PART IV

Oscillations and Waves

Wolves are social animals, and they howl to communicate over distances of several miles with other members of their pack. How are such sounds made? How do they travel through the air? And how are other wolves able to hear these sounds from such a great distance?

OVERVIEW

Motion That Repeats Again and Again

Up to this point in the book, we have generally considered processes that have a clear starting and ending point, such as a car accelerating from rest to a final speed, or a solid being heated from an initial to a final temperature. In Part IV, we begin to consider processes that are *periodic*—they repeat. A child on a swing, a boat bobbing on the water, and even the repetitive bass beat of a rock song are *oscillatory motions* that happen over and over without a starting or ending point. The *period*, the time for one cycle of the motion, will be a key parameter for us to consider as we look at oscillatory motion.

Our first goal will be to develop the language and tools needed to describe oscillations, ranging from the swinging of the bob of a pendulum clock to the bouncing of a car on its springs. Once we understand oscillations, we will extend our analysis to consider oscillations that travel—*waves*.

The Wave Model

We've had great success modeling the motion of complex objects as the motion of one or more particles. We were even able to explain the macroscopic properties of matter, such as pressure and temperature, in terms of the motion of the atomic particles that comprise all matter.

Now it's time to explore another way of looking at nature, the *wave model*. Familiar examples of waves include

- Ripples on a pond.
- The sound of thunder.
- The swaying ground of an earthquake.
- A vibrating guitar string.
- The colors of a rainbow.

Despite the great diversity of types and sources of waves, there is a single, elegant physical theory that is capable of describing them all. Our exploration of wave phenomena will call upon water waves, sound waves, and light waves for examples, but our goal will be to emphasize the unity and coherence of the ideas that are common to *all* types of waves. As was the case with the particle model, we will use the wave model to explain a wide range of phenomena.

When Waves Collide

The collision of two particles is a dramatic event. Energy and momentum are transferred as the two particles head off in different directions. Something much gentler happens when two waves come together—the two waves pass through each other unchanged. Where they overlap, we get a *superposition* of the two waves. We will finish our discussion of waves by analyzing the standing waves that result from the superposition of two waves traveling in opposite directions. The physics of standing waves will allow us to understand how your vocal tract can produce such a wide range of sounds, and how your ears are able to analyze them.

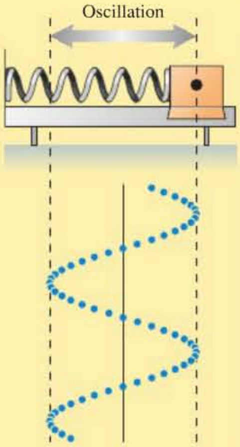
14 Oscillations

A gibbon swings below a branch, moving forward by pivoting about one handhold and then the next. How does an understanding of oscillatory motion allow us to analyze the swinging motion of a gibbon—and the walking motion of other animals?

LOOKING AHEAD ▶▶

The goal of Chapter 14 is to understand systems that oscillate with simple harmonic motion.

Simple Harmonic Motion



A cart attached to a spring oscillates back and forth. A graph of its motion is sinusoidal. This is a special kind of motion called **simple harmonic motion.**

An understanding of simple harmonic motion will build on many topics from past chapters.

Looking Back ◀◀

3.8 Period and frequency
6.2 Velocity and acceleration in circular motion
8.3 Springs and restoring forces
10.1 and10.6 Energy transformations and the conservation of energy

Spring Systems

Oscillations occur in any system having a restoring force that pushes the system back toward an equilibrium position.

A person bouncing up and down on elastic cords and the swaying of a tall building in the wind are both examples of simple harmonic motion.

An oscillation is characterized by the **period** (the time for one oscillation) and the **amplitude** (the size of the oscillation).

Pendulum Systems

A mass swinging on the end of a rod or a cord is a **pendulum**—and another example of simple harmonic motion. Its motion is mathematically the same as that of a mass on a spring.

The period of a pendulum is determined by the pendulum length and the strength of gravity; the amplitude doesn't affect the period. This makes a pendulum the ideal basis for a clock.

Damping and Resonance

As time goes on, oscillating systems may lose energy. This **damping** results in a slow decay of the oscillation.

The pendulum swings back and forth, tracing a pattern in the sand. The drag from the sand makes each oscillation smaller than the last.

The tuning fork oscillates at a particular frequency. There is one particular spot on a membrane in the inner ear that also oscillates at this exact frequency.

We'll see that the concept of **resonance** explains how your ear can distinguish different frequencies.

14.1 Equilibrium and Oscillation

Consider a marble that is free to roll inside a spherical bowl, as shown in **FIGURE 14.1**. The marble has an **equilibrium position** at the bottom of the bowl where it will rest with no net force on it. If you push the marble away from equilibrium, the marble's weight leads to a net force directed back toward the equilibrium position. We call this a **restoring force** because it acts to restore equilibrium. The magnitude of this restoring force increases if the marble is moved farther away from the equilibrium position.

If you pull the marble to the side and release it, it doesn't just roll back to the bottom of the bowl and stay put. It keeps on moving, rolling up and down each side of the bowl, repeatedly moving through its equilibrium position, as we see in **FIGURE 14.2**. We call such repetitive motion an **oscillation.** This oscillation is a result of an interplay between the restoring force and the marble's inertia, something we will see in all of the oscillations we consider.

We'll start our description by noting the most important fact about oscillatory motion: It repeats. Any oscillation is characterized by a *period,* the time for the motion to repeat. We met the concepts of period and frequency when we studied circular motion in Chapter 6. As a starting point then, let's review these ideas and see how they apply to oscillatory motion.

Frequency and Period

An electrocardiogram (ECG), such as the one shown in **FIGURE 14.3**, is a record of the electrical signals of the heart as it beats. We will explore the ECG in some detail in Chapter 21. Although the shape of a typical ECG is rather complex, notice that it has a *repeating pattern.* For this or any oscillation, the time to complete one full cycle is called the **period** of the oscillation. Period is given the symbol T.

An equivalent piece of information is the number of cycles, or oscillations, completed per second. If the period is $\frac{1}{10}$ s, then the oscillator can complete 10 cycles in 1 second. Conversely, an oscillation period of 10 s allows only $\frac{1}{10}$ of a cycle to be completed per second. In general, T seconds per cycle implies that $1/T$ cycles will be completed each second. The number of cycles per second is called the **frequency** f of the oscillation. The relationship between frequency and period is therefore

$$f=\frac{1}{T} \qquad \text{or} \qquad T=\frac{1}{f} \tag{14.1}$$

The units of frequency are **hertz,** abbreviated Hz. By definition,

$$1\ \text{Hz} = 1\ \text{cycle per second} = 1\ \text{s}^{-1}$$

We will frequently deal with very rapid oscillations and make use of the units shown in Table 14.1.

NOTE ▶ Uppercase and lowercase letters *are* important. 1 MHz is 1 megahertz $= 10^6$ Hz, but 1 mHz is 1 millihertz $= 10^{-3}$ Hz! ◀

EXAMPLE 14.1 Frequency and period of a radio station

An FM radio station broadcasts an oscillating radio wave at a frequency of 100 MHz. What is the period of the oscillation?

SOLVE The frequency f of oscillations in the radio transmitter is $100\ \text{MHz} = 1.0 \times 10^8$ Hz. The period is the inverse of the frequency; hence,

$$T=\frac{1}{f}=\frac{1}{1.0\times 10^8\ \text{Hz}}=1.0\times 10^{-8}\ \text{s}=10\ \text{ns}$$

FIGURE 14.1 Equilibrium and restoring forces for a ball in a bowl.

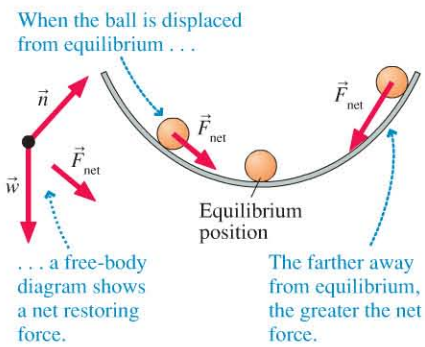


FIGURE 14.2 The motion of a ball rolling in a bowl.

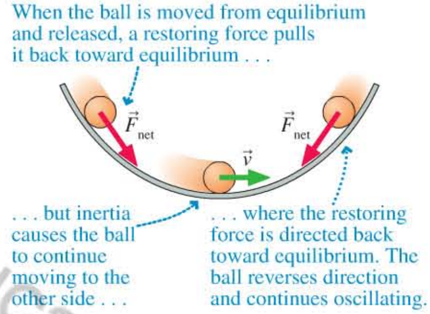


FIGURE 14.3 An electrocardiogram has a well-defined period.

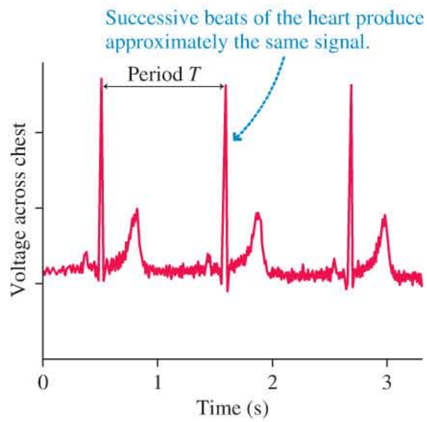


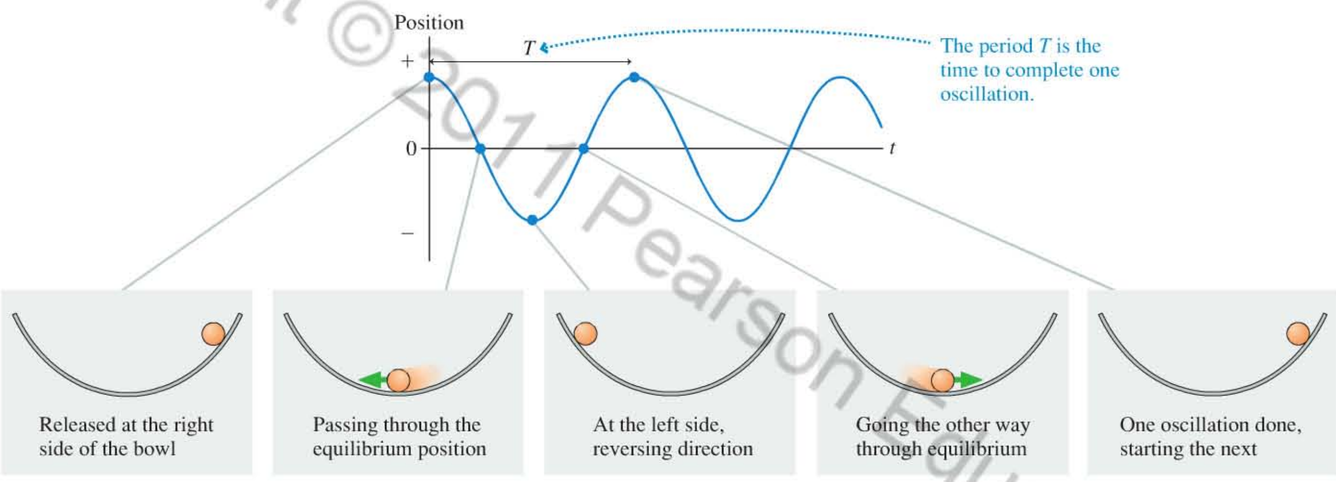
TABLE 14.1 Common units of frequency

Frequency	Period
10^3 Hz = 1 kilohertz = 1 kHz	1 ms
10^6 Hz = 1 megahertz = 1 MHz	1 μs
10^9 Hz = 1 gigahertz = 1 GHz	1 ns

Oscillatory Motion

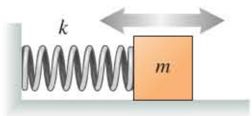
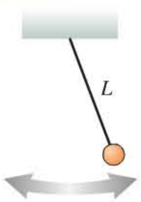
Let's return to the marble in the bowl and describe its motion in more detail. We'll start by making a graph of the motion, with positions to the right of equilibrium positive and positions to the left of equilibrium negative. **FIGURE 14.4** shows a series of "snapshots" of the motion and the corresponding points on the graph. This graph has the form of a *cosine function*. A graph or a function that has the form of a sine or cosine function is called **sinusoidal.** A sinusoidal oscillation is called **simple harmonic motion,** often abbreviated SHM.

FIGURE 14.4 Constructing a position-versus-time graph for a marble rolling in a bowl.



A marble rolling in the bottom of a bowl undergoes simple harmonic motion, as does a car bouncing up and down on its springs. SHM is very common, but we'll find that most cases of SHM can be modeled as one of two simple systems: a mass oscillating on a spring or a pendulum swinging back and forth. The following table shows two examples.

Examples of simple harmonic motion

Oscillating system		Related real-world example	
Mass on a spring 	The mass oscillates back and forth due to the restoring force of the spring. The period depends on the mass and the stiffness of the spring.	Vibrations in the ear 	Sound waves entering the ear cause the oscillation of a membrane in the cochlea. The vibration can be modeled as a mass on a spring. The period of oscillation of a segment of the membrane depends on mass (the thickness of the membrane) and stiffness (the rigidity of the membrane).
Pendulum 	The mass oscillates back and forth due to the restoring gravitational force. The period depends on the length of the pendulum and the free-fall acceleration g.	Motion of legs while walking 	The motion of a walking animal's legs can be modeled as pendulum motion. The rate at which the legs swing depends on the length of the legs and the free-fall acceleration g.

STOP TO THINK 14.1 Two oscillating systems have periods T_1 and T_2, with $T_1 < T_2$. How are the frequencies of the two systems related?

A. $f_1 < f_2$ B. $f_1 = f_2$ C. $f_1 > f_2$

14.2 Linear Restoring Forces and Simple Harmonic Motion

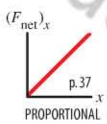
Simple harmonic motion can occur when a system has a restoring force that pushes it back toward equilibrium. We will begin our study with a very simple system, analyzing it in detail before moving on to other oscillatory systems.

FIGURE 14.5 shows a glider that rides with very little friction on an air track. There is a spring connecting the glider to the end of the track. When the spring is neither stretched nor compressed, the net force on the glider is zero. The glider just sits there—this is the equilibrium position.

If the glider is now displaced from this equilibrium position by Δx, the spring exerts a force back toward equilibrium—a restoring force. In Section 8.3, we found that the spring force is given by Hooke's law: $(F_{sp})_x = -k\Delta x$, where k is the spring constant. (Recall that a "stiffer" spring has a larger value of k.) If we set the origin of our coordinate system at the equilibrium position, the displacement from equilibrium Δx is equal to x; thus the spring force can be written as $(F_{sp})_x = -kx$.

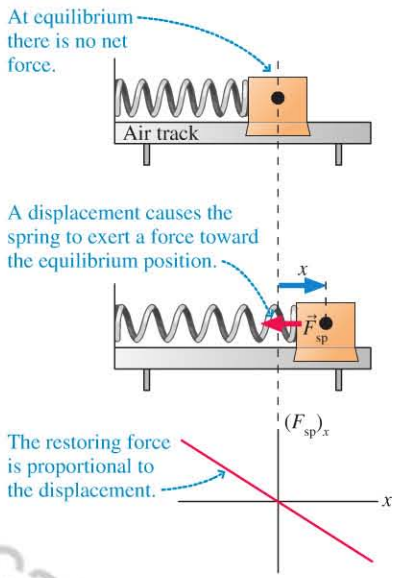
The net force on the glider is simply the spring force, so we can write

$$(F_{net})_x = -kx \tag{14.2}$$



The negative sign tells us that this is a restoring force because the force is in the direction opposite the displacement. If we pull the cart to the right (x is positive), the force is to the left (negative)—back toward equilibrium.

FIGURE 14.5 The restoring force on an air-track glider attached to a spring.



This is a **linear restoring force;** that is, **the net force is toward the equilibrium position and is proportional to the distance from equilibrium**.

Motion of a Mass on a Spring

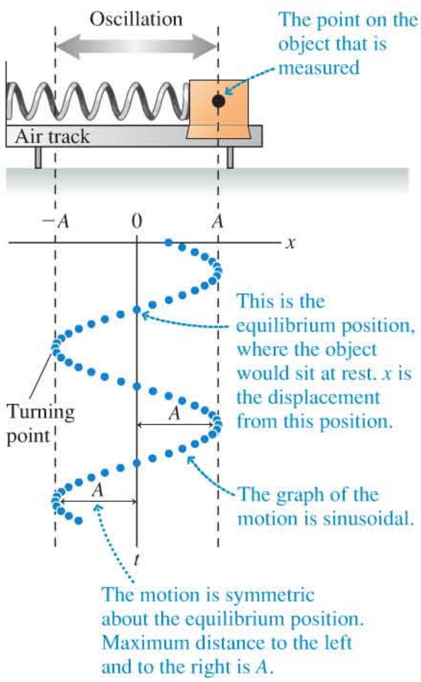
If we pull the air-track glider of Figure 14.5 a short distance to the right and release it, it will oscillate back and forth. **FIGURE 14.6** shows actual data from an experiment in which the position of a glider was measured 20 times every second. This is a position-versus-time graph that has been rotated 90° from its usual orientation in order for the x-axis to match the motion of the glider.

The object's maximum displacement from equilibrium is called the **amplitude** A of the motion. The object's position oscillates between $x = -A$ and $x = +A$.

NOTE ► When interpreting a graph, notice that the amplitude is the distance from equilibrium to the maximum, *not* the distance from the minimum to the maximum. ◄

The graph of the position is sinusoidal, so this is simple harmonic motion. **Oscillation about an equilibrium position with a linear restoring force is always simple harmonic motion.** There are many other circumstances in which a linear restoring force exists, and we will see simple harmonic motion in these cases as well.

FIGURE 14.6 An experiment showing the oscillation of an air-track glider.



Vertical Mass on a Spring

FIGURE 14.7 on the following page shows a block of mass m hanging from a spring with spring constant k. Will this mass-spring system have the same simple harmonic

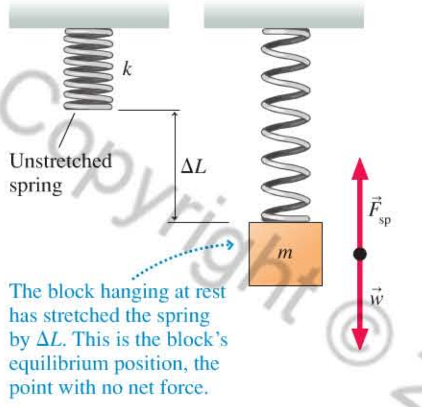


FIGURE 14.7 The equilibrium position of a mass on a vertical spring.

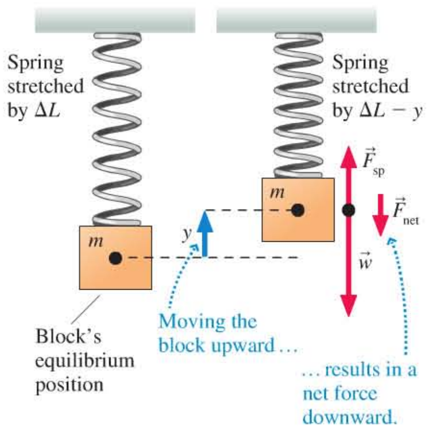


FIGURE 14.8 Displacing the block from its equilibrium position produces a restoring force.

Pendulum motion at the playground.

motion as the horizontal system we just saw, or will gravity add an additional complication? An important fact to notice is that the equilibrium position of the block is *not* where the spring is at its unstretched length. At the equilibrium position of the block, where it hangs motionless, the spring has stretched by ΔL.

Finding ΔL is a static-equilibrium problem in which the upward spring force balances the downward weight force of the block. The y-component of the spring force is given by Hooke's law:

$$(F_{sp})_y = k\,\Delta L \tag{14.3}$$

Newton's first law for the block in equilibrium is

$$(F_{net})_y = (F_{sp})_y + w_y = k\,\Delta L - mg = 0 \tag{14.4}$$

from which we can find

$$\Delta L = \frac{mg}{k} \tag{14.5}$$

This is the distance the spring stretches when the block is attached to it.

Suppose we now displace the block from this equilibrium position, as shown in FIGURE 14.8. We've placed the origin of the y-axis at the block's equilibrium position in order to be consistent with our analyses of oscillations throughout this chapter. If the block moves upward, as in the figure, the spring gets shorter compared to its equilibrium length, but the spring is still *stretched* compared to its unstretched length in Figure 14.7. When the block is at position y, the spring is stretched by an amount $\Delta L - y$ and hence exerts an *upward* spring force $F_{sp} = k(\Delta L - y)$. The net force on the block at this point is

$$(F_{net})_y = (F_{sp})_y + w_y = k(\Delta L - y) - mg = (k\,\Delta L - mg) - ky \tag{14.6}$$

But $k\,\Delta L - mg = 0$, from Equation 14.4, so the net force on the block is

$$(F_{net})_y = -ky \tag{14.7}$$

Equation 14.7 for a mass hung from a spring has the same form as Equation 14.2 for the horizontal spring, where we found $(F_{net})_x = -kx$. That is, the restoring force for vertical oscillations is identical to the restoring force for horizontal oscillations. **The role of gravity is to determine where the equilibrium position is, but it doesn't affect the restoring force for displacement from the equilibrium position.** Because it has a linear restoring force, **a mass on a vertical spring oscillates with simple harmonic motion.** The motion has the same form as that of the air-track glider.

The Pendulum

The chapter opened with a picture of a gibbon, whose body swings back and forth below a tree branch. The motion of the gibbon is essentially that of a **pendulum,** a mass suspended from a pivot point by a light string or rod. A pendulum oscillates about its equilibrium position, but is this simple harmonic motion? To answer this question, we need to examine the restoring force on the pendulum. If the restoring force is linear, the motion will be simple harmonic.

FIGURE 14.9a shows a mass m attached to a string of length L and free to swing back and forth. The pendulum's position can be described by either the arc of length s or the angle θ, both of which are zero when the pendulum hangs straight down. Because angles are measured counterclockwise, s and θ are positive when the pendulum is to the right of center, negative when it is to the left.

Two forces are acting on the mass: the string tension $\vec{T}$ and the weight $\vec{w}$. The motion is along a circular arc. We choose a coordinate system on the mass with one axis along the radius of the circle and the other tangent to the circle. We divide the forces into tangential components, parallel to the motion (denoted with a subscript t), and components directed toward the center of the circle. These are shown on the free-body diagram of **FIGURE 14.9b**.

The mass must move along a circular arc, as noted. As Figure 14.9b shows, the net force in this direction is the tangential component of the weight:

$$(F_{\text{net}})_t = \sum F_t = w_t = -mg\sin\theta \tag{14.8}$$

This is the restoring force pulling the mass back toward the equilibrium position.

This equation becomes much simpler if we restrict the pendulum's oscillations to *small angles* of about 10° (0.17 rad) or less. For such small angles, if the angle θ is in radians, it turns out that

$$\sin\theta \approx \theta$$

This result, which applies only when the angle is in radians, is called the **small-angle approximation.**

NOTE ► You can check this approximation on your calculator. Put your calculator in radian mode, enter some values that are less than 0.17 rad, take the sine, and see what you get. ◄

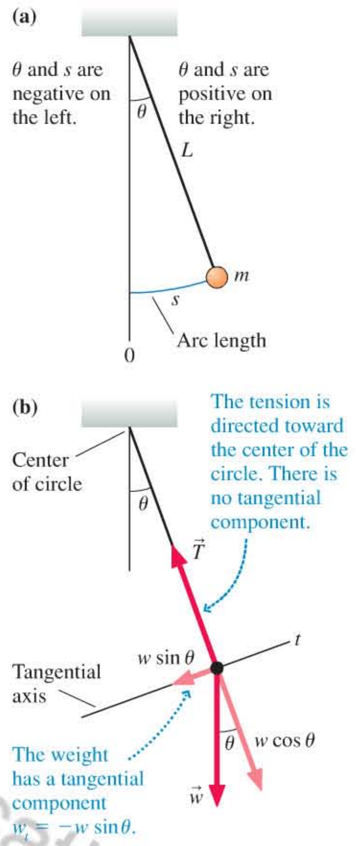
Recall from Chapter 6 that the angle is related to the arc length by $\theta = s/L$. Using this and the small-angle approximation, we can write the restoring force as

$$(F_{\text{net}})_t = -mg\sin\theta \approx -mg\theta = -mg\frac{s}{L} = -\left(\frac{mg}{L}\right)s \tag{14.9}$$

The net force is directed toward the equilibrium position, and it is linearly proportional to the displacement s from equilibrium. In other words, **the force on a pendulum is a linear restoring force for small angles, so the pendulum will undergo simple harmonic motion.**

Whenever there is a linear restoring force, there can be simple harmonic motion. There are many examples beyond the few we have considered so far, such as sloshing water in a cup or the vibration of a bridge or a building swaying in the wind. We will see these and other examples in the coming sections.

FIGURE 14.9 Describing the motion of and force on a pendulum.



STOP TO THINK 14.2 A ball is hung from a rope, making a pendulum. When it is pulled 5° to the side, the restoring force is 1.0 N. What will be the magnitude of the restoring force if the ball is pulled 10° to the side?

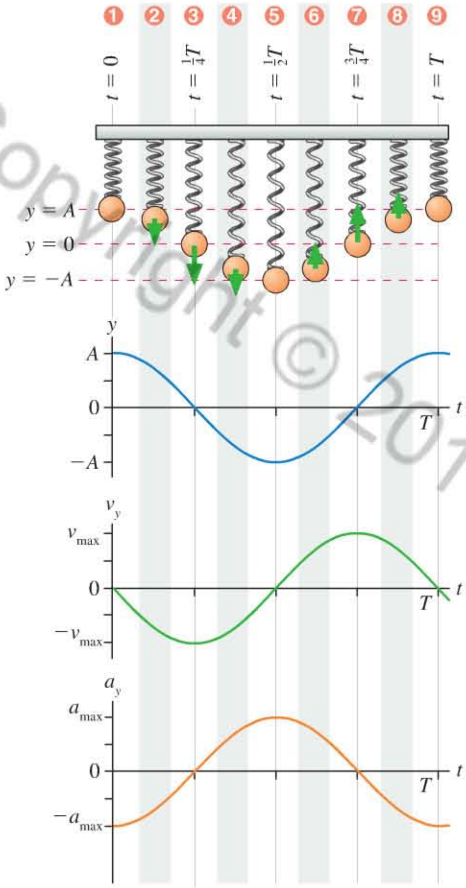
A. 0.5 N B. 1.0 N C. 1.5 N D. 2.0 N

14.3 Describing Simple Harmonic Motion

Now that we know what *causes* simple harmonic motion, we can continue to develop graphical and mathematical descriptions. If we do this for one system, we can adapt the description to any system. The details will vary, but the basic form of the motion will stay the same.

Let's look in detail at one period of the oscillation of a mass on a vertical spring. As we saw in the preceding section, the mass has an equilibrium position at which the net force is zero. If the mass is displaced from equilibrium, a linear restoring force causes the mass to undergo simple harmonic motion. In the table on the next page we present snapshots of the motion together with graphs of the position, velocity, and acceleration.

Details of oscillatory motion



A mass is suspended from a vertical spring with an equilibrium position at $y = 0$. The mass is then lifted upward by a distance A and released. We measure position with respect to the equilibrium position, with positive positions above the equilibrium point, negative below.

1. The mass starts at its maximum positive displacement, $y = A$. The velocity is zero at the instant the mass is released, but the acceleration is negative because there is a net downward force.
2. The mass is now moving downward, so the velocity is negative and the distance from equilibrium is decreasing. As the mass nears equilibrium, the restoring force, and thus the magnitude of the acceleration, decreases.
3. At this time the mass is at the equilibrium position, so the net force—and thus the acceleration—is zero. The speed is at a maximum, but the velocity is negative because the motion is downward.
4. The velocity is still negative but its magnitude is decreasing, so the acceleration is positive.
5. At this time, the mass has reached the lowest point of its motion, with $y = -A$. This is a **turning point** of the motion. The velocity is zero. The spring is at its maximum extension, so there is a net upward force and the acceleration is positive.
6. The mass has begun moving upward; the velocity is positive, and the acceleration is positive.
7. The mass is passing through the equilibrium position again, but in the opposite direction. The acceleration is zero because there is no net force; the upward velocity is positive.
8. The mass continues moving upward. The velocity is positive but its magnitude is decreasing, so the acceleration is negative.
9. The mass is now back at its starting position. This is another turning point; the mass is at rest but will soon begin moving downward, and the whole cycle will repeat.

There are three general points to note about the description of simple harmonic motion in the table above:

- The graphs are for an oscillation in which the object just happened to be at $y = A$ at $t = 0$. You can certainly imagine a different set of *initial conditions,* with the object at $y = -A$ or somewhere in the middle of an oscillation. But even if an oscillation begins at a different position, it will certainly pass through the $y = A$ position and we can simply choose to set $t = 0$ at this instant.
- The position, velocity, and acceleration graphs are all sinusoidal functions. The graphs all have the same general shape. We'll return to this point later.
- If we did a series of experiments on this system, we would find that **the frequency does not depend on the amplitude of the motion;** small oscillations and large oscillations have the same period. This is a key feature of simple harmonic motion that we will explore in more detail later in the chapter. Keep in mind that all these features of the motion of a mass on a spring also apply to other examples of simple harmonic motion.

Now that we have a complete graphical description of the motion, let's develop a mathematical description.

The position-versus-time graph in the above table is a cosine curve. We can write the object's position as

$$x(t) = A\cos\left(\frac{2\pi t}{T}\right) \tag{14.10}$$

where the notation $x(t)$ indicates that the position x is a *function* of time t. Because $\cos(2\pi \text{ rad}) = \cos(0 \text{ rad})$, we see that the position at time $t = T$ is the same as the position at $t = 0$. In other words, this is a cosine function with period T. We can write Equation 14.10 in an alternative form. Because the oscillation frequency is $f = 1/T$, we can write

$$x(t) = A\cos(2\pi f t) \qquad (14.11)$$

NOTE ► The argument $2\pi f t$ of the cosine function is in *radians*. That will be true throughout this chapter. Don't forget to set your calculator to radian mode before working oscillation problems. ◄

Just as the position graph was a cosine function, the velocity graph is an upside-down sine function with the same period T. The velocity v_x, which is a function of time, can be written as

$$v_x(t) = -v_{\max}\sin\left(\frac{2\pi t}{T}\right) = -v_{\max}\sin(2\pi f t) \qquad (14.12)$$

NOTE ► $v_{\max}$ is the maximum *speed* and thus is inherently a *positive* number. The minus sign in Equation 14.12 is needed to turn the sine function upside down. ◄

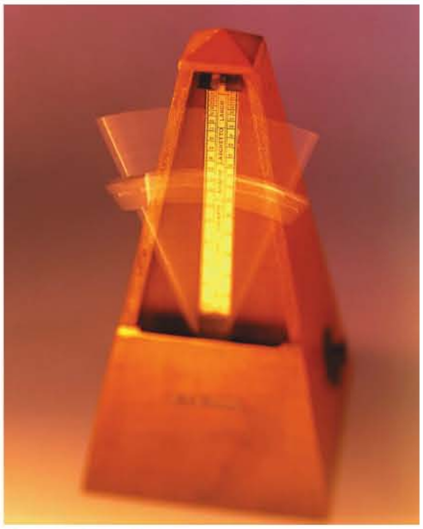
How about the acceleration? As we saw earlier in Equation 14.2, the restoring force that causes the mass to oscillate with simple harmonic motion is $(F_{\text{net}})_x = -kx$. Using Newton's second law, we see that this force causes an acceleration

$$a_x = \frac{(F_{\text{net}})_x}{m} = -\frac{k}{m}x \qquad (14.13)$$

Acceleration is proportional to the position x, but with a minus sign. Consequently we expect the acceleration-versus-time graph to be an inverted form of the position-versus-time graph. This is, indeed, what we find; the acceleration-versus-time graph in the table on the previous page is clearly an upside-down cosine function with the same period T. We can write the acceleration as

$$a_x(t) = -a_{\max}\cos\left(\frac{2\pi t}{T}\right) = -a_{\max}\cos(2\pi f t) \qquad (14.14)$$

In this and coming chapters, we will use sine and cosine functions extensively, so we will summarize some of the key aspects of mathematical relationships using these functions.



Keeping the beat The metal rod in a metronome swings back and forth, making a loud click each time it passes through the center. This is simple harmonic motion, so the motion repeats, cycle after cycle, with the same period. Musicians use this steady click to help them keep a steady beat. The photo—a long exposure—shows another interesting aspect of the motion: You can see the rod most clearly at those points where it was momentarily at rest—at each end of its swing.

Sinusoidal relationships

MP™

A quantity that oscillates in time and can be written

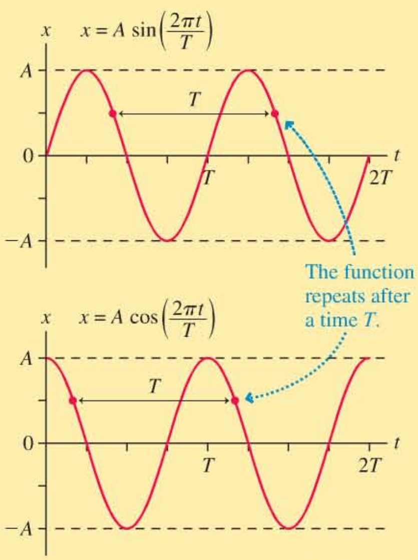
$$x = A\sin\left(\frac{2\pi t}{T}\right)$$

or

$$x = A\cos\left(\frac{2\pi t}{T}\right)$$

is called a **sinusoidal function** with **period** T. The argument of the functions, $2\pi t/T$, is in radians.

The graphs of both functions have the same shape, but they have different initial values at $t = 0$ s.



Continued

LIMITS If x is a sinusoidal function, then x is:

- *Bounded*—it can take only values between A and $-A$.
- *Periodic*—it repeats the same sequence of values over and over again. Whatever value x has at time t, it has the same value at $t + T$.

SPECIAL VALUES The function x has special values at certain times:

	$t = 0$	$t = \frac{1}{4}T$	$t = \frac{1}{2}T$	$t = \frac{3}{4}T$	$t = T$
$x = A\sin(2\pi t/T)$	0	A	0	$-A$	0
$x = A\cos(2\pi t/T)$	A	0	$-A$	0	A

Exercise 6

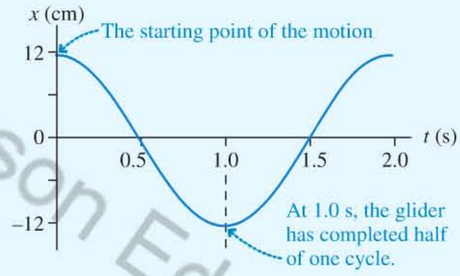
EXAMPLE 14.2 Motion of a glider on a spring

An air-track glider oscillates horizontally on a spring at a frequency of 0.50 Hz. Suppose the glider is pulled to the right of its equilibrium position by 12 cm and then released. Where will the glider be 1.0 s after its release? What is its velocity at this point?

PREPARE The glider undergoes simple harmonic motion with amplitude 12 cm. The frequency is 0.50 Hz, so the period is $T = 1/f = 2.0$ s. The glider is released at maximum extension from the equilibrium position, meaning that we can take this point to be $t = 0$.

SOLVE 1.0 s is exactly half the period. As the graph of the motion in FIGURE 14.10 shows, half a cycle brings the glider to its left turning point, 12 cm to the left of the equilibrium position. The velocity at this point is zero.

FIGURE 14.10 Position-versus-time graph for the glider.



ASSESS Drawing a graph was an important step that helped us make sense of the motion.

We were able to determine the velocity in Example 14.2 because $t = 1.0$ s was a turning point where the instantaneous velocity was zero. To determine the velocity at other points, we need to know v_{max} and use Equation 14.12. How does v_{max} depend on other variables of the motion? Basic reasoning about the motion tells us a few things. A large amplitude implies a high speed because the glider moves a large distance. A small period implies a high speed as well because the glider must complete its motion in a short time. Later in the chapter we will show that both of these assumptions are correct, and that the maximum speed is

$$v_{max} = 2\pi f A = \frac{2\pi A}{T} \qquad (14.15)$$

◀ **Small bird, fast wings** BIO As this rufous hummingbird flaps its wings, the motion of the wing tips is approximately simple harmonic. The frequency of the motion is about 45 Hz, so the period of the wingbeat is only a few hundredths of a second. As we see in Equation 14.15, this short period means that the tips of the hummingbird's tiny wings are moving at a high speed—over 15 m/s, or nearly 35 mph!

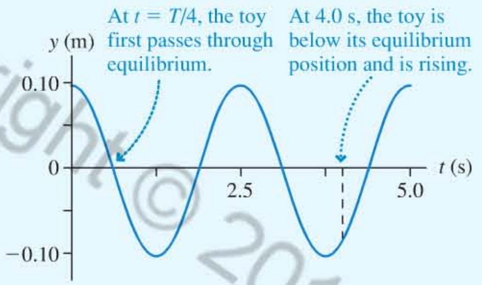
EXAMPLE 14.3 Analyzing the motion of a hanging toy

A classic children's toy consists of a wooden animal suspended from a spring. If you lift the toy up by 10 cm and let it go, it will gently bob up and down, completing 4 oscillations in 10 seconds.

a. What is the oscillation frequency?
b. When does the toy first reach its maximum speed, and what is this speed?
c. What are the position and velocity 4.0 s after you release the toy?

PREPARE The toy is a mass on a vertical spring, so it undergoes simple harmonic motion. The toy begins its motion at its maximum positive displacement of 10 cm, or 0.10 m; this is the amplitude of the motion. Because the toy is released at maximum positive displacement, we can set the moment of release as $t = 0$ s. The toy completes 4 oscillations in 10 seconds, so the period is 1/4 of 10 seconds, or $T = 2.5$ s. **FIGURE 14.11** shows the first two cycles of the oscillation.

FIGURE 14.11 A position graph for the spring toy.



SOLVE a. The period is 2.5 s, so the frequency is

$$f = \frac{1}{T} = \frac{1}{2.5\text{ s}} = 0.40\text{ oscillation/s} = 0.40\text{ Hz}$$

b. Figure 14.11 shows that the toy first passes through the equilibrium position at $t = T/4 = (2.5\text{ s})/4 = 0.62$ s. This is a point of maximum speed, with the value given by Equation 14.15:

$$v_{max} = 2\pi f A = 2\pi(0.40\text{ Hz})(0.10\text{ m}) = 0.25\text{ m/s}$$

c. 4.0 s after release is $t = 4.0$ s. Figure 14.11 shows that the toy is below the equilibrium position and rising at this time, so we expect that the position is negative and the velocity is positive. We can use Equations 14.11 and 14.12 to find the position and velocity at this time:

$$\begin{aligned} y(t = 4.0\text{ s}) &= A\cos(2\pi f t) \\ &= (0.10\text{ m})\cos[2\pi(0.40\text{ Hz})(4.0\text{ s})] \\ &= -0.081\text{ m} \end{aligned}$$

$$\begin{aligned} v_y(t = 4.0\text{ s}) &= -v_{max}\sin(2\pi f t) \\ &= -(0.25\text{ m/s})\sin[2\pi(0.40\text{ Hz})(4.0\text{ s})] \\ &= 0.15\text{ m/s} \end{aligned}$$

Note that your calculator *must* be in radian mode to do calculations like these.

ASSESS Our calculations for $t = 4.0$ s show that the toy is below the equilibrium position and moving upward, just as we expected from the graph in Figure 14.11. Drawing the graph provided a good check on our work.

Connection to Uniform Circular Motion

Both circular motion and simple harmonic motion are motions that repeat. Many of the concepts we have used for describing simple harmonic motion were introduced in our study of circular motion. We will use the close connection between the two to extend our knowledge of the kinematics of circular motion to simple harmonic motion.

We can demonstrate the relationship between circular and simple harmonic motion with a simple experiment. Suppose a turntable has a small ball glued to the edge. As **FIGURE 14.12a** shows, we can make a "shadow movie" of the ball by projecting a light past the ball and onto a screen. The ball's shadow oscillates back and forth as the turntable rotates.

If you place a real object on a real spring directly below the shadow, as shown in **FIGURE 14.12b**, and if you adjust the turntable to have the same period as the spring, you will find that the shadow's motion exactly matches the simple harmonic motion of the object on the spring. **Uniform circular motion projected onto one dimension is simple harmonic motion.**

We can show why the projection of circular motion is simple harmonic motion by considering the particle in **FIGURE 14.13** on the next page. As in Chapter 6, we can locate the particle by the angle ϕ measured counterclockwise from the x-axis. Projecting the ball's shadow onto a screen in Figure 14.12a is equivalent to observing just the x-component of the particle's motion. Figure 14.13 shows that the x-component, when the particle is at angle ϕ, is

$$x = A\cos\phi$$

If the particle starts from $\phi_0 = 0$ at $t = 0$, its angle at a later time t is

$$\phi = \omega t$$

where ω is the particle's *angular velocity,* as defined in Chapter 6. Recall that the angular velocity is related to the frequency f by

$$\omega = 2\pi f$$

FIGURE 14.12 A projection of the circular motion of a rotating ball matches the SHM of an object on a spring.

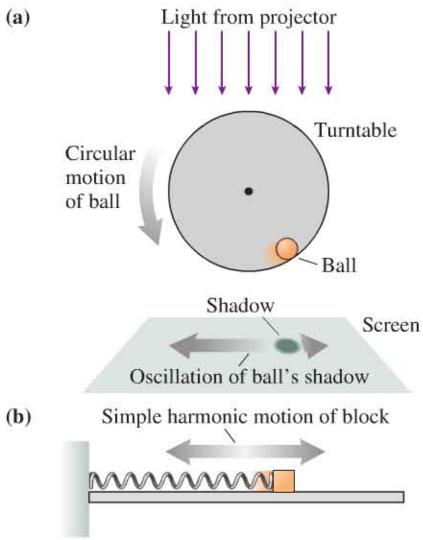
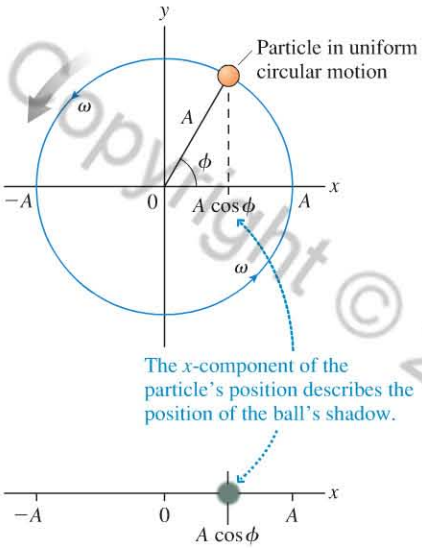


FIGURE 14.13 A particle in uniform circular motion with radius A and angular velocity ω.



TRY IT YOURSELF

SHM in your microwave It's possible to do something like the turntable demonstration right at home. Place a tall (microwave safe) glass or cup filled with water on the outside edge of the turntable in your microwave oven. Start the oven. The turntable will rotate, moving the cup in a circle. Stand in front of the oven with your eyes level with the cup. Watch the cup, paying attention to the side-to-side motion. This motion is the horizontal component of the circular motion, and so it is simple harmonic motion.

The particle's x-component can therefore be expressed as

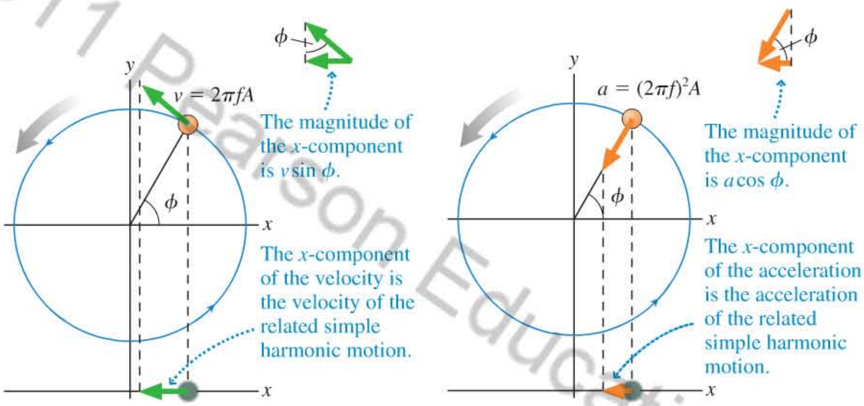
$$x(t) = A\cos(2\pi ft) \tag{14.16}$$

This is identical to Equation 14.11 for the position of a mass on a spring! **The x-component of a particle in uniform circular motion is simple harmonic motion.**

We can use this correspondence to deduce more details. **FIGURE 14.14** is the same motion we looked at in Figure 14.13, but here we've shown the velocity vector (tangent to the circle) and the acceleration vector (a centripetal acceleration pointing to the center of the circle). The magnitude of the velocity vector is the particle's speed. Recall from Chapter 6 that the speed of a particle in circular motion with radius A and frequency f is $v = 2\pi fA$. Thus the x-component of the velocity vector, which is pointing in the negative x-direction, is

$$v_x = -v\sin\phi = -(2\pi f)A\sin(2\pi ft)$$

FIGURE 14.14 Projection of the velocity and acceleration vectors.



According to the correspondence between circular motion and simple harmonic motion, this is the velocity of an object in simple harmonic motion. This is, indeed, exactly Equation 14.12, which we deduced from the graph of the motion, if we define the maximum speed, as we did in Equation 14.15, to be

$$v_{\text{max}} = 2\pi fA = \frac{2\pi A}{T}$$

Similarly, the magnitude of the acceleration vector is the centripetal acceleration $a = v^2/A = (2\pi f)^2A$. The x-component of the acceleration vector, which is the acceleration for simple harmonic motion, is

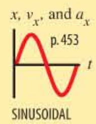
$$a_x = -a\cos\phi = -(2\pi f)^2A\cos(2\pi ft)$$

The maximum acceleration is

$$a_{\text{max}} = (2\pi f)^2A \tag{14.17}$$

We can now summarize our findings for the position, velocity, and acceleration of an object in simple harmonic motion:

$$\begin{aligned} x(t) &= A\cos(2\pi ft) \\ v_x(t) &= -(2\pi f)A\sin(2\pi ft) \\ a_x(t) &= -(2\pi f)^2A\cos(2\pi ft) \end{aligned} \tag{14.18}$$



Position, velocity, and acceleration for an object in simple harmonic motion with frequency f and amplitude A

Any simple harmonic motion follows these equations. **If you know the amplitude and the frequency, the motion is completely specified.**

EXAMPLE 14.4 Measuring the sway of a tall building

The John Hancock Center in Chicago is 100 stories high. Strong winds can cause the building to sway, as is the case with all tall buildings. On particularly windy days, the top of the building is known to oscillate with an amplitude of 40 cm ($\approx$16 in) and a period of 7.7 s. What are the maximum speed and acceleration of the top of the building?

PREPARE We will assume that the oscillation of the building is simple harmonic motion with amplitude $A = 0.40$ m. The frequency can be computed from the period:

$$f = \frac{1}{T} = \frac{1}{7.7\text{ s}} = 0.13\text{ Hz}$$

SOLVE We can use Equations 14.15 and 14.17 for the maximum velocity and acceleration to compute:

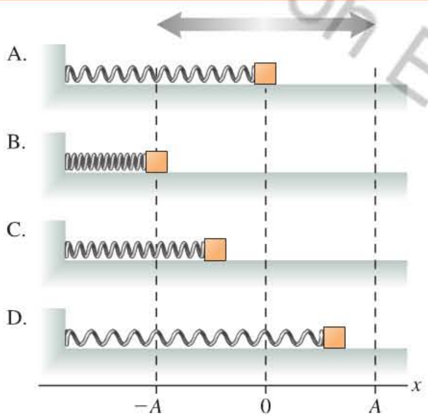
$$v_{max} = 2\pi fA = 2\pi(0.13\text{ Hz})(0.40\text{ m}) = 0.33\text{ m/s}$$

$$a_{max} = (2\pi f)^2 A = [2\pi(0.13\text{ Hz})]^2(0.40\text{ m}) = 0.27\text{ m/s}^2$$

In terms of the free-fall acceleration, the maximum acceleration is $a_{max} = 0.027g$.

ASSESS The free-fall acceleration is quite small, as you would expect; if it were large, building occupants would certainly complain! Even if they don't notice the motion directly, office workers on high floors of high buildings may experience a bit of nausea when the oscillations are large because the acceleration affects the equilibrium organ in the inner ear.

STOP TO THINK 14.3 The figures show four identical oscillators at different points in their motion. Which is moving fastest at the time shown?



14.4 Energy in Simple Harmonic Motion

A bungee jumper falls, increasing in speed, until the elastic cords attached to his ankles start to stretch. The kinetic energy of his motion is transformed into the elastic potential energy of the cords. Once the cords reach their maximum stretch, his velocity reverses. He rises as the elastic potential energy of the cords is transformed back into kinetic energy. If he keeps bouncing (simple harmonic motion), this transformation happens again and again.

This interplay between kinetic and potential energy is very important to understanding simple harmonic motion. The five diagrams in **FIGURE 14.15a** on the next page show the position and velocity of a mass on a spring at successive points in time.

The object begins at rest, with the spring at a maximum extension; the kinetic energy is zero, and the potential energy is a maximum. As the spring contracts, the object speeds up until it reaches the center point of its oscillation, the equilibrium point. At this point, the potential energy is zero and the kinetic energy is a maximum. As the object continues to move, it slows down as it compresses the

Elastic cords lead to the up-and-down motion of a bungee jump.

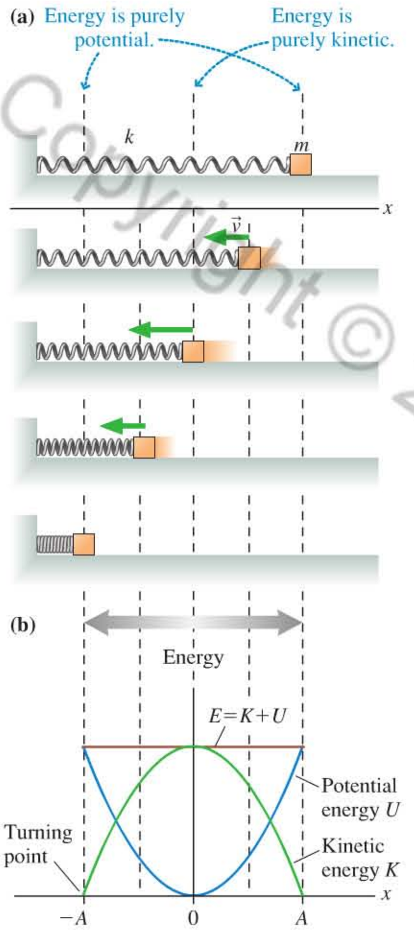


FIGURE 14.15 Energy transformations for a mass on a spring.

spring. Eventually, it reaches the turning point, where its instantaneous velocity is zero. At this point, the kinetic energy is zero and the potential energy is again a maximum.

Now we'll specify that the object has mass m, the spring has spring constant k, and the motion takes place on a frictionless surface. You learned in Chapter 10 that the elastic potential energy of a spring stretched by a distance x from its equilibrium position is

$$U = \frac{1}{2}kx^2 \tag{14.19}$$

The potential energy is zero at the equilibrium position and is a maximum when the spring is at its maximum extension or compression. There is no energy loss to thermal energy, so conservation of energy for this system can be written

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant} \tag{14.20}$$

FIGURE 14.15b shows a graph of the potential energy, kinetic energy, and total energy for the object as it moves. You can see that, as the object goes through its motion, energy is transformed from potential to kinetic and then back to potential. At maximum displacement, with $x = \pm A$ and $v_x = 0$, the energy is purely potential, so the potential energy has its maximum value:

$$E(\text{at } x = \pm A) = U_{\text{max}} = \frac{1}{2}kA^2 \tag{14.21}$$

At $x = 0$, where $v_x = \pm v_{\text{max}}$, the energy is purely kinetic, so the kinetic energy has its maximum value:

$$E(\text{at } x = 0) = K_{\text{max}} = \frac{1}{2}m(v_{\text{max}})^2 \tag{14.22}$$

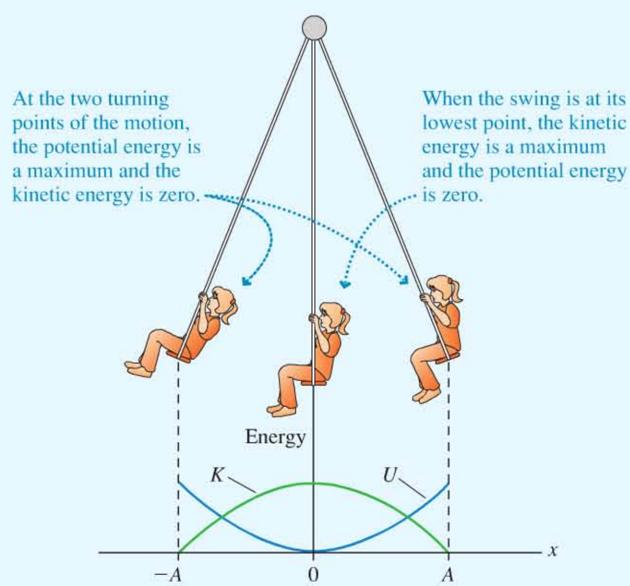
CONCEPTUAL EXAMPLE 14.5 Energy changes for a playground swing

You are at the park, undergoing simple harmonic motion on a swing. Describe the changes in energy that occur during one cycle of the motion, starting from when you are at the farthest forward point, motionless and just about to swing backward.

REASON The motion of the swing is that of a pendulum, with you playing the role of the pendulum bob. The energy at different points of the motion is illustrated in FIGURE 14.16. When you are motionless and at the farthest forward point, you are raised up; you have potential energy. As you swing back, your potential energy decreases and your kinetic energy increases, reaching a maximum when the swing is at the lowest point. The swing continues to move backward; you rise up, transforming kinetic energy into potential energy. The process then reverses as you move forward.

ASSESS This description matches the experience of anyone who has been on a swing. You know that you are momentarily motionless at the highest points, and that the motion is fastest at the lowest point.

FIGURE 14.16 Energy at different points of the motion of a swing.



Finding the Frequency for Simple Harmonic Motion

Now that we have an energy description of simple harmonic motion, we can use what we know about energy to deduce other details of the motion. Let's return to the mass on a spring of Figure 14.15. The graph in Figure 14.15b shows energy being transformed back and forth between kinetic and potential energy. At the turning points, the energy is purely potential; at the equilibrium point, the energy is purely kinetic. Because the total energy doesn't change, the maximum kinetic energy given in Equation 14.22 must be equal to the maximum potential energy given in Equation 14.21:

$$\frac{1}{2}m(v_{\text{max}})^2 = \frac{1}{2}kA^2 \tag{14.23}$$

By solving Equation 14.23 for the maximum speed, we can see that it is related to the amplitude by

$$v_{\text{max}} = \sqrt{\frac{k}{m}}A \tag{14.24}$$

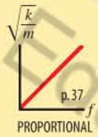
Earlier we found that

$$v_{\text{max}} = 2\pi f A \tag{14.25}$$

Comparing Equations 14.24 and 14.25, we see that the frequency, and thus the period, of an oscillating mass on a spring is determined by the spring constant k and the object's mass m:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{m}{k}} \tag{14.26}$$

Frequency and period of SHM
for mass m on a spring with spring constant k



We can make two observations about these equations:

- **The frequency and period of simple harmonic motion are determined by the physical properties of the oscillator.** The frequency and period of a mass on a spring are determined by (1) the mass and (2) the stiffness of the spring, as shown in FIGURE 14.17. This dependence of frequency and period on a force term and an inertia term will also apply to other oscillators.
- **The frequency and period of simple harmonic motion do not depend on the amplitude A.** A small oscillation and a large oscillation have the same frequency and period.

CONCEPTUAL EXAMPLE 14.6 Changing mass, changing period

An astronaut measures her mass each day using the Body Mass Measurement Device on the Space Shuttle, as described at right. During an 8-day flight, her mass steadily decreases. How does this change the frequency of her oscillatory motion on the device?

REASON The period and frequency of a mass-spring system depend on the mass of the object and the spring constant. The spring constant of the device won't change, so the only change that matters is the change in the astronaut's mass. Equation 14.26 shows that the frequency is proportional to $\sqrt{k/m}$, so a decrease in her mass will cause an increase in the frequency. The oscillation will be a bit more rapid.

ASSESS This makes sense. The force of the spring—which causes the oscillation—is the same, but the mass to be accelerated is less. We expect a higher frequency.

Now that we have a complete description of simple harmonic motion in one system, we will summarize the details in a Tactics Box on the next page, showing how to use this information to solve oscillation problems.

FIGURE 14.17 Frequency dependence on mass and spring stiffness.

High frequency
Stiff spring,
low mass

Low frequency
Soft spring,
high mass

Increasing the stiffness of the spring will increase the restoring force. This *increases* the frequency.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Increasing the mass will increase the inertia of the system. This *decreases* the frequency.

Measuring mass in space Astronauts on extended space flights monitor their mass to track the effects of weightlessness on their bodies. But because they are weightless, they can't just hop on a scale! Instead, they use an ingenious device in which an astronaut sitting on a platform oscillates back and forth due to the restoring force of a spring. The astronaut is the moving mass in a mass-spring system, so a measurement of the period of the motion allows a determination of an astronaut's mass.

9.3, 9.4, 9.6, 9.7, 9.8, 9.9 ActivPhysics

TACTICS BOX 14.1 Identifying and analyzing simple harmonic motion MP™

1. If the net force acting on a particle is a linear restoring force, the motion is simple harmonic motion around the equilibrium position.
2. The position, velocity, and acceleration as a function of time are given in Equations 14.18. The equations are given here in terms of x, but they can be written in terms of y, θ, or some other variable if the situation calls for it.
3. The amplitude A is the maximum value of the displacement from equilibrium. The maximum speed and the maximum magnitude of the acceleration are $v_{max} = 2\pi f A$ and $a_{max} = (2\pi f)^2 A$.
4. The frequency f (and hence the period $T = 1/f$) depends on the physical properties of the particular oscillator, but f does *not* depend on A. For a mass on a spring, the frequency is given by $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.
5. The sum of potential energy plus kinetic energy is constant. As the oscillation proceeds, energy is transformed from kinetic into potential energy and then back again.

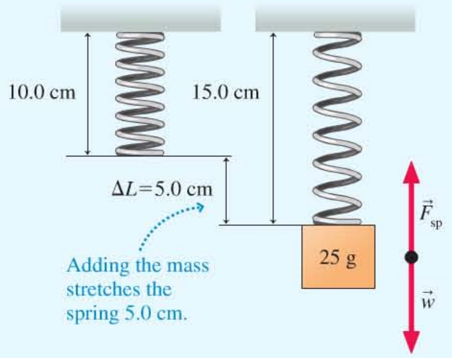
Exercise 11

EXAMPLE 14.7 Finding the frequency of an oscillator

A spring has an unstretched length of 10.0 cm. A 25 g mass is hung from the spring, stretching it to a length of 15.0 cm. If the mass is pulled down and released so that it oscillates, what will be the frequency of the oscillation?

PREPARE The spring provides a linear restoring force, so the motion will be simple harmonic, as noted in Tactics Box 14.1.

FIGURE 14.18 Visual overview of a mass suspended from a spring.



The oscillation frequency depends on the spring constant, which we can determine from the stretch of the spring. **FIGURE 14.18** gives a visual overview of the situation.

SOLVE When the mass hangs at rest, after stretching the spring to 15 cm, the net force on it must be zero. Thus the magnitude of the upward spring force equals the downward weight, giving $k\Delta L = mg$. The spring constant is thus

$$k = \frac{mg}{\Delta L} = \frac{(0.025\text{ kg})(9.8\text{ m/s}^2)}{0.050\text{ m}} = 4.9\text{ N/m}$$

Now that we know the spring constant, we can compute the oscillation frequency:

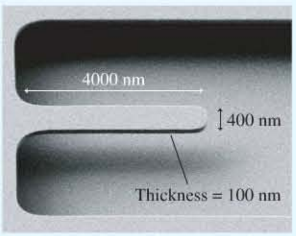
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.9\text{ N/m}}{0.025\text{ kg}}} = 2.2\text{ Hz}$$

ASSESS 2.2 Hz is 2.2 oscillations per second. This seems like a reasonable frequency for a mass on a spring. A frequency in the kHz range (thousands of oscillations per second) would have been suspect!

EXAMPLE 14.8 Weighing DNA molecules

It has recently become possible to "weigh" individual DNA molecules by measuring the influence of their mass on a nanoscale oscillator. **FIGURE 14.19** shows a thin rectangular cantilever etched out of silicon. The cantilever has a mass of 3.7×10^{-16} kg. If pulled down and released, the end of the cantilever vibrates with simple harmonic motion,

FIGURE 14.19 A nanoscale cantilever.



moving up and down like a diving board after a jump. When the end of the cantilever is bathed with DNA molecules whose ends have been modified to bind to a surface, one or more molecules may attach to the end of the cantilever. The addition of their mass causes a very slight—but measurable—decrease in the oscillation frequency.

A vibrating cantilever of mass M can be modeled as a simple block of mass $\frac{1}{3}M$ attached to a spring. (The factor of $\frac{1}{3}$ arises from the moment of inertia of a bar pivoted at one end: $I = \frac{1}{3}ML^2$.) Neither the mass nor the spring constant can be determined very accurately—perhaps only to two significant figures—but the oscillation frequency can be measured with very high precision simply by counting the oscillations. In one experiment, the cantilever was initially vibrating at exactly 12 MHz. Attachment of a DNA molecule caused the frequency to decrease by 50 Hz. What was the mass of the DNA molecule?

PREPARE We will model the cantilever as a block of mass $m = \frac{1}{3}M = 1.2 \times 10^{-16}$ kg oscillating on a spring with spring constant k. When the mass increases to $m + m_{\text{DNA}}$, the oscillation frequency decreases from $f_0 = 12{,}000{,}000$ Hz to $f_1 = 11{,}999{,}950$ Hz.

SOLVE The oscillation frequency of a mass on a spring is given by Equation 14.26. Addition of mass doesn't change the spring constant, so solving this equation for k allows us to write

$$k = m(2\pi f_0)^2 = (m + m_{\text{DNA}})(2\pi f_1)^2$$

The 2π terms cancel, and we can rearrange this equation to give

$$\frac{m + m_{\text{DNA}}}{m} = 1 + \frac{m_{\text{DNA}}}{m} = \left(\frac{f_0}{f_1}\right)^2 = \left(\frac{12{,}000{,}000\text{ Hz}}{11{,}999{,}950\text{ Hz}}\right)^2 = 1.0000083$$

Subtracting 1 from both sides gives

$$\frac{m_{\text{DNA}}}{m} = 0.0000083$$

and thus

$$m_{\text{DNA}} = 0.0000083m = (0.0000083)(1.2 \times 10^{-16}\text{ kg}) = 1.0 \times 10^{-21}\text{ kg} = 1.0 \times 10^{-18}\text{ g}$$

ASSESS This is a reasonable mass for a DNA molecule. It's a remarkable technical achievement to be able to measure a mass this small. With a slight further improvement in sensitivity, scientists will be able to determine the number of base pairs in a strand of DNA simply by weighing it!

EXAMPLE 14.9 **Slowing a mass with a spring collision**

A 1.5 kg mass slides across a horizontal, frictionless surface at a speed of 2.0 m/s until it collides with and sticks to the free end of a spring with spring constant 50 N/m. The spring's other end is anchored to a wall. How far has the spring compressed when the mass is, at least for an instant, at rest? How much time does it take for the spring to compress to this point?

PREPARE This is a collision problem, but we can solve it using the tools of simple harmonic motion. FIGURE 14.20 gives a visual overview of the problem. The motion is along the x-axis. We have set $x = 0$ at the uncompressed end of the spring, which is the point of collision. Once the mass hits and sticks, it will start oscillating with simple harmonic motion. The position-versus-time graph shows that the motion of the mass until it stops is $\frac{1}{4}$ of a cycle of simple harmonic motion—though the starting point is different from our usual choice. During this quarter cycle of motion, the kinetic energy of the mass is transformed into the potential energy of the compressed spring.

SOLVE Conservation of energy tells us that the potential energy of the spring when fully compressed is equal to the initial kinetic energy of the mass, so we write

$$\frac{1}{2}m(v_x)_i^2 = \frac{1}{2}kx_f^2$$

The final position of the mass is

$$x_f = (v_x)_i\sqrt{\frac{m}{k}} = (2.0\text{ m/s})\sqrt{\frac{1.5\text{ kg}}{50\text{ N/m}}} = 0.35\text{ m}$$

This is the compression of the spring.

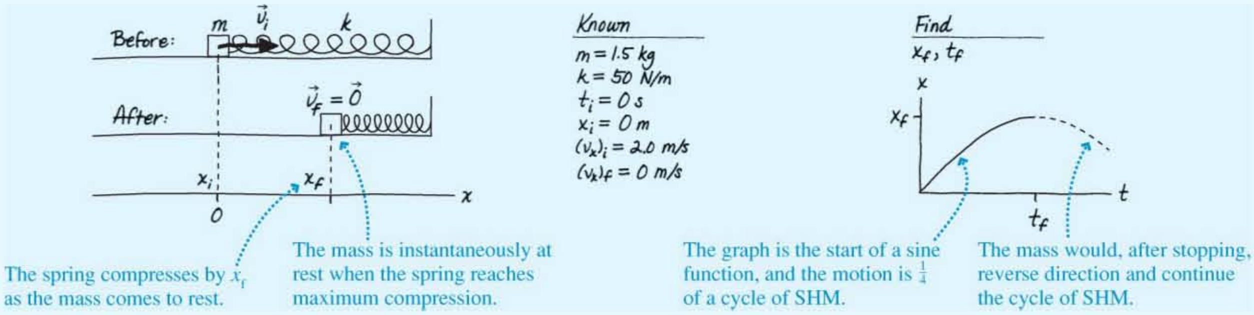
Because the compression is $\frac{1}{4}$ of a cycle of simple harmonic motion, the time needed to stop the mass is $t_f = \frac{1}{4}T$. The period is computed using Equation 14.26, so we write

$$t_f = \frac{1}{4}T = \frac{1}{4}2\pi\sqrt{\frac{m}{k}} = \frac{\pi}{2}\sqrt{\frac{1.5\text{ kg}}{50\text{ N/m}}} = 0.27\text{ s}$$

This is the time required for the spring to compress.

ASSESS Interestingly, the time needed to stop the mass does not depend on its initial velocity or on the distance the spring compresses. The motion is simple harmonic; thus the period depends only on the stiffness of the spring and the mass, not on the details of the motion.

FIGURE 14.20 Visual overview for the mass-spring collision.

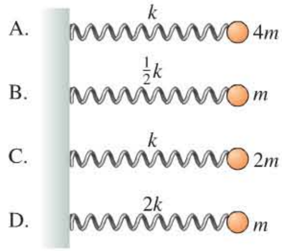


Automobile collision times When a car hits a stationary barrier, it takes approximately 0.1 s to come to rest, regardless of the initial speed, something we can understand with a simple model of the collision. The crumpling of the front of a car during a collision is quite complex, but for many cars the force is approximately proportional to the displacement during the compression of the front of the car. As long as we consider only this initial compression, we can model the body of the car as a mass and the front of the car as a spring. With this model, the collision is similar to that of Example 14.9, and, as in the example, the time for the car to come to rest does not depend on the initial speed.

9.10, 9.11, 9.12 ActivPhysics ONLINE

Many real-world collisions (such as a pole vaulter landing on a foam pad) involve a moving object that is stopped by an elastic object. In other cases (such as a bouncing rubber ball) the object itself is elastic. The time of the collision is reasonably constant, independent of the speed of the collision, for the reasons noted in the above example.

STOP TO THINK 14.4 Four mass-spring systems have masses and spring constants shown here. Rank in order, from highest to lowest, the frequencies of the oscillations.



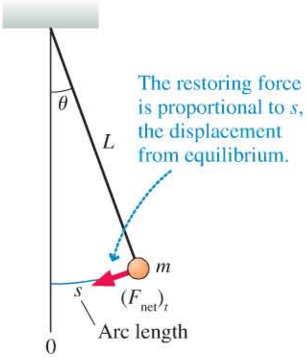
14.5 Pendulum Motion

As we've already seen, a simple pendulum—a mass at the end of a string or a rod that is free to pivot—is another system that exhibits simple harmonic motion. Everything we have learned about the mass on a spring can be applied to the pendulum as well.

In the first part of the chapter, we looked at the restoring force in the pendulum. For a pendulum of length L displaced by an arc length s, as in **FIGURE 14.21**, the tangential restoring force is

$$(F_{\text{net}})_t = -\frac{mg}{L}s \tag{14.27}$$

FIGURE 14.21 A simple pendulum.



NOTE ▶ Recall that this equation holds only for small angles. ◀

This linear restoring force has exactly the same form as the net force in a mass-spring system, but with the constants mg/L in place of the constant k. Given this, we can quickly deduce the essential features of pendulum motion by replacing k, wherever it occurs in the oscillating spring equations, with mg/L.

- The oscillation of a pendulum is simple harmonic motion; the equations of motion can be written for the arc length or the angle:

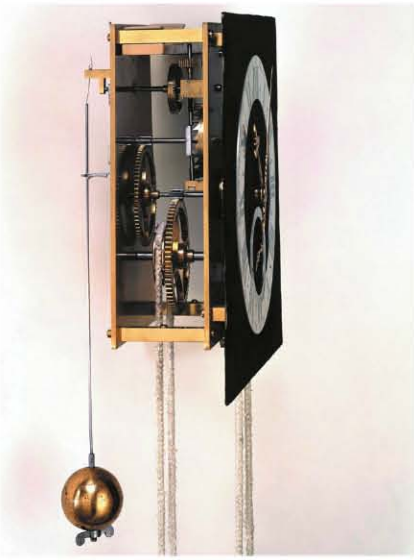
$$s(t) = A\cos(2\pi f t) \qquad \text{or} \qquad \theta(t) = \theta_{\max}\cos(2\pi f t)$$

- The frequency can be obtained from the equation for the frequency of the mass on a spring by substituting mg/L in place of k:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad \text{and} \qquad T = 2\pi\sqrt{\frac{L}{g}} \tag{14.28}$$

Frequency of a pendulum of length L with free-fall acceleration g

- As for a mass on a spring, the frequency does not depend on the amplitude. Note also that **the frequency, and hence the period, is independent of the mass.** It depends only on the length of the pendulum.



◀ **Pendulum prospecting** The period of a pendulum clock does not depend on the amplitude, but it does depend on the strength of gravity. Soon after Christiaan Huygens built an accurate pendulum clock in 1656 (the photo shows a replica), Jean Richer discovered that the clock ran more slowly near the equator. Richer correctly surmised that this was due to the weaker gravity near the equator because of the greater distance from the center of the earth. In later years, more accurate pendulums were built that could detect much smaller variations in gravity—small enough that they could sense the presence of dense mineral deposits or low-density strata containing petroleum.

Galileo was the first person to study the pendulum in detail. He realized that the pendulum's fixed frequency would serve as the basis of an accurate clock. Pendulum clocks were the most accurate timepieces available until well into the 20th century.

EXAMPLE 14.10 Designing a pendulum for a clock

A grandfather clock is designed so that one swing of the pendulum in either direction takes 1.00 s. What is the length of the pendulum?

PREPARE One period of the pendulum is two swings, so the period is $T = 2.00$ s.

SOLVE The period is independent of the mass and depends only on the length. From Equation 14.28,

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

Solving for L, we find

$$L = g\left(\frac{T}{2\pi}\right)^2 = (9.80\ \text{m/s}^2)\left(\frac{2.00\ \text{s}}{2\pi}\right)^2 = 0.993\ \text{m}$$

ASSESS A pendulum clock with a "tick" or "tock" each second requires a long pendulum of about 1 m—which is why these clocks were originally known as "tall case clocks."

Physical Pendulums and Locomotion

In Chapter 6, we computed maximum walking speed using the ideas of circular motion. We can also model the motion of your legs during walking as pendulum motion. When you walk, you push off with your rear leg and then let it swing forward for the next stride. At normal, comfortable walking speeds, you use very little force to bring your leg forward. Your leg swings forward under the influence of gravity—like a pendulum.

Try this: Stand on one leg, and gently swing your free leg back and forth. There is a certain frequency at which it will naturally swing. This is your leg's pendulum frequency. Now, try swinging your leg at twice this frequency. You can do it, but it is very difficult. The muscles that move your leg back and forth aren't very strong because under normal circumstances they don't need to apply much force.

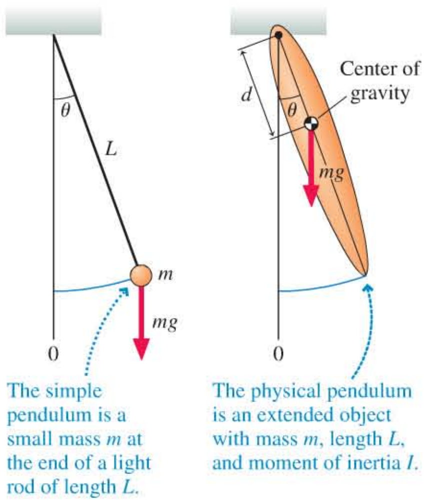
A pendulum, like your leg, whose mass is distributed along its length is known as a **physical pendulum**. The motion of a physical pendulum is similar to that of a simple pendulum, but its frequency depends on the distribution of mass.

FIGURE 14.22 shows a simple pendulum and a physical pendulum of the same length. The position of the center of gravity of the physical pendulum is at a distance d from the pivot.

What will be the frequency of a physical pendulum? This is really a rotational motion problem similar to those we considered in Chapter 7. We would expect the frequency to depend on the moment of inertia and the distance to the center of gravity as follows:

- The moment of inertia I is a measure of an object's resistance to rotation. Increasing the moment of inertia while keeping other variables equal should cause the frequency to decrease. In an expression for the frequency of the physical pendulum, we would expect I to appear in the denominator.
- When the pendulum is pushed to the side, a gravitational torque pulls it back. The greater the distance d of the center of gravity from the pivot point, the greater the torque. Increasing this distance while keeping the other variables constant should cause the frequency to increase. In an expression for the frequency of the physical pendulum, we would expect d to appear in the numerator.

FIGURE 14.22 A simple pendulum and a physical pendulum of equal length.

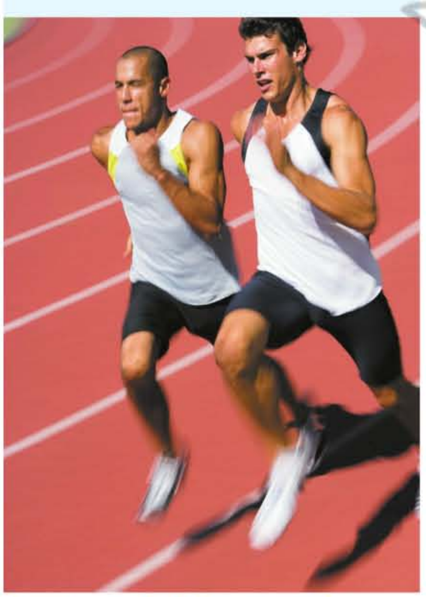


A careful analysis of the motion of the physical pendulum produces a result for the frequency that matches these expectations:

$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}} \tag{14.29}$$

Frequency of a physical pendulum of mass m, moment of inertia I, with center of gravity distance d from the pivot

TRY IT YOURSELF



How do you hold your arms? You maintain your balance when walking or running by moving your arms back and forth opposite the motion of your legs. You hold your arms so that the natural period of their pendulum motion matches that of your legs. At a normal walking pace, your arms are extended and naturally swing at the same period as your legs. When you run, your gait is more rapid. To decrease the period of the pendulum motion of your arms to match, you bend them at the elbows, shortening their effective length and increasing the natural frequency of oscillation. To test this for yourself, try running fast with your arms fully extended. It's quite awkward!

EXAMPLE 14.11 Finding the frequency of a swinging leg

A student in a biomechanics lab measures the length of his leg, from hip to heel, to be 0.90 m. What is the frequency of the pendulum motion of the student's leg? What is the period?

PREPARE We can model a human leg reasonably well as a rod of uniform cross section, pivoted at one end (the hip). Recall from Chapter 7 that the moment of inertia of a rod pivoted about its end is $\frac{1}{3}mL^2$. The center of gravity of a uniform leg is at the midpoint, so $d = L/2$.

SOLVE The frequency of a physical pendulum is given by Equation 14.29. Before we put in numbers, we will use symbolic relationships and simplify:

$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}} = \frac{1}{2\pi}\sqrt{\frac{mg(L/2)}{\frac{1}{3}mL^2}} = \frac{1}{2\pi}\sqrt{\frac{3}{2}\frac{g}{L}}$$

The expression for the frequency is similar to that for the simple pendulum, but with an additional numerical factor of 3/2 inside the square root. The numerical value of the frequency is

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{3}{2}\right)\left(\frac{9.8\ \text{m/s}^2}{0.90\ \text{m}}\right)} = 0.64\ \text{Hz}$$

The period is

$$T = \frac{1}{f} = 1.6\ \text{s}$$

ASSESS Notice that we didn't need to know the mass of the leg to find the period. The period of a physical pendulum does not depend on the mass, just as it doesn't for the simple pendulum. The period depends only on the *distribution* of mass. When you walk, swinging your free leg forward to take another stride corresponds to half a period of this pendulum motion. For a period of 1.6 s, this is 0.80 s. For a normal walking pace, one stride in just under one second sounds about right.

As you walk, your legs do swing as physical pendulums as you bring them forward. The frequency is fixed by the length of your legs and their distribution of mass; it doesn't depend on amplitude. Consequently, you don't increase your walking speed by taking more rapid steps—changing the frequency is quite difficult. You simply take longer strides, changing the amplitude but not the frequency.

Gibbons and other apes move through the forest canopy by a hand-over-hand swinging motion called *brachiation*. In this motion, the body swings under a pivot point where a hand grips a tree branch, a clear example of pendulum motion. A brachiating ape will increase its speed by taking bigger swings; it does this by "pumping" the swinging motion, much as you do when increasing your amplitude on a playground swing. But because the period of the pendulum motion is fixed, a brachiating gibbon can only go so fast. At some point a maximum speed is reached, and gibbons and other apes break into a different gait, launching themselves from branch to branch through the air.

STOP TO THINK 14.5 A pendulum clock is made with a metal rod. It keeps perfect time at a temperature of 20°C. At a higher temperature, the metal rod lengthens. How will this change the clock's timekeeping?

A. The clock will run fast; the dial will be ahead of the actual time.
B. The clock will keep perfect time.
C. The clock will run slow; the dial will be behind the actual time.

14.6 Damped Oscillations

A real pendulum clock must have some energy input; otherwise, the oscillation of the pendulum would slowly decrease in amplitude due to air resistance. If you strike a bell, the oscillation will soon die away as energy is lost to sound waves in the air and dissipative forces within the metal of the bell.

All real oscillators do run down—some very slowly but others quite quickly—as their mechanical energy is transformed into the thermal energy of the oscillator and its environment. An oscillation that runs down and stops is called a **damped oscillation.**

For a pendulum, the main energy loss is due to air resistance, which we called the *drag force* in Chapter 4. When we learned about the drag force, we noted that it depends on velocity: The faster the motion, the bigger the drag force. For this reason, the decrease in amplitude of an oscillating pendulum will be fastest at the start of the motion. For a pendulum or other oscillator with modest damping, we end up with a graph of motion like that in FIGURE 14.23a. The maximum displacement, x_{max}, decreases with time. As the oscillation decays, the *rate* of the decay decreases; the difference between successive peaks is less.

If we plot a smooth curve that connects the peaks of successive oscillations (we call such a curve an *envelope*), we get the dotted line shown in FIGURE 14.23b. It's possible, using calculus, to show that x_{max} decreases with time as

$$x_{max}(t) = Ae^{-t/\tau} \tag{14.30}$$

where $e \approx 2.718$ is the base of the natural logarithm and A is the *initial* amplitude. This steady decrease of x_{max} with time is called an **exponential decay.**

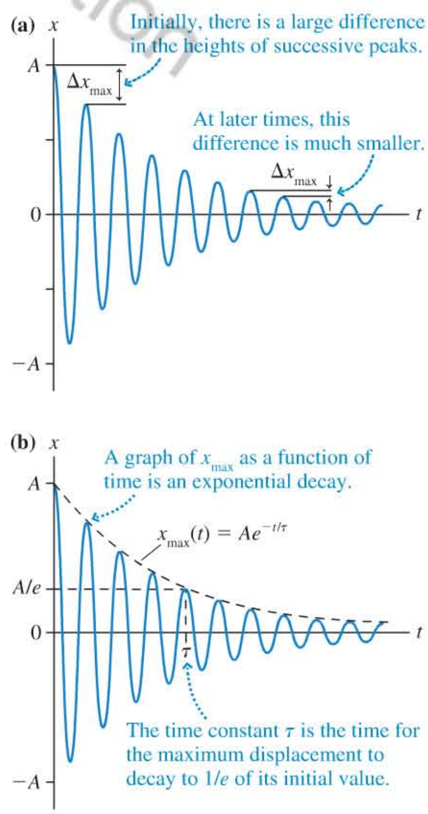
The constant τ (lowercase Greek tau) in Equation 14.30 is called the **time constant.** After one time constant has elapsed—that is, at $t = \tau$—the maximum displacement x_{max} has decreased to

$$x_{max}(\text{at } t = \tau) = Ae^{-1} = \frac{A}{e} \approx 0.37A$$

In other words, the oscillation has decreased after one time constant to about 37% of its initial value. The time constant τ measures the "characteristic time" during which damping causes the amplitude of the oscillation to decay away. An oscillation that decays quickly has a small time constant, whereas a "lightly damped" oscillator, which decays very slowly, has a large time constant.

Because we will see exponential decay again, we will look at it in more detail.

FIGURE 14.23 The motion of a damped oscillator.

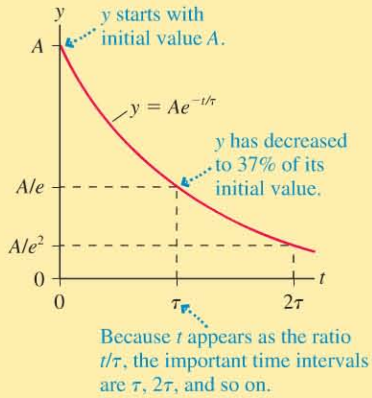


Exponential decay

Exponential decay occurs when a quantity y is proportional to the number e taken to the power $-t/\tau$. The quantity τ is known as the **time constant**. We write this mathematically as

$$y = Ae^{-t/\tau}$$

y is proportional to $e^{-t/\tau}$



SCALING Whenever t increases by one time constant, y decreases by a factor of $1/e$. For instance:

- At time $t = 0$, $y = A$.
- Increasing time to $t = \tau$ reduces y to A/e.
- A further increase to $t = 2\tau$ reduces y by another factor of $1/e$ to A/e^2.

Generally, we can say:

At $t = n\tau$, y has the value A/e^n.

LIMITS As t becomes large, y becomes very small and approaches zero.

Exercises 14–17

Damping smoothes the ride A car's wheels are attached to the car's body with springs so that the wheels can move up and down as the car moves over an uneven road. The car-spring system is a simple harmonic oscillator with a typical period of just under 1 second. You don't want the car to continue bouncing after hitting a bump, so a shock absorber provides damping. The time constant for the damping is about the same length as the period, so that the oscillation damps quickly.

Different Amounts of Damping

The damped oscillation shown in Figure 14.23 continues for a long time. The amplitude isn't zero after one time constant, or two, or three. . . . Mathematically, the oscillation never ceases, though the amplitude will eventually be so small as to be undetectable. For practical purposes, we can speak of the time constant τ as the *lifetime* of an oscillation—a measure of about how long it takes to decay. The best way to measure the relative size of the time constant is to compare it to the period. If $\tau \gg T$, the oscillation persists for many, many periods and the amplitude decrease from cycle to cycle is quite small. The oscillation of a bell after it is struck has $\tau \gg T$; the sound continues for a long time. Other oscillatory systems have very short time constants, as noted in the description of a car's suspension.

EXAMPLE 14.12 Finding a clock's decay time

The pendulum in a grandfather clock has a period of 1.00 s. If the clock's driving spring is allowed to run down, damping due to friction will cause the pendulum to slow to a stop. If the time constant for this decay is 300 s, how long will it take for the pendulum's swing to be reduced to half its initial amplitude?

PREPARE The time constant of 300 s is much greater than the 1.00 s period, so this is an example of modest damping as described by Equation 14.30.

SOLVE Equation 14.30 gives an expression for the decay of the maximum displacement of a damped harmonic oscillator:

$$x_{\max}(t) = Ae^{-t/\tau}$$

As noted in Tactics Box 14.1, we can write this equation equally well in terms of the pendulum's angle in a straightforward manner:

$$\theta_{\max}(t) = \theta_i e^{-t/\tau}$$

where θ_i is the initial angle of swing. At some time t, the time we wish to find, the amplitude has decayed to half its initial value. At this time,

$$\theta_{\max}(t) = \theta_i e^{-t/\tau} = \frac{1}{2}\theta_i$$

The θ_i cancels, giving $e^{-t/\tau} = \frac{1}{2}$.

To solve this for t, we take the natural logarithm of both sides and use the logarithm property $\ln(e^a) = a$:

$$\ln(e^{-t/\tau}) = -\frac{t}{\tau} = \ln\left(\frac{1}{2}\right) = -\ln 2$$

In the last step we used the property $\ln(1/b) = -\ln b$. Now we can solve for t:

$$t = \tau \ln 2$$

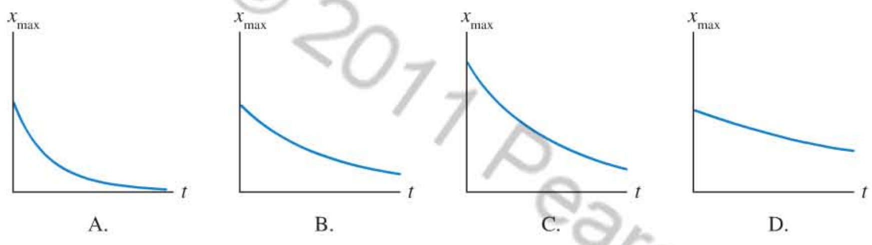
The time constant was specified as $\tau = 300$ s, so

$$t = (300 \text{ s})(0.693) = 208 \text{ s}$$

It will take 208 s, or about 3.5 min, for the oscillations to decay by half after the spring has run down.

ASSESS The time is less than the time constant, which makes sense. The time constant is the time for the amplitude to decay to 37% of its initial value; we are looking for the time to decay to 50% of its initial value, which should be a shorter time. The time to decay to $\frac{1}{2}$ of the initial amplitude, $t = \tau \ln 2$, could be called the *half-life.* We will see this expression again when we work with radioactivity, another example of an exponential decay.

STOP TO THINK 14.6 Rank in order, from largest to smallest, the time constants τ_A to τ_D of the decays in the figures. The scales on all the graphs are the same.



14.7 Driven Oscillations and Resonance

If you jiggle a cup of water, the water sloshes back and forth. This is an example of an oscillator (the water in the cup) subjected to a periodic external force (from your hand). This motion is called a **driven oscillation.**

We can give many examples of driven oscillations. The electromagnetic coil on the back of a loudspeaker cone provides a periodic magnetic force to drive the cone back and forth, causing it to send out sound waves. Earthquakes cause the surface of the earth to move back and forth; this motion causes buildings to oscillate, possibly producing damage or collapse.

Consider an oscillating system that, when left to itself, oscillates at a frequency f_0. We will call this the **natural frequency** of the oscillator. f_0 is simply the frequency of the system if it is displaced from equilibrium and released.

Suppose that this system is now subjected to a *periodic* external force of frequency f_{ext}. This frequency, which is called the **driving frequency,** is completely independent of the oscillator's natural frequency f_0. Somebody or something in the environment selects the frequency f_{ext} of the external force, causing the force to push on the system f_{ext} times every second. The external force causes the oscillation of the system, so it will oscillate at f_{ext}, the driving frequency, not at its natural frequency f_0.

Let's return to the example of the cup of water. If you nudge the cup, you will notice that the water sloshes back and forth at a particular frequency; this is the natural frequency f_0. Now shake the cup at some frequency; this is the driving frequency f_{ext}. As you shake the cup, the oscillation amplitude of the water depends very sensitively on the frequency f_{ext} of your hand. If the driving frequency is near the natural frequency of the system, the oscillation amplitude may become so large that water splashes out of the cup.

Any driven oscillator will show a similar dependence of amplitude on the driving frequency. Suppose a mass on a spring has a natural frequency $f_0 = 2$ Hz. We can use an external force to push and pull on the mass at frequency f_{ext}, measure the amplitude of the resulting oscillation, and then repeat this over and over for many different driving frequencies. A graph of amplitude versus driving frequency, such as the one in **FIGURE 14.24**, is called the oscillator's **response curve.**

Serious sloshing Water in a cup has a natural frequency at which it will slosh back and forth. The same is true of larger bodies of water. Water in Canada's Bay of Fundy would naturally move into or out of the bay with a period of 12 hours. This is nearly equal to the period of the tidal force of the moon; the two daily high tides are 12.5 hours apart. This *resonance,* a close match between the bay's natural frequency and the moon's driving frequency, produces a huge tidal amplitude. Low tide can be as much as 16 m below high tide, leaving boats high and dry.

FIGURE 14.24 The response curve shows the amplitude of a driven oscillator at frequencies near its natural frequency $f_0 = 2$ Hz.

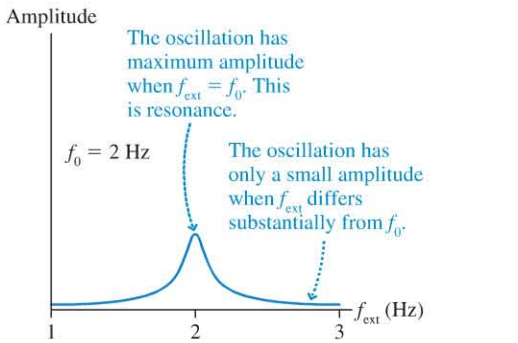
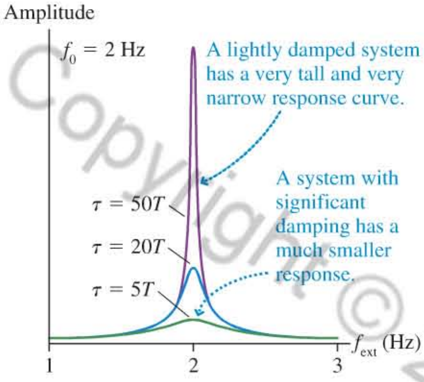


FIGURE 14.25 The response curve becomes taller and narrower as the damping is reduced.



At the right and left edges of Figure 14.24, the driving frequency is substantially different from the oscillator's natural frequency. The system oscillates, but its amplitude is very small. The system simply does not respond well to a driving frequency that differs much from f_0. As the driving frequency gets closer and closer to the natural frequency, the amplitude of the oscillation rises dramatically. After all, f_0 is the frequency at which the system "wants" to oscillate, so it is quite happy to respond to a driving frequency near f_0. Hence the amplitude reaches a maximum when the driving frequency matches the system's natural frequency: $f_{ext} = f_0$. This large-amplitude response to a driving force whose frequency matches the natural frequency of the system is a phenomenon called **resonance.** Within the context of driven oscillations, the natural frequency f_0 is often called the **resonance frequency.**

The amplitude can become exceedingly large when the frequencies match, especially if there is very little damping. FIGURE 14.25 shows the response curve of the oscillator of Figure 14.24 with different amounts of damping. Three different graphs are plotted, each with a different time constant for damping. The three graphs have damping that ranges from $\tau = 50T$ (very little damping) to $\tau = 5T$ (significant damping).

◀ **Simple harmonic music** A typical wine glass has a natural frequency of oscillation and a very small amount of damping. A tap on the rim of the glass causes it to "ring" like a bell. The time constant is hundreds of times longer than the period, so the sound will persist for several seconds. If you moisten your finger and slide it gently around the rim of the glass, it will stick and slip in quick succession. With some practice you can match the stick-slip to the frequency of oscillation of the glass. The resulting resonance creates a large amplitude and thus a very loud sound. You can tune the oscillation frequency by adding water, turning a set of glasses into an unusual musical instrument.

CONCEPTUAL EXAMPLE 14.13 Fixing an unwanted resonance

Railroad cars have a natural frequency at which they rock side to side. This can lead to problems on certain stretches of track that have bumps where the rails join. If the joints alternate sides, with a bump on the left rail and then on the right, a train car moving down the track is bumped one way and then the other. In some cases, bumps have caused rocking with amplitude large enough to derail the train. A train moving down the track at a certian speed is experiencing a large amplitude of oscillation due to alternating joints in the track. How can the driver correct this potentially dangerous situation?

REASON The large amplitude of oscillation is produced by a resonance, a match between the frequency at which the train car rocks back and forth and the frequency at which the car hits the bumps. To eliminate this resonance, the driver must either reduce the speed of the train—decreasing the driving frequency—or increase the speed of the train—thus increasing the driving frequency.

ASSESS It's perhaps surprising that increasing the speed of the train could produce a smoother ride. But increasing the frequency at which the train hits the bumps will eliminate the match with the natural rocking frequency just as surely as decreasing the speed.

Resonance and Hearing

Resonance in a system means that certain frequencies produce a large response and others do not. The phenomenon of resonance is responsible for the frequency discrimination of the ear.

As we will see in the next chapter, sound is a vibration in air. FIGURE 14.26 on the next page provides an overview of the structures by which sound waves that enter the ear produce vibrations in the cochlea, the coiled, fluid-filled, sound-sensing organ of the inner ear.

FIGURE 14.26 The structures of the ear.

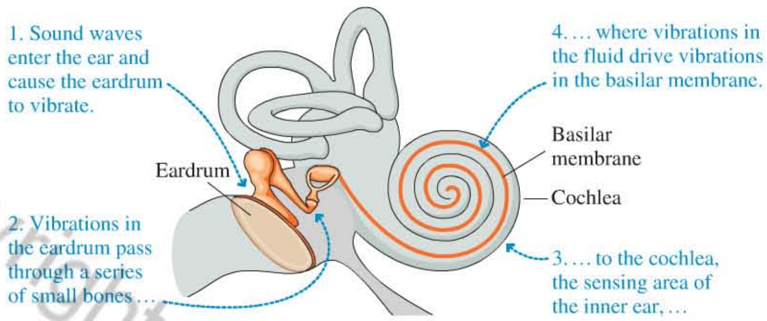


FIGURE 14.27 Resonance plays a role in determining the frequencies of sounds we hear.

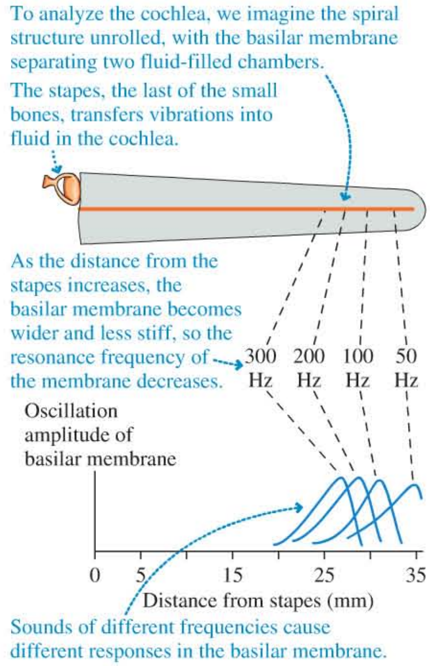


FIGURE 14.27 shows a very simplified model of the cochlea. As a sound wave travels down the cochlea, it causes a large-amplitude vibration of the basilar membrane at the point where the membrane's natural oscillation frequency matches the sound frequency—a resonance. Lower-frequency sound causes a response farther from the stapes. Sensitive hair cells on the membrane sense the vibration and send nerve signals to your brain. The fact that different frequencies produce maximal response at different positions allows your brain to very accurately determine frequency because a small shift in frequency causes a detectable change in the position of the maximal response. People with no musical training can listen to two notes and easily determine which is at a higher pitch.

We now know a bit about how your ear responds to the vibration of a sound wave—but how does this vibration get from a source to your ear? This is a topic we will consider in the next chapter, when we look at *waves,* oscillations that travel.

INTEGRATED EXAMPLE 14.14 Springboard diving

Flexible diving boards designed for large deflections are called springboards. If a diver jumps up and lands on the end of the board, the resulting deflection of the diving board produces a linear restoring force that launches him into the air. But if the diver simply bobs up and down on the end of the board, we can effectively model his motion as that of a mass oscillating on a spring.

A light and flexible springboard deflects by 15 cm when a 65 kg diver stands on its end. He then jumps and lands on the end of the board, depressing it by a total of 25 cm, after which he moves up and down with the oscillations of the end of the board.

a. What is the frequency of the oscillation?
b. What is the maximum speed of his up-and-down motion?

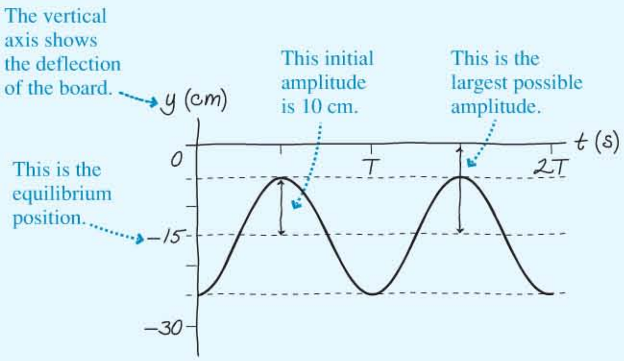
Suppose the diver then drives the motion of the board with his legs, gradually increasing the amplitude of the oscillation. At some point the oscillation becomes large enough that his feet leave the board.

c. What is the amplitude of the oscillation when the diver just becomes airborne at one point of the cycle? What is the acceleration at this point?
d. A diver leaving a springboard can achieve a much greater height than a diver jumping from a fixed platform. Use energy concepts to explain how the spring of the board allows a greater vertical jump.

PREPARE We will model the diver on the board as a mass on a spring. As we've seen, the oscillation frequency is determined by the spring constant and the mass. The mass of the diver is given; we can determine the spring constant from the deflection of the springboard when the diver stands on the end.

FIGURE 14.28 is a sketch of the oscillation that will help us visualize the motion. The equilibrium position, with the diver standing motionless on the end of the board, corresponds to a deflection of 15 cm. When the diver jumps on the board, the total deflection is 25 cm, which means a deflection of an additional 10 cm beyond the equilibrium position, so 10 cm is the amplitude of the subsequent oscillation. When the board rises, it won't bend beyond its undeflected position, so the maximum possible amplitude is 15 cm.

FIGURE 14.28 Position-versus-time graph for the springboard.



Continued

NOTE ► We've started the graph at the lowest point of the motion. Because we'll use only the equations for the maximum values of the speed and the acceleration, not the full equations that describe the motion, the exact starting point isn't critical. ◄

SOLVE a. **FIGURE 14.29** shows the forces on the diver as he stands motionless at the end of the board. The net force on him is zero, $\vec{F}_{net} = \vec{F}_{sp} + \vec{w} = \vec{0}$, so the two forces have equal magnitudes and we can write

$$F_{sp} = w$$

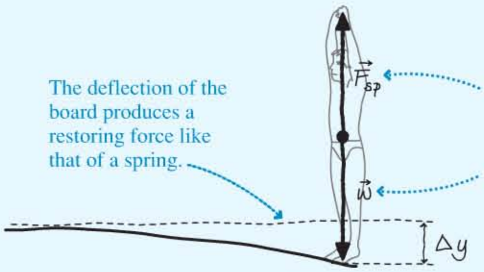
A linear restoring force means that the board obeys Hooke's law for a spring: $F_{sp} = k\Delta y$, where k is the spring constant. Thus the equilibrium equation is

$$k\Delta y = mg$$

Solving for the spring constant, we find

$$k = \frac{mg}{\Delta y} = \frac{(65\text{ kg})(9.8\text{ m/s}^2)}{0.15\text{ m}} = 4.2 \times 10^3\text{ N/m}$$

FIGURE 14.29 Forces on a springboard diver at rest at the end of the board.



The frequency of the oscillation depends on the diver's mass and the spring constant of the board:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.2 \times 10^3\text{ N/m}}{65\text{ kg}}} = 1.3\text{ Hz}$$

b. The maximum speed of the oscillation is given by Equation 14.15:

$$v_{max} = 2\pi f A = 2\pi(1.3\text{ Hz})(0.10\text{ m}) = 0.82\text{ m/s}$$

c. We can see from Figure 14.28 that an amplitude of 15 cm returns the board to its undeflected position. At this point, the board exerts no upward force—no supporting normal force—on the diver, so the diver loses contact with the board. (His apparent weight becomes zero.) The acceleration at this point in the motion has the maximum possible magnitude but is negative because the acceleration graph is an upside-down version of the position graph. For a 15 cm oscillation amplitude, the acceleration at this point is computed using Equation 14.17:

$$a = -a_{max} = -(2\pi f)^2 A = [2\pi(1.3\text{ Hz})]^2(0.15\text{ m})$$
$$= -10\text{ m/s}^2$$

d. The maximum jump height from a fixed platform is determined by the maximum speed at which a jumper leaves the ground. During a jump, chemical energy in the muscles is transformed into kinetic energy. This kinetic energy is transformed into potential energy as the jumper rises, back into kinetic energy as he falls, then into thermal energy as he hits the ground. The springboard recaptures and stores this kinetic energy rather than letting it degrade to thermal energy; a diver can jump up once and land on the board, storing the energy of his initial jump as elastic potential energy of the bending board. Then, as the board rebounds, turning the stored energy back into kinetic energy, the diver can push off from this moving platform, transforming even more chemical energy and thus further increasing his kinetic energy. This allows the diver to get "two jumps worth" of chemical energy in a single jump.

ASSESS The answer to part c, -10 m/s^2 is very close to $a = -g$; only rounding errors in early steps kept our result from being exactly $a = -g$. This is to be expected. We learned in earlier chapters that an object loses contact with a surface—like a car coming off the track in a loop-the-loop—when its apparent weight becomes zero: $w_{app} = 0$. And in Chapter 5 we found that the apparent weight of an object in vertical motion becomes zero when $a = -g$—that is, when it enters free fall. This correspondence is a good check on our work; because part c has the answer we expect, we have confidence in our earlier steps.

SUMMARY

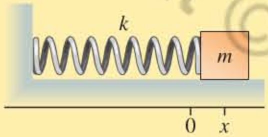
The goal of Chapter 14 has been to understand systems that oscillate with simple harmonic motion.

GENERAL PRINCIPLES

Restoring Forces

SHM occurs when a **linear restoring force** acts to return a system to an equilibrium position.

Mass on spring

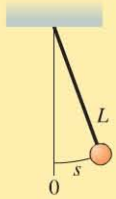


$$(F_{net})_x = -kx$$

The frequency of a mass on a spring depends on the mass and the spring constant:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Pendulum



$$(F_{net})_t = -\left(\frac{mg}{L}\right)s$$

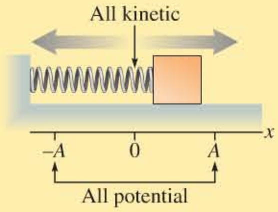
The frequency of a pendulum depends on the length and the free-fall acceleration:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Energy

If there is no friction or dissipation, kinetic and potential energies are alternately transformed into each other in SHM, with the sum of the two conserved.

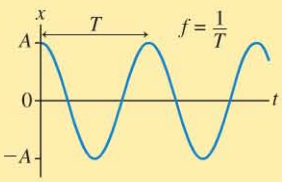
$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$$
$$= \frac{1}{2}mv_{max}^2$$
$$= \frac{1}{2}kA^2$$



IMPORTANT CONCEPTS

Oscillation

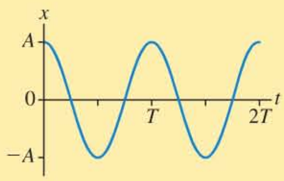
An **oscillation** is a repetitive motion about an equilibrium position. The **amplitude** A is the maximum displacement from equilibrium. The period T is the time for one cycle. We may also characterize an oscillation by its frequency f.



Simple Harmonic Motion (SHM)

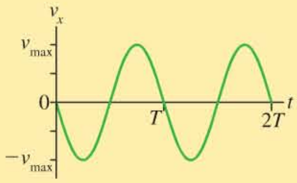
SHM is an oscillation that is described by a sinusoidal function. All systems that undergo SHM can be described by the same functional forms.

Position-versus-time is a cosine function.



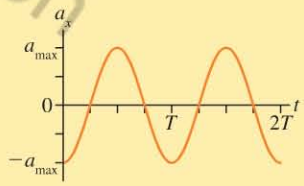
$$x(t) = A\cos(2\pi ft)$$
$$x_{max} = A$$

Velocity-versus-time is an inverted sine function.



$$v_x(t) = -v_{max}\sin(2\pi ft)$$
$$v_{max} = 2\pi fA$$

Acceleration-versus-time is an inverted cosine function.

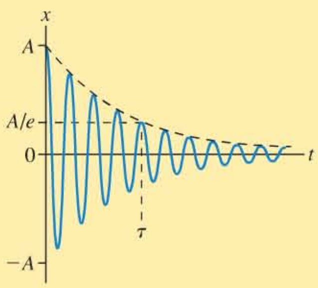


$$a_x(t) = -a_{max}\cos(2\pi ft)$$
$$a_{max} = (2\pi f)^2A$$

APPLICATIONS

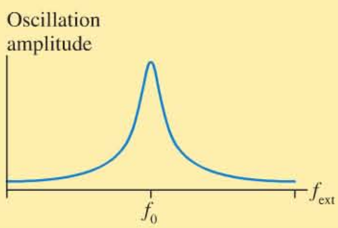
Damping

Simple harmonic motion with damping (due to drag) decreases in amplitude over time. The **time constant** τ determines how quickly the amplitude decays.



Resonance

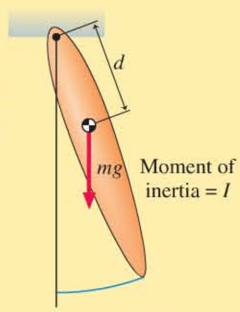
A system that oscillates has a **natural frequency** of oscillation f_0. **Resonance** occurs if the system is driven with a frequency f_{ext} that matches this natural frequency. This may produce a large amplitude of oscillation.



Physical pendulum

A **physical pendulum** is a pendulum with mass distributed along its length. The frequency depends on the position of the center of gravity and the moment of inertia.

The motion of legs during walking can be described using a physical pendulum model.



$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. Give three real-world examples of *oscillatory* motion. (Note that circular motion is similar to, but not the same as oscillatory motion.)
2. A person's heart rate is given in beats per minute. Is this a period or a frequency?
3. Figure Q14.3 shows the position-versus-time graph of a particle in SHM.
   a. At what time or times is the particle moving to the right at maximum speed?
   b. At what time or times is the particle moving to the left at maximum speed?
   c. At what time or times is the particle instantaneously at rest?

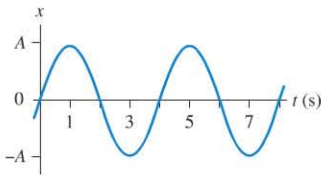


FIGURE Q14.3

4. A block oscillating on a spring has an amplitude of 20 cm. What will be the amplitude if the maximum kinetic energy is doubled?
5. A block oscillating on a spring has a maximum speed of 20 cm/s. What will be the block's maximum speed if the amplitude of the oscillation is doubled?
6. A block oscillating on a spring has a maximum kinetic energy of 2.0 J. What will be the maximum kinetic energy if the amplitude is doubled? Explain.
7. A block oscillating on a spring has a maximum speed of 30 cm/s. What will be the block's maximum speed if the initial elongation of the spring is doubled?
8. For the graph in Figure Q14.8, determine the frequency f and the oscillation amplitude A.

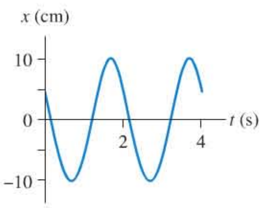


FIGURE Q14.8

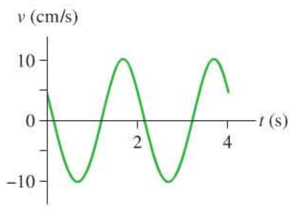


FIGURE Q14.9

9. For the graph in Figure Q14.9, determine the frequency f and the oscillation amplitude A.
10. A block oscillating on a spring has period $T = 2.0$ s.
    a. What is the period if the block's mass is doubled?
    b. What is the period if the value of the spring constant is quadrupled?
    c. What is the period if the oscillation amplitude is doubled while m and k are unchanged?

    Note: You do not know values for either m or k. Do *not* assume any particular values for them. The required analysis involves thinking about ratios.
11. A pendulum on Planet X, where the value of g is unknown, oscillates with a period of 2.0 s. What is the period of this pendulum if:
    a. Its mass is doubled?
    b. Its length is doubled?
    c. Its oscillation amplitude is doubled?

    Note: You do not know the values of m, L, or g, so do not assume any specific values.
12. BIO Flies flap their wings at frequencies much too high for pure muscle action. A hypothesis for how they achieve these high frequencies is that the flapping of their wings is the driven oscillation of a mass-spring system. One way to test this is to trim a fly's wings. If the oscillation of the wings can be modeled as a mass-spring system, how would this change the frequency of the wingbeats?
13. As we saw in Chapter 6, the free-fall acceleration is slightly less in Denver than in Miami. If a pendulum clock keeps perfect time in Miami, will it run fast or slow in Denver? Explain.
14. If you want to play a tune on wine glasses, you'll need to adjust the oscillation frequencies by adding water to the glasses. This changes the mass that oscillates (more water means more mass) but not the restoring force, which is determined by the stiffness of the glass itself. If you need to raise the frequency of a particular glass, should you add water or remove water?
15. BIO Sprinters push off from the ball of their foot, then bend their knee to bring their foot up close to the body as they swing their leg forward for the next stride. Why is this an effective strategy for running fast?
16. BIO Gibbons move through the trees by swinging from successive handholds, as we have seen. To increase their speed, gibbons may bring their legs close to their bodies. How does this help them move more quickly?

17. Describe the difference between the time constant τ and the period T. Don't just *name* them; say what is different about the physical concepts that they represent.

18. What is the difference between the driving frequency and the natural frequency of an oscillator?
19. BIO Humans have a range of hearing of approximately 20 Hz to 20 kHz. Mice have auditory systems similar to humans, but all of the physical elements are smaller. Given this, would you expect mice to have a higher or lower frequency range than humans? Explain.
20. A person driving a truck on a "washboard" road, one with regularly spaced bumps, notices an interesting effect: When the truck travels at low speed, the amplitude of the vertical motion of the car is small. If the truck's speed is increased, the amplitude of the vertical motion also increases, until it becomes quite unpleasant. But if the speed is increased yet further, the amplitude decreases, and at high speeds the amplitude of the vertical motion is small again. Explain what is happening.
21. BIO We've seen that stout tendons in the legs of hopping kangaroos store energy. When a kangaroo lands, much of the kinetic energy of motion is converted to elastic energy as the tendons stretch, returning to kinetic energy when the kangaroo again leaves the ground. If a hopping kangaroo increases its speed, it spends more time in the air with each bounce, but the contact time with the ground stays approximately the same. Explain why you would expect this to be the case.

Multiple-Choice Questions

22. | A spring has an unstretched length of 20 cm. A 100 g mass hanging from the spring stretches it to an equilibrium length of 30 cm.
    a. Suppose the mass is pulled down to where the spring's length is 40 cm. When it is released, it begins to oscillate. What is the amplitude of the oscillation?
       A. 5.0 cm B. 10 cm C. 20 cm D. 40 cm
    b. For the data given above, what is the frequency of the oscillation?
       A. 0.10 Hz B. 0.62 Hz C. 1.6 Hz D. 10 Hz
    c. Suppose this experiment were done on the moon, where the free-fall acceleration is approximately 1/6 of that on the earth. How would this change the frequency of the oscillation?
       A. The frequency would decrease.
       B. The frequency would increase.
       C. The frequency would stay the same.
23. | Figure Q14.23 represents the motion of a mass on a spring.
    a. What is the period of this oscillation?
       A. 12 s B. 24 s C. 36 s
       D. 48 s E. 50 s

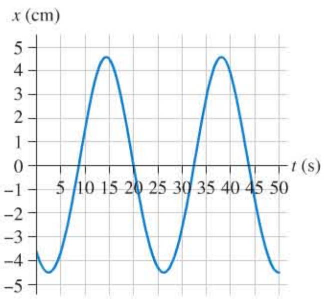


FIGURE Q14.23

    b. What is the amplitude of the oscillation?
       A. 1.0 cm B. 2.5 cm C. 4.5 cm
       D. 5.0 cm E. 9.0 cm
    c. What is the position of the mass at time $t = 30$ s?
       A. −4.5 cm B. −2.5 cm C. 0.0 cm
       D. 4.5 cm E. 30 cm
    d. When is the first time the velocity of the mass is zero?
       A. 0 s B. 2 s C. 8 s
       D. 10 s E. 13 s
    e. At which of these times does the kinetic energy have its maximum value?
       A. 0 s B. 8 s C. 13 s
       D. 26 s E. 30 s
24. | A ball of mass m oscillates on a spring with spring constant $k = 200$ N/m. The ball's position is $x = (0.350\text{ m})\cos(15.0t)$, with t measured in seconds.
    a. What is the amplitude of the ball's motion?
       A. 0.175 m B. 0.350 m C. 0.700 m
       D. 7.50 m E. 15.0 m
    b. What is the frequency of the ball's motion?
       A. 0.35 Hz B. 2.39 Hz C. 5.44 Hz
       D. 6.28 Hz E. 15.0 Hz
    c. What is the value of the mass m?
       A. 0.45 kg B. 0.89 kg C. 1.54 kg
       D. 3.76 kg E. 6.33 kg
    d. What is the total mechanical energy of the oscillator?
       A. 1.65 J B. 3.28 J C. 6.73 J
       D. 10.1 J E. 12.2 J
    e. What is the ball's maximum speed?
       A. 0.35 m/s B. 1.76 m/s C. 2.60 m/s
       D. 3.88 m/s E. 5.25 m/s
25. || If you carry heavy weights in your hands, how will this affect the natural frequency at which your arms swing back and forth?
    A. The frequency will increase.
    B. The frequency will stay the same.
    C. The frequency will decrease.
26. | A heavy brass ball is used to make a pendulum with a period of 5.5 s. How long is the cable that connects the pendulum ball to the ceiling?
    A. 4.7 m B. 6.2 m
    C. 7.5 m D. 8.7 m
27. | Suppose you travel to the moon, and you take with you two timepieces: a pendulum clock and a wristwatch that runs with a wheel and a mainspring. (The wheel and spring work, essentially, like a mass on a spring, but the wheel rotates back and forth rather than moving up and down.) Which will keep good time on the moon?
    A. Only the pendulum clock
    B. Only the wristwatch
    C. Both timepieces
    D. Neither timepiece
28. | BIO Very loud sounds can damage hearing by injuring the vibration-sensing hair cells on the basilar membrane. Suppose a person has injured hair cells on a segment of the basilar membrane close to the stapes. What type of sound is most likely to have produced this particular pattern of damage?
    A. Loud music with a mix of different frequencies
    B. A very loud, high-frequency sound
    C. A very loud, low-frequency sound

PROBLEMS

Section 14.1 Equilibrium and Oscillation

Section 14.2 Linear Restoring Forces and Simple Harmonic Motion

1. | When a guitar string plays the note "A," the string vibrates at 440 Hz. What is the period of the vibration?
2. | In the aftermath of an intense earthquake, the earth as a whole "rings" with a period of 54 minutes. What is the frequency (in Hz) of this oscillation?
3. | In taking your pulse, you count 75 heartbeats in 1 min. What are the period (in s) and frequency (in Hz) of your heart's oscillations? BIO
4. || Make a table with 3 columns and 8 rows. In row 1, label the columns θ (°), θ (rad), and $\sin\theta$. In the left column, starting in row 2, write 0, 2, 4, 6, 8, 10, and 12.
   a. Convert each of these angles, in degrees, to radians. Put the results in column 2. Show four decimal places.
   b. Calculate the sines. Put the results, showing four decimal places, in column 3.
   c. What is the first angle for which θ and $\sin\theta$ differ by more than 0.0010?
   d. Over what range of angles does the small-angle approximation appear to be valid?
5. | A heavy steel ball is hung from a cord to make a pendulum. The ball is pulled to the side so that the cord makes a 5° angle with the vertical. Holding the ball in place takes a force of 20 N. If the ball is pulled farther to the side so that the cord makes a 10° angle, what force is required to hold the ball?

Section 14.3 Describing Simple Harmonic Motion

6. || An air-track glider attached to a spring oscillates between the 10 cm mark and the 60 cm mark on the track. The glider completes 10 oscillations in 33 s. What are the (a) period, (b) frequency, (c) amplitude, and (d) maximum speed of the glider?
7. ||| An air-track glider is attached to a spring. The glider is pulled to the right and released from rest at $t = 0$ s. It then oscillates with a period of 2.0 s and a maximum speed of 40 cm/s.
   a. What is the amplitude of the oscillation?
   b. What is the glider's position at $t = 0.25$ s?
8. | What are the (a) amplitude and (b) frequency of the oscillation shown in Figure P14.8?

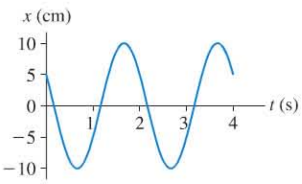


FIGURE P14.8

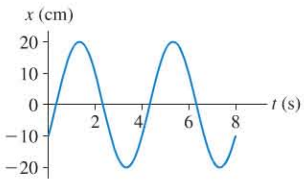


FIGURE P14.9

9. | What are the (a) amplitude and (b) frequency of the oscillation shown in Figure P14.9?
10. | An object in simple harmonic motion has an amplitude of 6.0 cm and a frequency of 0.50 Hz. Draw a position graph showing two cycles of the motion.
11. | During an earthquake, the top of a building oscillates with an amplitude of 30 cm at 1.2 Hz. What are the magnitudes of (a) the maximum displacement, (b) the maximum velocity, and (c) the maximum acceleration of the top of the building?
12. || Some passengers on an ocean cruise may suffer from motion sickness as the ship rocks back and forth on the waves. At one position on the ship, passengers experience a vertical motion of amplitude 1 m with a period of 15 s.
    a. To one significant figure, what is the maximum acceleration of the passengers during this motion?
    b. What fraction is this of g?
13. || A passenger car traveling down a rough road bounces up and down at 1.3 Hz with a maximum vertical acceleration of 0.20 m/s^2, both typical values. What are the (a) amplitude and (b) maximum speed of the oscillation?
14. ||| The New England Merchants Bank Building in Boston is 152 m high. On windy days it sways with a frequency of 0.17 Hz, and the acceleration of the top of the building can reach 2.0% of the free-fall acceleration, enough to cause discomfort for occupants. What is the total distance, side to side, that the top of the building moves during such an oscillation?

Section 14.4 Energy in Simple Harmonic Motion

15. ||| a. When the displacement of a mass on a spring is $\frac{1}{2}A$, what fraction of the mechanical energy is kinetic energy and what fraction is potential energy?
    b. At what displacement, as a fraction of A, is the energy half kinetic and half potential?
16. ||| A 1.0 kg block is attached to a spring with spring constant 16 N/m. While the block is sitting at rest, a student hits it with a hammer and almost instantaneously gives it a speed of 40 cm/s. What are
    a. The amplitude of the subsequent oscillations?
    b. The block's speed at the point where $x = \frac{1}{2}A$?
17. | A block attached to a spring with unknown spring constant oscillates with a period of 2.00 s. What is the period if
    a. The mass is doubled?
    b. The mass is halved?
    c. The amplitude is doubled?
    d. The spring constant is doubled?

    Parts a to d are independent questions, each referring to the initial situation.
18. || A 200 g air-track glider is attached to a spring. The glider is pushed 10.0 cm against the spring, then released. A student with a stopwatch finds that 10 oscillations take 12.0 s. What is the spring constant?
19. ||| The position of a 50 g oscillating mass is given by $x(t) = (2.0\text{ cm})\cos(10t)$, where t is in seconds. Determine:
    a. The amplitude.
    b. The period.
    c. The spring constant.
    d. The maximum speed.
    e. The total energy.
    f. The velocity at $t = 0.40$ s.

20. A 200 g mass attached to a horizontal spring oscillates at a frequency of 2.0 Hz. At one instant, the mass is at $x = 5.0$ cm and has $v_x = -30$ cm/s. Determine:
    a. The period. b. The amplitude.
    c. The maximum speed. d. The total energy.
21. A 507 g mass oscillates with an amplitude of 10.0 cm on a spring whose spring constant is 20.0 N/m. Determine:
    a. The period. b. The maximum speed.
    c. The total energy.
22. A 300 g oscillator has a speed of 95.4 cm/s when its displacement is 3.00 cm and 71.4 cm/s when its displacement is 6.00 cm. What is the oscillator's maximum speed?

Section 14.5 Pendulum Motion

23. A mass on a string of unknown length oscillates as a pendulum with a period of 4.00 s. What is the period if
    a. The mass is doubled?
    b. The string length is doubled?
    c. The string length is halved?
    d. The amplitude is halved?

    Parts a to d are independent questions, each referring to the initial situation.
24. A 200 g ball is tied to a string. It is pulled to an angle of 8.00° and released to swing as a pendulum. A student with a stopwatch finds that 10 oscillations take 12.0 s. How long is the string?
25. The angle of a pendulum is given by $\theta(t) = (0.10\text{ rad})\cos(5t)$, where t is in seconds. Determine:
    a. The amplitude. b. The frequency.
    c. The length of the string. d. The angle at $t = 2.0$ s.
26. It is said that Galileo discovered a basic principle of the pendulum—that the period is independent of the amplitude—by using his pulse to time the period of swinging lamps in the cathedral as they swayed in the breeze. Suppose that one oscillation of a swinging lamp takes 5.5 s. How long is the lamp chain?
27. The free-fall acceleration on the moon is 1.62 m/s^2. What is the length of a pendulum whose period on the moon matches the period of a 2.00-m-long pendulum on the earth?
28. Astronauts on the first trip to Mars take along a pendulum that has a period on earth of 1.50 s. The period on Mars turns out to be 2.45 s. What is the Martian free-fall acceleration?
29. A building is being knocked down with a wrecking ball, which is a big metal sphere that swings on a 10-m-long cable. You are (unwisely!) standing directly beneath the point from which the wrecking ball is hung when you notice that the ball has just been released and is swinging directly toward you. How much time do you have to move out of the way?
30. BIO Interestingly, there have been several studies using cadavers to determine the moment of inertia of human body parts by letting them swing as a pendulum about a joint. In one study, the center of gravity of a 5.0 kg lower leg was found to be 18 cm from the knee. When pivoted at the knee and allowed to swing, the oscillation frequency was 1.6 Hz. What was the moment of inertia of the lower leg?
31. A pendulum clock keeps time by the swinging of a uniform solid rod pivoted at one end. The angular position of the rod is given by $\theta(t) = (0.175\text{ rad})\sin(\pi t)$, where t is in seconds.
    a. What is the angular position of the rod at $t = 0.250$ s?
    b. What is the period of oscillation?
    c. How long is the rod?
32. You and your friends find a rope that hangs down 15 m from a high tree branch right at the edge of a river. You find that you can run, grab the rope, and swing out over the river. You run at 2.0 m/s and grab the rope, launching yourself out over the river. How long must you hang on if you want to stay dry?
33. A thin, circular hoop with a radius of 0.22 m is hanging from its rim on a nail. When pulled to the side and released, the hoop swings back and forth as a physical pendulum. The moment of inertia of a hoop for a rotational axis passing through its edge is $I = 2MR^2$. What is the period of oscillation of the hoop?
34. BIO An elephant's legs have a reasonably uniform cross section from top to bottom, and they are quite long, pivoting high on the animal's body. When an elephant moves at a walk, it uses very little energy to bring its legs forward, simply allowing them to swing like pendulums. For fluid walking motion, this time should be half the time for a complete stride; as soon as the right leg finishes swinging forward, the elephant plants the right foot and begins swinging the left leg forward.

    a. An elephant has legs that stretch 2.3 m from its shoulders to the ground. How much time is required for one leg to swing forward after completing a stride?
    b. What would you predict for this elephant's stride frequency? That is, how many steps per minute will the elephant take?

Section 14.6 Damped Oscillations

35. The amplitude of an oscillator decreases to 36.8% of its initial value in 10.0 s. What is the value of the time constant?
36. Calculate and draw an accurate displacement graph from $t = 0$ s to $t = 10$ s of a damped oscillator having a frequency of 1.0 Hz and a time constant of 4.0 s.
37. A small earthquake starts a lamppost vibrating back and forth. The amplitude of the vibration of the top of the lamppost is 6.5 cm at the moment the quake stops, and 8.0 s later it is 1.8 cm.
    a. What is the time constant for the damping of the oscillation?
    b. What was the amplitude of the oscillation 4.0 s after the quake stopped?
38. When you drive your car over a bump, the springs connecting the wheels to the car compress. Your shock absorbers then damp the subsequent oscillation, keeping your car from bouncing up and down on the springs. Figure P14.38 shows real data for a car driven over a bump. Estimate the frequency and the time constant for this damped oscillation.

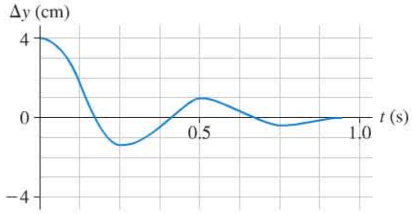


FIGURE P14.38

Section 14.7 Driven Oscillations and Resonance

39. A 25 kg child sits on a 2.0-m-long rope swing. You are going to give the child a small, brief push at regular intervals. If you want to increase the amplitude of her motion as quickly as possible, how much time should you wait between pushes?

40. Your car rides on springs, so it will have a natural frequency of oscillation. Figure P14.40 shows data for the amplitude of motion of a car driven at different frequencies. The car is driven at 20 mph over a washboard road with bumps spaced 10 feet apart; the resulting ride is quite bouncy. Should the driver speed up or slow down for a smoother ride?

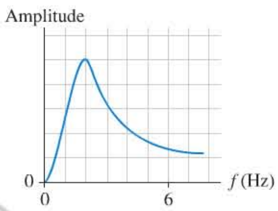


FIGURE P14.40

41. BIO Vision is blurred if the head is vibrated at 29 Hz because the vibrations are resonant with the natural frequency of the eyeball held by the musculature in its socket. If the mass of the eyeball is 7.5 g, a typical value, what is the effective spring constant of the musculature attached to the eyeball?

General Problems

42. A spring has an unstretched length of 12 cm. When an 80 g ball is hung from it, the length increases by 4.0 cm. Then the ball is pulled down another 4.0 cm and released.
   a. What is the spring constant of the spring?
   b. What is the period of the oscillation?
   c. Draw a position-versus-time graph showing the motion of the ball for three cycles of the oscillation. Let the equilibrium position of the ball be $y = 0$. Be sure to include appropriate units on the axes so that the period and the amplitude of the motion can be determined from your graph.

43. A 0.40 kg ball is suspended from a spring with spring constant 12 N/m. If the ball is pulled down 0.20 m from the equilibrium position and released, what is its maximum speed while it oscillates?

44. A spring is hanging from the ceiling. Attaching a 500 g mass to the spring causes it to stretch 20.0 cm in order to come to equilibrium.
   a. What is the spring constant?
   b. From equilibrium, the mass is pulled down 10.0 cm and released. What is the period of oscillation?
   c. What is the maximum speed of the mass? At what position or positions does it have this speed?

45. A spring with spring constant 15.0 N/m hangs from the ceiling. A ball is suspended from the spring and allowed to come to rest. It is then pulled down 6.00 cm and released. If the ball makes 30 oscillations in 20.0 s, what are its (a) mass and (b) maximum speed?

46. A spring is hung from the ceiling. When a coffee mug is attached to its end, the spring stretches 2.0 cm before reaching its new equilibrium length. The mug is then pulled down slightly and released. What is the frequency of oscillation?

47. On your first trip to Planet X you happen to take along a 200 g mass, a 40.0-cm-long spring, a meter stick, and a stopwatch. You're curious about the free-fall acceleration on Planet X, where ordinary tasks seem easier than on earth, but you can't find this information in your Visitor's Guide. One night you suspend the spring from the ceiling in your room and hang the mass from it. You find that the mass stretches the spring by 31.2 cm. You then pull the mass down 10.0 cm and release it. With the stopwatch you find that 10 oscillations take 14.5 s. Can you now satisfy your curiosity?

48. An object oscillating on a spring has the velocity graph shown in Figure P14.48. Draw a velocity graph if the following changes are made.
   a. The amplitude is doubled and the frequency is halved.
   b. The amplitude and spring constant are kept the same, but the mass is quadrupled.

   Parts a and b are independent questions, each starting from the graph shown.

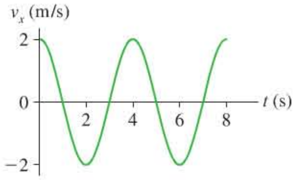


FIGURE P14.48

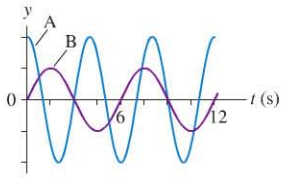


FIGURE P14.49

49. The two graphs in Figure P14.49 are for two different vertical mass-spring systems.
   a. What is the frequency of system A? What is the first time at which the mass has maximum speed while traveling in the upward direction?
   b. What is the period of system B? What is the first time at which the mechanical energy is all potential?
   c. If both systems have the same mass, what is the ratio k_A/k_B of their spring constants?

50. BIO As we've seen, astronauts measure their mass by measuring the period of oscillation when sitting in a chair connected to a spring. The Body Mass Measurement Device on Skylab, a 1970s space station, had a spring constant of 606 N/m. The empty chair oscillated with a period of 0.901 s. What is the mass of an astronaut who oscillates with a period of 2.09 s when sitting in the chair?

51. A 100 g ball attached to a spring with spring constant 2.50 N/m oscillates horizontally on a frictionless table. Its velocity is 20.0 cm/s when $x = -5.00$ cm.
   a. What is the amplitude of oscillation?
   b. What is the speed of the ball when $x = 3.00$ cm?

52. BIO The ultrasonic transducer used in a medical ultrasound imaging device is a very thin disk ($m = 0.10$ g) driven back and forth in SHM at 1.0 MHz by an electromagnetic coil.
   a. The maximum restoring force that can be applied to the disk without breaking it is 40,000 N. What is the maximum oscillation amplitude that won't rupture the disk?
   b. What is the disk's maximum speed at this amplitude?

53. A compact car has a mass of 1200 kg. Assume that the car has one spring on each wheel, that the springs are identical, and that the mass is equally distributed over the four springs.
   a. What is the spring constant of each spring if the empty car bounces up and down 2.0 times each second?
   b. What will be the car's oscillation frequency while carrying four 70 kg passengers?

54. Four people with a combined mass of 300 kg are riding in a 1100 kg car. When they drive down a washboard road with bumps spaced 5.0 m apart, they notice that the car bounces up and down with a maximum amplitude when the car is traveling at 6.0 m/s. The driver stops the car and everyone exits the vehicle. How much does the car rise up on its springs?

55. || INT A 500 g air-track glider attached to a spring with spring constant 10 N/m is sitting at rest on a frictionless air track. A 250 g glider is pushed toward it from the far end of the track at a speed of 120 cm/s. It collides with and sticks to the 500 g glider. What are the amplitude and period of the subsequent oscillations?

56. |||| INT A 1.00 kg block is attached to a horizontal spring with spring constant 2500 N/m. The block is at rest on a frictionless surface. A 10.0 g bullet is fired into the block, in the face opposite the spring, and sticks.
   a. What was the bullet's speed if the subsequent oscillations have an amplitude of 10.0 cm?
   b. Could you determine the bullet's speed by measuring the oscillation frequency? If so, how? If not, why not?

57. |||| Figure P14.57 shows two springs, each with spring constant 20 N/m, connecting a 2.5 kg block to two walls. The block slides on a frictionless surface. If the block is displaced from equilibrium, it will undergo simple harmonic motion. What is the frequency of that motion?

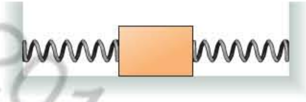


FIGURE P14.57

58. || Bungee Man is a superhero who does super deeds with the help of Super Bungee cords. The Super Bungee cords act like ideal springs no matter how much they are stretched. One day, Bungee Man stopped a school bus that had lost its brakes by hooking one end of a Super Bungee to the rear of the bus as it passed him, planting his feet, and holding on to the other end of the Bungee until the bus came to a halt. (Of course, he then had to quickly release the Bungee before the bus came flying back at him.) The mass of the bus, including passengers, was 12,000 kg, and its speed was 21.2 m/s. The bus came to a stop in 50.0 m.
   a. What was the spring constant of the Super Bungee?
   b. How much time after the Super Bungee was attached did it take the bus to stop?

59. |||| INT Two 50 g blocks are held 30 cm above a table. As shown in Figure P14.59, one of them is just touching a 30-cm-long spring. The blocks are released at the same time. The block on the left hits the table at exactly the same instant as the block on the right first comes to an instantaneous rest. What is the spring constant?

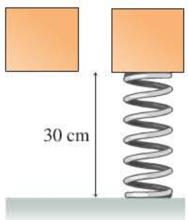


FIGURE P14.59

60. | The earth's free-fall acceleration varies from 9.78 m/s^2 at the equator to 9.83 m/s^2 at the poles. A pendulum whose length is precisely 1.000 m can be used to measure g. Such a device is called a *gravimeter.*
   a. How long do 100 oscillations take at the equator?
   b. How long do 100 oscillations take at the north pole?
   c. Suppose you take your gravimeter to the top of a high mountain peak near the equator. There you find that 100 oscillations take 201 s. What is g on the mountain top?

61. |||| INT A pendulum clock has a heavy bob supported on a very thin steel rod that is 1.00000 m long at 20°C.
   a. To 6 significant figures, what is the clock's period? Assume that g is 9.80 m/s^2 exactly.
   b. To 6 significant figures, what is the period if the temperature increases by 10°C?
   c. The clock keeps perfect time at 20°C. At 30°C, after how many hours will the clock be off by 1.0 s?

62. ||| INT A pendulum consists of a massless, rigid rod with a mass at one end. The other end is pivoted on a frictionless pivot so that it can turn through a complete circle. The pendulum is inverted, so the mass is directly above the pivot point, then released. The speed of the mass as it passes through the lowest point is 5.0 m/s. If the pendulum later undergoes small-amplitude oscillations at the bottom of the arc, what will the frequency be?

63. | Two side-by-side pendulum clocks have heavy bobs at the ends of rigid, very lightweight arms. One pendulum has a 38.8-cm-long rod, the other a 24.8-cm-long rod. Each clock makes one tick for each complete swing of its pendulum.
   a. Determine the frequencies and periods of the two clocks.
   b. Because the two pendulums have different frequencies, their ticks are usually "out of step." However, you notice that they do get back into step (tick at the same instant) at regular intervals. How much time elapses between such events?
   c. The getting-into-step phenomenon is, itself, periodic. What is the frequency of this phenomenon? Can you see a relationship between its frequency and the frequencies of the two clocks?

64. |||| BIO INT Orangutans can move by brachiation, swinging like a pendulum beneath successive handholds. If an orangutan has arms that are 0.90 m long and repeatedly swings to a 20° angle, taking one swing immediately after another, estimate how fast it is moving in m/s.

65. ||| The 15 g head of a bobble-head doll oscillates in SHM at a frequency of 4.0 Hz.
   a. What is the spring constant of the spring on which the head is mounted?
   b. Suppose the head is pushed 2.0 cm against the spring, then released. What is the head's maximum speed as it oscillates?
   c. The amplitude of the head's oscillations decreases to 0.50 cm in 4.0 s. What is the head's time constant?

66. || An oscillator with a mass of 500 g and a period of 0.50 s has an amplitude that decreases by 2.0% during each complete oscillation. If the initial amplitude is 10 cm, what will be the amplitude after 25 oscillations?

67. |||| INT An infant's toy has a 120 g wooden animal hanging from a spring. If pulled down gently, the animal oscillates up and down with a period of 0.50 s. His older sister pulls the spring a bit more than intended. She pulls the animal 30 cm below its equilibrium position, then lets go. The animal flies upward and detaches from the spring right at the animal's equilibrium position. If the animal does not hit anything on the way up, how far above its equilibrium position will it go?

68. |||| BIO A jellyfish can propel itself with jets of water pushed out of its bell, a flexible structure on top of its body. The elastic bell and the water it contains function as a mass-spring system, greatly increasing efficiency. Normally, the jellyfish emits one jet right after the other, but we can get some insight into the jet system by looking at a single jet thrust. Figure P14.68 shows a graph of the motion of one point in the wall of the bell for such a single jet; this is the pattern of a damped oscillation. The spring constant for the bell can be estimated to be 1.2 N/m.

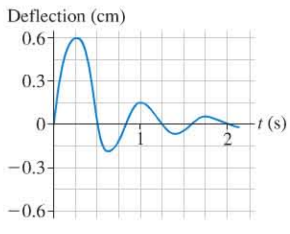


FIGURE P14.68

   a. What is the period for the oscillation?
   b. Estimate the effective mass participating in the oscillation. This is the mass of the bell itself plus the mass of the water.
   c. Consider the peaks of positive displacement in the graph. By what factor does the amplitude decrease over one period? Given this, what is the time constant for the damping?

69. |||| A 200 g oscillator in a vacuum chamber has a frequency of 2.0 Hz. When air is admitted, the oscillation decreases to 60% of its initial amplitude in 50 s. How many oscillations will have been completed when the amplitude is 30% of its initial value?

70. ||| BIO While seated on a tall bench, extend your lower leg a small amount and then let it swing freely about your knee joint, with no muscular engagement. It will oscillate as a damped pendulum. Figure P14.70 is a graph of the lower leg angle versus time in such an experiment. Estimate (a) the period and (b) the time constant for this oscillation.

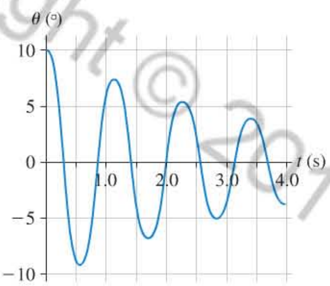


FIGURE P14.70

71. || A 2.0 kg block oscillates up and down on a spring with spring constant 220 N/m. Its initial amplitude is 15 cm. If the time constant for damping of the oscillation is 3.0 s, how much mechanical energy has been dissipated from the block-spring system after 6.0 s?

72. |||| BIO In Chapter 10, we saw that the Achilles tendon will stretch and then rebound, storing and returning energy during a step. We can model this motion as that of a mass on a spring. This is far from a perfect model, but it does give some insight. If a 60 kg person stands on a low wall with her full weight on the ball of one foot and the heel free to move, the stretch of the Achilles tendon will cause her center of gravity to lower by about 2.5 mm.
   a. What is the spring constant of her Achilles tendon?
   b. If she bounces a little, what is her oscillation period?
   c. When walking or running, the tendon spring begins to stretch as the ball of the foot takes the weight of a stride, transforming kinetic energy into elastic potential energy. Ideally, the cycle of the motion will have advanced so that potential energy has just finished being converted back to kinetic energy as the foot leaves the ground. What fraction of an oscillation period should the time between landing and liftoff correspond to? Given the period you calculated above, what is this time?
   d. Sprinters running a short race keep their foot in contact with the ground for about 0.10 s, some of which corresponds to the heel strike and subsequent rolling forward of the foot. Given this, does the answer to part c make sense?

Passage Problems

Web Spiders and Oscillations

All spiders have special organs that make them exquisitely sensitive to vibrations. Web spiders detect vibrations of their web to determine what has landed in their web, and where.

In fact, spiders carefully adjust the tension of strands to "tune" their web. Suppose an insect lands and is trapped in a web. The silk of the web serves as the spring in a spring-mass system while the body of the insect is the mass. The frequency of oscillation depends on the restoring force of the web and the mass of the insect. Spiders respond more quickly to larger—and therefore more valuable—prey, which they can distinguish by the web's oscillation frequency.

Suppose a 12 mg fly lands in the center of a horizontal spider's web, causing the web to sag by 3.0 mm.

73. | Assuming that the web acts like a spring, what is the spring constant of the web?
   A. 0.039 N/m   B. 0.39 N/m
   C. 3.9 N/m   D. 39 N/m

74. | Modeling the motion of the fly on the web as a mass on a spring, at what frequency will the web vibrate when the fly hits it?
   A. 0.91 Hz   B. 2.9 Hz   C. 9.1 Hz   D. 29 Hz

75. | If the web were vertical rather than horizontal, how would the frequency of oscillation be affected?
   A. The frequency would be higher.
   B. The frequency would be lower.
   C. The frequency would be the same.

76. | Spiders are more sensitive to oscillations at higher frequencies. For example, a low-frequency oscillation at 1 Hz can be detected for amplitudes down to 0.1 mm, but a high-frequency oscillation at 1 kHz can be detected for amplitudes as small as 0.1 μm. For these low- and high-frequency oscillations, we can say that
   A. The maximum acceleration of the low-frequency oscillation is greater.
   B. The maximum acceleration of the high-frequency oscillation is greater.
   C. The maximum accelerations of the two oscillations are approximately equal.

STOP TO THINK ANSWERS

Stop to Think 14.1: C. The frequency is inversely proportional to the period, so a shorter period implies a higher frequency.

Stop to Think 14.2: D. The restoring force is proportional to the displacement. If the displacement from equilibrium is doubled, the force is doubled as well.

Stop to Think 14.3: A. The maximum speed occurs when the mass passes through its equilibrium position.

Stop to Think 14.4: $f_D > f_C = f_B > f_A$. The frequency is determined by the ratio of k to m.

Stop to Think 14.5: C. The increase in length will cause the frequency to decrease and thus the period will increase. The time between ticks will increase, so the clock will run slow.

Stop to Think 14.6: $\tau_D > \tau_B = \tau_C > \tau_A$. The time constant is the time to decay to 37% of the initial height. The time constant is independent of the initial height.