Math 214 Homework 5

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Q4-5. Let $\mathbb{CP}^n$ denote the $n-$dimensional complex projective space.

(a) Show that the quotient map $\pi : \mathbb{C}^{n+1} \setminus \{0\} \to \mathbb{CP}^n$ is a surjective smooth submersion.

(b) Show that $\mathbb{CP}^n$ is diffeomorphic to $\mathbb{S}^n$.

Proof:

Q4-6. Let M be a nonempty smooth compact manifold. Show that there is no smooth submersion $F : M \to \mathbb{R}^k$ for any $k > 0$.

Proof:

Q4-7. Suppose M and N are smooth manifolds, and $\pi : M \to N$ is an injective smooth submersion. Show that there is no other smooth manifold structure on N that satisfies the conclusion of Theorem 4.29.

Proof:

Q4-8. Let $\pi : \mathbb{R}^2 \to \mathbb{R}$ be defined by $\pi(x, y) = xy$. Show that π is surjective and smooth, and that for each smooth manifold P, a map $F : \mathbb{R} \to P$ is smooth if and only if $F \circ \pi$ is smooth; but π is not a smooth submersion.

Proof:

Q4-9. Let M be a connected smooth manifold, and let $\pi : E \to M$ be a topological covering map. Complete the proof of proposition 4.40 by showing that there is only one smooth structure on E such that π is a smooth covering map.

Proof:

Q5-4. Show that the image of the curve $\beta : (-\pi, \pi) \to \mathbb{R}^2$ of Example 4.19 is not an embedded submanifold of $\mathbb{R}^2$.

Proof:

Q5-6. Suppose $M \subseteq \mathbb{R}^n$ is an embedded $m-$dimensional submanifold, and let $UM \subseteq T\mathbb{R}^n$ be the set of all *unit* tangent vectors to M:

$$UM = \{(x, v) \in T\mathbb{R}^n \ : \ x \in M, v \in T_xM, \ |v| = 1\}$$

This is called the ***Unit Tangent Bundle of*** M**.** Prove that UM is an embedded $(2n-1)-$dimensional submanifold of $T\mathbb{R}^n \approx \mathbb{R}^n \times \mathbb{R}^n$.

Proof:

Q5-7. Let $F : \mathbb{R}^2 \to \mathbb{R}$ be defined as $F(x, y) = x^3 + xy + y^3$. Which level sets of F are embedded submanifolds of $\mathbb{R}^2$? For each level set, prove either that it is or that it is not an embedded submanifold.

Proof:
