Physics 137B Homework 1

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Q1.Infinite square-well potential with oscillatory potentials:

Before perturbation, the potential is

$$V_0(x) = \begin{cases} 0, -\frac{L}{2} \leq x \leq \frac{L}{2} \\ \infty, \text{ elsewhere} \end{cases}$$

So, the unperturbed Hamiltonian can be written as

$$\hat{H}^{(0)} = \frac{\hat{P}^2}{2m} + V_0(\hat{X})$$

which as eigenstates

$$\langle x|n^{(0)}\rangle = \psi_n^{(0)}(x) = \begin{cases} \sqrt{\frac{2}{L}}\cos(nx\frac{\pi}{L}), n \text{ odd} \\ \sqrt{\frac{2}{L}}\sin(nx\frac{\pi}{L}), n \text{ even} \end{cases}$$

and eigenenergies

$$E_n^{(0)} = \left(\frac{\pi\hbar}{L}\right)^2 \frac{1}{2m}n^2 = E_1^{(0)}n^2$$

In this question we will consider two potentials:

$$V_1(x) = \alpha\cos\left(x\frac{\pi}{L}\right)$$
$$V_2(x) = \beta\cos\left(2x\frac{\pi}{L}\right)$$

(a) Before we begin, we need to evaluate some useful integrals:

(a)

$$I_1(n,n') = \int_{-L/2}^{L/2} dx\ \cos\left(nx\frac{\pi}{L}\right)\cos\left(n'x\frac{\pi}{L}\right)\cos\left(x\frac{\pi}{L}\right)$$