Physics Directed Reading Program

UG seminar misc. latex

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1 Abstract Definition of a group

Definition. A set G with some operation $* : G \times G \to G$ is said to be a group if it satisfies the following three conditions:

- There exists an **identity element** $e \in G$ such that for any other $g \in G$ we have
$$g * e = e * g = g$$
- For every $g \in G$, there exists an **inverse** i.e. some other element h such that $g * h = h * g = e$ We denote such an element as g^{-1}
- The map $*$ is **associative** i.e. for any $a, b, c \in G$ we have
$$a * (b * c) = (a * b) * c$$

Definition. Two groups $(G_1, \cdot)$ and $(G_2, \star)$ are **homomorphic** if there exists some function $f : G_1 \to G_2$ which preserves the group structure i.e. for any $x, y \in G_1$

$$f(x \cdot y) = f(x) \star f(y)$$

The map f is called a **Group Homomorphism.**

Definition. If we have a group homomorphism f between $(G_1, \cdot)$ and $(G_2, \star)$ are **homomorphic** which is **bijective** (i.e. one-to-one and onto), then the two groups are **isomorphic** and f is a **group isomorphism**.

Definition. A **Topological Manifold** M is a topological

Definition. We say two paths $\gamma_1, \gamma_2 : [0, 1] \to M$ on a space are **homotopic to each other** if there's a way to go from one to the other **continuously**.

We denote this as $\gamma_1 \sim \gamma_2$.

Definition. (More precise definition): Two maps $\gamma_0, \gamma_1 : [0, 1] \to M$ are homotopic to each other if there exists some continuous map $F : [0, 1] \times [0, 1] \to X$ such that

$$F(x, 0) = \gamma_0(x), \; F(x, 1) = \gamma_0(x)$$

Definition. Given a topological space X and some point $x_0 \in X$, we define the **Fundamental Group** to be the set of **equivalence classes** $[\gamma]$ of loops starting at x_0, $\gamma : [0, 1] \to M$

which are homotopic to each other. That is,

$$\pi_1(X, x_0) = \{[\gamma] \mid \alpha \in [\gamma] \text{ if } \alpha \sim \gamma\}$$

References