Physics 137B Notes

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These are notes taken from lectures on Quantum Mechanics delivered by Professor Raúl A. Briceño for UC Berekley's Physics 137B class in the Sprng 2024 semester.

Contents

1 January 18 - Review and Intro to Symmetries

1.1 Why Quantum?

- It's cool lmao
- Computers [Quantum Computing]
- Applications such as Condensed Matter Physics
- More accurate description of reality than Classical Mechanics
  - 3 of the 4 fundamental forces that we know of are Quantum Mechanical.
  - These are the Strong Nuclear, Weak Nuclear, and Quantum Electrodynamic.

1.2 Topics covered in 137A

- Review of Historical events such as the Photoelectric effect and other precursors to Quantum
- Postulates of QM
- Solve exactly some key examples – such as Free Particle, Particle in a Quantum Harmonic Oscillator, Hydrogen Atom.
  - Unfortunately, most of the problems in nature we want to solve are not solvable exactly.
  - In 137B, one of the key concepts we will introduce in that of *Perturbation Theory*, which will allow us to approximate solutions and their associated errors.

1.3 Review of 1D QM

- A particle is described by its wavefunction $\Psi(x,t)$ and the probability density is given by $P(x,t) = |\Psi(x,t)|^2$.
- The probability of finding the particle in a particular region of space is

$$dxP(x,t) = dx|\Psi(x,t)|^2 = \Psi(x,t)^*\Psi(x,t)$$

- Physically, we require $\int dx|\Psi(x,t)|^2 = 1$. Such a wavefunction is called *normalizable.*
- The wavefunction itself is not something we observe. Instead, our observables are the expectation values of operators. Expectation value of $\hat{\Theta}$ is

$$\langle\hat{\Theta}\rangle = \int dx\Psi(x,t)^*\hat{\Theta}\Psi(x,t)$$

- **Principle of Superposition:** If we have a set $\{\psi_1, \ldots, \psi_n\}$ of solutions which solve the Schroedinger equation, then any linear combination of the ψ_i's will also be a solutions

$$\Psi(x) = \sum_n c_n\psi(x,t)_n$$

- **Time-Dependent Schroedinger Equation:**

write the equation here later

- **Time Independent Schroedinger Equation:** (When the potential does not depend on time)

$$\left(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\psi(x) = E\psi(x)$$

- So, if $\{\psi_n\}$ satisfy TISE with wigenenergies $\{E_n\}$, then we know
  - The states $\{\psi_n\}$ are called *Stationary States*.
  - By the Principle of Superposition,

    $$\Psi(x,0) = \sum_n c_n \psi_n(x)$$

    describes the total wavefunction at time $t = 0$.
  - To verify that Ψ is a valid wavefunction, we can test its normalizability.

    $$\begin{aligned}
    1 &= \int dx \Psi(x)^* \Psi(x) \\
    &= \int dx \left(\sum_n c_n \psi_n(x) \right)^* \left(\sum_m c_m \psi_m(x) \right) \\
    &= \sum_n \sum_m \int dx c_n^* c_m \psi_n(x) \psi(m) \\
    &= \sum_n \sum_m \int dx c_n^* c_m \delta_{nm} \\
    &= \sum_n |c_n|^2
    \end{aligned}$$

  - At $t \neq 0$, we use the propagator to obtain the state

    $$\Psi(x,t) = \sum_n c_n e^{-E_n t/h} \psi_n(x)$$

- **Bra and Ket notation:**

  $$\begin{aligned}
  |\psi\rangle &= \sum_i |i\rangle \underbrace{\langle i|\psi\rangle}_{c_i} \\
  &= \sum_i |i\rangle c_i
  \end{aligned}$$

  $$\text{Also, recall that} \sum_i |i\rangle\langle i| = 1$$

  Or in the continuous case,

  $$\begin{aligned}
  |\psi\rangle &= \int dx |i\rangle \underbrace{\langle i|\psi\rangle}_{c_i} \\
  &= \int |i\rangle \psi(x)
  \end{aligned}$$

2 January 19 - Review (Continuation)

- Dirac Ket $|\psi\rangle$

$$\langle\psi_\alpha|\psi_\beta\rangle = \int dx \psi_\alpha^*(x)\psi_\beta(x)$$

- Observables are associated with Hermitian operators

$$\hat{\Theta}^\dagger = \hat{\Theta}$$

and the expermientally measured values are the expectation values of operators

$$\langle\hat{\Theta}\rangle = b$$

Hermitian operators have real eigenvalues.

- If $\underbrace{\hat{Q}}_{hermitian} |a\rangle = a|a\rangle$ and $\underbrace{\hat{Q}}_{hermitian} |b\rangle = b|b\rangle$ with $a \neq b$ then

$$\langle a|b\rangle = 0$$

- Eigenvectors of $\hat{Q}$ form a complete basis.

In this class we will primarily focus of operators that are Hermitian and/or unitary.

- Untiary Operators:

$$\hat{Q}^\dagger = \hat{Q}^{-1}$$

$$\begin{aligned} \implies \hat{Q}^\dagger\hat{Q} &= \hat{Q}^{-1}\hat{Q} = \mathbf{1} \\ &= \hat{Q}\underbrace{\hat{Q}^\dagger}_{\hat{Q}^{-1}} \end{aligned}$$

- Two observbles arae simultaneously diagonalizeable if ther operators textbfcommute i.e.

$$[\hat{A}, \hat{B}] = 0$$

such that

$$\hat{A}|a,b\rangle = a|a,b\rangle$$
$$\hat{A}|a,b\rangle = b|a,b\rangle$$

- Generalized Uncertainty principle:

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{\langle[A,B]\rangle}{2i}\right)^2$$

- Ehrenfest Theorem:
-

$$\frac{d}{dt}\langle\hat{Q}\rangle = \frac{i}{h}\langle[\hat{H},\hat{Q}]\rangle + \langle\frac{\partial\hat{Q}}{\partial t}\rangle$$

- THis will be important later when consdering symmetries and conservations.

2.1 Review of 3D QM:

- $|\vec{r}\rangle = |x\rangle \otimes |y\rangle \otimes |z\rangle$ where $\langle x|x'\rangle\delta(x-x')$.
- The inner product of general position space kets is just

$$\begin{aligned}\langle\vec{r}|\vec{r}'\rangle &= \langle x|x'\rangle\langle y|y'\rangle\langle z|z'\rangle \\ &= \delta(x-x')\delta(y-y')\delta(z-z').\end{aligned}$$

- $\psi(\vec{r},t) = \langle\vec{r}|\psi(t)\rangle$.
- The TISE in three dimensions is

$$writelater$$

- For a central potential, i.e. $V(\vec{r}) = V(r)$ (no angular dependence) we can get the general solution

$$\psi(\vec{r}) = R(r)Y(\theta,\phi)$$

- $Y_{lm}(\theta,\phi) = \langle\theta,\phi|l,m\rangle$.

2.2 Introduction to Symmetries

This is one of the most fundamental concepts in physics. Three of the four fundamental forces are heavily based on symmetry arguments. A theorem of great importance is **Noether's Theorem**.

Noether's Theorem

There is a correspondence between Symmetries and Conservation laws.

For instance, the facts that charge and energy are conversed in nuclear reactions are due to this theorem. Rather than being postulates, as they were in our earlier studies of Classical Mechanics, these conservations are the results of symmetries.

In 137B

We wll encounter many situations where the Hamiltonian is of the form

$$\hat{H}(\lambda) = \underbrace{\hat{H}_0}_{\text{known sol.}} + \lambda\delta\hat{H}$$

and generally there wll be some **degeneracies** i.e. spaces of Eigenvectors such that $|n_1\rangle, |n_2\rangle$ have the same energy $E_{n_1} = E_{n_2}$.

Then, we will usually introduce a **Perturbation**. [Insert graph]

What is a symmetry in Physics?

- A symmetry in Physics is an action/transformation that leaves a system unchanged or invariant.
- There can be **continuous** or **discrete** symmetries.
- Draw images and give example of continuous → circle and discrete → square.

Some more examples of continuous symmetries that we will be considering are

- Translations in space-time.

- momentum, energy.
- Rotations in space.
  - orbital angular momentum ($\vec{L}$).
  - spin.

and some discrete ones are

- Parity.
- Time reversal.

Active vs. Passive Transformations

Transformations can be thought of as acting on the state $|\psi\rangle$ or on the operator $\hat{\Theta}$, but physcially both are **equivalent**.

$$\implies \langle\psi'|\hat{\Theta}|\psi'\rangle = \langle\psi|\hat{0}'|\psi\rangle$$

[Draw images from lecture of function on graph example.]

- Let's define the transformation operator as $\hat{U}$ such that

$$\begin{aligned} |\psi'\rangle &= \hat{U}|\psi\rangle \\ \implies \langle\psi'| &= \langle\psi|\hat{U}^\dagger \end{aligned}$$

- This tells us that $\hat{U}$ is unitary. Why?

$$\begin{aligned} 1 &= \langle\psi|\psi\rangle \\ &= \langle\psi'|\psi'\rangle \\ &= \langle\psi|\underbrace{\hat{U}^\dagger\hat{U}}_{\mathbf{1}}|\psi\rangle \end{aligned}$$

  and

$$\langle\psi|\hat{U}^\dagger\hat{\Theta}\hat{U}|\psi\rangle = \langle\psi|\hat{Q}'|\psi\rangle$$

$$\implies \hat{\Theta}' = \hat{U}^\dagger\hat{\Theta}\hat{U}$$

  which tells us that $\hat{U}$ is a unitary operator.

- Now, the system is **invariant** under the transformation $|\psi\rangle \to \hat{U}|\psi\rangle$.