Professor: James Analitis

Physics 141A: Solid State Physics

Homework 2

kdeoskar@berkeley.edu

Question 1: Consider heat capacity data below for the compound $BaFe_2As_2$. THis is measured per mole of the formula unit, $BaFe_2As_2$. Note there is a phase transition at about $140K$, that you can ignore for now.

1. From the heat capacity, estimate how many degrees of freedom each formula unit contributes.
2. Fit the data using the low-temperature Debye model. Estimate the Debye Temperature.
3. Argue why the phase transition at $140K$ must affect the density of the atoms only weakly.

Solution:

text

Question 2: Physical Properties of the Free Electron Gas

(a) Give a sumple but approximate derivaiton of the Fermi gas prediction for heat capacity of the conduction electron in metals.

(b) Give a simple (not approximate) derivations of the Fermi gas prediction for magnetic susceptibility of the conduction electron in metals. Here susceptibility is $\chi = dM/dH = \mu_0 dM/dB$ at small H and is meant to consider the magnetization of the electron spins only.

(c) How are the results of (a) and (b) different from that of a classical gas of electrons? What other properties of metals may be different from the classical prediction?

(d) The experimental specific heat of potassium metal at low temperatures has the form

$$C = \gamma T + \alpha T^3$$

where $\gamma = 2.08\text{mJmol}^{-1}\text{K}^{-2}$ and $\alpha = 2.6\text{mJmol}^{-1}\text{K}^{-4}$.
Explain the origin of each of the two terms in this expression.
Make an estimate of the Fermi energy for potassium metal.

Solution:

text

Question 3:

(a) What is the relationship between the carrier density n and the Fermi momentum k_F in two dimensions?

(b) Show that in two dimensions the free electron density of states $g(E)$ is a constant independent of energy E for $E > 0$ and 0 for $E < 0$. What is the constant?

(c) Using the fact that the total number of particles $\langle N \rangle$ is given by

$$\langle N \rangle = \int_0^\infty g(E) f(E) dE$$

where $f(E)$ is the Fermi distribution function, show that in two-dimensions that

$$E_F = \mu + k_B T \ln \left(1 + e^{-\mu/k_B T}\right)$$

In order to solve this, you can loop up the integral describing n in a table, then note the relationship of n to E_F to get to the final expression.

(d) Estimate the amount that μ differeny from E_F. Explain how this shows that the chemical potential μ is essentially independent of temperature so long as $k_B T << \mu$. This is an important fundamental point about two-dimensional systems - over a wide range of temperature, the chemical potential can be essentially regarded as the Fermi energy. Indeed, even in higher dimensions, the difference is small.

Solution:

1. We know that, in the Sommerfeld model, the heat capacity of a metal is

$$C = \tilde{\gamma} \left(\frac{f N k_B}{2}\right) \left(\frac{T}{T_F}\right)$$

where f is the number of degrees of freedom and $\tilde{\gamma} = \pi^2/3$