1.

$\{0 < N\} \Rightarrow$

$\{0 < N \wedge \forall j \in [0, 1-1), A[0] \geq A[j]\}$

int m = A[0];

$\{0 < N \wedge \forall j \in [0, 1-1), m \geq A[j]\}$

int i = 1;

$\{i \leq N \wedge \forall j \in [0, i-1), m \geq A[j]\}$

while (i < N) {

$\{i \leq N \wedge \forall j \in [0, i-1), m \geq A[j] \wedge i < N\} \Rightarrow$

$\{i+1 \leq N \wedge \forall j \in [0, i+1), m \geq A[j]\}$

if (A[i] > m)

$\{i+1 \leq N \wedge \forall j \in [0, i-1), m \geq A[j] \wedge A[i] > m \wedge A[i] \geq A[i]\} \Rightarrow$

$\{i+1 \leq N \wedge \forall j \in [0, i-1), m \geq A[j] \wedge A[i] \geq A[i]\}$

m = A[i];

$\{i+1 \leq N \wedge \forall j \in [0, i-1), m \geq A[j] \wedge m \geq A[i]\}$

else

$\{i+1 \leq N \wedge \forall j \in [0, i-1), m \geq A[j] \wedge A[i] < m\} \Rightarrow$

$\{i+1 \leq N \wedge \forall j \in [0, i-1), m \geq A[j] \wedge m \geq A[i]\}$

skip;

$\{i+1 \leq N \wedge \forall j \in [0, i-1), m \geq A[j] \wedge m \geq A[i]\}$

i = i+1;

$\{i \leq N \wedge \forall j \in [0, i), m \geq A[j] \wedge m \geq A[j]\}$

}

$\{i \leq N \wedge \forall j \in [0, i), m \geq A[j] \wedge i \geq N\} \Rightarrow$

$\{\forall j \in [0, N), m \geq A[j]\} \Leftrightarrow \{m = \max(A[0], \cdots, A[N-1])\}$

2.

$\{x \geq 0 \wedge y > 0\} \Rightarrow$

$\{x \geq 0 \wedge y > 0 \wedge 0 < N \wedge x = x + 0\}$

int r = x;

$\{r \geq 0 \wedge y > 0 \wedge 0 < N \wedge x = r + 0\}$

int q = 0;

$\{r \geq 0 \wedge y > 0 \wedge q < N \wedge x = r + qy\}$

while (y <= r) {

$\{r \geq 0 \wedge y > 0 \wedge q < N \wedge y \leq r \wedge x = r + qy\} \Rightarrow$

$\{r - y \geq 0 \wedge q + 1 \leq N \wedge x = r - y + (q+1)y\}$

r = r - y;

$\{r \geq 0 \wedge q + 1 \leq N \wedge x = r + (q+1)y\}$

q = q + 1;

$\{r \geq 0 \wedge q \leq N \wedge x = r + qy\}$

}

$\{r \geq 0 \wedge q \leq N \wedge y > r \wedge x = r + qy\} \Rightarrow$

$\{x = qy + r \wedge 0 \leq r < y\}$

3.

$\{0 \le N\} \Rightarrow$

$\{0 \le 0 \le N \wedge \forall x,y \in [0, 1-1),\ x \le y \Rightarrow A[x] \le A[y]\}$

int i = 1;

$\{0 \le i \le N+1 \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y]\}$

while (i < N) {

$\{i \le N+1 \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge i < N\} \Rightarrow$

$\{0 \le i < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y \in [i-1, i-1),\ x \le y \Rightarrow A[x] \le A[y]\}$

int j = i

$\{0 \le i,j < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y \in [0, j-1),\ x \le y \Rightarrow A[x] \le A[y]\}$

while (j > 0 && A[j-1] > A[j]) {

$\{0 \le i,j < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y [0, j-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge j > 0 \wedge A[j-1] > A[j]\} \Rightarrow$

$\{0 \le i,j < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y \in [0, j-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge j > 0\}$

int t = A[j-1];

$\{0 \le i,j < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y \in [0, j-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge j > 0\}$

A[j-1] = A[j];

$\{0 \le i,j < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y \in [0, j-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge j > 0\}$

A[j] = t;

$\{0 \le i,j < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y \in [0, j-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge j > 0\}$

j = j-1;

$\{0 \le i,j < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y \in [0, j),\ x \le y \Rightarrow A[x] \le A[y] \wedge j > 0\}$

}

$\{0 \le i,j < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y] \wedge \forall x,y \in [0, j),\ x \le y \Rightarrow A[x] \le A[y] \wedge j > 0 \wedge (j \le 0 \vee A[j-1] \le A[j])\} \Rightarrow$

$\{0 \le i < N \wedge \forall x,y \in [0, i-1),\ x \le y \Rightarrow A[x] \le A[y]\}$

i = i+1;

$\{0 < i \le N \wedge \forall x,y \in [0, i),\ x \le y \Rightarrow A[x] \le A[y]\}$

}

$\{0 < i \le N \wedge \forall x,y \in [0, i),\ x \le y \Rightarrow A[x] \le A[y] \wedge i \ge N\} \Rightarrow$

$\{\forall x,y \in [0, N),\ x \le y \Rightarrow A[x] \le A[y]\} = \{\text{sorted}(A[0], \dots, A[N-1])\}$

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