Physics 137B Homework 9

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Question 1: Equations of motion:

(a) Using the generalized Ehrenfest theorem, show that

$$\frac{d\langle\vec{r}\rangle}{dt} = \frac{1}{m}\langle\left(\vec{p} - q\vec{A}\right)\rangle$$

where $\langle\vec{r}\rangle$ is the expectation value of the position of the particle.

(b) Using the fact that $\langle\vec{v}\rangle \equiv d\langle\vec{r}\rangle/dt$, show that

$$m\frac{d\langle\vec{v}\rangle}{dt} = q\langle\vec{E}\rangle + \frac{q}{2m}\langle\left(\vec{p}\times\vec{B} - \vec{B}\times\vec{p}\right)\rangle - \frac{q^2}{m}\langle\vec{A}\times\vec{B}\rangle$$

(c) Assuming a unfiorm electrinc and magnetic field, show that the expectation value of $d\langle\vec{r}\rangle/dt$ is consistent with the Lorentz force law,

$$m\frac{d\langle\vec{r}\rangle}{dt} = q\langle\vec{E}\rangle + q\langle\vec{v}\times\vec{B}\rangle$$

Solution:

Question 2: Selection Rules

(a) **Commutation relations:** Given the commutation relation $[\hat{L}_a, \hat{V}_b] = i\hbar\epsilon_{abc}\hat{V}_c$, show that these can be written as

fill these in later

where $\hat{V}_\pm \equiv \left(\hat{V}_x \pm i\hat{V}_y\right)$ and similarly for $\hat{L}_\pm$.

(b) **Selection rules:** Evaluate the value of these six commutators sandwiches by the $|nlm\rangle$ state. Show that these are consistent with the condition

Fill condition in later

(c) **Selection Rules continued:** Using the properties of the Clebsh-Gordon coefficient, explain the constraints places on $\delta l = l' - l$ and $\delta m = m' - m$.

(d) **Parity:** Let $\vec{V}_+$ and $\vec{V}_-$ be vectors that are even and odd under the parity transformations. What additional constraints are placed on δl and δm for matrix elements of the form

$$\langle n'l'm'|\hat{V}_\pm|nlm\rangle$$

due to parity, if any.

Solution:

Question 3: Dipole Appximation and Selection Rules

What are the diplole selection rules f or a one-dimensional harmonic oscillator potential?

Solution:
