

Professor: Alexander Givental

# Math 215A: Algebraic Topology

Homework 12

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**Question 1:** On a closed manifold, suppose that the cup product of  $l$  cohomology classes  $a_1, \dots, a_l$  (with coefficients in Ring  $R$ ) is non-zero. Prove that on this manifold, any function with only isolated (but not necessarily degenerate) critical points has at least  $l + 1$  *distinct* critical points.

**Solution:**

text

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**Question 2:** For a CW Complex  $X$ , show that the addition in  $H^n(X; G) = \pi(X, K(G, n))$  is induced by the map  $K(G, n) \times K(G, n) \rightarrow K(G, n)$  defined as the composition of loops  $K(G, n) = \Omega K(G, n+1)$ .

**Solution:**

text

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**Question 3:** Prove that two complex line-bundles over a CW-base are equivalent if and only if their first Chern classes coincide.

**Solution:**

text

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