Physics 137A: Quantum Mechanics	**Fall 2023**

PSET 06, Due October 16

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Problem 1:

We have a wavefunction

$$\psi(x) = \frac{A}{x^2 + a^2}, \;\; for \;\; (-\infty < x < \infty)$$

for positive constants A and a.

(a) First, we want to find A by normalizing the wavefunction.

We know

$$\int_{\infty}^{\infty} dx \, |\psi(x)|^2 = 1$$

So,

$$\begin{aligned}
&\int_{\infty}^{\infty} dx \, |\psi(x)|^2 = 1 \\
\implies &\int_{\infty}^{\infty} dx \, \psi(x)^* \psi(x) = 1 \\
\implies &\int_{\infty}^{\infty} dx \, \left(\frac{A}{x^2 + a^2}\right)^* \left(\frac{A}{x^2 + a^2}\right) = 1 \\
\implies &\int_{\infty}^{\infty} dx \, \left(\frac{A}{x^2 + a^2}\right)^2 = 1 \\
\implies A^2 \cdot &\int_{\infty}^{\infty} dx \, \left(\frac{1}{x^2 + a^2}\right)^2 = 1
\end{aligned}$$

To evaluate this integral, we employ a slightly round-about method. We have the following identity:

$$\int$$

Problem 2:

We have a particle in an infinite square well with the following position wavefunction

$$\Psi(x, 0) = \psi(x) = \begin{cases} Ax & 0 \leq x \leq L/2 \\ A(L - x) & L/2 \leq x \leq \end{cases}$$

(a) The wavefunction looks like:

insert figure

We can find the overall constant A using

$$\int_0^L dx\ \psi(x) = 1$$

So, we have

$$\begin{aligned}
&\int_0^{L/2} dx\ Ax + \int_{L/2}^{L} dx\ A(L-x) = 1 \\
\implies & A\left(\frac{x^2}{2}\Big|_0^{L/2}\right) + AL\cdot\left(x\Big|_{L/2}^{L}\right) - A\left(\frac{x^2}{2}\Big|_{L/2}^{L}\right) = 1 \\
\implies & \frac{A}{2}\cdot\left(L^2 - 0^2\right) + AL\cdot\left(L - \frac{L}{2}\right) - \frac{A}{2}\cdot\left(L^2 - \frac{L^2}{4}\right) = 1 \\
\implies & \frac{AL^2}{2} + \frac{AL^2}{8} = 1 \\
\implies & \frac{10AL^2}{16} = 1 \\
\implies & \boxed{A = \frac{8}{5L^2}}
\end{aligned}$$

So, the initial wavefunction is

$$\boxed{\Psi(x,0) = \begin{cases} \frac{8}{5L^2}x & 0 \le x \le L/2 \\ \frac{8}{5L^2}(L-x) & L/2 \le x \le \end{cases}}$$

(b) The system's wavefunction at time $t = 0$ can be written in the Energy Eigenbasis as

$$\mid \Psi(0)\rangle = \sum_0^\infty c_n \mid E_n\rangle$$

We can find these energy eigenstates by solving the TDSE:

$$\left(-\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right)\psi_E = E\psi_E$$

where $\psi_E \equiv \langle x|E\rangle$