22 Current and Resistance

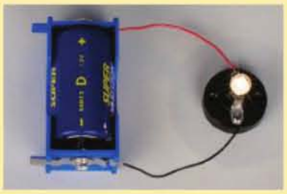
This woman is measuring her percentage body fat by gripping a device that passes a small electric current through her body. How does a measurement of the current reveal such details of the body's structure?

LOOKING AHEAD ▸▸

The goal of Chapter 22 is to learn how and why charge moves through a conductor as what we call a current.

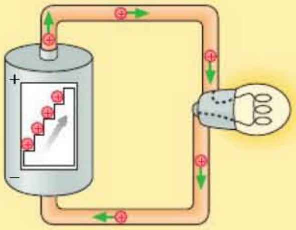
A Basic Circuit

Connecting a bulb to a battery makes the bulb glow. How is the chemical energy of the battery transferred to the bulb? In this chapter, you'll learn to explain this process in terms of electric current.



Looking Back ◂◂
20.1–20.2 Charges and conductors
20.7 Forces on charges in an electric field
21.3–21.5 Electric potential

In this simple circuit, the battery's chemical energy lifts charges "uphill" as if they were on a charge escalator. Charges then move "downhill" through the wire and bulb. This flow of charge—a **current**—warms the bulb until it glows.



Developing the model of current in a circuit will draw on many concepts from past chapters.

Current

Current is the flow of charge, and **charge is conserved:** it isn't used up in a circuit. These bulbs are connected one after the other. All of the current that goes through one bulb goes through the next and the next, so all the bulbs are equally bright.

You'll use this basic property of current to analyze simple circuits.

Energy and Power

Passing a current through a **resistor** converts electric energy to thermal energy, increasing the temperature of the resistor. Many practical devices are based on this principle.

How much current does it take to warm the hot air from this hair dryer? You'll find out how to do this calculation.

Ohm's Law

The current in a circuit depends not only on the resistance but also on the voltage. **Ohm's law** is a simple expression that relates current, voltage, and resistance.

Turning up the heat on the stove means turning up the voltage across the resistance of the burner.

Resistance

The wire filament in a lightbulb has a **resistance** to the flow of charge. Decreasing the resistance increases the current through the filament, leading to a brighter bulb.

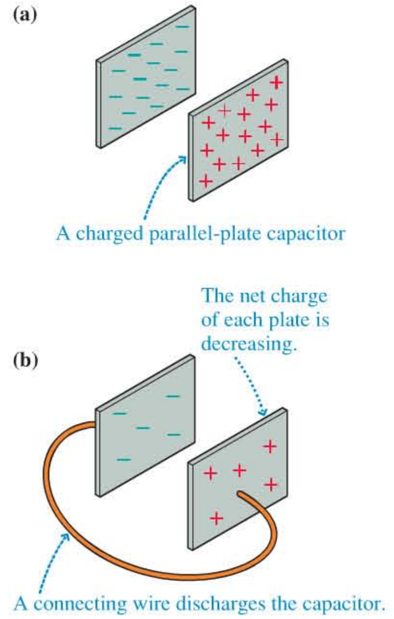
How long must the wire inside the bulb be to have the correct resistance? You'll learn how to figure this out.

22.1 A Model of Current

Let's start our exploration of current with a very simple experiment. **FIGURE 22.1a** shows a charged parallel-plate capacitor. If we connect the two capacitor plates to each other with a metal wire, as shown in **FIGURE 22.1b**, the plates quickly become neutral. We say that the capacitor has been *discharged.*

The wire is a conductor, a material through which charge easily moves. In Chapter 20, we defined a *current* as the motion of charges through a material. Later in this chapter we will develop a quantitative expression for current, but for now this definition will suffice. Apparently the excess charge on one capacitor plate is able to move through the wire to the other plate, neutralizing both plates. **The capacitor is discharged by a current in the connecting wire.**

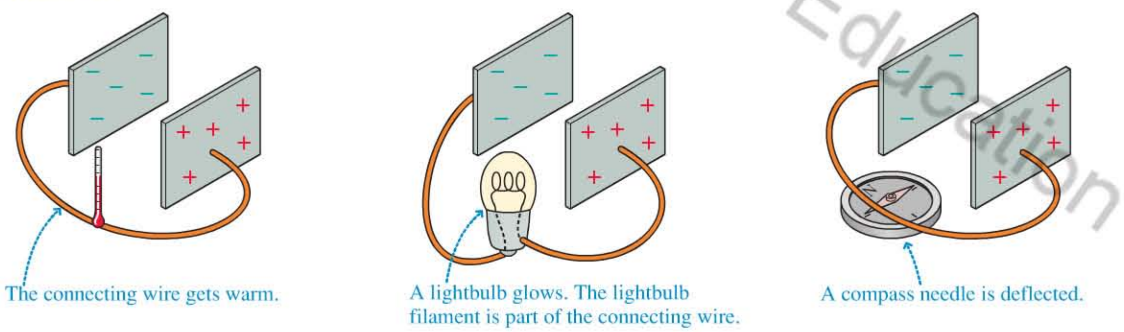
FIGURE 22.1 A capacitor is discharged by a metal wire.



NOTE ▶ Current is defined as the motion of charges, so we don't say that "current flows." It is *charges* that flow, not current. Current *is* the flow. ◀

If we observe the capacitor discharge, we see other effects. As **FIGURE 22.2** shows, the connecting wire gets warmer. If the wire is very thin in places, such as the thin filament in a lightbulb, the wire gets hot enough to glow. The current-carrying wire also deflects a compass needle. We will explore the connection between currents and magnetism in Chapter 24. For now, we will use "makes the wire warmer" and "deflects a compass needle" as *indicators* that a current is present in a wire. We can use the brightness of a lightbulb to tell us the magnitude of the current; **more current means a brighter bulb.**

FIGURE 22.2 Properties of a current.

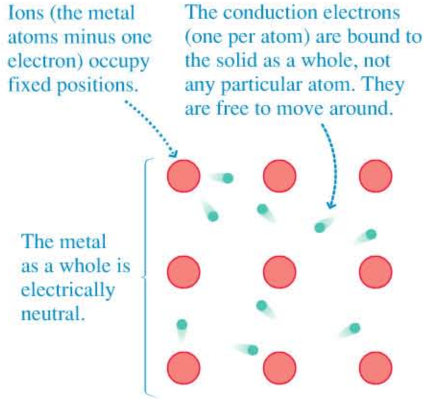


Charge Carriers

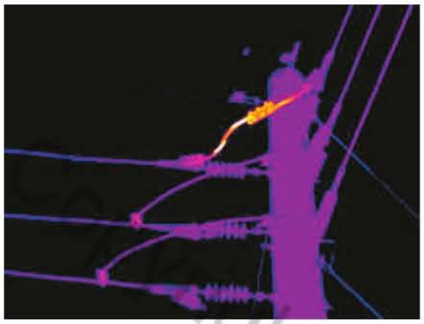
Opposite charges attract, but the oppositely charged plates of a capacitor don't spontaneously discharge because the charges can't leap from one plate to the other. A connecting wire discharges the capacitor by providing a pathway for charge to move from one side of the capacitor to the other. But does positive charge move toward the negative plate, or does negative charge move toward the positive plate?

The charges that move in a current are called the *charge carriers.* The first experiments that could distinguish between positive and negative charge carriers didn't take place until the early 20th century, but the experimental evidence is now clear: **The charge carriers in metals are electrons.** As **FIGURE 22.3** shows, it is the motion of the *conduction electrons,* which are free to move around, that forms a current—a flow of charge—in the metal. An *insulator* does not have such free charges and cannot carry a current. A *semiconductor* is an intermediate case, with relatively few charge carriers, which can be either positive or negative. It will carry a current, but not as easily as a conductor.

FIGURE 22.3 Conduction electrons in a metal.



NOTE ▶ Electrons are the charge carriers in *metals.* Other materials, such as semiconductors, may have different charge carriers. In ionic solutions, such as seawater, blood, and intercellular fluids, the charge carriers are ions, both positive and negative. ◀



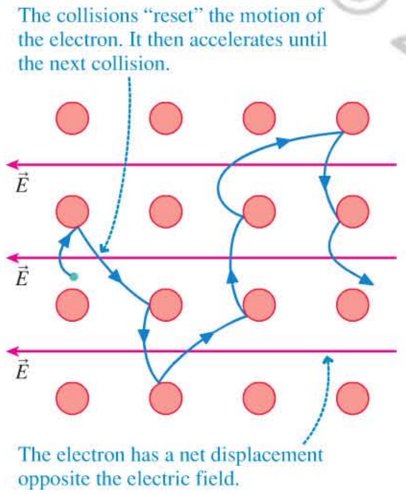
◀ **Hot wire** This thermal camera image of power lines shows that the lines are warm—as we'd expect, given the large currents that they carry. Spots where corrosion has thinned the wires get especially warm, making such images helpful for monitoring the condition of power lines.

Creating a Current

Suppose you want to slide a book across the table to your friend. You give it a quick push to start it moving, but it begins slowing down because of friction as soon as you take your hand off of it. The book's kinetic energy is transformed into thermal energy, leaving the book and the table slightly warmer. The only way to keep the book moving at a *constant* speed is to continue pushing it.

Something similar happens in a conductor. As we saw in Chapter 20, we can use an electric field to push on the electrons in a conductor. Suppose we take a piece of metal and apply an electric field, as in FIGURE 22.4. The field exerts a force on the electrons, and they begin to move. But the electrons aren't moving in a vacuum. Collisions between the electrons and the atoms of the metal slow them down, transforming the electrons' kinetic energy into the thermal energy of the metal, making the metal warmer. (Recall that "makes the wire warmer" is one of our indicators of a current.) The motion of the electrons will cease *unless you continue pushing*. To keep the electrons moving, we must maintain an electric field. In a constant field, an electron's average motion will be opposite the field. We call this motion the electron's *drift velocity*. If the field goes to zero, so does the drift velocity.

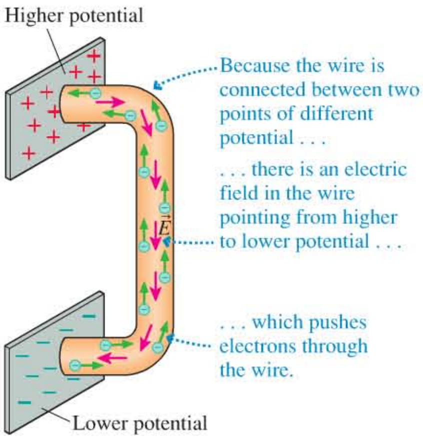
FIGURE 22.4 The motion of an electron in a conductor.



How can you have an electric field inside a conductor? One of the important conclusions of Chapter 20 was that $\vec{E} = \vec{0}$ inside a conductor in electrostatic equilibrium. But a conductor with electrons moving through it is *not* in electrostatic equilibrium. The charges are in motion, so the field need not be zero.

FIGURE 22.5 shows how a wire connected between the plates of a capacitor causes it to discharge. The separation of charges creates a potential difference between the two plates. We saw in Chapter 21 that whenever there's a potential difference, an electric field points from higher potential toward lower potential. Connecting a wire between the plates establishes an electric field in the wire, and this electric field causes electrons to flow from the negative plate (which has an excess of electrons) toward the positive plate. **The potential difference creates the electric field that drives the current in the wire.**

FIGURE 22.5 Creating a current in a wire.

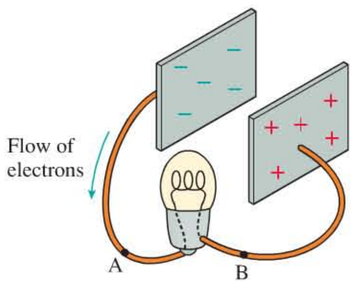


As the current continues, and the charges flow, the plates discharge and the potential difference decreases. At some point, the plates will be completely discharged, meaning no more potential difference, no more field—and no more current. Finally, $\vec{E} = \vec{0}$ inside the conducting wire, and we have equilibrium.

Conservation of Current

In FIGURE 22.6 a lightbulb has been added to the wire connecting two capacitor plates. The bulb glows while the current is discharging the capacitor. How does the current at point A compare to the current at point B? Are the currents at these points the same? Or is one larger than the other?

FIGURE 22.6 How does the current at A compare to the current at B?



You might have predicted that the current at B is less than the current at A because the bulb, in order to glow, must use up some of the current. It's easy to test this prediction; for instance, we could compare the currents at A and B by comparing how far two compass needles at these positions are deflected. Any such test gives the same result: The current at point B is *exactly equal* to the current at point A. **The current leaving a lightbulb is exactly the same as the current entering the lightbulb.**

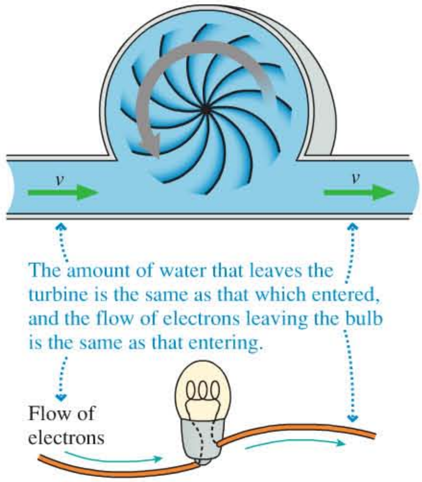
This is an important observation, one that demands an explanation. After all, "something" makes the bulb glow, so why don't we observe a decrease in the current? Electrons are charged particles. The lightbulb can't destroy electrons without violating both the law of conservation of mass and the law of conservation of charge. Thus the *number* of electrons is not changed by the lightbulb. Further, the lightbulb can't store electrons. Were it to do so, the bulb would become increasingly negative until its repulsive force stopped the flow of new electrons and the bulb would go out. This doesn't happen. Every electron entering the lightbulb must be matched by an electron leaving the bulb, and thus the current at B is the same as at A.

Let's consider an analogy with water flowing through a pipe. Suppose we put a turbine in the middle of the pipe so that the flow of the water turns the turbine, as in FIGURE 22.7. Water flows *through* the turbine. It is not consumed by the turbine, and the number of gallons of water per minute leaving the pipe is exactly the same as that entering. Nonetheless, the water must do work to turn the turbine, so there is an energy change as the water passes through.

Similarly, the lightbulb doesn't "use up" current, but, like the turbine, it *does* use energy. The energy is dissipated by atomic-level friction as the electrons move through the wire, making the wire hotter until, in the case of the lightbulb filament, it glows.

There are many other issues we'll need to examine, but we can draw a first important conclusion:

FIGURE 22.7 Water in a pipe turns a turbine.

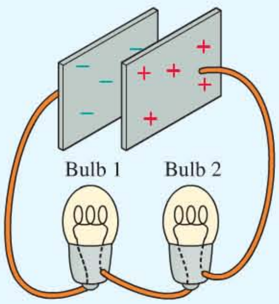


> **Law of conservation of current** The current is the same at all points in a current-carrying wire.

CONCEPTUAL EXAMPLE 22.1 Which bulb is brighter?

The discharge of a capacitor lights two identical bulbs, as shown in FIGURE 22.8. Compare the brightness of the two bulbs.

FIGURE 22.8 Two bulbs lit by the current discharging a capacitor.



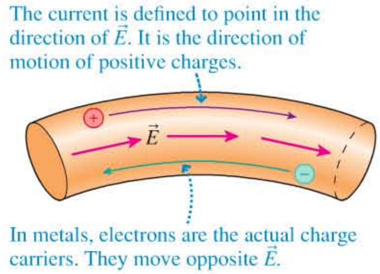
REASON Current is conserved, so any current that goes through bulb 1 must go through bulb 2 as well—the currents in the two bulbs are equal. We've noted that the brightness of a bulb is proportional to the current it carries. Identical bulbs carrying equal currents must have the same brightness.

ASSESS This result makes sense in terms of what we've seen about the conservation of current. No charge is "used up" by either bulb.

22.2 Defining and Describing Current

We have developed the idea of a current as the motion of electrons through metals, but currents were known and studied before the nature of charge carriers at an atomic level was understood. Current was *defined* as the flow of positive charges, as illustrated in FIGURE 22.9. Because the charge carriers turned out to be negative, at least for a metal, this definition of current is "wrong" in some sense. But at a macroscopic level, this isn't a problem. A capacitor is discharged regardless of whether positive charges move toward the negative plate or negative charges move

FIGURE 22.9 Current is defined as the flow of positive charge.

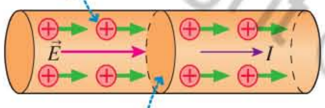


toward the positive plate. When we analyze circuits, we simply can't tell the sign of the charges that are moving through the wires. **All of our calculations will be correct and all of our circuits will work perfectly well if we define current to be the flow of positive charge.** The distinction is important only at the microscopic level.

Definition of Current

FIGURE 22.10 The current I.

The current I is due to the motion of charges in the electric field.



We imagine an area across the wire through which the charges move. In a time Δt, charge Δq moves through this area.

Because the coulomb is the SI unit of charge, and because currents are charges in motion, we define current as the *rate,* in coulombs per second, at which charge moves through a wire. **FIGURE 22.10** shows a wire in which the electric field is $\vec{E}$. This electric field causes charges to move through the wire. Because we are considering current as the motion of positive charges, the motion is in the direction of the field.

When traffic engineers measure the flow of traffic on a road, they place a wire across the road that detects the passage of cars. The flow of traffic is determined by the number of cars that pass this point in a certain time interval. We use a similar convention for current. As illustrated in Figure 22.10, we can measure the amount of charge Δq that passes through a cross section of the wire in a time interval Δt. We then define the current in the wire as

$$I = \frac{\Delta q}{\Delta t} \tag{22.1}$$

Definition of current

The current direction in a wire is from higher potential to lower potential or, equivalently, in the direction of the electric field $\vec{E}$. The SI unit for current is based on the units for charge and time according to Equation 22.1. Current is measured in coulombs per second, which we define as the **ampere,** with the abbreviation A:

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}$$

The current unit is named for the French scientist André Marie Ampère. The *amp* is an informal shortening of ampere. Household currents are typically ≈ 1 A. For example, the current through a 100 watt lightbulb is 0.83 A. The smaller currents in electronic devices are typically measured in milliamps ($1 \text{ mA} = 10^{-3}$ A) or microamps ($1\ \mu\text{A} = 10^{-6}$ A).

For a *steady current,* which will be our primary focus, the total amount of charge delivered by current I during the time interval Δt is

$$q = I\,\Delta t \tag{22.2}$$

EXAMPLE 22.2 Charge flow in a lightbulb

A 100 W lightbulb carries a current of 0.83 A. How much charge flows through the bulb in 1 minute?

SOLVE According to Equation 22.2, the total charge passing through the bulb in 1 min = 60 s is

$$q = I\,\Delta t = (0.83 \text{ A})(60 \text{ s}) = 50 \text{ C}$$

ASSESS The current corresponds to a flow of a bit less than 1 C per second, so our calculation seems reasonable, but the result is still somewhat surprising. That's a lot of charge! The enormous charge that flows through the bulb is a good check on the concept of conservation of current. If even a minuscule fraction of the charge stayed in the bulb, the bulb would become highly charged. For comparison, a Van de Graaff generator develops a potential of several hundred thousand volts due to an excess charge of just a few μC, a ten-millionth of the charge that flows through the bulb in 1 minute. Lightbulbs do not develop a noticeable charge, so the current into and out of the bulb must be exactly the same.

Conservation of Current at a Junction

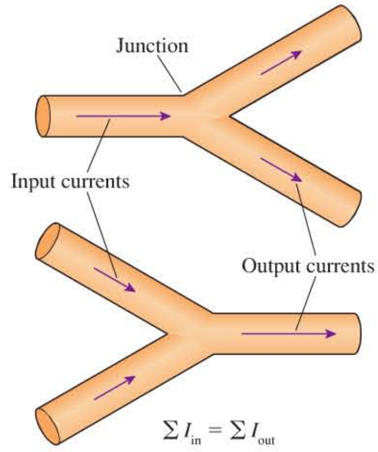
FIGURE 22.11 shows a wire splitting into two and two wires merging into one. A point where a wire branches is called a **junction.** The presence of a junction doesn't change the fact that current is conserved. We cannot create or destroy charges in the wire, and neither can we store them in the junction. The rate at which electrons flow into one *or many* wires must be exactly balanced by the rate at which they flow out of others. For a *junction,* the law of conservation of charge requires that

$$\sum I_{\text{in}} = \sum I_{\text{out}} \tag{22.3}$$

where, as usual, the $\sum$ symbol means "the sum of."

This basic conservation statement—that the sum of the currents into a junction equals the sum of the currents leaving—is called **Kirchhoff's junction law.** The junction law isn't a new law of physics; it is a consequence of the conservation of charge.

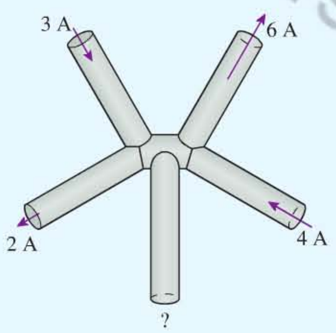
FIGURE 22.11 The sum of the currents into a junction must equal the sum of the currents leaving the junction.



EXAMPLE 22.3 Currents in a junction

Four wires have currents as noted in **FIGURE 22.12**. What are the direction and the magnitude of the current in the fifth wire?

FIGURE 22.12 The junction of five wires.



PREPARE This is a conservation of current problem. We compute the sum of the currents coming into the junction and the sum of the currents going out of the junction, and then compare these two sums. The unknown current is whatever is required to make the currents into and out of the junction "balance."

SOLVE Two of the wires have currents into the junction:

$$\sum I_{\text{in}} = 3\text{ A} + 4\text{ A} = 7\text{ A}$$

Two of the wires have currents out of the junction:

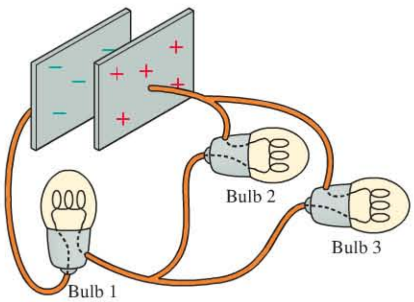
$$\sum I_{\text{out}} = 6\text{ A} + 2\text{ A} = 8\text{ A}$$

To conserve current, the fifth wire must carry a current of 1 A into the junction.

ASSESS If the unknown current is 1 A into the junction, a total of 8 A flows in—exactly what is needed to balance the current going out.

STOP TO THINK 22.1 The discharge of a capacitor lights three bulbs. Comparing the current in bulbs 1 and 2, we can say that

A. The current in bulb 1 is greater than the current in bulb 2.
B. The current in bulb 1 is less than the current in bulb 2.
C. The current in bulb 1 is equal to the current in bulb 2.

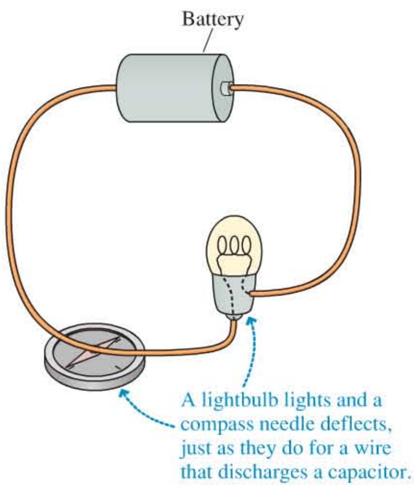


22.3 Batteries and emf

There are practical devices, such as a camera flash, that use the charge on a capacitor to drive a current. But a camera flash gives a single, bright flash of light; the capacitor discharges and the current ceases. If you want a light to illuminate your way along a dark path, you need a *continuous* source of light like a flashlight. Continuous light requires the current to be continuous as well.

FIGURE 22.13 shows a wire connecting the two terminals of a battery, much like the wire that connected the capacitor plates in Figure 22.1. Just like that wire, the wire

FIGURE 22.13 There is a current in a wire connecting the terminals of a battery.

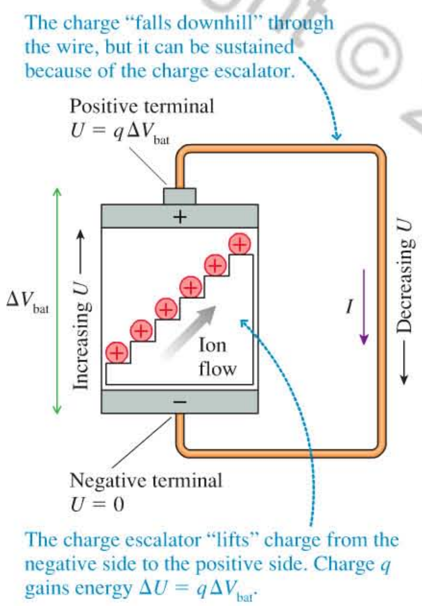


connecting the battery terminals gets warm, deflects a compass needle, and makes a lightbulb inserted into it glow brightly. These indicators tell us that charges flow through the wire from one terminal to the other. The current in the wire is the same whether it is supplied by a capacitor or a battery. Everything you've learned so far about current applies equally well to the current supplied by a battery, with one important difference—the duration of the current.

The wire connecting the battery terminals *continues* to deflect the compass needle and *continues* to light the lightbulb. The capacitor quickly runs out of excess charge, but the battery can keep the charges in motion.

How does a battery produce this sustained motion of charge? A real battery involves a series of chemical reactions, but **FIGURE 22.14** shows a simple model of a battery that illustrates the motion of charges. The inner workings of a battery act like a *charge escalator* between the two terminals. Charges are removed from the negative terminal and "lifted" to the positive terminal. It is the charge escalator that sustains the current in the wire by providing a continuously renewed supply of charges at the positive terminal.

FIGURE 22.14 The charge escalator model of a battery.



Once a charge reaches the positive terminal, it is able to flow downhill through the wire as a current until it reaches the negative terminal. The charge escalator then lifts the charge back to the positive terminal where it can start the loop all over again. This flow of charge in a continuous loop is what we call a **complete circuit**.

The charge escalator in the battery must be powered by some external source of energy. It is lifting the electrons "uphill" against an electric field. A battery consists of chemicals, called *electrolytes,* sandwiched between two electrodes made of different materials. The energy to move charges comes from chemical reactions between the electrolytes and the electrodes. These chemical reactions separate charge by moving positive ions to one electrode and negative ions to the other. In other words, chemical reactions, rather than a mechanical conveyor belt, transport charge from one electrode to the other.

As a battery creates a current in a circuit, the reactions that run the charge escalator deplete chemicals in the battery. A dead battery is one in which the supply of chemicals, and thus the supply of chemical energy, has been exhausted. You can "recharge" some types of batteries by forcing a current into the positive terminal, reversing the chemical reactions that move the charges, thus replenishing the chemicals and storing energy as chemical energy.

By separating charge, the charge escalator establishes the potential difference ΔV_{bat} that is shown between the terminals of the battery in Figure 22.14. Chemical reactions do work W_{chem} to move charge q from the negative to the positive terminal. If there are no internal energy losses, the charge gains electric potential energy $\Delta U = W_{chem}$.

The quantity W_{chem}/q, which is the work done *per charge* by the charge escalator, is called the **emf** of the battery. It is pronounced as the sequence of three letters "e-m-f." The symbol for emf is $\mathcal{E}$, a script E, and its units are those of the electric potential: joules per coulomb, or volts. The term emf was originally an abbreviation of "electromotive force." That is an outdated term, so today we just call it emf and it's not an abbreviation of anything.

The *rating* of a battery, such as 1.5 V, is the battery's emf. It is determined by the specific chemical reactions employed by the battery. An alkaline battery has an emf of 1.5 V; a rechargeable NiCd battery has an emf of 1.2 V. Larger emfs are created by using several smaller "cells" in a row, much like going from the first to the fourth floor by taking three separate escalators.

By definition, the electric potential is related to the electric potential energy of charge q by $\Delta V = \Delta U/q$. But $\Delta U = W_{chem}$ for the charges in the battery; hence the potential difference between the terminals is

$$\Delta V_{bat} = \frac{W_{chem}}{q} = \mathcal{E} \tag{22.4}$$

TRY IT YOURSELF

Listen to your potential Put on a set of earphones from a portable music player and place the plug on the table. Moisten your fingertips and hold a penny in one hand and a paper clip in the other. This makes a very weak battery; the penny and the clip are the electrodes and your moist skin the electrolyte. Touch the paper clip to the innermost contact on the headphone plug and the penny to the outermost. You will hear a *very* soft click as the potential difference causes a small current in the headphones.

The potential difference between the terminals of a battery, often called the **terminal voltage,** is ideally the battery's emf. In practice, inevitable energy losses within the battery cause the terminal voltage of a real battery to be slightly less than the emf. We'll overlook this small difference and assume $\Delta V_{bat} = \mathcal{E}$.

Electric generators, photocells, and power supplies use different means to separate charge, but otherwise they function much like a battery. The common feature of all such devices is that **they use some source of energy to separate charge and, thus, to create a potential difference.** The emf of the device is the work done per charge to separate the charge. In contrast, a capacitor stores separated charges, but a capacitor has no means to *do* the separation. Hence a capacitor has a potential difference, but not an emf.

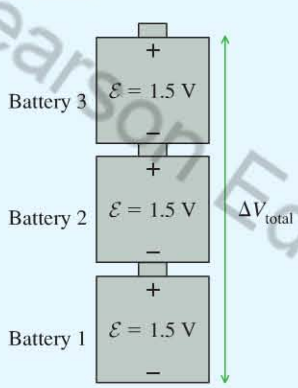
NOTE ► The term *emf,* often capitalized as EMF, is widely used in popular science articles in newspapers and magazines to mean "electromagnetic field." This is *not* how we will use the term *emf.* ◄

A shocking predator? BIO The torpedo ray captures and eats fish by paralyzing them with electricity. As we will see in Chapter 23, cells in the body use chemical energy to separate charge, just as in a battery. Special cells in the body of the ray called *electrocytes* produce an emf of a bit more than 0.10 V for a short time when stimulated. Such a small emf will not produce a large effect, but the torpedo ray has organs that contain clusters of hundreds of these electrocytes connected in a row. The total emf can be 50 V or more, enough to immobilize nearby prey.

CONCEPTUAL EXAMPLE 22.4 Potential difference for batteries in series

Three batteries are connected one after the other as shown in FIGURE 22.15; we say they are connected in *series.* What's the total potential difference?

FIGURE 22.15 Three batteries in series.



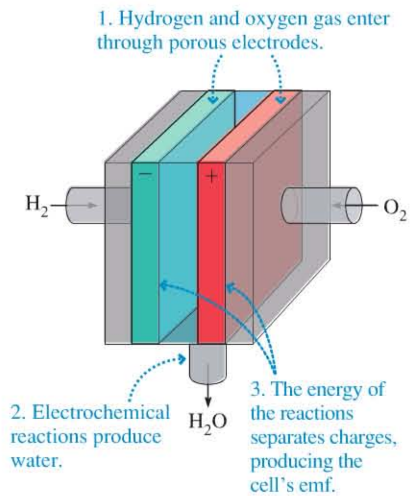
REASON We can think of this as three charge escalators, one after the other. Each one lifts charges to a higher potential. Because each battery raises the potential by 1.5 V, the total potential difference of the three batteries in series is 4.5 V.

ASSESS Common AA and AAA batteries are 1.5 V batteries. Many consumer electronics, such as digital cameras, use two or four of these batteries. Wires inside the device connect the batteries in series to produce a total 3.0 V or 6.0 V potential difference.

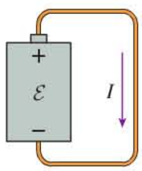
Electricity generators, such as coal-burning power plants, burn fuel (a source of chemical energy), transform the resulting thermal energy into the mechanical energy of a spinning turbine, and then use a generator, which we'll discuss in Chapter 24, to transform the mechanical energy into electricity. There are unavoidable thermodynamic inefficiencies associated with this process, as we learned in Chapter 11.

A more elegant solution to the generation of electricity is the *fuel cell.* A particular fuel cell, one that combines hydrogen fuel with oxygen, is illustrated in FIGURE 22.16. Rather than burning the fuel, as in a power plant, with the resulting thermal losses, the fuel cell's specially designed electrodes allow an electrochemical reaction that transforms the chemical energy of the hydrogen directly into electric energy. A fuel cell thus works like a battery with the chemicals supplied externally. As long as fuel and oxygen are coming in, a fuel cell can produce electricity.

FIGURE 22.16 The operation of a fuel cell.



STOP TO THINK 22.2 A battery produces a current in a wire. As the current continues, which of the following quantities (perhaps more than one) decreases?



A. The positive charge in the battery
B. The emf of the battery
C. The chemical energy in the battery

22.4 Connecting Potential and Current

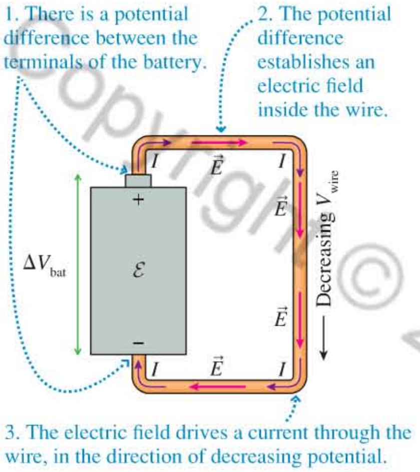


FIGURE 22.17 The electric field and the current inside the wire.

An important conclusion of the charge escalator model is that **a battery is a source of potential difference.** When charges flow through a wire that connects the battery terminals, this current is a *consequence* of the battery's potential difference. You can think of the battery's emf as being the *cause*. Current, heat, light, sound, and so on are all *effects* that happen when the battery is used in certain ways.

Figure 22.14 showed how the charge escalator model explained the motion of charges in a wire connected between the terminals of a battery: The charge escalator in the battery raises the charges "uphill," and then they flow "downhill" through the wire. Let's extend this analysis and look at the connection between the potential difference and the current.

In **FIGURE 22.17**, a battery produces a potential difference that causes a current in a wire connecting the terminals. You learned in Chapter 21 that the potential difference between any two points is independent of the path between them. Consequently, the potential difference between the two ends of the wire, along a path through the wire, is equal to the potential difference between the two terminals of the battery:

$$\Delta V_{\text{wire}} = \Delta V_{\text{bat}} \tag{22.5}$$

This potential difference causes a current in the direction of decreasing potential. Now, let's look at the factors that determine the magnitude of this current.

Resistance

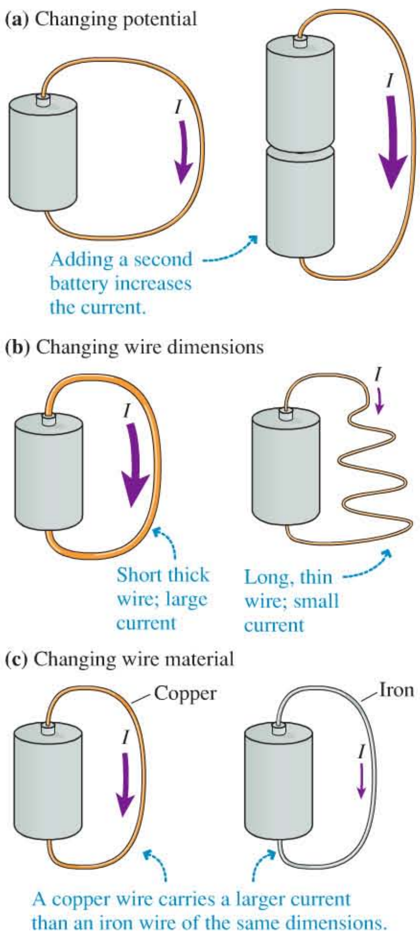


FIGURE 22.18 Factors affecting the current in a wire.

FIGURE 22.18 shows a series of experiments to determine what factors affect the current in a wire connected between the terminals of a battery. The experiments show that there are two factors that determine the current: the potential difference and the properties of the wire.

Figure 22.18a shows that adding a second battery in series increases the current, as you would expect. A larger potential difference creates a larger electric field that pushes charges through the wire faster. Careful measurements would show that the current I is proportional to ΔV_{wire}.

Figures 22.18b and 22.18c illustrate the two properties of the wire that affect the current: the dimensions and the material of which the wire is made. Figure 22.18b shows that increasing the length of the wire decreases the current, while increasing the thickness of the wire increases the current. This seems reasonable because it should be harder to push charges through a long wire than a short one, and an electric field should be able to push more charges through a fat wire than a skinny one. Figure 22.18c shows that wires of different materials will carry different currents—some materials are better conductors than others.

For any particular wire, we can define a quantity called the **resistance** that is a measure of how hard it is to push charges through the wire. We use the symbol R for resistance. A large resistance implies that it is hard to move the charges through the wire; in a wire with small resistance, the charges move much more easily. The current in the wire depends on the potential difference ΔV_{wire} between the ends of the wire and the wire's resistance R:

$$I = \frac{\Delta V_{\text{wire}}}{R} \tag{22.6}$$

Establishing a potential difference ΔV_{wire} between the ends of a wire of resistance R creates an electric field that, in turn, causes a current $I = \Delta V_{\text{wire}}/R$ in the wire. As we would expect, the smaller the resistance, the larger the current.

We can think of Equation 22.6 as the definition of resistance. If a potential difference V_{wire} causes current I in a wire, the wire's resistance is

$$R = \frac{\Delta V_{\text{wire}}}{I} \tag{22.7}$$

The SI unit of resistance is the **ohm,** defined as

$$1 \text{ ohm} = 1\ \Omega = 1 \text{ V/A}$$

where Ω is an uppercase Greek omega. The unit takes its name from the German physicist Georg Ohm. The ohm is the basic unit of resistance, although kilohms ($1\ \text{k}\Omega = 10^3\ \Omega$) and megohms ($1\ \text{M}\Omega = 10^6\ \Omega$) are widely used.

EXAMPLE 22.5 **Resistance of a lightbulb**

The glowing element in an incandescent lightbulb is the *filament,* a long, thin piece of tungsten wire that is heated by the electric current through it. When connected to the 120 V of an electric outlet, a 60 W bulb carries a current of 0.50 A. What is the resistance of the filament in the lamp?

SOLVE We can use Equation 22.7 to compute the resistance:

$$R = \frac{\Delta V_{\text{wire}}}{I} = \frac{120\ \text{V}}{0.50\ \text{A}} = 240\ \Omega$$

ASSESS As we will see below, the resistance of the filament varies with temperature. This value holds for the lightbulb only when the bulb is glowing and the filament is hot.

Resistivity

Figure 22.18c showed that the resistance of a wire depends on what it is made of. We define a quantity called **resistivity,** for which we use the symbol ρ (lowercase Greek rho), to characterize the electrical properties of materials. Materials that are good conductors have low resistivity; materials that are poor conductors (and thus that are good insulators) have high resistivity. The resistivity ρ has units of $\Omega \cdot \text{m}$. The resistivities of some common materials are listed in Table 22.1.

Metals are generally good conductors (and so have very low resistivity), but metals such as copper are much better conductors than metals such as nichrome, an alloy of nickel and chromium that is used to make heating wires. Water is a poor conductor, but the dissolved salts in seawater produce ions that can carry charge, so seawater is a good conductor, with a resistivity one million times less than that of pure water. Glass is an excellent insulator with a resistivity in excess of $10^{14}\ \Omega \cdot \text{m}$, 10^{22} times that of copper.

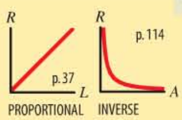
The resistivity of a material depends on the temperature, as you can see from the two values for tungsten listed in Table 22.1. As the temperature increases, so do the thermal vibrations of the atoms. This makes them "bigger targets" for the moving electrons, causing collisions to be more frequent. Thus the resistivity of a metal increases with increasing temperature.

The resistance of a wire depends both on the resistivity of its material and on the dimensions of the wire. A wire made of a material of resistivity ρ, with length L and cross-section area A, has resistance

TABLE 22.1 Resistivity of materials

Material	Resistivity ($\Omega \cdot \text{m}$)
Copper	1.7×10^{-8}
Tungsten (20°C)	5.6×10^{-8}
Tungsten (1500°C)	5.0×10^{-7}
Iron	9.7×10^{-8}
Nichrome	1.5×10^{-6}
Seawater	0.22
Blood (average)	1.6
Muscle	13
Fat	25
Pure water	2.4×10^5
Cell membrane	3.6×10^7

$$R = \frac{\rho L}{A} \tag{22.8}$$



Resistance of a wire in terms of resistivity and dimensions

Resistance is a property of a *specific* wire or conductor because it depends on the conductor's length and diameter as well as on the resistivity of the material from which it is made.

NOTE ▶ It is important to distinguish between resistivity and resistance. *Resistivity* is a property of the *material,* not any particular piece of it. All copper wires (at the same temperature) have the same resistivity. *Resistance* characterizes a specific piece of the conductor having a specific geometry. A short, thick copper wire has a smaller resistance than a long, thin copper wire. The relationship between resistivity and resistance is analogous to that between density and mass. ◀

EXAMPLE 22.6 The length of a lightbulb filament

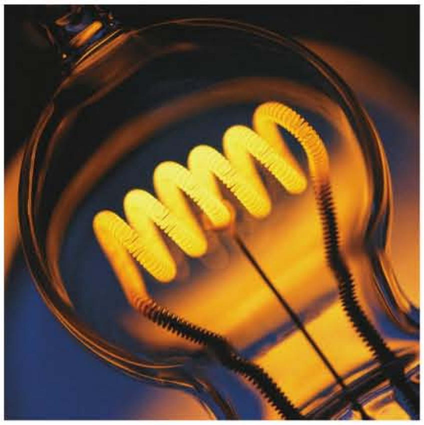
We calculated in Example 22.5 that a 60 W lightbulb has a resistance of 240 Ω. At the operating temperature of the tungsten filament, the resistivity is approximately $5.0 \times 10^{-7}\ \Omega \cdot \text{m}$. If the wire used to make the filament is 0.040 mm in diameter (a typical value), how long must the filament be?

PREPARE The resistance of a wire depends on its length, its cross-section area, and the material of which it is made.

SOLVE The cross-section area of the wire is $A = \pi r^2 = \pi(2.0 \times 10^{-5}\ \text{m})^2 = 1.26 \times 10^{-9}\ \text{m}^2$. Rearranging Equation 22.8 shows us that the filament must be of length

$$L = \frac{AR}{\rho} = \frac{(1.26 \times 10^{-9}\ \text{m}^2)(240\ \Omega)}{5.0 \times 10^{-7}\ \Omega \cdot \text{m}} = 0.60\ \text{m}$$

ASSESS This is quite long—nearly two feet. This result may seem surprising, but some reflection shows that it makes sense. The resistivity of tungsten is low, so the filament must be quite thin and long.



◀**Coils of coils** A close view of a typical lightbulb's filament shows that it is made of very thin wire that is coiled and then coiled again. The double-coil structure is necessary to fit the great length of the filament into the small space of the bulb's globe.

EXAMPLE 22.7 Making a heater

An amateur astronomer uses a heater to warm her telescope eyepiece so moisture does not collect on it. The heater is a 20-cm-long, 0.50-mm-diameter nichrome wire that wraps around the eyepiece. When the wire is connected to a 1.5 V battery, what is the current in the wire?

PREPARE The current in the wire depends on the emf of the battery and the resistance of the wire. The resistance of the wire depends on the resistivity of nichrome, given in Table 22.1, and the dimensions of the wire. Converted to meters, the relevant dimensions of the wire are $L = 0.20$ m and $r = 2.5 \times 10^{-4}$ m.

SOLVE The wire's resistance is

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{(1.5 \times 10^{-6}\ \Omega \cdot \text{m})(0.20\ \text{m})}{\pi(2.5 \times 10^{-4}\ \text{m})^2} = 1.5\ \Omega$$

The wire is connected to the battery, so $\Delta V_{\text{wire}} = \Delta V_{\text{bat}} = 1.5$ V. The current in the wire is

$$I = \frac{\Delta V_{\text{wire}}}{R} = \frac{1.5\ \text{V}}{1.5\ \Omega} = 1.0\ \text{A}$$

ASSESS The emf of the battery is small, but so is the resistance of the wire, so this is a reasonable current, enough to warm the wire and the eyepiece.

Electrical Measurements of Physical Properties

Measuring resistance is quite straightforward. Because resistance depends sensitively on the properties of materials, a measurement of resistance can be a simple but effective probe of other quantities of interest. For example, the resistivity of water is strongly dependent on dissolved substances in the water, so it is easy to make a quick test of water purity by making a measurement of resistivity.

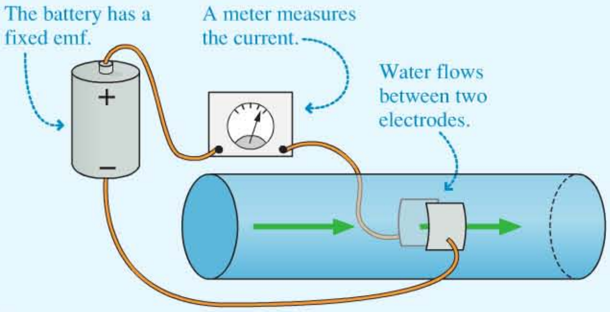
CONCEPTUAL EXAMPLE 22.8 Testing drinking water

A house gets its drinking water from a well that has an intermittent problem with salinity. Before the water is pumped into the house, it passes between two electrodes in the circuit shown in FIGURE 22.19. The current passing through the water is measured with a meter. Which corresponds to increased salinity—an increased current or a decreased current?

REASON Increased salinity causes the water's resistivity to decrease. This decrease causes a decrease in resistance between the electrodes. Current is inversely proportional to resistance, so this leads to an increase in current.

ASSESS Increasing salinity means more ions in solution and thus more charge carriers, so an increase in current is expected. Electrical systems similar to this can therefore provide a quick check of water purity.

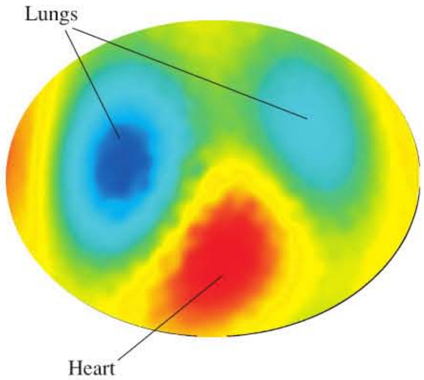
FIGURE 22.19 A water-testing circuit.



Different tissues in the body have different resistivities, as we see in Table 22.1. For example, fat has a higher resistivity than muscle. Consequently, a routine test to estimate the percentage of fat in a person's body is based on a measurement of the body's resistance, as illustrated in the photo at the start of the chapter. A higher resistance of the body means a higher proportion of fat.

More careful measurements of resistance can provide more detailed diagnostic information. Passing a small, safe current between pairs of electrodes on opposite sides of a person's torso permits a measurement of the resistance of the intervening tissue, a technique known as *electrical impedance tomography*. (Impedance is similar to resistance, but it also applies to AC circuits, which we will explore in Chapter 26.) FIGURE 22.20 shows an image of a patient's torso generated from measurements of resistance between many pairs of electrodes. The image shows the change in resistance between two subsequent measurements; decreasing resistance shows in red, increasing resistance in blue. This image was created during the resting phase of the heart, when blood was leaving the lungs and entering the heart. Blood is a better conductor than the tissues of the heart and lungs, so the motion of blood decreased the resistance of the heart and increased that of the lungs. This patient was healthy, but in a patient with circulatory problems any deviation from normal blood flow would lead to abnormal patterns of resistance that would be revealed in such an image.

FIGURE 22.20 An electrical impedance map showing the cross section of a healthy patient's torso.



STOP TO THINK 22.3 A wire connected between the terminals of a battery carries a current. The wire is removed and stretched, decreasing its cross-section area and increasing its length. When the wire is reconnected to the battery, the new current is

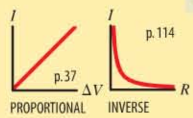
A. Larger than the original current.
B. The same as the original current.
C. Smaller than the original current.

22.5 Ohm's Law and Resistor Circuits

The relationship between the potential difference across a conductor and the current passing through it that we saw in the preceding section was first deduced by Georg Ohm and is known as **Ohm's law:**

$$I = \frac{\Delta V}{R} \tag{22.9}$$

Ohm's law for a conductor of resistance R



If we know that a wire of resistance R carries a current I, we can compute the potential difference between the ends of the wire as $\Delta V = IR$.

NOTE ► We could write the equation for Ohm's law as $\Delta V = IR$, but $I = \Delta V/R$ is a better description of cause and effect because it is the potential difference that causes the current. ◄

Ohmic and Nonohmic Materials

Despite its name, Ohm's law is *not* a law of nature. It is limited to those materials whose resistance R remains constant—or very nearly so—during use. Materials to which Ohm's law applies are called **ohmic.** FIGURE 22.21a shows that the current through an ohmic material is directly proportional to the potential difference; doubling the potential difference results in a doubling of the current. This is a linear relationship, and the resistance R can be determined from the slope of the graph.

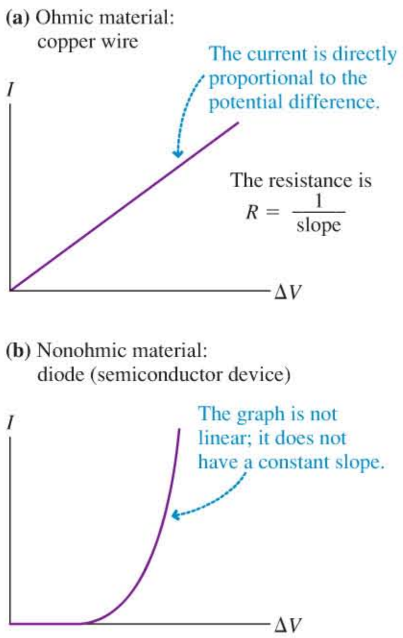
Many materials are ohmic over a reasonable range of operating conditions. The resistance of metals varies slightly with temperature, but a metal wire is ohmic as long as the temperature is reasonably constant, and we can give it a fixed resistance value.

Other materials and devices are **nonohmic,** meaning that the current through the device is *not* directly proportional to the potential difference. An example is a semiconductor device known as a *diode.* The graph in FIGURE 22.21b shows that a diode does not have a well-defined resistance. Three important examples of nonohmic devices are:

1. Batteries, where $\Delta V = \mathcal{E}$ is determined by chemical reactions, independent of I
2. Semiconductor devices, where the I-versus-ΔV curve is far from linear
3. Capacitors, where, as you'll learn in Chapter 26, the relationship between I and ΔV is very different from that of a resistor

The main point to remember is that Ohm's law does *not* apply to these nonohmic devices.

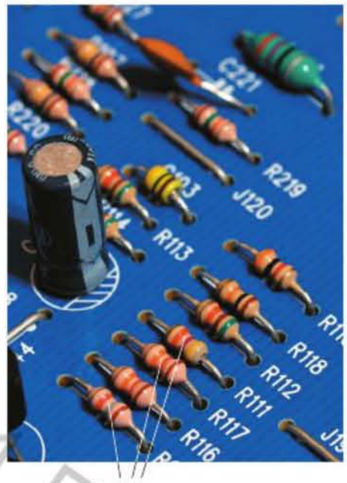
FIGURE 22.21 Current-versus-potential-difference graphs for ohmic and nonohmic materials.



Resistors

The word "resistance" may have negative connotations—who needs something that slows charges and robs energy? In some cases resistance *is* undesirable. But in many other cases, circuit elements are designed to have a certain resistance for very practical reasons. We call these circuit elements **resistors.** There are a few basic types that will be very important as we start to look at electric circuits in detail.

Types of resistors



Power resistors

We've seen that passing a current through a wire increases the temperature. This transformation of electric energy into thermal energy is the basis for many practical devices. The nichrome wire in this toaster is a resistor that gets hot enough to glow. Hotter wires produce the visible light of incandescent bulbs; cooler wires defrost the rear windows of cars.

Fixed resistors

If you open up an electronic device and look inside, you'll see a circuit board with many cylinders with colored bands. These cylinders are resistors. Each has a specified value of resistance that is revealed by the colors of the bands. We'll see examples of the use of resistors in timing circuits and other applications in the next chapter.

Variable resistors

Not all resistors have a fixed value of resistance; in many circuits and devices a variable resistor allows for changing circumstances. A volume control on a stereo is a variable resistor; so is the sensor on this nightlight. The resistance decreases when light shines on it. During the day, the resistance is low; at night, the resistance rises. A circuit monitors the resistance and switches on the light when the resistance is above a certain value.

CONCEPTUAL EXAMPLE 22.9 The changing current in a toaster

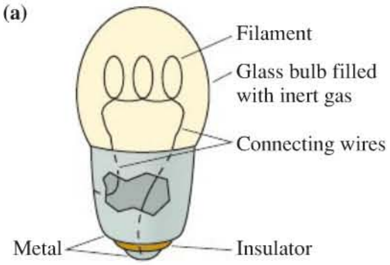
When you press the lever on a toaster, a switch connects the heating wires to 120 V. The wires are initially cool, but the current in the wires raises the temperature until they are hot enough to glow. As the wire heats up, how does the current in the toaster change?

REASON As the wire heats up, its resistivity increases, as noted above, so the resistance of the wires increases. Because the potential difference stays the same, an increasing resistance causes the current to decrease. The current through a toaster is largest when the toaster is first turned on.

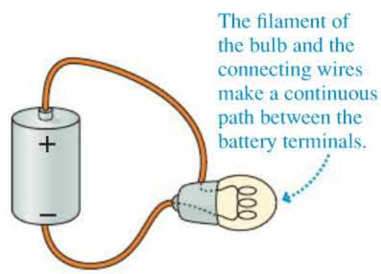
ASSESS This result makes sense. As the wire's temperature increases, the current decreases. This makes the system stable. If, instead, the current increased as the temperature increased, higher temperature could lead to more current, leading to even higher temperatures, and the toaster could overheat.

FIGURE 22.22 The basic circuit of a battery and a bulb.

(a)



(b)



Analyzing a Simple Circuit

FIGURE 22.22a shows the anatomy of a lightbulb. The important point is that a lightbulb, like a wire, has two "ends" and current passes *through* the bulb. Connections to the filament in the bulb are made at the tip and along the side of the metal cylinder. It is often useful to think of a lightbulb as a resistor that happens to give off light when a current is present. Now, let's look at a circuit using a battery, a lightbulb, and wires to make connections, as in FIGURE 22.22b. This is the basic circuit in a flashlight.

A typical flashlight bulb has a resistance of $\approx 3\ \Omega$, while a wire that one would use to connect such a bulb to a battery has a resistance of $\approx 0.01\ \Omega$. The resistance of the wires is so much less than that of the bulb that we can, with very little error,

FIGURE 22.23 The potential along a wire-resistor-wire combination.

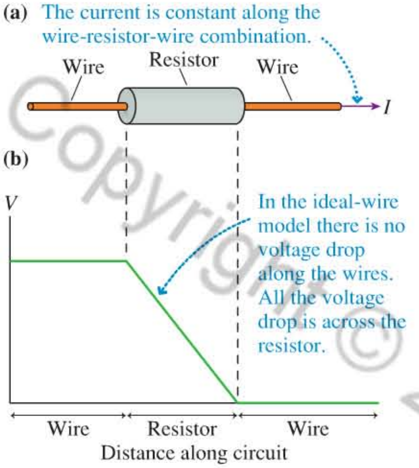
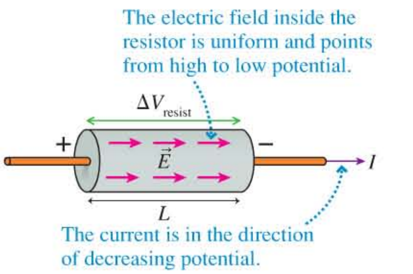


FIGURE 22.24 Electric field inside a resistor.



adopt the *ideal-wire model* and assume that any connecting wires in a circuit are ideal. An **ideal wire** has $R = 0\ \Omega$; hence the potential difference between the ends of an ideal wire is $\Delta V = 0$ V *even if there is a current in it.*

NOTE ► We know that, physically, the potential difference can't be zero. There must be an electric field in the wire for the charges to move, so it must have a potential difference. But in practice this potential difference is so small that we can assume it to be zero with little error. ◄

FIGURE 22.23 shows how the ideal-wire model is used in the analysis of circuits. The resistor in Figure 22.23a is connected at each end to a wire, and current I flows through all three. The current requires a potential difference $\Delta V_{resist} = IR_{resist}$ across the resistor, but there's no potential difference ($\Delta V_{wire} = 0$) for the ideal wires. Figure 22.23b shows this idea graphically by displaying the potential along the wire-resistor-wire combination. Current moves in the direction of decreasing potential, so there is a large *voltage drop*—a decrease in potential—across the resistor as we go from left to right, the direction of the current. The segments of the graph corresponding to the wires are horizontal because there's no voltage change along an ideal wire.

The linear variation in the potential across the resistor is similar to the linear variation in potential between the plates of a parallel-plate capacitor. In a capacitor, this linear variation in potential corresponds to a uniform electric field; the same will be true here. As we see in **FIGURE 22.24**, the electric field in a resistor carrying a current in a circuit is uniform; the strength of the electric field is

$$E = \frac{\Delta V}{L}$$

in analogy to Equation 21.6 for a parallel-plate capacitor. A larger potential difference corresponds to a larger field, as we would expect.

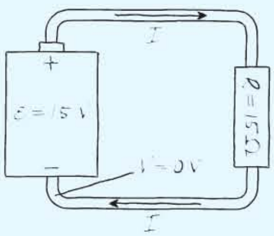
EXAMPLE 22.10 **Analyzing a single-resistor circuit**

A 15 Ω resistor is connected to the terminals of a 1.5 V battery.

a. Draw a graph showing the potential as a function of distance traveled through the circuit, starting from $V = 0$ V at the negative terminal of the battery.
b. What is the current in the circuit?

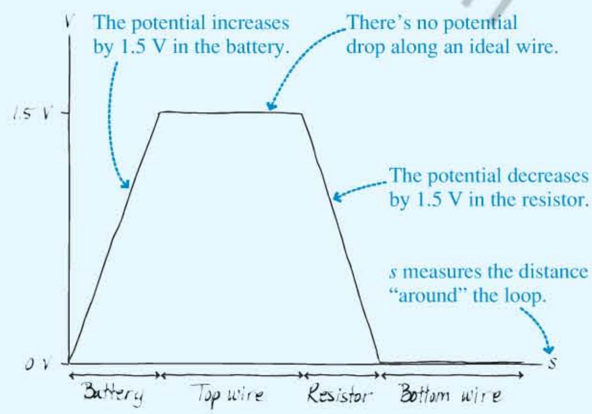
PREPARE To help us visualize the change in potential as charges move through the circuit, we begin with the sketch of the circuit in **FIGURE 22.25**. The zero point of potential is noted. We have drawn our sketch so that "up" corresponds to higher potential, which will help us make sense of the circuit. Charges are raised to higher potential in the battery, then travel "downhill" from the positive terminal through the resistor and back to the negative terminal. We assume ideal wires.

FIGURE 22.25 A single-resistor circuit.



SOLVE a. **FIGURE 22.26** is a graphical representation of the potential in the circuit. The distance s is measured from the battery's negative terminal, where $V = 0$ V. As we move around the circuit to the starting point, the potential must return to its original value. Because the wires are ideal, there is no change in potential along the wires. This means that the potential difference across the resistor must be equal to the potential difference across the battery: $\Delta V_R = \mathcal{E} = 1.5$ V.

FIGURE 22.26 Potential-versus-position graph.



b. Now that we know the potential difference of the resistor, we can compute the current in the resistor by using Ohm's law:

$$I = \frac{\Delta V_R}{R} = \frac{1.5\text{ V}}{15\ \Omega} = 0.10\text{ A}$$

Because current is conserved, this is the current at any point in the circuit. In other words, the battery's charge escalator lifts charge at the rate 0.10 C/s, and charge flows through the wires and the resistor at the rate 0.10 C/s.

ASSESS This is a reasonable value of the current in a battery-powered circuit.

As we noted, there are many devices whose resistance varies as a function of a physical variable that we might like to measure, such as light intensity, temperature, or sound intensity. As the following example shows, we can use resistance measurements to monitor a physical variable.

EXAMPLE 22.11 Using a thermistor

A thermistor is a device whose resistance varies with temperature in a well-defined way. A certain thermistor has a resistance of 2.8 kΩ at 20°C and 0.39 kΩ at 70°C. This thermistor is used in a water bath in a lab to monitor the temperature. The thermistor is connected in a circuit with a 1.5 V battery, and the current measured. What is the change in current in the circuit as the temperature rises from 20°C to 70°C?

SOLVE We can use Ohm's law to find the current in each case:

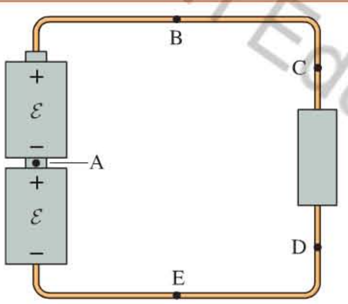
$$I(20°\mathrm{C}) = \frac{\Delta V}{R} = \frac{1.5\ \mathrm{V}}{2.8 \times 10^3\ \Omega} = 0.54\ \mathrm{mA}$$

$$I(70°\mathrm{C}) = \frac{\Delta V}{R} = \frac{1.5\ \mathrm{V}}{0.39 \times 10^3\ \Omega} = 3.8\ \mathrm{mA}$$

The change in current is thus 3.3 mA.

ASSESS A modest change in temperature leads to a large change in current, which is reasonable—this is a device intended to provide a sensitive indication of a temperature change.

STOP TO THINK 22.4 Two identical batteries are connected in series in a circuit with a single resistor. $V = 0$ V at the negative terminal of the lower battery. Rank in order, from highest to lowest, the potentials V_A to V_E at the labeled points, noting any ties. Assume the wires are ideal.



22.6 Energy and Power

When you flip the switch on a flashlight, a battery is connected to a lightbulb, which then begins to glow. The bulb is radiating energy. Where does this energy come from?

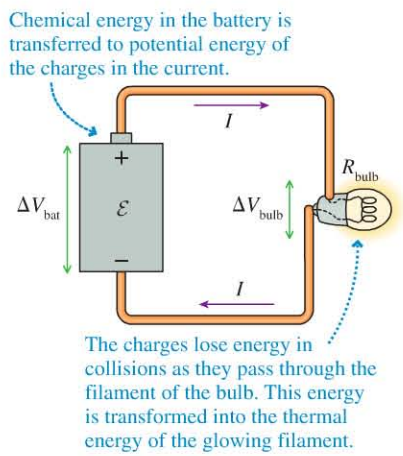
A battery not only supplies a potential difference but also supplies energy, as shown in the battery and bulb circuit of **FIGURE 22.27**. The charge escalator is an energy-transfer process, transferring the chemical energy E_{chem} stored in the battery to the electric potential energy U of the charges. That energy is then dissipated as the charges move through the lightbulb, keeping the filament warm and glowing.

Recall that charge q gains potential energy $\Delta U = q\,\Delta V$ as it moves through a potential difference ΔV. The potential difference of a battery is $\Delta V_{\mathrm{bat}} = \mathcal{E}$, so the battery supplies energy $\Delta U = q\mathcal{E}$ to charge q as it lifts the charge up the charge escalator from the negative to the positive terminal.

It's more useful to know the *rate* at which the battery supplies energy. You learned in Chapter 10 that the rate at which energy is transformed is *power,* measured in joules per second or *watts.* Suppose an amount of charge Δq moves through the battery in a time Δt. The charge Δq will increase its potential energy by $\Delta U = (\Delta q)\mathcal{E}$. The *rate* at which energy is transferred from the battery to the moving charges is

$$P_{\mathrm{bat}} = \text{rate of energy transfer} = \frac{\Delta U}{\Delta t} = \frac{\Delta q}{\Delta t}\mathcal{E} \qquad (22.10)$$

FIGURE 22.27 Energy transformations in a circuit with a battery and a lightbulb.



Hot dog resistors Before microwave ovens were common, there were devices that used a decidedly lower-tech approach to the rapid cooking of hot dogs. Prongs connected the hot dog to the 120 V of household electricity, making it the resistor in a circuit. The current through the hot dog dissipated energy as thermal energy, cooking the hot dog in about 2 minutes.

But $\Delta q/\Delta t$, the rate at which charge moves through the battery, is the current I. Hence the power supplied by a battery or any emf is

$$P_{\text{emf}} = I\mathcal{E} \tag{22.11}$$

Power delivered by an emf

$I\mathcal{E}$ has units of J/s, or W.

EXAMPLE 22.12 Power delivered by a car battery

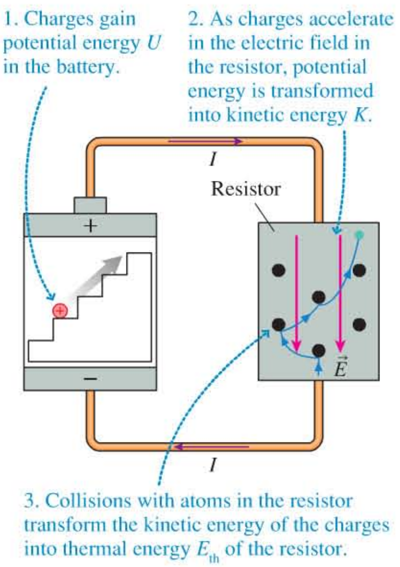
A car battery has $\mathcal{E} = 12$ V. When the car's starter motor is running, the battery current is 320 A. What power does the battery supply?

SOLVE The power is the product of the emf of the battery and the current:

$$P_{\text{bat}} = I\mathcal{E} = (320\text{ A})(12\text{ V}) = 3.8\text{ kW}$$

ASSESS This is a lot of power (about 5 hp), but this amount makes sense because turning over a car's engine is hard work. Car batteries are designed to reliably provide such intense bursts of power for starting the engine.

FIGURE 22.28 The power from the battery is dissipated in the resistor.



Suppose we consider a circuit consisting of a battery and a single resistor. $P_{\text{bat}} = I\mathcal{E}$ is the energy transferred per second from the battery's store of chemicals to the moving charges that make up the current. **FIGURE 22.28** shows the entire sequence of energy transformations, which looks like

$$E_{\text{chem}} : \ U : \ K : \ E_{\text{th}}$$

The net result is that **the battery's chemical energy is transferred to the thermal energy of the resistor,** raising its temperature.

In the resistor, the amount of charge Δq loses potential energy $\Delta U = (\Delta q)(\Delta V_R)$ as this energy is transformed into kinetic energy and then into the resistor's thermal energy. Thus the rate at which energy is transferred from the current to the resistor is

$$P_R = \frac{\Delta U}{\Delta t} = \frac{\Delta q}{\Delta t}\Delta V_R = I\,\Delta V_R \tag{22.12}$$

We say that this power—so many joules per second—is *dissipated* by the resistor as charge flows through it.

Our analysis of the single-resistor circuit in Example 22.10 found that $\Delta V_R = \mathcal{E}$. That is, the potential difference across the resistor is exactly the emf supplied by the battery. Because the current is the same in the battery and the resistor, a comparison of Equations 22.11 and 22.12 shows that

$$P_R = P_{\text{bat}} \tag{22.13}$$

The power dissipated in the resistor is exactly equal to the power supplied by the battery. The *rate* at which the battery supplies energy is exactly equal to the *rate* at which the resistor dissipates energy. This is, of course, exactly what we would have expected from energy conservation.

Most household appliances, such as a 100 W lightbulb or a 1500 W hair dryer, have a power rating. These appliances are intended for use at a standard household voltage of 120 V, and their rating is the power they will dissipate if operated with a potential difference of 120 V. Their power consumption will differ from the rating if they are operated at any other potential difference—for instance, if you use a lightbulb with a dimmer switch.

EXAMPLE 22.13 Finding the current in a lightbulb

How much current is "drawn" by a 75 W lightbulb connected to a 120 V outlet?

PREPARE We can model the lightbulb as a resistor.

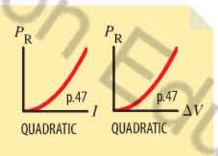
SOLVE Because the lightbulb is operating as intended, it will dissipate 75 W of power. We can rearrange Equation 22.12 to find

$$I = \frac{P_R}{\Delta V_R} = \frac{75\ \text{W}}{120\ \text{V}} = 0.63\ \text{A}$$

ASSESS We've said that we expect currents on the order of 1 A for lightbulbs and other household items, so our result seems reasonable.

A resistor obeys Ohm's law: $I = \Delta V_R / R$. This gives us two alternative ways of writing the power dissipated by a resistor. We can either substitute IR for ΔV_R or substitute $\Delta V_R / R$ for I. Thus

$$P_R = I\,\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R} \qquad (22.14)$$



Power dissipated by resistance R with current I and potential difference ΔV_R

It is worth writing the different forms of this equation to illustrate that the power varies as the square of both the current and the potential difference.

EXAMPLE 22.14 Finding the power of a dim bulb

How much power is dissipated by a 60 W (120 V) lightbulb when operated, using a dimmer switch, at 100 V?

PREPARE The 60 W rating is for operation at 120 V. We will assume that the resistance doesn't change if the bulb is run at a lower power—not quite right, but a reasonable approximation for this case in which the voltage is only slightly different from the rated value. We can compute the resistance for this case and then compute the power with the dimmer switch.

SOLVE The lightbulb dissipates 60 W at $\Delta V_R = 120$ V. Thus the filament's resistance is

$$R = \frac{(\Delta V_R)^2}{P_R} = \frac{(120\ \text{V})^2}{60\ \text{W}} = 240\ \Omega$$

The power dissipation when operated at $\Delta V_R = 100$ V is

$$P_R = \frac{(\Delta V_R)^2}{R} = \frac{(100\ \text{V})^2}{240\ \Omega} = 42\ \text{W}$$

ASSESS Reducing the voltage by 17% leads to a 30% reduction of the power. This makes sense; the power is proportional to the square of the voltage, so we expect a proportionally larger change in power.

EXAMPLE 22.15 Determining the voltage of a stereo

Most stereo speakers are designed to have a resistance of 8.0 Ω. If an 8.0 Ω speaker is connected to a stereo amplifier with a rating of 100 W, what is the maximum possible potential difference the amplifier can apply to the speakers?

PREPARE The rating of an amplifier is the *maximum* power it can deliver. Most of the time it delivers far less, but the maximum might be needed for brief, intense sounds. The maximum potential difference will occur when the amplifier is providing the maximum power, so we will make our computation with this figure. We can model the speaker as a resistor.

SOLVE The maximum potential difference occurs when the power is a maximum. At the maximum power of 100 W,

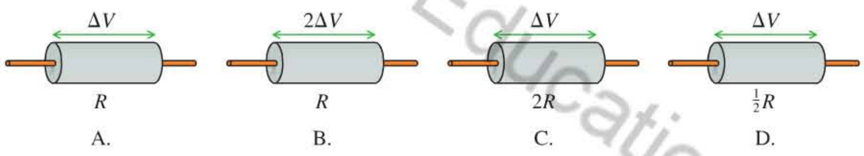
$$P_R = 100\text{ W} = \frac{(\Delta V_R)^2}{R} = \frac{(\Delta V_R)^2}{8.0\ \Omega}$$

$$\Delta V_R = \sqrt{(8.0\ \Omega)(100\text{ W})} = 28\text{ V}$$

This is the maximum potential difference the amplifier might provide.

ASSESS As a check on our result, we note that the resistance of the speaker is less than that of a lightbulb, so a smaller potential difference can provide 100 W of power.

STOP TO THINK 22.5 Rank in order, from largest to smallest, the powers P_A to P_D dissipated in resistors A to D.



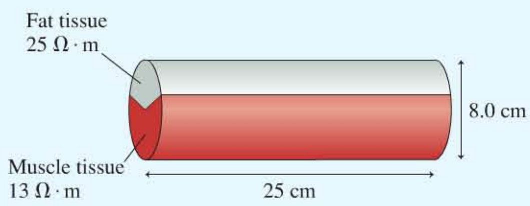
INTEGRATED EXAMPLE 22.16 Electrical measurements of body composition

The woman in the photo at the start of the chapter is gripping a device that passes a small current through her body. How does this permit a determination of body fat?

The exact details of how the device works are beyond the scope of this chapter, but the basic principle is quite straightforward: The device applies a small potential difference and measures the resulting current. Comparing multiple measurements allows the device to determine the resistance of one part of the body, the upper arm. The resistance of the upper arm depends sensitively on the percentage of body fat in the upper arm, and the percentage of body fat in the upper arm is a good predictor of the percentage of fat in the body overall. Let's make a simple model of the upper arm to show how the resistance of the upper arm varies with percentage body fat.

The model of a person's upper arm in FIGURE 22.29 ignores the nonconductive elements (such as the skin and the mineralized portion of the bone) and groups the conductive elements into two distinct sections—muscle and fat—that form two parallel segments. The resistivity of each tissue type is shown. This simple model isn't a good description of the actual structure of the arm, but it predicts the electrical character quite well.

FIGURE 22.29 A simple model of the resistance of the upper arm.



a. An experimental subject's upper arm, with the dimensions shown in the figure, is 40% fat and 60% muscle. A potential difference of 0.60 V is applied between the elbow and the shoulder. What current is measured?

b. A 0.60 V potential difference applied to the upper arm of a second subject with an arm of similar dimensions gives a current of 0.87 mA. What are the percentages of muscle and fat in this person's upper arm?

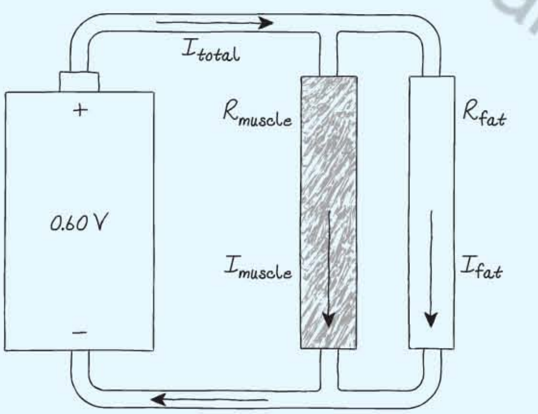
PREPARE **FIGURE 22.30** shows how we can model the upper arm as two resistors that are connected together at the ends. We use the ideal-wire model in which there's no "loss" of potential along the wires. Consequently, the potential difference across each of the two segments is the full 0.60 V of the battery. The current "splits" at the junction between the two resistors, and conservation of current tells us that

$$I_{\text{total}} = I_{\text{muscle}} + I_{\text{fat}}$$

The resistance of each segment depends on its resistivity (given in Figure 22.29) and on its length and cross-section area. The cross-section area of the whole arm is $A = \pi r^2 = \pi(0.040\ \text{m})^2 = 0.0050\ \text{m}^2$; the area of each segment is this number multiplied by the appropriate fraction.

SOLVE a. An object's resistance is related to its geometry and the resistivity of the material by $R = \rho L/A$. Thus the resistances of the muscle (60% of the area) and fat (40% of the area) segments are

FIGURE 22.30 Current through the tissues of the upper arm.



$$R_{\text{muscle}} = \frac{\rho_{\text{muscle}} L}{A_{\text{muscle}}} = \frac{(13\ \Omega \cdot \text{m})(0.25\ \text{m})}{(0.60)(0.0050\ \text{m}^2)} = 1100\ \Omega$$

$$R_{\text{fat}} = \frac{\rho_{\text{fat}} L}{A_{\text{fat}}} = \frac{(25\ \Omega \cdot \text{m})(0.25\ \text{m})}{(0.40)(0.0050\ \text{m}^2)} = 3100\ \Omega$$

The potential difference across each segment is 0.60 V. We can then use Ohm's law, $I = \Delta V/R$, to find that the current in each segment is

$$I_{\text{muscle}} = \frac{0.60\ \text{V}}{1100\ \Omega} = 0.55\ \text{mA}$$

$$I_{\text{fat}} = \frac{0.60\ \text{V}}{3100\ \Omega} = 0.19\ \text{mA}$$

The conservation of current equation then gives the total current as the sum of these two values:

$$I_{\text{total}} = 0.55\ \text{mA} + 0.19\ \text{mA} = 0.74\ \text{mA}$$

b. If we know the current, we can determine the amount of muscle and fat. Let the fraction of muscle tissue be x; the fraction of fat tissue is then $(1 - x)$. We repeat the steps of the above calculation with these expressions in place:

$$R_{\text{muscle}} = \frac{\rho_{\text{muscle}} L}{A} = \frac{(13\ \Omega \cdot \text{m})(0.25\ \text{m})}{(x)(0.0050\ \text{m}^2)} = \frac{650\ \Omega}{x}$$

$$R_{\text{fat}} = \frac{\rho_{\text{fat}} L}{A} = \frac{(25\ \Omega \cdot \text{m})(0.25\ \text{m})}{(1-x)(0.0050\ \text{m}^2)} = \frac{1250\ \Omega}{1-x}$$

In terms of these values, the current in each segment is:

$$I_{\text{muscle}} = \frac{0.60\ \text{V}}{650\ \Omega} x = 0.92(x)\ \text{mA}$$

$$I_{\text{fat}} = \frac{0.60\ \text{V}}{1250\ \Omega}(1-x) = 0.48(1-x)\ \text{mA}$$

The sum of these currents is the total current:

$$I_{\text{total}} = 0.87\ \text{mA} = 0.92(x)\ \text{mA} + 0.48(1-x)\ \text{mA}$$

Rearranging the terms on the right side gives

$$0.87\ \text{mA} = (0.48 + 0.44x)\ \text{mA}$$

Finally, we can solve for x:

$$x = 0.89$$

This person therefore has 89% muscle and 11% fatty tissue in the upper arm.

ASSESS A good check on our work is that the total current we find in part a is small—important for safety—and reasonably close to the value given in part b; the arms are the same size, and the variation in body fat between individuals isn't all that large, so we expect the numbers to be similar. The current given in part b is greater than we found in part a. Muscle has a lower resistance than fat, so the subject of part b must have a higher percentage of muscle—exactly what we found.

SUMMARY

The goal of Chapter 22 has been to learn how and why charge moves through a conductor as a current.

GENERAL PRINCIPLES

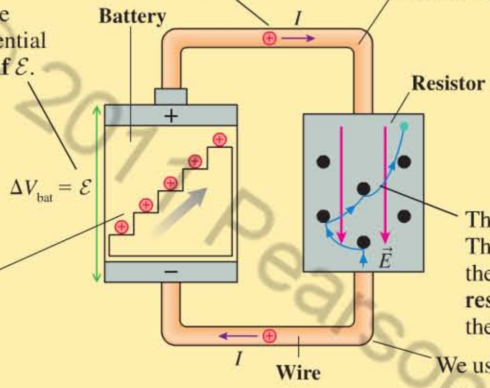
Current

The **current** is defined to be the motion of positive charges

$$I = \frac{\Delta q}{\Delta t}$$

Conservation of current dictates that the current is the same at all points in the circuit.

The battery does work to raise the potential of the charges. The potential difference of the battery is its **emf** $\mathcal{E}$.



The battery creates an electric field in the circuit that causes charges to move. Positive charges move in the direction of the electric field, which is the direction of decreasing potential.

A **battery** is a source of potential difference. Chemical processes in the battery separate charges. We use a **charge escalator** model to show the lifting of charges to higher potential.

The actual charge carriers are electrons. Their random collisions with atoms impede the flow of charge and are the source of **resistance.** The collisions increase the thermal energy of the **resistor.**

We use the **ideal-wire model** in which we assume that there is no resistance in the wires.

IMPORTANT CONCEPTS

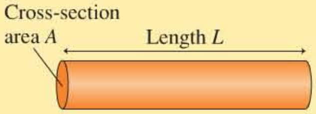
Resistance, resistivity, and Ohm's law

The **resistivity** ρ is a property of a material, a measure of how good a conductor the material is.

- Good conductors have low resistivity.
- Poor conductors have high resistivity.

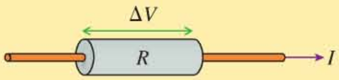
The **resistance** is a property of a particular wire or conductor. The resistance of a wire depends on its resistivity and dimensions.

$$R = \frac{\rho L}{A}$$



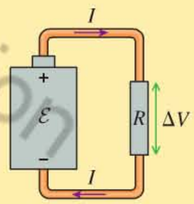
Ohm's law describes the relationship between potential difference and current in a resistor:

$$I = \frac{\Delta V}{R}$$



Energy and power

The energy used by a circuit is supplied by the emf of the battery through a series of energy transformations:



$$E_{chem} \rightarrow U_{elec} \rightarrow K \rightarrow E_{th}$$

Chemical energy in the battery | Potential energy of separated charges | Kinetic energy of moving charges | Thermal energy of atoms in the resistor

The battery *supplies* power at the rate

$$P_{emf} = I\mathcal{E}$$

The resistor *dissipates* power at the rate

$$P_R = I\,\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R}$$

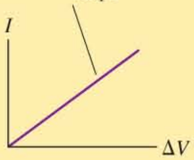
APPLICATIONS

Conducting materials

When a potential difference is applied to a wire, if the relationship between potential difference and current is linear, the material is **ohmic.**

The resistance is

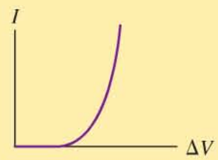
$$R = \frac{1}{\text{slope}}$$



Resistors are made of ohmic materials and have a well-defined value of resistance:

$$R = \frac{\Delta V}{I}$$

If the variation is not linear, the material is **nonohmic.**



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problems labeled INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

Problem difficulty is labeled as | (straightforward) to ||||| (challenging).

QUESTIONS

Conceptual Questions

1. Two wires connect a lightbulb to a battery, completing a circuit and causing the bulb to glow. Do the simple observations and measurements that you can make on this circuit prove that something is *flowing* through the wires? If so, state the observations and/or measurements that are relevant and the steps by which you can then infer that something must be flowing. If not, can you offer an alternative hypothesis about why the bulb glows that is at least plausible and that could be tested?
2. Two wires connect a lightbulb to a battery, completing a circuit and causing the bulb to glow. Are the simple observations and measurements you can make on this circuit able to distinguish a current composed of positive charge carriers from a current composed of negative charge carriers? If so, describe how you can tell which it is. If not, why not?
3. What *causes* electrons to move through a wire as a current?
4. A lightbulb is connected to a battery by two copper wires of equal lengths but different thicknesses. A thick wire connects one side of the lightbulb to the positive terminal of the battery and a thin wire connects the other side of the bulb to the negative terminal.
   a. Which wire carries a greater current? Or is the current the same in both? Explain.
   b. If the two wires are switched, will the bulb get brighter, dimmer, or stay the same? Explain.
5. All wires in Figure Q22.5 are made of the same material and have the same diameter. Rank in order, from largest to smallest, the currents I_1 to I_4. Explain.

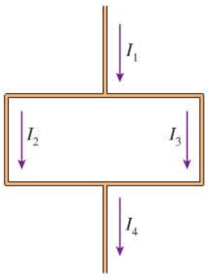


FIGURE Q22.5

6. A wire carries a 4 A current. What is the current in a second wire that delivers twice as much charge in half the time?
7. Metal 1 and metal 2 are each formed into 1-mm-diameter wires. The electric field needed to cause a 1 A current in metal 1 is larger than the electric field needed to cause a 1 A current in metal 2. Which metal has the larger resistivity? Explain.
8. BIO Cells in the nervous system have a potential difference of 70 mV across the cell membrane separating the interior of the cell from the extracellular fluid. This potential difference is maintained by ion pumps that move charged ions across the membrane. Is this an emf?
9. a. Which direction—clockwise or counterclockwise—does an electron travel through the wire in Figure Q22.9? Explain.
   b. Does an electron's electric potential energy increase, decrease, or stay the same as it moves through the wire? Explain.
   c. If you answered "decrease" in part b, where does the energy go? If you answered "increase" in part b, where does the energy come from?
   d. Which way—up or down—does an electron move through the *battery?* Explain.
   e. Does an electron's electric potential energy increase, decrease, or stay the same as it moves through the battery? Explain.
   f. If you answered "decrease" in part e, where does the energy go? If you answered "increase" in part e, where does the energy come from?

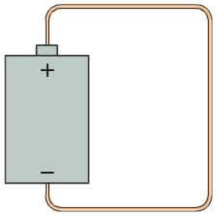


FIGURE Q22.9

10. If you change the temperature of a segment of metal wire, the dimensions change and the resistivity changes. How does each of these changes affect the resistance of the wire?
11. The wires in Figure Q22.11 are all made of the same material; the length and radius of each wire is noted. Rank in order, from largest to smallest, the resistances R_1 to R_5 of these wires. Explain.

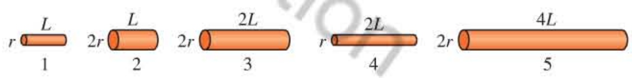


FIGURE Q22.11

12. The two circuits in Figure Q22.12 use identical batteries and wires of equal diameters. Rank in order, from largest to smallest, the currents I_1, I_2, I_3, and I_4 at points 1 to 4.

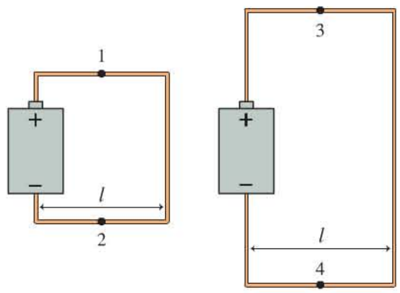


FIGURE Q22.12

13. The two circuits in Figure Q22.13 use identical batteries and wires of equal diameters. Rank in order, from largest to smallest, the currents I_1 to I_7 at points 1 to 7. Explain.

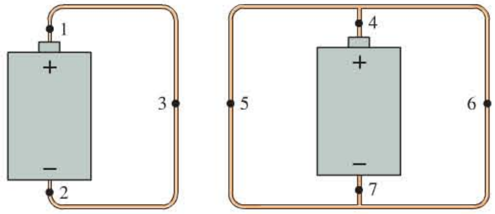


FIGURE Q22.13

14. Which, if any, of these statements are true? (More than one may be true.) Explain your choice or choices.
    a. A battery supplies energy to a circuit.
    b. A battery is a source of potential difference. The potential difference between the terminals of the battery is always the same.
    c. A battery is a source of current. The current leaving the battery is always the same.
15. Rank in order, from largest to smallest, the currents I_1 to I_4 through the four resistors in Figure Q22.15. Explain.

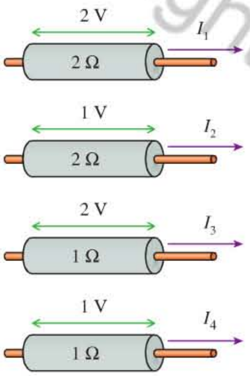


FIGURE Q22.15

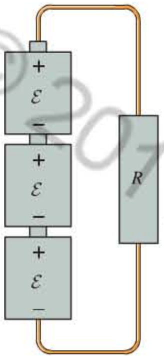


FIGURE Q22.16

16. The circuit in Figure Q22.16 has three batteries of emf $\mathcal{E}$ in series. Assuming the wires are ideal, sketch a graph of the potential as a function of distance traveled around the circuit, starting from $V = 0$ V at the negative terminal of the bottom battery. Note all important points on your graph.
17. When lightning strikes the ground, it generates a large electric field along the surface of the ground directed toward the point of the strike. People near a lightning strike are often injured not by the lightning itself but by a large current that flows up one leg and down the other due to this electric field. To minimize this possibility, you are advised to stand with your feet close together if you are trapped outside during a lightning storm. Explain why this is beneficial.
    Hint: The current path through your body, up one leg and down the other, has a certain resistance. The larger the current along this path, the greater the damage.
18. One way to find out if a wire has corroded is to measure its resistance. Explain why the resistance of a wire increases if it becomes corroded.
19. Over time, atoms "boil off" the hot filament in an incandescent bulb and the filament becomes thinner. How does this affect the brightness of the lightbulb?
20. Rank in order, from largest to smallest, the powers P_1 to P_4 dissipated by the four resistors in Figure Q22.20.

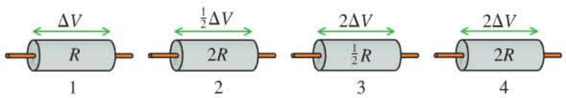


FIGURE Q22.20

21. We can model the rear window defroster in a car as a resistor that is connected to the car's 12 V battery. The defroster is made of a material whose resistance increases rapidly as the temperature increases. When the defroster is cold, its resistance is low; when the defroster is warm, its resistance is high. Why is it better to make a defroster with a material like this than with a material whose resistance is independent of temperature? Think about how the resistance, the current, and the power will change as the window warms.

Multiple-Choice Questions

22. | Lightbulbs are typically rated by their power dissipation when operated at a given voltage. Which of the following lightbulbs has the largest current through it when operated at the voltage for which it's rated?
    A. 0.8 W, 1.5 V B. 6 W, 3 V
    C. 4 W, 4.5 V D. 8 W, 6 V
23. || Lightbulbs are typically rated by their power dissipation when operated at a given voltage. Which of the following lightbulbs has the largest resistance when operated at the voltage for which it's rated?
    A. 0.8 W, 1.5 V B. 6 W, 3 V
    C. 4 W, 4.5 V D. 8 W, 6 V
24. | A copper wire is stretched so that its length increases and its diameter decreases. As a result,
    A. The wire's resistance decreases, but its resistivity stays the same.
    B. The wire's resistivity decreases, but its resistance stays the same.
    C. The wire's resistance increases, but its resistivity stays the same.
    D. The wire's resistivity increases, but its resistance stays the same.
25. | The potential difference across a length of wire is increased. Which of the following does *not* increase as well?
    A. The electric field in the wire
    B. The power dissipated in the wire
    C. The resistance of the wire
    D. The current in the wire
26. ||| A stereo amplifier creates a 5.0 V potential difference across a speaker. To double the power output of the speaker, the amplifier's potential difference must be increased to
    A. 7.1 V B. 10 V C. 14 V D. 25 V
27. | If a 1.5 V battery stores 5.0 kJ of energy (a reasonable value for an inexpensive C cell), for how many minutes could it sustain a current of 1.2 A?
    A. 2.7 B. 6.9 C. 9.0 D. 46
28. | Figure Q22.28 shows a side view of a wire of varying circular cross section. Rank in order the currents flowing in the three sections.

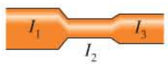


FIGURE Q22.28

    A. $I_1 > I_2 > I_3$
    B. $I_2 > I_3 > I_1$
    C. $I_1 = I_2 = I_3$
    D. $I_1 > I_3 > I_2$
29. ||| A person gains weight by adding fat—and therefore adding BIO girth—to his body and his limbs, with the amount of muscle remaining constant. How will this affect the electrical resistance of his limbs?
    A. The resistance will increase.
    B. The resistance will stay the same.
    C. The resistance will decrease.

PROBLEMS

Section 22.1 A Model of Current

Section 22.2 Defining and Describing Current

1. ||| The current in an electric hair dryer is 10 A. How much charge and how many electrons flow through the hair dryer in 5.0 min?
2. || 2.0×10^{13} electrons flow through a transistor in 1.0 ms. What is the current through the transistor?
3. | A wire carries a 1.0 A current for 30 s. How many electrons move past a point in the wire?
4. | BIO When a nerve cell depolarizes, charge is transferred across the cell membrane, changing the potential difference. For a typical nerve cell, 9.0 pC of charge flows in a time of 0.50 ms. What is the average current?
5. | A wire carries a 15 μA current. How many electrons pass a given point on the wire in 1.0 s?
6. || In a typical lightning strike, 2.5 C flows from cloud to ground in 0.20 ms. What is the current during the strike?
7. || A capacitor is charged to 6.0×10^{-4} C, then discharged by connecting a wire between the two plates. 40 μs after the discharge begins, the capacitor still holds 13% of its original charge. What was the average current during the first 40 μs of the discharge?
8. || In an ionic solution, 5.0×10^{15} positive ions with charge $+2e$ pass to the right each second while 6.0×10^{15} negative ions with charge $-e$ pass to the left. What are the magnitude and direction of current in the solution?
9. || The starter motor of a car engine draws a current of 150 A from the battery. The copper wire to the motor is 5.0 mm in diameter and 1.2 m long. The starter motor runs for 0.80 s until the car engine starts. How much charge passes through the starter motor?
10. | A car battery is rated at 90 A·hr, meaning that it can supply a 90 A current for 1 hr before being completely discharged. If you leave your headlights on until the battery is completely dead, how much charge leaves the positive terminal of the battery?
11. || What are the values of currents I_B and I_C in Figure P22.11? The directions of the currents are as noted.

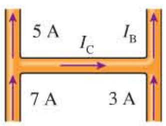


FIGURE P22.11

12. | The currents through several segments of a wire object are shown in Figure P22.12. What are the magnitudes and directions of the currents I_B and I_C in segments B and C?

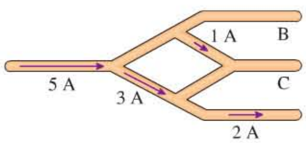


FIGURE P22.12

Section 22.3 Batteries and emf

13. || A battery supplies a steady 1.5 A current to a circuit. If the charges moving in the battery are positive ions with charge e, how many ions per second are transported from the negative terminal to the positive terminal?
14. | How much work is done to move 1.0 μC of charge from the negative terminal to the positive terminal of a 1.5 V battery?
15. | What is the emf of a battery that does 0.60 J of work to transfer 0.050 C of charge from the negative to the positive terminal?
16. || A 9.0 V battery supplies a 2.5 mA current to a circuit for 5.0 hr.
    a. How much charge has been transferred from the negative to the positive terminal?
    b. How much work has been done on the charges that passed through the battery?
17. | An individual hydrogen-oxygen fuel cell has an output of 0.75 V. How many cells must be connected in series to drive a 24.0 V motor?
18. | BIO An electric catfish can generate a significant potential difference using stacks of special cells called *electrocytes*. Each electrocyte develops a potential difference of 110 mV. How many cells must be connected in series to give the 350 V a large catfish can produce?

Section 22.4 Connecting Potential and Current

19. | A wire with resistance R is connected to the terminals of a 6.0 V battery. What is the potential difference ΔV_{ends} between the ends of the wire and the current I through it if the wire has the following resistances? (a) 1.0 Ω (b) 2.0 Ω (c) 3.0 Ω.
20. | Wires 1 and 2 are made of the same metal. Wire 2 has twice the length and twice the diameter of wire 1. What are the ratios (a) ρ_2/ρ_1 of the resistivities and (b) R_2/R_1 of the resistances of the two wires?
21. ||| A wire has a resistance of 0.010 Ω. What will the wire's resistance be if it is stretched to twice its original length without changing the volume of the wire?
22. ||| BIO Resistivity measurements on the leaves of corn plants are a good way to assess stress and overall health. The leaf of a corn plant has a resistance of 2.0 MΩ measured between two electrodes placed 20 cm apart along the leaf. The leaf has a width of 2.5 cm and is 0.20 mm thick. What is the resistivity of the leaf tissue? Is this greater than or less than the resistivity of muscle tissue in the human body?
23. || What is the resistance of
    a. A 1.0-m-long copper wire that is 0.50 mm in diameter?
    b. A 10-cm-long piece of iron with a 1.0 mm $\times$ 1.0 mm square cross section?
24. ||| A motorcyclist is making an electric vest that, when connected to the motorcycle's 12 V battery, will warm her on cold rides. She is using 0.25-mm-diameter copper wire, and she wants a current of 4.0 A in the wire. What length wire must she use?
25. || BIO The femoral artery is the large artery that carries blood to the leg. A person's femoral artery has an inner diameter of 1.0 cm. What is the resistance of a 20-cm-long column of blood in this artery?
26. || A 3.0 V potential difference is applied between the ends of a 0.80-mm-diameter, 50-cm-long nichrome wire. What is the current in the wire?
27. ||| A 1.0-mm-diameter, 20-cm-long copper wire carries a 3.0 A current. What is the potential difference between the ends of the wire?

28. |||| BIO The relatively high resistivity of dry skin, about $1 \times 10^6\ \Omega \cdot \text{m}$, can safely limit the flow of current into deeper tissues of the body. Suppose an electrical worker places his palm on an instrument whose metal case is accidently connected to a high voltage. The skin of the palm is about 1.5 mm thick. Estimate the area of skin on the worker's palm that would contact a flat panel, then calculate the approximate resistance of the skin of the palm.

29. | a. How long must a 0.60-mm-diameter copper wire be to carry a 0.50 A current when connected to the terminals of a 1.5 V flashlight battery?
b. What is the current if the wire is half this length?

Section 22.5 Ohm's Law and Resistor Circuits

30. | Figure P22.30 shows the current-versus-potential-difference graph for a resistor.
a. What is the resistance of this resistor?
b. Suppose the length of the resistor is doubled while keeping its cross section the same. (This requires doubling the amount of material the resistor is made of.) Copy the figure and add to it the current-versus-potential-difference graph for the longer resistor.

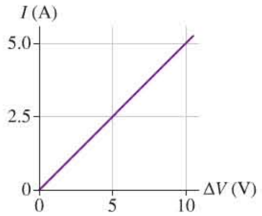


FIGURE P22.30

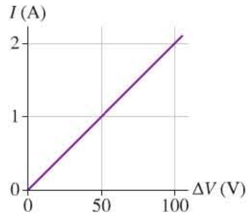


FIGURE P22.31

31. | Figure P22.31 is a current-versus-potential-difference graph for a cylinder. What is the cylinder's resistance?

32. | In Example 22.6 the length of a 60 W, 240 Ω lightbulb filament was calculated to be 60 cm.
a. If the potential difference across the filament is 120 V, what is the strength of the electric field inside the filament?
b. Suppose the length of the bulb's filament were doubled without changing its diameter or the potential difference across it. What would the electric field strength be in this case?
c. Remembering that the current in the filament is proportional to the electric field, what is the current in the filament following the doubling of its length?
d. What is the resistance of the filament following the doubling of its length?

33. | The electric field inside a 30-cm-long copper wire is 0.010 V/m. What is the potential difference between the ends of the wire?

34. || A small electric lap blanket contains a 40-foot-long wire wrapped back and forth inside. An 18 V supply creates a current in this wire, warming it and thus the blanket. What is the electric field strength inside this wire?

35. ||| Two identical lightbulbs are connected in series to a single 9.0 V battery.
a. Sketch the circuit.
b. Sketch a graph showing the potential as a function of distance through the circuit, starting with $V = 0\,\text{V}$ at the negative terminal of the battery.

Section 22.6 Energy and Power

36. | a. What is the resistance of a 1500 W (120 V) hair dryer?
b. What is the current in the hair dryer when it is used?

37. | You've brought your 1000 W (120 V) hair dryer on vacation to Europe, where the standard outlet voltages are 230 V. Assuming the hair dryer can operate safely at the higher voltage, can you actually use it if the outlet can provide at most 15 A, or will it draw more current than this?

38. || A 70 W electric blanket runs at 18 V.
a. What is the resistance of the wire in the blanket?
b. How much current does the wire carry?

39. | A 60-cm-long heating wire is connected to a 120 V outlet. If the wire dissipates 45 W, what are (a) the current in and (b) the resistance of the wire?

40. ||| BIO An electric eel develops a potential difference of 450 V, driving a current of 0.80 A for a 1.0 ms pulse. For this pulse, find (a) the power, (b) the total energy, and (c) the total charge that flows.

41. ||||| The total charge a household battery can supply is given in units of mA · hr. For example, a 9.0 V alkaline battery is rated 450 mA · hr, meaning that such a battery could supply a 1 mA current for 450 hr, a 2 mA current for 225 hr, etc. How much energy, in joules, is this battery capable of supplying?

General Problems

42. | A 3.0 V battery powers a flashlight bulb that has a resistance of 6.0 Ω. How much charge moves through the battery in 10 min?

43. |||| INT A sculptor has asked you to help electroplate gold onto a brass statue. You know that the charge carriers in the ionic solution are monovalent (charge e) gold ions, and you've calculated that you must deposit 0.50 g of gold to reach the necessary thickness. How much current do you need, in mA, to plate the statue in 3.0 hr?

44. |||| INT Older freezers developed a coating of ice inside that had to be melted periodically; an electric heater could speed this defrosting process. Suppose you're melting ice from your freezer using a heating wire that carries a current of 5.0 A when connected to 120 V.
a. What is the resistance of the wire?
b. How long will it take the heater to melt 720 g of accumulated ice at –10°C? Assume that all of the heat goes into warming and melting the ice, and that the melt water runs out and doesn't warm further.

45. |||| INT For a science experiment you need to electroplate a 100-nm-thick zinc coating onto both sides of a very thin, 2.0 cm × 2.0 cm copper sheet. You know that the charge carriers in the ionic solution are divalent (charge $2e$) zinc ions. The density of zinc is 7140 kg/m^3. If the electroplating apparatus operates at 1.0 mA, how long will it take the zinc to reach the desired thickness?

46. ||| INT The hot dog cooker described in the chapter heats hot dogs by connecting them to 120 V household electricity. A typical hot dog has a mass of 60 g and a resistance of 150 Ω. How long will it take for the cooker to raise the temperature of the hot dog from 20°C to 80°C? The specific heat of a hot dog is approximately 2500 J/kg · K.

47. |||| INT Air isn't a perfect electric insulator, but it has a very high resistivity. Dry air has a resistivity of approximately $3 \times 10^{13}\ \Omega \cdot \text{m}$. A capacitor has square plates 10 cm on a side

separated by 1.2 mm of dry air. If the capacitor is charged to 250 V, what fraction of the charge will flow across the air gap in 1 minute? Make the approximation that the potential difference doesn't change as the charge flows.

48. || The biochemistry that takes place inside cells depends on BIO various elements, such as sodium, potassium, and calcium, that are dissolved in water as ions. These ions enter cells through narrow pores in the cell membrane known as *ion channels*. Each ion channel, which is formed from a specialized protein molecule, is selective for one type of ion. Measurements with microelectrodes have shown that a 0.30-nm-diameter potassium ion (K^+) channel carries a current of 1.8 pA. How many potassium ions pass through if the ion channel opens for 1.0 ms?

49. || High-resolution measurements have shown that an ion channel BIO (see Problem 48) is a 0.30-nm-diameter cylinder with length of 5.0 nm. The intracellular fluid filling the ion channel has resistivity 0.60 $\Omega \cdot$ m. What is the resistance of the ion channel?

50. | When an ion channel opens in a cell wall (see Problem 48), BIO monovalent (charge e) ions flow through the channel at a rate of 1.0×10^7 ions/s.
   a. What is the current through the channel?
   b. The potential difference across the ion channel is 70 mV. What is the power dissipation in the channel?

51. ||| The total charge a battery can supply is rated in mA $\cdot$ hr, the product of the current (in mA) and the time (in hr) that the battery can provide this current. A battery rated at 1000 mA $\cdot$ hr can supply a current of 1000 mA for 1.0 hr, 500 mA current for 2.0 hr, and so on. A typical AA rechargeable battery has a voltage of 1.2 V and a rating of 1800 mA $\cdot$ hr. For how long could this battery drive current through a long, thin wire of resistance 22 Ω?

52. || The heating element of a simple heater consists of a 2.0-m-long, 0.60-mm-diameter nichrome wire. When plugged into a 120 V outlet, the heater draws 8.0 A of current when hot.
   a. What is the wire's resistance when it is hot?
   b. Use your answer to part a to calculate the resistivity of nichrome in this situation. Why is it not the same as the value of ρ given for nichrome in Table 22.1?

53. ||| Variations in the resistivity of blood can give valuable clues BIO to changes in the blood's viscosity and other properties. The resistivity is measured by applying a small potential difference and measuring the current. Suppose a medical device attaches electrodes into a 1.5-mm-diameter vein at two points 5.0 cm apart. What is the blood resistivity if a 9.0 V potential difference causes a 230 μA current through the blood in the vein?

54. |||| A 40 W (120 V) lightbulb has a tungsten filament of thickness 0.040 mm. The filament's operating temperature is 1500°C.
   a. How long is the filament?
   b. What is the resistance of the filament at 20°C?

55. || Wires aren't really ideal. The voltage drop across a current-carrying wire can be significant unless the resistance of the wire is quite low. Suppose a 50 ft extension cord is being used to provide power to an electric lawn mower. The cord carries a 10 A current. The copper wire in a typical extension cord has a 1.3 mm diameter. What is the voltage drop across a 50 ft length of wire at this current?

56. ||| When the starter motor on a car is engaged, there is a 300 A current in the wires between the battery and the motor. Suppose the wires are made of copper and have a total length of 1.0 m. What minimum diameter can the wires have if the voltage drop along the wires is to be less than 0.50 V?

57. ||| The electron beam inside a television picture tube is 0.40 mm INT in diameter and carries a current of 50 μA. This electron beam impinges on the inside of the picture tube screen.
   a. How many electrons strike the screen each second?
   b. The electrons move with a velocity of 4.0×10^7 m/s. What electric field strength is needed to accelerate electrons from rest to this velocity in a distance of 5.0 mm?
   c. Each electron transfers its kinetic energy to the picture tube screen upon impact. What is the *power* delivered to the screen by the electron beam?

   Hint: What potential difference produced the field that accelerated electrons? This is an emf.

58. | The two segments of the wire in Figure P22.58 have equal diameters and equal lengths but different resistivities ρ_1 and ρ_2. Current I passes through this wire. If the resistivities have the ratio $\rho_2/\rho_1 = 2$, what is the ratio $\Delta V_1/\Delta V_2$ of the potential differences across the two segments of the wire?

FIGURE P22.58

59. || A 15-cm-long nichrome wire is connected between the terminals of a 1.5 V battery. If the current in the wire is 2.0 A, what is the wire's diameter?

60. || A wire is 2.3 m long and has a diameter of 0.38 mm. When connected to a 1.2 V battery, there is a current of 0.61 A. What material is the wire likely made of?

61. | The filament of a 100 W (120 V) lightbulb is a tungsten wire 0.035 mm in diameter. At the filament's operating temperature, the resistivity is 5.0×10^{-7} $\Omega \cdot$ m. How long is the filament?

62. ||| You've made the finals of the Science Olympics! As one of INT your tasks, you're given 1.0 g of copper and asked to make a wire, using all the metal, with a resistance of 1.0 Ω. Copper has a density of 8900 kg/m^3. What length and diameter will you choose for your wire?

63. || Not too long ago houses were protected from excessive currents by fuses rather than circuit breakers. Sometimes a fuse blew out and a replacement wasn't at hand. Because a copper penny happens to have almost the same diameter as a fuse, some people replaced the fuse with a penny. Unfortunately, a penny never blows out, no matter how large the current, and the use of pennies in fuse boxes caused many house fires. Make the appropriate measurements on a penny, then calculate the resistance between the two faces of a solid-copper penny. (Modern pennies have the same dimensions, but are made of zinc with a copper coating.)

64. || An immersion heater used to boil water for a single cup of tea INT plugs into a 120 V outlet and is rated at 300 W.
   a. What is the resistance of the heater?
   b. Suppose your super-size, super-insulated tea mug contains 400 g of water at a temperature of 18°C. How long will this heater take to bring the water to a boil? You can ignore the energy needed to raise the temperature of the mug and the heater itself.

65. |||| The graph in Figure P22.65 shows the current through a 1.0 Ω resistor as a function of time.
   a. How much charge flowed through the resistor during the 10 s interval shown?
   b. What was the total energy dissipated by the resistor during this time?

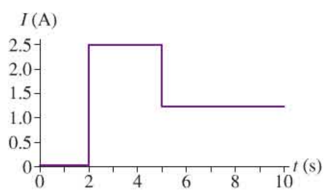


FIGURE P22.65

66. ||| It's possible to estimate the percentage of fat in the body by
BIO measuring the resistance of the upper leg rather than the upper arm; the calculation is similar. A person's leg measures 40 cm between the knee and the hip, with an average leg diameter (ignoring bone and other poorly conducting tissue) of 12 cm. A potential difference of 0.75 V causes a current of 1.6 mA. What are the fractions of (a) muscle and (b) fat in the leg?

67. | If you touch the two terminals of a power supply with your
BIO two fingertips on opposite hands, the potential difference will produce a current through your torso. The maximum safe current is approximately 5 mA.
   a. If your hands are completely dry, the resistance of your body from fingertip to fingertip is approximately 500 kΩ. If you accidentally touch both terminals of your 120 V household electricity supply with dry fingers, will you receive a dangerous shock?
   b. If your hands are moist, your resistance drops to approximately 1 kΩ. If you accidentally touch both terminals of your 120 V household supply with moist fingers, will you receive a dangerous shock?

68. | The average resistivity of the human body (apart from sur-
BIO face resistance of the skin) is about 5.0 $\Omega \cdot$m. The conducting path between the right and left hands can be approximated as a cylinder 1.6 m long and 0.10 m in diameter. The skin resistance can be made negligible by soaking the hands in salt water.
   a. What is the resistance between the hands if the skin resistance is negligible?
   b. If skin resistance is negligible, what potential difference between the hands is needed for a lethal shock current of 100 mA? Your result shows that even small potential differences can produce dangerous currents when skin is damp.

Passage Problems

Lightbulb Failure

You've probably observed that the most common time for an incandescent lightbulb to fail is the moment when it is turned on. Let's look at the properties of the bulb's filament to see why this happens.

The current in the tungsten filament of a lightbulb heats the filament until it glows. The filament is so hot that some of the atoms on its surface fly off and end up sticking on a cooler part of the bulb. Thus the filament gets progressively thinner as the bulb ages. There will certainly be one spot on the filament that is a bit thinner than elsewhere. This thin segment will have a higher resistance than the surrounding filament. More power will be dissipated at this spot, so it won't only be a thin spot, it also will be a hot spot.

Now, let's look at the resistance of the filament. The graph in Figure P22.69 shows data for the current in a lightbulb as a function of the potential difference across it. The graph is not linear, so the filament is not an ohmic material with a constant resistance. However, we can define the resistance at any particular potential difference ΔV to be $R = \Delta V/I$. This ratio, and hence the resistance, increases with ΔV and thus with temperature.

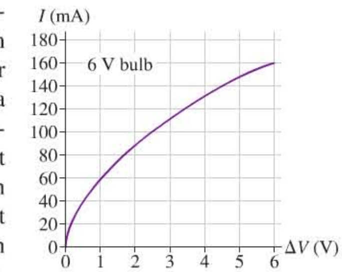


FIGURE P22.69

When the bulb is turned on, the filament is cold and its resistance is much lower than during normal, high-temperature operation. The low resistance causes a surge of higher-than-normal current lasting a fraction of a second until the filament heats up. Because power dissipation is I^2R, the power dissipated during this first fraction of a second is much larger than the bulb's rated power. This current surge concentrates the power dissipation at the high-resistance thin spot, perhaps melting it and breaking the filament.

69. | For the bulb in Figure P22.69, what is the approximate resistance of the bulb at a potential difference of 6.0 V?
   A. 7.0 Ω B. 17 Ω C. 27 Ω D. 37 Ω

70. | As the bulb ages, the resistance of the filament
   A. Increases. B. Decreases.
   C. Stays the same.

71. | Which of the curves in Figure P22.71 best represents the expected variation in current as a function of time in the short time interval immediately after the bulb is turned on?

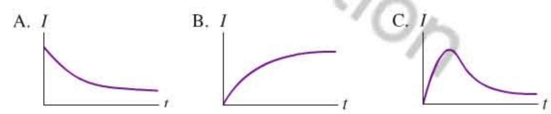


FIGURE P22.71

72. | There are devices to put in a light socket that control the current through a lightbulb, thereby increasing its lifetime. Which of the following strategies would increase the lifetime of a bulb without making it dimmer?
   A. Reducing the average current through the bulb
   B. Limiting the maximum current through the bulb
   C. Increasing the average current through the bulb
   D. Limiting the minimum current through the bulb

STOP TO THINK ANSWERS

Stop to Think 22.1: A. From Kirchhoff's junction law the current through bulb 1 is the sum of the currents through bulbs 2 and 3. Bulb 1 carries a larger current than bulb 2, so it will be brighter.

Stop to Think 22.2: C. Charge flows out of one terminal of the battery but back into the other; the amount of charge in the battery does not change. The emf is determined by the chemical reactions in the battery and is constant. But the chemical energy in the battery steadily decreases as the battery converts it to the potential energy of charges.

Stop to Think 22.3: C. Stretching the wire decreases the area and increases the length. Both of these changes increase the resistance of the wire. When the wire is reconnected to the battery, the resistance is greater but the potential difference is the same as in the original case, so the current will be smaller.

Stop to Think 22.4: $V_B = V_C > V_A > V_D = V_E$. There's no potential difference along ideal wires, so $V_B = V_C$ and $V_D = V_E$. Potential increases in going from the − to the + terminal of a battery, so $V_A > V_E$ and $V_B > V_A$. These imply $V_C > V_D$, which was expected because potential decreases as current passes through a resistor.

Stop to Think 22.5: $P_B > P_D > P_A > P_C$. The power dissipated by a resistor is $P_R = (\Delta V_R)^2/R$. Increasing R decreases P_R; increasing ΔV_R increases P_R. But changing the potential has a larger effect because P_R depends on the square of ΔV_R.