3 Vectors and Motion in Two Dimensions

Once the leopard jumps, its trajectory is fixed by the initial speed and angle of the jump. How can we work out where the leopard will land?

LOOKING AHEAD ▶▶

The goals of Chapter 3 are to learn more about vectors and to use vectors as a tool to analyze motion in two dimensions.

Tools for Describing Motion

In the last chapter, we discussed motion along a line. In this chapter, we'll look at motion in which the direction changes. We'll need new tools.

Types of Motion

There are a few basic types of motion that we'll consider. In each case there is an acceleration due to a change in speed or direction—or both.

Looking Back ◀◀
1.1–1.2 Basic motion concepts

Vectors

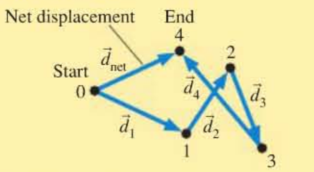
We use vectors to describe quantities, like velocity, for which both the magnitude and direction are important.

Vectors specify a direction as well as a magnitude.

Looking Back ◀◀
1.5 Vectors

Vector Math

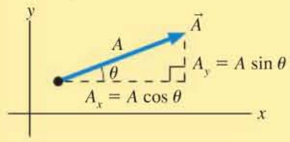
Our basic kinematic variables are vectors. Working with them means learning how to work with vectors.



You'll learn how to add, subtract, and perform other mathematical operations on vectors.

Vector Components

We make measurements in a coordinate system. How do vectors fit in?



You'll learn how to find the components of vectors and how to add and subtract vectors using components.

Motion on a Ramp

Gravity causes the motion, but it's not straight down—it's at an angle.

A speed skier is accelerating down a ramp. How fast is he moving at the end of his run?

Looking Back ◀◀
2.5 Motion with constant acceleration

Projectile Motion

Objects launched through the air follow a parabolic path. Water going over the falls, the leaping salmon, footballs, and jumping leopards all follow similar paths.

The salmon is moving horizontally and vertically—as do all objects undergoing projectile motion.

Looking Back ◀◀
2.7 Free fall

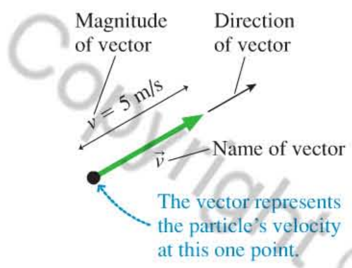
Circular Motion

Motion in a circle at a constant speed involves acceleration, but not the constant acceleration you studied in Chapter 2.

The riders are moving at a constant speed but with an ever-changing direction. It is the changing direction that causes the acceleration and makes the ride fun.

3.1 Using Vectors

FIGURE 3.1 The velocity vector $\vec{v}$ has both a magnitude and a direction.

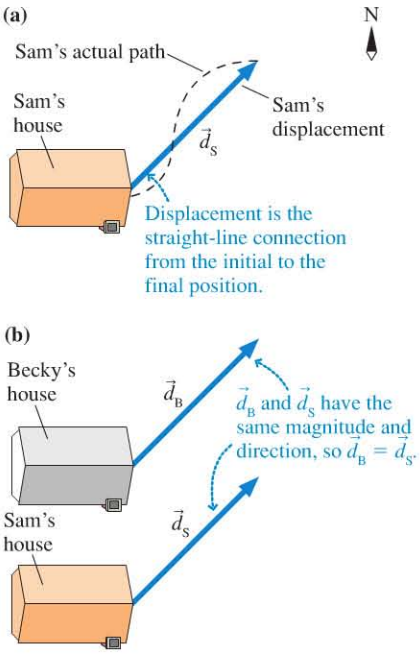


In the previous chapter, we solved many problems in which an object moved in a straight-line path. In this chapter, we will look at particles that take curving paths—motion in two dimensions. Because the direction of motion will be so important, we need to develop an appropriate mathematical language to describe it—the language of vectors.

We introduced the concept of a vector in Chapter 1, and in the next few sections we will develop that concept into a useful and powerful tool. We will practice using vectors by analyzing a problem of motion in one dimension (that of motion on a ramp) and by studying the interesting notion of relative velocity. We will then be ready to analyze the two-dimensional motion of projectiles and of objects moving in a circle.

Recall from Chapter 1 that a vector is a quantity with both a size (magnitude) and a direction. FIGURE 3.1 shows how to represent a particle's velocity as a vector $\vec{v}$. The particle's speed at this point is 5 m/s *and* it is moving in the direction indicated by the arrow. The magnitude of a vector is represented by the letter without an arrow. In this case, the particle's speed—the magnitude of the velocity vector $\vec{v}$—is $v = 5$ m/s. The magnitude of a vector, a *scalar* quantity, cannot be a negative number.

FIGURE 3.2 Displacement vectors.



NOTE ▶ Although the vector arrow is drawn across the page, from its tail to its tip, this arrow does *not* indicate that the vector "stretches" across this distance. Instead, the arrow tells us the value of the vector quantity only at the one point where the tail of the vector is placed. ◀

We found in Chapter 1 that the displacement of an object is a vector drawn from its initial position to its position at some later time. Because displacement is an easy concept to think about, we can use it to introduce some of the properties of vectors. However, **all the properties we will discuss in this chapter (addition, subtraction, multiplication, components) apply to all types of vectors, not just to displacement.**

Suppose that Sam, our old friend from Chapter 1, starts from his front door, walks across the street, and ends up 200 ft to the northeast of where he started. Sam's displacement, which we will label $\vec{d}_S$, is shown in FIGURE 3.2a. The displacement vector is a straight-line connection from his initial to his final position, not necessarily his actual path. The dashed line indicates a possible route Sam might have taken, but his displacement is the vector $\vec{d}_S$.

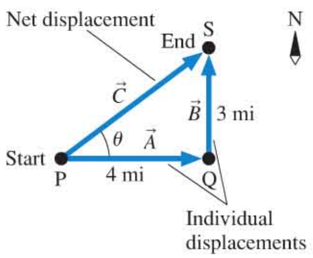
To describe a vector we must specify both its magnitude and its direction. We can write Sam's displacement as

$$\vec{d}_S = (200 \text{ ft, northeast})$$

where the first number specifies the magnitude and the second item gives the direction. The magnitude of Sam's displacement is $d_S = 200$ ft, the distance between his initial and final points.

Sam's next-door neighbor Becky also walks 200 ft to the northeast, starting from her own front door. Becky's displacement $\vec{d}_B = (200$ ft, northeast) has the same magnitude and direction as Sam's displacement $\vec{d}_S$. Because vectors are defined by their magnitude and direction, **two vectors are equal if they have the same magnitude and direction.** This is true regardless of the individual starting points of the vectors. Thus the two displacements in FIGURE 3.2b are equal to each other, and we can write $\vec{d}_B = \vec{d}_S$.

FIGURE 3.3 The net displacement $\vec{C}$ resulting from two displacements $\vec{A}$ and $\vec{B}$.



Vector Addition

As we saw in Chapter 1, we can combine successive displacements by vector addition. Let's review and extend this concept. FIGURE 3.3 shows the displacement of a hiker who starts at point P and ends at point S. She first hikes 4 miles to the east, then 3 miles to the north. The first leg of the hike is described by the displacement vector $\vec{A} = (4$ mi, east). The second leg of the hike has displacement $\vec{B} = (3$ mi, north). By definition, a vector from her initial position P to her final position S is also a displacement. This is vector $\vec{C}$ on the figure. $\vec{C}$ is the *net displacement* because it describes the net result of the hiker's having first displacement $\vec{A}$, then displacement $\vec{B}$.

The word "net" implies addition. The net displacement $\vec{C}$ is an initial displacement $\vec{A}$ *plus* a second displacement $\vec{B}$, or

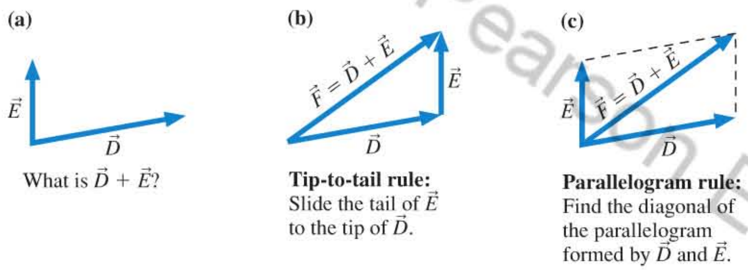
$$\vec{C} = \vec{A} + \vec{B} \tag{3.1}$$

The sum of two vectors is called the **resultant vector.** Vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. You can add vectors in any order you wish.

Look back at Tactics Box 1.4 on page 19 to review the three-step procedure for adding two vectors. This tip-to-tail method for adding vectors, which is used to find $\vec{C} = \vec{A} + \vec{B}$ in Figure 3.3, is called *graphical addition.* Any two vectors of the same type—two velocity vectors or two force vectors—can be added in exactly the same way.

When two vectors are to be added, it is often convenient to draw them with their tails together, as shown in **FIGURE 3.4a**. To evaluate $\vec{D} + \vec{E}$, you could move vector $\vec{E}$ over to where its tail is on the tip of $\vec{D}$, then use the tip-to-tail rule of graphical addition. This gives vector $\vec{F} = \vec{D} + \vec{E}$ in **FIGURE 3.4b**. Alternatively, **FIGURE 3.4c** shows that the vector sum $\vec{D} + \vec{E}$ can be found as the diagonal of the parallelogram defined by $\vec{D}$ and $\vec{E}$. This method is called the *parallelogram rule* of vector addition.

FIGURE 3.4 Two vectors can be added using the tip-to-tail rule or the parallelogram rule.

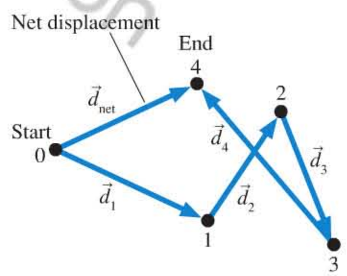


Vector addition is easily extended to more than two vectors. **FIGURE 3.5** shows the path of a hiker moving from initial position 0 to position 1, then position 2, then position 3, and finally arriving at position 4. These four segments are described by displacement vectors $\vec{d}_1$, $\vec{d}_2$, $\vec{d}_3$, and $\vec{d}_4$. The hiker's *net* displacement, an arrow from position 0 to position 4, is the vector $\vec{d}_{net}$. In this case,

$$\vec{d}_{net} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4 \tag{3.2}$$

The vector sum is found by using the tip-to-tail method three times in succession.

FIGURE 3.5 The net displacement after four individual displacements.



Multiplication by a Scalar

The hiker in Figure 3.3 started with displacement $\vec{A}_1 = (4 \text{ mi, east})$. Suppose a second hiker walks twice as far to the east. The second hiker's displacement will then certainly be $\vec{A}_2 = (8 \text{ mi, east})$. The words "twice as" indicate a multiplication, so we can say

$$\vec{A}_2 = 2\vec{A}_1$$

Multiplying a vector by a positive scalar gives another vector of *different magnitude* but pointing in the *same direction*.

Let the vector $\vec{A}$ be specified as a magnitude A and a direction θ_A; that is, $\vec{A} = (A, \theta_A)$. Now let $\vec{B} = c\vec{A}$, where c is a positive scalar constant. Then

$$\vec{B} = c\vec{A} \text{ means that } (B, \theta_B) = (cA, \theta_A) \tag{3.3}$$

The vector is stretched or compressed by the factor c (i.e., vector $\vec{B}$ has magnitude $B = cA$), but $\vec{B}$ points in the same direction as $\vec{A}$. This is illustrated in **FIGURE 3.6**.

Suppose we multiply $\vec{A}$ by zero. Using Equation 3.3, we get

$$0 \cdot \vec{A} = \vec{0} = (0 \text{ m, direction undefined}) \tag{3.4}$$

The product is a vector having zero length or magnitude. This vector is known as the **zero vector,** denoted $\vec{0}$. The direction of the zero vector is irrelevant; you cannot describe the direction of an arrow of zero length!

FIGURE 3.6 Multiplication of a vector by a positive scalar.

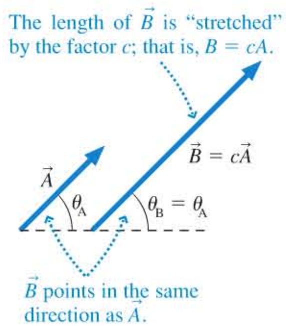


FIGURE 3.7 Vector $-\vec{A}$.

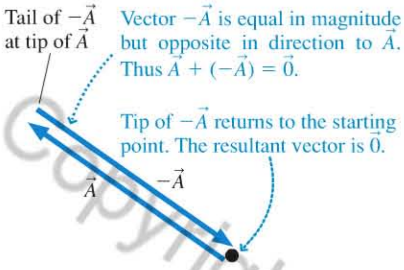
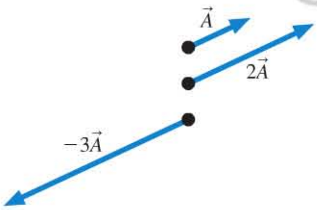


FIGURE 3.8 Vectors $\vec{A}$, $2\vec{A}$, and $-3\vec{A}$.



What happens if we multiply a vector by a negative number? Equation 3.3 does not apply if $c < 0$ because vector $\vec{B}$ cannot have a negative magnitude. Consider the vector $-\vec{A}$, which is equivalent to multiplying $\vec{A}$ by -1. Because

$$\vec{A} + (-\vec{A}) = \vec{0} \tag{3.5}$$

The vector $-\vec{A}$ must be such that, when it is added to $\vec{A}$, the resultant is the zero vector $\vec{0}$. In other words, the *tip* of $-\vec{A}$ must return to the *tail* of $\vec{A}$, as shown in **FIGURE 3.7**. This will be true only if $-\vec{A}$ is equal in magnitude to $\vec{A}$ but opposite in direction. Thus we can conclude that

$$-\vec{A} = (A, \text{direction opposite } \vec{A}) \tag{3.6}$$

Multiplying a vector by -1 reverses its direction without changing its length.

As an example, **FIGURE 3.8** shows vectors $\vec{A}$, $2\vec{A}$, and $-3\vec{A}$. Multiplication by 2 doubles the length of the vector but does not change its direction. Multiplication by -3 stretches the length by a factor of 3 *and* reverses the direction.

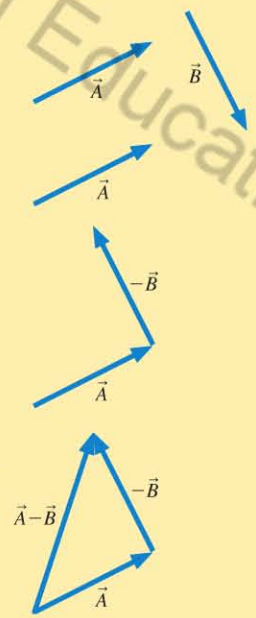
Vector Subtraction

How might we *subtract* vector $\vec{B}$ from vector $\vec{A}$ to form the vector $\vec{A} - \vec{B}$? With numbers, subtraction is the same as the addition of a negative number. That is, $5 - 3$ is the same as $5 + (-3)$. Similarly, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$. We can use the rules for vector addition and the fact that $-\vec{B}$ is a vector opposite in direction to $\vec{B}$ to form rules for vector subtraction.

TACTICS BOX 3.1 **Subtracting vectors**

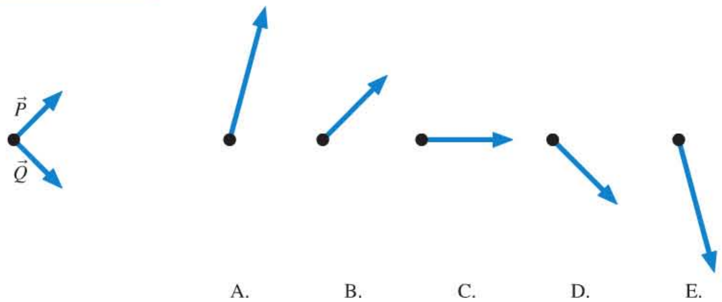
To subtract $\vec{B}$ from $\vec{A}$:

1. Draw $\vec{A}$.
2. Place the tail of $-\vec{B}$ at the tip of $\vec{A}$.
3. Draw an arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. This is vector $\vec{A} - \vec{B}$.



Exercises 5–8

STOP TO THINK 3.1 Which figure shows $2\vec{P} - \vec{Q}$?



3.2 Using Vectors on Motion Diagrams

In Chapter 2, we defined velocity for one-dimensional motion as an object's displacement—the change in position—divided by the time interval in which the change occurs:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

In two dimensions, an object's displacement is a vector. Suppose an object undergoes displacement $\vec{d}$ during the time interval Δt. Let's define an object's velocity *vector* to be

$$\vec{v} = \frac{\vec{d}}{\Delta t} = \left(\frac{d}{\Delta t}, \text{ same direction as } \vec{d}\right) \tag{3.7}$$

Definition of velocity in two or more dimensions

Notice that we've multiplied a vector by a scalar: The velocity vector is simply the displacement vector multiplied by the scalar $1/\Delta t$. Consequently, as we found in Chapter 1, **the velocity vector points in the direction of the displacement.** As a result, we can use the dot-to-dot vectors on a motion diagram to visualize the velocity.

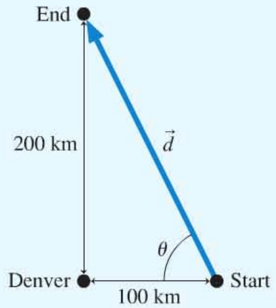
NOTE ► Strictly speaking, the velocity defined in Equation 3.7 is the *average* velocity for the time interval Δt. This is adequate for using motion diagrams to visualize motion. As we did in Chapter 2, when we make Δt very small, we get an *instantaneous* velocity we can use in performing some calculations. ◄

EXAMPLE 3.1 Finding the velocity of an airplane

A small plane is 100 km due east of Denver. After 1 hour of flying at a constant speed in the same direction, it is 200 km due north of Denver. What is the plane's velocity?

PREPARE The initial and final positions of the plane are shown in FIGURE 3.9; the displacement $\vec{d}$ is the vector that points from the initial to the final position.

FIGURE 3.9 Displacement vector for an airplane.



SOLVE The length of the displacement vector is the hypotenuse of a right triangle:

$$d = \sqrt{(100\text{ km})^2 + (200\text{ km})^2} = 224\text{ km}$$

The direction of the displacement vector is described by the angle θ in Figure 3.9. From trigonometry, this angle is

$$\theta = \tan^{-1}\left(\frac{200\text{ km}}{100\text{ km}}\right) = \tan^{-1}(2.00) = 63.4°$$

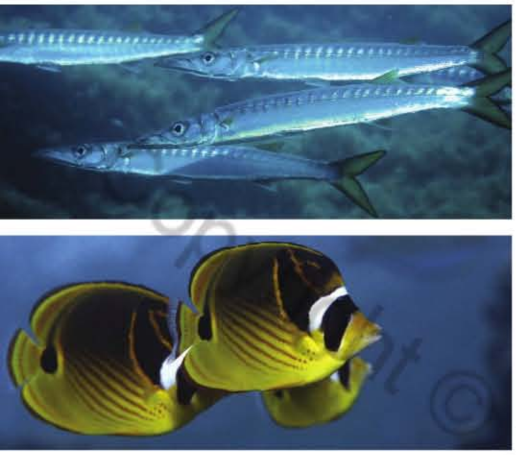
Thus the plane's displacement vector is

$$\vec{d} = (224\text{ km}, 63.4° \text{ north of west})$$

Because the plane undergoes this displacement during 1 hour, its velocity is

$$\vec{v} = \left(\frac{d}{\Delta t}, \text{ same direction as } \vec{d}\right) = \left(\frac{224\text{ km}}{1\text{ h}}, 63.4° \text{ north of west}\right)$$
$$= (224\text{ km/h}, 63.4° \text{ north of west})$$

ASSESS The plane's *speed* is the magnitude of the velocity, $v = 224$ km/h. This is approximately 140 mph, which is a reasonable speed for a small plane.



Lunging versus veering BIO The top photo shows a barracuda, a type of fish that catches prey with a rapid linear acceleration, a quick change in speed. The barracuda's body shape is optimized for such a straight-line strike. The butterfly fish in the bottom photo has a very different appearance. It can't rapidly change its speed, but its body shape lets it quickly change its direction. Once the barracuda gets up to speed, it can't change its direction very easily, so the butterfly fish can, by employing this other type of acceleration, avoid capture.

We defined an object's acceleration in one dimension as $a_x = \Delta v_x/\Delta t$. In two dimensions, we need to use a vector to describe acceleration. The vector definition of acceleration is a straightforward extension of the one-dimensional version:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta\vec{v}}{\Delta t} \tag{3.8}$$

Definition of acceleration in two or more dimensions

There is an acceleration whenever there is a *change* in velocity. Because velocity is a vector, it can change in either or both of two possible ways:

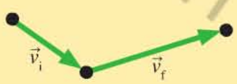
1. The magnitude can change, indicating a change in speed.
2. The direction of motion can change.

In Chapter 2 we saw how to compute an acceleration vector for the first case, in which an object speeds up or slows down while moving in a straight line. In this chapter we will examine the second case, in which an object changes its direction of motion.

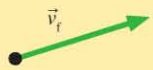
Suppose an object has an initial velocity $\vec{v}_i$ at time t_i and later, at time t_f, has velocity $\vec{v}_f$. The fact that the velocity *changes* tells us the object undergoes an acceleration during the time interval $\Delta t = t_f - t_i$. We see from Equation 3.8 that the acceleration points in the same direction as the vector $\Delta\vec{v}$. This vector is the change in the velocity $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$, so to know which way the acceleration vector points, we have to perform the vector subtraction $\vec{v}_f - \vec{v}_i$. Tactics Box 3.1 showed how to perform vector subtraction. Tactics Box 3.2 shows how to use vector subtraction to find the acceleration vector.

TACTICS BOX 3.2 **Finding the acceleration vector**

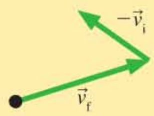
To find the acceleration between velocity $\vec{v}_i$ and velocity $\vec{v}_f$:



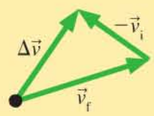
❶ Draw the velocity vector $\vec{v}_f$.



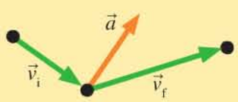
❷ Draw $-\vec{v}_i$ at the tip of $\vec{v}_f$.



❸ Draw $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$
$= \vec{v}_f + (-\vec{v}_i)$
This is the direction of $\vec{a}$.



❹ Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_i$ and $\vec{v}_f$.



Exercises 11,12

Now that we know how to determine acceleration vectors, we can make a complete motion diagram with dots showing the position of the object, average velocity vectors found by connecting the dots with arrows, and acceleration vectors found using Tactics Box 3.2. Note that there is *one* acceleration vector linking each *two* velocity vectors, and $\vec{a}$ is drawn at the dot between the two velocity vectors it links. Let's look at two examples, one with changing speed and one with changing direction.

EXAMPLE 3.2 Drawing the acceleration for a Mars descent

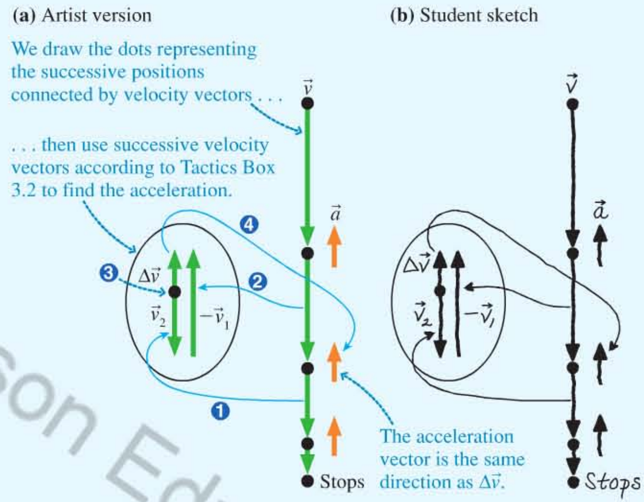
A spacecraft slows as it safely descends to the surface of Mars. Draw a complete motion diagram for the last few seconds of the descent.

PREPARE **FIGURE 3.10** shows two versions of a motion diagram: a professionally drawn version like you generally find in this text and a simpler version similar to what you might draw for a homework assignment. As the spacecraft slows in its descent, the dots get closer together and the velocity vectors get shorter.

SOLVE The inset in Figure 3.10 shows how Tactics Box 3.2 is used to determine the acceleration at one point. All the other acceleration vectors will be similar, because for each pair of velocity vectors the earlier one is longer than the later one.

ASSESS As the spacecraft slows, the acceleration vectors and velocity vectors point in opposite directions, consistent with what we learned about the sign of the acceleration in Chapter 2.

FIGURE 3.10 Motion diagram for a descending spacecraft.

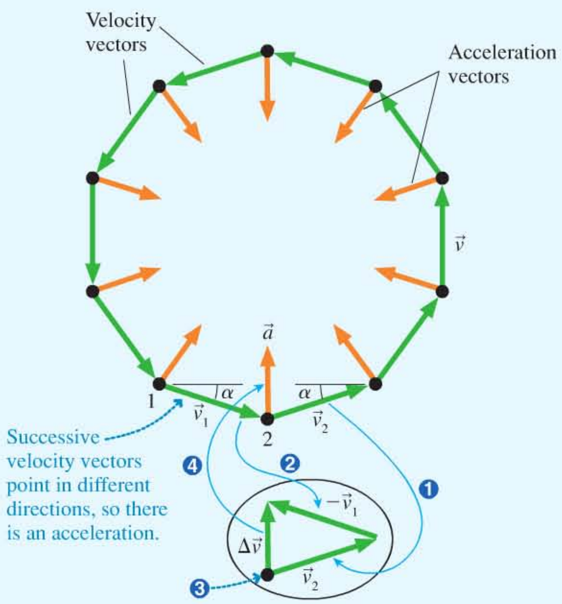


EXAMPLE 3.3 Drawing the acceleration for a Ferris wheel ride

Anne rides a Ferris wheel at an amusement park. Draw a complete motion diagram for Anne's ride.

PREPARE **FIGURE 3.11** shows 10 points of the motion during one complete revolution of the Ferris wheel. A person riding a Ferris wheel moves in a circle at a constant speed,

FIGURE 3.11 Motion diagram for Anne on a Ferris wheel.



Continued

so we've shown equal distances between successive dots. As before, the velocity vectors are found by connecting each dot to the next. Note that the velocity vectors are *straight lines,* not curves.

We see that all the velocity vectors have the same length, but each has a different *direction,* and that means Anne is accelerating. This is not a "speeding up" or "slowing down" acceleration, but is, instead, a "change of direction" acceleration.

SOLVE The inset to Figure 3.11 shows how to use the steps of Tactics Box 3.2 to find the acceleration at one particular position, at the bottom of the circle. Vector $\vec{v}_1$ is the velocity vector that leads into this dot, while $\vec{v}_2$ moves away from it. From the circular geometry of the main figure, the two angles marked α are equal. Thus we see that $\vec{v}_2$ and $-\vec{v}_1$ form an isosceles triangle and vector $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ is exactly vertical, toward the center of the circle. If we did a similar calculation for each point of the motion, we'd find a similar result: In each case, the acceleration points toward the center of the circle.

ASSESS The speed is constant but the direction is changing, so there is an acceleration, as we expect.

No matter which dot you select on the motion diagram of Figure 3.11, the velocities change in such a way that the acceleration vector $\vec{a}$ points directly toward the center of the circle. An acceleration vector that always points toward the center of a circle is called a *centripetal acceleration.* We will have much more to say about centripetal acceleration later in this chapter.

3.3 Coordinate Systems and Vector Components

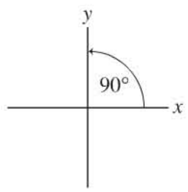
Archaeologists establish a coordinate system so that they can precisely determine the positions of objects they excavate.

In the past two sections, we have seen how to add and subtract vectors graphically, using these operations to deduce important details of motion. But the graphical combination of vectors is not an especially good way to find quantitative results. In this section we will introduce a *coordinate description* of vectors that will be the basis for doing vector calculations.

Coordinate Systems

As we saw in Chapter 1, the world does not come with a coordinate system attached to it. A coordinate system is an artificially imposed grid that you place on a problem in order to make quantitative measurements. The right choice of coordinate system will make a problem easier to solve. We will generally use **Cartesian coordinates,** the familiar rectangular grid with perpendicular axes, as illustrated in FIGURE 3.12.

FIGURE 3.12 A Cartesian coordinate system.



Coordinate axes have a positive end and a negative end, separated by zero at the origin where the two axes cross. When you draw a coordinate system, it is important to label the axes. This is done by placing x and y labels at the *positive* ends of the axes, as in Figure 3.12. The purpose of the labels is twofold:

- To identify which axis is which.
- To identify the positive ends of the axes.

This will be important when you need to determine whether the quantities in a problem should be assigned positive or negative values.

Component Vectors

FIGURE 3.13 shows a vector $\vec{A}$ and an *xy*-coordinate system that we've chosen. Once the directions of the axes are known, we can define two new vectors *parallel to the axes* that we call the **component vectors** of $\vec{A}$. Vector $\vec{A}_x$, called the *x-component vector,* is the projection of $\vec{A}$ along the *x*-axis. Vector $\vec{A}_y$, the *y-component vector,* is the projection of $\vec{A}$ along the *y*-axis. Notice that the component vectors are perpendicular to each other.

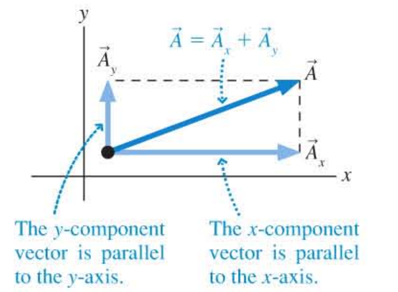
You can see, using the parallelogram rule, that $\vec{A}$ is the vector sum of the two component vectors:

$$\vec{A} = \vec{A}_x + \vec{A}_y \tag{3.9}$$

In essence, we have "broken" vector $\vec{A}$ into two perpendicular vectors that are parallel to the coordinate axes. We say that we have **decomposed** or **resolved** vector $\vec{A}$ into its component vectors.

NOTE ► It is not necessary for the tail of $\vec{A}$ to be at the origin. All we need to know is the *orientation* of the coordinate system so that we can draw $\vec{A}_x$ and $\vec{A}_y$ parallel to the axes. ◄

FIGURE 3.13 Component vectors $\vec{A}_x$ and $\vec{A}_y$ are drawn parallel to the coordinate axes such that $\vec{A} = \vec{A}_x + \vec{A}_y$.



Components

You learned in Chapter 2 to give the one-dimensional kinematic variable v_x a positive sign if the velocity vector $\vec{v}$ points toward the positive end of the *x*-axis and a negative sign if $\vec{v}$ points in the negative *x*-direction. The basis of this rule is that v_x is the *x-component* of $\vec{v}$. We need to extend this idea to vectors in general.

Suppose we have a vector $\vec{A}$ that has been decomposed into component vectors $\vec{A}_x$ and $\vec{A}_y$ parallel to the coordinate axes. We can describe each component vector with a single number (a scalar) called the **component.** The *x-component* and *y-component* of vector $\vec{A}$, denoted A_x and A_y, are determined as follows:

TACTICS BOX 3.3 Determining the components of a vector MP™

1. The absolute value $|A_x|$ of the *x*-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The *sign* of A_x is positive if $\vec{A}_x$ points in the positive *x*-direction, negative if $\vec{A}_x$ points in the negative *x*-direction.
3. The *y*-component A_y is determined similarly.

Exercises 16–18

In other words, the component A_x tells us two things: how big $\vec{A}_x$ is and which end of the axis $\vec{A}_x$ points toward. **FIGURE 3.14** shows three examples of determining the components of a vector.

NOTE ► $\vec{A}_x$ and $\vec{A}_y$ are *component vectors;* they have a magnitude and a direction. A_x and A_y are simply *components.* The components A_x and A_y are scalars—just numbers (with units) that can be positive or negative. ◄

Much of physics is expressed in the language of vectors. We will frequently need to decompose a vector into its components or to "reassemble" a vector from its components, moving back and forth between the graphical and the component representations of a vector.

Let's start with the problem of decomposing a vector into its *x*- and *y*-components. **FIGURE 3.15a** on the next page shows a vector $\vec{A}$ at an angle θ above horizontal. It is *essential* to use a picture or diagram such as this to define the angle you are using to describe a vector's direction. $\vec{A}$ points to the right and up, so Tactics Box 3.3 tells us that the components A_x and A_y are both positive.

FIGURE 3.14 Determining the components of a vector.

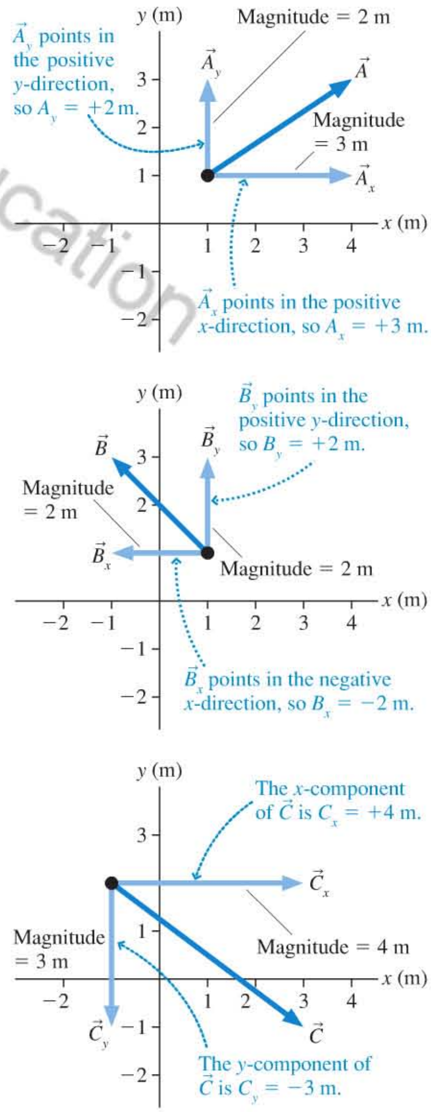


FIGURE 3.15 Breaking a vector into components.

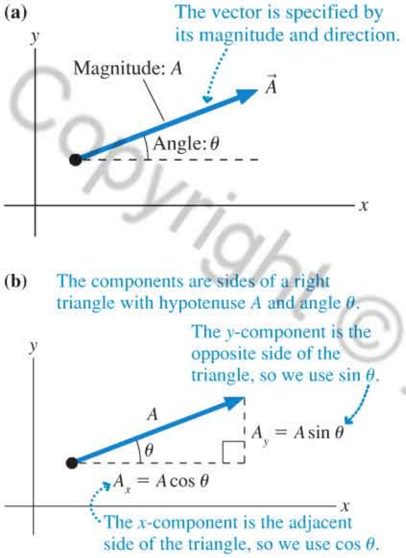


FIGURE 3.16 Specifying a vector from its components.

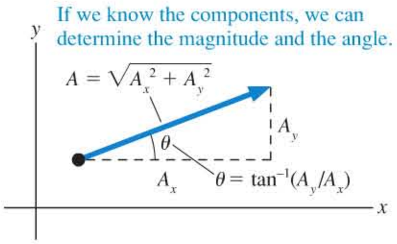
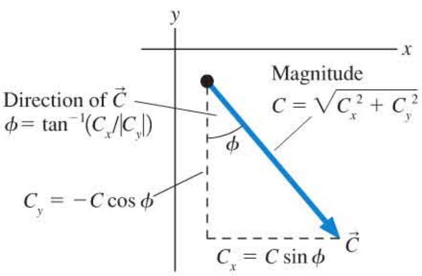


FIGURE 3.17 Relationships for a vector with a negative component.



We can find the components using trigonometry, as illustrated in **FIGURE 3.15b**. For this case, we find that

$$
\begin{aligned}
A_x &= A\cos\theta \\
A_y &= A\sin\theta
\end{aligned}
\tag{3.10}
$$

where A is the magnitude, or length, of $\vec{A}$. These equations convert the length and angle description of vector $\vec{A}$ into the vector's components, but they are correct *only* if θ is measured from horizontal.

Alternatively, if we are given the components of a vector, we can determine the length and angle of the vector from the x- and y-components, as shown in **FIGURE 3.16**. Because A in Figure 3.16 is the hypotenuse of a right triangle, its length is given by the Pythagorean theorem:

$$A = \sqrt{A_x^2 + A_y^2} \tag{3.11}$$

Similarly, the tangent of angle θ is the ratio of the opposite side to the adjacent side, so

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \tag{3.12}$$

Equations 3.11 and 3.12 can be thought of as the "inverse" of Equations 3.10.

How do things change if the vector isn't pointing to the right and up—that is, if one of the components is negative? **FIGURE 3.17** shows vector $\vec{C}$ pointing to the right and down. In this case, the component vector $\vec{C}_y$ is pointing *down,* in the negative y-direction, so the y-component C_y is a *negative* number. The angle ϕ is drawn measured from the y-axis, so the components of $\vec{C}$ are

$$
\begin{aligned}
C_x &= C\sin\phi \\
C_y &= -C\cos\phi
\end{aligned}
\tag{3.13}
$$

The roles of sine and cosine are reversed from those in Equations 3.10 because the angle ϕ is measured with respect to vertical, not horizontal.

NOTE ► Whether the x- and y-components use the sine or cosine depends on how you define the vector's angle. As noted above, you *must* draw a diagram to define the angle that you use, and you must be sure to refer to the diagram when computing components. Don't use Equations 3.10 or 3.13 as general rules—they aren't! They appear as they do because of how we defined the angles. ◄

Next, let's look at the "inverse" problem for this case: determining the length and direction of the vector given the components. The signs of the components don't matter for determining the length; the Pythagorean theorem always works to find the length or magnitude of a vector because the squares eliminate any concerns over the signs. The length of the vector in Figure 3.17 is simply

$$C = \sqrt{C_x^2 + C_y^2} \tag{3.14}$$

When we determine the direction of the vector from its components, we must consider the signs of the components. Finding the angle of vector $\vec{C}$ in Figure 3.17 requires the length of C_y *without* the minus sign, so vector $\vec{C}$ has direction

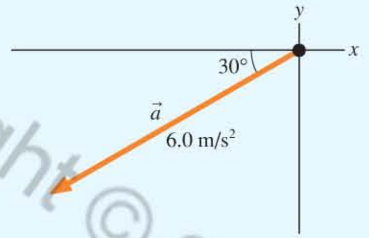
$$\phi = \tan^{-1}\left(\frac{C_x}{|C_y|}\right) \tag{3.15}$$

Notice that the roles of x and y differ from those in Equation 3.12.

EXAMPLE 3.4 **Finding the components of an acceleration vector**

Find the x- and y-components of the acceleration vector $\vec{a}$ shown in **FIGURE 3.18**.

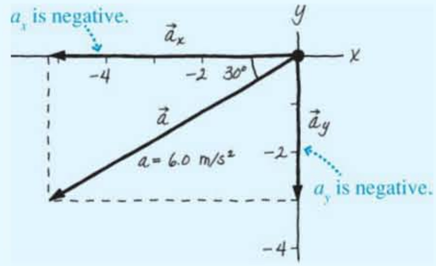
FIGURE 3.18 Acceleration vector $\vec{a}$ of Example 3.4.



PREPARE It's important to *draw* the vectors. Making a sketch is crucial to setting up this problem. **FIGURE 3.19** shows the original vector $\vec{a}$ decomposed into component vectors parallel to the axes.

SOLVE The acceleration vector $\vec{a} = (6.0\text{ m/s}^2,\ 30°$ below the negative x-axis) points to the left (negative x-direction) and down (negative y-direction), so the components a_x and a_y are both negative:

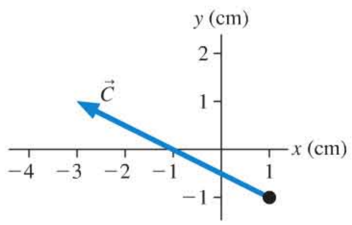
FIGURE 3.19 The components of the acceleration vector.



$$a_x = -a\cos 30° = -(6.0\text{ m/s}^2)\cos 30° = -5.2\text{ m/s}^2$$
$$a_y = -a\sin 30° = -(6.0\text{ m/s}^2)\sin 30° = -3.0\text{ m/s}^2$$

ASSESS The magnitude of the y-component is less than that of the x-component, as seems to be the case in Figure 3.19, a good check on our work. The units of a_x and a_y are the same as the units of vector $\vec{a}$. Notice that we had to insert the minus signs manually by observing that the vector points down and to the left.

STOP TO THINK 3.2 What are the x- and y-components C_x and C_y of vector $\vec{C}$?



Working with Components

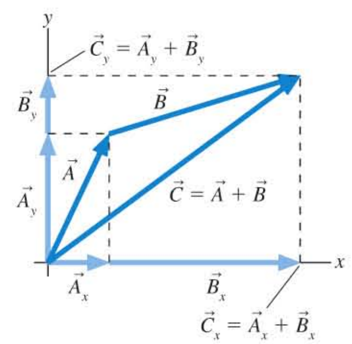
We've seen how to add vectors graphically, but there's an easier way: using components. To illustrate, let's look at the vector sum $\vec{C} = \vec{A} + \vec{B}$ for the vectors shown in **FIGURE 3.20**. You can see that the component vectors of $\vec{C}$ are the sums of the component vectors of $\vec{A}$ and $\vec{B}$. The same is true of the components: $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

In general, if $\vec{D} = \vec{A} + \vec{B} + \vec{C} + \cdots$, then the x- and y-components of the resultant vector $\vec{D}$ are

$$D_x = A_x + B_x + C_x + \cdots$$
$$D_y = A_y + B_y + C_y + \cdots \qquad (3.16)$$

This method of vector addition is called *algebraic addition.*

FIGURE 3.20 Using components to add vectors.



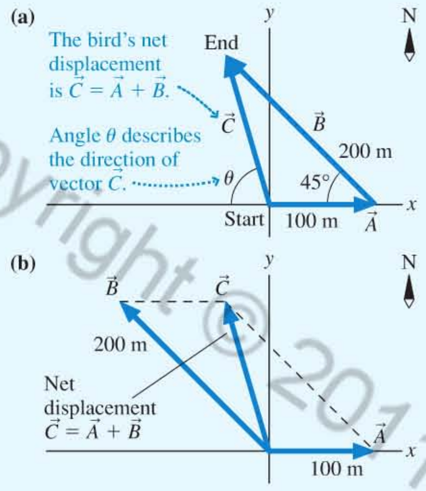
EXAMPLE 3.5 **Using algebraic addition to find a bird's displacement**

A bird flies 100 m due east from a tree, then 200 m northwest (that is, 45° north of west). What is the bird's net displacement?

PREPARE **FIGURE 3.21a** on the next page shows the displacement vectors $\vec{A}$ = (100 m, east) and $\vec{B}$ = (200 m, northwest) and also the net displacement $\vec{C}$. We draw vectors tip-to-tail if we are going to add them graphically, but it's usually easier to draw them all from the origin if we are going to use algebraic addition. **FIGURE 3.21b** redraws the vectors with their tails together.

Continued

FIGURE 3.21 Finding the net displacement.



SOLVE To add the vectors algebraically we must know their components. From the figure these are seen to be

$$A_x = 100 \text{ m}$$
$$A_y = 0 \text{ m}$$
$$B_x = -(200 \text{ m})\cos 45° = -141 \text{ m}$$
$$B_y = (200 \text{ m})\sin 45° = 141 \text{ m}$$

We learned *from the figure* that $\vec{B}$ has a negative x-component. Adding $\vec{A}$ and $\vec{B}$ by components gives

$$C_x = A_x + B_x = 100 \text{ m} - 141 \text{ m} = -41 \text{ m}$$
$$C_y = A_y + B_y = 0 \text{ m} + 141 \text{ m} = 141 \text{ m}$$

The magnitude of the net displacement $\vec{C}$ is

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-41 \text{ m})^2 + (141 \text{ m})^2} = 147 \text{ m}$$

The angle θ, as defined in Figure 3.21, is

$$\theta = \tan^{-1}\left(\frac{C_y}{|C_x|}\right) = \tan^{-1}\left(\frac{141 \text{ m}}{41 \text{ m}}\right) = 74°$$

Thus the bird's net displacement is $\vec{C} = (147 \text{ m}, 74° \text{ north of west})$.

ASSESS The final values of C_x and C_y match what we would expect from the sketch in Figure 3.21. The geometric addition was a valuable check on the answer we found by algebraic addition.

When you look at a trail map for a hike in a mountainous region, it will give the length of a trail *and* the elevation gain—an important variable! The elevation gain is simply d_y, the vertical component of the displacement for the hike.

Vector subtraction and the multiplication of a vector by a scalar are also easily performed using components. To find $\vec{D} = \vec{P} - \vec{Q}$ we would compute

$$D_x = P_x - Q_x \qquad (3.17)$$
$$D_y = P_y - Q_y$$

Similarly, $\vec{T} = c\vec{S}$ is

$$T_x = cS_x \qquad (3.18)$$
$$T_y = cS_y$$

The next few chapters will make frequent use of *vector equations*. For example, you will learn that the equation to calculate the net force on a car skidding to a stop is

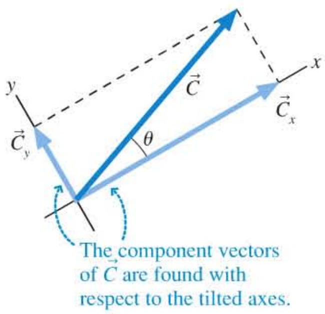
$$\vec{F} = \vec{n} + \vec{w} + \vec{f} \qquad (3.19)$$

Equation 3.19 is really just a shorthand way of writing the two simultaneous equations:

$$F_x = n_x + w_x + f_x \qquad (3.20)$$
$$F_y = n_y + w_y + f_y$$

In other words, a vector equation is interpreted as meaning: Equate the x-components on both sides of the equals sign, then equate the y-components. Vector notation allows us to write these two equations in a more compact form.

FIGURE 3.22 A coordinate system with tilted axes.



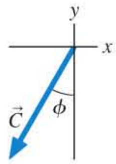
Tilted Axes

Although we are used to having the x-axis horizontal, there is no requirement that it has to be that way. In Chapter 1, we saw that for motion on a slope, it is often most convenient to put the x-axis along the slope. When we add the y-axis, this gives us a tilted coordinate system such as that shown in FIGURE 3.22.

Finding components with tilted axes is no harder than what we have done so far. Vector $\vec{C}$ in Figure 3.22 can be decomposed into component vectors $\vec{C}_x$ and $\vec{C}_y$, with $C_x = C\cos\theta$ and $C_y = C\sin\theta$.

STOP TO THINK 3.3 Angle ϕ that specifies the direction of $\vec{C}$ is computed as

A. $\tan^{-1}(C_x/C_y)$.
B. $\tan^{-1}(C_x/|C_y|)$.
C. $\tan^{-1}(|C_x|/|C_y|)$.
D. $\tan^{-1}(C_y/C_x)$.
E. $\tan^{-1}(C_y/|C_x|)$.
F. $\tan^{-1}(|C_y|/|C_x|)$.



3.4 Motion on a Ramp

In this section, we will examine the problem of motion on a ramp or incline. There are three reasons to look at this problem. First, it will provide good practice at using vectors to analyze motion. Second, it is a simple problem for which we can find an exact solution. Third, this seemingly abstract problem has real and important applications.

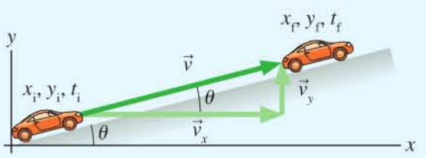
We begin with a constant-velocity example to give us some practice with vectors and components before moving on to the more general case of accelerated motion.

EXAMPLE 3.6 Finding the height gained on a slope

A car drives up a steep 10° slope at a constant speed of 15 m/s. After 10 s, how much height has the car gained?

PREPARE FIGURE 3.23 is a visual overview, with x- and y-axes defined. The velocity vector $\vec{v}$ points up the slope. We are interested in the vertical motion of the car, so we decompose $\vec{v}$ into component vectors $\vec{v}_x$ and $\vec{v}_y$ as shown.

FIGURE 3.23 Visual overview of a car moving up a slope.



Known
$x_i = y_i = 0$ m
$t_i = 0$ s, $t_f = 10$ s
$v = 15$ m/s
$\theta = 10°$

Find
Δy

SOLVE The velocity component we need is v_y; this describes the vertical motion of the car. Using the rules for finding components outlined above, we find

$$v_y = v\sin\theta = (15 \text{ m/s})\sin(10°) = 2.6 \text{ m/s}$$

Because the velocity is constant, the car's vertical displacement (i.e., the height gained) during 10 s is

$$\Delta y = v_y\,\Delta t = (2.6 \text{ m/s})(10 \text{ s}) = 26 \text{ m}$$

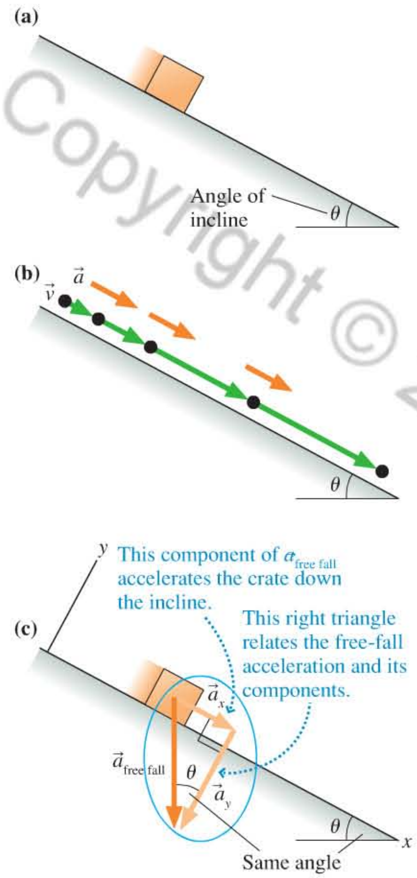
ASSESS The car is traveling at a pretty good clip—15 m/s is a bit faster than 30 mph—up a steep slope, so it should climb a respectable height in 10 s. 26 m, or about 80 ft, seems reasonable.

Accelerated Motion on a Ramp

FIGURE 3.24a on the next page shows a crate sliding down a frictionless (i.e., smooth) ramp tilted at angle θ. The crate accelerates due to the action of gravity, but it is *constrained* to accelerate parallel to the surface. What is the acceleration?

A motion diagram for the crate is drawn in FIGURE 3.24b. There is an acceleration because the velocity is changing, with both the acceleration and velocity vectors parallel to the ramp. We can take advantage of the properties of vectors to find the crate's acceleration. To do so, FIGURE 3.24c sets up a coordinate system with the x-axis along the ramp and the y-axis perpendicular. All motion will be along the x-axis.

FIGURE 3.24 Acceleration on an inclined plane.



If the incline suddenly vanished, the object would have a free-fall acceleration $\vec{a}_{\text{free fall}}$ straight down. As Figure 3.24c shows, this acceleration vector can be decomposed into two component vectors: a vector $\vec{a}_x$ that is *parallel* to the incline and a vector $\vec{a}_y$ that is *perpendicular* to the incline. The vector addition rules studied earlier in this chapter tell us that $\vec{a}_{\text{free fall}} = \vec{a}_x + \vec{a}_y$.

The motion diagram shows that the object's actual acceleration $\vec{a}_x$ is parallel to the incline. The surface of the incline somehow "blocks" the other component of the acceleration $\vec{a}_y$, through a process we will examine in Chapter 5, but $\vec{a}_x$ is unhindered. It is this component of $\vec{a}_{\text{free fall}}$, parallel to the incline, that accelerates the object.

We can use trigonometry to work out the magnitude of this acceleration. Figure 3.24c shows that the three vectors $\vec{a}_{\text{free fall}}$, $\vec{a}_y$, and $\vec{a}_x$ form a right triangle with angle θ as shown; this angle is the same as the angle of the incline. By definition, the magnitude of $\vec{a}_{\text{free fall}}$ is g. This vector is the hypotenuse of the right triangle. The vector we are interested in, $\vec{a}_x$, is opposite angle θ. Thus the value of the acceleration along a frictionless slope is

$$a_x = \pm g \sin\theta \tag{3.21}$$

NOTE ► The correct sign depends on the direction in which the ramp is tilted. The acceleration in Figure 3.24 is $+g\sin\theta$, but upcoming examples will show situations in which the acceleration is $-g\sin\theta$. ◄

Let's look at Equation 3.21 to verify that it makes sense. A good way to do this is to consider some **limiting cases** in which the angle is at one end of its range. In these cases, the physics is clear and we can check our result. Let's look at two such possibilities:

1. Suppose the plane is perfectly horizontal, with $\theta = 0°$. If you place an object on a horizontal surface, you expect it to stay at rest with no acceleration. Equation 3.21 gives $a_x = 0$ when $\theta = 0°$, in agreement with our expectations.
2. Now suppose you tilt the plane until it becomes vertical, with $\theta = 90°$. You know what happens—the object will be in free fall, parallel to the vertical surface. Equation 3.21 gives $a_x = g$ when $\theta = 90°$, again in agreement with our expectations.

NOTE ► Checking your answer by looking at such limiting cases is a very good way to see if your answer makes sense. We will often do this in the Assess step of a solution. ◄

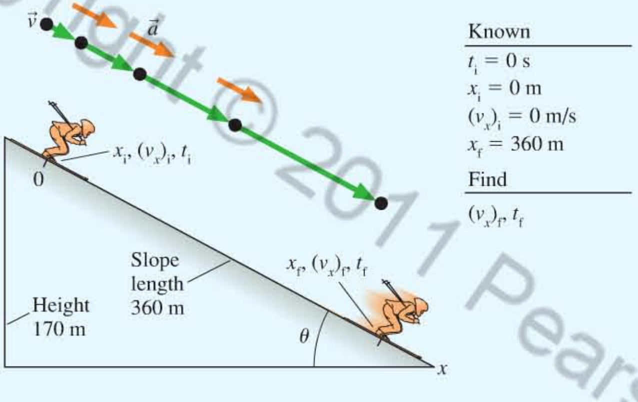
◄ **Extreme physics** A speed skier, on wide skis with little friction, wearing an aerodynamic helmet and crouched low to minimize air resistance, moves in a straight line down a steep slope—pretty much like an object sliding down a frictionless ramp. There is a maximum speed that a skier could possibly achieve at the end of the slope. Course designers set the starting point to keep this maximum speed within reasonable (for this sport!) limits.

EXAMPLE 3.7 **Maximum possible speed for a skier**

The Willamette Pass ski area in Oregon was the site of the 1993 U.S. National Speed Skiing Competition. The skiers started from rest and then accelerated down a stretch of the mountain with a reasonably constant slope, aiming for the highest possible speed at the end of this run. During this acceleration phase, the skiers traveled 360 m while dropping a vertical distance of 170 m. What is the fastest speed a skier could achieve at the end of this run? How much time would this fastest run take?

PREPARE We begin with the visual overview in FIGURE 3.25. The motion diagram shows the acceleration of the skier and the pictorial representation gives an overview of the problem including the dimensions of the slope. As before, we put the x-axis along the slope.

FIGURE 3.25 Visual overview of a skier accelerating down a slope.



SOLVE The fastest possible run would be one without any friction or air resistance, meaning the acceleration down the slope is given by Equation 3.21. The acceleration is in the positive x-direction, so we use the positive sign. What is the angle in Equation 3.21? Figure 3.25 shows that the 360-m-long slope is the hypotenuse of a triangle of height 170 m, so we use trigonometry to find

$$\sin\theta = \frac{170\text{ m}}{360\text{ m}}$$

which gives $\theta = \sin^{-1}(170/360) = 28°$. Equation 3.21 then gives

$$a_x = +g\sin\theta = (9.8\text{ m/s}^2)(\sin 28°) = 4.6\text{ m/s}^2$$

For linear motion with constant acceleration, we can use the third of the kinematic equations in Table 2.4: $(v_x)_f^2 = (v_x)_i^2 + 2a_x\,\Delta x$. The initial velocity $(v_x)_i$ is zero; thus:

This is the distance along the slope, the length of the run.

$$(v_x)_f = \sqrt{2a_x\Delta x} = \sqrt{2(4.6\text{ m/s}^2)(360\text{ m})} = 58\text{ m/s}$$

This is the fastest that any skier could hope to be moving at the end of the run. Any friction or air resistance would decrease this speed. Because the acceleration is constant and the initial velocity $(v_x)_i$ is zero, the time of the fastest-possible run is

$$\Delta t = \frac{(v_x)_f}{a_x} = \frac{58\text{ m/s}}{4.6\text{ m/s}^2} = 13\text{ s}$$

A speed skiing event is a quick affair!

ASSESS The final speed we calculated is 58 m/s, which is about 130 mph, reasonable because we expect a high speed for this sport. In the competition noted, the actual winning speed was 111 mph, not much slower than the result we calculated. Obviously, the efforts to minimize friction and air resistance are working!

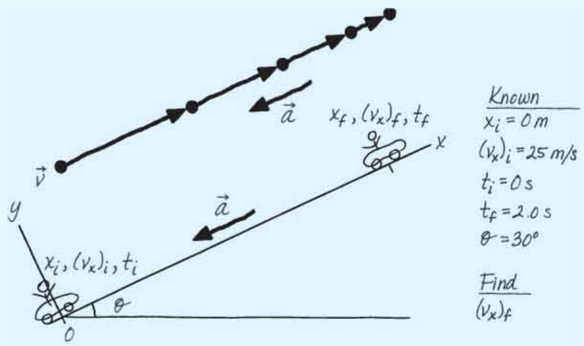
Skis on snow have very little friction, but there are other ways to reduce the friction between surfaces. For instance, a roller coaster car rolls along a track on low-friction wheels. No drive force is applied to the cars after they are released at the top of the first hill: the speed changes due to gravity alone. The cars speed up as they go down hills and slow down as they climb.

EXAMPLE 3.8 Speed of a roller coaster

A classic wooden coaster has cars that go down a big first hill, gaining speed. The cars then ascend a second hill with a slope of 30°. If the cars are going 25 m/s at the bottom and it takes them 2.0 s to climb this hill, how fast are they going at the top?

PREPARE We start with the visual overview in FIGURE 3.26, which includes a motion diagram, a pictorial representation, and a list of values. We've done this with a sketch such as you might draw for your homework. Notice how the motion diagram of Figure 3.26 differs from that of the previous example: The velocity decreases as the car moves up the hill, so the acceleration vector is opposite the direction of the velocity vector. The motion is along the x-axis, as before, but the acceleration vector points in the negative x-direction, so the component a_x is negative. In the motion diagram, notice that we drew only a single acceleration vector—a reasonable shortcut because we know that the acceleration is constant. One vector can represent the acceleration for the entire motion.

FIGURE 3.26 The coaster's speed decreases as it goes up the hill.



Continued

SOLVE To determine the final speed, we need to know the acceleration. We will assume that there is no friction or air resistance, so the magnitude of the roller coaster's acceleration is given by Equation 3.21 using the minus sign, as noted:

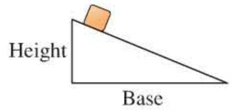
$$a_x = -g\sin\theta = -(9.8\ \mathrm{m/s^2})\sin 30° = -4.9\ \mathrm{m/s^2}$$

The speed at the top of the hill can then be computed using our kinematic equation for velocity:

$$(v_x)_\mathrm{f} = (v_x)_\mathrm{i} + a_x\,\Delta t = 25\ \mathrm{m/s} + (-4.9\ \mathrm{m/s^2})(2.0\ \mathrm{s}) = 15\ \mathrm{m/s}$$

ASSESS The speed is less at the top of the hill than at the bottom, as it should be, but the coaster is still moving at a pretty good clip at the top—almost 35 mph. This seems reasonable; a fast ride is a fun ride.

STOP TO THINK 3.4 A block of ice slides down a ramp. For which height and base length is the acceleration the greatest?



A. Height 4 m, base 12 m
B. Height 3 m, base 6 m
C. Height 2 m, base 5 m
D. Height 1 m, base 3 m

3.5 Relative Motion

You've now dealt many times with problems that say something like "A car travels at 30 m/s" or "A plane travels at 300 m/s." But, as we will see, we may need to be a bit more specific.

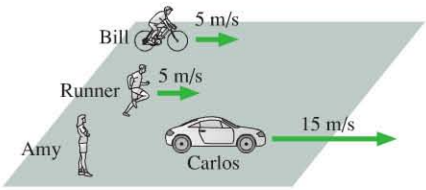


FIGURE 3.27 Amy, Bill, and Carlos each measure the velocity of the runner. The velocities are shown relative to Amy.

In FIGURE 3.27, Amy, Bill, and Carlos are watching a runner. According to Amy, the runner's velocity is $v_x = 5$ m/s. But to Bill, who's riding alongside, the runner is lifting his legs up and down but going neither forward nor backward relative to Bill. As far as Bill is concerned, the runner's velocity is $v_x = 0$ m/s. Carlos sees the runner receding in his rearview mirror, in the *negative* x-direction, getting 10 m farther away from him every second. According to Carlos, the runner's velocity is $v_x = -10$ m/s. Which is the runner's *true* velocity?

Velocity is not a concept that can be true or false. The runner's velocity *relative to Amy* is 5 m/s; that is, his velocity is 5 m/s in a coordinate system attached to Amy and in which Amy is at rest. The runner's velocity relative to Bill is 0 m/s, and the velocity relative to Carlos is −10 m/s. These are all valid descriptions of the runner's motion.

Throwing for the gold An athlete throwing the javelin does so while running. It's harder to throw the javelin on the run, but there's a very good reason to do so. The distance of the throw will be determined by the velocity of the javelin with respect to the ground—which is the sum of the velocity of the throw plus the velocity of the athlete. A faster run means a longer throw.

Relative Velocity

Suppose we know that the runner's velocity relative to Amy is 5 m/s; we will call this velocity $(v_x)_\mathrm{RA}$. The second subscript "RA" means "**R**unner relative to **A**my." We also know that the velocity of **C**arlos relative to **A**my is 15 m/s; we write this as $(v_x)_\mathrm{CA} = 15$ m/s. It is equally valid to compute Amy's velocity relative to Carlos. From Carlos's point of view, Amy is moving to the left at 15 m/s; we write Amy's velocity relative to **C**arlos as $(v_x)_\mathrm{AC} = -15$ m/s; note that $(v_x)_\mathrm{AC} = -(v_x)_\mathrm{CA}$.

Given the runner's velocity relative to Amy and Amy's velocity relative to Carlos, we can compute the runner's velocity relative to Carlos by combining the two velocities we know. The subscripts as we have defined them are our guide for this combination:

$$(v_x)_\mathrm{RC} = (v_x)_\mathrm{RA} + (v_x)_\mathrm{AC} \tag{3.22}$$

The "A" appears on the right of the first expression and on the left of the second; when we combine these velocities, we "cancel" the A to get $(v_x)_\mathrm{RC}$.

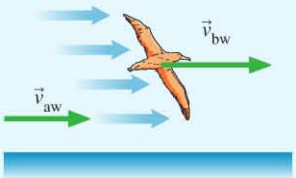
Generally, you can add two relative velocities in this manner, by "canceling" subscripts as in Equation 3.22. In Chapter 27, when we learn about relativity, we will have a more rigorous scheme for computing relative velocities, but this technique will serve our purposes at present.

EXAMPLE 3.9 Speed of a seabird

Researchers doing satellite tracking of albatrosses in the Southern Ocean observed a bird maintaining sustained flight speeds of 35 m/s—nearly 80 mph! This seems surprisingly fast until you realize that this particular bird was flying with the wind, which was moving at 23 m/s. What was the bird's airspeed—its speed relative to the air? This is a truer measure of its flight speed.

PREPARE **FIGURE 3.28** shows the wind and the albatross moving to the right, so all velocities will be positive. We've shown the velocity $(v_x)_{bw}$ of the **b**ird with respect to the **w**ater, which is the measured flight speed, and the velocity $(v_x)_{aw}$ of the **a**ir with respect to the **w**ater, which is the known wind speed. We want to find the bird's airspeed—the speed of the **b**ird with respect to the **a**ir.

FIGURE 3.28 Relative velocities for the albatross and the wind for Example 3.9.



SOLVE We need the subscript for the water to "cancel," so, according to Equation 3.22, we write

$$(v_x)_{ba} = (v_x)_{bw} + (v_x)_{wa}$$

The term $(v_x)_{wa}$, is the opposite of the second of our known values, so we use $(v_x)_{wa} = -(v_x)_{aw} = -23$ m/s to find

$$(v_x)_{ba} = (35 \text{ m/s}) + (-23 \text{ m/s}) = 12 \text{ m/s}$$

ASSESS 12 m/s—about 25 mph—is a reasonable airspeed for a bird. And it's slower than the observed flight speed, which makes sense because the bird is flying with the wind.

This technique for finding relative velocities also works for two-dimensional situations, as we see in the next example. Relative motion in two dimensions is another good exercise in working with vectors.

EXAMPLE 3.10 Finding the ground speed of an airplane

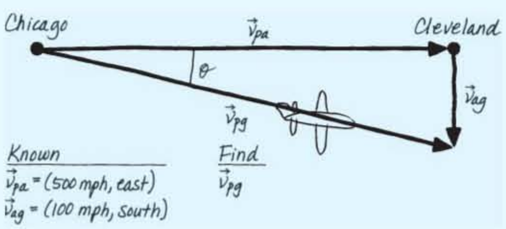
Cleveland is approximately 300 miles east of Chicago. A plane leaves Chicago flying due east at 500 mph. The pilot forgot to check the weather and doesn't know that the wind is blowing to the south at 100 mph. What is the plane's velocity relative to the ground?

PREPARE **FIGURE 3.29** is a visual overview of the situation. We are given the speed of the **p**lane relative to the **a**ir ($\vec{v}_{pa}$) and the speed of the **a**ir relative to the **g**round ($\vec{v}_{ag}$); the speed of the **p**lane relative to the **g**round will be the vector sum of these velocities:

$$\vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag}$$

This vector sum is shown in Figure 3.29.

FIGURE 3.29 The wind causes a plane flying due east in the air to move to the southeast relative to the ground.



SOLVE The plane's speed relative to the ground is the hypotenuse of the right triangle in Figure 3.29; thus:

$$v_{pg} = \sqrt{{v_{pa}}^2 + {v_{ag}}^2} = \sqrt{(500 \text{ mph})^2 + (100 \text{ mph})^2} = 510 \text{ mph}$$

The plane's direction can be specified by the angle θ measured from due east:

$$\theta = \tan^{-1}\left(\frac{100 \text{ mph}}{500 \text{ mph}}\right) = \tan^{-1}(0.20) = 11°$$

The velocity of the plane relative to the ground is thus

$$\vec{v}_{pg} = (510 \text{ mph}, 11° \text{ south of east})$$

ASSESS The good news is that the wind is making the plane move a bit faster relative to the ground; the bad news is that the wind is making the plane move in the wrong direction!

3.6 Motion in Two Dimensions: Projectile Motion

Balls flying through the air, long jumpers, and cars doing stunt jumps are all examples of the two-dimensional motion that we call *projectile motion.* Projectile motion is an extension to two dimensions of the free-fall motion we studied in Chapter 2. **A projectile is an object that moves in two dimensions under the influence of gravity and nothing else.** Although real objects are also influenced by air resistance, the effect of air resistance is small for reasonably dense objects moving at modest speeds, so we can ignore it for the cases we consider in this chapter. As long as we can neglect air resistance, any projectile will follow the same type of path: a trajectory with the mathematical form of a parabola. Because the form of the motion will always be the same, the strategies we develop to solve one projectile problem can be applied to others as well.

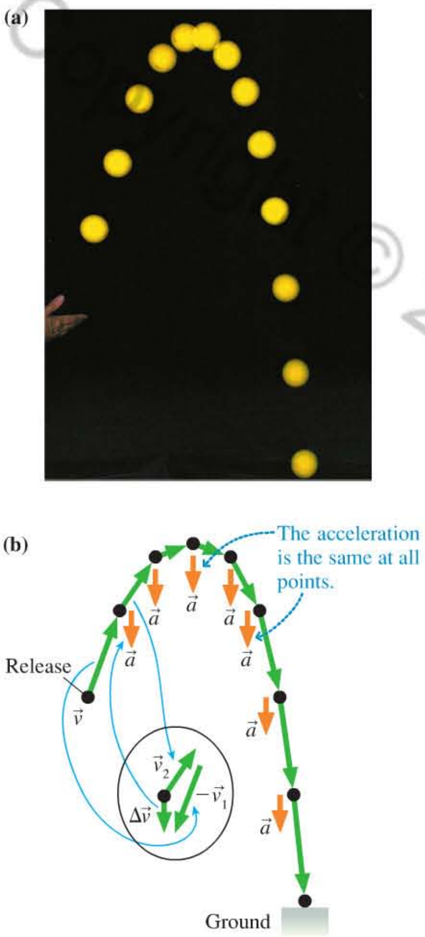


FIGURE 3.30 The motion of a tossed ball. The inset shows how to find the direction of $\Delta\vec{v}$, the change in velocity. This is the direction in which the acceleration $\vec{a}$ points.

FIGURE 3.30a shows the parabolic arc of a ball tossed into the air; the camera has captured its position at equal intervals of time. In FIGURE 3.30b we show the motion diagram for this toss, with velocity vectors connecting the points. The acceleration vector points in the same direction as the change in velocity $\Delta\vec{v}$, which we can compute using the techniques of Tactics Box 3.2. You can see that the acceleration vector points straight down; a careful analysis would show that it has magnitude 9.80 m/s^2. Consequently, the acceleration of a projectile is the same as the acceleration of an object falling straight down—namely, the free-fall acceleration:

$$\vec{a}_{\text{free fall}} = (9.80\ \text{m/s}^2,\ \text{straight down})$$

Because the free-fall acceleration is the same for all objects, it is no wonder that the shape of the trajectory—a parabola—is the same as well.

As the projectile moves, the free-fall acceleration will change the vertical component of the velocity, but there will be no change to the horizontal component of the velocity. Therefore, the vertical and horizontal components of the acceleration are

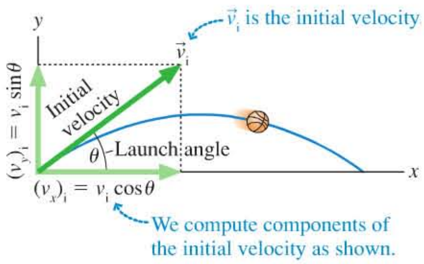
$$\begin{aligned} a_x &= 0\ \text{m/s}^2 \\ a_y &= -g = -9.80\ \text{m/s}^2 \end{aligned} \tag{3.23}$$

The vertical component of acceleration a_y for all projectile motion is just the familiar $-g$ of free fall, while the horizontal component a_x is zero.

Analyzing Projectile Motion

Suppose you toss a basketball down the court, as shown in FIGURE 3.31. To study this projectile motion, we've established a coordinate system with the x-axis horizontal and the y-axis vertical. The start of a projectile's motion is called the *launch,* and the angle θ of the initial velocity $\vec{v}_i$ above the horizontal (i.e., above the x-axis) is the **launch angle.** As you learned in Section 3.3, the initial velocity vector $\vec{v}_i$ can be expressed in terms of the x- and y-components $(v_x)_i$ and $(v_y)_i$. You can see from the figure that

FIGURE 3.31 The launch and motion of a projectile.



$$\begin{aligned} (v_x)_i &= v_i \cos\theta \\ (v_y)_i &= v_i \sin\theta \end{aligned} \tag{3.24}$$

where v_i is the initial speed.

NOTE ► The components $(v_x)_i$ and $(v_y)_i$ are not always positive. A projectile launched at an angle *below* the horizontal (such as a ball thrown downward from the roof of a building) has *negative* values for θ and $(v_y)_i$. However, the *speed* v_i is always positive. ◄

To see how the acceleration determines the subsequent motion, FIGURE 3.32 shows a projectile launched at a speed of 22.0 m/s at an angle of 63° from the horizontal.

In Figure 3.32a, the initial velocity vector is broken into its horizontal and vertical components. In Figure 3.32b, the velocity vector and its component vectors are shown every subsequent 1.0 s. Because there is no horizontal acceleration ($a_x = 0$), the value of v_x never changes. In contrast, v_y decreases by 9.8 m/s every second. This is what it *means* to accelerate at $a_y = -9.8\ \text{m/s}^2 = (-9.8\ \text{m/s})$ per second. Nothing *pushes* the projectile along the curve. Instead, the downward acceleration changes the velocity vector as shown, causing it to increase downward as the motion proceeds. At the end of the motion, when the ball is at the same height as it started, v_y is -19.6 m/s, the negative of its initial value. **The ball finishes its motion moving downward at the same speed as it started moving upward,** just as we saw in the case of one-dimensional free fall in Chapter 2.

You can see from Figure 3.32 that **projectile motion is made up of two independent motions: uniform motion at constant velocity in the horizontal direction and free-fall motion in the vertical direction.** In Chapter 2, we saw kinematic equations for constant-velocity and constant-acceleration motion. We can adapt these general equations to this current case: The horizontal motion is constant-velocity motion at $(v_x)_i$; the vertical motion is constant-acceleration motion with initial velocity $(v_y)_i$ and an acceleration of $a_y = -g$.

$$x_f = x_i + (v_x)_i\,\Delta t \qquad y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$
$$(v_x)_f = (v_x)_i = \text{constant} \qquad (v_y)_f = (v_y)_i - g\,\Delta t \tag{3.25}$$

Equations of motion for the parabolic trajectory of a projectile

A close look at these equations reveals a surprising fact: **The horizontal and vertical components of projectile motion are independent of each other.** The initial horizontal velocity has *no* influence over the vertical motion, and vice versa. This independence of the horizontal and vertical motions is illustrated in **FIGURE 3.33**, which shows a strobe photograph of two balls, one shot horizontally and the other released from rest at the same instant. The *vertical* motions of the two balls are identical, and they hit the floor simultaneously. Neither ball has any initial motion in the vertical direction, so both fall distance h in the same amount of time.

Let's extend these ideas to consider a "classic" problem in physics:

A hungry hunter in the jungle wants to shoot down a coconut that is hanging from the branch of a tree. He aims the gun directly at the coconut, but as luck would have it the coconut falls from the branch at the exact instant the hunter pulls the trigger. Does the bullet hit the coconut?

FIGURE 3.34 shows a useful way to analyze this problem. Figure 3.34a shows the trajectory of a projectile. Without gravity, a projectile would follow a

FIGURE 3.32 The velocity and acceleration vectors of a projectile.

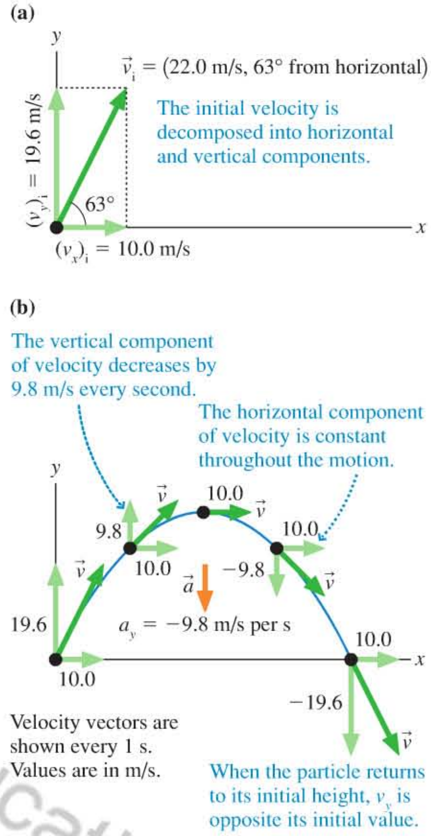


FIGURE 3.33 A projectile launched horizontally falls in the same time as a projectile that is released from rest.

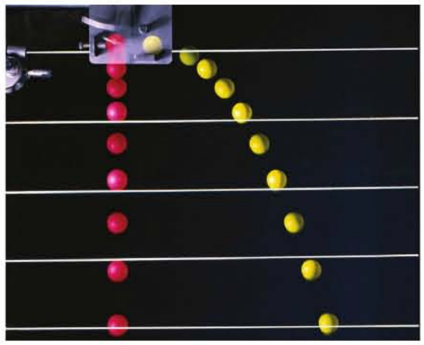
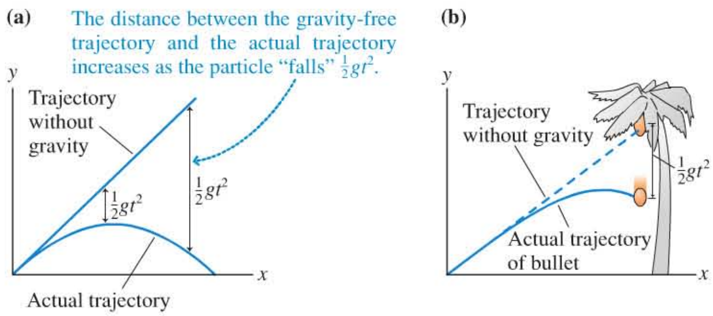


FIGURE 3.34 A projectile follows a parabolic trajectory because it "falls" a distance $\frac{1}{2}gt^2$ below a straight-line trajectory.



TRY IT YOURSELF

A game of catch in a moving vehicle
While riding in a car moving at a constant speed, toss a ball or a coin into the air. You can easily catch it! The ball and you continue to move forward at a constant speed during the ball's up-and-down vertical motion. The vertical motion is completely independent of, and unaffected by, the horizontal motion. From the point of a view of a person watching you drive by, the ball's motion would be a parabolic arc.

straight line. Because of gravity, the particle at time t has "fallen" a distance $\frac{1}{2}gt^2$ below this line. The separation grows as $\frac{1}{2}gt^2$, giving the trajectory its parabolic shape. Figure 3.34b applies this reasoning to the bullet and coconut. Although the bullet travels very fast, it follows a slightly curved trajectory, not a straight line. Had the coconut stayed on the tree, the bullet would have curved under its target because gravity causes it to fall a distance $\frac{1}{2}gt^2$ below the straight line. But $\frac{1}{2}gt^2$ is also the distance the coconut falls while the bullet is in flight. Thus, as Figure 3.34b shows, the bullet and the coconut fall the same distance and meet at the same point!

STOP TO THINK 3.5 A 100 g ball rolls off a table and lands 2 m from the base of the table. A 200 g ball rolls off the same table with the same speed. How far does it land from the base of the table?

A. <1 m.
B. 1 m.
C. Between 1 m and 2 m.
D. 2 m.
E. Between 2 m and 4 m.
F. 4 m.

3.7 Projectile Motion: Solving Problems

Now that we have a good idea of how projectile motion works, we can use that knowledge to solve some true two-dimensional motion problems.

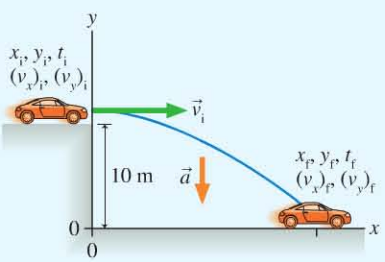
EXAMPLE 3.11 **Planning a Hollywood stunt**

To get the shots of cars flying through the air in movies, it is sometimes necessary to drive a car off a cliff and film it. Suppose a stunt man drives a car off a 10-m-high cliff at a speed of 20 m/s How far does the car land from the base of the cliff?

PREPARE We start with a visual overview of the situation in **FIGURE 3.35**. Note that we have chosen to put the origin of the coordinate system at the base of the cliff. We assume that the car is moving horizontally as it leaves the cliff. In this case, the x- and y-components of the initial velocity are

$$(v_x)_i = v_i = 20 \text{ m/s}$$
$$(v_y)_i = 0 \text{ m/s}$$

FIGURE 3.35 Visual overview for Example 3.11.



Known
$x_i = 0$ m
$(v_y)_i = 0$ m/s
$t_i = 0$ s
$y_i = 10$ m, $y_f = 0$ m
$(v_x)_i = v_i = 20$ m/s
$a_x = 0$ m/s^2
$a_y = -g$

Find
x_f

SOLVE Each point on the trajectory has x- and y-components of position, velocity, and acceleration but only *one* value of time. The time needed to move horizontally to the final position x_f is the *same* time needed to fall 10 m vertically. **Although the horizontal and vertical motions are independent, they are both related to the time t.** This is a critical observation for solving projectile motion problems. We will call the time interval between the car leaving the cliff and landing on the ground Δt. In this problem, we'll analyze the vertical motion first. We can solve the vertical-motion equations for the time interval Δt. We'll then use that value of Δt in the equation for the horizontal motion.

The vertical motion is just free fall. The initial vertical velocity is zero; the car falls from $y_i = 10$ m to $y_f = 0$ m. We can analyze this motion using the vertical-position equation from Equations 3.25:

$$y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$0 \text{ m} = 10 \text{ m} + (0 \text{ m/s})(\Delta t) - \tfrac{1}{2}(9.8 \text{ m/s}^2)(\Delta t)^2$$

Rearranging the terms and then solving for Δt give

$$-10 \text{ m} = -\tfrac{1}{2}(9.8 \text{ m/s}^2)(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2(10 \text{ m})}{9.8 \text{ m/s}^2}} = 1.43 \text{ s}$$

Now that we have the time, we can use the horizontal-position equation from Equations 3.25 to find out where the car lands:

$$x_f = x_i + (v_x)_i\,\Delta t$$
$$x_f = 0 \text{ m} + (20 \text{ m/s})(1.43 \text{ s}) = 29 \text{ m}$$

ASSESS The cliff height is $h \approx 33$ ft and the initial horizontal velocity is $(v_x)_i \approx 40$ mph. At this speed, a car moves faster than 60 feet per second, so traveling $x_f = 29 \text{ m} \approx 95$ ft before hitting the ground seems quite reasonable.

The approach of Example 3.11 is a general one. We can condense the relevant details into a problem-solving strategy.

PROBLEM-SOLVING STRATEGY 3.1 **Projectile motion problems** MP™

PREPARE There are a number of steps that you should go through in setting up the solution to a projectile motion problem:

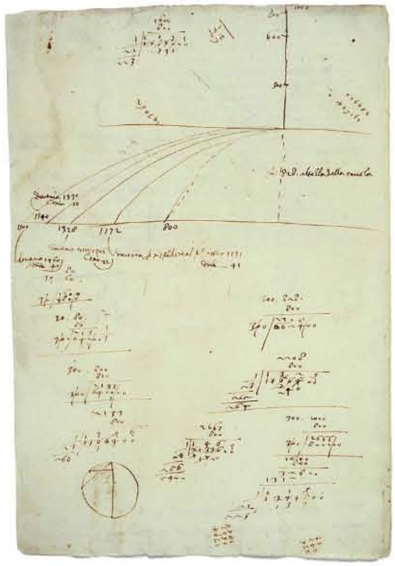
- Make simplifying assumptions. Whether the projectile is a car or a basketball, the motion will be the same.
- Draw a visual overview including a pictorial representation showing the beginning and ending points of the motion.
- Establish a coordinate system with the x-axis horizontal and the y-axis vertical. In this case, you know that the horizontal acceleration will be zero and the vertical acceleration will be free fall: $a_x = 0$ and $a_y = -g$.
- Define symbols and write down a list of known values. Identify what the problem is trying to find.

SOLVE There are two sets of kinematic equations for projectile motion, one for the horizontal component and one for the vertical:

Horizontal	Vertical
$x_f = x_i + (v_x)_i\,\Delta t$	$y_f = y_i + (v_y)_i\,\Delta t - \frac{1}{2}g(\Delta t)^2$
$(v_x)_f = (v_x)_i = \text{constant}$	$(v_y)_f = (v_y)_i - g\,\Delta t$

Δt is the same for the horizontal and vertical components of the motion. Find Δt by solving for the vertical or the horizontal component of the motion; then use that value to complete the solution for the other component.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.



Galileo was the first person to make a serious study of projectile motion, deducing the independence of the horizontal and vertical components. This page from his notes shows his analysis of a projectile launched horizontally. In his day, this topic was cutting-edge science; now it is in Chapter 3 of a 30-chapter book!

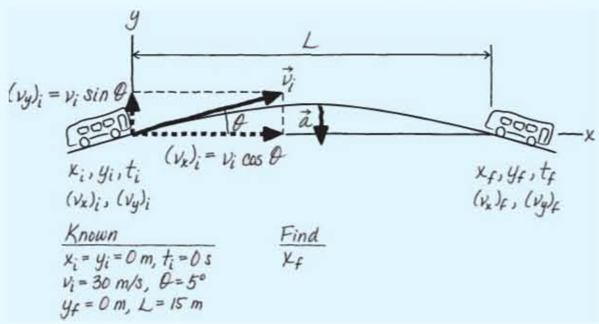
ActivPhysics ONLINE 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7

EXAMPLE 3.12 **Checking the feasibility of a Hollywood stunt**

The main characters in the movie *Speed* are on a bus that has been booby-trapped to explode if its speed drops below 50 mph. But there is a problem ahead: A 50 ft section of a freeway overpass is missing. They decide to jump the bus over the gap. The road leading up to the break has an angle of about 5°. A view of the speedometer just before the jump shows that the bus is traveling at 67 mph. The movie bus makes the jump and survives. Is this realistic, or movie fiction?

PREPARE We begin by converting speed and distance to SI units. The initial speed is $v_i = 30$ m/s and the size of the gap is $L = 15$ m. Next, following the problem-solving strategy, we make a sketch, the visual overview shown in **FIGURE 3.36**, and a list of

FIGURE 3.36 Visual overview of the bus jumping the gap.



values. In choosing our axes, we've placed the origin at the point where the bus starts its jump. The initial velocity vector is tilted 5° above horizontal, so the components of the initial velocity are

$$(v_x)_i = v_i\cos\theta = (30\text{ m/s})(\cos 5°) = 30\text{ m/s}$$

$$(v_y)_i = v_i\sin\theta = (30\text{ m/s})(\sin 5°) = 2.6\text{ m/s}$$

How do we specify the "end" of the problem? By setting $y_f = 0$ m, we'll solve for the horizontal distance x_f at which the bus returns to its initial height. If x_f exceeds 50 ft, the bus successfully clears the gap. We have optimistically drawn our diagram as if the bus makes the jump, but . . .

SOLVE Problem-Solving Strategy 3.1 suggests using one component of the motion to solve for Δt. We will begin with the vertical motion. The kinematic equation for the vertical position is

$$y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

We know that $y_f = y_i = 0$ m. If we factor out Δt, the position equation becomes

$$0 = \Delta t\left((v_y)_i - \tfrac{1}{2}g\,\Delta t\right)$$

One solution to this equation is $\Delta t = 0$ s. This is a legitimate solution, but it corresponds to the instant when $y = 0$ at the beginning

Continued

of the trajectory. We want the second solution, for $y = 0$ at the end of the trajectory, which is when

$$0 = (v_y)_i - \tfrac{1}{2}g\,\Delta t = (2.6\text{ m/s}) - \tfrac{1}{2}(9.8\text{ m/s}^2)\,\Delta t$$

which gives

$$\Delta t = \frac{2 \times (2.6\text{ m/s})}{9.8\text{ m/s}^2} = 0.53\text{ s}$$

During the 0.53 s that the bus is moving vertically it is also moving horizontally. The final horizontal position of the bus is $x_f = x_i + (v_x)_i\,\Delta t$, or

$$x_f = 0\text{ m} + (30\text{ m/s})(0.53\text{ s}) = 16\text{ m}$$

This is how far the bus has traveled horizontally when it returns to its original height. 16 m is a bit more than the width of the gap, so a bus coming off a 5° ramp at the noted speed would make it—just barely!

ASSESS We can do a quick check on our math by noting that the bus takes off and lands at the same height. This means, as we saw in Figure 3.32b, that the y-velocity at the landing should be the negative of its initial value. We can use the velocity equation for the vertical component of the motion to compute the final value and see that the final velocity value is as we predict:

$$(v_y)_f = (v_y)_i - g\,\Delta t$$
$$= (2.6\text{ m/s}) - (9.8\text{ m/s}^2)(0.53\text{ s}) = -2.6\text{ m/s}$$

During the filming of the movie, the filmmakers really did jump a bus over a gap in an overpass! The actual jump was a bit more complicated than our example because a real bus, being an extended object rather than a particle, will start rotating as the front end comes off the ramp. The actual stunt jump used an extra ramp to give a boost to the front end of the bus. Nonetheless, our example shows that the filmmakers did their homework and devised a situation in which the physics was correct.

The Range of a Projectile

When the quarterback throws a football down the field, how far will it go? What will be the **range** for this particular projectile motion, the horizontal distance traveled?

Example 3.12 was a range problem—for a given speed and a given angle, we wanted to know how far the bus would go. The speed and the angle are the two variables that determine the range. A higher speed means a greater range, of course. But how does angle figure in?

FIGURE 3.37 shows the trajectory that a projectile launched at 100 m/s will follow for different launch angles. At very small or very large angles, the range is quite small. If you throw a ball at a 75° angle, it will do a great deal of up-and-down motion, but it won't achieve much horizontal travel. If you throw a ball at a 15° angle, the ball won't be in the air long enough to go very far. These cases both have the same range, as Figure 3.37 shows.

If the angle is too small or too large, the range is shorter than it could be. The "just right" case that gives the maximum range when landing at the same elevation as the launch is a launch angle of 45°, as Figure 3.37 shows.

If that's true, why does a long jumper take off at an angle that is so much less than 45°, as shown in FIGURE 3.38? One reason is that he changes the position of his legs as he jumps—he doesn't really land at the same height as that from which he took off, which changes things a bit. But there's a more important reason. In Figure 3.37 we looked at the range of projectiles that were launched at the *same speed* to see that a 45° angle gave the longest range. But the biomechanics of running and jumping don't allow you to keep the same launch speed as you increase the angle of your jump. Any increase in your launch angle comes at the sacrifice of speed, so the situation of Figure 3.37 doesn't apply. The optimum angle for jumping is less than 45° because your faster jump speed outweighs the effect of a smaller jump angle.

For other projectiles, such as golf balls and baseballs, the optimal angle is less than 45° for a different reason: air resistance. Up to this point we've ignored air resistance, but for small objects traveling at high speeds, air resistance is critical. Aerodynamic forces come into play, causing the projectile's trajectory to deviate from a parabola. The maximum range for a golf ball comes at an angle much less than 45°, as you no doubt know if you have ever played golf.

FIGURE 3.37 Trajectories of a projectile for different launch angles, assuming air resistance can be neglected.

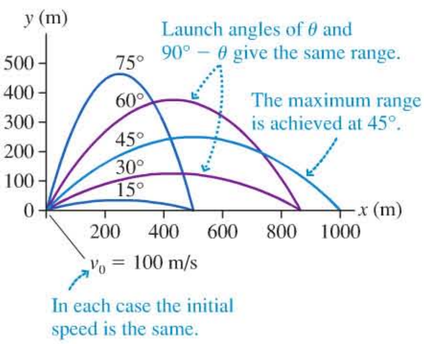


FIGURE 3.38 The trajectory of a long jumper.

▶ **Physics of fielding** BIO The batter hits a high fly ball, and the fielder makes a graceful arc to the exact spot where it lands, catching it on the run. He didn't estimate velocity and calculate the ball's trajectory, so how did he do it? The key is that the fielder is in constant motion. He monitors the relative motion of the ball as he runs and makes adjustments in his velocity to keep the ball at a constant angle with respect to him. By doing this, he'll be at the right spot when the ball lands. He doesn't know where the ball will land—just how to be there when it does!

3.8 Motion in Two Dimensions: Circular Motion

The 32 cars on the London Eye Ferris wheel move at a constant speed of about 0.5 m/s in a vertical circle of radius 65 m. The cars may move at a constant speed, but they do *not* move with constant velocity. Velocity is a vector that depends on both an object's speed *and* its direction of motion, and the direction of circular motion is constantly changing. This is the hallmark of **uniform circular motion:** constant speed, but continuously changing direction. We will introduce some basic ideas about circular motion in this section, then return to treat it in considerably more detail in Chapter 6. For now, we will consider only objects that move around a circular trajectory at constant speed.

Period, Frequency, and Speed

The time interval it takes an object to go around a circle one time, completing one revolution (abbreviated rev), is called the **period** of the motion. Period is represented by the symbol T.

Rather than specify the time for one revolution, we can specify circular motion by its **frequency,** the number of revolutions per second, for which we use the symbol f. An object with a period of one-half second completes 2 revolutions each second. Similarly, an object can make 10 revolutions in 1 s if its period is one-tenth of a second. This shows that frequency is the inverse of the period:

$$f = \frac{1}{T} \tag{3.26}$$

Although frequency is often expressed as "revolutions per second," *revolutions* are not true units but merely the counting of events. Thus the SI unit of frequency is simply inverse seconds, or s^{-1}. Frequency may also be given in revolutions per minute (rpm) or another time interval, but these usually need to be converted to s^{-1} before doing calculations.

The London Eye Ferris wheel.

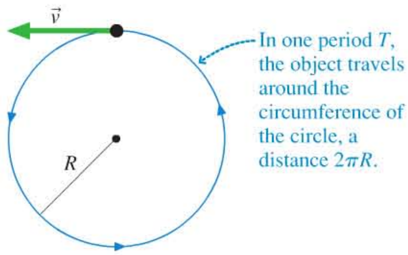
FIGURE 3.39 shows an object moving at a constant speed in a circular path of radius R. We know the time for one revolution—one period T—and we know the distance traveled, so we can write an equation relating the period, the radius, and the speed:

$$v = \frac{2\pi R}{T} \tag{3.27}$$

Given Equation 3.26 relating frequency and period, we can also write this equation as

$$v = 2\pi f R \tag{3.28}$$

FIGURE 3.39 Relating frequency and speed.



EXAMPLE 3.13 **Spinning some tunes**

An audio CD has a diameter of 120 mm and spins at up to 540 rpm. When a CD is spinning at its maximum rate, how much time is required for one revolution? If a speck of dust rides on the outside edge of the disk, how fast is it moving?

PREPARE Before we get started, we need to do some unit conversions. The diameter of a CD is given as 120 mm, which is 0.12 m. The radius is 0.060 m. The frequency is given in rpm; we need to convert this to s^{-1}:

$$f = 540\ \frac{\text{rev}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} = 9.0\ \frac{\text{rev}}{\text{s}} = 9.0\ \text{s}^{-1}$$

Continued

SOLVE The time for one revolution is the period; this is given by Equation 3.26:

$$T = \frac{1}{f} = \frac{1}{9.0\ \text{s}^{-1}} = 0.11\ \text{s}$$

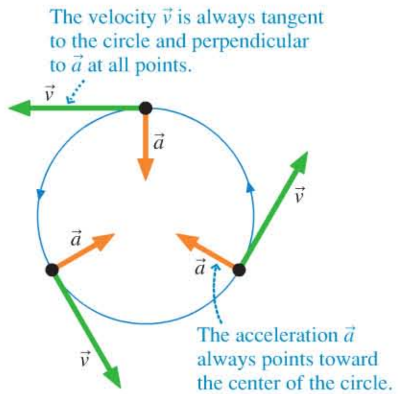
The dust speck is moving in a circle of radius 0.12 m at a frequency of $9.0\ \text{s}^{-1}$. We can use Equation 3.28 to find the speed:

$$v = 2\pi f R = 2\pi(9.0\ \text{s}^{-1})(0.060\ \text{m}) = 3.4\ \text{m/s}$$

ASSESS If you've watched a CD spin, you know that it takes much less than a second to go around, so the value for the period seems reasonable. The speed we calculate for the dust speck is nearly 8 mph, but for a point on the edge of the CD to go around so many times in a second, it must be moving pretty fast.

Acceleration in Circular Motion

FIGURE 3.40 The velocity and acceleration vectors for circular motion.



It may seem strange to think that an object moving with constant speed can be accelerating, but that's exactly what an object in uniform circular motion is doing. It is accelerating because its velocity is changing as its direction of motion changes. What is the acceleration in this case? We saw in Example 3.3 that for circular motion at a constant speed, **the acceleration vector $\vec{a}$ points toward the center of the circle.** This is an idea that is worth reviewing. As you can see in FIGURE 3.40, the velocity is always tangent to the circle, so $\vec{v}$ and $\vec{a}$ are perpendicular to each other at all points on the circle.

An acceleration that always points directly toward the center of a circle is called a **centripetal acceleration.** The word "centripetal" comes from a Greek root meaning "center seeking."

NOTE ▶ Centripetal acceleration is not a new type of acceleration; all we are doing is *naming* an acceleration that corresponds to a particular type of motion. The magnitude of the centripetal acceleration is constant because each successive $\Delta\vec{v}$ in the motion diagram has the same length. ◀

To complete our description of circular motion, we need to find a quantitative relationship between the magnitude of the acceleration a and the speed v. Let's return to the case of the Ferris wheel. During a time Δt in which a car on the Ferris wheel moves around the circle from point 1 to point 2, the car moves through an angle θ and undergoes a displacement $\vec{d}$, as shown in FIGURE 3.41a. We've chosen a relatively large angle θ for our drawing so that angular relationships can be clearly seen, but for a small angle the displacement is essentially identical to the actual distance traveled, and we'll make this approximation.

FIGURE 3.41b shows how the velocity changes as the car moves, and FIGURE 3.41c shows the vector calculation of the change in velocity. The triangle we use to make this calculation is geometrically *similar* to the one that shows the displacement, as

FIGURE 3.41 Changing position and velocity for an object in circular motion.

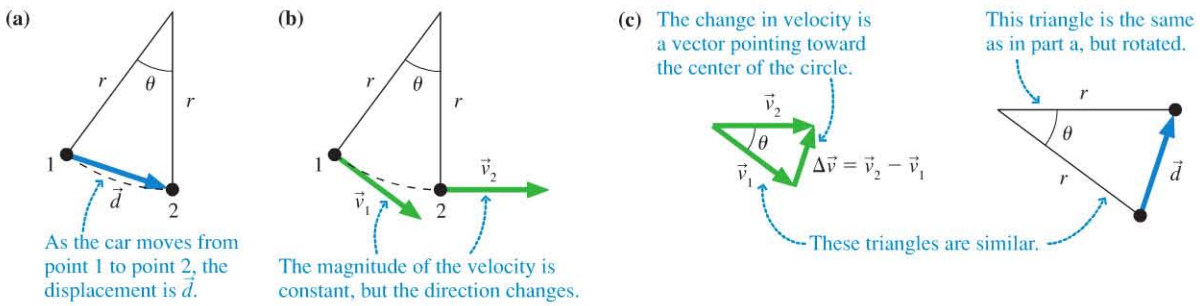


Figure 3.41c shows. This is a key piece of information: You'll remember from geometry that similar triangles have equal ratios of their sides, so we can write

$$\frac{\Delta v}{v} = \frac{d}{r} \tag{3.29}$$

where Δv is the magnitude of the velocity-change vector $\Delta\vec{v}$. We've used the unsubscripted speed v for the length of a side of the first triangle because it is the same for velocities $\vec{v}_1$ and $\vec{v}_2$.

Now we're ready to compute the acceleration. The displacement is just the speed v times the time interval Δt, so we can write

$$d = v\Delta t$$

We can substitute this for d in Equation 3.29 to obtain

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

which we can rearrange like so:

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

We recognize the left-hand side of the equation as the acceleration, so this becomes

$$a = \frac{v^2}{r}$$

Combining this magnitude with the direction we noted above, we can write the centripetal acceleration as

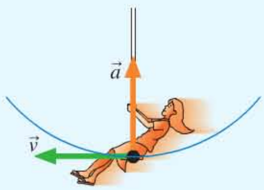
$$\vec{a} = \left(\frac{v^2}{r}, \text{toward center of circle}\right) \tag{3.30}$$

Centripetal acceleration of object moving in a circle of radius r at speed v

CONCEPTUAL EXAMPLE 3.14 **Acceleration on a swing**

A child is riding a playground swing. The swing rotates in a circle around a central point where the rope or chain for the swing is attached. The speed isn't changing at the lowest point of the motion, but the direction is—this is circular motion, with an acceleration directed upward, as shown in **FIGURE 3.42**. More acceleration will mean a more exciting ride. What change could the child make to increase the acceleration she experiences?

FIGURE 3.42 A child at the lowest point of motion on a swing.



REASON The acceleration the child experiences is the "changing direction" acceleration of circular motion, given by Equation 3.30. The acceleration depends on the speed and the radius of the circle. The radius of the circle is determined by the length of the chain or rope, so the only easy way to change the acceleration is to change the speed, which she could do by swinging higher. Because the acceleration is proportional to the square of the speed, doubling the speed means a fourfold increase in the acceleration.

ASSESS If you have ever ridden a swing, you know that the acceleration you experience is greater the faster you go—so our answer makes sense.

EXAMPLE 3.15 Finding the acceleration of a Ferris wheel

A typical carnival Ferris wheel has a radius of 9.0 m and rotates 6.0 times per minute. What magnitude acceleration do the riders experience?

PREPARE The cars on a Ferris wheel move in a circle at constant speed; the acceleration the riders experience is a centripetal acceleration.

SOLVE In order to use Equation 3.30 to compute an acceleration, we need to know the speed v of a rider on the Ferris wheel. The wheel rotates 6.0 times per minute; therefore, the time for one rotation (i.e., the period) is 10 s. We can use Equation 3.27 to find the speed:

$$v = \frac{2\pi R}{T} = \frac{(2\pi)(9.0\text{ m})}{10\text{ s}} = 5.7\text{ m/s}$$

Knowing the speed, we can use Equation 3.30 to find the magnitude of the acceleration:

$$a = \frac{v^2}{r} = \frac{(5.7\text{ m/s})^2}{9.0\text{ m}} = 3.6\text{ m/s}^2$$

ASSESS This is about 1/3 of the free-fall acceleration; the acceleration, in units of g, is $0.37g$. This is enough to notice, but not enough to be scary! Our answer seems reasonable.

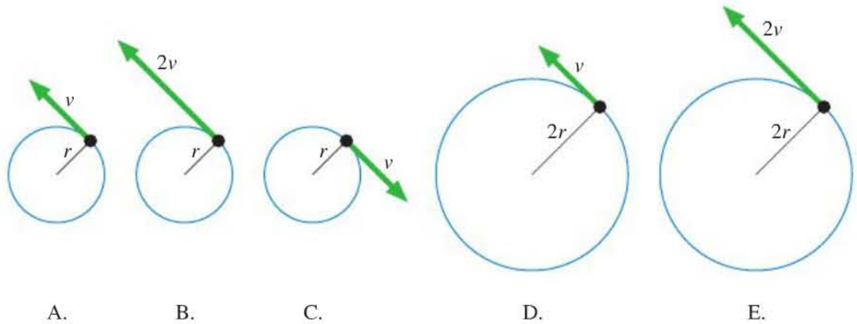
What Comes Next: Forces

So far we have been studying motion without saying too much about what actually *causes* motion. Kinematics, the mathematical description of motion, is a good place to start because motion is very visible and very familiar. And in our study of motion we have introduced many of the basic tools, such as vectors, that we will use in the rest of the book.

But now it's time to look at what causes motion: forces. By learning about forces, which you will do in the next several chapters, you will be able to explore a much wider range of problems in much more depth. As an example, think about the picture of a roller coaster with an inverted loop. How is it that riders can go through the loop and not fall out of their seats? This is just one of the problems that you will study once you know a bit about forces and the connection between forces and motion.

◀ **Amusement park kinematics** Acceleration is fun—at least that's what the designer of this roller coaster seems to think! The coaster has ramps that give linear acceleration, parabolic segments in which the coaster follows a projectile path with a free-fall acceleration, and circular arcs in which the centripetal acceleration is greater than g. All of this acceleration means there are forces on the riders—and the coaster must be carefully designed so that these forces are well within safe limits.

STOP TO THINK 3.6 Which of the following particles has the greatest centripetal acceleration?



INTEGRATED EXAMPLE 3.16 World-record jumpers

Frogs, with their long, strong legs, are excellent jumpers. And thanks to the good folks of Calaveras County, California, who have a jumping frog contest every year in honor of a Mark Twain story, we have very good data as to just how far a determined frog can jump. The current record holder is Rosie the Ribeter, a bullfrog who made a leap of 6.5 m from a standing start. This compares favorably with the world record for a human, which is a mere 3.7 m.

Typical data for a serious leap by a bullfrog look like this: The frog goes into a crouch, then rapidly extends its legs by 15 cm as it pushes off, leaving the ground at an angle of 30° to the horizontal. It's in the air for 0.68 s before landing at the same height from which it took off. Given this leap, what is the acceleration while the frog is pushing off? How far does the frog jump?

PREPARE The problem really has two parts: the leap through the air and the acceleration required to produce this leap. We'll need to analyze the leap—the projectile motion—first, which will give us the frog's launch speed and the distance of the jump. Once we know the velocity with which the frog leaves the ground, we can calculate its acceleration while pushing off the ground. Let's start with a visual overview of the two parts, as shown in **FIGURE 3.43**. Notice that the second part of the problem uses a different x-axis, tilted as we did earlier for motion on a ramp.

SOLVE The "flying through the air" part of Figure 3.43a is projectile motion. The frog lifts off at a 30° angle with a speed v_i; the x- and y-components of the initial velocity are

$$(v_x)_i = v_i \cos(30°)$$
$$(v_y)_i = v_i \sin(30°)$$

The vertical motion can be analyzed as we did in Example 3.12. The kinematic equation is

$$y_f = y_i + (v_y)_i \Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

We know that $y_f = y_i = 0$, so this reduces to

$$(v_y)_i = -\tfrac{1}{2} a_y \Delta t = -\tfrac{1}{2}(-9.8 \text{ m/s}^2)(0.68 \text{ s}) = 3.3 \text{ m/s}$$

We know the y-component of the velocity and the angle, so we can find the magnitude of the velocity and the x-component:

$$v_i = \frac{(v_y)_i}{\sin 30°} = \frac{3.3 \text{ m/s}}{\sin 30°} = 6.6 \text{ m/s}$$

$$(v_x)_i = v_i \cos 30° = (6.6 \text{ m/s}) \cos 30° = 5.7 \text{ m/s}$$

The horizontal motion is uniform motion, so the frog's horizontal position when it returns to the ground is

$$x_f = x_i + (v_x)_i \Delta t = 0 + (5.7 \text{ m/s})(0.68 \text{ s}) = 3.9 \text{ m}$$

This is the length of the jump.

Now that we know how fast the frog is going when it leaves the ground, we can calculate the acceleration necessary to produce this jump—the "pushing off the ground" part of Figure 3.43b. We've drawn the x-axis along the direction of motion, as we did for problems of motion on a ramp. We know the displacement Δx of the jump but not the time, so we can use the third equation in Table 2.4:

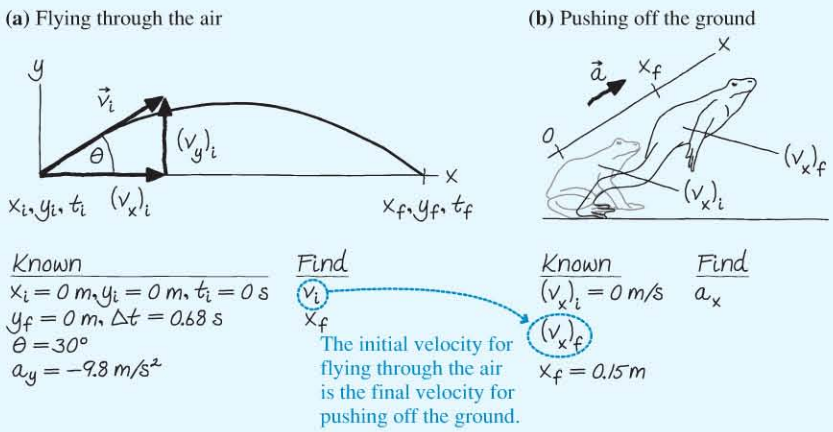
$$(v_x)_f^2 = (v_x)_i^2 + 2a_x \Delta x$$

The initial velocity is zero, the final velocity is $(v_x)_f = 6.6$ m/s, and the displacement is the 15 cm (or 0.15 m) stretch of the legs during the jump. Thus the frog's acceleration while pushing off is

$$a_x = \frac{(v_x)_f^2}{2\Delta x} = \frac{(6.6 \text{ m/s})^2}{2(0.15 \text{ m})} = 150 \text{ m/s}^2$$

ASSESS A 3.9 m jump is more than a human can achieve, but it's less than the record for a frog, so the final result for the distance seems reasonable. Such a long jump must require a large acceleration during the pushing-off phase, which is what we found.

FIGURE 3.43 A visual overview for the leap of a frog.



SUMMARY

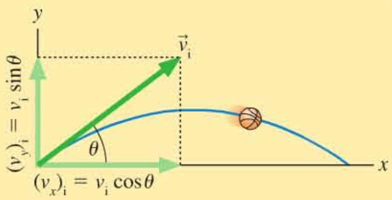
The goals of Chapter 3 have been to learn more about vectors and to use vectors as a tool to analyze motion in two dimensions.

GENERAL PRINCIPLES

Projectile Motion

A projectile is an object that moves through the air under the influence of gravity and nothing else.

The path of the motion is a parabola.



The motion consists of two pieces:

1. Vertical motion with free-fall acceleration, $a_y = -g$.
2. Horizontal motion with constant velocity.

Kinematic equations:

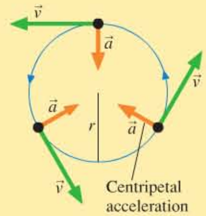
$$x_f = x_i + (v_x)_i\,\Delta t$$
$$(v_x)_f = (v_x)_i = \text{constant}$$
$$y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$
$$(v_y)_f = (v_y)_i - g\,\Delta t$$

Circular Motion

For an object moving in a circle at a constant speed:

- The period T is the time for one rotation.
- The frequency $f = 1/T$ is the number of revolutions per second.
- The velocity is tangent to the circular path.
- The acceleration points toward the center of the circle and has magnitude

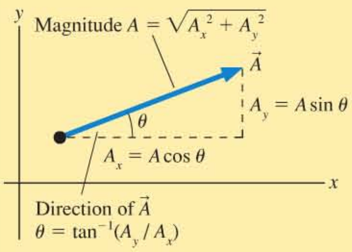
$$a = \frac{v^2}{r}$$



IMPORTANT CONCEPTS

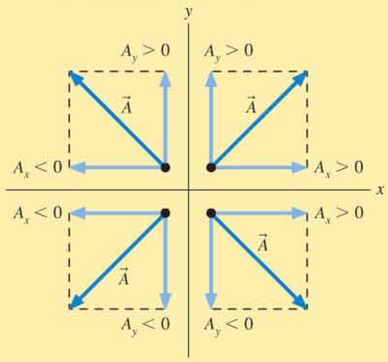
Vectors and Components

A vector can be decomposed into x- and y-**components.**



The magnitude and direction of a vector can be expressed in terms of its components.

The sign of the components depends on the direction of the vector:

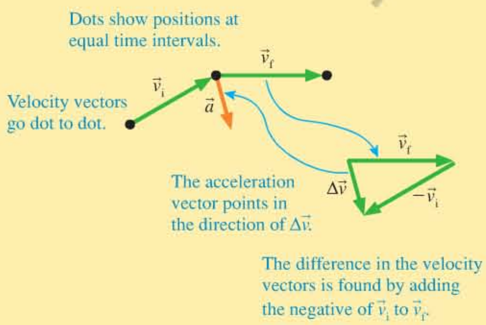


The Acceleration Vector

We define the acceleration vector as

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta\vec{v}}{\Delta t}$$

We find the acceleration vector on a motion diagram as follows:

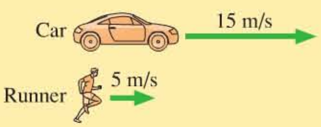


APPLICATIONS

Relative motion

Velocities can be expressed relative to an observer. We can add relative velocities to convert to another observer's point of view.

c = car, r = runner, g = ground



The speed of the car with respect to the runner is:

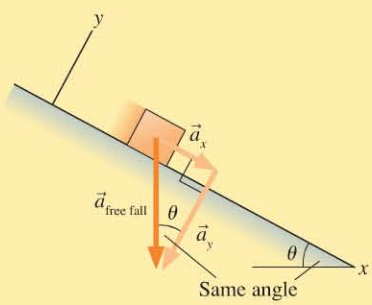
$$(v_x)_{cr} = (v_x)_{cg} + (v_x)_{gr}$$

Motion on a ramp

An object sliding down a ramp will accelerate parallel to the ramp:

$$a_x = \pm g\sin\theta$$

The correct sign depends on the direction in which the ramp is tilted.



MP™ For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. a. Can a vector have nonzero magnitude if a component is zero? If no, why not? If yes, give an example.
   b. Can a vector have zero magnitude and a nonzero component? If no, why not? If yes, give an example.
2. Is it possible to add a scalar to a vector? If so, demonstrate. If not, explain why not.
3. Suppose two vectors have unequal magnitudes. Can their sum be $\vec{0}$? Explain.
4. Suppose $\vec{C} = \vec{A} + \vec{B}$.
   a. Under what circumstances does $C = A + B$?
   b. Could $C = A - B$? If so, how? If not, why not?
5. For a projectile, which of the following quantities are constant during the flight: x, y, v_x, v_y, v, a_x, a_y? Which of the quantities are zero throughout the flight?
6. A baseball player throws a ball at a 40° angle to the ground. The ball lands on the ground some distance away.
   a. Is there any point on the trajectory where $\vec{v}$ and $\vec{a}$ are parallel to each other? If so, where?
   b. Is there any point where $\vec{v}$ and $\vec{a}$ are perpendicular to each other? If so, where?
7. An athlete performing the long jump tries to achieve the maximum distance from the point of takeoff to the first point of touching the ground. After the jump, rather than land upright, she extends her legs forward as in the photo. How does this affect the time in the air? How does this give the jumper a longer range?

8. A person trying to throw a ball as far as possible will run forward during the throw. Explain why this increases the distance of the throw.
9. A passenger on a jet airplane claims to be able to walk at a speed in excess of 500 mph. Can this be true? Explain.
10. If you go to a ski area, you'll likely find that the beginner's slope has the smallest angle. Use the concept of acceleration on a ramp to explain why this is so.
11. In an amusement-park ride, cars rolling along at high speed suddenly head up a long, straight ramp. They roll up the ramp, reverse direction at the highest point, then roll backward back down the ramp. In each of the following segments of the motion, are the cars accelerating, or is their acceleration zero? If accelerating, which way does their acceleration vector point?
    a. As the cars roll up the ramp.
    b. At the highest point on the ramp.
    c. As the cars roll back down the ramp.
12. There are competitions in which pilots fly small planes low over the ground and drop weights, trying to hit a target. A pilot flying low and slow drops a weight; it takes 2.0 s to hit the ground, during which it travels a horizontal distance of 100 m. Now the pilot does a run at the same height but twice the speed. How much time does it take the weight to hit the ground? How far does it travel before it lands?
13. A cyclist goes around a level, circular track at constant speed. Do you agree or disagree with the following statement: "Because the cyclist's speed is constant, her acceleration is zero." Explain.
14. You are driving your car in a circular path on level ground at a constant speed of 20 mph. At the instant you are driving north, and turning left, are you accelerating? If so, toward what point of the compass (N, S, E, W) does your acceleration vector point? If not, why not?
15. An airplane has been directed to fly in a clockwise circle, as seen from above, at constant speed until another plane has landed. When the plane is going north, is it accelerating? If so, in what direction does the acceleration vector point? If not, why not?
16. When you go around a corner in your car, your car follows a path that is a segment of a circle. To turn safely, you should keep your car's acceleration below some safe upper limit. If you want to make a "tighter" turn—that is, turn in a circle with a smaller radius—how should you adjust your speed? Explain.

Multiple-Choice Questions

17. || Which combination of the vectors shown in Figure Q3.17 has the largest magnitude?
    A. $\vec{A} + \vec{B} + \vec{C}$
    B. $\vec{B} + \vec{A} - \vec{C}$
    C. $\vec{A} - \vec{B} + \vec{C}$
    D. $\vec{C} - \vec{A} - \vec{C}$

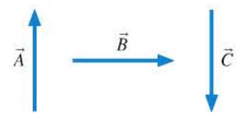


FIGURE Q3.17

18. || Two vectors appear as in Figure Q3.18. Which combination points directly to the left?
    A. $\vec{P} + \vec{Q}$
    B. $\vec{P} - \vec{Q}$
    C. $\vec{Q} - \vec{P}$
    D. $-\vec{Q} - \vec{P}$

FIGURE Q3.18

19. | The gas pedal in a car is sometimes referred to as "the accelerator." Which other controls on the vehicle can be used to produce acceleration?
    A. The brakes.
    B. The steering wheel.
    C. The gear shift.
    D. All of the above.

20. | A car travels at constant speed along the curved path shown from above in Figure Q3.20. Five possible vectors are also shown in the figure; the letter E represents the zero vector. Which vector best represents
   a. The car's *velocity* at position 1?
   b. The car's *acceleration* at point 1?
   c. The car's *velocity* at position 2?
   d. The car's *acceleration* at point 2?
   e. The car's *velocity* at position 3?
   f. The car's *acceleration* at point 3?

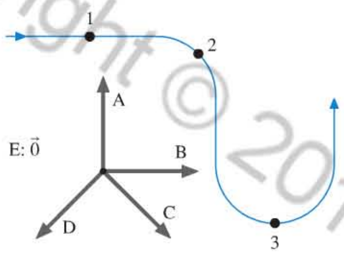


FIGURE Q3.20

21. | A ball is fired from a cannon at point 1 and follows the trajectory shown in Figure Q3.21. Air resistance may be neglected. Five possible vectors are also shown in the figure; the letter E represents the zero vector. Which vector best represents
   a. The ball's *velocity* at position 2?
   b. The ball's *acceleration* at point 2?
   c. The ball's *velocity* at position 3?
   d. The ball's *acceleration* at point 3?

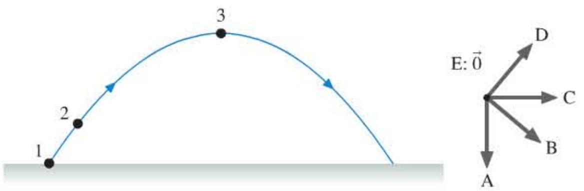


FIGURE Q3.21

22. | A ball thrown at an initial angle of 37.0° and initial velocity of 23.0 m/s reaches a maximum height h, as shown in Figure Q3.22. With what initial speed must a ball be thrown *straight up* to reach the same maximum height h?
   A. 13.8 m/s B. 17.3 m/s
   C. 18.4 m/s D. 23.0 m/s

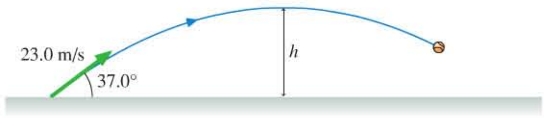


FIGURE Q3.22

23. | A cannon, elevated at 40° is fired at a wall 300 m away on level ground, as shown in Figure Q3.23. The initial speed of the cannonball is 89 m/s

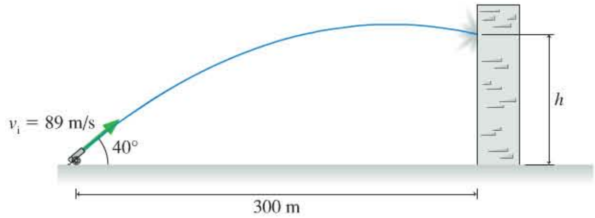


FIGURE Q3.23

   a. How long does it take for the ball to hit the wall?
   A. 1.3 s B. 3.3 s C. 4.4 s
   D. 6.8 s E. 7.2 s
   b. At what height h does the ball hit the wall?
   A. 39 m B. 47 m C. 74 m
   D. 160 m E. 210 m

24. | A car drives horizontally off a 73-m-high cliff at a speed of 27 m/s. Ignore air resistance.
   a. How long will it take the car to hit the ground?
   A. 2.0 s B. 3.2 s C. 3.9 s
   D. 4.9 s E. 5.0 s
   b. How far from the base of the cliff will the car hit?
   A. 74 m B. 88 m C. 100 m
   D. 170 m E. 280 m

25. | A football is kicked at an angle of 30° with a speed of 20 m/s. To the nearest second, how long will the ball stay in the air?
   A. 1 s B. 2 s C. 3 s D. 4 s

26. | A football is kicked at an angle of 30° with a speed of 20 m/s. To the nearest 5 m, how far will the ball travel?
   A. 15 m B. 25 m C. 35 m D. 45 m

27. | Riders on a Ferris wheel move in a circle with a speed of 4.0 m/s. As they go around, they experience a centripetal acceleration of 2.0 m/s^2. What is the diameter of this particular Ferris wheel?
   A. 4.0 m B. 6.0 m C. 8.0 m
   D. 16 m E. 24 m

PROBLEMS

Section 3.1 Using Vectors

1. || Trace the vectors in Figure P3.1 onto your paper. Then use graphical methods to draw the vectors (a) $\vec{A}+\vec{B}$ and (b) $\vec{A}-\vec{B}$.

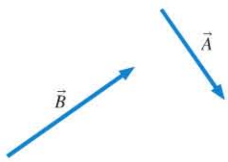


FIGURE P3.1

2. ||| Trace the vectors in Figure P3.2 onto your paper. Then use graphical methods to draw the vectors (a) $\vec{A}+\vec{B}$ and (b) $\vec{A}-\vec{B}$.

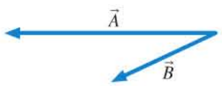


FIGURE P3.2

Section 3.2 Using Vectors on Motion Diagrams

3. | A car goes around a corner in a circular arc at constant speed. Draw a motion diagram including positions, velocity vectors, and acceleration vectors.

4. | a. Is the object's average speed between points 1 and 2 greater than, less than, or equal to its average speed between points 0 and 1? Explain how you can tell.
   b. Find the average acceleration vector at point 1 of the three-point motion diagram in Figure P3.4.

FIGURE P3.4

5. Figure 3.11 showed the motion diagram for Anne as she rode a Ferris wheel that was turning at a constant speed. The inset to the figure showed how to find the acceleration vector at the lowest point in her motion. Use a similar analysis to find Anne's acceleration vector at the 12 o'clock, 4 o'clock, and 8 o'clock positions of the motion diagram. Use a ruler so that your analysis is accurate.

Section 3.3 Coordinate Systems and Vector Components

6. A position vector with magnitude 10 m points to the right and up. Its x-component is 6.0 m. What is the value of its y-component?
7. A velocity vector 40° above the positive x-axis has a y-component of 10 m/s. What is the value of its x-component?
8. Jack and Jill ran up the hill at 3.0 m/s. The horizontal component of Jill's velocity vector was 2.5 m/s.
   a. What was the angle of the hill?
   b. What was the vertical component of Jill's velocity?
9. A cannon tilted upward at 30° fires a cannonball with a speed of 100 m/s. At that instant, what is the component of the cannonball's velocity parallel to the ground?
10. a. What are the x- and y-components of vector $\vec{E}$ of Figure P3.10 in terms of the angle θ and the magnitude E?
    b. For the same vector, what are the x- and y-components in terms of the angle ϕ and the magnitude E?

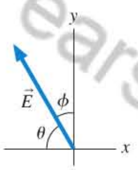


FIGURE P3.10

11. Draw each of the following vectors, then find its x- and y-components.
    a. $\vec{d} = (100\text{ m}, 45° \text{ below } +x\text{-axis})$
    b. $\vec{v} = (300\text{ m/s}, 20° \text{ above } +x\text{-axis})$
    c. $\vec{a} = (5.0\text{ m/s}^2, -y\text{-direction})$
12. Draw each of the following vectors, then find its x- and y-components.
    a. $\vec{d} = (2.0\text{ km}, 30° \text{ left of } +y\text{-axis})$
    b. $\vec{v} = (5.0\text{ cm/s}, -x\text{-direction})$
    c. $\vec{a} = (10\text{ m/s}^2, 40° \text{ left of } -y\text{-axis})$
13. Each of the following vectors is given in terms of its x- and y-components. Draw the vector, label an angle that specifies the vector's direction, then find the vector's magnitude and direction.
    a. $v_x = 20$ m/s, $v_y = 40$ m/s
    b. $a_x = 2.0$ m/s^2, $a_y = -6.0$ m/s^2
14. Each of the following vectors is given in terms of its x- and y-components. Draw the vector, label an angle that specifies the vector's direction, then find the vector's magnitude and direction.
    a. $v_x = 10$ m/s, $v_y = 30$ m/s
    b. $a_x = 20$ m/s^2, $a_y = 10$ m/s^2
15. While visiting England, you decide to take a jog and find yourself in the neighborhood shown on the map in Figure P3.15. What is your displacement after running 2.0 km on Strawberry Fields, 1.0 km on Penny Lane, and 4.0 km on Abbey Road?

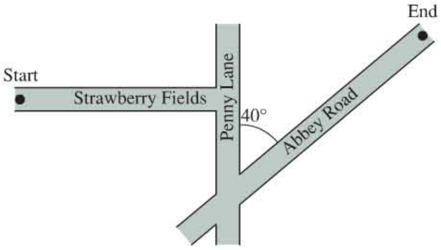


FIGURE P3.15

Section 3.4 Motion on a Ramp

16. You begin sliding down a 15° ski slope. Ignoring friction and air resistance, how fast will you be moving after 10 s?
17. A car traveling at 30 m/s runs out of gas while traveling up a 5.0° slope. How far will it coast before starting to roll back down?
18. In the Soapbox Derby, young participants build non-motorized cars with very low-friction wheels. Cars race by rolling down a hill. The track at Akron's Derby Downs, where the national championship is held, begins with a 55-ft-long section tilted 13° below horizontal.

    a. What is the maximum possible acceleration of a car moving down this stretch of track?
    b. If a car starts from rest and undergoes this acceleration for the full 55 ft, what is its final speed in m/s?
19. A piano has been pushed to the top of the ramp at the back of a moving van. The workers think it is safe, but as they walk away, it begins to roll down the ramp. If the back of the truck is 1.0 m above the ground and the ramp is inclined at 20°, how much time do the workers have to get to the piano before it reaches the bottom of the ramp?
20. Starting from rest, several toy cars roll down ramps of differing lengths and angles. Rank them according to their speed at the bottom of the ramp, from slowest to fastest. Car A goes down a 10 m ramp inclined at 15°, car B goes down a 10 m ramp inclined at 20°, car C goes down an 8.0 m ramp inclined at 20°, and car D goes down a 12 m ramp inclined at 12°.

Section 3.5 Relative Motion

21. Anita is running to the right at 5 m/s, as shown in Figure P3.21. Balls 1 and 2 are thrown toward her at 10 m/s by friends standing on the ground. According to Anita, what is the speed of each ball?

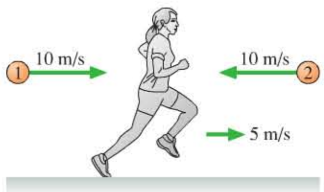


FIGURE P3.21

22. Anita is running to the right at 5 m/s, as shown in Figure P3.22. Balls 1 and 2 are thrown toward her by friends standing on the ground. According to Anita, both balls are approaching her at 10 m/s. According to her friends, with what speeds were the balls thrown?

FIGURE P3.22

23. A boat takes 3.0 h to travel 30 km down a river, then 5.0 h to return. How fast is the river flowing?

24. Two children who are bored while waiting for their flight at the airport decide to race from one end of the 20-m-long moving sidewalk to the other and back. Phillippe runs on the sidewalk at 2.0 m/s (relative to the sidewalk). Renee runs on the floor at 2.0 m/s. The sidewalk moves at 1.5 m/s relative to the floor. Both make the turn instantly with no loss of speed.
   a. Who wins the race?
   b. By how much time does the winner win?

25. A skydiver deploys his parachute when he is 1000 m directly above his desired landing spot. He then falls through the air at a steady 5.0 m/s. There is a breeze blowing to the west at 2.0 m/s.
   a. At what angle with respect to vertical does he fall?
   b. By what distance will he miss his desired landing spot?

Section 3.6 Motion in Two Dimensions: Projectile Motion

Section 3.7 Projectile Motion: Solving Problems

26. An object is launched with an initial velocity of 50.0 m/s at a launch angle of 36.9° above the horizontal.
   a. Make a table showing values of x, y, v_x, v_y, and the speed v every 1 s from $t = 0$ s to $t = 6$ s.
   b. Plot a graph of the object's trajectory during the first 6 s of motion.

27. A ball is thrown horizontally from a 20-m-high building with a speed of 5.0 m/s.
   a. Make a sketch of the ball's trajectory.
   b. Draw a graph of v_x, the horizontal velocity, as a function of time. Include units on both axes.
   c. Draw a graph of v_y, the vertical velocity, as a function of time. Include units on both axes.
   d. How far from the base of the building does the ball hit the ground?

28. A ball with a horizontal speed of 1.25 m/s rolls off a bench 1.00 m above the floor.
   a. How long will it take the ball to hit the floor?
   b. How far from a point on the floor directly below the edge of the bench will the ball land?

29. King Arthur's knights use a catapult to launch a rock from their vantage point on top of the castle wall, 12 m above the moat. The rock is launched at a speed of 25 m/s and an angle of 30° above the horizontal. How far from the castle wall does the launched rock hit the ground?

30. Two spheres are launched horizontally from a 1.0-m-high table. Sphere A is launched with an initial speed of 5.0 m/s. Sphere B is launched with an initial speed of 2.5 m/s.
   a. What are the times for each sphere to hit the floor?
   b. What are the distances that each travels from the edge of the table?

31. A rifle is aimed horizontally at a target 50 m away. The bullet hits the target 2.0 cm below the aim point.
   a. What was the bullet's flight time?
   b. What was the bullet's speed as it left the barrel?

32. BIO A gray kangaroo can bound across a flat stretch of ground with each jump carrying it 10 m from the takeoff point. If the kangaroo leaves the ground at a 20° angle, what are its (a) take-off speed and (b) horizontal speed?

33. On the Apollo 14 mission to the moon, astronaut Alan Shepard hit a golf ball with a golf club improvised from a tool. The free-fall acceleration on the moon is 1/6 of its value on earth. Suppose he hit the ball with a speed of 25 m/s at an angle 30° above the horizontal.
   a. How long was the ball in flight?
   b. How far did it travel?
   c. Ignoring air resistance, how much farther would it travel on the moon than on earth?

Section 3.8 Motion in Two Dimensions: Circular Motion

34. An old-fashioned LP record rotates at $33\frac{1}{3}$ rpm.
   a. What is its frequency, in rev/s?
   b. What is its period, in seconds?

35. A typical hard disk in a computer spins at 5400 rpm.
   a. What is the frequency, in rev/s?
   b. What is the period, in seconds?

36. BIO Racing greyhounds are capable of rounding corners at very high speeds. A typical greyhound track has turns that are 45-m-diameter semicircles. A greyhound can run around these turns at a constant speed of 15 m/s. What is its acceleration in m/s^2 and in units of g?

37. A CD-ROM drive in a computer spins the 12-cm-diameter disks at 10,000 rpm.
   a. What are the disk's period (in s) and frequency (in rev/s)?
   b. What would be the speed of a speck of dust on the outside edge of this disk?
   c. What is the acceleration in units of g that this speck of dust experiences?

38. BIO To withstand "g-forces" of up to 10 g's, caused by suddenly pulling out of a steep dive, fighter jet pilots train on a "human centrifuge." 10 g's is an acceleration of 98 m/s^2. If the length of the centrifuge arm is 12 m, at what speed is the rider moving when she experiences 10 g's?

39. A particle rotates in a circle with centripetal acceleration $a = 8.0\ m/s^2$. What is a if
   a. The radius is doubled without changing the particle's speed?
   b. The speed is doubled without changing the circle's radius?

40. Entrance and exit ramps for freeways are often circular stretches of road. As you go around one at a constant speed, you will experience a constant acceleration. Suppose you drive through an entrance ramp at a modest speed and your acceleration is 3.0 m/s^2. What will be the acceleration if you double your speed?

41. BIO A peregrine falcon in a tight, circular turn can attain a centripetal acceleration 1.5 times the free-fall acceleration. If the falcon is flying at 20 m/s, what is the radius of the turn?

General Problems

42. Suppose $\vec{C} = \vec{A} + \vec{B}$ where vector $\vec{A}$ has components $A_x = 5$, $A_y = 2$ and vector $\vec{B}$ has components $B_x = -3$, $B_y = -5$.
   a. What are the x- and y-components of vector $\vec{C}$?
   b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.
   c. What are the magnitude and direction of vector $\vec{C}$?

43. Suppose $\vec{D} = \vec{A} - \vec{B}$ where vector $\vec{A}$ has components $A_x = 5$, $A_y = 2$ and vector $\vec{B}$ has components $B_x = -3$, $B_y = -5$.
   a. What are the x- and y-components of vector $\vec{D}$?
   b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{D}$.
   c. What are the magnitude and direction of vector $\vec{D}$?
44. Suppose $\vec{E} = 2\vec{A} + 3\vec{B}$ where vector $\vec{A}$ has components $A_x = 5$, $A_y = 2$ and vector $\vec{B}$ has components $B_x = -3$, $B_y = -5$.
   a. What are the x- and y-components of vector $\vec{E}$?
   b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{E}$.
   c. What are the magnitude and direction of vector $\vec{E}$?
45. For the three vectors shown in Figure P3.45, the vector sum $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ has components $D_x = 2$ and $D_y = 0$.
   a. What are the x- and y-components of vector $\vec{B}$?
   b. Write $\vec{B}$ as a magnitude and a direction.

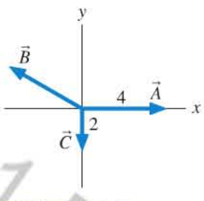


FIGURE P3.45

46. Let $\vec{A} = (3.0\text{ m, }20°\text{ south of east})$, $\vec{B} = (2.0\text{ m, north})$, and $\vec{C} = (5.0\text{ m, }70°\text{ south of west})$.
   a. Draw and label $\vec{A}$, $\vec{B}$, and $\vec{C}$ with their tails at the origin. Use a coordinate system with the x-axis to the east.
   b. Write the x- and y-components of vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.
   c. Find the magnitude and the direction of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$.
47. A typical set of stairs is angled at 38°. You climb a set of stairs at a speed of 3.5 m/s.
   a. How much height will you gain in 2.0 s?
   b. How much horizontal distance will you cover in 2.0 s?
48. The minute hand on a watch is 2.0 cm long. What is the displacement vector of the tip of the minute hand
   a. From 8:00 to 8:20 A.M.?
   b. From 8:00 to 9:00 A.M.?
49. A field mouse trying to escape a hawk runs east for 5.0 m, darts southeast for 3.0 m, then drops 1.0 m down a hole into its burrow. What is the magnitude of the net displacement of the mouse?
50. A pilot in a small plane encounters shifting winds. He flies 26.0 km northeast, then 45.0 km due north. From this point, he flies an additional distance in an unknown direction, only to find himself at a small airstrip that his map shows to be 70.0 km directly north of his starting point. What were the length and direction of the third leg of his trip?
51. A small plane is 100 km south of the equator. The plane is flying at 150 km/h at a heading of 30° to the west of north. In how many minutes will the plane cross the equator?
52. BIO The bacterium *Escherichia coli* (or *E. coli*) is a single-celled organism that lives in the gut of healthy humans and animals. When grown in a uniform medium rich in salts and amino acids, these bacteria swim along zig-zag paths at a constant speed of 20 μm/s. Figure P3.52 shows the trajectory of an *E. coli* as it moves from point A to point E. Each segment of the motion can be identified by two letters, such as segment BC.
   a. For each of the four segments in the bacterium's trajectory, calculate the x- and y-components of its displacement and of its velocity.
   b. Calculate both the total distance traveled and the magnitude of the net displacement for the entire motion.
   c. What are the magnitude and the direction of the bacterium's average velocity for the entire trip?

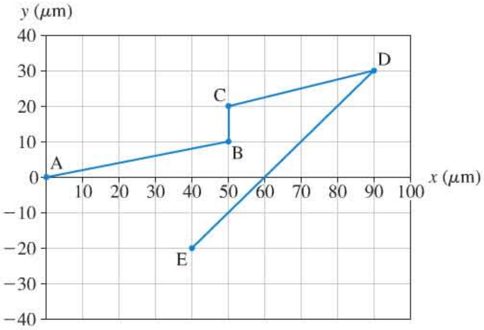


FIGURE P3.52

53. A skier is gliding along at 3.0 m/s on horizontal, frictionless snow. He suddenly starts down a 10° incline. His speed at the bottom is 15 m/s.
   a. What is the length of the incline?
   b. How long does it take him to reach the bottom?
54. A block slides along the frictionless track shown in Figure P3.54 with an initial speed of 5.0 m/s. Assume it turns all the corners smoothly, with no loss of speed.

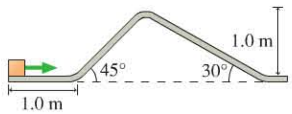


FIGURE P3.54

   a. What is the block's speed as it goes over the top?
   b. What is its speed when it reaches the level track on the right side?
   c. By what percentage does the block's final speed differ from its initial speed? Is this surprising?
55. One game at the amusement park has you push a puck up a long, frictionless ramp. You win a stuffed animal if the puck, at its highest point, comes to within 10 cm of the end of the ramp without going off. You give the puck a push, releasing it with a speed of 5.0 m/s when it is 8.5 m from the end of the ramp. The puck's speed after traveling 3.0 m is 4.0 m/s. Are you a winner?
56. When the moving sidewalk at the airport is broken, as it often seems to be, it takes you 50 s to walk from your gate to the baggage claim. When it is working and you stand on the moving sidewalk the entire way, without walking, it takes 75 s to travel the same distance. How long will it take you to travel from the gate to baggage claim if you walk while riding on the moving sidewalk?
57. Ships A and B leave port together. For the next two hours, ship A travels at 20 mph in a direction 30° west of north while ship B travels 20° east of north at 25 mph.
   a. What is the distance between the two ships two hours after they depart?
   b. What is the speed of ship A as seen by ship B?
58. Mary needs to row her boat across a 100-m-wide river that is flowing to the east at a speed of 1.0 m/s. Mary can row the boat with a speed of 2.0 m/s relative to the water.
   a. If Mary rows straight north, how far downstream will she land?
   b. Draw a picture showing Mary's displacement due to rowing, her displacement due to the river's motion, and her net displacement.
59. A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 12 m/s. A wise elder duck finally realizes that the solution is to fly at an angle to the wind. If the ducks can fly at 16 m/s relative to the air, in what direction should they head in order to move directly south?

60. ||| A kayaker needs to paddle north across a 100-m-wide harbor. The tide is going out, creating a tidal current flowing east at 2.0 m/s. The kayaker can paddle with a speed of 3.0 m/s.
   a. In which direction should he paddle in order to travel straight across the harbor?
   b. How long will it take him to cross?

61. |||| A plane has an airspeed of 200 mph. The pilot wishes to reach a destination 600 mi due east, but a wind is blowing at 50 mph in the direction 30° north of east.
   a. In what direction must the pilot head the plane in order to reach her destination?
   b. How long will the trip take?

62. ||| The Gulf Stream off the east coast of the United States can flow at a rapid 3.6 m/s to the north. A ship in this current has a cruising speed of 10 m/s. The captain would like to reach land at a point due west from the current position.
   a. In what direction with respect to the water should the ship sail?
   b. At this heading, what is the ship's speed with respect to land?

63. || A physics student on Planet Exidor throws a ball, and it follows the parabolic trajectory shown in Figure P3.63. The ball's position is shown at 1.0 s intervals until $t = 3.0$ s. At $t = 1.0$ s, the ball's velocity has components $v_x = 2.0$ m/s, $v_y = 2.0$ m/s.

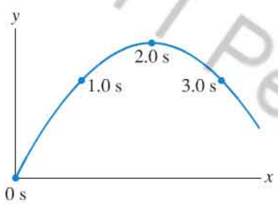


FIGURE P3.63

   a. Determine the x- and y-components of the ball's velocity at $t = 0.0$ s, 2.0 s, and 3.0 s.
   b. What is the value of g on Planet Exidor?
   c. What was the ball's launch angle?

64. ||| A ball thrown horizontally at 25 m/s travels a horizontal distance of 50 m before hitting the ground. From what height was the ball thrown?

65. ||| BIO In 1780, in what is now referred to as "Brady's Leap," Captain Sam Brady of the U.S. Continental Army escaped certain death from his enemies by running over the edge of the cliff above Ohio's Cuyahoga River, which is confined at that spot to a gorge. He landed safely on the far side of the river. It was reported that he leapt 22 ft across while falling 20 ft.

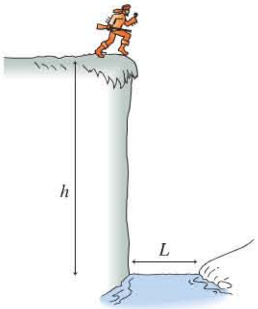


FIGURE P3.65

   a. Representing the distance jumped as L and the vertical drop as h, as shown in Figure P3.65, derive an expression for the minimum speed v he would need to make his leap if he ran straight off the cliff.
   b. Evaluate your expression for a 22 ft jump with a 20 ft drop to the other side.
   c. Is it reasonable that a person could make this leap? Use the fact that the world record for the 100 m dash is approximately 10 s to estimate the maximum speed such a runner would have.

66. ||| The longest recorded pass in an NFL game traveled 83 yards in the air from the quarterback to the receiver. Assuming that the pass was thrown at the optimal 45° angle, what was the speed at which the ball left the quarterback's hand?

67. ||| A spring-loaded gun, fired vertically, shoots a marble 6.0 m straight up in the air. What is the marble's range if it is fired horizontally from 1.5 m above the ground?

68. || In a shot-put event, an athlete throws the shot with an initial speed of 12.0 m/s at a 40.0° angle from the horizontal. The shot leaves her hand at a height of 1.80 m above the ground.
   a. How far does the shot travel?
   b. Repeat the calculation of part (a) for angles 42.5°, 45.0°, and 47.5°. Put all your results, including 40.0°, in a table. At what angle of release does she throw the farthest?

69. ||| A tennis player hits a ball 2.0 m above the ground. The ball leaves his racquet with a speed of 20 m/s at an angle 5.0° above the horizontal. The horizontal distance to the net is 7.0 m, and the net is 1.0 m high. Does the ball clear the net? If so, by how much? If not, by how much does it miss?

70. |||| Water at the top of Horseshoe Falls (part of Niagara Falls) is moving horizontally at 9.0 m/s as it goes off the edge and plunges 53 m to the pool below. If you ignore air resistance, at what angle is the falling water moving as it enters the pool?

71. ||| Figure 3.37 shows that the range of a projectile launched at a 60° angle has the same range as a projectile launched at a 30° angle—but they won't be in the air for the same amount of time. Suppose a projectile launched at a 30° angle is in the air for 2.0 s. How long will the projectile be in the air if it is launched with the same speed at a 60° angle?

72. || A supply plane needs to drop a package of food to scientists working on a glacier in Greenland. The plane flies 100 m above the glacier at a speed of 150 m/s. How far short of the target should it drop the package?

73. ||| A child slides down a frictionless 3.0-m-long playground slide tilted upward at an angle of 40°. At the end of the slide, there is an additional section that curves so that the child is launched off the end of the slide horizontally.
   a. How fast is the child moving at the bottom of the slide?
   b. If the end of the slide is 0.40 m above the ground, how far from the end does she land?

74. ||| INT A sports car is advertised to be able to "reach 60 mph in 5 seconds flat, corner at $0.85g$, and stop from 70 mph in only 168 feet."
   a. In which of those three situations is the magnitude of the car's acceleration the largest? In which is it the smallest?
   b. At 60 mph, what is the smallest turning radius that this car can navigate?

75. || INT A Ford Mustang can accelerate from 0 to 60 mph in a time of 5.6 s. A Mini Cooper isn't capable of such a rapid start, but it can turn in a very small circle 34 ft in diameter. How fast would you need to drive the Mini Cooper in this tight circle to match the magnitude of the Mustang's acceleration?

76. || The "Screaming Swing" is a carnival ride that is—not surprisingly—a giant swing. It's actually two swings moving in opposite directions. At the bottom of its arc, riders are moving at 30 m/s with respect to the ground in a 50-m-diameter circle.
   a. What is the acceleration, in m/s^2 and in units of g, that riders experience?
   b. At the bottom of the ride, as they pass each other, how fast do the riders move with respect to each other?

77. || On an otherwise straight stretch of road near Moffat, Colorado, the road suddenly turns. This bend in the road is a segment of a circle with radius 110 m. Drivers are cautioned to slow down to 40 mph as they navigate the curve.
   a. If you heed the sign and slow to 40 mph, what will be your acceleration going around the curve at this constant speed? Give your answer in m/s^2 and in units of g.
   b. At what speed would your acceleration be double that at the recommended speed?

Passage Problems

Riding the Water Slide

A rider on a water slide goes through three different kinds of motion, as illustrated in Figure P3.78. Use the data and details from the figure to answer the following questions.

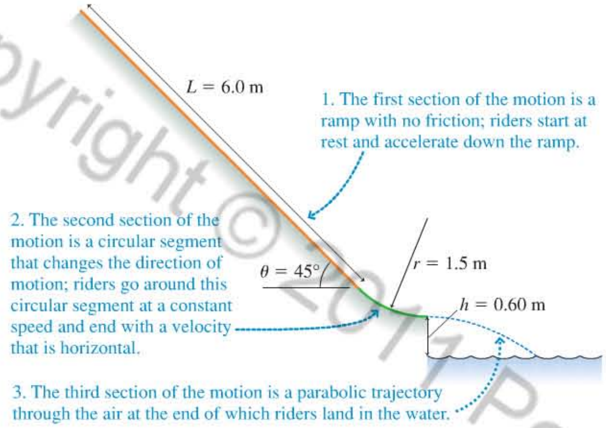


FIGURE P3.78

78. | At the end of the first section of the motion, riders are moving at what approximate speed?
A. 3 m/s B. 6 m/s C. 9 m/s D. 12 m/s

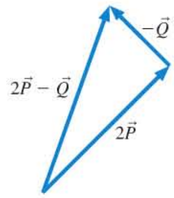
79. | Suppose the acceleration during the second section of the motion is too large to be comfortable for riders. What change could be made to decrease the acceleration during this section?
A. Reduce the radius of the circular segment.
B. Increase the radius of the circular segment.
C. Increase the angle of the ramp.
D. Increase the length of the ramp.

80. | What is the vertical component of the velocity of a rider as he or she hits the water?
A. 2.4 m/s B. 3.4 m/s C. 5.2 m/s D. 9.1 m/s

81. | Suppose the designers of the water slide want to adjust the height h above the water so that riders land twice as far away from the bottom of the slide. What would be the necessary height above the water?
A. 1.2 m B. 1.8 m C. 2.4 m D. 3.0 m

82. | During which section of the motion is the magnitude of the acceleration experienced by a rider the greatest?
A. The first. B. The second.
C. The third. D. It is the same in all sections.

STOP TO THINK ANSWERS



Stop to Think 3.1: A. The graphical construction of $2\vec{P} - \vec{Q}$ is shown at right.

Stop to Think 3.2: From the axes on the graph, we can see that the x- and y-components are -4 cm and $+2$ cm, respectively.

Stop to Think 3.3: C. Vector $\vec{C}$ points to the left and down, so both C_x and C_y are negative. C_x is in the numerator because it is the side opposite ϕ.

Stop to Think 3.4: B. The angle of the slope is greatest in this case, leading to the greatest acceleration.

Stop to Think 3.5: D. Mass does not appear in the kinematic equations, so the mass has no effect; the balls will follow the same path.

Stop to Think 3.6: B. The magnitude of the acceleration is v^2/r. Acceleration is largest for the combination of highest speed and smallest radius.