Instructor: James Analytis

Physics 141B: Solid State Physics II

Homework 1

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Code for questions 2-4 can be found here.

Question 1: SSH Model

(a). We have a unitary operator Θ such that, for hamiltonian H,

$$\Theta H \Theta^{-1} = -H$$

i.e.

$$\Theta H = -H\Theta$$

Consider an energy eigenstate

$$H|\psi\rangle = E|\psi\rangle$$

and then consider the state $\Theta|\psi\rangle$. Then,

$$
\begin{aligned}
H\left(\Theta|\psi\rangle\right) &= \left(H\Theta\right)|\psi\rangle \\
&= -\Theta H|\psi\rangle \\
&= -\Theta\left(E|\psi\rangle\right) \\
&= -E\left(\Theta|\psi\rangle\right)
\end{aligned}
$$

Thus, for eigenstate $|\psi\rangle$ with energy E there exists eigenstate $\Theta|\psi\rangle$ with energy $-E$.

(b). The SSH Model with nearest-neighbor-only hopping has Chiral symmetry

$$\sigma_z H \sigma_z^{-1} = -H$$

because its hamiltonian can be expressed as

$$H(k) = \vec{\sigma} \cdot \vec{b}(k)$$

where

$$\vec{b_x} = \begin{pmatrix} \Delta_1(k) \\ 0 \\ 0 \end{pmatrix}, \ \vec{b_y} = \begin{pmatrix} 0 \\ \Delta_2(k) \\ 0 \end{pmatrix}, \ \vec{b_z} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and $\vec{b}(k) = b_x \hat{x} + b_y \hat{y} + b_z \hat{z}$

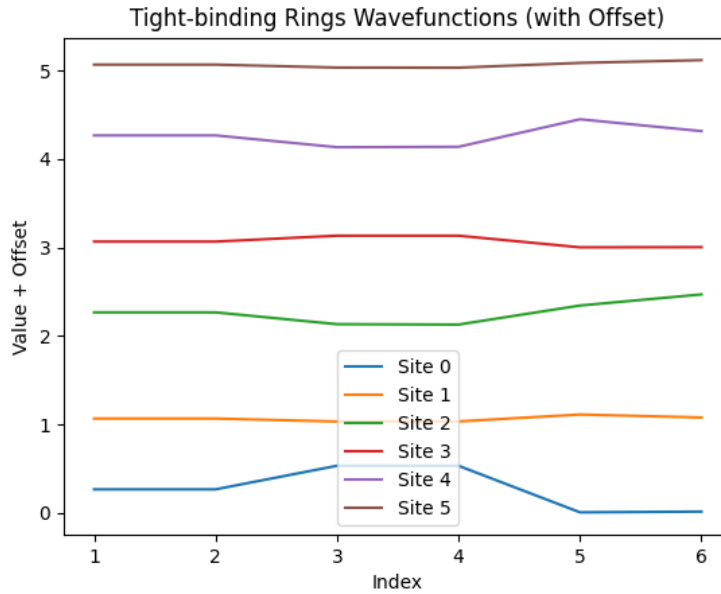More specifically, the $z-$component of $\vec{b}$ is zero and so our hamiltonian has no σ_z terms and only terms with σ_x, σ_y. This allows it to anticommute with σ_z from the canonical commutation rules of the Pauli matrices. The statement that H anticommutes with σ_z is equivalent to the Chiral symmetry statement:

$$\{\sigma_z, H\} = 0$$
$$\implies \sigma_z H + H \sigma_z = 0$$
$$\implies \sigma_z H = -H \sigma_z$$
$$\implies \sigma_z H \sigma_z^{-1} = -H$$

If we allowed for second-neighbor hoppings as well, then we would have non-zero σ_z terms and our hamiltonian would no longer anticommute with σ_z. Thus, it would also no longer have Chiral Symmetry.
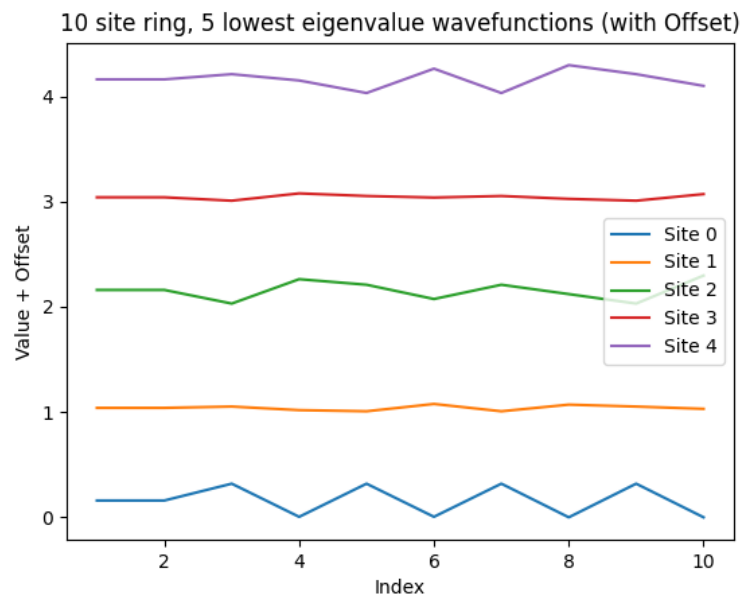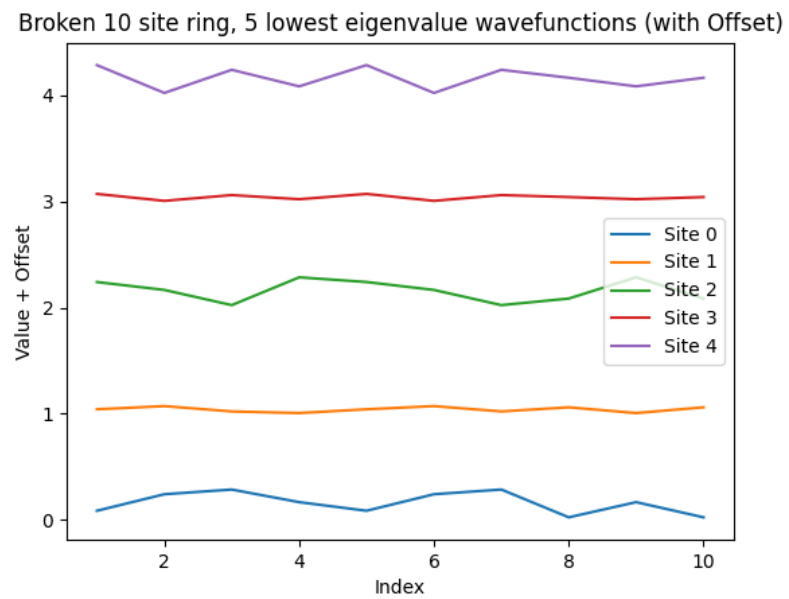
Question 2

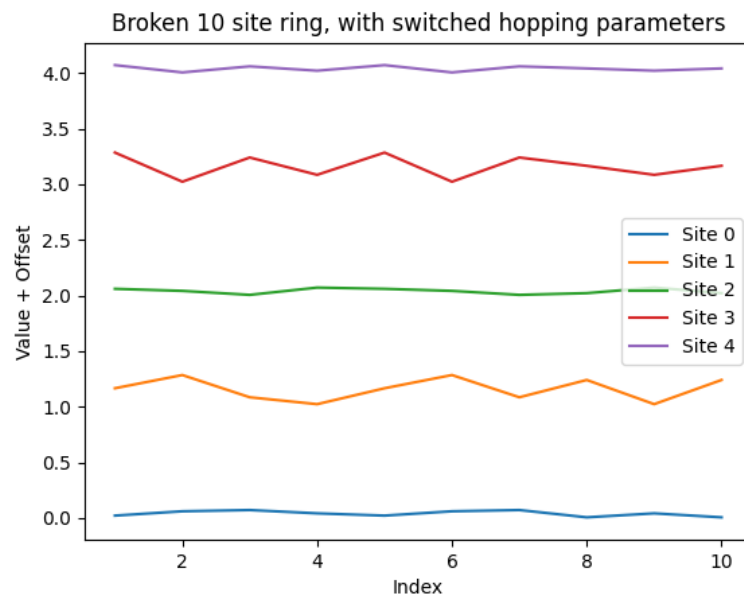For the 10 site ring (zero-indexed), we have

Question 3

(i). For the five smallest eigenenergy states:

10 site ring, 5 lowest eigenvalue wavefunctions (with Offset)

(ii). Breaking the ring:

Broken 10 site ring, 5 lowest eigenvalue wavefunctions (with Offset)

(iii). Switching the hopping parameters:

Broken 10 site ring, with switched hopping parameters

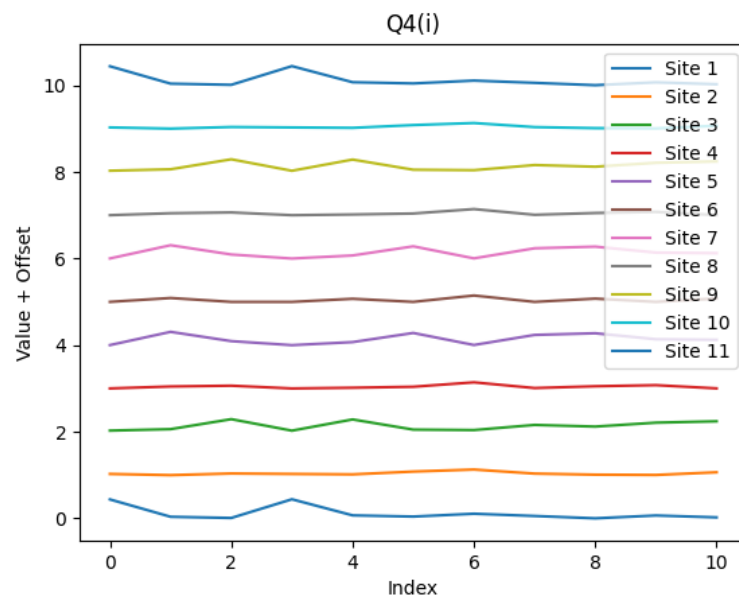## Question 4

(i). For the 11 site ring:

Q4(i)

(ii). Switching the hopping parameters: