Physics 137B Lecture 6

Keshav Balwant Deoskar

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These are notes taken from lectures on Quantum Mechanics delivered by Professor Raúl A. Briceño for UC Berekley's Physics 137B class in the Sprng 2024 semester.

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Recap

- We've been trying to develop ***Perturbation Theory*** in order to solve problems with Hamiltonians of the form:
$$\hat{H} = \underbrace{\hat{H}_0}_{\text{known sol.}} + \lambda \hat{H}'$$
- We did so by assuming we could parametrize the Eigen-states and Eigen-energies as functions of some small parameter λ
$$|n^{(\lambda)}\rangle = \sum_j \lambda^j |n^{(j)}\rangle$$
$$E_n^{(\lambda)} = \sum_j \lambda^j E_n^{(j)}$$

  (If this cannot be done, then the system we are studying is non-perturbative).
- So far, we've found the ***Leading order corrections*** for ***non-degenerate systems*** to be
$$E_n^{(1)} = \langle n^{(0)}|\hat{H}'|n^{(0)}\rangle$$
$$|n^{(1)}\rangle = \sum_{k\neq n} |k^{(0)}\rangle \frac{\langle k^{(0)}|\hat{H}'|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}}$$

1.1 Second Order Perturbation Theory

In second order PT, we also grab the λ^2 coefficients:

$$\mathcal{O}(\lambda^2) : \lambda^2 \left[(\hat{H}_0 - E_n^{(0)})|n^{(2)}\rangle + (\hat{H}' - E_n^{(1)})|n^{(1)}\rangle - E_n^{(2)}|n^{(0)}\rangle \right] = 0$$

Once again, we act on the equation with $|n^{(0)}\rangle$:

$$\implies \underbrace{\langle n^{(0)}|(\hat{H}_0 - E_n^{(0)})|n^{(2)}\rangle}_{=0} + \langle n^0|\hat{H}'|n^{(1)}\rangle\rangle - E_n^{(1)} \underbrace{\langle n^{(0)}|n^{(1)}\rangle}_{=0} = 0$$
$$\implies \boxed{E_n^{(2)} = \langle n^0|\hat{H}'|n^{(1)}\rangle\rangle}$$

Using the expression we get for $|n^{(1)}\rangle$:

$$\implies E_n^{(2)} = \sum_{n\neq k} \frac{\langle n^{(0)}|\hat{H}'|k^{(0)}\rangle\langle k^{(0)}|\hat{H}'|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}}$$

[Write about the possible interpretation as the state benig propagated and then returning back above by watching recording]

Interesting fact: When we were finding the states using 1st order PT we wrote

$$|n\rangle = |n^{(0)}\rangle + \lambda|n^{(1)}\rangle$$

and said we wouldn't worry about normalization, but in fact they are normalized up to second order corrections:

$$\begin{aligned}\implies \langle n|n\rangle &= \langle n^{(0)}|n^{(0)}\rangle + \lambda\left(\underbrace{\langle n^{(0)}|n^{(1)}\rangle}_{0} + \underbrace{\langle n^{(1)}|n^{(0)}\rangle}_{0}\right) + \mathcal{O}(\lambda^2) \\ &= 1 + \mathcal{O}(\lambda^2)\end{aligned}$$

This came abbout because we had $|n^{(1)}\rangle = \sim_{k\neq n} (stuff)$ summing over states k *not equal to* n.

Now, what about $|n^{(2)}\rangle$? We have two choices:

Choice #1: Once again sum over states $k \neq n$

$$|n^{(2)}\rangle = \sum_{k\neq n} |k^{(0)}\rangle c_{nk}^{(2)}$$

In this case, we yet again get normalized states

Choice #2: Include $k = n$

$$|n^{(2)}\rangle = \sum_{k} |k^{(0)}\rangle c_{nk}^{(2)}$$

In this case, we can determine $c_{nn}^{(2)}$ from the normalization condition

$$\langle n|n\rangle = 1 + \mathcal{O}(\lambda^3)$$

For now, we will follow Choice #1. So, let

$$|n^{(2)}\rangle = \sum_{k\neq n} |^{(0)}\rangle c_{nk}^{(2)}$$

Then

$$\implies \left(\hat{H}_0 - E_n^{(0)}\right)\sum_{k\neq n}|k^{(0)}\rangle c_{nk}^{(2)} + \left(\hat{H}' - E_n^{(0)}\right)|n^{(1)}\rangle 0 E_n^{(2)}|n^{(0)}\rangle = 0$$

Let $l \neq n$. Then,

$$\begin{aligned}&\sum_{k\neq n}\langle l^{(0)}|k^{(0)}\rangle c_{nk}^{(2)}\left(E_k^{(0)} - E_n^{(0)}\right) + \langle l^{(0)}|\left(\hat{H}' - E_n^{(0)}\right)|k^{(0)}\rangle - E_n^{(0)}\underbrace{\langle l^{(0)}|n^{(0)}\rangle}_{0} = 0 \\ \implies c_{nk}^{(2)} &= \sum_{k\neq n}\frac{E_n^{(1)}}{E_n^{(0)} - E_l^{(0)}} + \sum_{k\neq n}\sum_{l\neq n}\frac{\langle k^{(0)}|\hat{H}'|l^{(0)}\rangle\langle l^{(0)}|\hat{H}'|n^{(0)}\rangle}{(E_n^{(0)} - E_k^{(0)})(E_n^{(2)} - E_l^{(0)})}\end{aligned}$$

[THE ABOVE ARE DUBIOUS; DOUBLE-CHECK THE CALCULATIONS LATER]

1.2 Degenerate Perturbation Theory

So far, we've assumed that our systems have stationary states which are non-degerate. However, in many situations we have orthogonal states $|a^{(0)}\rangle$ and $|b^{(0)}\rangle$ with $\langle a^{(0)}|b^{(0)}\rangle = 0$ but with

$$\hat{H}^{(0)}|a^{(0)}\rangle = E^{(0)}|a^{(0)}\rangle$$
$$\hat{H}^{(0)}|b^{(0)}\rangle = E^{(0)}|b^{(0)}\rangle$$

So, we'll have to tweak our approach. We will try to use the parameter λ to split the energies.