Physics 137B Lecture (Not sure)

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These are notes taken from lectures on Quantum Mechanics delivered by Professor Raúl A. Briceño for UC Berekley's Physics 137B class in the Sprng 2024 semester.

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Recap

- Last time we discussed two particle states:

$$|a,b\rangle_B = \frac{1}{\sqrt{2}}\left(|a\rangle \otimes |b\rangle + |b\rangle \otimes |a\rangle\right)$$
$$|a,b\rangle_F = \frac{1}{\sqrt{2}}\left(|a\rangle \otimes |b\rangle - |b\rangle \otimes |a\rangle\right)$$

- Since the Fermionic wavefunction has to be antisysmmetric, it must be the case that

$$|a,a\rangle_F = 0$$

1.1 Slatter Determinant

If, instead of 2, we have N particles auseful tool for constructing antisymmetric states $\langle\phi_i|\phi_j\rangle = S_{ij}$ is the ***Slatter Determinant***.

$$|\psi\rangle_{A,N} = \frac{1}{\sqrt{N!}}\begin{bmatrix} |\phi_1(\vec{r}_1)\rangle & |\phi_2(\vec{r}_1)\rangle & \cdots & |\phi_N(\vec{r}_1)\rangle \\ |\phi_N(\vec{r}_1)\rangle & |\phi_N(\vec{r}_2)\rangle \cdots |\phi_N(\vec{r}_3)\rangle & & \\ \vdots & & \ddots & \\ |\phi_1(\vec{r}_N)\rangle & |\phi_2(\vec{r}_N)\rangle & \cdots & |\phi_N(\vec{r}_N)\rangle \end{bmatrix}$$

1.2 Non-interacting N particles

[Fill in some stuff from recording]

$$\text{Let's consider :} \quad \begin{array}{c} \text{distinguishable} \\ \text{spinless bosons} \\ \text{spin-1/2 fermions} \end{array}$$

1.2.1 Distinguishable

[Write later]

1.2.2 Spinless Bosons

[Fill later]

Note: This is the starting point for Bose-Einstein condensates (at low temperatures, all of the bosons neter the same energy state.)

1.2.3 Spin-1/2 fermions

This time, it depends on the whether N is odd or even.

Next, let's take the large N limit of a system with N fermions. This gives us the *Fermi Gas Model*.

1.3 Fermi Gas Model

- Large N limit of N non-interacting fermions
- Metals
- Heavy Nuclei
- White Dwarfs and Neutron Stars

Let's assume our fermions live in a cube of side-length L which is so large that our boundary conditions *don't quite matter*. i.e. it doesn't matter much to the behavior in the interior whether we apply periodic boundary conditions or something else.

Let's impose preiodic boundary conditions. Then,

$$\phi(x+L,y,z) = \phi(x,y,z)$$
$$\phi(x,y,z) \sim e^{i(k_x x + k_y y + k_z z)} \text{ where } k_x L = 2\pi n_x$$

It's useful to define the vector

$$\vec{k} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Let's try to count the number of modes which can fit in this box, with the goal of finding the total number of spin-1/2 particles inside the box.

We have

$$\begin{aligned} dn &= 2 \times \mathrm{d}n_x \mathrm{d}n_y \mathrm{d}n_z \\ &= 2 \times \left(\frac{L}{2\pi}\right)^2 \mathrm{d}k_x \mathrm{d}k_y \mathrm{d}k_z \end{aligned}$$

We're going to engage in a slight *oopsie* by assuming that even though our momenta are discrete, the steps between consecutive momenta are infinitessimally small. We're going to take the large N limit, where this assumption does hurt, so it should work out.

[Write motivation for Fermi Energy and Fermi Momentum]

Fermi Energy and Fermi Momentum

> We define the ***Fermi Energy*** to be the largest energy that any particle in the gas takes.

[Write the rest from recording]