Physics 137B Notes

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These are notes taken from lectures on Quantum Mechanics delivered by Professor Raúl A. Briceño for UC Berekley's Physics 137B class in the Sprng 2024 semester.

Contents

1 January 18 - Review and Intro to Symmetries

1.1 Why Quantum?

- It's cool lmao
- Computers [Quantum Computing]
- Applications such as Condensed Matter Physics
- More accurate description of reality than Classical Mechanics
  - 3 of the 4 fundamental forces that we know of are Quantum Mechanical.
  - These are the Strong Nuclear, Weak Nuclear, and Quantum Electrodynamic.

1.2 Topics covered in 137A

- Review of Historical events such as the Photoelectric effect and other precursors to Quantum
- Postulates of QM
- Solve exactly some key examples – such as Free Particle, Particle in a Quantum Harmonic Oscillator, Hydrogen Atom.
  - Unfortunately, most of the problems in nature we want to solve are not solvable exactly.
  - In 137B, one of the key concepts we will introduce in that of *Perturbation Theory*, which will allow us to approximate solutions and their associated errors.

1.3 Review of 1D QM

- A particle is described by its wavefunction $\Psi(x,t)$ and the probability density is given by $P(x,t) = |\Psi(x,t)|^2$.
- The probability of finding the particle in a particular region of space is

$$dxP(x,t) = dx|\Psi(x,t)|^2 = \Psi(x,t)^*\Psi(x,t)$$

- Physically, we require $\int dx|\Psi(x,t)|^2 = 1$. Such a wavefunction is called *normalizable.*
- The wavefunction itself is not something we observe. Instead, our observables are the expectation values of operators. Expectation value of $\hat{\Theta}$ is

$$\langle\hat{\Theta}\rangle = \int dx\Psi(x,t)^*\hat{\Theta}\Psi(x,t)$$

- **Principle of Superposition:** If we have a set $\{\psi_1, \ldots, \psi_n\}$ of solutions which solve the Schroedinger equation, then any linear combination of the ψ_i's will also be a solutions

$$\Psi(x) = \sum_n c_n\psi(x,t)_n$$

- **Time-Dependent Schroedinger Equation:**

write the equation here later

- **Time Independent Schroedinger Equation:** (When the potential does not depend on time)

$$\left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\psi(x) = E\psi(x)$$

- So, if $\{\psi_n\}$ satisfy TISE with wigenenergies $\{E_n\}$, then we know
  - The states $\{\psi_n\}$ are called *Stationary States*.
  - By the Principle of Superposition,

$$\Psi(x,0) = \sum_n c_n \psi_n(x)$$

    describes the total wavefunction at time $t = 0$.
  - To verify that Ψ is a valid wavefunction, we can test its normalizability.

$$\begin{aligned}
1 &= \int dx \Psi(x)^* \Psi(x) \\
&= \int dx \left(\sum_n c_n \psi_n(x) \right)^* \left(\sum_m c_m \psi_m(x) \right) \\
&= \sum_n \sum_m \int dx c_n^* c_m \psi_n(x) \psi(m) \\
&= \sum_n \sum_m \int dx c_n^* c_m \delta_{nm} \\
&= \sum_n |c_n|^2
\end{aligned}$$

  - At $t \neq 0$, we use the propagator to obtain the state

$$\Psi(x,t) = \sum_n c_n e^{-E_n t/h} \psi_n(x)$$

- **Bra and Ket notation:**

$$\begin{aligned}
|\psi\rangle &= \sum_i |i\rangle \underbrace{\langle i|\psi\rangle}_{c_i} \\
&= \sum_i |i\rangle c_i
\end{aligned}$$

  Also, recall that $\sum_i |i\rangle\langle i| = 1$

  Or in the continuous case,

$$\begin{aligned}
|\psi\rangle &= \int dx |i\rangle \underbrace{\langle i|\psi\rangle}_{c_i} \\
&= \int |i\rangle \psi(x)
\end{aligned}$$