Physics 137B Lecture 5

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These are notes taken from lectures on Quantum Mechanics delivered by Professor Raúl A. Briceño for UC Berekley's Physics 137B class in the Sprng 2024 semester.

Contents

1 January 26 - Perturbation Theory continued...

Last time, we left off while trying to extract $|n^{(0)}\rangle$.

To extract $|n^{(0)}\rangle$, we make use of the fact that $\{|l^{(0)}\rangle\}$ is orthonormal. Act using another stationary state $|l^{(0)}\rangle$ where $l \neq n$.

$$\implies \sum_{k\neq n} c_{nk}^{(1)} \left(E_k^{(0)} - E_n^{(0)}\right) \underbrace{\langle l^{(0)}|k^{(0)}\rangle}_{\delta_{lk}} + \langle l^{(0)}|\hat{H}'|n^{(0)}\rangle - E_n^{(1)} \underbrace{\langle l^{(0)}|n^{(0)}\rangle}_{0} = 0$$

$$\implies \left(E_l^{(0)} - E_n^{(0)}\right) c_{nl}^{(1)} = -\langle l^{(0)}|\hat{H}'|n^{(0)}\rangle$$

Again, for emphasis, we are currently dealing with a special case: **Non-degenerate Perturbation Theory** wherein $E_n^{(0)} = E_l^{(0)}$ iff $n = l$. So, rearraning the equation, we finally get

$$\boxed{c_{nl}^{(1)} = \frac{\langle l^{(0)}|\hat{H}'|n^{(0)}\rangle}{\left(E_n^{(0)} - E_l^{(0)}\right)}}$$

This gives us the first order correction for the state i.e.

$$\boxed{|n^{(1)}\rangle = \sum_{k\neq n} \frac{\langle l^{(0)}|\hat{H}'|n^{(0)}\rangle}{\left(E_n^{(0)} - E_l^{(0)}\right)|k^{(0)}\rangle}}$$

Before moving on to second-order PT and degenerate PT, let's see some nice examples.

1.1 Example: 1D Infinite Square Well with a $\delta-$potential

The known potential is

$$\hat{H}_0 = \frac{\hat{P}^2}{2m} + V_0(x); \quad V_0(x) = \begin{cases} 0 & -\frac{L}{2} \leq x \leq \frac{L}{2} \\ \infty & \text{otherwise} \end{cases}$$

and we have $\hat{H}' = \alpha\delta(x)$

The goal is to find $E_n^{(1)}$ and $|n^{(1)}\rangle$. We follow the standard procedure:

1. Find $|n^{(0)}\rangle$ and $E_n^{(0)}$
2. Use $E_n^{(1)} = \langle n^{(0)}|\hat{H}'|n^{(0)}\rangle$
3. Use $|n^{(1)}\rangle = \sum_{k\neq n} \frac{\langle k^{(0)}|\hat{H}'|n^{(0)}\rangle}{\left(E_n^{(0)} - E_k^{(0)}\right)}|k^{(0)}\rangle$
4. "think" : do your solutions make sense?

<u>**Step 1:**</u> To get the *unperturbed* eigenstates and eigenenergies, we set up the TISE:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_n^{(0)}(x) = E_n^{(0)}\psi_n^{(0)}$$

and impose the *boundary conditions*.

Doing so, we obtain

$$\psi_n^{(0)}(x) = \begin{cases} \frac{2}{L}\cos\left(\frac{n\pi x}{L}\right) & n = 2n'+1 \\ \frac{2}{L}\sin\left(\frac{n\pi x}{L}\right) & n = 2n' \end{cases}$$

and

$$E_n^{(0)} = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 n^2$$

Step 2:

The energy correction is

$$\begin{aligned} E_n^{(1)} &= \langle n^{(1)}|\hat{H}'|n^{(0)}\rangle \\ &= \int dx\ \langle n^{(0)}|x\rangle\langle x|\hat{H}'|x'\rangle\langle x'|n^{(0)}\rangle \\ &= \int dx\psi(x)^*\alpha\delta(x)\psi(x) \\ &= \alpha|\psi_n(0)|^2 \\ &= \begin{cases} \frac{\alpha 2}{L} & n = \text{ odd} \\ 0 & n = \text{ even} \end{cases} \end{aligned}$$

So, the overall eigenenergy is given by

$$E_n^{(0)} = \left(\frac{\pi\hbar}{L}\right)^2\frac{n^2}{2m} + \begin{cases} \alpha\frac{L}{2} \text{ iff } n = \text{ odd} \\ 0 \text{ iff } n = \text{ even} \end{cases}$$

Note that this makes sense if and only if

$$\begin{aligned} \alpha\frac{L}{2} &< \left(\frac{\pi\hbar}{L}\right)^2\frac{n^2}{2m} \\ \implies \alpha &< \frac{2}{L}\left(\frac{\pi h}{L}\right)^2\frac{n^2}{2m} \end{aligned}$$

So, if experimental values of α tell us that the 1st order correction is bigger than the 0-order corrrection, we most definitely have a problem and our system may be *non-perturbative.*

Step 3:

The eigenstate correction is

$$|n^{(1)}\rangle = \sum_{k\neq n}\frac{\langle k^{(0)}|\hat{H}'|n^{(0)}\rangle}{\left(E_n^{(0)} - E_k^{(0)}\right)}|k^{(0)}\rangle$$