential, estimate the total number of sodium ion channels in the membrane of a nerve cell.

78. BIO INT The giant axon of a squid is 0.5 mm in diameter, 10 cm long, and not myelinated. Unmyelinated cell membranes behave as capacitors with 1 μF of capacitance per square centimeter of membrane area. When the axon is charged to the -70 mV resting potential, what is the energy stored in this capacitance?

79. BIO A cell has a 7.0-nm-thick membrane with a total membrane area of 6.0×10^{-9} m^2.
   a. We can model the cell as a capacitor, as we have seen. What is the magnitude of the charge on each "plate" when the membrane is at its resting potential of -70 mV?
   b. How many sodium ions does this charge correspond to?

Passage Problems

The Defibrillator BIO

A defibrillator is designed to pass a large current through a patient's torso in order to stop dangerous heart rhythms. Its key part is a capacitor that is charged to a high voltage. The patient's torso plays the role of a resistor in an RC circuit. When a switch is closed, the capacitor discharges through the patient's torso. A jolt from a defibrillator is intended to be intense and rapid; the maximum current is very large, so the capacitor discharges quickly. This rapid pulse depolarizes the heart, stopping all electrical activity. This allows the heart's internal nerve circuitry to reestablish a healthy rhythm.

A typical defibrillator has a 32 μF capacitor charged to 5000 V. The electrodes connected to the patient are coated with a conducting gel that reduces the resistance of the skin to where the effective resistance of the patient's torso is 100 Ω.

80. Which pair of graphs in Figure P23.80 best represents the capacitor voltage and the current through the torso as a function of time after the switch is closed?

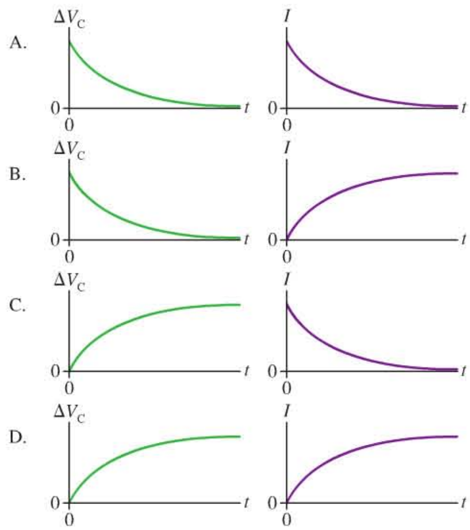


FIGURE P23.80

81. | For the values noted in the passage above, what is the time constant for the discharge of the capacitor?
A. 3.2 μs B. 160 μs C. 3.2 ms D. 160 ms
82. | If a patient receives a series of jolts, the resistance of the torso may increase. How does such a change affect the initial current and the time constant of subsequent jolts?
A. The initial current and the time constant both increase.
B. The initial current decreases, the time constant increases.
C. The initial current increases, the time constant decreases.
D. The initial current and the time constant both decrease.
83. | In some cases, the defibrillator may be charged to a lower voltage. How will this affect the time constant of the discharge?
A. The time constant will increase.
B. The time constant will not change.
C. The time constant will decrease.

Electric Fish BIO INT

The voltage produced by a single nerve or muscle cell is quite small, but there are many species of fish that use multiple action potentials in series to produce significant voltages. The electric organs in these fish are composed of specialized disk-shaped cells called *electrocytes*. The cell at rest has the usual potential difference between the inside and the outside, but the net potential difference *across* the cell is zero. An electrocyte is connected to nerve fibers that initially trigger a depolarization in one side of the cell but not the other. For the very short time of this depolarization, there is a net potential difference across the cell, as shown in Figure P23.84. Stacks of these cells connected in series can produce a large total voltage. Each stack can produce a small current; for more total current, more stacks are needed, connected in parallel.

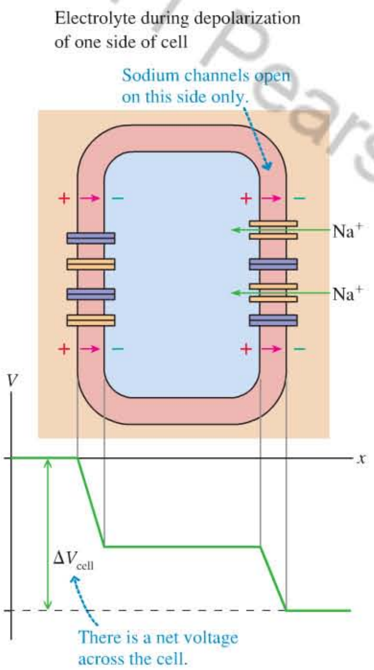


FIGURE P23.84

84. | In an electric eel, each electrocyte can develop a voltage of 150 mV for a short time. For a total voltage of 450 V, how many electrocytes must be connected in series?
A. 300
B. 450
C. 1500
D. 3000
85. | An electric eel produces a pulse of current of 0.80 A at a voltage of 500 V. For the short time of the pulse, what is the instantaneous power?
A. 400 W
B. 500 W
C. 625 W
D. 800 W
86. | Electric eels live in fresh water. The torpedo ray is an electric fish that lives in salt water. The electrocytes in the ray are grouped differently than in the eel; each stack of electrocytes has fewer cells, but there are more stacks in parallel. Which of the following best explains the ray's electrocyte arrangement?
A. The lower resistivity of salt water requires more current but lower voltage.
B. The lower resistivity of salt water requires more voltage but lower current.
C. The higher resistivity of salt water requires more current but lower voltage.
D. The higher resistivity of salt water requires more voltage but lower current.
87. | The electric catfish is another electric fish that produces a voltage pulse by means of stacks of electrocytes. As the fish grows in length, the magnitude of the voltage pulse the fish produces grows as well. The best explanation for this change is that, as the fish grows,
A. The voltage produced by each electrocyte increases.
B. More electrocytes are added to each stack.
C. More stacks of electrocytes are added in parallel to the existing stacks.
D. The thickness of the electrocytes increases.

STOP TO THINK ANSWERS

Stop to Think 23.1: A, B, and **D.** These three are the same circuit because the logic of the connections is the same. In each case, there is a junction that connects one side of each circuit element and a second junction that connects the other side. In C, the functioning of the circuit is changed by the extra wire connecting the two sides of the capacitor.

Stop to Think 23.2: C = D > A = B. The two bulbs in series are of equal brightness, as are the two bulbs in parallel. But the two bulbs in series have a larger resistance than a single bulb, so there will be less current through the bulbs in series than the bulbs in parallel.

Stop to Think 23.3: C. The voltmeter must be connected in parallel with the resistor, and the ammeter in series.

Stop to Think 23.4: A > B > C = D. All the current from the battery goes through A, so it is brightest. The current divides at the junction, but not equally. Because B is in parallel with C + D, but has half the resistance of the two bulbs together, twice as much current travels through B as through C + D. So B is dimmer than A but brighter than C and D. C and D are equally bright because of conservation of current.

Stop to Think 23.5: $(C_{eq})_B > (C_{eq})_A > (C_{eq})_C$. Two capacitors in parallel have a larger capacitance than either alone; two capacitors in series have a smaller capacitance than either alone.

Stop to Think 23.6: B. The two 2 Ω resistors are in series and equivalent to a 4 Ω resistor. Thus $\tau = RC = 4$ s.