Instructor: James Analytis

Physics 141B: Solid State Physics II

Homework 1

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Question 1: SSH Model

(a). We have a unitary operator Θ such that, for hamiltonian H,

$$\Theta H \Theta^{-1} = -H$$

i.e.

$$\Theta H = -H\Theta$$

Consider an energy eigenstate

$$H|\psi\rangle = E|\psi\rangle$$

and then consider the state $\Theta|\psi\rangle$. Then,

$$\begin{aligned} H\left(\Theta|\psi\rangle\right) &= \left(H\Theta\right)|\psi\rangle \\ &= -\Theta H|\psi\rangle \\ &= -\Theta\left(E|\psi\rangle\right) \\ &= -E\left(\Theta|\psi\rangle\right) \end{aligned}$$

Thus, for eigenstate $|\psi\rangle$ with energy E there exists eigenstate $\Theta|\psi\rangle$ with energy $-E$.

(b). The SSH Model with nearest-neighbor-only hopping has Chiral symmetry

$$\sigma_z H \sigma_z^{-1} = -H$$

because its hamiltonian can be expressed as

$$H(k) = \vec{\sigma} \cdot \vec{b}(k)$$

where

$$\vec{b}_x = \begin{pmatrix} \Delta_1(k) \\ 0 \\ 0 \end{pmatrix},\ \vec{b}_y = \begin{pmatrix} 0 \\ \Delta_2(k) \\ 0 \end{pmatrix},\ \vec{b}_z = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{and } \vec{b}(k) = b_x\hat{x} + b_y\hat{y} + b_z\hat{z}$$

More specifically, the $z-$component of $\vec{b}$ is zero and so our hamiltonian has no σ_z terms and only terms with σ_x, σ_y. This allows it to anticommute with σ_z from the canonical commutation

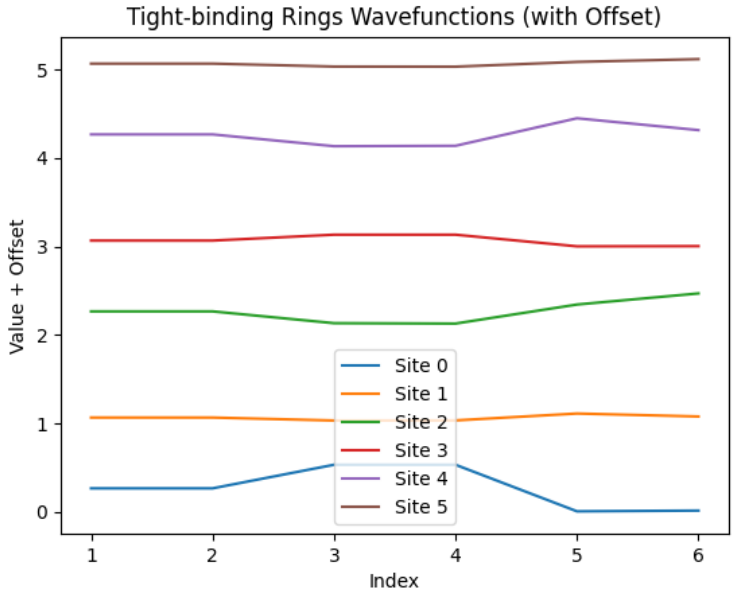
rules of the Pauli matrices. The statement that H anticommutes with σ_z is equivalent to the Chiral symmetry statement:

$$\begin{aligned} &\{\sigma_z, H\} = 0 \\ \implies &\sigma_z H + H\sigma_z = 0 \\ \implies &\sigma_z H = -H\sigma_z \\ \implies &\sigma_z H \sigma_z^{-1} = -H \end{aligned}$$

If we allowed for second-neighbor hoppings as well, then we would have non-zero σ_z terms and our hamiltonian would no longer anticommute with σ_z. Thus, it would also no longer have Chiral Symmetry.

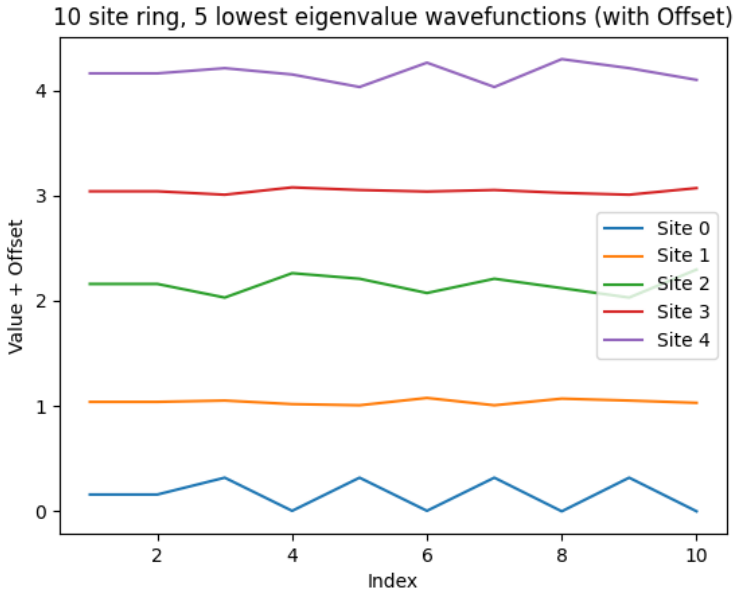
Question 2

For the 10 site ring (zero-indexed), we have

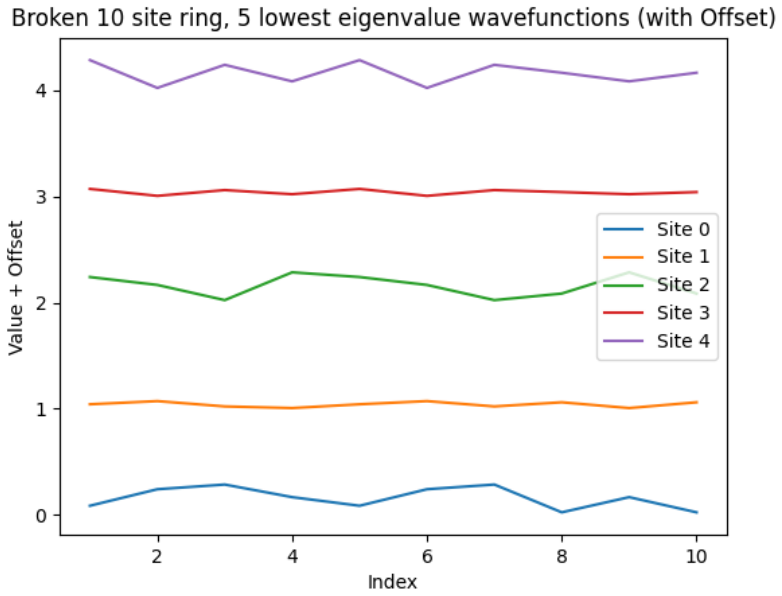


Question 3

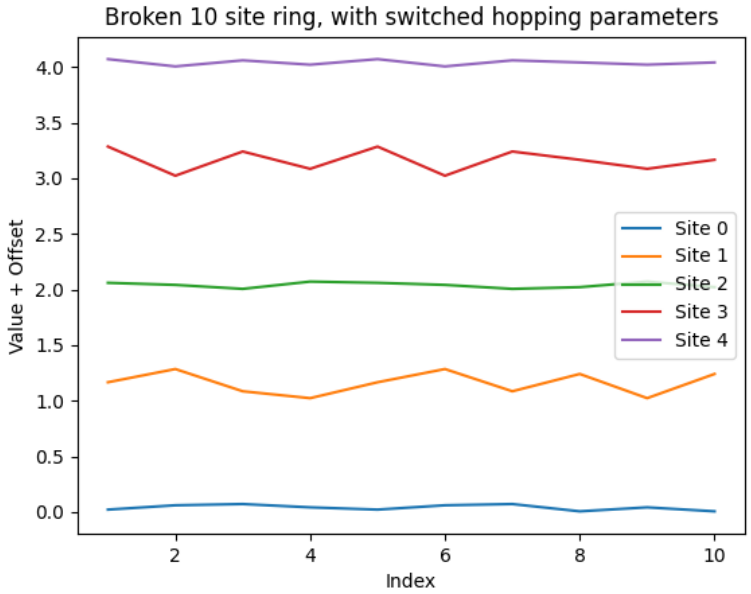
(i). For the five smallest eigenenergy states:



(ii). Breaking the ring:

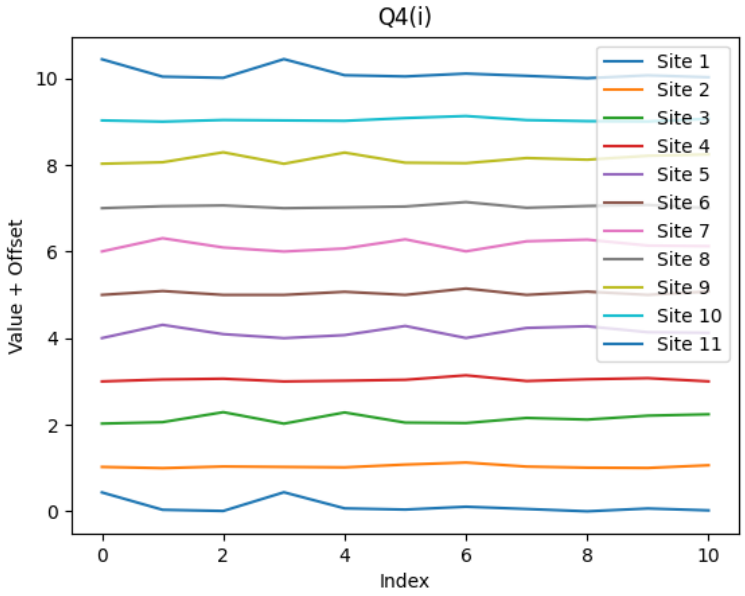


(iii). Switching the hopping parameters:

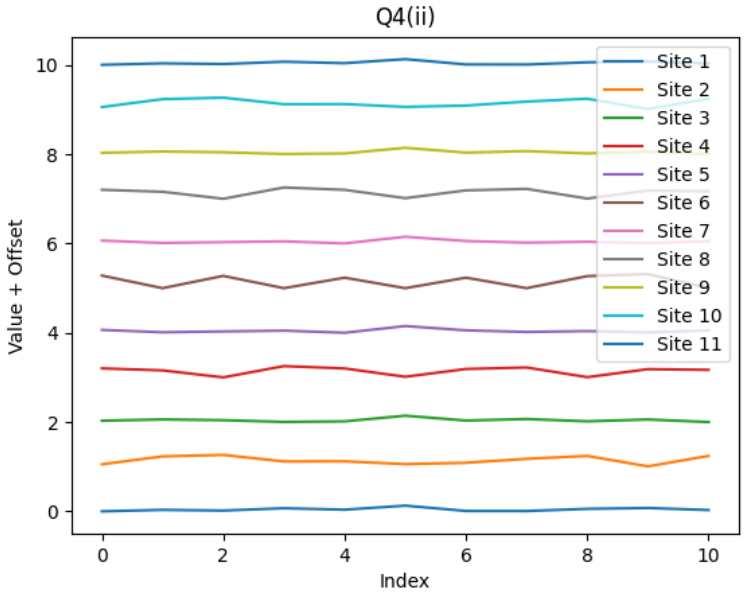


Question 4

(i). For the 11 site ring:



(ii). Switching the hopping parameters:


Q4(ii)
Site 1
Site 2
Site 3
Site 4
Site 5
Site 6
Site 7
Site 8
Site 9
Site 10
Site 11
Value + Offset
Index
0
2
4
6
8
10