18 Ray Optics

These thin beams of light at this laser show are well described using ray optics. How do these light beams behave when they reflect from shiny surfaces or pass through transparent materials?

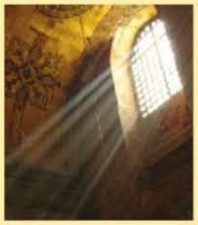
LOOKING AHEAD ➤➤

The goal of Chapter 18 is to understand and apply the ray model of light.

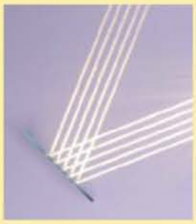
The Ray Model of Light

The ray model applies when light interacts with objects that are large compared to its wavelength. It describes how shadows are formed and how mirrors and lenses work. You'll learn that ...

Looking Back ◀◀
17.1 Models of light



... light rays travel in straight lines unless they are ...



... *reflected* by a surface or ...

... *refracted* at the surface of a transparent material.

Reflection

Light rays can bounce, or **reflect**, off of a surface. The ray model explains what we see in two important cases.

Light rays reflecting from a smooth surface, such as that of still water, can form an image of objects.

Light rays reflect in all directions when they strike a rough surface such as paper, allowing us to see the page.

Refraction

When light rays travel from one transparent material to another, they can change direction, or **refract**, at the interface.

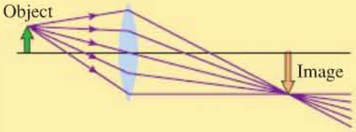
Looking Back ◀◀
17.1 Index of refraction

A diamond sparkles as light reflects and refracts at its many facets.

Light refracting at the water's surface is responsible for the two images of the turtle.

Images Formed by Lenses and Mirrors

You'll learn how lenses form images, starting with a graphical method—**ray tracing**—to find image positions, sizes, and orientations. We'll then develop the **thin-lens equation** for more accurate results.



We can use ray tracing to show how this lens creates a **real image** on the opposite side of the lens from the object.

The **virtual image** seen through this lens is on the same side of the lens as the object.

Curved mirrors also can be used to create images. Again, we'll use both graphical and mathematical methods to understand image formation by mirrors.

Your passenger-side rearview mirror is curved, allowing you to see a wider field of view.

18.1 The Ray Model of Light

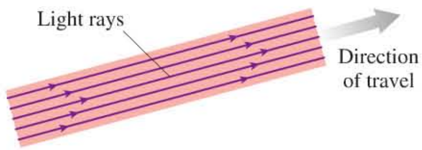
A flashlight makes a beam of light through the night's darkness. Sunbeams stream into a darkened room through a small hole in the shade, and laser beams are even more well defined. Our everyday experience that light travels in straight lines is the basis of the ray model of light.

The ray model is an oversimplification of reality, but nonetheless is very useful within its range of validity. As we saw in Chapter 17, diffraction and other wave aspects of light are important only for apertures and objects comparable in size to the wavelength of light. Because the wavelength is so small, typically 0.5 μm, the wave nature of light is not apparent when light interacts with ordinary-sized objects. The ray model of light, which ignores diffraction, is valid as long as any apertures through which the light passes (lenses, mirrors, holes, and the like) are larger than about 1 mm.

To begin, let us define a **light ray** as a line in the direction along which light energy is flowing. A light ray is an abstract idea, not a physical entity or a "thing." Any narrow beam of light, such as the laser beam in **FIGURE 18.1**, is actually a bundle of many parallel light rays. You can think of a single light ray as the limiting case of a laser beam whose diameter approaches zero. Laser beams are good approximations of light rays, certainly adequate for demonstrating ray behavior, but any real laser beam is a bundle of many parallel rays.

The following table outlines five basic ideas and assumptions of the ray model of light.

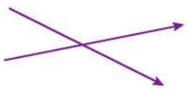
FIGURE 18.1 A laser beam is a bundle of parallel light rays.



The ray model of light

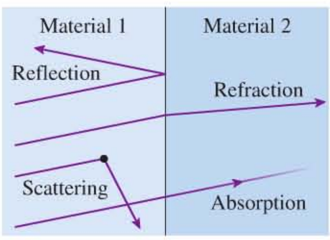
Light rays travel in straight lines.

Light travels through a vacuum or a transparent material in straight lines called light rays. The speed of light in a material is $v = c/n$, where n is the index of refraction of the material.



Light rays can cross.

Light rays do not interact with each other. Two rays can cross without either being affected in any way.



A light ray travels forever unless it interacts with matter.

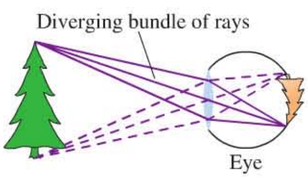
A light ray continues forever unless it has an interaction with matter that causes the ray to change direction or to be absorbed. Light interacts with matter in four different ways:

- At an interface between two materials, light can be *reflected, refracted,* or both.
- Within a material, light can be either *scattered* or *absorbed.*

These interactions are discussed later in the chapter.

An object is a source of light rays.

An **object** is a source of light rays. Rays originate from *every* point on the object, and each point sends rays in *all* directions. Objects may be self-luminous—they create light rays—or they may be reflective objects that only reflect rays that originate elsewhere.



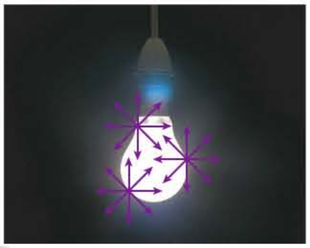
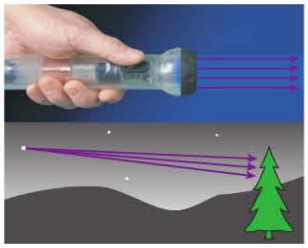
The eye sees by focusing a bundle of rays.

The eye sees an object when *diverging* bundles of rays from each point on the object enter the pupil and are focused to an image on the retina. Imaging is discussed later in the chapter, and the eye will be treated in much greater detail in Chapter 19.

Sources of Light Rays

In the ray model, there are two kinds of objects. **Self-luminous objects** (or *sources*) directly create light rays. Self-luminous objects include lightbulbs and the sun. Other objects, such as a piece of paper or a tree, are **reflective objects** that reflect rays originating in self-luminous objects. The table below shows four important kinds of self-luminous sources. Note that although ray and point sources are idealizations, they are useful in understanding the propagation of rays.

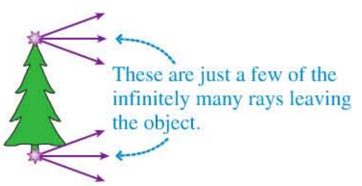
Self-luminous objects

A ray source	A point source	An extended source	A parallel-ray source
			
Since a light ray is an idealization, there are no true ray sources. Still, the thin beam of a laser is often a good approximation of a single ray.	A point source is also an idealized source of light. It is infinitely small and emits light rays in every direction. The tiny filaments of these bulbs approximate point sources.	This is the most common light source. The *entire surface* of an extended source is luminous, so that **every point of an extended source acts as a point source.** Lightbulbs, flames, and the sun are extended sources.	Certain sources, such as flashlights and movie projectors, produce a bundle of parallel rays. Rays from a very distant object, such as a star, are very nearly parallel.

Reflective objects, such as a newspaper, a face, or a mirror, can also be considered as sources of light rays. However, the origin of these rays is not in the object itself. Instead, light rays from a self-luminous object strike a reflective object and "bounce" off of it. These rays can then illuminate other objects, or enter our eyes and form images of the reflective object, just as rays from self-luminous objects do.

Ray Diagrams

FIGURE 18.2 A ray diagram simplifies the situation by showing only a few rays.



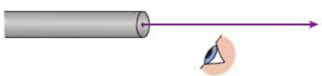
Rays originate from *every* point on an object and travel outward in *all* directions, but a diagram trying to show all these rays would be hopelessly messy and confusing. To simplify the picture, we usually use a **ray diagram** that shows only a few rays. For example, **FIGURE 18.2** is a ray diagram showing only a few rays leaving the top and bottom points of the object and traveling to the right. These rays will be sufficient to show us how the object is imaged by lenses or mirrors.

NOTE ► Ray diagrams are the basis for a *visual overview* that we'll use throughout this chapter. Be careful not to think that a ray diagram shows all of the rays. The rays shown on the diagram are just a subset of the infinitely many rays leaving the object. ◄

Seeing Objects

FIGURE 18.3 A laser beam traveling through air is invisible.

You can't see a laser beam crossing the room because no light ray enters your eye.



How do we *see* an object? The eye works by focusing an image of an object on the retina, a process we'll examine in Chapter 19. For now, we'll ignore the details of image formation and instead make use of a simpler fact from the figure in the ray model table above: **In order for our eye to see an object, rays from that object must enter the eye.** This idea helps explain some subtle points about seeing.

Consider a ray source such as a laser. Can you see a laser beam traveling across the room? Under ordinary circumstances, the answer is no. As we've seen, a laser beam is a good approximation of a single ray. This ray travels in a straight line from the laser to whatever it eventually strikes. As **FIGURE 18.3** shows, no light ray enters the eye, so the beam is invisible. The same argument holds for a parallel-ray source.

A point source and an extended source behave differently, as shown in FIGURE 18.4. Because a point source emits rays in *every* direction, some of these rays will enter the eye no matter where it is located. Thus a point source is visible to everyone looking at it. And, since every point on the surface of an extended source is itself a point source, all parts of an extended source (not blocked by something else) can be viewed by all observers as well.

We can also use our simple model of seeing to explain how we see non-luminous objects. As we've already mentioned, such objects *reflect* rays that strike them. Most ordinary objects—paper, skin, grass—reflect incident light in every direction, a process called **diffuse reflection.** FIGURE 18.5 illustrates the idea. Single rays are broken into many weaker rays that leave in all directions, a process called **scattering.**

Scattered light is what allows you to read a book by lamplight. As shown in Figure 18.5, every point on the surface of the page is struck by a ray (or rays) from the lamp. Then, because of diffuse reflection, these rays scatter in every direction; some of the scattered rays reach your eye, allowing you to see the page.

It is possible to make a laser beam visible by scattering it from very small particles suspended in air. These particles can be dust, smoke, or water droplets such as fog. FIGURE 18.6 shows that as the beam strikes such a particle, it scatters rays in every direction. Some of these rays enter the eye, making each particle in the path of the beam visible and outlining the beam's path across the room.

FIGURE 18.4 Point and extended sources can be seen by all observers.

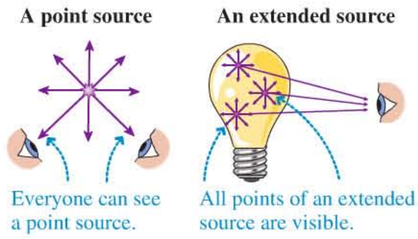


FIGURE 18.5 Reading a book by scattered light.

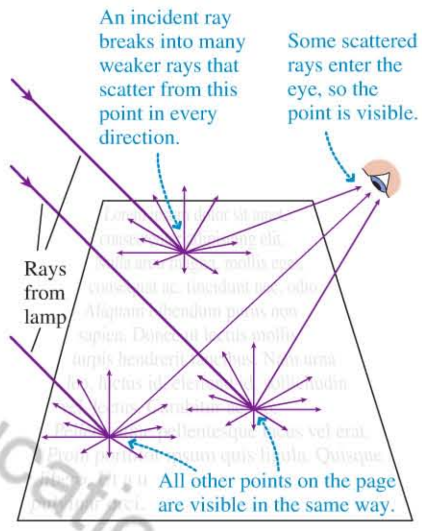
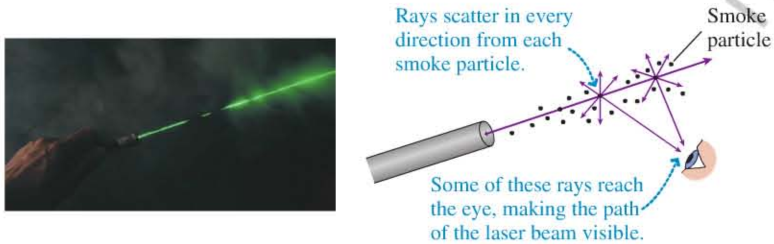


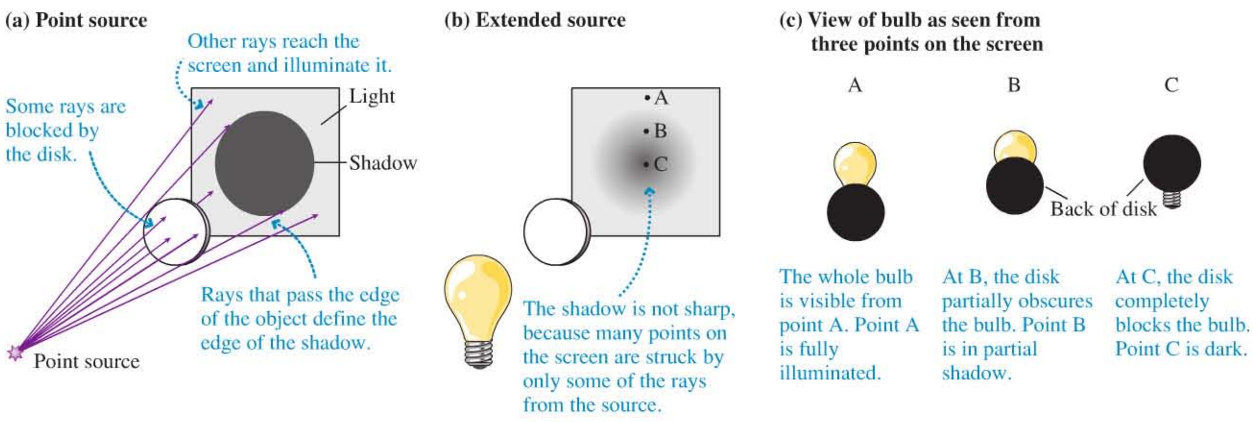
FIGURE 18.6 A laser beam is visible if it travels through smoke or dust.



Shadows

Our ray model of light also explains the common phenomenon of *shadows.* Suppose an opaque object (such as a cardboard disk) is placed between a source of light and a screen. The object intercepts some of the rays, leaving a dark area behind it. Other rays travel on to a screen and illuminate it. The simplest shadows are those cast by a point source of light. This process is shown in FIGURE 18.7a. With a point source, the shadow is completely dark, and the edges of the shadow are sharp.

FIGURE 18.7 Shadows produced by point and extended sources of light.



During a solar eclipse, the sun—a small but extended source—casts a shadow of the moon on the earth. The moon's shadow has a dark center surrounded by a region of increasing brightness, just as in Figure 18.7b.

Shadows cast by extended sources are more complicated. An extended source is a collection of a large number of point sources, each of which casts its own shadow. However, as shown in FIGURE 18.7b, the patterns of shadow and light from each point overlap and thus the shadow region is no longer sharp. FIGURE 18.7c shows the view of the *bulb* as seen from three points on the screen. Depending on the size of the source, there is often a true shadow that no light reaches, surrounded by a fuzzy region of increasing brightness.

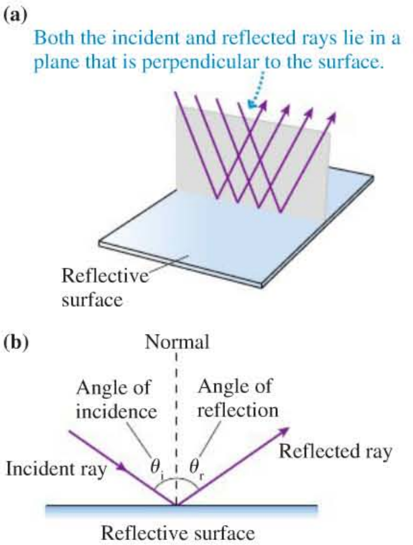
18.2 Reflection

Reflection of light is a familiar, everyday experience. You see your reflection in the bathroom mirror first thing every morning, reflections in your car's rearview mirror as you drive to school, and the sky reflected in puddles of standing water. Reflection from a smooth, shiny surface, such as a mirror or a piece of polished metal, is called **specular reflection.**

FIGURE 18.8a shows a bundle of parallel light rays reflecting from a mirror-like surface. You can see that the incident and reflected rays are both in a plane that is normal, or perpendicular, to the reflective surface. A three-dimensional perspective accurately shows the relation between the light rays and the surface, but figures such as this are hard to draw by hand. Instead, it is customary to represent reflection with the simpler visual overview of FIGURE 18.8b. In this figure,

- The incident and reflected rays are in the plane of the page. The reflective surface extends into and out of the page.
- A *single* light ray represents the entire bundle of parallel rays. This is oversimplified, but it keeps the figure and the analysis clear.

FIGURE 18.8 Specular reflection of light.



The angle θ_i between the incident ray and a line perpendicular to the surface—the *normal* to the surface—is called the **angle of incidence.** Similarly, the **angle of reflection** θ_r is the angle between the reflected ray and the normal to the surface. The **law of reflection,** easily demonstrated with simple experiments, states that

1. The incident ray and the reflected ray are both in the same plane, which is perpendicular to the surface, and
2. The angle of reflection equals the angle of incidence: $\theta_r = \theta_i$.

NOTE ► Optics calculations *always* use the angle measured from the normal, not the angle between the ray and the surface. ◄

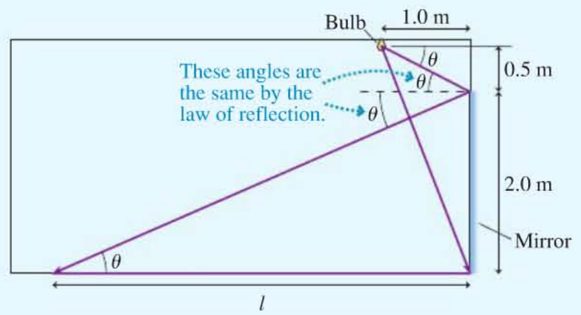
Reflection is an everyday experience.

EXAMPLE 18.1 Light reflecting from a mirror

A full-length mirror on a closet door is 2.0 m tall. The bottom touches the floor. A bare lightbulb hangs 1.0 m from the closet door, 0.5 m above the top of the mirror. How long is the streak of reflected light across the floor?

PREPARE Treat the lightbulb as a point source and use the ray model of light. FIGURE 18.9 is a visual overview of the light rays. We need to consider only the two rays that strike the edges of the mirror. All other reflected rays will fall between these two.

FIGURE 18.9 Visual overview of the light rays reflecting from a mirror.



SOLVE The ray that strikes the bottom of the mirror reflects from it and hits the floor just where the mirror meets the floor. For the top ray, Figure 18.9 has used the law of reflection to set the angle of reflection equal to the angle of incidence; we call both θ. By simple geometry, the other angles shown are also equal to θ. From the small triangle at the upper right,

$$\theta = \tan^{-1}\left(\frac{0.5\text{ m}}{1.0\text{ m}}\right) = 26.6°$$

But we also have $\tan\theta = (2.0\text{ m})/l$, or

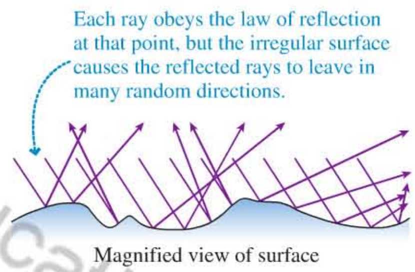
$$l = \frac{2.0\text{ m}}{\tan\theta} = \frac{2.0\text{ m}}{\tan 26.6°} = 4.0\text{ m}$$

Since the lower ray struck right at the mirror's base, the total length of the reflected streak is 4.0 m.

Diffuse Reflection

We've already discussed diffuse reflection, the reflection of light rays off of a surface such as paper or cloth that is not shiny like a mirror. If you magnify the surface of a diffuse reflector, you'll find that on the microscopic scale it is quite rough. The law of reflection $\theta_r = \theta_i$ is still obeyed at each point, but the irregularities of the surface cause the reflected rays to leave in many random directions. This situation is shown in **FIGURE 18.10**. Diffuse reflection is actually much more common than the mirror-like specular reflection.

FIGURE 18.10 Diffuse reflection from an irregular surface.



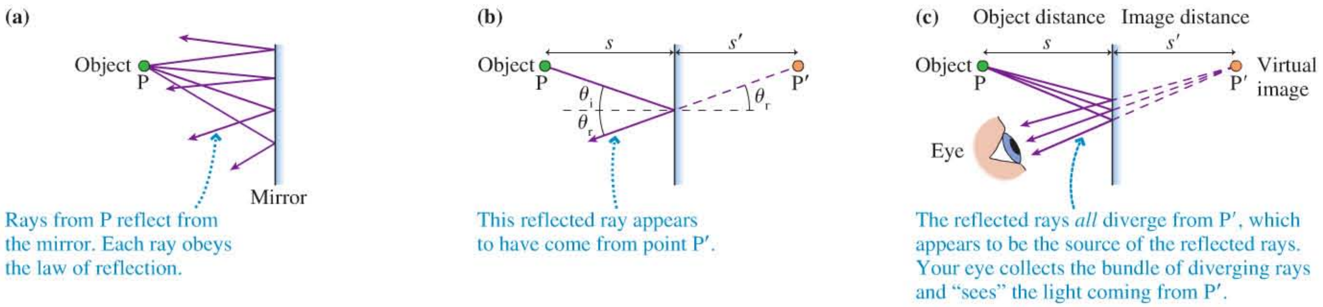
ActivPhysics ONLINE 15.4

The Plane Mirror

One of the most commonplace observations is that you can see yourself in a mirror. How? **FIGURE 18.11a** shows rays from point source P reflecting from a flat mirror, called a **plane mirror.** Consider the particular ray shown in **FIGURE 18.11b**. The reflected ray travels along a line that passes through point P′ on the "back side" of the mirror. Because $\theta_r = \theta_i$, simple geometry dictates that P′ is the same distance behind the mirror as P is in front of the mirror. That is, the **image distance** s' is equal to the **object distance** s:

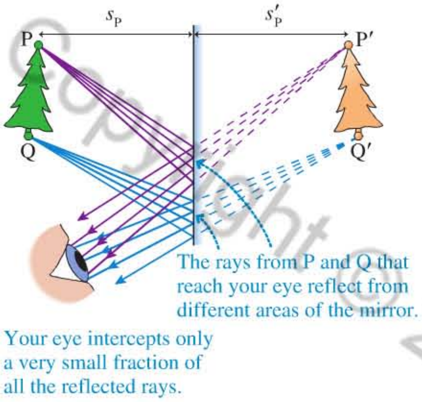
$$s' = s \qquad \text{(plane mirror)} \qquad (18.1)$$

FIGURE 18.11 The light rays reflecting from a plane mirror.



The reflected ray in Figure 18.11b appears to have come from point P′. But because our argument applies to any incoming ray, all reflected rays appear to be coming from point P′, as **FIGURE 18.11c** shows. We call P′, the point from which the reflected rays diverge, the **virtual image** of P. The image is "virtual" in the sense that no rays actually leave P′, which is in darkness behind the mirror. But as far as your eye is concerned, the light rays act exactly *as if* the light really originated at P′. So while you may say "I see P in the mirror," what you are actually seeing is the virtual image of P.

FIGURE 18.12 Each point on the extended object has a corresponding image point an equal distance on the opposite side of the mirror.



A floating image The student stands on his leg that is hidden behind the mirror. Because every point of his body to the left of the mirror has its image to the right, when he raises his other leg he appears to float above the floor.

For an extended object, such as the one in FIGURE 18.12, each point on the object has a corresponding image point an equal distance on the opposite side of the mirror. The eye captures and focuses diverging bundles of rays from each point of the image in order to see the full image in the mirror. Two facts are worth noting:

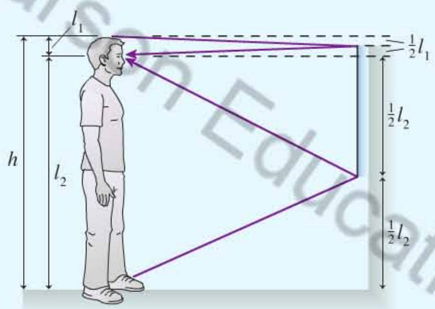
1. Rays from each point on the object spread out in all directions and strike *every point* on the mirror. Only a very few of these rays enter your eye, but the other rays are very real and might be seen by other observers.
2. Rays from points P and Q enter your eye after reflecting from *different* areas of the mirror. This is why you can't always see the full image of an object in a very small mirror.

EXAMPLE 18.2 **How high is the mirror?**

If your height is h, what is the shortest mirror on the wall in which you can see your full image? Where must the top of the mirror be hung?

PREPARE Use the ray model of light. FIGURE 18.13 is a visual overview of the light rays. We need to consider only the two rays that leave the top of your head and your feet and reflect into your eye.

FIGURE 18.13 Visual overview of light rays from your head and feet reflecting into your eye.

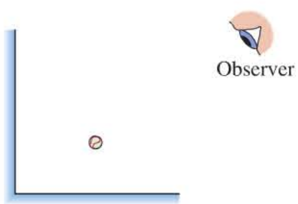


SOLVE Let the distance from your eyes to the top of your head be l_1 and the distance to your feet be l_2. Your height is $h = l_1 + l_2$. A light ray from the top of your head that reflects from the mirror at $\theta_r = \theta_i$ and enters your eye must, by congruent triangles, strike the mirror a distance $\frac{1}{2}l_1$ above your eyes. Similarly, a ray from your foot to your eye strikes the mirror a distance $\frac{1}{2}l_2$ below your eyes. The distance between these two points on the mirror is $\frac{1}{2}l_1 + \frac{1}{2}l_2 = \frac{1}{2}h$. A ray from anywhere else on your body will reach your eye if it strikes the mirror between these two points. Pieces of the mirror outside these two points are irrelevant, not because rays don't strike them but because the reflected rays don't reach your eye. Thus the shortest mirror in which you can see your full reflection is $\frac{1}{2}h$. But this will work only if the top of the mirror is hung midway between your eyes and the top of your head.

ASSESS It is interesting that the answer does not depend on how far you are from the mirror.

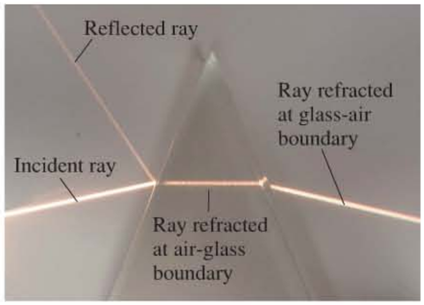
STOP TO THINK 18.1 Two plane mirrors form a right angle. How many images of the ball can you see in the mirrors?

A. 1
B. 2
C. 3
D. 4



18.3 Refraction

FIGURE 18.14 A light beam refracts twice in passing through a glass prism.



In FIGURE 18.14, two things happen when a light ray crosses the boundary between the air and the glass:

1. Part of the light *reflects* from the boundary, obeying the law of reflection. This is how you see reflections from pools of water or storefront windows, even though water and glass are transparent.
2. Part of the light continues into the second medium. It is *transmitted* rather than reflected, but the transmitted ray changes direction as it crosses the boundary. The transmission of light from one medium to another, but with a change in direction, is called **refraction.**

In Figure 18.14, notice the refraction of the light beam as it passes through the prism. Notice also that the ray direction changes as the light enters and leaves the glass. You can also see a weak reflection leaving the first surface of the prism.

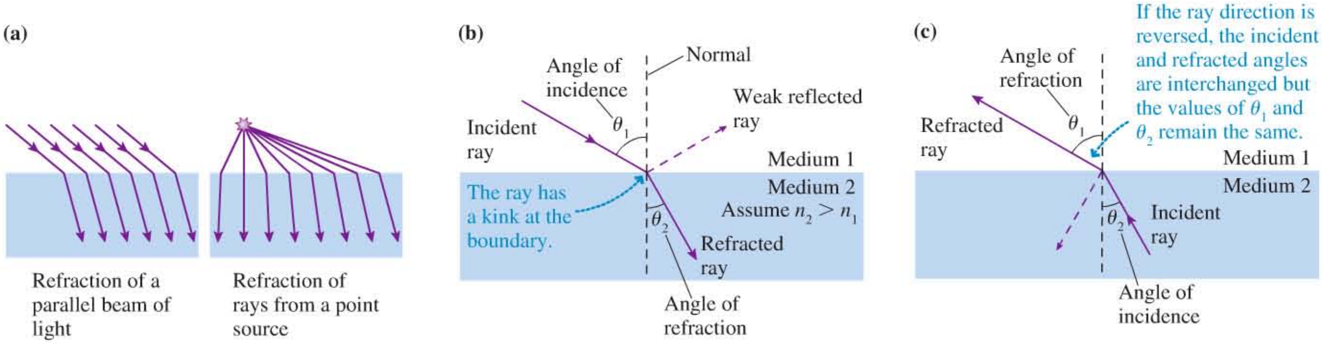
Reflection from the boundary between transparent media is usually weak. Typically 95% of the light is transmitted and only 5% is reflected. Our goal in this section is to understand refraction, so we will usually ignore the weak reflection and focus on the transmitted light.

NOTE ► The transparent material through which light travels is called the *medium* (plural *media*). ◄

ActivPhysics Online 15.1, 15.2, 15.3

FIGURE 18.15a shows the refraction of light rays from a parallel beam of light, such as a laser beam, and rays from a point source. These pictures remind us that an infinite number of rays are incident on the boundary, but our analysis will be simplified if we focus on a single light ray. FIGURE 18.15b is a ray diagram showing the refraction of a single ray at a boundary between medium 1 and medium 2. Let the angle between the ray and the normal be θ_1 in medium 1 and θ_2 in medium 2. Just as for reflection, the angle between the incident ray and the normal is the *angle of incidence*. The angle on the transmitted side, *measured from the normal,* is called the **angle of refraction.** Notice that θ_1 is the angle of incidence in Figure 18.15b but is the angle of refraction in FIGURE 18.15c, where the ray is traveling in the opposite direction.

FIGURE 18.15 Refraction of light rays.



Refraction was first studied experimentally by the Arab scientist Ibn al-Haitham in about the year 1000. His work arrived in Europe six hundred years later, where it influenced the Dutch scientist Willebrord Snell. In 1621, Snell proposed a mathematical statement of the "law of refraction" or, as we know it today, Snell's law. If a ray refracts between medium 1 and medium 2, having indices of refraction n_1 and n_2, the ray angles θ_1 and θ_2 in the two media are related by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{18.2}$$

Snell's law for refraction between two media

Notice that Snell's law does not mention which is the incident angle and which the refracted angle.

TABLE 18.1 Indices of refraction

Medium	n
Vacuum	1 exactly
Air (actual)	1.0003
Air (accepted)*	1.00
Water	1.33
Ethyl alcohol	1.36
Oil	1.46
Glass (typical)	1.50
Polystyrene plastic	1.59
Cubic zirconia	2.18
Diamond	2.42
Silicon (infrared)	3.50

*Use this value in problems.

Table 18.1 lists the indices of refraction for several media. It is interesting to note that the n in the law of refraction, Equation 18.2, is the same index of refraction n we studied in Chapter 17. There we found that the index of refraction determines the speed of a light wave in a medium according to $v = c/n$. Here, it appears to play a different role, determining by how much a light ray is bent when crossing the boundary between two different media. Although we won't do so here, it is possible to use Huygens' principle to show that Snell's law is a *consequence* of the change in the speed of light as it moves across a boundary between media.

Examples of Refraction

Look back at Figure 18.15. As the ray in Figure 18.15b moves from medium 1 to medium 2, where $n_2 > n_1$, it bends closer to the normal. In Figure 18.15c, where the ray moves from medium 2 to medium 1, it bends away from the normal. This is a general conclusion that follows from Snell's law:

- When a ray is transmitted into a material with a higher index of refraction, it bends to make a smaller angle with the normal.
- When a ray is transmitted into a material with a lower index of refraction, it bends to make a larger angle with the normal.

This rule becomes a central idea in a procedure for analyzing refraction problems.

TACTICS BOX 18.1 **Analyzing refraction** MP™

1. **Draw a ray diagram.** Represent the light beam with one ray.
2. **Draw a line normal (perpendicular) to the boundary.** Do this at each point where the ray intersects a boundary.
3. **Show the ray bending in the correct direction.** The angle is larger on the side with the smaller index of refraction. This is the qualitative application of Snell's law.
4. **Label angles of incidence and refraction.** Measure all angles from the normal.
5. **Use Snell's law.** Calculate the unknown angle or unknown index of refraction.

Exercises 10–13

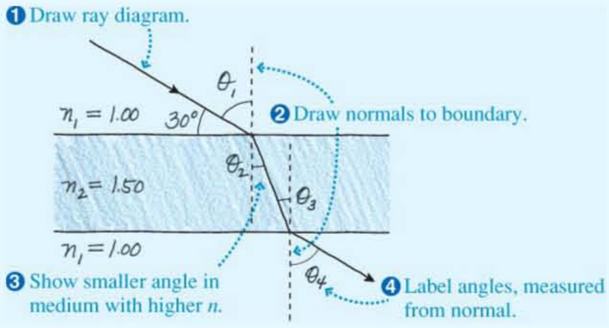
EXAMPLE 18.3 **Deflecting a laser beam**

A laser beam is aimed at a 1.0-cm-thick sheet of glass at an angle 30° above the glass.

a. What is the laser beam's direction of travel in the glass?
b. What is its direction in the air on the other side of the glass?

PREPARE Represent the laser beam with a single ray and use the ray model of light. **FIGURE 18.16** is a visual overview in which the first four steps of Tactics Box 18.1 have been identified. Notice that the angle of incidence must be measured from the normal, so $\theta_1 = 60°$, not the 30° value given in the problem. The index of refraction of glass was taken from Table 18.1.

FIGURE 18.16 The ray diagram of a laser beam passing through a sheet of glass.



SOLVE

a. Snell's law, the final step in the Tactics Box, is $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Using $\theta_1 = 60°$, we find that the direction of travel in the glass is

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 60°}{1.5}\right)$$
$$= \sin^{-1}(0.577) = 35.3°$$

or 35° to two significant figures.

b. Snell's law at the second boundary is $n_2 \sin\theta_3 = n_1 \sin\theta_4$. You can see from Figure 18.16 that the interior angles are equal: $\theta_3 = \theta_2 = 35.3°$. Thus the ray emerges back into the air traveling at angle

$$\theta_4 = \sin^{-1}\left(\frac{n_2 \sin\theta_3}{n_1}\right) = \sin^{-1}(1.5 \sin 35.3°)$$
$$= \sin^{-1}(0.867) = 60°$$

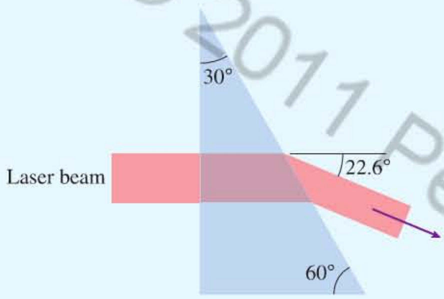
This is the same as θ_1, the original angle of incidence.

ASSESS As expected, the laser beam bends toward the normal as it moves into the higher-index glass, and away from the normal as it moves back into air. The beam exits the glass still traveling in the same direction as it entered, but its path is *displaced.* This is a general result for light traveling through a medium with parallel sides. As the glass becomes thinner, the displacement becomes less; there is no displacement as the glass thickness becomes zero. This will be an important observation when we later study lenses.

EXAMPLE 18.4 Measuring the index of refraction

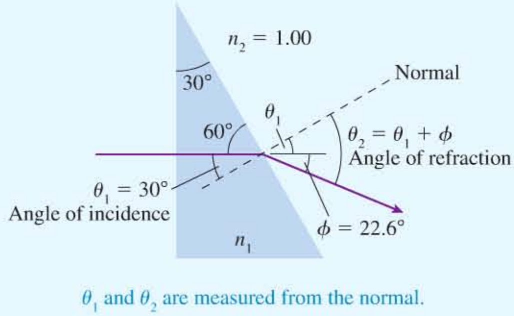
FIGURE 18.17 shows a laser beam deflected by a 30°-60°-90° prism. What is the prism's index of refraction?

FIGURE 18.17 A prism deflects a laser beam.



PREPARE Represent the laser beam with a single ray and use the ray model of light. **FIGURE 18.18** uses the steps of Tactics Box 18.1 to draw a ray diagram. The ray is incident perpendicular to the front face of the prism ($\theta_i = 0°$); thus it is transmitted through the first boundary without deflection. At the second boundary it is especially important to *draw the normal to the surface* at the point of incidence and to *measure angles from the normal.*

FIGURE 18.18 Visual overview of a laser beam passing through the prism.

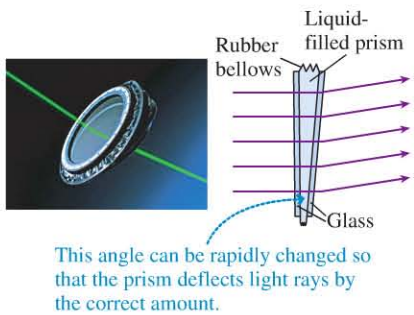


SOLVE From the geometry of the triangle you can find that the laser's angle of incidence on the hypotenuse of the prism is $\theta_1 = 30°$, the same as the apex angle of the prism. The ray exits the prism at angle θ_2 such that the deflection is $\phi = \theta_2 - \theta_1 = 22.6°$. Thus $\theta_2 = 52.6°$. Knowing both angles and $n_2 = 1.00$ for air, we can use Snell's law to find n_1:

$$n_1 = \frac{n_2 \sin\theta_2}{\sin\theta_1} = \frac{1.00 \sin 52.6°}{\sin 30°} = 1.59$$

ASSESS Referring to the indices of refraction in Table 18.1, we see that the prism is made of polystyrene plastic.

▶ **Optical image stabilization** When you make a video with a handheld video camera, the inevitable movement of your hands shows up as unwanted motion in the video itself. High-end video cameras largely eliminate this motion using *optical image stabilization.* The angle of an internal liquid-filled prism is automatically adjusted so that rays entering the prism are refracted at the second glass plate to always hit the camera's light sensor in the correct spot.



Total Internal Reflection

What would have happened in Example 18.4 if the prism angle had been 45° rather than 30°? The light rays would approach the rear surface of the prism at an angle of incidence $\theta_1 = 45°$. When we try to calculate the angle of refraction at which the ray emerges into the air, we find

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1.59}{1.00}\sin 45° = 1.12$$

$$\theta_2 = \sin^{-1}(1.12) = ???$$

Angle θ_2 cannot be computed because the sine of an angle can't be greater than 1. The ray is unable to refract through the boundary. Instead, 100% of the light *reflects* from the boundary back into the prism. This process is called **total internal reflection,** often abbreviated TIR. That it really happens is illustrated in **FIGURE 18.19**. Here, three light beams strike the surface of the water at increasing angles of incidence. The

FIGURE 18.19 One of the three beams of light undergoes total internal reflection.

FIGURE 18.20 Refraction and reflection of rays as the angle of incidence increases.

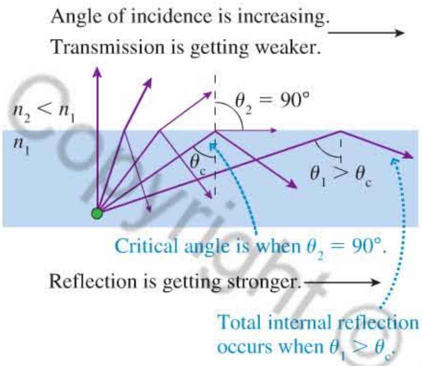
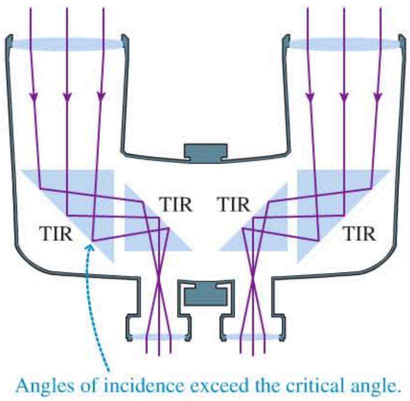


FIGURE 18.21 Binoculars and other optical instruments make use of total internal reflection (TIR).



two beams with the smallest angles of incidence refract out of the water, but the beam with the largest angle of incidence undergoes total internal reflection at the water's surface.

FIGURE 18.20 shows several rays leaving a point source in a medium with index of refraction n_1. The medium on the other side of the boundary has $n_2 < n_1$. As we've seen, crossing a boundary into a material with a lower index of refraction causes the ray to bend away from the normal. Two things happen as angle θ_1 increases. First, the refraction angle θ_2 approaches 90°. Second, the fraction of the light energy that is transmitted decreases while the fraction reflected increases.

A **critical angle** θ_c is reached when $\theta_2 = 90°$. Snell's law becomes $n_1 \sin\theta_c = n_2 \sin 90°$, or

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{18.3}$$

Critical angle of incidence for total internal reflection

The refracted light vanishes at the critical angle and the reflection becomes 100% for any angle $\theta_1 \geq \theta_c$. The critical angle is well defined because of our assumption that $n_2 < n_1$. **There is no critical angle and no total internal reflection if $n_2 > n_1$.**

We can compute the critical angle in a typical piece of glass at the glass-air boundary as

$$\theta_{\text{c glass}} = \sin^{-1}\left(\frac{1.00}{1.50}\right) = 42°$$

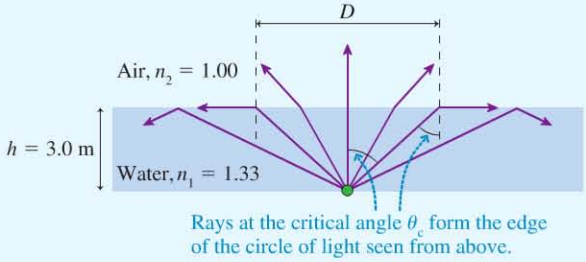
The fact that the critical angle is smaller than 45° has important applications. For example, **FIGURE 18.21** shows a pair of binoculars. The lenses are much farther apart than your eyes, so the light rays need to be brought together before exiting the eyepieces. Rather than using mirrors, which get dirty, are easily scratched, and require alignment, binoculars use a pair of prisms on each side. Thus the light undergoes two TIRs and emerges from the eyepiece. (The actual prism arrangement in binoculars is a bit more complex, but this illustrates the basic idea.)

EXAMPLE 18.5 Seeing a submerged light

A lightbulb is set in the bottom of a 3.0-m-deep swimming pool. What is the diameter of the circle inside which a duck swimming on the surface could see the bulb?

PREPARE Represent the lightbulb as a point source and use the ray model of light. **FIGURE 18.22** is a visual overview of the light rays. The lightbulb emits rays at all angles, but only some of the rays refract into the air where they can be seen from above. Rays striking the surface at greater than the critical angle undergo TIR back down into the water. The diameter of the circle of light is the distance D between the two points at which rays strike the surface at the critical angle.

FIGURE 18.22 Visual overview of the rays leaving a lightbulb at the bottom of a swimming pool.

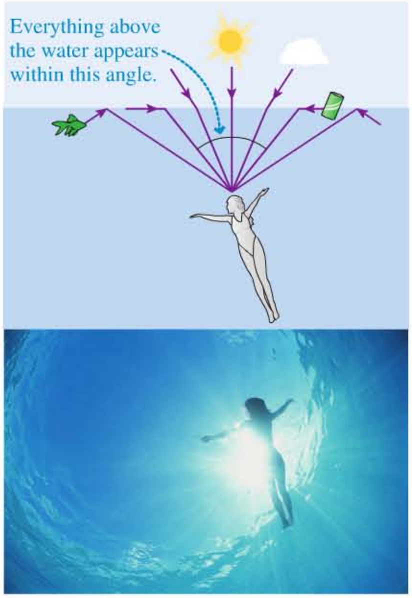


SOLVE From trigonometry, the circle diameter is $D = 2h\tan\theta_c$, where h is the depth of the water. The critical angle for a water-air boundary is $\theta_c = \sin^{-1}(1.00/1.33) = 48.7°$. Thus

$$D = 2(3.0\text{ m})\tan 48.7° = 6.8\text{ m}$$

ASSESS Light rays emerging at the edge of the circle actually skim the surface of the water. By reversing the direction of the rays, we can understand what a diver sees when she's underwater. This idea is explored further in the discussion on the next page.

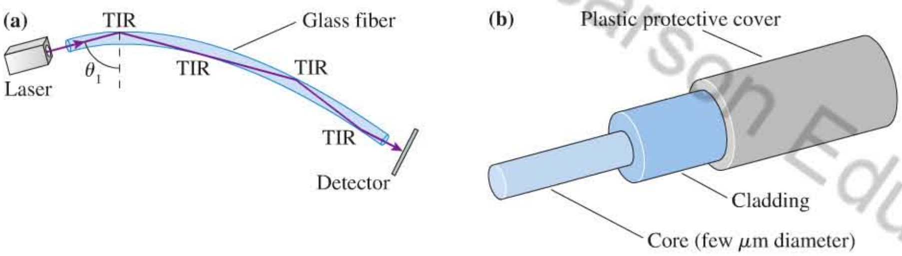
▶ Snell's window We can understand what a diver sees when she's underwater by reversing the direction of all the rays in Figure 18.22 in the preceding example. The drawing shows that she can see the sun overhead, and clouds at larger angles. She can even see objects sitting at the waterline—but they appear at the edge of a circle as she looks up. Anything outside of this circle is a reflection of something in the water. The photo shows what she sees: a bright circle from the sky above—*Snell's window*—surrounded by the dark reflection of the water below.



Fiber Optics

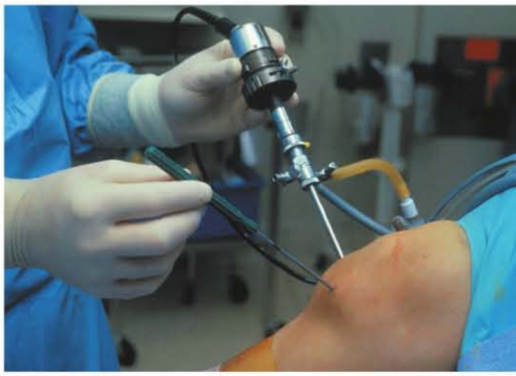
The most important modern application of total internal reflection is the transmission of light through optical fibers. **FIGURE 18.23a** shows a laser beam shining into the end of a long, narrow-diameter glass fiber. The light rays pass easily from the air into the glass, but they then strike the inside wall of the fiber at an angle of incidence θ_1 approaching 90°. This is much larger than the critical angle, so the laser beam undergoes TIR and remains inside the glass. The laser beam continues to "bounce" its way down the fiber as if the light were inside a pipe. Indeed, optical fibers are sometimes called "light pipes." The rays have an angle of incidence *smaller* than the critical angle ($\theta_1 \approx 0$) when they finally reach the flat end of the fiber; thus they refract out without difficulty and can be detected.

FIGURE 18.23 Light rays are confined within an optical fiber by total internal reflection.



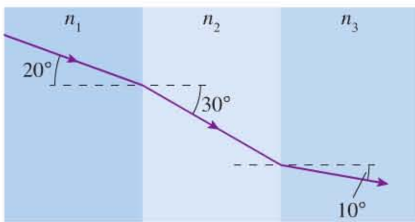
While a simple glass fiber can transmit light, reliance on a glass-air boundary is not sufficiently reliable for commercial use. Any small scratch on the side of the fiber alters the rays' angle of incidence and allows leakage of light. **FIGURE 18.23b** shows the construction of a practical optical fiber. A small-diameter glass *core* is surrounded by a layer of glass *cladding*. The glasses used for the core and the cladding have $n_{core} > n_{cladding}$. Thus, light undergoes TIR at the core-cladding boundary and remains confined within the core. This boundary is not exposed to the environment and hence retains its integrity even under adverse conditions.

Optical fibers have found important applications in medical diagnosis and treatment. Thousands of small fibers can be fused together to make a flexible bundle capable of transmitting high-resolution images along its length. Such *endoscopes* are used for minimally invasive inspection of body cavities, joints, and internal organs. One end of the endoscope bundle has a lens that allows an image of what's in front of it to be sent up the bundle where it can be observed on a television monitor by the physician. Optical fibers are also used to send light *down* the bundle to provide illumination, and small instruments such as forceps and clamps can be operated along its length to retrieve samples for biopsy.



Arthroscopic surgery BIO Operations on injured joints can often be performed using an endoscope inserted through a small incision. The endoscope allows the surgeon to observe the procedure, which is performed with instruments inserted through another incision. The recovery time for such surgery is usually much shorter than for conventional operations requiring a full incision to expose the interior of the joint.

STOP TO THINK 18.2 A light ray travels from medium 1 to medium 3 as shown. For these media,

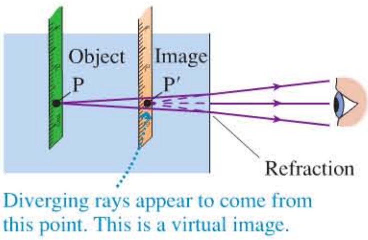


A. $n_3 > n_1$
B. $n_3 = n_1$
C. $n_3 < n_1$
D. We can't compare n_1 to n_3 without knowing n_2.

FIGURE 18.24 Refraction causes an object in an aquarium to appear closer than it really is.

(a) A ruler in an aquarium

(b) Finding the image of the ruler



18.4 Image Formation by Refraction

FIGURE 18.24a shows a photograph of a ruler as seen through the front of an aquarium tank. The part of the ruler below the waterline appears *closer* than the part that is above water. FIGURE 18.24b shows why this is so. Rays that leave point P on the ruler refract away from the normal at the water-air boundary. (The thin glass wall of the aquarium has little effect on the refraction of the rays and can be ignored.) To your eye, outside the aquarium, these rays appear to diverge not from the object at point P, but instead from point P′ that is *closer* to the boundary. The same argument holds for every point on the ruler, so that the **ruler appears closer than it really is because of refraction of light at the boundary.**

We found that the rays reflected from a mirror diverge from a point that is not the object point. We called that point a *virtual image.* Similarly, if rays from an object point P refract at a boundary between two media such that the rays then diverge from a point P′ and *appear* to come from P′, we call P′ a virtual image of point P. The virtual image of the ruler is what you see.

Let's examine this image formation a bit more carefully. FIGURE 18.25 shows a boundary between two transparent media having indices of refraction n_1 and n_2. Point P, a source of light rays, is the object. Point P′, from which the rays *appear* to diverge, is the virtual image of P. The figure assumes $n_1 > n_2$, but this assumption isn't necessary. Distance s, measured from the boundary, is the object distance. Our goal is to determine the image distance s'.

The line through the object and perpendicular to the boundary is called the **optical axis.** Consider a ray that leaves the object at angle θ_1 with respect to the optical axis. θ_1 is also the angle of incidence at the boundary, where the ray refracts into the second medium at angle θ_2. By tracing the refracted ray backward, you can see that θ_2 is also the angle between the refracted ray and the optical axis at point P′.

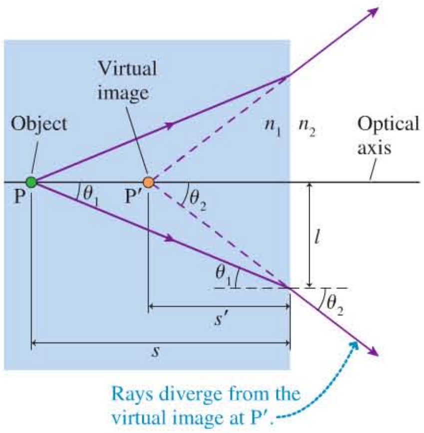
The distance l is common to both the incident and the refracted rays, and you can see that $l = s\tan\theta_1 = s'\tan\theta_2$. Thus

$$s' = \frac{\tan\theta_1}{\tan\theta_2}s \tag{18.4}$$

Snell's law relates the sines of angles θ_1 and θ_2; that is,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \tag{18.5}$$

FIGURE 18.25 Finding the virtual image P′ of an object at P.



In practice, the angle between any of these rays and the optical axis is very small because the pupil of your eye is very much smaller than the distance between the object and your eye. (The angles in the figure have been greatly exaggerated.) The small-angle approximation $\sin\theta \approx \tan\theta \approx \theta$, where θ is in radians, is therefore applicable. Consequently,

$$\frac{\tan\theta_1}{\tan\theta_2} \approx \frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \tag{18.6}$$

Using this result in Equation 18.4, we find that the image distance is

$$s' = \frac{n_2}{n_1}s \tag{18.7}$$

NOTE ▶ The fact that the result for s' is independent of θ_1 implies that *all* rays appear to diverge from the same point P′. This property of the diverging rays is essential in order to have a well-defined image. ◀

This section has given us a first look at image formation via refraction. We will extend this idea to image formation with lenses in the next section.

EXAMPLE 18.6 **An air bubble in a window**

A fish and a sailor look at each other through a 5.0-cm-thick glass porthole in a submarine. There happens to be a small air bubble right in the center of the glass. How far behind the surface of the glass does the air bubble appear to the fish? To the sailor?

PREPARE Represent the air bubble as a point source and use the ray model of light. Light rays from the bubble refract into the air on one side and into the water on the other. The ray diagram looks like Figure 18.25.

SOLVE The index of refraction of the glass is $n_1 = 1.50$. The bubble is in the center of the window, so the object distance from either side of the window is $s = 2.5$ cm. From the water side, the fish sees the bubble at an image distance

$$s' = \frac{n_2}{n_1}s = \frac{1.33}{1.50}(2.5 \text{ cm}) = 2.2 \text{ cm}$$

This is the apparent depth of the bubble. The sailor, in air, sees the bubble at an image distance

$$s' = \frac{n_2}{n_1}s = \frac{1.00}{1.50}(2.5 \text{ cm}) = 1.7 \text{ cm}$$

ASSESS The image distance is *shorter* for the sailor because of the *larger* difference between the two indices of refraction.

18.5 Thin Lenses: Ray Tracing

A **lens** is a transparent material that uses refraction of light rays at *curved* surfaces to form an image. In this section we want to establish a pictorial method of understanding image formation. This method is called **ray tracing.** We will defer a mathematical analysis of the image formation by lenses until the next section.

FIGURE 18.26 Converging and diverging lenses.

(a) Converging lenses, which are thicker in the center than at the edges, refract parallel rays toward the optical axis.

(b) Diverging lenses, which are thinner in the center than at the edges, refract parallel rays away from the optical axis.

FIGURE 18.26 shows parallel light rays entering two different lenses. The lens in Figure 18.26a, called a **converging lens,** causes the rays to refract *toward* the optical axis. **FIGURE 18.27** shows how, for a converging lens, a ray refracts toward the optical axis at both the first, air-to-glass boundary and the second, glass-to-air boundary. The common point through which initially parallel rays pass is called the **focal point** of the lens. The distance of the focal point from the lens is called the **focal length** f of the lens. The lens in Figure 18.26b, called a **diverging lens,** refracts parallel rays *away from* the optical axis. This lens also has a focal point, but it is not as obvious in the figure.

FIGURE 18.28 on the next page clarifies the situation. In the case of a diverging lens, a backward projection of the diverging rays shows that they all *appear* to have started from the same point. This is the focal point of a diverging lens, and its distance from the lens is the focal length of the lens. For both types of lenses, **the focal length is the distance from the lens to the point at which rays parallel to the optical axis converge or from which they appear to diverge.**

FIGURE 18.27 Both surfaces of a converging lens bend an incident ray toward the optical axis.

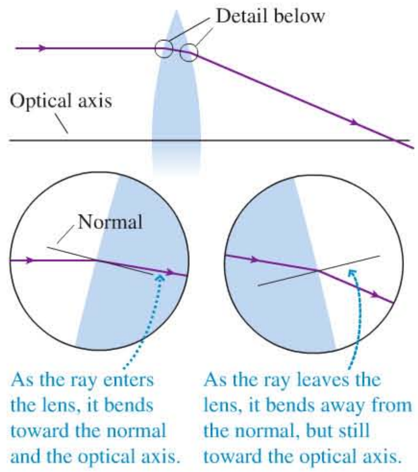
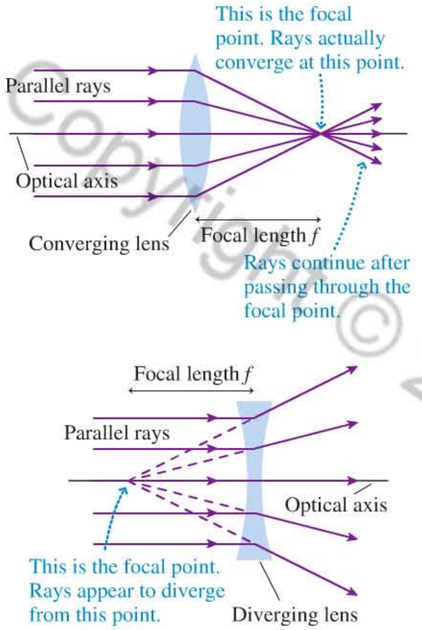


FIGURE 18.28 The focal point and focal length of converging and diverging lenses.



NOTE ► The focal length f is a property *of the lens,* independent of how the lens is used. The focal length characterizes a lens in much the same way that a mass m characterizes an object or a spring constant k characterizes a spring. ◄

Converging Lenses

These basic observations about lenses are enough to understand image formation by a **thin lens,** an idealized lens whose thickness is zero and that lies entirely in a plane called the **lens plane.** Within this *thin-lens approximation*, **all refraction occurs as the rays cross the lens plane, and all distances are measured from the lens plane.** Fortunately, the thin-lens approximation is quite good for most practical applications of lenses.

NOTE ► We'll *draw* lenses as if they have a thickness, because that is how we expect lenses to look, but our analysis will not depend on the shape or thickness of a lens. ◄

FIGURE 18.29 shows three important situations of light rays passing through a thin, converging lens. Part (a) is familiar from Figure 18.28. If the direction of each of the rays in Figure 18.29a is reversed, Snell's law tells us that each ray will exactly retrace its path and emerge from the lens parallel to the optical axis. This leads to Figure 18.29b, which is the "mirror image" of part (a). Notice that the lens actually has *two* focal points, located at distances f on either side of the lens.

FIGURE 18.29 Three important sets of rays passing through a thin, converging lens.

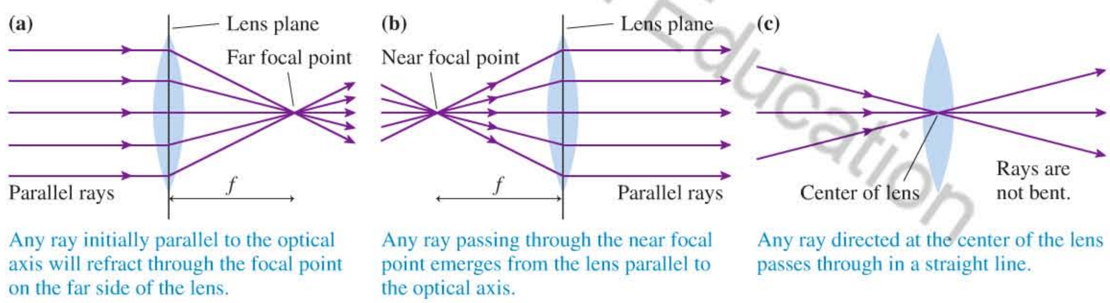


Figure 18.29c shows three rays passing through the *center* of the lens. At the center, the two sides of a lens are very nearly parallel to each other. Earlier, in Example 18.3, we found that a ray passing through a piece of glass with parallel sides is *displaced* but *not bent* and that the displacement becomes zero as the thickness approaches zero. Consequently, a ray through the center of a thin lens, which has zero thickness, is neither bent nor displaced but travels in a straight line.

These three situations form the basis for ray tracing.

Real Images

FIGURE 18.30 on the next page shows a lens and an object whose distance s from the lens is larger than the focal length. Rays from point P on the object are refracted by the lens so as to converge at point P′ on the opposite side of the lens, at a distance s' from the lens. If rays diverge from an object point P and interact with a lens such that the refracted rays *converge* at point P′, actually meeting at P′, then we call P′ a **real image** of point P. Contrast this with our prior definition of a *virtual image* as a point from which rays appear to *diverge.*

All points on the object that are in the same plane, the **object plane,** converge to image points in the **image plane.** Points Q and R in the object plane of Figure 18.30 have image points Q′ and R′ in the same plane as point P′. Once we locate *one* point in the image plane, such as point P′, we know that the full image lies in the same plane.

There are two important observations to make about Figure 18.30. First, as also seen in FIGURE 18.31, the image is upside down with respect to the object. This is

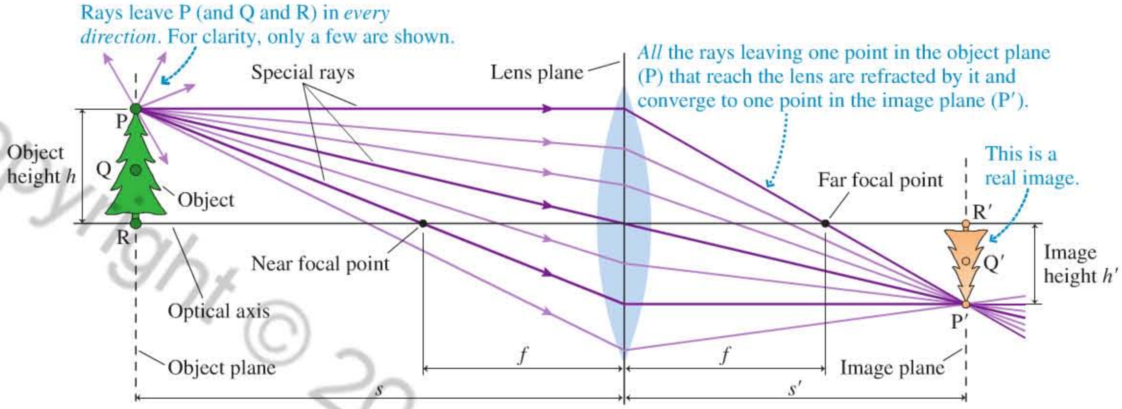


FIGURE 18.30 Rays from an object point P are refracted by the lens and converge to a real image at point P′.

called an **inverted image,** and it is a standard characteristic of real-image formation with a converging lens. Second, rays from point P *fill* the entire lens surface, so that all portions of the lens contribute to the image. A larger lens will "collect" more rays and thus make a brighter image.

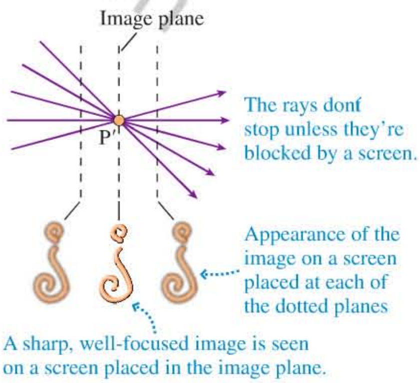
FIGURE 18.32 is a close-up view of the rays and images very near the image plane. The rays don't stop at P′ unless we place a screen in the image plane. When we do so, we see a sharp, well-focused image on the screen. If a screen is placed other than in the image plane, an image is produced on the screen, but it's blurry and out of focus.

NOTE ► Our ability to see a real image on a screen sets real images apart from *virtual* images. But keep in mind that we need not *see* a real image in order to *have* an image. A real image exists at a point in space where the rays converge even if there's no viewing screen in the image plane. ◄

Figure 18.30 highlights the three "special rays" that are based on the three situations of Figure 18.29. Notice that these three rays alone are sufficient to locate the image point P′. That is, we don't need to draw all the rays shown in Figure 18.30. The procedure known as *ray tracing* consists of locating the image by the use of just these three rays.

FIGURE 18.31 The lamp's image is upside down.

FIGURE 18.32 A close-up look at the rays and images near the image plane.



TACTICS BOX 18.2 Ray tracing for a converging lens

❶ **Draw an optical axis.** Use graph paper or a ruler! Establish an appropriate scale.

❷ **Center the lens on the axis.** Mark and label the focal points at distance f on either side.

❸ **Represent the object with an upright arrow at distance s.** It's usually best to place the base of the arrow on the axis and to draw the arrow about half the radius of the lens.

❹ **Draw the three "special rays" from the tip of the arrow.** Use a straight-edge or a ruler.

a. A ray initially parallel to the axis refracts through the far focal point.

b. A ray that enters the lens along a line through the near focal point emerges parallel to the axis.

c. A ray through the center of the lens does not bend.

❺ **Extend the rays until they converge.** The rays converge at the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.

❻ **Measure the image distance s'.** Also, if needed, measure the image height relative to the object height. The magnification can be found from Equation 18.8.

Exercise 19

EXAMPLE 18.7 Finding the image of a flower

A 4.0-cm-diameter flower is 200 cm from the 50-cm-focal-length lens of a camera. How far should the plane of the camera's light detector be placed behind the lens to record a well-focused image? What is the diameter of the image on the detector?

PREPARE The flower is in the object plane. Use ray tracing to locate the image.

SOLVE FIGURE 18.33 shows the ray-tracing diagram and the steps of Tactics Box 18.2. The image has been drawn in the plane where the three special rays converge. You can see *from the drawing* that the image distance is $s' \approx 65$ cm. This is where the detector needs to be placed to record a focused image. The heights of the object and image are labeled h and h'. The ray through the center of the lens is a straight line; thus the object and image both subtend the same angle θ. Using similar triangles,

$$\frac{h'}{s'} = \frac{h}{s}$$

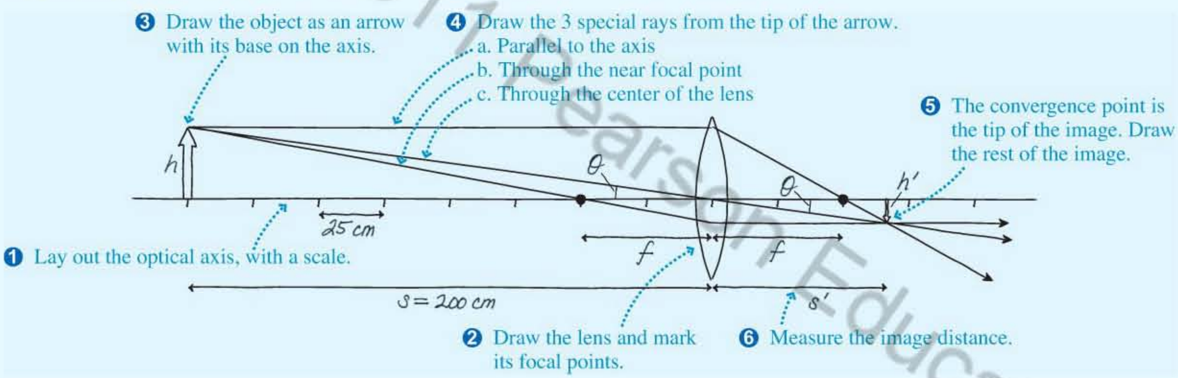
Solving for h' gives

$$h' = h\frac{s'}{s} = (4.0\text{ cm})\frac{65\text{ cm}}{200\text{ cm}} = 1.3\text{ cm}$$

The flower's image has a diameter of 1.3 cm.

ASSESS We've been able to learn a great deal about the image from a simple geometric procedure.

FIGURE 18.33 Ray-tracing diagram for the image of a flower.



Magnification

The image can be either larger or smaller than the object, depending on the location and focal length of the lens. Because the image height scales with that of the object, we're usually interested in the *ratio* h'/h of the image height to the object height. This ratio is greater than 1 when the image is taller than the object, and less than 1 when the image is shorter than the object.

But there's more to a description of the image than just its size. We also want to know its *orientation* relative to the object; that is, is the image upright or inverted? It is customary to combine image size and orientation information in a single number, the **magnification** m, defined as

$$m = -\frac{s'}{s} \tag{18.8}$$

Magnification of a lens or mirror

You saw in Example 18.7 that $s'/s = h'/h$. Consequently, we interpret the magnification m as follows:

1. The absolute value of m gives the ratio of image height to object height: $h'/h = |m|$.
2. A positive value of m indicates that the image is upright relative to the object. A negative value of m indicates that the image is inverted relative to the object.

The magnification in Example 18.7 would be

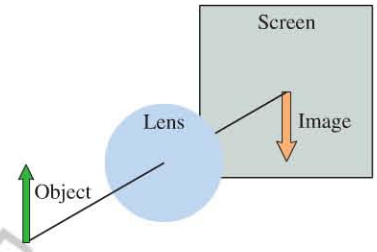
$$m = -\frac{s'}{s} = -\frac{65\text{ cm}}{200\text{ cm}} = -0.33$$

indicating that the image is 33% the size of the object and, because of the minus sign, is inverted.

NOTE ► Equation 18.8 applies to real or virtual images produced by both lenses and mirrors. Although s and s' are both positive in Example 18.7, leading to a negative magnification, we'll see that this is *not* the case for virtual images. ◄

STOP TO THINK 18.3 A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if the lens is removed?

A. The image will be inverted and blurry.
B. The image will be upright and sharp.
C. The image will be upright and blurry.
D. The image will be much dimmer but otherwise unchanged.
E. There will be no image at all.



Virtual Images

The preceding section considered a converging lens with the object at distance $s > f$; that is, the object was outside the focal point. What if the object is inside the focal point, at distance $s < f$? **FIGURE 18.34** shows just this situation.

The special rays initially parallel to the axis and through the center of the lens present no difficulties. However, a ray through the near focal point would travel toward the left and would never reach the lens! Referring back to Figure 18.29b, you can see that the rays emerging parallel to the axis entered the lens *along a line* passing through the near focal point. It's the angle of incidence on the lens that is important, not whether the light ray actually passes through the focal point. This was the basis for the wording of step 4b in Tactics Box 18.2 and is the third special ray shown in Figure 18.34.

The three refracted rays don't converge. Instead, all three rays appear to *diverge* from point P′. This is the situation we found for rays reflecting from a mirror and for the rays refracting out of an aquarium. Point P′ is a *virtual image* of the object point P. Furthermore, it is an **upright image,** having the same orientation as the object.

The refracted rays, which are all to the right of the lens, *appear* to come from P′, but none of the rays were ever at that point. No image would appear on a screen placed in the image plane at P′. So what good is a virtual image?

Your eye collects and focuses bundles of diverging rays. Thus, as **FIGURE 18.35a** shows, you can "see" a virtual image by looking *through* the lens. This is exactly what you do with a magnifying glass, producing a scene like the one in **FIGURE 18.35b**. In fact, you view a virtual image any time you look *through* the eyepiece of an optical instrument such as a microscope or binoculars.

FIGURE 18.34 Rays from an object at distance $s < f$ are refracted by the lens and diverge to form a virtual image at point P′.

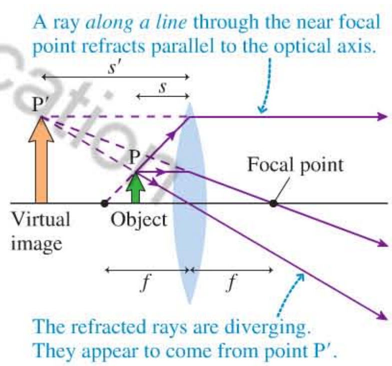
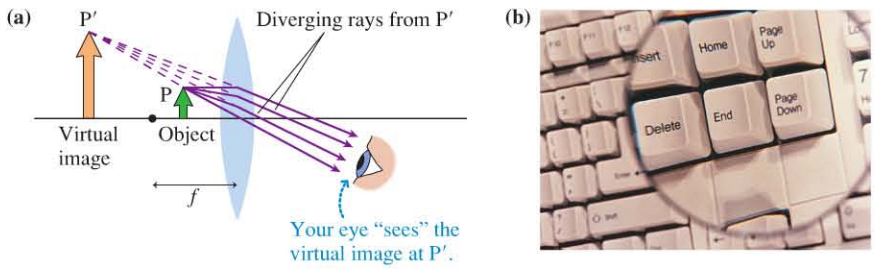


FIGURE 18.35 A converging lens is a magnifying glass when the object distance is $< f$.



NOTE ▶ Recall that a lens thicker in the middle than at the edges is classified as a converging lens. The light rays from an object *can* converge to form a real image after passing through such a lens, but only if the object distance is greater than the focal length of the lens: $s > f$. If $s < f$, the rays leaving a converging lens diverge to produce a virtual image. ◀

Because a virtual image is upright, the magnification $m = -s'/s$ is positive. This means that the ratio s'/s must be *negative*. We can ensure this if **we define the image distance s' to be negative for a virtual image,** indicating that the image is on the *same* side of the lens as the object. This is our first example of a **sign convention** for the various distances that appear in understanding image formation from lenses and mirrors. We'll have more to say about sign conventions when we study the thin-lens equation in a later section.

EXAMPLE 18.8 Magnifying a flower

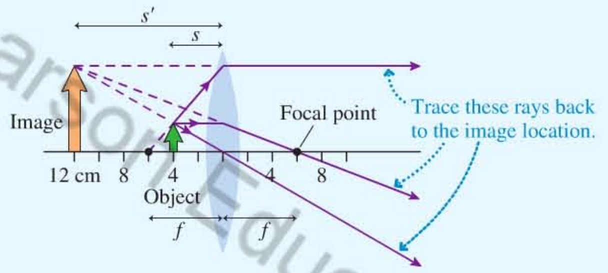
To see a flower better, you hold a 6.0-cm-focal-length magnifying glass 4.0 cm from the flower. What is the magnification?

PREPARE The flower is in the object plane. Use ray tracing to locate the image. Once the image distance is known, Equation 18.8 can be used to find the magnification.

SOLVE FIGURE 18.36 shows the ray-tracing diagram. The three special rays diverge from the lens, but we can use a straightedge to extend the rays backward to the point from which they diverge. This point, the image point, is seen to be 12 cm to the left of the lens. Because this is a virtual image, the image distance is $s' = -12$ cm. From Equation 18.8 the magnification is

$$m = -\frac{s'}{s} = -\frac{-12\text{ cm}}{4.0\text{ cm}} = 3.0$$

FIGURE 18.36 Ray-tracing diagram for a magnifying glass.

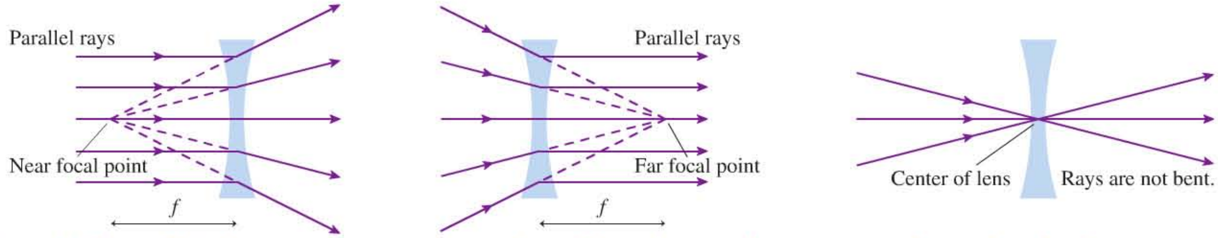


ASSESS The image is three times as large as the object and, as we see from the ray-tracing diagram and the fact that $m > 0$, upright.

Diverging Lenses

As Figure 18.26b showed, a *diverging lens* is one that is thinner at its center than at its edge. FIGURE 18.37 shows three important sets of rays passing through a diverging lens. These are based on Figures 18.26 and 18.28, where you saw that rays initially parallel to the axis diverge after passing through a diverging lens.

FIGURE 18.37 Three important sets of rays passing through a thin, diverging lens.



Any ray initially parallel to the optical axis diverges along a line through the near focal point.

Any ray directed along a line toward the far focal point emerges from the lens parallel to the optical axis.

Any ray directed at the center of the lens passes through in a straight line.

15.9 ActivPhysics ONLINE

Ray tracing follows the steps of Tactics Box 18.2 for a converging lens *except* that two of the three special rays in step 4 are different.

TACTICS BOX 18.3 Ray tracing for a diverging lens

❶–❸ **Follow steps 1 through 3 of Tactics Box 18.2.**

❹ **Draw the three "special rays" from the tip of the arrow.** Use a straightedge or a ruler.

a. A ray parallel to the axis diverges along a line through the near focal point.
b. A ray along a line toward the far focal point emerges parallel to the axis.
c. A ray through the center of the lens does not bend.

❺ **Trace the diverging rays backward.** The point from which they are diverging is the image point, which is always a virtual image.

❻ **Measure the image distance s'**, which, because the image is virtual, we will take as a negative number. Also, if needed, measure the image height relative to the object height. The magnification can be found from Equation 18.8.

EXAMPLE 18.9 Demagnifying a flower

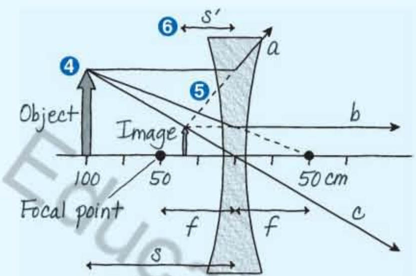
A diverging lens with a focal length of 50 cm is placed 100 cm from a flower. Where is the image? What is its magnification?

PREPARE The flower is in the object plane. Use ray tracing to locate the image. Then Equation 18.8 can be used to find the magnification.

SOLVE FIGURE 18.38 shows the ray-tracing diagram. The three special rays (labeled a, b, and c to match the Tactics Box) do not converge. However, they can be traced backward to an intersection ≈33 cm to the left of the lens. Because the rays appear to diverge from the image, this is a virtual image and s' is < 0. The magnification is

$$m = -\frac{s'}{s} = -\frac{-33\text{ cm}}{100\text{ cm}} = 0.33$$

FIGURE 18.38 Ray-tracing diagram for demagnifying.



The image, which can be seen by looking *through* the lens, is one-third the size of the object and upright.

ASSESS Ray tracing with a diverging lens is somewhat trickier than with a converging lens, so this example is worth careful study.

Diverging lenses *always* make virtual images and, for this reason, are rarely used alone. However, they have important applications when used in combination with other lenses. Cameras, eyepieces, and eyeglasses often incorporate diverging lenses.

18.6 Image Formation with Spherical Mirrors

Curved mirrors can also be used to form images. Such mirrors are commonly used in telescopes, security and rearview mirrors, and searchlights. Their images can be analyzed with ray diagrams similar to those used with lenses. We'll consider only the important case of **spherical mirrors,** whose surface is a section of a sphere.

FIGURE 18.39 shows parallel light rays approaching two spherical mirrors. The upper mirror, where the edges curve toward the light source, is called a **concave mirror.** Parallel rays reflect off the shiny front surface of the mirror and pass through a single point on the optical axis. This is the focal point of the mirror. The lower mirror, where the edges curve away from the light source, is called a **convex mirror.** Parallel rays that reflect off its surface appear to have come from a point behind the mirror. This is the focal point for a convex mirror. For both mirrors, the focal length is the distance from the mirror surface to the focal point.

FIGURE 18.39 The focal point and focal length of concave and convex mirrors.

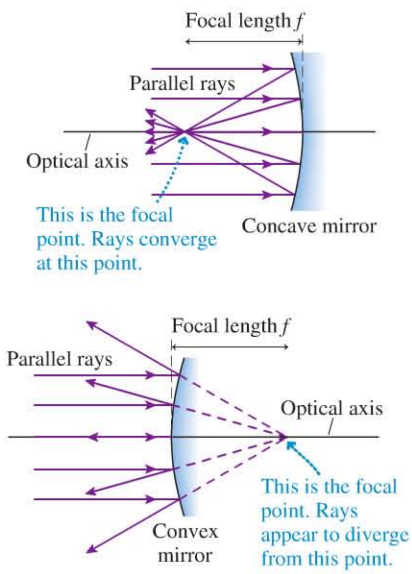
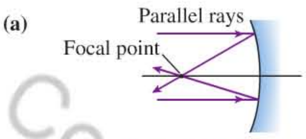
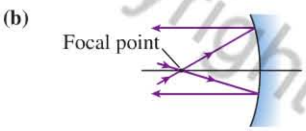


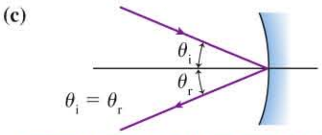
FIGURE 18.40 Three special rays for a concave mirror.



Any ray initially parallel to the optical axis will reflect through the focal point.



Any ray passing through the focal point will, after reflection, emerge parallel to the optical axis.



Any ray directed at the center of the mirror will reflect at an equal angle on the opposite side of the optical axis.

Concave Mirrors

To understand image formation by a concave mirror, consider the three special rays shown in **FIGURE 18.40**. These rays are closely related to those used for ray tracing with lenses. Figure 18.40a shows two incoming rays parallel to the optical axis. As Figure 18.39 showed, these rays reflect off the mirror and pass through the focal point.

Figure 18.40b is the same as Figure 18.40a, but with the directions of the rays reversed. Here we see that rays passing through the focal point emerge parallel to the axis. Finally, Figure 18.40c shows what happens to a ray that is directed toward the center of the mirror. Right at its center, the surface of the mirror is perpendicular to the optical axis. The law of reflection then tells us that the incoming ray will reflect at the same angle, but on the opposite side of the optical axis.

Let's begin by considering the case where the object's distance s from the mirror is greater than the focal length ($s > f$), as shown in **FIGURE 18.41**. The three special rays just discussed are enough to locate the position and size of the image. Recall that when ray tracing a thin lens, although we drew the lens as having an actual thickness, the rays refracted at an imaginary plane centered on the lens. Similarly, when ray tracing mirrors, the incoming rays reflect off the **mirror plane** as shown in Figure 18.41, not off the curved surface of the mirror. We see that the image is *real* because rays converge at the image point P′. Further, the image is *inverted.*

FIGURE 18.41 A real image formed by a concave mirror.

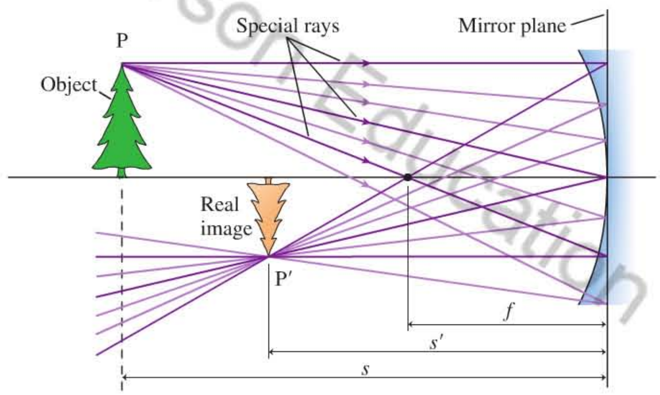


Figure 18.41 suggests the following Tactics Box for using ray tracing with a concave mirror:

TACTICS BOX 18.4 Ray tracing for a concave mirror MP™

❶ **Draw an optical axis.** Use graph paper or a ruler! Establish an appropriate scale.

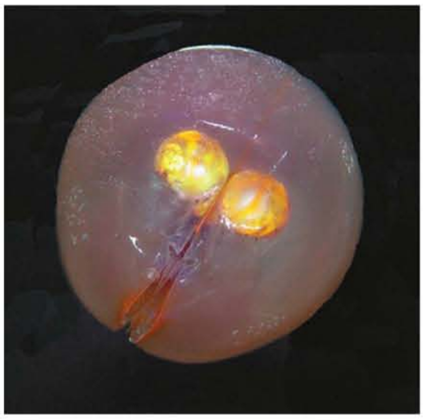
❷ **Center the mirror on the axis.** Mark and label the focal point at distance f from the mirror's surface.

❸ **Represent the object with an upright arrow at distance s.** It's usually best to place the base of the arrow on the axis and to draw the arrow about half the radius of the mirror.

❹ **Draw the three "special rays" from the tip of the arrow.** Use a straightedge or a ruler. The rays should reflect off the mirror plane.

a. A ray parallel to the axis reflects through the focal point.
b. An incoming ray that passes through the focal point emerges parallel to the axis.
c. A ray that strikes the center of the mirror reflects at an equal angle on the opposite side of the optical axis.

Continued



Look into my eyes BIO The eyes of most animals use lenses to focus an image. The *gigantocypris,* a deep-sea crustacean, is unusual in that it uses two concave mirrors to focus light onto its retina. Because it lives at depths where no sunlight penetrates, it is believed that gigantocypris uses its mirror eyes to hunt bioluminescent animals.

❺ **Extend the rays until they converge.** The rays converge at the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.

❻ **Measure the image distance s'.** Also, if needed, measure the image height relative to the object height. The magnification can be found from Equation 18.8.

Exercises 21a, 22

EXAMPLE 18.10 **Analyzing a concave mirror**

A 3.0-cm-high object is located 60 cm from a concave mirror. The mirror's focal length is 40 cm. Use ray tracing to find the position, height, and magnification of the image.

PREPARE FIGURE 18.42 shows the ray-tracing diagram and the steps of Tactics Box 18.4.

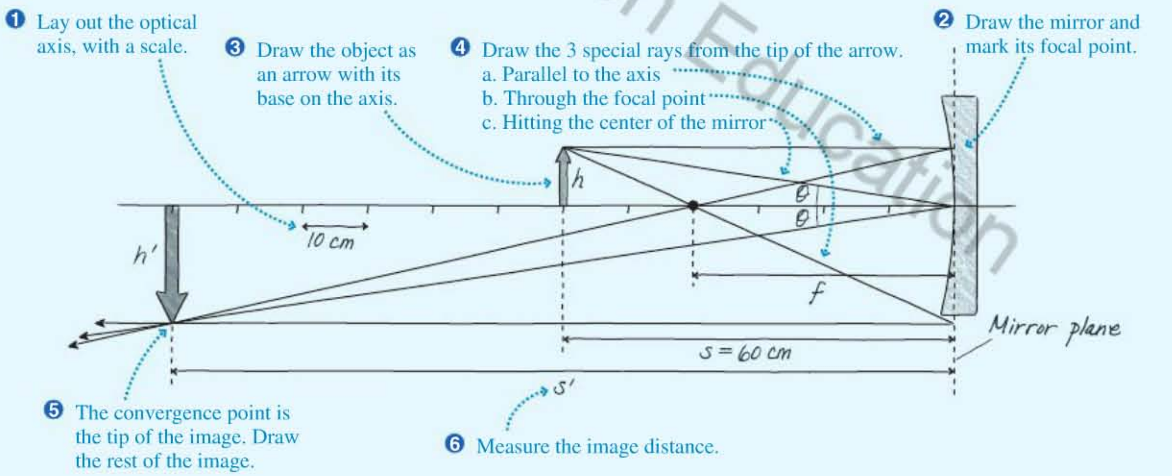
SOLVE After preparing a careful drawing, we can use a ruler to find that the image position is $s' \approx 120$ cm. The magnification is thus

$$m = -\frac{s'}{s} \approx -\frac{120\text{ cm}}{60\text{ cm}} = -2.0$$

The negative sign indicates that the image is inverted. The image height is thus twice the object height, or $h' \approx 6$ cm.

ASSESS The image is a *real* image because light rays converge at the image point.

FIGURE 18.42 Ray-tracing diagram for a concave mirror.



If the object is inside the focal point ($s < f$), ray tracing can be used to show that the image is a virtual image. This situation is analogous to the formation of a virtual image by a lens when the object is inside the focal point.

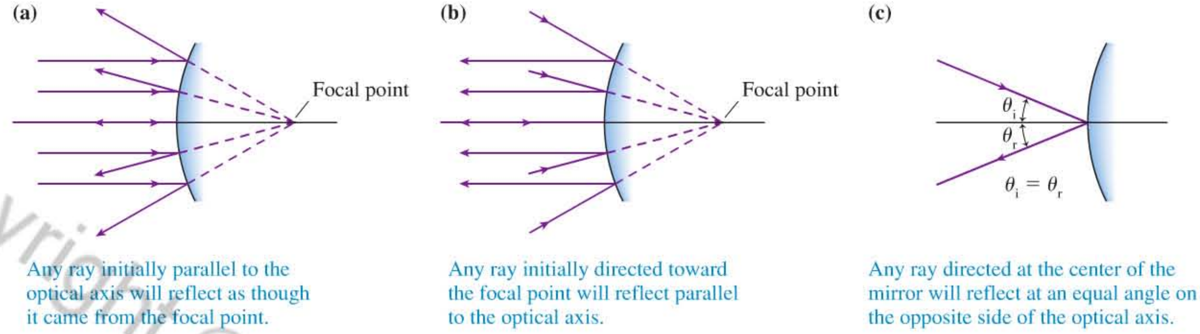
Convex Mirrors

A common example of a convex mirror is a silvered ball, such as a tree ornament. You may have noticed that if you look at your reflection in such a ball, your image appears right side up but is quite small. FIGURE 18.43 shows a self-portrait of the Dutch artist M. C. Escher that illustrates these observations. Let's use ray tracing to see why the image appears this way.

Once more, there are three special rays we can use to find the location of the image. These rays are shown in FIGURE 18.44 on the next page; they are similar to the special rays we've already studied in other situations.

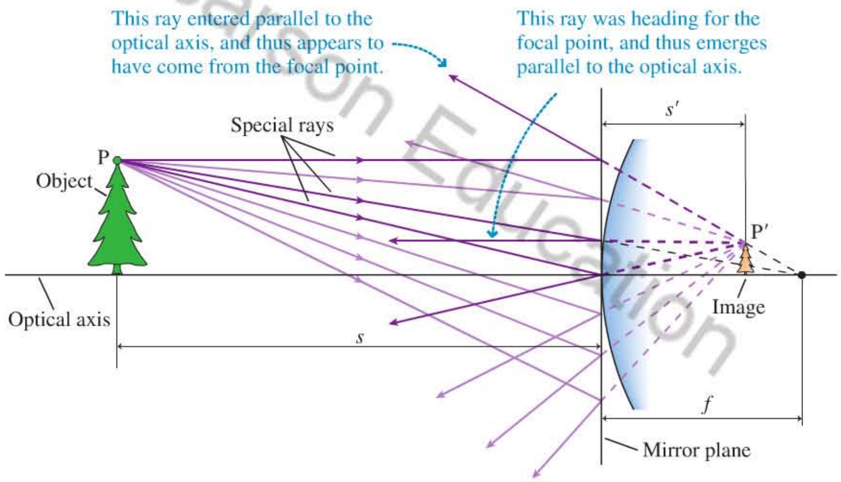
FIGURE 18.43 Self-portrait of M. C. Escher.

FIGURE 18.44 Three special rays for a convex mirror.



We can use these three special rays to find the image of an object, as shown in **FIGURE 18.45**. We see that the image is virtual—no actual rays converge at the image point P′. Instead, diverging rays *appear* to have come from this point. The image is also upright and much smaller than the object, in accord with our experience and the drawing of Figure 18.43.

FIGURE 18.45 Rays from point P reflect from the mirror and appear to have come from point P′.



15.5–15.8 Activ Online Physics

These observations form the basis of the following Tactics Box.

The small image in a convex mirror allows a wide-angle view of the store to be visible.

TACTICS BOX 18.5 **Ray tracing for a convex mirror**

❶–❸ **Follow steps 1 through 3 of Tactics Box 18.4.**

❹ **Draw the three "special rays" from the tip of the arrow.** Use a straightedge or a ruler.
  a. A ray parallel to the axis reflects as though it came from the focal point.
  b. A ray initially directed toward the focal point reflects parallel to the axis.
  c. A ray that strikes the center of the mirror reflects at an equal angle on the opposite side of the optical axis.

❺ **Extend the emerging rays *behind the mirror* until they converge.** The point of convergence is the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.

❻ **Measure the image distance s'.** Also, if needed, measure the image height relative to the object height. The magnification can be found from Equation 18.8.

Exercises 21b, 23

Convex mirrors are used for a variety of safety and monitoring applications, such as passenger-side rearview mirrors and the round mirrors used in stores to keep an eye on the customers. The idea behind such mirrors can be understood from Figure 18.45. When an object is reflected in a convex mirror, the image appears smaller. Because the image is, in a sense, a miniature version of the object, you can *see much more of it* within the edges of the mirror than you could with an equal-sized flat mirror. This wide-angle view is clearly useful for checking traffic behind you, or for checking up on your store.

CONCEPTUAL EXAMPLE 18.11 **Driver and passenger mirrors**

The rearview mirror on the driver's side of a car is a plane (flat) mirror, while the mirror on the passenger's side is convex. Why is this?

REASON It is important for the driver to have a wide field of view from either mirror. He sits close to the driver-side mirror, so it appears large and can reflect a fairly wide view of what's behind. The passenger-side mirror is quite far from the driver, so it appears relatively small. If it were flat, it would offer only a narrow view of what's behind. Making it convex, like the security mirror discussed above, provides a wider field of view, but the trade-off is a smaller image. That's why the passenger-side mirror usually contains a warning: Objects in mirror are closer than they appear!

STOP TO THINK 18.4 A concave mirror of focal length f forms an image of the moon. Where is the image located?

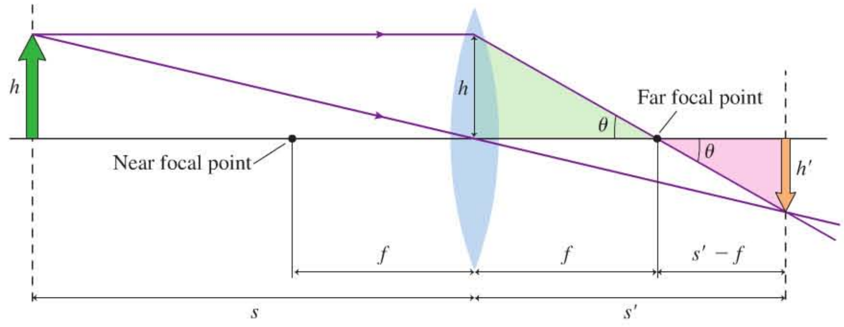
A. At the mirror's surface
B. Almost exactly a distance f behind the mirror
C. Almost exactly a distance f in front of the mirror
D. At a distance behind the mirror equal to the distance of the moon in front of the mirror

18.7 The Thin-Lens Equation

Ray tracing is an important tool for quickly grasping the overall positions and sizes of an object and its image. For more precise work, however, we would like a mathematical expression that relates the three fundamental quantities of an optical system: the focal length f of the lens or mirror, the object distance s, and the image distance s'. We can find such an expression by considering the converging lens in **FIGURE 18.46**. Two of the special rays are shown: one initially parallel to the optical axis that then passes through the far focal point, and the other traveling undeviated through the center of the lens.

ActivPhysics Online 15.10, 15.11

FIGURE 18.46 Deriving the thin-lens equation.



Consider the two right triangles highlighted in green and pink. Because they both have one 90° angle and a second angle θ that is the same for both, the two triangles are *similar.* This means that they have the same shape, although their sizes may be different. For similar triangles, the ratios of any two similar sides are the same. Thus we have

$$\frac{h'}{h}=\frac{s'-f}{f} \tag{18.9}$$

Further, we found in Example 18.7 that

$$\frac{h'}{h}=\frac{s'}{s} \tag{18.10}$$

Combining Equations 18.9 and 18.10 gives

$$\frac{s'}{s}=\frac{s'-f}{f}$$

Dividing both sides by s' gives

$$\frac{1}{s}=\frac{s'-f}{s'f}=\frac{1}{f}-\frac{1}{s'}$$

which we can write as

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \tag{18.11}$$

Thin-lens equation (also works for mirrors) relating object and image distances to focal length

This equation, the **thin-lens equation,** relates the three important quantities f, s, and s'. In particular, if we know the focal length of a lens and the object's distance from the lens, we can use the thin-lens equation to find the position of the image.

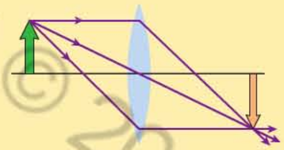
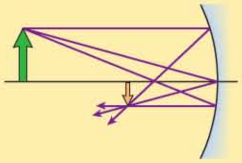
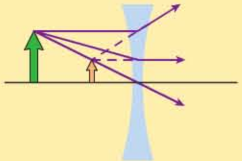
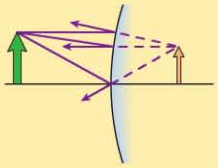
NOTE ► Although we derived the thin-lens equation for a converging lens that produced a real image, it works equally well for *any* image—real or virtual—produced by both converging and diverging lenses. And, in spite of its name, the thin-lens equation also describes the images formed by *mirrors.* ◄

It's worth checking that the thin-lens equation describes what we already know about lenses. In Figure 18.29a, we saw that rays initially parallel to the optical axis are focused at the focal point of a converging lens. Initially parallel rays come from an object extremely far away, with $s \to \infty$. Because $1/\infty = 0$, the thin-lens equation tells us that the image distance is $s' = f$, as we expected. Or suppose an object is located right at the focal point, with $s = f$. Then, according to Equation 18.11, $1/s' = 1/f - 1/s = 0$. This implies that the image distance is infinitely far away ($s' = \infty$), so the rays leave the lens parallel to the optical axis. Indeed, this is what Figure 18.29b showed. Now, it's true that no real object or image can be at infinity. But if either the object or image is more than several focal lengths from the lens ($s \gg f$ or $s' \gg f$), then it's an excellent approximation to consider the distance to be infinite, the rays to be parallel to the axis, and the reciprocal ($1/s$ or $1/s'$) in the thin-lens equation to be zero.

Sign Conventions for Lenses and Mirrors

We've already noted that the image distance s' is positive for real images and negative for virtual images. In the thin-lens equation, the sign of the focal length can also be either positive or negative, depending on the type of lens or mirror. Tactics Box 18.6 summarizes the sign conventions that we will use in this text.

TACTICS BOX 18.6 **Using sign conventions for lenses and mirrors**

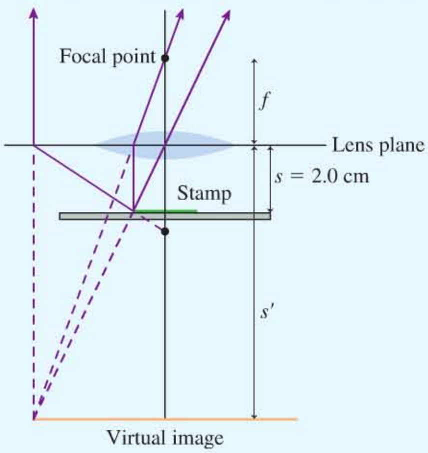
Quantity	Positive when		Negative when	
Object distance s	Always		We won't treat this case, which can occur for two lenses or mirrors in combination.	
Image distance s'	**Real image**		**Virtual image**	
	Image is on the opposite side of the lens from the object. 	Image is in front of the mirror. 	Image is on the same side of the lens as the object. 	Image is behind the mirror. 
Focal length f	Converging lens or concave mirror		Diverging lens or convex mirror	
Magnification m	Image is upright.		Image is inverted.	

NOTE ► When using the thin-lens equation, the focal length must be taken as positive or negative according to Tactics Box 18.6. In the pictorial method of ray tracing, however, focal lengths are just *distances* and are therefore always positive. ◄

EXAMPLE 18.12 Analyzing a magnifying lens

A stamp collector uses a magnifying lens that sits 2.0 cm above the stamp. The magnification is 4. What is the focal length of the lens?

FIGURE 18.47 Ray-tracing diagram of a magnifying lens.



PREPARE A magnifying lens is a converging lens with the object distance less than the focal length ($s < f$). Assume it is a thin lens. The user looks *through* the lens and sees a virtual image. **FIGURE 18.47** shows the lens and a ray-tracing diagram.

SOLVE A virtual image is upright, so $m = +4$. The magnification is $m = -s'/s$; thus

$$s' = -4s = -4(2.0\text{ cm}) = -8.0\text{ cm}$$

We can use s and s' in the thin-lens equation to find the focal length:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{2.0\text{ cm}} + \frac{1}{-8.0\text{ cm}} = 0.375\text{ cm}^{-1}$$

Thus

$$f = \frac{1}{0.375\text{ cm}^{-1}} = 2.7\text{ cm}$$

ASSESS $f > 2$ cm, as expected because the object has to be inside the focal point.

EXAMPLE 18.13 What is the focal length?

An object is 38.0 cm to the left of a lens. Its image is found to be 22.0 cm from the lens on the same side as the object. What is the focal length of the lens? Draw a ray diagram for the lens and object.

PREPARE We know the object distance is $s = 38.0$ cm. According to Tactics Box 18.6, the image distance is a negative number because the image is on the same side of the lens as the object, so $s' = -22.0$ cm.

Continued

SOLVE The thin-lens equation, Equation 18.11, is

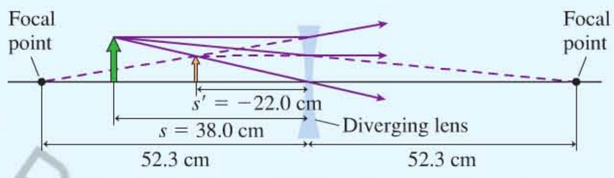
$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{38.0\text{ cm}}+\frac{1}{-22.0\text{ cm}}=-0.0191\text{ cm}^{-1}$$

from which we find

$$f=\frac{1}{-0.0191\text{ cm}^{-1}}=-52.3\text{ cm}$$

Because f is negative, Tactics Box 18.6 indicates that the lens is a diverging lens. We can use the techniques learned in Section 18.5 to draw the ray diagram shown in FIGURE 18.48. Notice that the negative value of s' corresponds to a virtual image.

FIGURE 18.48 Ray-tracing diagram of a diverging lens.



ASSESS This problem would be difficult to solve graphically because you need to locate the focal points in order to draw a ray diagram. Using the thin-lens equation gives rapid and accurate results. Nonetheless, sketching a ray diagram once f is known is very helpful in fully understanding the situation.

EXAMPLE 18.14 Finding the mirror image of a candle

A concave mirror has a focal length of 2.4 m. Where should a candle be placed so that its image is inverted and twice as large as the object?

PREPARE Even though this is a mirror, we can use the thin-lens equation. We have $f = +2.4$ m because, according to Tactics Box 18.6, a concave mirror has a positive focal length. We also know that the magnification is $m = -2$ because an inverted image implies a negative magnification.

SOLVE Equation 18.8, $m = -s'/s$, relates s' and s:

$$s' = -ms$$

We can insert this expression for s' into the thin-lens equation:

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{s}+\frac{1}{-ms}=\frac{1}{s}+\frac{1}{-(-2)s}=\frac{1}{s}+\frac{1}{2s}=\frac{3}{2s}$$

from which we get

$$s=\frac{3}{2}f=\frac{3}{2}(2.4\text{ m})=3.6\text{ m}$$

The candle should be placed 3.6 m in front of the mirror.

ASSESS The magnification equation tells us that $s' = 2s = 7.2$ m. This is positive, so the image is real and on the same side of the mirror as the object.

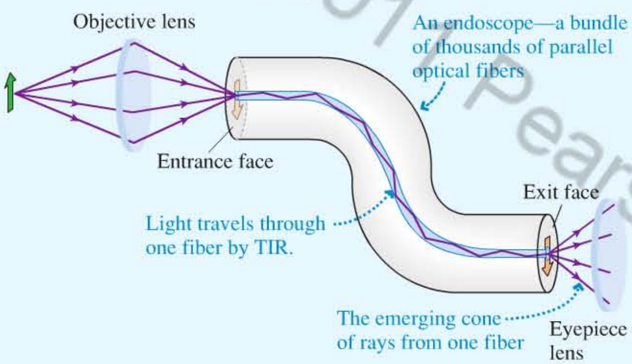
STOP TO THINK 18.5 A candle is placed in front of a converging lens. A well-focused image of the flame is seen on a screen on the opposite side of the lens. If the candle is moved farther away from the lens, how must the screen be adjusted to keep showing a well-focused image?

A. The screen must be moved closer to the lens.
B. The screen must be moved farther away from the lens.
C. The screen does not need to be moved.

INTEGRATED EXAMPLE 18.15 Optical fiber imaging

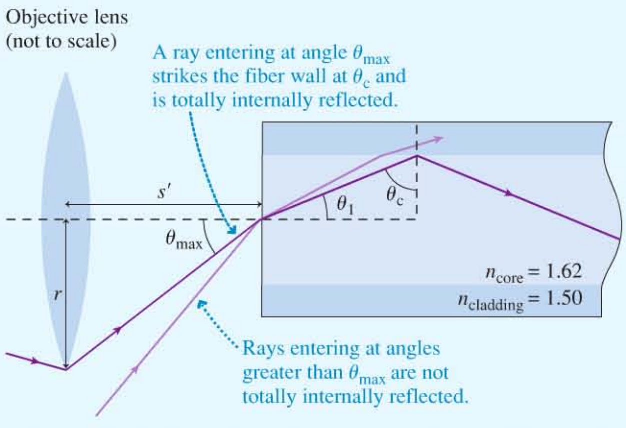
An *endoscope* is a narrow bundle of optical fibers that can be inserted through a bodily opening or a small incision to view the interior of the body. As **FIGURE 18.49** shows, an *objective* lens focuses a real image onto the entrance face of the fiber bundle. Individual fibers—using total internal reflection—transport the light to the exit face, where it emerges. The doctor observes a magnified image of the exit face by viewing it through an *eyepiece* lens.

FIGURE 18.49 An endoscope.



A single optical fiber, as discussed in Section 18.3 and shown in **FIGURE 18.50**, consists of a lower-index glass core surrounded by a higher-index cladding layer. To remain in the fiber, light rays propagating through the core must strike the cladding boundary at angles of incidence greater than the critical angle θ_c. As Figure 18.50 shows, this means that rays that enter the core at angles of incidence smaller than θ_{max} are totally internally reflected down the fiber; those that enter at angles larger than θ_{max} are not totally internally reflected, and escape from the fiber.

FIGURE 18.50 Cross section of an optical fiber (greatly magnified).



The objective lens of an endoscope must be carefully matched to the fiber. Ideally, the lens diameter is such that rays from the edge of the lens enter the fiber at θ_{max}, as shown in Figure 18.50. A lens larger than this is not useful because rays from its outer regions will enter the fiber at an angle larger than θ_{max} and will thus escape from the fiber. A lens smaller than this will suffer from a reduced light-gathering power.

a. What is θ_{max} for the fiber shown in Figure 18.50?
b. A typical objective lens is 3.0 mm in diameter and can focus on an object 3.0 mm in front of it. What focal length should the lens have so that rays from its edge just enter the fiber at angle θ_{max}?
c. What is the magnification of this lens?

PREPARE From Figure 18.50 we can find the critical angle θ_c and then use geometry to find θ_1. Snell's law can then be used to find θ_{max}. Once θ_{max} is known, we can find the image distance s', and, because we know that the object distance is $s = 3$ mm, we can use the thin-lens equation to solve for f.

SOLVE

a. The critical angle for total internal reflection is given by Equation 18.3:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{n_{cladding}}{n_{core}}\right) = \sin^{-1}\left(\frac{1.50}{1.62}\right) = 68°$$

Because of the right triangle, this ray's angle of refraction from the fiber's face is $\theta_1 = 90° - 68° = 22°$. Then, by Snell's law, this ray enters from the air at angle θ_{max} such that

$$n_{air}\sin\theta_{max} = 1.00\sin\theta_{max} = n_{core}\sin\theta_1 = 1.62(\sin 22°) = 0.61$$

Thus

$$\theta_{max} = \sin^{-1}(0.61) = 38°$$

b. As Figure 18.50 shows, the distance of the lens from the fiber's face—the image distance s'—is related to the lens radius $r = 1.5$ mm by

$$\frac{r}{s'} = \tan\theta_{max}$$

Thus the image distance is

$$s' = \frac{r}{\tan\theta_{max}} = \frac{1.5\text{ mm}}{\tan 38°} = 1.9\text{ mm}$$

Then the thin-lens equation gives

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{3.0\text{ mm}} + \frac{1}{1.9\text{ mm}} = 0.85\text{ mm}^{-1}$$

so that

$$f = \frac{1}{0.85\text{ mm}^{-1}} = 1.2\text{ mm}$$

c. The magnification is

$$m = -\frac{s'}{s} = -\frac{1.9\text{ mm}}{3.0\text{ mm}} = -0.63$$

ASSESS The object distance of 3.0 mm is greater than the 1.2 mm focal length we calculated, as must be the case when a converging lens produces a real image.

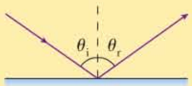
SUMMARY

The goal of Chapter 18 has been to understand and apply the ray model of light.

GENERAL PRINCIPLES

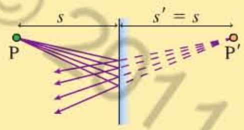
Reflection

Law of reflection: $\theta_r = \theta_i$



Reflection can be **specular** (mirror-like) or **diffuse** (from rough surfaces).

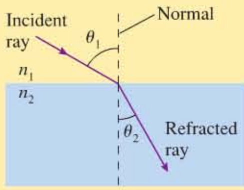
Plane mirrors: A virtual image is formed at P′ with $s' = s$, where s is the **object distance** and s' is the **image distance.**



Refraction

Snell's law of refraction:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Index of refraction is $n = c/v$. The ray is closer to the normal on the side with the larger index of refraction.

If $n_2 < n_1$, **total internal reflection** (TIR) occurs when the angle of incidence θ_1 is greater than $\theta_c = \sin^{-1}(n_2/n_1)$.

IMPORTANT CONCEPTS

The ray model of light

Light travels along straight lines, called **light rays,** at speed $v = c/n$.

A light ray continues forever unless an interaction with matter causes it to reflect, refract, scatter, or be absorbed.

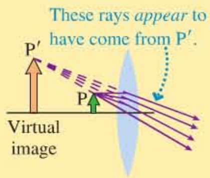
Light rays come from self-luminous or reflective **objects.** Each point on the object sends rays in all directions.

Ray diagrams represent all the rays emitted by an object by only a few select rays.

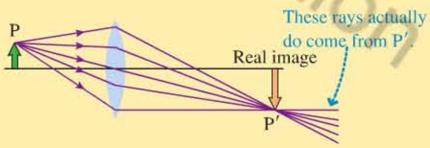
In order for the eye to see an object (or image), rays from the object or image must enter the eye.

Image formation

If rays diverge from P and, after interacting with a lens or mirror, *appear* to diverge from P′ without actually passing through P′, then P′ is a **virtual image** of P.



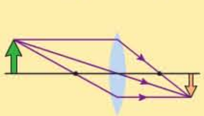
If rays diverge from P and interact with a lens or mirror so that the refracted rays *converge* at P′, then P′ is a **real image** of P. Rays actually pass through a real image.



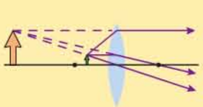
APPLICATIONS

Ray tracing for lenses

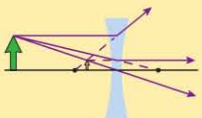
Three special rays in three basic situations:



Converging lens
Real image



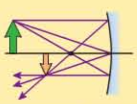
Converging lens
Virtual image



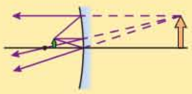
Diverging lens
Virtual image

Ray tracing for mirrors

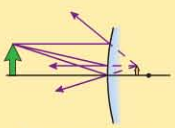
Three special rays in three basic situations:



Concave mirror
Real image



Concave mirror
Virtual image



Convex mirror
Virtual image

The thin-lens equation

For a lens or curved mirror, the object distance s, the image distance s', and the focal length f are related by the thin-lens equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

The **magnification** of a lens or mirror is $m = -s'/s$.

Sign conventions for the thin-lens equation:

Quantity	Positive when	Negative when
s	Always	Not treated here
s'	*Real* image; on opposite side of a lens from object, or in front of a mirror	*Virtual* image; on same side of a lens as object, or behind a mirror
f	Converging lens or concave mirror	Diverging lens or convex mirror
m	Image is upright.	Image is inverted.

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

QUESTIONS

Conceptual Questions

1. The idea of light rays goes back to the ancient Greeks. However, they believed that "visual rays" were *emitted* by eyes. If you were transported back in time, what arguments would you present to those early scientists to convince them that vision has something to do with rays going into, rather than out of, eyes?
2. Is there any property that distinguishes a light ray emitted by a light bulb and one that has been diffusely reflected by the page of a book? Explain.
3. If you turn on your car headlights during the day, the road ahead of you doesn't appear to get brighter. Why not?
4. Can you see the rays from the sun on a clear day? Why or why not? How about when they stream through a forest on a foggy morning? Why or why not?
5. If you take a walk on a summer night along a dark, unpaved road in the woods, with a flashlight pointing at the ground several yards ahead to guide your steps, any water-filled potholes are noticeable because they appear much darker than the surrounding dry road. Explain why.
6. You are looking at the image of a pencil in a mirror, as shown in Figure Q18.6.
   a. What happens to the image you see if the top half of the mirror, down to the midpoint, is covered with a piece of cardboard? Explain.
   b. What happens to the image you see if the bottom half of the mirror is covered with a piece of cardboard?

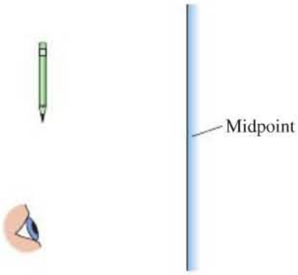


FIGURE Q18.6

7. In *The Toilet of Venus* by Velázquez (see Figure Q18.7), we can see the face of Venus in the mirror. Can she see her own face in the mirror, when the mirror is held as shown in the picture? If yes, explain why; if not, what does she see instead?

FIGURE Q18.7

Diego de Silva Velazquez (1599-1660), "Venus and Cupid," 1650. Oil on canvas. National Gallery, London. Erich Lessing/Art Resource, N.Y.

8. In Manet's *A Bar at the Folies-Bergère* (see Figure Q18.8) the reflection of the barmaid is visible in the mirror behind her. Is this the reflection you would expect if the mirror's surface is parallel to the bar? Where is the man seen facing her in the mirror actually standing?

FIGURE Q18.8

Edouard Manet 1832-1883, "Bar at the Folies-Bergere". 1881/82. Oil on Canvas. 37 13/16″ × 51″ (90 × 130 cm). Courtauld Institute Galleries, London. AKG-Images.

9. Explain why ambulances have the word "AMBULANCE" written backward on the front of them.
10. a. Consider *one* point on an object near a lens. What is the minimum number of rays needed to locate its image point?
    b. For each point on the object, how many rays from this point actually strike the lens and refract to the image point?
11. When you look at your reflection in the bowl of a spoon, it is upside down. Why is this?
12. A concave mirror brings the sun's rays to a focus at a distance of 30 cm from the mirror. If the mirror were submerged in a swimming pool, would the sun's rays be focused nearer to, further from, or at the same distance from the mirror?
13. A student draws the ray diagram shown in Figure Q18.13 but forgets to label the object, the image, or the type of lens used. Using the diagram, explain whether the lens is converging or diverging, which arrow represents the object, and which represents the image.

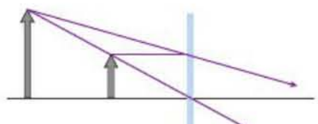


FIGURE Q18.13

14. An object at distance s from a concave mirror of focal length f produces a real image at distance s' from the mirror. Suppose the mirror is replaced by a new mirror, at the same location, with focal length $\frac{1}{2}f$. Will the new image be real or virtual? Will its distance from the mirror be more or less than s'? Explain.
15. A lens can be used to start a fire by focusing an image of the sun onto a piece of flammable material. All other things being equal, would a lens with a short focal length or a long focal length be better as a fire starter? Explain.

Multiple-Choice Questions

Questions 16 through 18 are concerned with the situation sketched in Figure Q18.16, in which a beam of light in the air encounters a transparent block with index of refraction $n = 1.53$. Some of the light is reflected and some is refracted.

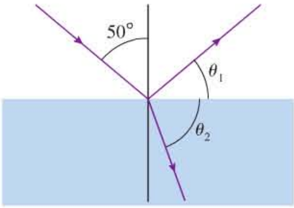


FIGURE Q18.16

16. | What is θ_1?
   A. 40° B. 45°
   C. 50° D. 90°
17. | What is θ_2?
   A. 20° B. 30°
   C. 50° D. 60°
18. | Is there an angle of incidence between 0° and 90° such that all of the light will be reflected?
   A. Yes, at an angle greater than 50°
   B. Yes, at an angle less than 50° C. No
19. | A 2.0-m-tall man is 5.0 m from the converging lens of a camera. His image appears on a detector that is 50 mm behind the lens. How tall is his image on the detector?
   A. 10 mm B. 20 mm
   C. 25 mm D. 50 mm
20. || You are 2.4 m from a plane mirror, and you would like to take a picture of yourself in the mirror. You need to manually adjust the focus of the camera by dialing in the distance to what you are photographing. What distance do you dial in?
   A. 1.2 m B. 2.4 m C. 3.6 m D. 4.8 m
21. || Figure Q18.21 shows an object and lens positioned to form a well-focused, inverted image on a viewing screen. Then a piece of cardboard is lowered just in front of the lens to cover the *top half* of the lens. What happens to the image on the screen?

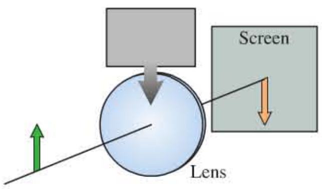


FIGURE Q18.21

   A. Nothing.
   B. The upper half of the image will vanish.
   C. The lower half of the image will vanish.
   D. The image will become fuzzy and out of focus.
   E. The image will become dimmer.
22. || A real image of an object can be formed by
   A. A converging lens.
   B. A plane mirror.
   C. A convex mirror.
   D. Any of the above.
23. | An object is 40 cm from a converging lens with a focal length of 30 cm. A real image is formed on the other side of the lens, 120 cm from the lens. What is the magnification?
   A. 2.0 B. 3.0 C. 4.0
   D. 1.33 E. 0.33
24. | The lens in Figure Q18.24 is used to produce a real image of a candle flame. What is the focal length of the lens?
   A. 9.0 cm
   B. 12 cm
   C. 24 cm
   D. 36 cm
   E. 48 cm

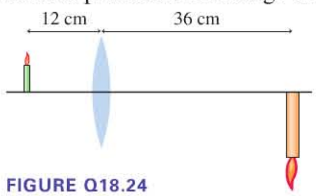


FIGURE Q18.24

25. | A converging lens of focal length 20 cm is used to form a real image 1.0 m away from the lens. How far from the lens is the object?
   A. 20 cm B. 25 cm C. 50 cm D. 100 cm
26. | You look at yourself in a convex mirror. Your image is
   A. Erect. B. Inverted.
   C. It's impossible to tell without knowing how far you are from the mirror and its focal length.
27. || An object is 50 cm from a diverging lens with a focal length of −20 cm. How far from the lens is the image, and on which side of the lens is it?
   A. 14 cm, on the same side as the object
   B. 14 cm, on the opposite side from the object
   C. 30 cm, on the same side as the object
   D. 33 cm, on the same side as the object
   E. 33 cm, on the opposite side from the object

VIEW ALL SOLUTIONS

PROBLEMS

Section 18.1 The Ray Model of Light

1. || A 5.0-ft-tall girl stands on level ground. The sun is 25° above the horizon. How long is her shadow?
2. |||| A 10-cm-diameter disk emits light uniformly from its surface. 20 cm from this disk, along its axis, is an 8.0-cm-diameter opaque black disk; the faces of the two disks are parallel. 20 cm beyond the black disk is a white viewing screen. The lighted disk illuminates the screen, but there's a shadow in the center due to the black disk. What is the diameter of the *completely dark* part of this shadow?
3. ||||| A point source of light illuminates an aperture 2.00 m away. A 12.0-cm-wide bright patch of light appears on a screen 1.00 m behind the aperture. How wide is the aperture?

Section 18.2 Reflection

4. | The mirror in Figure P18.4 deflects a horizontal laser beam by 60°. What is the angle ϕ?

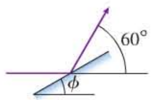


FIGURE P18.4

5. | Figure P18.5 shows an object O in front of a plane mirror. Use ray tracing to determine from which locations A–D the object's image is visible.

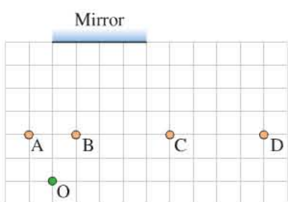


FIGURE P18.5

6. ||| A light ray leaves point A in Figure P18.6, reflects from the mirror, and reaches point B. How far below the top edge does the ray strike the mirror?

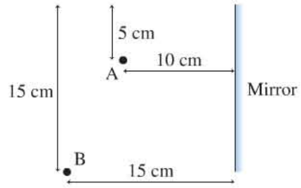


FIGURE P18.6

7. ||| It is 165 cm from your eyes to your toes. You're standing 200 cm in front of a tall mirror. How far is it from your eyes to the image of your toes?
8. ||| A ray of light impinges on a mirror as shown in Figure P18.8. A second mirror is fastened at 90° to the first.
   a. After striking both mirrors, at what angle relative to the incoming ray does the outgoing ray emerge?
   b. What is the answer if the incoming angle is 30°?

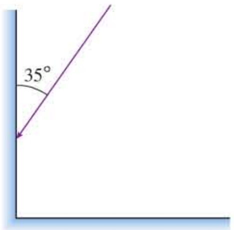


FIGURE P18.8

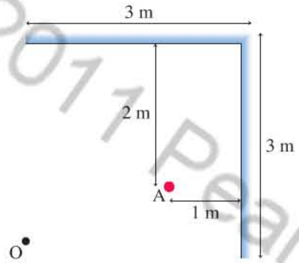


FIGURE P18.9

9. ||| A red ball is placed at point A in Figure P18.9.
   a. How many images are seen by an observer at point O?
   b. Where is each image located?
   c. Draw a ray diagram showing the formation of each image.

Section 18.3 Refraction

10. ||| An underwater diver sees the sun 50° above horizontal. How high is the sun above the horizon to a fisherman in a boat above the diver?
11. || A laser beam in air is incident on a liquid at an angle of 37° with respect to the normal. The laser beam's angle in the liquid is 26°. What is the liquid's index of refraction?
12. ||| A 1.0-cm-thick layer of water stands on a horizontal slab of glass. A light ray in the air is incident on the water 60° from the normal. After entering the glass, what is the ray's angle from the normal?
13. ||| A 4.0-m-wide swimming pool is filled to the top. The bottom of the pool becomes completely shaded in the afternoon when the sun is 20° above the horizon. How deep is the pool?
14. || A diamond is underwater. A light ray enters one face of the diamond, then travels at an angle of 30° with respect to the normal. What was the ray's angle of incidence on the diamond?
15. || A thin glass rod is submerged in oil. What is the critical angle for light traveling inside the rod?

Section 18.4 Image Formation by Refraction

16. ||| A biologist keeps a specimen of his favorite beetle embedded in a cube of polystyrene plastic. The hapless bug appears to be 2.0 cm within the plastic. What is the beetle's actual distance beneath the surface?
17. || A fish in a flat-sided aquarium sees a can of fish food on the counter. To the fish's eye, the can looks to be 30 cm outside the aquarium. What is the actual distance between the can and the aquarium? (You can ignore the thin glass wall of the aquarium.)
18. | A swim mask has a pocket of air between your eyes and the flat glass front.

   a. If you look at a fish while swimming underwater with a swim mask on, does the fish appear closer or farther than it really is? Draw a ray diagram to explain.
   b. Does the fish see your face closer or farther than it really is? Draw a ray diagram to explain.

Section 18.5 Thin Lenses: Ray Tracing

19. | An object is 30 cm in front of a converging lens with a focal length of 10 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted? Is it real or virtual?
20. | An object is 6.0 cm in front of a converging lens with a focal length of 10 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted? Is it real or virtual?
21. || An object is 20 cm in front of a diverging lens with a focal length of 10 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted? Is it real or virtual?
22. | An object is 15 cm in front of a diverging lens with a focal length of 10 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted? Is it real or virtual?

Section 18.6 Image Formation with Spherical Mirrors

23. | A concave cosmetic mirror has a focal length of 40 cm. A 5-cm-long mascara brush is held upright 20 cm from the mirror. Use ray tracing to determine the location and height of its image. Is the image upright or inverted? Is it real or virtual?
24. | A light bulb is 60 cm from a concave mirror with a focal length of 20 cm. Use ray tracing to determine the location of its image. Is the image upright or inverted? Is it real or virtual?
25. | BIO The illumination lights in an operating room use a concave mirror to focus an image of a bright lamp onto the surgical site. One such light has a mirror with a focal length of 15.0 cm. Use ray tracing to find the position of its lamp when the patient is positioned 1.0 m from the mirror (you'll need a careful drawing to get a good answer).

26. || BIO A dentist uses a curved mirror to view the back side of teeth on the upper jaw. Suppose she wants an erect image with a magnification of 2.0 when the mirror is 1.2 cm from a tooth. (Treat this problem as though the object and image lie along a straight line.) Use ray tracing to decide whether a concave or convex mirror is needed, and to estimate its focal length.
27. | A convex mirror, like the passenger-side rearview mirror on a car, has a focal length of 2.0 m. An object is 4.0 m from the mirror. Use ray tracing to determine the location of its image. Is the image upright or inverted? Is it real or virtual?
28. | An object is 6 cm in front of a convex mirror with a focal length of 10 cm. Use ray tracing to determine the location of the image. Is the image upright or inverted? Is it real or virtual?

Section 18.7 The Thin-Lens Equation

For Problems 29 through 38, calculate the image position and height.

29. | A 2.0-cm-tall object is 40 cm in front of a converging lens that has a 20 cm focal length.
30. || A 1.0-cm-tall object is 10 cm in front of a converging lens that has a 30 cm focal length.
31. || A 2.0-cm-tall object is 15 cm in front of a converging lens that has a 20 cm focal length.
32. | A 1.0-cm-tall object is 75 cm in front of a converging lens that has a 30 cm focal length.
33. | A 2.0-cm-tall object is 15 cm in front of a diverging lens that has a −20 cm focal length.
34. | A 1.0-cm-tall object is 60 cm in front of a diverging lens that has a −30 cm focal length.
35. || A 3.0-cm-tall object is 15 cm in front of a convex mirror that has a −25 cm focal length.
36. || A 3.0-cm-tall object is 45 cm in front of a convex mirror that has a −25 cm focal length.
37. | A 3.0-cm-tall object is 15 cm in front of a concave mirror that has a 25 cm focal length.
38. | A 3.0-cm-tall object is 45 cm in front of a concave mirror that has a 25 cm focal length.

General Problems

39. ||| Starting 3.5 m from a department store mirror, Suzanne INT walks toward the mirror at 1.5 m/s for 2.0 s. How far is Suzanne from her image in the mirror after 2.0 s?
40. || You slowly back away from a plane mirror at a speed of INT 0.10 m/s. With what speed does your image appear to be moving away from you?
41. || At what angle ϕ should the laser beam in Figure P18.41 be aimed at the mirrored ceiling in order to hit the midpoint of the far wall?

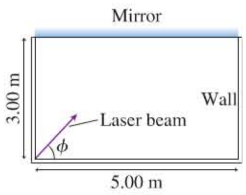


FIGURE P18.41

42. |||| You're helping with an experiment in which a vertical cylinder will rotate about its axis by a very small angle. You need to devise a way to measure this angle. You decide to use what is called an *optical lever*. You begin by mounting a small mirror on top of the cylinder. A laser 5.0 m away shoots a laser beam at the mirror. Before the experiment starts, the mirror is adjusted to reflect the laser beam directly back to the laser. Later, you measure that the reflected laser beam, when it returns to the laser, has been deflected sideways by 2.0 mm. How many degrees has the cylinder rotated?
43. || Figure P18.43 shows a light ray incident on a polished metal cylinder. At what angle θ will the ray be reflected?

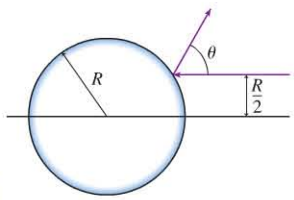


FIGURE P18.43

44. ||| The place you get your hair cut has two nearly parallel mirrors 5.0 m apart. As you sit in the chair, your head is 2.0 m from the nearer mirror. Looking toward this mirror, you first see your face and then, farther away, the back of your head. (The mirrors need to be slightly nonparallel for you to be able to see the back of your head, but you can treat them as parallel in this problem.) How far away does the back of your head appear to be? Neglect the thickness of your head.
45. | You shine your laser pointer through the flat glass side of a rectangular aquarium at an angle of incidence of 45°. The index of refraction of this type of glass is 1.55.
    a. At what angle from the normal does the beam from the laser pointer enter the water inside the aquarium?
    b. Does your answer to part a depend on the index of refraction of the glass?
46. || A ray of light traveling through air encounters a 1.2-cm-thick sheet of glass at a 35° angle of incidence. How far does the light ray travel in the glass before emerging on the far side?
47. || What is the angle of incidence in air of a light ray whose angle of refraction in glass is half the angle of incidence?
48. |||| Figure P18.48 shows a light ray incident on a glass cylinder. What is the angle α of the ray after it has entered the cylinder?

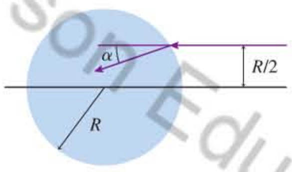


FIGURE P18.48

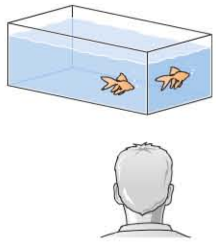


FIGURE P18.49

49. | If you look at a fish through the corner of a rectangular aquarium you sometimes see two fish, one on each side of the corner, as shown in Figure P18.49. Sketch some of the light rays that reach your eye from the fish to show how this can happen.
50. ||||| It's nighttime, and you've dropped your goggles into a swimming pool that is 3.0 m deep. If you hold a laser pointer 1.0 m directly above the edge of the pool, you can illuminate the goggles if the laser beam enters the water 2.0 m from the edge. How far are the goggles from the edge of the pool?
51. |||| One of the contests at the school carnival is to throw a spear at an underwater target lying flat on the bottom of a pool. The water is 1.0 m deep. You're standing on a small stool that places your eyes 3.0 m above the bottom of the pool. As you look at the target, your gaze is 30° below horizontal. At what angle below horizontal should you throw the spear in order to hit the target? Your raised arm brings the spear point to the level of your eyes as you throw it, and over this short distance you can assume that the spear travels in a straight line rather than a parabolic trajectory.
52. ||| Figure P18.52 shows a meter stick lying on the bottom of a 100-cm-long tank with its zero mark against the left edge. You look into the tank at a 30° angle, with your line of sight just grazing the upper left edge of the tank. What mark do you see on the meter stick if the tank is (a) empty, (b) half full of water, and (c) completely full of water?

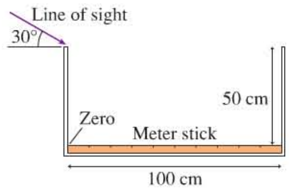


FIGURE P18.52

53. ||| There is just one angle of incidence β onto a prism for which the light inside an isosceles prism travels parallel to the base and emerges at that same angle β, as shown in Figure P18.53.
   a. Find an expression for β in terms of the prism's apex angle α and index of refraction n.
   b. A laboratory measurement finds that $\beta = 52.2°$ for a prism that is shaped as an equilateral triangle. What is the prism's index of refraction?

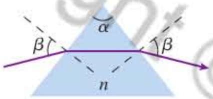


FIGURE P18.53

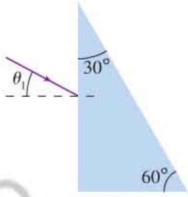


FIGURE P18.54

54. |||| What is the smallest angle θ_1 for which a laser beam will undergo total internal reflection on the hypotenuse of the glass prism in Figure P18.54?
55. ||| A 1.0-cm-thick layer of water stands on a horizontal slab of glass. Light from within the glass is incident on the glass-water boundary. What is the maximum angle of incidence for which a light ray can emerge into the air above the water?
56. ||||| The glass core of an optical fiber has index of refraction 1.60. The index of refraction of the cladding is 1.48. What is the maximum angle between a light ray and the wall of the core if the ray is to remain inside the core?
57. || A swimmer looks upward from the bottom of a 3.0-m-deep swimming pool. The end of the diving board is directly above him, 2.0 m above the water's surface. How far from the swimmer does the board appear to be?
58. |||| A 150-cm-tall diver is standing completely submerged on the bottom of a swimming pool full of water. You are sitting on the end of the diving board, almost directly over her. How tall does the diver appear to be?
59. || To a fish, the 4.00-mm-thick aquarium walls appear only 3.50 mm thick. What is the index of refraction of the walls?
60. || A microscope is focused on an amoeba. When a 0.15-mm-
BIO thick cover glass ($n = 1.50$) is placed over the amoeba, by how far must the microscope objective be moved to bring the organism back into focus? Must it be raised or lowered?
61. || A ray diagram can be used to find the location of an object if you are given the location of its image and the focal length of the mirror. Draw a ray diagram to find the height and position of an object that makes a 2.0-cm-high upright virtual image that appears 8.0 cm behind a convex mirror of focal length 20 cm.
62. | A 2.0-cm-tall object is located 8.0 cm in front of a converging lens with a focal length of 10 cm. Use ray tracing to determine the location and height of the image. Is the image upright or inverted? Is it real or virtual?
63. || The image produced by a converging lens is typically a different size from the object itself. However, for a lens with focal length f there is one object distance that will yield an image the same size as the object. What is that object distance?
64. |||| A near-sighted person might correct his vision by wearing
BIO diverging lenses with focal length $f = -50$ cm. When wearing his glasses, he looks not at actual objects but at the virtual images of those objects formed by his glasses. Suppose he looks at a 12-cm-long pencil held vertically 2.0 m from his glasses. Use ray tracing to determine the location and height of the image.
65. | A 1.0-cm-tall object is 20 cm in front of a converging lens that has a 10 cm focal length. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.
66. || A 2.0-cm-tall object is 20 cm in front of a converging lens that has a 60 cm focal length. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.
67. || A 1.0-cm-tall object is 7.5 cm in front of a diverging lens that has a 10 cm focal length. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.
68. || A 1.5-cm-tall object is 90 cm in front of a diverging lens that has a 45 cm focal length. Use ray tracing to find the position and height of the image. To do this accurately, use a ruler or paper with a grid. Determine the image distance and image height by making measurements on your diagram.
69. | A 1.6-m-tall woman stands 2.0 m in front of a convex funhouse mirror with a focal length of 2/3 m. Use ray tracing to determine the location and height of her image.
70. ||| A 2.0-cm-tall candle flame is 2.0 m from a wall. You happen to have a lens with a focal length of 32 cm. How many places can you put the lens to form a well-focused image of the candle flame on the wall? For each location, what are the height and orientation of the image?
71. || A 2.0-cm-diameter spider is 2.0 m from a wall. Determine the focal length and position (measured from the wall) of a lens that will make a half-size image of the spider on the wall.
72. ||| Figure P18.72 shows a meter stick held lengthwise along the optical axis of a concave mirror. How long is the image of the meter stick?

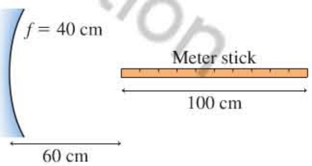


FIGURE P18.72

73. || A slide projector needs to create a 98-cm-high image of a 2.0-cm-tall slide. The screen is 300 cm from the slide.
   a. What focal length does the lens need? Assume that it is a thin lens.
   b. How far should you place the lens from the slide?
74. ||| The writing on the passenger-side mirror of your car says "Warning! Objects in mirror are closer than they appear." There is no such warning on the driver's mirror. Consider a typical convex passenger-side mirror with a focal length of −80 cm. A 1.5-m-tall cyclist on a bicycle is 25 m from the mirror. You are 1.0 m from the mirror, and suppose, for simplicity, that the mirror, you, and the cyclist all lie along a line.

   a. How far are you from the image of the cyclist?
   b. How far would you have been from the image if the mirror were flat?
   c. What is the image height?
   d. What would the image height have been if the mirror were flat?
   e. Why is there a label on the passenger-side mirror?

Passage Problems

Mirages

There is an interesting optical effect you have likely noticed while driving along a flat stretch of road on a sunny day. A small, distant dip in the road appears to be filled with water. You may even see the reflection of an oncoming car. But, as you get closer, you find no puddle of water after all; the shimmering surface vanishes, and you see nothing but empty road. It was only a *mirage,* the name for this phenomenon.

The mirage is due to the different index of refraction of hot and cool air. The actual bending of the light rays that produces the mirage is subtle, but we can make a simple model as follows. When air is heated, its density decreases and so does its index of refraction. Consequently, a pocket of hot air in a dip in a road has a lower index of refraction than the cooler air above it. Incident light rays with large angles of incidence (that is, nearly parallel to the road, as shown in Figure P18.75) experience total internal reflection. The mirage that you see is due to this reflection. As you get nearer, the angle goes below the critical angle and there is no more total internal reflection; the "water" disappears!

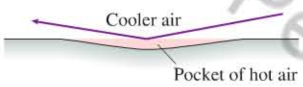


FIGURE P18.75

75. | The pocket of hot air appears to be a pool of water because
   A. Light reflects at the boundary between the hot and cool air.
   B. Its density is close to that of water.
   C. Light refracts at the boundary between the hot and cool air.
   D. The hot air emits blue light that is the same color as the day-time sky.
76. | Which of these changes would allow you to get closer to the mirage before it vanishes?
   A. Making the pocket of hot air nearer in temperature to the air above it
   B. Looking for the mirage on a windy day, which mixes the air layers
   C. Increasing the difference in temperature between the pocket of hot air and the air above it
   D. Looking at it from a greater height above the ground
77. | If you could clearly see the image of an object that was reflected by a mirage, the image would appear
   A. Magnified.
   B. With up and down reversed.
   C. Farther away than the object.
   D. With right and left reversed.

STOP TO THINK ANSWERS

Stop to Think 18.1: C. There's one image behind the vertical mirror and a second behind the horizontal mirror. A third image in the corner arises from rays that reflect twice, once off each mirror.

Stop to Think 18.2: A. The ray travels closer to the normal in both media 1 and 3 than in medium 2, so n_1 and n_3 are both larger than n_2. The angle is smaller in medium 3 than in medium 1, so $n_3 > n_1$.

Stop to Think 18.3: E. The rays from the object are diverging. Without a lens, the rays cannot converge to form any kind of image on the screen.

Stop to Think 18.4: C. For a converging mirror, the focal length f is the distance from the mirror at which incoming parallel rays meet. The moon is so distant that rays from any point on the moon are very nearly parallel. Thus the image of the moon would be very nearly at a distance f in front of the mirror.

Stop to Think 18.5: A. The thin-lens equation is $1/s + 1/s' = 1/f$. The focal length of the lens is fixed. Because $1/s$ gets smaller as s is increased, $1/s'$ must get larger to compensate. Thus s' must decrease.