

3.1 Show that $\log(P(n))$ is $\Theta(\log n)$

$$P(n) = \sum_{i=0}^k c_i n^i \quad c_i \geq 1 \text{ for } 0 \leq i \leq k$$

$$\log(c n^k) \geq \log n$$

$$0 \leq n \leq n^k$$

$$\log(n^k) \geq \log(n)$$

if $k \geq 1$ \therefore valid

$$\log(c n^k) \leq \log(n)$$

$$0 \leq c \leq k$$

$$\log(0 n^k) \leq \log(n) \therefore \text{valid}$$

$\therefore \log(P(n))$ is $\Theta(\log n)$

3.2 $\sum_{k=1}^n \log k$ is $\Theta(n \log n)$

$$\sum_{k=1}^n \log k = \log k + \log k+1 + \dots + \log n$$

$$\lim_{k \rightarrow n} \sum_{k=1}^n \log k = \log((1/n))$$

$$0 \leq \log((1/n)) \leq n \log n \therefore \text{valid}$$

3.3

$$f(n) = n^2$$

$$cnt \leq n^2$$

$$\text{if } n=5 \text{ } cnt=15$$

$$15 \leq 25 \therefore \text{valid}$$

$$cnt \leq n^2$$

$$\text{if } n=0 \text{ } cnt=0$$

$$0 \leq 0 \therefore \text{valid}$$

$\therefore cnt$ is $\Theta(n^2)$