Professor: James Analitis

Physics 141A: Solid State Physics

Homework 5

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Question 1: Consider a one-dimensional spring and mass model of a crystal. Generalize this model to include springs not only between neighbors but also between second nearest neighbors. Let the spring constant between neighbors be called κ_1 and the spring constant between second neighbors be called κ_2. Let the mass of each atom be m.

(a) Calculate the dispersion curve $\omega(k)$ for this model.

(b) Determine the sound wave velocity. Show the group velocity vanishes at the Brillouin zone boundary.

Solution:

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Question 2: Normal modes of a One-Dimensional Diatomic Chain

(a) What is the difference between an acoustic mode and an optical mode?
▷ Describe how particles move in each case.

(b) Derive the dispersion relation for the longitudinal oscillations of a one-dimensional diatomic mass-and-spring crystal where the unit cell is of length a and each unit cell contains one atom of mass m_1 and one atom of mass m_2 connected together by springs with spring constant κ.

(c) Determine the frequences of the acoustic and optical modes at $k = 0$ as well as at the Brillouin zone boundary.
▷ Describe the motion of the masses in each case.
▷ Determine the sound velocity and show that the group velocity is zero at the zone boundary.
▷ Show that the sound velocity is also given by $v_s = \sqrt{\beta^{-1}/\rho}$ where ρ is the chain density and β is the compressibility.

(d) Sketch the dispersion in both reduced and extended zone scheme.
▷ If there are N unit cells, how many different normal modes are there?
▷ How many *branches* of excitations are there? I.e., in reduced zone scheme, how many modes are there at each k?

(e) What happens when $m_1 = m_2$?

<u>Solution:</u>

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