Physics 137B Lecture 9

Keshav Balwant Deoskar

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Contents

1 February 5 - Hydrogen Atom

1.1 Review of the Hydrogen Atom

Super quickly, let's go over what we know about the Hydrogen Atom. If any of this is unfamiliar, refer to a Griffiths Intro to QM Chapter 4 or any other popular textbook.

- Recall that the Hydrogen Atom is an example of a ***central potential*** i.e. a Potential with Spherical Symmetry
$$V(\vec{r}) = V(r)$$
- In general, when we have central potentials, we can assume ***separable solutions***
$$\underbrace{\psi(\vec{r})}_{nlm} = \underbrace{R(r)}_{nl}\underbrace{Y(\theta,\phi)}_{lm}$$
where the Radial Wavefunction is characterized by the quantum numbers n, l while the Spherical Wavefunction is characterized by l, m.
- The Schrodinger Equation then separates into radial and spherical parts
$$\text{Radial: } \frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}R(r)\right) - \frac{2mr^2}{\hbar^2}\left(V(r)-E\right)R(r) = l(l+1)R(r)$$
$$\text{Spherical: } \frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial}{\partial\theta}Y(\theta,\phi)\right)\frac{1}{\sin^2(\theta)}\frac{\partial^2 Y(\theta,\phi)}{\partial\phi^2} = -l(l+1)Y(\theta,\phi)$$
- The solutions to the Spherical Equation are called the ***Spherical Harmonics*** and are given by
$$Y_{lm}(\theta,\phi) = \sqrt{\frac{(l-m)!(2l+1)}{(l+m)!}}\frac{e^{i\phi m}}{\sqrt{4\pi}}P_l^m(\theta,\phi)$$
where $P_l^m(\theta,\phi)$ is an associated Legendre Polynomial. They have the following Normalizataion condition
$$\int \mathrm{d}\Omega\; Y^*_{lm}(\theta,\phi)Y_{l'm'}(\theta,\phi) = \delta_{l,l'}\cdot\delta_{m,m'}$$
- The particular central potential which describes the Hydrogen atom is the ***Coulombic Potential***
$$V(r) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r}$$
- Solving the Schrodinger Equation, we find the Eigen-energies are given by
$$E_n = -\frac{n}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)\frac{1}{n^2} = \frac{E_1}{n^2}$$
Notice that the energies only depend on n, so we have degeneracies due to *both* l and n.
- The natural lengthscale for this problem isthe ***Bohr Radius***, and we can express the energies in terms of the Bohr Radius, $a = 4\pi\epsilon_0\hbar^2/(m_e e^2)$ as
$$E_1 = -\frac{\hbar^2}{a^2}\frac{1}{n^2}$$

- The general solutions to the Schrodinger Equation with the Coulombic Potential are given by

$$\psi_{nlm}(r,\theta,\phi) = R_{nl}(r)Y_{lm}(\theta,\phi)$$

  $m \in [-l, l]$ and $0 \leq l < n$

1.2 Degeneracies

We label the state $\psi_{nlm}(r,\theta,\phi)$ as $|nlm\rangle$