5 Applying Newton's Laws

Why does this sky surfer fall at a constant speed? And why does he suddenly slow down when his parachute opens?

LOOKING AHEAD

The goal of Chapter 5 is to learn how to solve problems about motion along a straight line.

Equilibrium Problems

An object at rest or moving at a constant velocity has zero acceleration. According to Newton's second law, this means that the net force on it must also be zero.

This boulder is in *static equilibrium:* It remains at rest.

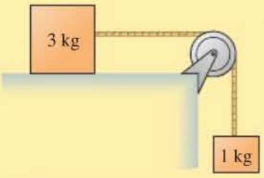
A ski lift, moving at a constant velocity, is in *dynamic equilibrium*.

Looking Back
4.4, 4.7 Identifying forces, free-body diagrams
4.6 Newton's second law

Applying Newton's Second Law

In this chapter, you'll learn how to use Newton's second law in component form to solve a variety of problems in mechanics.

What are the blocks' accelerations?



Friction, tension, gravity, and the pulley all act in this problem. You'll learn explicit strategies to solve problems like this.

Looking Back
2.5, 2.7 Constant acceleration and free fall
3.2–3.3 Vectors and components

Interacting Objects

When two objects interact with each other, each exerts a force on the other. By Newton's third law, these forces are equal in magnitude but oppositely directed.

The barge pushes back on the tug just as hard as the tug pushes on the barge. Newton's third law will help you understand the motion of objects that are in contact

A common way for two objects to interact is with ropes or strings under tension. You'll learn how to solve problems involving tension and how pulleys act to change the direction of the tension force.

Forces and Newton's Second Law

Chapter 4 introduced several important forces. Now you'll need to understand these forces in more detail, so that you can use them in Newton's second law. For example, you'll learn . . .

. . . that **mass and weight are not the same thing.** However, you'll find that there is a simple relationship between the two.

. . . a simple model for friction that provides a reasonably accurate description of how static and kinetic friction behave.

This astronaut on the moon weighs only 1/6 of what he does on earth. His *mass*, however, is the same on both the moon and the earth.

Static friction adjusts its magnitude as needed to keep the sofa from slipping.

Kinetic (sliding) friction does not depend on an object's speed.

This human tower is in equilibrium because the net force on each man is zero.

5.1 Equilibrium

Chapter 4 introduced Newton's three laws of motion. Now, in Chapter 5, we want to use these laws to solve force and motion problems. This chapter focuses on objects that are at rest or that move in a straight line, such as runners, bicycles, cars, planes, and rockets. Circular motion and rotational motion will be treated in Chapters 6 and 7.

The simplest applications of Newton's second law, $\vec{F}_{\text{net}} = m\vec{a}$, are those for which the acceleration $\vec{a}$ is *zero*. In such cases, the net force acting on the object must be zero as well. One way an object can have $\vec{a} = \vec{0}$ is to be at rest. An object that remains at rest is said to be in **static equilibrium.** A second way for an object to have $\vec{a} = \vec{0}$ is to move in a straight line at a constant speed. Such an object is in **dynamic equilibrium.** The key property of both these cases of **equilibrium** is that the net force acting on the object is $\vec{F}_{\text{net}} = \vec{0}$.

To use Newton's laws, we have to identify all the forces acting on an object and then evaluate $\vec{F}_{\text{net}}$. Recall that $\vec{F}_{\text{net}}$ is the vector sum

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

where $\vec{F}_1$, $\vec{F}_2$, and so on are the individual forces, such as tension or friction, acting on the object. We found in Chapter 3 that vector sums can be evaluated in terms of the x- and y-components of the vectors; that is, the x-component of the net force is $(F_{\text{net}})_x = F_{1x} + F_{2x} + F_{3x} + \cdots$. If we restrict ourselves to problems where all the forces are in the xy-plane, then the equilibrium requirement $\vec{F}_{\text{net}} = \vec{a} = \vec{0}$ is a shorthand way of writing two simultaneous equations:

$$(F_{\text{net}})_x = F_{1x} + F_{2x} + F_{3x} + \cdots = 0$$
$$(F_{\text{net}})_y = F_{1y} + F_{2y} + F_{3y} + \cdots = 0$$

Recall from your math classes that the Greek letter Σ (sigma) stands for "the sum of." It will be convenient to abbreviate the sum of the x-components of all forces as

$$F_{1x} + F_{2x} + F_{3x} + \cdots = \sum F_x$$

With this notation, Newton's second law for an object in equilibrium, with $\vec{a} = \vec{0}$, can be written as the two equations

$$\sum F_x = ma_x = 0 \quad \text{and} \quad \sum F_y = ma_y = 0 \tag{5.1}$$

In equilibrium, the sums of the x- and y-components of the force are zero

Although this may look a bit forbidding, we'll soon see how to use a free-body diagram of the forces to help evaluate these sums.

When an object is in equilibrium, we are usually interested in finding the forces that keep it in equilibrium. Newton's second law is the basis for a strategy for solving equilibrium problems.

PROBLEM-SOLVING STRATEGY 5.1 **Equilibrium problems** MP™

PREPARE First check that the object is in equilibrium: Does $\vec{a} = \vec{0}$?

- An object at rest is in static equilibrium.
- An object moving at a constant velocity is in dynamic equilibrium.

Then identify all forces acting on the object and show them on a free-body diagram. Determine which forces you know and which you need to solve for.

Continued

SOLVE An object in equilibrium must satisfy Newton's second law for the case where $\vec{a} = \vec{0}$. In component form, the requirement is

$$\sum F_x = ma_x = 0 \qquad \text{and} \qquad \sum F_y = ma_y = 0$$

You can find the force components that go into these sums directly from your free-body diagram. From these two equations, solve for the unknown forces in the problem.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

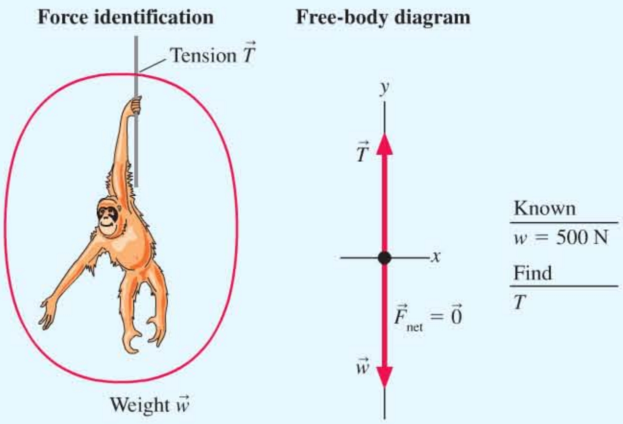
Static Equilibrium

EXAMPLE 5.1 Forces supporting an orangutan

An orangutan weighing 500 N hangs from a vertical vine. What is the tension in the vine?

PREPARE The orangutan is at rest, so it is in static equilibrium. The net force on it must then be zero. **FIGURE 5.1** first identifies the forces acting on the orangutan: the upward force of the tension in the vine and the downward, long-range force of gravity. These forces are then shown on a free-body diagram, where it's noted that equilibrium requires $\vec{F}_{net} = \vec{0}$.

FIGURE 5.1 The forces on an orangutan.



SOLVE Neither force has an x-component, so we need to examine only the y-components of the forces. In this case, the y-component of Newton's second law is

$$\sum F_y = T_y + w_y = ma_y = 0$$

You might have been tempted to write $T_y - w_y$ because the weight force points down. But remember that T_y and w_y are *components* of vectors, and can thus be positive (for a vector such as $\vec{T}$ that points up) or negative (for a vector such as $\vec{w}$ that points down). The fact that $\vec{w}$ points down is taken into account when we *evaluate* the components—that is, when we write them in terms of the *magnitudes* T and w of the vectors $\vec{T}$ and $\vec{w}$.

Because the tension vector $\vec{T}$ points straight up, in the positive y-direction, its y-component is $T_y = T$. Because the weight vector $\vec{w}$ points straight down, in the negative y-direction, its y-component is $w_y = -w$. This is where the signs enter. With these components, Newton's second law becomes

$$T - w = 0$$

This equation is easily solved for the tension in the vine:

$$T = w = 500 \text{ N}$$

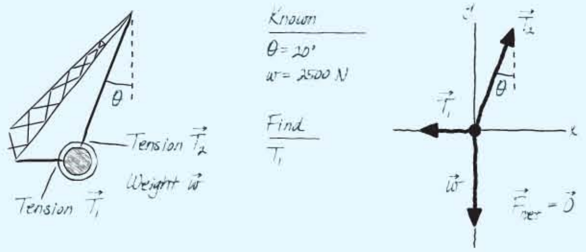
ASSESS It's not surprising that the tension in the vine equals the weight of the orangutan. However, we'll soon see that this is *not* the case if the object is accelerating.

EXAMPLE 5.2 Readying a wrecking ball

A wrecking ball weighing 2500 N hangs from a cable. Prior to swinging, it is pulled back to a 20° angle by a second, horizontal cable. What is the tension in the horizontal cable?

PREPARE Because the ball is not moving, it hangs in static equilibrium, with $\vec{a} = \vec{0}$, until it is released. In **FIGURE 5.2**, we start by identifying all the forces acting on the ball: a tension force from each cable and the ball's weight. We've used different symbols $\vec{T}_1$ and $\vec{T}_2$ for the two different tension forces. We then construct a free-body diagram for these three forces, noting that $\vec{F}_{net} = m\vec{a} = \vec{0}$. We're looking for the magnitude T_1 of the tension force $\vec{T}_1$ in the horizontal cable.

FIGURE 5.2 Visual overview of a wrecking ball just before release.



Continued

SOLVE The requirement of equilibrium is $\vec{F}_{\text{net}} = m\vec{a} = \vec{0}$. In component form, we have the two equations:

$$\sum F_x = T_{1x} + T_{2x} + w_x = ma_x = 0$$
$$\sum F_y = T_{1y} + T_{2y} + w_y = ma_y = 0$$

As always, we *add* the force components together. Now we're ready to write the components of each force vector in terms of the magnitudes and directions of those vectors. We learned how to do this in Section 3.3 of Chapter 3. With practice you'll learn to read the components directly off the free-body diagram, but to begin it's worthwhile to organize the components into a table.

Force	Name of x-component	Value of x-component	Name of y-component	Value of y-component
$\vec{T}_1$	T_{1x}	$-T_1$	T_{1y}	0
$\vec{T}_2$	T_{2x}	$T_2 \sin\theta$	T_{2y}	$T_2 \cos\theta$
$\vec{w}$	w_x	0	w_y	$-w$

We see from the free-body diagram that $\vec{T}_1$ points along the negative x-axis, so $T_{1x} = -T_1$ and $T_{1y} = 0$. We need to be careful with our trigonometry as we find the components of $\vec{T}_2$. Remembering that the side adjacent to the angle is related to the cosine, we see that the vertical (y) component of $\vec{T}_2$ is $T_2 \cos\theta$. Similarly, the horizontal (x) component is $T_2 \sin\theta$. The weight vector points straight down, so its y-component is $-w$. Notice that negative signs enter as we evaluate the components of the vectors, *not* when we write Newton's second law. This is a critical aspect of solving force and motion problems. With these components, Newton's second law now becomes

$$-T_1 + T_2 \sin\theta + 0 = 0 \quad \text{and} \quad 0 + T_2 \cos\theta - w = 0$$

We can rewrite these equations as

$$T_2 \sin\theta = T_1 \quad \text{and} \quad T_2 \cos\theta = w$$

These are two simultaneous equations with two unknowns: T_1 and T_2. To eliminate T_2 from the two equations, we solve the second equation for T_2, giving $T_2 = w/\cos\theta$. Then we insert this expression for T_2 into the first equation to get

$$T_1 = \frac{w}{\cos\theta} \sin\theta = \frac{\sin\theta}{\cos\theta} w = w \tan\theta = (2500\ \text{N}) \tan 20° = 910\ \text{N}$$

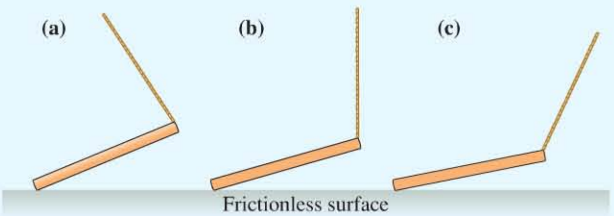
where we made use of the fact that $\tan\theta = \sin\theta / \cos\theta$.

ASSESS It seems reasonable that to pull the ball back to this modest angle, a force substantially less than the ball's weight will be required.

CONCEPTUAL EXAMPLE 5.3 Forces in static equilibrium

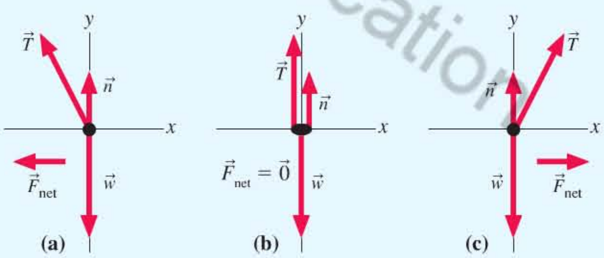
A rod is free to slide on a frictionless sheet of ice. One end of the rod is lifted by a string. If the rod is at rest, which diagram in FIGURE 5.3 shows the correct angle of the string?

FIGURE 5.3 Which is the correct angle of the string?



REASON If the rod is to hang motionless, it must be in static equilibrium with $\sum F_x = ma_x = 0$ and $\sum F_y = ma_y = 0$. FIGURE 5.4 shows free-body diagrams for the three string orientations. Remember that tension always acts along the direction of the string and that the weight force always points straight down. The ice pushes up with a normal force perpendicular to the surface, but frictionless ice cannot exert any horizontal force. If the string is angled, we see that its horizontal component exerts a net force on the rod. Only in case b, where the tension and the string are vertical, can the net force be zero.

FIGURE 5.4 Free-body diagrams for three angles of the string.



ASSESS If friction were present, the rod could in fact hang as in cases a or c. But without friction, the rods in these cases would slide until they came to rest as in case b.

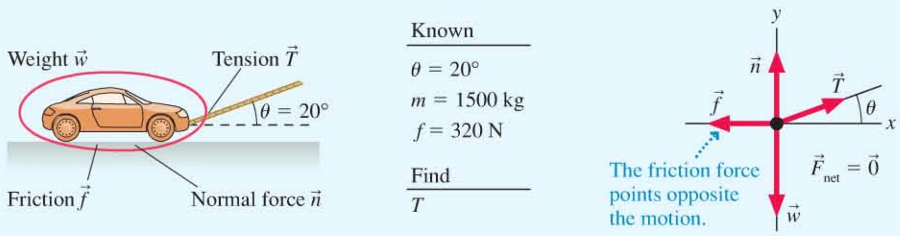
Dynamic Equilibrium

EXAMPLE 5.4 Tension in towing a car

A car with a mass of 1500 kg is being towed at a steady speed by a rope held at a 20° angle. A friction force of 320 N opposes the car's motion. What is the tension in the rope?

PREPARE The car is moving in a straight line at a constant speed ($\vec{a} = \vec{0}$) so it is in dynamic equilibrium and must have $\vec{F}_{\text{net}} = m\vec{a} = \vec{0}$. FIGURE 5.5 shows three contact forces acting on the car—the tension force $\vec{T}$, friction $\vec{f}$, and the normal force $\vec{n}$—and the long-range force of gravity $\vec{w}$. These four forces are shown on the free-body diagram.

FIGURE 5.5 Visual overview of a car being towed.



SOLVE This is still an equilibrium problem, even though the car is moving, so our problem-solving procedure is unchanged. With four forces, the requirement of equilibrium is

$$\sum F_x = n_x + T_x + f_x + w_x = ma_x = 0$$
$$\sum F_y = n_y + T_y + f_y + w_y = ma_y = 0$$

We can again determine the horizontal and vertical components of the forces by "reading" the free-body diagram. The results are shown in the table.

Force	Name of x-component	Value of x-component	Name of y-component	Value of y-component
$\vec{n}$	n_x	0	n_y	n
$\vec{T}$	T_x	$T\cos\theta$	T_y	$T\sin\theta$
$\vec{f}$	f_x	$-f$	f_y	0
$\vec{w}$	w_x	0	w_y	$-w$

With these components, Newton's second law becomes

$$T\cos\theta - f = 0$$
$$n + T\sin\theta - w = 0$$

The first equation can be used to solve for the tension in the rope:

$$T = \frac{f}{\cos\theta} = \frac{320\text{ N}}{\cos 20°} = 340\text{ N}$$

to two significant figures. It turned out that we did not need the y-component equation in this problem. We would need it if we wanted to find the normal force $\vec{n}$.

ASSESS Had we pulled the car with a horizontal rope, the tension would need to exactly balance the friction force of 320 N. Because we are pulling at an angle, however, part of the tension in the rope pulls *up* on the car instead of in the forward direction. Thus we need a little more tension in the rope when it's at an angle.

5.2 Dynamics and Newton's Second Law

ActivPhysics Online 2.1–2.4

Newton's second law is the essential link between force and motion. The essence of Newtonian mechanics can be expressed in two steps:

- The forces acting on an object determine its acceleration $\vec{a} = \vec{F}_{net}/m$.
- The object's motion can be found by using $\vec{a}$ in the equations of kinematics.

We want to develop a strategy to solve a variety of problems in mechanics, but first we need to write the second law in terms of its components. To do so, let's first rewrite Newton's second law in the form

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = m\vec{a}$$

where $\vec{F}_1, \vec{F}_2, \vec{F}_3$, and so on are the forces acting on an object. To write the second law in component form merely requires that we use the x- and y-components of the acceleration. Thus Newton's second law, $\vec{F}_{net} = m\vec{a}$, is

$$\sum F_x = ma_x \quad \text{and} \quad \sum F_y = ma_y \tag{5.2}$$

Newton's second law in component form

The first equation says that **the component of the acceleration in the x-direction is determined by the sum of the x-components of the forces acting on the object.** A similar statement applies to the y-direction.

There are two basic types of problems in mechanics. In the first, you use information about forces to find an object's acceleration, then use kinematics to determine the object's motion. In the second, you use information about the object's motion to determine its acceleration, then solve for unknown forces. Either way, the two equations of Equation 5.2 are the link between force and motion, and they form the basis of a problem-solving strategy. The primary goal of this chapter is to illustrate the use of this strategy.

PROBLEM-SOLVING STRATEGY 5.2 **Dynamics problems**

PREPARE Sketch a visual overview consisting of:

- A list of values that identifies known quantities and what the problem is trying to find.
- A force identification diagram to help you identify all the forces acting on the object.
- A free-body diagram that shows all the forces acting on the object.

If you'll need to use kinematics to find velocities or positions, you'll also need to sketch:

- A motion diagram to determine the direction of the acceleration.
- A pictorial representation that establishes a coordinate system, shows important points in the motion, and defines symbols.

It's OK to go back and forth between these steps as you visualize the situation.

SOLVE Write Newton's second law in component form as

$$\sum F_x = ma_x \quad \text{and} \quad \sum F_y = ma_y$$

You can find the components of the forces directly from your free-body diagram. Depending on the problem, either:

- Solve for the acceleration, then use kinematics to find velocities and positions.
- Use kinematics to determine the acceleration, then solve for unknown forces.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

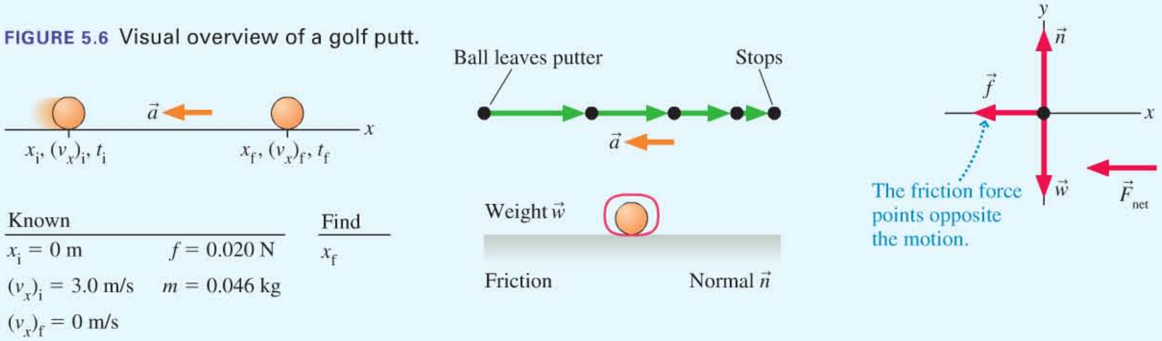
Exercise 24

EXAMPLE 5.5 **Putting a golf ball**

A golfer putts a 46 g ball with a speed of 3.0 m/s. Friction exerts a 0.020 N retarding force on the ball, slowing it down. Will her putt reach the hole, 10 m away?

PREPARE FIGURE 5.6 is a visual overview of the problem. We've collected the known information, drawn a sketch, and identified what we want to find. The motion diagram shows that the ball is slowing down as it rolls to the right, so the acceleration vector points to the left. Next, we identify the forces acting on the ball and show them on a free-body diagram. Note that the net force points to the left, as it must because the acceleration points to the left.

FIGURE 5.6 Visual overview of a golf putt.



SOLVE Newton's second law in component form is

$$\sum F_x = n_x + f_x + w_x = 0 - f + 0 = ma_x$$
$$\sum F_y = n_y + f_y + w_y = n + 0 - w = ma_y = 0$$

We've written the equations as sums, as we did with equilibrium problems, then "read" the values of the force components from the free-body diagram. The components are simple enough in this problem that we don't really need to show them in a table. It is particularly important to notice that we set $a_y = 0$ in the second equation. This is because the ball does not move in the y-direction, so it can't have any acceleration in the y-direction. This will be an important step in many problems.

The first equation is $-f = ma_x$, from which we find

$$a_x = -\frac{f}{m} = \frac{-(0.020\ \text{N})}{0.046\ \text{kg}} = -0.43\ \text{m/s}^2$$

The negative sign shows that the acceleration is directed to the left, as expected.

Now that we know the acceleration, we can use kinematics to find how far the ball will roll before stopping. We don't have any information about the time it takes for the ball to stop, so we'll use the kinematic equation $(v_x)_f^2 = (v_x)_i^2 + 2a_x(x_f - x_i)$. This gives

$$x_f = x_i + \frac{(v_x)_f^2 - (v_x)_i^2}{2a_x} = 0\ \text{m} + \frac{(0\ \text{m/s})^2 - (3.0\ \text{m/s})^2}{2(-0.43\ \text{m/s}^2)} = 10.5\ \text{m}$$

If her aim is true, the ball will just make it into the hole.

ASSESS It seems reasonable that a ball putted on grass with an initial speed of 3 m/s—about jogging speed—would travel roughly 10 m.

EXAMPLE 5.6 Finding a rocket cruiser's acceleration

A rocket cruiser with a mass of 2200 kg and weighing 5000 N is flying horizontally over the surface of a distant planet. At its present speed, a 3000 N drag force acts on the cruiser. The cruiser's engines can be tilted so as to provide a thrust angled up or down. The pilot turns the thrust up to 14,000 N while pivoting the engines to continue flying horizontally. What is the cruiser's acceleration?

PREPARE FIGURE 5.7 is a visual overview in which we've listed the known information, identified the forces on the cruiser, and drawn a free-body diagram. (Because kinematics is not needed to find the acceleration, we don't need a pictorial diagram.) As discussed in Chapter 4, the thrust force points *opposite* the direction of the rocket exhaust, which we've shown at angle θ. The thrust must have an upward vertical component to balance the weight force; otherwise, the cruiser would fall. To continue flying horizontally requires the net force to be directed forward.

SOLVE Newton's second law in component form is

$$\sum F_x = (F_{\text{thrust}})_x + D_x + w_x = ma_x$$
$$\sum F_y = (F_{\text{thrust}})_y + D_y + w_y = ma_y$$

From the free-body diagram, we see that $(F_{\text{thrust}})_x = F_{\text{thrust}}\cos\theta$, $(F_{\text{thrust}})_y = F_{\text{thrust}}\sin\theta$, $D_x = -D$, $D_y = 0$, $w_x = 0$, and $w_y = -w$. We know that a_y must be zero because the cruiser is to accelerate *horizontally*. Thus the second law becomes

$$F_{\text{thrust}}\cos\theta - D = ma_x$$
$$F_{\text{thrust}}\sin\theta - w = 0$$

The first of these equations contains a_x, the quantity we want to find, but we can't solve for a_x without knowing what θ is. Fortunately, we can use the second equation to find θ, then use this value of θ in the first equation to find a_x.

The second equation gives

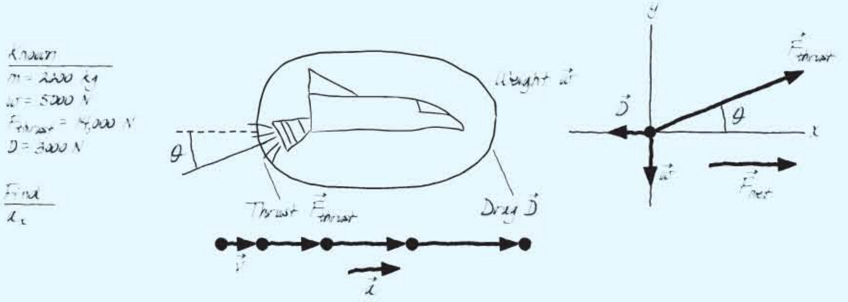
$$\sin\theta = \frac{w}{F_{\text{thrust}}} = \frac{5000\ \text{N}}{14{,}000\ \text{N}} = 0.357$$
$$\theta = \sin^{-1}(0.357) = 20.9°$$

Now we can use this value in the first equation to get

$$a_x = \frac{1}{m}(F_{\text{thrust}}\cos\theta - D)$$
$$= \frac{1}{2200\ \text{kg}}[(14{,}000\ \text{N})\cos(20.9°) - 3000\ \text{N}] = 4.6\ \text{m/s}^2$$

ASSESS An important key to solving this problem was to use the information that the cruiser accelerates only in the horizontal direction. Mathematically, this means that $a_y = 0$. Because the thrust is much greater than the weight, we need only a modest downward component of the thrust to cancel the weight and let the cruiser accelerate horizontally. So our engine tilt seems reasonable.

FIGURE 5.7 Visual overview of a rocket cruiser.



EXAMPLE 5.7 Towing a car with acceleration

A car with a mass of 1500 kg is being towed by a rope held at a 20° angle. A friction force of 320 N opposes the car's motion. What is the tension in the rope if the car goes from rest to 12 m/s in 10 s?

PREPARE You should recognize that this problem is almost identical to Example 5.4. The difference is that the car is now accelerating, so it is no longer in equilibrium. This means, as shown in FIGURE 5.8, that the net force is not zero. We've already identified all the forces in Example 5.4.

SOLVE Newton's second law in component form is

$$\sum F_x = n_x + T_x + f_x + w_x = ma_x$$
$$\sum F_y = n_y + T_y + f_y + w_y = ma_y = 0$$

We've again used the fact that $a_y = 0$ for motion that is purely along the x-axis. The components of the forces were worked out in Example 5.4. With that information, Newton's second law in component form is

$$T\cos\theta - f = ma_x$$
$$n + T\sin\theta - w = 0$$

Because the car speeds up from rest to 12 m/s in 10 s, we can use kinematics to find the acceleration:

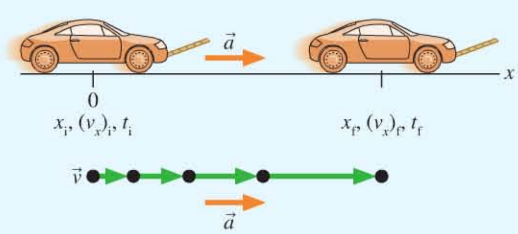
$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{t_f - t_i} = \frac{(12\text{ m/s}) - (0\text{ m/s})}{(10\text{ s}) - (0\text{ s})} = 1.2\text{ m/s}^2$$

We can now use the first Newton's-law equation above to solve for the tension. We have

$$T = \frac{ma_x + f}{\cos\theta} = \frac{(1500\text{ kg})(1.2\text{ m/s}^2) + 320\text{ N}}{\cos 20°} = 2300\text{ N}$$

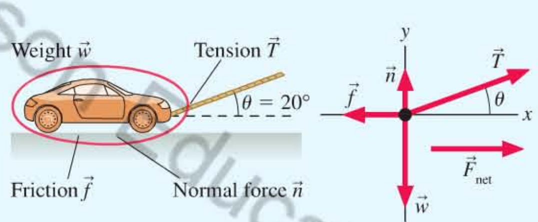
ASSESS The tension is substantially more than the 340 N found in Example 5.4. It takes a much more force to accelerate the car than to keep it rolling at a constant speed.

FIGURE 5.8 Visual overview of a car being towed.



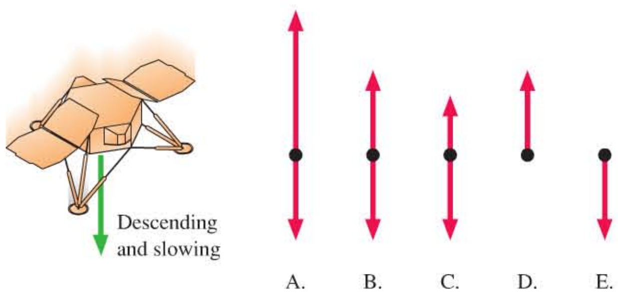
Known
$x_i = 0$ m
$(v_x)_i = 0$ m/s
$t_i = 0$ s, $\theta = 20°$
$m = 1500$ kg
$f = 320$ N
$(v_x)_f = 12$ m/s
$t_f = 10$ s

Find
T



These first examples have shown all the details of our problem-solving strategy. Our purpose has been to demonstrate how the strategy is put into practice. Future examples will be briefer, but the basic *procedure* will remain the same.

STOP TO THINK 5.1 A Martian lander is approaching the surface. It is slowing its descent by firing its rocket motor. Which is the correct free-body diagram for the lander?



5.3 Mass and Weight

When the doctor asks what you weigh, what does she really mean? We do not make much distinction in our ordinary use of language between the terms "weight" and "mass," but in physics their distinction is of critical importance.

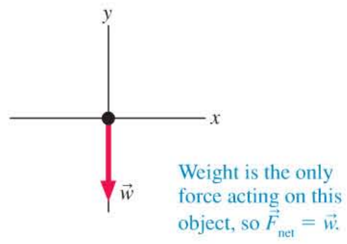
Mass, you'll recall from Chapter 4, is a quantity that describes an object's inertia, its tendency to resist being accelerated. Loosely speaking, it also describes the amount of matter in an object. Mass, measured in kilograms, is an intrinsic property of an object; it has the same value wherever the object may be and whatever forces might be acting on it.

Weight, on the other hand, is a *force*. Specifically, it is the gravitational force exerted on an object by a planet. Weight is a vector, not a scalar, and the vector's direction is always straight down. Weight is measured in newtons.

Mass and weight are not the same thing, but they are related. We can use Galileo's discovery about free fall to make the connection. FIGURE 5.9 shows the free-body diagram of an object in free fall. The *only* force acting on this object is its weight $\vec{w}$ the downward pull of gravity. Newton's second law for this object is

$$\vec{F}_{\text{net}} = \vec{w} = m\vec{a} \tag{5.3}$$

FIGURE 5.9 The free-body diagram of an object in free fall.



Recall Galileo's discovery that *any* object in free fall, regardless of its mass, has the same acceleration:

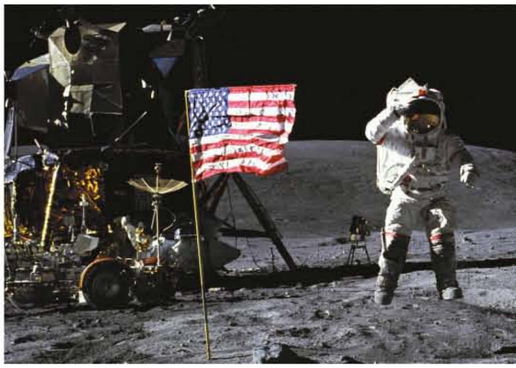
$$\vec{a}_{\text{free fall}} = (g, \text{downward}) \tag{5.4}$$

where $g = 9.80\ \text{m/s}^2$ is the free-fall acceleration at the earth's surface. So a_y in Equation 5.3 is equal to $-g$, and we have $-w = -mg$, or

$$w = mg \tag{5.5}$$

The magnitude of the weight force, which we call simply "the weight," is directly proportional to the mass, with g as the constant of proportionality. Thus, for example, the weight of a 3.6 kg book is $w = (3.6\ \text{kg})(9.8\ \text{m/s}^2) = 35\ \text{N}$.

NOTE ► Although we derived the relationship between mass and weight for an object in free fall, the weight of an object is *independent* of its state of motion. Equation 5.5 holds for an object at rest on a table, sliding horizontally, or moving in any other way. ◄



On the moon, astronaut John Young jumps 2 feet straight up, despite his spacesuit that weighs 370 pounds on earth. On the moon, where $g = 1.6\ \text{m/s}^2$, he and his suit together weighed only 90 pounds.

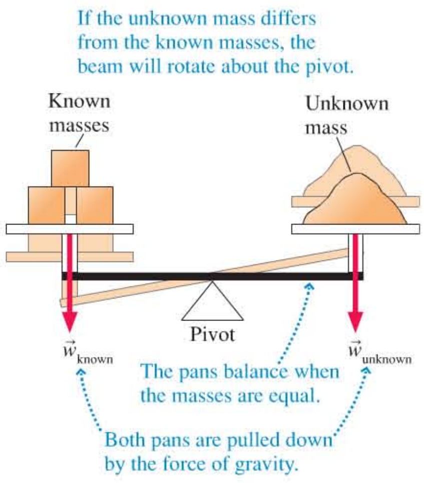
Because an object's weight depends on g, and the value of g varies from planet to planet, weight is not a fixed, constant property of an object. The value of g at the surface of the moon is about one-sixth its earthly value, so an object on the moon would have only one-sixth its weight on earth. The object's weight on Jupiter would be greater than its weight on earth. Its mass, however, would be the same. The amount of matter has not changed, only the gravitational force exerted on that matter.

So, when the doctor asks what you weigh, she really wants to know your *mass*. That's the amount of matter in your body. You can't really "lose weight" by going to the moon, even though you would weigh less there!

Measuring Mass and Weight

A *pan balance,* shown in FIGURE 5.10, is a device for measuring *mass.* You may have used a pan balance to "weigh" chemicals in a chemistry lab. An unknown mass is placed in one pan, then known masses are added to the other until the pans balance. Gravity pulls down on both sides, effectively *comparing* the masses, and the unknown mass equals the sum of the known masses that balance it. Although a pan balance requires gravity in order to function, it does not depend on the value of g. Consequently, the pan balance would give the same result on another planet.

FIGURE 5.10 A pan balance measures mass.



A *spring scale* measures weight, not mass. A spring scale can be understood on the basis of Newton's second law. The object being weighed compresses the springs, as shown in FIGURE 5.11 on the following page, which then push up with force $\vec{F}_{\text{sp}}$. But because the object is at rest, in static equilibrium, the net force on it must be zero. Thus the upward spring force must exactly balance the downward weight force:

$$F_{\text{sp}} = w = mg \tag{5.6}$$

The *reading* of a spring scale is F_{sp}, the magnitude of the force that the spring is exerting. If the object is in equilibrium, then F_{sp} is exactly equal to the object's weight w. The scale does not "know" the weight of the object. All it can do is measure how much its spring is stretched or compressed. On a different planet, with a

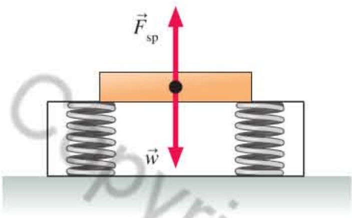


FIGURE 5.11 A spring scale measures weight.

different value for g, the expansion or compression of the spring would be different and the scale's reading would be different.

The unit of force in the English system is the *pound*. We noted in Chapter 4 that the pound is defined as 1 lb = 4.45 N. An object whose weight $w = mg$ is 4.45 N has a mass

$$m = \frac{w}{g} = \frac{4.45\ \text{N}}{9.80\ \text{m/s}^2} = 0.454\ \text{kg} = 454\ \text{g}$$

You may have learned in previous science classes that "1 pound = 454 grams" or, equivalently, "1 kg = 2.2 lb." Strictly speaking, these well-known "conversion factors" are not true. They are comparing a weight (pounds) to a mass (kilograms). The correct statement is: "A mass of 1 kg has a weight on *earth* of 2.2 pounds." On another planet, the weight of a 1 kg mass would be something other than 2.2 pounds.

EXAMPLE 5.8 **Masses of people**

What is the mass, in kilograms, of a 90 pound gymnast, a 160 pound professor, and a 240 pound football player?

SOLVE We must convert their weights into newtons; then we can find their masses from $m = w/g$:

$$w_{\text{gymnast}} = 90\ \text{lb} \times \frac{4.45\ \text{N}}{1\ \text{lb}} = 400\ \text{N} \qquad m_{\text{gymnast}} = \frac{w_{\text{gymnast}}}{g} = \frac{400\ \text{N}}{9.80\ \text{m/s}^2} = 41\ \text{kg}$$

$$w_{\text{prof}} = 160\ \text{lb} \times \frac{4.45\ \text{N}}{1\ \text{lb}} = 710\ \text{N} \qquad m_{\text{prof}} = \frac{w_{\text{prof}}}{g} = \frac{710\ \text{N}}{9.80\ \text{m/s}^2} = 72\ \text{kg}$$

$$w_{\text{football}} = 240\ \text{lb} \times \frac{4.45\ \text{N}}{1\ \text{lb}} = 1070\ \text{N} \qquad m_{\text{football}} = \frac{w_{\text{football}}}{g} = \frac{1070\ \text{N}}{9.80\ \text{m/s}^2} = 110\ \text{kg}$$

ASSESS It's worth remembering that a *typical* adult has a mass in the range of 60 to 80 kg.

This popular amusement park ride shoots you straight up with an acceleration of $4g$. As a result, you feel five times as heavy as usual.

Apparent Weight

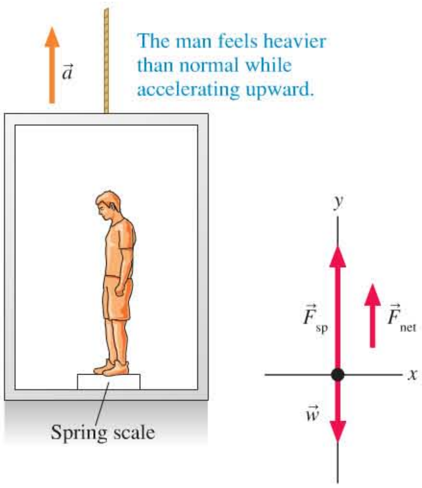
The weight of an object is the force of gravity on that object. You may never have thought about it, but gravity is not a force that you can feel or sense directly. Your *sensation* of weight—how heavy you *feel*—is due to *contact forces* pressing against you. Surfaces touch you and activate nerve endings in your skin. As you read this, your sensation of weight is due to the normal force exerted on you by the chair in which you are sitting. When you stand, you feel the contact force of the floor pushing against your feet.

When you stand on a scale, the contact force is the upward spring force F_{sp} acting on your feet. If you and the scale are in equilibrium, with $\vec{a} = \vec{0}$, this spring force and thus the scale reading are equal to your weight.

What would happen, however, if you stood on a scale while *accelerating* upward in an elevator? What would the scale read then? Recall the sensations you feel while being accelerated. You feel "heavy" when an elevator suddenly accelerates upward, and you feel lighter than normal as the upward-moving elevator brakes to a halt. Your true weight $w = mg$ has not changed during these events, but your *sensation* of your weight has.

To investigate this, imagine a man weighing himself by standing on a spring scale in an elevator as it accelerates upward. What does the scale read? As FIGURE 5.12 shows, the only forces acting on the man are the upward spring force of the scale and the downward weight force. Because the man now has an acceleration $\vec{a}$, according to Newton's second law there must be a net force acting on the man in the direction of $\vec{a}$.

FIGURE 5.12 A man weighing himself in an accelerating elevator.



Looking at the free-body diagram in Figure 5.12, we see that the y-component of Newton's second law is

$$\sum F_y = (F_{sp})_y + w_y = F_{sp} - w = ma_y \tag{5.7}$$

where m is the man's mass. Solving Equation 5.7 for F_{sp} gives

$$F_{sp} = w + ma_y \tag{5.8}$$

If the elevator is either at rest or moving with constant velocity, then $a_y = 0$ and the man is in equilibrium. In that case, $F_{sp} = w$ and the scale correctly reads his weight. But if $a_y \neq 0$, the scale's reading is *not* the man's true weight. If the elevator is accelerating upward, then $a_y = +a$, and Equation 5.8 reads $F_{sp} = w + a$. Thus $F_{sp} > w$ and the man *feels* heavier than normal. If the elevator is accelerating downward, the acceleration vector $\vec{a}$ points downward and $a_y = -a$. Thus $F_{sp} < w$ and the man feels lighter.

Let's define a person's **apparent weight** w_{app} as the magnitude of the contact force that supports him; in this case, this is the spring force F_{sp}. From Equation 5.8, the apparent weight is

$$w_{app} = w + ma_y = mg + ma_y = m(g + a_y) \tag{5.9}$$

Thus, when the man accelerates upward, his apparent weight is greater than his true weight; when accelerating downward, his apparent weight is less than his true weight.

An object doesn't have to be on a scale for its apparent weight to differ from its true weight. An object's apparent weight is the magnitude of the contact force supporting it. It makes no difference whether this is the spring force of the scale or simply the normal force of the floor.

The idea of apparent weight has important applications. Astronauts are nearly crushed by their apparent weight during a rocket launch when a is much greater than g. Much of the thrill of amusement park rides, such as roller coasters, comes from rapid changes in your apparent weight.

TRY IT YOURSELF

Physics students can't jump The next time you ride up in an elevator, try jumping in the air just as the elevator starts to rise. You'll feel like you can hardly get off the ground. This is because with $a_y > 0$ your apparent weight is *greater* than your actual weight; for an elevator with a large acceleration it's like trying to jump while carrying an extra 20 pounds. What will happen if you jump as the elevator slows at the top?

EXAMPLE 5.9 **Apparent weight in an elevator**

Anjay's mass is 70 kg. He's standing on a scale in an elevator. As the elevator stops, the scale reads 750 N. Had the elevator been moving up or down? If the elevator had been moving at 5.0 m/s, how long does it take to stop?

PREPARE The scale reading as he stops is his apparent weight, so $w_{app} = 750$ N. Because we know his mass m, we can use Equation 5.9 to find the elevator's acceleration a_y. Then we can use kinematics to find the time it takes to stop the elevator.

SOLVE From Equation 5.9 we have $w_{app} = m(g + a_y)$, so that

$$a_y = \frac{w_{app}}{m} - g = \frac{750\ \text{N}}{70\ \text{kg}} - 9.80\ \text{m/s}^2 = 0.91\ \text{m/s}^2$$

This is a *positive* acceleration. If the elevator is stopping with a positive acceleration, it must have been moving *down*, with a negative velocity.

To find the stopping time, we can use the kinematic equation

$$(v_y)_f = (v_y)_i + a_y\,\Delta t$$

to get

$$\Delta t = \frac{(v_y)_f - (v_y)_i}{a_y} = \frac{(0\ \text{m/s}) - (-5.0\ \text{m/s})}{0.91\ \text{m/s}^2} = 5.5\ \text{s}$$

Notice that we used -5.0 m/s as the initial velocity because the elevator was moving down before it stopped.

ASSESS Anjay's true weight is $mg = (70\ \text{kg})(9.8\ \text{m/s}^2) = 670$ N. Thus his apparent weight is *greater* than his true weight. You have no doubt experienced this sensation in an elevator that is stopping as it reaches the ground floor. If it had stopped while going up, you'd feel *lighter* than your true weight.

Weightlessness

One last issue before leaving this topic: Suppose the elevator cable breaks and the elevator, along with the man and his scale, plunges straight down in free fall! What will the scale read? The acceleration in free fall is $a_y = -g$. When this acceleration is used in Equation 5.9, we find that $w_{app} = 0$! In other words, the man has *no sensation* of weight.

A weightless experience You probably wouldn't want to experience weightlessness in a falling elevator. But, as we learned in Chapter 3, objects undergoing projectile motion are in free fall as well. The special plane shown flies in the same parabolic trajectory as would a projectile with no air resistance. Objects inside, such as these passengers, are then moving along a perfect free-fall trajectory. Just as for the man in the elevator, they then float with respect to the plane's interior. Such flights can last up to 30 seconds.

Think about this carefully. Suppose, as the elevator falls, the man inside releases a ball from his hand. In the absence of air resistance, as Galileo discovered, both the man and the ball would fall at the same rate. From the man's perspective, the ball would appear to "float" beside him. Similarly, the scale would float beneath him and not press against his feet. He is what we call *weightless.*

Surprisingly, "weightless" does *not* mean "no weight." An object that is **weightless** has no *apparent* weight. The distinction is significant. The man's weight is still mg because gravity is still pulling down on him, but he has no *sensation* of weight as he free falls. The term "weightless" is a very poor one, likely to cause confusion because it implies that objects have no weight. As we see, that is not the case.

But isn't this exactly what happens to astronauts orbiting the earth? You've seen films of astronauts and various objects floating inside the Space Shuttle. If an astronaut tries to stand on a scale, it does not exert any force against her feet and reads zero. She is said to be weightless. But if the criterion to be weightless is to be in free fall, and if astronauts orbiting the earth are weightless, does this mean that they are in free fall? This is a very interesting question to which we shall return in Chapter 6.

STOP TO THINK 5.2 You're bouncing up and down on a trampoline. At the very highest point of your motion, your apparent weight is

A. More than your true weight.
B. Less than your true weight.
C. Equal to your true weight.
D. Zero.

5.4 Normal Forces

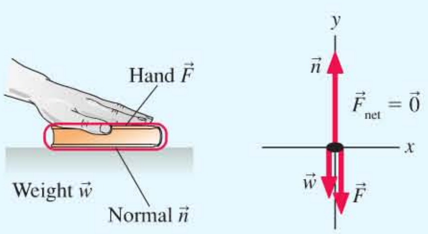
In Chapter 4 we saw that an object at rest on a table is subject to an upward force due to the table. This force is called the *normal force* because it is always directed normal, or perpendicular, to the surface of contact. As we saw, the normal force has its origin in the atomic "springs" that make up the surface. The harder the object bears down on the surface, the more these springs are compressed and the harder they push back. Thus the normal force *adjusts* itself so that the object stays on the surface without penetrating it. This fact is key in solving for the normal force.

EXAMPLE 5.10 Normal force on a pressed book

A 1.2 kg book lies on a table. The book is pressed down from above with a force of 15 N. What is the normal force acting on the book from the table below?

PREPARE The book is not moving and is thus in static equilibrium. We need to identify the forces acting on the book, and prepare a free-body diagram showing these forces. These steps are illustrated in FIGURE 5.13.

FIGURE 5.13 Finding the normal force on a book pressed from above.



SOLVE Because the book is in static equilibrium, the net force on it must be zero. The only forces acting are in the y-direction, so Newton's second law is

$$\sum F_y = n_y + w_y + F_y = n - w - F = ma_y = 0$$

We learned in the last section that the weight force is $w = mg$. The weight of the book is thus

$$w = mg = (1.2 \text{ kg})(9.8 \text{ m/s}^2) = 12 \text{ N}$$

With this information, we see that the normal force exerted by the table is

$$n = F + w = 15 \text{ N} + 12 \text{ N} = 27 \text{ N}$$

ASSESS The magnitude of the normal force is *larger* than the weight of the book. From the table's perspective, the extra force from the hand pushes the book further into the atomic springs of the table. These springs then push back harder, giving a normal force that is greater than the weight of the book.

A common situation is an object on a ramp or incline. If friction is neglected, there are only two forces acting on the object: gravity and the normal force. However, we need to carefully work out the components of these two forces in order to solve dynamics problems. **FIGURE 5.14a** shows how. Be sure you avoid the two common errors shown in **FIGURE 5.14b**.

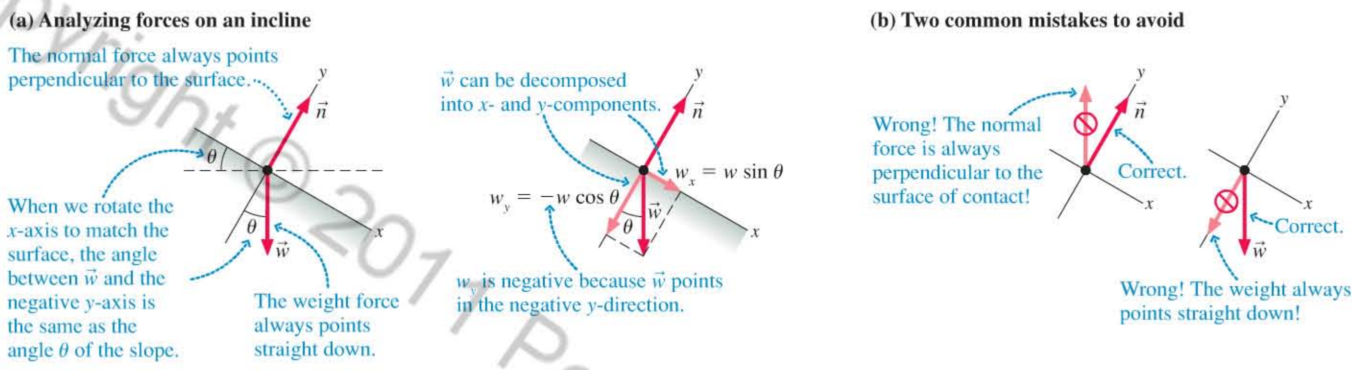


FIGURE 5.14 The forces on an object on an incline.

EXAMPLE 5.11 Acceleration of a downhill skier

A skier slides down a steep slope of 27° on ideal, frictionless snow. What is his acceleration?

PREPARE **FIGURE 5.15** is a visual overview. We choose a coordinate system tilted so that the x-axis points down the slope. This greatly simplifies the analysis, because with this choice $a_y = 0$ (the skier does not move in the y-direction at all). The free-body diagram is based on the information in Figure 5.14.

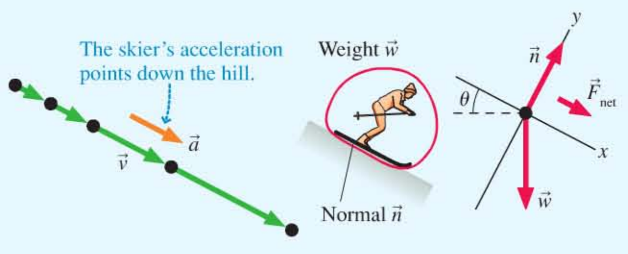


FIGURE 5.15 Visual overview of a downhill skier.

SOLVE We can now use Newton's second law in component form to find the skier's acceleration:

$$\sum F_x = w_x + n_x = ma_x$$
$$\sum F_y = w_y + n_y = ma_y$$

Because $\vec{n}$ points directly in the positive y-direction, $n_y = n$ and $n_x = 0$. Figure 5.14a showed the important fact that the angle between $\vec{w}$ and the negative y-axis is the *same* as the slope angle θ. With this information, the components of $\vec{w}$ are $w_x = w\sin\theta = mg\sin\theta$ and $w_y = -w\cos\theta = -mg\cos\theta$, where we used the fact that $w = mg$. With these components in hand, Newton's second law becomes

$$\sum F_x = w_x + n_x = mg\sin\theta = ma_x$$
$$\sum F_y = w_y + n_y = -mg\cos\theta + n = ma_y = 0$$

In the second equation we used the fact that $a_y = 0$. The m cancels in the first of these equations, leaving us with

$$a_x = g\sin\theta$$

This is the expression for acceleration on a frictionless surface that we presented, without proof, in Chapter 3. Now we've justified our earlier assertion. We can use this to calculate the skier's acceleration:

$$a_x = g\sin\theta = (9.8\text{ m/s}^2)\sin(27°) = 4.4\text{ m/s}^2$$

ASSESS Our result shows that when $\theta = 0$, so that the slope is horizontal, the skier's acceleration is zero, as it should be. Further, when $\theta = 90°$ (a vertical slope), his acceleration is g, which makes sense because he's in free fall when $\theta = 90°$. Notice that the mass canceled out, so we didn't need to know the skier's mass.

5.5 Friction

In everyday life, friction is everywhere. Friction is absolutely essential for many things we do. Without friction you could not walk, drive, or even sit down (you would slide right off the chair!). It is sometimes useful to think about idealized frictionless situations, but it is equally necessary to understand a real world where friction is present. Although friction is a complicated force, many aspects of friction can be described with a simple model.

FIGURE 5.16 Static friction keeps an object from slipping.

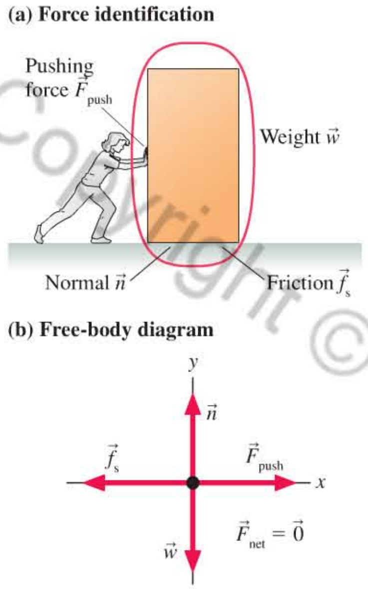


FIGURE 5.17 Static friction acts in *response* to an applied force.

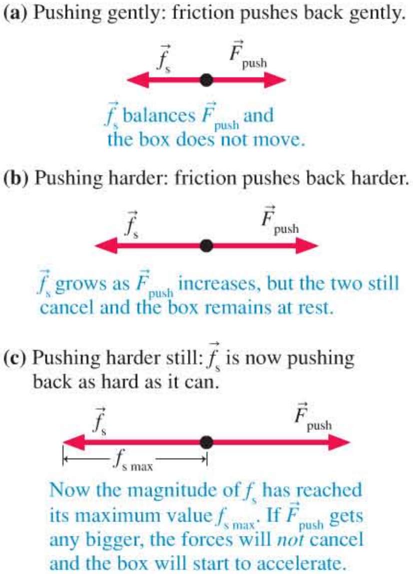


TABLE 5.1 Coefficients of friction

Materials	Static μ_s	Kinetic μ_k	Rolling μ_r
Rubber on concrete	1.00	0.80	0.02
Steel on steel (dry)	0.80	0.60	0.002
Steel on steel (lubricated)	0.10	0.05	
Wood on wood	0.50	0.20	
Wood on snow	0.12	0.06	
Ice on ice	0.10	0.03	

Static Friction

Chapter 4 defined static friction $\vec{f}_s$ as the force that a surface exerts on an object to keep it from slipping across that surface. Consider the woman pushing on the box in FIGURE 5.16a. Because the box is not moving with respect to the floor, the woman's push to the right must be balanced by a static friction force $\vec{f}_s$ pointing to the left. This is the general rule for finding the *direction* of $\vec{f}_s$: Decide which way the object *would* move if there were no friction. The static friction force $\vec{f}_s$ then points in the opposite direction, to prevent motion relative to the surface.

Determining the *magnitude* of $\vec{f}_s$ is a bit trickier. Because the box is at rest, it's in static equilibrium. From the free-body diagram of FIGURE 5.16b, this means that the static friction force must exactly balance the pushing force, so that $f_s = F_{push}$. As shown in FIGURES 5.17a and 5.17b, the harder the woman pushes, the harder the friction force from the floor pushes back. If she reduces her pushing force, the friction force will automatically be reduced to match. Static friction acts in *response* to an applied force.

But there's clearly a limit to how big $\vec{f}_s$ can get. If she pushes hard enough, the box will slip and start to move across the floor. In other words, the static friction force has a *maximum* possible magnitude $f_{s\,max}$, as illustrated in FIGURE 5.17c. Experiments with friction (first done by Leonardo da Vinci) show that $f_{s\,max}$ is proportional to the magnitude of the normal force between the surface and the object; that is,

$$f_{s\,max} = \mu_s n \qquad (5.10)$$

where μ_s is called the **coefficient of static friction.** The coefficient is a number that depends on the materials from which the object and the surface are made. The higher the coefficient of static friction, the greater the "stickiness" between the object and the surface, and the harder it is to make the object slip. Table 5.1 lists some approximate values of coefficients of friction.

NOTE ▶ Equation 5.10 does *not* say $f_s = \mu_s n$. The value of f_s depends on the force or forces that static friction has to balance to keep the object from moving. It can have any value from zero up to, but not exceeding, $\mu_s n$. ◀

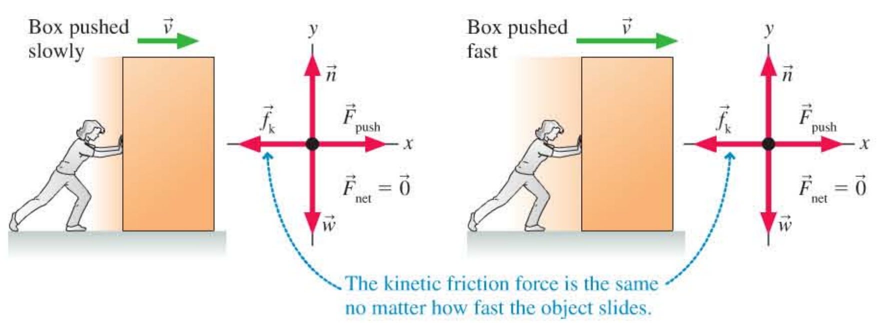
So our rules for static friction are:

- The direction of static friction is such as to oppose motion.
- The magnitude f_s of static friction adjusts itself so that the net force is zero and the object doesn't move.
- The magnitude of static friction cannot exceed the maximum value $f_{s\,max}$ given by Equation 5.10. If the friction force needed to keep the object stationary is greater than $f_{s\,max}$, the object slips and starts to move.

Kinetic Friction

Once the box starts to slide, as in FIGURE 5.18, the static friction force is replaced by a kinetic (or sliding) friction force $\vec{f}_k$. Kinetic friction is in some ways simpler than static friction: The direction of $\vec{f}_k$ is always opposite the direction in which an object

FIGURE 5.18 The kinetic friction force is *opposite* the direction of motion.



slides across the surface, and experiments show that kinetic friction, unlike static friction, has a nearly *constant* magnitude, given by

$$f_k = \mu_k n \tag{5.11}$$

where μ_k is called the **coefficient of kinetic friction.** Equation 5.11 also shows that kinetic friction, like static friction, is proportional to the magnitude of the normal force n. Notice that **the magnitude of the kinetic friction force does not depend on how fast the object is sliding.**

Table 5.1 includes approximate values of μ_k. You can see that $\mu_k < \mu_s$, which explains why it is easier to keep a box moving than it was to start it moving.

FIGURE 5.19 The bottom of the wheel is stationary.

Rolling Friction

If you slam on the brakes hard enough, your car tires slide against the road surface and leave skid marks. This is kinetic friction because the tire and the road are *sliding* against each other. A wheel *rolling* on a surface also experiences friction, but not kinetic friction: The portion of the wheel that contacts the surface is stationary with respect to the surface, not sliding. The photo in FIGURE 5.19 was taken with a stationary camera. Note how the part of the wheel touching the ground is not blurred, indicating that this part of the wheel is not moving with respect to the ground.

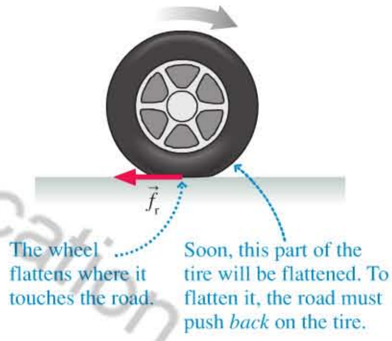
Textbooks draw wheels as circles, but no wheel is perfectly round. The weight of the wheel, and of any object supported by the wheel, causes the bottom of the wheel to flatten where it touches the surface, as FIGURE 5.20 shows. As a wheel rolls forward, the leading part of the tire must become deformed. This requires that the road push *backward* on the tire. In this way the road causes a backward force, even without slipping between the tire and the road.

The force of this *rolling friction* can be calculated in terms of a **coefficient of rolling friction** μ_r:

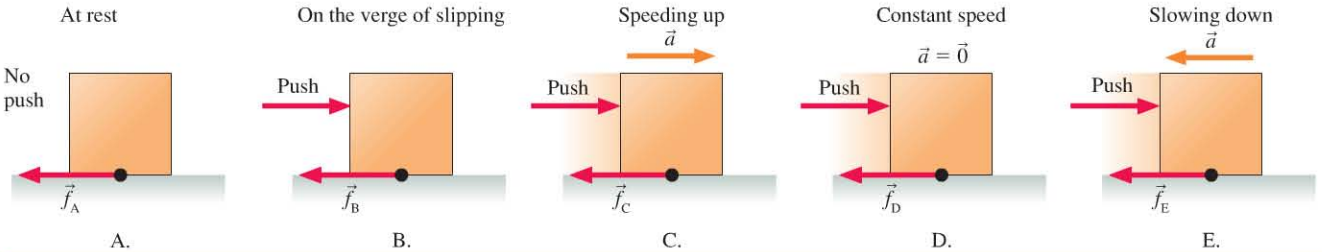
$$f_r = \mu_r n \tag{5.12}$$

with the *direction* of the force opposing the direction of motion. Thus rolling friction acts very much like kinetic friction, but values of μ_r (see Table 5.1) are much lower than values of μ_k. This is why it is easier to roll an object on wheels than to slide it.

FIGURE 5.20 Rolling friction is due to deformation of a wheel.



STOP TO THINK 5.3 Rank in order, from largest to smallest, the size of the friction forces $\vec{f}_A$ to $\vec{f}_E$ in the five different situations (one or more friction forces could be zero). The box and the floor are made of the same materials in all situations.

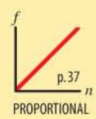


Working with Friction Forces

ActivPhysics ONLINE 2.5, 2.6

These ideas can be summarized in a *model* of friction:

$$\begin{aligned}
&\text{Static: } \vec{f}_s = (\text{magnitude } \le f_{s\max} = \mu_s n, \\
&\qquad\qquad \text{direction as necessary to prevent motion}) \\
&\text{Kinetic: } \vec{f}_k = (\mu_k n, \text{ direction opposite the motion}) \\
&\text{Rolling: } \vec{f}_r = (\mu_r n, \text{ direction opposite the motion})
\end{aligned} \tag{5.13}$$



Optimized braking If you slam on your brakes, your wheels will lock up and you'll go into a skid. Then it is the *kinetic* friction force between the road and your tires that slows your car to a halt. If, however, you apply the brakes such that you don't quite skid and your tires continue to roll, the force stopping you is the *static* friction force between the road and your tires. This is a better way to brake, because the maximum static friction force is always greater than the kinetic friction force. *Antilock braking systems* (ABS) automatically do this for you when you slam on the brakes, stopping you in the shortest possible distance.

Here "motion" means "motion relative to the surface." The maximum value of static friction $f_{s\,max} = \mu_s n$ occurs at the point where the object slips and begins to move. Note that only one kind of friction force at a time can act on an object.

NOTE ► Equations 5.13 are a "model" of friction, not a "law" of friction. These equations provide a reasonably accurate, but not perfect, description of how friction forces act. They are a simplification of reality that works reasonably well, which is what we mean by a "model." They are not a "law of nature" on a level with Newton's laws. ◄

TACTICS BOX 5.1 **Working with friction forces** MP™

1. If the object is *not moving* relative to the surface it's in contact with, then the friction force is **static friction.** Draw a free-body diagram of the object. The *direction* of the friction force is such as to oppose sliding of the object relative to the surface. Then use Problem-Solving Strategy 5.1 to solve for f_s. If f_s is greater than $f_{s\,max} = \mu_s n$, then static friction cannot hold the object in place. The assumption that the object is at rest is not valid, and you need to redo the problem using kinetic friction.
2. If the object is *sliding* relative to the surface, then **kinetic friction** is acting. From Newton's second law, find the normal force n. Equation 5.13 then gives the magnitude and direction of the friction force.
3. If the object is *rolling* along the surface, then **rolling friction** is acting. From Newton's second law, find the normal force n. Equation 5.13 then gives the magnitude and direction of the friction force.

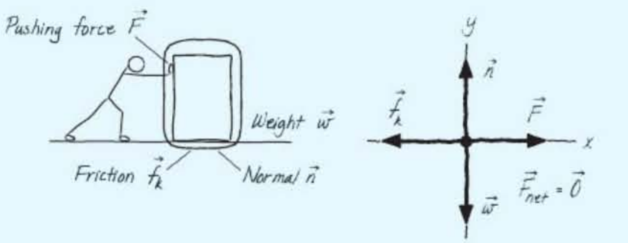
Exercises 20, 21

EXAMPLE 5.12 Finding the force to push a box

Carol pushes a 10.0 kg wood box across a wood floor at a steady speed of 2.0 m/s. How much force does Carol exert on the box?

PREPARE Let's assume the box slides to the right. In this case, a kinetic friction force $\vec{f}_k$, opposes the motion by pointing to the left. In **FIGURE 5.21** we identify the forces acting on the box and construct a free-body diagram.

FIGURE 5.21 Forces on a box being pushed across a floor.



SOLVE The box is moving at a constant speed, so it is in dynamic equilibrium with $\vec{F}_{net} = \vec{0}$. This means that the x- and y-components of the net force must be zero:

$$\sum F_x = n_x + w_x + F_x + (f_k)_x = 0 + 0 + F - f_k = 0$$

$$\sum F_y = n_y + w_y + F_y + (f_k)_y = n - w + 0 + 0 = 0$$

In the first equation, the x-component of $\vec{f}_k$ is equal to $-f_k$ because $\vec{f}_k$ is directed to the left. Similarly, $w_y = -w$ because the weight force points down.

From the first equation, we see that Carol's pushing force is $F = f_k$. To evaluate this, we need f_k. Here we can use our model for kinetic friction:

$$f_k = \mu_k n$$

Because the friction is wood sliding on wood, we can use Table 5.1 to find $\mu_k = 0.20$. Further, we can use the second Newton's-law equation to find that the normal force is $n = w = mg$. Thus

$$F = f_k = \mu_k n = \mu_k mg$$
$$= (0.20)(10.0\text{ kg})(9.80\text{ m/s}^2) = 20\text{ N}$$

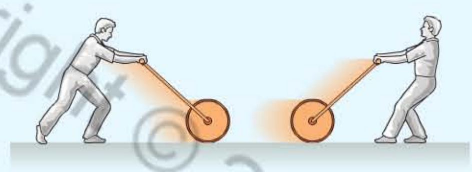
This is the force that Carol needs to apply to the box to keep it moving at a steady speed.

ASSESS The speed of 2.0 m/s with which Carol pushes the box does not enter into the answer. This is because our model of kinetic friction does not depend on the speed of the sliding object.

CONCEPTUAL EXAMPLE 5.13 To push or pull a lawn roller?

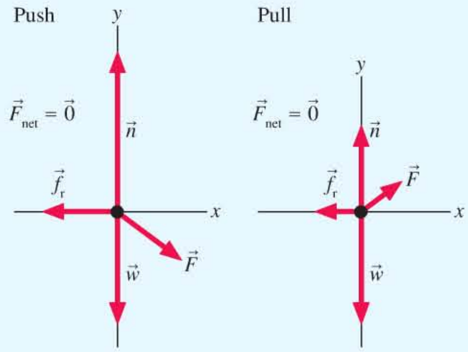
A lawn roller is a heavy cylinder used to flatten a bumpy lawn, as shown in FIGURE 5.22. Is it easier to push or pull such a roller? Which is more effective for flattening the lawn: pushing or pulling? Assume that the pushing or pulling force is directed along the handle of the roller.

FIGURE 5.22 Pushing and pulling a lawn roller.



REASON FIGURE 5.23 shows free-body diagrams for the two cases. We assume that the roller is pushed at a constant speed so that it is in dynamic equilibrium with $\vec{F}_{\text{net}} = \vec{0}$. Because the roller does not move in the y-direction, the y-component of the net force must be zero. According to our model, the magnitude f_r of rolling friction is proportional to the magnitude n of the normal force. If we *push* on the roller, our pushing force $\vec{F}$ will have a downward y-component. To compensate for this, the normal force must increase and, because $f_r = \mu_r n$, the rolling friction will increase as well. This makes the roller harder to move. If we *pull* on the roller, the now upward y-component of $\vec{F}$ will lead to a *reduced* value of n and hence of f_r. Thus the roller is easier to pull than to push.

FIGURE 5.23 Free-body diagrams for the lawn roller.



However, the purpose of the roller is to flatten the soil. If the normal force $\vec{n}$ of the ground on the roller is greater, then by Newton's third law the force of the roller on the ground will be greater as well. So for smoothing your lawn, it's better to push.

ASSESS You've probably experienced this effect while using an upright vacuum cleaner. The vacuum is harder to push on the forward stroke than when drawing it back.

EXAMPLE 5.14 How to dump a file cabinet

A 50.0 kg steel file cabinet is in the back of a dump truck. The truck's bed, also made of steel, is slowly tilted. What is the magnitude of the static friction force on the cabinet when the bed is tilted 20°? At what angle will the file cabinet begin to slide?

PREPARE We'll use our model of static friction. The file cabinet will slip when the static friction force reaches its maximum possible value $f_{s\,\text{max}}$. FIGURE 5.24 shows the visual overview when the truck bed is tilted at angle θ. We can make the analysis easier if we tilt the coordinate system to match the bed of the truck. To prevent the file cabinet from slipping, the static friction force must point *up* the slope.

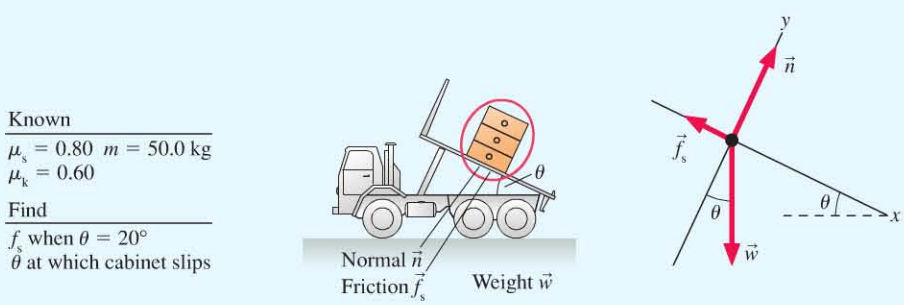
SOLVE Before it slips, the file cabinet is in static equilibrium. Newton's second law gives

$$\sum F_x = n_x + w_x + (f_s)_x = 0$$
$$\sum F_y = n_y + w_y + (f_s)_y = 0$$

From the free-body diagram we see that f_s has only a negative x-component and that n has only a positive y-component. We also have $w_x = w\sin\theta$ and $w_y = -w\cos\theta$. Thus the second law becomes

$$\sum F_x = w\sin\theta - f_s = mg\sin\theta - f_s = 0$$
$$\sum F_y = n - w\cos\theta = n - mg\cos\theta = 0$$

FIGURE 5.24 Visual overview of a file cabinet in a tilted dump truck.



Continued

The x-component equation allows us to determine the magnitude of the static friction force when $\theta = 20°$:

$$f_s = mg\sin\theta = (50.0 \text{ kg})(9.80 \text{ m/s}^2)\sin 20° = 168 \text{ N}$$

This value does not require that we know μ_s. The coefficient of static friction enters only when we want to find the angle at which the file cabinet slips. Slipping occurs when the static friction force reaches its maximum value:

$$f_s = f_{s\max} = \mu_s n$$

From the y-component of Newton's second law we see that $n = mg\cos\theta$. Consequently,

$$f_{s\max} = \mu_s mg\cos\theta$$

The x-component of the second law gave

$$f_s = mg\sin\theta$$

Setting $f_s = f_{s\max}$ then gives

$$mg\sin\theta = \mu_s mg\cos\theta$$

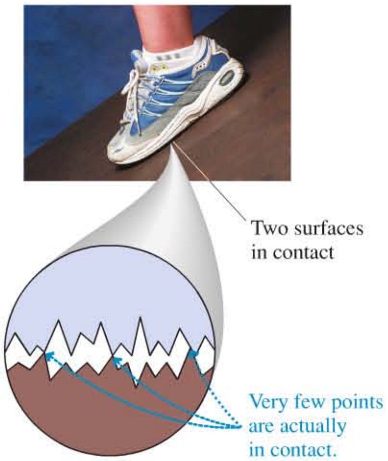
The mg in both terms cancels, and we find

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \mu_s$$

$$\theta = \tan^{-1}\mu_s = \tan^{-1}(0.80) = 39°$$

ASSESS Steel doesn't slide all that well on unlubricated steel, so a fairly large angle is not surprising. The answer seems reasonable. It is worth noting that $n = mg\cos\theta$ in this example. A common error is to use simply $n = mg$. Be sure to evaluate the normal force within the context of each particular problem.

FIGURE 5.25 A microscopic view of friction.



Causes of Friction

It is worth a brief pause to look at the *causes* of friction. All surfaces, even those quite smooth to the touch, are very rough on a microscopic scale. When two objects are placed in contact, they do not make a smooth fit. Instead, as **FIGURE 5.25** shows, the high points on one surface become jammed against the high points on the other surface, while the low points are not in contact at all. Only a very small fraction (typically 10^{-4}) of the surface area is in actual contact. The amount of contact depends on how hard the surfaces are pushed together, which is why friction forces are proportional to n.

For an object to slip, you must push it hard enough to overcome the forces exerted at these contact points. Once the two surfaces are sliding against each other, their high points undergo constant collisions, deformations, and even brief bonding that lead to the resistive force of kinetic friction.

5.6 Drag

The air exerts a drag force on objects as they move through it. You experience drag forces every day as you jog, bicycle, ski, or drive your car. The drag force $\vec{D}$:

- Is opposite in direction to the velocity $\vec{v}$.
- Increases in magnitude as the object's speed increases.

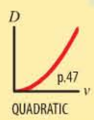
At relatively low speeds, the drag force in air is small and can usually be neglected, but drag plays an important role as speeds increase. Fortunately, we can use a fairly simple *model* of drag if the following three conditions are met:

- The object's size (diameter) is between a few millimeters and a few meters.
- The object's speed is less than a few hundred meters per second.
- The object is moving through the air near the earth's surface.

These conditions are usually satisfied for balls, people, cars, and many other objects in our everyday experience. Under these conditions, the drag force can be written:

$$\vec{D} = \left(\tfrac{1}{2}C_D\rho A v^2, \text{ direction opposite the motion}\right) \tag{5.14}$$

Drag force on an object of cross-section area A moving at speed v



Here, ρ is the density of air ($\rho = 1.22\ \text{kg/m}^3$ at sea level), A is the cross-section area of the object (in m^2), and the **drag coefficient** C_D depends on the details of the object's shape. However, the value of C_D for everyday moving objects is roughly 1/2, so a good approximation to the drag force is

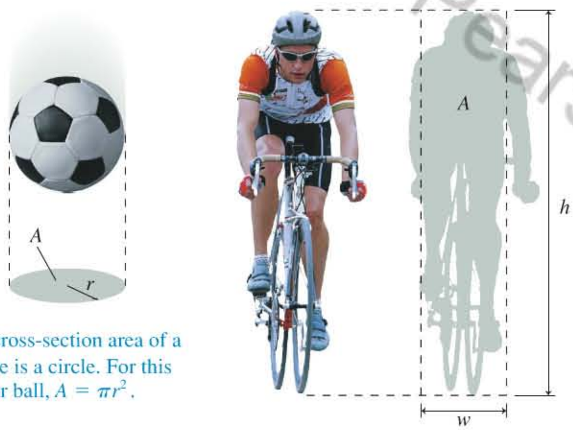
$$D \approx \tfrac{1}{4}\rho A v^2 \tag{5.15}$$

This is the expression for the magnitude of the drag force that we'll use in this chapter.

The size of the drag force in air is proportional to the *square* of the object's speed: If the speed doubles, the drag increases by a factor of 4. This model of drag fails for objects that are very small (such as dust particles) or very fast (such as jet planes) or that move in other media (such as water).

FIGURE 5.26 shows that the area A in Equation 5.14 is the cross section of the object as it "faces into the wind." It's interesting to note that the magnitude of the drag force depends on the object's *size and shape* but not on its *mass.* This has important consequences for the motion of falling objects.

FIGURE 5.26 How to calculate the cross-section area A.



Terminal Speed

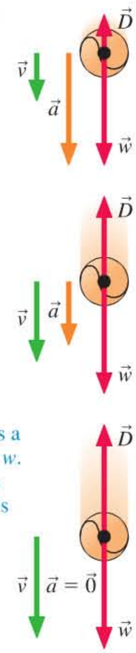
Just after an object is released from rest, its speed is low and the drag force is small (as shown in FIGURE 5.27a). Because the net force is nearly equal to the weight, the object will fall with an acceleration only a little less than g. As it falls farther, its speed and hence the drag force increase. Now the net force is smaller, so the acceleration is smaller (as shown in FIGURE 5.27b). It's still speeding up, but at a lower *rate.* Eventually the speed will increase to a point such that the magnitude of the drag force *equals* the weight (as shown in FIGURE 5.27c). The net force and hence the acceleration at this speed are then *zero,* and the object falls with a *constant* speed. The speed at which the exact balance between the upward drag force and the downward weight force causes an object to fall without acceleration is called the **terminal speed** v_{term}. **Once an object has reached terminal speed, it will continue falling at that speed until it hits the ground.**

It's straightforward to compute the terminal speed. It is the speed, by definition, at which $D = w$ or, equivalently, $\frac{1}{4}\rho A v^2 = mg$. This speed is then

$$v_{\text{term}} \approx \sqrt{\frac{4mg}{\rho A}} \tag{5.16}$$

This equation shows that a more massive object has a greater terminal speed than a less massive object of equal size. A 10-cm-diameter lead ball, with a mass of 6 kg, has a terminal speed of 150 m/s, while a 10-cm-diameter Styrofoam ball, with a mass of 50 g, has a terminal speed of only 14 m/s.

FIGURE 5.27 A falling object eventually reaches terminal speed.

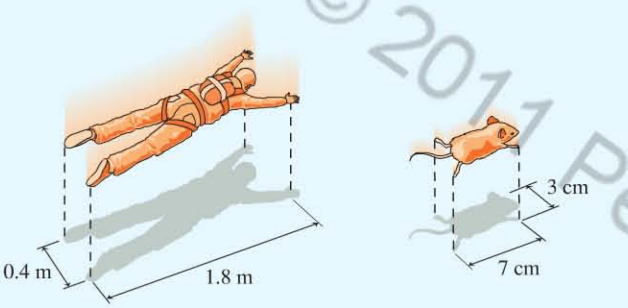


EXAMPLE 5.15 **Terminal speeds of a skydiver and a mouse**

A skydiver and his pet mouse jump from a plane. Estimate their terminal speeds.

PREPARE To use Equation 5.16 we need to estimate the mass m and cross-section area A of both man and mouse. **FIGURE 5.28** shows how. A typical skydiver might be 1.8 m long and 0.4 m wide ($A = 0.72\ \text{m}^2$) with a mass of 75 kg, while a mouse has a mass of perhaps 20 g (0.020 kg) and is 7 cm long and 3 cm wide ($A = 0.07\ \text{m} \times 0.03\ \text{m} = 0.0021\ \text{m}^2$).

FIGURE 5.28 The cross-section areas of a skydiver and a mouse.



SOLVE We can use Equation 5.16 to find that for the skydiver

$$v_{\text{term}} \approx \sqrt{\frac{4mg}{\rho A}} = \sqrt{\frac{4(75\ \text{kg})(9.8\ \text{m/s}^2)}{(1.22\ \text{kg/m}^3)(0.72\ \text{m}^2)}} = 58\ \text{m/s}$$

This is roughly 130 mph. A higher speed can be reached by falling feet first or head first, which reduces the area A. Fortunately the skydiver can open his parachute, greatly increasing A. This brings his terminal speed down to a safe value.

For the mouse we have

$$v_{\text{term}} \approx \sqrt{\frac{4mg}{\rho A}} = \sqrt{\frac{4(0.020\ \text{kg})(9.8\ \text{m/s}^2)}{(1.22\ \text{kg/m}^2)(0.0021\ \text{m}^2)}} = 17\ \text{m/s}$$

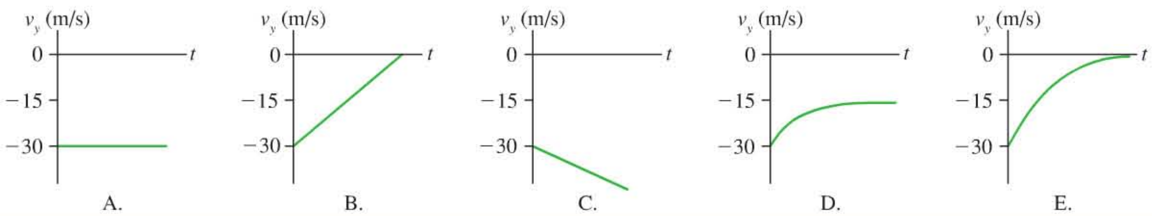
The mouse has no parachute—nor does he need one! A mouse's terminal speed is slow enough that he can fall from any height, even out of an airplane, and survive. Cats, too, have relatively low terminal speeds. In a study of cats that fell from high rises, over 90% survived—including one that fell 45 stories!

ASSESS The mouse survives the fall not only because of its lower terminal speed. The smaller an animal's body, the proportionally more robust it is. Further, a small animal's low mass and terminal speed mean that it has a very small *kinetic energy,* an idea we'll study in Chapter 10.

Although we've focused our analysis on falling objects, the same ideas apply to objects moving horizontally. If an object is thrown or shot horizontally, $\vec{D}$ causes the object to slow down. An airplane reaches its maximum speed, which is analogous to the terminal speed, when the drag is equal to and opposite the thrust: $D = F_{\text{thrust}}$. The net force is then zero and the plane cannot go any faster.

We will continue to neglect drag unless a problem specifically calls for drag to be considered.

STOP TO THINK 5.4 The terminal speed of a Styrofoam ball is 15 m/s. Suppose a Styrofoam ball is shot straight down with an initial speed of 30 m/s. Which velocity graph is correct?



5.7 Interacting Objects

2.7–2.9 Activ Online Physics

Up to this point we have studied the dynamics of a single object subject to forces exerted on it by other objects. In Example 5.11, for instance, the box was acted upon by friction, normal, weight, and pushing forces that came from the floor, the earth, and the person pushing. As we've seen, such problems can be solved by an application of Newton's second law after all the forces have been identified.

But in Chapter 4 we found that real-world motion often involves two or more objects interacting with each other. We further found that forces always come in action/reaction *pairs* that are related by Newton's third law. To remind you, Newton's third law states:

- Every force occurs as one member of an action/reaction pair of forces. The two members of the pair always act on *different* objects.
- The two members of an action/reaction pair point in *opposite* directions and are *equal* in magnitude.

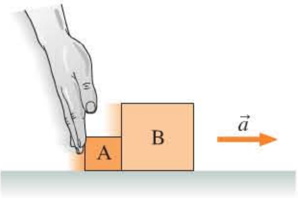
Our goal in this section is to learn how to apply the second *and* third laws to interacting objects.

Objects in Contact

One common way that two objects interact is via direct contact forces between them. Consider, for example, the two blocks being pushed across a frictionless table in **FIGURE 5.29**. To analyze block A's motion, we need to identify all the forces acting on it and then draw its free-body diagram. We repeat the same steps to analyze the motion of block B. However, the forces on A and B are *not* independent: Forces $\vec{F}_{\text{B on A}}$ acting on block A and $\vec{F}_{\text{A on B}}$ acting on block B are an action/reaction pair and thus have the same magnitude. Furthermore, because the two blocks are in contact, their *accelerations* must be the same, so that $a_{Ax} = a_{Bx} = a_x$. Because the accelerations of both blocks are equal, we can drop the subscripts A and B and call both accelerations a_x.

These observations suggest that we can't solve for the motion of one block without considering the motion of the other. The following revised version of our basic Problem-Solving Strategy 5.2 that was developed earlier in this chapter shows how to do this.

FIGURE 5.29 Two boxes moving together have the same acceleration.



PROBLEM-SOLVING STRATEGY 5.3 **Objects-in-contact problems**

PREPARE Identify those objects whose motion you wish to study. Make simplifying assumptions.

Prepare a visual overview:

- Make a sketch of the situation. Define symbols and identify what the problem is trying to find. You may want to give each object its own coordinate system.
- Draw each object separately and prepare a *separate* force identification diagram for each object.
- Identify the action/reaction pairs of forces. If object A acts on object B with force $\vec{F}_{\text{A on B}}$, then identify the force $\vec{F}_{\text{B on A}}$ that B exerts on A.
- Draw a *separate* free-body diagram for each object. Use subscript labels to distinguish forces, such as $\vec{n}$ and $\vec{w}$, that act independently on more than one object.

SOLVE Use Newton's second and third laws:

- Write Newton's second law in component form for each object. Find the force components from the free-body diagrams.
- Equate the magnitudes of the two forces in each action/reaction pair.
- Determine how the accelerations of the objects are related to each other. Objects in contact will have the *same* acceleration; Section 5.8 shows how accelerations are related for objects connected by ropes or strings.
- Solve for the unknown forces or acceleration.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

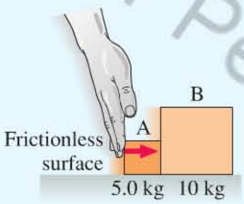
NOTE ► Two steps are especially important when drawing the free-body diagrams. First, draw a *separate* diagram for each object. They need not have the same coordinate system. Second, show only the forces acting *on* that object. The force $\vec{F}_{\text{A on B}}$ goes on the free-body diagram of object B, but $\vec{F}_{\text{B on A}}$ goes on the diagram of object A. The two members of an action/reaction pair *always* appear on two different free-body diagrams—*never* on the same diagram. ◄

You might be puzzled that the Solve step calls for the use of the third law to equate just the *magnitudes* of action/reaction forces. What about the "opposite in direction" part of the third law? You have already used it! Your free-body diagrams should show the two members of an action/reaction pair to be opposite in direction, and that information will have been utilized in writing the second-law equations. Because the directional information has already been used, all that is left is the magnitude information.

EXAMPLE 5.16 Pushing two blocks

FIGURE 5.30 shows a 5.0 kg block A being pushed with a 3.0 N force. In front of this block is a 10 kg block B; the two blocks move together. What force does block A exert on block B?

FIGURE 5.30 Two blocks are pushed by a hand.



PREPARE The visual overview of **FIGURE 5.31** lists the known information and identifies $F_{\text{A on B}}$ as what we're trying to find. Then, following the steps of Problem-Solving Strategy 5.3, we've drawn *separate* force identification diagrams and *separate* free-body diagrams for the two blocks. Both blocks have a weight force and a normal force, so we've used subscripts A and B to distinguish between them.

The force $\vec{F}_{\text{A on B}}$ is the contact force that block A exerts on B; it forms an action/reaction pair with the force $\vec{F}_{\text{B on A}}$ that block B exerts on A. Notice that force $\vec{F}_{\text{A on B}}$ is drawn acting on block B; it is the force *of* A *on* B. **Force vectors are always drawn on the free-body diagram of the object that *experiences* the force,** not the object exerting the force. Because action/reaction pairs act in opposite directions, force $\vec{F}_{\text{B on A}}$ pushes backward on block A and appears on A's free-body diagram.

SOLVE We begin by writing Newton's second law in component form for each block. Because the motion is only in the x-direction, we need only the x-component of the second law. For block A,

$$\sum F_x = (F_{\text{H}})_x + (F_{\text{B on A}})_x = m_{\text{A}} a_{\text{A}x}$$

The force components can be "read" from the free-body diagram, where we see $\vec{F}_{\text{H}}$ pointing to the right and $\vec{F}_{\text{B on A}}$ pointing to the left. Thus

$$F_{\text{H}} - F_{\text{B on A}} = m_{\text{A}} a_{\text{A}x}$$

For B, we have

$$\sum F_x = (F_{\text{A on B}})_x = F_{\text{A on B}} = m_{\text{B}} a_{\text{B}x}$$

We have two additional pieces of information: First, Newton's third law tells us that $F_{\text{B on A}} = F_{\text{A on B}}$. Second, the boxes are in contact and must have the same acceleration a_x: that is, $a_{\text{A}x} = a_{\text{B}x} = a_x$. With this information, the two x-component equations become

$$F_{\text{H}} - F_{\text{A on B}} = m_{\text{A}} a_x$$
$$F_{\text{A on B}} = m_{\text{B}} a_x$$

Our goal is to find $F_{\text{A on B}}$, so we need to eliminate the unknown acceleration a_x. From the first equation, $a_x = F_{\text{A on B}}/m_{\text{B}}$. Substituting this into the second equation gives

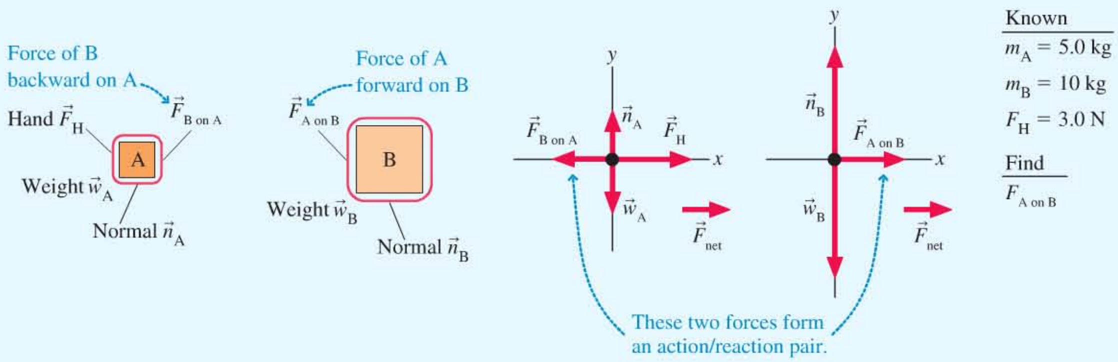
$$F_{\text{H}} - F_{\text{A on B}} = \frac{m_{\text{A}}}{m_{\text{B}}} F_{\text{A on B}}$$

This can be solved for the force of block A on block B, giving

$$F_{\text{A on B}} = \frac{F_{\text{H}}}{1 + m_{\text{A}}/m_{\text{B}}} = \frac{3.0\text{ N}}{1 + (5.0\text{ kg})/(10\text{ kg})} = \frac{3.0\text{ N}}{1.5} = 2.0\text{ N}$$

ASSESS Force F_{H} accelerates both blocks, a total mass of 15 kg, but force $F_{\text{A on B}}$ on B accelerates only block B, with a mass of 10 kg. Thus it makes sense that $F_{\text{A on B}} < F_{\text{H}}$.

FIGURE 5.31 A visual overview of the two blocks.



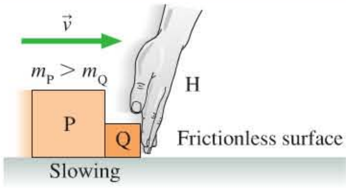
STOP TO THINK 5.5 Boxes P and Q are sliding to the right across a frictionless table. The hand H is slowing them down. The mass of P is larger than the mass of Q. Rank in order, from largest to smallest, the *horizontal* forces on P, Q, and H.

A. $F_{\text{Q on H}} = F_{\text{H on Q}} = F_{\text{P on Q}} = F_{\text{Q on P}}$

B. $F_{\text{Q on H}} = F_{\text{H on Q}} > F_{\text{P on Q}} = F_{\text{Q on P}}$

C. $F_{\text{Q on H}} = F_{\text{H on Q}} < F_{\text{P on Q}} = F_{\text{Q on P}}$

D. $F_{\text{H on Q}} = F_{\text{H on P}} > F_{\text{P on Q}}$



5.8 Ropes and Pulleys

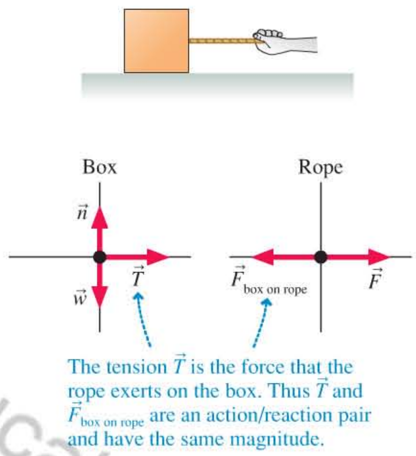
Many objects are connected by strings, ropes, cables, and so on. In single-particle dynamics, we defined *tension* as the force exerted on an object by a rope or string. We can learn several important facts about ropes and tension by considering the box being pulled by a rope in **FIGURE 5.32**; the rope in turn is being pulled by a hand that exerts a force $\vec{F}$ on the rope.

The box is pulled by the rope, so the box's free-body diagram shows a tension force $\vec{T}$. The *rope* is subject to two horizontal forces: the force $\vec{F}$ of the hand on the rope, and the force $\vec{F}_{\text{box on rope}}$ with which the box pulls back on the rope. $\vec{T}$ and $\vec{F}_{\text{box on rope}}$ form an action/reaction pair, so their magnitudes are equal: $F_{\text{box on rope}} = T$. Newton's second law *for the rope* is thus

$$\sum F_x = F - F_{\text{box on rope}} = F - T = m_{\text{rope}} a_x \qquad (5.17)$$

where m_{rope} is the rope's mass.

FIGURE 5.32 A box being pulled by a rope.



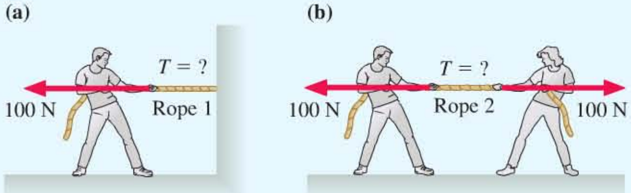
In many problems, the mass of a string or rope is significantly less than the mass of the objects it pulls on. In that case, it's reasonable to make the approximation—called the **massless string approximation**—that $m_{\text{rope}} = 0$. If $m_{\text{rope}} = 0$, then the right side of Equation 5.17 is zero and so $T = F$. In other words, **the tension in a massless string or rope equals the magnitude of the force pulling on the end of the string or rope.** As a result:

- A massless string or rope "transmits" a force undiminished from one end to the other: If you pull on one end of a rope with force F, the other end of the rope pulls on what it's attached to with a force of the same magnitude F.
- The tension in a massless string or rope is the same from one end to the other.

CONCEPTUAL EXAMPLE 5.17 Pulling a rope

FIGURE 5.33a shows a student pulling horizontally with a 100 N force on a rope that is attached to a wall. In **FIGURE 5.33b**, two students in a tug-of-war pull on opposite ends of a rope with 100 N each. Is the tension in the second rope larger, smaller, or the same as that in the first?

FIGURE 5.33 Pulling on a rope. Which produces a larger tension?



REASON Surely pulling on a rope from both ends causes more tension than pulling on one end. Right? Before jumping to conclusions, let's analyze the situation carefully. We found above that the force pulling on the end of a rope—here, the 100 N force exerted by the student—and the tension in the rope have the same magnitude. Thus, the tension in rope 1 is 100 N, the force with which the student pulls on the rope.

To find the tension in the second rope, consider the force that the *wall* exerts on the *first* rope. The first rope is in equilibrium, so the 100 N force exerted by the student must be balanced by a 100 N force on the rope from the wall. The first rope is being pulled from *both* ends by a 100 N force—the exact same situation as for the second rope, pulled by the students. A rope doesn't care whether it's being pulled on by a wall or by a person, so the tension in the second rope is the *same* as that in the first, or 100 N.

ASSESS This example reinforces what we just learned about ropes: A rope pulls on the objects at each of its ends with a force equal in magnitude to the tension, and the external force applied to each end of the rope and the rope's tension have equal magnitude.

FIGURE 5.34 An ideal pulley changes the direction in which a tension force acts, but not its magnitude.

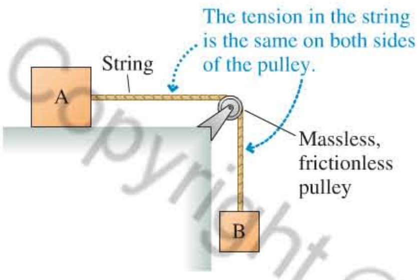
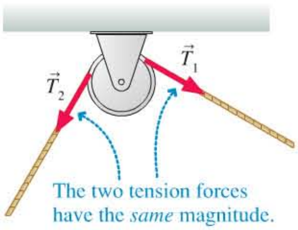


FIGURE 5.35 Tension forces on a pulley.



Pulleys

Strings and ropes often pass over pulleys. FIGURE 5.34 shows a simple situation in which block B drags block A across a table as it falls. As the string moves, static friction between the string and the pulley causes the pulley to turn. If we assume that

- The string *and* the pulley are both massless, and
- There is no friction where the pulley turns on its axle,

then no net force is needed to accelerate the string or turn the pulley. In this case, **the tension in a massless string is unchanged by passing over a massless, frictionless pulley.** We'll assume such an ideal pulley for problems in this chapter. Later, when we study rotational motion in Chapter 8, we'll consider the effect of the pulley's mass.

In some situations we are interested in the force exerted on the pulley by the rope. Even though the pulley in FIGURE 5.35 is frictionless, the tension force still pulls on the pulley. Because the rope is under tension on *both* sides of the pulley, the pulley is subject to *two* tension forces, one from each side of the rope. The net force on the pulley due to the rope is then the vector sum of these two tension forces, both of which have the same magnitude, equal to the rope's tension.

We can collect all these observations about ropes, pulleys, and tension into a Tactics Box. We'll use these three rules extensively in solving problems with ropes, strings, and pulleys.

TACTICS BOX 5.2 Working with ropes and pulleys

For massless ropes or strings and massless, frictionless pulleys:

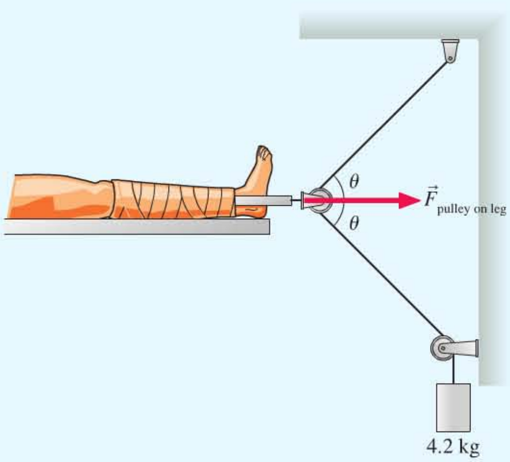
- If a force pulls on one end of a rope, the tension in the rope equals the magnitude of the pulling force.
- If two objects are connected by a rope, the tension is the same at both ends.
- If the rope passes over a pulley, the tension in the rope is unaffected.

Exercises 29–32

EXAMPLE 5.18 Placing a leg in traction

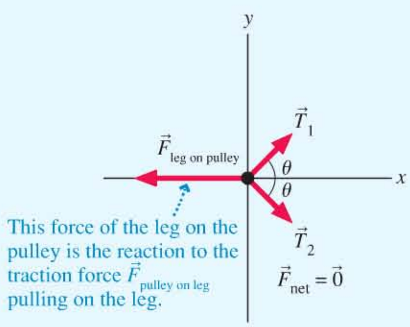
For serious fractures of the leg, the leg may need to have a stretching force applied to it to keep contracting leg muscles from forcing the broken bones together too hard. This is often done using *traction*, an arrangement of a rope, a weight, and pulleys as shown in FIGURE 5.36. The rope must make the same angle θ on both sides of the pulley so that the net force of the rope on the pulley is horizontally to the right, but θ can be adjusted to control the amount of traction. The doctor has specified 50 N of traction for this patient, with a 4.2 kg hanging mass. What is the proper angle θ?

FIGURE 5.36 A leg in traction.



PREPARE The pulley attached to the patient's leg is in static equilibrium, so the net force on it must be zero. FIGURE 5.37 shows a free-body diagram for the pulley, which we'll assume to be frictionless. Forces $\vec{T}_1$ and $\vec{T}_2$ are the tension forces of the rope as it

FIGURE 5.37 Free-body diagram for the pulley.



pulls on the pulley. These forces are equal in magnitude for a frictionless pulley, and their combined pull is to the right. This force is balanced by the force $\vec{F}_{\text{leg on pulley}}$ of the patient's leg pulling to the left. The traction force $\vec{F}_{\text{pulley on leg}}$ forms an action/reaction pair with $\vec{F}_{\text{leg on pulley}}$, so 50 N of traction means that $\vec{F}_{\text{leg on pulley}}$ also has a magnitude of 50 N.

SOLVE Two important properties of ropes, given in Tactics Box 5.2, are that (1) the tension equals the magnitude of the force pulling on its end and (2) the tension is the same throughout the rope. Thus, if a hanging mass m pulls on the rope with its weight mg, the tension along the entire rope is $T = mg$. For a 4.2 kg hanging mass, the tension is then $T = mg = 41$ N.

The pulley, in equilibrium, must satisfy Newton's second law for the case where $\vec{a} = \vec{0}$. Thus

$$\sum F_x = T_{1x} + T_{2x} + (F_{\text{leg on pulley}})_x = ma_x = 0$$

The tension forces both have the same magnitude T, and both are at angle θ from horizontal. The x-component of the leg force is negative because it's directed to the left. Then Newton's law becomes

$$2T\cos\theta - F_{\text{leg on pulley}} = 0$$

so that

$$\cos\theta = \frac{F_{\text{leg on pulley}}}{2T} = \frac{50\text{ N}}{82\text{ N}} = 0.61$$

$$\theta = \cos^{-1}(0.61) = 52°$$

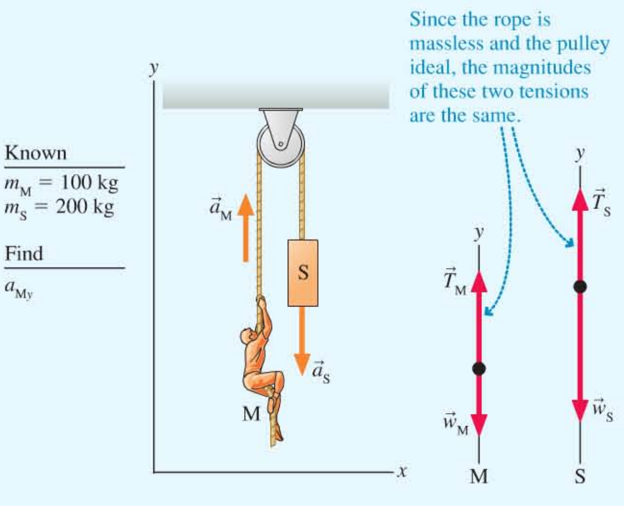
ASSESS The traction force would approach $2mg = 82$ N if angle θ approached zero because the two tensions would pull in parallel. Conversely, the traction force would approach 0 N if θ approached 90°. Because the desired traction force is roughly halfway between 0 N and 82 N, an angle near 45° is reasonable.

EXAMPLE 5.19 Lifting a stage set

A 200 kg set used in a play is stored in the loft above the stage. The rope holding the set passes up and over a pulley, then is tied backstage. The director tells a 100 kg stagehand to lower the set. When he unties the rope, the set falls and the unfortunate man is hoisted into the loft. What is the stagehand's acceleration?

PREPARE **FIGURE 5.38** shows the visual overview. The objects of interest are the stagehand M and the set S, for which we've drawn separate free-body diagrams. Assume a massless rope and a massless, frictionless pulley. Tension forces $\vec{T}_S$ and $\vec{T}_M$ are due to a massless rope going over an ideal pulley, so their magnitudes are the same.

FIGURE 5.38 Visual overview for the stagehand and set.



SOLVE From the two free-body diagrams, we can write Newton's second law in component form. For the man we have

$$\sum F_{My} = T_M - w_M = T_M - m_M g = m_M a_{My}$$

For the set we have

$$\sum F_{Sy} = T_S - w_S = T_S - m_S g = m_S a_{Sy}$$

Only the y-equations are needed. Because the stagehand and the set are connected by a rope, the upward distance traveled by one is the *same* as the downward distance traveled by the other. Thus the *magnitudes* of their accelerations must be the same, but, as Figure 5.38 shows, their *directions* are opposite. We can express this mathematically as $a_{Sy} = -a_{My}$. We also know that the two tension forces have equal magnitudes, which we'll call T. Inserting this information into the above equations gives

$$T - m_M g = m_M a_{My}$$

$$T - m_S g = -m_S a_{My}$$

These are simultaneous equations in the two unknowns T and a_{My}. We can solve for T in the first equation to get

$$T = m_M a_{My} + m_M g$$

Inserting this value of T into the second equation then gives

$$m_M a_{My} + m_M g - m_S g = -m_S a_{My}$$

which we can rewrite as

$$(m_S - m_M)g = (m_S + m_M)a_{My}$$

Finally, we can solve for the hapless stagehand's acceleration:

$$a_{My} = \frac{m_S - m_M}{m_S + m_M} g = \left(\frac{100\text{ kg}}{300\text{ kg}}\right) \times 9.80\text{ m/s}^2 = 3.3\text{ m/s}^2$$

This is also the acceleration with which the set falls. If the rope's tension was needed, we could now find it from $T = m_M a_{My} + m_M g$.

ASSESS If the stagehand weren't holding on, the set would fall with free-fall acceleration g. The stagehand acts as a *counterweight* to reduce the acceleration.

EXAMPLE 5.20 **A not-so-clever bank robbery**

Bank robbers have pushed a 1000 kg safe to a second-story floor-to-ceiling window. They plan to break the window, then lower the safe 3.0 m to their truck. Not being too clever, they stack up 500 kg of furniture, tie a rope between the safe and the furniture, and place the rope over a pulley. Then they push the safe out the window. What is the safe's speed when it hits the truck? The coefficient of kinetic friction between the furniture and the floor is 0.50.

PREPARE The visual overview in **FIGURE 5.39** establishes a coordinate system and defines the symbols that will be needed to calculate the safe's motion. The objects of interest are the safe S and the furniture F, which we will model as particles. We will assume a massless rope and a massless, frictionless pulley; the tension is then the same everywhere in the rope.

SOLVE We can write Newton's second law directly from the free-body diagrams. For the furniture,

$$\sum F_{Fx} = T_F - f_k = T - f_k = m_F a_{Fx}$$

$$\sum F_{Fy} = n - w_F = n - m_F g = 0$$

And for the safe,

$$\sum F_{Sy} = T_S - w_S = T - m_S g = m_S a_{Sy}$$

The safe and the furniture are tied together, so their accelerations have the same magnitude. But as the furniture slides to the right with positive acceleration a_{Fx}, the safe falls in the negative y-direction, so its acceleration a_{Sy} is negative; we can express this mathematically as $a_{Fx} = -a_{Sy}$. We also have made use of the fact that $T_S = T_F = T$. We have one additional piece of information, the model of kinetic friction:

$$f_k = \mu_k n = \mu_k m_F g$$

where we used the y-equation of the furniture to deduce that $n = m_F g$. Substitute this result for f_k into the x-equation of the furniture, then rewrite the furniture's x-equation and the safe's y-equation:

$$T - \mu_k m_F g = -m_F a_{Sy}$$

$$T - m_S g = m_S a_{Sy}$$

We have succeeded in reducing our knowledge to two simultaneous equations in the two unknowns a_{Sy} and T. We subtract the second equation from the first to eliminate T:

$$(m_S - \mu_k m_F)g = -(m_S + m_F)a_{Sy}$$

Finally, we can solve for the safe's acceleration:

$$a_{Sy} = -\left(\frac{m_S - \mu_k m_F}{m_S + m_F}\right)g$$

$$= -\frac{1000\text{ kg} - 0.5(500\text{ kg})}{1000\text{ kg} + 500\text{ kg}} \times 9.80\text{ m/s}^2 = -4.9\text{ m/s}^2$$

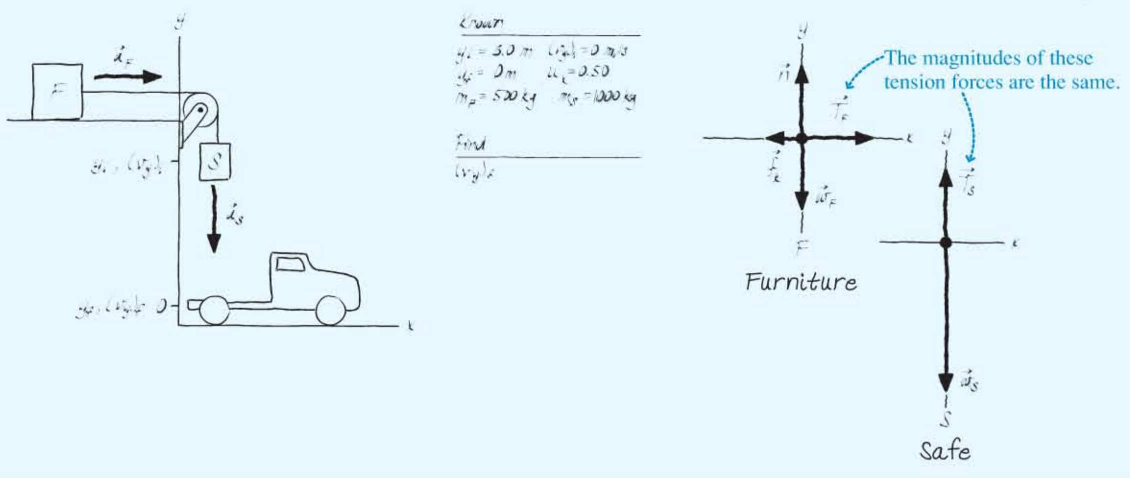
Now we need to calculate the kinematics of the falling safe. Because the time of the fall is not known or needed, we can use

$$(v_y)_f^2 = (v_y)_i^2 + 2a_{Sy}\,\Delta y = 0 + 2a_{Sy}(y_f - y_i) = -2a_{Sy}y_i$$

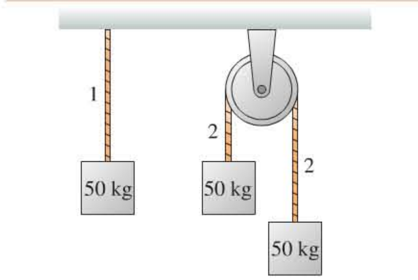
$$(v_y)_f = \sqrt{-2a_{Sy}y_i} = \sqrt{-2(-4.9\text{ m/s}^2)(3.0\text{ m})} = 5.4\text{ m/s}$$

The value of $(v_y)_f$ is negative, but we only needed to find the speed, so we took the absolute value. It seems unlikely that the truck will survive the impact of the 1000 kg safe!

FIGURE 5.39 Visual overview of the furniture and falling safe.



Newton's three laws form the cornerstone of the science of mechanics. These laws allowed scientists to understand many diverse phenomena, from the motion of a raindrop to the orbits of the planets. These laws were so precise at predicting motion that they went unchallenged for well over two hundred years. At the beginning of the twentieth century, however, it began to be apparent that the laws of mechanics and the laws of electricity and magnetism were somehow inconsistent. Bringing these two apparently disconnected theories into harmony required the genius of a young patent clerk named Albert Einstein. In doing so, he shook the foundations not only of Newtonian mechanics but also of our very notions of space and time. We will continue to develop Newtonian mechanics for the next few chapters because of its tremendous importance to the physics of everyday life. But it's worth keeping in the back of your mind that Newton's laws aren't the ultimate statement about motion. Later in this textbook we'll reexamine motion and mechanics from the perspective of Einstein's theory of relativity.



STOP TO THINK 5.6 All three 50 kg blocks are at rest. Is the tension in rope 2 greater than, less than, or equal to the tension in rope 1?

INTEGRATED EXAMPLE 5.21 Stopping distances

A 1500 kg car is traveling at a speed of 30 m/s when the driver slams on the brakes and skids to a halt. Determine the stopping distance if the car is traveling up a 10° slope, down a 10° slope, or on a level road.

PREPARE We'll represent the car as a particle and we'll use the model of kinetic friction. We want to solve the problem only once, not three separate times, so we'll leave the slope angle θ unspecified until the end.

FIGURE 5.40 shows the visual overview. We've shown the car sliding uphill, but these representations work equally well for a level or downhill slide if we let θ be zero or negative, respectively. We've used a tilted coordinate system so that the motion is along the x-axis. The car *skids* to a halt, so we've taken the coefficient of *kinetic* friction for rubber on concrete from Table 5.1.

SOLVE Newton's second law and the model of kinetic friction are

$$\sum F_x = n_x + w_x + (f_k)_x$$
$$= 0 - mg\sin\theta - f_k = ma_x$$
$$\sum F_y = n_y + w_y + (f_k)_y$$
$$= n - mg\cos\theta + 0 = ma_y = 0$$

We've written these equations by "reading" the motion diagram and the free-body diagram. Notice that both components of the weight vector $\vec{w}$ are negative. $a_y = 0$ because the motion is entirely along the x-axis.

The second equation gives $n = mg\cos\theta$. Using this in the friction model, we find $f_k = \mu_k mg\cos\theta$. Inserting this result back into the first equation then gives

$$ma_x = -mg\sin\theta - \mu_k mg\cos\theta$$
$$= -mg(\sin\theta + \mu_k\cos\theta)$$
$$a_x = -g(\sin\theta + \mu_k\cos\theta)$$

This is a constant acceleration. Constant-acceleration kinematics gives

$$(v_x)_f^2 = 0 = (v_x)_i^2 + 2a_x(x_f - x_i) = (v_x)_i^2 + 2a_x x_f$$

which we can solve for the stopping distance x_f:

$$x_f = -\frac{(v_x)_i^2}{2a_x} = \frac{(v_x)_i^2}{2g(\sin\theta + \mu_k\cos\theta)}$$

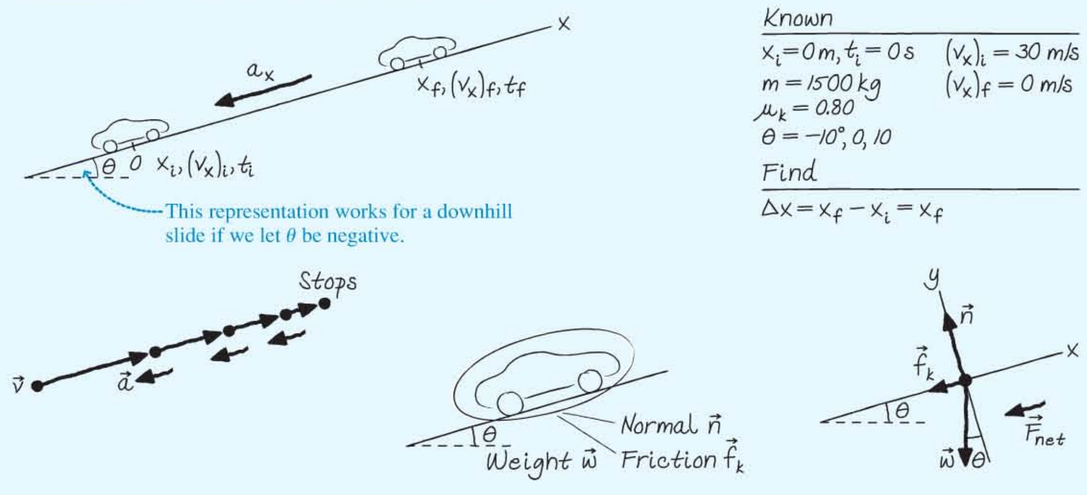
Notice how the minus sign in the expression for a_x canceled the minus sign in the expression for x_f. Evaluating our result at the three different angles gives the stopping distances:

$$x_f = \begin{cases} 48\text{ m} & \theta = 10° & \text{uphill} \\ 57\text{ m} & \theta = 0° & \text{level} \\ 75\text{ m} & \theta = -10° & \text{downhill} \end{cases}$$

The implications are clear about the danger of driving downhill too fast!

ASSESS 30m/s $\approx$ 60 mph and 57m $\approx$ 180 feet on a level surface. These are similar to the stopping distances you learned when you got your driver's license, so the results seem reasonable. Additional confirmation comes from noting that the expression for a_x becomes $-g\sin\theta$ if $\mu_k = 0$. This is what you learned in Chapter 3 for the acceleration on a frictionless inclined plane.

FIGURE 5.40 Visual overview for a skidding car.



SUMMARY

The goal of Chapter 5 has been to learn how to solve problems about motion in a straight line.

GENERAL STRATEGY

All examples in this chapter follow a three-part strategy. You'll become a better problem solver if you adhere to it as you do the homework problems. The *Dynamics Worksheets* in the *Student Workbook* will help you structure your work in this way.

Equilibrium Problems

Object at rest or moving at constant velocity.

PREPARE Make simplifying assumptions.

- Check that the object is either at rest or moving with constant velocity ($\vec{a} = \vec{0}$).
- Identify forces and show them on a free-body diagram.

SOLVE Use Newton's second law in component form:

$$\sum F_x = ma_x = 0$$
$$\sum F_y = ma_y = 0$$

"Read" the components from the free-body diagram.

ASSESS Is your result reasonable?

Dynamics Problems

Object accelerating.

PREPARE Make simplifying assumptions. Make a **visual overview:**

- Sketch a pictorial representation.
- Identify known quantities and what the problem is trying to find.
- Identify all forces and show them on a free-body diagram.

SOLVE Use Newton's second law in component form:

$$\sum F_x = ma_x \text{ and } \sum F_y = ma_y$$

"Read" the components of the vectors from the free-body diagram. If needed, use kinematics to find positions and velocities.

ASSESS Is your result reasonable?

Objects in Contact

Two or more objects interacting.

PREPARE Make a **visual overview:**

- Sketch a pictorial representation.
- Identify all forces acting on *each* object.
- Identify action/reaction pairs of forces acting on objects in the system.
- Draw a *separate* free-body diagram for each object.

SOLVE Write Newton's second law for each object. Use Newton's third law to equate the magnitudes of action/reaction pairs. Determine how the accelerations of the objects are related to each other.

ASSESS Is your result reasonable?

IMPORTANT CONCEPTS

Specific information about three important forces:

Weight $\vec{w} = (mg, \text{downward})$

Friction $\vec{f}_s = (0 \text{ to } \mu_s n, \text{direction as necessary to prevent motion})$

$\vec{f}_k = (\mu_k n, \text{direction opposite the motion})$

$\vec{f}_r = (\mu_r n, \text{direction opposite the motion})$

Drag $\vec{D} \approx (\frac{1}{4}\rho A v^2, \text{direction opposite the motion})$ for motion in air

Newton's laws are vector expressions. You must write them out by **components:**

$$(F_{net})_x = \sum F_x = ma_x$$
$$(F_{net})_y = \sum F_y = ma_y$$

For equilibrium problems, $a_x = 0$ and $a_y = 0$.

APPLICATIONS

Apparent weight is the magnitude of the contact force supporting an object. It is what a scale would read, and it is your sensation of weight:

$$w_{app} = m(g + a_y)$$

Apparent weight equals your true weight $w = mg$ only when $a_y = 0$.

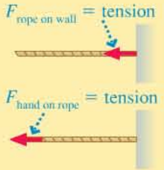
A falling object reaches **terminal speed**

$$v_{term} \approx \sqrt{\frac{4mg}{\rho A}}$$

Terminal speed is reached when the drag force exactly balances the weight force: $\vec{a} = \vec{0}$.

Strings and pulleys

- A string or rope pulls what it's connected to with a force equal to its tension.
- The tension in a rope is equal to the force pulling on the rope.
- The tension in a massless rope is the same at all points in the rope.
- Tension does not change when a rope passes over a massless, frictionless pulley.



For homework assigned on MasteringPhysics, go to www.masteringphysics.com

Problem difficulty is labeled as | (straightforward) to |||| (challenging).

Problems labeled can be done on a Workbook Dynamics Worksheet; INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

QUESTIONS

Conceptual Questions

1. An object is subject to two forces that do not point in opposite directions. Is it possible to choose their magnitudes so that the object is in equilibrium? Explain.
2. Are the objects described here in static equilibrium, dynamic equilibrium, or not in equilibrium at all?
   a. A girder is lifted at constant speed by a crane.
   b. A girder is lowered by a crane. It is slowing down.
   c. You're straining to hold a 200 lb barbell over your head.
   d. A jet plane has reached its cruising speed and altitude.
   e. A rock is falling into the Grand Canyon.
   f. A box in the back of a truck doesn't slide as the truck stops.
3. What forces are acting on you right now? What net force is acting on you right now?
4. Decide whether each of the following is true or false. Give a reason!
   a. The mass of an object depends on its location.
   b. The weight of an object depends on its location.
   c. Mass and weight describe the same thing in different units.
5. An astronaut takes his bathroom scale to the moon and then stands on it. Is the reading of the scale his true weight? Explain.
6. A light block of mass m and a heavy block of mass M are attached to the ends of a rope. A student holds the heavier block and lets the lighter block hang below it, as shown in Figure Q5.6. Then she lets go. Air resistance can be neglected.
   a. What is the tension in the rope while the blocks are falling, before either hits the ground?
   b. Would your answer be different if she had been holding the lighter block initially?

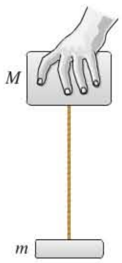


FIGURE Q5.6

7. Four balls are thrown straight up. Figure Q5.7 is a "snapshot" showing their velocities. They have the same size but different mass. Air resistance is negligible. Rank in order, from largest to smallest, the magnitudes of the net forces, $F_{net\,1}$, $F_{net\,2}$, $F_{net\,3}$, $F_{net\,4}$, acting on the balls. Some may be equal. Give your answer in the form A > B = C > D, and state your reasoning.

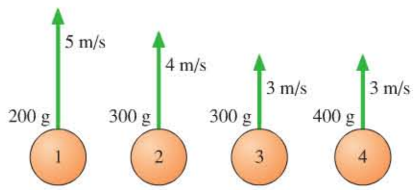


FIGURE Q5.7

8. Suppose you attempt to pour out 100 g of salt, using a pan balance for measurements, while in an elevator that is accelerating upward. Will the quantity of salt be too much, too little, or the correct amount? Explain.
9. a. Can the normal force on an object be directed horizontally? If not, why not? If so, provide an example.
   b. Can the normal force on an object be directed downward? If not, why not? If so, provide an example.
10. A ball is thrown straight up. Taking the drag force of air into account, does it take longer for the ball to travel to the top of its motion or for it to fall back down again?
11. Three objects move through the air as shown in Figure Q5.11. Rank in order, from largest to smallest, the three drag forces D_1, D_2, and D_3. Some may be equal. Give your answer in the form A > B = C and state your reasoning.

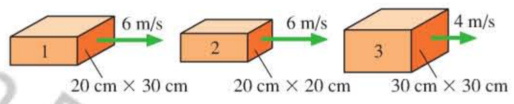


FIGURE Q5.11

12. A skydiver is falling at her terminal speed. Right after she opens her parachute, which has a very large area, what is the direction of the net force on her?
13. Raindrops can fall at different speeds; some fall quite quickly, others quite slowly. Why might this be true?
14. An airplane moves through the air at a constant speed. The jet engine's thrust applies a force in the direction of motion. Reducing thrust will cause the plane to fly at a slower—but still constant—speed. Explain why this is so.
15. Is it possible for an object to travel in air faster than its terminal speed? If not, why not? If so, explain how this might happen.

For Questions 16 through 19, determine the tension in the rope at the point indicated with a dot.

- All objects are at rest.
- The strings and pulleys are massless, and the pulleys are frictionless.

16.

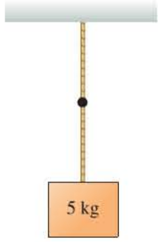


FIGURE Q5.16

17.

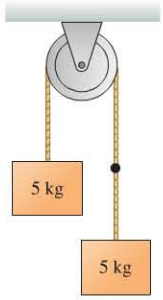


FIGURE Q5.17

18.

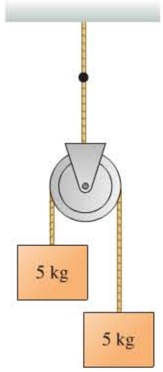


FIGURE Q5.18

19.

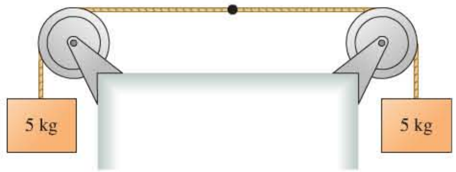


FIGURE Q5.19

20. The floor is frictionless. In which direction is the kinetic friction force on block 1 in Figure Q5.20? On block 2? Explain.

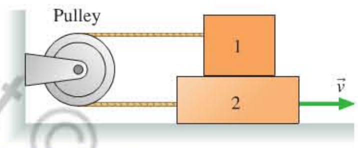


FIGURE Q5.20

Multiple-Choice Questions

21. The wood block in Figure Q5.21 is at rest on a wood ramp. In which direction is the static friction force on block 1?
   A. Up the slope.
   B. Down the slope.
   C. The friction force is zero.
   D. There's not enough information to tell.

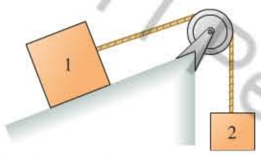


FIGURE Q5.21

22. A 2.0 kg ball is suspended by two light strings as shown in Figure Q5.22. What is the tension T in the angled string?
   A. 9.5 N B. 15 N C. 20 N
   D. 26 N E. 30 N

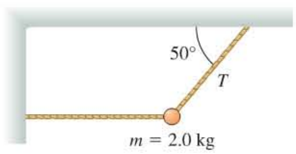


FIGURE Q5.22

23. While standing in a low tunnel, you raise your arms and push against the ceiling with a force of 100 N. Your mass is 70 kg.
   a. What force does the ceiling exert on you?
      A. 10 N B. 100 N C. 690 N
      D. 790 N E. 980 N
   b. What force does the floor exert on you?
      A. 10 N B. 100 N C. 690 N
      D. 790 N E. 980 N

24. A 5.0 kg dog sits on the floor of an elevator that is accelerating *downward* at 1.20 m/s^2.
   a. What is the magnitude of the normal force of the elevator floor on the dog?
      A. 34 N B. 43 N C. 49 N
      D. 55 N E. 74 N
   b. What is the magnitude of the force of the dog on the elevator floor?
      A. 4.2 N B. 49 N C. 55 N
      D. 43 N E. 74 N

25. A 3.0 kg puck slides due east on a horizontal frictionless surface at a constant speed of 4.5 m/s. Then a force of magnitude 6.0 N, directed due north, is applied for 1.5 s. Afterward,
   a. What is the northward component of the puck's velocity?
      A. 0.50 m/s B. 2.0 m/s C. 3.0 m/s
      D. 4.0 m/s E. 4.5 m/s
   b. What is the speed of the puck?
      A. 4.9 m/s B. 5.4 m/s C. 6.2 m/s
      D. 7.5 m/s E. 11 m/s

26. A rocket in space, initially at rest, fires its main engines at a constant thrust. As it burns fuel, the mass of the rocket decreases. Which of the graphs in Figure Q5.26 best represents the velocity of the rocket as a function of time?

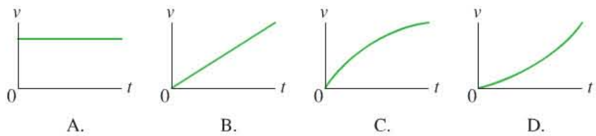


FIGURE Q5.26

27. Eric has a mass of 60 kg. He is standing on a scale in an elevator that is accelerating downward at 1.7 m/s^2. What is the approximate reading on the scale?
   A. 0 N B. 400 N C. 500 N D. 600 N

28. The two blocks in Figure Q5.28 are at rest on frictionless surfaces. What must be the mass of the right block in order that the two blocks remain stationary?
   A. 4.9 kg B. 6.1 kg C. 7.9 kg
   D. 9.8 kg E. 12 kg

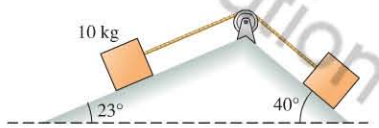


FIGURE Q5.28

29. A football player at practice pushes a 60 kg blocking sled across the field at a constant speed. The coefficient of kinetic friction between the grass and the sled is 0.30. How much force must he apply to the sled?
   A. 18 N B. 60 N C. 180 N D. 600 N

30. Two football players are pushing a 60 kg blocking sled across the field at a constant speed of 2.0 m/s. The coefficient of kinetic friction between the grass and the sled is 0.30. Once they stop pushing, how far will the sled slide before coming to rest?
   A. 0.20 m B. 0.68 m C. 1.0 m D. 6.6 m

31. Land Rover ads used to claim that their vehicles could climb a slope of 45°. For this to be possible, what must be the minimum coefficient of static friction between the vehicle's tires and the road?
   A. 0.5 B. 0.7 C. 0.9 D. 1.0

32. A truck is traveling at 30 m/s on a slippery road. The driver slams on the brakes and the truck starts to skid. If the coefficient of kinetic friction between the tires and the road is 0.20, how far will the truck skid before stopping?
   A. 230 m B. 300 m C. 450 m D. 680 m

PROBLEMS

Section 5.1 Equilibrium

1. | The three ropes in Figure P5.1 are tied to a small, very light ring. Two of the ropes are anchored to walls at right angles, and the third rope pulls as shown. What are T_1 and T_2, the magnitudes of the tension forces in the first two ropes?

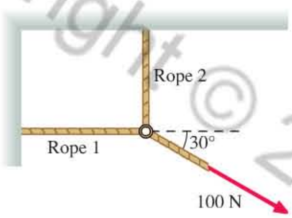


FIGURE P5.1

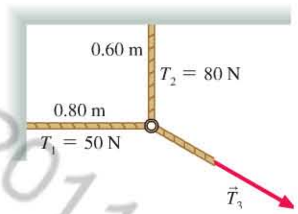


FIGURE P5.2

2. ||| The three ropes in Figure P5.2 are tied to a small, very light ring. Two of these ropes are anchored to walls at right angles with the tensions shown in the figure. What are the magnitude and direction of the tension $\vec{T}_3$ in the third rope?

3. |||| A 20 kg loudspeaker is suspended 2.0 m below the ceiling by two cables that are each 30° from vertical. What is the tension in the cables?

4. || A 1000 kg steel beam is supported by the two ropes shown in Figure P5.4. Each rope can support a maximum sustained tension of 5600 N. Do the ropes break?

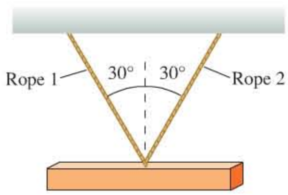


FIGURE P5.4

5. | A cable is used to raise a 25 kg urn from an underwater archeological site. There is a 25 N drag force from the water as the urn is raised at a constant speed. What is the tension in the cable?

6. |||| BIO When you bend your knee, the quadriceps muscle is stretched. This increases the tension in the quadriceps tendon attached to your kneecap (patella), which, in turn, increases the tension in the patella tendon that attaches your kneecap to your lower leg bone (tibia). Simultaneously, the end of your upper leg bone (femur) pushes outward on the patella. Figure P5.6 shows how these parts of a knee joint are arranged. What size force does the femur exert on the kneecap if the tendons are oriented as in the figure and the tension in each tendon is 60 N?

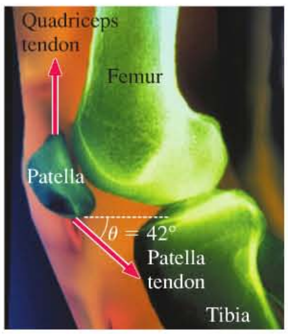


FIGURE P5.6

7. || The two angled ropes used to support the crate in Figure P5.7 can withstand a maximum tension of 1500 N before they break. What is the largest mass the ropes can support?

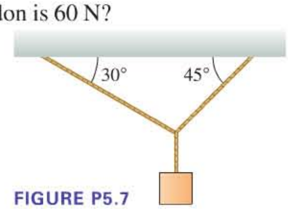


FIGURE P5.7

Section 5.2 Dynamics and Newton's Second Law

8. || A force with x-component F_x acts on a 500 g object as it moves along the x-axis. The object's acceleration graph (a_x versus t) is shown in Figure P5.8. Draw a graph of F_x versus t.

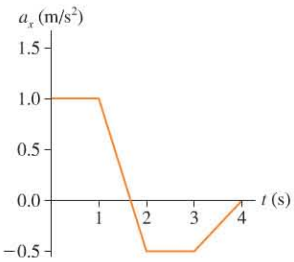


FIGURE P5.8

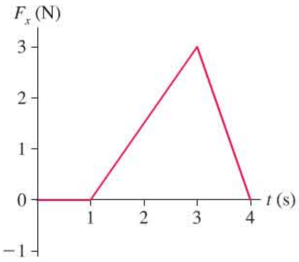


FIGURE P5.9

9. || A force with x-component F_x acts on a 2.0 kg object as it moves along the x-axis. A graph of F_x versus t is shown in Figure P5.9. Draw an acceleration graph (a_x versus t) for this object.

10. | A force with x-component F_x acts on a 500 g object as it moves along the x-axis. A graph of F_x versus t is shown in Figure P5.10. Draw an acceleration graph (a_x versus t) for this object.

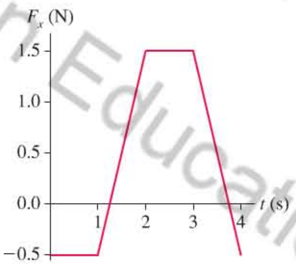


FIGURE P5.10

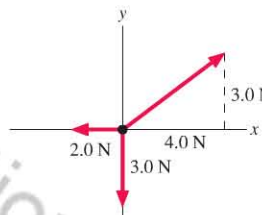


FIGURE P5.11

11. || The forces in Figure P5.11 are acting on a 2.0 kg object. Find the values of a_x and a_y, the x- and y-components of the object's acceleration.

12. | The forces in Figure P5.12 are acting on a 2.0 kg object. Find the values of a_x and a_y, the x- and y-components of the object's acceleration.

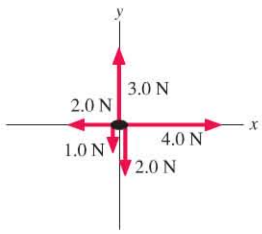


FIGURE P5.12

13. | A horizontal rope is tied to a 50 kg box on frictionless ice. What is the tension in the rope if
    a. The box is at rest?
    b. The box moves at a steady 5.0 m/s?
    c. The box has $v_x = 5.0$ m/s and $a_x = 5.0$ m/s^2?

14. |||| A crate pushed along the floor with velocity $\vec{v}_i$ slides a distance d after the pushing force is removed.
    a. If the mass of the crate is doubled but the initial velocity is not changed, what distance does the crate slide before stopping? Explain.
    b. If the initial velocity of the crate is doubled to $2\vec{v}_i$ but the mass is not changed, what distance does the crate slide before stopping? Explain.

15. || In a head-on collision, a car stops in 0.10 s from a speed of 14 m/s. The driver has a mass of 70 kg, and is, fortunately, tightly strapped into his seat. What force is applied to the driver by his seat belt during that fraction of a second?

Section 5.3 Mass and Weight

16. An astronaut's weight on earth is 800 N. What is his weight on Mars, where $g = 3.76\ \text{m/s}^2$?
17. A woman has a mass of 55.0 kg.
    a. What is her weight on earth?
    b. What are her mass and her weight on the moon, where $g = 1.62\ \text{m/s}^2$?
18. A box with a 75 kg passenger inside is launched straight up into the air by a giant rubber band. After the box has left the rubber band but is still moving *upward,*
    a. What is the passenger's true weight?
    b. What is the passenger's apparent weight?
19. a. How much force does an 80 kg astronaut exert on his chair while sitting at rest on the launch pad?
    b. How much force does the astronaut exert on his chair while accelerating straight up at $10\ \text{m/s}^2$?
20. It takes the elevator in a skyscraper 4.0 s to reach its cruising speed of 10 m/s. A 60 kg passenger gets aboard on the ground floor. What is the passenger's apparent weight
    a. Before the elevator starts moving?
    b. While the elevator is speeding up?
    c. After the elevator reaches its cruising speed?
21. Zach, whose mass is 80 kg, is in an elevator descending at 10 m/s. The elevator takes 3.0 s to brake to a stop at the first floor.
    a. What is Zach's apparent weight before the elevator starts braking?
    b. What is Zach's apparent weight while the elevator is braking?
22. INT Figure P5.22 shows the velocity graph of a 75 kg passenger in an elevator. What is the passenger's apparent weight at $t = 1.0$ s? At 5.0 s? At 9.0 s?

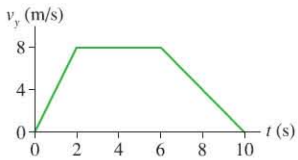


FIGURE P5.22

Section 5.4 Normal Forces

23. a. A 0.60 kg bullfrog is sitting at rest on a level log. How large is the normal force of the log on the bullfrog?
    b. A second 0.60 kg bullfrog is on a log tilted 30° above horizontal. How large is the normal force of the log on this bullfrog?
24. A 23 kg child goes down a straight slide inclined 38° above horizontal. The child is acted on by his weight, the normal force from the slide, and kinetic friction.
    a. Draw a free-body diagram of the child.
    b. How large is the normal force of the slide on the child?

Section 5.5 Friction

25. Bonnie and Clyde are sliding a 300 kg bank safe across the floor to their getaway car. The safe slides with a constant speed if Clyde pushes from behind with 385 N of force while Bonnie pulls forward on a rope with 350 N of force. What is the safe's coefficient of kinetic friction on the bank floor?
26. A 4000 kg truck is parked on a 15° slope. How big is the friction force on the truck?
27. A 1000 kg car traveling at a speed of 40 m/s skids to a halt on wet concrete where $\mu_k = 0.60$. How long are the skid marks?
28. A stubborn 120 kg mule sits down and refuses to move. To drag the mule to the barn, the exasperated farmer ties a rope around the mule and pulls with his maximum force of 800 N. The coefficients of friction between the mule and the ground are $\mu_s = 0.80$ and $\mu_k = 0.50$. Is the farmer able to move the mule?
29. A 10 kg crate is placed on a horizontal conveyor belt. The materials are such that $\mu_s = 0.50$ and $\mu_k = 0.30$.
    a. Draw a free-body diagram showing all the forces on the crate if the conveyer belt runs at constant speed.
    b. Draw a free-body diagram showing all the forces on the crate if the conveyer belt is speeding up.
    c. What is the maximum acceleration the belt can have without the crate slipping?
    d. If acceleration of the belt exceeds the value determined in part c, what is the acceleration of the crate?
30. What is the minimum downward force on the box in Figure P5.30 that will keep it from slipping? The coefficients of static and kinetic friction between the box and the floor are 0.35 and 0.25, respectively.

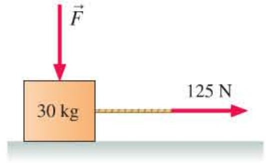


FIGURE P5.30

Section 5.6 Drag

31. What is the drag force on a 1.6-m-wide, 1.4-m-high car traveling at
    a. 10 m/s (≈22 mph)? b. 30 m/s (≈65 mph)?
32. A 22-cm-diameter bowling ball has a terminal speed of 77 m/s. What is the ball's mass?
33. A 75 kg skydiver can be modeled as a rectangular "box" with dimensions 20 cm × 40 cm × 1.8 m. What is his terminal speed if he falls feet first?

Section 5.7 Interacting Objects

34. A 1000 kg car pushes a 2000 kg truck that has a dead battery. When the driver steps on the accelerator, the drive wheels of the car push backward against the ground with a force of 4500 N.
    a. What is the magnitude of the force of the car on the truck?
    b. What is the magnitude of the force of the truck on the car?
35. Blocks with masses of 1.0 kg, 2.0 kg, and 3.0 kg are lined up in a row on a frictionless table. All three are pushed forward by a 12 N force applied to the 1.0 kg block. How much force does the 2.0 kg block exert on (a) the 3.0 kg block and (b) the 1.0 kg block?

Section 5.8 Ropes and Pulleys

36. What is the tension in the rope of Figure P5.36?

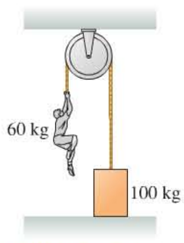


FIGURE P5.36

37. A 2.0-m-long, 500 g rope pulls a 10 kg block of ice across a horizontal, frictionless surface. The block accelerates at $2.0\ \text{m/s}^2$. How much force pulls forward on (a) the block of ice, (b) the rope?
38. Figure P5.38 shows two 1.00 kg blocks connected by a rope. A second rope hangs beneath the lower block. Both ropes have a mass of 250 g. The entire assembly is accelerated upward at $3.00\ \text{m/s}^2$ by force $\vec{F}$.
    a. What is F?
    b. What is the tension at the top end of rope 1?
    c. What is the tension at the bottom end of rope 1?
    d. What is the tension at the top end of rope 2?

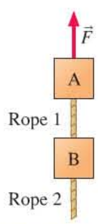


FIGURE P5.38

39. Each of 100 identical blocks sitting on a frictionless surface is connected to the next block by a massless string. The first block is pulled with a force of 100 N.
   a. What is the tension in the string connecting block 100 to block 99?
   b. What is the tension in the string connecting block 50 to block 51?

40. Two blocks on a frictionless table, A and B, are connected by a massless string. When block A is pulled with a certain force, dragging block B, the tension in the string is 24 N. When block B is pulled by the same force, dragging block A, the tension is 18 N. What is the ratio m_A/m_B of the blocks' masses?

General Problems

41. A 500 kg piano is being lowered into position by a crane while two people steady it with ropes pulling to the sides. Bob's rope pulls to the left, 15° below horizontal, with 500 N of tension. Ellen's rope pulls toward the right, 25° below horizontal.
   a. What tension must Ellen maintain in her rope to keep the piano descending vertically at constant speed?
   b. What is the tension in the vertical main cable supporting the piano?

42. Dana has a sports medal suspended by a long ribbon from her rearview mirror. As she accelerates onto the highway, she notices that the medal is hanging at an angle of 10° from the vertical.
   a. Does the medal lean toward or away from the windshield? Explain.
   b. What is her acceleration?

43. INT Figure P5.43 shows the velocity graph of a 2.0 kg object as it moves along the x-axis. What is the net force acting on this object at $t = 1$ s? At 4 s? At 7 s?

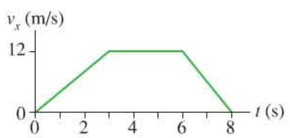


FIGURE P5.43

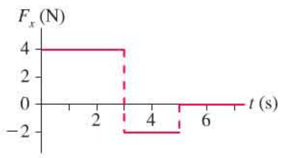


FIGURE P5.44

44. INT Figure P5.44 shows the net force acting on a 2.0 kg object as it moves along the x-axis. The object is at rest at the origin at $t = 0$ s. What are its acceleration and velocity at $t = 6.0$ s?

45. A 50 kg box hangs from a rope. What is the tension in the rope if
   a. The box is at rest?
   b. The box has $v_y = 5.0$ m/s and is speeding up at 5.0 m/s^2?

46. A 50 kg box hangs from a rope. What is the tension in the rope if
   a. The box moves up at a steady 5.0 m/s?
   b. The box has $v_y = 5.0$ m/s and is slowing down at 5.0 m/s^2?

47. BIO Your forehead can withstand a force of about 6.0 kN before fracturing, while your cheekbone can only withstand about 1.3 kN.
   a. If a 140 g baseball strikes your head at 30 m/s and stops in 0.0015 s, what is the magnitude of the ball's acceleration?
   b. What is the magnitude of the force that stops the baseball?
   c. What force does the baseball apply to your head? Explain.
   d. Are you in danger of a fracture if the ball hits you in the forehead? In the cheek?

48. BIO Seat belts and air bags save lives by reducing the forces exerted on the driver and passengers in an automobile collision. Cars are designed with a "crumple zone" in the front of the car. In the event of an impact, the passenger compartment decelerates over a distance of about 1 m as the front of the car crumples. An occupant restrained by seat belts and air bags decelerates with the car. By contrast, an unrestrained occupant keeps moving forward with no loss of speed (Newton's first law!) until hitting the dashboard or windshield, as we saw in Figure 4.2. These are unyielding surfaces, and the unfortunate occupant then decelerates over a distance of only about 5 mm.
   a. A 60 kg person is in a head-on collision. The car's speed at impact is 15 m/s. Estimate the net force on the person if he or she is wearing a seat belt and if the air bag deploys.
   b. Estimate the net force that ultimately stops the person if he or she is not restrained by a seat belt or air bag.
   c. How do these two forces compare to the person's weight?

49. INT Bob, who has a mass of 75 kg, can throw a 500 g rock with a speed of 30 m/s. The distance through which his hand moves as he accelerates the rock forward from rest until he releases it is 1.0 m.
   a. What constant force must Bob exert on the rock to throw it with this speed?
   b. If Bob is standing on frictionless ice, what is his recoil speed after releasing the rock?

50. INT An 80 kg spacewalking astronaut pushes off a 640 kg satellite, exerting a 100 N force for the 0.50 s it takes him to straighten his arms. How far apart are the astronaut and the satellite after 1.0 min?

51. What thrust does a 200 g model rocket need in order to have a vertical acceleration of 10.0 m/s^2
   a. On earth?
   b. On the moon, where $g = 1.62$ m/s^2?

52. A 20,000 kg rocket has a rocket motor that generates 3.0×10^5 N of thrust.
   a. What is the rocket's initial upward acceleration?
   b. At an altitude of 5.0 km the rocket's acceleration has increased to 6.0 m/s^2. What mass of fuel has it burned?

53. You've always wondered about the acceleration of the elevators in the 101-story-tall Empire State Building. One day, while visiting New York, you take your bathroom scales into the elevator and stand on them. The scales read 150 lb as the door closes. The reading varies between 120 lb and 170 lb as the elevator travels 101 floors.
   a. What is the magnitude of the acceleration as the elevator starts upward?
   b. What is the magnitude of the acceleration as the elevator brakes to a stop?

54. A 23 kg child goes down a straight slide inclined 38° above horizontal. The child is acted on by his weight, the normal force from the slide, kinetic friction, and a horizontal rope exerting a 30 N force as shown in Figure P5.54. How large is the normal force of the slide on the child?

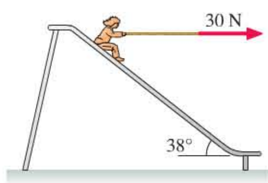


FIGURE P5.54

55. INT Josh starts his sled at the top of a 3.0-m-high hill that has a constant slope of 25°. After reaching the bottom, he slides across a horizontal patch of snow. The hill is frictionless, but the coefficient of kinetic friction between his sled and the snow is 0.05. How far from the base of the hill does he end up?

56. A wood block, after being given a starting push, slides down a wood ramp at a constant speed. What is the angle of the ramp above horizontal?

57. || Researchers often use *force plates* to measure the forces BIO INT that people exert against the floor during movement. A force plate works like a bathroom scale, but it keeps a record of how the reading changes with time. Figure P5.57 shows the data from a force plate as a woman jumps straight up and then lands.
   a. What was the vertical component of her acceleration during push-off?
   b. What was the vertical component of her acceleration while in the air?
   c. What was the vertical component of her acceleration during the landing?
   d. What was her speed as her feet left the force plate?
   e. How high did she jump?

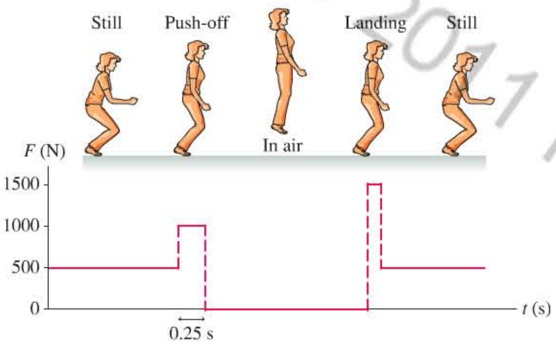


FIGURE P5.57

58. |||| A 77 kg sprinter is running the 100 m dash. At one instant, BIO early in the race, his acceleration is $4.7\ m/s^2$.
   a. What *total* force does the track surface exert on the sprinter? Assume his acceleration is parallel to the ground. Give your answer as a magnitude and an angle with respect to the horizontal.
   b. This force is applied to one foot (the other foot is in the air), which for a fraction of a second is stationary with respect to the track surface. Because the foot is stationary, the net force on it must be zero. Thus the force of the lower leg bone on the foot is equal but opposite to the force of the track on the foot. If the lower leg bone is 60° from horizontal, what are the components of the leg's force on the foot in the directions parallel and perpendicular to the leg? (Force components perpendicular to the leg can cause dislocation of the ankle joint.)

59. |||| Sam, whose mass is 75 kg, takes off across level snow on his jet-powered skis. The skis have a thrust of 200 N and a coefficient of kinetic friction on snow of 0.10. Unfortunately, the skis run out of fuel after only 10 s.
   a. What is Sam's top speed?
   b. How far has Sam traveled when he finally coasts to a stop?

60. ||| A person with compromised pinch strength in his fingers can only exert a normal BIO force of 6.0 N to either side of a pinch-held object, such as the book shown in Figure P5.60. What is the heaviest book he can hold onto vertically before it slips out of his fingers? The coefficient of static friction of the surface between the fingers and the book cover is 0.80.

FIGURE P5.60

61. ||| A 1.0 kg wood block is pressed against a vertical wood wall by a 12 N force as shown in Figure P5.61. If the block is initially at rest, will it move upward, move downward, or stay at rest?

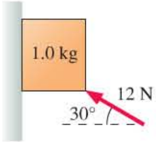


FIGURE P5.61

62. |||| A 50,000 kg locomotive, with steel wheels, is traveling at 10 m/s on steel rails when its engine and brakes both fail. How far will the locomotive roll before it comes to a stop?

63. || An Airbus A320 jetliner has a takeoff mass of 75,000 kg. It reaches its takeoff speed of 82 m/s (180 mph) in 35 s. What is the thrust of the engines? You can neglect air resistance but not rolling friction.

64. |||| A 2.0 kg wood block is launched up a wooden ramp that is inclined at a 35° angle. The block's initial speed is 10 m/s.
   a. What vertical height does the block reach above its starting point?
   b. What speed does it have when it slides back down to its starting point?

65. ||| Two blocks are at rest on a frictionless incline, as shown in Figure P5.65. What are the tensions in the two strings?

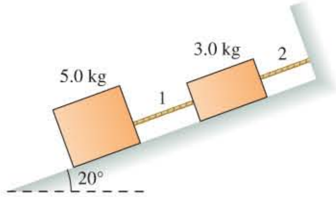


FIGURE P5.65

66. || Two identical blocks are stacked one on top of the other. The bottom block is free to slide on a frictionless surface. The coefficient of static friction between the blocks is 0.35. What is the maximum horizontal force that can be applied to the lower block without the upper block slipping?

67. ||| A wood block is sliding up a wood ramp. If the ramp is very steep, the block will reverse direction at its highest point and slide back down. If the ramp is shallow, the block will stop when it reaches its highest point. What is the smallest ramp angle, measured from the horizontal, for which the block will slide back down?

68. |||| The fastest recorded skydive was by an Air Force officer who jumped from a helium balloon at an elevation of 103,000 ft, three times higher than airliners fly. Because the density of air is so low at these altitudes, he reached a speed of 614 mph at an elevation of 90,000 ft, then gradually slowed as the air became more dense. Assume that he fell in the spread-eagle position of Example 5.15 and that his low-altitude terminal speed is 125 mph. Use this information to determine the density of air at 90,000 ft.

69. ||| A 2.7 g Ping-Pong ball has a diameter of 4.0 cm.
   a. The ball is shot straight up at twice its terminal speed. What is its initial acceleration?
   b. The ball is shot straight down at twice its terminal speed. What is its initial acceleration?

70. ||| Two blocks are connected by a string as in Figure P5.70. What is the upper block's acceleration if the coefficient of kinetic friction between the block and the table is 0.20?

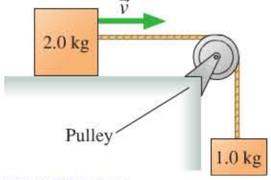


FIGURE P5.70

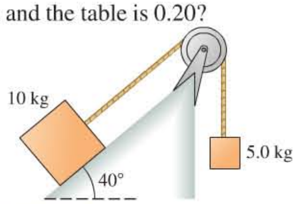


FIGURE P5.71

71. The 10 kg block in Figure P5.71 slides down a frictionless ramp. What is its acceleration?

72. A 2.0 kg wood block is pulled along a wood floor at a steady speed. A second wood block, with mass 3.0 kg, is attached to the first by a horizontal string. What is the magnitude of the force pulling on the first block?

73. A magician pulls a tablecloth out from under some dishes. How far do the dishes move during the 0.25 s it takes to pull out the tablecloth? The coefficient of kinetic friction between the cloth and the dishes is $\mu_k = 0.12$.

74. The 100 kg block in Figure P5.74 takes 6.0 s to reach the floor after being released from rest. What is the mass of the block on the left?

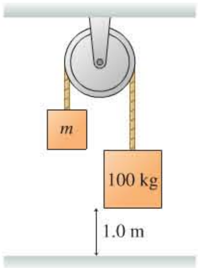


FIGURE P5.74

Problems 75 and 76 show free-body diagrams. For each,

a. Write a realistic dynamics problem for which this is the correct free-body diagram. Your problem should ask a question that can be answered with a value of position or velocity (such as "How far?" or "How fast?"), and should give sufficient information to allow a solution.

b. Solve your problem!

75.

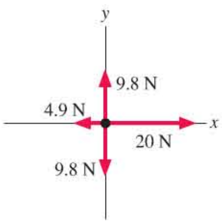


FIGURE P5.75

76.

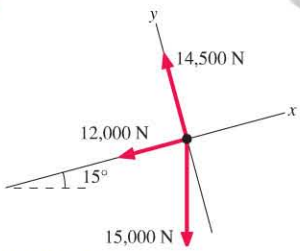


FIGURE P5.76

In Problems 77 through 79 you are given the dynamics equations that are used to solve a problem. For each of these, you are to

a. Write a realistic problem for which these are the correct equations.

b. Draw the free-body diagram and the pictorial representation for your problem.

c. Finish the solution of the problem.

77. $-0.80n = (1500 \text{ kg})a_x$
$n - (1500 \text{ kg})(9.8 \text{ m/s}^2) = 0$

78. $T - 0.2n - (20 \text{ kg})(9.8 \text{ m/s}^2)\sin 20° = (20 \text{ kg})(2.0 \text{ m/s}^2)$
$n - (20 \text{ kg})(9.8 \text{ m/s}^2)\cos 20° = 0$

79. $(100 \text{ N})\cos 30° - f_k = (20 \text{ kg})a_x$
$n + (100 \text{ N})\sin 30° - (20 \text{ kg})(9.8 \text{ m/s}^2) = 0$
$f_k = 0.20n$

Passage Problems

Sliding on the Ice

In the winter sport of curling, players give a 20 kg stone a push across a sheet of ice. The stone moves approximately 40 m before coming to rest. The final position of the stone, in principle, only depends on the initial speed at which it is launched and the force of friction between the ice and the stone, but team members can use brooms to sweep the ice in front of the stone to adjust its speed and trajectory a bit; they must do this without touching the stone. Judicious sweeping can lengthen the travel of the stone by 3 m.

80. A curler pushes a stone to a speed of 3.0 m/s over a time of 2.0 s. Ignoring the force of friction, how much force must the curler apply to the stone to bring it up to speed?
A. 3.0 N B. 15 N C. 30 N D. 150 N

81. The sweepers in a curling competition adjust the trajectory of the stone by
A. Decreasing the coefficient of friction between the stone and the ice.
B. Increasing the coefficient of friction between the stone and the ice.
C. Changing friction from kinetic to static.
D. Changing friction from static to kinetic.

82. Suppose the stone is launched with a speed of 3 m/s and travels 40 m before coming to rest. What is the *approximate* magnitude of the friction force on the stone?
A. 0 N B. 2 N C. 20 N D. 200 N

83. Suppose the stone's mass is increased to 40 kg, but it is launched at the same 3 m/s. Which one of the following is true?
A. The stone would now travel a longer distance before coming to rest.
B. The stone would now travel a shorter distance before coming to rest.
C. The coefficient of friction would now be greater.
D. The force of friction would now be greater.

STOP TO THINK ANSWERS

Stop to Think 5.1: A. The lander is descending and slowing. The acceleration vector points upward, and so $\vec{F}_{net}$ points upward. This can be true only if the thrust has a larger magnitude than the weight.

Stop to Think 5.2: D. When you are in the air, there is *no* contact force supporting you, so your apparent weight is zero: You are weightless.

Stop to Think 5.3: $f_B > f_C = f_D = f_E > f_A$. Situations C, D, and E are all kinetic friction, which does not depend on either velocity or acceleration. Kinetic friction is less than the maximum static friction that is exerted in B. $f_A = 0$ because no friction is needed to keep the object at rest.

Stop to Think 5.4: D. The ball is shot *down* at 30 m/s, so $v_{0y} = -30$ m/s. This exceeds the terminal speed, so the upward drag force is *greater* than the downward weight force. Thus the ball *slows down* even though it is "falling." It will slow until $v_y = -15$ m/s, the terminal velocity, then maintain that velocity.

Stop to Think 5.5: B. $F_{Q\,on\,H} = F_{H\,on\,Q}$ and $F_{P\,on\,Q} = F_{Q\,on\,P}$ because these are action/reaction pairs. Box Q is slowing down and therefore must have a net force to the left. So from Newton's second law we also know that $F_{H\,on\,Q} > F_{P\,on\,Q}$.

Stop to Think 5.6: Equal to. Each block is hanging in equilibrium, with no net force, so the upward tension force is mg.