Math 214 Notes

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These are notes taken from lectures on Differential Topology delivered by Eric C. Chen for UC Berekley's Math 214 class in the Sprng 2024 semester. Any errors that may have crept in are solely my fault.

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Recap

- Last time, we finished proving the Whitney Extension theorems and started studying Transversality.
- Write more
- The second version of the Transverality Theorem gave us a generalization for the Regular Level Set Theorem.

Today, we'll explore Transversality more. We'll see that if we hae non transverse intersections, we can often perturb our objects gently to make the intersection transverse.

Theorem: (Parametric Transversality Theorem)
Let $F : N \times S \to M$ be a smooth map viewed as $F_s = F(\ , s)$ transverse to msooth submanifold $X \subseteq M$, then F_S os trammsverse tp X for almost every $s \in S$.

Proof: Write from image.

Next, we have an application of this theorem which tells us that every smooth map can be deformed to have transverse intersection.

Theorem: (Transversality Homotopy Theorem) Write from image.

Proof: Write from image.

This marks the end of our section on Sard's Theorem. Some closing remraks:

1. Write from picture. Might not require Sard's Theorem.
2. Poincare Duality connects tbe (n-1) homology group with the first cohomology group, and then there's something whcih connects that to the set of homotopy classes of maps $M \to S^1$.
3. More from image.

1.1 Onto Lie Groups! Chapter 7 begins.

Lie Group

A ***Lie Group*** is a smooth manifold G with group structure such that

$$m : G \times G \to G, \quad (g, h) \mapsto gh$$
$$i : G \to G, \quad \mapsto g^{-1}$$

are smooth.

Examples:

- Write from image

Left/Right translations

Fix $g \in G$. Then, left and right translation refer to the smooth maps

$$write\ from\ image$$

Remark:

- $L_{g_1} \circ R_{g_2} = R_{g_2} \circ L_{g_1}$