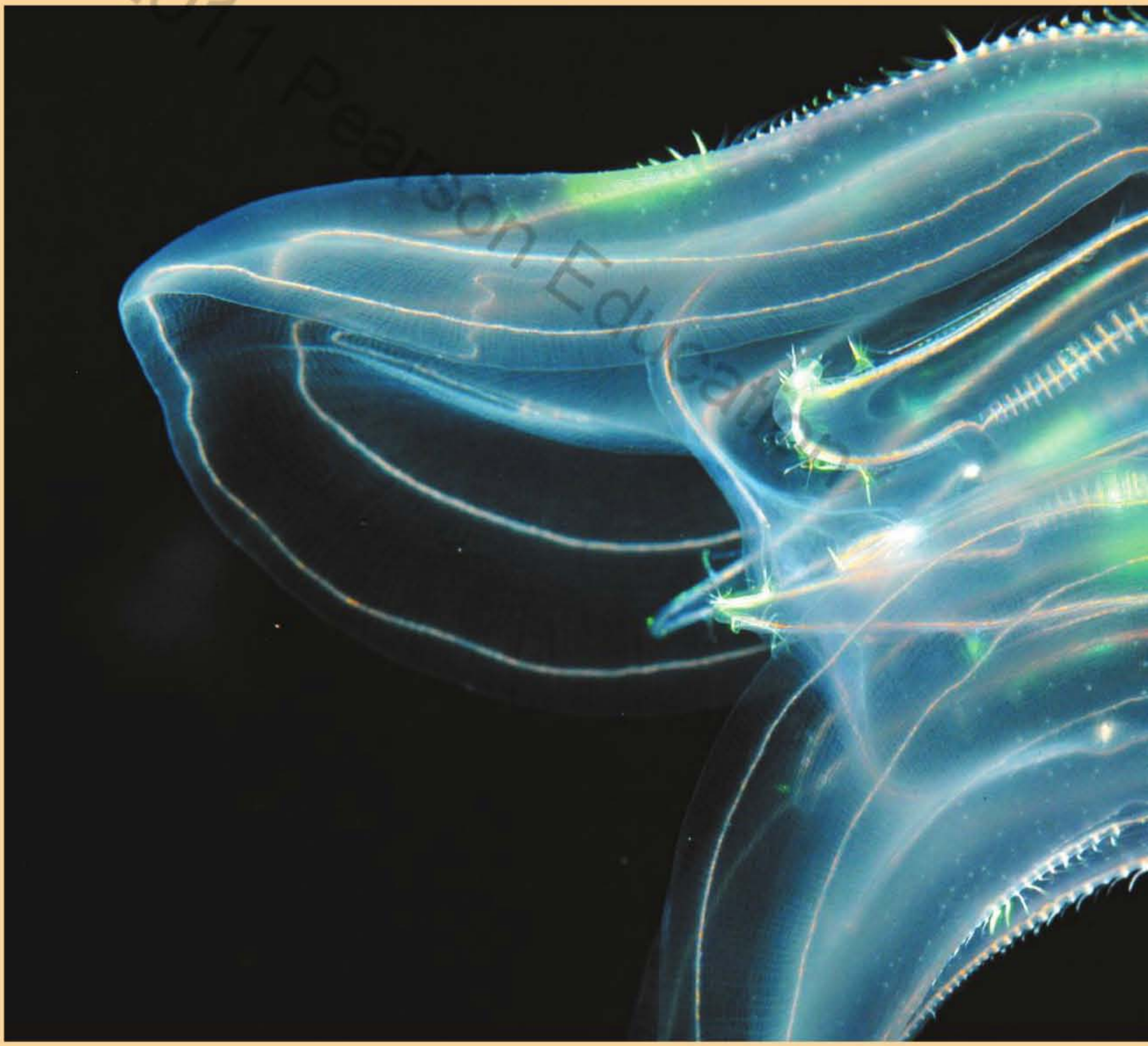
PART

VII

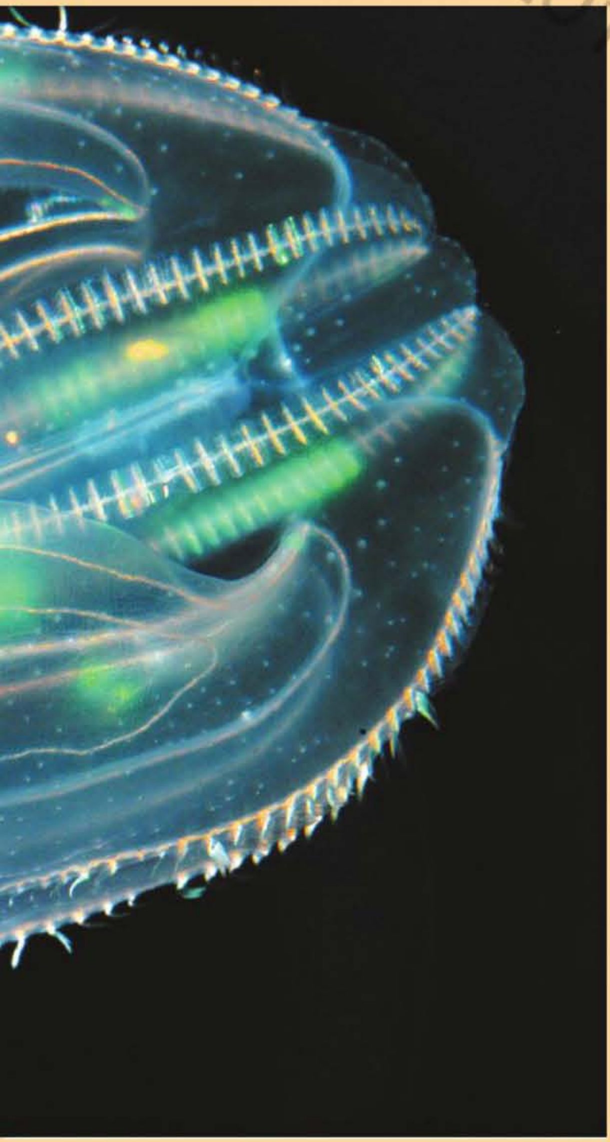
Modern Physics



The eerie glow of this comb jelly is due to *bioluminescence*. Energy released in chemical reactions in special cells is turned directly into a cool blue light. How is this light different from the light from a hot, incandescent filament?

New Ways of Looking at the World

Newton's mechanics and Maxwell's electromagnetism are remarkable theories that explain a wide range of physical phenomena, as we have seen in the past 26 chapters—but our story doesn't stop there. In the early 20th century, a series of discoveries profoundly altered our understanding of the universe at the most fundamental level, forcing scientists to reconsider the very nature of space and time and to develop new models of light and matter.



Relativity

The idea of measuring distance with a meter stick and time with a clock or stopwatch seems self-evident. But Albert Einstein, as a young, unknown scientist, realized that Maxwell's theory of electromagnetism could be consistent only if an additional rather odd assumption was made: that the speed of light is the same for all observers, no matter how they might be moving with respect to each other or to the source of the light. This assumption changes the way that we think about space and time. When we study Einstein's theory of *relativity,* you will see how different observers can disagree about lengths and time intervals. We need to go beyond stopwatches and meter sticks. Time can pass at different rates for different observers; time is, as you will see, relative. Our exploration will end with the most famous equation in physics, Einstein's $E = mc^2$. Matter can be converted to energy, and energy to matter.

Quantum Physics

We've seen that light, a wave, sometimes acts like a particle, a photon. We'll now find that particles such as electrons or atoms sometimes behave like waves. All of the characteristic of waves, such as diffraction and interference, will also apply to particles. This odd notion—that there's no clear distinction between particles and waves—is the core of a new model of light and matter called *quantum physics.* The wave nature of particles will lead to the *quantization* of energy. A particle confined in a box—or an electron in an atom—can have only certain energies. This idea will be a fruitful one for us, allowing us to understand the spectra of gases and phenomena such as bioluminescence.

Atoms, Nuclei, and Particles

We have frequently used the atomic model, explaining the properties of matter by considering the behavior of the atoms that comprise it. But when you get right down to it, what *is* an atom? And what's inside an atom? As you know, an atom has a tiny core called a *nucleus.* We'll look at what goes on inside the nucleus. One remarkable discovery will be that the nuclei of certain atoms spontaneously decay, turning the atom from one element to another. This phenomenon of *radioactivity* will give us a window into the nature of atoms and the particles that comprise them.

Once we know that the nucleus of an atom is composed of protons and neutrons, there's a natural next question to ask: What's inside a proton or neutron? There is an answer to this question, an answer we will learn in the final chapter of this book.

27 Relativity

For a wildlife conservation study, this turtle has been fitted with a collar containing a global positioning system (GPS) receiver, allowing the turtle to be tracked over thousands of miles with an accuracy of ±15 m. How does the theory of relativity affect the accuracy of a GPS receiver?

LOOKING AHEAD ▸▸

The goal of Chapter 27 is to understand how Einstein's theory of relativity changes our concepts of space and time.

What's Relativity All About?

Einstein's theory of relativity is based on a very simple-sounding principle: All the laws of physics are the same in all **inertial reference frames**—frames that move at a constant velocity.

One important consequence of this principle is that light travels at the same speed c in all inertial reference frames. This seemingly innocuous result will lead us to completely rethink our ideas of space and time.

The effects of relativity are large only for objects moving at *relativistic* speeds, close to the speed of light.

Looking Back ◂◂
3.5 Relative motion

For everyday motion, velocities add: The speed of the javelin equals the speed at which the athlete throws it *plus* her running speed.

Light and objects with speeds near c do not obey this simple rule. The speed of the light from this plane's landing lights is the *same* whether the plane is moving or not.

Simultaneity

We'll find that two events that occur at the same time in one reference frame occur at *different* times in another frame moving with respect to the first.

To you, the lightning strikes hit the ground simultaneously. The strikes are not simultaneous to someone moving relative to you.

Time Dilation

Relativity shows that *a moving clock runs slow* compared to a stationary clock.

Your GPS receiver receives signals from some of 24 highly accurate clocks in satellites orbiting the earth at high speeds. These clocks must be corrected for relativistic effects in order for the GPS system to work.

Length Contraction

The length of a moving object is *shortened* compared to a stationary object.

In the reference frame of an electron moving along this 2-mile-long particle accelerator at nearly the speed of light, the accelerator appears to be only 3.3 cm long!

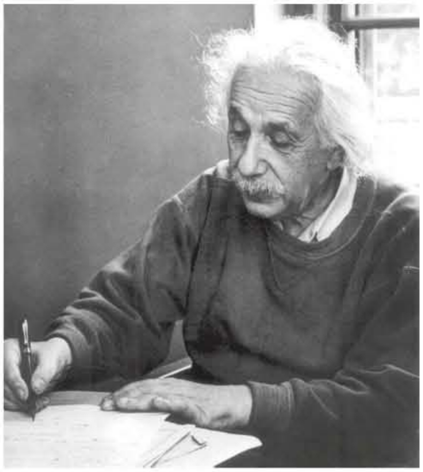
Mass and Energy

Einstein showed that mass and energy are equivalent, according to his famous equation $E = mc^2$.

The sun's energy comes from the conversion of 4 billion kilograms of matter into energy every second. Even so, the sun has enough mass to keep burning for another 4 billion years.

Looking Back ◂◂
10.3 Kinetic energy

27.1 Relativity: What's It All About?



Albert Einstein (1879–1955) was one of the most influential thinkers in history.

What do you think of when you hear the phrase "theory of relativity"? A white-haired Einstein? $E = mc^2$? Black holes? Time travel? Perhaps you've heard that the theory of relativity is so complicated and abstract that only a handful of people in the whole world really understand it.

There is, without doubt, a certain mystique associated with relativity, an aura of the strange and exotic. The good news is that understanding the ideas of relativity is well within your grasp. Einstein's *special theory of relativity,* the portion of relativity we'll study, is not mathematically difficult at all. The challenge is conceptual because relativity questions deeply held assumptions about the nature of space and time. In fact, that's what relativity is all about—space and time.

In one sense, relativity is not a new idea at all. Certain ideas about relativity are part of Newtonian mechanics: Newton's laws appear to hold just as well on a fast-moving train as they do in a stationary laboratory. Einstein, however, thought that relativity should apply to *all* the laws of physics, not just mechanics. The difficulty, as you'll see, is that some aspects of relativity appear to be incompatible with the laws of electromagnetism, particularly the laws governing the propagation of light waves.

Lesser scientists might have concluded that relativity simply doesn't apply to electromagnetism. Einstein's genius was to see that the incompatibility arises from *assumptions* about space and time, assumptions no one had ever questioned because they seem so obviously true. Rather than abandon the ideas of relativity, Einstein changed our understanding of space and time.

Fortunately, you need not be a genius to follow a path that someone else has blazed. However, we will have to exercise the utmost care with regard to logic and precision. We will need to state very precisely just how it is that we know things about the physical world and then ruthlessly follow the logical consequences. The challenge is to stay on this path, not to let our prior assumptions—assumptions that are deeply ingrained in all of us—lead us astray.

What's Special About Special Relativity?

Einstein's first paper on relativity, in 1905, dealt exclusively with inertial reference frames, reference frames that move relative to each other with constant velocity. Ten years later, Einstein published a more encompassing theory of relativity that considers accelerated motion and its connection to gravity. The second theory, because it's more general in scope, is called *general relativity.* General relativity is the theory that describes black holes, curved spacetime, and the evolution of the universe. It is a fascinating theory but, alas, very mathematical and outside the scope of this textbook.

Motion at constant velocity is a "special case" of motion—namely, motion for which the acceleration is zero. Hence Einstein's first theory of relativity has come to be known as *special relativity.* It is special in the sense of being a restricted, special case of his more general theory, not special in the everyday sense of meaning distinctive or exceptional. Special relativity, with its conclusions about time dilation and length contraction, is what we will study.

27.2 Galilean Relativity

Galileo was the first to understand how the laws of physics depended on the relative motion between different observers. A firm grasp of *Galilean relativity* is necessary if we are to appreciate and understand what is new in Einstein's theory. Thus we begin with the ideas of relativity that are embodied in Newtonian mechanics.

You may have had the experience of sitting on a train and looking up to see another train moving slowly past. It can be hard to tell if the other train is moving past your stationary train, or if you're moving in the opposite direction past a stationary train. Only the *relative* velocity between the trains has meaning.

Reference Frames

Suppose you're passing me as we both drive in the same direction along a freeway. My car's speedometer reads 55 mph while your speedometer shows 60 mph. Is 60 mph your "true" speed? That is certainly your speed relative to someone standing beside the road, but your speed relative to me is only 5 mph. Your speed is 120 mph relative to a driver approaching from the other direction at 60 mph.

An object does not have a "true" speed or velocity. The very definition of velocity, $v = \Delta x/\Delta t$, assumes the existence of a coordinate system in which, during some time interval Δt, the displacement Δx is measured. The best we can manage is to specify an object's velocity relative to, or with respect to, the coordinate system in which it is measured.

Let's define a **reference frame** to be a coordinate system in which experimenters equipped with meter sticks, stopwatches, and any other needed equipment make position and time measurements on moving objects. Three ideas are implicit in our definition of a reference frame:

- A reference frame extends infinitely far in all directions.
- The experimenters are at rest in the reference frame.
- The number of experimenters and the quality of their equipment are sufficient to measure positions and velocities to any level of accuracy needed.

The first two points are especially important. It is often convenient to say "the laboratory reference frame" or "the reference frame of the rocket." These are shorthand expressions for "a reference frame, infinite in all directions, in which the laboratory (or the rocket) and a set of experimenters happen to be at rest."

NOTE ► A reference frame is not the same thing as a "point of view." That is, each person or each experimenter does not have his or her own private reference frame. **All experimenters at rest relative to each other share the same reference frame.** ◄

FIGURE 27.1 The standard reference frames S and S′.

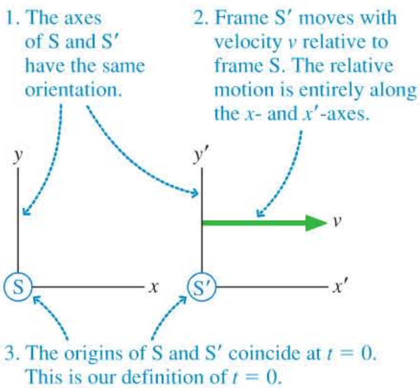
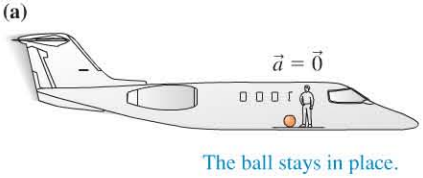


FIGURE 27.1 shows how we represent two reference frames, S and S′, that are in relative motion. The coordinate axes in S are x, y, z and those in S′ are x', y', z'. Reference frame S′ moves with velocity v relative to S or, equivalently, S moves with velocity $-v$ relative to S′. There's no implication that either reference frame is "at rest." Notice that the zero of time, when experimenters start their stopwatches, is the instant when the origins of S and S′ coincide.

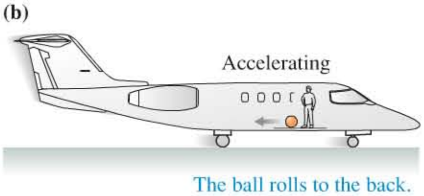
FIGURE 27.2 Two reference frames.

(a)



A ball with no horizontal forces stays at rest in an airplane cruising at constant velocity. The airplane is an inertial reference frame.

(b)



The ball rolls to the back of the plane during takeoff. An accelerating plane is not an inertial reference frame.

Inertial Reference Frames

Certain reference frames are especially simple. FIGURE 27.2a shows a physics student cruising at constant velocity in an airplane. If the student places a ball on the floor, it stays there. There are no horizontal forces, and the ball remains at rest relative to the airplane. That is, $\vec{a} = \vec{0}$ in the airplane's coordinate system when $\vec{F}_{net} = \vec{0}$, so Newton's first law is satisfied. Similarly, if the student drops the ball, it falls straight down—relative to the student—with an acceleration of magnitude g, satisfying Newton's second law.

We define an **inertial reference frame** as one in which Newton's first law is valid. That is, an inertial reference frame is one in which an isolated particle, one on which there are no forces, either remains at rest or moves in a straight line at constant speed, as measured by experimenters at rest in that frame.

Not all reference frames are inertial. The physics student in FIGURE 27.2b conducts the same experiment during takeoff. He carefully places the ball on the floor just as the airplane starts to accelerate down the runway. You can imagine what happens. The ball rolls to the back of the plane as the passengers are being pressed back into their seats. If the student measures the ball's motion using a meter stick attached to the plane, he will find that the ball accelerates *in the plane's reference frame.* Yet he would be unable to identify any force on the ball that would act to accelerate it toward the back of the plane. This violates Newton's first law, so the plane is *not* an inertial reference frame during takeoff. **In general, accelerating reference frames are not inertial reference frames.**

NOTE ► An inertial reference frame is an idealization. A true inertial reference frame would need to be floating in deep space, far from any gravitational influence. In practice, an earthbound laboratory is a good approximation of an inertial reference frame because the accelerations associated with the earth's rotation and motion around the sun are too small to influence most experiments. ◄

These ideas are in accord with your everyday experience. If you're in a jet flying smoothly at 600 mph—an inertial reference frame—Newton's laws are valid: You can toss and catch a ball, or pour a cup of coffee, exactly as you would on the ground. But if the plane were diving, or shaking from turbulence, simple "experiments" like these would fail. A ball thrown straight up would land far from your hand, and the stream of coffee would bend and turn on its way to missing the cup. These apparently simple observations can be stated as the *Galilean principle of relativity:*

This flight attendant pours wine on a smoothly flying airplane moving at 600 mph just as easily as she does at the terminal. These ideas were first discussed by Galileo in 1632 in the context of pouring water while on a moving ship.

Galilean principle of relativity Newton's laws of motion are valid in all inertial reference frames.

In our study of relativity, we will restrict our attention to inertial reference frames. This implies that the relative velocity v between two reference frames is constant. Any reference frame that moves at constant velocity with respect to an inertial reference frame is itself an inertial reference frame. Conversely, a reference frame that accelerates with respect to an inertial reference frame is not an inertial reference frame. Although special relativity can be used for accelerating reference frames, we will confine ourselves here to the simple case of inertial reference frames moving with respect to each other at constant velocity.

STOP TO THINK 27.1 Which of these is an inertial reference frame (or a very good approximation)?

A. Your bedroom
B. A car rolling down a steep hill
C. A train coasting along a level track
D. A rocket being launched
E. A roller coaster going over the top of a hill
F. A sky diver falling at terminal speed

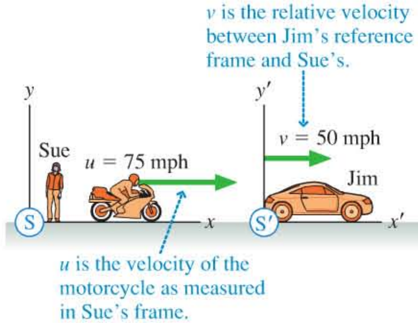
The Galilean Velocity Transformation

Special relativity is largely concerned with how physical quantities such as position and time are measured by experimenters in different reference frames. Let's begin by studying how, within Galilean relativity, the *velocity* of an object is measured in different reference frames. We have already touched on these ideas back in Section 3.5.

Suppose Sue is standing beside a highway as Jim drives by at 50 mph, as shown in **FIGURE 27.3**. Let S be Sue's reference frame—a reference frame attached to the ground—and let S′ be the reference frame moving with Jim, attached to his car. We see that the velocity of reference frame S′ relative to S is $v = 50$ mph.

Now suppose a motorcyclist blasts down the highway, traveling in the same direction as Jim. Sue measures the motorcycle's velocity to be $u = 75$ mph. What is

FIGURE 27.3 A motorcycle's velocity as seen by Sue and by Jim.



the cycle's velocity u' measured relative to Jim? We can answer this on the basis of common sense. If you're driving at 50 mph, and someone passes you going 75 mph, then his speed *relative to you* is 25 mph. This is the *difference* between his speed relative to the ground and your speed relative to the ground. Thus Jim measures the motorcycle's velocity to be $u' = 25$ mph.

NOTE ► In this chapter, we will use v to represent the velocity of one reference frame relative to another. We will use u and u' to represent the velocities of objects with respect to reference frames S and S′. In addition, we will assume that all motion is parallel to the x-axis. ◄

We can state this idea as a general rule. If u is the velocity of an object as measured in reference frame S, and u' its velocity as measured in S′, then the two velocities are related by

$$u' = u - v \qquad \text{and} \qquad u = u' + v \tag{27.1}$$

Galilean transformation of velocity, where v is the velocity of S′ relative to S

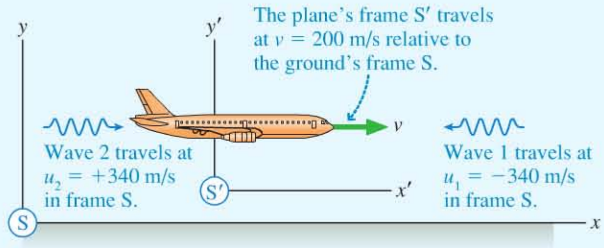
Equations 27.1 are the *Galilean transformation of velocity.* If you know the velocity of a particle as measured by the experimenters in one inertial reference frame, you can use Equations 27.1 to find the velocity that would be measured by experimenters in any other inertial reference frame.

EXAMPLE 27.1 **Finding the speed of sound**

An airplane is flying at speed 200 m/s with respect to the ground. Sound wave 1 is approaching the plane from the front, while sound wave 2 is catching up from behind. Both waves travel at 340 m/s relative to the ground. What is the velocity of each wave relative to the plane?

PREPARE Assume that the earth (frame S) and the airplane (frame S′) are inertial reference frames. Frame S′, in which the airplane is at rest, moves with velocity $v = 200$ m/s relative to frame S. **FIGURE 27.4** shows the airplane and the sound waves.

FIGURE 27.4 Experimenters in the plane measure different speeds for the sound waves than do experimenters on the ground.



SOLVE The speed of a mechanical wave, such as a sound wave or a wave on a string, is its speed *relative to its medium.* Thus the *speed of sound* is the speed of a sound wave through a reference frame in which the air is at rest. This is reference frame S, where wave 1 travels with velocity $u_1 = -340$ m/s and wave 2 travels with velocity $u_2 = +340$ m/s. Notice that the Galilean transformations use *velocities,* with appropriate signs, not just speeds.

The airplane travels to the right with reference frame S′ at velocity v. We can use the Galilean transformations of velocity to find the velocities of the two sound waves in frame S′:

$$u_1' = u_1 - v = -340 \text{ m/s} - 200 \text{ m/s} = -540 \text{ m/s}$$
$$u_2' = u_2 - v = 340 \text{ m/s} - 200 \text{ m/s} = 140 \text{ m/s}$$

Thus wave 1 approaches the plane with a *speed* of 540 m/s, while wave 2 approaches with a speed of 140 m/s.

ASSESS This isn't surprising. If you're driving at 50 mph, a car coming the other way at 55 mph is approaching you at 105 mph. A car coming up behind you at 55 mph seems to be gaining on you at the rate of only 5 mph. Wave speeds behave the same. Notice that a mechanical wave would appear to be stationary to a person moving at the wave speed. To a surfer, the crest of the ocean wave remains at rest under his or her feet.

STOP TO THINK 27.2 Ocean waves are approaching the beach at 10 m/s. A boat heading out to sea travels at 6 m/s. How fast are the waves moving in the boat's reference frame?

A. 16 m/s B. 10 m/s C. 6 m/s D. 4 m/s

27.3 Einstein's Principle of Relativity

The 19th century was an era of optics and electromagnetism. Thomas Young demonstrated in 1801 that light is a wave, and by midcentury scientists had devised techniques for measuring the speed of light. Faraday discovered electromagnetic induction in 1831, setting in motion a train of events leading to Maxwell's conclusion, in 1864, that light is an electromagnetic wave.

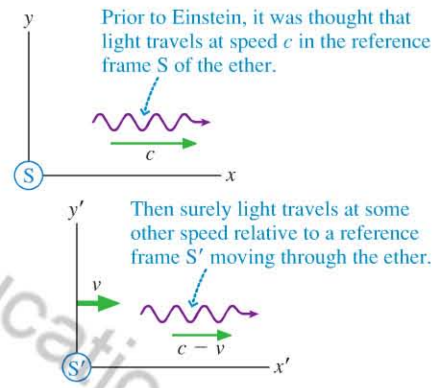
If light is a wave, what is the medium in which it travels? This was perhaps the most important scientific question in the second half of the 19th century. The medium in which light waves were assumed to travel was called the *ether.* Experiments to measure the speed of light were assumed to be measuring its speed through the ether. But just what is the ether? What are its properties? Can we collect a jar full of ether to study? Despite the significance of these questions, experimental efforts to detect the ether or measure its properties kept coming up empty-handed.

Maxwell's theory of electromagnetism didn't help the situation. The crowning success of Maxwell's theory was his prediction that light waves travel with speed

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}$$

This is a very specific prediction with no wiggle room. The difficulty with such a specific prediction was the implication that Maxwell's laws of electromagnetism are valid *only* in the reference frame of the ether. After all, as FIGURE 27.5 shows, the light speed should certainly be faster or slower than c in a reference frame moving through the ether, just as the sound speed is different to someone moving through the air.

FIGURE 27.5 It seems as if the speed of light should differ from c in a reference frame moving through the ether.



As the 19th century closed, it appeared that Maxwell's theory did not obey the classical principle of relativity. There was just one reference frame, the reference frame of the ether, in which the laws of electromagnetism seemed to be true. And to make matters worse, the fact that no one had been able to detect the ether meant that no one could identify the one reference frame in which Maxwell's equations "worked."

It was in this muddled state of affairs that a young Albert Einstein made his mark on the world. Even as a teenager, Einstein had wondered how a light wave would look to someone "surfing" the wave, traveling alongside the wave at the wave speed. You can do that with a water wave or a sound wave, but light waves seemed to present a logical difficulty. An electromagnetic wave sustains itself by virtue of the fact that a changing magnetic field induces an electric field and a changing electric field induces a magnetic field. But to someone moving with the wave, *the fields would not change.* How could there be an electromagnetic wave under these circumstances?

Several years of thinking about the connection between electromagnetism and reference frames led Einstein to the conclusion that *all* the laws of physics, not just the laws of mechanics, should obey the principle of relativity. In other words, the principle of relativity is a fundamental statement about the nature of the physical universe. The Galilean principle of relativity stated only that Newton's laws hold in any inertial reference frame. Einstein was able to state a much more general principle:

In certain rivers, tides send waves upriver that can be surfed for miles. From the reference frame of these surfers, the waves are standing still. If you could move along with a light wave, would the electric and magnetic fields appear motionless?

> **Principle of relativity** All the laws of physics are the same in all inertial reference frames.

All of the results of Einstein's theory of relativity flow from this one simple statement.

The Constancy of the Speed of Light

If Maxwell's equations of electromagnetism are laws of physics, and there's every reason to think they are, then, according to the principle of relativity, Maxwell's equations must be true in *every* inertial reference frame. On the surface this seems to be an innocuous statement, equivalent to saying that the law of conservation of momentum is true in every inertial reference frame. But follow the logic:

1. Maxwell's equations are true in all inertial reference frames.
2. Maxwell's equations predict that electromagnetic waves, including light, travel at speed $c = 3.00 \times 10^8$ m/s.
3. Therefore, **light travels at speed c in all inertial reference frames.**

FIGURE 27.6 Light travels at speed c in all inertial reference frames, regardless of how the reference frames are moving with respect to the light source.

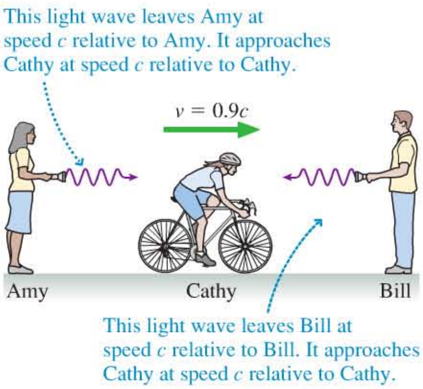
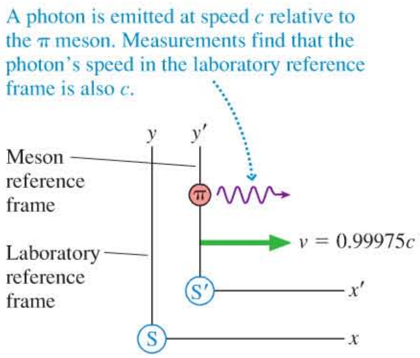


FIGURE 27.6 shows the implications of this conclusion. *All* experimenters, regardless of how they move with respect to each other, find that *all* light waves, regardless of the source, travel in their reference frame with the *same* speed c. If Cathy's velocity toward Bill and away from Amy is $v = 0.9c$, Cathy finds, by making measurements in her reference frame, that the light from Bill approaches her at speed c, not at $c + v = 1.9c$. And the light from Amy, which left Amy at speed c, catches up from behind at speed *c relative to Cathy,* not the $c - v = 0.1c$ you would have expected.

Although this prediction goes against all shreds of common sense, the experimental evidence for it is strong. Laboratory experiments are difficult because even the highest laboratory speed is insignificant in comparison to c. In the 1930s, however, the physicists R. J. Kennedy and E. M. Thorndike realized that they could use the earth itself as a laboratory. The earth's speed as it circles the sun is about 30,000 m/s. The velocity of the earth in January differs by 60,000 m/s from its velocity in July, when the earth is moving in the opposite direction. Kennedy and Thorndike were able to use a very sensitive and stable interferometer to show that the numerical values of the speed of light in January and July differ by less than 2 m/s.

FIGURE 27.7 Experiments find that the photons travel through the laboratory with speed c, not the speed $1.99975c$ that you might expect.



More recent experiments have used unstable elementary particles, called π mesons, that decay into high-energy photons, or particles of light. The π mesons, created in a particle accelerator, move through the laboratory at 99.975% the speed of light, or $v = 0.99975c$, as they emit photons at speed c in the π meson's reference frame. As **FIGURE 27.7** shows, you would expect the photons to travel through the laboratory with speed $c + v = 1.99975c$. Instead, the measured speed of the photons in the laboratory was, within experimental error, 3.00×10^8 m/s.

In summary, *every* experiment designed to compare the speed of light in different reference frames has found that light travels at 3.00×10^8 m/s in every inertial reference frame, regardless of how the reference frames are moving with respect to each other.

How Can This Be?

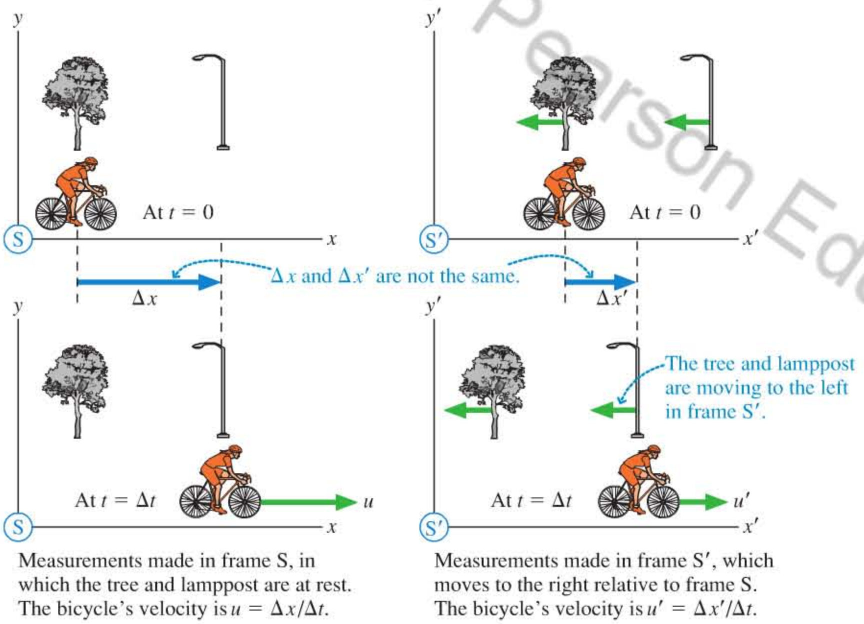
You're in good company if you find this impossible to believe. Suppose I shot a ball forward at 50 m/s while driving past you at 30 m/s. You would certainly see the ball traveling at 80 m/s relative to you and the ground. What we're saying with regard to light is equivalent to saying that the ball travels at 50 m/s relative to my car and *at the same time* travels at 50 m/s relative to the ground, even though the car is moving across the ground at 30 m/s. It seems logically impossible.

You might think that this is merely a matter of semantics. If we can just get our definitions and use of words straight, then the mystery and confusion will disappear. Or perhaps the difficulty is a confusion between what we "see" versus what "really happens." In other words, a better analysis, one that focuses on what really happens, would find that light "really" travels at different speeds in different reference frames.

Alas, what "really happens" is that light travels at 3.00×10^8 m/s in every inertial reference frame, regardless of how the reference frames are moving with respect to each other. It's not a trick. There remains only one way to escape the logical contradictions.

The definition of velocity is $u = \Delta x/\Delta t$, the ratio of a distance traveled to the time interval in which the travel occurs. Suppose you and I both make measurements on an object as it moves, but you happen to be moving relative to me. Perhaps I'm standing on the corner, you're jogging to the right, and we're both trying to measure the velocity of a bicycle moving to the right as it passes both of us. Further, suppose we have agreed in advance to measure the bicycle as it moves from the tree to the lamppost in **FIGURE 27.8**. Your $\Delta x'$ differs from my Δx because of your motion relative to me, causing you to calculate a bicycle velocity u' in your reference frame that differs from its velocity u in my reference frame. This is just the Galilean transformations showing up again.

FIGURE 27.8 Measuring the velocity of an object by appealing to the basic definition $u = \Delta x/\Delta t$.



Now let's repeat the measurements, but this time let's measure the velocity of a light wave as it travels from the tree to the lamppost. Once again, your $\Delta x'$ differs from my Δx, although the difference will be pretty small unless you happen to be jogging at a speed that is an appreciable fraction of the speed of light. The obvious conclusion is that your light speed u' differs from my light speed u. But it doesn't. The experiments show that, for a light wave, we'll get the *same* values: $u' = u = c$.

The only way this can be true is if your Δt is not the same as my Δt. If the time it takes the light to move from the tree to the lamppost in your reference frame, a time we'll now call $\Delta t'$, differs from the time Δt it takes the light to move from the tree to the lamppost in my reference frame, then we might find that $\Delta x'/\Delta t' = \Delta x/\Delta t$. That is, $u' = u$ even though you are moving with respect to me.

We've assumed, since the beginning of this textbook, that time is simply time. It flows along like a river, and all experimenters in all reference frames simply use it. For example, suppose the tree and the lamppost both have big clocks that we both can see. Shouldn't we be able to agree on the time interval Δt the light needs to move from the tree to the lamppost?

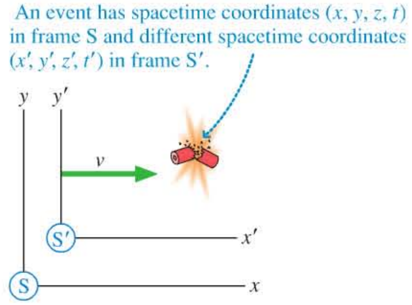
Perhaps not. It's demonstrably true that $\Delta x' \neq \Delta x$. It's experimentally verified that $u' = u$ for light waves. Something must be wrong with *assumptions* that we've made about the nature of time. The principle of relativity has painted us into a corner, and our only way out is to reexamine our understanding of time.

27.4 Events and Measurements

To question some of our most basic assumptions about space and time requires extreme care. We need to be certain that no assumptions slip into our analysis unnoticed. Our goal is to describe the motion of a particle in a clear and precise way, making the barest minimum of assumptions.

Events

FIGURE 27.9 The location and time of an event are described by its spacetime coordinates.



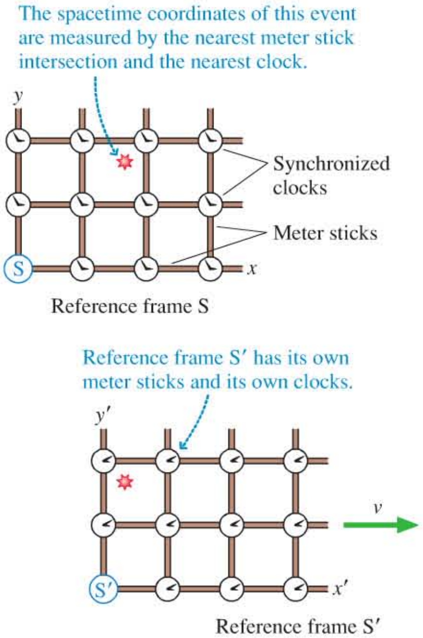
The fundamental entity of relativity is called an **event.** An event is a physical activity that takes place at a definite point in space and at a definite instant of time. A firecracker exploding is an event. A collision between two particles is an event. A light wave hitting a detector is an event.

Events can be observed and measured by experimenters in different reference frames. An exploding firecracker is as clear to you as you drive by in your car as it is to me standing on the street corner. We can quantify where and when an event occurs with four numbers: the coordinates (x, y, z) and the instant of time t. These four numbers, illustrated in FIGURE 27.9, are called the **spacetime coordinates** of the event.

The spatial coordinates of an event measured in reference frames S and S′ may differ. But it now appears that the instant of time recorded in S and S′ may also differ. Thus the spacetime coordinates of an event measured by experimenters in frame S are (x, y, z, t), and the spacetime coordinates of the *same event* measured by experimenters in frame S′ are (x', y', z', t').

The motion of a particle can be described as a sequence of two or more events. We introduced this idea in the preceding section when we agreed to measure the velocity of a bicycle and then of a light wave by comparing the object passing the tree (first event) to the object passing the lamppost (second event).

FIGURE 27.10 The spacetime coordinates of an event are measured by a lattice of meter sticks and clocks.



Measurements

Events are what "really happen," but how do we learn about an event? That is, how do the experimenters in a reference frame determine the spacetime coordinates of an event? This is a problem of *measurement.*

We defined a reference frame to be a coordinate system in which experimenters can make position and time measurements. That's a good start, but now we need to be more precise as to *how* the measurements are made. Imagine that a reference frame is filled with a cubic lattice of meter sticks, as shown in FIGURE 27.10. At every intersection is a clock, and all the clocks in a reference frame are *synchronized.* We'll return in a moment to consider how to synchronize the clocks, but assume for the moment it can be done.

Now, with our meter sticks and clocks in place, we can use a two-part measurement scheme:

- The (x, y, z) coordinates of an event are determined by the intersection of meter sticks closest to the event.
- The event's time t is the time displayed on the clock nearest the event.

You can imagine, if you wish, that each event is accompanied by a flash of light to illuminate the face of the nearest clock and make its reading known.

Several important issues need to be noted:

1. The clocks and meter sticks in each reference frame are imaginary, so they have no difficulty passing through each other.
2. Measurements of position and time made in one reference frame must use only the clocks and meter sticks in that reference frame.
3. There's nothing special about the sticks being 1 m long and the clocks 1 m apart. The lattice spacing can be altered to achieve whatever level of measurement accuracy is desired.
4. We'll assume that the experimenters in each reference frame have assistants sitting beside every clock to record the position and time of nearby events.
5. Perhaps most important, t is the time at which the event *actually happens,* not the time at which an experimenter sees the event or at which information about the event reaches an experimenter.
6. All experimenters in one reference frame agree on the spacetime coordinates of an event. In other words, **an event has a unique set of spacetime coordinates in each reference frame.**

STOP TO THINK 27.3 A carpenter is working on a house two blocks away. You notice a slight delay between seeing the carpenter's hammer hit the nail and hearing the blow. At what time does the event "hammer hits nail" occur?

A. At the instant you hear the blow
B. At the instant you see the hammer hit
C. Very slightly before you see the hammer hit
D. Very slightly after you see the hammer hit

Clock Synchronization

It's important that all the clocks in a reference frame be **synchronized,** meaning that all clocks in the reference frame have the same reading at any one instant of time. We would not be able to use a sequence of events to track the motion of a particle if the clocks differed in their readings. Thus we need a method of synchronization. One idea that comes to mind is to designate the clock at the origin as the *master clock.* We could then carry this clock around to every clock in the lattice, adjust that clock to match the master clock, and finally return the master clock to the origin.

This would be a perfectly good method of clock synchronization in Newtonian mechanics, where time flows along smoothly, the same for everyone. But we've been driven to reexamine the nature of time by the possibility that time is different in reference frames moving relative to each other. Because the master clock would *move,* we cannot assume that the master clock keeps time in the same way as the stationary clocks.

It's easy to synchronize clocks that are all in one place, but synchronizing distant clocks takes some care.

We need a synchronization method that does not require moving the clocks. Fortunately, such a method is easy to devise. Each clock is resting at the intersection of meter sticks, so by looking at the meter sticks, the assistant knows, or can calculate, exactly how far each clock is from the origin. Once the distance is known, the assistant can calculate exactly how long a light wave will take to travel from the origin to each clock. For example, light will take 1.00 μs to travel to a clock 300 m from the origin.

NOTE ► It's handy for many relativity problems to know that the speed of light is $c = 300\ \text{m}/\mu\text{s}$. ◄

To synchronize the clocks, the assistants begin by setting each clock to display the light travel time from the origin, but they don't start the clocks. Next, as

FIGURE 27.11 Synchronizing the clocks.

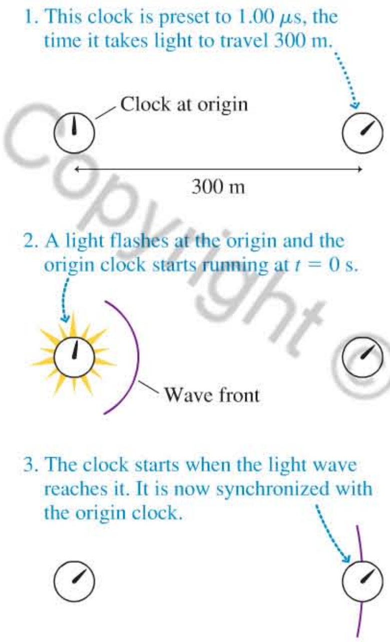


FIGURE 27.11 shows, a light flashes at the origin and, simultaneously, the clock at the origin starts running from $t = 0$ s. The light wave spreads out in all directions at speed c. A photodetector on each clock recognizes the arrival of the light wave and, without delay, starts the clock. The clock had been preset with the light travel time, so each clock as it starts reads exactly the same as the clock at the origin. Thus all the clocks will be synchronized after the light wave has passed by.

Events and Observations

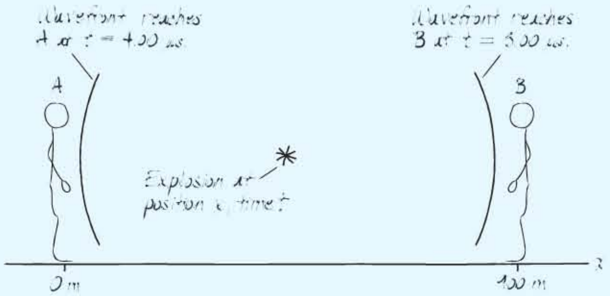
We noted above that t is the time the event *actually happens*. This is an important point, one that bears further discussion. Light waves take time to travel. Messages, whether they're transmitted by light pulses, telephone, or courier on horseback, take time to be delivered. An experimenter *observes* an event, such as an exploding firecracker, only *at a later time* when light waves reach his or her eyes. But our interest is in the event itself, not the experimenter's observation of the event. The time at which the experimenter sees the event or receives information about the event is not when the event actually occurred.

Suppose at $t = 0$ s a firecracker explodes at $x = 300$ m. The flash of light from the firecracker will reach an experimenter at the origin at $t_1 = 1.0\ \mu$s. The sound of the explosion will reach the experimenter at $t_2 = 0.88$ s. Neither of these is the time t_{event} of the explosion, although the experimenter can work backward from these times, using known wave speeds, to determine t_{event}. In this example, the spacetime coordinates of the event—the explosion—are (300 m, 0 m, 0 m, 0 s).

EXAMPLE 27.2 Finding the time of an event

Experimenter A in reference frame S stands at the origin looking in the positive x-direction. Experimenter B stands at $x = 900$ m looking in the negative x-direction. A firecracker explodes somewhere between them. Experimenter B sees the light flash at $t = 3.00\ \mu$s. Experimenter A sees the light flash at $t = 4.00\ \mu$s. What are the spacetime coordinates of the explosion?

PREPARE Experimenters A and B are in the same reference frame and have synchronized clocks. FIGURE 27.12 shows the two experimenters and the explosion at unknown position x.



SOLVE The two experimenters observe light flashes at two different instants, but there's only one event. Light travels at 300 m/μs, so the additional 1.00 μs needed for the light to reach experimenter A implies that distance $(x - 0\text{ m})$ from x to A is 300 m longer than distance $(900\text{ m} - x)$ from B to x; that is,

$$(x - 0\text{ m}) = (900\text{ m} - x) + 300\text{ m}$$

This is easily solved to give $x = 600$ m as the position coordinate of the explosion. The light takes 1.00 μs to travel 300 m to experimenter B and 2.00 μs to travel 600 m to experimenter A. The light is received at 3.00 μs and 4.00 μs, respectively; hence it was emitted by the explosion at $t = 2.00\ \mu$s. The spacetime coordinates of the explosion are (600 m, 0 m, 0 m, 2.00 μs).

ASSESS Although the experimenters *see* the explosion at different times, they agree that the explosion actually *happened* at $t = 2.00\ \mu$s.

◀ FIGURE 27.12 The light wave reaches the experimenters at different times. Neither of these is the time at which the event actually happened.

Simultaneity

Two events 1 and 2 that take place at different positions x_1 and x_2 but at the *same time* $t_1 = t_2$, as measured in some reference frame, are said to be **simultaneous** in that reference frame. Simultaneity is determined by when the events actually happen, not when they are seen or observed. In general, simultaneous events are not *seen* at the same time because of the difference in light travel times from the events to an experimenter.

EXAMPLE 27.3 Are the explosions simultaneous?

An experimenter in reference frame S stands at the origin looking in the positive x-direction. At $t = 3.0\ \mu s$ she sees firecracker 1 explode at $x = 600$ m. A short time later, at $t = 5.0\ \mu s$, she sees firecracker 2 explode at $x = 1200$ m. Are the two explosions simultaneous? If not, which firecracker exploded first?

PREPARE Light from both explosions travels toward the experimenter at 300 m/μs.

SOLVE The experimenter *sees* two different explosions, but perceptions of the events are not the events themselves. When did the explosions *actually* occur? Using the fact that light travels 300 m/μs, it's easy to see that firecracker 1 exploded at $t_1 = 1.0\ \mu s$ and firecracker 2 also exploded at $t_2 = 1.0\ \mu s$. The events *are* simultaneous.

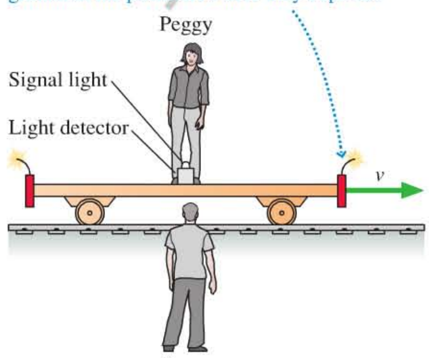
STOP TO THINK 27.4 A tree and a pole are 3000 m apart. Each is suddenly hit by a bolt of lightning. Mark, who is standing at rest midway between the two, sees the two lightning bolts at the same instant of time. Nancy is at rest under the tree. Define event 1 to be "lightning strikes tree" and event 2 to be "lightning strikes pole." For Nancy, does event 1 occur before, after, or at the same time as event 2?

27.5 The Relativity of Simultaneity

We've now established a means for measuring the time of an event in a reference frame, so let's begin to investigate the nature of time. The following "thought experiment" is very similar to one suggested by Einstein.

FIGURE 27.13 shows a long railroad car traveling to the right with a velocity v that may be an appreciable fraction of the speed of light. A firecracker is tied to each end of the car, right above the ground. Each firecracker is powerful enough that, when it explodes, it will make a burn mark on the ground at the position of the explosion.

FIGURE 27.13 A railroad car traveling to the right with velocity v.



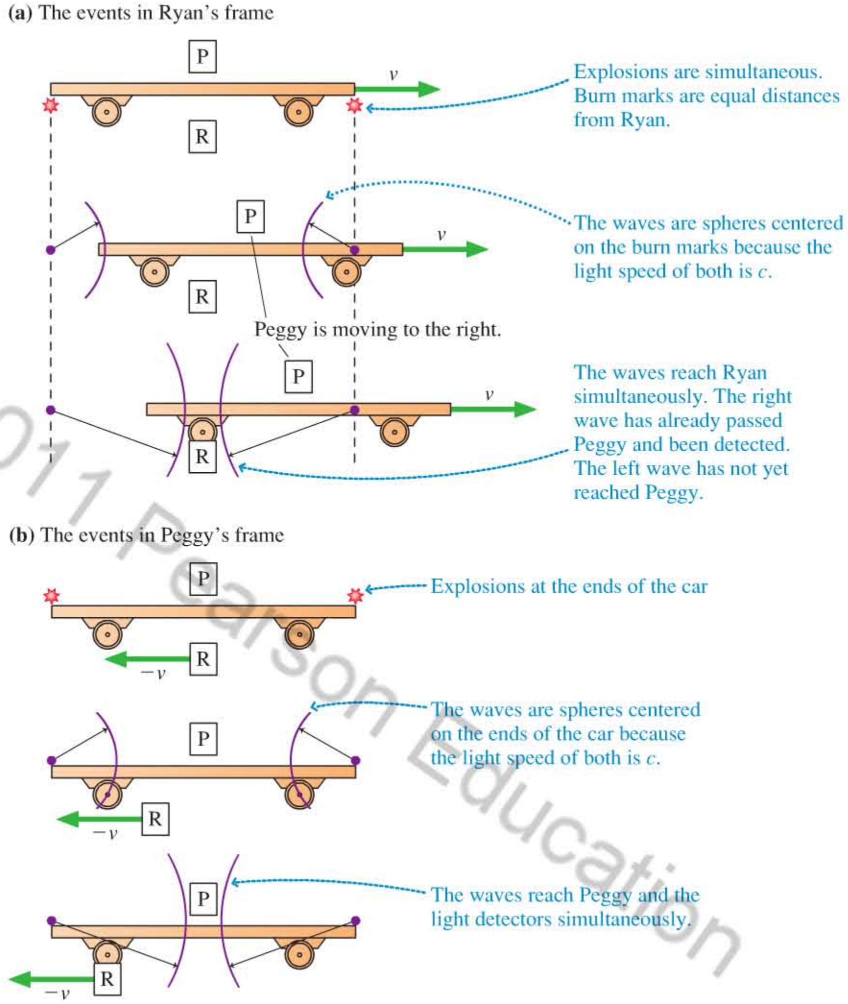
Ryan is standing on the ground, watching the railroad car go by. Peggy is standing in the exact center of the car with a special box at her feet. This box has two light detectors, one facing each way, and a signal light on top. The box works as follows:

1. If a flash of light is received at the right detector before a flash is received at the left detector, then the light on top of the box will turn green.
2. If a flash of light is received at the left detector before a flash is received at the right detector, or if two flashes arrive simultaneously, the light on top will turn red.

The firecrackers explode as the railroad car passes Ryan, and he sees the two light flashes from the explosions simultaneously. He then measures the distances to the two burn marks and finds that he was standing exactly halfway between the marks. Because light travels equal distances in equal times, Ryan concludes that the two explosions were simultaneous in his reference frame, the reference frame of the ground. Further, because he was midway between the two ends of the car, he was directly opposite Peggy when the explosions occurred.

FIGURE 27.14a on the next page shows the sequence of events in Ryan's reference frame. Light travels at speed c in all inertial reference frames, so, although the firecrackers were moving, the light waves are spheres centered on the burn marks. Ryan determines that the light wave coming from the right reaches Peggy and the box before the light wave coming from the left. Thus, according to Ryan, the signal light on top of the box turns green.

FIGURE 27.14 Exploding firecrackers seen in two different reference frames.



How do things look in Peggy's reference frame, a reference frame moving to the right at velocity v relative to the ground? As FIGURE 27.14b shows, Peggy sees Ryan moving to the left with speed v. Light travels at speed c in all inertial reference frames, so the light waves are spheres centered on the ends of the car. If the explosions are simultaneous, as Ryan has determined, the two light waves reach her and the box simultaneously. Thus, according to Peggy, the signal light on top of the box turns red!

Now the light on top must be either green or red. *It can't be both!* Later, after the railroad car has stopped, Ryan and Peggy can place the box in front of them. It has either a red light or a green light. Ryan can't see one color while Peggy sees the other. Hence we have a paradox. It's impossible for Peggy and Ryan both to be right. But who is wrong, and why?

What do we know with absolute certainty?

1. Ryan detected the flashes simultaneously.
2. Ryan was halfway between the firecrackers when they exploded.
3. The light from the two explosions traveled toward Ryan at equal speeds.

The conclusion that the explosions were simultaneous in Ryan's reference frame is unassailable. The light is green.

Peggy, however, made an assumption. It's a perfectly ordinary assumption, one that seems sufficiently obvious that you probably didn't notice, but an assumption nonetheless. Peggy assumed that the explosions were simultaneous.

Didn't Ryan find them to be simultaneous? Indeed, he did. Suppose we call Ryan's reference frame S, the explosion on the right event R, and the explosion on the left event L. Ryan found that $t_R = t_L$. But Peggy has to use a different set of clocks, the clocks in her reference frame S′, to measure the times t'_R and t'_L at which the explosions occurred. The fact that $t_R = t_L$ in frame S does *not* allow us to conclude that $t'_R = t'_L$ in frame S′.

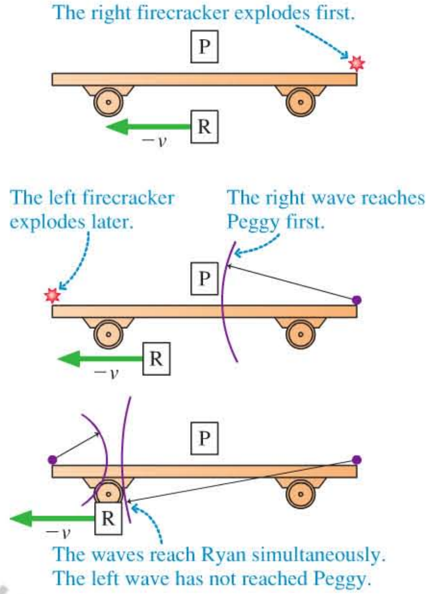
In fact, the right firecracker must explode *before* the left firecracker in frame S′. Figure 27.14b, with its assumption about simultaneity, was incorrect. **FIGURE 27.15** shows the situation in Peggy's reference frame with the right firecracker exploding first. Now the wave from the right reaches Peggy and the box first, as Ryan had concluded, and the light on top turns green.

One of the most disconcerting conclusions of relativity is that **two events occurring simultaneously in reference frame S are *not* simultaneous in any reference frame S′ that is moving relative to S.** This is called the **relativity of simultaneity.**

The two firecrackers *really* explode at the same instant of time in Ryan's reference frame. And the right firecracker *really* explodes first in Peggy's reference frame. It's not a matter of when they see the flashes. Our conclusion refers to the times at which the explosions actually occur.

The paradox of Peggy and Ryan contains the essence of relativity, and it's worth careful thought. First, review the logic until you're certain that there *is* a paradox, a logical impossibility. Then convince yourself that the only way to resolve the paradox is to abandon the assumption that the explosions are simultaneous in Peggy's reference frame. If you understand the paradox and its resolution, you've made a big step toward understanding what relativity is all about.

FIGURE 27.15 The real sequence of events in Peggy's reference frame.



STOP TO THINK 27.5 A tree and a pole are 3000 m apart. Each is suddenly hit by a bolt of lightning. Mark, who is standing at rest midway between the two, sees the two lightning bolts at the same instant of time. Nancy is flying her rocket at $v = 0.5c$ in the direction from the tree toward the pole. The lightning hits the tree just as she passes by it. Define event 1 to be "lightning strikes tree" and event 2 to be "lightning strikes pole." For Nancy, does event 1 occur before, after, or at the same time as event 2?

27.6 Time Dilation

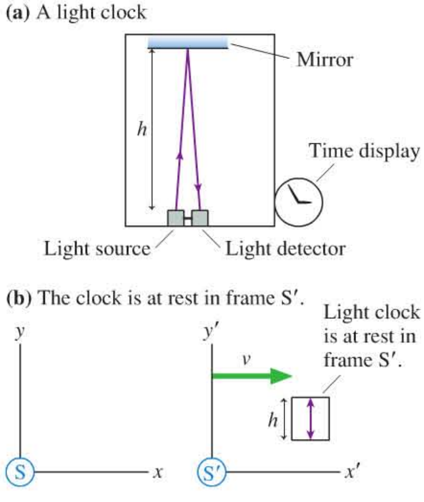
The principle of relativity has driven us to the logical conclusion that the time at which an event occurs may not be the same for two reference frames moving relative to each other. Our analysis thus far has been mostly qualitative. It's time to start developing some quantitative tools that will allow us to compare measurements in one reference frame to measurements in another reference frame.

FIGURE 27.16a shows a special clock called a **light clock.** The light clock is a box of height h with a light source at the bottom and a mirror at the top. The light source emits a very short pulse of light that travels to the mirror and reflects back to a light detector beside the source. The clock advances one "tick" each time the detector receives a light pulse, and it immediately, with no delay, causes the light source to emit the next light pulse.

Our goal is to compare two measurements of the interval between two ticks of the clock: one taken by an experimenter standing next to the clock and the other by an experimenter moving with respect to the clock. To be specific, **FIGURE 27.16b** shows the clock at rest in reference frame S′. We call this the **rest frame** of the clock. Reference frame S′ moves to the right with velocity v relative to reference frame S.

Relativity requires us to measure *events,* so let's define event 1 to be the emission of a light pulse and event 2 to be the detection of that light pulse. Experimenters in

FIGURE 27.16 The ticking of a light clock can be measured by experimenters in two different reference frames.



both reference frames are able to measure where and when these events occur *in their frame*. In frame S, the time interval $\Delta t = t_2 - t_1$ is one tick of the clock. Similarly, one tick in frame S′ is $\Delta t' = t_2' - t_1'$.

It's simple to calculate the tick interval $\Delta t'$ observed in frame S′, the rest frame of the clock, because the light simply goes straight up and back down. The total distance traveled by the light is $2h$, so the time of one tick is

$$\Delta t' = \frac{2h}{c} \tag{27.2}$$

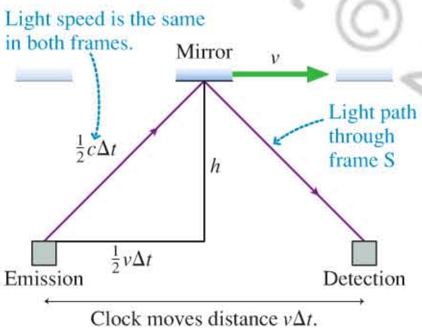


FIGURE 27.17 A light clock analysis in which the speed of light is the same in all reference frames.

FIGURE 27.17 shows the light clock as seen in frame S. As seen in S, the clock is moving to the right at speed v. Thus the mirror has moved a distance $\frac{1}{2}v(\Delta t)$ during the time $\frac{1}{2}(\Delta t)$ in which the light pulse moves from the source to the mirror. To move from the source to the mirror, as seen from frame S the light must move along the *diagonal path* shown. That is, the light must travel *farther* from source to mirror than it did in the rest frame of the clock.

The length of this diagonal is easy to calculate because, according to special relativity, the speed of light is equal to c in all inertial frames. Thus the diagonal length is simply

$$\text{distance} = \text{speed} \times \text{time} = c\left(\frac{1}{2}\,\Delta t\right) = \frac{1}{2}c\,\Delta t$$

We can then apply the Pythagorean theorem to the right triangle in Figure 27.17 to find that

$$h^2 + \left(\frac{1}{2}v\,\Delta t\right)^2 = \left(\frac{1}{2}c\,\Delta t\right)^2 \tag{27.3}$$

We can solve for Δt by first rewriting Equation 27.3 as

$$h^2 = \left(\frac{1}{2}c\,\Delta t\right)^2 - \left(\frac{1}{2}v\,\Delta t\right)^2 = \left[\left(\frac{1}{2}c\right)^2 - \left(\frac{1}{2}v\right)^2\right]\Delta t^2 = \left(\frac{1}{2}\right)^2(c^2 - v^2)\,\Delta t^2$$

so that

$$\Delta t^2 = \frac{h^2}{\left(\frac{1}{2}\right)^2(c^2 - v^2)} = \frac{(2h)^2}{c^2 - v^2} = \frac{(2h/c)^2}{1 - v^2/c^2}$$

from which we find

$$\Delta t = \frac{2h/c}{\sqrt{1 - v^2/c^2}} = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} \tag{27.4}$$

where we have used Equation 27.2 to write $2h/c$ as $\Delta t'$. The time interval between two ticks in frame S is *not* the same as in frame S′.

It's useful to define $\beta = v/c$, the speed as a fraction of the speed of light. For example, a reference frame moving with $v = 2.4 \times 10^8$ m/s has $\beta = 0.80$. In terms of β, Equation 27.4 is

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \beta^2}} \tag{27.5}$$

If reference frame S′ is at rest relative to frame S, then $\beta = 0$ and $\Delta t = \Delta t'$. In other words, experimenters in both reference frames measure time to be the same. But two experimenters moving relative to each other will measure *different* time intervals between the same two events. We're unaware of these differences in our everyday lives because our typical speeds are so small compared to c. But the differences are easily measured in a laboratory, and they do affect the precise timekeeping needed to make accurate location measurements with a GPS receiver.

Proper Time

ActivPhysics Online 17.1

Frame S′ has one important distinction. It is the *one and only* inertial reference frame in which the clock is at rest. Consequently, it is the one and only inertial reference frame in which the times of both events—the emission of the light and the detection of the light—are measured at the *same* position. You can see that the light pulse in Figure 27.16, the rest frame of the clock, starts and ends at the same position, while in Figure 27.17 the emission and detection take place at *different* positions in frame S.

The time interval between two events that occur at the *same position* is called the **proper time** $\Delta\tau$. Only one inertial reference frame measures the proper time, and it can do so with a single clock that is present at both events. Experimenters in an inertial reference frame moving with speed $v = \beta c$ relative to the proper-time frame must use two clocks to measure the time interval because the two events occur at different positions. We can rewrite Equation 27.5, where the time interval $\Delta t'$ is the proper time $\Delta\tau$, to find that the time interval in any other reference frame is

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}} \geq \Delta\tau \qquad (27.6)$$

Time dilation in terms of proper time $\Delta\tau$ (where $\beta = v/c$)

Because $\beta = v/c$, and v is always less than c, β is always less than 1. This means that the factor $1/\sqrt{1-\beta^2}$ appearing in Equation 27.6 is always *equal to* (when $v = 0$) *or greater than 1*. Thus $\Delta t \geq \Delta\tau$. Recalling that Δt is the time between clock ticks in a frame such as S in which the clock is moving, while the proper time $\Delta\tau$ is the time between ticks in a frame at which the clock is at rest, we can interpret Equation 27.6 as saying that **the time interval between two ticks is the shortest in the reference frame in which the clock is at rest.** The time interval between two ticks is longer when it is measured in any reference frame in which the clock is moving. Because a longer tick interval implies a clock that runs more slowly, we can also say that a **moving clock runs slowly compared to an identical clock at rest.** This "stretching out" of the time interval implied by Equation 27.6 is called **time dilation.**

NOTE ► If $v > c$, so that $\beta > 1$, the factor $1 - \beta^2$ appearing in Equation 27.6 would be *negative*. In this case Δt would contain the square root of a negative number; that is, it would be an *imaginary* number. Time intervals certainly have to be real numbers, suggesting that $v > c$ is not physically possible. One of the predictions of relativity, as you've undoubtedly heard, is that nothing can travel faster than the speed of light. Now you can begin to see why. We'll examine this issue more closely later in the chapter. ◄

Equation 27.6 was derived using a light clock because the operation of a light clock is clear and easy to analyze. But the conclusion is really about time itself. Any clock, regardless of how it operates, behaves the same. For example, suppose you and a light clock are traveling in a very fast spaceship. The light clock happens to tick at the same rate as your heart beats—say, 60 times a minute, or once a second. Because the light clock is at rest in your frame, it measures the proper time between two successive beats of your heart; that is, $\Delta\tau = 1$ s. But to an experimenter stationed on the ground, watching you pass by at an enormous speed, the time interval $\Delta t = \Delta\tau/\sqrt{1-\beta^2}$ between two ticks of the clock—and hence two beats of your heart—would be *longer.* If, for instance, $\Delta t = 2$ s, the ground-based experimenter would conclude that your heart is beating only 30 times per minute. To the experimenter, *all* processes on your spaceship, including all your biological processes, would appear to run slowly.

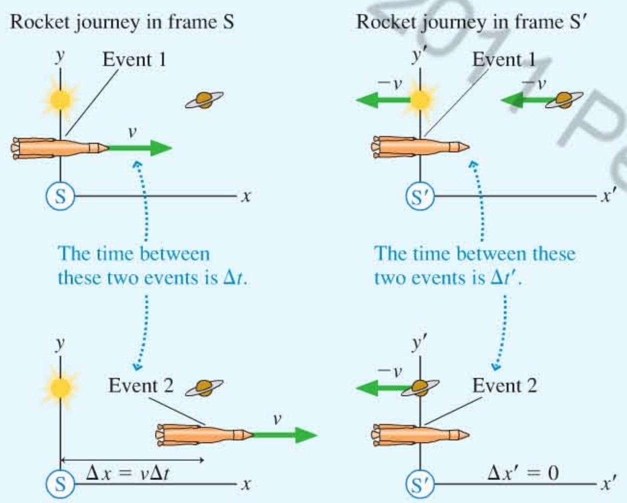
EXAMPLE 27.4 Journey time from the sun to Saturn

Saturn is 1.43×10^{12} m from the sun. A rocket travels along a line from the sun to Saturn at a constant speed of exactly $0.9c$ relative to the solar system. How long does the journey take as measured by an experimenter on earth? As measured by an astronaut on the rocket?

PREPARE Let the solar system be in reference frame S and the rocket be in reference frame S′ that travels with velocity $v = 0.9c$ relative to S. Relativity problems must be stated in terms of *events*. Let event 1 be "the rocket and the sun coincide" (the experimenter on earth says that the rocket passes the sun; the astronaut on the rocket says that the sun passes the rocket) and event 2 be "the rocket and Saturn coincide."

FIGURE 27.18 shows the two events as seen from the two reference frames. Notice that the two events occur at the *same position* in S′, the position of the rocket.

FIGURE 27.18 Visual overview of the trip as seen in frames S and S′.



SOLVE The time interval measured in the solar system reference frame, which includes the earth, is simply

$$\Delta t = \frac{\Delta x}{v} = \frac{1.43 \times 10^{12}\ \text{m}}{0.9 \times (3.00 \times 10^{8}\ \text{m/s})} = 5300\ \text{s}$$

Relativity hasn't abandoned the basic definition $v = \Delta x/\Delta t$, although we do have to be sure that Δx and Δt are measured in just one reference frame and refer to the same two events.

How are things in the rocket's reference frame? The two events occur at the *same position* in S′. Thus the time measured by the astronauts is the *proper time* $\Delta\tau$ between the two events. We can then use Equation 27.6 with $\beta = 0.9$ to find

$$\Delta\tau = \sqrt{1-\beta^2}\,\Delta t = \sqrt{1-0.9^2}(5300\ \text{s}) = 2310\ \text{s}$$

ASSESS The time interval measured between these two events by the astronauts is less than half the time interval measured by experimenters on earth. The difference has nothing to do with when earthbound astronomers *see* the rocket pass the sun and Saturn. Δt is the time interval from when the rocket actually passes the sun, as measured by a clock at the sun, until it actually passes Saturn, as measured by a synchronized clock at Saturn. The interval between *seeing* the events from earth, which would have to allow for light travel times, would be something other than 5300 s. Δt and $\Delta\tau$ are different because *time is different* in two reference frames moving relative to each other.

The global positioning system (GPS) If you've ever used a GPS receiver, you know it can pinpoint your location anywhere in the world. The system uses a set of orbiting satellites whose positions are very accurately known. The satellites orbit at a speed of about 14,000 km/h, enough to make the moving satellite's clocks run slow by about 7 μs a day. This may not seem like much, but it would introduce an error of 2000 m into your position! To function properly, the clocks are carefully corrected for effects due to relativity (including effects due to general relativity).

STOP TO THINK 27.6 Molly flies her rocket past Nick at constant velocity v. Molly and Nick both measure the time it takes the rocket, from nose to tail, to pass Nick. Which of the following is true?

A. Both Molly and Nick measure the same amount of time.
B. Molly measures a shorter time interval than Nick.
C. Nick measures a shorter time interval than Molly.

Experimental Evidence

Is there any evidence for the crazy idea that clocks moving relative to each other tell time differently? Indeed, there's plenty. An experiment in 1971 sent an atomic clock around the world on a jet plane while an identical clock remained in the laboratory. This was a difficult experiment because the traveling clock's speed was so small compared to c, but measuring the small differences between the time intervals was just barely within the capabilities of atomic clocks. It was also a more complex experiment than we've analyzed because the clock accelerated as it moved around a circle. Nonetheless, the traveling clock, upon its return, was 200 ns behind the clock that stayed at home, which was exactly as predicted by relativity.

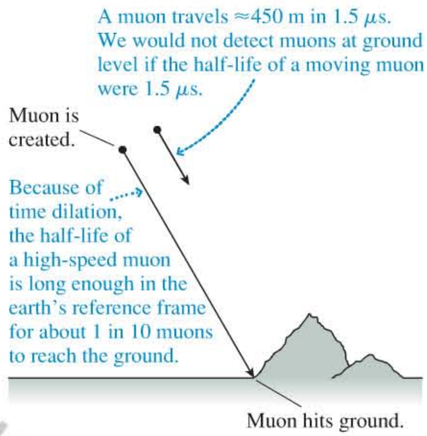
Very detailed studies have been done on unstable particles called *muons* that are created at the top of the atmosphere, at a height of about 60 km, when high-energy cosmic rays collide with air molecules. It is well known, from laboratory studies, that stationary muons decay with a *half-life* of 1.5 μs. That is, half the muons decay within 1.5 μs, half of those remaining decay in the next 1.5 μs, and so on. The decays can be used as a clock.

The muons travel down through the atmosphere at very nearly the speed of light. The time needed to reach the ground, assuming $v \approx c$, is $\Delta t \approx (60{,}000 \text{ m})/(3 \times 10^8 \text{ m/s}) = 200\ \mu\text{s}$. This is 133 half-lives, so the fraction of muons reaching the ground should be $\approx (1/2)^{133} = 10^{-40}$. That is, only 1 out of every 10^{40} muons should reach the ground. In fact, experiments find that about 1 in 10 muons reach the ground, an experimental result that differs by a factor of 10^{39} from our prediction!

The discrepancy is due to time dilation. In **FIGURE 27.19**, the two events "muon is created" and "muon hits ground" take place at two different places in the earth's reference frame. However, these two events occur at the *same position* in the muon's reference frame. (The muon is like the rocket in Example 27.4.) Thus the muon's internal clock measures the proper time. The time-dilated interval $\Delta t = 200\ \mu\text{s}$ in the earth's reference frame corresponds to a time of only $\Delta t' \approx 5\ \mu\text{s}$ in the muon's reference frame. That is, in the muon's reference frame it takes only 5 μs from its creation at the top of the atmosphere until the ground runs into it. This is 3.3 half-lives, so the fraction of muons reaching the ground is $(1/2)^{3.3} = 0.1$, or 1 out of 10. We wouldn't detect muons at the ground at all if not for time dilation.

The details are beyond the scope of this textbook, but dozens of high-energy particle accelerators around the world that study quarks and other elementary particles have been designed and built on the basis of Einstein's theory of relativity. The fact that they work exactly as planned is strong testimony to the reality of time dilation.

FIGURE 27.19 We wouldn't detect muons at the ground if not for time dilation.



The Twin Paradox

The most well-known relativity paradox is the twin paradox. George and Helen are twins. On their 25th birthday, Helen departs on a starship voyage to a distant star. Let's imagine, to be specific, that her starship accelerates almost instantly to a speed of $0.95c$ and that she travels to a star that is 9.5 light years (9.5 ly) from earth. Upon arriving, she discovers that the planets circling the star are inhabited by fierce aliens, so she immediately turns around and heads home at $0.95c$.

A **light year,** abbreviated ly, is the distance that light travels in one year. A light year is vastly larger than the diameter of the solar system. The distance between two neighboring stars is typically a few light years. For our purpose, we can write the speed of light as $c = 1$ ly/year. That is, light travels 1 light year per year.

This value for c allows us to determine how long, according to George and his fellow earthlings, it takes Helen to travel out and back. Her total distance is 19 ly and, due to her rapid acceleration and rapid turn around, she travels essentially the entire distance at speed $v = 0.95c = 0.95$ ly/year. Thus the time she's away, as measured by George, is

$$\Delta t_G = \frac{19 \text{ ly}}{0.95 \text{ ly/year}} = 20 \text{ years} \tag{27.7}$$

George will be 45 years old when his sister Helen returns with tales of adventure.

While she's away, George takes a physics class and studies Einstein's theory of relativity. He realizes that time dilation will make Helen's clocks run more slowly than his clocks, which are at rest relative to him. Her heart—a clock—will beat fewer times and the minute hand on her watch will go around fewer times. In other words, she's aging more slowly than he is. Although she is his twin, she will be younger than he is when she returns.

Alpha Centauri (arrow) is one of the closest stars to the sun, at a distance of 4.3 ly. If you traveled there and back at $0.99c$, your earthbound friends would be 8.6 years older, while you would have aged only 1.2 years.

Calculating Helen's age is not hard. We simply have to identify Helen's clock, because it's always with Helen as she travels, as the clock that measures proper time $\Delta\tau$. From Equation 27.6,

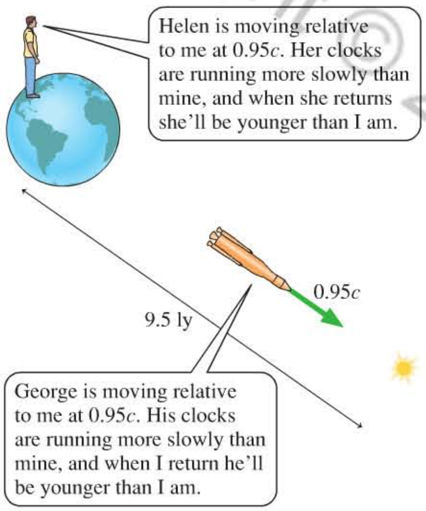
$$\Delta t_{\mathrm{H}} = \Delta\tau = \sqrt{1-\beta^2}\,\Delta t_{\mathrm{G}} = \sqrt{1-0.95^2}\,(20 \text{ years}) = 6.25 \text{ years} \qquad (27.8)$$

George will have just celebrated his 45th birthday as he welcomes home his 31-year-and-3-month-old twin sister.

This may be unsettling, because it violates our commonsense notion of time, but it's not a paradox. There's no logical inconsistency in this outcome. So why is it called "the twin paradox"? Read on.

Helen, knowing that she had quite of bit of time to kill on her journey, brought along several physics books to read. As she learns about relativity, she begins to think about George and her friends back on earth. Relative to her, they are all moving away at $0.95c$. Later they'll come rushing toward her at $0.95c$. Time dilation will cause their clocks to run more slowly than her clocks, which are at rest relative to her. In other words, as **FIGURE 27.20** shows, Helen concludes that people on earth are aging more slowly than she is. Alas, she will be much older than they when she returns.

FIGURE 27.20 The twin paradox.



Finally, the big day arrives. Helen lands back on earth and steps out of the starship. George is expecting Helen to be younger than he is. Helen is expecting George to be younger than she is.

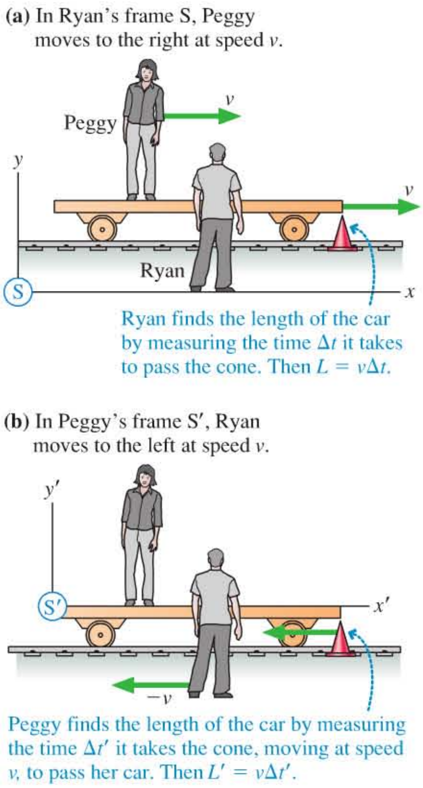
Here's the paradox! It's logically impossible for each to be younger than the other at the time when they are reunited. Where, then, is the flaw in our reasoning? It seems to be a symmetrical situation—Helen moves relative to George and George moves relative to Helen—but symmetrical reasoning has led to a conundrum.

But are the situations really symmetrical? George goes about his business day after day without noticing anything unusual. Helen, on the other hand, experiences three distinct periods during which the starship engines fire, she's crushed into her seat, and free dust particles that had been floating inside the starship are no longer, in the starship's reference frame, at rest or traveling in a straight line at constant speed. In other words, George spends the entire time in an inertial reference frame, *but Helen does not.* The situation is *not* symmetrical.

The principle of relativity applies *only* to inertial reference frames. Our discussion of time dilation was for inertial reference frames. Thus George's analysis and calculations are correct. Helen's analysis and calculations are *not* correct because she was trying to apply an inertial reference frame result to a noninertial reference frame.

Helen is younger than George when she returns. This is strange, but not a paradox. It is a consequence of the fact that time flows differently in two reference frames moving relative to each other.

FIGURE 27.21 The length of a train car as measured by Ryan and by Peggy.



27.7 Length Contraction

We've seen that relativity requires us to rethink our idea of time. Now let's turn our attention to the concepts of space and distance. Consider again Peggy on her train car, which is reference frame S′, moving past Ryan, who is at rest in frame S, at relative speed v. Ryan wants to measure the length L of Peggy's car as it moves past him. As shown in **FIGURE 27.21a**, he can do so by measuring the time Δt that it takes the car to pass the fixed cone; he then calculates that $L = v\,\Delta t$.

FIGURE 27.21b shows the situation in Peggy's reference frame S′, where the car is at rest. Peggy wants to measure the length L' of her car; we'll soon see that L' need not be the same as L. Peggy can measure L' by finding the time $\Delta t'$ that the cone, moving at speed v, takes to move from one end of the car to the other. In this way, she finds that $L' = v\,\Delta t'$.

Speed v is the relative speed between S and S′ and is the same for both Ryan and Peggy. From Ryan's and Peggy's measurements of the car's length we can then write

$$v = \frac{L}{\Delta t} = \frac{L'}{\Delta t'} \tag{27.9}$$

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The time interval Δt measured in Ryan's frame S is the proper time $\Delta\tau$ because the two events that define the time intervals—the front end and back end of the car passing the cone—occur at the same position (the cone) in Ryan's frame. We can use the time-dilation result, Equation 27.6, to relate $\Delta\tau$ measured by Ryan to $\Delta t'$ measured by Peggy. Equation 27.9 then becomes

$$\frac{L}{\Delta\tau} = \frac{L'}{\Delta t'} = \frac{L'}{\Delta\tau/\sqrt{1-\beta^2}} \tag{27.10}$$

The $\Delta\tau$ cancels, so the car's length L in Ryan's frame is related to its length L' in Peggy's frame by

$$L = \sqrt{1-\beta^2}\,L' \tag{27.11}$$

Surprisingly, we find that **the length of the car in Ryan's frame is different from its length in Peggy's frame.**

Peggy's frame S′, in which the car's length is L', has one important distinction. It is the *one and only* inertial reference frame in which the car is at rest. Experimenters in frame S′ can take all the time they need to measure L' because the car isn't moving. The length of an object measured in the reference frame in which the object is at rest is called the **proper length** ℓ. When you measure the length of an everyday object, such as a curtain rod or tabletop, it is usually at rest in your reference frame, so everyday length measurements are of proper length.

We can use the proper length ℓ to write Equation 27.11 as

$$L = \sqrt{1-\beta^2}\,\ell \le \ell \tag{27.12}$$

Length contraction in terms of proper length ℓ

Because $\beta \ge 0$, the factor $\sqrt{1-\beta^2}$ is less than or equal to 1. This means that $L \le \ell$. Because ℓ, the proper length, is measured in a reference frame in which the object is at rest while L is measured in a frame in which the object is moving, we see that **the length of an object is greatest in the reference frame in which the object is at rest.** This "shrinking" of the length of an object or the distance between two objects, as measured by an experimenter moving with respect to the object(s), is called **length contraction.**

NOTE ► A moving object's length is contracted only in the direction in which it's moving (its length along the x-axis in Figure 27.21). The object's length in the y- and z-directions doesn't change. ◄

Two perspectives on a relativistic trip
The Stanford Linear Accelerator is a 3.2-km-long electron accelerator that accelerates electrons to a speed of 0.99999999995c. From our perspective, the electrons take a time Δt = (3200 m)/c = 11 μs to make the trip. However, we see their "clocks" run slowly, ticking off only 110 ps during the trip. What do the electrons see? In their reference frame the end of the accelerator is coming toward them at 0.99999999995c, but its length is contracted to only 3.3 cm. Thus it arrives in a time (3.3 cm)/c = 110 ps. The same result, but from a different perspective.

EXAMPLE 27.5 Length contraction of a ladder

Dan holds a 5.0-m-long ladder parallel to the ground. He then gets up to a good sprint, eventually reaching 98% of the speed of light. How long is the ladder according to Dan, once he is running, and according to Carmen, who is standing on the ground as Dan goes by?

PREPARE Let reference frame S′ be attached to Dan. The ladder is at rest in this reference frame, so Dan measures the proper length of the ladder: ℓ = 5.0 m. Dan's frame S′ moves relative to Carmen's frame S with velocity $v = 0.98c$.

SOLVE We can find the length of the ladder in Carmen's frame from Equation 27.12. We have

$$L = \sqrt{1-\beta^2}\,\ell = \sqrt{1-0.98^2}\,(5.0\text{ m}) = 1.0\text{ m}$$

ASSESS The length of the moving ladder as measured by Carmen is only one-fifth its length as measured by Dan. These lengths are different because *space is different* in two reference frames moving relative to each other.

The conclusion that space is different in reference frames moving relative to each other is a direct consequence of the fact that time is different. Experimenters in both reference frames agree on the relative velocity v, leading to Equation 27.9: $v = L/\Delta t = L'/\Delta t'$. Because of time dilation, Ryan (who measures proper time) finds that $\Delta t < \Delta t'$. Thus L *has* to be less than L'. That is the only way Ryan and Peggy can reconcile their measurements.

The Binomial Approximation

A useful mathematical tool is the **binomial approximation.** Suppose we need to evaluate the quantity $(1 + x)^n$. If x is much less than 1, it turns out that an excellent approximation is

$$(1 + x)^n \approx 1 + nx \qquad \text{if} \qquad x \ll 1 \tag{27.13}$$

You can try this on your calculator. Suppose you need to calculate 1.01^2. Comparing this expression with Equation 27.13, we see that $x = 0.01$ and $n = 2$. Equation 27.13 then tells us that

$$1.01^2 \approx 1 + 2 \times 0.01 = 1.02$$

The exact result, using a calculator, is 1.0201. The approximate answer is good to about 99.99%! The smaller the value of x, the better the approximation.

The binomial approximation is very useful when we need to calculate a relativistic expression for a speed much less than c, so that $v \ll c$. Because $\beta = v/c$, a reference frame moving with $v^2/c^2 \ll 1$ has $\beta^2 \ll 1$. In these cases, we can write

$$\begin{aligned} \sqrt{1-\beta^2} &= (1 - v^2/c^2)^{1/2} \approx 1 - \frac{1}{2}\frac{v^2}{c^2} \\ \frac{1}{\sqrt{1-\beta^2}} &= (1 - v^2/c^2)^{-1/2} \approx 1 + \frac{1}{2}\frac{v^2}{c^2} \end{aligned} \tag{27.14}$$

The following example illustrates the use of the binomial approximation.

EXAMPLE 27.6 The shrinking school bus

An 8.0-m-long school bus drives past at 30 m/s. By how much is its length contracted?

PREPARE The school bus is at rest in an inertial reference frame S′ moving at velocity $v = 30$ m/s relative to the ground frame S. The given length, 8.0 m, is the proper length ℓ in frame S′.

SOLVE In frame S, the school bus is length contracted to

$$L = \sqrt{1-\beta^2}\,\ell$$

The bus's speed v is much less than c, so we can use the binomial approximation to write

$$L \approx \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)\ell = \ell - \frac{1}{2}\frac{v^2}{c^2}\ell$$

The *amount* of the length contraction is

$$\begin{aligned} \ell - L &= \frac{1}{2}\frac{v^2}{c^2}\ell = \left(\frac{30 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}}\right)^2 (4.0 \text{ m}) \\ &= 4.0 \times 10^{-14} \text{ m} = 40 \text{ fm} \end{aligned}$$

where 1 fm = 1 femtometer = 10^{-15} m.

ASSESS The amount the bus "shrinks" is only slightly larger than the diameter of the nucleus of an atom. It's no wonder that we're not aware of length contraction in our everyday lives. If you had tried to calculate this number exactly, your calculator would have shown $\ell - L = 0$. The difficulty is that the difference between ℓ and L shows up only in the 14th decimal place. A scientific calculator determines numbers to 10 or 12 decimal places, but that isn't sufficient to show the difference. The binomial approximation provides an invaluable tool for finding the very tiny difference between two numbers that are nearly identical.

27.8 Velocities of Objects in Special Relativity

In Section 27.2 we discussed Galilean relativity, which is applicable to objects that are moving at speeds much less than the speed of light. We found that if the velocity of an object is u in reference frame S, then its velocity measured in frame S′, moving at velocity v relative to frame S, is $u' = u - v$.

But we soon learned that this expression is invalid for objects moving at an appreciable fraction of the speed of light. In particular, light itself moves at speed c as measured by *all* observers, independent of their relative velocities. The Galilean transformation of velocity needs to be modified for objects moving at relativistic speeds.

Although a proof is beyond the scope of this text, Einstein's relativity includes a velocity-addition expression valid for *any* velocities. If u is the velocity of an object as measured in reference frame S, and u' its velocity as measured in S′, moving at velocity v relative to frame S, then the two velocities are related by

$$u' = \frac{u - v}{1 - uv/c^2} \quad \text{or} \quad u = \frac{u' + v}{1 + u'v/c^2} \tag{27.15}$$

Lorentz transformation of velocity

These equations were discovered by Dutch physicist H. A. Lorentz a few years before Einstein published his theory of relativity, but Lorentz didn't completely understand their implications for space and time. Notice that the denominator is ≈ 1 if either u or v is much less than c. In other words, these equations agree with the Galilean velocity transformation when velocities are nonrelativistic (i.e., $\ll c$), but they differ as velocities approach the speed of light.

NOTE ► It is important to distinguish carefully between v, which is the relative velocity of the reference frames in which measurements are carried out, and u and u', which are the velocities of an *object* as measured in two different reference frames. ◄

EXAMPLE 27.7 **A speeding bullet**

A rocket flies past the earth at precisely $0.9c$. As it goes by, the rocket fires a bullet in the forward direction at precisely $0.95c$ with respect to the rocket. What is the bullet's speed with respect to the earth?

PREPARE The rocket and the earth are inertial reference frames. Let the earth be frame S and the rocket be frame S′. The velocity of frame S′ relative to frame S is $v = 0.9c$. The bullet's velocity in frame S′ is $u' = 0.95c$.

SOLVE We can use the Lorentz velocity transformation to find

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.95c + 0.90c}{1 + (0.95c)(0.90c)/c^2} = 0.997c$$

The bullet's speed with respect to the earth is 99.7% of the speed of light.

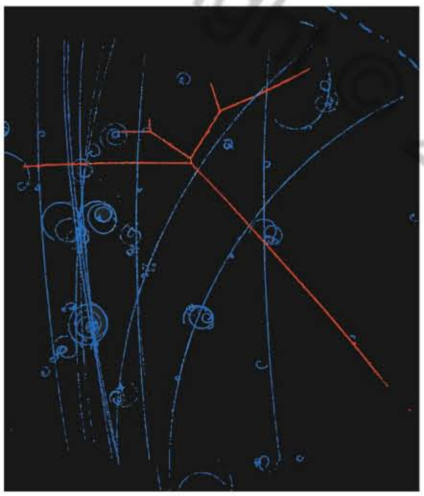
NOTE ► Many relativistic calculations are much easier when velocities are specified as a fraction of c. ◄

ASSESS The Galilean transformation of velocity would give $u = 1.85c$. Now, despite the very high speed of the rocket and of the bullet with respect to the rocket, the bullet's speed with respect to the earth remains less than c. This is yet more evidence that objects cannot exceed the speed of light.

Suppose the rocket in Example 27.7 fired a laser beam in the forward direction as it traveled past the earth at velocity v. The laser beam would travel away from the rocket at speed $u' = c$ in the rocket's reference frame S′. What is the laser beam's speed in the earth's frame S? According to the Lorentz velocity transformation, it must be

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{c + v}{1 + cv/c^2} = \frac{c + v}{1 + v/c} = \frac{c + v}{(c + v)/c} = c$$

Light travels at speed c in both frame S and frame S′. This important consequence of the principle of relativity is "built into" the Lorentz velocity transformation.



In this photograph, the track (shown in red) of a high-energy proton enters from the lower right. The proton then collides with other protons, sending them in all directions, where further collisions occur. Momentum is conserved, but at these high speeds the relativistic expression for momentum must be used.

27.9 Relativistic Momentum

In Newtonian mechanics, the total momentum of a system is a conserved quantity. Further, the law of conservation of momentum, $P_f = P_i$, is true in all inertial reference frames *if* the particle velocities in different reference frames are related by the Galilean velocity transformation.

The difficulty, of course, is that the Galilean transformation is not consistent with the principle of relativity. It is a reasonable approximation when all velocities are much less than c, but the Galilean transformation fails dramatically as velocities approach c. It's not hard to show that $P'_f \neq P'_i$ if the particle velocities in frame S′ are related to the particle velocities in frame S by the Lorentz velocity transformation.

There are two possibilities:

1. The so-called law of conservation of momentum is not really a law of physics. It is approximately true at low velocities but fails as velocities approach the speed of light.
2. The law of conservation of momentum really is a law of physics, but the expression $p = mu$ is not the correct way to calculate momentum when the particle velocity u becomes a significant fraction of c.

Momentum conservation is such a central and important feature of mechanics that it seems unlikely to fail in relativity. How else might the momentum of a particle be defined?

The classical momentum, for one-dimensional motion, is $p = mu = m(\Delta x/\Delta t)$. Δt is the time needed to move a displacement Δx. That seemed clear enough within a Newtonian framework, but now we've learned that experimenters in different reference frames disagree about the amount of time needed. So whose Δt should we use?

One possibility is to use the time measured *by the particle*. This is the proper time $\Delta\tau$ because the particle is at rest in its own reference frame. With this in mind, let's redefine the momentum of a particle of mass m moving with velocity $u = \Delta x/\Delta t$ to be

$$p = m\frac{\Delta x}{\Delta\tau} \tag{27.16}$$

We can relate this new expression for p to the familiar Newtonian expression by using the time-dilation result $\Delta\tau = (1 - u^2/c^2)^{1/2}\,\Delta t$ to relate the proper time interval measured by the particle to the more practical time interval Δt measured by experimenters in frame S. With this substitution, Equation 27.16 becomes

$$p = m\frac{\Delta x}{\Delta\tau} = m\frac{\Delta x}{\sqrt{1 - u^2/c^2}\,\Delta t} = \frac{mu}{\sqrt{1 - u^2/c^2}} \tag{27.17}$$

You can see that Equation 27.17 reduces to the classical expression $p = mu$ when the particle's speed $u \ll c$. That is an important requirement, but whether this is the "correct" expression for p depends on whether the total momentum P is conserved when the velocities of a system of particles are transformed with the Lorentz velocity transformation equations. The proof is rather long and tedious, so we will assert, without actual proof, that the momentum defined in Equation 27.17 is, indeed, conserved. **The law of conservation of momentum is still valid in all inertial reference frames *if* the momentum of each particle is calculated with Equation 27.17.**

To simplify our notation, let's define the quantity

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \tag{27.18}$$

With this definition of γ, the momentum of a particle is

$$p = \gamma m u \tag{27.19}$$

Relativistic momentum for a particle with mass m and speed u

EXAMPLE 27.8 **Momentum of a subatomic particle**

Electrons in a particle accelerator reach a speed of $0.999c$ relative to the laboratory. One collision of an electron with a target produces a muon that moves forward with a speed of $0.950c$ relative to the laboratory. The muon mass is 1.90×10^{-28} kg. What is the muon's momentum in the laboratory frame and in the frame of the electron beam?

PREPARE Let the laboratory be reference frame S. The reference frame S′ of the electron beam (i.e., a reference frame in which the electrons are at rest) moves in the direction of the electrons at $v = 0.999c$. The muon velocity in frame S is $u = 0.95c$.

SOLVE γ for the muon in the laboratory reference frame is

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \frac{1}{\sqrt{1 - 0.95^2}} = 3.20$$

Thus the muon's momentum in the laboratory is

$$\begin{aligned} p &= \gamma m u \\ &= (3.20)(1.90 \times 10^{-28}\ \text{kg})(0.95 \times 3.00 \times 10^8\ \text{m/s}) \\ &= 1.73 \times 10^{-19}\ \text{kg} \cdot \text{m/s} \end{aligned}$$

The momentum is a factor of 3.2 larger than the Newtonian momentum mu. To find the momentum in the electron-beam reference frame, we must first use the velocity transformation equation to find the muon's velocity in frame S′:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.95c - 0.999c}{1 - (0.95c)(0.999c)/c^2} = -0.962c$$

In the laboratory frame, the faster electrons are overtaking the slower muon. Hence the muon's velocity in the electron-beam frame is negative. γ' for the muon in frame S′ is

$$\gamma' = \frac{1}{\sqrt{1 - u'^2/c^2}} = \frac{1}{\sqrt{1 - 0.962^2}} = 3.66$$

The muon's momentum in the electron-beam reference frame is

$$\begin{aligned} p' &= \gamma' m u' \\ &= (3.66)(1.90 \times 10^{-28}\ \text{kg})(-0.962 \times 3.00 \times 10^8\ \text{m/s}) \\ &= -2.01 \times 10^{-19}\ \text{kg} \cdot \text{m/s} \end{aligned}$$

ASSESS From the laboratory perspective, the muon moves only slightly slower than the electron beam. But it turns out that the muon moves faster with respect to the electrons, although in the opposite direction, than it does with respect to the laboratory.

The Cosmic Speed Limit

FIGURE 27.22a on the next page is a graph of momentum versus velocity. For a Newtonian particle, with $p = mu$, the momentum is directly proportional to the velocity. The relativistic expression for momentum agrees with the Newtonian value if $u \ll c$, but p approaches ∞ as $u \to c$.

The implications of this graph become clear when we relate momentum to force. Consider a particle subjected to a constant force, such as a rocket that never runs out of fuel. From the impulse-momentum theorem we have $\Delta p = F\,\Delta t$, or $p = mu = Ft$ if the rocket starts from rest at $t = 0$. If Newtonian physics were correct, a particle would go faster and faster as its velocity $u = p/m = (F/m)t$ increased without limit. But the relativistic result, shown in **FIGURE 27.22b**, is that the particle's velocity approaches the speed of light ($u \to c$) as p approaches ∞. Relativity gives a very different outcome than Newtonian mechanics.

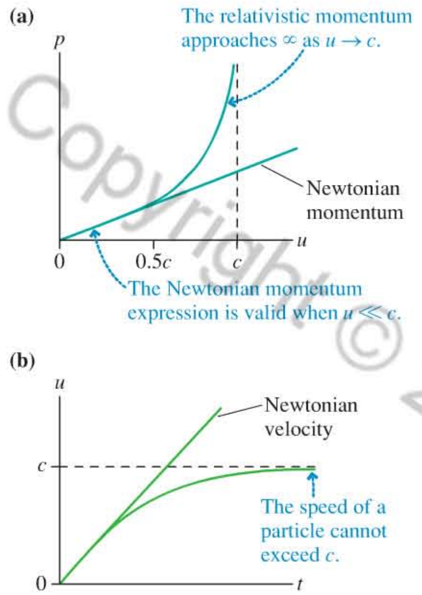


FIGURE 27.22 The speed of a particle cannot reach the speed of light.

The speed c is a "cosmic speed limit" for material particles. A force cannot accelerate a particle to a speed higher than c because the particle's momentum becomes infinitely large as the speed approaches c. The amount of effort required for each additional increment of velocity becomes larger and larger until no amount of effort can raise the velocity any higher.

Actually, at a more fundamental level, c is a speed limit for *any* kind of **causal influence.** If you throw a rock and break a window, your throw is the *cause* of the breaking window and the rock is the causal influence. A causal influence can be any kind of particle, wave, or information that travels from A to B and allows A to be the cause of B.

For two unrelated events—a firecracker explodes in Tokyo and a balloon bursts in Paris—the relativity of simultaneity tells us that in one reference frame the firecracker may explode before the balloon bursts, but in some other reference frame the balloon may burst first.

However, for two causally related events—A causes B—it would be nonsense for an experimenter in any reference frame to find that B occurs before A. No experimenter in any reference frame, no matter how it is moving, will find that you are born before your mother is born.

But according to relativity, a causal influence traveling faster than light could result in B causing A, a logical absurdity. Thus **no causal influence of any kind—a particle, wave, or other influence—can travel faster than c.**

The existence of a cosmic speed limit is one of the most interesting consequences of the theory of relativity. "Hyperdrive," in which a spaceship suddenly leaps to faster-than-light velocities, is simply incompatible with the theory of relativity. Rapid travel to the stars must remain in the realm of science fiction.

◀ In the *Star Wars* movies, the heroes escaped the imperial forces by jumping to hyperspace, a process in which the spaceship evidently exceeds the speed of light. According to relativity, this would allow Luke to go back before he was born, and maybe convince his dad to be a better guy.

27.10 Relativistic Energy

Energy is our final topic in this chapter on relativity. Space, time, velocity, and momentum are changed by relativity, so it seems inevitable that we'll need a new view of energy. Indeed, one of the most profound results of relativity, and perhaps the one with the most far-reaching consequences, was Einstein's discovery of the fundamental relationship between energy and mass.

Consider an object of mass m moving at speed u. Einstein found that the **total energy** of such an object is

$$E = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = \gamma mc^2 \qquad (27.20)$$

Total energy of an object of mass m moving at speed u

where $\gamma = 1/(1 - u^2/c^2)^{1/2}$ was defined in Equation 27.18.

To understand this expression, let's start by examining its behavior for objects traveling at speeds much less than the speed of light. In this case we can use the binomial approximation you learned in Section 27.7 to write

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$$

For low speeds u, then, the object's total energy is

$$E \approx mc^2 + \frac{1}{2}mu^2 \qquad (27.21)$$

▶ This fuel rod for a nuclear power reactor contains about 5 kg of uranium. Its usable energy content, which comes from the conversion of a small fraction of the uranium's mass to energy, is equivalent to that of about 10 million kg of coal.

The second term in this expression is the familiar Newtonian kinetic energy $K = \frac{1}{2}mu^2$, written here in terms of velocity u rather than v. But there is an additional term in the total energy, the **rest energy** given by

$$E_0 = mc^2 \tag{27.22}$$

When a particle is at rest, with $u = 0$, it still has energy E_0. Indeed, because c is so large, the rest energy can be enormous. Equation 27.22 is, of course, Einstein's famous $E = mc^2$, perhaps the most famous equation in all of physics. It tells us that there is a fundamental equivalence between mass and energy, an idea we'll explore later in this section.

EXAMPLE 27.9 The rest energy of an apple

What is the rest energy of a 200 g apple?

SOLVE From Equation 27.22 we have

$$E_0 = mc^2 = (0.20\text{ kg})(3.0 \times 10^8\text{ m/s})^2 = 1.8 \times 10^{16}\text{ J}$$

ASSESS This is an enormous energy, enough to power a medium-sized city for about a year.

Equation 27.21 suggests that the total energy of an object is the sum of a rest energy, which is a new idea, and the familiar kinetic energy. But Equation 27.21 is valid only when the object's speed is low compared to c. For higher speeds, we need to use the full energy expression, Equation 27.20. We can use Equation 27.20 to find a relativistic expression for the kinetic energy K by subtracting the rest energy E_0 from the total energy. Doing so gives

$$K = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2 = (\gamma - 1)E_0 \tag{27.23}$$

Thus we can write the total energy of an object of mass m as

$$E = \underbrace{mc^2}_{\text{Rest energy } E_0} + \underbrace{(\gamma - 1)mc^2}_{\text{Kinetic energy } K} \tag{27.24}$$

EXAMPLE 27.10 Comparing energies of a ball and an electron

Calculate the rest energy and the kinetic energy of (a) a 100 g ball moving with a speed of 100 m/s and (b) an electron with a speed of $0.999c$.

PREPARE The ball, with $u \ll c$, is a classical particle. We don't need to use the relativistic expression for its kinetic energy. The electron is highly relativistic.

SOLVE

a. For the ball, with $m = 0.100$ kg,

$$E_0 = mc^2 = 9.00 \times 10^{15}\text{ J}$$

$$K = \frac{1}{2}mu^2 = 500\text{ J}$$

b. For the electron, we start by calculating

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = 22.4$$

Then, using $m_e = 9.11 \times 10^{-31}$ kg,

$$E_0 = mc^2 = 8.20 \times 10^{-14}\text{ J}$$

$$K = (\gamma - 1)E_0 = 175 \times 10^{-14}\text{ J}$$

ASSESS The ball's kinetic energy is a typical kinetic energy. Its rest energy, by contrast, is a staggeringly large number. For a relativistic electron, on the other hand, the kinetic energy is more important than the rest energy.

STOP TO THINK 27.7 An electron moves through the lab at a speed such that $\gamma = 1.5$. The electron's kinetic energy is

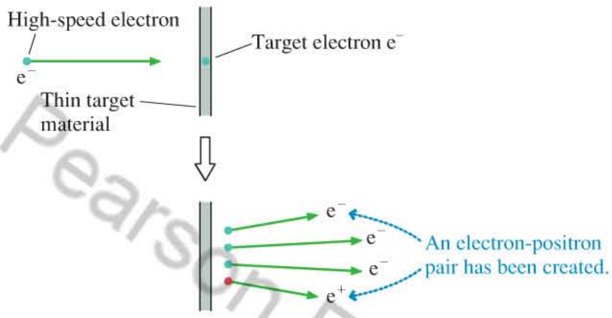
A. Greater than its rest energy.
B. Equal to its rest energy.
C. Less than its rest energy.

The Equivalence of Mass and Energy

The tracks of elementary particles in a bubble chamber show the creation of an electron-positron pair. The negative electron and positive positron spiral in opposite directions in the magnetic field.

Now we're ready to explore the significance of Einstein's famous equation $E = mc^2$. FIGURE 27.23 shows an experiment that has been done countless times in the last 50 years at particle accelerators around the world. An electron that has been accelerated to $u \approx c$ is aimed at a target material. When a high-energy electron collides with an atom in the target, it can easily knock one of the electrons out of the atom. Thus we would expect to see two electrons leaving the target: the incident electron and the ejected electron. Instead, *four* particles emerge from the target: three electrons and a positron. A *positron,* or positive electron, is the antimatter version of an electron, identical to an electron in all respects other than having charge $q = +e$. In particular, a positron has the same mass m_e as an electron.

FIGURE 27.23 An inelastic collision between electrons can create an electron-positron pair.



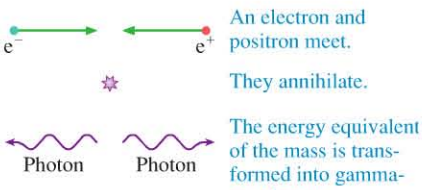
In chemical-reaction notation, the collision is

$$e^-(\text{fast}) + e^-(\text{at rest}) \rightarrow e^- + e^- + e^- + e^+$$

An electron and a positron have been created, apparently out of nothing. Mass $2m_e$ before the collision has become mass $4m_e$ after the collision. (Notice that charge has been conserved in this collision.)

Although the mass has increased, it wasn't created "out of nothing." If you measured the energies before and after the collision, you would find that the kinetic energy before the collision was *greater* than the kinetic energy after. In fact, the decrease in kinetic energy is exactly equal to the rest energy of the two particles that have been created: $\Delta K = 2m_e c^2$. The new particles have been created *out of energy!*

FIGURE 27.24 The annihilation of an electron-positron pair.



Not only can particles be created from energy, particles can return to energy. FIGURE 27.24 shows an electron colliding with a positron, its antimatter partner. When a particle and its antiparticle meet, they *annihilate* each other. The mass disappears, and the energy equivalent of the mass is transformed into two high-energy photons. Photons have no mass and represent pure energy. Positron-electron annihilation is also the basis of the medical procedure known as a positron-emission tomography, or PET scans. We'll study this important diagnostic tool in detail in Chapter 30.

Conservation of Energy

The creation and annihilation of particles with mass, processes strictly forbidden in Newtonian mechanics, are vivid proof that neither mass nor the Newtonian definition of energy is conserved. Even so, the *total* energy—the kinetic energy *and* the energy equivalent of mass—remains a conserved quantity.

Law of conservation of total energy The energy $E = \sum E_i$ of an isolated system is conserved, where $E_i = \gamma_i m_i c^2$ is the total energy of particle i.

Mass and energy are not the same thing, but, as the last few examples have shown, they are *equivalent* in the sense that mass can be transformed into energy and energy can be transformed into mass as long as the total energy is conserved.

Probably the most well-known application of the conservation of total energy is nuclear fission. The uranium isotope ^{236}U, containing 236 protons and neutrons, does not exist in nature. It can be created when a ^{235}U nucleus absorbs a neutron, increasing its atomic mass from 235 to 236. The ^{236}U nucleus quickly fragments into two smaller nuclei and several extra neutrons, a process known as **nuclear fission**. The nucleus can fragment in several ways, but one is

$$\mathrm{n} + {}^{235}\mathrm{U} \rightarrow {}^{236}\mathrm{U} \rightarrow {}^{144}\mathrm{Ba} + {}^{89}\mathrm{Kr} + 3\mathrm{n} \tag{27.25}$$

Ba and Kr are the atomic symbols for barium and krypton.

This reaction seems like an ordinary chemical reaction—until you check the masses. The masses of atomic isotopes are known with great precision from many decades of measurement in instruments called mass spectrometers. As shown in Table 27.1, if you add up the masses on both sides, you find that the mass of the products is 0.186 u less than the mass of the initial neutron and ^{235}U, where, you will recall, $1\ \mathrm{u} = 1.66 \times 10^{-27}$ kg is the atomic mass unit. Converting to kilograms gives us the mass loss of 3.07×10^{-28} kg.

Mass has been lost, but the energy equivalent of the mass has not. As **FIGURE 27.25** shows, the mass has been converted to kinetic energy, causing the two product nuclei and three neutrons to be ejected at very high speeds. The kinetic energy is easily calculated: $\Delta K = m_{\text{lost}}c^2 = 2.8 \times 10^{-11}$ J.

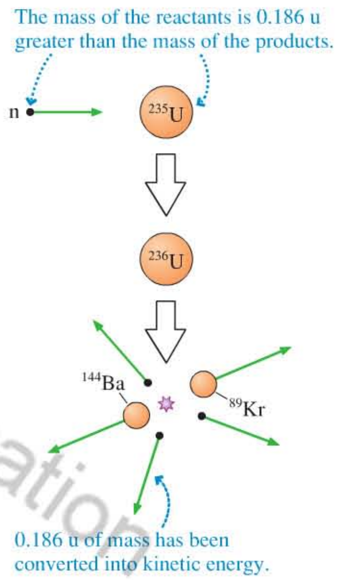
This is a very tiny amount of energy, but it is the energy released from *one* fission. The number of nuclei in a macroscopic sample of uranium is on the order of N_A, Avogadro's number. Hence the energy available if *all* the nuclei fission is enormous. This energy, of course, is the basis for both nuclear power reactors and nuclear weapons.

We started this chapter with an expectation that relativity would challenge our basic notions of space and time. We end by finding that relativity changes our understanding of mass and energy. Most remarkable of all is that each and every one of these new ideas flows from one simple statement: The laws of physics are the same in all inertial reference frames.

TABLE 27.1 Mass before and after fission of ^{235}U

Initial nucleus	Initial mass (u)	Final nucleus	Final mass (u)
^{235}U	235.0439	^{144}B	143.9229
n	1.0087	^{89}K	88.9176
		3n	3.0260
Total	236.0526		235.8665

FIGURE 27.25 In nuclear fission, the energy equivalent of lost mass is converted into kinetic energy.



INTEGRATED EXAMPLE 27.11 The global positioning system

The turtle in the photo that opens this chapter is being tracked using the global positioning system (GPS), a system of 24 satellites in circular orbits high above the earth. Each satellite, with an orbital speed of 3900 m/s, carries an atomic clock whose time is accurate to ±1 ns per day. Every 30 s, each satellite sends out a radio signal giving its precise location in space and the exact time the signal was sent.

The turtle's GPS receiver records the time at which the signal is received. Because the signal specifies the time at which it was sent, the receiver can easily calculate how long it took the signal to reach the turtle. Then, because radio waves are electromagnetic waves traveling at the speed of light, the precise distance to the satellite can be calculated.

The signal from one satellite actually locates the turtle's position only along a large sphere centered on the satellite. To pinpoint the location exactly requires the signals from four or more satellites. In this problem, we'll ignore these complicating effects.

a. As the satellite passes overhead, suppose two lights flash: one at the front of the satellite (the side toward which the satellite is moving) and one at the rear (the side opposite the direction of motion). In the reference frame of the satellite, the two flashes are simultaneous. Are they simultaneous to an experimenter on the earth? If not, which flash occurs first?
b. In one day, how much time does the clock running on the satellite gain or lose compared to an identical clock on earth?
c. If the clock error in part b were not properly taken into account, by how much would the turtle's position on earth be in error after one day?

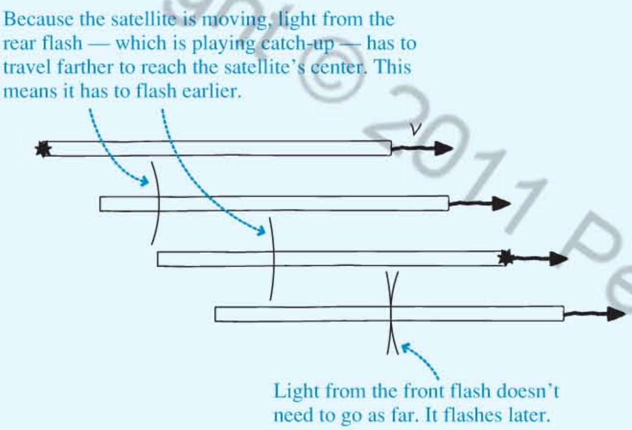
PREPARE Consider an astronaut standing on the satellite halfway between the lights. Because the two flashes are simultaneous in the reference frame of the satellite, the light from these flashes will reach the astronaut at the same time. An experimenter on the earth will also see these flashes reaching the astronaut at the same time, but will not agree that the flashes occurred simultaneously. We can use these observations to decide which flash occurred first to an earthbound experimenter.

In part b, because of time dilation the clock on the moving satellite runs slow compared to one on the earth, so this clock will *lose* time.

Continued

SOLVE a. **FIGURE 27.26** shows the satellite (here represented by a rod) as seen by an experimenter on earth. When the light waves from the two flashes meet at the satellite's center, the one from the rear of the satellite will have traveled farther than the one from the front. This is because the flash coming from the rear needs to "catch up" with the center of the moving satellite. Since both waves travel at speed c, but the wave from the rear travels farther, the rear light must have flashed *earlier* according to an earthbound experimenter.

FIGURE 27.26 The satellite as seen by an observer on earth.



b. The clock on the satellite measures proper time $\Delta\tau$ because this clock is at rest in the reference frame of the satellite. We want to know how much time the satellite's clock measures during an interval $\Delta t = 24\text{ h} = 86{,}400\text{ s}$ measured on the earth. We can rearrange Equation 27.6 as

$$\Delta\tau = \Delta t\sqrt{1-\beta^2}$$

Even for a fast-moving satellite, $\beta = v/c$ is so small that the term $\sqrt{1-\beta^2}$ will be exactly 1 on most calculators. We must therefore use the binomial expansion, Equation 27.14, to write

$$\Delta\tau = \Delta t\sqrt{1-\beta^2} \approx \Delta t\left(1 - \frac{1}{2}\frac{v^2}{c^2}\right) = \Delta t - \frac{1}{2}\frac{v^2}{c^2}\Delta t$$

The *difference* between the satellite and earth clocks is

$$\Delta t - \Delta\tau = \frac{1}{2}\frac{v^2}{c^2}\Delta t = \frac{1}{2}\left(\frac{3900\text{ m/s}}{3.0\times10^8\text{ m/s}}\right)^2(86{,}400\text{ s}) = 7.3\ \mu\text{s}$$

Because this result is positive, $\Delta\tau$ is less than Δt: The moving clock *loses* 7.3 μs per day compared to a clock on earth.

c. The radio signal from the satellite travels at speed c. In 7.3 μs, the signal travels a distance

$$\Delta x = c\Delta t = (3.0\times10^8\text{ m/s})(7.3\times10^{-6}\text{ s}) = 2200\text{ m}$$

If the satellite clocks were not corrected for this relativistic effect, all GPS receivers on earth would miscalculate their positions by 2.2 km after just one day.

ASSESS Even for a very fast-moving object like a satellite, the corrections due to relativity are small. But these small corrections can have large effects on high-precision measurements. Interestingly, the relativistic correction is implemented *before* the satellites are launched by setting the clocks to run slightly fast, by a factor of 1.000000000447. Once in orbit, the clocks slow down to match an earthbound clock exactly.

SUMMARY

The goal of Chapter 27 has been to understand how Einstein's theory of relativity changes our concepts of space and time.

GENERAL PRINCIPLES

Principle of Relativity

All the laws of physics are the same in all inertial reference frames.

- The speed of light c is the same in all inertial reference frames.
- No particle or causal influence can travel at a speed greater than c.

IMPORTANT CONCEPTS

Time

Time measurements depend on the motion of the experimenter relative to the events.

Proper time $\Delta\tau$ is the time interval between two events measured in a reference frame in which the events occur at the same position. The time interval Δt between the same two events, in a frame moving with relative velocity v, is

$$\Delta t = \Delta\tau/\sqrt{1-\beta^2} \geq \Delta\tau$$

where $\beta = v/c$. This is called **time dilation.**

Space

Spatial measurements depend on the motion of the experimenter relative to the events.

Proper length ℓ is the length of an object measured in a reference frame in which the object is at rest. The length L in a frame in which the object moves with velocity v is

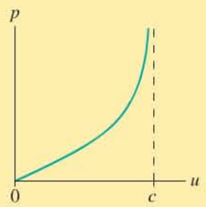
$$L = \sqrt{1-\beta^2}\,\ell \leq \ell$$

This is called **length contraction.**

Momentum

The law of conservation of momentum is valid in all inertial reference frames if the momentum of a particle with velocity u is $p = \gamma m u$, where

$$\gamma = 1/\sqrt{1-u^2/c^2}$$



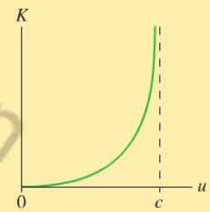
The momentum approaches ∞ as $u \to c$.

Energy

The **total energy** of a particle is $E = \gamma mc^2$. This can be written as

$$E = \underbrace{mc^2}_{\text{Rest energy } E_0} + \underbrace{(\gamma - 1)mc^2}_{\text{Kinetic energy } K}$$

K approaches ∞ as $u \to c$.



The total energy of an isolated system is conserved.

Simultaneity

Events that are simultaneous in reference frame S are not simultaneous in frame S′ moving relative to S.

Mass-energy equivalence

Mass m can be transformed into energy $E = mc^2$.

Energy can be transformed into mass $m = E/c^2$.

APPLICATIONS

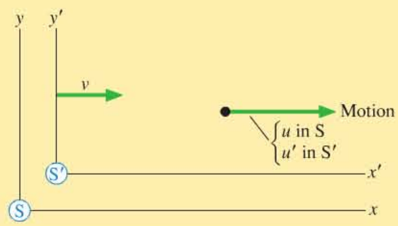
An **event** happens at a specific place in space and time. Spacetime coordinates are (x, t) in frame S and (x', t') in frame S′.

A **reference frame** is a coordinate system with meter sticks and clocks for measuring events. Experimenters at rest relative to each other share the same reference frame.

If an object has velocity u in frame S and u' in frame S′, the two velocities are related by the **Lorentz velocity transformation:**

$$u' = \frac{u - v}{1 - uv/c^2} \qquad u = \frac{u' + v}{1 + u'v/c^2}$$

where v is the relative velocity between the two frames.



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. You are in an airplane cruising smoothly at 600 mph. What experiment, if any, could you do that would demonstrate that you are moving, while those on the ground are at rest?
2. Frame S′ moves relative to frame S as shown in Figure Q27.2.
   a. A ball is at rest in frame S′. What are the speed and direction of the ball in frame S?
   b. A ball is at rest in frame S. What are the speed and direction of the ball in frame S′?

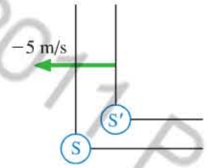


FIGURE Q27.2

3. a. Two balls move as shown in Figure Q27.3. What are the speed and direction of each ball in a reference frame that moves with ball 1?
   b. What are the speed and direction of each ball in a reference frame that moves with ball 2?

FIGURE Q27.3

4. A lighthouse beacon alerts ships to the danger of a rocky coastline.
   a. According to the lighthouse keeper, with what speed does the light leave the lighthouse?
   b. A boat is approaching the coastline at speed $0.5c$. According to the captain, with what speed is the light from the beacon approaching her boat?
5. As a rocket passes the earth at $0.75c$, it fires a laser perpendicular to its direction of travel as shown in Figure Q27.5.
   a. What is the speed of the laser beam relative to the rocket?
   b. What is the speed of the laser beam relative to the earth?

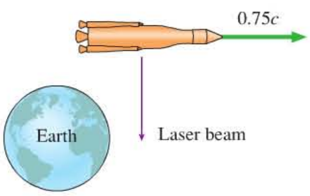


FIGURE Q27.5

6. At the instant that a clock standing next to you reads $t = 1.0\ \mu s$, you look at a second clock, 300 m away, and see that it reads $t = 0\ \mu s$. Are the two clocks synchronized? If not, which one is ahead?
7. Firecracker 1 is 300 m from you. Firecracker 2 is 600 m from you in the same direction. You see both explode at the same time. Define event 1 to be "firecracker 1 explodes" and event 2 to be "firecracker 2 explodes." Does event 1 occur before, after, or at the same time as event 2? Explain.
8. Firecrackers 1 and 2 are 600 m apart. You are standing exactly halfway between them. Your lab partner is 300 m on the other side of firecracker 1. You see two flashes of light, from the two explosions, at exactly the same instant of time. Define event 1 to be "firecracker 1 explodes" and event 2 to be "firecracker 2 explodes." According to your lab partner, based on measurements he or she makes, does event 1 occur before, after, or at the same time as event 2? Explain.
9. Your clocks and calendars are synchronized with the clocks and calendars in a star system exactly 10 ly from earth that is at rest relative to the earth. You receive a TV transmission from the star system that shows a date and time display. The date it shows is June 17, 2050. When you glance over at your own wall calendar, what date does it show?
10. Two trees are 600 m apart. You are standing exactly halfway between them and your lab partner is at the base of tree 1. Lightning strikes both trees.
    a. Your lab partner, based on measurements he makes, determines that the two lightning strikes were simultaneous. What did you see? Did you see the lightning hit tree 1 first, hit tree 2 first, or hit them both at the same instant of time? Explain.
    b. Lightning strikes again. This time your lab partner sees both flashes of light at the same instant of time. What did you see? Did you see the lightning hit tree 1 first, hit tree 2 first, or hit them both at the same instant of time? Explain.
    c. In the scenario of part b, were the lightning strikes simultaneous? Explain.
11. Figure Q27.11 shows Peggy standing at the center of her railroad car as it passes Ryan on the ground. Firecrackers attached to the ends of the car explode. A short time later, the flashes from the two explosions arrive at Peggy at the same time.
    a. Were the explosions simultaneous in Peggy's reference frame? If not, which exploded first? Explain.
    b. Were the explosions simultaneous in Ryan's reference frame. If not, which exploded first? Explain.

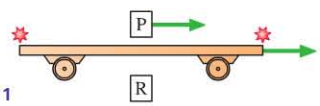


FIGURE Q27.11

12. In Figure Q27.12, clocks C_1 and C_2 in frame S are synchronized. Clock C′ moves at speed v relative to frame S. Clocks C′ and C_1 read exactly the same as C′ goes past. As C′ passes C_2, is the time shown on C′ earlier, later, or the same as the time shown on C_2? Explain.

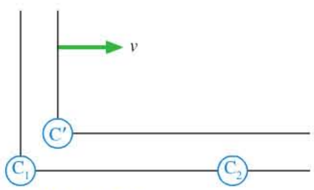


FIGURE Q27.12

13. A meter stick passes you at a speed of $0.5c$. Explain clearly how you would measure the length of this fast-moving object.

14. You're passing a car on the highway. You want to know how much time is required to completely pass the car, from no overlap between the cars to no overlap between the cars. Call your car A, the car you are passing B.
   a. Specify two events that can be given spacetime coordinates. In describing the events, refer to cars A and B and to their front bumpers and rear bumpers.
   b. In either reference frame, is there *one* clock that is present at both events?
   c. Who, if anyone, measures the proper time between the events?
15. Your friend flies from Los Angeles to New York. He determines the distance using the tried-and-true $d = vt$. You and your assistants on the ground also measure the distance using meter sticks and surveying equipment.
   a. Who, if anyone, measures the proper length?
   b. Who, if anyone, measures the shorter distance?
16. A 100-m-long train is heading for an 80-m-long tunnel. If the train moves sufficiently fast, is it possible, according to experimenters on the ground, for the entire train to be inside the tunnel at one instant of time? Explain.
17. Dan picks up a 15-m-long pole and begins running very fast, holding the pole horizontally and pointing in the direction he's running. He heads toward a barn that is 12 m long and has open doors at each end. Dan runs so fast that, to Farmer Brown standing by his barn, the ladder is only 5 m long. As soon as the pole is completely inside the barn, Farmer Brown closes both doors so that Dan and the pole are inside with both doors shut. Then, just before Dan reaches the far door, Farmer Brown opens both doors and Dan emerges, still moving at high speed. According to Dan, however, the barn is contracted to only 4 m and the pole has its full 15 m length. Farmer Brown sees the pole completely inside the barn with both doors closed. What does Dan see happening?
18. The rocket speeds shown in Figure Q27.18 are relative to the earth. Is the speed of A relative to B greater than, less than, or equal to $0.8c$?

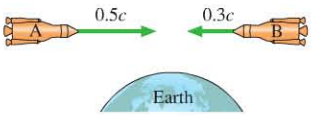


FIGURE Q27.18

19. Can a particle of mass m have total energy less than mc^2? Explain.
20. In your chemistry classes, you have probably learned that, in a chemical reaction, the mass of the products is equal to the mass of the reactants. That is, the mass of the substances produced in a chemical reaction is equal to the mass of the substances consumed in the reaction. Is this absolutely true, or is there actually a small difference? Explain.

Multiple-Choice Questions

21. Lee and Leigh are twins. At their first birthday party, Lee is placed on a spaceship that travels away from the earth and back at a steady 0.866c. The spaceship eventually returns, landing in the swimming pool at Leigh's eleventh birthday party. When Lee emerges from the ship, it is discovered that
   A. He is still only 1 year old. B. He is 6 years old.
   C. He is also 11 years old. D. He is 21 years old.
22. A space cowboy wants to eject from his spacecraft 100,000 km after passing a space buoy, as seen by spectators at rest with respect to the buoy. To do this, the cowboy sets a timer on his craft that will start as he passes the buoy. He plans to cruise by the buoy at $0.300c$. How much time should he allow between passing the buoy and ejecting?
   A. 1.01 s B. 1.06 s C. 1.11 s
   D. 1.33 s E. 1.58 s
23. Event 1 occurs at (0 m, 0 s) in reference frame S. The following events occur, also measured in reference frame S. Which of them could have been caused by event 1?
   A. (1 m, 2 ns) B. (−1 m, −2 ns) C. (−1 m, 2 ns)
   D. All of them
   E. None of them
24. Energy in the sun is produced by the fusion of four protons into a helium nucleus. The process involves several steps, but the net reaction is simply $4p \rightarrow {}^4He$ + energy. Given this, you can say that
   A. One helium atom has more mass than four hydrogen atoms.
   B. One helium atom has less mass than four hydrogen atoms.
   C. One helium atom has the same mass as four hydrogen atoms.
25. A particle moving at speed $0.40c$ has momentum p_0. The speed of the particle is increased to $0.80c$. Its momentum is now
   A. Less than $2p_0$ B. Exactly $2p_0$ C. Greater than $2p_0$
26. A particle moving at speed $0.40c$ has kinetic energy K_0. The speed of the particle is increased to $0.80c$. The kinetic energy is now
   A. Less than $4K_0$ B. Exactly $4K_0$ C. Greater than $4K_0$

PROBLEMS

Section 27.2 Galilean Relativity

1. A sprinter crosses the finish line of a race. The roar of the crowd in front approaches her at a speed of 360 m/s. The roar from the crowd behind her approaches at 330 m/s. What are the speed of sound and the speed of the sprinter?
2. A baseball pitcher can throw a ball with a speed of 40 m/s. He is in the back of a pickup truck that is driving away from you. He throws the ball in your direction, and it floats toward you at a lazy 10 m/s. What is the speed of the truck?
3. A boy on a skateboard coasts along at 5 m/s. He has a ball that he can throw at a speed of 10 m/s. What is the ball's speed relative to the ground if he throws the ball (a) forward or (b) backward?
4. A boat takes 3.0 hours to travel 30 km down a river, then 5.0 hours to return. How fast is the river flowing?
5. When the moving sidewalk at the airport is broken, as it often seems to be, it takes you 50 s to walk from your gate to baggage claim. When it is working and you stand on the moving sidewalk the entire way, without walking, it takes 75 s to travel the same distance. How long will it take you to travel from the gate to baggage claim if you walk while riding on the moving sidewalk?

6. An assembly line has a staple gun that rolls to the left at 1.0 m/s while parts to be stapled roll past it to the right at 3.0 m/s. The staple gun fires 10 staples per second. How far apart are the staples in the finished part?

Section 27.3 Einstein's Principle of Relativity

7. An out-of-control alien spacecraft is diving into a star at a speed of 1.0×10^8 m/s. At what speed, relative to the spacecraft, is the starlight approaching?
8. A starship blasts past the earth at 2.0×10^8 m/s. Just after passing the earth, the starship fires a laser beam out its back. With what speed does the laser beam approach the earth?
9. A positron moving in the positive x-direction at 2.0×10^8 m/s collides with an electron at rest. The positron and electron annihilate, producing two gamma-ray photons. Photon 1 travels in the positive x-direction and photon 2 travels in the negative x-direction. What is the speed of each photon?

Section 27.4 Events and Measurements

Section 27.5 The Relativity of Simultaneity

10. Your job is to synchronize the clocks in a reference frame. You are going to do so by flashing a light at the origin at $t = 0$ s. To what time should the clock at $(x, y, z) = (30\text{ m}, 40\text{ m}, 0\text{ m})$ be preset?
11. Bjorn is standing at $x = 600$ m. Firecracker 1 explodes at the origin and firecracker 2 explodes at $x = 900$ m. The flashes from both explosions reach Bjorn's eye at $t = 3.0\ \mu$s. At what time did each firecracker explode?
12. Bianca is standing at $x = 600$ m. Firecracker 1, at the origin, and firecracker 2, at $x = 900$ m, explode simultaneously. The flash from firecracker 1 reaches Bianca's eye at $t = 3.0\ \mu$s. At what time does she see the flash from firecracker 2?
13. You are standing at $x = 9.0$ km. Lightning bolt 1 strikes at $x = 0$ km and lightning bolt 2 strikes at $x = 12.0$ km. Both flashes reach your eye at the same time. Your assistant is standing at $x = 3.0$ km. Does your assistant see the flashes at the same time? If not, which does she see first and what is the time difference between the two?

14. A light flashes at position $x = 0$ m. One microsecond later, a light flashes at position $x = 1000$ m. In a second reference frame, moving along the x-axis at speed v, the two flashes are simultaneous. Is this second frame moving to the right or to the left relative to the original frame?
15. Jose is looking to the east. Lightning bolt 1 strikes a tree 300 m from him. Lightning bolt 2 strikes a barn 900 m from him in the same direction. Jose sees the tree strike $1.0\ \mu$s before he sees the barn strike. According to Jose, were the lightning strikes simultaneous? If not, which occurred first and what was the time difference between the two?
16. You are flying your personal rocketcraft at $0.90c$ from Star A toward Star B. The distance between the stars, in the stars' reference frame, is 1.0 ly. Both stars happen to explode simultaneously in your reference frame at the instant you are exactly halfway between them. Do you see the flashes simultaneously? If not, which do you see first and what is the time difference between the two?

Section 27.6 Time Dilation

17. A cosmic ray travels 60 km through the earth's atmosphere in $400\ \mu$s, as measured by experimenters on the ground. How long does the journey take according to the cosmic ray?
18. At what speed relative to a laboratory does a clock tick at half the rate of an identical clock at rest in the laboratory? Give your answer as a fraction of c.
19. An astronaut travels to a star system 4.5 ly away at a speed of $0.90c$. Assume that the time needed to accelerate and decelerate is negligible.
    a. How long does the journey take according to Mission Control on earth?
    b. How long does the journey take according to the astronaut?
    c. How much time elapses between the launch and the arrival of the first radio message from the astronaut saying that she has arrived?
20. A starship voyages to a distant planet 10 ly away. The explorers stay 1 yr, return at the same speed, and arrive back on earth 26 yr after they left. Assume that the time needed to accelerate and decelerate is negligible.
    a. What is the speed of the starship?
    b. How much time has elapsed on the astronauts' chronometers?

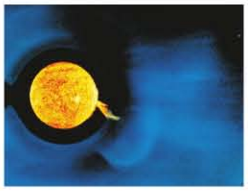
Section 27.7 Length Contraction

21. At what speed, as a fraction of c, will a moving rod have a length 60% that of an identical rod at rest?
22. Jill claims that her new rocket is 100 m long. As she flies past your house, you measure the rocket's length and find that it is only 80 m. Should Jill be cited for exceeding the $0.5c$ speed limit?
23. A muon travels 60 km through the atmosphere at a speed of $0.9997c$. According to the muon, how thick is the atmosphere?
24. The Stanford Linear Accelerator (SLAC) accelerates electrons to $v = 0.99999997c$ in a 3.2-km-long tube. If they travel the length of the tube at full speed (they don't, because they are accelerating), how long is the tube in the electrons' reference frame?
25. Our Milky Way galaxy is 100,000 ly in diameter. A spaceship crossing the galaxy measures the galaxy's diameter to be a mere 1.0 ly.
    a. What is the speed of the spaceship relative to the galaxy?
    b. How long is the crossing time as measured in the galaxy's reference frame?
26. An optical interferometer can detect a displacement of about 50 nm. At what speed would a meter stick "shrink" by 50 nm? **Hint:** Use the binomial approximation.

Section 27.8 Velocities of Objects in Special Relativity

27. A rocket cruising past earth at $0.800c$ shoots a bullet out the back door, opposite the rocket's motion, at $0.900c$ relative to the rocket. What is the bullet's speed relative to the earth?
28. A base on Planet X fires a missile toward an oncoming space fighter. The missile's speed according to the base is $0.85c$. The space fighter measures the missile's speed as $0.96c$. How fast is the space fighter traveling relative to Planet X?

29. ||| A solar flare blowing out from the sun at $0.90c$ is overtaking a rocket as it flies away from the sun at $0.80c$. According to the crew on board, with what speed is the flare gaining on the rocket?



Section 27.9 Relativistic Momentum

30. | A proton is accelerated to $0.999c$.
    a. What is the proton's momentum?
    b. By what factor does the proton's momentum exceed its Newtonian momentum?
31. ||| A 1.0 g particle has momentum 400,000 kg · m/s. What is the particle's speed?
32. | At what speed is a particle's momentum twice its Newtonian value?
33. || What is the speed of a particle whose momentum is mc?

Section 27.10 Relativistic Energy

34. || What are the kinetic energy, the rest energy, and the total energy of a 1.0 g particle with a speed of $0.80c$?
35. | A quarter-pound hamburger with all the fixings has a mass of 200 g. The food energy of the hamburger (480 food calories) is 2 MJ.
    a. What is the energy equivalent of the mass of the hamburger?
    b. By what factor does the energy equivalent exceed the food energy?
36. || How fast must an electron move so that its total energy is 10% more than its rest mass energy?
37. | At what speed is a particle's kinetic energy twice its rest energy?

General Problems

38. ||| A firecracker explodes at $x = 0$ m, $t = 0$ μs. A second explodes at $x = 300$ m, $t = 2.0$ μs. What is the proper time between these events?
39. ||| You're standing on an asteroid when you see your best friend rocketing by in her new spaceship. As she goes by, you notice that the front and rear of her ship coincide exactly with the 400-m-diameter of another nearby asteroid that is stationary with respect to you. However, you happen to know that your friend's spaceship measured 500 m long in the showroom. What is your friend's speed relative to you?
40. || A subatomic particle moves through the laboratory at $0.90c$. Laboratory experimenters measure its lifetime, from creation to annihilation, to be 2.3 ps (1 ps = 1 picosecond = 10^{-12} s). According to the particle, how long did it live?
41. || You and Maria each own identical spaceships. As you fly past Maria, you measure her ship to be 90 m long and your own ship to be 100 m long.
    a. How long does Maria measure your ship to be?
    b. How fast is Maria moving relative to you?
42. ||| A very fast-moving train car passes you, moving to the right at $0.50c$. You measure its length to be 12 m. Your friend David flies past you to the right at a speed relative to you of $0.80c$. How long does David measure the train car to be?
43. || Two events in reference frame S occur 10 μs apart at the same point in space. Frame S′ travels at speed $v = 0.90c$ relative to frame S.
    a. What is the time interval between the events in reference frame S′?
    b. What is the distance between the events in frame S′?
44. ||| A 30-m-long rocket train car is traveling from Los Angeles to New York at $0.50c$ when a light at the center of the car flashes. When the light reaches the front of the car, it immediately rings a bell. Light reaching the back of the car immediately sounds a siren.
    a. Are the bell and siren simultaneous events for a passenger seated in the car? If not, which occurs first and by how much time?
    b. Are the bell and siren simultaneous events for a bicyclist waiting to cross the tracks? If not, which occurs first and by how much time?
45. ||| INT Because of the earth's rotation, a person living on top of a mountain moves at a faster speed than someone at sea level. The mountain dweller's clocks thus run slowly compared to those at sea level. If the average life span of a hermit is 80 years, on average how much longer would a hermit dwelling on the top of a 3000-m-high mountain live compared to a sea-level hermit?
46. ||| Two clocks are synchronized. One is placed in a race car that drives around a 2.0-km-diameter track at 100 m/s for 24 h. Afterward, by how much do the two clocks differ?
    Hint: Use the binomial approximation.
47. ||| You fly 5000 km across the United States on an airliner at 250 m/s. You return two days later at the same speed.
    a. Have you aged more or less than your friends at home?
    b. By how much?
    Hint: Use the binomial approximation.
48. || INT A cube has a density of 2000 kg/m^3 while at rest in the laboratory. What is the cube's density as measured by an experimenter in the laboratory as the cube moves through the laboratory at 90% of the speed of light in a direction perpendicular to one of its faces?
49. ||| In Section 27.6 we explained that muons can reach the ground because of time dilation. But how do things appear in the muon's reference frame, where the muon's half-life is only 1.5 μs? How can a muon travel the 60 km to reach the earth's surface before decaying? Resolve this apparent paradox. Be as quantitative as you can in your answer.
50. || A spaceship flies past an experimenter who measures its length to be one-half the length he had measured when the spaceship was at rest. An astronaut aboard the spaceship notes that his clock ticks at 1-second intervals. What is the time between ticks as measured by the experimenter?
51. ||| Marissa's spaceship approaches Joseph's at a speed of $0.99c$. As Marissa passes Joseph, they synchronize their clocks to both read $t = 0$ s. When Marissa's clock reads 100 s, she sends a light signal back to Joseph. According to his clock, when does he receive this signal?
52. ||| At a speed of $0.90c$, a spaceship travels to a star that is 9.0 ly distant.
    a. According to a scientist on earth, how long does the trip take?
    b. According to a scientist on the spaceship, how long does the trip take?
    c. According to the scientist on the spaceship, what is the distance traveled during the trip?
    d. At what speed do observers on the spaceship see the star approaching them?

53. In an attempt to reduce the extraordinarily long travel times for voyaging to distant stars, some people have suggested traveling at close to the speed of light. Suppose you wish to visit the red giant star Betelgeuse, which is 430 ly away, and that you want your 20,000 kg rocket to move so fast that you age only 20 years during the round trip.
   a. How fast must the rocket travel relative to earth?
   b. How much energy is needed to accelerate the rocket to this speed?
   c. How many times larger is this energy than the total energy used by the United States in the year 2000, which was roughly 1.0×10^{20} J?

54. A rocket traveling at $0.500c$ sets out for the nearest star, Alpha Centauri, which is 4.25 ly away from earth. It will return to earth immediately after reaching Alpha Centauri. What distance will the rocket travel and how long will the journey last according to (a) stay-at-home earthlings and (b) the rocket crew? (c) Which answers are the correct ones, those in part a or those in part b?

55. A distant quasar is found to be moving away from the earth at $0.80c$. A galaxy closer to the earth and along the same line of sight is moving away from us at $0.20c$. What is the recessional speed of the quasar as measured by astronomers in the other galaxy?

56. Two rockets approach each other. Each is traveling at $0.75c$ in the earth's reference frame. What is the speed of one rocket relative to the other?

57. A military jet traveling at 1500 m/s has engine trouble and the pilot must bail out. Her ejection seat shoots her forward at 300 m/s relative to the jet. According to the Lorentz velocity transformation, by how much is her velocity relative to the ground less than the 1800 m/s predicted by Galilean relativity? **Hint:** Use the binomial approximation.

58. James, Daniella, and Tara all possess identical clocks. As Daniella passes James in her rocket, James observes that her clock runs at 80% the rate of his clock. As Tara passes in her rocket, in the same direction as Daniella, James observes that her clock runs at 70% the rate of his clock. At what rate, relative to her clock, does Daniella observe Tara's clock to run?

59. Two rockets approach earth from opposite directions at equal speeds relative to the earth. A scientist on earth notes that it takes 1 h and 10 min, according to her watch, for the clocks on the rockets to advance by 1 h.
   a. How fast are the rockets moving with respect to the earth?
   b. According to an astronaut on one rocket, how long does it take the clock on the other rocket to advance by 1 h?

60. Two rockets, A and B, approach the earth from opposite directions at speed $0.800c$. The length of each rocket measured in its rest frame is 100 m. What is the length of rocket A as measured by the crew of rocket B?

61. The highest-energy cosmic ray ever detected had an energy of about 3.0×10^{20} eV. Assume that this cosmic ray was a proton.
   a. What was the proton's speed as a fraction of c?
   b. If this proton started at the same time and place as a photon traveling at the speed of light, how far behind the photon would it be after traveling for 1 ly?

62. What is the speed of an electron after being accelerated from rest through a 20×10^6 V potential difference?

63. What is the speed of a proton after being accelerated from rest through a 50×10^6 V potential difference?

64. The half-life of a muon at rest is 1.5 μs. Muons that have been accelerated to a very high speed and are then held in a circular storage ring have a half-life of 7.5 μs.
   a. What is the speed of the muons in the storage ring?
   b. What is the total energy of a muon in the storage ring? The mass of a muon is 207 times the mass of an electron.

65. What is the momentum of a particle with speed $0.95c$ and total energy 2.0×10^{-10} J?

66. What is the momentum of a particle whose total energy is four times its rest energy? Give your answer as a multiple of mc.

67. What is the total energy, in MeV, of
   a. A proton traveling at 99.0% of the speed of light?
   b. An electron traveling at 99.0% of the speed of light?

68. What is the velocity, as a fraction of c, of
   a. A proton with 500 GeV total energy?
   b. An electron with 2.0 GeV total energy?

69. At what speed is the kinetic energy of a particle twice its Newtonian value?

70. What is the speed of an electron whose total energy equals the rest energy of a proton?

71. The factor γ appears in many relativistic expressions. A value $\gamma = 1.01$ implies that relativity changes the Newtonian values by approximately 1% and that relativistic effects can no longer be ignored. At what kinetic energy, in MeV, is $\gamma = 1.01$ for (a) an electron, and (b) a proton?

72. The chemical energy of gasoline is 46 MJ/kg. If gasoline's mass could be completely converted into energy, what mass of gasoline would be needed to equal the chemical energy content of 1.0 kg of gasoline?

73. The sun radiates energy at the rate 3.8×10^{26} W. The source of this energy is fusion, a nuclear reaction in which mass is transformed into energy. The mass of the sun is 2.0×10^{30} kg.
   a. How much mass does the sun lose each year?
   b. What percentage is this of the sun's total mass?
   c. Estimate the lifetime of the sun.

74. The radioactive element radium (Ra) decays by a process known as *alpha decay,* in which the nucleus emits a helium nucleus. (These high-speed helium nuclei were named alpha particles when radioactivity was first discovered, long before the identity of the particles was established.) The reaction is $^{226}Ra \rightarrow {}^{222}Rn + {}^{4}He$, where Rn is the element radon. The accurately measured atomic masses of the three atoms are 226.025, 222.017, and 4.003. How much energy is released in each decay? (The energy released in radioactive decay is what makes nuclear waste "hot.")

75. The nuclear reaction that powers the sun is the fusion of four protons into a helium nucleus. The process involves several steps, but the net reaction is simply $4p \rightarrow {}^{4}He + \text{energy}$. The mass of a helium nucleus is known to be 6.64×10^{-27} kg.
   a. How much energy is released in each fusion?
   b. What fraction of the initial rest mass energy is this energy?

76. || When antimatter (which we'll learn more about in Chapter 30) INT interacts with an equal mass of ordinary matter, both matter and antimatter are converted completely into energy in the form of photons. In an antimatter-fueled spaceship, a staple of science fiction, the newly created photons are shot from the back of the ship, propelling it forward. Suppose such a ship has a mass of 2.0×10^6 kg, and carries a mass of fuel equal to 1% of its mass, or 1.0×10^4 kg of matter and an equal mass of antimatter.
   a. What is the final speed of the ship, assuming it starts from rest, if all energy released in the matter-antimatter annihilation is transformed into the kinetic energy of the ship?
   b. Not only do photons have energy, as you learned in Chapter 25, they also have momentum. Explain why, when energy and momentum conservation are both considered, the final speed of the ship will be less than you calculated in part a.

Passage Problems

Pion Therapy BIO

Subatomic particles called *pions* are created when protons, accelerated to speeds very near c in a particle accelerator, smash into the nucleus of a target atom. Charged pions are unstable particles that decay into muons with a half-life of 1.8×10^{-8} s. Pions have been investigated for use in cancer treatment because they pass through tissue doing minimal damage until they decay, releasing significant energy at that point. The speed of the pions can be adjusted so that the most likely place for the decay is in a tumor.

Suppose pions are created in an accelerator, then directed into a medical bay 30 m away. The pions travel at the very high speed of $0.99995c$. Without time dilation, half of the pions would have decayed after traveling only 5.4 m, not far enough to make it to the medical bay. Time dilation allows them to survive long enough to reach the medical bay, enter tissue, slow down, and then decay where they are needed, in a tumor.

77. | What is the half-life of a pion in the reference frame of the patient undergoing pion therapy?
   A. 1.8×10^{-10} s B. 1.8×10^{-8} s
   C. 1.8×10^{-7} s D. 1.8×10^{-6} s
78. | According to the pion, what is the distance it travels from the accelerator to the medical bay?
   A. 0.30 m B. 3.0 m C. 30 m D. 3000 m
79. | The proton collision that creates the pion also creates a gamma-ray photon traveling in the same direction as the pion. The photon will get to the medical bay first because it is moving faster. What is the speed of the photon in the pion's reference frame?
   A. $0.00005c$ B. $0.5c$
   C. $0.99995c$ D. c
80. | If the pion slows down to $0.99990c$, about what percentage of its kinetic energy is lost?
   A. 0.03% B. 0.3% C. 3% D. 30%

STOP TO THINK ANSWERS

Stop to Think 27.1: A, C, and **F.** These move at constant velocity, or very nearly so. The others are accelerating.

Stop to Think 27.2: A. $u' = u - v = -10 \text{ m/s} - 6 \text{ m/s} = -16 \text{ m/s}$. The *speed* is 16 m/s.

Stop to Think 27.3: C. Even the light has a slight travel time. The event is the hammer hitting the nail, not your seeing the hammer hit the nail.

Stop to Think 27.4: At the same time. Mark is halfway between the tree and the pole, so the fact that he *sees* the lightning bolts at the same time means they *happened* at the same time. It's true that Nancy *sees* event 1 before event 2, but the events actually occurred before she sees them. Mark and Nancy share a reference frame, because they are at rest relative to each other, and all experimenters in a reference frame, after correcting for any signal delays, *agree* on the spacetime coordinates of an event.

Stop to Think 27.5: After. This is the same as the case of Peggy and Ryan. In Mark's reference frame, as in Ryan's, the events are simultaneous. Nancy *sees* event 1 first, but the time when an event is seen is not when the event actually happens. Because all experimenters in a reference frame agree on the spacetime coordinates of an event, Nancy's position in her reference frame cannot affect the order of the events. If Nancy had been passing Mark at the instant the lightning strikes occur in Mark's frame, then Nancy would be equivalent to Peggy. Event 2, like the firecracker at the front of Peggy's railroad car, occurs first in Nancy's reference frame.

Stop to Think 27.6: C. Nick measures proper time because Nick's clock is present at both the "nose passes Nick" event and the "tail passes Nick" event. Proper time is the smallest measured time interval between two events.

Stop to Think 27.7: C. The kinetic energy is $(\gamma - 1)mc^2 = 0.5mc^2$, which is less than the rest energy mc^2.