Higher Order Epistasis Networks Tables and Figures

Table 1 Simulation results when the truth obeys strong heredity

Model	T1_tpr	T1_tnr	T1_fpr	T1_fnr	T2_tpr	T2_tnr	T2_fpr	T2_fnr	T3_tpr	T3_tnr	T3_fpr	T3_fnr	Train_MSE	Train_Rsq	Test_MSE	Test_Rsq	Run Time
forward_select	1.000	0.999	0.001	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	3.330	0.727	3.490	0.711	0.757
iform_order_2_weak	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.128	0.907	1.252	0.895	5.896
iform_order_2_strong	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.102	0.909	1.198	0.900	1.557
forward_select_2	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.086	0.910	1.198	0.900	25.481
forward_select_3	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.992	0.918	1.121	0.906	471.881
iform_order_3_weak	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.020	0.916	1.135	0.905	11.346
iform_order_3_strong	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.968	0.920	1.060	0.911	1.872
glinternet	1.000	0.559	0.441	0.000	1.000	0.982	0.018	0.000	0.000	1.000	0.000	1.000	1.246	0.898	1.446	0.880	208.167
hierNet	1.000	0.697	0.303	0.000	1.000	0.976	0.024	0.000	0.000	1.000	0.000	1.000	0.906	0.925	1.421	0.882	27.521
Oracle	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	0.953	0.921	1.050	0.912	NA

Table 2 Simulation results when the truth obeys weak heredity

Model	T1_tpr	T1_tnr	T1_fpr	T1_fnr	T2_tpr	T2_tnr	T2_fpr	T2_fnr	T3_tpr	T3_tnr	T3_fpr	T3_fnr	Train_MSE	Train_Rsq	Test_MSE	Test_Rsq	Run Time
forward_select	1.000	0.999	0.001	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	3.326	0.731	3.480	0.716	4.355
iform_order_2_weak	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.119	0.910	1.200	0.901	8.342
iform_order_2_strong	1.000	0.992	0.008	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.580	0.872	1.707	0.859	2.952
forward_select_2	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.083	0.912	1.167	0.904	38.872
forward_select_3	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.979	0.921	1.089	0.910	569.983
iform_order_3_weak	1.000	1.000	0.000	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.003	0.919	1.079	0.911	13.054
iform_order_3_strong	1.000	0.992	0.008	0.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.578	0.872	1.705	0.859	2.787
glinternet	1.000	0.469	0.531	0.000	1.000	0.980	0.020	0.000	0.000	1.000	0.000	1.000	0.906	0.927	1.425	0.883	29.975
hierNet	1.000	0.657	0.343	0.000	1.000	0.973	0.027	0.000	0.000	1.000	0.000	1.000	0.856	0.931	1.412	0.884	33.302
Oracle	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	0.940	0.924	1.034	0.915	NA

Table 3 Simulation results when the truth is anti-heredity

Model	T1_tpr	T1_tnr	T1_fpr	T1_fnr	T2_tpr	T2_tnr	T2_fpr	T2_fnr	T3_tpr	T3_tnr	T3_fpr	T3_fnr	Train_MSE	Train_Rsq	Test_MSE	Test_Rsq	Run Time
forward_select	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	3.284	0.729	3.510	0.714	1.005
iform_order_2_weak	1.0000	0.9996	0.0004	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	3.140	0.741	3.435	0.719	7.866
iform_order_2_strong	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	3.284	0.729	3.510	0.714	2.386
forward_select_2	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	1.081	0.911	1.171	0.904	29.095
forward_select_3	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.989	0.918	1.095	0.910	548.617
iform_order_3_weak	1.0000	0.997	0.003	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	3.155	0.739	3.448	0.719	13.216
iform_order_3_strong	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	3.284	0.729	3.510	0.714	2.703
glinternet	1.0000	0.290	0.710	0.0000	1.0000	0.971	0.029	0.0000	0.0000	1.0000	0.0000	1.0000	0.844	0.931	1.578	0.871	26.564
hierNet	1.0000	0.142	0.858	0.0000	1.0000	0.915	0.085	0.0000	0.0000	1.0000	0.0000	1.0000	0.307	0.975	2.216	0.819	3.417
Oracle	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	0.952	0.921	1.031	0.915	NA

Table 4 Simulation results when the truth is constructed of pure interactions

Model	T1_tpr	T1_tnr	T1_fpr	T1_fnr	T2_tpr	T2_tnr	T2_fpr	T2_fnr	T3_tpr	T3_tnr	T3_fpr	T3_fnr	Train_MSE	Train_Rsq	Test_MSE	Test_Rsq	Run Time
forward_select	NaN	0.980	0.020	NaN	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	3.316	0.025	3.445	-0.039	1.177
iform_order_2_weak	NaN	0.972	0.028	NaN	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	3.007	0.115	3.181	0.040	5.840
iform_order_2_strong	NaN	0.979	0.021	NaN	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	3.294	0.031	3.429	-0.034	2.081
forward_select_2	NaN	1.000	0.000	NaN	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.117	0.669	1.170	0.644	26.396
forward_select_3	NaN	1.000	0.000	NaN	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	1.005	0.703	1.081	0.671	530.362
iform_order_3_weak	NaN	0.975	0.025	NaN	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	3.043	0.106	3.209	0.032	9.461
iform_order_3_strong	NaN	0.979	0.021	NaN	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	3.294	0.031	3.429	-0.034	2.265
glinternet	NaN	0.429	0.571	NaN	1.000	0.983	0.017	0.000	0.000	1.000	0.000	1.000	1.002	0.699	1.445	0.561	145.078
hierNet	NaN	0.147	0.853	NaN	1.000	0.955	0.045	0.000	0.000	1.000	0.000	1.000	0.672	0.802	1.758	0.467	4.491
Oracle	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	0.968	0.713	1.022	0.689	NA

Table 5 The detection of epistasis for the relative growth rate (*r*) of shoot length in the full-sib family of mei tree by a low-order epistatic model

Coefficient	Estimate	SE	T-value	P-value
(Intercept)	0.18285	0.07613	2.402	0.0174 *
AATTC_nn_np_2517_a	0.40013	0.06509	6.147	5.13e-09 ***
AATTC_nn_np_2815_a	0.15792	0.06837	2.310	0.0221 *
CATG_nn_np_3479_a	0.23433	0.05285	4.434	1.63e-05 ***
CATG_nn_np_1284_a	0.22200	0.05313	4.179	4.61e-05 ***
AATTC_nn_np_2815_a×AATTC_lm_ll_3034_a	0.45783	0.09244	4.953	1.71e-06 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3504 on 176 degrees of freedom
Multiple R-squared: 0.3428, Adjusted R-squared: 0.3241
F-statistic: 18.36 on 5 and 176 DF, p-value: 1.189e-14

Table The detection of epistasis for the relative growth rate (*r*) of shoot length in the full-sib family of mei tree by a high-order epistatic model

Coefficient	Estimate	SE	T-value	P-value
(Intercept)	0.16859	0.05801	2.906	0.00415 **
AATTC_nn_np_2517_a	0.27773	0.04396	6.318	2.27e-09 ***
AATTC_nn_np_2815_a	0.26382	0.05295	4.983	1.54e-06 ***
CATG_nn_np_3479_a	0.20767	0.03467	5.990	1.23e-08 ***
CATG_nn_np_1284_a	0.04522	0.04265	1.060	0.29055
AATTC_nn_np_2815_a×AATTC_lm_ll_3034_a	1.82572	0.17925	10.185	< 2e-16 ***
AATTC_nn_np_2815_a×AATTC_hk_hk_278_a	0.25935	0.03888	6.671	3.48e-10 ***
CATG_lm_ll_3153_a	0.14877	0.03491	4.262	3.36e-05 ***
CATG_nn_np_1284_a×AATTC_nn_np_554_a	0.22994	0.05104	4.505	1.23e-05 ***
AATTC_nn_np_2815_a.AATTC_lm_ll_3034_a× AATTC_nn_np_1615_a	-1.51714	0.19060	-7.960	2.39e-13 ***
AATTC_nn_np_2815_a×AATTC_nn_np_929_a	-0.30805	0.05477	-5.624	7.57e-08 ***
AATTC_hk_hk_479_d	0.16044	0.03443	4.660	6.37e-06 ***
AATTC_nn_np_2517_a×CATG_hk_hk_648_a	0.14537	0.02840	5.118	8.33e-07 ***

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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2268 on 169 degrees of freedom
Multiple R-squared: 0.7356, Adjusted R-squared: 0.7168
F-statistic: 39.19 on 12 and 169 DF, p-value: < 2.2e-16
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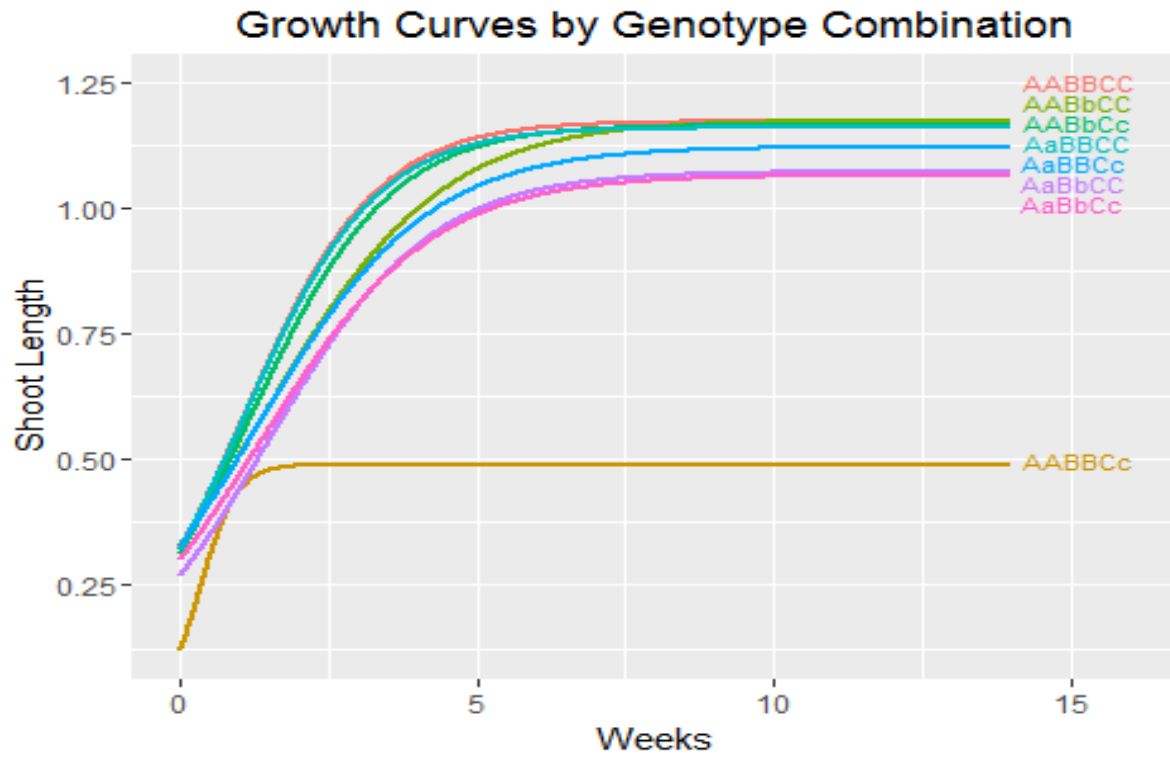
Figure 1 Growth curves of shoot length in mei drawn from estimated growth parameters at three loci of significant high-order epistasis
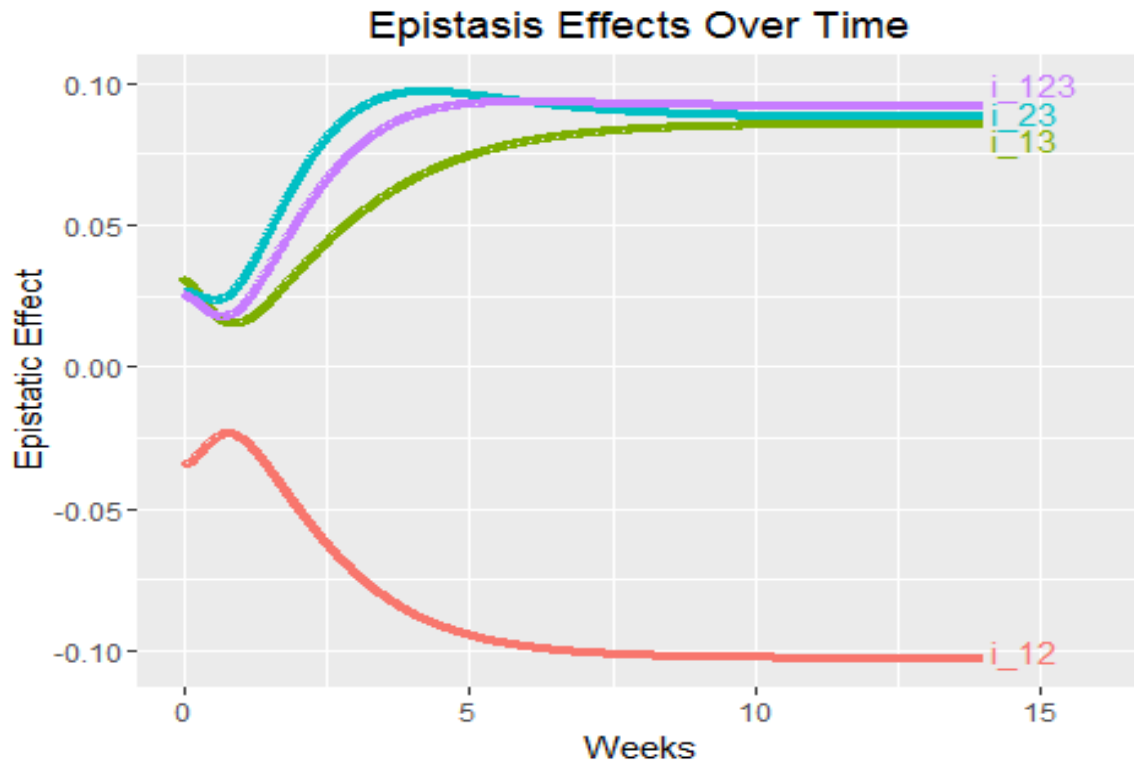
Figure 2 Curves of epistatic effects on shoot length growth in mei at three significant loci